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VERTICAL MOVEMENT OF SALTWATER-FRESHWATER INTERFACE IN A THICK GROUNDWATER SYSTEM

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ABSTRACT

A theoretical function is derived to predict the transient position of the saltwater-freshwater interface set in vertical motion from an initially static equilibrium state. The function is found to be proportional to the square root of time after commencement. The proportionality is governed by the porosity and permeability of the aquifer, the difference in density of the two liquids, and the amount of head change. Laboratory sand-column experiments conducted generally validated the model. Precise experiments are desirable to elucidate a theoretical constant.

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INTRODUCTION

In a groundwater aquifer that adjoins the ocean, salt water and fresh water mix but remain discrete because of their different densities. The zone where they mix is called an interface. The location and configuration of this zone depend on the waters' densities and the rate at which the fresh water flows through the zone. As noted independently by Ghyben and Herzberg in their studies of Holland (Todd 1960) and Andrews in his study of O'ahu, Hawai'i (Palmer 1956), the zonal configuration of waters of different densities is stable when they are in equilibrium. Later investigations of coastal aquifers indicated that the density gradients of the interface depend on the nature of the flow and the geometry of the aquifer. For example, coastal aquifers in Florida exhibit a large mixed zone that departs from the predictions of the equilibrium theory, while some O'ahu aquifers exhibit a zone small in comparison to the thickness of the freshwater lens. Such differences may be attributed to the flow pattern, which depends, in Florida, on the presence of a lower aquiclude and, on O'ahu, on a coastal-caprock aquiclude. In the O'ahu lens, the depth at which salinity is increased by 50% of the salinity difference between the fresh water and the salt water is predicted accurately by the Ghyben-Herzberg equilibrium theory and has been relatively stable for the last twenty years.

Partly in response to increasing concern about freshwater resources, numerous studies have been reported in the literature on the movement of this mixed-water zone (Todd 1960). Important studies were summarized in a book by Bear (1972). In early studies, the zone was characterized by a surface, and linearized approximations of the movement of the interface in response to changes in the groundwater surface (the water table, for example) were derived. The exact mathematical description of the surface representing the interface requires a solution of the flow equation with a nonlinear free boundary, precluding the use of classical analytical methods. The introduction of the advection-dispersion equation eliminates the nonlinear boundary, so the system can be described as a continuous linear one; however, the exact mathematical treatment involves the simultaneous solution of the flow and transport equations. Thus, except for instances in which groundwater flows are independent of the space dimension, approximate numerical methods are the only alternative.

With high-speed computers, these equations for large-scale systems can be solved efficiently using numerical modeling techniques. Examples of such models now available include those by Pinder and Cooper (1970), da Costa and Wilson (1979), and Voss (1984). In all available models, the movement of the interface in response to changes in the groundwater level is assumed to be instantaneous. Pinder, Cooper, and Voss treat the system as the inland transport of salt from the ocean, while da Costa and Wilson treat it as the interaction of two immiscible fluids, tracking the moving interface by refining the discretized net near the interface. Because of the nature of the numerical approximation of the convection-dispersion equation and the need to compute the equations at specific nodes or points, such models produce large mixed zones. In the models of the immiscible fluids in which a sharp discontinuous front exists, criteria need to be imposed for the time it takes the front to respond. The simplest is based on the assumption that the discontinuous surface responds instantaneously to any changes in surface forces.

Thick Honolulu freshwater lenses are initially in dynamic equilibrium. Wentworth (1942) hypothesized a substantial "lag" time for the interface to respond fully to a step and sustained change in the freshwater head. However, Wentworth did not formulate mathematical equations describing the hydraulics of either his "reservoir-pipe" analogy of the groundwater flow system or the system itself (Lau 1981).

