

Market Monitoring and Managerial Incentives under Dynamic
Contracting

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Abstract

I study the impact of stock price informativeness on incentives and contractual efficiency in a dynamic agency setting. In the model, the firm's stock price is the outcome of speculative trading that efficiently impounds dispersed information in the market. The stock price is an *aggregate* and *rational* forecast of future firm value, but as a managerial performance measure, it includes incentive irrelevant noise and is insensitive to the actual effort decision of the manager. Under multi-period contracting, the interim stock price also acts as the performance benchmark for the subsequent period. When contracts are renegotiated in light of updated beliefs about future firm value, these two conflicting roles of stock price – current performance measure and future performance benchmark – are not optimally coordinated over time. Consequently, there are many situations where stock price informativeness negatively impacts the manager's long-term effort incentive and contractual efficiency. Furthermore, when firm insiders can form their own more accurate forecast of future cash flow, a long-term renegotiable contract contingent only on realized cash flow can outperform a long-term full commitment contract contingent on both interim stock price and realized cash flow.

1 Introduction

Financial markets have been long recognized as an important means of monitoring the management of publicly traded firms. By aggregating information from dispersed market participants, stock prices provide firms with quantifiable measures of managerial performance that cannot be extracted from other sources (e.g., accounting data). Given the large body of empirical evidence demonstrating the predictive ability of stock price,¹ tying top management compensation to the firm's stock price seems to be a simple way of mitigating managerial myopia. This monitoring function of stock market crucially depends on the information impounded in market prices through speculative trading. With an informationally efficient equity market, the standard view is that as speculators gather more signals about future firm fundamentals, stock price becomes a more accurate measure of managerial performance, and price-based compensation becomes more effective in motivating far-sighted managerial actions (Holmstrom and Tirole (1993)).

Studies that examine the impact of stock price informativeness on contractual efficiency have focused on settings in which compensation terms are fixed throughout the agency relationship. In the multi-task moral hazard model of Holmstrom and Tirole (1993), the contract terms and efforts are chosen once and for all at the outset of the agency.² However, real world contractual relationships are ongoing processes in changing environments. For instance, managers are usually free to quit and, in light of new information, the contracting parties can renegotiate and replace the remaining portion of the contract with a new one by mutual consent. In light of the dynamic nature of most managerial employment contracts, this paper takes another look at the incentive impact of stock price informativeness when efforts and contracts interact and react to updated beliefs about firm value.

Specifically, the paper adopts the two-period LEN setting of Holmstrom and Tirole (1993) but allows sequential choices of managerial efforts and contract terms. The manager undertakes an effort in the first period that results in an *uncertain* payoff in the second period. In addition, the aggregate cash flow (liquidating firm value) is affected by the manager's effort in the second period and some exogenous noise in the firm's operation. The contracting parties have limited commitment capacity to contract terms. The firm has all the bargaining power and once the

¹Seminal work on the information production role of stock markets includes Grossman (1976), Grossman and Stiglitz (1980), and Verrecchia (1982).

²Several recent studies (Bolton, Scheinkman, and Xiong (2006), Edmans, Gabaix, Sadzik, and Sannikov (2012), Peng and Roell (2014) and Gopalan, Milbourn, Song, and Thakor (2014)) have examined the mix of short- and long-term equity based pay for corporate executives that is optimal from the ex-ante perspective.

manager's first-period effort is sunk, the firm's objective shifts to motivate the desired second-period effort at minimal costs. Relevant information about the uncertain payoff of the manager's first-period effort also trickles in once the effort is sunk. There is a public unverifiable signal that reveals some noise in the payoff from the manager's first-period effort. In addition, some informed traders in the stock market privately learn the manager's first-period effort payoff. The stock price summarizes dispersed information in the market to form an *aggregate and rational* forecast of the future cash flow.

In this dynamic agency setting, stock price informativeness turns out to have more subtle incentive implications than the established view suggests. This subtlety arises from two *conflicting* roles of stock price under multi-period contracting. On one hand, because interim stock price reflects the consequences of present decisions on future firm value, it acts as a *performance measure* and facilitates rewarding the manager's far-sighted actions through price based compensation. On the other hand, since comparing future cash flow against the interim stock price filters out irrelevant outcomes of past actions, the interim stock price acts as a *performance benchmark* and enhances incentive provision for the remaining agency relationship. This performance benchmarking function makes the manager's future wage sensitive to his future actions but insensitive to his past actions. Clearly, as the stock price becomes a more accurate forecast of future cash flow, both roles of the stock price – current performance measure and future performance benchmark – are strengthened. The question, however, is how they interact to determine incentive and efficiency when they cannot be optimally coordinated over time due to limited commitment?

Coupling performance measurement and performance benchmark roles of the stock price in a limited commitment agency setting, this model emphasizes the following tensions. First, as an *aggregate and rational* forecast of future firm value, the stock price has inherent weaknesses when used as a managerial performance measure. As an *aggregate* forecast, the stock price does not separate out the random component of value and information assessing the manager's contribution to it. Furthermore, as a *rational* forecast, the stock price relies on conjectured action and is inherently weak at detecting shirking. These two features increase the noise-to-signal ratio of the interim stock price in reflecting managerial effort and thus the cost of price-based compensation.

Second, even absent any reward attached to stock price, the manager as a long-run player in the agency relationship is still motivated to behave in a far-sighted manner so as to increase his

future performance based pay. This “implicit” incentive embedded in long-term employment, as opposed to “explicit” bonus conditioned on concurrent stock price, does not impose any risk on the manager: it arises from the impact of his current effort on his *expected* future compensation, which is deterministic. By contrast, explicit incentive contingent on the stock price always exposes the risk-averse manager to the inherent uncertainty in the firm’s fundamentals (i.e., the risk in the payoffs from the manager’s earlier effort). It is therefore costly for the firm to offset any reduction in the implicit incentive with higher powered price-based explicit incentive.

Third, exploiting the stock price as a future performance benchmark weakens the manager’s implicit incentive. Anticipating that the stock price is an endogenous moving target against which his future performance is evaluated, the manager will be reluctant to put in early effort that increases the long-run firm value. In the extreme case where the stock price perfectly reveals the future cash flow, the manager’s future wage is independent of his earlier effort choice and the implicit incentive is completely destroyed. Since it is costly to counter the loss of implicit incentive with explicit incentive, using interim stock price as a performance benchmark imposes a negative externality on incentive provision for the early part of the agency relationship. An inefficiency thus arises if the firm owner sets contract terms in a sequentially optimal fashion.

The equilibrium analysis of the model shows that limited commitment limits the benefit of market monitoring. A key parameter in the characterization of equilibrium outcomes is the payoff uncertainty of the manager’s effort undertaken early in the agency relationship. As this uncertainty increases, the interim stock price becomes more volatile and covaries more with future total cash flow, which makes the stock price more valuable as a future performance benchmark but less valuable as a current performance measure. Furthermore, because using stock price as a performance benchmark impairs the manager’s implicit incentive, the manager’s first-period effort can be lower when the stock price resolves more uncertainty in future cash flow and thus is more heavily exploited as a performance benchmark.

Firm profit equals the total surplus from the two-period agency relationship. While stock price informativeness results in lower effort early on when the payoff from such effort is relatively uncertain, it always improves the effort incentive for the later part of the agency relationship. When the payoff from the first-period effort is sufficiently uncertain, the efficiency gain due to improved performance benchmarking can outweigh the efficiency loss due to impaired implicit incentive. Taken together, equilibrium firm profit can decrease with market monitoring when the payoff risk from the manager’s first-period effort is within an intermediate range. This range

enlarges as the stock price contains more value relevant but incentive-irrelevant noise so that its relative merit as a performance measure is weakened. Within this range, it is possible that as stock price aggregates more information, the manager works harder in both periods, but managerial compensation increases even more, reducing firm profit and undermining efficiency.

The paper's main analyses examine a setting in which the manager can quit after the first period, because the two conflicting roles of stock price is most transparent in the presence of the manager's interim participation constraint. Specifically, higher (lower) interim stock price raises the expectation about future cash flow and will be followed by lower (higher) fixed pay, so that for the second period, the manager is effectively only rewarded for the realized cash flow in excess of the expectation captured by the stock price. This type of intertemporal performance benchmarking undermines the manager's first-period effort incentive. As the stock price becomes more informative of future cash flow, the firm owner relies more on the stock price in assessing the to-be-realized cash flow. The second-period fixed pay therefore responds more negatively to the stock price, and offsets more of the manager's expected second-period bonus associated with the payoff from his first-period effort. Consequently, the implicit incentive for the first-period effort decreases as the stock price becomes more informative. In the extreme case where the stock price perfectly reveals the first-period effort payoff, the manager's effort has no impact on his future compensation and the implicit incentive is completely destroyed.

