JOHNSON, Rockne Hart, 1930-

THE SYNTHESIS OF POINT DATA AND PATH DATA IN ESTIMATING SOFAR SPEED.

University of Hawaii, Ph.D., 1968
Geophysics

University Microfilms, Inc., Ann Arbor, Michigan

THE SYNTHESIS OF POINT DATA

AND PATH DATA IN ESTIMATING

SOFAR SPEED

A DISSERTATION SUBMITTED TO THE GRADUATE DIVISION OF THE
UNIVERSITY OF HAWAII IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

IN GEOSCIENCES

June 1968

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PREFACE

Geophysical interest in the deep ocean sound (sofar) channel centers on its use as a tool for the detection and location of remote events. Its potential application to oceanography may lie in the monitoring of variations of physical properties averaged over long paths by sofar travel-time measurements.

As the accuracy of computed event locations is generally dependent on the accuracy of travel-time calculations, the spatial and temporal variation of sofar speed is a matter of fundamental interest. The author's interest in this problem has grown out of a practical need for such information for application to the problem of locating the sources of earthquake _T_ waves and submarine volcanic sounds. Although an extensive body of sound-speed data is available from hydrographic casts, considerably more precise measurements can be made of explosion travel times over long paths. This dissertation develops a novel procedure for analytically combining these two types of data to produce a functional description of the spatial variation of sofar speed.

Data used in this paper has been collected from many sources. To name the more recent, a large number of explosion travel-time measurements were acquired in 1967 through a cooperative effort with John Ewing and David Epp of the Lamont Geological Observatory and with the Pacific Missile Range. Hydrographic measurements from Scorpio Expedition (1967) in the South Pacific were provided prior to publication by Joseph Reid of the Scripps Institution of Oceanography.

This work has been partially supported by the Advanced Research Projects Agency through contract Nonr 3748(01) with the Office of Naval Research.

ABSTRACT

Although an extensive body of data on the speed of sound in the ocean is available from hydrographic casts, considerably more precise measurements can be made of explosion travel times over long paths through the deep ocean sound (sofar) channel. A novel method is presented for analytically combining the two types of data to produce a functional description of the spatial variation of sofar speed. This method is based on the fact that the integral, over a path, of a series representation of the reciprocal of speed yields a series representation of the travel time over that path, the same set of coefficients entering linearly into both series. Both types of observations may then be combined in the same matrix equation for estimating the coefficients. An example is computed for the Pacific Ocean in the form of a spherical harmonic function of degree 6. Hydrographic data consisted of values averaged for 4013 one-degree squares. Approximately 400 temporally independent travel-time measurements, over paths ranging in length from 17° to 110°, were used. The paths were concentrated in the northeast Pacific. The estimated variance of a single point observation of sofar speed was 1.56 $(\text{meter/sec})^2$ while the variance of a single observation of harmonic-mean sofar speed was 0.016 $(\text{meter/sec})^2$. These values were used, where appropriate, for weighting the data during least-squares estimation of the coefficients of the spherical harmonic function.

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CHAPTER I

BACKGROUND

The relationship of the sofar channel to seismology was established by Tolstoy and Ewing[1] who showed that earthquake _T_ waves travel through the sofar channel. They pointed out that the lower speed of sound propagation through the ocean, compared to the speeds of body waves, might permit more accurate epicenter determinations than conventional methods.

The detection by sofar of volcanic eruptions was predicted by Ewing et al.[2] and demonstrated by Dietz and Sheehy.[3] More recent discoveries by sofar of previously unknown submarine volcanoes have been by Kibblewhite[4] and by Norris and Johnson.[5]

A program of routine _T_-phase source location was conducted at the Hawaii Institute of Geophysics from August 1964 to July 1967.[6] Using

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[1] Ivan Tolstoy and Maurice Ewing, "The _T_ Phase of Shallow-Focus Earthquakes," Seismological Society of America, _Bulletin_, XL (1950), 25-52.

[2] Maurice Ewing, George P. Woollard, Allyn C. Vine, and J. Lamar Worzel, "Recent Results in Submarine Geophysics," Geological Society of America, _Bulletin_, LVII (1946), 909-934.

[3] Robert S. Dietz and S. H. Sheehy, "Transpacific Detection of Myojin Volcanic Explosions by Underwater Sound," Geological Society of America, _Bulletin_, LXV (1954), 941-956.

[4] A. C. Kibblewhite, "The Acoustic Detection and Location of an Underwater Volcano," _New Zealand Journal of Science_, IX (1966), 178-199.

[5] Roger A. Norris and Rockne H. Johnson, "Volcanic Eruptions Recently Located in the Pacific by Sofar Hydrophones," Hawaii Institute of Geophysics rpt. 67-22 (1968), 16 pp.

[6] Rockne H. Johnson, "Routine Location of _T_-Phase Sources in the Pacific," Seismological Society of America, _Bulletin_, LVI (1966), 109-118.

sofar depth hydrophones in the North Pacific, locations were computed for earthquake T-phase sources distributed nearly throughout the Pacific.[7]

The method used in this program for obtaining sofar speed was as follows. First, a contour chart of sofar speed in the Pacific was drawn from hydrographic station data.[8] Harmonic mean values were then obtained graphically along a sampling of great circle paths on this chart. The paths connected the hydrophone stations with points arbitrarily selected along seismicly active regions. These data together with such explosion data as was available, were used to estimate the coefficients of a polynomial representation of harmonic mean sofar speed as a function of latitude and longitude. A different set of coefficients was required for each hydrophone station.

This procedure suffered from several defects. The labor of graphic integration discouraged taking a large sample thereby biasing the applicability of the functions to regions of presupposed interest. An explosion measurement could only be used for the hydrophone station at which it was measured. As the functions for the various hydrophone stations were based on different paths, they exhibited a degree of internal inconsistency. For example, the least-squares computation of the location for some events showed excessively large time deviations of opposite sign at stations on neighboring azimuths.

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[7] Frederick K. Duennebier and Rockne H. Johnson, "T-Phase Sources and Earthquake Epicenters in the Pacific," Hawaii Institute of Geophysics rpt. 67-24 (1967), 17 pp.

[8] Rockne H. Johnson and Roger A. Norris, "Sofar Velocity Chart of the Pacific Ocean," Hawaii Institute of Geophysics rpt. 64-4 (1964), 12 pp.

There were other indications of a need for an improved method. Explosion travel time measurements offer considerably greater precision than measurements at points, as many of the perturbing factors in point measurements are averaged out along the paths. As much of the ocean is as yet inadequately explored, and with the advent of precision navigation systems, it is to be expected that many measurements of both types will accumulate in the future. Efficient use of these data requires that they be combined within a statistical framework where each piece of information is used but weighted according to its variance.

