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A Granular Computing-Based Model for Group Decision-Making in Multi-Criteria and Heterogeneous Environments

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Abstract

Granular computing is a growing computing paradigm of information processing that covers any techniques, methodologies, and theories employing information granules in complex problem solving. Within the recent past, it has been applied to solve group decision-making processes and different granular computing-based models have been constructed, which focus on some particular aspects of these decision-making processes. This study presents a new granular computing-based model for group decision-making processes defined in multi-criteria and heterogeneous environments that is able to improve with minimum adjustment both the consistency associated with individual decision-makers and the consensus related to the group. Unlike the existing granular computing-based approaches, this new one is able to take into account a higher number of features when dealing with this kind of decision-making processes.

1. Introduction

Granular Computing (GrC) [1] has arisen as a sound and consistent methodology of processing, describing and constructing information granules, which are the underlying concept that has far achieving implications by causing non-specific, semantically significant entities being essential to the understanding of real-world [2].

Despite of the fact that the evident variety of the existing approaches in which the information granules can be formalized, there exist several underlying shared features articulated with regards to the ensuing application domains, algorithmic developments and fundamentals. A requirement to start with the processing of the information granules, irrespective of the way in which they are formalized, is to search for certain general ways of forming them [3]. First, the principle of justifiable granularity provides an option. Second, by bringing into a picture the issue of non-numeric data, the information granularity helps accomplish more suitable rapport with reality by quantifying the nature of the data by means of information granules.

In group decision-making processes [4, 5], the distribution of information granularity has been used to make the models dealing with such processes more reflective of reality and to capture the variety of knowledge sources and viewpoints expressed by individual decision-makers. A group decision-making process is an example of granularity distribution in which the assignment of granularity arises as a crucial element to build consensus, a crucial question in this kind of decision-making processes. Building consensus is about arriving at the decision that every decision-maker is at ease with [6]. To achieve this situation, obviously each decision-maker must exercise a certain flexibility degree and be ready to soften her or his posture, and, here, information granularity becomes involved [7]. In a nutshell, by admitting a granular realization of the decision-makers' assessments, instead of numeric, the collaboration is facilitated.

In the literature, we can find a number of approaches constructed from the perspective of the GrC, in particular by using the concept of information granularity, to build consensus in a group decision-making process. In addition to the consensus, a number of approaches have been developed to deal with consistency [8], another important point that must be taken into account in a group decision-making process. For example, in [7] a model dealing with consensus and consistency in the context of the analytic hierarchy process (AHP) was proposed; in [9] a model managing both consensus and consistency was also presented, but in this case fuzzy preference relations were used to model the assessments; in [10] a model improving multiplicative consistency in reciprocal preference relations was developed; in [11] a model handling consensus with intuitionistic reciprocal preference relations was introduced; in [12] a modified consensus model also handling consistency in the AHP was proposed; in [13] a model managing consensus

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with minimum adjustment in multi-criteria contexts was implemented; and in [14] a model dealing with consistency in multi-criteria and heterogeneous contexts was introduced, to cite some examples. It is needless to say that there exist a great number of approaches based on GrC that deal with different features of group decision-making processes. However, any of them is able to deal with all those features at the same time.

The aim of this study is to make use of the GrC paradigm, in particular the concept of distribution of information granularity, by giving rise to a granular model managing group decision-making processes. First, this new granular model has the ability to deal with multi-criteria environments, i.e., decision-making processes in which different criteria must be kept in mind to assess the alternatives. Second, it has the ability to handle heterogeneous environments, i.e., decision-making processes in which every decision-maker has a different knowledge about the alternatives and criteria. Third, it is able to improve the consistency of the individual decision-makers and the consensus achieved among the group. And, fourth, it is able to make this improvement with minimum adjustment, i.e., by modifying the minimum possible the assessments given by the decision-makers.

The material is structured as follows. In Section 2, we recall the formal definition of a multi-criteria group decision-making problem and the fundamentals aspects to keep in mind when facing them. Section 3 elaborates on the new granular computing-based model for group decision-making in multi-criteria and heterogeneous environments that is able to improve consistency and consensus with minimum adjustment. Section 4 conducts a case study to illustrate the proposed granular computing-based model. Its advantages, shortcomings and performance are discussed in Section 5. Finally, Section 6 covers conclusions and future research studies.

2. Multi-criteria group decision-making

Because the wisdom of a number of decision-makers is assumed to be better than any individual decision-maker in political forecasting, public policies, and so forth [15], group decision-making processes have been widely investigated. These processes define scenarios where various decision-makers collectively select the best alternative (course of action, option) from a collection of them [16, 7]. To do so, the decision-makers must provide their assessments of the alternatives, being usual the consideration of a number of criteria [17]. In such a case, a multi-criteria group decision-making problem is faced [18], which is formally characterized by a finite set of alternatives, $A = \{a_1, a_2, \ldots, a_n\}$, being $n \geq 2$; a finite group of decision-makers, $DM = \{dm_1, dm_2, \ldots, dm_m\}$, being $m \geq 2$; and a finite set of criteria, $C = \{c_1, c_2, \ldots, c_q\}$, being $q \geq 2$, that are used to assess the alternatives. In addition, an importance weight $w_l \in [0, 1]$ is associated with every criterion $c_l \in C$. Generally, the weights are normalized, i.e., $\sum_{l=1}^{q} w_l = 1$.

