Platform Pricing in the Presence of Cross-platform Network Effects

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Abstract

As platform businesses are booming, different platforms are connected through multi-homing users. Thus, cross-platform network effects arise and have become a strategic concern among business practitioners. However, extant literature has largely ignored cross-platform network effects. We use a stylized model to examine the network effects through the feedback loop and the impact of cross-platform network effects on platform pricing through two levels of impact: the number of multi-homing users and service complementarity. We find that platforms charge less on both sides in the presence of cross-platform network effects when they are separately owned. Moreover, platforms charge even lower prices on both sides when services are complementary. An important caveat is that platforms are willing to sacrifice prices to attract more users when cross-platform network effects are present. We also extend our model to analyze the impact of cross-platform network effects on platform pricing when two platforms are integrated.

Keywords: Two-sided markets, Network Effects, Multi-homing, Platform Pricing.

1. Introduction

Platform businesses are integrated into our daily lives, facilitating services in various aspects, from entertainment (e.g., Netflix, YouTube, Amazon Prime), online shopping (e.g., Amazon, Temu, Shopify), and food delivery (e.g., Uber Eat, DoorDash, Instacart), to vacation and trips (e.g., Expedia, Uber, Airbnb), medical services (e.g., Sharecare, Maple), and business operations (e.g., Zoom, MS Teams). Consumers are inevitably using more than one platform. In many cases, some consumers, mentally or by contract, bundle services from different platforms together. For example, a family entertainment plan may include both Netflix and ESPN channels; some Uber drivers register on both Uber X and Uber Eats.

The literature on two-sided platforms defines users who use more than one platform as *multi-homing users* or *multi-homers*. When sharing a significant number of multi-homers, two platforms are connected and influence each other through multi-homers. From this connection, cross-platform network effects arise and affect the platforms' decisions.

Cross-platform network effects influence platforms through two levels of impact. One level of impact is from the number of multi-homing users. Multi-homing users interact with users on the other side of two platforms. Therefore, the number of multi-homers affects and are affected by the other side users through cross-side network externality of both platforms simultaneously. Consequently, changes on one platform will lead to changes on the other platform through the number of multi-homers. In particular, an increasing number of buyers on a platform will attract more sellers, including multi-homing sellers; more multi-homing sellers benefit and attract more buyers on the other platform. For example, many drivers register for both Uber X and Uber Eats. High passenger demand on Uber X attracts a large pool of multi-homing drivers, subsequently attracting more restaurants to use Uber Eats.

Another level of impact comes from the relationship between services facilitated by the two platforms. If the services are complementary, multi-homing users gain additional benefits. For example, Apple TV and Apple Podcast facilitate complementary services; multi-homing users benefit from accessing various content on both platforms. However, when two platforms facilitate substitutive services, multi-homers bear a utility loss when consuming similar services. The boom of one platform may cause a threat to the other platform. For instance, many restaurants may use both DoorDash and Uber Eats for food delivery, but a burst of DoorDash limits the growth of Uber Eats.

Two levels of impact complicate the understanding of cross-platform network effects, which have also become a strategic concern for business practitioners. Even when platforms are separately owned, they may be connected through multi-homers and affected by cross-platform network effects. However, how to adjust pricing in the presence of cross-platform network effects is still unexplored. Some platforms attempt to leverage the cross-platform network effects by acquisition or expansion. For example, in 2018, Alibaba acquired Ele.me, a food delivery platform in China, so that an increasing number of consumers would use both Taobao for online shopping and Ele.me for takeout food ordering. In 2016, Uber launched Uber

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Eats to expand its business from transportation to food delivery. Since then, Uber has encouraged drivers to multi-home on both platforms. These practices indicate that platform businesses indeed benefit from cross-platform network effects. However, it remains unclear how platforms should optimize their pricing strategies in the presence of cross-platform network effects after acquisition or expansion.

We define cross-platform network effects as the overall impact of network effects through various feedback loops. The feedback loop is the dynamic impact of the demand on both sides of the platform. A change in the number of users on one side will affect the number of users on the other side of the platform. This change then goes back to the side where the change is initiated (Rysman 2004; Evans and Noel 2008). In the presence of cross-platform network effects, various feedback loops are formed. One feedback loop is between single-homing users on both sides of one platform, while another is between multi-homing users and single-homing users on this platform. For example, as the number of single-homing users on Apple Podcast increases, the number of podcasters increases. Meanwhile, another feedback loop emerges as the growing number of podcasters attracts more multi-homers who enjoy the services facilitated by both Apple Podcast and Apple TV. These are only two examples of many feedback loops. More importantly, these feedback loops are interrelated. One feedback loop also impacts the other ones, and vice versa. Therefore, cross-platform network effects and their impacts on platform strategic decisions are intricated.

Despite how important cross-platform network effects are in platform business practices, extant literature has largely ignored the impacts of cross-platform network effects on platforms' pricing strategies. Some studies (Anderson et al., 2018; Armstrong, 2006; Belleflamme & Peitz, 2019) examine the pricing strategies of competing platforms when users on one side of the platform multi-home. In those models, competing platforms facilitate only substitutive services; therefore, complementary services facilitated by two platforms have been ignored. Although Eisenmann et al. (2011) advocate considering different types of services facilitated by platforms, to our best knowledge, no study has investigated the case where multi-homers use two platforms that facilitate substitutive or complementary services. In addition, these studies have not clearly defined and examined cross-platform network effects. Overall, there is a lack of academic understanding of how to systematically demonstrate the economic mechanism of cross-platform network effects and how cross-platform network effects affect platform pricing decisions.