An alternative to the above setting is one wherein both contracting parties commit to stay for long-term employment but renegotiate contract terms upon observing new information. In this case, it is without loss of generality to study the *renegotiation-proof* contract that spans both periods. When stock price is the best forecast of future cash flow at the interim date, it can be verified that the optimal long-term renegotiation-proof contract is performance equivalent to the sequentially optimal one-period contracts (See also Gibbons and Murphy (1998) and Christensen et al. (2003)). However, it is worth noting that with commitment to long-term employment, the contracting parties can, through renegotiation, effectively contract on *any* forecast of future cash flow. This implies that the optimal long-term renegotiable contract conditioned only on the realized cash flow can *at least* achieve the same efficiency as the optimal renegotiation-proof contract conditioned on both stock price and realized cash flow. A natural conclusion then follows that if firm insiders obtains more accurate but unverifiable forecast of future firm value than stock price, the optimal *long-term renegotiable contract* contingent *only on realized cash flow* can outperform the *full commitment* contract contingent on *both the interim stock price*

and realized cash flow.

The paper’s analyses relate to the literature on optimal aggregation of performance measures in incentive contracts. Gjesdal (1981) shows that aggregation for valuation purposes typically differs from aggregation for stewardship/monitoring purpose. Feltham and Xie (1994) demonstrate that contracting on a market price that aggregates accounting information only, is likely to be inferior to a contract that uses the very same accounting measures directly (see also Paul (1992)). Dutta and Reichelstein (2005) examine the optimal aggregation of earnings and stock price in repeated agency setting with long-term full commitment contract, where they explicitly articulated the performance measure and performance benchmarking roles of stock price. However, none of these studies allow contract renegotiation.

It is well known that when long-term commitment is infeasible, the equilibrium outcome is determined by sequentially rational contracting decisions. Ex post efficient contracts, however, may well be inefficient from an ex ante perspective. In a two-period LEN-setting Indjejikian and Nanda (1999) and Christensen et al. (2003, 2005) show that limited commitment generally creates a welfare loss if performance measures are inter-temporarily correlated. Some recent work further examines the inclusion of non-financial variables in managerial performance measures.³ In the current model, the firm’s stock price reflects the market’s rational and aggregate forecast of future cash flow, rather than a mechanical signal of future payoff resulting from the manager’s far-sighted actions.

2 Model Setup

Consider a publicly traded firm that is established in period 0 and hires a manager to operate the firm for two more periods spanning three dates, $t = 0, 1, 2$. In period 1 (between $t = 0$ and 1), the manager engages in some actions whose cash flow impact extends into the second period. For example, he searches for investment opportunities, organizes the firm’s operations, recruits employees, and the myriad of other actions that will determine future firm profitability. These start-up activities are modeled as a choice of an unobservable effort e_1 that results in an uncertain future payoff v_1 . In period 2 (between $t = 1$ and 2), the manager undertakes another effort e_2 relating to operation and control activities, such as cost cutting or safeguarding assets, that results in an uncertain payoff v_2 .

We assume that shareholders and potential investors are risk neutral, but the manager is

³See, for example, Sliwka (2002), Dutta and Reichelstein (2003), and Fan and Li (2018).

risk averse with a negative exponential utility function:

$$U = -\exp \left[-\rho \cdot \sum_{t=1}^2 (s_t - \psi(e_t)) \right], \quad (1)$$

where s_t is the compensation payment received by the manager at date t and $\psi(e_t) = \frac{e_t^2}{2}$ is the manager's personal cost of effort. These effort costs may represent the manager's private utility cost in undertaking these actions. Alternatively, these efforts may be thought of as discretionary investments made by the manager, in which case $\psi(e_t)$ is the manager's opportunity cost of not diverting e_t into his private benefits. For simplicity, we ignore discounting. Without loss of generality, the manager's reservation utility from alternative employment is assumed to have a certainty equivalent of zero.⁴

The firm liquidates at date 2 when the total cash flow $V = v_1 + v_2$ is realized. V is verifiable for contracting purposes but its individual components v_1 and v_2 cannot be separately identified. Given managerial actions e_1 and e_2 , v_1 and v_2 are determined as

$$v_1 = e_1 + \theta + \tau; \text{ and} \quad (2)$$

$$v_2 = e_2 + \varepsilon, \quad (3)$$

where θ , τ , and ε are independent normally distributed noise terms that represent random fluctuations in firm value beyond the manager's control:

$$\theta \sim N(0, \sigma_\theta^2), \quad \tau \sim N(0, \delta \cdot \sigma_\theta^2), \quad \text{and} \quad \varepsilon \sim N(0, \sigma_\varepsilon^2).$$

Once the manager starts to take on the effort e_1 , relevant information about v_1 trickles in and partially resolves the payoff uncertainty. To characterize the unfolding of uncertain events over time, the random component in v_1 is split into two parts, θ and τ . The random component τ is *publicly* observed prior to second-period contract renegotiation, which captures the impact of certain macro economic conditions, e.g., industry shocks, political events, etc., on the payoff v_1 . To make the problem nontrivial, τ is not verifiable for contracting purposes. The remaining random component, $e_1 + \theta$, captures the uncertain component of firm value (e.g., relationships with large suppliers and customers) that is affected by the manager's effort. The constant δ measures the variability of τ relative to that of θ .

⁴I assume that the manager's outside option is independent of his observed performance. This is different from career concern models (Holmstrom (1999) and Gibbons and Murphy (1992)) in which managers have ex-post bargaining power and can capture the value of their reputations. Meyer and Vickers (1997) demonstrate that the incentive from such "career concerns" can be costlessly neutralized with explicit monetary incentives and therefore has no impact on effective effort choice or contractual efficiency.

Similar to Holmstrom and Tirole (1993), the paper explicitly models the process by which the stock market aggregates firm specific information from dispersed investors. There are n informed individuals who privately observe the remaining unknown payoff $e_1 + \theta$. Following Kyle (1985), the firm’s equity market consists of these n (risk-neutral) informed traders, some liquidity traders, and a risk-neutral market maker who sets the price. Based on her private information, informed trader i , $i \in \{1, 2, \dots, n\}$, places a market order z_i for the firm’s shares. Each informed investor’s order is designed to maximize her expected trading profits at the expense of liquidity traders. Let $u \sim N(0, \sigma_u^2)$ denote the quantity demanded by the liquidity traders with u uncorrelated to all other variables in the model. A market maker observes the aggregate order flow $f = \sum_{i=1}^n z_i + u$ for the firm’s shares. Based on the order flow f and the publicly observed noise term τ , the market maker sets date 1 price P to break even in expectation and provides the liquidity necessary to clear the market. With an efficient equity market, this interim stock price aggregates dispersed information from investors and would represent the best estimator of firm value.

Unlike Holmstrom and Tirole (1993) in which contracts and efforts are chosen at the outset, managerial efforts and contracts evolve and interact over time to determine firm value. Since the two parties cannot commit not to renegotiate the original contract upon observing updated information, e.g., the stock price P and the publicly observed noise τ , any second-period contract will be renegotiated to a sequentially Pareto optimal contract at the beginning of the second period.

For tractability, I focus on the LEN framework with linear contracts, exponential utility, and normally distributed noise terms (Holmstrom and Milgrom (1987, 1991)). Following Fudenberg and Tirole (1990), the firm owner has all the bargaining power in renegotiation. At the beginning of the second period, she makes a take-it-or-leave-it offer regarding a new contract, which the agent can choose to accept or reject. Because the manager cannot be forced to stay when his future expected payoff is negative, the contracting parties need to “settle up” each period. When the stock price P represents the best interim predictor of the future cash flow V , period-1 compensation would be optimally conditioned on the forward-looking stock price. However, once the long-term effort e_1 is sunk, the firm’s attention shifts to motivate the desired operational effort e_2 at minimal cost. With the manager’s interim participation constraint, it is equivalent to analyze the scenario in which the firm owner offers a new linear contract in every period that

holds the manager to his reservation utility at each (re)contracting date. Therefore,

$$s_1 = \alpha_1 + \lambda \cdot P, \text{ and} \tag{4}$$

$$s_2 = \alpha_2 + \beta \cdot V, \tag{5}$$

where α_1 and α_2 are the fixed payments in periods 1 and 2, respectively, λ is the period-1 bonus rate on the stock price P , and β is the period-2 bonus rate on the liquidating firm value V . Similar to Holmstrom and Tirole (1993), I assume that the compensation is made by inside owners. I also assume the manager is prohibited from trading in the firm's stock, which is consistent with insider trading laws in most countries given that the manager has privileged information regarding his own actions.

Expressing managerial compensation as a sequence of one-period contracts makes the two contracting roles of the stock price P transparent. In the first period, the forward-looking stock price reflects the consequences of the manager's choice of e_1 and acts as a *performance measure* for such effort. However, in the second period, the same stock price P acts as a *performance benchmark* because the firm owner would want to only reward the manager for the realized cash flow in excess of the expectation captured by the stock price. As the stock price is more informative of performance, it becomes a better performance measure for past managerial actions and a better performance benchmark for evaluating his future performance. While the performance measurement role better motivates the manager's effort, the performance benchmarking role discourages effort as the manager realizes that exhibiting good performance results in a more demanding performance benchmark in the future. When contract terms are chosen to be sequentially optimal, the performance benchmarking role of the interim stock price tends to be over-emphasized, making it difficult to motivate first period effort and undermining equilibrium contractual efficiency. Section 3 examines incentive and welfare implications of market monitoring in the limited commitment setting.