To solve a multi-criteria group decision-making problem, the following steps are usually carried out:

- Expressing the assessments. The decision-makers provide their assessments of the alternatives using a particular representation domain and a given representation format [19].
- Consensus reaching process. In order to reach an enough agreement, a dynamic and iterative process of debating and revision of assessments by the decision-makers is carried out [20, 21].
- Selection process. A collective assessment is determined by aggregating all the individual assessments expressed by the decision-makers. Then, the information contained in this collective assessment is exploited to rank the alternatives. The first alternative of this ranking is selected as solution to the decision-making process [22].

The decision-makers' assessments of the alternatives can determine the preference degree of an alternative over other according to a criterion or the degree up to which the alternative satisfies the criterion. In the first case, a pairwise comparison is used as representation format. In the second case, a utility value (non-pairwise comparison) is used as representation format. Even though both have been used in decision-making processes, the pairwise comparisons better model such processes [23]. However, when pairwise comparisons are used to model the assessments, it is produced more information than the one required. In addition, the global understanding of the alternatives by the decision-makers is restricted. This can lead to some contradictory (inconsistent) assessments [8]. Therefore, consistency in the assessments expressed by the decision-makers must be analyzed [24].

Once the representation format has been chosen, the representation domain in which the assessment are modeled must be established. In multi-criteria group decision-making processes, which are cognitive processes where decision-makers (humans) participate, the fuzzy set theory has demonstrated to be a useful tool in modeling assessments pervaded with human uncertainty [25]. In particular, this theory and its extensions have been widely used [26].

Another aspect that must be considered is that of the different decision-makers' knowledge about the criteria and alternatives, which is generally characterized by assuming that every decision-maker, $dm_z \in DM$, has a different importance weight, $v_{zl} \in [0,1]$, for every criterion, $c_l \in C$. In such a case, we face a multi-criteria group-decision making process defined in a heterogeneous (non-homogeneous) environment [27].

In this study, we suppose that both the weights related to the criteria and the weights associated with the decision-makers for every criterion are assigned directly by the person in charge of the decision problem. However, any other approach could be used. In the case of the criteria, any of the existing subjective weighting methods, objective weighting methods, or hybrid weighting methods could be used [28]. In the case of the decision-makers, the weights can also be obtained automatically from the assessments provided by the decision-makers. For instance, the most consistent decision-makers could receive a higher weight than inconsistent ones [29].

3. Granular computing-based model

This section describes in detail the new granular computing-based model for group decision-making in multi-criteria and heterogeneous environments. Being based on the concept of information granularity, it aims to improve both consistency and consensus with minimum adjustment, i.e., by adjusting as little as possible the initial assessments communicated by the decision-makers.

This new granular computing-based model is divided into three steps: (i) expressing assessments, (ii) improving of consistency and consensus with minimum adjustment, and (iii) selection process. These steps are presented in detail in the next subsections.

3.1. Expressing assessments

In Section 2, we have mentioned that there exist different representation domains and representation formats to characterize the assessments [19]. Here, we assume [0,1]-values as representation domain and preference relations as representation format. It means that fuzzy preference relations are used to model the assessments [30]. They have been chosen because they are the most used in group decision-making processes.

The function $\mu_{P^{zl}} : A \times A \rightarrow [0,1]$ characterizes the fuzzy preference relation P^{zl} given by the decision-maker dm_z on the set of alternatives A for the criterion c_l. To symbolize the fuzzy preference relation P^{zl} in an understanding way, the matrix $P^{zl} = (p_{jk}^{zl})$ can be employed, being $n \times n$ the size of this matrix. Here $p_{jk}^{zl} = \mu_{P^{zl}}(a_j, a_k)$ indicates the preference degree of a_j over a_k ($j \neq k$) according to dm_z for c_l. Particularly, a value equal or greater than 0.5 is assigned to p_{jk}^{zl} whether the decision-maker dm_z prefers a_j over a_k for the criterion c_l; a value equal to 0.5 is assigned to p_{jk}^{zl} whether the decision-maker dm_z equally prefers a_j and a_k for the criterion c_l; and a value lower than 0.5 is assigned to p_{jk}^{zl} whether the decision-maker dm_z prefers a_k over a_j for the criterion c_l. In a nutshell, the starting point of the granular computing-based model is a set of $m \times q$ fuzzy preference relations, P^{zl}, $z = 1, \ldots, m$, $l = 1, \ldots, q$, of dimension $n \times n$.

3.2. Improving of consistency and consensus with minimum adjustment

Before applying the selection process, both the consistency levels associated with individual decision-makers and the consensus achieved among them must be as higher as possible. This improvement requires that the decision-makers modify their initial assessments. It means that the decision-makers accept the adjustment of their assessments to some extent. Nevertheless, the decision-makers could not accept the modified assessments if they are far from the initial ones expressed.

The flexibility allowed by the decision-makers in their assessments can be modeled by the concept of information granularity that transforms the entries of the fuzzy preference relations, which are composed of a numerical value, into information granules of higher abstraction [31], leading to granular fuzzy preference relations [9]. As formalism of granulation, intervals are used. It means that the entries of the fuzzy preference relations are interpreted as intervals instead of precise numeric values. Particularly, the information granularity level α determines the length of the intervals and can be used to improve both the consistency and the consensus.