We build a stylized model to examine cross-platform network effects and platform pricing in the presence of cross-platform network effects. In our model, we incorporate two levels of impacts from cross-platform network effects: two platforms have single-homing users on both sides of the platforms and share a pool of multi-homing users on one side. The relationship between services on the two platforms can be either complementary or substitutive; multi-homers' utilities are adjusted accordingly. We then solve and interpret the equilibrium pricing using network effects through various feedback loops in two scenarios: a) when two platforms are separately owned and b) when two platforms are integrated.

We find some interesting and meaningful results. When two platforms are separately owned, cross-platform network effects induce the platforms to reduce charges on both sides. Moreover, platforms further reduce the prices on both sides when the services on the two platforms are complementary, compared with substitutive services. After the two platforms are integrated, the impact of service complementarity/substitutivity is side-dependent. Specifically, on the multi-homing side, the relationship between services on platforms has no direct impact on pricing strategy in the presence of cross-platform network effects. However, on the side with only single-homing users, platforms charge less when two platforms facilitate complementary services than when two platforms facilitate substitutive services. One important caveat from our findings is that platforms are willing to sacrifice prices to attract more users on both sides.

The remainder of this paper is organized as follows. We summarize the relevant literature in Section 2. In Section 3, we re-examine Armstrong (2006)'s model to show network effects through the feedback loop. In section 4, we analyze the optimal platform pricing strategies by two separately owned platforms in the presence of cross-platform network effects. Specifically, Section 4.1 establishes the theoretical model. Section 4.2 solves and explains network effects through various feedback loops. In section 5, we analyze and interpret platform pricing in the presence of cross-platform network effects. In section 6, we examine the platform pricing when the two platforms are integrated. We summarize the case where platforms charge separate prices for multi-homers and the case where the integrated platform company offers a bundle price specifically for multi-homing users. We conclude our research in Section 7.

2. Related Literature

Our research primarily contributes to three streams of literature.

The first stream is on network effects and network externalities. Network effects and network externalities are inherent in platform business models. Although network effects and network externalities have been used interchangeably in papers on two-sided markets, a few studies strived to distinguish the two concepts. Katz and Shapiro (1985) first define network externalities in a one-side market as "the utility that a given user derives from the goods depends upon the number of other users who are in the same network." They then define the positive network effects as "the value of membership to one user is positively affected when another user joins and enlarges the network" (Katz and Shapiro 1994). Liebowitz and Margolis (1994) stress that network externality denotes a specific kind of network effects that is associated with "unexploited gains." Along this line, Armstrong (2006) and Rochet and Tirole (2003) introduced the network effects and network externalities to two-sided markets. Cross-side network externality refers to the marginal benefit that a user on one side gets from interactions with users on the other side. In other words, cross-side network externalities are one-way impacts from one side to the other side. In contrast, network effects are the overall impacts after interactions between the two sides. The current paper builds on these studies that distinguish network effects and network externalities and moves forward to provide insights on cross-platform network effects.

Another stream is feedback loops. The feedback loop is an important but overlooked concept related to network effects in a two-sided market. Rysman (2004) proposes and explicitly estimates the feedback loop using the yellow page directory data. He shows that an increase in the usage of the yellow page directory leads to an increase in the demand for advertisements. Then, the increase in the quantity of advertising implies a further increase in usage. This is an example of the feedback loop. Evans and Noel (2008) further examine the feedback loop in merger analysis between two platforms. They also find that any changes on one side of the platform will lead to changes on the other side; then, the impact comes back to the same side. In a more general setting, Tremblay (2017) suggests that feedback loops exist due to the interrelated relationships between demand and price on one platform and those of other platforms in multi-sided markets. However, these studies have ignored the dynamics of feedback loops. It is still unclear how network effects through the feedback loops affect the platform pricing in the presence of cross-platform network effects.

Literature on two-sided platforms examines platform pricing with multi-homing users based on two dimensions: pure multi-homing users on one side (Armstrong 2006; Athey et al. 2018) and multi-homing users on both sides (Armstrong and Wright 2007; Gabszewicz and Wauthy 2004; Belleflamme and Peitz 2019; Liu et al. 2019). Armstrong (2006) finds that platforms may ignore the interests of multi-homing users and set excessive pricing on the multi-homers under the assumption that all the users on one side multi-home while users on the other side all single-home. Although Ambrus et al. (2016) study the case where both overlapping viewers and exclusive viewers are on one side in media platforms, they assume that overlapping viewers value less to the media platform compared with the viewers that are exclusive to the platform and ignore the network effects and network externalities. In addition, most scholars study platform pricing with multi-homing users between competing platforms, facilitating substitutive services. Therefore, we expand the model by including substitutive and complementary services in our model and examine platform pricing with mixed single-homing users and multi-homing users on one side of the platforms.

