Essays on Central Bank Digital Currency and Monetary Policy

by

Adib J. Rahman

A Dissertation

In

Economics

Submitted to the Graduate Faculty
of University of Hawaii at Manoa in
Partial Fulfillment of
the Requirements for
the Degree of

DOCTOR OF PHILOSOPHY

Approved

Liang Wang
Chair of Committee

Miroslav Gabrovski

Nori Tarui

Steven Bond-Smith

Wei (Victor) Huang
University Representative

June, 2025

ACKNOWLEDGEMENTS

I am immensely grateful to my advisor, Liang Wang, for his patience and continued support throughout the course of my PhD. For their generous use of time and encouragement, I am deeply thankful to the members of my dissertation committee: Miroslav Gabrovski, Nori Tarui, and Steven Bond-Smith. I would also like to extend my gratitude to the external committee member, Wei (Victor) Huang, for agreeing to serve in this role.

I am particularly grateful to the UHERO team, where I have spent almost three years working as a Graduate Assistant on the forecast team. Special thanks go to Peter Fuleky for giving me the opportunity to work on the forecast reports alongside him and for our engaging conversations about financial markets, writing, and various other topics. I am also thankful to Ekaterina Sherstyuk for being an exceptional teacher, for teaching me game theory, and for her guidance on the proof of the existence of a competitive equilibrium.

I extend my warm thanks to all my colleagues in the Department of Economics at UH Manoa for their help and support during the course of my studies, particularly: Ivan Rivadeneyra Camino, Nicole Siegal, Hosuk Choi, Binh Le, Hamid Bouaicha, Dongkyu Park, Minh Nguyen, and Joseph Eshun. I would also like to thank Mindy Scordilis for her exceptional administrative support.

I am also thankful to friends outside the department. During my time at the East-West Center, I have met many interesting and friendly people from all walks of life in my dormitory, Hale Manoa. Finally, I would like to thank my parents and my sister for their patience, understanding, and support. Their endless sacrifices have made it possible for me to achieve this doctoral degree.

TABLE OF CONTENTS

ABSTRACT

This dissertation consists of three essays examining various aspects of central bank digital currency (CBDC) and monetary policy.

The first essay, coauthored with Liang Wang, investigates the effects of CBDC issuance in an economy with tax evasion and agent heterogeneity. We present a tractable model where all trades are voluntary, factoring in agent heterogeneity with unobservable idiosyncratic shocks. In the model, CBDC and cash compete as means of payment. The government can utilize CBDC to collect a labor tax, which appears to be non-distortionary. While agents with a lower marginal utility might challenge the government's ability to finance its CBDC expenditure, we conjecture that a class of feasible policies can be identified for the optimal design of CBDC. Such policies could potentially involve higher nominal interest rates and lower inflation compared to the inflation rate associated with cash. In summary, the introduction of CBDC could enhance output and aggregate welfare by disincentivizing tax evasion.

The second essay explores interest-bearing money, illiquid bonds, and banking in a news economy. Private currencies can facilitate intertemporal exchange under limited commitment, but they may exhibit excessive price volatility when backed by productive assets whose expected short-run return is subject to news shocks or flows of information. In the model, banks act as intermediaries by supplying private currencies in the form of bank deposits, backed by short-run expected returns on firms' output pledged as collateral. Adverse news events about firm productivity in the short run can induce price volatility in bank deposits, potentially leading to a liquidity shortage. Adding household heterogeneity with news shocks results in bank deposits being priced at a premium in economies where liquidity matters, particularly during a liquidity shortage. Interest-bearing money, not backed by productive assets, can help alleviate this shortage. I show how interest-bearing money offers an additional policy tool that can help lift depressed asset prices. The interest rate affects asset prices via the investment channel, with banks using interest-bearing reserves as insurance against risk shocks. The relative insensitivity of the value of money to news events means social welfare could be improved, even if lump-sum taxation is not permitted. However, I find that the power of interest-bearing money is limited by the assumption

that household preferences are publicly observable, allowing type-contingent transfers. Extending the model, an illiquid bond and a cash-in-advance constraint enable the coexistence of government debt and private currencies in the absence of news.

The third essay analyzes the optimal design and implementation of CBDC in an economy where CBDC and bank deposits coexist as competing payment instruments. I develop a general equilibrium model to study the optimal design and implementation of central bank digital currencies (CBDCs) in an economy where CBDCs and private bank deposits coexist as competing payment instruments. The findings suggest that the welfare consequences of CBDCs and the policies required for first-best implementation depend on specific parameters. In sufficiently patient economies, a passive monetary policy with non-interest bearing CBDC can achieve the first-best allocation without crowding out bank deposits. Conversely, in more impatient economies, active policies with positive inflation and nominal interest rates are necessary, and interest-bearing CBDC could potentially crowd out bank deposits if the interest rate on CBDC is too high relative to deposit rates. The results highlight the importance of carefully designing CBDCs with incentive-feasible policies intended to maximize welfare and minimize risks to financial intermediation and stability. In a calibrated model representing the US economy, I find that increasing the CBDC interest rate from 0% to 5% leads to a decline of about 5% in bank deposits, while improving overall welfare by 2%.

LIST OF FIGURES

CHAPTER 1
CENTRAL BANK DIGITAL CURRENCY (CBDC) AND MONETARY POLICY WITH HETEROGENEOUS AGENTS

1.1 Introduction

Recent advances in payment technologies have sparked a wave of interest in the introduction of central bank digital currency (CBDC) among central banks worldwide. At present, over 80% of central banks are engaging in CBDC-related research, with 10% having developed pilot projects (Boar et al. (2020)). One potential driving factor for central banks investigating CBDCs is the reduction of illicit activities associated with paper currency. Tax evasion, a prevalent illicit activity in many countries, is often associated with cash usage. According to estimates from the Internal Revenue Service (2019), tax evasion accounted for a tax revenue loss of around $441 billion between 2011 and 2013, representing approximately 1% of the US GDP during that period. Our paper focuses on how CBDC can address the issue of tax evasion, specifically, we study how CBDC can compete with cash as a medium of exchange and improve welfare by minimizing tax evasion.

We develop a general equilibrium model by extending the framework of Xiang (2013) and Andolfatto (2011). Our model is a simplified version of the celebrated Lagos and Wright (2005) (LW) framework without search frictions, where trade among individuals is restricted to occur in competitive markets, similar to the competitive equilibrium version in Rocheteau and Wright (2005). We include private information by incorporating heterogeneity among agents in their need for liquidity. Specifically, one type of agents experiences a higher marginal utility than the other after receiving an idiosyncratic liquidity shock. We study optimal monetary policy when only cash or CBDC or both cash and CBDC are available to agents as payment instruments. Both cash and CBDC have some distinguishing features that make them appealing as means of payments. Cash is anonymous and provides agents with the opportunity to evade taxes by hiding their cash balances.