In a nutshell, by modifying the $[0,1]$-assessments provided by the decision-makers within the limits of the intervals formed according to the information granularity level α, we aim to improve both the consistency and the consensus. In addition to it, we aim to obtain modified assessments as close as possible to the ones provided by the decision-makers. To perform this task, the particle swarm optimization (PSO) algorithm is used [32]. We use this algorithm as it is easy to execute via programming, it has a fast convergence rate, and it needs a low number of parameters that require to be adjusted [33]. In addition, this algorithm has successfully been applied in similar problems [7, 9, 13]. Anyway, any other optimization technique such as differential evolution could also be applied [34].

3.2.1. Algorithm PSO starts with a swarm consisting of a number of particles depicted as positions in a search-space of d dimensions, i.e., the particles represent potential solutions of the optimization task. By moving to a new position based on their previous positions and a new velocity, the particles try to discover, in a iterative way, a solution that optimizes a fitness function f.

Even though there exist several variants of the PSO algorithm [33], which could be applied, we assume the generic form in this study. Considering that every particle i is composed of three d-dimensional vectors, $\mathbf{v}_i = (v_{i,1}, v_{i,2}, \ldots, v_{i,d})$, $\mathbf{x}_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,d})$ and $\mathbf{xb}_i = (xb_{i,1}, xb_{i,2}, \ldots, xb_{i,d})$, representing its velocity, current position and best position achieved so far, respectively, the following expressions control the next velocity and position of the particle [32]:

$$\begin{aligned} v_{i,h}(t+1) = \omega(t) \cdot v_{i,h}(t) \\ + c_1 \cdot r_h \cdot (xb_{i,h} - x_{i,h}(t)) \\ + c_2 \cdot s_h \cdot (xg_h - x_{i,h}(t)) \end{aligned} \tag{1}$$

$$x_{i,h}(t+1) = x_{i,h}(t) + v_{i,h}(t+1) \tag{2}$$

being t the current iteration and h the dimension of the particle i; ω, called inertia weight, serves to scale the current velocity (a small value means local exploration and a high value means global exploration); c_1 and c_2 denote two acceleration coefficients modeling the step size the particle takes in the direction of its best position and in the direction of the best global position, respectively; $\mathbf{r} = (r_1, r_2, \ldots, r_d)$ and $\mathbf{s} = (s_1, s_2, \ldots, s_d)$ are two vectors of values from two random sequences in $[0,1]$; and $\mathbf{xg} = (xg_1, xg_2, \ldots, xg_d)$ is a vector characterizing the global best position achieved by a particle of the swarm.

3.2.2. Particle's representation According to the distinguishing features of the group decision-making processes in multi-criteria and heterogeneous environments that we model, every particle is characterized by a vector whose components belong to the interval $[0,1]$. Particularly, whether the decision-making problem is set up with q criteria, m decision-makers and n alternatives, every particle consists in a vector of $q \cdot (n-1) \cdot n \cdot m$ dimensions.

Let $p_{jk}^{zl} \in [0,1]$ be an element of the fuzzy preference relation P^{zl} expressed by the decision-maker dm_z on the criterion c_l. Whether α is the information granularity level established, the element, p_{jk}^{zl}, can take values within the interval $[\mathrm{L}_{jk}, \mathrm{U}_{jk}]$, whose boundaries are computed as follows:

$$[\mathrm{L}_{jk}, \mathrm{U}_{jk}] = [\max(0, p_{jk}^{zl} - \frac{\alpha}{2}), \min(p_{jk}^{zl} + \frac{\alpha}{2}, 1)] \tag{3}$$

For illustrative purposes, let p_{jk}^{zl} and α be 0.6 and 0.3, respectively. Let $x_{i,h} = 0.4$ be the corresponding component of the particle i. Using (3), the corresponding interval to p_{jk}^{zl} is $[0.45, 0.75]$. Finally, using (4), we get that the adjusted value $\overline{p}_{jk}^{zl}$ associated with p_{jk}^{zl} is 0.57.

$$\overline{p}_{jk}^{zl} = \mathrm{L}_{jk} + (\mathrm{U}_{jk} - \mathrm{L}_{jk}) \cdot x_{i,h} \tag{4}$$

3.2.3. Fitness function The particles aim to maximize the value returned by the fitness function f at their positions, i.e., by modifying the initial values of the entries of the fuzzy preference relations provided by the decision-makers, first, we aim to improve the individual consistency associated with the decision-makers, second, we aim to improve the consensus among the group and, third, we aim to improve the similarity between the modified values of the entries of the fuzzy preference relations and the initial ones. Therefore, f is defined as follows:

$$f = \beta \cdot (\gamma \cdot f_1 + (1-\gamma) \cdot f_2) + (1-\beta) \cdot f_3 \tag{5}$$

being $\gamma \in [0,1]$ a parameter establishing a trade-off between the consistency, f_1, and the consensus, f_2, and $\beta \in [0,1]$ a parameter establishing a trade-off between these two values, f_1 and f_2, and f_3, which measures the similarity between the fuzzy preference relations given by the decision-makers, P^{zl}, and the modified ones, $\overline{P}^{zl}$, $z = 1, \ldots, m$, $l = 1, \ldots, q$.