Our research also fits into the literature on platform mergers and how platforms with multi-homing users adjust pricing after mergers or acquisitions. Some scholars study platform pricing before and after mergers with single-homing users on both sides (Baranes et al., 2019; Chandra and Collard-Wexler, 2009; Correia-da-Silva et al., 2019). Only a few scholars specifically examine the platform pricing with multi-homing users after two platforms are integrated. Cayseele and Vanormelingen (2019) empirically study the post-merger platform prices by analyzing the data from the Belgian newspaper industry. They find that platform merger has limited impact on the side with single-homing readers but increases the price on multi-homing advertisers. However, they only consider one-way network effects. By accounting for two-way network effects, Ivaldi and Zhang (2020) find that post-integrated platforms also increase prices on the side with multi-homing advertisers. They also estimate whether a channel is a substitute or complementary to another channel for multi-homing advertisers. However, no extant theoretical studies investigate two platforms facilitating substitutive or complementary services with multi-homing users. Therefore, it is still unclear how the two levels of impact from cross-platform network effects would affect platform pricing when two platforms are separately owned and integrated.

3. Network Effects through the Feedback Loops

Due to cross-side network externalities between the two sides of a platform, any change in the number of users on one side will affect the number of users on the other side. The change on the other side will then lead to an additional change on the original side. Extant studies (Rysman 2004, Evans and Noel 2008) define the process as a *feedback loop*. However, how feedback loops function in the platform economy has not been examined. In this section, we use Armstrong (2006)' model setup to delineate feedback loops and interpret network effects on a monopoly platform through the feedback loops. We will also solve the platforms' equilibrium prices through feedback loops. The analysis in this section will lay the foundations for our subsequent study on cross-platform network effects when two platforms are connected through multi-homers on one side.

Consider the model setup of Armstrong (2006), where a monopoly platform connects users on two sides, denoted as side *i* and side *j*, respectively; the utility function of a representative user on each side is:

$$u_i = \alpha_i n_j - p_i \qquad u_j = \alpha_j n_i - p_j$$

Where p_i and p_j are the prices charged by the platform on *i*-side and *j*-side users, respectively; n_i and n_j are the number of users on the two sides, respectively. α_i denotes the marginal benefit by each *j*-side user on the utility of *i*-side users and α_j denotes the benefit by each *i*-side user on the utility of *j*-side users.

Moreover, the number of users on both sides is a function of the users' utility.

$$n_i = \phi_i(u_i) \qquad n_j = \phi_j(u_j)$$

Under this model setup, we can depict a feedback loop — a change on one side of the platform will affect the other side and circle back with a further change on the original side — on the platform. For example, an increase in the number of users on side *i* will enhance side-*j* users' utilities. When the utility of *j*-side users increases, the platform attracts more users on side *j*. The increase in the number of *j*-side users will boost the utility of *i*-side users and consequently cause a further increase in the number of users on side *i*. Therefore, a feedback loop generates a network effect on the same side. We denote the network effect through a feedback loop as $T \equiv \frac{\partial u_i}{\partial n_j} \cdot \frac{\partial n_i}{\partial u_i} \cdot \frac{\partial u_j}{\partial n_i} \cdot \frac{\partial n_j}{\partial u_j}$

Definition 1: $T \equiv \frac{\partial u_i}{\partial n_j} \cdot \frac{\partial n_i}{\partial u_i} \cdot \frac{\partial u_j}{\partial n_i} \cdot \frac{\partial n_j}{\partial u_j} = \alpha_i \alpha_j \phi_i'(u_i) \phi_j'(u_j)$ *is defined as the network effect through a feedback loop between users on side i and side j.*

Lemma 1. The network effect through one feedback loop remains the same no matter where the loop starts.

As shown in Figure 1, a complete feedback loop is a circle comprised of two cross-side network externalities, from side *i* to side *j* and vice versa. It causes a network effect on the original side, which quantitively equals the multiply of the two cross-side network externalities. Therefore, whether the circle starts from side *i* or side *j* does not alter the magnitude of the network effect. In other words, if there is a change on one side of a platform due to a policy adjustment or a demand shock, a feedback loop will generate an additional change on the same side, which is proportional to the original change. The proportion is the same regardless of the initial change on which side.

Moreover, the feedback process is continuous. The change generated by a feedback loop will trigger the second-round feedback loop and change, the third round, and so on. It is a recursive and, theoretically, infinite process. If the network effect by a feedback loop is greater than one, T >1, the change will be magnified by each round of the feedback loop, and the accumulated changes will burst. In practice, when platforms are rapidly growing, recalling the rise of Uber and Airbnb, the network effect by a feedback loop is high due to strong cross-side externalities between the seller and buyer sides. Platform companies often attract users with high subsidies. The number of platform users, initialed by the subsidies, soon skyrocketed.

However, for platforms running steadily, any change on one side of a platform will converge through the feedback process, and the platform economy transforms between steady states. A necessary condition is required as follows. We assume this condition holds throughout the paper.

Condition 1: $T < 1$, or, $\alpha_i \alpha_j \phi_i'(u_i) \phi_j'(u_j) < 1$

Lemma 2: The accumulated network effect through infinite recursive feedback loops is $\frac{1}{1-T}$.

Lemma 2 is intuitive. The network effect by the first round of the feedback loop is T, and that by the second round of the feedback loop is T^2, and so on. The accumulated network effect through infinite feedback loops is the sum of network effects through all the feedback loops, $\frac{1}{1-T}$.

The comprehension of the recursive feedback process and engendered network effect can help us understand how a platform's strategies, such as pricing strategies, impact the demands on different sides of the platform and shape the equilibrium outcome of the platform economy. For example, in Armstrong (2006)'s model, we can calculate the network effect by a marginal price change and use this approach to solve the profit-maximizing prices. Armstrong(2006) omitted the detailed network effects but used the envelop theorem in his solution. We provide an alternative approach to the solution as follows.