In the paper, we define CBDC as government-issued money in digital format that can be used for retail transactions. An individual can use CBDC to purchase goods and services by opening an online account with the central bank, thereby ensuring widespread accessibility. We presume that tax evasion is unlikely with CBDC as the government can partially track the CBDC balances of agents through centralized blockchain technology. Agents can benefit from using CBDC by earning interest, akin to interest-bearing treasury securities, but must bear a fixed fee to open their CBDC accounts with the central bank. The fixed CBDC fee can be thought of as a cost that summarizes in a reduced form the cost of losing anonymity for the agents, adopting an electronic device, or working with the CBDC application (Davoodalhosseini (2021)).

In the literature, it is common practice to assume that the government possesses a lump-sum tax instrument. Optimal monetary policy is usually conducted with deflation (Friedman rule) and zero nominal interest rate; where the requisite deflation is financed by lump-sum taxes. Since agents can evade taxes with cash, this means that lump-sum taxation is not feasible. We restrict our model to voluntary trades. Optimal policy in our model is inflationary, which is intended to insure individuals against idiosyncratic liquidity shocks. The government does not have the ability to levy lump-sum taxes, but can levy a voluntary labor tax if agents use CBDC. The government can influence the relative rate of return on the two currencies by adjusting the supplies of cash and CBDC.

Our findings suggest that in a quasi-linear environment, the labor tax that can only be extracted when individuals use CBDC is non-distortionary. In fact, a positive tax on labor can be interpreted as a positive income tax, with no impact on efficiency. The Friedman rule can be optimal regardless of whether labor income is positively taxed. However, in the absence of lump-sum taxation, the Friedman rule does not implement a first-best solution. Our model also examines how the potential for tax evasion with cash imposes restrictions in the absence of a lump-sum tax instrument. A key component in our model is how tax evasion with cash imposes a restriction in the absence of a lump-sum tax instrument. To dissuade individuals from using cash, the only option for the government is positive inflation. If individuals use CBDC, then there will still be inflation, however they must be compensated with positive nominal interest rates. If CBDC has to have a higher relative rate of return than cash, then inflation in a CBDC regime must be lower than in a cash regime. This is our conjecture, as we do not have a proof. But the ingredients of our argument are fairly standard.

Our paper also provides insights into the optimal design of CBDC when tax evasion is a major concern in an economy. We argue that agents with a higher marginal utility will have the "right" incentives to use CBDC when their debt-constraint is binding. Conversely, agents with a lower marginal utility may have an incentive to misreport their types to acquire a higher money balance, and thus are more likely to prefer cash. We further explore how the implementation of CBDC can redistribute the purchasing power of agents to improve welfare. The specifics of this redistribution depend on policy parameters; individuals with lower marginal utility may be incentivized to use CBDC if they pay the fixed fee, assuming that inflation from CBDC is lower than that of cash. A lower inflation from CBDC will allow CBDC to offer a higher rate of return. Additionally, we derive the equilibrium fixed cost of CBDC that the government can collect.

1.2 Literature and Background

Our paper contributes to the burgeoning literature on CBDC in the context of tax evasion. Specifically, our work closely aligns with Wang (2020), Kwon et al. (2020), and Bajaj and Damodaran (2022). Wang (2020) explores CBDC design while considering tax evasion, portraying agents and the government in a dynamic game where the former is audited by the latter, with inflation dissuading agents from using cash to evade taxes. The author finds that the introduction of an interest-bearing CBDC decreases tax evasion, thereby increasing output and welfare. Kwon et al. (2020) study tax evasion and CBDC in relation to central bank independence. They introduce a proportional sales tax as a cost associated with CBDC that can potentially lead to distortion. In contrast, the labor tax in our model is non-distortionary and is also tied to the fixed cost of CBDC. Bajaj and Damodaran (2022) examine tax evasion and the informal economy within a Lagos and Wright (2005) framework, where the government expends effort in collecting taxes. They include preference heterogeneity to characterize equilibrium conditions that may result from agents choosing multiple currencies. Unlike our paper, they assume that the government can observe all cash transactions. Other papers that exclusively study tax evasion and the shadow economy within an LW framework include Gomis-Porqueras et al. (2014), Aruoba (2021), and Lahcen (2020).

There are also several papers that examine the welfare implications of CBDC issuance. Williamson (2019) finds that CBDC can reduce crime associated with cash in an environment where banks have limited commitment. The paper posits that CBDC can also economize on the scarcity of safe collateral by paying interest, a point that aligns with our research. Davoodalhosseini (2021) illustrates that CBDC can enhance welfare when the central bank can cross-subsidize between different types of agents, a feature not possible with cash. While the author includes the concept of nonlinear interest-bearing CBDC, our study demonstrates the possible linkage between the interest rate on CBDC and its associated cost.

Several papers have examined the impact of CBDC on the banking sector and monetary policy. Assuming a perfectly competitive banking sector, Keister and Sanches (2021) find that while CBDC can promote exchange efficiency, it may also increase the funding costs of financial intermediaries and crowd out financial intermediation, thereby preventing an efficient level of investment. Andolfatto (2021) and Chiu et al. (2019) explore the impact of CBDC issuance on banking in imperfectly competitive markets. On the other hand, Whited et al. (2022) use U.S. bank data to quantify the impact of CBDC on bank lending. They find that if a CBDC pays interest, this may amplify the effect of monetary policy shocks on bank lending.

Issues around financial stability with CBDC issuance have also been studied by several authors, including Brunnermeier and Niepelt (2019), who derive equilibrium conditions in which CBDC can lead to financial stability. Similarly, Kim and Kwon (2019) use a general equilibrium model of bank liquidity provision, resembling Diamond and Dybvig (1983), to find that CBDC does not result in a credit crunch and hinder financial stability. More works that tackle this topic include Williamson (2021), Keister and Monnet (2020), and Fernández-Villaverde et al. (2021). In addition to these, a number of papers have explored the use of multiple means of payments in an LW model, such as Dong and Jiang (2010), Zhu and Hendry (2019), and Chiu and Wong (2015). Lastly, other papers that examine CBDC and monetary policy in a DSGE framework include Barrdear and Kumhof (2021), Ferrari et al. (2022), and Niepelt (2022).