Based on (5), the computation of three values, namely f_1, f_2 and f_3, is required to get the value of the fitness function f.

First, to compute f_1, the average of the individual consistency associated with the decision-makers is calculated:

$$f_1 = \frac{1}{m} \sum_{z=1}^{m} cl^z \tag{6}$$

where cl^z denotes the global consistency level associated with the decision-maker dm_z. This value is calculated by means of the weighted average of the consistency levels associated with that decision-maker in every criterion c_l:

$$cl^z = \frac{1}{\sum_{l=1}^{q} v_{zl}} \sum_{l=1}^{q} v_{zl} \cdot cl^{zl} \tag{7}$$

where cl^{zl} denotes the consistency level associated with the decision-maker dm_z in the criterion c_l. This value is calculated by means of the procedure developed in [29]. Refer to it for a description in detail of this procedure.

Second, to compute f_2, a new approach based on the coincidence concept is implemented [35]. It is as follows:

- Computation of a matrix, $SM = (sm_{jk}^{zyl})$, for every pair of decision-makers, dm_z and dm_y, and every criterion, c_l, determining the similarity between the assessments communicated by them on that criterion:

$$sm_{jk}^{zyl} = 1 - |p_{jk}^{zl} - p_{jk}^{yl}| \tag{8}$$

- Computation of a matrix, $CM^l = (cm_{jk}^l)$, for every criterion, c_l, determining the consensus reached by the group on that criterion:

$$cm_{jk}^l = \frac{2}{m \cdot (m-1)} \sum_{z=1}^{m-1} \sum_{y=z+1}^{m} sm_{jk}^{zyl} \tag{9}$$

- Computation, for every matrix CM^l, of a global consensus measure, gc^l, associated with the criterion c_l:

$$gc^l = \frac{1}{n \cdot (n-1)} \sum_{j=1}^{n} \sum_{k=1;k\neq j}^{n} cm_{jk}^l \tag{10}$$

- Computation of f_2 via the weighted average of the measures of global consensus associated with the criteria:

$$f_2 = \sum_{l=1}^{q} w_l \cdot gc^l \tag{11}$$

And third, to compute f_3, the similarity between the fuzzy preference relations expressed by the decision-makers and the adjusted ones must be obtained. This is carried out as follows:

- Computation of a similarity index, si^{zl}, related to every decision-maker, dm_z, on every criterion, c_l, determining the similarity between the fuzzy preference relation communicated by that decision-maker and the suggested one on that criterion:

$$si^{zl} = \frac{1}{n \cdot (n-1)} \sum_{j=1}^{n} \sum_{\substack{k=1 \\ k\neq j}}^{n} 1 - |p_{jk}^{zl} - \overline{p}_{jk}^{zl}| \tag{12}$$

- Computation of a similarity index, si^l, for every criterion, c_l, determining the similarity between the fuzzy preference relations communicated by the group of individuals and the suggested ones on that criterion:

$$si^l = \frac{1}{m} \sum_{z=1}^{m} si^{zl} \tag{13}$$

- Computation of f_3 via the weighted average of the similarity indexes related to the criteria:

$$f_3 = \sum_{l=1}^{q} w_l \cdot si^l \tag{14}$$

In this study, we have used the Manhattan distance to calculate the similarity measures. However, other distance functions such as the Euclidean or the Cosine distances, to cite some of them, could be used depending of the characteristics sought. Refer to [36] for a comparative study of the effect of the application of some different similarity measures.

3.3. Selection process

This step returns the best alternative (or alternatives) by means of an aggregation process and an exploitation process, which are elaborated on next.

3.3.1. Aggregation The collective assessments is calculated by fusing all the individual decision-makers' assessments. It means that the individual fuzzy preference relations must be aggregated to obtain a collective fuzzy preference relation, which is done by using a certain aggregation function [37]. Here, as we consider that both the criteria and the decision-makers have associated weights of importance, the weighted arithmetic mean is used as aggregation function. In particular, the procedure for obtaining the collective fuzzy preference relation is the following:

- A collective fuzzy preference relation, $\overline{P}^{cl} = (\overline{p}_{jk}^{cl})$, is obtained for each criterion c^l as follows:

$$\overline{p}_{jk}^{cl} = \frac{1}{\sum_{z=1}^{m} v_{zl}} \cdot \sum_{z=1}^{m} v_{zl} \cdot \overline{p}_{jk}^{zl} \tag{15}$$

- Using the information contained in the collective fuzzy preference relations related to the criteria, the collective fuzzy preference relation, $\overline{P}^c = (\overline{p}^c_{jk})$, is calculated as follows:

$$\overline{p}^c_{jk} = \sum_{l=1}^{q} w_l \cdot \overline{p}^{cl}_{jk} \tag{16}$$

3.3.2. Exploitation To select the best alternative (or alternatives) solving the decision-making problem, the information included in the collective fuzzy preference relation is exploited. Different functions can be applied to carried out it [22]. Among then, we use the following two choice degrees of alternatives:

- The quantifier-guided dominance degree, $QGDD_j$, determining the dominance that the alternative a_j has over the other alternatives. It is computed as:

$$QGDD_j = \frac{1}{n-1} \sum_{k=1;k\neq j}^{n} \overline{p}^c_{jk} \tag{17}$$

- The quantifier-guided non-dominance degree, $QGNDD_j$, determining the degree in which the alternative a_j is not dominated by the other alternatives. It is computed as:

$$QGDND_j = \frac{1}{n-1} \sum_{k=1;k\neq j}^{n} 1 - d_{kj} \tag{18}$$

being $d_{kj} = \max\{\overline{p}^c_{kj} - \overline{p}^c_{jk}, 0\}$, which determines the degree in which the alternative a_j is dominated by the alternative a_k.