A method for inverting travel time observations to obtain local speeds was proposed by Adams.[9] In Adams' model the region sampled by the observations was partitioned into a set of sub-regions within each of which the speed was constant. If the observations were for paths which adequately crisscrossed the region, then a matrix which specified the paths could be inverted to yield the speed in each subregion. The assignment of constant properties to sub-regions is perhaps appropriate in the solid earth where geologically distinct regions can be recognized. In most of the ocean, however, the variation is quite gradual and a continuous function of speed is more appropriate. A further modification is to include point observations. Such modifications are made in this paper.

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[9] Wm. Mansfield Adams, "Estimating the Spacial Dependence of the Transfer Function of a Continuum," Hawaii Institute of Geophysics rpt. 64-22 (1964), 13 pp.

CHAPTER II

SOFAR SLOWNESS

The computation of travel time by a series representation of speed would place the series in the denominator, an unwieldy location. It is much more convenient to work with the reciprocal of the speed.[1] Furthermore, as the ellipticity of the earth must be taken into account, it is more convenient to reduce speed measurements at points to the equivalent angular speed about the earth's center than it is to reduce the arc length of a travel path to linear measure. Consequently, the analysis to follow will be in terms of the reciprocal of the geocentric angular speed of sound at the sofar axis. This will be called the sofar slowness.

Distances are calculated for arcs of great circles. The departure of a great circle from the geodesic on an ellipsoid is probably of less consequence than the neglect of horizontal refraction. Over most of the Pacific the horizontal refraction at the sofar axis is small. For example, along a travel path between Hawaii and California the horizontal radius of curvature of a ray would be roughly 100 earth radii. However, a ray crossing the convergence between the Kuroshio Extension and the Oyashio would be subjected to quite strong contrasts of sound speed. For a plane boundary between the two currents the maximum refraction would be about 8°. However, the actual boundary is one of meanders and eddies[2]

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[1] George E. Backus, "Geographical Interpretation of Measurements of Average Phase Velocities of Surface Waves over Great Circular and Great Semi-Circular Paths," Seismological Society of America, Bulletin, LIV (1964), 571-610.

[2] Michitaka Uda, "Oceanography of the Subarctic Pacific Ocean," Journal Fisheries Research Board of Canada, XX (1963), 127.

and transmission across it is probably multipathed and diffuse, as in optically viewing a region beyond a hot stove. The present treatment will ignore these effects.

CHAPTER III

STATISTICAL MODEL

Let the slowness S be represented as a function of colatitude θ and east longitude ϕ in the form

$$S(\theta, \phi) = \sum_{i=1}^{m} b_i H_i(\theta, \phi) \qquad (1)$$

where the H_i are linearly independent functions of position and the b_i are coefficients to be determined. Travel time t is the integral of (1) over the path p, here assumed to be an arc of a great circle.

$$t_p = \int_p S(\theta, \phi) = \sum_{i=1}^{m} b_i \int_p H_i(\theta, \phi) \qquad (2)$$

The fundamental point of this paper is that the same set of coefficients b_i occurs linearly in both (1) and (2). Therefore, the coefficients may be estimated, as by the method of least squares, either from a set of slowness observations at points or from a set of travel time observations along paths or from a combination of both. The matrix equation for the estimation is

$$Y = H B + e \qquad (3)$$

where

$$Y' = (S_1, S_2, \ldots, S_j, t_1, t_2, \ldots, t_k),$$

a vector of observations,

$$H = \begin{bmatrix} H_1(\theta_1, \phi_1) & H_2(\theta_1, \phi_1) & \cdots & H_i(\theta_1, \phi_1) & \cdots & H_m(\theta_1, \phi_1) \\ H_1(\theta_2, \phi_2) & H_2(\theta_2, \phi_2) & \cdots & H_i(\theta_2, \phi_2) & \cdots & H_m(\theta_2, \phi_2) \\ \vdots & \vdots & & \vdots & & \vdots \\ H_1(\theta_j, \phi_j) & H_2(\theta_j, \phi_j) & \cdots & H_i(\theta_j, \phi_j) & \cdots & H_m(\theta_j, \phi_j) \\ \int_1 H_1(\theta, \phi) & \int_1 H_2(\theta, \phi) & \cdots & \int_1 H_i(\theta, \phi) & \cdots & \int_1 H_m(\theta, \phi) \\ \int_2 H_1(\theta, \phi) & \int_2 H_2(\theta, \phi) & \cdots & \int_2 H_i(\theta, \phi) & \cdots & \int_2 H_m(\theta, \phi) \\ \vdots & \vdots & & \vdots & & \vdots \\ \int_k H_1(\theta, \phi) & \int_k H_2(\theta, \phi) & \cdots & \int_k H_i(\theta, \phi) & \cdots & \int_k H_m(\theta, \phi) \end{bmatrix}$$

a $(j + k)$ by m condition matrix,

$$B' = (b_1, b_2, \ldots, b_i, \ldots, b_m)$$

a vector of coefficients, and e is a vector of residual errors.

By the ordinary method of least squares the coefficients are estimated as

$$\hat{B} = (H'H)^{-1} H'Y.$$

(the circumflex accent denotes an estimate of a random variable.)

Now the value of m is a matter of choice; a larger value will usually provide a better fit of the function to the data but the function will be more cumbersome and take longer to compute. A method of estimating the coefficients of a series term by term, thereby postponing the decision on the number of terms, has been worked out by Fougere.[1] Parts of his method will be applied to the present study.

Fougere's method involves replacing the term HB in (3) with an equal term XA in which the columns of the matrix X are mutually orthogonal. Then when A is estimated as

$$\hat{A} = (X'X)^{-1} X'Y,$$

X'X is a diagonal matrix. Not only is its inversion trivial but the components of A are independently determined as

$$\hat{A}_i = (X_i'X_i)^{-1} X_i'Y$$

where X_i is the ith column vector of X. Therefore, if one can construct the matrix X column by column, then the coefficients A_i can be computed one by one.

The well known Gram-Schmidt process is a method for constructing a set of mutually orthogonal vectors in the vector space spanned by any set of linearly independent vectors (such as the columns of H). The procedure is to first specify one column vector in H, say the first, to be a column vector in X. Then the next column vector in X is obtained by subtracting, from a second column vector in H, its projection onto the first column vector of X. Each succeeding column of X is then generated by subtracting, from a corresponding column of H, all components parallel to previous columns of X. X, then, is built up in the desired manner.

It remains to determine $\hat{B}$ from $\hat{A}$. The Gram-Schmidt process may be considered as a linear transformation C such that

$$HC = X.$$

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[1] Paul F. Fougere, "SPherical Harmonic Analysis. 1. A New Method and Its Verification," Journal of Geophysical Research, LXVIII (1963), 1131-1139.

