Integration of Two Types of School Districts

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Abstract

In Japan, compulsory education consists of six years in elementary school and three years in junior high school. But recently, the government has pushed to introduce integrated schools for nine years that combine the functions of elementary and junior high schools. Becuase the current elementary and junior high school districts are so intricate that it is not easy to determine which schools should be converted to integrated schools. Moreover, to make it easier for residents to agree, school districts similar to both elementary and junior high school districts should be created for integrated schools. Therefore, we first propose to use Adjusted Rand Index (ARI) as a measure of similarity of two partitions of districts. Then, we formulate the problem of deciding which schools should be converted to integrated elementary and junior high schools as a linear programming problem and propose a procedure for creating a school district reorganization plan. In a simulation experiment using the Nara City dataset, we confirm that the proposed method creates school districts for integrated schools that are similar to those of existing elementary and junior high schools.

Keywords: Linear programming, School district optimization, Spatial similarity, Adjusted rand index

1. Introduction

There are 50 states in the United States. Within them are 3,244 counties and county-equivalents [1]. Such a partition of the country is often operated in a stable manner, but if the states are too large and the counties are too small, and we want to reorganize the country into an intermediate partition between states and counties, what kind of partition should we make? This paper examines methods to find the optimal partition when integrating two types of districts.

In Japan, compulsory education consists of six years of elementary school (abbreviated as ES) and three years of junior high school (JS). To facilitate the smooth progression from ES to JS, the government has promoted *elementary-junior high cooperation*, which includes school families and integrated schools. School family is a policy promoted by several local governments, in which ES and JS form a hierarchical parent-child relationship and jointly hold events to promote friendship among students and teachers. Methods for determining school districts through linear programming for these school families have been proposed (Shimizu et al., 2023).

However, Japan is experiencing a declining birthrate, leading to an increase in vacant classrooms. This makes it possible to convert existing ES and JS into integrated schools (IS) without the burden of land acquisition. Such conversions have been observed nationwide[2]. According to Japanese government's School Basic Survey[3], the number of IS is increasing, and becoming more widespread.

When introducing integrated schools under

[1]Wikipedia, browsed on June 12, 2024. https://en.wikipedia.org/wiki/List_of_United_States_counties_and_county_equivalents

[2]For example, https://www.city.nara.lg.jp/site/kyouiku/7515.html

[3]https://www.mext.go.jp/b_menu/toukei/chousa01/kihon/1267995.htm

URI: https://hdl.handle.net/10125/109489
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declining birthrate conditions, school mergers are inevitable. Deciding which schools to convert and which to close is not straightforward. Since current ES districts may span multiple JS districts, determining the districts for IS is also not obvious. In particular, if the change from the original ES and JS district is large, it will be difficult for the local government to explain the situation to the residents, so it is desirable to determine a school district where the difference from both is as small as possible. Therefore, we first propose to use Adjusted Rand Index (ARI) as a measure of similarity of two partitions of districts. Then, this paper proposes methods for reorganizing school districts by formulating the linear programming problem of deciding districts for new integrated schools. Using data from Nara City, the simulation experiment shows the optimal placement of integrated schools when all schools are converted. We confirmed that the proposed method creates school districts that are similar to those of existing ES and JS.

2. Related Work

2.1. Reorganization of school districts

The method of determining school districts has been studied as a variant of a facility location problem (FLP) from various perspectives (Roodman & Schwarz, 1975)(Pizzolato et al., 2004)(Diamond & Wright, 1987)(November et al., 1996). Examples of the application of FLP to optimize school placement have been reported in countries around the world (Bhatnagar & Bolia, 2019)(Yang et al., 2018)(Araya et al., 2012). In particular, problems that involve not only optimizing the initial location but also changing the location pose additional difficulties and have been studied even in recent years (Albareda-Sambola et al., 2009)(Arabani & Farahani, 2012)(Delmelle et al., 2014).