Based on these two choice degrees of alternatives, the procedure to select the best alternative (or alternatives) is as follows:

- Both choice degrees of alternatives are applied over A to get the next sets of alternatives:

$$DD = \{a_j \in A \mid QGDD_j = \sup_{a_k \in A} QGDD_k\} \tag{19}$$

$$NDD = \{a_j \in A \mid QGNDD_j = \sup_{a_k \in A} QGNDD_k\} \tag{20}$$

- The intersection is applied to the above sets to get the following set of alternatives:

$$QG = DD \bigcap NDD \tag{21}$$

If $\#QG = 1$, then this is alternative chosen as solution to the problem. Otherwise, continue.

- If $\#DD = 1$, then the alternative located in this set is chosen as solution to the problem. If not, the alternative, a_j, of this set having the higher $QGNDD_j$ is selected as solution to the problem.

4. Case study

A company wants to invest in the stock market. To do it, the company asks to four stock market investors (decision-makers), dm_1, dm_2, dm_3 and dm_4, who must choose the best choice between four possible stocks, a_1, a_2, a_3 and a_4, considering four criteria, earnings momentum (c_1), conservative capital structure (c_2), favorable asset utilization (c_3) and good current and projected profitability (c_4), whose weights of importance are 0.2, 0.3, 0.1 and 0.4, respectively. Based on their background and knowledge, the weights of importance of the stock market investors related to the criteria are:

$$\begin{array}{llll} v_{11} = 0.2 & v_{12} = 0.3 & v_{13} = 0.3 & v_{14} = 0.2 \\ v_{21} = 0.3 & v_{22} = 0.3 & v_{23} = 0.2 & v_{24} = 0.2 \\ v_{31} = 0.4 & v_{32} = 0.1 & v_{33} = 0.2 & v_{34} = 0.3 \\ v_{41} = 0.1 & v_{42} = 0.3 & v_{43} = 0.3 & v_{44} = 0.3 \end{array}$$

Initially, the stock market investors give these fuzzy preference relations on the different criteria:

$$P^{11} = \begin{bmatrix} - & 0.3 & 0.5 & 0.5 \\ 0.5 & - & 0.7 & 0.7 \\ 0.5 & 0.3 & - & 0.7 \\ 0.5 & 0.3 & 0.3 & - \end{bmatrix} P^{12} = \begin{bmatrix} - & 0.3 & 0.9 & 0.9 \\ 0.7 & - & 0.1 & 0.1 \\ 0.1 & 0.9 & - & 0.7 \\ 0.1 & 0.9 & 0.3 & - \end{bmatrix}$$

$$P^{13} = \begin{bmatrix} - & 0.3 & 0.3 & 0.7 \\ 0.9 & - & 0.3 & 0.3 \\ 0.9 & 0.7 & - & 0.3 \\ 0.5 & 0.7 & 0.9 & - \end{bmatrix} P^{14} = \begin{bmatrix} - & 0.7 & 0.9 & 0.9 \\ 0.3 & - & 0.6 & 0.7 \\ 0.2 & 0.3 & - & 0.8 \\ 0.2 & 0.2 & 0.1 & - \end{bmatrix}$$

$$P^{21} = \begin{bmatrix} - & 0.5 & 0.3 & 0.9 \\ 0.5 & - & 0.9 & 0.9 \\ 0.7 & 0.1 & - & 0.5 \\ 0.1 & 0.1 & 0.5 & - \end{bmatrix} P^{22} = \begin{bmatrix} - & 0.7 & 0.7 & 0.7 \\ 0.1 & - & 0.9 & 0.3 \\ 0.5 & 0.3 & - & 0.9 \\ 0.5 & 0.5 & 0.3 & - \end{bmatrix}$$

$$P^{23} = \begin{bmatrix} - & 0.5 & 0.5 & 0.3 \\ 0.5 & - & 0.5 & 0.7 \\ 0.5 & 0.5 & - & 0.5 \\ 0.9 & 0.1 & 0.5 & - \end{bmatrix} P^{24} = \begin{bmatrix} - & 0.8 & 0.7 & 0.2 \\ 0.2 & - & 0.4 & 0.4 \\ 0.1 & 0.6 & - & 0.8 \\ 0.9 & 0.6 & 0.3 & - \end{bmatrix}$$

$$P^{31} = \begin{bmatrix} - & 0.1 & 0.7 & 0.3 \\ 0.7 & - & 0.5 & 0.3 \\ 0.3 & 0.5 & - & 0.5 \\ 0.7 & 0.7 & 0.5 & - \end{bmatrix} P^{32} = \begin{bmatrix} - & 0.9 & 0.9 & 0.7 \\ 0.3 & - & 0.7 & 0.5 \\ 0.3 & 0.1 & - & 0.7 \\ 0.3 & 0.5 & 0.3 & - \end{bmatrix}$$