An alternative setting to the above is one wherein both contracting parties commit to stay for long-term employment but renegotiate contract terms upon observing new information. In this case, it is without loss of generality to study the *renegotiation-proof* contract that spans both periods. When stock price is the best forecast of future cash flow at the interim date, it can be verified that the optimal long-term renegotiation-proof contract is performance equivalent to the sequentially optimal one-period contracts (See also Gibbons and Murphy (1998) and Christensen et al. (2003)). However, it is worth noting that with commitment to long-term employment, the contracting parties can, through renegotiation, effectively contract on *any* forecast of future

cash flow. This naturally leads to the question of whether firm insiders can achieve the same (or even better) contractual efficiency with a long-term renegotiable contract based only on realized cash flow as the full commitment contract based on both stock price and realized cash flow. Section 4 examines this setting.

3 Main Analyses

This section investigates how incentives and contractual efficiency are affected by the forecast accuracy of the stock price. Since the two parties cannot commit not to renegotiate the original contract after observing the stock market price P , any second-period contract will be renegotiated to a sequentially Pareto optimal contract at the beginning of the second period. Following Fudenberg and Tirole (1990), the firm owner has all the bargaining power in renegotiation. At the beginning of the second period, she makes a take-it-or-leave-it offer regarding a new contract, which the agent can choose to accept or reject. Because this renegotiation is anticipated by both parties at the initial date, Fudenberg and Tirole (1990) show that the principal cannot do better than to offer a sequentially optimal (i.e., renegotiation-proof) contract when contracts are unrestricted. Christensen et al. (2005) and Sabac (2007) show that this result continues to hold for dynamic LEN models in which contracts are restricted to be linear.

3.1 Price formation

As a first step, I examine how the interim stock price aggregates dispersed information to reflect the manager's early effort choice, while taking the manager's future contract and action as given. The endogenous determination of these variables and their effect on the interim stock price are then derived in subsequent analysis. Let (\hat{e}_1, \hat{e}_2) denote the conjecture of the manager's private effort choices. Given the conjectured efforts, let $\pi_2(\hat{e}_2)$ denote the expected second period firm profit, i.e., cash flow v_2 net of compensation cost s_2 . In the equity market equilibrium, the market maker sets a fair price for the share given observed τ and the order flow $f = \sum_{i=1}^n z_i + u$ so that stock price values the firm at its expected value of future cash flows. The stock price at date 1 equals

$$P = \mathbb{E}[V - s_1 - s_2 | f, \tau, \hat{e}_1, \hat{e}_2]. \quad (6)$$

Lemma 1 *The equity market equilibrium is characterized as*

$$(1 + \lambda) \cdot P = \pi_2(\hat{e}_2) + \tau + \hat{e}_1 + \gamma \cdot \left(f - \frac{n \cdot \hat{e}_1}{(n + 1) \cdot \gamma} \right), \text{ where}$$

$$f = \sum_{i=1}^n z_i + u,$$

$$z_i = \frac{e_1 + \theta}{(n + 1) \cdot \gamma}, \text{ and}$$

$$\gamma^2 = \frac{n \cdot \sigma_\theta^2}{(n + 1)^2 \cdot \sigma_u^2}.$$

Given the order flow $f = \sum_{i=1}^n z_i + u$, the equilibrium stock price P can be expressed as

$$(1 + \lambda) \cdot P = \pi_2(\hat{e}_2) + \tau + \frac{\hat{e}_1}{n + 1} + \frac{n}{n + 1} (e_1 + \theta + \xi), \quad (7)$$

where $\xi \sim N\left(0, \frac{\sigma_\theta^2}{n}\right)$ is trading noise that is uncorrelated to θ , τ and ε .

Similar to standard Kyle model, γ denotes the sensitivity of stock price to order flow. Stock price aggregates all available information to provide the best unbiased estimate of firm value. The volatility in stock price, i.e., $\left(\frac{n}{n+1}\right)^2 \cdot (\sigma_\xi^2 + \sigma_\theta^2) + \sigma_\tau^2 = \left(\frac{n}{n+1} + \delta\right) \cdot \sigma_\theta^2$, is proportional to informed investors' informational advantage σ_θ^2 . This is because the impact of liquidity trading on stock price is directly tied to γ , the sensitivity of stock price to order flow. In the equilibrium described in Proposition 1, as traders' informational advantage increases, γ increases correspondingly, and this allows liquidity trading to have a bigger impact on stock price. In contrast when n goes up, γ decreases, and this limits the impact of liquidity trading on stock price.

The above result shows that in an informationally efficient market, the equilibrium stock price aggregates dispersed information to form an unbiased estimator of the liquidating firm value. However, as an *aggregate* forecast, the stock price does not separate out the random component of firm value (τ) and information assessing the manager's contribution (f) to firm value. Furthermore, as a *rational* forecast, the stock price relies on the prior belief of e_1 and is inherently weak at detecting shirking. In fact, the stock price is more sensitive to value-relevant noise τ than to the actual effort decision of the manager: as is evident from (7), P reacts one-to-one with the public noise term τ , but a unit increase in e_1 , the manager's actual effort choice, increases the expected stock price by $\frac{n}{n+1}$ unit only. This observation is consistent with Paul

(1992) who argues that an efficient equity market does not weight information properly for the purpose of managerial performance evaluation. This paper examines how managerial incentive changes when stock price incorporates more information about managerial performance. Notice that the more the information based trading (larger n), the more sensitive the expected price is to change in the actual effort e_1 . The key parameter for subsequent analysis is the intensity of information-based trading, sometimes referred to as the intensity of “market monitoring”:

$$\kappa_P = \frac{n}{n+1} = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\xi^2}. \quad (8)$$

Notice that while stock price aggregates value-relevant noise with incentive-relevant information, firm insiders can “read” from the stock price and “back out” the signal $e_1 + \theta + \xi$ of the manager’s effort. Because $e_1 + \theta + \xi$ best measures the manager’s effort, period 1 contract should ideally be conditioned on this signal instead of the stock price P . However, given that τ is unverifiable, this signal is not available for contracting purposes.

To conclude, the stock market aggregates all value-relevant information from dispersed investors in the market. With the equity market acting as an efficient information aggregator, the interim stock price becomes the best estimator of future firm value. Upon observing the stock price on date 1, the conditional distribution of the random cash flow component, $v_1 - \tau$, is:

$$E_1[v_1 - \tau] = (1 + \lambda)P - \pi_2(\hat{e}_2) - \tau = \frac{\hat{e}_1}{n+1} + \frac{n}{n+1}(e_1 + \theta + \xi); \text{ and} \quad (9)$$

$$\text{Var}_1[v_1 - \tau] = \text{Var}[\theta|\theta + \xi] = (1 - \kappa_P) \cdot \sigma_\theta^2, \quad (10)$$

where the subscript in the expectation and variance operators indicates that the time (date 1) at which the expectation is taken. The model’s main analysis focuses on the setting in which stock price is the best information available to firm insiders. Section 3 then examines the long-term renegotiable contract and allows for the possibility that firm insiders might have better but unverifiable information about future cash flow than that embedded in stock price.

3.2 Interim stock price as a performance benchmark

By backward induction, the analysis starts with the characterization of optimal (linear) contract for the second period. After the early effort e_1 is sunk and the interim stock price P has been observed, the manager’s expected utility from a given contract $s_2 = \alpha_2 + \beta \cdot V$ is $E_1 U = -\Gamma \cdot \exp(-\rho \cdot \text{CE}_2^m)$, where $\Gamma \equiv \exp\left\{-\rho \cdot \left(\alpha_1 + \lambda \cdot P - \frac{e_1^2}{2}\right)\right\}$ and

$$\text{CE}_2^m = \alpha_2 + \beta \cdot E_1^m[V] - \frac{e_2^2}{2} - \frac{\rho}{2} \cdot \beta^2 \cdot \text{Var}_1^m[V]. \quad (11)$$

In the above expression, the superscripts m indicates that the expectation is taken by the manager who knows his private effort choice e_1 .

Recall that the aggregate cash flow, $V = v_1 + v_2$, reflects the total payoffs from the manager's sequentially chosen efforts. Since the early effort e_1 is already sunk, the sequentially optimal contract s_2 must maximize the remaining surplus, denoted as $\pi_2(\beta) \equiv \mathbb{E}_1[v_2 - s_2]$, subject to the constraint that the manager's effort choice e_2 maximizes his continuation payoff characterized by (11).