The monopoly platform optimizes the prices on the two sides, p_i and p_j to maximize its profit.

$$\max_{p_i, p_j} \pi = n_i(p_i - f_i) + n_j(p_j - f_j)$$

Hence, the optimal prices are defined by the first-order conditions with respect to p_i *and* p_j

$$\frac{\partial n_i}{\partial p_i}(p_i - f_i) + n_i + \frac{\partial n_j}{\partial p_i}(p_j - f_j) = 0 \quad (1)$$

$$\frac{\partial n_i}{\partial p_j}(p_i - f_i) + n_j + \frac{\partial n_j}{\partial p_j}(p_j - f_j) = 0 \quad (2)$$

(1) shows that at the optimal level, the marginal impact of *i*-side price (p_i) on the platform, profit equals zero. Specifically, the marginal impact on the platform profit comprises the multiply of the marginal impact of p_i on the number of *i*-side users ($\frac{\partial n_i}{\partial p_i}$) and *i*-side margin $(p_i - f_i)$, the number of *i*-side users, and the multiply of the marginal impact of p_i on the number of *j*-side users ($\frac{\partial n_j}{\partial p_i}$) and *j*-side margin $(p_j - f_j)$.

Considering the marginal impact of p_i on the number of *i*-side users, we have

$$\frac{\partial n_i}{\partial p_i} = -\phi_i'(u_i) \cdot \left(\frac{1}{1-T}\right) \quad (3)$$

Intuitively, if p_i increases, the *i*-side users' utility declines, and the number of *i*-side users decreases. Thus, the initial marginal impact of p_i on the number of *i*-side users is $-\phi_i'(u_i)$. According to Lemma 2, recursive feedback loops amplify the initial marginal impact by $\frac{1}{1-T}$. Thus, the overall marginal impact of p_i on the number of *i*-side users is $-\phi_i'(u_i) \cdot \left(\frac{1}{1-T}\right)$. Similarly, we have the marginal impact of p_i on the number of *j*-side users as

$$\frac{\partial n_j}{\partial p_i} = -\alpha_j \phi_j'(u_j)\phi_i'(u_i) \cdot \left(\frac{1}{1-T}\right) \quad (4)$$

The *i*-side price p_i alters *i*-side users' utility, then changes the number of *i*-side users. This change, through cross-side network externality, affects *j*-side users' utility and the number of *j*-side users. Thus, the initial marginal impact of p_i on the number of *j*-side users is $= -\alpha_j \phi_j'(u_j)\phi_i'(u_i)$. Though the recursive feedback loops start from the *j*-side this time, the initial marginal impact will be magnified at the same level $\frac{1}{1-T}$. The overall marginal impact of p_i on the number of *j*-side users is $-\alpha_j \phi_j'(u_j)\phi_i'(u_i) \cdot \left(\frac{1}{1-T}\right)$. Using feedback loops, we can derive the formulas of optimal prices, which are the same as those in Armstrong (2006). Due to the space limit, I omit the derivation process.

4. Cross-platform Network Effects

When multiple platforms share common users, a change on one platform may be transferred through the common users to other platforms. Cross-platform network effects emerge. Cross-platform network effects connect the economies on these platforms and complicate the platforms' pricing decisions. To demonstrate the formation and impact of cross-platform network effects through the multi-homing users, we examine a model of two platforms sharing a group of users on one side. It is common in business practice. For example, many drivers register and serve on both Uber X and Uber Eats. Although passengers using Uber X and restaurants using Uber Eats are two different groups of service buyers, their benefits depend on the number of Uber drivers. The prosperity of one group of buyers favors the other group.

In section 4.1, we first set up our model to incorporate two levels of cross-platform network effects: multi-homing users on one side and the relationship between services facilitated by two platforms. Then, we will examine various feedback loops and network effects through various feedback loops in section 4.2. Section 4.3 analyses the optimal platform pricing in the presence of cross-platform network effects by using network effects through various feedback loops.

4.1 Model Setup

We consider two two-sided platforms connected through multi-homing users on one side. Specifically, platform A and platform B share a group of multi-homing users on one side, denoted as side *i*. The utility of a representative multi-homing user is u_i^m, and the number of multi-homing users is n_i^m. Besides multi-homing users, both platforms have a group of single-homing users on side *i*. The utilities of representative single-homing users on the *i*-side of Platforms A and B are u_i^a and u_i^b respectively, and the numbers of the single-homing users on the *i*-side of the two platforms are n_i^a and n_i^b. We denote the total numbers of *i*-side users of the two platforms as n_i^A and n_i^B respectively. Hence, $n_i^A = n_i^a + n_i^m$ and $n_i^B = n_i^b + n_i^m$. On the other side, denoted as side *j*, the two platforms have separated groups of single-homing users but no multi-homing users. The utilities of representative *j*-side single-homing users on Platforms A and B are u_j^A and u_j^B and the number of *j*-side single-homing users are n_j^A and n_j^B respectively.