$$P^{33} = \begin{bmatrix} - & 0.3 & 0.3 & 0.7 \\ 0.5 & - & 0.5 & 0.7 \\ 0.3 & 0.5 & - & 0.5 \\ 0.3 & 0.7 & 0.9 & - \end{bmatrix} P^{34} = \begin{bmatrix} - & 0.5 & 0.6 & 0.3 \\ 0.5 & - & 0.2 & 0.5 \\ 0.3 & 0.6 & - & 0.7 \\ 0.7 & 0.5 & 0.4 & - \end{bmatrix}$$

$$P^{41} = \begin{bmatrix} - & 0.8 & 0.9 & 0.7 \\ 0.3 & - & 0.7 & 0.5 \\ 0.3 & 0.2 & - & 0.7 \\ 0.2 & 0.5 & 0.2 & - \end{bmatrix} P^{42} = \begin{bmatrix} - & 0.2 & 0.2 & 0.7 \\ 0.8 & - & 0.5 & 0.6 \\ 0.7 & 0.5 & - & 0.6 \\ 0.3 & 0.4 & 0.3 & - \end{bmatrix}$$

$$P^{43} = \begin{bmatrix} - & 0.5 & 0.3 & 0.7 \\ 0.5 & - & 0.9 & 0.7 \\ 0.8 & 0.1 & - & 0.5 \\ 0.1 & 0.3 & 0.5 & - \end{bmatrix} P^{44} = \begin{bmatrix} - & 0.1 & 0.1 & 0.2 \\ 0.8 & - & 0.2 & 0.8 \\ 0.9 & 0.7 & - & 0.9 \\ 0.8 & 0.1 & 0.1 & - \end{bmatrix}$$

Before presenting the values returned by the proposed granular computing-based model, we show the values given to the parameters of the PSO. These values, which are assigned because of an intense experimentation, are the following: c_1 and c_2 are set to 2 in (1); the swarm contains 100 particles; 300 iterations are carried out; and ω is linearly decreased from 0.9 to 0.4 in (1) as follows:

$$\omega(t) = (0.9 - 0.4) \cdot \frac{300 - t}{300} + 0.4f \tag{22}$$

being t the current iteration. In addition, α, β and γ, are established as 0.2, 0.75 and 0.5, respectively.

The values of 0.917, 0.825 and 0.915, are returned by the PSO for f_1, f_2 and f_3, respectively. It means the global consistency is 0.917, the consensus achieved is 0.825 and the similarity between the original fuzzy preference relations and the adjusted ones is 0.915. In addition, the adjusted fuzzy preference relations generated by the PSO are:

$$\overline{P}^{11} = \begin{bmatrix} - & 0.4 & 0.6 & 0.6 \\ 0.6 & - & 0.6 & 0.6 \\ 0.4 & 0.4 & - & 0.6 \\ 0.4 & 0.4 & 0.4 & - \end{bmatrix} \overline{P}^{12} = \begin{bmatrix} - & 0.4 & 0.8 & 0.8 \\ 0.6 & - & 0.2 & 0.2 \\ 0.2 & 0.8 & - & 0.6 \\ 0.2 & 0.8 & 0.4 & - \end{bmatrix}$$

$$\overline{P}^{13} = \begin{bmatrix} - & 0.4 & 0.4 & 0.8 \\ 0.8 & - & 0.4 & 0.3 \\ 0.8 & 0.68 & - & 0.4 \\ 0.4 & 0.6 & 0.8 & - \end{bmatrix} \overline{P}^{14} = \begin{bmatrix} - & 0.6 & 0.8 & 0.8 \\ 0.4 & - & 0.6 & 0.6 \\ 0.3 & 0.4 & - & 0.7 \\ 0.3 & 0.3 & 0.2 & - \end{bmatrix}$$

$$\overline{P}^{21} = \begin{bmatrix} - & 0.6 & 0.4 & 0.8 \\ 0.4 & - & 0.8 & 0.8 \\ 0.6 & 0.2 & - & 0.6 \\ 0.2 & 0.2 & 0.4 & - \end{bmatrix} \overline{P}^{22} = \begin{bmatrix} - & 0.6 & 0.6 & 0.69 \\ 0.2 & - & 0.8 & 0.4 \\ 0.49 & 0.4 & - & 0.8 \\ 0.6 & 0.5 & 0.4 & - \end{bmatrix}$$

$$\overline{P}^{23} = \begin{bmatrix} - & 0.6 & 0.6 & 0.4 \\ 0.4 & - & 0.6 & 0.69 \\ 0.4 & 0.49 & - & 0.6 \\ 0.8 & 0.2 & 0.4 & - \end{bmatrix} \overline{P}^{24} = \begin{bmatrix} - & 0.7 & 0.6 & 0.3 \\ 0.3 & - & 0.5 & 0.5 \\ 0.2 & 0.5 & - & 0.7 \\ 0.8 & 0.5 & 0.4 & - \end{bmatrix}$$

$$\overline{P}^{31} = \begin{bmatrix} - & 0.2 & 0.6 & 0.4 \\ 0.6 & - & 0.5 & 0.4 \\ 0.2 & 0.5 & - & 0.6 \\ 0.6 & 0.6 & 0.4 & - \end{bmatrix} \overline{P}^{32} = \begin{bmatrix} - & 0.8 & 0.8 & 0.6 \\ 0.4 & - & 0.6 & 0.5 \\ 0.2 & 0.2 & - & 0.6 \\ 0.4 & 0.6 & 0.4 & - \end{bmatrix}$$