For a given bonus rate β , the first-order condition for the manager's optimal effort choice yields:

$$e_2(\beta) = \beta. \quad (12)$$

The interim stock price provides a signal $e_1 + \theta + \xi$ of the unknown cash flow $v_1 - \tau$. Because e_1 is his private knowledge, the manager's expectation of the liquidating firm value is

$$\mathbb{E}_1^m[V] = e_2(\beta) + \mathbb{E}_1^m[v_1] = e_2(\beta) + \tau + e_1 + \kappa_P \cdot (\theta + \xi). \quad (13)$$

In contrast, the firm owner holds a conjecture \hat{e}_1 about the manager's past effort choice. Her expectation of the firm value is

$$\mathbb{E}_1[V] = e_2(\beta) + \mathbb{E}_1[v_1] = e_2(\beta) + \tau + (1 - \kappa_P) \cdot \hat{e}_1 + \kappa_P \cdot (e_1 + \theta + \xi).$$

The firm owner and the manager start with the same belief at date 0, but the manager's private effort choice in the first period leads to a potential belief difference about the liquidating firm value. This potential belief difference is critical to determine the actions along the equilibrium path. However, the manager's equilibrium actions have no impact on the stock price informativeness, so that the optimal dynamic contract can be solved while taking the the intensity of market monitoring as exogenous. In the current LEN framework,

$$\text{Var}_1^m[V] = \text{Var}_1[V] = \text{Var}[\theta|\theta + \xi] + \text{Var}[\varepsilon] = (1 - \kappa_P) \cdot \sigma_\theta^2 + \sigma_\varepsilon^2. \quad (14)$$

Anticipating the manager's optimal effort choice $e_2(\beta)$, the fixed pay α_2 is chosen to satisfy the manager's interim participation constraint. Because the manager cares about the total level of compensation but not its timing, a positive portion of the first-period fixed pay can always be deferred to the second period without affecting the manager's interim participation constraint. The *class* of the second-period fixed pay that satisfies the manager's interim participation constraint is

$$\alpha_2 = \Delta - \beta \cdot \mathbb{E}_1[V] + \frac{e_2(\beta)^2}{2} + \frac{\rho}{2} \cdot \beta^2 \cdot \text{Var}_1[V], \quad (15)$$

where the first term $\Delta \geq 0$ represents the part of the first-period fixed pay deferred to the second period. Notice if $\Delta = 0$, then the manager just breaks even for the second period. This deferred compensation Δ can act as a “lock-in” mechanism to prevent the manager from quitting after the first period. The second term, $-\beta \cdot E_1[V]$, reflects the performance benchmarking role of stock price. With this fixed pay adjustment, the manager’s pay effectively increases with the excess of realized aggregate cash flow over its expectation as captured by the stock price. The larger the bonus rate β , the more sensitive the second period wage is to this “performance surprise” relative to the stock price.

Let $s_2^* = \alpha_2^* + \beta^* \cdot v_2$ denote the sequentially optimal contract. Given (15), the manager’s certainty equivalent (11) becomes:⁵

$$\begin{aligned} CE_2^*(e_1) &= \Delta + \beta^* \cdot (E_1^m[V] - E_1[V]) \\ &= \Delta + \beta^* \cdot (1 - \kappa_P) \cdot (e_1 - \hat{e}_1). \end{aligned} \quad (16)$$

The certainty equivalent in (16) depends on the difference between the manager’s actual effort choices e_1 and the firm owner’s conjecture \hat{e}_1 . This is because the manager updates his beliefs knowing his private effort choice e_1 , whereas the firm owner sets the fixed pay α_2^* given her conjecture of the manager’s effort. In choosing e_1 , the manager takes the firm’s conjectures as given. As a result, his continuation utility increases in e_1 *deterministically*. In equilibrium, the firm’s conjectures must be correct ($e_1 = \hat{e}_1$) and the manager’s second-period certainty equivalent is simply equal to Δ . Even though there is no belief difference in equilibrium, the manager’s effort incentive is affected by the possibility of influencing his future performance expectation and thus his continuation payoff. I refer to the impact of the manager’s effort choice e_1 on his continuation payoff $CE_2^*(e_1)$ as the *implicit* incentive, denoted as λ^I .

The sequentially optimal bonus rate β^* can be solved from the first-order condition for the maximization of the second-period joint surplus $\pi_2(\beta)$:

$$\max_{\beta} \pi_2(\beta) \equiv e_2(\beta) - \frac{e_2(\beta)^2}{2} - \frac{\rho}{2} \cdot \beta^2 \cdot \text{Var}_1[V], \quad (17)$$

which leads to the following proposition:

⁵For illustrative purpose, I assume here that the manager can commit to stay for the second period as long as the second-period contract satisfies their reservation utilities. In the absence of this commitment, the manager will have an incentive to quit after maximizing their first-period utility, i.e., shirking in the first period, while anticipating that he will leave in the second. To prevent this “take-the-money-and-run” strategy, the firm can defer some first-period fixed pay to the second period, given that the manager only care about the level of total compensation but not its timing. This deferred compensation acts as “lock-in” mechanism but has no impact on incentives or welfare (see also Dutta and Fan (2016) and Dutta and Li (2018)).

Proposition 1 (i) *The optimal renegotiation-proof contract for the second period varies κ_P and entails*

$$\beta^*(\kappa_P) = \frac{1}{1 + \rho \cdot \text{Var}_1[V]} \text{ and } \pi_2^*(\kappa_P) = \frac{\beta^*(\kappa_P)}{2}. \quad (18)$$

Both the sequentially optimal bonus rate $\beta^(\kappa_P)$ and equilibrium second period profit $\beta^*(\kappa_P)$ increase with κ_P .*

(ii) *The implicit incentive λ^I for the manager's early effort e_1 varies with κ_P and is given by*

$$\lambda^I(\kappa_P) = \frac{1 - \kappa_P}{1 + \rho \cdot \text{Var}_1[V]}, \quad (19)$$

which decreases in κ_P and $\lambda^I(\kappa_P) \rightarrow 0$ as $\kappa_P \rightarrow 1$. In words, the implicit incentive decreases in market monitoring and approaches zero as the stock price perfectly resolves the remaining uncertainty of the manager's effort payoff.

To understand the intuition of why the implicit incentive λ^I decreases with the market monitoring parameter κ_P , notice that the conditional expectation of the future cash flow increases in the observed stock price P . Because $E_1[v_1] = \tau + (1 - \kappa_P) \cdot \hat{e}_1 + \kappa_P \cdot (e_1 + \theta + \xi)$ and the fixed pay adjusts with this updated expectation to meet the manager's outside options, for a unit increase in the long-term effort e_1 , the manager's expected total pay will increase by only $\beta^* \cdot (1 - \kappa_P)$ units. As information-based trading makes the stock price more informative of v_1 , the firm relies more on the stock price in estimating the long-term effort return. Consequently, the expected fixed pay α_2 is more inversely related to the actual long-term effort choice e_1 . In the extreme case of $\kappa_P = 1$, the stock price perfectly reveals the payoff from the long-term effort and the firm owner completely captures the bonus pay associated with the manager's long-term effort, in which case the manager's continuation payoff is independent of e_1 . Therefore, a more informative leading indicator reduces the implicit incentive for the long-term effort e_1 . The component $-\beta^* \cdot \kappa_P$ in the implicit incentive $\beta^* \cdot (1 - \kappa_P)$ for the long-term effort b reflects this performance benchmarking effect.

The above analyses have taken β^* as fixed. It is obvious from (18) that β^* increases in the informativeness of the stock price. As a result, as κ_P increases, the second-period incentive becomes higher-powered, which would improve the positive implicit incentive on e_1 . However, because the effect of κ_P on β^* is of second order compared with its effect on $1 - \kappa_P$, the overall implicit incentive on e_1 (i.e., $\beta^* \cdot (1 - \kappa_P)$) always decreases in κ_P .

3.3 Interim stock price as a performance measure

Stock price acts as a performance measure for the manager's long-term effort in the first period contract. To facilitate the analysis, it is convenient to normalize P to the following equivalent performance measure:

$$\begin{aligned}\bar{P} &= \kappa_P^{-1} \cdot (1 + \lambda) \cdot P \\ &= e_1 + \kappa_P^{-1} \cdot \tau + \xi + \theta + \Gamma,\end{aligned}\tag{20}$$

where Γ is a constant that depends on future contract parameters. Then the ex ante variance of \bar{P} is

$$\text{Var}_0[\bar{P}] = (\kappa_P^{-1} \cdot \delta + 1) \cdot \kappa_P^{-1} \cdot \sigma_\theta^2.\tag{21}$$

The first-period compensation contract $s_1 \equiv \alpha_1 + \lambda \cdot P$ can thus be re-written as

$$s_1 = \alpha_1 + \bar{\lambda} \cdot \bar{P},\tag{22}$$

where

$$\bar{\lambda} = \kappa_P \cdot \frac{\lambda}{1 + \lambda}.\tag{23}$$

There is a one-to-one correspondence between $\{\lambda, P\}$ and $\{\bar{\lambda}, \bar{P}\}$. Specifically, λ measures the *nominal* impact on bonus from a unit increase in stock price P , and $\bar{\lambda}$ measures the *real* impact on bonus from a unit increase in the manager's effort e_1 . Because stock price incorporates rational conjecture and doesn't move one-to-one with the manager's effort choice, the real incentive $\bar{\lambda}$ is less than the nominal incentive λ . The variance $\text{Var}_0[\bar{P}]$ is thus the noise-to-signal ratio of stock price in reflecting the manager's first-period effort.

The manager's expected utility at the time of choosing the long-term effort is characterized as

$$\text{CE}_1 = \alpha_1 + \bar{\lambda} \cdot e_1 - \frac{e_1^2}{2} - \frac{\rho}{2} \cdot \bar{\lambda}^2 \cdot \text{Var}_0[\bar{P}] + \text{CE}_2^*(e_1).\tag{24}$$

The manager's effort choice e_1 impacts both his current period bonus through explicit incentive as well as his continuation payoff through implicit incentive. The first order condition for the optimal effort is

$$e_1(\Lambda) = \Lambda \equiv \bar{\lambda} + \lambda^I,\tag{25}$$

where

$$\Lambda \equiv \bar{\lambda} + \lambda^I\tag{26}$$

can be viewed as the *effective* incentive rate for e_1 . This effective rate consists of two components. The first term $\bar{\lambda}$ is the *explicit* incentive rate that captures the sensitivity of the manager's period-1 price-based compensation with his actual effort choice. The second term

$$\lambda^I = \frac{1 - \kappa_P}{1 + \rho \cdot \text{Var}_1[V]} \quad (27)$$

is the *implicit* incentive from the sequentially optimal second-period contract.

