Proceedings of the 57th Hawaii International Conference on System Sciences | 2024

A Study of Different Protocols of Distribution of Information Granularity to Build Consensus in Fuzzy Group Decision-Making

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Abstract

Information granularity has been regarded as a crucial design asset whose careful application becomes essential to create more realistic models. In processes of group decision-making, by admitting an average information granularity level, models can capture the diversity of knowledge sources, which allow them to be more reflective of reality. Concretely, a distribution of information granularity along with an optimization of the distribution process have been applied to build consensus while limiting the information loss. Given that there exist different protocols of distribution of information granularity, viz. a symmetric and uniform distribution, an asymmetric but uniform distribution, a symmetric but non-uniform distribution, and an asymmetric and non-uniform distribution, this study aims to discuss how we can take advantage of them to build consensus in group decision-making with fuzzy preference relations. Some numerical experiments are also conducted to analyze the performance and effectiveness of these protocols to build consensus.

Keywords: Information granularity, group decision-making, consensus, fuzzy preference relations.

1. Introduction

The processes of group decision-making (GDM) consist in a sequence of actions conducted by two or more individuals to decide between several alternatives the best one meeting the problem requirements (Carlsson, 2010; Hwang & Lin, 1987). Although the participation of more than one individual presents a number of advantages (each one brings diverse expertise to the group along with unique ways of thinking on the problem (Hirokawa & Poole, 1999)), it requires additional effort to reach a consensual decision (Truman, 2017). When a decision is made based upon the knowledge of different individuals, consensus has been understood as a way of reaching agreement creatively and dynamically between the individuals (Zhu et al., 2022). Several individuals being part of a consensus reaching process try to make decisions that all of them can accept as an alternative to just voting for a choice and permitting the majority decide (Butler & Rothstein, 2006). This ensures that all positions are borne in mind.

Given that the achievement of agreement is very difficult at the beginning of the GDM process, diverse models intending to build consensus have been developed (Herrera-Viedma et al., 2014), notably in fuzzy environments, i.e., scenarios where the fuzzy sets and their extensions are utilized to represent the individuals' opinions (Bustince et al., 2016). Although these methods strive for building consensus, most of them achieve it so that they lead to high differences between the individuals' initial opinions and the modified ones, which produces a remarkable information loss. To resolve this issue, diverse restrictions have been integrated into the modification process. Concretely, information granularity has recently been employed to integrate them (Pedrycz & Song, 2011).

Within the Granular Computing paradigm (Bargiela & Pedrycz, 2003), a fundamental principle is that of a distribution of information granularity and its optimization process (Pedrycz, 2014). In system modelling, an existing model, regardless of its origin, is elevated by an optimal distribution of information granularity to a new level that can be denoted as granular model. Although ideal models do not exist, the precise outcome generated by a model is unrealistic.

URI: https://hdl.handle.net/10125/106599
978-0-9981331-7-1

To quantify the lack of numeric precision, information granularity has been involved in somehow or other. One consents to an average level of information granularity to quantify a limited understanding about the phenomenon that the model addresses, to make the model reflective of reality, and to capture the heterogeneity of knowledge sources expressed by individuals in GDM (Pedrycz, 2014).

To control the difference between the initial and modified positions when building consensus by means of an average level of information granularity, the key notion consists in modeling the evaluations in terms of information granules, which make the required flexibility degree available with the aim of increasing the agreement. In line with it, various GDM models have been proposed (Qin, Martinez, et al., 2023), e.g., Pedrycz and Song (2011) and Liu et al. (2018) presented methods building consensus in the analytic hierarchy process, Cabrerizo et al. (2014) introduced a model building consensus with fuzzy preference relations, and Cabrerizo et al. (2018) developed a model building consensus with intuitionistic reciprocal preference relations, to cite some examples. The common characteristic of these models is the adoption of a uniform and symmetric distribution of information granularity. Although they allow to build consensus while they restrict the range where the evaluations can be changed, other works have demonstrated that the performance and flexibility of those models can be enhanced through an optimal (non-uniform) distribution of information granularity (Qin, Ma, & Liang, 2023; Qin, Ma, & Pedrycz, 2023; B. Zhang et al., 2023).

However, there are unresolved issues that need to be addressed. In addition to a symmetric and uniform distribution of information granularity and a symmetric but non-uniform distribution of information granularity, there exist other protocols of distribution of information granularity that can be adopted (Pedrycz, 2014). To enhance the existing works, this study aims to discuss how to build consensus in GDM with fuzzy preference relations by means of all the existing protocols of information granularity distribution and the ensuing optimization.

The material is structured as follows. Section 2 offers some background knowledge. Section 3 describes how we can take advantage of the existing protocols of distribution of information granularity and their ensuing optimization. A numerical experiment is completed in Section 4 to illustrate the essence of the existing protocols. A comparative analysis is also conducted to show the flexibility and performance of each protocol. Finally, concluding remarks and future research directions are pointed out in Section 5.

2. Preliminaries

We introduce some prerequisites that help understand the proposed study. This includes the formal description of a process of GDM with fuzzy preference relations, the importance of the consensus in such kind of decision-making scenarios, and the algorithm of particle swarm optimization (PSO).

2.1. Fuzzy GDM

In these decision-making environments, a number of alternatives, which are possible solutions to a decision-making problem, are evaluated by a group of experts (Cabrerizo et al., 2014). Formally, let a_i and e_l be the alternative and the expert, respectively, where $i = 1, 2, \ldots, m$ $(m \geq 2)$ and $l = 1, 2, \ldots, n$ $(n \geq 2)$. The objective is to assign a value representing the collective evaluation (opinion) of the experts to each alternative.