$$\overline{P}^{33} = \begin{bmatrix} - & 0.3 & 0.4 & 0.6 \\ 0.5 & - & 0.5 & 0.6 \\ 0.2 & 0.5 & - & 0.6 \\ 0.4 & 0.6 & 0.8 & - \end{bmatrix} \overline{P}^{34} = \begin{bmatrix} - & 0.5 & 0.6 & 0.4 \\ 0.5 & - & 0.3 & 0.4 \\ 0.2 & 0.5 & - & 0.6 \\ 0.6 & 0.6 & 0.4 & - \end{bmatrix}$$

$$\overline{P}^{41} = \begin{bmatrix} - & 0.7 & 0.8 & 0.6 \\ 0.4 & - & 0.6 & 0.6 \\ 0.4 & 0.3 & - & 0.8 \\ 0.3 & 0.4 & 0.3 & - \end{bmatrix} \overline{P}^{42} = \begin{bmatrix} - & 0.3 & 0.3 & 0.6 \\ 0.7 & - & 0.6 & 0.7 \\ 0.8 & 0.41 & - & 0.7 \\ 0.3 & 0.5 & 0.3 & - \end{bmatrix}$$

$$\overline{P}^{43} = \begin{bmatrix} - & 0.45 & 0.3 & 0.6 \\ 0.6 & - & 0.8 & 0.8 \\ 0.8 & 0.2 & - & 0.6 \\ 0.2 & 0.3 & 0.4 & - \end{bmatrix} \overline{P}^{44} = \begin{bmatrix} - & 0.2 & 0.2 & 0.3 \\ 0.7 & - & 0.3 & 0.8 \\ 0.8 & 0.6 & - & 0.8 \\ 0.7 & 0.2 & 0.2 & - \end{bmatrix}$$

Once both the consistency and consensus have been improved with minimum adjustment, the selection process is applied to obtain the best stock (or stocks). Using (15), we get these collective fuzzy preference relations for the criteria:

$$\overline{P}^{c1} = \begin{bmatrix} - & 0.41 & 0.56 & 0.58 \\ 0.52 & - & 0.62 & 0.58 \\ 0.38 & 0.37 & - & 0.62 \\ 0.41 & 0.42 & 0.39 & - \end{bmatrix}$$

$$\overline{P}^{c2} = \begin{bmatrix} - & 0.47 & 0.59 & 0.69 \\ 0.49 & - & 0.54 & 0.44 \\ 0.47 & 0.50 & - & 0.69 \\ 0.37 & 0.60 & 0.37 & - \end{bmatrix}$$

$$\overline{P}^{c3} = \begin{bmatrix} - & 0.44 & 0.41 & 0.62 \\ 0.60 & - & 0.58 & 0.59 \\ 0.60 & 0.46 & - & 0.54 \\ 0.42 & 0.43 & 0.60 & - \end{bmatrix}$$

$$\overline{P}^{c4} = \begin{bmatrix} - & 0.47 & 0.52 & 0.43 \\ 0.50 & - & 0.40 & 0.58 \\ 0.40 & 0.51 & - & 0.70 \\ 0.61 & 0.40 & 0.30 & - \end{bmatrix}$$

The collective fuzzy preference relation $\overline{P}^{c}$ is calculated by fusing $\overline{P}^{c1}$, $\overline{P}^{c2}$, $\overline{P}^{c3}$ and $\overline{P}^{c4}$, which is done by using (16):

$$\overline{P}^{c} = \begin{bmatrix} - & 0.46 & 0.54 & 0.56 \\ 0.51 & - & 0.50 & 0.54 \\ 0.44 & 0.47 & - & 0.67 \\ 0.48 & 0.47 & 0.37 & - \end{bmatrix}$$

By means of (17) and (18), we get these quantifier-guided dominance degrees and the quantifier-guided non-dominance degrees associated with the stocks, respectively:

$$QGDD_1 = 0.52 \qquad QGNDD_2 = 0.98$$
$$QGDD_2 = 0.52 \qquad QGNDD_2 = 1.00$$
$$QGDD_3 = 0.53 \qquad QGNDD_3 = 0.96$$
$$QGDD_4 = 0.44 \qquad QGNDD_4 = 0.85$$

Considering these values of the choice degrees, $DD = \{a_3\}$, $NDD = \{a_2\}$, and $QG = \{\emptyset\}$. Therefore, as $\#DD = 1$, it means that the company should invest in the stock a_3 according to the opinions of the four stock market investors.

5. Discussion

When the group decision-making models deal with different features of the decision-making process, a comparison of the results of a model with others is not a straightforward task. The aspects considered by the models are different and, as a consequence, a quantitative comparison would be not meaningful. However, as mentioned previously, the proposed granular computing-based model is able to deal with more features related to group decision-making processes than the existing approaches based on granular computing. On the contrary, it presents a higher number of parameters that must be established. In any case, in the following, we analyze the performance of the model from several viewpoints.