The principal's objective function becomes choosing Λ to maximize

$$\pi_1 = \Lambda - \frac{\Lambda^2}{2} - \frac{\rho}{2} \cdot \bar{\lambda}^2 \cdot \text{Var}_0[\bar{P}]. \quad (28)$$

In the above expression, the long-term effort incentive is determined by the *effective* bonus rate Λ , whereas the risk premium is determined by the explicit bonus rate $\bar{\lambda}$. In constructing the optimal first period contract $s_1^* = \alpha_1^* + \bar{\lambda}^* \cdot \bar{P}$, the firm owner chooses Λ^* (or equivalently $\bar{\lambda}^*$ and λ^*) to maximize the above expression, taking κ_P as well as the implicit incentive λ^I as given. The optimal α_1^* is then set to satisfy the manager's participation constraint.

Proposition 2 *In the optimal renegotiation-proof contract, the effective incentive rate Λ^* for the manager's long-term effort varies with κ_P and is given by*

$$\Lambda^*(\kappa_P) = \frac{1 + \lambda^I \cdot \rho \cdot \text{Var}_0[\bar{P}]}{1 + \rho \cdot \text{Var}_0[\bar{P}]} \quad (29)$$

The effective incentive $\Lambda^(\kappa_P)$ is maximized at an interior value of $\kappa_P < 1$ whenever σ_θ^2 is above a threshold value. Specifically,*

(i) *When $\delta = 0$,*

(a) *if $\sigma_\theta^2 < \sigma_\varepsilon^2$, $\Lambda^*(\kappa_P)$ increases in κ_P for all $\kappa_P \in (0, 1)$;*

(b) *if $\sigma_\theta^2 > \sigma_\varepsilon^2$, $\Lambda^*(\kappa_P)$ first increases and then decreases in κ_P ;*

(ii) *For all $\delta > 0$, there exists a unique threshold $\hat{\sigma}_\theta^2 > \sigma_\varepsilon^2$ s.t. for all $\sigma_\theta^2 > \hat{\sigma}_\theta^2$, the effective incentive $\Lambda^*(\kappa_P)$ decreases in κ_P for κ_P sufficiently close to one. The threshold $\hat{\sigma}_\theta^2$ increases in σ_ε^2 .*

This result follows from Proposition 1 which demonstrates that more information-based trading (larger κ_P) improves the explicit incentive provision but impairs the implicit incentive. When $\kappa_P = 0$, the interim stock price P contains no incentive relevant information, and the

implicit incentive λ^I is the sole motivator for the manager's long-term effort. When $\kappa_P = 1$, the interim P fully resolves the uncertainty in v_1 , the implicit incentive is destroyed, and the long-term effort can only be induced with explicit bonus on the stock price P . The explicit incentive is more expensive as the payoff v_1 becomes more uncertain ex ante (larger σ_θ^2). On the other hand, when contracting parties better understand the context of future actions (smaller σ_ε^2), the future bonus rate β^* increases, leading to stronger implicit incentive λ^I . Thus when σ_θ^2 is large relative to σ_ε^2 , long-term effort incentive should ideally be provided through a combination of implicit and effective incentives, implying that the effective effort incentive is highest when $\kappa_P < 1$; that is, once the market monitoring becomes sufficiently intense, additional increase in κ_P lowers the implicit incentive more than the offsetting improvement in explicit incentive. Proposition 2 shows that in the special case of $\delta = 0$, if $\sigma_\theta^2 > \sigma_\varepsilon^2$, then the manager's long-term effort incentive first increases and then decreases in κ_P . Because τ represents value-relevant but incentive-irrelevant noise in stock price, the larger is this noise (larger δ), the more costly is the explicit incentive and the more likely that effort incentive is maximized at an interior $\kappa_P < 1$. The analysis shows that for all $\delta > 0$, as long as σ_θ^2 is larger than a threshold $\hat{\sigma}_\theta^2 > \sigma_\varepsilon^2$, effort incentive is maximized at $\kappa_P < 1$. Not surprisingly, this threshold is larger than σ_ε^2 and increases in σ_ε^2 .

3.4 Equilibrium firm profit and market monitoring

Now I investigate how market monitoring as captured by the intensity of information-based trading in stock market affects contractual efficiency. Let

$$\pi_1^* = \Lambda^* - \frac{(\Lambda^*)^2}{2} - \frac{\rho}{2} \cdot (\Lambda^* + \lambda^I)^2 \cdot \text{Var}_0[\bar{P}] \quad (30)$$

denote the equilibrium surplus from the manager's long-term effort and $\Pi(\lambda^I, \kappa_P)$ denote expected total firm profit in equilibrium. Then

$$\Pi = \pi_1^* + \pi_2^*. \quad (31)$$

To maximize the expected firm profit, the optimal Λ^* are chosen taking κ_P as well as the implicit incentives λ^I as given. By the Envelope Theorem, the derivative of the equilibrium firm profit with respect to κ_P is

$$\frac{d\Pi}{d\kappa_P} = \underbrace{\frac{\partial\pi_2^*}{\partial\kappa_P} + \frac{\partial\pi_1^*}{\partial\kappa_P}}_{\text{direct effect (+)}} + \underbrace{\frac{\partial\pi_1^*}{\partial\lambda^I} \times \frac{d\lambda^I}{d\kappa_P}}_{\text{indirect effect (-)}} \quad (32)$$

In the above expression, $\frac{\partial \pi_2^*}{\partial \kappa_P}$ and $\frac{\partial \pi_1^*}{\partial \kappa_P}$ capture the *direct* effects of the forecast accuracy κ_P on the surplus from the operational- and long-term efforts, respectively. Both of these effects are positive because, *ceteris paribus*, a higher κ_P improves the functions of stock price as both a performance benchmark for period 2 and as a performance measure for period 1. On the other hand, $\frac{\partial \pi_1^*}{\partial \lambda^I} \times \frac{d\lambda^I}{d\kappa_P}$ capture the *indirect* effects of κ_P on firm profit through its impact on the implicit incentives λ^I . It is obvious that $\frac{\partial \pi_1^*}{\partial \lambda^I} > 0$ from (29), because the manager's long-term effort incentives increase in the implicit incentives, which reduces the required risk premium for inducing any given level of efforts. However, as shown in Proposition 1, $\frac{d\lambda^I}{d\kappa_P}$ is negative. Hence, the net effect of κ_P on firm profit is ambiguous.

The following proposition characterizes how Π , the equilibrium firm profit changes with the intensity of market monitoring κ_P :

Proposition 3 *If v_1 , the cash flow from the first-period effort, is subject to moderate risk, contractual efficiency may be maximized with less than perfect market monitoring ($\kappa_P < 1$). Furthermore, the more the noise in v_1 is composed of the publicly observable noise τ , or the more the noise in v_2 , the more likely that firm profit is maximized at an interior $\kappa_P < 1$. Formally,*

- (i) *When $\sigma_\theta^2 \in \left(\frac{\sigma_\varepsilon^2}{1+\delta}, \left(1 + \frac{\delta}{1+\delta}\right) \sigma_\varepsilon^2 + \frac{2\delta}{\rho \cdot (1+\delta)} \right)$, Π is maximized at an interior $\kappa_P \in (0, 1)$.*
- (ii) *As δ or σ_ε^2 increase, $\frac{\sigma_\varepsilon^2}{1+\delta}$ decreases and $\left(1 + \frac{\delta}{1+\delta}\right) \sigma_\varepsilon^2 + \frac{2\delta}{\rho \cdot (1+\delta)}$ increases so that the above range enlarges..*

More information-based trading in the stock market always improves the incentive provision for the second-period effort e_2 but can hurt the incentive of the first-period effort e_1 . Therefore, as κ_P increases, the firm profit from e_2 always improves, but the firm profit from e_1 can decrease. As demonstrated in Proposition 2, when σ_θ^2 is small relative to σ_ε^2 , e_1 tends to increase in κ_P . It thus follows that for relatively small σ_θ^2 , both π_1^* and π_2^* increase in κ_P so that total firm profit Π increases with κ_P for all $\kappa_P \in (0, 1)$. One might conjecture that firm profit is maximized at an interior $\kappa_P < 1$ when σ_θ^2 is relatively large so that explicit incentive is more costly. This turns out to be only partially correct, because for sufficiently large σ_θ^2 (relative to σ_ε^2), the second-period efficiency gain from a more informative interim price can be large enough to outweigh the efficiency loss from reduced first-period effort. Therefore, only when the first-period effort payoff v_1 is subject to moderate risk, it is possible that firm profit is maximized at an intermediate level of information based trading. This range enlarges as δ increases, i.e., when the stock price

contains more value-relevant but incentive irrelevant noise, or as σ_ε^2 increases, i.e., when the second period operational environment is noisy so that the implicit incentive is less important.