The rest of the paper proceeds as follows. Section 2 describes the environment. Section 3 describes the decision-making problems of the agents. Section 4 characterizes the competitive monetary equilibrium in which cash and CBDC can either coexist or exist independently. The redistributive policy with CBDC is examined in details. Section 5 concludes.

1.3 Environment

The model is similar to that of Xiang (2013) and Andolfatto (2011). There is a continuum of infinitely-lived households consisting of consumer-producer pairs, distributed uniformly on the unit interval. Time is discrete and goes forever, indexed by $t = 0, 1, 2, ..., \infty$. In the spirit of Lagos and Wright (2005), each time-period t is divided into two subperiods, labeled *day* and *night*. Households belong to one of two permanent groups: Group 1 and Group 2. Each group is of equal measure. Denote by A and B the set of Group 1 and Group 2, respectively.

All households meet at a central location during the day. Let $x_t(i) \in \mathbb{R}$ denote consumption (production, if negative) of output during the day by household $i \in A \cup B$ at date t. Linear preferences in $x_t(i)$ implies that utility is transferable. Assuming that the day good is perishable, an aggregate resource constraint implies

$$X \equiv \int_{A\cup B} x_t(i)di \leq 0 \tag{1.1}$$

for all $t \geq 0$.

Let $\{c_t(i), y_t(i)\} \in \mathbb{R}^2_+$ denote consumption and production, respectively, output at night household $i \in A \cup B$ at date t. The utility from night consumption is given by $\delta_t u(c_t(i))$, where $u'' < 0 < u'$, $u'(0) = \infty$ and $u(0) = 0$. The utility from night production is given by $g(y_t(i))$, where $g' > 0$ for $y > 0$, $g'' \geq 0$. Following Kocherlakota (2003), we impose a spatial structure for night transactions, labeled *location 1* and *location 2*. After the shock to consumer type is realized, producers in group 1(2) households travel to location 2(1), while consumers in group 1(2) travel to location 1(2). This implies that a household cannot consume its own output at night. Perishability of the night good implies another aggregate resource constraint

$$\textstyle\int_A c_t(i)di \leq \int_B y_t(i)di \quad \text{and} \quad \int_B c_t(i)di \leq \int_A y_t(i)di. \tag{1.2}$$

For consumer heterogeneity, we introduce an idiosyncratic shock on consumer types that captures the differences in their marginal utilities. More specifically, at the beginning of each night, consumers experience an idiosyncratic preference shock represented by the parameter $\delta_t(i)$, where $\delta_t(i) \in \left\{\delta^l = 1, \delta^h = \eta\right\}$ and $\eta > 1$. The shock is *i.i.d.* across consumers within each group and across time. In this paper, we assume that the realization of these preference shocks δ_t is private information.

Households discount payoffs across period with the discount factor $\beta \in (0,1)$, so that their preferences can be represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{x_t(i) + \delta_t(i)u(c_t(i)) - g(y_t(i))\right\}. \tag{1.3}$$

We focus on symmetric stationary allocations, where all agents are equally weighted and agents of the same type are treated the same and the two types are treated symmetrically. During the night, the social planner instructs consumers of a representative household to consume $c^j \in \left\{c^l, c^h\right\}$ conditional on type realization. Symmetric locations at night implies that there is a measure $1/4$ of type h consumers, a measure $1/4$ of type l consumers, and a measure $1/2$ of producers; so that the resource constraint (1.2) can be expressed in another way

$$\frac{1}{4}c^l + \frac{1}{4}c^h = \frac{1}{2}y. \tag{1.4}$$

The ex-ante lifetime utility of households at a stationary allocation $\left(c^l, c^h, y\right)$ is expressed as

$$W = \frac{1}{(1-\beta)}\left\{\frac{1}{2}\left[u(c^l) + \eta u(c^h)\right] - g(y)\right\}. \tag{1.5}$$

Linear utility in the day good x implies that any lottery over $\{x_t(i)\}$ for all $t \geq 0$ such that $E_0\left[x_t(i)\right] = 0$ can be a solution. A planner may set $x_t(i) = 0$ for all i households at date $t \geq 0$ as a trivial solution. The first-best allocation $\left(c^{l^*}, c^{h^*}, y^*\right)$ maximizes (1.5) subject to the aggregate resource constraints (1.1) and (1.4).

The first-best allocation is characterized by

$$\begin{aligned} u'(c^{l^*}) &= \eta u'(c^{h^*}), \\ u'(c^{l^*}) &= g'(y^*), \\ c^{l^*} + c^{h^*} &= 2y^*. \end{aligned} \tag{1.6}$$

In what follows, we impose restrictions on this environment that will make a medium of exchange essential. We assume that households lack commitment and are anonymous. Limited commitment implies that all trades are voluntary satisfying sequential rationality (individually rational at every period t). Anonymity means that it is impossible to monitor the past action of agents pertaining to their trading histories. Given these assumptions, trade by credit is infeasible so that renders money—in the form of cash and CBDC—essential, as stated by Kocherlakota (1998). Furthermore, we restrict all trades to occur in a sequence of competitive spot markets with cash and CBDC being exchanged for goods in the day and night. This still preserves the essentiality of money even without search frictions as shown in Lagos and Wright (2005).

Money (or a medium of exchange) is essential because society would not be able to achieve desirable outcomes otherwise. Individuals would not be trading as a result of these frictions mentioned before. In contrast to Andolfatto (2011) where attention is restricted to linear mechanisms, we show how interest-bearing CBDC may improve welfare in nonlinear environments, much akin to Andolfatto (2010).

1.4 Decision-making of households

1.4.1 Government policy

The central bank and the government are a consolidated entity who issue intrinsically worthless tokens called cash and CBDC. Denote by $\{(v_1, v_2), (w_1, w_2)\}$ the price of cash and CBDC in the day and night, respectively. Let (M_c, M_e) denote the cash and CBDC supply for next period, which evolve over time, according to $M_c = \gamma_c M_c^-$ and $M_e = \gamma_e M_e^-$, respectively, where γ_c and γ_e denote (gross) growth rates of cash and CBDC, respectively (a superscript '-' stands for the previous period's cash and CBDC supply). The government's policy rule is to pay nominal interest rate R on CBDC balances to those individuals who are willing to pay the fixed fee, K. In addition, the government can also collect taxes, T_x, on labor x from each household that uses CBDC as a payment instrument during the day. This is because we are assuming that a household can hide cash balances to avoid paying the labor tax.