The validity of assuming that the bottom response is instantaneous needs to be proven. However, the difficulties of obtaining field or laboratory data have prevented this. On O'ahu four deep wells have been drilled to monitor the zone, but, because of the difficulties of observing such wells, only one has been monitored continuously. The data collected from this monitoring show that on a long-term basis the freshwater lens is relatively stable and that the Ghyben-Herzberg equilibrium theory predicts quite accurately the depth of the 50% salinity difference. Data also indicate that there is some response to seasonal changes in the rate at which water is pumped from the aquifer, but the short-term response cannot be analyzed due to the nature of the data. Laboratory data collected on the response of the interface to change in the saltwater head was submitted by Yamamoto (1985) in his thesis "Vertical Movement of the Freshwater-Saltwater Interface of a Thick Freshwater Lens." The laboratory work was part of a master's degree project conducted at the University of Hawaii under the direction of L. S. Lau. The technique used in the hydraulic simulation, such as the need to withdraw water to determine salt concentration, might affect the response of the interface, especially in small-scale models. However, Yamamoto's results seem consistent enough that analysis of the resulting data may yield an experimental assessment of the theoretical movement of the interface.

Yamamoto, in his analysis of the laboratory-generated data, chose a semi-empirical comparison with an exponential function based on the head distribution produced by the flow from an impounded groundwater reservoir. This analysis was not an adequate fit of the theory to the data because the location of the interface in such a one-dimensional system is due, at any time, to the kinematics of the flow rather than the dynamics of the system. As indicated previously, classical analytical techniques cannot be applied to this problem because of the nonlinearity of the boundary condition at the interface. However, the response of the interface can be examined by looking at the interface's continuity condition. The semi-analytical similarity method, in conjunction with dimensional analysis, enables the determination of the location of the interface without having to solve the complete groundwater-flow and transport equations.

ANALYSIS OF SALTWATER-FRESHWATER INTERFACE MOVEMENT

The equations used to determine the movement of a one-dimensional, saltwater-freshwater interface are similar to those used to calculate the heat transferred where changes in phase occur, such as the melting of ice or the evaporation of water. These problems involve a discontinuous front and are classified generally as Stefan problems (Rubinshtefin 1971). Our analysis will be limited to the examination of the mass balance equation at the interface. The system in which the vertical movement of the interface occurs is depicted in Figure 1.

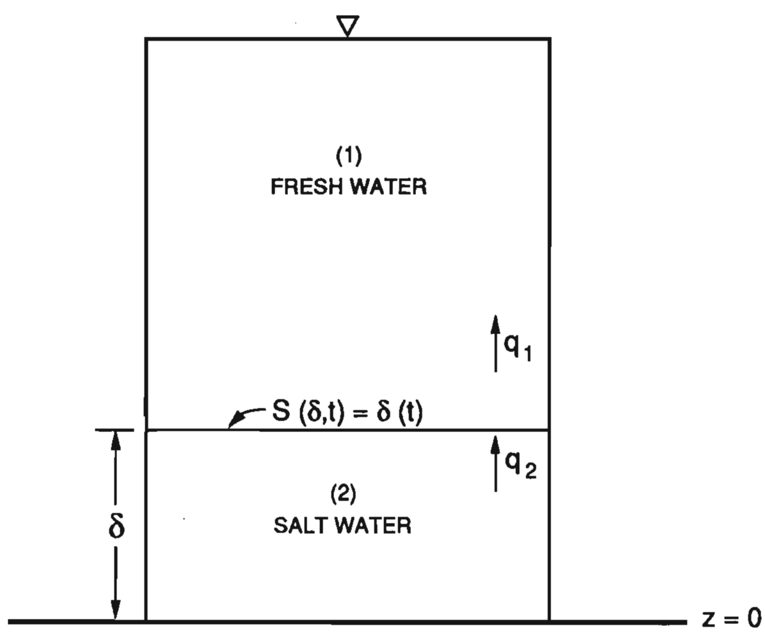

Figure 1. Vertical movement of saltwater-freshwater front

The equations of flow in the two regions may be written as follows,

$$\nabla \cdot (\nabla K_i h_i) = n \frac{\partial h_i}{\partial t}, \quad i = 1, 2 \tag{1}$$

where i = 1 (fresh water) and 2 (salt water), K_i = hydraulic conductivity, h_i = head, n = porosity, z_i = space dimension in the ith zone, and t = time. The relevant boundary conditions are

$$h_1(\delta, t) = H_S$$
$$h_2(0, t) = H_S \tag{2}$$

where H_S is the saltwater head, with the initial conditions

$$\delta(0) = 0$$
$$h_1(z, 0) = f(z) = H_0$$

where H_0 is the initial head. Because the similarity method imposes certain conditions, all of the boundary conditions have not been specified.