To model the evaluations provided by the experts, different structures have been proposed (Millet, 1997), namely, an ordered vector of alternatives, a utility value determining the fulfilment degree of the alternative as solution to the problem, or a pairwise comparison representing the preference degree of an alternative over other, which, when applying repeatedly, produces a preference relation. Among the diverse structures and domains of evaluation representation, in this study, we assume fuzzy preference relations (Herrera-Viedma et al., 2021). It means that we make use of preference relations as representation structure and values in-between $[0, 1]$ as representation domain. We have selected them as they are the most popular ones in fuzzy GDM processes and because most of the models based upon a distribution of information granularity to build consensus have been developed for GDM problems with fuzzy preference relations.

Definition 2.1 *(Kacprzyk & Fedrizzi, 1986) "An individual fuzzy preference relation of expert* e_l*,* P^l*, is given by its membership function* $\mu_{P^l} : A \times A \to [0, 1]$*."*

In GDM, each expert e_l must provide a fuzzy preference relation modeled by a matrix $P^l = [p_{ij}^l]$ whose entry $p_{ij}^l = \mu_{P^l}(a_i, a_j)$ is such that the greater p_{ij}^l, the higher the expert's preference of a_i over a_j: from $p_{ij}^l = 1$ representing an expert's total preference of a_i over a_j, through $p_{ij}^l = 0.5$ denoting expert's indifference between a_i over a_j, to $p_{ij}^l = 0$ that indicates an expert's definite preference of a_j over a_i. Due to the components of the leading diagonal, i.e., p_{ii}^l, are not taken into account, they are denoted as "–" (Kacprzyk & Fedrizzi, 1986).

Given that a consensus reaching process leads to a high-quality decision having a strong engagement to application, uncountable consensus reaching processes have been designed for fuzzy GDM (H. Zhang et al., 2020). In them, to evaluate the agreement, consensus measures must be defined. Particularly, soft consensus measures assuming values in-between $[0,1]$ have been proposed (Herrera-Viedma et al., 2014). To compute them, the similitude between the values contained in the entries of the fuzzy preference relations has been measured by using three different approaches (Herrera-Viedma et al., 2014): (i) Coincidence between solutions, (ii) soft coincidence between preferences, and (iii) strict coincidence between preferences. The second one has been the most used in GDM under fuzzy preference relations as it allows to consider diverse partial coincidence levels that can be assessed in $[0,1]$. It means a gradual conception of the coincidence concept is assumed. Let P^l $(l = 1, \ldots, n)$ be a set of fuzzy preference relations, using the soft coincidence concept, the consensus c achieved by the experts can be measured as (Herrera-Viedma et al., 2014):

$$c = \frac{2\sum_{l=1}^{n-1}\sum_{q=l+1}^{n}\frac{\sum_{i=1}^{m}\sum_{j=1;\, j\neq i}^{m} 1-|p_{ij}^{l}-p_{ij}^{q}|}{m^2-m}}{n^2-n} \tag{1}$$

2.2. PSO algorithm

The goal of this algorithm, created by Eberhart and Kennedy (1995), is to emulate the social behavior of bird flocks. It is a stochastic optimization method that, in a series of iterations, tries to solve an optimization problem. In each iteration, it tries to find the most promising candidate solutions according to a certain quality measure (fitness function f). It starts by considering a swarm (population) of particles (candidate solutions in a d-dimensional search space). The particles are moved to different directions of the search space according to a series of not complex mathematical formulas. In each iteration, the displacement of each particle to a new position is guided by a certain velocity, which depends on its best location so far in the search space and the best location reached by any of the other particles. The goal is to guide the particles towards the best location (solution) (Kennedy & Eberhart, 1995).

Although there are various variants of this algorithm (Wang et al., 2018), we will use the generic version, in which the particle's velocity, $\mathbf{v}_i = (v_{i,1}, \ldots, v_{i,d})$, is updated according to $\mathbf{v}_i(\mathrm{t}+1) = \omega\mathbf{v}_i(\mathrm{t}) + c_1\mathbf{r}(\mathbf{xl}_i - \mathbf{x}_i) + c_2\mathbf{s}(\mathbf{xg} - \mathbf{x}_i)$, where $\mathbf{r} = (r_1, \ldots, r_d)$ and $\mathbf{s} = (s_1, \ldots, s_d)$ are two vectors of random numbers obtained from the uniform distribution over $[0,1]$, and "t" designates the index of the current iteration. In terms of the coefficients, c_1 and c_2 are two acceleration coefficients that influence the step size taken by the particle i towards its best position, $\mathbf{xl}_i = (xl_{i,1}, \ldots, xl_{i,d})$, and the global best position, $\mathbf{xg} = (xg_1, \ldots, xg_d)$, respectively, and ω stands for the inertia weight (Kennedy & Eberhart, 1995; Wang et al., 2018). The assignment of low values to ω implies exploitation (local search). Due to this, the value assigned to ω is usually decremented as the number of iterations increases. This ensures global search at the beginning of the algorithm and local search at the end. Its value is commonly altered as:

$$\omega(t) = (\omega(1) - \omega(o))\frac{o-t}{o} + \omega(o) \tag{2}$$

where $\omega(1)$, $\omega(t)$, and $\omega(o)$ are the initial, the current iteration, and the final values assigned to ω; o and t represent the maximum number of iterations and the current iteration. The next position of the particle is obtained according to $\mathbf{x}_i(\mathrm{t}+1) = \mathbf{x}_i(\mathrm{t}) + \mathbf{v}_i(\mathrm{t}+1)$.