Following Armstrong 2006, we assume the number of a group of platform users is an increasing function of the users' utility.

$$n_k^h = \phi_{kh}(u_k^h)$$

where $k = i, j$ and $h = a, b, m, A, B$

The users' utilities are determined by cross-side network externalities and prices charged by the platform. Specifically, single-homing users on the side *i* of platform A pay p_i^A to join the platform and benefit from interactions with *j*-side users on platform A:

$$u_i^a = \alpha_i^A n_j^A - p_i^A$$

Where α_i^A denotes the cross-side network externality of each *j*-side user on *i*-side users on platform

A. In the same vein, *i*-side single-homing users on platform B pay p_i^B to join the platform and gain benefits, $\alpha_i^B n_j^B$, from interactions with all the *j*-side single-homing users on platform B. α_i^B is the cross-side network externality from *j*-side users to *i*-side users on platform B. The utilities of *i*-side single-homing users on platform B are:

$$u_i^b = \alpha_i^B n_j^B - p_i^B$$

i-side multi-homing users, as they join in both platforms, pay two prices, $p_i^A \; and \; p_i^B$. They gain benefits from interactions with *j*-side users on both platforms, $\alpha_i^A n_j^A$ and $\alpha_i^B n_j^B$, respectively. In addition, there are synergies or conflicts when multi-homers use both platforms. [brief examples]. We define the additional impact on the users' utility when using both platforms as $\delta \cdot \alpha_i^A n_j^A \cdot \alpha_i^B n_j^B$. δ depends on the relationship between the services facilitated on two platforms. If the two platforms facilitate complementary services, multi-homing users earn extra benefit from using both platforms, $\delta > 0$. While $\delta < 0$, if the services on the two platforms are substitutive. For example, PlayStation and Xbox both facilitate the interactions between gamers and game developers, multi-homing users may have a negative impact due to the substitutive services facilitated by PlayStation and Xbox. On the other hand, Priceline.com connects travelers with lodging providers while OpenTable connects restaurants with diners; multi-homing users on Priceline.com and OpenTable gain synergy due to complementary services facilitated by two platforms. Therefore, the utility function of *i*-side multi-homing users is:

$$u_i^m = \alpha_i^A n_j^A - p_i^A + \alpha_i^B n_j^B - p_i^B + \delta \alpha_i^A \alpha_i^B n_j^A n_j^B$$

$\delta \alpha_i^A \alpha_i^B n_j^A n_j^B$ is multiplication because the magnitudes of benefits from cross-platform network effects change in a non-linear manner. By individual rationality condition, the utilities of multi-homing users should always be positive; otherwise, multi-homers will not use two platforms simultaneously. We ensure this by imposing Condition 2.

Condition 2: $\delta \alpha_i^A n_j^A + 1 > 0$ and $1 + \delta \alpha_i^B n_j^B > 0$

On side *j*, each platform has single-homing users that are exclusive to each platform due to technology limitations or the platform's exclusive dealing policy. On platform A, *j*-side users pay p_j^A to join the platform and to interact with two segments of users on side *i*: single-homing users on platform A and multi-homing users. Side-*j* users gains cross-side network externality, α_j^A. The utilities of *j*-side users on platform A are:

$$u_j^A = \alpha_j^A n_i^A - p_j^A, \text{ where } n_i^A = n_i^a + n_i^m$$

n_i^a is the number of *i*-side single-homing users; n_i^m is the number of *i*-side multi-homing users. In addition, n_i^m reflects the first level of impact from cross-platform network effects. The total number of *i*-side users, n_i^A, includes both multi-homing users and single-homing users on platform A. Similarly, *j*-side single-homing users on platform B pay p_j^B to join the platform and gain total benefits, $\alpha_j^B n_i^B$, from interacting with two segments of *i*-side users: single-homing users on platform B and multi-homing users. The utilities are denoted as:

$$u_j^B = \alpha_j^B n_i^B - p_j^B, where \; n_i^B = n_i^b + n_i^m$$

n_i^B refers to the total number of *i*-side users on platform B.

We specify the number of *i*-side users in each segment of two platforms as an increasing function of users' utilities in the respective segment:

$$n_i^a = \phi_{ia}(u_i^a); \; n_i^m = \phi_{im}(u_i^m); \; n_i^b = \phi_{ib}(u_i^b)$$

For simplicity, we denote functions that represent the number of users on either side of two platforms as ϕ_{ia}, ϕ_{im}, ϕ_{ib}, ϕ_{jA} and ϕ_{jB} respectively thereafter.

We complete the model set-up by stating each platform's profit maximization goals. Each platform incurs per-user costs, f_i^k and $f_j^k, k \in (A, B)$, to maintain the users on each side of each platform k. Despite two platforms are connected through *i*-side multi-homing users, platform A and platform B are still separately owned. Therefore, each platform maximizes its own profit, and its profit maximization objective function is:

$$\max_{p_i^A, p_j^A} \pi^A = n_i^A(p_i^A - f_i^A) + n_j^A(p_j^A - f_j^A); \quad \max_{p_i^B, p_j^B} \pi^B = n_i^B(p_i^B - f_i^B) + n_j^B(p_j^B - f_j^B)$$

Where $n_i^A = n_i^a + n_i^m$ and $n_i^B = n_i^b + n_i^m$

The solutions of these two objective functions determine the equilibrium market outcomes. Since platform A and platform B are symmetric, for clarity, we omit model set-up for platform B.