Antunes and Peeters (2000) reported a facility reallocation in Portugal when the duration of elementary school was extended from 6 to 9 years, which is probably the most similar precedent to this paper. They used simulated annealing to obtain the solution because of limited computer and solver capabilities at the time. However, since the performance of computers and solvers has improved dramatically, we will only use a solver that finds the exact solution. Antunes and Peeters (2000) constructed their model by considering the budget for the construction of school facilities as the main inhibitor of change. However, since the elementary school district is now an important unit of the local community, we would consider a model that penalizes the division of school districts rather than the financial cost of building facilities. Although different from school district reorganization, Sadahiro (2011) proposed to use the percentage of agreement between whether two points belong to the same area or not as the similarity for the division of administrative districts. However, no study has ever proposed a method to include it in the objective function of optimization.

2.2. Adjusted Rand Index (ARI)

The Rand Index is a measure of the similarity of two clusterings in statistics, especially data clustering, named after William M. Rand (Rand, 1971). The adjusted Rand index is defined as the adjusted grouping of elements by chance (Hubert & Arabie, 1985). ARI is often used in machine learning to evaluate clustering. And research on ARI itself continues to advance (Chacón & Rastrojo, 2023). ARI is also implemented in scikit-learn (Pedregosa et al., 2011), a library available in the Python language, which we used in our experiments in this paper.

Consider a given set N consisting of $|N|$ elements. And consider X_1 and X_2, which are exhaustive partitions of the set N without duplication. Then, define a, b, c, and d as follows:

a : The number of pairs of element $n \in N$ that are in the same subset in X_1 and in the same subset in X_2.

b : The number of pairs of element $n \in N$ that are in the different subset in X_1 and in the different subset in X_2.

c : The number of pairs of element $n \in N$ that are in the same subset in X_1 and in the different subset in X_2.

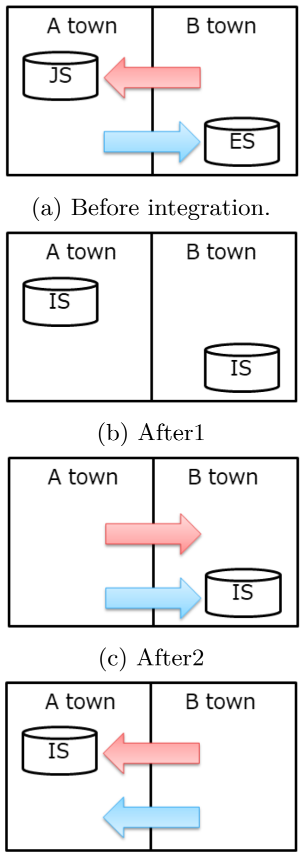
e : The number of pairs of element $n \in N$ that are in the different subset in X_1 and in the same subset in X_2.

Rand index (RI) is deined as $\frac{a+b}{a+b+c+d}$.

RI takes values between 0 and 1, but can be high even for random clusters depending on the number of clusters. This is inconvenient when comparing values for different sets, so ARI corrects RI using expected values that are assigned to the same cluster by chance (Pedregosa et al., 2011): $ARI =$

$\frac{RI-E(RI)}{\max(RI)-E(RI)}$, where $E(RI)$ is the expected value of RI. And ARI can be negative.

3. Simple Example



(a) Before integration.

(b) After1

(c) After2

(d) After3

Figure 1: Three patterns of integration in a local government consisting of two towns.

Table 1: Number of students in each town.

# Students	A town	B town
JS	10	15
ES	20	30

For simplicity, consider a local government with only two city sections (Figure 1(a)). The JS is located in town A and the ES in town B. Each city section has the number of students as shown in Table 1, and there is a burden for students to commute across the city section to school. There are three ways to integrate schools in this local government, as shown in Figures 1(b) through 1(d).

Table 2: The evaluation of three integrations.

