# Stages in Teaching Mathematics in the Elementary School

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The work of Jean Piaget is causing many changes in the theory of learning. One area in which these changes are being felt and in which the Swiss psychologist has personally conducted experiments is the area of the learning of mathematical concepts by children. This paper will review the theory of the development of knowledge as proposed by Piaget and then consider the implications of this theory for the teaching of mathematics in the elementary school.

The concept which Piaget considers fundamental to the development of knowledge is that of an operation. An operation is "a set of actions modifying the object, and enabling the knower to get at the structure of the transformation."[1] Notice that this definition requires that the learner carry out his actions on an object or a set of objects and that the learner must be cognizant of the nature of the change he has produced i.e., he must be able to "reason" about the actions he is performing. Some examples of operations would be ordering objects, putting objects in a series, counting objects or measuring objects.

The criteria for determining whether a particular behavior is or is not an operation in terms of the definition above are twofold. First, the action must be intereorized to the learner i.e., it is initiated by the learner and not some external agent. Second, the action must be reversible i.e., the learner must be aware of the fact that a second action exists which when applied to the object following his first action restores the object to its original condition.

It is important to keep in mind the fact that an operation is never isolated, it is always linked to other operations with the framework of a total structure.

Piaget lists four stages through which the learner proceeds in the development of knowledge. These stages are based upon the development, in the learner, of a well defined set of operations.

The first stage is the sensory-motor which usually lasts from birth to the age of two years. During this period occur the fundamental learnings upon which all knowledge is built. For example, the child learns that physical objects have a permanence independent of his perception. This is evidenced by the fact that very young children do not hunt for an object once it is out of sight and later learn to look for an object which has been hidden. The child also learns that physical motions have a cause and his own actions may produce a chain of events.

The second or pre-operational stage which usually lasts from age two to age seven is characterized by the development of language and the use of symbolic referents for objects but the criteria for operations are still not satisfied. For example, at this stage if a pile of coins is placed on a table and then the coins are spread out, a child will tell you there are more coins on the table after they have been spread. The criterion of reversibility has not been satisfied.

The third stage is that of concrete operations and usually occurs from age seven to age eleven. In this stage, the child is able to perform true operations, such as classifying or ordering but these operations are carried out in terms of concrete physical objects. It is at this time that the idea of number, the idea of spatial and temporal relations, the fundamental operations of elementary mathematics, elementary geometry, and even elementary physics make their appearance.

The final stage or formal operational usually occurs from age twelve to age fifteen. By this time, the child is no longer limited to the manipulation of physical objects but can successfully carry out his operations on abstract ideas. He is able to reason from hypotheses rather than from objects. He is even able to construct new knowledge by reorganizing knowledge which he already possesses.

Piaget stresses that the order of appearance of these four stages is fixed but the "timetable" suggested for their appearance varies with the individual learner.

We can identify four factors which contribute to the development of knowledge as proposed by Piaget: nervous maturation, encounters with experience, social transmission, and equilibration or auto-regulation.[2] Perhaps a simple analogy at this point will help to explain the role these four factors play in the development of knowledge. A pile of stones and some mortar do not of themselves form a wall. The stones must be arranged properly and the mortar used to keep them in place before the wall can take shape. The first three factors (nervous maturation, encounters with experience, and social transmission) may be thought of as supplying the child with the raw materials for the development of knowledge, the stones and mortar. The fourth factor (equilibration) corresponds to the process of arranging the stones and applying the mortar, the construction by the child of a "wall" of knowledge.

What does all of this have to do with teaching mathematics in the elementary school?

First, the young child must be given ample opportunity to carry out his mathematical operations with actual physical objects. He should count, order, arrange, add, or subtract real oranges or apples or toys or with whatever he is performing the operation. While the textbook may offer attractive pictures to look at, these should not be substituted for reality until the operations on physical objects have been mastered by the child.

Second, learning doesn't result from talking to the child. The child must be actively involved in creating the mathematics he is to learn. For example, knowledge of the relationship of the lengths of the diagonals of a square should come about as the result of the child's measuring the diagonals of several squares.

Third, since one of the factors affecting the development of knowledge is social transmission, it is important for the child to compare his answers with those of other children. As a result, he comes to accept the possibility that there may be more than one solution to a problem. In this way, unique or creative solutions are something he can look for each time he approaches a problem. He also comes to accept the possibility that a situation may be viewed in different ways. Thus, seemingly unrelated problems can now be grouped together and a general solution developed for that class of problems.

Fourth, there should be many opportunities when the child is presented with collections of mathematical data (for example, sets of numbers or collections of geometric shapes) and asked to list everything he observes about the data. In effect, he must be allowed to draw his own generalizations, simple or profound, without being told to look for specific ones.

Fifth, after determining where the child's mathematical strengths and weaknesses lie, it would be beneficial to determine his stage of development as defined by Piaget. Then educational experiences appropriate for that stage of development could be prescribed for him.

Sixth, since the concrete precedes the formal operation, new content should be based on experience with physical objects. This applies not only to the young; but also to the older child as well. Symbolic statements, whether of number properties such as the commutative law, space properties such as the relationship among the number of surfaces, vertices, and edges of a cube, or algebraic properties such as the structure of a group, take on fuller meaning if the child has had several concrete experiences with the objects he is considering.

Finally, since we can partially control the factors of maturation, experience, and social transmission, we may be able to speed up the stages of development. This would mean then we can profitably expose the child to more mathematics than is presently incorporated into the elementary school program. The crucial problem then becomes one of deciding what mathematics the child should be exposed to and what order of presentation will produce the greatest mathematical maturity.

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[1]Jean Piaget: Development and Learning, Journal of Research in Science Teaching, Vol. 2, Issue 3, 1964. Pp. 176-185.

[2]Duckworth, Eleanor—Piaget Rediscovered Journal of Research in Science Teaching, Vol. 2, Issue 3, 1964. Pp. 172-175.