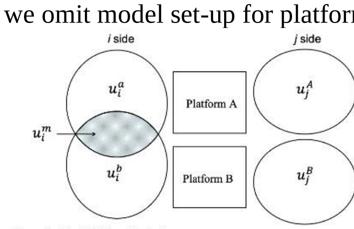

Figure 1. Model Setup Illustration

4.2 Network Effects Through Various Feedback Loops

In our stylized model, we incorporate two levels of impacts from cross-platform network effects. One level of impact is the number of multi-homers, while the other is the relationship between services facilitated by two platforms. In the presence of cross-platform network effects, various feedback loops exist. For example, there may be a feedback loop between *i*-side single-homing users and *j*-side single-homing users on platform A. Side *j* single-homing users on platform A can also include in a feedback loop with *i*-side multi-homers, who may affect *j*-side single-homing users on platform B through another feedback loop. These feedback loops are infinite, continuous,, and are also interrelated. Therefore, network effects through the feedback loops are more complicated in the presence of cross-platform network effects.

In this section, we define various feedback loops and explain network effects through various feedback loops in the presence of cross-platform network effects. We build our analysis upon the discussions in Section 3. Since we have two platforms, platform A and platform B, we use platform A as our focal platform for clarity and consistency in our analysis.

Definition 2: $T^{mB} = \left(\delta\alpha_i^A n_j^A + 1\right)\alpha_i^B \alpha_j^B \phi'_{im}\phi'_{jB}$ *is the network effects through one feedback loop between i-side multi-homing users and j-side single-homing users on platform B.*

$\left(\delta\alpha_i^A n_j^A + 1\right)\alpha_i^B$ can be rewritten as $\frac{\partial u_i^m}{\partial n_j^B}$. It refers to the marginal benefit *i*-side multi-homing users get from interactions with additional *j*-side single-homing users on platform B. $\phi'_{im} = \frac{\partial n_i^m}{\partial u_i^m}$ is the marginal change in the number of *i*-side multi-homing users due to marginal change in the utilities of multi-homers. Similarly, $\phi'_{jB} = \frac{\partial n_j^B}{\partial u_j^B}$ is the marginal change in the number of *j*-side single-homing users on platform B due to marginal change in the utilities of these single-homing users. T^{mB} can be re-written as $\frac{\partial n_i^m}{\partial u_i^m} \cdot \frac{\partial u_j^B}{\partial n_i^m} \cdot \frac{\partial n_j^B}{\partial u_j^B} \cdot \frac{\partial u_i^m}{\partial n_j^B} \cdot$. One interpretation of T^{mB} is that a change in multi-homing users' utilities will lead to a change in the number of *i*-side multi-homing users. This change then causes the utilities of *j*-side single-homing users on platform B to change. Consequently, the number of *j*-side single-homing users on platform B changes, which goes back to side *i* to cause changes in the utilities of multi-homing users. Therefore, T^{mB} reflects the network effects through the feedback loop between *i*-side multi-homing users and *j*-side single-homing users on platform B. Another way to interpret T^{mB} is to start from side *j*. Since Lemma 1 highlights that network effects through one feedback loop remain the same no matter where it starts, we omit the discussion on an alternative interpretation of T^{mB} .

As condition 2 indicates, $\left(\delta\alpha_i^A n_j^A + 1\right) > 0$ ensures that multi-homers gain positive benefits by using two platforms as a bundle. Otherwise, multi-homers will not use any platforms at all. Therefore, T^{mB} is always positive regardless of the sign of δ. In other words, despite the relationship between services facilitated by two platforms, network effects through one feedback loop between multi-homing users and single-homing users on *j* side of platform B are always positive. Although $\left(\delta\alpha_i^A n_j^A + 1\right)$ is always positive, the value of $\left(\delta\alpha_i^A n_j^A + 1\right)$ is greater when δ>0 than it is when δ<0. This is because multi-homing users gain more additional benefits through cross-platform network effects when complementary services are facilitated by two platforms. The network effects through the feedback loop between *i*-side multi-homing users and *j*-side single-homing users on platform B are stronger when services are complementary. For example, when users use both Amazon Prime Video and Wondery, users fulfill different needs. Users can watch shows on Amazon Prime Video while listening to podcasts on Wondery. In the case where multi-homers use both Play Station and Xbox, multi-homers access to similar services (i.e., gaming) facilitated by two platforms.

Following Condition 1, we have $0 < T^{mB} < 1$ to ensure the platform B sustains. If this condition fails, say, $T^{mB} > 1$, then the platform may collapse because network effects though the feedback loop between *i*-side multi-homing users and *j*-side single-homing users on platform B may enlarge at an exponential rate.

Definition 3. $T^{bB} = \alpha_i^B \alpha_j^B \phi'_{ib}\phi'_{jB}$ *is the network effects through a feedback loop between i-side single-homing users on platform B and j-side single-homing users on platform B.*

$\phi'_{ib} = \frac{\partial n_i^b}{\partial u_i^b}$ is the marginal change in the number of *i*-side single-homing users on platform B due to the marginal change in the utilities of these single-homing users. T^{bB} can be rewritten as $\frac{\partial u_i^b}{\partial n_j^B} \cdot \frac{\partial n_i^b}{\partial u_i^b} \cdot \frac{\partial u_j^B}{\partial n_i^b} \cdot \frac{\partial n_j^B}{\partial u_j^B}$. T^{bB} reflects the network effects through the feedback loop between *j*-side single-homing users and *i*-side single-homing users on platform B. When the number of *j*-side single-homing users on platform B increases, the utilities of *i*-side single-homing users on platform B also increase. As a result, the number of *i*-side single-homing users on platform B increases. More *i*-side single-homing users on platform B will also attract more single-homing users on side *j* due to the increasing utilities of *j*-side single-homing users on platform B. Since T^{bB} reflects the network effects through single-homing users on both sides of platform B, the relationship between services of two platforms (i.e., δ) have no impact in T^{bB}. Following Condition 1, we have $0 < T^{bB} < 1$ to ensure that platform B sustains.