In what follows, the government has an aggregate interest obligation $(R-1)M_e$. The government can also print new money in the form of cash and CBDC $M_c - M_c^- +$

$M_e - M_e^-$. The government can only collect the fixed CBDC fee $K^j \geq 0$ at night. A household enters the night market and decides whether to pay the fixed fee. If a household pays the fixed fee then he earns the nominal interest R at night from its CBDC holdings. Both K^j and R will affect the future CBDC balances carried forward into the next day. If a household declines to pay the fixed fee, then CBDC here works like cash.

We simplify matters by applying a result that holds in this class of quasilinear models. We design the government policy so that only the mass $1/4$ of agents will voluntarily pay the fixed fee at night and conditional on their initial real CBDC balances a_e (explained in more details below) will pay the labor tax T_x in the day. Thus, a feasible government policy will have to satisfy the government budget constraint,

$$(R-1)M_e = M_c - M_c^- + M_e - M_e^- + T_x X 1_{a_e} + \frac{1}{4}K^j,$$

where $1_{a_e} \geq 0$ is an indicator function, for the labor tax that can be collected only when CBDC is used as a means of payments and X is the aggregate labor. By multiplying both sides of this latter expression by w_2, the government budget constraint may alternatively be expressed in real terms by

$$[(R-2)\gamma_e + 1]\, w_2 M_e^- - (\gamma_c - 1) w_2 M_c^- = \tau_x X + \frac{1}{4}\kappa^j, \tag{1.7}$$

where $\tau \equiv w_2 T$ and $\kappa^j \equiv w_2 K^j$. Furthermore, assume the cash and CBDC supply, respectivley, is expanded at a constant growth rate, that is, $\gamma_c \geq 1$ and $\gamma_e \geq 1$. We define a *zero intervention policy* as a policy when $\gamma_c = \gamma_e = R = 1$, so that $\tau_x = \kappa^j = 0$.

1.4.2 The day market

Households enter the day with $(m_1, e_1) \geq 0$ of nominal balances of cash and CBDC and the night market with $(m_2, e_2) \geq 0$. Households can trade x_c of the day good with cash and x_e of the day good with CBDC. The day budget constraint with cash is given by $x_c = v_1 m_1 - v_1 m_2$ and the day budget constraint with CBDC is given by $x_e = {w_1 e_1}/{1+T_x} - {w_1 e_2}/{1+T_x}$. Denote by $a_c \equiv v_1 m_1$ and $q_c \equiv v_2 m_2$ the real cash balances at day and the night, respectively. Similarly, denote by $a_e \equiv w_1 e_1$ and $q_e \equiv w_2 e_2$ the real CBDC balances at day and night, respectively. Define $\phi \equiv {v_1}/{v_2}$

and $\psi \equiv {}^{w_1}/_{w_2}$. Since $x = x_c + x_e$, the day-market budget constraint can now be expressed as

$$x = a_c - \phi q_c + \frac{w_2}{w_2 + \tau_x} a_e - \frac{w_1}{w_2 + \tau_x} q_e. \tag{1.8}$$

Denote by $W(a_c, a_e)$ the utility value of a household entering a day with real cash and CBDC balances, (a_c, a_e). Also denote by $V(q_c, q_e)$ the utility value of beginning the night with (q_c, q_e) cash and CBDC balances. Note that $V(q_c, q_e)$ denotes the value before a household knows its consumer type. The value functions W and V must satisfy the recursive relationship

$$W(a_c, a_e) \equiv \max_{q_c, q_e} \left\{ a_c - \phi q_c + \frac{w_2}{w_2 + \tau_x} a_e - \frac{w_1}{w_2 + \tau_x} q_e + V(q_c, q_e) \right\}. \tag{1.9}$$

We will later impose assumptions on V so the demand for both cash and CBDC are determined by the first-order conditions:

$$\frac{\partial V(q_c, q_e)}{\partial q_c} = \phi \tag{1.10}$$

and

$$\frac{\partial V(q_c, q_e)}{\partial q_e} = \frac{w_1}{w_2 + \tau_x}. \tag{1.11}$$

Note that the demand for CBDC decreases with labor tax, τ_x (see also Gomis-Porqueras et al. (2014)). Moreover, all households enter the night with identical real cash and CBDC balances $q_c \in (0, \infty)$ and $q_e \in (0, \infty)$. In other words, cash and CBDC demand are independent of the initial cash and CBDC holdings (a_c, a_e). This is often highlighted in the Lagos-Wright (LW) models. Applying the envelope theorem yields

$$\frac{\partial W(a_c, a_e)}{\partial a_c} = 1, \tag{1.12}$$

$$\frac{\partial W(a_c, a_e)}{\partial a_e} = \frac{w_1}{w_2 + \tau_x}. \tag{1.13}$$

1.4.3 The night market

A household carries over a nominal cash-CBDC portfolio of (m_2, e_2) at night. Consumer preference shock is realized at the beginning of the night market. Consumers and producers in a household travel to either *location 1* or *location 2*. A household makes the consumption and production decisions, which are carried out by consumers and producers by traveling into different locations.

Denote by $c^j = c_c^j + c_e^j$ for consumption of a household with realized consumer type j, where c_c^j is type j's consumption by using cash and c_e^j is type j's consumption by using CBDC. Similarly, $y^j = y_c^j + y_e^j$ is the output produced where a mixture of cash and CBDC can be used for its purchase. Hence, future cash balances are given by $m_1^+(j) = m_2 + v_2^{-1}(y_c^j - c_c^j)$. Expressed in real terms, this constraint is given by

$$a_c^+(j) = \frac{\phi}{\gamma_c}\left(q_c - c_c^j + y_c^j\right).$$

Future CBDC balances will depend on whether type j consumer will pay the fixed fee K^j to open a CBDC account with the central bank. A payment of K^j is required for a type j consumer to earn a nominal interest rate R. The interpretation of K^j and R is quite similar to that in Andolfatto (2010), except that these parameters are introduced into the night market after the realization of consumer types. Define the indicator function for a type j household by $\sigma^j \in [0,1]$, where $\sigma^j = 1$ means that a type j household pays the fixed fee at date t. In the event a household of type j opts not to pay the fixed fee, they do not accrue any interest on their CBDC holdings. In this scenario, CBDC functions similarly to cash, except that households pay the labor tax without earning any interest on their CBDC holdings. Given a type j household is willing to pay the fixed fee K^j, future CBDC balances are given by $e_1^+(j) = Re_2 - K^j + w_2^{-1}(y_e^j - c_e^j)$.[1] Alternatively, in real terms,

$$a_e^+(j) = \frac{\psi}{\gamma_e}\left(Rq_e - \kappa^j - c_e^j + y_e^j\right).$$

For a household with realized consumer type $j \in \{l, h\}$, there are two cash and CBDC constraints for consumption of night output; that is,

$$c_c^j \leq q_c, \tag{1.14}$$

$$c_e^j \leq \sigma^j(Rq_e - \kappa^j) + (1 - \sigma^j)q_e. \tag{1.15}$$

[1]Note that our CBDC fee structure has a nonlinear mechanism, which could be taken advantage of by a coalition of agents. However, the presumed absence of commitment makes such coalitions infeasible.

