

Information Design in Financial Markets*

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Abstract

We study the optimal disclosure policy of a firm that wishes to maximize its expected stock price in the classic setting in which its stock is traded by risk-averse investors and noise traders. We find that the optimal disclosure policy is imprecise and leads to skewed posterior beliefs. This policy subjects short positions to tail risk, causing investors to demand a large increase in price to absorb noise-trader purchases and leading to overvaluation. Despite providing purely firm-specific information, this policy impacts the firm's expected returns. We further show the firm can inflate its price even when restricted to simple policies that withhold news lying above or below a threshold.

Keywords: Short-selling, Disclosure, Bayesian Persuasion, Downside Risk, Trading.

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1 Introduction

A vast and influential literature analyzes the optimal disclosure choices of firms that aim to maximize their stock price. This research has traditionally focused on the quantity of information firms provide as captured by the precision with which their disclosures reflect their fundamental value (Diamond (1985), Diamond and Verrecchia (1991), Easley and O’Hara (2004)). However, in many contexts, firms have flexibility not only in the quantity or precision of the information they disclose, but also the type of information they disclose.¹ In this paper, we ask whether firms can exploit the ability to tailor their disclosure policies in order to raise the expected prices they receive for their shares.

We begin with the classic setting in which a firm’s shares are traded in a competitive market composed of risk-averse investors and noise traders (Grossman and Stiglitz (1980), De Long et al. (1990)). Prior to trade, the firm, which seeks to maximize its expected stock price, has the power to commit to releasing a signal of firm value. Drawing on the Bayesian persuasion literature, our key modeling innovation is to allow the firm to choose a signal that possesses any statistical relationship with its idiosyncratic value (Kamenica and Gentzkow (2011)). This captures firms’ ability to tailor both the precision and nature of the information they disclose. Given that an individual firm is unlikely to produce significant novel information regarding the market’s future performance, we constrain the firm to selecting among signals that aid investors only in estimating the firm’s idiosyncratic payoffs.²

Conventional wisdom from the prior literature that studies public disclosure in exchange economies holds that the provision of additional information reduces information asymmetry and, on average, increases prices. As a result, a firm’s optimal disclosure policy is to reveal as much information as possible (Diamond and Verrecchia (1991), Easley and O’Hara (2004)).

¹As examples, firms may tailor investors’ posterior beliefs by applying accounting policies that require the disclosure of bad news but withhold or delay the release of good news (Watts (2003)), smoothing performance metrics over time (Acharya and Lambrecht (2015)), disclosing extreme or unusual outcomes separately, or releasing information concerning economic risks.

²Consistent with the typical firm providing little information on market-wide performance, Bonsall et al. (2013) finds the average response to even large “bellweather” firms’ earnings forecasts is no more than two basis points.

However, this literature generally confines attention to a limited range of disclosure policies, such as signals that equal the firm’s true value plus normally-distributed noise. We find that simple disclosure policies of this nature are not optimal upon generalizing the firm’s choice set. Instead, the optimal policy is imprecise and leaves investors with positively-skewed posterior beliefs regarding the firm’s value. Such a policy essentially discourages short-selling.

As a preliminary step towards deriving this result, we assess how the firm’s expected stock price depends upon the distribution of its idiosyncratic payoffs – which, given that the firm has complete flexibility its disclosure policy, may take any form (Kamenica and Gentzkow (2011)). In the absence of noise trade, the market prices idiosyncratic payoffs at their expected value. As a result, independent of their distribution, disclosure regarding these payoffs has, on average, no impact on prices. In contrast, we show that the presence of noise trade can create a divergence between the pricing of idiosyncratic payoffs and their expected value, which, in turn, creates room for disclosure to influence the expected price.

The impact of noise trade on the pricing of idiosyncratic payoffs stems from the preferences of the risk-averse investors, who generally long the stock, but short the stock when noise traders buy in sufficiently large amounts. Given their risk aversion, these investors demand compensation to hold these positions, and thus noise-trader purchases (sales) drive the stock price up (down). When the firm’s cash flows are normally distributed, as in prior work that studies the price-impact of noise trade, short and long positions of the same magnitude are identically risky. This implies that the market is “symmetrically” liquid: noise-trader purchases inflate the firm’s stock price to precisely the same extent that noise-trader sales depress this price, such that, on average, noise traders have no effect on price.

The result that noise trade has no expected impact on prices appears strong, and we show, in fact, that it generally does not hold when moving beyond symmetric payoff distributions. Intuitively, consider a firm with highly positively-skewed payoffs, i.e., one with a small probability of a large spike in value. When investors short such a firm’s stock, they suffer large

losses if such a spike occurs, that is, they face downside tail risk. In contrast, when investors long the stock, they do not face downside risk, but rather perceive a lottery-like distribution of payoffs with a small probability of a large gain. Under most commonly-used formulations of investor preferences, which exhibit “prudence,” the risk premium that investors require to hold a position with downside risk is greater than that for a lottery-like position (e.g., [Menezes et al. \(1980\)](#)).³ As a result, the market exhibits “asymmetric” liquidity: noise-trader purchases inflate the firm’s stock price to greater extent than noise-trader sales depress this price. This implies that noise trade – even when, on average, non-directional – drives up the firm’s stock price. Applying this reasoning, we show that, when investors have prudent preferences, there exist asymmetric distributions over the firm’s payoffs that cause it to be, in expectation, overvalued.

Using these findings, we next construct a disclosure policy that causes the firm’s expected price to exceed that given perfect disclosure. As perfect disclosure eliminates all risk and causes the firm’s price to equal its fundamental value, this implies that the optimal policy causes the firm’s expected price to exceed its expected value. In our construction, we exploit the fact that an information signal can be crafted to ensure that skewness moves in any desired direction. This property of skewness distinguishes it from the first two central moments, which, according to the laws of iterated expectations and total variance, always on average remain the same and decline, respectively, upon the arrival of information. Specifically, independent of the skewness of the prior, the policy we construct leaves investors, in expectation, with a positively-skewed posterior, thereby creating downside risk to potential short sellers. We further show that the *optimal* disclosure policy never leaves investors with a symmetric posterior such as the normal distribution, and never fully reveals the firm’s payoffs along an interval.

To provide additional insight into the nature of the optimal disclosure policy, we next

³Such an aversion to downside risk is consistent with experimental evidence and the tail-risk premia implied by stock and derivative prices (e.g., [Harvey and Siddique \(2000\)](#), [Ang et al. \(2006\)](#), [Bollerslev and Todorov \(2011\)](#)).

assume that investors possess constant absolute risk aversion (CARA) utility, which incorporates an aversion to downside risk (Eeckhoudt and Schlesinger (2006)) and derive the optimal disclosure policy when firm’s payoff takes on one of two values (e.g., Ottaviani and Sørensen (2015), Breon-Drish (2015)). This policy reveals that the firm’s payoff is high with some probability, and otherwise reveals nothing. When the payoff is not revealed, this policy leaves investors with a posterior belief that the probability of the high state falls short of one half, i.e., a positively-skewed belief. This policy can be thought of as providing “noisy” bad news and “precise” good news. We further show that the optimal disparity between the precision of good and bad news reflects a trade off between the frequency with which investors are left with a skewed posterior belief versus the extent of skewness in this belief.

Finally, we analyze the case in which the firm is confined to selecting among policies that either recognize or do not recognize the firm’s value. This analysis is motivated by the fact that such “recognition” policies arise in equilibrium in models of voluntary disclosure, are frequently optimal in persuasion settings, and may be easier to implement than generic disclosure policies.⁴ We show that the firm is able to generate overvaluation using a simple recognition policy in which they disclose their value when it lies below a threshold, and withhold otherwise, as this policy leads to a positively-skewed posterior. However, we find that more complex recognition policies that recognize the firms value on multiple disjoint intervals can generate greater skewness and thus further increase expected price.

While our analysis is framed in terms of deriving the optimal disclosure policy, it also provides new insights into the price-impact of mandated disclosures and the equilibrium disclosures that arise in settings where firms do not have commitment power. For example, many accounting procedures, such as the lower of cost or market rule applied to inventories, apply a higher standard to recognizing good than bad news, thereby reducing the precision of bad news. Our analysis suggests that such policies may, counter-intuitively, lead to overvaluation. Furthermore, models of voluntary disclosure suggest that, in equilibrium, firms

⁴See, for example, Verrecchia (1983), Göx and Wagenhofer (2010), Bouvard et al. (2015), Goldstein and Leitner (2018), Dworzak and Martini (2019), and Szydlowski (2020).

may provide more good news than bad news. Given that investors' posteriors given non-disclosure are negatively skewed in such equilibria, our results suggest that they are likely to lead firms to be, on average, undervalued.

Related literature. Our paper fits into the growing Bayesian persuasion literature. From a technical perspective, the problem of persuasion in an asset-pricing setting is difficult, as equilibrium prices depend upon all moments of an asset's payoffs. As a result, no general form for the optimal policy can be derived; this policy depends upon both investors' utility functions and their priors, and, even under specific distributional assumptions, is both analytically and computationally difficult to solve for (Dughmi and Xu (2019)). In contrast, existing models of persuasion typically assume an objective function that depends only upon the posterior expected value; see Kolotilin (2018) and Dworzak and Martini (2019) for discussions of this assumption. Given these challenges, in our general analysis, our focus is on characterizing properties of the optimal disclosure policy that distinguish it from the policies that are the focus of the existing disclosure literature.

Our paper further relates to prior work that finds investor risk aversion can cause imperfect and/or asymmetric disclosures to be optimal. Suijs (2008) show in an overlapping-generations model that disclosure that is more precise given negative than positive news can reduce price volatility and increase expected prices by lowering the risk premium when negative news arrives. Armstrong et al. (2016) demonstrate that risk-averse managers issue more precise disclosures when their firm performs poorly when increasing disclosure precision demands that they incur a personal cost. Gollier and Schlee (2011) consider the impact of information regarding aggregate consumption on stock prices, finding that additional information can, on average, reduce these prices. Note this work operates under the assumption that the precision of information a firm releases influences its risk premium, which, in a large economy, requires that this information aids investors in assessing the market's performance. In contrast, our motivation for imperfect disclosure holds even when firms' information is

specific to their own performance.⁵

Our results also have implications for the empirical and theoretical asset-pricing literature that considers how skewness influences prices. Extensions of the CAPM to consider investor preferences for skewness suggest that only coskewness with the market portfolio should be priced (Kraus and Litzenberger (1976)). This has led much of the empirical research on skewness to focus on its systematic component (Harvey and Siddique (2000), Ang et al. (2006)). In contrast, our paper suggests that idiosyncratic skewness influences prices in the presence of noise trade, providing a potential explanation for the empirically documented negative relationship between idiosyncratic skewness and returns (Boyer et al. (2010)). Finally, our paper contributes to the literature on short-sale constraints. Notably, Lamont (2012) finds that firms take legal and regulatory actions to harm short sellers in order to maintain inflated prices. Our model suggests that an empirically-unexplored means through which firms may impede short-selling and thus foster overvaluation is through their disclosure policies.

Structure of the paper The paper is organized as follows. In Section 2, we introduce our baseline model. In Section 3, we present distribution-free properties of the optimal disclosure policy. In Section 4, we explicitly solve for the optimal policy in a binary model and in Section 5, we analyze recognition policies and a subset of these policies that disclose the firms value when it lies on either above or below a threshold. Section 6 extends the model to consider a multi-asset economy, disclosure about systematic risks, and private information. Section 7 concludes.

2 Model

We analyze the optimal disclosure policy of a firm whose stock is traded in a market consisting noise traders and risk-averse investors. We conduct this analysis within a generalized version

⁵Similarly, Jiang and Yang (2017) study the optimal disclosure policy when liquidity motives may force a firm to inefficiently retain its shares, and found that disclosing a lower bound is the optimal means to mitigate these inefficiencies.

of the standard model of competitive trade between homogeneous, uninformed investors and noise traders (e.g., De Long et al. (1990)). Formally, consider a firm that has a random payoff \tilde{v} distributed according to a prior distribution $\mu_0(\cdot)$ with support $\mathcal{V} \subseteq \mathbb{R}$.⁶ A continuum of (rational) investors, indexed by $i \in [0, 1]$, trade in the firm’s stock, submitting demand orders of D_i . Because each of these traders is small, they behave as price takers. In addition to the stock, these investors trade in a risk-free asset, in unlimited supply, with rate of return normalized to 1 (thus, the risk-free asset is the numeraire). The investors possess homogeneous initial wealth and preferences $u(w)$ defined over their terminal wealth, w . For notational simplicity, but without loss of generality, we set the investors’ initial wealth to zero.⁷ We assume that preferences are smooth, increasing (i.e., $u' > 0$), risk-averse (i.e., $u'' < 0$), and prudent (i.e., $u''' > 0$). Note many commonly used utility function including CARA and CRRA exhibit prudence (Eeckhoudt and Schlesinger (2006)). We return to the role of prudence role later in this section.

The investors trade alongside noise traders who, in the aggregate, demand $-\tilde{z}$ units of stock, where \tilde{z} is independent of the firm’s payoff \tilde{v} and has variance $\sigma_z^2 \equiv \text{Var}[\tilde{z}] > 0$. We assume that \tilde{z} is symmetrically distributed around zero, such that $m_z \equiv \text{E}[\tilde{z}] = 0$. Moreover, we assume that the stock is in zero net supply, such that the residual supply that must be absorbed by the risk-averse investors is \tilde{z} . The assumption that the stock is in zero net supply plays an important role in our analysis, as it rules out a direct effect of disclosure on the risk premium. To be precise, given this assumption, if there were no noise trade in our model, the firm’s price would simply equal its expected payoffs, leaving no room for disclosure to impact price, on average.

In Section 6.1, we show that the zero net supply assumption implies that our model, which is cast in terms of a single firm, is equivalent to one in which a firm is embedded in a large economy and discloses information about an idiosyncratic component of its value.⁸

⁶Throughout, we use a tilde to denote a random variable (e.g., “ \tilde{v} ”) and we drop the tilde to indicate a realization of that random variable (e.g., “ v ”).

⁷Non-zero initial wealth W can be absorbed into the utility function by redefining $u^*(x) = u(W + x)$.

⁸Note we do not require that the firm’s disclosure explicitly concerns only components of its value that

The notion that the firm’s disclosure provides investors with novel information regarding its idiosyncratic performance, as opposed to aggregate performance, appears to match individual firms’ financial disclosures, which lead to large firm-specific returns but very small market returns (e.g., [Bonsall et al. \(2013\)](#)).⁹ If the average supply were instead positive, which would correspond to the firm disclosing information about market-wide risks, our results would trade off against the conventional impact of disclosure on the risk premium ([Gollier and Schlee \(2011\)](#)). We extend the model to analyze this case in [Section 6.2](#).

The model’s timeline is as follows. There are four dates, $t \in \{1, 2, 3, 4\}$. At $t = 1$, the firm publicly commits to a disclosure policy, that is, it chooses a signal \tilde{s} about its payoff \tilde{v} . At $t = 2$, the firm publicly discloses the realization s of the signal \tilde{s} and the risk-averse investors form a posterior belief $\mu(\cdot)$ about the firm’s payoff \tilde{v} . At $t = 3$, conditional on the realized signal, risk-averse investors submit their orders alongside the noise traders, and the equilibrium price $P(z, \mu)$ is set to clear the market. Finally, at $t = 4$, the firm’s payoff \tilde{v} realizes and is paid out as a dividend to the firm’s shareholders.

We allow the firm to choose any disclosure policy subject only to the constraint that the investors’ expected utility induced by the firm’s policy is finite. Full commitment is a strong assumption. However, the ability of the firm to choose among arbitrary signals may be interpreted either as the choice of what information to gather and present in the financial statements and/or as a decision regarding how to disclose information that is readily available to the firm. For example, the firm’s decision may be thought of as a selection among accounting methods. Note also that, while our analysis is cast in terms of characterizing the optimal disclosure policy, our results also provide insight into how an exogenous disclosure

are idiosyncratic. Instead, we only require that the information in a firm’s disclosures on systematic risk could be gleaned from other sources such as other firms’ disclosures or macroeconomic indicators.

⁹Note that [Savor and Wilson \(2016\)](#) find more significant market-wide responses to early announcing firms’ financial disclosures. However, the information in early announcers’ earnings regarding the market’s performance, if removed, would likely come out soon after through other news events, such as other firms’ earnings announcements. Consequently, it is unlikely these firms have a long-lived impact on the amount of information known about the market.

⁹For example, if the disclosure policy induced posteriors beliefs characterized by a t-distribution, then the expected utility of CARA investors would not be finite.

policy – such as an accounting standard – influences stock prices in the presence of noise trade. Finally, in Section 5, we consider a more restricted commitment environment where the firm is confined to selecting among policies that either reveal or withhold its value.

Formally, a disclosure policy is captured by a signal \tilde{s} that is statistically related to \tilde{v} , such that upon observing the signal’s realization $\tilde{s} = s$, investors update their beliefs about the firm’s payoff \tilde{v} according to Bayes’ rule. Equivalently, we can identify the realization of a signal, $\tilde{s} = s$, with the posterior distribution that it induces, denoted $\mu(\cdot)$, and the disclosure policy with the distribution $\tau(\cdot)$ of posterior distributions that are obtained upon conditioning on the realized signal. As shown in [Kamenica and Gentzkow \(2011\)](#), letting $\Delta(\mathcal{V})$ denote the set of all probability distributions on the support \mathcal{V} , there exists a signal that leads to the distribution of posteriors $\tau(\cdot)$ if and only if $\tau(\cdot)$ satisfies the *Bayesian plausibility constraint*,

$$\int_{\Delta(\mathcal{V})} \tau(\mu) d\mu = \mu_0. \tag{1}$$

This constraint states that the average posterior probability of any realization of the firm’s value is equal to its prior probability.

We assume that the firm selects its signal ex ante to maximize its expected price. Formally, the firm’s optimization problem at $t = 1$ can be written as:

$$\begin{aligned} \max_{\tau(\cdot)} \int_{\Delta(\mathcal{V})} E[P(\tilde{z}, \mu)] d\tau(\mu) \\ \text{subject to (1)}. \end{aligned} \tag{2}$$

In other words, the firm’s objective function is obtained by taking two expectations. First, for every posterior $\mu(\cdot)$ on the support of the disclosure policy $\tau(\cdot)$, the firm evaluates the expected price $E[P(\tilde{z}, \mu)]$, which averages out all possible realizations of the noise-trader demand. Then, the firm takes the expectation of this average price across all possible realizations of the posterior $\mu(\cdot)$.

In our analyses, we reference a few notable examples of disclosure policies that are com-

mon in prior literature. First, we refer to a *perfect* disclosure policy as one in which the signal $\tilde{s} = \tilde{v}$ almost surely. That is, disclosure is perfect if it always perfectly reveals the firm’s payoff. Second, we refer to a policy of *additive noise* as one for which $\tilde{s} = \tilde{v} + \tilde{\varepsilon}$, where $\tilde{\varepsilon}$ is independent of \tilde{v} . In addition, we refer to *recognition policies*, which map probabilistically each realization $\tilde{v} = v$ of the firm’s value into one of two messages: either that particular realization is disclosed perfectly, i.e., $\tilde{s} = v$; or it is withheld, i.e., $\tilde{s} = \emptyset$. A recognition policy can be represented by a function $\omega : \mathcal{V} \rightarrow [0, 1]$, which gives the probability $\omega(v)$ that each realization of firm value $\tilde{v} = v$ is withheld. Finally, when discussing the case in which the firm’s payoff is binary, we consider binary signal structures, $\tilde{s} \in \{s_B, s_G\}$, where s_G (s_B) stands for “good” (“bad”) news, in the sense that the posterior is more optimistic conditional on good news than on bad news. Binary signals are fully characterized by the probability that the signal is high conditional on each realized payoff, $q_v \equiv \Pr[\tilde{s} = s_G | \tilde{v} = v]$.