Corollary 1 For $\sigma_\theta^2 \in \left(\frac{\sigma_\varepsilon^2}{1+\delta}, \min \left(\left(1 + \frac{\delta}{1+\delta}\right) \sigma_\varepsilon^2 + \frac{2\delta}{\rho \cdot (1+\delta)}, \hat{\sigma}_\theta^2 \right) \right)$, where $\hat{\sigma}_\theta^2$ is the threshold value identified in Proposition 2, equilibrium firm profit decreases with the price informativeness κ_P whereas the manager's equilibrium efforts e_1^* and e_2^* increase with κ_P .

This corollary follows by comparing Propositions 2 and 3. It highlights that under limited commitment, firm profit and managerial effort moves in a non-synchronized manner. As information based trading intensifies (κ_P increases) so that the interim price impounds more incentive- and value-relevant information, although the firm owner may choose to exploit the cheaper marginal cost of the explicit incentive and induce more long-term effort, the net firm profit can decrease due to the impaired implicit incentive. This is because managerial effort is determined by the trade-off between marginal risk premium and marginal effort cost. However, compared with the marginal risk premium that determines the long-term effort incentive, the total risk premium – due to its convexity – is relatively more sensitive to changes in the implicit incentive rate than to changes in price informativeness. Consequently, for $w \in \left(\frac{\sigma_\varepsilon^2}{1+\delta}, \min \left(\left(1 + \frac{\delta}{1+\delta}\right) \sigma_\varepsilon^2 + \frac{2\delta}{\rho \cdot (1+\delta)}, \hat{\sigma}_\theta^2 \right) \right)$, an equilibrium emerges in which as κ_P increases, the manager becomes more productive, but his wage increases even more, reducing firm profits and undermining contractual efficiency.

4 Long-term renegotiable contract

In this section, I examine whether the impact of a more informative stock price on managerial incentives and welfare can be extended to alternative long-term contract settings. The main setting examines the sequence of sequentially optimal contracts wherein the second-period fixed pay is adjusted based on updated information. The manager's implicit incentive arises from the impact of the long-term effort on future cash flow, and is amplified by future period pay-performance sensitivity. A natural question is that whether the same levels of managerial incentives and welfare can be obtained when both the firm and the manager can commit to the two-period employment relationship but the contract terms are subject to renegotiation. It is without loss of generality to restrict the analysis to a long-term renegotiation-proof contract that exactly meets the manager's date 0 reservation utility. By sequential optimality, the renegotiation-proof second-period bonus rate must again be fixed at β^* . Let $\tilde{s} = \tilde{\alpha} + \tilde{\lambda} \cdot P + \beta^* \cdot V$

denote the long-term renegotiation-proof contract. Given the correlation between P and V , the variance of the total compensation from \tilde{s} can be written as

$$\begin{aligned}\text{Var}_0[\tilde{s}] &= \text{Var}_0[\tilde{\lambda} \cdot \bar{P} + \beta^* \cdot V] \\ &= \text{Var}_0\left[\left(\tilde{\lambda} + \kappa_P \cdot \beta^*\right) \cdot \bar{P} + \beta^* \cdot (V - \kappa_P \cdot \bar{P})\right].\end{aligned}\quad (33)$$

Notice that the random variables \bar{P} and $V - \kappa_P \cdot \bar{P}$ are uncorrelated, and the variance of the second-period “value innovation” ($V - \kappa_P \cdot \bar{P}$) is simply $\text{Var}_1[V] = \sigma_\varepsilon^2 + (1 - \kappa_P) \cdot \sigma_\theta^2$. This variance reduction effect captures the “performance benchmarking” role of the stock price for the second period. The manager’s ex ante expected utility under the long-term renegotiation-proof contract is then characterized by the following certainty equivalent expression:

$$\begin{aligned}\text{CE}_1 &= \tilde{\alpha} + \tilde{\lambda} \cdot e_1 + \beta^* \cdot (e_2 + e_1) - \frac{1}{2} (e_1^2 + e_2^2) \\ &\quad - \frac{\rho}{2} \cdot \left[\left(\tilde{\lambda} + \kappa_P \cdot \beta^*\right)^2 \cdot \text{Var}_0[\bar{P}] + (\beta^*)^2 \cdot \text{Var}_1[V] \right].\end{aligned}\quad (34)$$

It is easy to see from the above expression that the incentive provisions are intertemporally related due to the correlation between the performance measures. Because the second-period incentive rate has to be sequentially optimal, more informative stock price increases the second-period bonus rate but impairs the implicit incentive for the long-term effort. Therefore, the effective bonus rates and welfare are exactly the same as those obtained in the main setting. Since similar equivalence result has been established by prior studies on renegotiation with correlated performances, I keep the discussion brief and state the following Lemma:⁶

Lemma 2 *When both the firm and the manager commit to the two-period employment relationship, the optimal long-term renegotiation-proof contract $\tilde{s}^* = \tilde{\alpha}^* + \tilde{\lambda}^* \cdot \bar{P} + \beta^* \cdot V$ entails*

$$\tilde{\lambda}^*(\kappa_P) = \Lambda^*(\kappa_P) - \beta^*(\kappa_P), \quad (35)$$

where $\Lambda^*(\kappa_P)$ and $\beta^*(\kappa_P)$ are functions of κ_P as derived in Proposition 1 and 2. Moreover, \tilde{s}^* induces the same level of managerial efforts and gives the firm the same level of expected profit as the the sequence of sequentially optimal contracts $\{s_1^*, s_2^*\}$ in the main setting that satisfies the manager’s interim participation constraint.

⁶See, among others, Gibbons and Murphy (1992), Christensen et al. (2003), and Sabac (2007). However, in these studies, there is no forward-looking information that is informative about the manager’s long-term effort and future cash flow.

In the above renegotiation-proof contract, the stock price and the realized cash flow combine to provide incentives for the long-term effort. Due to the correlation between P and V , the same level of efficiency can be obtained by allowing the two parties to initially sign a bonus contract based only on V and then negotiate away the correlated risk given the observed price P since at that point the long-term effort is already sunk. Thus firm insiders can effectively make any information observed about firm value effectively contractible by signing a long-term renegotiable contract. In particular, if they observe additional unverifiable signals besides stock price, they can effectively contract on their better but unverifiable information through renegotiation. Such an arrangement mimics the outcome under the complete contracting when their information is contractible. A result of this type was first explored by Hermalin and Katz (1991), who show in a one-period setting that renegotiation based on imperfect signals of the manager's action improves contractual efficiency. The following analysis extends their conclusion into a multiperiod setting with long-term effort and sequentially optimal contracts.

The model's main analysis focuses on the setting in which stock price is the best information available to firm insiders. However, firm insiders might have better but unverifiable information about future cash flow than that embedded in stock price. Specifically, firm insiders also can combine the effort signal inferred from stock price with their own information to form a more accurate signal of $e_1 + \theta$. Denote the firm insider's more accurate signal as $v \equiv e_1 + \theta + \xi'$, where ξ' is a normally distributed mean zero error term. Analogous to κ_P , let $\kappa_v \equiv \frac{\sigma_v^2}{\sigma_v^2 + \sigma_{\xi'}^2}$. Then the firm owner' updated belief of the future total cash flow at date 1 becomes:

$$\mathbb{E}_1[V] = \pi_2^* + \tau + (1 - \kappa_v) \cdot \hat{e}_1 + \kappa_v \cdot v; \text{ and} \quad (36)$$

$$\text{Var}_1[V] = \sigma_{\xi}^2 + (1 - \kappa_v) \cdot \sigma_{\theta}^2. \quad (37)$$

Let $\bar{s} = \bar{\alpha} + \bar{\beta} \cdot V$ denote the initial long-term renegotiable contract, and let v denote the best information firm insiders have with regard to $e_1 + \theta$ upon observing stock price, the public signal τ , and their own piece of unverifiable information. Notice that this initial contract is not necessarily renegotiation-proof. Its role is to set up the manager's reservation utility at the time of renegotiation as a function of the realized v . After the firm insiders form their own assessment of $e_1 + \theta$, the expected utility to the manager from the remaining portion of the contract, i.e., the second-period bonus pay $\bar{\beta} \cdot V$, is characterized by the certainty equivalent:

$$\text{CE}_2^m = \bar{\beta} \cdot [\bar{e}_2 + \mathbb{E}_1^m(v_1)] - \frac{(\bar{e}_2)^2}{2} - \frac{1}{2} \cdot \rho \cdot (\bar{\beta})^2 \cdot \text{Var}_1[V], \quad (38)$$

where $\bar{e}_2 = e_2(\bar{\beta})$ denotes the induced effort under the initial contract. Because the manager's

certainty equivalent under the initial contract depends on v , renegotiation occurs in equilibrium, which makes v effectively contractible.