Let $\lambda_c^j \geq 0$ and $\lambda_e^j \geq 0$ denote the Lagrange multipliers associated with the constraints (1.14) and (1.15), respectively. In what follows, the choice problem at night for a type j household is given by

$$
\begin{aligned}
V^j(q_c, q_e) \equiv \max_{\substack{\sigma^j, c_c^j, c_e^j, \\ y_c^j, y_e^j}} \Bigg\{ & \delta^j u(c^j) - g(y^j) \\
& + \beta W\left(\frac{\phi}{\gamma_c}\left[q_c - c_c^j + y_c^j\right], \frac{\psi}{\gamma_e}\left[\sigma^j(Rq_e - \kappa^j - c_e^j + y_e^j) + (1-\sigma^j)(q_e - c_e^j + y_e^j)\right]\right) \\
& + \lambda_c^j(q_c - c_c^j) + \lambda_e^j\left[\sigma^j(Rq_e - \kappa^j) + (1-\sigma^j)q_e\right]\Bigg\}.
\end{aligned} \tag{1.16}
$$

We now make the following assumptions on the function V:

Assumption 1.1. *i)* $V_{q_c,q_c} < 0 < V_{q_c}$ *and* $\phi < \frac{\partial V(0,q_e)}{\partial q_c}$;

ii) $V_{q_e,q_e} < 0 < V_{q_e}$ *and* $\frac{w_1}{w_2+\tau_x} < \frac{\partial V(q_c,0)}{\partial q_e}$;

iii) V *is non-differentiable at* $(R-1)q_e = \kappa^j$ *for* $\sigma^j \in [0,1]$.

In fact, *Assumption 1* captures the properties of a stationary monetary equilibrium. By applying (1.13) and (1.14), all producers regardless of household types, produce identical output y_c with cash and y_e with CBDC satisfying

$$g'(y_c) = \frac{\beta\phi}{\gamma_c}, \tag{1.17}$$

$$g'(y_e) = \frac{\beta w_1}{\gamma_e(w_2+\tau_x)}. \tag{1.18}$$

The demand for desired consumption c_c^j with cash and the desired consumption c_e^j with CBDC is characterized by the following first-order conditions:

$$\delta^j u'(c_c^j) = \frac{\beta\phi}{\gamma_c} + \lambda_c^j, \tag{1.19}$$

$$\delta^j u'(c_e^j) = \frac{\beta w_1}{\gamma_e(w_2+\tau_x)} + \lambda_e^j. \tag{1.20}$$

If the CBDC constraint binds ($\lambda_e^j > 0$), then a household with realized type j consumer will pay the fixed CBDC fee ($\sigma^j = 1$), as they would like to relinquish their money or discharge their debt for consumption the following day. But if the CBDC constraint is slack ($\lambda_e^j = 0$) then there could be three possiblities, so that the optimal decision to pay the fixed fee κ for a type j consumer satisfies

$$\sigma^j = \begin{cases} 1 \\ [0,1] \\ 0 \end{cases} \text{if } \kappa^j \begin{cases} < \\ = (R-1)q_e. \\ > \end{cases} \tag{1.21}$$

As highlighted in Andolfatto (2010), only consumers with sufficiently large money holdings q_e will find it optimal to pay the fixed fee κ^j given that $R > 1$ and $\kappa^j > 0$. That is, CBDC is used in large-scale transactions and cash is used in small-scale transactions, as there is intrinsically no difference in how both these payment instruments are used if there is no fixed fee. Moreover, by the envelope theorem

$$\frac{\partial V^j(q_c, q_e)}{\partial q_c} = \frac{\beta\phi}{\gamma_c} + \lambda_c^j, \tag{1.22}$$

$$\frac{\partial V^j(q_c, q_e)}{\partial q_e} = \begin{cases} \frac{\beta R w_1}{\gamma_e(w_2+\tau_x)} + R\lambda_e^j \\ \\ \frac{\beta w_1}{\gamma_e(w_2+\tau_x)} + \lambda_e^j \end{cases} \text{if } \kappa^j \begin{cases} < (R-1)q_e \\ > (R-1)q_e. \end{cases} \tag{1.23}$$

Given the uncertainty of whether cash or CBDC will be utilized as a payment instrument, we can employ equations (1.17) and (1.18) to derive the following rate-of-return equality condition:

$$\frac{\phi}{\gamma_c} = \frac{w_1}{\gamma_e(w_2 + \tau_x)}. \tag{1.24}$$

Condition (1.24) restricts attention to equilibria where cash and CBDC must have the same rate of return from the night to the next day, if they are to be accepted as payment. Alternatively, condition (1.24) can also be stated as a no-arbitrage condition. It follows that at the individual level, the cash-CBDC portfolio composition is indeterminate in equilibrium. We define $\zeta = {}^{\gamma_e}/_{\gamma_c}$ and rewrite this condition as

$$\zeta = \frac{w_1}{\phi\left(\frac{w_1}{\psi} + \tau_x\right)}. \tag{1.25}$$

The above condition ensures the co-existence of cash and CBDC, as they have the same rate of return. If $\zeta > {}^{w_1}/_{\phi(\frac{w_1}{\psi}+\tau_x)}$, then the rate of return on cash is higher than that of CBDC, so that all individuals will use cash. Conversely, if $\zeta < {w_1}/{\phi(\frac{w_1}{\psi}+\tau_x)}$ then all individuals will use CBDC.