		(a) Before	(b) After1	(c) After2	(d) After3
# Students Commuting	JS	15	**0**	10	15
	ES	20	**0**	20	30
# Students Transferred	JS	-	15	**15**	0
	ES	-	20	**0**	20
ARI	JS	-	0.0	**1.0**	**1.0**
	ES	-	0.0	**1.0**	**1.0**

The first method is to convert both the ES and JS into an IS (Figure 1(b)). The second method is to abolish the JS and convert the ES to an IS (Figure 1(c)). The third method is to convert the JS into an IS while eliminating the ES (Figure 1(d)).

The first method may look the best, since there will be no students commuting across city sections. However, there are two problems with this method. The first is that the number of students who would have to be transferred when the schools are integrated is larger than in the other methods. As shown in Table 2, the After2 method results in fewer students being transferred than the other methods. Second, the school district is divided into two while there is only one district before the integration. The ARI proposed in this paper can be used to evaluate the second impact by quantifying the high degree of similarity with the existing school district. As shown in Table 2, the school districts in After2 and After3 match the previous school districts. Method 1, described in the next section, attempts to control the first problem, and Method 2 attempts to control the second problem.

4. Problem Setting and Formulation

The study assumes a local government context where ES and JS districts and the number of students in the city sections are given. The problem is to determine the locations of integrated schools and to assign addresses to schools. We minimize commuting distances and maximize the similarity ARI between the IS districts and ES and JS at the same time, given school capacity constraints.

ARI evaluates whether all pairs of elements of a set are in the same cluster or not. But linear programming problems tend to be computationally expensive when the number of terms in the objective function becomes too large, so we decided to evaluate only adjacent city sections pairs. This is because a boundary is always set between

Table 3: Notation

Symbol	Definition
N	Set of city sections: $n \in N$
$\mathrm{adj}(n)$	Set of adjacent city sections to city section n. $\mathrm{adj}(n) \subset N$
S_e	Set of potential ES sites.
S_j	Set of potential JS sites.
S	Set of potential school sites. $s \in S = S_e \cup S_j$
s_n	a school assigned city section n in the current school district
$\bar{s}$	a school paired with school s in the same location to form an integrated school
C_s^+	Upper bound of students in school s
C_s^-	Lower bound of students in school s
W_n^e	# students in city section n commuting to an ES
W_n^j	# students in city section n commuting to a JS
$D_{n\to s}$	Weight of commuting cost for a student from city section n to school s
G	Weight of changing the school enrollment of one student
H	Weight of changing a pair of city sections from the same school district to another
$x_{n\to s}$	Indicator variable that students in city section n commute to school s
y_s	Indicator variable that school s is operated
$\ell_{nn'}^s$	Indicator variable indicating that city section n and adjacent n' are assigned to the same school s
$z_{n\to n'}^s$	Flow from city section n to city section n' toward school s

city sections when a school district is divided, Therefore, we propose the following formulation. The definitions of the symbols in this paper are shown in Table 3.

Minimize:

$$\sum_{n\in N}\sum_{s\in S} D_{n\to s} W_n x_{n\to s} \quad (1)$$

$$+\sum_{n\in N}\sum_{s\in S} G\mathbb{1}(s \neq s_n) W_n x_{n\to s} \quad (2)$$

$$-\sum_{n'\in \mathrm{adj}(n)}\sum_{n\in N}\sum_{s\in S} H\mathbb{1}(s_{n'} = s_n)\ell_{nn'}^s \quad (3)$$

Equation (1) is the total commuting distance for students in the city. Equation (2) is the total number of students in the city who would need to transfer due to school district reorganization. By making this as small as possible, we expect to reduce the amount of change from ES and JS districts. Equation (3) is the quantity corresponding to term a in the ARI equation. Notice that the term has only adjacent city section pairs. By making it as large as possible, we can expect the ARI to increase. The terms b, c, and d were omitted from the linear programming objective function, because the large number of terms in this function makes the computation too heavy.