This is the algorithm that will be used on the grounds that it has been employed in most of the proposals building consensus by an optimal distribution of information granularity (Cabrerizo et al., 2014; Liu et al., 2022; Pedrycz & Song, 2011). Nevertheless, any other optimization algorithm such as the differential evolution could be applied as it has also been employed to these kind of problems (Cabrerizo et al., 2023).

3. Building consensus by distributing an information granularity level

At the beginning of the GDM process, it is unusual that the individual experts' judgments coincide. For this reason, the experts must have no objection to change them whether they want to work together with the aim of achieving an agreement. This requires a specific flexibility degree accepted by the experts, who must freely agree to "soften" their positions. To model it, several works have demonstrated that the information granularity concept is an excellent tool (Pedrycz & Song, 2011). In brief, to facilitate the cooperation between the experts and the achievement of consensus, the pairwise comparisons contained in the fuzzy preference relations should be considered as granular realizations instead of precise numerical values in-between $[0,1]$. The advantage of using the concept of information granularity is that it controls the loss of information (the difference between the initial experts' evaluations and the modified ones) to some extent (Pedrycz & Song, 2011).

The existing approaches have modeled the granular realizations as intervals, being their length determined by the information granularity level. It means that interval-valued preference relations, $IV^l(P^l)$ $(l = 1, 2, \ldots, n)$, are built, where $IV^l(\cdot)$ denotes an interval-valued preference relation family. The flexibility provided by the length of the intervals has been exploited to optimize an optimization criterion that, in this case, is associated with the consensus.

In the rest of this section, we discuss how the existing protocols of distribution of information granularity, and their ensuing optimization, can be applied to build consensus. These protocols are: (i) A uniform and symmetric distribution of information granularity (d_1), (ii) a uniform but asymmetric distribution of information granularity (d_2), (iii) a non-uniform but symmetric distribution of information granularity (d_3), and (iv) a non-uniform and asymmetric distribution of information granularity (d_4).

3.1. Protocol d_1

It is the simplest one. All numeric values of the fuzzy preference relations are similarly treated. They are substituted by intervals that have the same length and are symmetrically allocated around the numeric values provided by the experts. Let $\varepsilon^l \in [0, 1]$ be the average level of information granularity admitted by expert e_l. Considering the fuzzy preference relation expressed by this expert, we focus on each component p^l_{ij}, whose value must be altered within the following interval:

$$iv^l_{ij} = [\max(0, p^l_{ij} - 0.5\varepsilon^l), \min(1, p^l_{ij} + 0.5\varepsilon^l)] \quad (3)$$

Let $\overline{P}^l \in IV^l(P^l)$, then the following optimization model can be defined:

$$\begin{cases} \max\limits_{\overline{p}^l_{ij}} c \\ \text{s.t.} \begin{cases} |\overline{p}^l_{ij} - p^l_{ij}| < 0.5\varepsilon^l \\ \overline{p}^l_{ij} \in [0, 1] \end{cases} \end{cases} \quad (4)$$

To resolve model (4), the PSO algorithm is applied to obtain a set of optimal (modified) fuzzy preference relations $\overline{P}^l$ $(l = 1, 2, \ldots, n)$ maximizing the consensus. In light of the number of decision variables is $n(m^2 - m)$ in model (4), this number corresponds to the dimension d of the particles. Let $\mathbf{x}_i(t) = (x_{i,1}(t), \ldots, x_{i,d}(t))$ be the ith particle at iteration t. Here, we assume that the search space is $x_{i,h}(t) \in [0, 1]$, $h = 1, 2, \ldots, d$. Then, to manage the constraints, the following expression is used to convert the value of $x_{i,h}$ to its corresponding value within the allowed interval:

$$\overline{p}^l_{ij} = y + (z - y)x_{i,h} \quad (5)$$

where y and z are the lower and upper boundaries of the interval iv^l_{ij}.

3.2. Protocol d_2

This protocol provides more flexibility than the previous one. Although the intervals have the same length, their asymmetric location around the numeric values provided by the experts brings a certain flexibility level that can be taken advantage of during the process of optimization. Given a_i, a_j, and e_l, we focus on p^l_{ij}, whose value must be adjusted within:

$$iv^l_{ij} = [\max(0, p^l_{ij} - \gamma^l_{ij}\varepsilon^l), \min(1, p^l_{ij} + (1 - \gamma^l_{ij})\varepsilon^l)] \quad (6)$$

where $\gamma^l_{ij} \in [0, 1]$ controls the asymmetry of the location of the interval associated with p^l_{ij} and whose length is ε^l. That is, different asymmetric locations of the intervals are allowed from one component to other, which increases the available flexibility level.