Using (1.25) we can also derive an expression for the labor tax τ_x,

$$\tau_x = \frac{(\psi - \zeta\phi)\, w_1}{\zeta\phi\psi}. \tag{1.26}$$

If only cash is used by agents, then a tax on labor income is not attainable for the government, so $\tau_x = 0$. If CBDC is used then $\tau_x \geq 0$. We can obtain an upper bound on τ_x by defining $\overline{\tau_x} \equiv \tau_x < {(\psi-\zeta\phi)w_1}/{\zeta\phi\psi}$. Therefore, the range of values for labor tax is feasible within the interval $[0, \overline{\tau_x}]$, that is, $\tau_x \in [0, \overline{\tau_x}]$. Figure 1.1 depicts the cases in which cash and CBDC equilibria are separated by the labor tax τ_x. Note that $\zeta = 1$ implies $\phi = \psi$, so that the rate of return on cash and CBDC is exactly equal.

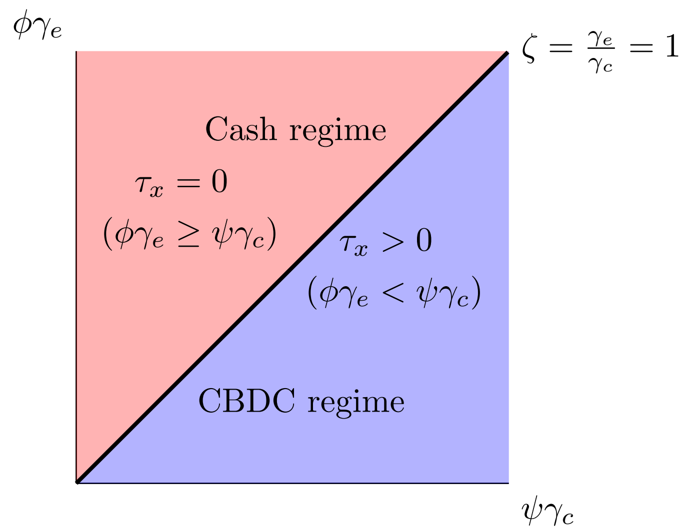

Figure 1.1: Separation of cash and CBDC equilbria

1.5 Competitive equilibrium

We now seek to characterize steady-state equilibria that can arise from the conditions in the model. As mentioned earlier, we can have economies where only cash is used, economies where only CBDC is used, and economies where a mixture of cash and CBDC is used by households.

Before deriving the equilibrium allocations, we first make some assumptions. On the one hand, since type h consumers receive a higher marginal utility from consumption, they are more likely to discharge their debt so that both their cash and CBDC constraints will bind, that is, $\lambda_c^h > 0$ and $\lambda_e^h > 0$. For this reason, we assume that type h consumers will pay the fixed fee $\kappa^h = \kappa$. On the other hand, type l consumers are likely to misrepresent themselves as type h consumers by hiding their money balances to acquire the high money balance at the beginning of period 0. For this reason, we can assume that their debt constraints will remain slack, that is, $\lambda_c^l = \lambda_e^l = 0$. Therefore, type l consumers will not be paying the fixed fee, that is, $\kappa^l = 0$. Later on, we will consider the case where both the cash and CBDC constraints for type l consumers will bind.

1.5.1 CBDC-only economy ($\zeta < {}^{w_1}/_{\phi(\frac{w_1}{\psi}+\tau_x)}$)

1.5.1.1 Market clearing

Suppose that CBDC has a higher rate of return than cash, so that agents only use CBDC as a means of payment. We have two market-clearing conditions. For the night goods market, the clearing condition is (1.4). The market-clearing condition for the money market involving CBDC is given by

$$q_e = w_2 M_e^-. \tag{1.27}$$

1.5.1.2 CBDC Equilibrium

In what follows, we restrict attention to stationary equilibria; which entails ${}^{w_1}/_{w_1^+} = {}^{w_2^+}/_{w_2^+} = \gamma_e$. If the CBDC constraint for type h constraint binds then for any $\gamma_e > \beta$, we must have

$$c_e^h = Rq_e - \kappa. \tag{1.28}$$

First, note that since

$$\frac{\partial V(q_c, q_e)}{\partial q_e} = \frac{1}{2}\frac{\partial V^l(q_c, q_e)}{\partial q_e} + \frac{1}{2}\frac{\partial V^h(q_c, q_e)}{\partial q_e}, \tag{1.29}$$

combining (1.23) with $\sigma^l = 0$ and $\sigma^h = 1$ yields

$$\frac{\partial V(q_c, q_e)}{\partial q_e} = \frac{1}{2}\left[\frac{\beta w_1}{\gamma_e(w_2 + \tau_x)}\right] + \frac{1}{2}\left[\frac{\beta R w_1}{\gamma_e(w_2 + \tau_x)} + R\lambda_j^e\right]. \tag{1.30}$$

Combining (1.20) and (1.11) leads to

$$\frac{w_1}{w_2 + \tau_x} = \frac{1}{2}u'(c_e^l) + \frac{1}{2}R\eta u'(c_e^h). \tag{1.31}$$

Now, combining (1.31) with the market clearing conditions (1.4) and (1.27) leads to

$$\left[2\left(\frac{\gamma_e}{R\beta}\right) - \frac{1}{R}\right] u'(c_e^l) = \eta u'(c_e^h). \tag{1.32}$$

Appealing to (1.18), one obtains

$$g'(y_e) = u'(c_e^l). \tag{1.33}$$

The equilibrium allocation $\left(c_e^l, c_e^h, y_e\right)$ for a CBDC economy is then characterized by (1.32), (1.33) and (1.4) when type l consumers are not debt constrained. Note that the "standard" Friedman rule prescription of setting $(R, \gamma_e) = (1, \beta)$ will result in the competitive monetary equilibrium corresponding to the first-best allocation. However, since type l consumers do not willingly pay the fixed fee κ, implementing a deflationary policy according to the Friedman rule is not feasible. Moreover, the labor tax τ_x does not affect the equilibrium allocation, as such a tax is voluntary in the sense that individuals can opt out of paying this tax by using cash instead of using CBDC for transactions. We have the following proposition.