Note that the set up to this point places few constraints on investors’ preferences and no constraints on the prior distribution over the firm’s payoffs. We start, in the next section, by characterizing properties of the optimal disclosure policy in this general case. However, no generic form for the optimal policy can be derived without making further assumptions regarding distributions and preferences. To provide further insight into the optimal disclosure policy, we then consider a binary model in which the policy can be explicitly derived.

3 General Results

In this section, we characterize general properties of the optimal disclosure policy. Most notably, we show that this policy is neither perfect, nor is it, in general, uninformative. Moreover, the optimal policy leads the firm’s expected price to exceed its expected cash flows by causing the market to be more liquid for noise-trader sales than purchases (Proposition 1). These findings stand in contrast to much of the prior theoretical literature on disclosure, which typically finds that prices are maximized when firms provide full information and that,

on average, disclosure brings the firm’s price closer to its true value (see, e.g., [Goldstein and Yang \(2017\)](#)).

Next, we establish two stronger results concerning the optimal policy. When investors’ prior is continuous, not only is the optimal policy imperfect, but also it never reveals the firm’s value with positive probability ([Proposition 2](#)). Likewise, when the prior possesses mass points, this policy reveals the firm’s value when it takes on at most a single value with positive mass. These results illustrate that recognition policies in which the state is revealed when it falls above or below a threshold, while frequently optimal in persuasion settings, are not optimal in a financial-market setting given the dependence of prices on higher moments.

Finally, we show that the optimal policy never leaves investors with symmetric posterior beliefs such as the normal – despite the common focus on normally-distributed signals in the disclosure literature ([Corollary 1](#)). This result suggests that firms with greater flexibility in their disclosure policies will exhibit more skewed returns.

3.1 Equilibrium pricing and overvaluation

As a first step, we derive the firm’s price and demonstrate that, given the presence of noise trade, this price need not equate to the firm’s expected cash flows, on average. To be precise, we show that, while symmetric distributions lead the firm to be accurately priced, there exist asymmetric distributions such that the market price, on average, is *greater* than the expected payoff, which we refer to as “overvaluation.”

Note if the market price is P , the terminal wealth of an investor i who submits an order D_i is $D_i(\tilde{v} - P)$. When the investor possesses the posterior $\mu(\cdot)$ over the firm’s payoff, they choose their demand to maximize:

$$\max_{D_i} E_\mu [u(D_i(\tilde{v} - P))],$$

where E_μ is the expectation with respect to the posterior distribution $\mu(\cdot)$ as induced by

the signal realization $\tilde{s} = s$. Market clearing requires the aggregate demand by investors be equal to the aggregate supply of noise traders, that is, $\int D_i di = z$. Because all investors are homogeneous, their equilibrium demands are identical, $D_i = D$ for all i , and therefore the market-clearing condition reduces to $D = z$. The following familiar fixed-point equation for the market-clearing price is a straightforward rearranging of the investor's first-order condition evaluated at the market-clearing condition:¹⁰

$$P(z, \mu) = \frac{\mathbb{E}_\mu[\tilde{v}u'(z(\tilde{v} - P(z, \mu)))]}{\mathbb{E}_\mu[u'(z(\tilde{v} - P(z, \mu)))]}. \quad (3)$$

Observe that the price is a function both of the realized noise-trader demand, z , and of investors' posterior, $\mu(\cdot)$; in order to clear the market given a supply z , the price adjusts so as to ensure that investors are willing to take the other side of noise traders' demands. Moving forward, we assume that equation (3) has a unique solution $P(z, \mu)$, which simplifies the exposition; however, the essence of our results does not depend upon this assumption.¹¹

We seek to identify posterior distributions that lead to overvaluation, in the sense that they cause the average price across all realizations of noise-trader demand to exceed the expected payoff. The following definition formalizes this notion.

Definition 1. A posterior $\mu(\cdot)$ induces overvaluation (undervaluation) if and only if $\mathbb{E}[P(\tilde{z}, \mu)] > \mathbb{E}_\mu[\tilde{v}]$ (resp., $\mathbb{E}[P(\tilde{z}, \mu)] < \mathbb{E}_\mu[\tilde{v}]$). Otherwise, we say that $\mu(\cdot)$ induces accurate valuation.

We next state two lemmas that will be used as building blocks for our main results concerning the optimal disclosure policy. First, we show that for any symmetric prior distribution the firm is accurately valued, where a random variable \tilde{v} is symmetrically distributed if \tilde{v} and its rotation $2\mathbb{E}_\mu[\tilde{v}] - \tilde{v}$ have the same distribution.¹²

¹⁰The second-order condition for a maximum is satisfied because the utility is concave.

¹¹In the end of this section, we show that a sufficient condition for this to hold is that investors have CARA utility. Note further that, if this assumption is dropped, our results upon focusing on, for any given $\mu(\cdot)$, a specific solution for price as a function of \tilde{z} . Technical details are available upon request.

¹²In particular, if the distribution of \tilde{v} admits a density $f(\cdot)$, then \tilde{v} is symmetrically distributed if and only if $f(\mathbb{E}_\mu[\tilde{v}] + x) = f(\mathbb{E}_\mu[\tilde{v}] - x)$ for all $x \in \mathbb{R}$.

Lemma 1. *If the distribution $\mu(\cdot)$ is symmetric, then $\mu(\cdot)$ induces accurate valuation, i.e., $E[P(\tilde{z}, \mu)] = E_\mu[\tilde{v}]$.*

To understand this lemma, note that, given a posterior $\mu(\cdot)$, when noise traders sell z shares, the price is driven below expected firm value, i.e., $P(z, \mu) \leq E_\mu[\tilde{v}]$, and vice versa when these traders buy z shares. Importantly, when the firm's value is symmetric, price is driven up by the exact same amount when noise traders buy z as it is driven down when they sell z . Intuitively, under a symmetric payoff distribution, investors face *identical* risk whether they long or short the stock, and thus require price move by the same amount in order to take the other side of noise traders' demands. Furthermore, because noise trading is symmetrically distributed, there is an equal likelihood that noise traders buy and sell z , such that noise trade has no on-average impact on price. This result is familiar from the linear equilibria that commonly arise in noisy rational-expectations models such as [Grossman and Stiglitz \(1980\)](#) and [Hellwig \(1980\)](#). Given that price is a linear function of noise trade in such equilibria, sell and buy orders have precisely offsetting impacts on the price.

In contrast, when the distribution of payoffs is not symmetric, i.e., when it exhibits skewness, it is no longer the case that longing and shorting expose a trader to identical risk. Intuitively, short and long positions are exposed to the skewness of the underlying distribution in opposing directions. If payoffs are positively skewed, investors who are long (short) face positively (negatively) skewed returns. Consequently, if investors have preferences for skewness, the premium they require to absorb noise-trader purchases differs than the premium they require to absorb noise-trader sales. As such, the firm's expected price will deviate from its expected payoffs in the presence of a skewed distribution.

Given our assumption that $u''' > 0$, which implies investors prefer positive skewness ([Menezes et al. \(1980\)](#)), this intuitive argument suggests that positively-skewed payoff distributions lead to overvaluation in the presence of noise trade. However, the link between payoff skewness and investor demands, and thus equilibrium prices, is difficult to formalize without placing specific assumptions on investor preferences and payoff distributions. This is

analogous to the link between payoff dispersion and prices: while return variability is almost ubiquitously posited to decrease prices and increase returns, even under strict risk aversion, distributional orderings such as increasing variance and second-order stochastic dominance are not sufficient to ensure a decline in prices (e.g., Gollier (2001)).

In the next section, we make the link between skewness and prices explicit under a binary formulation of the model. For the purposes of this section, however, we need only to demonstrate that there always exists *some* asymmetric distribution that leads to overvaluation.

Lemma 2. *There exists a distribution $\hat{\mu}(\cdot)$ with support $\hat{\mathcal{V}} \subseteq \mathcal{V}$ that induces overvaluation.*

The intuition for this result is that, upon deviating slightly away from a symmetric distribution towards an asymmetric distribution, under prudent preferences, the firm is no longer accurately valued. Again, this follows as investors face different risks when longing and shorting the stock under an asymmetric distribution.¹³ In general, such a deviation may just as well lead to undervaluation as it will lead to overvaluation. However, we show that one can always find a deviation in a direction that leads to overvaluation.

3.2 Characteristics of the optimal disclosure policy

We are now in a position to analyze the optimal disclosure policy. To start, we define the notion of a disclosure policy that creates overvaluation. This definition is analogous to Definition 1, but averages over the posterior beliefs that are created by a given policy.

Definition 2. A disclosure policy $\tau(\cdot)$ induces overvaluation if the ex ante expected price is greater than the ex ante expectation of the firm's payoff, $\int_{\Delta(\mathcal{V})} E[P(\tilde{z}, \mu)] d\tau(\mu) > E_{\mu_0}[\tilde{v}]$.

While this notion of overvaluation might appear to predict that a firm would exhibit negative expected returns, this need not be the case. Recall that, for sake of simplicity, our

¹³By contrast, when utility is quadratic in wealth (so that $u''' = 0$), not only symmetric, but also asymmetric distributions induce accurate valuation (see Lemma D.1 in Appendix D). This occurs because under quadratic utility investors are insensitive to the third and higher moments of the distribution. Note the case in which $u''' < 0$ can be ruled out as it is inconsistent with a globally concave, increasing utility function.

baseline model assumes that the firm’s payoff is purely idiosyncratic. The model is easily extended to allow for the more realistic scenario in which firm value includes a systematic risk, and thus the firm’s price exhibits a risk premium – again, as long as the novel information in firm’s disclosure concerns the idiosyncratic component of its value. Together with investors’ risk aversion, if this risk premium is sufficiently large, even a firm that is overvalued in accordance with Definition 2 will earn positive expected returns; we conduct such an analysis in Section 6.1. Thus, this notion of overvaluation should be interpreted as relative to the firm’s risk-adjusted valuation, i.e., the valuation that would arise in the absence of noise trade.

Given this definition, we have the following result.

Proposition 1. *There exists a disclosure policy that induces overvaluation.*

We prove this result by explicitly constructing a disclosure policy that leads to overvaluation. The policy we construct is a recognition policy, revealing the underlying payoff with some probability that depends upon this payoff. The probability that each payoff is recognized is set such that the policy, when it does not reveal the firm’s payoff, leaves investors with an asymmetric posterior belief that leads to overvaluation (according to Lemma 2, such a posterior belief always exists). Following the intuition provided in the previous section, this posterior belief generally exhibits positive skewness. Thus, independent of the skewness of the prior, the policy leads investors to have, on average, positively-skewed posteriors. The ability of information to generate skewness represents a key departure from the first two moments: on average, the laws of total expectation and variance imply that the mean remains unchanged upon the release of new information, and the variance falls.

The next proposition characterizes additional properties of the optimal policy.

Proposition 2. *An optimal disclosure policy, τ^* :*

- (i) assigns positive probability to at most one degenerate distribution; and*
- (ii) zero probability to non-degenerate symmetric distributions.*

Put simply, part (i) of the proposition states that, when the firm’s value is continuously distributed, the optimal disclosure policy never perfectly reveals the firm’s value with positive probability. Likewise, if the firm’s value has mass points, the optimal policy reveals the firm’s value with positive probability when it takes on at most one specific value. Intuitively, under a policy that perfectly reveals multiple realizations of firm value, the firm is accurately valued when any of these realizations occur. As a result, such a policy gives up an opportunity to create overvaluation. We formalize this notion by showing that, starting from a policy that reveals multiple payoffs, there is an alternative policy that leads to a greater expected price. Conditional upon one of these payoffs occurring, rather than revealing the payoff, the policy randomly either reveals this payoff or sends a new signal (where the probabilities of these two events depend upon the payoff). Upon observing this new signal, investors are left with asymmetric posterior beliefs that lead to an inflated price.

Part (ii) of the proposition states that the optimal policy never leaves investors with a symmetric posterior. The intuition for this result is similar to part (i): because symmetric distributions lead the firm to be accurately valued, leaving investors with such a posterior again gives up the opportunity to create overvaluation. We show this in a similar manner to part (i), beginning with a policy that leaves investors with a symmetric posterior, and constructing an alternative policy that leads to a higher expected price. Rather than sending the signal that leads to an symmetric posterior, this alternative policy randomly either reveals the firm’s value or sends a new signal that leads to an inflated price.

Proposition 2 is relevant because it suggests that, if the firm could optimally design its disclosure policy ex ante, then investors would never trade under symmetric beliefs, such as the normal distribution – which is widespread in the asset-pricing and disclosure literatures. Furthermore, it implies that recognition polices, which are commonly optimal in persuasion settings, are not optimal in the financial-market setting. We summarize these implications of the proposition in the next corollary.

Corollary 1. *Both perfect disclosure and recognition policies are suboptimal. Furthermore,*

if the firm's payoff is normally distributed, then a signal that equals the firm's true value plus normally-distributed independent noise is suboptimal.

In the remainder of the paper, we conduct further analyses of the model under the assumption that the risk-averse investors have CARA utility, $u(w) = \exp(-\rho w)$, where $\rho > 0$ is the coefficient of absolute risk aversion. Given this assumption, the equation for price (equation (3)) can be solved explicitly:

$$P(z, \mu) = E_\mu[\tilde{v}] + \frac{E_\mu[(\tilde{v} - E_\mu[\tilde{v}]) \exp(-\rho z \tilde{v})]}{E_\mu[\exp(-\rho z \tilde{v})]}. \quad (4)$$

We will also reference a second-order Taylor expansion of equation (4) to illustrate the ideas underlying our results. Specifically, expanding equation (4) as a function of the noise-trader demand and taking its expectation yields:

$$E[P(\tilde{z}, \mu)] \approx E_\mu[\tilde{v}] + (\text{Var}_\mu[\tilde{v}])^{\frac{3}{2}} \text{Skew}_\mu[\tilde{v}] \frac{\rho^2 \sigma_z^2}{2}, \quad (5)$$

where $\text{Skew}_\mu[\tilde{v}] \equiv E_\mu \left[\left(\frac{\tilde{v} - E[\tilde{v}]}{\sqrt{\text{Var}[\tilde{v}]}} \right)^3 \right]$ (see Appendix B for a formal derivation). This approximation makes transparent the ideas underlying Proposition 2. Specifically, observe from equation (5) that, as the posterior variance shrinks, the average price converges towards the expected firm payoff. Roughly speaking, this is the reason why the optimal disclosure policy almost never fully resolves investor uncertainty (except, potentially, at a mass point). Furthermore, equation (5) demonstrates that, if the posterior distribution is symmetric, the average price again converges towards the expected firm payoff because, in this case, the posterior skewness $\text{Skew}_\mu[\tilde{v}]$ goes to zero. Hence, the optimal disclosure policy does not lead to symmetric posteriors.

4 Optimal Disclosure Policy Under Binary Fundamentals

In this section, we derive the firm’s optimal policy when the firm’s payoff \tilde{v} belongs to $\mathcal{V} = \{m_v - \sigma_v, m_v + \sigma_v\} \equiv \{v_L, v_H\}$. We denote investors’ posterior given the signal \tilde{s} by $q \equiv \Pr[\tilde{v} = v_H | \tilde{s} = s]$ and assume now that noise trade \tilde{z} is normally distributed with mean $m_z = 0$ and variance $\sigma_z^2 > 0$. We start by deriving an explicit expression for the firm’s stock price, $P(z, q)$, as a function of realized noisy trade $\tilde{z} = z$ and a given posterior belief q , which is familiar from prior work that employs CARA utility with binary payoffs (e.g., [Kurlat and Veldkamp \(2015\)](#), [Smith \(2019\)](#)).

Lemma 3. *Suppose that the firm’s payoff is binary. Then, the firm’s stock price equals*

$$P(z, q) = m_v + \sigma_v \frac{q(\exp(-2\rho z \sigma_v) + 1) - 1}{q(\exp(-2\rho z \sigma_v) - 1) + 1}. \quad (6)$$

The firm’s equilibrium price possesses several intuitive properties. First, it is bounded between the low and high values of the firm’s payoff, $m_v - \sigma_v$ and $m_v + \sigma_v$, which is necessary to ensure the absence of an arbitrage opportunity. Furthermore, when investors are risk neutral (i.e., when $\rho \rightarrow 0$), the price converges to the expected firm payoff $m_v + \sigma_v(2q - 1)$. However, when investors are risk averse (i.e., when $\rho \neq 0$) and $\tilde{z} > 0$ ($\tilde{z} < 0$), the price is greater (less) than the expected payoff; that is, there is a negative (positive) “risk” premium.

The case of a binary payoff starkly illustrates the difference in risk to shorting and longing a stock under an asymmetric payoff distribution and the resultant effect on the equilibrium price. Suppose first that $q = 0.1$. Then, when a trader takes the other side of noise-trader sales, they perceive a lottery-like payoff, with a 10% probability of a large gain. In contrast, when a trader takes the other side of noise-trader purchases, they perceive a downside risk with a 10% probability of a large loss. The converse holds when $q = 0.9$.

Because investors are averse to downside risk, markets are asymmetrically liquid for

purchases relative to sales when $q = 0.1$, and vice versa when $q = 0.9$. This can be seen in the price, which is a nonlinear function of the realized noise-trader demand z . Specifically, the price function reacts more strongly to a buy order of z than a sell order of z when $q < 1/2$, and vice versa when $q > 1/2$. Therefore, noise trade increases (decreases) price, on average, if and only if $q < 1/2$ ($q > 1/2$).

Armed with Lemma 3, we now derive the optimal disclosure policy. Let $q_0 \equiv \Pr[\tilde{v} = v_H]$ denote the prior probability of the high payoff, with $q_0 \in (0, 1)$. Ex ante, the firm chooses a disclosure policy $\tau(\cdot)$ to solve the program (2). For binary payoffs, this program is equivalent to choosing a distribution over the posterior probability of the high payoff, \tilde{q} , subject to the requirement that, in expectation, the posterior equals the prior q_0 :

$$\mathbb{E}[\Pr[\tilde{v} = v_H|\tilde{s}]] = q_0. \quad (7)$$

The following result characterizes the firm's optimal disclosure policy.

Proposition 3. *Suppose that the firm's payoff is binary. Then, there exists a cutoff $\hat{q} \in (0, \frac{1}{2})$ such that:*

- (i) *If $q_0 \leq \hat{q}$, then the optimal disclosure policy is uninformative (i.e., $\tilde{s} = \emptyset$ a.e.);*
- (ii) *If $q_0 > \hat{q}$, then a binary signal $\tilde{s} \in \{s_B, s_G\}$ is an optimal disclosure policy, where*

$$\begin{aligned} \Pr[\tilde{s} = s_B|\tilde{v} = v_L] &= 1, \\ \Pr[\tilde{s} = s_B|\tilde{v} = v_H] &= \kappa \in (0, 1), \end{aligned}$$

and κ is chosen so that $\Pr[\tilde{v} = v_H|\tilde{s} = s_B] = \hat{q}$.¹⁴

The proposition states that, for q_0 below a threshold $\hat{q} \in (0, \frac{1}{2})$, the optimal disclosure is uninformative. In contrast, for $q_0 > \hat{q}$, a binary signal is optimal. This policy always sends the low signal when $\tilde{v} = v_L$ and sends the low signal given $\tilde{v} = v_H$ with probability κ , where

¹⁴Note there also exist other optimal policies in which more than two signals sent. However, all such policies are essentially equivalent to the binary signal in the sense that they induce the same distribution over posterior beliefs, and thus it is redundant to separately study these policies. As an example, consider an alternative policy that, whenever the optimal binary signal would send signal s_B , instead randomly sends either signal $s_{B,1}$ or $s_{B,2}$. Under this alternative policy, both the signals $s_{B,1}$ and $s_{B,2}$ lead to the same posterior belief as s_B , and thus the distribution over posteriors is unchanged.