As Y = XA + e, substituting HC for X gives

$$Y = HCA + e$$

Comparison with (3) shows that B = CA. A method for computing C is given by Fougere.[2]

The ordinary method of least squares applies to the case where the observations are uncorrelated and of constant variance. In the present case, although the assumption of independence will be made, a significant range of variance is encountered. Treatment by the generalized method of least squares is therefore required.[3]

In the uncorrelated case the generalized method accomplishes weighting. The procedure is to premultiply (3) by a nonsingular matrix T such that

$$(T'T)^{-1} = V = E\,(ee')$$

where E is the expectation operator. In the uncorrelated case, T is a diagonal matrix the elements of which are the reciprocals of the square roots of the variances.[4] One then has

$$TY = THB + Te$$

from which the least squares estimate of B is

$$\hat{B} = (H'V^{-1}H)^{-1}H'V^{-1}Y.$$

By Fougere's method however, one substitutes XA for THB where X is now an orthogonal matrix derived from TH. The elements of A are now

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[2] Ibid., p. 1135.

[3] A. C. Aitken, "On Least Squares and Linear Combinations of Observations," Royal Society of Edinburgh, Proceedings, LV (1935), 42-48.

[4] J. Johnston, Econometric Methods (New York, 1960), pp. 186, 208.

independently determined as

$$A_i = (X_i{}'X_i)^{-1} X_i{}'TY \qquad (4)$$

As before, one has CA = B where C is the transformation in X = THC.

CHAPTER IV

DATA

The point slowness data, derived from hydrographic casts, consist of values assigned to the centers of 4013 one-degree squares distributed throughout the Pacific. These include the data used by Johnson and Norris[1] in constructing a hand-drawn sofar-speed contour map (reproduced here as Figure 1). Those data have been supplemented by the hydrographic data from Scorpio Expeditions I and II (Eltanin Cruises 28 and 29, March-July 1967) which crossed the South Pacific along 43^{o}S and 28^{o}S.

Each value is the average of those computed from hydrographic casts taken within that one-degree square. Sofar speed was computed from hydrographic measurements by Wilson's formula.[2] Conversion from linear speed to geocentric angular speed was by the formula

$$1 \text{ minute arc/second} = (1852.26 - 3.11 \cos 2\theta) \text{ meters/second}$$

where θ is colatitude. This formula is an approximat fit to the International ellipsoid.

The variance of the data was estimated by comparing them with the sofar-speed contour map (Figure 1). For about 1400 of the one-degree squares, the slowness assigned was based on a single hydrographic station in each square. The difference between these values and the value from the contour map yielded an estimate of variance of 13.2 $(\text{sec/radian})^2$. This is equivalent to a speed variance of 1.56 $(\text{meters/second})^2$.

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[1] Rockne H. Johnson and Roger A. Norris, "Geographic Variation of Sofar Speed and Axis Depth in the Pacific," Hawaii Institute of Geophysics rpt. 67-7, (1967), pp. A1-A83.

[2] Wayne D. Wilson, "Speed of Sound in Sea Water as a Function of Temperature, Pressure, and Salinity," Acoustical Society of America, Journal, XXXII (1960), 641-644.

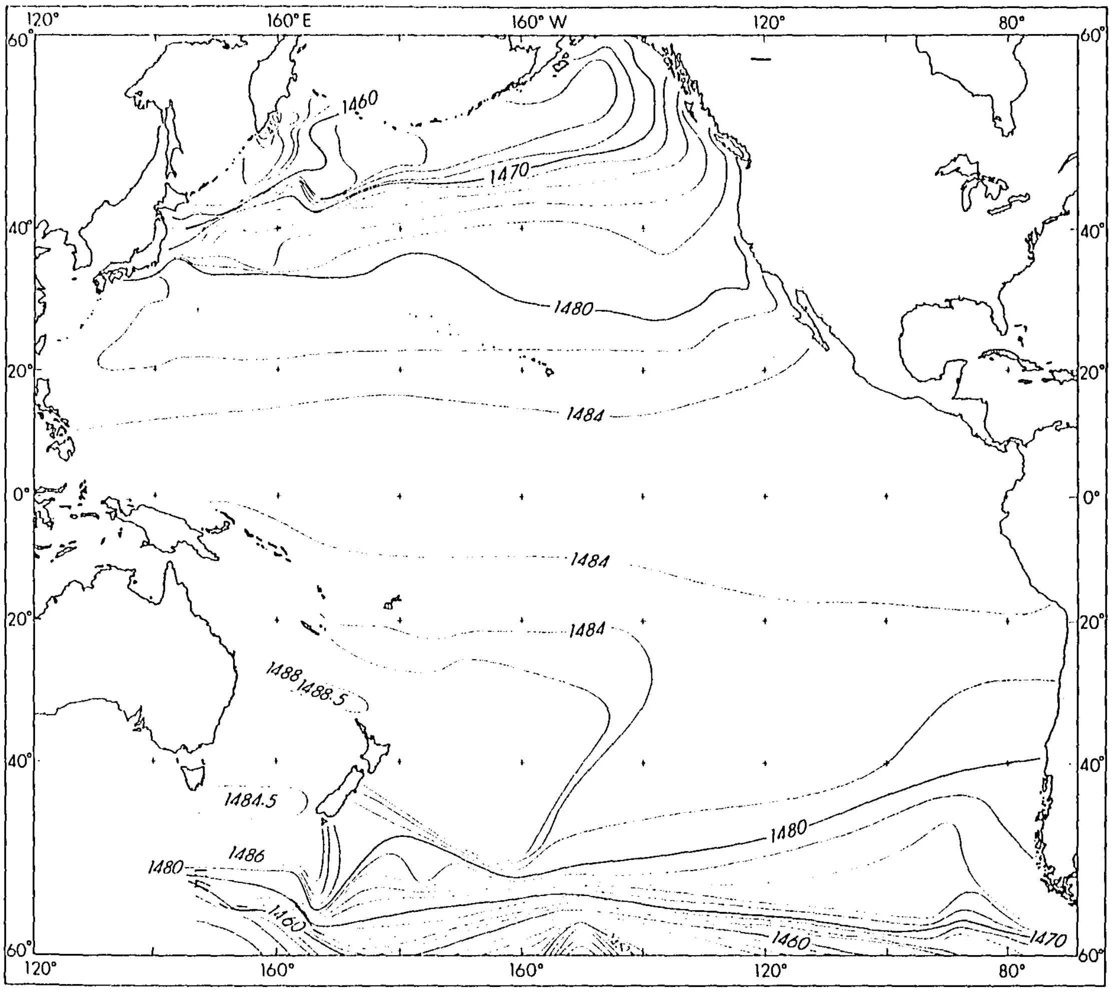

Fig. 1. Contour map of sofar speed, in meters per second, hand-drawn from plotted point data.

This is the estimate of variance for a single measurement. For those squares represented by n hydrographic casts, the variance of the mean is smaller by a factor of 1/n.

The travel-time data represent 124 paths crisscrossing the northeast Pacific region shown in Figure 2. These data have accumulated from experiments conducted by a number of organizations. The most recent, and spatially most extensive, was conducted in connection with the cruise of the Conrad from Victoria, B. C., to Tahiti, to Panama during August to October 1967. This vessel was equipped with a satellite navigation system which provided positional precision of about one-half kilometer.[3] Travel times were successfully measured from 33 Conrad stations, the ship's position being fixed by satellite navigation for 23 of these.