Proposition 1.1. *When $\zeta < {}^{w_1}/_{\phi(\frac{w_1}{\psi}+\tau_x)}$, in a pure CBDC economy, the labor tax $\tau_x \in (0, \overline{\tau_x}]$ is non-distortionary and does not affect the equilibrium allocation $\left(c_e^l, c_e^h, y_e\right)$.*

1.5.2 Cash-only economy ($\zeta > {}^{w_1}/_{\phi(\frac{w_1}{\psi}+\tau_x)}$)

1.5.2.1 Market clearing

Assuming that the rate of return on cash is higher than that of CBDC, agents always demand physical currency and do not accept electronic means of payment. In addition to the night goods market clearing (given by condition (1.4)), the clearing condition for the money market in the form of cash is

$$q_c = v_2 M_c^-. \tag{1.34}$$

1.5.2.2 Cash Equilibrium

Similar to the CBDC-only economy, we focus on stationary equilibria in a cash-only economy; in which case $v_1/v_1^+ = v_2^+/v_2^+ = \gamma_c$. Assuming that the cash constraint for type h binds, we will have $c_c^h = q_c$ for any $\gamma_c > \beta$. Once again, since

$$\frac{\partial V(q_c, q_e)}{\partial q_c} = \frac{1}{2}\frac{\partial V^l(q_c, q_e)}{\partial q_c} + \frac{1}{2}\frac{\partial V^h(q_c, q_e)}{\partial q_c}, \tag{1.35}$$

combining (1.10), (1.19), and (1.22) yields

$$\phi = \frac{1}{2}u'(c_c^l) + \frac{1}{2}\eta u'(c_c^h). \tag{1.36}$$

Combining (1.19) when type l consumers are not cash constrained with (1.36) along with the market-clearing conditions (1.4) and (1.34) leads to

$$\left[2\left(\frac{\gamma_c}{\beta}\right) - 1\right] u'(c_c^l) = \eta u'(c_c^h). \tag{1.37}$$

Considering (1.17), one obtains

$$g'(y_c) = u'(c_c^l). \tag{1.38}$$

Now, the equilibrium allocation $\left(c_c^l, c_c^h, y_c\right)$ for a cash economy is characterized by (1.37), (1.38) and (1.4). Once again, while the Friedman rule ($\gamma_c = \beta$) could potentially result in the first-best allocation, its implementation is unfeasible in this economy due to the lack of a lump-sum tax instrument associated with cash. This limitation arises from the fact that individuals have the option to conceal their cash balances should they desire to consume more of the day good in the following day.

The above equilibrium allocation is assuming that the cash constraint for type l consumers is slack. We now consider when type l consumers are cash constrained, that is, $\lambda_c^l > 0$. Using (1.14) and the market-clearing condition (1.4), one obtains

$$c_c^l = c_c^h = y_c. \tag{1.39}$$

Furthermore, combining (1.17) and (1.36) gives rise to

$$g'(y_c) = \frac{\beta}{2\gamma_c}(1+\eta)\,u'(y_c). \tag{1.40}$$

The equations (1.39) and (1.40) fully characterize the equilibrium allocation $\left(c_c^l, c_c^h, y_c\right)$ for a cash economy when the debt-constraints for both type l and type h consumers bind. As the night good y_c and the ex ante welfare W are strictly decreasing in γ_c in both these scenarios, the optimal policy is to set a zero intervention policy with a fixed money supply.

Proposition 1.2. *When $\zeta > \psi/\phi$ with $\tau_x^* = 0$, in a pure cash economy, the optimal policy is to set $\kappa^* = 0$, which implies $\gamma_c^* = 1$.*

Owing to the limited commitment and anonymity that make lump-sum taxation unfeasible, setting the lower limit to $\gamma_c = 1$ optimizes welfare. The second-best solution, as outlined in Xiang (2013), is to maintain a passive policy that minimizes inflation and simultaneously maximizes the rate of return on currency.[2]

1.5.3 Redistributive policy with CBDC

In the event that the CBDC constraint for type l consumers is not binding, a sufficiently low rate of inflation will be necessary to redirect purchasing power in a socially favorable manner. This is because type l saving falls as inflation rate rises. Since inflation effectively serves as a tax on currency, the purchasing power of the unconstrained type l consumers diminishes when they face higher inflation. Depending on the specific parameters, these unconstrained l types could become constrained and that would not decrease their saving.

[2] Indeed, this discovery closely aligns with Proposition 1 in Xiang (2013).

To this end, we consider a mixed cash and CBDC economy with a policy of zero intervention; so that $\tau_x = \kappa = 0$ and $\gamma_e^1 = \gamma_c^1 = R = 1$. Using condition (1.32), the CBDC constraint for both types of consumers will bind when $\beta < 2/(1+\eta)$ conditional on $\eta_0 \equiv \eta > 1$ and $\gamma_e^1 = \gamma_c^1 = 1$ and $R^1 = 1$. As in Andolfatto (2011), we refer to this economy as an impatient economy. Then along with another market-clearing condition for the mixed regime $q_c = w_2 M_c^-$, the equilbrium allocation must satisfy (1.28) and

$$c_e^l = q_e. \tag{1.41}$$

Note that first we have to solve for the equilibrium fixed fee, κ, to simplify the equations above. Making use of the government budget constraint (1.7) and combining with (1.27), we can obtain

$$\kappa = 4\left\{\left[(R-2)\gamma_e + 1\right] q_e - \tau_x X\right\}. \tag{1.42}$$

Applying the aggregate resource constraint (1.1), the latter expression can be further reduced to

$$\kappa^* = 4\left\{\left[(R-2)\gamma_e + 1\right] q_e\right\}. \tag{1.43}$$

Condition (1.43) is the optimal κ^* that is necessary to make the debt-constraints bind for both types of consumers. Furthermore, the linear utility in the day good x yields a result that is immediately apparent.

Proposition 1.3. *κ^* is determined independently of τ_x.*

In other words, given that agents have linear preferences in $x_t(i)$, they are indifferent across any lottery over $\{x_t(i) : t \geq 0\}$ that delivers a specific expected value. As a result, the fixed CBDC fee is not dependent on the labor tax, τ_x. Though the labor tax does not directly generate revenue for the government, it serves as a device to influence individual behavior.

Next, we will derive the equilibrium allocation when agents use CBDC. Using the night goods market-clearing condition (1.4) along with (1.28) and (1.41), this implies

$$\frac{1}{4}q_e + \frac{1}{4}(Rq_e - \kappa) = \frac{1}{2}y;$$

so that

$$q_e = \frac{2\zeta\phi\psi y_e}{\zeta\phi\psi\left\{1+R-4\left[(R-2)\gamma_e+1\right]\right\}}. \tag{1.44}$$

Now, plugging back everything, the equilibrium allocation is, in this case, given by

$$c_e^l = \frac{2\zeta\phi\psi y_e}{\zeta\phi\psi\left\{1+R-4\left[(R-2)\gamma_e+1\right]\right\}}, \tag{1.45}$$

$$c_e^h = \frac{2R\zeta\phi\psi y_e - 8\left[(R-2)\gamma_e+1\right]\zeta\phi\psi y_e}{\zeta\phi\psi\left\{1+R-4\left[(R-2)\gamma_e+1\right]\right\}}. \tag{1.46}$$

From the above conditions, we can ensure that the saving of type l consumers, $s_e^l(\gamma_e,\gamma_c,R) \equiv q_e(\gamma_e,\gamma_c,R) - c_e^l(\gamma_e,\gamma_c,R)$, goes to zero if the CBDC constraint for l types binds. Alternatively, for a binding cash constraint, type l saving $s_c^l(\gamma_e,\gamma_c,R) \equiv q_c(\gamma_e,\gamma_c,R) - c_c^l(\gamma_e,\gamma_c,R) = 0$ from condition (1.39).