Let $\overline{P}^l \in IV^l(P^l)$, for this protocol, this optimization model can be constructed:

$$\begin{cases} \max\limits_{\overline{p}^l_{ij}} c \\ \text{s.t.} \begin{cases} |\overline{p}^l_{ij} - p^l_{ij}| < \varepsilon^l \\ \overline{p}^l_{ij} \in [0, 1] \end{cases} \end{cases} \quad (7)$$

To resolve model (7), the PSO algorithm is applied like in protocol d_1. However, to manage the constraints of model (7), the following expression is used:

$$\overline{p}^l_{ij} = y + (z - y)x_{i,h} \quad (8)$$

where y and z are $\max(0, p^l_{ij} - \varepsilon^l)$ and $\min(1, p^l_{ij} + \varepsilon^l)$, respectively. Then, the specific intervals in which the values of the optimal fuzzy preference relations, $\overline{P}^l$ $(l = 1, 2, \ldots, n)$, are located can be computed by using (6), where:

$$\gamma^l_{ij} = \begin{cases} \dfrac{\varepsilon^l - |p^l_{ij} - \overline{p}^l_{ij}|}{2\varepsilon^l}, & \text{if } p^l_{ij} \leq \overline{p}^l_{ij} \\ 1 - \dfrac{\varepsilon^l - |p^l_{ij} - \overline{p}^l_{ij}|}{2\varepsilon^l}, & \text{otherwise} \end{cases} \quad (9)$$

3.3. Protocol d_3

This protocol replaces the numeric values of the fuzzy preference relations by intervals of different length that are symmetrically allocated around them. It means a different information granularity level ε_{ij}^l is distributed to each p_{ij}^l. Given a_i, a_j, and e_l, we focus on p_{ij}^l, whose value must be adjusted within the interval:

$$iv_{ij}^l = [\max(0, p_{ij}^l - 0.5\varepsilon_{ij}^l), \min(1, p_{ij}^l + 0.5\varepsilon_{ij}^l)] \tag{10}$$

Let $\overline{P}^l \in IV^l(P^l)$, in this case, this optimization model can be constructed:

$$\begin{cases} \max\limits_{\overline{p}_{ij}^l} c \\ \text{s.t.} \begin{cases} \sum_{i=1}^m \sum_{j=1;j\neq i}^m 2|p_{ij}^l - \overline{p}_{ij}^l| \leq (m^2-m)\varepsilon^l \\ \overline{p}_{ij}^l \in [0,1] \end{cases} \end{cases} \tag{11}$$

where ε^l is the average level of information granularity allowed by expert e_l, i.e.:

$$\varepsilon^l = \frac{1}{m^2-m} \sum_{i=1}^m \sum_{j=1;j\neq i}^m \varepsilon_{ij}^l \tag{12}$$

According to model (11), we calculate the value that corresponds to $\overline{p}_{ij}^l$. Then, we can calculate the value that corresponds to ε_{ij}^l. Let $\Delta_1^l = (m^2-m)\varepsilon^l - \sum_{i=1}^m \sum_{j=1;j\neq i}^m 2|p_{ij}^l - \overline{p}_{ij}^l|$, without loss of generality, ε_{ij}^l can be distributed as follows:

$$\varepsilon_{ij}^l = \begin{cases} 2|p_{ij}^l - \overline{p}_{ij}^l| + \frac{1}{\#\Upsilon_1^l}\Delta_1^l, & \text{if } (i,j) \in \Upsilon_1^l \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

where $\Upsilon_1^l = \{(i,j) \mid \overline{p}_{ij}^l \neq p_{ij}^l\}$ and $\#\Upsilon_1^l$ is its cardinality. Remarkably, if $\Delta_1^l = 0$, then the unique solution is $\varepsilon_{ij}^l = 2|\overline{p}_{ij}^l - p_{ij}^l|$.

To resolve model (11), the PSO algorithm is applied. The dimension d of the particles is also $n(m^2-m)$, which is the number of decision variables in model (11). Here, the search space is $x_{i,h}(t) \in [0,1]$, $h = 1, 2, \ldots, d$. In model (11), to manage the constraints, the fitness function f of the PSO is defined as:

$$f(\mathbf{x}_i(t)) = \begin{cases} c, & \text{if } g(\mathbf{x}_i(t)) = 1 \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

where c is the value of the consensus achieved considering the fuzzy preference relations generated by the vector received as parameter and computed using (1), and g is a function that controls that the information granularity level injected is lower than the one allowed by the experts. If one of the modified fuzzy preference relations constructed based upon the vector received as parameter has an average information granularity level higher than the one allowed by that expert, this function returns 0 as there exists a fuzzy preference relation that is not valid. In other case, it returns 1.

Using the PSO, the entries of the fuzzy preference relations are modified to maximize the value of c and, then, using (13), the value of ε_{ij}^l can be computed.

3.4. Protocol d_4

This is the most flexible protocol because it substitutes the numeric values of the fuzzy preference relations by intervals of different length that are asymmetrically allocated around them. Given a_i, a_j, and e_l, we focus on p_{ij}^l, whose value must be adjusted within:

$$iv_{ij}^l = [\max(0, p_{ij}^l - \gamma_{ij}^l\varepsilon_{ij}^l), \min(1, p_{ij}^l + (1-\gamma_{ij}^l)\varepsilon_{ij}^l)] \tag{15}$$

where $\gamma_{ij}^l \in [0,1]$ controls the asymmetry of the location of the interval of p_{ij}^l and whose length is ε_{ij}^l.

Let $\overline{P}^l \in IV^l(P^l)$, for this protocol, this optimization model can be constructed:

$$\begin{cases} \max\limits_{\overline{p}_{ij}^l} c \\ \text{s.t.} \begin{cases} \sum_{i=1}^m \sum_{j=1;j\neq i}^m |p_{ij}^l - \overline{p}_{ij}^l| \leq (m^2-m)\varepsilon^l \\ \overline{p}_{ij}^l \in [0,1] \end{cases} \end{cases} \tag{16}$$

where ε^l is the average level of information granularity allowed by expert e_l (see (12)).