Under sequential optimality, the bonus rate for the second period will be renegotiated to β^* . To induce contract acceptance, the firm will also offer an additional fixed pay $\Delta\bar{\alpha}$ such that, given the updated assessment v , the manager has the same certainty equivalent under the renegotiated contract as the initial contract. In other words,

$$\begin{aligned}\Delta\bar{\alpha} &= \bar{\beta} \cdot [\bar{e}_2 + E_1(v_1)] - \frac{\bar{\beta}^2}{2} - \frac{1}{2} \cdot \rho \cdot (\bar{\beta})^2 \cdot \text{Var}_1[V] \\ &\quad - \beta^* \cdot [e_2^* + E_1(v_1)] + \frac{(\beta^*)^2}{2} + \frac{1}{2} \cdot \rho \cdot (\beta^*)^2 \cdot \text{Var}_1[V] \\ &= (\bar{\beta} - \beta^*) \cdot (\kappa_v \cdot v + \tau + (1 - \kappa_v) \cdot \hat{e}_1) + \Phi,\end{aligned}\tag{39}$$

where Φ is a constant that is unrelated to the firm owner's updated expectation about V .

Since the manager knows his actual choice of e_1 , the manager's second-period certainty equivalent from the renegotiated contract $\Delta\bar{\alpha} + \beta^* \cdot V$ is given by:

$$CE_2^m(e_1) = \Phi' + \bar{\beta} \cdot (\kappa_v \cdot v + \tau) + \beta^* \cdot (1 - \kappa_v) \cdot (e_1 - \hat{e}_1)\tag{40}$$

where Φ' is again a constant that is unrelated to either party's updated expectation. Notice that the last term in the above expression capture the same implicit incentives as in the main setting, while the second term shows that the anticipated change in the fixed pay provides incentives for e_1 . Therefore, offering a renegotiable long-term contract when v is not contractible can mimic the outcome of the long-term renegotiation-proof contract with a contractible v . Given that $\kappa_v \geq \kappa_P$, it is possible that the optimal long-term renegotiable contract contingent only on realized cash flow V can outperform the optimal long-term full commitment contract contingent on both the realized cash flow V and the interim stock price P :

Proposition 4 *When the insiders' information v is not verifiable for contracting purposes, the following results hold:*

(i) *The optimal long-term renegotiable contract $\bar{s}^* = \bar{\alpha}^* + \bar{\beta}^* \cdot V$ entails*

$$\bar{\beta}^* = \kappa_v^{-1} \cdot [\Lambda^*(\kappa_v) - \lambda^I(\kappa_v)],\tag{41}$$

and induces the same level of managerial efforts and gives the firm the same level of expected profit as when v is contractible.

- (ii) When κ_v is sufficiently larger than κ_P , the optimal long-term renegotiable contract \bar{s}^* conditioned on the realized cash flow V outperforms the optimal long-term full commitment contract conditioned on both P and V .

5 Conclusion

In this paper, I study the impact of stock price informativeness on incentives and contractual efficiency in a dynamic agency setting where the manager is held to his reservation utility at the beginning of each contracting period. In the model, the firm's stock price is the outcome of self-interested and speculative trading. However, the stock price, as an *aggregate* and *rational* forecast of future firm value, is insensitive to the actual effort decision of the manager and includes incentive irrelevant noise. Under multi-period contracting, the interim stock price acts as both the performance measure for the current period and the performance benchmark for the subsequent period. When contracts are renegotiated in light of updated beliefs of future firm value, these two conflicting roles are not optimally coordinated over time. Consequently, there are many situations where stock price informativeness negatively impacts the manager's long-term effort incentive and contractual efficiency.

With these results in place, I further examine the long-term renegotiable contract that, through renegotiation, allows the contracting parties to contract on any unverifiable signals of future cash flow at the interim date. When firm insiders can combine stock price with their private signals to form a more accurate assessment of terminal firm value, a long-term renegotiable contract based solely on terminal firm value can outperform the long-term full commitment contract contingent on both terminal firm value and interim stock price.

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Appendix

This appendix contains the proofs of all formal results stated in the paper.

Proof of Lemma 1. The manager's first-period compensation increases with stock price – for every dollar of the interim stock price, the manager will be paid λ . Let

$$P' = (1 + \lambda) \cdot P.$$

Conjecture a linear pricing rule of the form

$$P' = \pi_2(\hat{e}_2) + \tau + \gamma \cdot f,$$

and a linear trading strategy of the form

$$z_i = b_0 + b_1 \cdot (e_1 + \theta).$$

The i^{th} informed trader's maximization problem is

$$\max_{z_i} E[\{V - P'\} z_i | \tau, e_1 + \theta].$$

Taking the conjectured pricing rule P' and each of the other $n - 1$ informed traders' conjectured orders, and noting that $E[u] = 0$, the profit maximization condition is:

$$\max_{z_i} \{V - \tau - \pi_2(\hat{e}_2) - \gamma \cdot ([n - 1] \cdot [b_0 + b_1 \cdot (e_1 + \theta)] + z_i)\} \cdot z_i.$$

Noting that $V - \tau - \pi_2(\hat{e}_2) = e_1 + \theta$ and solving the informed investors' problem yields:

$$z_i = \frac{e_1 + \theta - \gamma \cdot [n - 1] \cdot [b_0 + b_1 \cdot (e_1 + \theta)]}{2\gamma}.$$

Consider the proposed linear pricing rule $z_i = b_0 + b_1 \cdot (e_1 + \theta)$, compare coefficients with the above, and simplifying,

$$b_0 = 0 \text{ and } b_1 = \frac{1}{(n + 1) \cdot \gamma}.$$

Next consider the market maker. His pricing rule is chosen to ensure zero expected profit:

$$P' = \pi_2(\hat{e}_2) + \tau + \gamma \cdot f.$$

with

$$\gamma = \frac{\text{Cov}(V - \tau, f)}{\text{Var}(f)}.$$

Substituting for the conjectured trades:

$$\gamma = \frac{nb_1\sigma_\theta^2}{n^2b_1^2\sigma_\theta^2 + \sigma_u^2}.$$

Jointly solving the expression for b_1 and λ yields

$$\gamma^2 = \frac{n\sigma_\theta^2}{(n+1)^2 \cdot \sigma_u^2};$$

and

$$b_1 = \frac{1}{(n+1) \cdot \gamma} = \sqrt{\frac{\sigma_u^2\sigma_\theta^2}{n}}.$$

Using the equilibrium trading strategies z_i in the pricing function derived previously,

$$P' = \pi_2(\hat{e}_2) + \tau + \gamma \cdot \left[\frac{n \cdot (e_1 + \theta)}{(n+1) \cdot \gamma} + u \right].$$

Denoting $\frac{n+1}{n}\gamma u = \xi$ and noting that ξ is uncorrelated to all other variables (follows from properties of u), the above can be rewritten as

$$P' \equiv (1 + \lambda) \cdot P = \pi_2(\hat{e}_2) + \tau + \frac{n}{n+1} \cdot [e_1 + \theta + \xi].$$

Using the expression for γ , the variance of ξ is given by

$$\text{Var}(\xi) = \frac{(n+1)^2 \gamma^2 \sigma_u^2}{n^2} = \frac{\sigma_\theta^2}{n}.$$

■

Proof of Proposition 1. Substituting the optimal second period effort response $e_2(\beta) = \beta$ into the expression of $\pi_2(\beta)$, the optimal β^* can then be solved from the first order condition for the maximization of $\pi_2(\beta)$:

$$1 - (1 + \rho \cdot \text{Var}_1[V]) \cdot \beta^* = 0.$$

Given that

$$\begin{aligned} \text{Var}_1[V] &= \text{Var}[\varepsilon] + \text{Var}_1[v_1] \\ &= \sigma_\varepsilon^2 + (1 - \kappa_P) \cdot \sigma_\theta^2, \end{aligned}$$

the expression for the sequentially optimal bonus coefficient in the second period β^* is:

$$\begin{aligned} \beta^* &= \frac{1}{1 + \rho \cdot \text{Var}_1[V]} \\ &= \frac{1}{1 + \rho \cdot [\sigma_\varepsilon^2 + (1 - \kappa_P) \cdot \sigma_\theta^2]}. \end{aligned}$$

It can be easily verified that

$$\begin{aligned}\lambda^I &= \beta^* \cdot (1 - \kappa_P) \\ &= \frac{1}{1 + \rho \cdot [\sigma_\varepsilon^2 + (n+1)^{-1} \cdot \sigma_\theta^2]} \frac{1}{n+1}\end{aligned}$$

strictly decreases in n and approaches zero as $n \rightarrow \infty$; that is, $\frac{d\lambda^I}{d\kappa_P} < 0$ and $\lambda^I \rightarrow 0$ as $\kappa_P \rightarrow 1$.