In order to weight these data against the point observations it is necessary to estimate their variance. An experiment involving repeated measurements over the same paths was conducted from February 1966 to July 1967. This experiment consisted of dropping ten sofar bombs (4-lb. TNT) each month into an array of hydrophones near Midway Island and recording arrival times at a number of other hydrophone stations around the North Pacific. As the experiment involved fixed hydrophones at both ends of the paths, the distances could be quite precisely computed. The resulting sofar speed calculations, for paths to Oahu, Wake, and Eniwetok, are plotted in Figure 3. Each plotted vertical bar is centered at the mean for approximately ten explosions fired at two-minute intervals. The lengths of the bars indicate a range of plus

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[3] Manik Talwani, H. James Dorman, J. Lamar Worzel, and George M. Bryan, "Navigation at Sea by Satellite," Journal of Geophysical Research, LXXI (1966), 5891-5902.

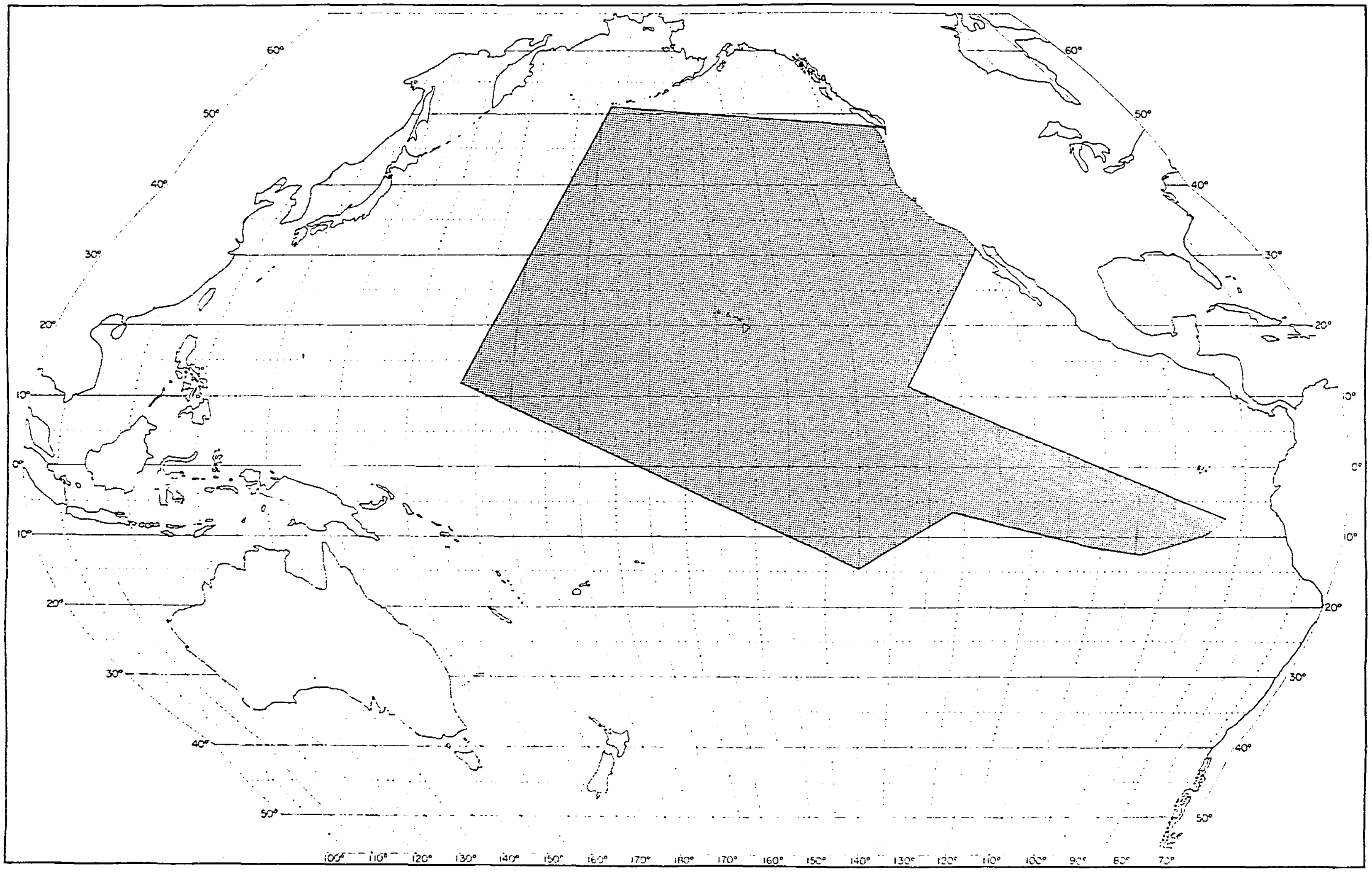

Fig. 2. Region crisscrossed by precisely measured sofar travel paths.