At the interface, the mass balance equation for the discontinuous surface may be written as

$$\rho_2\left(q_2 + n\frac{dS}{dt}\right) - \rho_1\left(q_1 + n\frac{dS}{dt}\right) = 0 \text{ on } S$$

where $S(\delta,t)$ is the surface separating the two regions. The equation written in another form is

$$\frac{\rho_2}{\rho_1}q_2 - q_1 = -\left(\frac{\rho_2 - \rho_1}{\rho_1}\right) n\frac{dS}{dt}\,. \tag{3}$$

And Darcy's law specifies that

$$q_1 = -K_1\frac{\partial h_1}{\partial Z_1} \text{ and } q_2 = -K_2\frac{\partial h_2}{\partial Z_2}\,.$$

Substituting these in equation (3) results in the expression,

$$\rho' K_2\frac{\partial h_2}{\partial Z_2} - K_1\frac{\partial h_1}{\partial Z_1} = \bar{\rho}\, n\frac{dS}{dt} \text{ where } \rho' = \frac{\rho_2}{\rho_1} \text{ and } \bar{\rho} = \frac{\rho_2 - \rho_1}{\rho_1}\,.$$

Flow equations (1) written in terms of regions z_1 ($\delta \le z_1 < \infty$) and z_2 ($0 \le z_2 \le \delta$) are

$$\frac{\partial}{\partial Z_2}\left(K_2\frac{\partial h_2}{\partial Z_2}\right) = n\frac{\partial h_2}{\partial t}$$
$$\frac{\partial}{\partial Z_1}\left(K\frac{\partial h_1}{\partial Z_1}\right) = n\frac{\partial h_1}{\partial t}\,. \tag{4}$$

The assumption of a thick aquifer allows the redefinition of the independent variables, or let

$$\xi_i = \frac{z_i}{\delta(t)} \text{ and } t = \tau\,.$$

Substitution of the new variables in the flow equations produces the transformed equation

$$\frac{1}{\delta^2}\left[\frac{\partial}{\partial \xi_i}\left(K_i\frac{\partial h_i}{\partial \xi_i}\right) + n\xi_i\frac{d\delta^2}{d\tau}\frac{\partial h_i}{\partial \xi_i}\right] = n\frac{\partial h_i}{\partial \tau} \quad i = 1, 2 \tag{5}$$

and the balance equation at the discontinuous boundary becomes

$$\rho' K_2\frac{\partial h_2}{\partial \xi_2} - K_1\frac{\partial h_1}{\partial \xi_1} = \bar{\rho} n\delta\frac{d\delta}{d\tau} = \frac{1}{2}\bar{\rho} n\frac{d\delta^2}{d\tau}\,. \tag{6}$$

The boundary conditions are transformed into

$h_1\ (1,\tau) \ = \ H_S$

$h_2\ (0,\tau) \ = \ H_S\,.$

The boundary mass balance equation is examined for the condition that the heads are independent of time or,

$$\frac{\partial h_i}{\partial t} = 0$$

in equation (5). Clearly h_1 and h_2 are functions of ξ_1 and ξ_2, respectively. Thus, the only function that satisfies equation (6) is

$$\frac{d\delta^2}{d\tau} = \text{constant} = 2\alpha^2$$

(Ames 1965). This specifies that the location of the interface is always proportional to the square root of time or $\delta = \alpha\sqrt{(\tau)}$. It may be necessary to analyze equation (5) to determine the nature of constant α. However, since the primary interest at this time is the validation of the available laboratory data on the interface location, the flow equations in the two regions will not be examined further.

To determine the nature of τ, equation (6) is written in a dimensionless form. For $K_1 = K_2 = K$, equation (6) becomes

$$\rho'\frac{\partial h_2}{\partial \xi_2} - \frac{\partial h_1}{\partial \xi_1} = \frac{1}{2}\,\bar{\rho}\,\frac{n}{K}\,\frac{d\delta^2}{dt}\ .$$

Let

$$h_2' = \frac{h_2}{H_{2f} - H_{20}} = \frac{h_2}{\Delta H_2}\,,\quad h_1' = \frac{h_1}{H_{1f} - H_{10}} = \frac{h_1}{\bar{\rho}\Delta H_2}$$

where H_{if} and H_{io} represent the values obtained from the final and initial measurements of the head.