Lemma 3: In the presence of cross-platform network effects, the overall network effects through various feedback loops on platform B are $\frac{1}{1-(T^{bB}+T^{mB})}$.

We define the network effects through two types of feedback loops: T^{mB} refers to the network effects through the feedback loop between multi-homing users and *j*-side single-homing users on platform B; T^{bB} refers to the network effects through the feedback loop between single-homing users on both sides of platform B. In addition, both T^{mB} and T^{bB} are connected through the *j*-side users on platform B. When the utilities of multi-homing users change, T^{mB} changes due to the change in the number or the utilities of *j*-side users on platform B. The impact continues through side *j* to *i*-side single-homing users on platform B through T^{bB}. The intuition behind is that any change in either the utilities or the number of *j*-side users on platform B goes across to *i*-side users on platform B, including multi-homing users and single-homing users on *i* side. The impact, therefore, comes back to side *j* continuously. Since feedback loops are infinite and continuous, the overall network effects are the sum of all the network effects through infinite feedback loops, denoted as $\frac{1}{1-(T^{bB}+T^{mB})}$. To ensure the impact from network effects through infinite feedback loops on platform B can be sustained and the increasing rate as a result of feedback loops is bounded, we impose the following condition:

Condition 3: $0 < T^{bB} + T^{mB} < 1$.

If $T^{bB} + T^{mB} > 1$, a slight change in the number of users on either side of platform B magnifies the growth of users exponentially.

In this section, we draw upon the analysis in Section 3 to define and interpret the network effects through various feedback loops in the presence of cross-platform network effects. We now use the network effects through various feedback loops to analyze platform pricing in the following section.

5. Equilibrium Pricing When Two Platforms are Separately Owned

We now extend our analysis to elaborate on how cross-platform network effects affect platform pricing when two platforms are integrated. In practice, platform businesses leverage cross-platform network effects through acquisition (e.g., Expedia and HomeAway) or expansion (e.g., Uber X and Uber Eats).

We use the same utility functions for users on each side of two platforms as established in Section 3. The only difference lies in the profit maximization objective: post-integration platforms aim to maximize total profits by setting prices on both sides of each platform simultaneously. Therefore, the profit maximization objective function for integrated platforms is:

$$\max_{p_i^A, p_i^B, p_j^A, p_j^B} \pi_{nb}^{A+B} = (n_i^a + n_i^m)(p_i^A - f_i^A) + (n_i^b + n_i^m)(p_i^B - f_i^B) + n_j^A(p_j^A - f_j^A) + n_j^B(p_j^B - f_j^B)$$

Solutions to this objective function are the equilibrium prices for post-integrated platforms.

Lemma 3: The post-integration optimal pricing strategy on i side of platform A is:

$$p_i^A = -\alpha_j^A n_j^A + f_i^A + \frac{n_i^A \cdot \frac{\partial n_i^B}{\partial p_i^B} - n_i^B \cdot \frac{\partial n_i^B}{\partial p_i^A}}{\frac{\partial n_i^A}{\partial p_i^A} \cdot \frac{\partial n_i^B}{\partial p_i^B} - \frac{\partial n_i^A}{\partial p_i^B} \cdot \frac{\partial n_i^B}{\partial p_i^A}}$$

Proposition 1. On i side of integrated platforms, optimal pricing is not affected by the relationship between services facilitated by two platforms.

At equilibrium, platform A's pricing on *i* side is the per-user cost on *i* side of platform A, f_i^A, and lowered by total external benefits to the single-homing users on *j* side of platform A, $\alpha_j^A n_j^A$. The equilibrium price is also adjusted by interdependence between demand elasticities on *i* side of two platforms, $\frac{n_i^A \cdot \frac{\partial n_i^B}{\partial p_i^B} - n_i^B \cdot \frac{\partial n_i^B}{\partial p_i^A}}{\frac{\partial n_i^A}{\partial p_i^A} \cdot \frac{\partial n_i^B}{\partial p_i^B} - \frac{\partial n_i^A}{\partial p_i^B} \cdot \frac{\partial n_i^B}{\partial p_i^A}}$. When setting the price on *i* side, platform A needs to consider not only users on *j* side of two platforms and all the users on its own platform but also the single-homing users on the same side of platform B. Relationship between services on two platforms have no impact on post-integration pricing strategies on *i* side. One possible explanation is that integrated platforms internalize the cross-platform network effects since integrated platforms share the same profit objective. Therefore, each platform sets price accounting for the decisions of the other platform.

Lemma 4: The post-integration optimal pricing on j side of platform A is:

$$p_j^A = f_j^A + \frac{n_j^A}{\phi'_{jA}} - \alpha_i^A(n_i^a + n_i^m) - \delta\alpha_i^A\alpha_i^B n_j^B \cdot \left(\frac{n_i^a + n_i^m(\phi'_{im}\phi'_{ia} + \phi'_{im}\phi'_{ib}) + n_i^b}{\phi'_{ia}\phi'_{im} + \phi'_{ia}\phi'_{ib} + \phi'_{im}\phi'_{ib}}\right)$$

Proposition 2. On j side, the platform charges lower prices when services are complementary.