$$\text{Baseline objective} = (1)$$
$$\text{Method 1 objective} = (1) + (2)$$
$$\text{Method 2 objective} = (1) + (3)$$

We propose an Method 2 objective to minimize commuting distance while maximizing ARI. To evaluate the effect of Eq. (3), we compare it to a baseline that minimizes only the commuting distance to school. Since minimizing the number of students who change schools is also considered to be effective in increasing the ARI, we compare the results using Method 1 objective and Method 2 Objective in the experiment section. The weight of Eqs. (1), (2), and (3) can then be set by D, G, and H to adjust the balance of how much weight is given to each term.

Constraints:

$$\sum_{s\in S_e} x_{n\to s} = 1 \qquad \forall n \in N \quad (4)$$

$$\sum_{s\in S_j} x_{n\to s} = 1 \qquad \forall n \in N \quad (5)$$

$$\sum_{n\in N} W_n^e x_{n\to s} \leq C_s^+ y_s \qquad \forall s \in S_e \quad (6)$$

$$\sum_{n\in N} W_n^e x_{n\to s} \geq C_s^- y_s \qquad \forall s \in S_e \quad (7)$$

$$\sum_{n\in N} W_n^j x_{n\to s} \leq C_s^+ y_s \qquad \forall s \in S_j \quad (8)$$

$$\sum_{n\in N} W_n^j x_{n\to s} \geq C_s^- y_s \qquad \forall s \in S_j \quad (9)$$

$$\sum_{n'\in \mathcal{N}_n} (z_{n\to n'}^s - z_{n'\to n}^s) \geq x_{n\to s} - |N|\mathbb{1}(s_n = s) y_s$$

$$\forall n \in N, \forall s \in S \quad (10)$$

$$\sum_{n' \in \mathcal{N}_n} z^s_{n \to n'} \leq |N| x_{n \to s} \quad \forall n \in N \quad (11)$$

$$\sum_{n' \in \mathcal{N}_n} z^s_{n' \to n} \leq |N| x_{n \to s} \quad \forall n \in N \quad (12)$$

$$x_{n_s \to s} = y_s \quad \forall s \in S \quad (13)$$

$$\ell^s_{nn'} \leq x_{n \to s} \quad \forall s \in S \quad (14)$$

$$\ell^s_{nn'} \leq x_{n' \to s} \quad \forall s \in S \quad (15)$$

$$y_s = 1 \quad \forall s \in S_j \quad (16)$$

$$y_{\bar{s}} = 1 \quad \forall s \in S_j \quad (17)$$

$$y_s = y_{\bar{s}} \quad \forall s \in S_e \quad (18)$$

$$x_{n \to s} - x_{n \to \bar{s}} \leq 1 - y_{\bar{s}} \quad \forall s \in S \quad (19)$$

Variables:

$$x_{n \to s} \in \{0, 1\} \quad \forall n \in N, \forall s \in S \quad (20)$$

$$y_s \in \{0, 1\} \quad \forall s \in S \quad (21)$$

$$\ell^s_{nn'} \in \{0, 1\} \quad \forall n' \in \mathcal{N}_n, \forall n \in N, \forall s \in S \quad (22)$$

$$z^s_{n' \to n} \geq 0 \quad \forall n' \in \mathcal{N}_n, \forall n \in N, \forall s \in S \quad (23)$$

The IS are represented in the equation as one ES and one JS located at the same address. Equations (4) and (5) are constraints that assign one ES and one JS per city section. Equations (6), (7), (8), and (9) are constraints to keep the number of students allocated to a school between the upper and lower bounds. An IS has an upper and a lower bound on the number of students for each of its ES and JS. Equations (10), (11), and (12) are constraints that allow the school district to be a contiguous district without exclaves (Caro et al., 2004). Equation (13) is a constraint for students of a school district to commute to a school located inside the school district. Without this constraint, ES students would have to change schools when they become JS students, and the school would not be an IS. Equations (14) and (15) are linear constraints to satisfy $\ell^s_{nn'} = x_{n \to s} \times x_{n' \to s}$. Equations (16) and (17) are constraints to convert all JS to IS. This constraints were added because preliminary experiments have shown that the computational complexity would be significant if JS were also included in the list of schools to be closed. Equation (18) is a constraint expressing the condition that an ES can either be closed or converted to an IS. Equation (19) is a constraint that the ES and JS districts must be the same when schools are integrated.