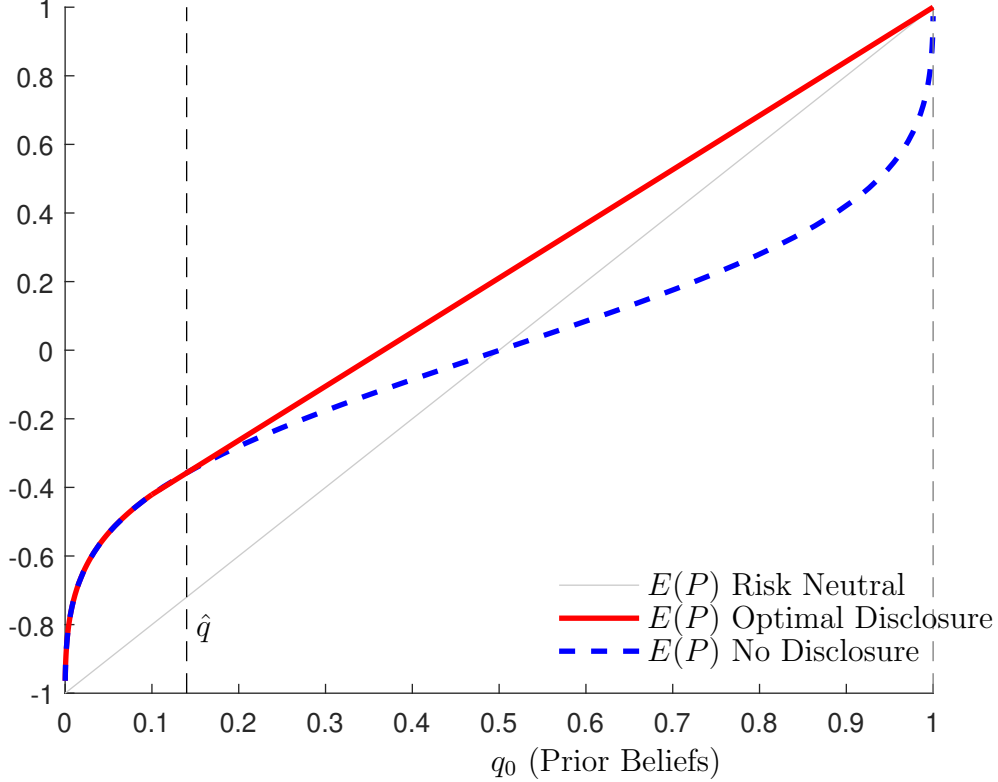


Figure 1: The optimal disclosure policy (red line) concavifies the expected price as a function of beliefs q_0 . Parameters: $\rho = 2, m_z = 0, \sigma_z^2 = 1$.

κ is such that the posterior belief that $\tilde{v} = v_H$ given the low signal is \hat{q} . In this case, the optimal disclosure policy is more precise when it reveals good news ($\tilde{s} = s_G$) than when it reveals bad news ($\tilde{s} = s_B$): upon observing $\tilde{s} = s_G$, investors know for sure that payoff is high, whereas upon observing $\tilde{s} = s_B$, investors perceive, with the non-zero probability \hat{q} , that the payoff is actually high. Note that, as $\hat{q} < 1/2$, given the low signal, investors assign a relatively small probability to the firm’s payoff being high – hence, the posterior is positively skewed. The parameter κ can be thought of as the extent of asymmetry in the precision of the disclosure given that it reveals positive versus negative news. For brevity, we refer to it as the disclosure’s imprecision, but emphasize it specifically refers to the imprecision of the disclosure when it reveals bad news.

To illustrate why the optimal policy takes this form, Figure 1 depicts [Kamenica and Gentzkow \(2011\)](#)’s method to deriving the optimal policy. The blue curve depicts the average price, $E[P(\tilde{z}, q)]$, as a function of investors’ posterior q and the grey curve depicts the risk-

neutral price $P = E[\tilde{v}]$. Consistent with Lemma 1, the average price intersects the risk-neutral price exactly at $q = 1/2$, when the payoff distribution is symmetric. In the absence of disclosure, the firm is overvalued for low posteriors, and undervalued for high posteriors. The reason, again, is that the firm's payoff is positively skewed for low posteriors ($q \leq 1/2$) and negatively skewed for high posteriors ($q \geq 1/2$).

The red curve in Figure 1 is the concave closure of the average price as a function of the posterior q . This curve reflects the maximum expected payoffs that can be obtained by either a policy of non-disclosure or a policy that mixes over two posterior beliefs, and characterizes the firm's value function for a given prior belief q_0 . The cutoff \hat{q} is the tangency point between the red and blue curves. For priors below the cutoff \hat{q} , an uninformative disclosure policy is optimal, because the prior lies on the concave closure. Intuitively, when $q_0 < \hat{q}$, the prior is already significantly positively skewed and thus leads to overvaluation; disclosure of any sort would only serve to reduce this overvaluation. For priors above the cutoff \hat{q} , an expected price on the concave closure can be obtained by a disclosure policy that mixes between the two posterior beliefs $q = 1$ and $q = \hat{q}$.

To provide intuition for the impact of varying the disclosure's imprecision κ on the ex ante expected price, we now apply the approximate pricing function (5) to the binary case. When $\tilde{s} = s_G$, it is common knowledge that the fundamental is high, and hence, price equals $m_v + \sigma_v$ for any realization of noise trade. By contrast, when $\tilde{s} = s_B$, the approximate pricing equation (5) becomes:

$$E[P(\tilde{z}, q(\kappa))] \approx E[\tilde{v} | \tilde{s} = s_B] + 8\sigma_v^3 q(\kappa)[1 - q(\kappa)][1 - 2q(\kappa)] \frac{\rho^2 \sigma_z^2}{2},$$

where $q(\kappa) \equiv \Pr[\tilde{v} = m_v + \sigma_v | \tilde{s} = s_B]$ is the posterior conditional on bad news as a function of κ . Taking the ex ante expectation of the price (with respect to both the posterior \tilde{q} and

noise-trader demand \tilde{z}) yields an approximate ex ante price of:

$$E[P(\tilde{z}, \tilde{q})] \approx m_v + \underbrace{(1 - q_0 + \kappa q_0)}_{\text{probability of overvaluation}} \underbrace{8\sigma_v^3 q(\kappa)[1 - q(\kappa)][1 - 2q(\kappa)] \frac{\rho^2 \sigma_z^2}{2}}_{\text{magnitude of overvaluation}}. \quad (8)$$

Equation (8) reveals that imprecision κ has two effects on the ex ante price. First, a higher κ directly increases the probability of realizing bad news, $\Pr[\tilde{s} = s_B] = 1 - q_0 + \kappa q_0$, by increasing the probability that the high payoff is misclassified. Therefore, an increase in κ raises the probability of realizing a posterior that leads to overvaluation. Second, κ has a non-monotonic effect on the magnitude of overvaluation conditional on bad news. In particular, in the extreme cases where $q(\kappa) = 0$ or $q(\kappa) = 1/2$, the distribution is degenerate and symmetric, respectively, and thus the firm is accurately valued (in accordance with Lemma 1). In contrast, when $q(\kappa) \in (0, 1/2)$, the distribution is positively skewed, leading to overvaluation. Thus, the optimal κ sets $q(\kappa) \in (0, 1/2)$. Note further that the optimal κ exceeds the value of κ that maximizes overvaluation conditional on bad news, $\bar{\kappa}$; otherwise, an increase to $\bar{\kappa}$ would both increase the frequency and the magnitude of overvaluation.¹⁵ Above $\bar{\kappa}$, the optimal level of κ trades off the frequency and magnitude of overvaluation.

We next show that the optimal degree of imprecision κ decreases in the the prior q_0 . To understand this inverse relation, recall that under the optimal policy, independent of the prior, bad news leaves investors with the same posterior belief: $\Pr(\tilde{v} = v_H | \tilde{s} = s_B) = \hat{q}$. Now, as the prior q_0 increases, fixing the disclosure policy, the posterior $\Pr(\tilde{v} = v_H | \tilde{s} = s_B)$ also increases. Thus, as q_0 increases, in order to ensure that the posterior given negative news remains at \hat{q} , the optimal disclosure policy provides negative news that is more informative by misclassifying the high payoff less frequently. The general intuition underlying this result is that, when a firm's fundamentals are less positively skewed (which corresponds to a greater q_0 in the binary model), then to induce overvaluation, it must create more skewness through its disclosure policy.

¹⁵Note that κ is, in fact, strictly greater than $\bar{\kappa}$, as a marginal increase in κ from $\bar{\kappa}$ has no effect on overvaluation, but has a linear effect on the probability of overvaluation.

Corollary 2. *Suppose that the firm’s payoff is binary. The optimal degree of imprecision κ is decreasing in the prior probability that the payoff is high, q_0 .*

Before concluding this section we note that the optimal level of imprecision in the disclosure also depends on the amount of noise trading, as captured by σ_z . At the same time, imprecision affects the sensitivity of prices with respect to z , and thus influences liquidity. We leave it to future research to explore this interaction.

5 Recognition and Truncation Policies

We next turn our attention to the case in which the firm is restricted in the disclosure policies it can choose among. Specifically, we examine two types of policies: recognition policies and the subset of these policies whereby the firm withholds its value whenever it lies either above or below a threshold, which we refer to as truncation policies. While we previously established that recognition, and hence truncation policies, are not optimal among the broad class of all disclosure policies, these two types of policies are nevertheless of interest for two reasons. First, several accounting rules resemble recognition policies in that they withhold news that does not pass a criterion, such as lowering the firm’s earnings or exceeding a materiality threshold. Threshold policies in particular resemble accounting standards that prescribe asset impairments when information is unfavorable, but do not allow for positive revaluations when information is favorable.¹⁶ Thus, these policies may be more feasible for a firm to implement than a generic disclosure policy. Second, such policies are frequently optimal in other persuasion settings and arise endogenously in models of voluntary disclosure. Hence, analyzing these policies offers insight into how disclosure influences prices in the presence of noise trade in these settings.

We demonstrate three results regarding these policies. First, truncation policies are sufficient to enable a firm to create overvaluation, as they typically lead to asymmetric

¹⁶These rules, which reflect “conservative” accounting standards, have been studied extensively in prior literature (e.g., Bertomeu and Cheynel (2015), Göx and Wagenhofer (2010), and Friedman et al. (2019)).

posteriors. Second, more general recognition policies can increase the firm's expected price relative to truncation policies by generating additional skewness in investors' beliefs. Finally, a recognition policy in which the firm withholds on two disjoint intervals maximizes the expected third moment in investors' posteriors and thus approximately maximizes the firm's expected price.

We begin by analyzing truncation policies. Note a recognition policy that deterministically either withholds or discloses each realization (i.e., $\omega(v) \in \{0, 1\}$) can be equivalently represented by the set of realizations that are withheld, $\Omega \equiv \{v \in \mathcal{V} : \omega(v) = 1\}$. Given any threshold k , we define an *upper truncation policy* as one for which $\Omega = [k, \infty)$ and a *lower truncation policy* as one for which $\Omega = (-\infty, k]$. When studying these policies, we further assume that the prior distribution over the firm's payoff is normal, $\tilde{v} \sim N(m_v, \sigma_v^2)$. In addition to offering tractability, the normal distribution has a desirable feature for studying truncations: it is single peaked. Not only is this feature realistic, but it also implies that, when the lower (upper) truncated normal is negatively (positively) skewed. This appears to be a general feature of single-peaked distributions that we are able to capture by employing the normal prior.

As an intermediate result, we compute the ex ante expected price that arises under truncation rules. In stating the following lemma, we let $\phi(\cdot)$ and $\Phi(\cdot)$ denote the p.d.f. and c.d.f. of a standard normal, respectively.

Lemma 4. *Suppose that the firm's payoff is normally distributed and let $\lambda(x) \equiv \frac{\phi(x)}{1-\Phi(x)}$.*

(i) **Lower truncation.** *If the withholding set is $\Omega = (-\infty, k]$, then the ex ante expected price is given by:*

$$\mathbb{E}[P(\tilde{z}, \tilde{\mu})] = m_v - \Phi\left(\frac{k - m_v}{\sigma_v}\right) \sigma_v \left\{ \mathbb{E}\left[\lambda\left(\frac{k - m_v}{\sigma_v} + \sigma_v \rho \tilde{z}\right)\right] - \lambda\left(\frac{k - m_v}{\sigma_v}\right) \right\}.$$

(ii) **Upper truncation.** *If, instead, the withholding set is $\Omega = [k, \infty)$, then the ex ante expected price is given by:*

$$\mathbb{E}[P(\tilde{z}, \tilde{\mu})] = m_v + \Phi\left(-\frac{k - m_v}{\sigma_v}\right) \sigma_v \left\{ \mathbb{E}\left[\lambda\left(\frac{k - m_v}{\sigma_v} + \sigma_v \rho \tilde{z}\right)\right] - \lambda\left(\frac{k - m_v}{\sigma_v}\right) \right\}.$$

Observe that under either truncation rule, the ex ante expected price deviates from m_v by a factor proportional to:

$$\sigma_v \left\{ \mathbb{E} \left[\lambda \left(\frac{k - m_v}{\sigma_v} + \sigma_v \rho \tilde{z} \right) \right] - \lambda \left(\frac{k - m_v}{\sigma_v} \right) \right\} > 0. \quad (9)$$

This term embodies the pricing implications of a posterior that has a non-zero third central moment: for any finite k , (9) is positive as $\lambda(\cdot)$ is convex. This effect on the ex ante price arises only when the firm's value is withheld, and is thus multiplied by the probability of withholding under the threshold policy. Whether expression (9) enters the pricing equation with a positive or negative sign depends on whether the truncation is upper or lower. Under a lower truncation, the impact on the ex ante price is negative because, conditional on withholding, the posterior is negatively skewed. Conversely, under an upper truncation, the impact is positive because, conditional on withholding, the posterior is positively skewed. It thus follows that a lower truncation leads to undervaluation, while an upper truncation always leads to overvaluation. The next proposition formalizes this result and is illustrated in Figure 2.

Proposition 4. *Suppose that the firm's value is normally distributed. Then,*

- (i) *Any lower truncation policy leads to an ex ante price that is strictly lower than the ex ante expectation of the firm's payoff, m_v .*
- (ii) *Any upper truncation policy leads to an ex ante price that is strictly higher than m_v .*

Proposition 4 implies that disclosure can lead to overvaluation even when a firm is restricted to simple truncation policies, because such truncation policies are sufficient to generate positive skewness. This result demonstrates that our main finding – that firms can generate overvaluation via disclosure – continues to hold even when firms are significantly restricted in their commitment power. Note that, similar to the binary model, the optimal disclosure policy (here, the threshold k) strikes a balance between the probability of creating overvaluation, as captured by $1 - \Phi \left(\frac{k - m_v}{\sigma_v} \right)$, and the degree of overvaluation, as captured

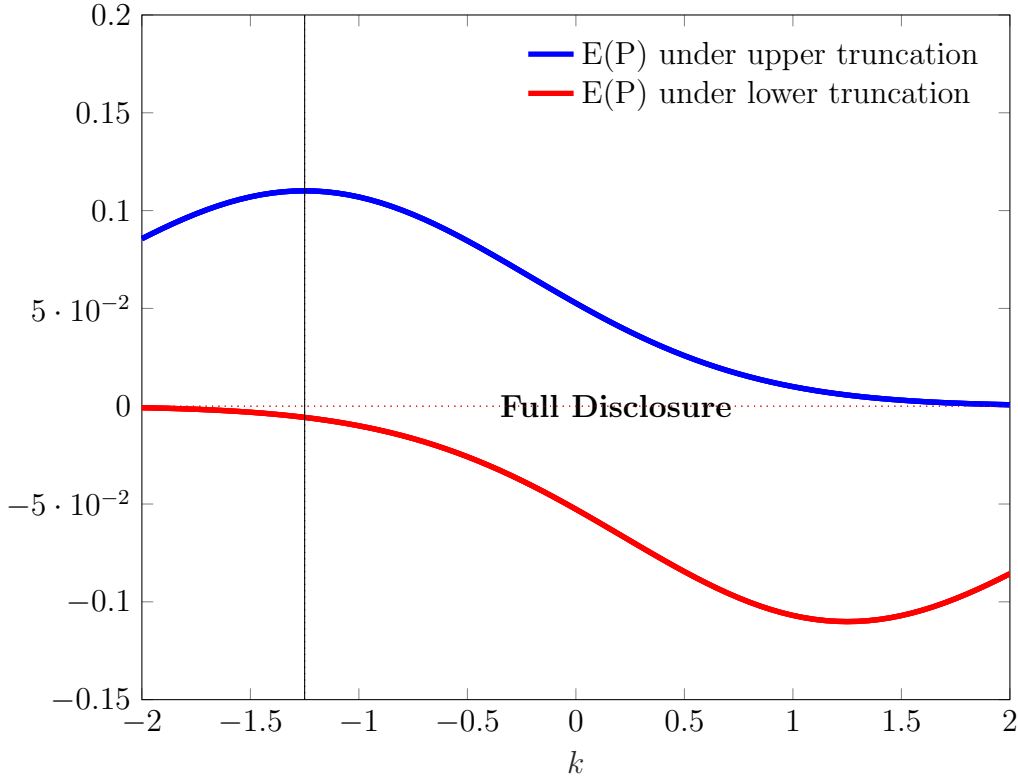


Figure 2: Truncation policies and expected prices. Parameters: $\mu_v = 0, m_z = 0, \sigma_z = 1, \sigma_v = 1, \rho = 1$

by (9). Decreasing the threshold k increases the probability of skewed posteriors, but, in a neighborhood of the optimum, also lowers the degree of skewness.

In general, truncation rules are not optimal among the class of recognition policies. This is true even in the case of a normal prior, in which truncation policies can generate significant skewness.¹⁷ However, it is most apparent when considering relatively flat distributions such as the uniform. In the uniform case, any truncation policy leads to a symmetric posterior, which, in accordance with Lemma 1, induces accurate valuation. In contrast, following the logic underlying Proposition 1, there exist recognition policies that generate positive skewness and thus overvaluation.

To further illustrate the dominance of recognition policies over truncation policies, we conclude by deriving the policy that maximizes the approximate price in equation (5). This

¹⁷We have found numerically that under a normal prior, there exists a recognition policy characterized by a withholding set $\Omega = [a, b] \cup [k, \infty)$ (as in Proposition 5) that leads to a greater expected price than the optimal upper truncation policy.