According to model (16), we calculate the value that corresponds to $\overline{p}_{ij}^l$. Then, we can calculate the value that corresponds to ε_{ij}^l. Let $\Delta_2^l = (m^2-m)\varepsilon^l - \sum_{i=1}^m \sum_{j=1;j\neq i}^m |p_{ij}^l - \overline{p}_{ij}^l|$, without loss of generality, ε_{ij}^l can be distributed as follows:

$$\varepsilon_{ij}^l = \begin{cases} |p_{ij}^l - \overline{p}_{ij}^l| + \frac{1}{\#\Upsilon_2^l}\Delta_2^l, & \text{if } (i,j) \in \Upsilon_2^l \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

where $\Upsilon_2^l = \{(i,j) \mid \overline{p}_{ij}^l \neq p_{ij}^l\}$ and $\#\Upsilon_2^l$ is its cardinality. If $\Delta_2^l = 0$, then the unique solution is $\varepsilon_{ij}^l = |\overline{p}_{ij}^l - p_{ij}^l|$.

Using the PSO as in the protocol d_3, the entries of the fuzzy preference relations are modified to maximize the value of c and, then, using (17), the value of each ε^l_{ij} can be computed. Then, the specific intervals in which the values of the optimal fuzzy preference relations, $\overline{P}^l$ ($l = 1, 2, \ldots, n$), are located can be computed by using (15), where:

$$\gamma^l_{ij} = \begin{cases} \dfrac{\varepsilon^l_{ij} - |p^l_{ij} - \overline{p}^l_{ij}|}{2\varepsilon^l_{ij}}, & \text{if } p^l_{ij} \leq \overline{p}^l_{ij} \\ 1 - \dfrac{\varepsilon^l_{ij} - |p^l_{ij} - \overline{p}^l_{ij}|}{2\varepsilon^l_{ij}}, & \text{otherwise} \end{cases} \tag{18}$$

4. Numerical experiments

We offer a numerical first experiment to examine the effectiveness and performance of the protocols of information granularity distribution described in Section 3 to build consensus in fuzzy GDM. Let us suppose a GDM problem consisting in four experts, $E = \{e_1, e_2, e_3, e_4\}$, that must choose between four alternatives, $A = \{a_1, a_2, a_3, a_4\}$. Initially, we assume the experts provide these fuzzy preference relations:

$$P^1 = \begin{bmatrix} - & 0.10 & 0.80 & 0.20 \\ 0.90 & - & 0.60 & 0.70 \\ 0.40 & 0.10 & - & 0.20 \\ 0.60 & 0.30 & 0.80 & - \end{bmatrix}$$

$$P^2 = \begin{bmatrix} - & 0.20 & 0.90 & 0.80 \\ 0.80 & - & 0.90 & 0.30 \\ 0.10 & 0.30 & - & 0.50 \\ 0.10 & 0.70 & 0.50 & - \end{bmatrix}$$

$$P^3 = \begin{bmatrix} - & 0.90 & 0.20 & 0.10 \\ 0.30 & - & 0.80 & 0.80 \\ 0.50 & 0.10 & - & 0.90 \\ 0.60 & 0.10 & 0.20 & - \end{bmatrix}$$

$$P^4 = \begin{bmatrix} - & 0.80 & 0.10 & 0.80 \\ 0.40 & - & 0.50 & 0.20 \\ 0.80 & 0.40 & - & 0.90 \\ 0.40 & 0.80 & 0.20 & - \end{bmatrix}$$

Through (1), the consensus according to these fuzzy preference relations is 0.630. To increase this value, we make use of the distribution of the information granularity along with its optimization. Concretely, we apply the four protocols described in Section 3. Apropos of the average information granularity level, we assume it is 0.1. Regarding the PSO, it was executed by setting the inertia weight coefficients, $\omega(1)$ and $\omega(o)$, to 0.9 and 0.4, respectively, and the acceleration parameters, c_1 and c_2, to 2 (in the literature, these values are commonly encountered). Concerning the size of the swarm and the number of iterations they were set to $10n(m^2 - m)$ and 1000, respectively, as no improvement was achieved with higher values.

4.1. Protocol d_1

For each component of the fuzzy preference relation, this protocol builds intervals whose length is 0.1, being symmetrically distributed around the value provided by the expert. As an example, for p^1_{12}, the interval built is $[0.05, 0.15]$. Considering it, the PSO produces the following modified fuzzy preference relations:

$$\overline{P}^1 = \begin{bmatrix} - & 0.14 & 0.76 & 0.23 \\ 0.85 & - & 0.64 & 0.69 \\ 0.44 & 0.12 & - & 0.22 \\ 0.55 & 0.29 & 0.76 & - \end{bmatrix}$$

$$\overline{P}^2 = \begin{bmatrix} - & 0.22 & 0.86 & 0.80 \\ 0.78 & - & 0.85 & 0.26 \\ 0.13 & 0.29 & - & 0.46 \\ 0.14 & 0.69 & 0.45 & - \end{bmatrix}$$

$$\overline{P}^3 = \begin{bmatrix} - & 0.85 & 0.22 & 0.14 \\ 0.34 & - & 0.75 & 0.80 \\ 0.46 & 0.14 & - & 0.85 \\ 0.55 & 0.14 & 0.22 & - \end{bmatrix}$$

$$\overline{P}^4 = \begin{bmatrix} - & 0.75 & 0.14 & 0.80 \\ 0.43 & - & 0.52 & 0.23 \\ 0.75 & 0.38 & - & 0.86 \\ 0.42 & 0.75 & 0.23 & - \end{bmatrix}$$

With these optimal fuzzy preference relations, the consensus reached is 0.674.