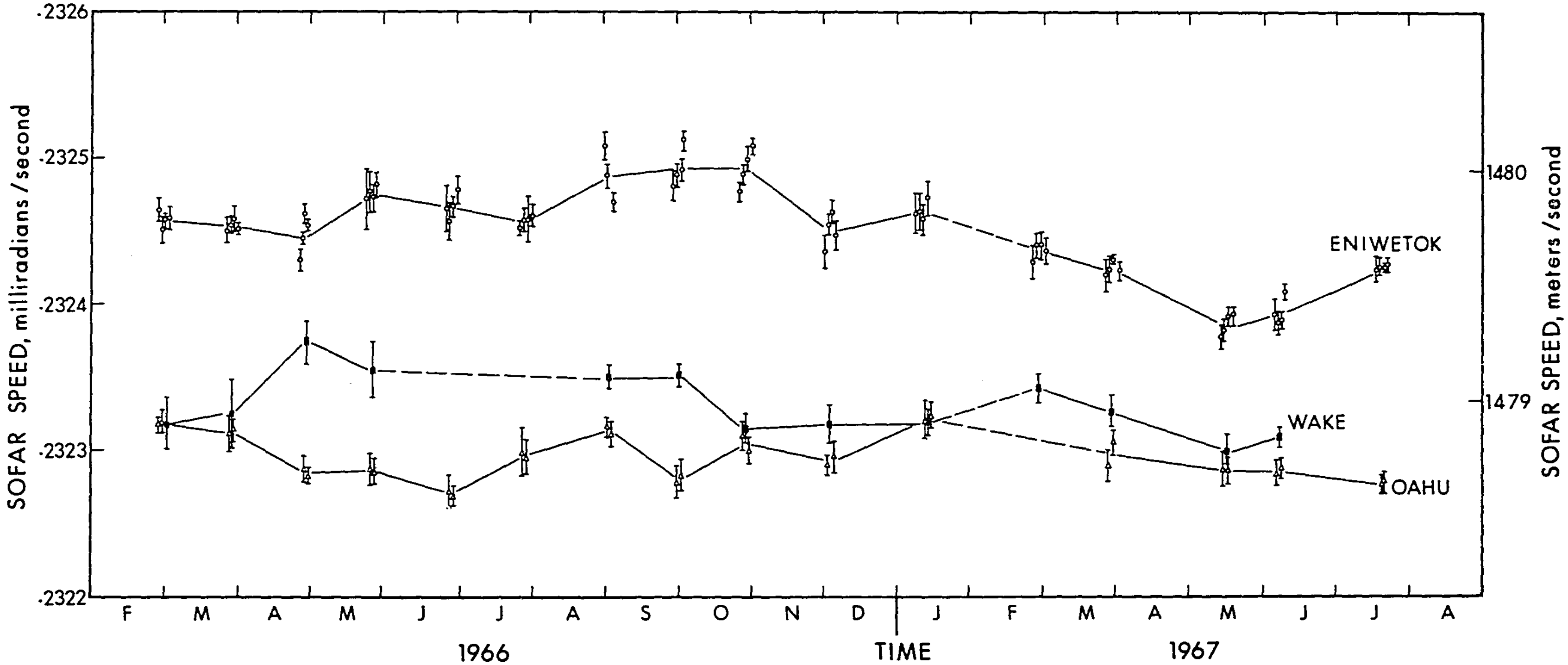

Fig. 3. Temporal variation of sofar speed over paths from Midway to Eniwetok, Wake, and Oahu.

and minus one standard deviation. At Oahu and Eniwetok where signals were received at more than one hydrophone the positions of the plotted bars have been offset along the time axis for clarity. The standard deviation of the harmonic-mean sofar speed, for a set of ten bombs dropped at two-minute intervals, was typically about 5 centimeters per second. However, the range of sofar speed for a 15-month period over each path was about 50 centimeters per second.

The observed temporal variations, then, indicate an upper limit on the precision obtainable from travel-time measurements. They will be employed here to estimate the variance of a single path slowness measurement taken under conditions of precise navigation.

Given a set of n paths for which the covariances of sofar slowness have been estimated, consider a path composed of all those paths laid end to end. The composite path is now a single path for which an estimate of variance of sofar slowness will incorporate all the available information. As the travel time over the composite path is equal to the sum of the travel times over the individual paths,

$$\overline{S} \sum_{i=1}^{n} \Delta_i = \sum_{i=1}^{n} \overline{S}_i \Delta_i .$$

Here $\overline{S}$ is the mean slowness over the composite path, and the Δ_i and $\overline{S}_i$ are the distances and mean slownesses over the individual paths.

The variance of $\overline{S}$ is

$$\sigma^2 = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \Delta_i \Delta_j \sigma_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n} \Delta_i \Delta_j}$$

for covariance σ_{ij} between paths _i_ and _j_. The estimate of σ^2 obtained from the experiment just described was 0.134 $(\text{second/radian})^2$. This is about one hundredth that of a point slowness observation. This value is used in this paper to weight those path observations for which the sampling was not adequately distributed through the year.

All path data used, except for ten of the _Conrad_ stations, were obtained under such conditions that the distances could be measured with high precision. The situation in each case was one of the following:

a) Loran-C navigation of the shooting ship in a region of good control,

b) satellite navigation of the shooting ship,

c) the shooting ship approximately on a great circle path between fixed hydrophones (travel time was for total path between fixed hydrophones),

d) explosive dropped into a fixed array,

e) microwave navigation of the shooting ship.

The receiving hydrophones were always fixed.

During the last part of the _Conrad_ cruise the satellite navigation equipment was inoperative. However, as the paths were long, _circa_ 90 degrees, the less precise navigation had a proportionately small effect on the precision of the path length computation. The variance for these observations was estimated by treating them as repeated observations over the same path. This appeared feasible as the plotted values of mean sofar slowness versus _Conrad_ station number showed no significant drift over this range of stations. The variance estimated was 1.0 $(\text{sec/rad})^2$.

CHAPTER V

SLOWNESS FUNCTION

The choice of the form of the functional representation for sofar slowness is arbitrary, depending on the extent and character of the region and the expected application. Features such as the Antarctic convergence and the Oyashio-Kuroshio Convergence suggest that discontinuities may be appropriate; however, such an approach has the added requirement of defining boundaries. Again, the relative utility may be debated of computing a continuous function for the entire ocean versus computing a set of less complicated functions for smaller areas.

A major expected application of this function is to the computation of travel times over long paths. Such a process is conceptually simplified when the function is specified by a single equation throughout the domain. A set of such functions for which the properties on a spherical surface are well known is the spherical surface harmonics. With these substituted into equation (1) in place of the H_i, the slowness function of degree r takes the form

$$S_r = \sum_{n=o}^{r} \sum_{n=o}^{n} P_n^m (\cos\theta)\,(a_n^m \cos m\phi + b_n^m \sin m\phi) \qquad (5)$$

where

$$P_n^m (\cos\theta) = \sin^m\theta \, \frac{d^m P_n (\cos\theta)}{d(\cos\theta)^m}$$

Equation (2) requires that spherical harmonic functions be integrated over the travel paths. An exact method would involve rotation of the pole of the coordinate system to a pole of the path, followed by integration over the path.[1] As an expediency in the present case, however, the integrals were approximated by summations, with the harmonic functions being evaluated at increments of 0.08 radian or less. This increment was found to yield an accuracy of 0.01% or better over the paths involved.

By dividing equation (2) by distance Δ_p one has the mean slowness

$$\overline{S}_p = \frac{t_p}{\Delta_p} = \Sigma\, b_i \frac{\int_p H_i}{\Delta_p}$$

The *ith* term in the summation is simply the average value of the corresponding harmonic over the path. This modification of equation (2) allows one to combine repeated measurements over nearly the same path by direct averaging of the mean slowness values.

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[1] Backus, *op. cit.*

CHAPTER VI

RESULTS

A decision is required as to the degree, r, to which the function should be computed. Some degree of subjectivity is inevitable, whether it takes the form of choosing a confidence level or of designing a utility function. Under Fougere's method the decision on the number of terms in the series may be made at any point in the calculations. The progressive improvement in the fit of the function to the data, as well as the computing time, may then influence the decision to terminate. The fit of the function to the data is indicated by the residual variance,

$$\hat{\sigma}_m^2 = \frac{n}{n-m} \; \frac{Y'T'TY - \sum_{i=1}^{m} X_i'X_i \hat{A}_i^2}{\mathrm{tr}\; T'T}$$

where n is the total number of observations, m is the number of coefficients computed, and vector notation is as in equation (4). This formula, adapted from Fougere, is convenient for sequential computation but cannot be handled with sufficient accuracy by the computer when, as in the present case, A_1 is orders of magnitude larger than any of the other A_i. The difficulty is avoided by treating the first term in the summation in a more elementary manner as follows

$$\hat{\sigma}_m^2 = \frac{n}{n-m} \; \frac{(TY - X_1 \hat{A}_1)' \, (TY - X_1 \hat{A}_1) - \sum_{i=2}^{m} X_i'X_i \hat{A}_i^2}{\mathrm{tr}\; T'T}$$

During the computing of the coefficients, $\hat{A}_i$, a graph of $\hat{\sigma}_m^2$ versus m was printed by the computer at values of m = 1, 4, 9, . . ., corresponding to integer values of the degree of the spherical harmonic series. By monitoring this graph and the computing time a subjective determination was made of the utility of continued computing.