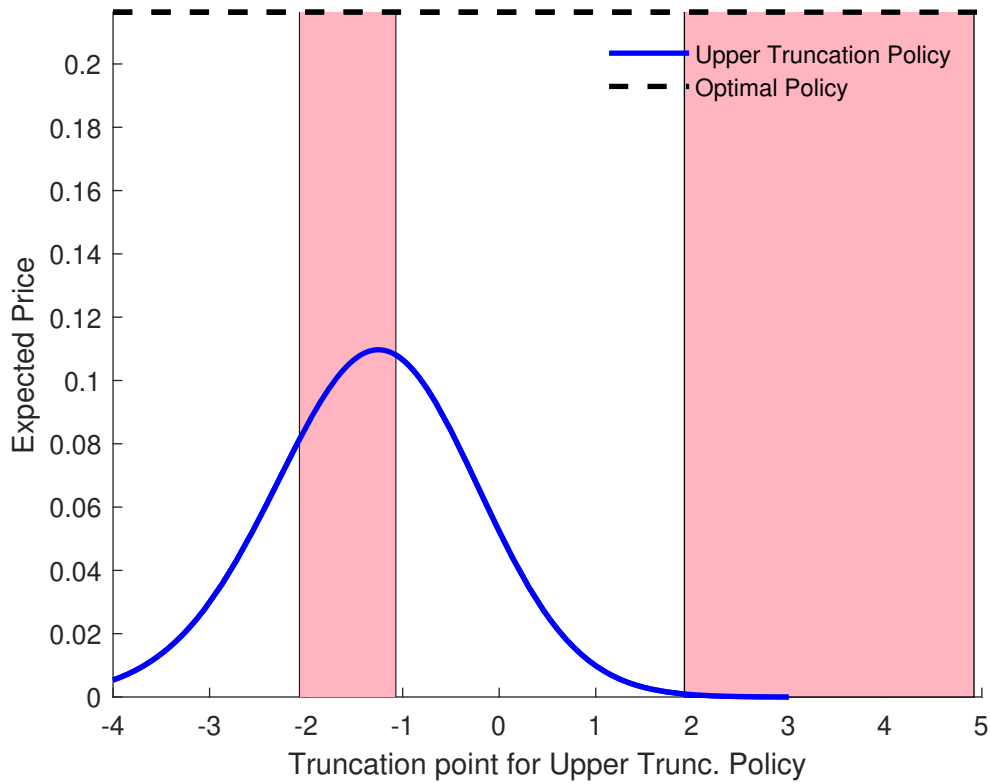


Figure 3: Optimal policy vs upper truncation policy. The blue curve is the ex ante expected price as a function of the threshold. The pink area represents the non-disclosure region under the optimal recognition policy (using the approximate price). The black dashed line is the value of the ex ante expected price evaluated at the optimal recognition policy. Parameters: $\sigma_x = 1, \sigma_z = 1$.

analysis is instructive because the expectation of this approximate price is exactly equal to the expected firm value adjusted by the expected third central moment in investors' posteriors. Thus, the policy that maximizes this approximate price is precisely the one that maximizes the expected third central moment in investors' posteriors, $E[E_\mu[(\tilde{v} - E_\mu[\tilde{v}])^3]]$. We find that, given any prior with unbounded support, a policy that withholds realizations above a threshold k and in a second interval $[a, b] < k$ maximizes this price, generating more positive skewness than any truncation policy. Intuitively, by picking appropriately the boundaries of these intervals, one can ensure that, given non-disclosure, most of the probability mass is in $[a, b]$ and, at the same time, that there is still a small probability of large realizations in $[k, \infty)$. This leaves investors with a posterior that is significantly more skewed than an upper truncation. Figure 3 illustrates these results in the case of the normal prior.

Proposition 5. *Suppose $\mathcal{V} = \mathbb{R}$. Within the class of recognition policies, the disclosure policy that maximizes the expected third central moment and thus the ex ante expectation of the approximate price in (5) withholds realizations of the payoff in a set of the form $\Omega = [a, b] \cup [k, \infty)$, where $a < b < k$.*

6 Extensions

6.1 Multi-firm economy

In this section, we demonstrate how the case studied in the main model, in which the firm is in zero net supply, is equivalent to studying an economy consisting of a large number of firms in which a specific firm chooses the nature of the information it discloses about its idiosyncratic value. This extension further shows how a firm can exhibit a positive risk premium even under the optimal disclosure policy.

We now assume that there are N firms. We will let N grow large to consider the “large economy limit,” but begin with a finite number of firms for demonstrative purposes. Firm i

produces cash flows \tilde{v}_i where:

$$\tilde{v}_i = \tilde{\alpha}_i + \tilde{f}.$$

The term $\tilde{f} \sim N(\mu_f, \sigma_f^2)$ represents market-wide cash flows. The terms $\tilde{\alpha}_i$ are independent of each other and of \tilde{f} , representing the firms' idiosyncratic cash flows. Each firm has a supply of $m_z = 1$.

We further posit now that there are J risk-averse investors in the model who possess CARA utility with risk aversion ρ . Furthermore, in each stock, there are J noise traders who sell $\tilde{z}_i \sim N(0, \sigma_z^2)$ shares of firm i . We assume that \tilde{z}_i are independent across firms and independent of $\{\tilde{\alpha}_i\}_{i \in \{1, \dots, N\}}$ and \tilde{f} . When analyzing the large economy limit, we will also let J approach infinity. Note that both the number of risk-averse traders and noise traders must move to infinity alongside N in the large economy limit. If the number of investors did not grow, there would be an infinite amount of per-capita risk in the limit, and if the number of noise traders did not grow, they would be washed out in the limit. Note it is also necessary to assume that the demands of the noise traders within each firm are correlated to ensure they do not wash away in the limit. For simplicity, we assume these demands are perfectly correlated, so that the aggregate demand from noise traders in each firm i is $-J\tilde{z}_i$.

We next establish the firms' limiting prices in equilibrium.

Proposition 6. *As $N, J \rightarrow \infty$ at the same rate, firm i 's expected price $E[\tilde{P}_i]$ satisfies:*

$$E[\tilde{P}_i] = E \left\{ \frac{E[\tilde{\alpha}_i \exp(-\rho \tilde{z}_i \tilde{\alpha}_i) | \tilde{z}_i]}{E[\exp(-\rho \tilde{z}_i \tilde{\alpha}_i) | \tilde{z}_i]} \right\} + \mu_f - \rho \sigma_f^2. \quad (10)$$

An important feature of the price in equation (10) is that the pricing of the firm's idiosyncratic and systematic risks are additively separable. This feature has three noteworthy implications. First, the pricing of the idiosyncratic component of the firm's cash flows, $\frac{E[\tilde{\alpha}_i \exp(-\rho \tilde{z}_i \tilde{\alpha}_i)]}{E[\exp(-\rho \tilde{z}_i \tilde{\alpha}_i)]}$, is identical to the pricing of \tilde{v}_i in the main model. Consequently, if we assume that firm i chooses an information system that does not provide information on the macro-factor \tilde{f} that is not available from other sources, i.e., that provides novel information

on $\tilde{\alpha}_i$ only, the optimization problem is identical to the one we have analyzed above. Second, as long as the risk premium created by the term $-\rho\sigma_f^2$ is sufficiently large, it need not be the case that the firm is literally overvalued. And third, unless investors' posterior about $\tilde{\alpha}_i$ is symmetric, the term in (10) corresponding to the pricing of the idiosyncratic component will be different from zero. This implies that disclosure about idiosyncratic risks can have an effect on the risk premium even in a large economy, provided that it leaves investors with an asymmetric posterior.

6.2 Disclosure about a systematic risk

To reiterate, our main analysis is founded upon the notion that most individual firms do not possess significant information regarding systematic risk that is incremental to other publicly available information. However, it is plausible that a limited number of large firms may have such information in their possession. To provide insight into the optimal disclosure policy of such firms, we now consider a variant of the binary model in which the firm's shares are in positive supply, i.e., $m_z > 0$, which implies that the firm's disclosure provides information to investors that enables them to update on terminal aggregate consumption. In this case, a second-order approximation of the average price analogous to expression (5) now yields:

$$E[P(\tilde{z}, \mu)] \approx E_\mu[\tilde{v}] - \underbrace{\text{Var}_\mu[\tilde{v}]\rho m_z}_{\text{variance effect}} + \underbrace{(\text{Var}_\mu[\tilde{v}])^{\frac{3}{2}} \text{Skew}_\mu[\tilde{v}] \frac{\rho^2 \sigma_z^2}{2}}_{\text{third-moment effect}}.^{18} \quad (11)$$

Observe that the approximate price now includes an additional risk-premium term that is proportional to m_z and the conditional variance of the firm's value. This term arises because, in expectation, when investors hold a positive amount of the stock, they hold undiversifiable risk. Thus, the firm faces an additional trade-off when determining the optimal disclosure policy: additional information tends to lower this risk premium by reducing the conditional variance. While the firm could completely eliminate this risk premium by perfectly revealing

¹⁸See Appendix B for the derivation of this expression.

the firm's payoff, this would render them unable to generate positive skewness, which increases the firm's price via the third term of (11). In general, the presence of positive supply pushes the firm towards the provision of additional information, and, in certain cases, may lead perfect disclosure to be optimal.

Equation (11) further illustrates that the trade-off between the variance and third-moment effects depend upon the relative magnitudes of the average supply m_z and the extent of noise trade σ_z^2 . Intuitively, the average supply controls the magnitude of the risk premium, while the extent of noise trade determines the degree to which skewness inflates the firm's price. The importance of the relative magnitudes of m_z and σ_z^2 in determining the optimal policy is particularly transparent in the binary model introduced in Section 4. In this model, when the average supply m_z grows large relative to the variance of noise trading σ_z^2 , rather than the imprecise disclosure policy discussed in Proposition 3, perfect disclosure is optimal. The proposition below states formally this result.

Proposition 7. *Suppose that the firm's payoff is binary. There exists a $c > 0$ such that:*

- (i) *perfect disclosure is optimal if and only if $\sigma_z^2 \leq cm_z$; whereas*
- (ii) *if $\sigma_z^2 > cm_z$, then the optimal disclosure policy is characterized as in Proposition 3.*

This proposition suggests that large firms traded primarily by sophisticated investors should be inclined towards providing more information, and firms in general should be inclined towards providing more information regarding the systematic components of their performance.

6.3 Private information

Our baseline model assumes that the firm's disclosure is the only source of information that investors possess, and that trade among investors is driven entirely by the presence of noise traders. In contrast, a large literature in finance studies the impact of private information on trade and prices, as well as its interaction with disclosure (e.g., [Diamond \(1985\)](#)). In

Appendix E, we explore the robustness of our main results to the presence of trade on private information. To do so, we apply the techniques of Breon-Drish (2015), which enable characterizing a rational expectations equilibrium under non-normal distributions. Moreover, as in Section 4, we assume that $\tilde{v} \in \{m_v - \sigma_v, m_v + \sigma_v\}$. In line with Hellwig (1980), in addition to the public disclosure, each risk-averse investor i privately observes a noisy signal about the firm's payoff, $\tilde{y}_i = \tilde{v} + \tilde{\varepsilon}_i$. Letting the prior be denoted $q = \Pr(\tilde{v} = m_v + \sigma_v)$, we show that the price function in this setting equals:

$$P(\tilde{v}, \tilde{z}) = m_v + \sigma_v \frac{\frac{q}{1-q} \exp\left(\frac{2\sigma_v}{\sigma_\varepsilon^2} \left[\left(1 + \frac{1}{\rho^2 \sigma_\varepsilon^2 \sigma_z^2}\right) (\tilde{v} - \rho \sigma_\varepsilon^2 \tilde{z})\right]\right) - 1}{\frac{q}{1-q} \exp\left(\frac{2\sigma_v}{\sigma_\varepsilon^2} \left[\left(1 + \frac{1}{\rho^2 \sigma_\varepsilon^2 \sigma_z^2}\right) (\tilde{v} - \rho \sigma_\varepsilon^2 \tilde{z})\right]\right) + 1}. \quad (12)$$

Notice that the price directly depends on the fundamental \tilde{v} , as the market aggregates investors' noisy private signals. In this setting, our numerical analyses, depicted in Figure 4, indicate that expected price exhibits similar features as a function of the prior as in our baseline case: this price exceeds (falls short of) the firm's expected cash flows when the prior is positively skewed $q < 1/2$ (negatively skewed $q > 1/2$). As a result, the firm's optimal policy is to induce a posterior that is positively skewed through imprecise disclosure of negative news.

The presence of private information, however, does affect the quantitative nature of our results. Figure 5 depicts the optimal imprecision of the disclosure policy as a function of the noise in investors' signals. As can be gleaned from the plot, the optimal degree of imprecision increases as investors' private information becomes more noisy. This occurs because if investors are already well informed about the firm's payoff, then there is little role left for persuasion. By contrast, the firm's ability to manipulate investors' belief through disclosure is enhanced when the quality of investors' information drops.

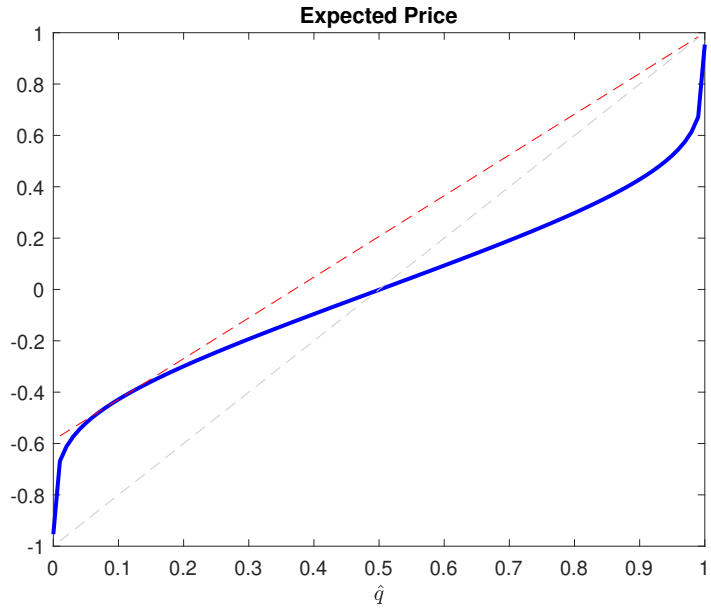


Figure 4: Information design in a noisy rational expectations equilibrium. Parameters: $\sigma_v = 1, \rho = 2, \sigma_z = 1.5, \sigma_\varepsilon = 1$

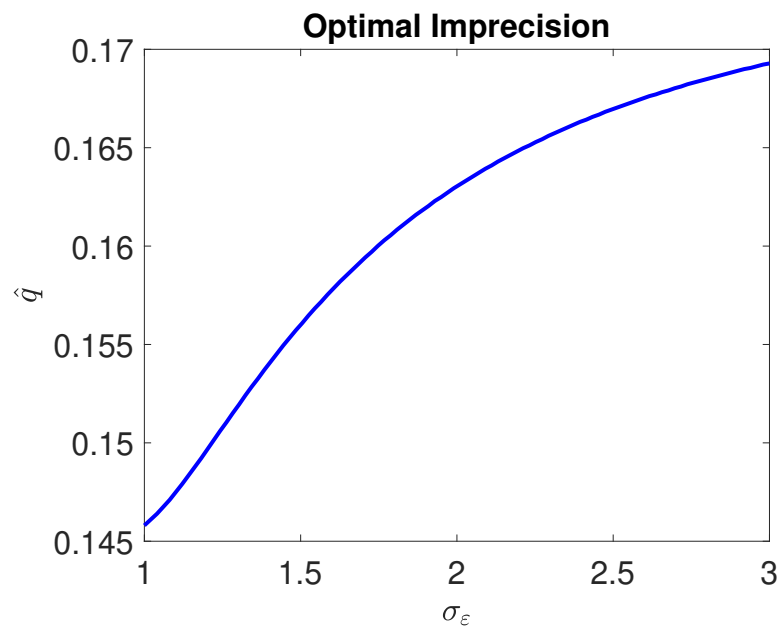


Figure 5: The effect of private information (σ_ε) on optimal imprecision. Parameters: $\sigma_v = 1, \rho = 1, \sigma_z = 1$.

7 Conclusion

We study a firm's optimal choice of disclosure policy in a competitive market with noise traders. Relative to the vast prior literature that studies disclosure in models of competitive markets, our innovation is to generalize the firm's choice set by allowing them to commit to any disclosure policy. We find that, upon endowing the firm with this flexibility, their optimal policy does not resemble the policies studied in the prior literature. Specifically, this policy is imprecise and induces positively-skewed market beliefs regarding the firm's value. This policy, in turn, causes the firm's price to exceed its expected value – independent of investors' prior beliefs over the firm's value. These findings stand in contrast to the majority of prior work on disclosure and pricing, which typically finds that the provision of additional information enhances price efficiency. Our results also indicate that, in the presence of noise trade, even when disclosure concerns a firm's idiosyncratic payoffs, it can influence the firm's cost of capital. Taken together, our findings suggest that future work studying on disclosure's effect on prices in models with noise trade may benefit from considering a broader range of disclosure policies.

Appendix

A Proofs of Section 3

Proof of Lemma 1. Evaluate the pricing equation (3) at the noise-trader demand $\tilde{z} = -z$,

$$P(-z, \mu) = \frac{\mathbb{E}_\mu[\tilde{v}u'(-z(\tilde{v} - P(-z, \mu)))]}{\mathbb{E}_\mu[u'(-z(\tilde{v} - P(-z, \mu)))]}.$$

Substitute $-z(\tilde{v} - P(-z, \mu)) = z(2\mathbb{E}_\mu[\tilde{v}] - \tilde{v} - (2\mathbb{E}_\mu[\tilde{v}] - P(-z, \mu)))$ inside the argument of u' to get

$$\begin{aligned} P(-z, \mu) &= \frac{\mathbb{E}_\mu[\tilde{v}u'(z(2\mathbb{E}_\mu[\tilde{v}] - \tilde{v} - (2\mathbb{E}_\mu[\tilde{v}] - P(-z, \mu)))))]}{\mathbb{E}_\mu[u'(z(2\mathbb{E}_\mu[\tilde{v}] - \tilde{v} - (2\mathbb{E}_\mu[\tilde{v}] - P(-z, \mu)))))]} \\ \iff 2\mathbb{E}_\mu[\tilde{v}] - P(-z, \mu) &= \frac{\mathbb{E}_\mu[(2\mathbb{E}_\mu[\tilde{v}] - \tilde{v})u'(z(2\mathbb{E}_\mu[\tilde{v}] - \tilde{v} - (2\mathbb{E}_\mu[\tilde{v}] - P(-z, \mu)))))]}{\mathbb{E}_\mu[u'(z(2\mathbb{E}_\mu[\tilde{v}] - \tilde{v} - (2\mathbb{E}_\mu[\tilde{v}] - P(-z, \mu)))))]}, \end{aligned} \quad (\text{A.1})$$

where we have obtained the second line by multiplying each side by -1 and adding $2\mathbb{E}_\mu[\tilde{v}]$ to each side. Note that (A.1) is the pricing equation for the mirror random variable $2\mathbb{E}_\mu[\tilde{v}] - \tilde{v}$. Therefore, letting μ^m denote the distribution this mirror random variable, we have that

$$P(z, \mu^m) = 2\mathbb{E}_\mu[\tilde{v}] - P(-z, \mu) \quad (\text{A.2})$$

is a market-clearing price when the distribution is μ^m and noise traders sell z units.

Then, taking expectations with respect to the noise-trader demand on both sides of (A.2) yields

$$\begin{aligned} \mathbb{E}[P(\tilde{z}, \mu^m)] &= 2\mathbb{E}_\mu[\tilde{v}] - \mathbb{E}[P(-\tilde{z}, \mu)] \\ \iff \mathbb{E}[P(\tilde{z}, \mu^m)] &= 2\mathbb{E}_\mu[\tilde{v}] - \mathbb{E}[P(\tilde{z}, \mu)] \\ \iff \mathbb{E}[P(\tilde{z}, \mu^m)] + \mathbb{E}[P(\tilde{z}, \mu)] &= 2\mathbb{E}_\mu[\tilde{v}], \end{aligned}$$

where on the right-hand side of the second line we have substituted $-\tilde{z}$ with \tilde{z} (because they are identically distributed) and in the third line we have moved $\mathbb{E}[P(\tilde{z}, \mu)]$ from the right-hand side to the left-hand side. Evaluating the third line for symmetric distributions (i.e., such that $\mu = \mu^m$) yields the desired result that $\mathbb{E}[P(\tilde{z}, \mu)] = \mathbb{E}_\mu[\tilde{v}]$. ■

Proof of Lemma 2. Because $|\mathcal{V}| \geq 2$, we can find $v_L, v_H \in \mathcal{V}$ such that $v_L < v_H$. We construct a binary distribution with support $\hat{\mathcal{V}} \equiv \{v_L, v_H\}$. With a slight abuse of notation, we identify the distribution μ with the probability of the high payoff, that is, we write $\mu \equiv \Pr[\tilde{v} = v_H]$.