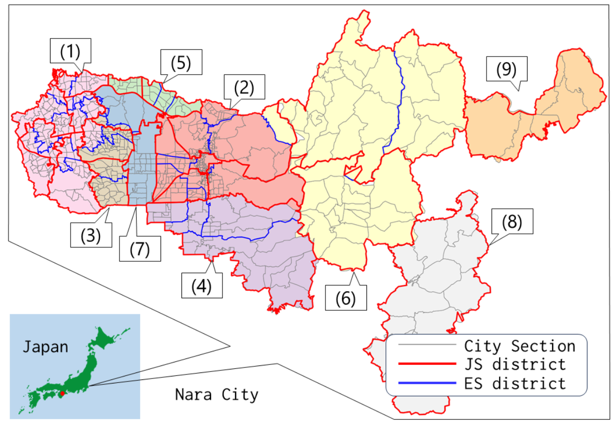


Figure 2: The nine subareas in Nara city are color-coded. Features of the nine subareas are shown in Table 4.

5. Dataset

We used publicly available TopoJSON files to extract the city sections in Nara City[4]. In order to represent the current school district in terms of city sections, we extract the overlapping portions of school districts publicly available[5], and city sections above. As a result, the total number of city sections became 699. However, the data for city sections with enclaves were manually edited to eliminate the enclaves as much as possible by merging them into adjacent city sections. The adjacency of city sections was determined using PySAL(Rey & Anselin, 2007).

The integrated school is assumed to provide a school building, either by adding a JS at the same location as the ES or by adding an ES at the same location as the JS. Schools that are not converted to IS are assumed to be closed. Preliminary experiments showed that including all schools as candidates for closure could not be computed. Therefore, all JS shall be assumed to be converted to IS, and only ES shall be candidates for closure. The school capacities were set based on current data shown in Table 5. The capacity of the unopened IS was set to the same value as the latest newly established IS (No. 56). And No. 31 is scheduled to be closed and will not be converted to an IS.

Based on our preliminary experiments, we decided to optimize each subarea because it would be computationally expensive to optimize the entire

[4]Geoshape Repository Census Boundary Data Set 2020 edition. https://geoshape.ex.nii.ac.jp/ka/resource/

[5]Open data of ES and JS districts in Nara City. https://www.city.nara.lg.jp/soshiki/131/61239.html

Table 4: Features of the nine subareas. The numbers next to the names of the subareas correspond to the numbers in the callouts in Figure 2. The rows are sorted by the subarea with the largest number of schools. Note that the current number of integrated schools (IS) includes both the number of elementary schools (ES) and the number of junior high schools (JS). The number of students was estimated as of 2024.

No.	Name of subarea	# city sections	# schools $\|S_e\|$	# schools $\|S_j\|$	# curent schools ES	# curent schools JS	# curent schools (IS)	# students ES	# students JS	ARI ES vs JS
(1)	West	186	17	17	12	6	1	5,412	2,360	0.493
(2)	Central City	244	15	14	11	4	0	3,182	1,475	0.461
(3)	Hills	94	6	6	4	2	0	2,230	1,049	0.487
(4)	South	43	5	5	4	1	0	691	382	0.000
(5)	North	28	5	5	4	2	1	1,121	497	0.495
(6)	East	45	4	4	3	2	1	64	69	0.630
(7)	Center	36	4	4	2	2	0	828	568	0.858
(8)	SouthEast	17	2	2	1	1	0	159	101	1.000
(9)	Far East	6	1	1	1	1	1	47	21	1.000
	Total	699	59	58	42	21	4	13,734	6,522	0.552

city area at once. The city has divided the area into nine sub-areas for planning urban development, which are publicly announced [6] shown in Figure 2. We therefore divided the dataset according to these subareas. For the number of students, we used estimates for the year 2024. For reference, ARI shows how similar ES and JS districts are. The ARI value varies from 0 to 1 from subarea to subarea. The overall ARI value is 0.552, which is fairly similar.