Computing was terminated with the computation of 49 coefficients. The residual variance at this point was 14 $(\text{sec/rad})^2$. The graph of residual variance versus number of terms is Figure 4. The coefficients, both for the orthogonal basis (A_i of equation (4)) and for the spherical harmonic bases of degree 4, 5, and 6 (25, 36, and 49 terms), are listed in the Appendix. A contour map corresponding to the 49-term function is shown in Figure 5. For ease of comparison with the hand-drawn sofar-speed contour map, Figure 1, the contours in Figure 5 have been converted to speed. Figure 6 shows the regional distribution of residuals for the degree-six function. The spacing of the regions in this figure suggests that harmonics as high as degree twelve might be required to significantly reduce the residual variance.

While the coefficients in the orthogonal basis are independently determined, those in the spherical harmonic basis change with the degree of the series representation. The zeroth-degree term of the higher-degree representations departs significantly from the constant term in the orthogonal basis (A_1). This behavior is a result of having the data confined to less than one hemisphere of the earth's surface. The function takes on widely ranging values on other parts of the sphere as the least-squares process accommodates the data within the region of the observations. Of course the function is intended for application only in the Pacific and the global mean of the function need not represent the average slowness in the Pacific. Fougere, on the other hand, in dealing with a global phenomenon (geomagnetism), found such so-called instability

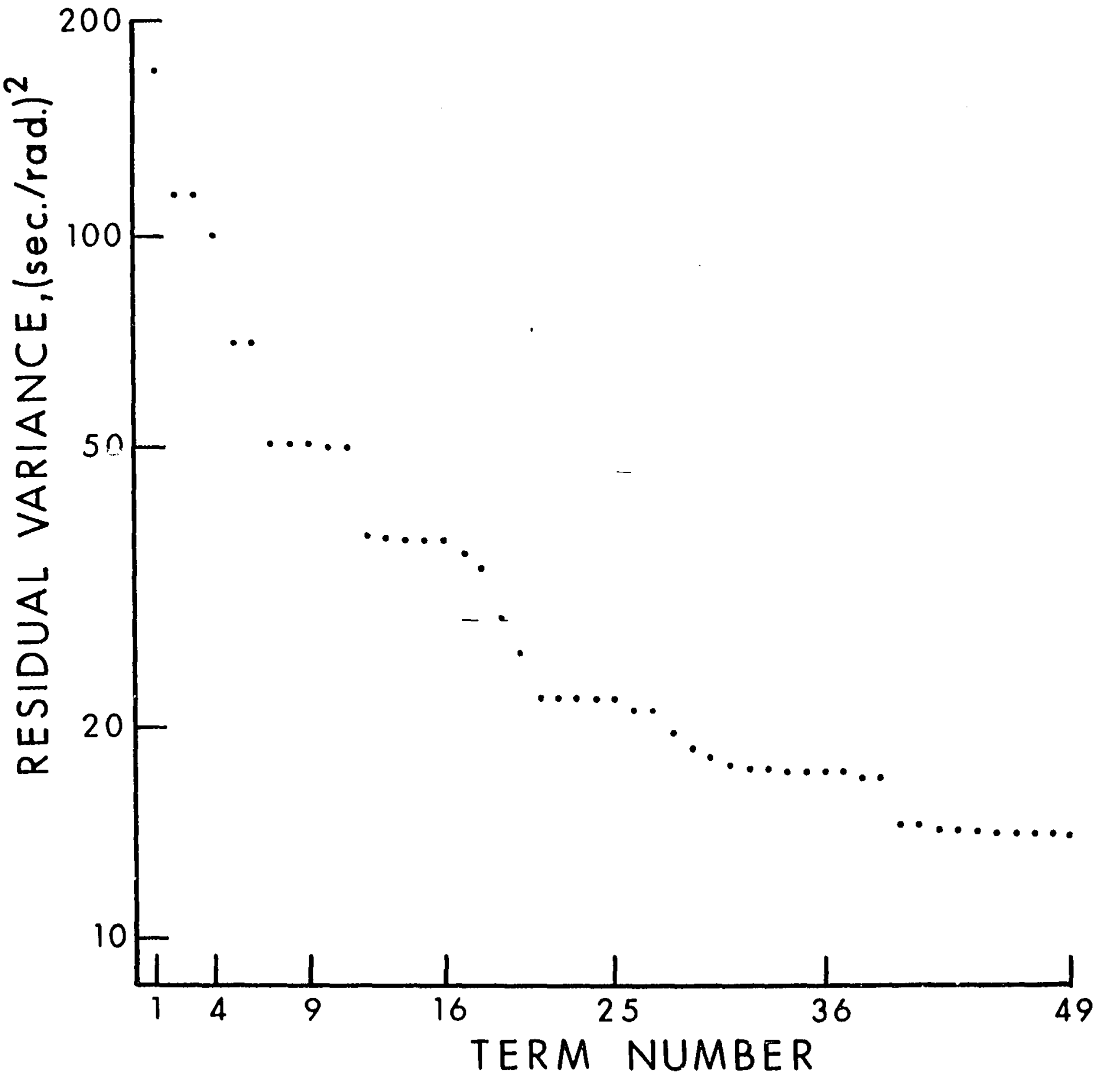

Fig. 4. Residual variance versus number of terms for orthogonal function.