At $\mu = 0$ the distribution is degenerate, and hence symmetric. Therefore, by Lemma 1, we have that $E[P(\tilde{z}, \mu)]|_{\mu=0} = E_\mu[\tilde{v}]|_{\mu=0} = v_L$. To show that there exists a $\mu \in (0, 1)$ that induces overvaluation, we increase μ at the margin in a neighborhood of $\mu = 0$.

Fix $z > 0$. For this specific case, the market-clearing price is implicitly defined by

$$(v_H - P) \mu u'(-z(v_H - P)) + (v_L - P) (1 - \mu) u'(-z(v_L - P)) = 0,$$

Substituting $\phi \equiv P - E_\mu[\tilde{v}] = P - v_L - \mu(v_H - v_L)$ into the previous equation, we get

$$\Gamma(\mu, \phi) \equiv ((1 - \mu)k - \phi) \mu u'(z(-(1 - \mu)k + \phi)) - (\mu k + \phi) (1 - \mu) u'(z(\mu k + \phi)),$$

where $k \equiv v_H - v_L$. In this way, we can directly solve for ϕ .

By the implicit function theorem,

$$\frac{\partial \phi}{\partial \mu} = -\frac{\Gamma_\mu(\mu, \phi)}{\Gamma_\phi(\mu, \phi)},$$

where

$$\begin{aligned} \Gamma_\phi(\mu, \phi) &= -[\mu u'(z(-(1 - \mu)k + \phi)) + (1 - \mu) u'(z(\mu k + \phi))] \\ &\quad + z[((1 - \mu)k - \phi) \mu u''(z(-(1 - \mu)k + \phi)) - (\mu k + \phi) (1 - \mu) u''(z(\mu k + \phi))], \\ \Gamma_\phi(0, \phi) &= -u'(z\phi) - z\phi u''(z\phi), \end{aligned}$$

and

$$\begin{aligned} \Gamma_\mu(\mu, \phi) &= -k\mu u'(z(-(1 - \mu)k + \phi)) + ((1 - \mu)k - \phi) u'(z(-(1 - \mu)k + \phi)) \\ &\quad + zk((1 - \mu)k - \phi) \mu u''(z(-(1 - \mu)k + \phi)) \\ &\quad - k(1 - \mu) u'(z(\mu k + \phi)) + (\mu k + \phi) u'(z(\mu k + \phi)) - zk(\mu k + \phi) (1 - \mu) u''(z(\mu k + \phi)), \\ \Gamma_\mu(0, \phi) &= (k - \phi) u'(z(-k + \phi)) - ku'(z\phi) + \phi u'(z\phi) - zk\phi u''(z\phi). \end{aligned}$$

Therefore,

$$\frac{\partial \phi}{\partial \mu} \Big|_{\mu=0} = -\frac{\Gamma_\mu(0, \phi)}{\Gamma_\phi(0, \phi)} = \frac{(k - \phi) u'(z(-k + \phi)) - ku'(z\phi) + \phi u'(z\phi) - zk\phi u''(z\phi)}{u'(z\phi) + z\phi u''(z\phi)}.$$

Because at $\mu = 0$ we have $\phi(z) = 0$ for all z , the expression above for the derivative simplifies to

$$\frac{\partial \phi(z)}{\partial \mu} \Big|_{\mu=0} = \frac{k}{u'(0)} [u'(-zk) - u'(0)].$$

The second derivative of $\frac{\partial\phi(z)}{\partial\mu}|_{\mu=0}$ with respect to z is

$$\partial^2 \left[\frac{\partial\phi(z)}{\partial\mu} \Big|_{\mu=0} \right] / \partial z^2 = \frac{k^3}{u'(0)} u'''(-zk) > 0.$$

Thus $\frac{\partial\phi(z)}{\partial\mu}|_{\mu=0}$ is strictly convex if $u''' > 0$. Then, by Jensen's inequality we have

$$\mathbb{E} \left[\frac{\partial\phi(\tilde{z})}{\partial\mu} \Big|_{\mu=0} \right] = \frac{k}{u'(0)} \mathbb{E}[u'(-\tilde{z}k) - u'(0)] > \frac{k}{u'(0)} \mathbb{E}[u'(0) - u'(0)] = 0.$$

The inequality above is strict because \tilde{z} is assumed to be non-degenerate. This shows that by increasing μ at the margin starting from $\mu = 0$ one generates overvaluation. Hence, for any binary support there exists a $\hat{\mu} \in (0, 1)$ that induces overvaluation. ■

Proof of Proposition 1. The claim is a special case of the proof of Proposition 2(i), which can be found below. Under the disclosure policy that always perfectly reveals the firm's payoff, the firm is accurately valued. Because the policy of perfect disclosure randomizes only over degenerate distributions, it assigns probability one to degenerate distributions. Therefore, as shown in the proof of Proposition 2, we can construct a disclosure policy such that the ex ante expected price is strictly greater than the ex ante expected value. ■

Proof of Proposition 2(i). Let τ^* be an optimal disclosure policy and $\mathcal{D} \equiv \{\delta_v : v \in \mathcal{V}\}$ be the set of all degenerate distributions δ_v on the support of the prior, \mathcal{V} . We want to show that, at the optimum, $\tau^*(\mathcal{D}) = 0$ or $\tau^*(\mathcal{D}) = \tau^*(\{\delta_{v^*}\}) > 0$ for a single v^* . To ease the notational burden, henceforth we simply write τ in place of τ^* for the optimal disclosure policy.

By contradiction, suppose that $\tau(\mathcal{D}) > 0$ but $\tau(\{\delta_v\}) < \tau(\mathcal{D})$ for all $\delta_v \in \mathcal{D}$. We show that there exists a disclosure policy $\hat{\tau}$, which we define below, that strictly increases the ex ante expected price. Let $\tau_{|\mathcal{D}} \in \Delta(\mathcal{D})$ denote the distribution of posterior distributions conditional on the posterior distribution being degenerate, that is,

$$\tau_{|\mathcal{D}}(S) \equiv \frac{\tau(S)}{\tau(\mathcal{D})}$$

for all measurable subsets S of \mathcal{D} . Because $\tau_{|\mathcal{D}}(\{\delta_v\}) < 1$ for all $\delta_v \in \mathcal{D}$, there must exist (at least) two distributions δ_{v_1} and δ_{v_2} in the support of $\tau_{|\mathcal{D}}(S)$. Without loss of generality,

let us use the convention that $v_1 < v_2$. Take small non-intersecting neighborhoods around each of these two points $\mathcal{N}_{j,\eta} \equiv \{\delta_v \in \mathcal{D} : |v - v_j| \leq \eta\}$, for $\eta > 0$ and $j = 1, 2$. Because δ_{v_1} and δ_{v_2} are in the support of $\tau_{|\mathcal{D}}(S)$, we have $\tau_{|\mathcal{D}}(\mathcal{N}_{i,\eta}) > 0$, and therefore $\tau(\mathcal{N}_{j,\eta}) > 0$, for $j = 1, 2$ and all $\eta > 0$.

At this point, define the disclosure policy $\hat{\tau}_\eta$ (parametrized by η) as follows: for any measurable subset S of $\Delta(\mathcal{V})$, let

$$\begin{aligned} \hat{\tau}_\eta(S) &= \tau(S \setminus [\mathcal{N}_{1,\eta} \cup \mathcal{N}_{2,\eta}]) + (1 - \varepsilon_{1,\eta}) \tau(S \cap \mathcal{N}_{1,\eta}) + (1 - \varepsilon_{2,\eta}) \tau(S \cap \mathcal{N}_{2,\eta}) \\ &\quad + \mathbb{I}[S \cap \{\hat{\mu}_\eta\} \neq \emptyset] \cdot [\varepsilon_{1,\eta} \tau(\mathcal{N}_{1,\eta}) + \varepsilon_{2,\eta} \tau(\mathcal{N}_{2,\eta})], \end{aligned}$$

where $\mathbb{I}[\cdot]$ is the indicator function, $\varepsilon_{1,\eta}, \varepsilon_{2,\eta} \in (0, 1)$ are chosen appropriately as will be described below and $\hat{\mu}_\eta$ is given by

$$\hat{\mu}_\eta \equiv \frac{\varepsilon_{1,\eta} \int_{\mathcal{N}_{1,\eta}} \mu d\tau(\mu) + \varepsilon_{2,\eta} \int_{\mathcal{N}_{2,\eta}} \mu d\tau(\mu)}{\varepsilon_{1,\eta} \tau(\mathcal{N}_{1,\eta}) + \varepsilon_{2,\eta} \tau(\mathcal{N}_{2,\eta})}.$$

In words, starting from the original disclosure policy τ , we are constructing another disclosure policy $\hat{\tau}_\eta$ such that realizations of the firm's payoff that previously were perfectly disclosed, now are pooled into a new signal realization $\tilde{s} = \hat{s}_\eta$. Specifically, whenever the realized payoff v is in $\mathcal{N}_{j,\eta}$, the firm randomizes between perfect disclosure $\tilde{s} = v$, with probability $1 - \varepsilon_{j,\eta}$, and sending the pooling message $\tilde{s} = \hat{s}_\eta$, with probability $\varepsilon_{j,\eta}$. Conditional on $\tilde{s} = \hat{s}_\eta$, investors' posterior belief is $\hat{\mu}_\eta$.

Note that the conditional distribution $\hat{\mu}_\eta$ is well-defined, because the conditioning probability is strictly positive: $\varepsilon_{1,\eta}, \varepsilon_{2,\eta} \in (0, 1)$ and $\tau(\mathcal{N}_{1,\eta}), \tau(\mathcal{N}_{2,\eta}) > 0$. Further, one verifies that the disclosure policy $\hat{\tau}_\eta$ satisfies the Bayesian plausibility constraint (1) as follows:

$$\begin{aligned} \int_{\Delta(\mathcal{V})} \mu d\hat{\tau}(\mu) &= \int_{\Delta(\mathcal{V}) \setminus [\mathcal{N}_{1,\eta} \cup \mathcal{N}_{2,\eta}]} \mu d\hat{\tau}(\mu) + \int_{\mathcal{N}_{1,\eta} \cup \mathcal{N}_{2,\eta}} \mu d\hat{\tau}(\mu) \\ &= \int_{\Delta(\mathcal{V}) \setminus [\mathcal{N}_{1,\eta} \cup \mathcal{N}_{2,\eta}]} \mu d\tau(\mu) + [\varepsilon_{1,\eta} \tau(\mathcal{N}_{1,\eta}) + \varepsilon_{2,\eta} \tau(\mathcal{N}_{2,\eta})] \hat{\mu}_\eta \\ &\quad + (1 - \varepsilon_{1,\eta}) \int_{\mathcal{N}_{1,\eta}} \mu d\tau(\mu) + (1 - \varepsilon_{2,\eta}) \int_{\mathcal{N}_{2,\eta}} \mu d\tau(\mu) \\ &= \int_{\Delta(\mathcal{V})} \mu d\tau = \mu_0, \end{aligned}$$

where the first equality follows from the definition of $\hat{\tau}$, the second equality from the definition of $\hat{\mu}_\eta$, and the third equality from the fact that the original disclosure policy τ was Bayes plausible.

There remains to pick suitable values for $\varepsilon_{1,\eta}$ and $\varepsilon_{2,\eta}$. We have shown in the proof of Lemma 2 that for any binary support, there exists a distribution that induces overvaluation. Here, we take the support $\{v_1, v_2\}$ and let $\hat{q} \equiv \Pr[\tilde{v} = v_2]$ describe a distribution that induces overvaluation. For any $\eta > 0$, choose $\varepsilon_{1,\eta}, \varepsilon_{2,\eta} \in (0, 1)$ such that

$$\frac{\varepsilon_{2,\eta} \tau(\mathcal{N}_{2,\eta})}{\varepsilon_{1,\eta} \tau(\mathcal{N}_{1,\eta}) + \varepsilon_{2,\eta} \tau(\mathcal{N}_{2,\eta})} = \hat{q}.^{19}$$

With this choice of $\varepsilon_{1,\eta}$ and $\varepsilon_{2,\eta}$, the price given that \tilde{v} follows the distribution $\hat{\mu}_\eta$ solves

$$P(z, \hat{\mu}_\eta) = \frac{\left\{ \begin{array}{l} (1 - \hat{q}) \mathbb{E}[\tilde{v} u'(-z(\tilde{v} - P(z, \hat{\mu}_\eta))) | \tilde{v} \in [v_1 - \eta, v_1 + \eta]] \\ + \hat{q} \mathbb{E}[\tilde{v} u'(-z(\tilde{v} - P(z, \hat{\mu}_\eta))) | \tilde{v} \in [v_2 - \eta, v_2 + \eta]] \end{array} \right\}}{\left\{ \begin{array}{l} (1 - \hat{q}) \mathbb{E}[u'(-z(\tilde{v} - P(z, \hat{\mu}_\eta))) | \tilde{v} \in [v_1 - \eta, v_1 + \eta]] \\ + \hat{q} \mathbb{E}[u'(-z(\tilde{v} - P(z, \hat{\mu}_\eta))) | \tilde{v} \in [v_2 - \eta, v_2 + \eta]] \end{array} \right\}},$$

whereas the expectation given $\hat{\mu}_\eta$ is

$$\mathbb{E}_{\hat{\mu}_\eta}[\tilde{v}] = (1 - \hat{q}) \mathbb{E}_{\hat{\mu}_\eta}[\tilde{v} | \tilde{v} \in [v_1 - \eta, v_1 + \eta]] + \hat{q} \mathbb{E}_{\hat{\mu}_\eta}[\tilde{v} | \tilde{v} \in [v_2 - \eta, v_2 + \eta]].$$

Letting $P(z, \hat{\mu}_0) \equiv \lim_{\eta \downarrow 0} P(z, \hat{\mu}_\eta)$ and $\mathbb{E}_{\hat{\mu}_0}[\tilde{v}] \equiv \lim_{\eta \downarrow 0} \mathbb{E}_{\hat{\mu}_\eta}[\tilde{v}]$, by the squeeze theorem we have

$$\begin{aligned} P(z, \hat{\mu}_0) &= \frac{(1 - \hat{q})v_1 u'(-z(v_1 - P(z, \hat{\mu}_0))) + \hat{q}v_2 u'(-z(v_2 - P(z, \hat{\mu}_0)))}{(1 - \hat{q})u'(-z(v_1 - P(z, \hat{\mu}_0))) + \hat{q}u'(-z(v_2 - P(z, \hat{\mu}_0)))}, \\ \mathbb{E}_{\hat{\mu}_0}[\tilde{v}] &= (1 - \hat{q})v_1 + \hat{q}v_2. \end{aligned}$$

Recall that distribution \hat{q} leads to overvaluation, that is, $\mathbb{E}[P(\tilde{z}, \hat{\mu}_0)] > \mathbb{E}_{\hat{\mu}_0}[\tilde{v}]$. Because this inequality is strict, there exists an $\eta' > 0$ sufficiently small such that $\mathbb{E}[P(\tilde{z}, \hat{\mu}_{\eta'})] > \mathbb{E}_{\hat{\mu}_{\eta'}}[\tilde{v}]$.

Let us now compute the ex ante expected price under τ and $\hat{\tau}_{\eta'}$. Under τ the ex ante expected price is

$$\begin{aligned} \int_{\Delta(\mathcal{V})} \mathbb{E}[P(\tilde{z}, \mu)] d\tau(\mu) &= \int_{\Delta(\mathcal{V}) \setminus [\mathcal{N}_{1,\eta'} \cup \mathcal{N}_{2,\eta'}]} \mathbb{E}[P(\tilde{z}, \mu)] d\tau(\mu) + \int_{\mathcal{N}_{1,\eta'} \cup \mathcal{N}_{2,\eta'}} \mathbb{E}[P(\tilde{z}, \mu)] d\tau(\mu) \\ &= \int_{\Delta(\mathcal{V}) \setminus [\mathcal{N}_{1,\eta'} \cup \mathcal{N}_{2,\eta'}]} \mathbb{E}[P(\tilde{z}, \mu)] d\tau(\mu) + \int_{\mathcal{N}_{1,\eta'} \cup \mathcal{N}_{2,\eta'}} \mathbb{E}_\mu[\tilde{v}] d\tau(\mu) \quad (\text{A.3}) \end{aligned}$$

where the second line uses the fact that $\mathbb{E}[P(\tilde{z}, \mu)] = \mathbb{E}_\mu[\tilde{v}]$ for all degenerate μ . Similarly,

¹⁹A solution $\varepsilon_{1,\eta}, \varepsilon_{2,\eta} \in (0, 1)$ exists, because at $\varepsilon_{2,\eta} = 0$ the left-hand side is $0 < h$, whereas at $\varepsilon_{2,\eta} = 1, \varepsilon_{1,\eta} = 0$ the left-hand side is $1 > h$.

under $\hat{\tau}_{\eta'}$ the ex ante expected price is

$$\begin{aligned}
& \int_{\Delta(\mathcal{V})} \mathbb{E}[P(\tilde{z}, \mu)] d\hat{\tau}_{\eta'}(\mu) \\
= & \int_{\Delta(\mathcal{V}) \setminus [\mathcal{N}_{1,\eta'} \cup \mathcal{N}_{2,\eta'}]} \mathbb{E}[P(\tilde{z}, \mu)] d\tau(\mu) + [\varepsilon_{1,\eta'} \tau(\mathcal{N}_{1,\eta'}) + \varepsilon_{2,\eta'} \tau(\mathcal{N}_{2,\eta'})] \mathbb{E}[P(\tilde{z}, \hat{\mu}_{\eta'})] \\
& + (1 - \varepsilon_{1,\eta'}) \int_{\mathcal{N}_{1,\eta'}} \mathbb{E}_{\mu}[\tilde{v}] d\tau(\mu) + (1 - \varepsilon_{2,\eta'}) \int_{\mathcal{N}_{2,\eta'}} \mathbb{E}_{\mu}[\tilde{v}] d\tau(\mu) \\
= & \int_{\Delta(\mathcal{V}) \setminus [\mathcal{N}_{1,\eta'} \cup \mathcal{N}_{2,\eta'}]} \mathbb{E}[P(\tilde{z}, \mu)] d\tau(\mu) + \int_{\mathcal{N}_{1,\eta'} \cup \mathcal{N}_{2,\eta'}} \mathbb{E}_{\mu}[\tilde{v}] d\tau(\mu) \\
& + [\varepsilon_{1,\eta'} \tau(\mathcal{N}_{1,\eta'}) + \varepsilon_{2,\eta'} \tau(\mathcal{N}_{2,\eta'})] \left\{ \mathbb{E}[P(\tilde{z}, \hat{\mu}_{\eta'})] - \mathbb{E}_{\hat{\mu}_{\eta'}}[\tilde{v}] \right\}, \tag{A.4}
\end{aligned}$$

Taking the difference between (A.4) and (A.3) yields

$$\begin{aligned}
& \int_{\Delta(\mathcal{V})} \mathbb{E}[P(\tilde{z}, \mu)] d\hat{\tau}_{\eta'}(\mu) - \int_{\Delta(\mathcal{V})} \mathbb{E}[P(\tilde{z}, \mu)] d\tau(\mu) \\
= & [\varepsilon_{1,\eta'} \tau(\mathcal{N}_{1,\eta'}) + \varepsilon_{2,\eta'} \tau(\mathcal{N}_{2,\eta'})] \left\{ \mathbb{E}[P(\tilde{z}, \hat{\mu}_{\eta'})] - \mathbb{E}_{\hat{\mu}_{\eta'}}[\tilde{v}] \right\} > 0,
\end{aligned}$$

which is strictly positive because it is the product of two strictly positive terms.²⁰ Therefore, the disclosure policy $\hat{\tau}$ yields an ex ante expected price that is strictly greater than τ , contradicting optimality of τ . ■

Proof of Proposition 2(ii). Let $\mathcal{S} \subset \Delta(\mathcal{V}) \setminus \mathcal{D}$ be the set of all symmetric non-degenerate distributions on the support of the prior, \mathcal{V} . We want to show that, at the optimum, $\tau^*(\mathcal{S}) = 0$.