6. Experiment

The proposed method was implemented in Python and the mathematical optimization solver Gurobi Optimizer 11.0.1 (Gurobi Optimization, LLC, 2022), and the solution was obtained on a Ryzen 5 4600H 3.00GHz CPU with 6 cores (12 threads) and 8GB memory. Computation time was not limited, but the tolerance was set to 10%.

The optimization calculation was successfully completed in 20 minutes at most. Table 6 shows the results when G (Method 1) and H (Method 2) are varied. Baseline results are also shown on the far left for comparison. For both Method 1 and Method 2, ARI increased as the penalty weights were increased. However, ARI decreased as weights increased further. The maximum value of ARI is larger for Method 2 than Method 1. This is due to the fact that the objective function of Method 2 is closer to the ARI. However, since Method 2 does not focus on city section pairs that have different original school districts, the objective function tends to be larger the larger the school district. Therefore, increasing the penalty weights too much results in a decrease in the ARI. Note that there is not much difference in the number of city sections that need to change the schools to which they are assigned under any of the methods. Similarly, the number of students who need to transfere schools does not differ much among the methods. The objective function of Method 1 is expected to have the effect of reducing this number, but it does not work well. Method 2 has fewer schools than Method 1 and therefore longer commuting distances. From this point of view, there is room for improvement of the objective function.

To analyze these results, the results for the parameters that yielded the largest ARI for each method are extracted and partitioned into subareas and shown in Table 7. In the subareas with a small number of schools, there is little difference between methods. However, when we look at the top three rows in terms of the number of schools, we see that Method 2 has a higher ARI. The ARI of ES tends to be larger than that of middle schools. This may be related to the fact that the subareas are determined based on the ES districts.

Finally, the school districts obtained with each method are shown in Figures 3(a), 3(b), 3(c). Although it may be difficult to compare the details due to space limitation, all three IS school districts depicted in pastel colors seem to resemble both the ES school district (blue line) and the JS school

[6] https://www.city.nara.lg.jp/uploaded/attachment/12154.pdf

district (red line). Method 2, however, seems to retain the shape of the original school district a little better than the other methods.

7. Conclusion and Future Work

This paper first proposed the use of ARI as the similarity of school districts when integrating ES and JS to create school districts that are similar to both. Then, we proposed an objective function to increase the ARI in the linear programming formulation. Simulation experiments showed that the proposed method can achieve a larger ARI than other methods. Since this paper focused on implementing IS and experimented with only one local government dataset, there remains room for further investigation of generalizability and scalability. Taking into account private schools and analyzing the relationship between school performance and transportation accessibility in more areas would be the next research direction.

In the following, we discuss three other issues to be addressed in the future. The first issue is the order in which to introduce IS. The introduction of IS is expected to take time. Therefore, it is necessary to determine the order in which the schools should be converted to IS. Especially during the transitional period, it will be difficult to decide when to close schools. We also need to consider the situation where the number of students fluctuates due to the declining birthrate.

The second issue is the need to develop a method for setting the parameter H appropriately. Basically, it is a question of which of the objective functions is more important: commuting distance or similarity. Although we have defined similarity, it is unknown how much importance local government and residents place on that measure. The parameters may need to be determined after comparing the outputs of the optimization.