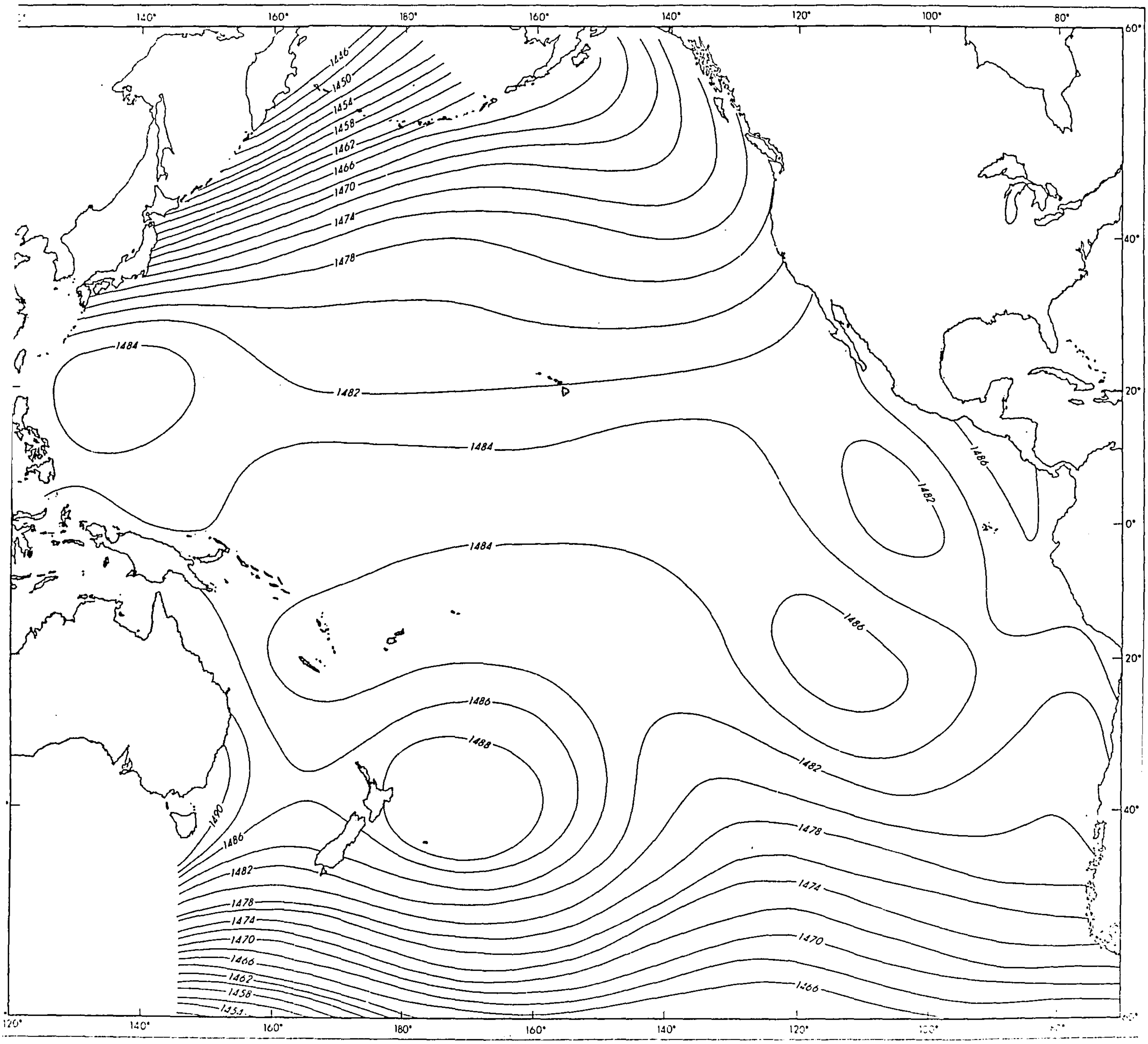

Fig. 5. Contour map of sofar speed, in meters per second, derived from slowness function of degree 6.

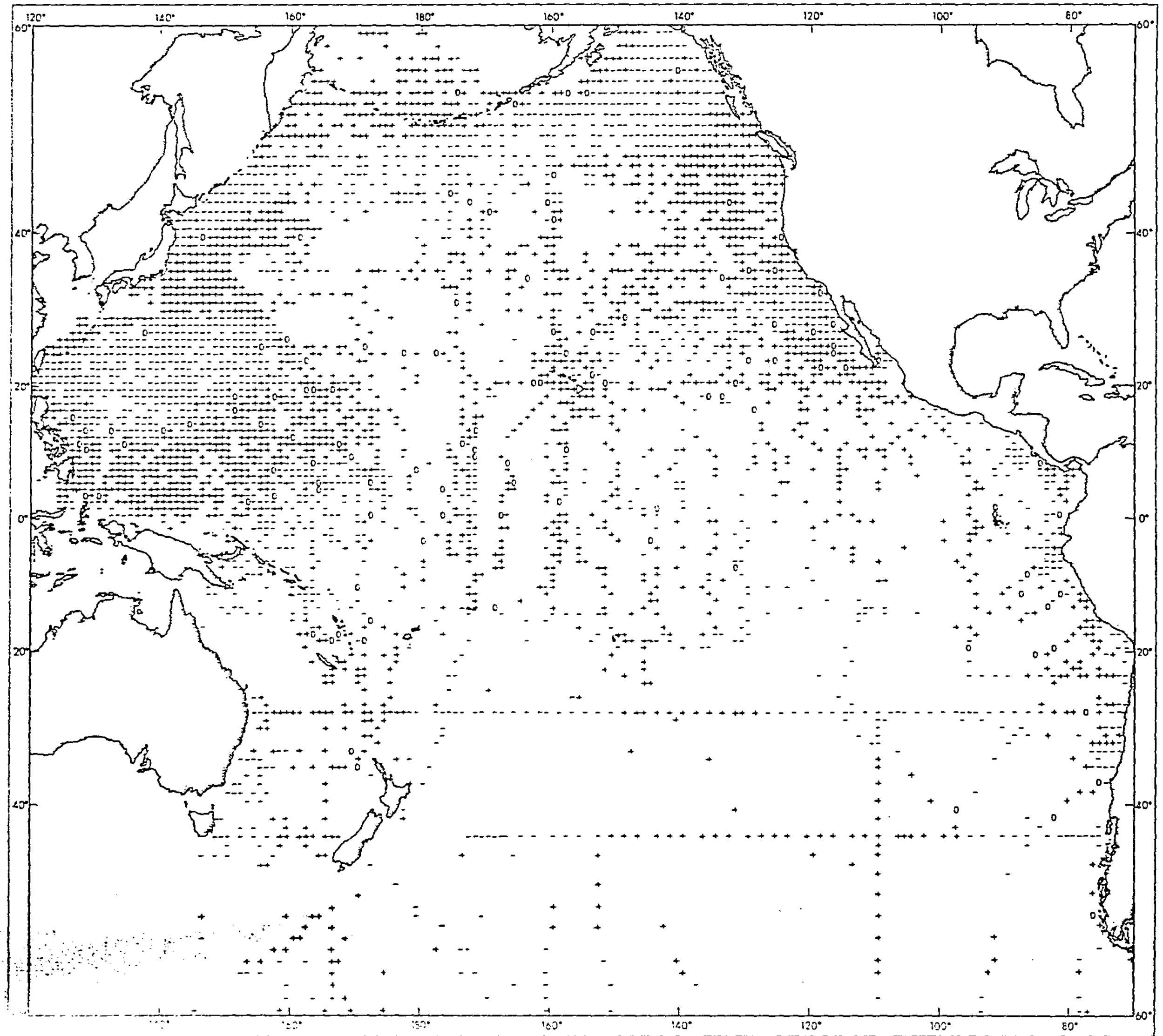

Fig. 6. Algebraic sign of the residuals for the point data and the degree-six function (observed minus computed value of speed).

objectionable.[1] As indicated by Figure 6, the largest gaps in the data in the Pacific are on the order of 15 to 20 degrees in breadth. As the shortest wave length in the degree-six spherical harmonic series is 60 degrees, it may be assumed that no gross departures from probable values of the sofar slowness occur within these gaps.