Again, we simply write τ in place of τ^* . By contradiction, suppose that at the optimum $\tau(\mathcal{S}) > 0$. Let the function $T : \mathcal{V} \rightarrow \mathcal{D}$ be given by

$$T(v) = \delta_v$$

and, for all measurable subsets S of \mathcal{D} , let

$$(\mu T^{-1})(S) = \mu(T^{-1}(S)) = \mu(\{v : \delta_v \in S\}).$$

²⁰The proof would be simpler if we knew that $\tau(\delta_{v_1}), \tau(\delta_{v_2}) > 0$, because in that case we could just take these two distributions instead of intervals around them (i.e., we could set $\eta = 0$). However, in general δ_{v_1} and δ_{v_2} may not be mass points of τ , and taking intervals around them ensures that $\hat{\tau}_{\eta}$ improves upon τ with positive probability.

be the measure induced by μ on \mathcal{D} . In other words, we have performed a change of variable $T(v)$ from the realization of firm value v to the distribution δ_v that is degenerate at that specific realization. (μT^{-1}) thus represents the same distribution as μ , but (μT^{-1}) projects it onto the space $\Delta(\mathcal{V})$ of all distributions on the support, whereas μ is a distribution defined on the support \mathcal{V} . Consider the alternative disclosure policy, which for all measurable subsets S of $\Delta(\mathcal{V})$ is defined as

$$\hat{\tau}(S) = \tau(S \setminus \mathcal{S}) + \int_{\mathcal{S}} (\mu T^{-1})(S \cap \mathcal{D}) d\tau(\mu).$$

In words, we construct the new disclosure policy $\hat{\tau}$ with the following logic: whenever the original policy τ realizes some symmetric distribution μ^s , the new policy τ draws a realization of the firm's payoff according to μ^s and discloses it perfectly. One sees that $\hat{\tau}(S)$ is Bayes plausible as follows,

$$\begin{aligned} \int_{\Delta(\mathcal{V})} \mu d\hat{\tau}(\mu) &= \int_{\Delta(\mathcal{V}) \setminus \mathcal{S}} \mu d\hat{\tau}(\mu) + \int_{\mathcal{S}} \mu d\hat{\tau}(\mu) \\ &= \int_{\Delta(\mathcal{V}) \setminus \mathcal{S}} \mu d\tau(\mu) + \int_{\mathcal{S}} \left[\int_{\mathcal{D}} \delta d(\mu T^{-1})(\delta) \right] d\tau(\mu) \\ &= \int_{\Delta(\mathcal{V}) \setminus \mathcal{S}} \mu d\tau(\mu) + \int_{\mathcal{S}} \left[\int_{\mathcal{V}} \delta_v d\mu(v) \right] d\tau(\mu) \\ &= \int_{\Delta(\mathcal{V})} \mu d\tau(\mu) = \mu_0, \end{aligned}$$

where: the second equality follows from the definition of $\hat{\tau}$ (namely, $\int_{\mathcal{S}} \mu d\hat{\tau}(\mu) = 0$ because, by construction, $\hat{\tau}(\mathcal{S}) = 0$); the third equality from the change of variable $v = T^{-1}(\delta_v)$ (e.g., see Theorem 12.46 in [Aliprantis and Border \(1999\)](#)); the fourth inequality by

$$\int_{\mathcal{V}} \delta_v d\mu(v) = \mu,$$

because

$$\left(\int_{\mathcal{V}} \delta_v d\mu(v) \right)(V) = \int_{\mathcal{V}} \mathbb{I}[v \in V] d\mu(v) = \mu(V)$$

for all measurable subsets V of \mathcal{V} ; and the last inequality by the fact that τ satisfied the Bayesian plausibility constraint (1).

The ex ante expected price is the same under τ and $\hat{\tau}$, because we are substituting symmetric distributions on the support of τ with degenerate distributions. Formally, the ex

ante expected price under τ is

$$\begin{aligned} \int_{\Delta(\mathcal{V})} \mathbb{E}[P(\tilde{z}, \mu)] d\tau(\mu) &= \int_{\Delta(\mathcal{V}) \setminus \mathcal{S}} \mathbb{E}[P(\tilde{z}, \mu)] d\tau(\mu) + \int_{\mathcal{S}} \mathbb{E}[P(\tilde{z}, \mu)] d\tau(\mu) \\ &= \int_{\Delta(\mathcal{V}) \setminus \mathcal{S}} \mathbb{E}[P(\tilde{z}, \mu)] d\tau(\mu) + \int_{\mathcal{S}} \mathbb{E}_{\mu}[\tilde{v}] d\tau(\mu), \end{aligned} \quad (\text{A.5})$$

where the second line uses the fact that all $\mu \in \mathcal{S}$ are symmetric distributions. Similarly, The ex ante expected price under $\hat{\tau}$ is

$$\begin{aligned} \int_{\Delta(\mathcal{V})} \mathbb{E}[P(\tilde{z}, \mu)] d\hat{\tau}(\mu) &= \int_{\Delta(\mathcal{V}) \setminus \mathcal{S}} \mathbb{E}[P(\tilde{z}, \mu)] d\hat{\tau}(\mu) + \int_{\mathcal{S}} \left[\int_{\mathcal{D}} \mathbb{E}[P(\tilde{z}, \delta)] d(\mu T^{-1})(\delta) \right] d\tau(\mu) \\ &= \int_{\Delta(\mathcal{V}) \setminus \mathcal{S}} \mathbb{E}[P(\tilde{z}, \mu)] d\hat{\tau}(\mu) + \int_{\mathcal{S}} \left[\int_{\mathcal{V}} \mathbb{E}[P(\tilde{z}, \delta_v)] d\mu(v) \right] d\tau(\mu) \\ &= \int_{\Delta(\mathcal{V}) \setminus \mathcal{S}} \mathbb{E}[P(\tilde{z}, \mu)] d\hat{\tau}(\mu) + \int_{\mathcal{S}} \mathbb{E}_{\mu}[\tilde{v}] d\tau(\mu), \end{aligned} \quad (\text{A.6})$$

where the second line uses the change of variable $v = T^{-1}(\delta_v)$ and the third line follows from $\mathbb{E}[P(\tilde{z}, \delta_v)] = \mathbb{E}_{\mu}[\tilde{v}]$. Inspection of (A.5) and (A.6) reveals that the ex ante expected price is the same under τ and $\hat{\tau}$.

To conclude the proof, we argue that $\hat{\tau}$ cannot be an optimal disclosure policy. Indeed,

$$\begin{aligned} \hat{\tau}(\mathcal{D}) &= \tau(\mathcal{D}) + \int_{\mathcal{S}} (\mu T^{-1})(\mathcal{D}) d\tau(\mu) \\ &\geq \int_{\mathcal{S}} \mu(\mathcal{V}) d\tau(\mu) = \int_{\mathcal{S}} d\tau(\mu) = \tau(\mathcal{S}) > 0, \end{aligned}$$

that is, $\hat{\tau}$ assigns a positive probability mass to degenerate distributions. Moreover, we must have $\hat{\tau}(\{\delta_v\}) < 1$ for all $\delta_v \in \mathcal{D}$, because all distributions in \mathcal{S} are non-degenerate. But then, by Proposition 2(i) $\hat{\tau}$ cannot be an optimal disclosure policies. This contradicts optimality of τ , which we have shown to be payoff-equivalent to $\hat{\tau}$. ■

B Proofs of Section 4

To begin with, we derive the price under CARA utility (4) and its second-order approximations (5) and (11) around $z = 0$. Under CARA utility, $u(w) = -\exp(-\rho w)$, $u'(w) =$

$\exp(-\rho w)$, and hence (3) boils down to

$$\begin{aligned}
P(z, \mu) &= \frac{\mathbb{E}_\mu[\tilde{v} \exp(-\rho z(\tilde{v} - P(z, \mu)))]}{\mathbb{E}_\mu[\exp(-\rho z(\tilde{v} - P(z, \mu)))]} \\
\iff P(z, \mu) &= \mathbb{E}_\mu[\tilde{v}] + \frac{\mathbb{E}_\mu[(\tilde{v} - \mathbb{E}_\mu[\tilde{v}]) \exp(-\rho z(\tilde{v} - P(z, \mu)))]}{\mathbb{E}_\mu[\exp(-\rho z(\tilde{v} - P(z, \mu)))]} \\
\iff P(z, \mu) &= \mathbb{E}_\mu[\tilde{v}] + \frac{\mathbb{E}_\mu[(\tilde{v} - \mathbb{E}_\mu[\tilde{v}]) \exp(-\rho z(\tilde{v} - \mathbb{E}_\mu[\tilde{v}]))]}{\mathbb{E}_\mu[\exp(-\rho z(\tilde{v} - \mathbb{E}_\mu[\tilde{v}]))]},
\end{aligned}$$

where to obtain the first equivalence we have added and subtracted $\mathbb{E}_\mu[\tilde{v}]$ on the right-hand side of the equation, and to obtain the second equivalence we have multiplied both the numerator and the denominator by $\exp(-\rho z(P(z, \mu) - \mathbb{E}_\mu[\tilde{v}]))$. The last line is (4) in the main text.

As in Gromb and Vayanos (2002) (p. 371), define

$$h(z) \equiv \mathbb{E}_\mu[\exp(-\rho z(\tilde{v} - \mathbb{E}_\mu[\tilde{v}]))],$$

which is the moment generating function of the random variable $\tilde{v} - \mathbb{E}_\mu[\tilde{v}]$ evaluated at $-\rho z$. In this way, the price can be expressed as

$$P(z, \mu) = \mathbb{E}_\mu[\tilde{v}] - \rho^{-1} \frac{h'(z)}{h(z)}. \quad (\text{B.1})$$

To approximate (4) around $z = 0$, note that

$$\begin{aligned}
h(0) &= 1 \\
h'(0) &= 0 \\
h''(0) &= \rho^2 \mathbb{E}_\mu[(\tilde{v} - \mathbb{E}_\mu[\tilde{v}])^2] \\
h'''(0) &= -\rho^3 \mathbb{E}_\mu[(\tilde{v} - \mathbb{E}_\mu[\tilde{v}])^3],
\end{aligned}$$

therefore,

$$\begin{aligned}
P(0, \mu) &= \mathbb{E}_\mu[\tilde{v}] \\
P'(z, \mu) &= -\rho^{-1} \frac{h''(z)h(z) - [h'(z)]^2}{[h(z)]^2} \implies P'(0, \mu) = -\rho \mathbb{E}_\mu[(\tilde{v} - \mathbb{E}_\mu[\tilde{v}])^2] \\
P''(z, \mu) &= -\rho^{-1} \frac{\{h'''(z)h(z) + h''(z)h'(z) - 2h'(z)h''(z)\}[h(z)]^2 - \{h''(z)h(z) - [h'(z)]^2\}2h(z)h'(z)}{[h(z)]^4} \\
&\implies P''(0, \mu) = \rho^2 \mathbb{E}_\mu[(\tilde{v} - \mathbb{E}_\mu[\tilde{v}])^3].
\end{aligned}$$

Plugging these expressions into the second-order Taylor approximation gives

$$\begin{aligned} P(\tilde{z}, \mu) &\approx P(0, \mu) + P'(0, \mu)\tilde{z} + P''(0, \mu)\frac{\tilde{z}^2}{2} \\ &= \mathbb{E}_\mu[\tilde{v}] - \mathbb{E}_\mu[(\tilde{v} - \mathbb{E}_\mu[\tilde{v}])^2]\rho\tilde{z} + \mathbb{E}_\mu[(\tilde{v} - \mathbb{E}_\mu[\tilde{v}])^3]\frac{\rho^2\tilde{z}^2}{2}. \end{aligned}$$

Taking the expectation with respect to \tilde{z} yields (11). Imposing $m_z = 0$ yields (5).

B.1 Proofs of Section 4

Proof of Lemma 3. Specializing (4) to the binary setting, we have

$$\begin{aligned} P(z, q) &= m_v + (2q - 1)\sigma_v + \frac{2q(1 - q)\sigma_v \exp(-\rho z(m_v + \sigma_v)) - 2q(1 - q)\sigma_v \exp(-\rho z(m_v - \sigma_v))}{q \exp(-\rho z(m_v + \sigma_v)) + (1 - q) \exp(-\rho z(m_v - \sigma_v))} \\ &= m_v + \sigma_v \frac{q \exp(-\rho z(m_v + \sigma_v)) - (1 - q) \exp(-\rho z(m_v - \sigma_v))}{q \exp(-\rho z(m_v + \sigma_v)) + (1 - q) \exp(-\rho z(m_v - \sigma_v))} \\ &= m_v + \sigma_v \frac{q \exp(-2\rho z\sigma_v) - (1 - q)}{q \exp(-2\rho z\sigma_v) + (1 - q)}, \end{aligned}$$

where in the second line we have added the second and third terms, and in the third line we have divided both the numerator and the denominator by $\exp(-\rho z(m_v - \sigma_v))$. The expression in (6) follows from a straightforward rearranging of the last line. ■

Proof of Proposition 3. This is a special case of the proof of Proposition 7 below, with $m_z = 0$. ■

Proof of Proposition 7. Define $\tilde{a} \equiv \exp(-2\rho\tilde{z}\sigma_v)$ and

$$\phi(q) \equiv \mathbb{E}[P(\tilde{z}, q)] - \mathbb{E}_q[\tilde{v}] = \sigma_v \mathbb{E} \left[\frac{q(\tilde{a} + 1) - 1}{q(\tilde{a} - 1) + 1} - (2q - 1) \right].$$

The first, second, and third derivatives of $\phi(q)$ with respect to q are, respectively,

$$\begin{aligned}\phi'(q) &= \sigma_v \mathbb{E} \left[\frac{2\tilde{a}}{(1 - (1 - \tilde{a})q)^2} - 2 \right], \\ \phi''(q) &= \sigma_v \mathbb{E} \left[\frac{4(1 - \tilde{a})\tilde{a}}{(1 - (1 - \tilde{a})q)^3} \right], \\ \phi'''(q) &= \sigma_v \mathbb{E} \left[\frac{12(1 - \tilde{a})^2\tilde{a}}{(1 - (1 - \tilde{a})q)^4} \right].\end{aligned}$$

For future reference, note that the third derivative

$$\phi'''(q) > 0 \text{ for all } q.$$

The second derivative evaluated at $q = 0$ and $q = 1$ equals, respectively,

$$\begin{aligned}\phi''(0) &= 4\mathbb{E} [\tilde{a} - \tilde{a}^2] = 4 \exp(-2\sigma_v \rho m_z + 2\sigma_v^2 \rho^2 \sigma_z^2) [1 - \exp(-2\sigma_v \rho m_z + 6\sigma_v^2 \rho^2 \sigma_z^2)], \\ \phi''(1) &= 4\mathbb{E} [\tilde{a}^{-2} - \tilde{a}^{-1}] = 4 \exp(4\sigma_v \rho m_z + 2\sigma_v^2 \rho^2 \sigma_z^2) [\exp(6\sigma_v^2 \rho^2 \sigma_z^2) - \exp(-2\sigma_v \rho m_z)].\end{aligned}$$

The first derivative evaluated at $q = 0$ and $q = 1$ equals, respectively,

$$\begin{aligned}\phi'(0) &= 2\mathbb{E} [\tilde{a}] - 2 = 2 \exp(-2\sigma_v \rho m_z + 2\sigma_v^2 \rho^2 \sigma_z^2) - 2, \\ \phi'(1) &= 2\mathbb{E} [\tilde{a}^{-1}] - 2 = 2 \exp(2\sigma_v \rho m_z + 2\sigma_v^2 \rho^2 \sigma_z^2) - 2.\end{aligned}$$

Observe that

$$\begin{aligned}\phi'(0) > 0 &\iff \sigma_z^2 > \frac{m_z}{\sigma_v \rho}, \\ \phi''(0) < 0 &\iff \sigma_z^2 > \frac{m_z}{3\sigma_v \rho},\end{aligned}$$

and

$$\phi'(1), \phi''(1) > 0$$

for all parameter values.

Proof of Part (i) Let the constant c in the statement of the proposition be given by $c \equiv (\sigma_v \rho)^{-1}$. Under the condition of this part of the proposition, that is, $\sigma_z^2 \leq \frac{m_z}{\sigma_v \rho}$, we have $\phi(q) < 0$ for all $q \in (0, 1)$. This is a consequence of the following two facts.

Fact I. For $q \approx 0$ we have $\phi(q) < 0$.

Proof of Fact I. If $\sigma_z^2 = \frac{m_z}{\sigma_v \rho}$, then this follows from $\phi'(0) = 0$ and $\phi''(0) < 0$. If $\sigma_z^2 < \frac{m_z}{\sigma_v \rho}$,

then this follows from $\phi'(0) < 0$. ■

Fact II. There does not exist any $q \in (0, 1)$ such that $\phi(q) = 0$.

Proof of Fact II. By contradiction, suppose that there existed points $q \in (0, 1)$ such that $\phi(q) = 0$. Let q_1 denote the lowest of such points. Because $\phi(q) < 0$ for $q \approx 0$ (by Fact I), at $q = q_1$ the function $\phi(q)$ must cross the 0-line from below, i.e., $\phi'(q_1) \geq 0$ (otherwise, we would have $\phi(q) > 0$ for q in a small neighborhood to the left of q_1 , which together with Fact I and continuity would contradict q_1 being the lowest q such that $\phi(q) = 0$). Also, let $q^\#$ denote the point at which $\phi(q)$ switches from concavity to convexity, i.e., $\phi''(q^\#) = 0$. Because the third derivative is always positive, there can be at most one such a point $q^\#$. If $\phi(q)$ is convex everywhere (which occurs when $\sigma_z^2 \leq \frac{m_z}{3\sigma_v\rho}$), then let $q^\# = 0$. Observe that we must have $q_1 \geq q^\#$, because $\phi'(0) \leq 0$ and $\phi'(q_1) \geq 0$ imply that at $q = q_1$ the function $\phi(q)$ has already switched from concavity to convexity. Then, $\phi(q) > 0$ for all $q > q_1$, because all these values of q are in the region where $\phi(q)$ is convex. But this conclusion contradicts $\phi(1) = 0$. ■

Combining Facts I and Facts II, and applying the concavification argument of [Kamenica and Gentzkow \(2011\)](#), we conclude that the disclosure policy that maximizes the ex ante price is one of perfect disclosure. The optimal distribution of posteriors conditional on public information is, therefore,

$$\tilde{q}^* = \begin{cases} 1 & \text{with prob. } q_0 \\ 0 & \text{with prob. } 1 - q_0 \end{cases},$$

where $q_0 \equiv \Pr[\tilde{v} = m_v + \sigma_v]$ is the prior probability that the firm's payoff is high. This disclosure policy achieves an ex ante expected price $E[P(\tilde{z}, \tilde{q}^*)] = m_v$.