Since $\Delta H_2 = \frac{1}{\bar{\rho}}\,\Delta H_1$ at equilibrium, equation (6) may be written in terms of the difference in the values obtained from measuring the saltwater head, or

$$\rho'\frac{\partial h_2'}{\partial \xi_2} - \bar{\rho}\,\frac{\partial h_1'}{\partial \xi_1} = \frac{1}{2}\,\frac{\bar{\rho}n}{K\Delta H_2}\,\frac{d\delta^2}{dt} = \frac{1}{2}\,\frac{d\delta^2}{d\tau}$$

where $\tau = K\Delta H_2 t/\bar{\rho}n$. The alternative equation, based on the difference in freshwater head, can be obtained by dividing through by ΔH_1 or

$$\rho'\frac{\partial h_2'}{\partial \xi_2} - \bar{\rho}\,\frac{\partial h_1'}{\partial \xi_1} = \frac{1}{2}\,\frac{\bar{\rho}^2 n}{K\Delta H_1}\,\frac{d\delta^2}{dt} = \beta_2\,\frac{d\delta^2}{d\tau}$$

where $\tau = K\Delta H_1 t/\bar{\rho}^2 n$.

Since the left side of the above equation is a constant, integration results in the expression

$$S = \delta = \alpha\sqrt{(\tau)} = \alpha\sqrt{(K\Delta H_2/\bar{\rho}n)\,t} = \alpha\sqrt{(K\Delta H_1/\bar{\rho}^2 n)\,t} \qquad (7)$$

for the initial condition $\delta(0) = 0$. The constant α as it appears in this equation is left as an undetermined constant, to be analyzed later.

COMPARISON WITH LABORATORY TESTS

The results obtained by Yamamoto were plotted using the expression given in equation (7), as shown in Figure 2. Test 1, which measured the response to the largest difference in initial and final measurements of the saltwater head, was used to determine the relationship of the best-fit or theoretical line. The constant α was then determined to be approximately 0.01. In the first three tests, fresh water and salt water flowed vertically, and in the last two, fresh water was allowed to flow horizontally along the top of the model. It is notable that, when plotted, all of the points obtained in the laboratory were located to the right of the best-fit line equation (7). Thus, it can be concluded that the lateral flow in the upper part of the hydraulic model had no effect on the movement of the interface. It also should be noted that in tests 2, 4, and 5, the final interface position exceeded the calculated value, indicating that there was some difficulty in obtaining accurate measurements. However, the grouping of the measured points indicates that the linear relationship of the location of the interface to the square root of time is a valid representation.

An illustration of this is the following rough calculation of the time required for the Koolau basalt aquifer to reach equilibrium after a 1-ft change in the saltwater head. For a permeability value (K) of 1200 ft/day and a porosity value of 10%,

$$t = \left(\frac{\delta}{\alpha}\right)^2 \left(\frac{\bar{\rho}n}{K\Delta H_2}\right) = \left(\frac{40}{0.01}\right)^2 \left(\frac{0.025 \times 0.1}{1200 \times 1}\right) = 33.3 \text{ days} .$$

Thus, when the sea level rises a foot, a period of 33 days is required for the interface to reach equilibrium again. The significance of the 30-day value cannot be evaluated at this time because the coefficient α is unknown.