4.2. Protocol d_2

For each component of the fuzzy preference relation, this protocol builds intervals whose length is 0.1, being asymmetrically distributed around the value provided by the expert. Considering it, the PSO generates the following optimal (modified) fuzzy preference relations:

$$\overline{P}^1 = \begin{bmatrix} - & 0.19 & 0.70 & 0.26 \\ 0.80 & - & 0.68 & 0.63 \\ 0.44 & 0.18 & - & 0.29 \\ 0.52 & 0.38 & 0.71 & - \end{bmatrix}$$

$$\overline{P}^2 = \begin{bmatrix} - & 0.29 & 0.80 & 0.71 \\ 0.80 & - & 0.80 & 0.31 \\ 0.19 & 0.20 & - & 0.58 \\ 0.19 & 0.71 & 0.51 & - \end{bmatrix}$$

$$\overline{P}^3 = \begin{bmatrix} - & 0.80 & 0.29 & 0.19 \\ 0.35 & - & 0.70 & 0.70 \\ 0.44 & 0.19 & - & 0.80 \\ 0.52 & 0.19 & 0.29 & - \end{bmatrix}$$

$$\overline{P}^4 = \begin{bmatrix} - & 0.71 & 0.18 & 0.70 \\ 0.40 & - & 0.58 & 0.29 \\ 0.70 & 0.30 & - & 0.80 \\ 0.49 & 0.70 & 0.29 & - \end{bmatrix}$$

These optimal fuzzy preference relations coming from the following interval-valued fuzzy preference relations, which are computed by using (6) and (9):

$$IV^1 = \begin{bmatrix} - & [0.095, 0.195] & [0.700, 0.800] & [0.180, 0.280] \\ [0.800, 0.900] & - & [0.590, 0.690] & [0.615, 0.715] \\ [0.370, 0.470] & [0.090, 0.190] & - & [0.195, 0.295] \\ [0.510, 0.610] & [0.290, 0.390] & [0.705, 0.805] & - \end{bmatrix}$$

$$IV^2 = \begin{bmatrix} - & [0.195, 0.295] & [0.800, 0.900] & [0.705, 0.805] \\ [0.750, 0.850] & - & [0.800, 0.900] & [0.255, 0.355] \\ [0.095, 0.195] & [0.200, 0.300] & - & [0.490, 0.590] \\ [0.095, 0.195] & [0.655, 0.755] & [0.455, 0.555] & - \end{bmatrix}$$

$$IV^3 = \begin{bmatrix} - & [0.800, 0.900] & [0.195, 0.295] & [0.095, 0.195] \\ [0.275, 0.375] & - & [0.700, 0.800] & [0.700, 0.800] \\ [0.420, 0.520] & [0.095, 0.195] & - & [0.800, 0.900] \\ [0.510, 0.610] & [0.095, 0.195] & [0.195, 0.295] & - \end{bmatrix}$$

$$IV^4 = \begin{bmatrix} - & [0.705, 0.805] & [0.090, 0.190] & [0.700, 0.800] \\ [0.350, 0.450] & - & [0.490, 0.590] & [0.195, 0.295] \\ [0.700, 0.800] & [0.300, 0.400] & - & [0.800, 0.900] \\ [0.395, 0.495] & [0.700, 0.800] & [0.195, 0.295] & - \end{bmatrix}$$

With the above optimal fuzzy preference relations, the consensus reached is 0.744.

4.3. Protocol d_3

For each component of the fuzzy preference relations, this protocol builds intervals of different length symmetrically distributed around the value provided by the experts. Because $\varepsilon^l = 0.1$, it must be considered that the average of the lengths of the intervals must be 0.1. Considering it, the PSO returns the following optimal fuzzy preference relations:

$$\overline{P}^1 = \begin{bmatrix} - & 0.17 & 0.76 & 0.20 \\ 0.77 & - & 0.65 & 0.69 \\ 0.42 & 0.11 & - & 0.28 \\ 0.56 & 0.30 & 0.66 & - \end{bmatrix}$$

$$\overline{P}^2 = \begin{bmatrix} - & 0.19 & 0.77 & 0.77 \\ 0.78 & - & 0.77 & 0.38 \\ 0.13 & 0.27 & - & 0.51 \\ 0.19 & 0.69 & 0.48 & - \end{bmatrix}$$

$$\overline{P}^3 = \begin{bmatrix} - & 0.85 & 0.23 & 0.17 \\ 0.40 & - & 0.77 & 0.71 \\ 0.49 & 0.13 & - & 0.86 \\ 0.56 & 0.14 & 0.26 & - \end{bmatrix}$$

$$\overline{P}^4 = \begin{bmatrix} - & 0.79 & 0.17 & 0.78 \\ 0.40 & - & 0.54 & 0.38 \\ 0.75 & 0.26 & - & 0.87 \\ 0.40 & 0.79 & 0.26 & - \end{bmatrix}$$

Based upon these optimal fuzzy preference relations, the matrices, D^l $(l = 1, 2, 3, 4)$, are constructed, which contains the information granularity distributed to each entry of $\overline{P}^l$ according to (13):

$$D^1 = \begin{bmatrix} - & 0.142 & 0.082 & 0.000 \\ 0.262 & - & 0.102 & 0.022 \\ 0.042 & 0.022 & - & 0.162 \\ 0.082 & 0.000 & 0.282 & - \end{bmatrix}$$

$$D^2 = \begin{bmatrix} - & 0.022 & 0.262 & 0.062 \\ 0.042 & - & 0.262 & 0.162 \\ 0.062 & 0.062 & - & 0.022 \\ 0.182 & 0.022 & 0.042 & - \end{bmatrix}$$

$$D^3 = \begin{bmatrix} - & 0.102 & 0.062 & 0.142 \\ 0.202 & - & 0.062 & 0.182 \\ 0.022 & 0.062 & - & 0.082 \\ 0.082 & 0.082 & 0.122 & - \end{bmatrix}$$

$$D^4 = \begin{bmatrix} - & 0.020 & 0.140 & 0.040 \\ 0.000 & - & 0.080 & 0.360 \\ 0.100 & 0.280 & - & 0.060 \\ 0.000 & 0.020 & 0.120 & - \end{bmatrix}$$