Proof of Part (ii) We have seen before that if $\sigma_z^2 > \frac{m_z}{\sigma_v\rho}$, then $\phi'(0) > 0$, and therefore $\phi(q) > 0$ for $q \approx 0$. Further, $\phi'(1) > 0$ implies $\phi(q) < 0$ for $q \approx 1$. Hence, $\phi(q)$ has both a minimum and a maximum. The minimum (maximum) must be in the convex (concave) region. That is, the minimum must be to the right of $q^\#$ and the maximum to its left (note that $q^\#$ is in the interior of the interval because $\phi''(0) < 0$ and $\phi''(1) > 0$). Further, no maximum (minimum) can exist in the convex (concave) region, and hence these minimum and maximum are unique.

Applying the concavification approach of [Kamenica and Gentzkow \(2011\)](#), we concavify $\phi(q)$ by taking the line passing through the point $(q, \phi) = (1, 0)$ that is tangent to $\phi(q)$.

Denote this line by $\phi = \alpha + \beta q$. Then, the parameters (α, β) and the tangency point \hat{q} satisfy

$$\begin{cases} \alpha + \beta = 0 \\ \alpha + \beta \hat{q} - \phi(\hat{q}) - \phi(1) = 0 \\ \beta - \phi'(\hat{q}) = 0 \end{cases} ,$$

Using the first equation to get $\alpha = -\beta$ (because $\phi(1) = 0$), and the third equation to get $\beta = \phi'(\hat{q})$, the system of equations simplifies to a single equation in one unknown,

$$\Psi(\hat{q}) = 0, \tag{B.2}$$

where

$$\Psi(q) \equiv \phi'(q)(1 - q) + \phi(q).$$

Let q_{\max} denote the point such that $\phi(q)$ is maximized. This equation has a solution in $(q_{\max}, q^\#)$. This can be shown as follows: $\Psi(q_{\max}) = \phi(q_{\max}) > 0$; and $\Psi(q^\#) = \phi'(q^\#)(1 - q^\#) + \phi(q^\#) < \phi(1) = 0$ by convexity of $\phi(q)$ to the right of $q^\#$. Uniqueness of the solution follows from $\Psi'(q) = \phi''(q)(1 - q) < 0$ by concavity to the left of $q^\#$.

We are now in a position to derive the optimal disclosure policy. If the prior $q_0 \leq \hat{q}$ defined in (B.2), then the ex ante expected price is maximized at $q = q_0$. The optimal disclosure policy in this case is

$$\tilde{q}^* = q_0 \text{ with prob. } 1.$$

By contrast, if the prior $q_0 > \hat{q}$, then the ex ante expected price is maximized by randomizing between $q = 1$ and $q = \hat{q}$. The optimal disclosure policy is therefore

$$\tilde{q}^* = \begin{cases} 1 & \text{with prob. } \pi^* \\ \hat{q} & \text{with prob. } 1 - \pi^* \end{cases} ,$$

where π^* satisfies the Bayesian plausibility constraint (7),

$$\pi^* + (1 - \pi^*) \hat{q} = q_0.$$

A solution for π^* exists whenever $q_0 > \hat{q}$, which is the case under consideration. This distribution of posteriors can be implemented through a binary disclosure policy of the form

in part (ii) of Proposition 3, where κ solves

$$\begin{cases} \Pr[\tilde{v} = m_v + \sigma_v | \tilde{q}^* = \hat{q}] &= \hat{q} \\ \Pr[\tilde{v} = m_v + \sigma_v | \tilde{q}^* = \hat{q}] &= \frac{q_0 \kappa}{q_0 \kappa + (1 - q_0)} \end{cases}. \quad (\text{B.3})$$

Combining the two equations of the system (B.3), we obtain that the optimal degree of asymmetry is

$$\kappa^*(\hat{q}) = \frac{(1 - q_0) \hat{q}}{q_0 (1 - \hat{q})}. \quad (\text{B.4})$$

■

Proof of Corollary 2. Observe that \hat{q} is independent of the prior q_0 . We have seen that for $q_0 \leq \hat{q}$ the optimal policy is one of full non-disclosure, which corresponds to $\kappa = 1$. For $q_0 > \hat{q}$, we can see from (B.4) that $\kappa^*(\hat{q})$ is decreasing in q_0 , with $\kappa^*(\hat{q})|_{q_0=\hat{q}} = 1$ (i.e., full non-disclosure) and $\kappa^*(\hat{q})|_{q_0=1} = 0$ (i.e., perfect disclosure). ■

B.2 Proofs of Section 5

Proof of Lemma 4. To derive the equilibrium prices under each type of truncation, we make use of the expression (B.1) above, which involves the moment generating function of the posterior distribution. It is convenient to rewrite equivalently (B.1) as follows,

$$P(z, \mu) = \frac{MGF'(-\rho z)}{MGF(-\rho z)}, \quad (\text{B.5})$$

where $MGF(t) \equiv E_\mu[\exp(t\tilde{v})]$ is the moment generating function of the random variable \tilde{v} conditional on a posterior μ .

Conditional on perfect disclosure $\tilde{s} = v$, the price is $P(z, v) = v$ for all z . Conditional on withholding $s = \emptyset$, the posterior distribution is truncated normal. Let us consider a normal that is truncated both from below at some α and from above at some β . We will then specialize the formula depending on whether we solve for the price under lower or upper truncation. For a truncated normal, the moment generating function equals

$$MGF(t) = \exp\left(m_v t + \frac{\sigma_v^2}{2} t^2\right) \frac{\Phi\left(\frac{\beta - m_v}{\sigma_v} - \sigma_v t\right) - \Phi\left(\frac{\alpha - m_v}{\sigma_v} - \sigma_v t\right)}{\Phi\left(\frac{\beta - m_v}{\sigma_v}\right) - \Phi\left(\frac{\alpha - m_v}{\sigma_v}\right)},$$

(e.g., Tallis, 1961). The derivative of the moment generating function equals

$$\begin{aligned}
MGF'(t) &= (m_v + \sigma_v^2 t) \exp\left(m_v t + \frac{\sigma_v^2}{2} t^2\right) \frac{\Phi\left(\frac{\beta - m_v}{\sigma_v} - \sigma_v t\right) - \Phi\left(\frac{\alpha - m_v}{\sigma_v} - \sigma_v t\right)}{\Phi\left(\frac{\beta - m_v}{\sigma_v}\right) - \Phi\left(\frac{\alpha - m_v}{\sigma_v}\right)} \\
&\quad - \sigma_v \exp\left(m_v t + \frac{\sigma_v^2}{2} t^2\right) \frac{\phi\left(\frac{\beta - m_v}{\sigma_v} - \sigma_v t\right) - \phi\left(\frac{\alpha - m_v}{\sigma_v} - \sigma_v t\right)}{\Phi\left(\frac{\beta - m_v}{\sigma_v}\right) - \Phi\left(\frac{\alpha - m_v}{\sigma_v}\right)} \\
&= \frac{\exp\left(m_v t + \frac{\sigma_v^2}{2} t^2\right)}{\Phi\left(\frac{\beta - m_v}{\sigma_v}\right) - \Phi\left(\frac{\alpha - m_v}{\sigma_v}\right)} \left\{ \begin{aligned} &(m_v + \sigma_v^2 t) \left[\Phi\left(\frac{\beta - m_v}{\sigma_v} - \sigma_v t\right) - \Phi\left(\frac{\alpha - m_v}{\sigma_v} - \sigma_v t\right) \right] \\ &- \sigma_v \left[\phi\left(\frac{\beta - m_v}{\sigma_v} - \sigma_v t\right) - \phi\left(\frac{\alpha - m_v}{\sigma_v} - \sigma_v t\right) \right] \end{aligned} \right\}.
\end{aligned}$$

Hence, (B.1) becomes

$$P(z, \emptyset) = m_v - \sigma_v^2 \rho z - \sigma_v \frac{\phi\left(\frac{\beta - m_v}{\sigma_v} + \sigma_v \rho z\right) - \phi\left(\frac{\alpha - m_v}{\sigma_v} + \sigma_v \rho z\right)}{\Phi\left(\frac{\beta - m_v}{\sigma_v} + \sigma_v \rho z\right) - \Phi\left(\frac{\alpha - m_v}{\sigma_v} + \sigma_v \rho z\right)}$$

and the ex ante expectation of the price becomes

$$\begin{aligned}
\mathbb{E}[P(\tilde{z}, \tilde{s})] &= \int_{\mathbb{R} \setminus (\alpha, \beta)} v \frac{1}{\sigma_v} \phi\left(\frac{v - m_v}{\sigma_v}\right) dv + \left[\Phi\left(\frac{\beta - m_v}{\sigma_v}\right) - \Phi\left(\frac{\alpha - m_v}{\sigma_v}\right) \right] \mathbb{E}[P(\tilde{z}, \emptyset)] \\
&= m_v - \left[\Phi\left(\frac{\beta - m_v}{\sigma_v}\right) - \Phi\left(\frac{\alpha - m_v}{\sigma_v}\right) \right] \left\{ \sigma_v^2 \rho m_z + \sigma_v \mathbb{E} \left[\frac{\phi\left(\frac{\beta - m_v}{\sigma_v} + \sigma_v \rho \tilde{z}\right) - \phi\left(\frac{\alpha - m_v}{\sigma_v} + \sigma_v \rho \tilde{z}\right)}{\Phi\left(\frac{\beta - m_v}{\sigma_v} + \sigma_v \rho \tilde{z}\right) - \Phi\left(\frac{\alpha - m_v}{\sigma_v} + \sigma_v \rho \tilde{z}\right)} - \frac{\phi\left(\frac{\beta - m_v}{\sigma_v}\right) - \phi\left(\frac{\alpha - m_v}{\sigma_v}\right)}{\Phi\left(\frac{\beta - m_v}{\sigma_v}\right) - \Phi\left(\frac{\alpha - m_v}{\sigma_v}\right)} \right] \right\}.
\end{aligned} \tag{B.6}$$

Recall that we are assuming $m_z = 0$. Then, setting $\alpha = -\infty$ and $\beta = k$, (B.6) boils down to the expression in part (i) of the lemma. Setting $\alpha = k$ and $\beta = \infty$, (B.6) boils down to the expression in part (ii) of the lemma. ■

Proof of Proposition 4. Proof of Part (i) The claim is a consequence of the fact that the function $\lambda\left(\frac{k - m_v}{\sigma_v} + \sigma_v \rho z\right)$ in equation (9) is convex in z . Because of convexity, Jensen's inequality implies

$$\mathbb{E} \left[\lambda \left(\frac{k - m_v}{\sigma_v} + \sigma_v \rho \tilde{z} \right) \right] - \lambda \left(\frac{k - m_v}{\sigma_v} \right) > \lambda \left(\frac{k - m_v}{\sigma_v} + \underbrace{\sigma_v \rho m_z}_{=0} \right) - \lambda \left(\frac{k - m_v}{\sigma_v} \right) = 0.$$

Inspection of the expression in Lemma 4(i) reveals that this term enters the price with a

negative sign. Thus, $E[P(\tilde{z}, \tilde{\mu})]$ is bounded above by m_v for any lower truncation threshold.

Proof of Part (ii) An argument analogous to that of part (i), but with inverted sign, shows that $E[P(\tilde{z}, \tilde{\mu})]$ is bounded below by m_v for any upper truncation threshold.

■

Proof of Proposition 5. Inspection of (5) reveals that the objective function is the ex ante expectation of the third central moment, $E[E_{\tilde{\mu}}[(\tilde{v} - E_{\tilde{\mu}}[\tilde{v}])^3]]$. Let $m_{\Omega} \equiv E[\tilde{v}|\tilde{s} = \emptyset]$ denote the posterior expectation conditional on withholding. The firm chooses a function $\omega : \mathbb{R} \rightarrow [0, 1]$, which for each realization v of the firm's payoff assigns a probability $\omega(v)$ of withholding conditional on that realization. Finally, let $f(v)$ denote the p.d.f. of the prior. The firm solves the program

$$\begin{aligned} & \max_{m_{\Omega}, \{\omega(v)\}_{v \in \mathbb{R}}} \int (\tilde{v} - m_{\Omega})^3 \omega(v) f(v) dv \\ & s.t. \\ & \int (m_{\Omega} - v) \omega(v) f(v) dv = 0, \\ & \omega(v) \in [0, 1] \text{ for all } v \in \mathbb{R}. \end{aligned}$$

The Lagrangian is

$$\mathcal{L} = \int (\tilde{v} - m_{\Omega})^3 \omega(v) f(v) dv - \lambda \int (v - m_{\Omega}) \omega(v) f(v) dv.$$

Differentiate the Lagrangian with respect to each $\omega(v)$:

$$\psi(v) \equiv \frac{1}{f(v)} \frac{\partial \mathcal{L}}{\partial \omega(v)} = (v - m_{\Omega})^3 - \lambda (v - m_{\Omega} (ND))$$

At the optimum, $\psi(v) > 0$ ($\psi(v) < 0$) implies $\omega(v) = 1$ ($\omega(v) = 0$). Next, differentiate the Lagrangian with respect to m_{Ω} :

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial m_{\Omega}} &= -3 \int (\tilde{v} - m_{\Omega})^2 \omega(v) f(v) dv + \lambda \int \omega(v) f(v) dv = 0 \\ \implies \lambda &= 3 \frac{\int (\tilde{v} - m_{\Omega})^2 \omega(v) f(v) dv}{\int \omega(v) f(v) dv} = 3 \text{Var}[\tilde{v}|\tilde{s} = \emptyset] > 0. \end{aligned}$$

Summing up, the function $\psi(v)$ has the following properties. First, it has three roots $v \in \{m_{\Omega} - \sqrt{\lambda}, m_{\Omega}, m_{\Omega} + \sqrt{\lambda}\}$, such that $\psi(v) = 0$. Second, it has a local maximum at $v =$

$m_\Omega - \sqrt{\frac{\lambda}{3}}$ and a local minimum at $v = m_\Omega + \sqrt{\frac{\lambda}{3}}$. Third, it satisfies $\lim_{v \rightarrow -\infty} \psi(v) = -\lim_{v \rightarrow \infty} \psi(v) = \infty$. Thanks to these properties, we know that $\psi(v) \geq 0$ if and only if $v \in [m_\Omega - \sqrt{\lambda}, m_\Omega] \cup [m_\Omega + \sqrt{\lambda}, \infty)$. We conclude that the withholding set of the optimal recognition policy satisfies the fixed point equation $\Omega^* = [m_{\Omega^*} - \sqrt{\lambda}, m_{\Omega^*}] \cup [m_{\Omega^*} + \sqrt{\lambda}, \infty)$, which proves the proposition. ■

C Proof of Proposition 6

An investor's first-order condition for firm i for any given vector of prices $\{P_h\}_{h \in \{1, \dots, N\}}$ satisfies

$$\begin{aligned} \frac{\partial}{\partial D_i} \mathbb{E} \left[\exp \left(-\rho \sum_{h=1}^N D_h (\tilde{v}_h - P_h) \right) \right] &= 0 \\ \iff \mathbb{E} \left[(\tilde{v}_i - P_i) \exp \left(-\rho \sum_{h=1}^N D_h (\tilde{v}_h - P_h) \right) \right] &= 0. \end{aligned}$$

Because investors are homogenous, they hold the same positions in equilibrium. Market clearing requires that the i^{th} investor holds $\frac{1}{J}$ of the total supply of the i^{th} stock. The total supply of this stock equals its innate supply, 1, plus noise trader sales, \tilde{z}_i . So, conditional on the vector of noise trade $\tilde{\mathbf{z}} \equiv \{\tilde{z}_i\}_{i \in \{1, \dots, J\}}$, market clearing in stock i requires that:

$$D_i = \frac{1}{J} (1 + J\tilde{z}_i) = \frac{1}{J} + \tilde{z}_i,$$

Let $P_i(\tilde{\mathbf{z}})$ denote the firm's price for a given outcome of noise trade. Then, solving for price from the investors' first-order conditions yields:

$$\begin{aligned} P_i(\tilde{\mathbf{z}}) &= \frac{\mathbb{E} \left[\tilde{v}_i \exp \left(-\rho \sum_{h=1}^N (J^{-1} + \tilde{z}_h) \tilde{v}_h \right) \mid \tilde{\mathbf{z}} \right]}{\mathbb{E} \left[\exp \left(-\rho \sum_{h=1}^N (J^{-1} + \tilde{z}_h) \tilde{v}_h \right) \mid \tilde{\mathbf{z}} \right]} \\ &= \frac{\mathbb{E} \left[\tilde{v}_i \exp \left(-\rho \left(\sum_{h=1}^N (J^{-1} + \tilde{z}_h) \tilde{\alpha}_h + \tilde{f} \sum_{h=1}^N (J^{-1} + \tilde{z}_h) \right) \right) \mid \tilde{\mathbf{z}} \right]}{\mathbb{E} \left[\exp \left(-\rho \left(\sum_{h=1}^N (J^{-1} + \tilde{z}_h) \tilde{\alpha}_h + \tilde{f} \sum_{h=1}^N (J^{-1} + \tilde{z}_h) \right) \right) \mid \tilde{\mathbf{z}} \right]}. \end{aligned}$$

Now, note that:

$$\begin{aligned} \mathbb{E}[P_i(\tilde{\mathbf{z}})] &= \mathbb{E} \left\{ \frac{\mathbb{E} \left[\tilde{v}_i \exp \left(-\rho \left(\sum_{h=1}^N (J^{-1} + \tilde{z}_h) \tilde{\alpha}_h + \tilde{f} \sum_{h=1}^N (J^{-1} + \tilde{z}_h) \right) \right) \middle| \tilde{\mathbf{z}} \right]}{\mathbb{E} \left[\exp \left(-\rho \left(\sum_{h=1}^N (J^{-1} + \tilde{z}_h) \tilde{\alpha}_h + \tilde{f} \sum_{h=1}^N (J^{-1} + \tilde{z}_h) \right) \right) \middle| \tilde{\mathbf{z}} \right]} \right\} \\ &= \mathbb{E} \left\{ \frac{\mathbb{E} \left[\tilde{v}_i \exp \left(\sum_{h \neq i} (J^{-1} + \tilde{z}_h) \tilde{\alpha}_h \right) \exp \left(-\rho \left((J^{-1} + \tilde{z}_i) \tilde{\alpha}_i + \tilde{f} \sum_{h=1}^N (J^{-1} + \tilde{z}_h) \right) \right) \middle| \tilde{\mathbf{z}} \right]}{\mathbb{E} \left[\exp \left(\sum_{h \neq i} (J^{-1} + \tilde{z}_h) \tilde{\alpha}_h \right) \exp \left(-\rho \left((J^{-1} + \tilde{z}_i) \tilde{\alpha}_i + \tilde{f} \sum_{h=1}^N (J^{-1} + \tilde{z}_h) \right) \right) \middle| \tilde{\mathbf{z}} \right]} \right\}. \end{aligned}$$