5.1. Information granularity level α

With the purpose of putting the returned results by the proposed granular computing-based model in a certain context, the results achieved when using the initial assessments expressed by the decision-makers are reported (it means that $\alpha = 0$). In such a case, the consistency, f_1, reached is 0.916 and the consensus, f_2, achieved is 0.711. If we compare with the results achieved when $\alpha = 0.2$ (the value used in the case study illustrated in Section 4), i.e., a certain flexibility in the initial assessments is allowed by the decision-makers, both the consistency and the consensus achieved by the proposed model are higher. Therefore, the analysis of how the consistency and the consensus are improved and how the similarity is deteriorated according to the value of the granularity level α is a matter of interest.

Table 1 shows the values of f, f_1, f_2 and f_3, for chosen values of α, having β and γ the same values that the ones used in the case study, i.e., $\beta = 0.75$ and $\gamma = 0.5$. We can observe that the higher the information granularity level allowed, the higher the probability of achieving better values for f_1 and f_2, and the higher the probability of achieving worst values for f_3. It is natural because a certain flexibility level to be exploited is injected into the assessments. It means that a higher interval is allowed, which implies that the probability of improving the consistency and the consensus is higher, but the probability of obtaining and adjusted assessment more different from the initial one is also higher. Notably the improvement of the consensus achieved as, in this case, the global consistency is already high for the initial assessments.

Table 1. f, f_1, f_2 and f_3x for given values of α.

	f	f_1	f_2	f_3
$\alpha = 0.2$	0.882	0.917	0.825	0.915
$\alpha = 0.4$	0.904	0.917	0.916	0.868
$\alpha = 0.6$	0.912	0.918	0.950	0.847
$\alpha = 0.8$	0.915	0.918	0.961	0.842
$\alpha = 1.0$	0.916	0.919	0.967	0.836

Table 2. f_1 and f_2 for given values of γ.

	f_1	f_2
$\gamma = 0.00$	0.915	0.924
$\gamma = 0.25$	0.916	0.917
$\gamma = 0.50$	0.917	0.916
$\gamma = 0.75$	0.918	0.827
$\gamma = 1.00$	0.919	0.712

5.2. Parameter γ

Table 2 shows the impact of the parameter γ, which stands in the composite fitness function f (these values are obtained by using the same fuzzy preference relations provided initially by the decision-makers in the case study and being $\alpha = 0.4$ and $\beta = 0.75$). As this parameter determines a trade-off between the consistency and the consensus, we focus on the values reached by f_1 and f_2. On one hand, whether γ is equal to 1, the PSO focuses on the maximization of the consistency associated with the individual decision-makers. As a result, higher values of f_1 are reached. On the other, whether γ is set to values lower than 1, the PSO focuses on both the consistency and the consensus and, therefore, f_1 achieves lower values, which is expected. Particularly, whether γ is equal to 0, the PSO focuses on only the consensus achieved among the decision-makers. It means that higher values of f_2 are obtained.

Table 3. $0.5 \cdot f_1 + 0.5 \cdot f_2$ **and** f_3 **for given values of** β**.**

	$0.5 \cdot f_1 + 0.5 \cdot f_2$	f_3
$\beta = 0.00$	0.815	0.994
$\beta = 0.25$	0.821	0.992
$\beta = 0.50$	0.841	0.978
$\beta = 0.75$	0.917	0.868
$\beta = 1.00$	0.921	0.830

5.3. Parameter β

Table 3 shows the impact of the parameter β, which also stands in the composite fitness function f (these values are obtained by using the same fuzzy preference relations provided initially by the decision-makers in the case study and being $\alpha = 0.4$ and $\gamma = 0.5$). As this parameter determines a trade-off between the combination of the consistency and the consensus ($\gamma \cdot f_1 + (1 - \gamma) \cdot f_2$) and the similarity f_3, we focus on these values. On one hand, whether β is equal to 0, the PSO focuses on the maximization of the similarity. As a result, a higher value of f_3 is achieved (0.994 in this case). On the other, whether β is set to values higher than 0, the PSO focuses on both the combination of the consistency and the consensus and the similarity. Therefore, as it is expected, f_3 reaches lower values. Notably, whether β is equal to 1, the PSO focuses on only the combination of the consistency and the consensus. It means that higher values of this combination are obtained (0.921 in this case) whereas f_3 achieves its lower value (0.830 in this case).

6. Concluding remarks

In this study, a new granular computing-based model for group decision-making has been introduced. Similar to the existing models based on the GrC paradigm, it yields information granules in the form of intervals by allocating a given information granularity level through the assessments given by the decision-makers in the form of fuzzy preference relations. Unlike the existing granular models, it allows to reach a sound balance between the generation of assessments as close as possible to the initial ones given by the decision-makers and the improvement of both the consistency associated with individual decision-makers and the consensus related to the group in decision-making processes defined in multi-criteria and heterogeneous environments.

This research may be continued in the following ways. First, here, we have used a uniform distribution of information granularity. i.e, the information granularity level α distributed across the assessments has been equal for all the decision-makers. Nevertheless, we could consider a non-uniform distribution of the information granularity. In addition to it, the information granularity levels could be optimized so that a particular value of α could become available to every decision-maker. Second, it would be interesting to analyze how the computation time increases with a larger number of criteria, alternatives and decision-makers, and what number of components can still be handled in a reasonable time.

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