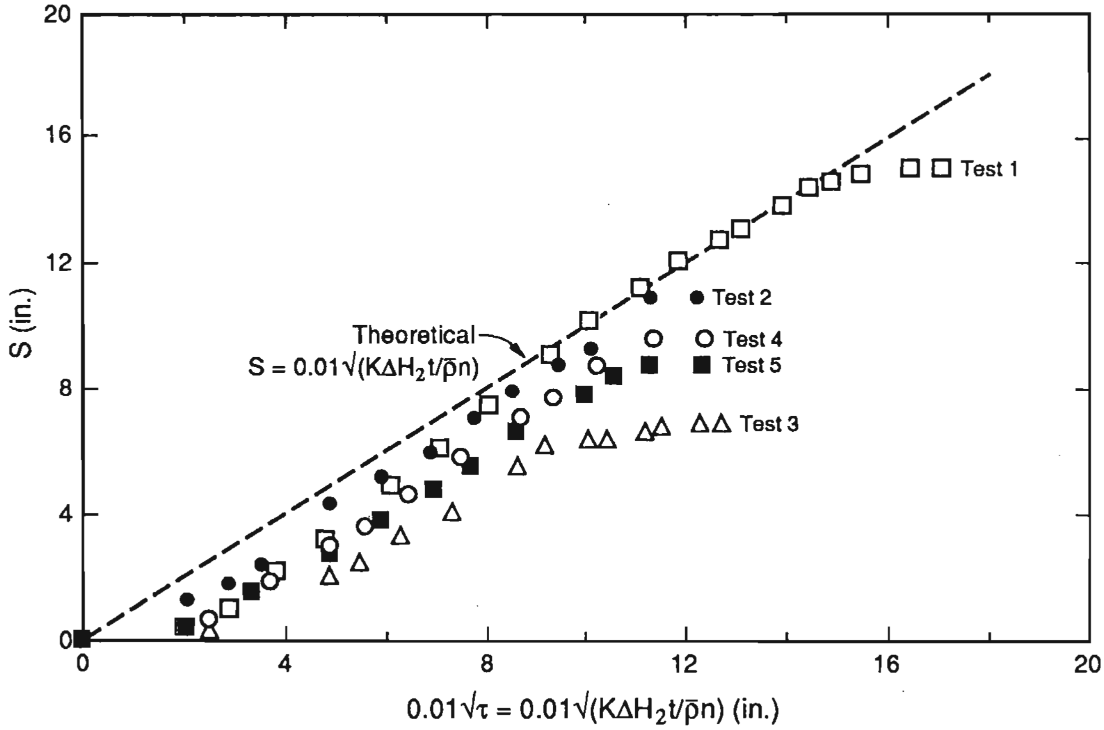

Figure 2. Plot of interface location at various times

DISCUSSION

The results of the laboratory tests, although consistent, demonstrate some of the difficulty of using the hydraulic simulation model to determine the movement of the fluids. The head locations were measured in increments of one-sixteenth of an inch (0.0625 in.), and ΔH is 0.188 in. for tests 2 through 5. The discrepancy in the calculated and observed values of the equilibrium position of the interface is not unexpected. Minor inaccuracies in measuring hydraulic conductivity, porosity, and the location of the interface in equilibrium have a minimal effect on the rightward shift of the experimental points.

The main source of the deviation from the theoretical line for this small-scale model may be the withdrawal, for the measurement of salt content, of fluid near the interface. The delay due to this extraction can be approximated by comparing the total sample volume to the flow of salt water into the hydraulic model. Yamamoto tracked the movement of the interface by taking a 5-ml sample at each of three points at various times. The total sample volume each time was thus 15 ml. Since the rate of extraction was not specified, it is assumed that it was short; in the simple calculation below, it is assumed to be instantaneous. (Although this calculation was performed, no attempt was made to use the value as a correction in the presentation of the laboratory measurements.)

For illustration purposes only, the data from test 1 will be discussed. Based on the average rate of the movement of the interface, it was approximated that salt water was flowing in at 12 ml/hr. This is based on the estimate that equilibrium was reached in a period of 131 hr. Thus, an average of 1.25 hr would be required for the sample amount of 15 ml to flow into the model. Because this calculation is approximate, no correction for time will be attempted. However, it is apparent that any delay would result in a shift of the observed points to the right of the theoretical line. This effect could be large at times. The extraction of fluid might also explain partially the atypical behavior of the interface in reaching equilibrium.

Although these results demonstrate that the location of the interface is proportional to the square root of time, additional laboratory testing is desirable. If possible, further tests should use methods other than the extraction of fluid to measure salt content. When this cannot be avoided, the use of larger-scale hydraulic models should be considered. The ratio of the sample to the inflow volume must be as small as possible so that any delay in the movement of the interface is minimized.

CONCLUSIONS

The analysis presented here establishes a function that describes the transient position of the interface over time. Using this function, the position is found to be proportional to the square root of time. Proportionality is governed by the porosity and permeability of the aquifer, the difference in density of the two liquids, and the amount of head change. As predicted by the Ghyben-Herzberg relation, a saltwater-freshwater interface moves vertically to achieve a steady state. To arrive at a theoretical constant, precise laboratory experiments are needed, but the results described here may serve as a guide in determining the sustainable yield of coastal aquifers and in estimating the impact of rises in sea level on freshwater lenses.

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