Using the independence of \tilde{v}_i from $\tilde{\alpha}_h \forall h \neq i$, this reduces to:

$$\begin{aligned} &\mathbb{E} \left\{ \frac{\mathbb{E} \left[\exp \left(\sum_{h \neq i} (J^{-1} + \tilde{z}_h) \tilde{\alpha}_h \right) \middle| \tilde{\mathbf{z}} \right] \mathbb{E} \left[\tilde{v}_i \exp \left(-\rho \left((J^{-1} + \tilde{z}_i) \tilde{\alpha}_i + \tilde{f} \sum_{h=1}^N (J^{-1} + \tilde{z}_h) \right) \right) \middle| \tilde{\mathbf{z}} \right]}{\mathbb{E} \left[\exp \left(\sum_{h \neq i} (J^{-1} + \tilde{z}_h) \tilde{\alpha}_h \right) \middle| \tilde{\mathbf{z}} \right] \mathbb{E} \left[\exp \left(-\rho \left((J^{-1} + \tilde{z}_i) \tilde{\alpha}_i + \tilde{f} \sum_{h=1}^N (J^{-1} + \tilde{z}_h) \right) \right) \middle| \tilde{\mathbf{z}} \right]} \right\} \\ &= \mathbb{E} \left\{ \frac{\mathbb{E} \left[\tilde{v}_i \exp \left(-\rho \left((J^{-1} + \tilde{z}_i) \tilde{\alpha}_i + \tilde{f} \sum_{h=1}^N (J^{-1} + \tilde{z}_h) \right) \right) \middle| \tilde{\mathbf{z}} \right]}{\mathbb{E} \left[\exp \left(-\rho \left((J^{-1} + \tilde{z}_i) \tilde{\alpha}_i + \tilde{f} \sum_{h=1}^N (J^{-1} + \tilde{z}_h) \right) \right) \middle| \tilde{\mathbf{z}} \right]} \right\}. \end{aligned}$$

Using the independence of $\tilde{\alpha}_i$ from \tilde{f} , this further reduces to:

$$\mathbb{E} \left\{ \frac{\mathbb{E} [\tilde{\alpha}_i \exp(-\rho(J^{-1} + \tilde{z}_i) \tilde{\alpha}_i) | \tilde{\mathbf{z}}]}{\mathbb{E} [\exp(-\rho(J^{-1} + \tilde{z}_i) \tilde{\alpha}_i) | \tilde{\mathbf{z}}]} + \frac{\mathbb{E} [\tilde{f} \exp(-\rho \tilde{f} \sum_{h=1}^N (J^{-1} + \tilde{z}_h)) | \tilde{\mathbf{z}}]}{\mathbb{E} [\exp(-\rho \sum_{h=1}^N (J^{-1} + \tilde{z}_h)) | \tilde{\mathbf{z}}]} \right\}. \quad (\text{C.1})$$

Calculating the expectation given that $\tilde{f} \sim N(\mu_f, \sigma_f^2)$, the second component of expression (C.1) reduces to:

$$\frac{\mathbb{E} \left[\tilde{f} \exp \left(-\rho \tilde{f} \sum_{h=1}^N (J^{-1} + \tilde{z}_h) \right) \middle| \tilde{\mathbf{z}} \right]}{\mathbb{E} \left[\exp \left(-\rho \sum_{h=1}^N (J^{-1} + \tilde{z}_h) \right) \middle| \tilde{\mathbf{z}} \right]} = \mu_f - \rho \left(\sum_{h=1}^N (J^{-1} + \tilde{z}_h) \right) \sigma_f^2.$$

Thus, expression (C.1) reduces to:

$$\begin{aligned} &\mathbb{E} \left\{ \frac{\mathbb{E} [\tilde{\alpha}_i \exp(-\rho(J^{-1} + \tilde{z}_i) \tilde{\alpha}_i) | \tilde{\mathbf{z}}]}{\mathbb{E} [\exp(-\rho(J^{-1} + \tilde{z}_i) \tilde{\alpha}_i) | \tilde{\mathbf{z}}]} + \mu_f - \rho \left(\sum_{h=1}^N (J^{-1} + \tilde{z}_h) \right) \sigma_f^2 \right\} \\ &= \mathbb{E} \left\{ \frac{\mathbb{E} [\tilde{\alpha}_i \exp(-\rho(J^{-1} + \tilde{z}_i) \tilde{\alpha}_i) | \tilde{\mathbf{z}}]}{\mathbb{E} [\exp(-\rho(J^{-1} + \tilde{z}_i) \tilde{\alpha}_i) | \tilde{\mathbf{z}}]} \right\} + \mu_f - \rho N J^{-1} \sigma_f^2. \end{aligned}$$

Taking the large economy limit as N and J approach ∞ at the same rate, we have:

$$\begin{aligned}
\lim_{N, J \rightarrow \infty} \mathbb{E}[P_i(\tilde{\mathbf{z}})] &= \lim_{N, J \rightarrow \infty} \left[\mathbb{E} \left\{ \frac{\mathbb{E}[\tilde{\alpha}_i \exp(-\rho(J^{-1} + \tilde{z}_i)\tilde{\alpha}_i) | \tilde{\mathbf{z}}]}{\mathbb{E}[\exp(-\rho(J^{-1} + \tilde{z}_i)\tilde{\alpha}_i) | \tilde{\mathbf{z}}]} \right\} + \mu_f - \rho N J^{-1} \sigma_f^2 \right] \\
&= \mathbb{E} \left\{ \frac{\mathbb{E}[\tilde{\alpha}_i \lim_{N, J \rightarrow \infty} \exp(-\rho(J^{-1} + \tilde{z}_i)\tilde{\alpha}_i) | \tilde{\mathbf{z}}]}{\mathbb{E}[\lim_{N, J \rightarrow \infty} \exp(-\rho(J^{-1} + \tilde{z}_i)\tilde{\alpha}_i) | \tilde{\mathbf{z}}]} \right\} + \mu_f - \rho \sigma_f^2 \\
&= \mathbb{E} \left\{ \frac{\mathbb{E}[\tilde{\alpha}_i \exp(-\rho \tilde{z}_i \tilde{\alpha}_i) | \tilde{\mathbf{z}}]}{\mathbb{E}[\exp(-\rho \tilde{z}_i \tilde{\alpha}_i) | \tilde{\mathbf{z}}]} \right\} + \mu_f - \rho \sigma_f^2,
\end{aligned}$$

which is equation (10) in the main text.

D Proof that quadratic utility yields accurate valuation

In this appendix, we consider the case where investors preferences are quadratic in terminal wealth w (i.e., $u''' = 0$). That is, we are going to assume that $u(\cdot)$ takes the form

$$u(w) = w - \frac{\rho}{2} w^2 \text{ for } \rho > 0.$$

For quadratic utility to satisfy $u' > 0$, the distributional and parametric assumptions of the model have to be such that $w < 1/\rho$ for all realizations of the terminal wealth $\tilde{w} = w$.

Lemma D.1 below shows that the firm cannot induce overvaluation through its disclosure policy, because the firm will always be accurately value regardless of investors' posterior distribution.

Lemma D.1. *Suppose that investors' utility is quadratic in terminal wealth. Then, any distribution μ induces accurate valuation, $\mathbb{E}[P(\tilde{z}, \mu)] = \mathbb{E}_\mu[\tilde{v}]$.*

Proof of Lemma D.1. Take any distribution μ . Since the distribution is fixed, we drop μ from the notation. Equation (3) for the market-clearing price can be written equivalently as

$$0 = \mathbb{E}[(\tilde{v} - P)u'(z(\tilde{v} - P))].$$

In the case of quadratic utility, this boils down to

$$\begin{aligned}
0 &= \mathbb{E}[\tilde{v}] - P - \mathbb{E}[(\tilde{v} - P)^2] \rho z \\
&= -\phi - (\text{Var}[\tilde{v}] + \phi^2) \rho z,
\end{aligned}$$

where in the second line we have defined $\phi \equiv P - \mathbb{E}[\tilde{v}]$. Solving for ϕ yields two solutions

(whenever they exist),

$$\phi_1(z) = \frac{-1 - \sqrt{1 - 4\text{Var}[\tilde{v}]\rho^2 z^2}}{2\rho z} \text{ and } \phi_2(z) = \frac{-1 + \sqrt{1 - 4\text{Var}[\tilde{v}]\rho^2 z^2}}{2\rho z}.$$

Each of these solutions $j \in \{1, 2\}$ has the property that $\phi_j(z) = -\phi_j(-z)$ for all z , hence

$$\mathbb{E}[\phi_j(\tilde{z})] = -\mathbb{E}[\phi_j(-\tilde{z})] \iff \mathbb{E}[\phi_j(\tilde{z})] = -\mathbb{E}[\phi_j(\tilde{z})] \implies \mathbb{E}[\phi_j(\tilde{z})] = 0,$$

where the “if and only if” exploits the fact that \tilde{z} is symmetrically distributed around zero. By definition of ϕ , $\mathbb{E}[\phi_j(\tilde{z})] = 0$ is equivalent to $\mathbb{E}[P(\tilde{z})] = \mathbb{E}[\tilde{v}]$. Therefore, we have accurate valuation regardless of the distribution. ■

E Extension of the binary model where investors have private information

In this extension of the model with binary fundamentals (Section 4), we solve for the equilibrium price, and the ex ante expected price, when risk-averse investors not only observe the firm’s public disclosure \tilde{s} , but also a private signal. In particular, we assume that each investor i privately observes the noisy signal $\tilde{y}_i = \tilde{v} + \tilde{\varepsilon}_i$, where the error terms $\tilde{\varepsilon}_i$ are mutually independent across investors and independent of the firm’s payoff \tilde{v} . Also, we assume that $\tilde{\varepsilon}_i \sim N(0, \sigma_\varepsilon^2)$. All other assumptions are unaltered.

Our derivations follow Breon-Drish (2015). Let $q \equiv \Pr[\tilde{v} = m_v + \sigma_v | \tilde{s} = s]$ be the posterior that the firm’s payoff is high conditional on public information only. Also, let $q_i \equiv \Pr[\tilde{v} = m_v + \sigma_v | \tilde{s} = s, \tilde{y}_i = y_i]$ be investor i ’s posterior conditional on his information set, which consists of the public signal \tilde{s} and of the private signal \tilde{y}_i . Investor i ’s optimization problem is

$$\max_{D_i} [q_i \exp(-\rho D_i (m_v + \sigma_v - P)) + (1 - q_i) \exp(-\rho D_i (m_v - \sigma_v - P))].$$

Thus, investor i ’s optimal demand D_i^* solves the first-order condition

$$\begin{aligned} q_i (m_v + \sigma_v - P) \exp(-\rho D_i (m_v + \sigma_v - P)) &= -(1 - q_i) (m_v - \sigma_v - P) \exp(-\rho D_i (m_v - \sigma_v - P)) \\ \iff \log \left(\frac{q_i (m_v + \sigma_v - P)}{-(1 - q_i) (m_v - \sigma_v - P)} \right) &= 2\rho \sigma_v D_i \\ \implies D_i^* &= \frac{1}{2\rho \sigma_v} \log \left(-\frac{q_i (m_v + \sigma_v - P)}{(1 - q_i) (m_v - \sigma_v - P)} \right). \end{aligned}$$

Market clearing requires that

$$\begin{aligned} \frac{1}{2\rho\sigma_v} \int_0^1 \log \left(-\frac{q_i (m_v + \sigma_v - P)}{(1 - q_i) (m_v - \sigma_v - P)} \right) di &= z \\ -\frac{m_v - \sigma_v - P}{m_v + \sigma_v - P} &= \exp \left(\int_0^1 \log \left(\frac{q_i}{1 - q_i} \right) di - 2\rho\sigma_v z \right). \end{aligned}$$

Simplifying yields:

$$P = m_v + \sigma_v \frac{\exp \left(\int_0^1 \log \left(\frac{q_i}{1 - q_i} \right) di - 2\rho\sigma_v z \right) - 1}{\exp \left(\int_0^1 \log \left(\frac{q_i}{1 - q_i} \right) di - 2\rho\sigma_v z \right) + 1}. \quad (\text{E.1})$$

Note that the price in (E.1) is monotonic in $\int_0^1 \log \left(\frac{q_i}{1 - q_i} \right) di - 2\rho\sigma_v z$. Breon-Drish (2015) proves that any equilibrium must take the following form in this model:

$$\tilde{P} = g(\tilde{v} + \beta\tilde{z})$$

such that the observable statistic $\tilde{\chi} = g^{-1}(\tilde{P})$ is a “truth plus noise” signal about \tilde{v} with conditional variance $\beta^2\sigma_z^2$. The next step is to calculate the investor’s belief in equilibrium, for a conjectured coefficient β . A useful result is that the log-odds ratios in price are linear in the signals, given the normally distributed errors. Next, we will prove this and then solve for the equilibrium. Let $\phi(\cdot)$ denote the standard normal p.d.f. We have:

$$\begin{aligned} \log \left(\frac{q_i}{1 - q_i} \right) &= \log \left(\frac{\Pr[\tilde{v} = m_v + \sigma_v | y_i, P]}{\Pr[\tilde{v} = m_v - \sigma_v | y_i, P]} \right) \\ &= \log \left(\frac{q \phi \left(\frac{y_i - m_v - \sigma_v}{\sigma_\varepsilon} \right) \phi \left(\frac{\chi - m_v - \sigma_v - \beta m_z}{\beta \sigma_z} \right)}{(1 - q) \phi \left(\frac{y_i - m_v + \sigma_v}{\sigma_\varepsilon} \right) \phi \left(\frac{\chi - m_v + \sigma_v - \beta m_z}{\beta \sigma_z} \right)} \right). \end{aligned}$$

Further, notice:

$$\begin{aligned}
\log\left(\frac{q_i}{1-q_i}\right) &= \log\left(\frac{q}{1-q}\right) + \log\left(\frac{\phi\left(\frac{y_i-m_v-\sigma_v}{\sigma_\varepsilon}\right)}{\phi\left(\frac{y_i-m_v+\sigma_v}{\sigma_\varepsilon}\right)}\right) + \log\left(\frac{\phi\left(\frac{\chi-m_v-\sigma_v-\beta m_z}{\beta\sigma_z}\right)}{\phi\left(\frac{\chi-m_v+\sigma_v-\beta m_z}{\beta\sigma_z}\right)}\right) \\
&= \log\left(\frac{q}{1-q}\right) + \log\left(\frac{\exp\left(-\frac{(y_i-m_v-\sigma_v)^2}{2\sigma_\varepsilon^2}\right)}{\exp\left(-\frac{(y_i-m_v+\sigma_v)^2}{2\sigma_\varepsilon^2}\right)}\right) + \log\left(\frac{\exp\left(-\frac{(\chi-m_v-\sigma_v-\beta m_z)^2}{2\beta^2\sigma_z^2}\right)}{\exp\left(-\frac{(\chi-m_v+\sigma_v-\beta m_z)^2}{2\beta^2\sigma_z^2}\right)}\right) \\
&= \log\left(\frac{q}{1-q}\right) - \frac{(y_i-m_v-\sigma_v)^2}{2\sigma_\varepsilon^2} + \frac{(y_i-m_v+\sigma_v)^2}{2\sigma_\varepsilon^2} \\
&\quad - \frac{(\chi-m_v-\sigma_v-\beta m_z)^2}{2\beta^2\sigma_z^2} + \frac{(\chi-m_v+\sigma_v-\beta m_z)^2}{2\beta^2\sigma_z^2} \\
&= \log\left(\frac{q}{1-q}\right) + 2\frac{\sigma_v}{\sigma_\varepsilon^2}(y_i-m_v) + \frac{2\sigma_v}{\beta^2\sigma_z^2}(\chi-m_v-\beta m_z). \tag{E.2}
\end{aligned}$$

Reinjecting (E.2) into (E.1) gives us the equilibrium price as a function of the unknown coefficient β :

$$\begin{aligned}
\tilde{P} &= m_v + \sigma_v \frac{\exp\left(\int_0^1 \left[\log\left(\frac{q}{1-q}\right) + 2\frac{\sigma_v}{\sigma_\varepsilon^2}(\tilde{y}_i - m_v) + 2\frac{\sigma_v}{\beta^2\sigma_z^2}(\tilde{\chi} - m_v - \beta m_z)\right] di - 2\rho\sigma_v\tilde{z}\right) - 1}{\exp\left(\int_0^1 \left[\log\left(\frac{q}{1-q}\right) + 2\frac{\sigma_v}{\sigma_\varepsilon^2}(\tilde{y}_i - m_v) + 2\frac{\sigma_v}{\beta^2\sigma_z^2}(\tilde{\chi} - m_v - \beta m_z)\right] di - 2\rho\sigma_v\tilde{z}\right) + 1} \\
&= m_v + \sigma_v \frac{\exp\left(\log\left(\frac{q}{1-q}\right) + 2\frac{\sigma_v}{\sigma_\varepsilon^2}(\tilde{v} - m_v) + 2\frac{\sigma_v}{\beta^2\sigma_z^2}(\tilde{\chi} - m_v - \beta m_z) - 2\rho\sigma_v\tilde{z}\right) - 1}{\exp\left(\log\left(\frac{q}{1-q}\right) + 2\frac{\sigma_v}{\sigma_\varepsilon^2}(\tilde{v} - m_v) + 2\frac{\sigma_v}{\beta^2\sigma_z^2}(\tilde{\chi} - m_v - \beta m_z) - 2\rho\sigma_v\tilde{z}\right) + 1}. \tag{E.3}
\end{aligned}$$

In equilibrium, we must have that the ratio of the coefficient on \tilde{z} to that on \tilde{v} in the above equation is β (we can safely ignore the fact that $\tilde{\chi}$ appears in the above equation when solving for this ratio, as we know that the ratio of the coefficient on \tilde{z} to that on \tilde{v} in $\tilde{\chi}$ is β). Thus, the equilibrium conditions yield:

$$\beta = -\frac{\rho\sigma_v}{\frac{\sigma_v}{\sigma_\varepsilon^2}} = -\rho\sigma_\varepsilon^2$$

and we can write

$$\tilde{\chi} = \tilde{v} - \rho\sigma_\varepsilon^2\tilde{z}.$$

This resembles the standard noisy rational expectations equilibrium (e.g., [Hellwig \(1980\)](#)).

Using this, we can simplify the expression for the equilibrium price (E.3) as follows:

$$P(\tilde{v}, \tilde{z}, q) = m_v + \sigma_v \frac{\frac{q}{1-q} \exp\left(\frac{2\sigma_v}{\sigma_\varepsilon^2} \left[\left(1 + \frac{1}{\rho^2 \sigma_\varepsilon^2 \sigma_z^2}\right) (\tilde{v} - \rho \sigma_\varepsilon^2 \tilde{z} - m_z) + \frac{m_z}{\rho \sigma_z^2}\right]\right) - 1}{\frac{q}{1-q} \exp\left(\frac{2\sigma_v}{\sigma_\varepsilon^2} \left[\left(1 + \frac{1}{\rho^2 \sigma_\varepsilon^2 \sigma_z^2}\right) (\tilde{v} - \rho \sigma_\varepsilon^2 \tilde{z} - m_z) + \frac{m_z}{\rho \sigma_z^2}\right]\right) + 1}.$$

Contrary to the model of Section 4 without private information, here the equilibrium price is a function of the firm's payoff, because it aggregates investors' private signals, which depend on the firm's payoff. We can still compute the ex ante expected price using the law of iterated expectations. Conditional on a realized posterior $\tilde{q} = q$, the firm's payoff is high with probability q and low with probability $1 - q$. Therefore, conditional on $\tilde{q} = q$ the firm expects an average price equal to

$$\bar{P}(q) \equiv q \mathbb{E}[P(m_v + \sigma_v, \tilde{z}, q)] + (1 - q) \mathbb{E}[P(m_v - \sigma_v, \tilde{z}, q)].$$

The ex ante expected price can be computed as the expectation of the average price $\bar{P}(\tilde{q})$ with respect to the posterior \tilde{q} . ■

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