Another question is whether the sample spacing is everywhere adequate to prevent aliasing. The features most likely to give trouble in this regard are the Kuroshio-Oyashio Convergence and the Antarctic Convergence. These are practically discontinuities at any instant, however the time-averaged variation is somewhat less abrupt. Except for a seasonal bias, the sampling in the northwest Pacific is probably adequate as nearly every one-degree square is sampled and the temporal meanders of the Kuroshio no doubt exceeds this spacing. The region of the Antarctic Convergence, however, has been quite sparsely sampled and one may expect some misrepresentation on this account. The only remedy is to collect more data.

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[1] Paul F. Fougere, "Spherical Harmonic Analysis. 2. A New Model Derived from Magnetic Observatory Data for Epoch 1960.0," Journal of Geophysical Research, LXX (1965), 2171-2179.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

Sofar travel-time measurements may be analytically combined with local measurements of sofar speed to obtain a functional representation of sofar slowness, provided that estimates of the precision of the various measurements can be made. Employment of such a representation in source-location computations will, at least, render the computations internally consistent. Furthermore, the representation here provided for the Pacific Ocean utilizes nearly all the presently available data.

As, in the central North Pacific at least, the long-term fluctuations of sofar speed show no strong correlation with the seasons an attempt to predict temporal variations is not presently feasible. In high latitudes seasonal variations are, no doubt, more pronounced. It should be noted that there has been an understandable tendency to collect such data during the summer months and the present representation is biased in this respect. For high latitude paths there are other problems such as scattering and refraction at convergences of strongly contrasting currents. Also the vector field of near-surface currents may significantly alter sofar travel times where the sound channel is shallow.

The largest gaps in the point data occur in the South Pacific. These are being rapidly reduced by oceanographic expeditions, however, insufficient attention has been paid to the concurrent collection of path data. The relatively high precision of such measurements make them quite valuable although they require coordination between the shooting ship and the receiving stations. It is recommended that sofar shots be carried out routinely by future expeditions, especially on ships equipped for satellite navigation.

Considerably greater benefit would be derived from such shots were not the locations of sofar hydrophone stations confined to the North Pacific. Hydrophone stations in the South Pacific would also be of value for monitoring of volcanic and seismic activity. The East Pacific Rise and Pacific Antarctic Ridge are poorly covered by the present seismograph network. Even the hydrophone network in the North Pacific detects and provides locations for many more seismic events.[1] The conventional seismograph station at Macquarie Island has also demonstrated its effectiveness in detecting T waves from these regions.[2] Perhaps advances in buoy technology will reduce the expense of sofar hydrophone installations for geophysical purposes to a feasible level. If so, their deployment in the South Pacific is recommended.

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[1] Duennebier and Johnson, op. cit.

[2] R. J. S. Cooke, "Observations of the Seismic T Phase at Macquarie Island," New Zealand Journal of Geology and Geophysics, X (1967), 1212-1225.

APPENDIX

TABLE I. COEFFICIENTS OF SLOWNESS FUNCTION BASED ON COMBINED POINT AND PATH DATA, SECONDS PER RADIAN

Term	Orthogonal Basis	Term	Spherical Harmonic Bases		
			Degree 4	Degree 5	Degree 6
1	4307.2383	a_0^0	4124.566445	2820.407869	- 2419.200896
2	32.355392	a_1^0	- 209.015785	-1463.627584	- 5042.197530
3	6.244447	a_1^1	- 498.327824	-3157.376289	-14747.381176
4	11.311575	b_1^1	8.226817	- 717.479687	- 5161.292183
5	41.614227	a_2^0	349,489875	820.844368	6165.064522
6	- 1.649976	a_2^1	- 175.732320	-1372.345257	- 4721.846920
7	19.403915	b_2^1	- 42.970575	- 271.430561	- 1724.798658
8	.147446	a_2^2	- 125.344011	- 688.645953	- 3220.198352
9	.113214	b_2^2	- 28.076486	- 354.621521	- 2587.788679
10	13.181213	a_3^0	168.690813	1111.508249	3995.825719
11	- 2.161591	a_3^1	78.072820	- 24.077136	1400.004158
12	17.553970	b_3^1	69.856480	37.433786	597.491722
13	- 1.165633	a_3^2	- 12.058124	- 185.278502	- 634.077873
14	1.932945	b_3^2	- 6.772749	- 87.002587	- 569.893991
15	- 1.036626	a_3^3	- 9.662373	- 57.739663	- 259.022498
16	- .751462	b_3^3	- 5.912725	- 54.607345	- 417.820124
17	25.196075	a_4^0	- 22.977012	339.095683	- 400.699071
18	- 10.847346	a_4^1	13.455176	166.188434	664.154607
19	12.212422	b_4^1	15.297564	73.485307	313.931306
20	- 4.501008	a_4^2	0.252737	- 16.062215	55.844508
21	6.441711	b_4^2	7.311346	- 1.930994	62.836863
22	- .103810	a_4^3	- 0.345259	- 9.612403	- 28.925778

TABLE I. (Continued) COEFFICIENTS OF SLOWNESS FUNCTION BASED ON COMBINED POINT AND PATH DATA, SECONDS PER RADIAN

Term	Orthogonal Basis	Term	Spherical Harmonic Bases		
			Degree 4	Degree 5	Degree 6
23	.559683	b_4^3	0.233893	- 7.814688	- 61.520180
24	- .199757	a_4^4	- 0.234734	- 1.878086	- 6.588870
25	- .216948	b_4^4	- 0.216948	- 3.414519	- 30.098130
26	- 21.341675	a_5^0		- 78.944696	- 316.266577
27	- 2.608631	a_5^1		34.783559	- 26.068659
28	9.983725	b_5^1		14.812887	26.696707
29	- 2.236276	a_5^2		2.430671	15.905405
30	2.310547	b_5^2		4.594276	24.304727
31	- .589166	a_5^3		- 0.682360	0.134804
32	- .330528	b_5^3		- 0.214661	2.097999
33	- .132765	a_5^4		- 0.189583	- 0.121911
34	- .115480	b_5^4		- 0.225368	- 2.655991
35	- .019713	a_5^5		- 0.014022	0.126363
36	- .077437	b_5^5		- 0.077437	- 0.999484
37	- 13.859641	a_6^0			12.592135
38	6.980897	a_6^1			- 4.594095
39	- .974733	b_6^1			- 5.646851
40	- 3.836010	a_6^2			- 2.325553
41	1.105536	b_6^2			1.233674
42	- .240722	a_6^3			- 0.086915
43	- .410920	b_6^3			0.401331
44	- .048518	a_6^4			- 0.044545
45	- .046032	b_6^4			- 0.000065

TABLE I. (Continued) COEFFICIENTS OF SLOWNESS FUNCTION BASED ON COMBINED POINT AND PATH DATA, SECONDS PER RADIAN

Term	Orthogonal Basis	Term	Spherical Harmonic Bases		
			Degree 4	Degree 5	Degree 6
46	.014759	a_6^5			0.012322
47	- .022780	b_6^5			- 0.040884
48	.002026	a_6^6			0.006057
49	- .012400	b_6^6			- 0.012400

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