The third issue is to make it easy to implement the proposed procedures. For example, we would like to develop a tool that can be incorporated into a GIS so that the procedures described in this paper can be applied to any local government, so that not only city officials but also residents can operate the same tool for exchanging opinions. The tool would be more useful if it could not only display the results of the optimization, but also obtain the evaluation values when the school district is changed manually. GIS visualization could provide useful insights about the complexity of a study area, unique yet practical attributes such as natural barriers or boundaries, which may impact generalizability and scalability of proposed methods. Quick computation time is necessary for such use, but using a general-purpose solver has scalability limitations. Heuristic solutions may be needed.

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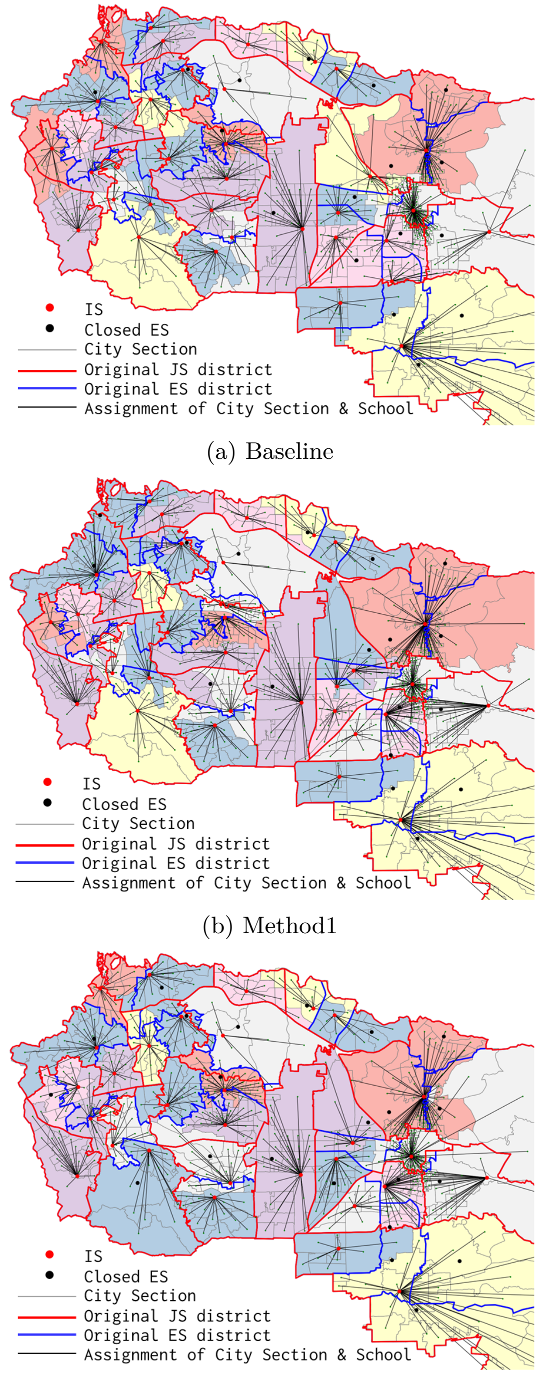

(a) Baseline

(b) Method1

(c) Method2

Figure 3: The maps are color-coded according to the optimal solution of the three methods for introducing IS (integrated schools). Note the IS district in the lower left. In baseline and Method 1, the yellow school district divides the current school district, but in Method 2, the boundaries of the current school district are directly the boundaries of the new school district.

Table 5: Upper (max) and lower (min) bounds for the number of students in ES and JS. The gray shading indicates the bounds of the school if it is converted to an IS. No. 31 is scheduled to be closed and will not be converted to an IS.