Considering the information granularity level distributed to each component, the following interval-valued fuzzy preference relations, which contains the intervals related to each component of $\overline{P}^l$, are constructed:

$$IV^1 = \begin{bmatrix} - & [0.029, 0.171] & [0.759, 0.841] & [0.200, 0.200] \\ [0.769, 1.000] & - & [0.549, 0.651] & [0.689, 0.711] \\ [0.379, 0.421] & [0.089, 0.111] & - & [0.119, 0.281] \\ [0.559, 0.641] & [0.300, 0.300] & [0.659, 0.941] & - \end{bmatrix}$$

$$IV^2 = \begin{bmatrix} - & [0.189, 0.211] & [0.769, 1.000] & [0.769, 0.831] \\ [0.779, 0.821] & - & [0.769, 1.000] & [0.219, 0.381] \\ [0.069, 0.131] & [0.269, 0.331] & - & [0.489, 0.511] \\ [0.009, 0.191] & [0.689, 0.711] & [0.479, 0.521] & - \end{bmatrix}$$

$$IV^3 = \begin{bmatrix} - & [0.849, 0.951] & [0.169, 0.231] & [0.029, 0.171] \\ [0.199, 0.401] & - & [0.769, 0.831] & [0.709, 0.891] \\ [0.489, 0.511] & [0.069, 0.131] & - & [0.859, 0.941] \\ [0.559, 0.641] & [0.059, 0.141] & [0.139, 0.261] & - \end{bmatrix}$$

$$IV^4 = \begin{bmatrix} - & [0.790, 0.810] & [0.030, 0.170] & [0.780, 0.820] \\ [0.400, 0.400] & - & [0.460, 0.540] & [0.020, 0.380] \\ [0.750, 0.850] & [0.260, 0.540] & - & [0.870, 0.930] \\ [0.400, 0.400] & [0.790, 0.810] & [0.140, 0.260] & - \end{bmatrix}$$

With the above optimal fuzzy preference relations, the consensus reached is 0.713.

4.4. Protocol d_4

For each component of the fuzzy preference relation, this protocol builds intervals of different length asymmetrically distributed around the value provided by the experts. Similar to the above protocol, because $\varepsilon^l = 0.1$, then the average of the lengths of the intervals must be 0.1. Considering it, the PSO generates the following optimal fuzzy preference relations:

$$\overline{P}^1 = \begin{bmatrix} - & 0.26 & 0.67 & 0.26 \\ 0.86 & - & 0.73 & 0.61 \\ 0.41 & 0.18 & - & 0.50 \\ 0.47 & 0.30 & 0.73 & - \end{bmatrix}$$

$$\overline{P}^2 = \begin{bmatrix} - & 0.25 & 0.68 & 0.69 \\ 0.80 & - & 0.74 & 0.38 \\ 0.23 & 0.29 & - & 0.50 \\ 0.43 & 0.62 & 0.47 & - \end{bmatrix}$$

$$\overline{P}^3 = \begin{bmatrix} - & 0.83 & 0.28 & 0.26 \\ 0.44 & - & 0.74 & 0.61 \\ 0.46 & 0.20 & - & 0.87 \\ 0.45 & 0.26 & 0.22 & - \end{bmatrix}$$

$$\overline{P}^4 = \begin{bmatrix} - & 0.79 & 0.27 & 0.68 \\ 0.44 & - & 0.74 & 0.43 \\ 0.68 & 0.40 & - & 0.87 \\ 0.43 & 0.63 & 0.22 & - \end{bmatrix}$$

Based upon these optimal fuzzy preference relations, the matrices, D^l ($l = 1, 2, 3, 4$), are constructed, which contains the information granularity distributed to each entry of $\overline{P}^l$ according to (17):

$$D^1 = \begin{bmatrix} - & 0.160 & 0.130 & 0.060 \\ 0.040 & - & 0.130 & 0.090 \\ 0.010 & 0.080 & - & 0.300 \\ 0.130 & 0.000 & 0.070 & - \end{bmatrix}$$

$$D^2 = \begin{bmatrix} - & 0.050 & 0.220 & 0.110 \\ 0.000 & - & 0.160 & 0.080 \\ 0.130 & 0.010 & - & 0.000 \\ 0.330 & 0.080 & 0.030 & - \end{bmatrix}$$

$$D^3 = \begin{bmatrix} - & 0.070 & 0.080 & 0.160 \\ 0.140 & - & 0.060 & 0.190 \\ 0.040 & 0.100 & - & 0.030 \\ 0.150 & 0.160 & 0.020 & - \end{bmatrix}$$

$$D^4 = \begin{bmatrix} - & 0.012 & 0.172 & 0.122 \\ 0.042 & - & 0.242 & 0.232 \\ 0.122 & 0.000 & - & 0.032 \\ 0.032 & 0.172 & 0.022 & - \end{bmatrix}$$