■

Proof of Proposition 2. From the FOC for maximizing (28) and given that $\Lambda^* = \bar{\lambda} + \lambda^I$, the first-order condition yields

$$\Lambda^* = \frac{1 + \lambda^I \cdot \rho \cdot \text{Var}_0[\bar{P}]}{1 + \rho \cdot \text{Var}_0[\bar{P}]}.$$

Because $\kappa_P = \frac{n}{n+1}$ and $\text{Var}[\bar{P}] = (\kappa_P^{-1} \cdot \delta + 1) \cdot \kappa_P^{-1} \cdot \sigma_\theta^2$, when $\delta = 0$,

$$\Lambda^* = \frac{1 + \frac{(1-\kappa_P) \cdot \rho \sigma_\theta^2 \cdot \kappa_P^{-1}}{1 + \rho(\sigma_\varepsilon^2 + (1-\kappa_P) \cdot \sigma_\theta^2)}}{1 + \rho \cdot \kappa_P^{-1} \cdot \sigma_\theta^2},$$

so that

$$\begin{aligned}\frac{\partial \Lambda^*}{\partial \kappa_P} &\stackrel{\text{sign}}{=} \frac{\partial \Lambda^*}{\partial n} \\ &\stackrel{\text{sign}}{=} -(\sigma_\theta^2 - \sigma_\varepsilon^2) (\rho \sigma_\varepsilon^2 + 1) n^2 + (2\rho \sigma_\varepsilon^4 + 2\sigma_\varepsilon^2) n + (\rho \sigma_\theta^4 + \rho \sigma_\theta^2 \sigma_\varepsilon^2 + \rho \sigma_\varepsilon^4 + \sigma_\varepsilon^2).\end{aligned}$$

When $\sigma_\theta^2 < \sigma_\varepsilon^2$, the above is positive for all $\kappa_P \in (0, 1)$. When $\sigma_\theta^2 > \sigma_\varepsilon^2$, the above is positive (negative) for small (large) κ_P . Therefore, when $\delta = 0$, if $\sigma_\theta^2 < \sigma_\varepsilon^2$, Λ^* increases in κ_P for all $\kappa_P \in (0, 1)$, and if $\sigma_\theta^2 > \sigma_\varepsilon^2$, Λ^* first increases and then decreases in κ_P .

For any $\delta > 0$, it can be verified that

$$\lim_{\kappa_P \rightarrow 1} \left(\frac{\partial \Lambda^*}{\partial \kappa_P} \right) \stackrel{\text{sign}}{=} -\rho(\delta + 1)^2 \sigma_\theta^2 + (\delta + \rho \sigma_\varepsilon^2 + 2\delta \rho \sigma_\varepsilon^2).$$

Therefore, as long as σ_θ^2 is larger than $\frac{\delta + \rho \sigma_\varepsilon^2 + 2\delta \rho \sigma_\varepsilon^2}{\rho(\delta + 1)^2}$, Λ^* is maximized at an interior $\kappa_P \in (0, 1)$.

Because $\frac{(\delta + \rho \sigma_\varepsilon^2 + 2\delta \rho \sigma_\varepsilon^2)}{\rho(\delta + 1)^2}$ reaches the maximum of $\frac{(2\rho \sigma_\varepsilon^2 + 1)^2}{4\rho(\rho \sigma_\varepsilon^2 + 1)}$ at $\delta = (2\rho \sigma_\varepsilon^2 + 1)^{-1}$. It follows that as long as σ_θ^2 is larger than the threshold value $\hat{\sigma}_\theta^2 \equiv \frac{(2\rho \sigma_\varepsilon^2 + 1)^2}{4\rho(\rho \sigma_\varepsilon^2 + 1)}$, Λ^* decreases in κ_P as κ_P is sufficiently close to one. It can be easily verified that $\hat{\sigma}_\theta^2 > \sigma_\varepsilon^2$ and increases in σ_ε^2 . ■

Proof of Proposition 3.

$$\begin{aligned}
\frac{d\Pi}{d\kappa_P} &= \frac{d\pi_1^*}{d\kappa_P} + \frac{d\pi_2^*}{d\kappa_P} \\
&= \underbrace{\frac{\partial\pi_2^*}{\partial\kappa_P} + \frac{\partial\pi_1^*}{\partial\kappa_P}}_{\text{direct effect (+)}} + \underbrace{\frac{\partial\pi_1^*}{\partial\lambda^I} \times \frac{d\lambda^I}{d\kappa_P}}_{\text{indirect effect (-)}}
\end{aligned} \tag{42}$$

Because

$$\begin{aligned}
\frac{\partial\pi_1}{\partial\kappa_P} &= -\frac{\rho(\Lambda^* - \lambda^I)^2}{2} \times \frac{\partial\text{Var}_0[\bar{P}]}{\partial\kappa_P}, \\
\frac{\partial\pi_1}{\partial\lambda^I} \frac{d\lambda^I}{d\kappa_P} &= \rho\text{Var}_0[\bar{P}](\Lambda^* - \lambda^I) \times \frac{d\lambda^I}{d\kappa_P}, \text{ and} \\
\frac{d\pi_2^*}{d\kappa_P} &= -\frac{\rho\beta^2}{2} \times \frac{\partial\text{Var}_0[\bar{P}]}{\partial\kappa_P}
\end{aligned}$$

It can be verified that

$$\lim_{\kappa_P \rightarrow 1} \frac{d\Pi}{d\kappa_P} \stackrel{\text{sign}}{=} (\sigma_\theta^2(1+\delta) - \sigma_\varepsilon^2) (\rho(2\delta+1)\sigma_\varepsilon^2 + 2\delta - \rho(1+\delta)\sigma_\theta^2)$$

Therefore, when $\sigma_\theta^2 \in \left(\frac{\sigma_\varepsilon^2}{1+\delta}, \left(1 + \frac{\delta}{1+\delta}\right)\sigma_\varepsilon^2 + \frac{2\delta}{\rho(1+\delta)}\right)$, the above is less than zero so that Π is maximized at an interior $\kappa_P \in (0, 1)$. It is obvious that as δ or σ_ε^2 increase, $\frac{\sigma_\varepsilon^2}{1+\delta}$ decreases and $\left(1 + \frac{\delta}{1+\delta}\right)\sigma_\varepsilon^2 + \frac{2\delta}{\rho(1+\delta)}$ increases. ■

Proof of Propostion 4. Given that $\Lambda = \lambda + \beta$, the firm owner's maximization problem is equivalent to choosing $\{\beta_2, \Lambda\}$ to maximize the expected net firm profit given by the following expression:

$$\begin{aligned}
\Pi &= \max_{\{\beta_1, \beta_2, \Lambda\}} \left\{ -\frac{(\lambda + \beta)^2}{2} + \beta + (\lambda + \beta) - \frac{\beta^2}{2} \right. \\
&\quad \left. - \frac{1}{2}\rho((\lambda + \kappa_P \cdot \beta)^2 \text{Var}_0[\bar{P}] + \beta^2 \text{Var}_1[V]) \right\}, \\
\Pi &= \max_{\{\beta_1, \beta_2, \Lambda\}} \left\{ -\frac{(\lambda + \beta)^2}{2} + (\lambda + \beta) - \frac{\beta^2}{2} + \beta - \frac{1}{2}\rho [(\lambda + \kappa_P \cdot \beta)^2 \text{Var}_0[\bar{P}] + \beta^2 \text{Var}_1[V]] \right\},
\end{aligned}$$

The FOCs for the optimal $\{\beta^0, \Lambda^0\}$ are as follows:

$$\begin{aligned}
0 &= 1 - \Lambda - \rho(\Lambda - (1 - \kappa_P) \cdot \beta) \text{Var}_0[\bar{P}], \text{ and} \\
0 &= 1 - \beta + 1 - \Lambda - \rho\kappa_P(\Lambda - (1 - \kappa_P)\beta) \text{Var}[P] - \rho\beta \text{Var}_1[V].
\end{aligned}$$

Solving for the optimal β and Λ from the above first order conditions and plugging into the firm's objective function, the efficiency loss under long-term full commitment contract becomes

$$\frac{1}{2} \rho \frac{\text{Var}_0[\bar{P}] \kappa_P^2 + \text{Var}_1[V] + 2\rho \text{Var}_1[V] \text{Var}_0[\bar{P}]}{\rho \text{Var}_1[V] + 2\rho \text{Var}_0[\bar{P}] + \rho \kappa_P^2 \text{Var}_0[\bar{P}] + \rho^2 \text{Var}_1[V] \text{Var}_0[\bar{P}] - 2\rho \kappa_P \text{Var}_0[\bar{P}] + 1}.$$

Efficiency loss under the renegotiation-proof contract when the firm insiders' signal quality is κ_v

$$\frac{1}{2} \frac{\rho \cdot (\delta + \kappa_v^{-1}) \sigma_\theta^2}{1 + \rho \cdot (\delta + \kappa_v^{-1}) \sigma_\theta^2} \cdot \frac{1 - \kappa_v}{1 + \rho \cdot (\sigma_\varepsilon^2 + (1 - \kappa_v) \sigma_\theta^2)} + \frac{1}{2} \frac{\rho (\sigma_\varepsilon^2 + (1 - \kappa_v) \sigma_\theta^2)}{1 + \rho (\sigma_\varepsilon^2 + (1 - \kappa_v) \sigma_\theta^2)}.$$

As $\kappa_P \rightarrow 0$ and $\kappa_v \rightarrow 1$, the difference between the above two approaches

$$\frac{1}{2} \frac{\rho}{\rho \sigma_\varepsilon^2 + 1} \frac{\rho \sigma_\varepsilon^2 + \rho \sigma_\theta^2 \sigma_\varepsilon^2 + 2\sigma_\theta^2}{\rho \sigma_\varepsilon^2 + \rho \sigma_\theta^2 + 2} > 0.$$

Therefore, the long-term renegotiable contract contingent on terminal firm value only outperforms the full commitment contract contingent on both terminal firm value and interim stock price. ■