Current status: Elementary schools

No.	ES max	capacity min	JS max	capacity min	No.	ES max	capacity min	JS max	capacity min
1	780	150	245	159	20	480	150	245	159
2	750	150	245	159	21	450	150	245	159
3	690	150	245	159	22	450	150	245	159
4	630	150	245	159	23	420	150	245	159
5	600	150	245	159	24	420	150	245	159
6	600	150	245	159	25	420	150	245	159
7	600	150	245	159	26	390	150	245	159
8	600	150	245	159	27	390	150	245	159
9	600	150	245	159	28	373	150	245	159
10	600	150	245	159	29	360	150	245	159
11	600	150	245	159	30	360	150	245	159
12	600	150	245	159	31	330	85	-	-
13	570	150	245	159	32	210	150	245	159
14	570	150	245	159	33	210	115	245	159
15	540	150	245	159	34	210	0	245	159
16	540	150	245	159	35	180	95	245	159
17	510	150	245	159	36	180	150	245	159
18	480	150	245	159	37	90	0	245	159
19	480	150	245	159	38	90	0	245	159

Current status: Junior high schools

No.	ES max	capacity min	JS max	capacity min	No.	ES max	capacity min	JS max	capacity min
39	540	150	1050	175	48	540	150	490	175
40	540	150	805	175	49	540	150	455	175
41	540	150	735	175	50	540	150	455	175
42	540	150	665	175	51	540	150	420	175
43	540	150	665	175	52	540	150	373	175
44	540	150	630	175	53	540	150	350	175
45	540	150	560	175	54	540	150	280	0
46	540	150	560	175	55	540	150	105	0
47	540	150	522	175					

Current status: Integrated schools

No.	ES max	capacity min	JS max	capacity min	No.	ES max	capacity min	JS max	capacity min
56	540	150	245	159	58	180	0	105	0
57	420	150	210	175	59	120	0	70	0

Table 6: Results of baseline, method 1 and method 2

	Baseline	Method 1				Method 2			
weight	-	10^1	10^2	10^3	10^4	10^1	10^2	10^3	10^4
commute distance [m]	1,031	1,036	1,056	1,084	1,205	1,026	1,084	1,123	1,185
# schools	42	42	41	40	38	42	36	34	34
# city sections (ES change)	393	383	383	391	401	383	424	449	494
# city sections (JS change)	379	380	362	292	299	376	315	263	263
# students transfered (ES)	7,689	7,699	7,991	7,850	7,998	7,527	8,585	8,574	8,511
# students transfered (JS)	3,369	3,323	3,230	3,026	3,133	3,277	3,034	2,832	2,810
ARI (ES)	0.538	0.529	0.644	**0.696**	0.657	0.554	0.665	**0.737**	0.697
ARI (JS)	0.467	0.471	0.544	**0.646**	0.589	0.483	0.613	**0.674**	0.629
Elapsed Time (sec)	90	240	303	79	542	1,247	703	129	721

Table 7: Results of baseline, method 1 and method 2 in subareas

	ARI (ES)			ARI (JS)			Commute distance [m]			# Schools		
Method	Base	1	2	Base	1	2	Base	1	2	Base	1	2
(1) West	0.534	0.572	**0.749**	0.402	0.569	**0.601**	877	916	1,014	15	14	12
(2) CentralCity	0.358	0.675	**0.676**	0.271	0.607	**0.634**	979	1,110	1,077	8	8	7
(3) Hills	0.630	0.516	**0.688**	0.309	0.360	**0.405**	927	958	967	5	5	4
(4) South	**0.334**	0.302	0.302	0	0	0	1,776	1,777	1,777	2	2	2
(5) North	**0.452**	**0.452**	**0.452**	**0.368**	**0.368**	**0.368**	962	962	962	3	3	3
(6) East	0.679	**0.782**	0.569	0.499	0.596	**0.707**	2,966	2,986	4,006	4	4	2
(7) Center	**0.858**	**0.858**	**0.858**	**1**	**1**	**1**	1,168	1,168	1,168	2	2	2
(8) SouthEast	0	**1**	**1**	0	**1**	**1**	2,812	2,963	2,963	2	1	1
(9) FarEast	**1**	**1**	**1**	**1**	**1**	**1**	4,278	4,278	4,278	1	1	1