Considering the information granularity level distributed to each component, the following interval-valued fuzzy preference relations, which contains the intervals related to each component of $\overline{P}^l$, are constructed based on (15) and (18):

$$IV^1 = \begin{bmatrix} - & [0.100, 0.260] & [0.670, 0.800] & [0.200, 0.260] \\ [0.860, 0.900] & - & [0.600, 0.730] & [0.610, 0.700] \\ [0.400, 0.410] & [0.100, 0.180] & - & [0.200, 0.500] \\ [0.470, 0.600] & [0.300, 0.300] & [0.730, 0.800] & - \end{bmatrix}$$

$$IV^2 = \begin{bmatrix} - & [0.200, 0.250] & [0.680, 0.900] & [0.690, 0.800] \\ [0.800, 0.800] & - & [0.740, 0.900] & [0.300, 0.380] \\ [0.100, 0.230] & [0.290, 0.300] & - & [0.500, 0.500] \\ [0.100, 0.430] & [0.620, 0.700] & [0.470, 0.500] & - \end{bmatrix}$$

$$IV^3 = \begin{bmatrix} - & [0.830, 0.900] & [0.200, 0.280] & [0.100, 0.260] \\ [0.300, 0.440] & - & [0.740, 0.800] & [0.610, 0.800] \\ [0.460, 0.500] & [0.100, 0.200] & - & [0.870, 0.900] \\ [0.450, 0.600] & [0.100, 0.260] & [0.200, 0.220] & - \end{bmatrix}$$

$$IV^4 = \begin{bmatrix} - & [0.789, 0.801] & [0.099, 0.271] & [0.679, 0.801] \\ [0.399, 0.441] & - & [0.499, 0.741] & [0.199, 0.431] \\ [0.679, 0.801] & [0.400, 0.400] & - & [0.869, 0.901] \\ [0.399, 0.431] & [0.629, 0.801] & [0.199, 0.221] & - \end{bmatrix}$$

With the above optimal fuzzy preference relations, the consensus reached is 0.790.

4.5. Comparative analysis

Although the average level of information granularity allowed has been low (0.1), the results achieved by each protocol show that a distribution of information granularity along with its optimization can improve the consensus significantly compared to when precise numerical values are utilized (in such a case, the consensus is 0.630). In addition, to better evaluate the performance of the four protocols, an experiment in which 200 GDM problems with different number of experts ($l = 4, \ldots, 7$) and alternatives ($i = 4, \ldots, 8$) are randomly created is also conducted. Assuming $\varepsilon^l = 0.1$, the results show that the consensus reached when precise numerical values are used has been improved by d_1, d_2, d_3, and d_4, with an average percentage of 6.5%, 16.4%, 12.1%, and 20.3%, respectively.

Table 1. Values of c for chosen values of ε^l.

	$\varepsilon^l = 0.1$	$\varepsilon^l = 0.2$	$\varepsilon^l = 0.3$
d_1	0.674	0.744	0.804
d_2	0.744	0.864	0.960
d_3	0.713	0.790	0.862
d_4	0.790	0.929	0.991

Finally, with the objective of investigating the impact of the average level of information granularity on the performance of the different protocols, we execute them with different values of ε^l. Particularly, we set ε^l to 0.1, 0.2 and 0.3. Table 1 shows the evolution of the values for the consensus for different granule sizes. It seems easy to understand that the greater the information granularity level injected into the model, the higher the probability of reaching higher values of c (consensus). This can be observed in Table 1, where a clear increasing trend in the values of c is visible in line with the increment of the values of ε^l. This is due to the greater the value of ε^l, the higher the length of the intervals in which the preference degree can be located and, therefore, the possibility of maximizing the consensus is greater. On the other hand, independently of the information granularity level allowed, the protocol based on an asymmetric and non-uniform distribution of information granularity is the one that achieves greater values of the consensus. It seems natural because, between all the protocols analyzed, this is the one exhibiting the highest flexibility level and, therefore, the probability of achieving a greater value of consensus is also higher.

5. Concluding remarks

To build consensus in fuzzy GDM via a distribution of an average level of information granularity, this study has been the first one that has analyzed the existing four protocols to do it. While there have been several analyses, particularly those coming from the studies of a symmetric and uniform distribution and a symmetric but non-uniform distribution of information granularity, a complete analysis considering all the protocols had not been fully developed. The numerical experiments conducted have illustrated the performance and effectiveness of each protocol. In comparison with the protocols based upon a symmetric distribution of information granularity, the ones based upon an asymmetric distribution have achieved greater values of consensus. Between them, the protocol based upon a non-uniform distribution of information granularity has achieved the best performance. It is due to this protocol, which is the most flexible one, allows for a more efficient use of the resources by distributing higher granularity levels to the components of the fuzzy preference relations requiring it the most.

As a first step at attempting to improve this study, the next research directions emerge. Firstly, even though the computational time required to complete the experiments conducted in Section 4 has been low (few seconds), it would be interesting to study how the computational time increases in real-life situations, as, for example, in social networks, which have modified the context in which GDM problems are conducted by allowing the participation of thousands of individuals in the GDM process. Secondly, the expert's flexibility required to build consensus during the GDM process is simulated by the average level of information granularity. Its value, determined by each expert on each problem, has the interpretation that values close to 1 indicate higher expert's cooperation to build consensus. In this study, we have assumed the same value of the average level of information granularity for all experts. Therefore, it would be interesting to analyze both the performance and the computational aspects of the protocols with different values of the average level of information granularity for different experts.

Acknowledgements

This work was supported by the grant PID2019-103880RB-I00 funded by MCIN / AEI / 10.13039/501100011033, and by the grant PID2022-139297OB-I00 funded by MCIN / AEI / 10.13039/501100011033 and by "ERDF A way of making Europe".

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