Post-integration, platform A's pricing strategy on *j* side is the per-user cost on *j* side of platform A, f_j^A, adjusted upward by the participation elasticity of users on *j* side of platform A, and adjusted downward by external benefits to all the users on *i* side of platform A, $-\alpha_i^A(n_i^a + n_i^m) \cdot \frac{n_i^a+n_i^m(\phi'_{im}\phi'_{ia}+\phi'_{im}\phi'_{ib})+n_i^b}{\phi'_{ia}\phi'_{im}+\phi'_{ia}\phi'_{ib}+\phi'_{im}\phi'_{ib}}$ is the adjusted participation elasticity on *i* side of both platforms. The positive adjusted participation elasticity modifies the additional benefits multi-homers on *i* side gain from interactions with users on *j* side of platform A. In the presence of cross-platform network effects, post-integrated platforms are willing to sacrifice price for more users on *j* side, so that more users on *i* side will be attracted when two platforms facilitate complementary services.

6. Equilibrium Platform Pricing When Two Platforms Are Integrated

Now, we extend our analysis to the case where two platforms are integrated. Post-integrated platforms still have three segments of users on side *i*: single-homing users on platform A, single-homing users on platform B, and multi-homing users. Nonetheless, the integrated platforms may remain the same charge for multi-homers (i.e., $p_i^A + p_i^B$), or change to bundle price (i.e., p_i^m) for multi-homers. For example, after the acquisition, Expedia and HomeAway charged two different prices for multi-homing users, while Amazon charges a bundle price for users who use both Amazon Prime and Twitch. On side *j*, single-homing users are exclusive to each platform. After two platforms are integrated, network effects through feedback loops may change in the presence of cross-platform network effects. We elaborate on how two levels of impacts from the cross-platform network effects affect platform pricing in two scenarios. In section 5.1, we analyze the equilibrium platform pricing for the first scenario in which the integrated platforms remain the same charge to multi-homers. In section 5.2, we discuss the optimal pricing in the second scenario in which integrated platforms charge multi-homing users a bundle price.

We discussed a benchmark case in Section 5, where the two platforms are separately owned and set prices independently, and single-homing users and multi-homing users pay the same prices when using a platform. In this section, we extend the study to three additional applications of platform pricing with cross-platform network effects. We named the benchmark case "*Separately Owned Without Price Discrimination.*" The three extension cases are as follows.

a) "*Separately Owned with Price Discrimination*" — The two platforms are separately owned and set prices independently, but they can distinguish multi-homing users from single-homing users and charge them differently. As multi-homing users play a vital role in cross-platform network effects, which crucially affect the platforms' profits and pricing strategies, platform firms in practice often provide discounts to their users who also use another platform, even though they are financially independent to the latter. For example, Amazon Music and Disney+ both offer discounts to multi-homing users of the two platforms. The multi-homing users essentially pay at different prices from the single-homing users.

b) "*Integrated Without Price Discrimination*" — The two platforms are integrated through mergers or acquisitions, and they set prices to maximize their joint profit. However, the multi-homing users do not receive any special treatment in price. For example, after Alibaba acquired Ele.me, each platform still charged multi-homing users the same price as that for the single-homing users.

c) "*Integrated with Price Discrimination*" — The two platforms are integrated and set prices to maximize their joint profit. They can easily identify multi-homing users and set a unique price to maximize the benefit from cross-platform network effects by the multi-homing users. For example, Amazon expanded the Amazon Prime Membership coverage to include the access to Twitch after the acquisition. Due to page limitations, we skip the detailed analyses and discussions about the above extension cases.

7. Conclusion

The conventional view on platform pricing strategies is to consider only cross-side network effects on its own platform. Nevertheless, such a view is incomplete with the recent boom in platform businesses. As more and more platform businesses emerge, an increasing number of consumers are using more than one platform. These multi-homing users connect two platforms and form cross-platform network effects through two root causes-- the number of multi-homing users and the synergies or conflicts multi-homing users experience from using both platforms. These two root causes of cross-platform network effects complicate platforms' strategic decisions. Platforms need to consider not only the cross-side network effects, but also the cross-platform network effects when making strategic decisions. In this paper, we develop a stylized model to accommodate these two levels of impacts from cross-platform network effects and examine how cross-platform network effects affect the platform pricing strategies.

This paper produces several meaningful findings that offer managerial guidance to platforms. First, platforms will charge less on the side with multi-homing users to boost the positive cross-platform network effects between the two platforms when two platforms are connected through the multi-homing users on one side. However, the platforms' prices on the other side can increase or decrease, depending on the synergies or conflicts that the multi-homing users have on the original side. Second, if the multi-homing users receive synergies from using the two platforms, the two platforms prefer to charge less on the other side so that more users on the other side will significantly enhance the multi-homing users' utilities and willingness to pay on the original side. As a result, the platforms' revenue from the original size will increase substantially. However, if the multi-homing users face conflicts in using the two platforms, boosting the number of users on the other side becomes worthless, and the two platforms will raise the price on the other side.

These insights show that platforms should re-think pricing strategies when cross-platform network effects are present. In the presence of cross-platform network effects, two platforms are connected by the multi-homing users. Hence, platforms should pay attention to the multi-homing users, especially when platforms are trying to attract more users. This paper also offers a guideline for platforms to understand the underlying impacts of cross-platform network effects and improve their decision-making. Ignorance of cross-platform network effects may result in biased pricing.

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