We now consider an economy that is sufficiently patient, that is, $\beta \geq 2/(1+\eta)$. In this scenario, with sufficiently low inflation and high nominal interest rates, the cash and CBDC constraints for the l types continue to be slack. Under these conditions, the equilibrium allocation in a mixed cash-CBDC regime is characterized by (1.32) and (1.37). This means that inflation hurts efficiency as long as both the cash and CBDC constraints remain slack. It is easy to verify that $s_c^l(\gamma_e,\gamma_c,R) \equiv q_c(\gamma_e,\gamma_c,R) - c_c^l(\gamma_e,\gamma_c,R)$ is monotonically decreasing in γ_c and that $s_c^l(\gamma_e,\gamma_c,R) = 0$ for some $\gamma_c^0 \geq \gamma_c^1$. Considering type l saving with CBDC, we have

$$s_e^l(\gamma_e,\gamma_c,R) = \frac{2y_e + \kappa(\gamma_e,\gamma_c,R) - (1+R)c_e^l(\gamma_e,\gamma_c,R)}{R}. \tag{1.47}$$

For $\beta > 2/(1+\eta)$, we know that $s_e^l(1,1,1) = 2y_e > 0$. Note that $s_e^l(\gamma_e,\gamma_c,R)$ is increasing in κ. The policy parameters (γ_e,γ_c,R) will have an indirect effect on type l saving with CBDC. Observe that by condition (1.32), $c_e^l(\gamma_e,\gamma_c,R)$ is monotonicaly increasing in γ_e and monotonically decreasing in R. However, the indirect effect of κ on s_e^l is difficult for us to establish. This is because the effect of these policy parameters on the fixed fee κ is ambiguous. If κ were decreasing (increasing) in γ_e (γ_c) and increasing in R, then $s_e^l(\gamma_e,\gamma_c,R)$ would be decreasing (increasing) in γ_e (γ_c) and increasing in R. If that were to be the case, then there would exist a CBDC inflation rate $1 < \gamma_e^0 < \infty$, a cash inflation rate $1 < \gamma_c^0 < \infty$ and a nominal interest rate

$R^0 > 1$ such that $s_e^0(\gamma_e^0, \gamma_c^0, R^0) = 0$. This would guarantee that the optimal transfer creates an inflation rate that is high enough and a nominal interest rate that is low enough to make the type l consumers debt-constrained. However, it ensures that the inflation and nominal interest rates are not too excessive to deter the consumers from accumulating an adequate amount of real cash and CBDC balances, denoted by q_c and q_e, respectively.

When $\beta < 2/(1+2\eta)$, payment of fixed fee κ by type h consumers will increase their purchasing power when both consumer types are debt constrained. This is assuming $\psi \geq \phi\zeta$, a condition that will guarantee the coexistence of CBDC alongside cash as they have the same rate of return. Given this assumption, type h consumers would be using CBDC and type l would be using cash. The interesting question is how to induce type l consumers to switch to CBDC and extract the fixed fee κ. When $\beta \geq 2/(1+2\eta)$, type l consumers would be willing to pay the fixed fee if the saving from CBDC is at least as high as the saving from cash, that is, $s_e^l(\gamma_e^1, \gamma_c^1, R^1) \geq s_c^l(\gamma_e^1, \gamma_c^1, R^1)$. We summarize our argument by stating the following conjecture.

Conjecture 1.1. *When $\beta < 2/(1+2\eta)$, the optimal policy is characterized by $(\gamma_c^*, \gamma_e^*) > 1$ that satisfies* (1.39), (1.40), (1.43), (1.45), *and* (1.46) *in a mixed cash-CBDC economy with the condition $\psi \geq \phi\zeta$. When $\beta \geq 2/(1+2\eta)$, the optimal policy must satisfy $\gamma_c^* \geq \gamma_c^0$, $\gamma_e^* \geq \gamma_e^0$ and a nominal interest rate $R^0 \geq R^1$ so that $s_e^l(\gamma_e^1, \gamma_c^1, R^1) \geq s_c^l(\gamma_e^1, \gamma_c^1, R^1)$, with an equilibrium allocation characterized by* (1.32), (1.33), (1.37), *and* (1.38). *For both these conditions, $\tau_x \in [0, \overline{\tau_x}]$ as defined by* (1.26).

In the conjecture, a binding CBDC constraint for the l types results in the net positive revenue for the government, as the fixed CBDC fee and the labor tax can be positive. Since the labor tax in the CBDC economy is non-distortionary, the introduction of a CBDC may improve welfare. This is because with cash, the labor tax is not feasible as agents can hide their money balance to evade taxes. If CBDC has to have a higher rate of return than cash, then CBDC inflation will need to be lower than cash inflation. Since $\beta \geq 2/(1+\eta)$, a relatively lower inflation rate with CBDC than with cash ($\gamma_e^0 \leq \gamma_c^0$) can increase the saving for l types if they use CBDC instead of cash. By potentially attaining a higher equilibrium level of output y, a CBDC economy can improve welfare compared to a cash economy.

1.5.4 Numerical Example

In this section, we provide some numerical examples for our model. We first assume that the utility functions are given by $u(c) = \ln c$ and $g(y, \omega) = {}^{\omega y^2}/_2$, where $\omega > 1$.

To analyze the welfare implications of introducing a CBDC compared to a cash-based economy, we conduct a numerical exercise by fixing $\beta = 0.96$, $\eta = 1.05$, and $\omega = 0.35$. We vary the inflation rates for both the cash economy (γ_c) and the CBDC economy (γ_e) from 1 to 10 percent, along with the nominal interest rates (R) in the CBDC economy over the same range. For each inflation rate in the cash economy, we solve for the equilibrium consumption levels and output and compute the welfare level. Similarly, for each combination of inflation rate and nominal interest rate in the CBDC economy, we solve for the equilibrium values and compute the welfare level. Figure 1.2 compares welfare directly between the cash and CBDC economies at different inflation rates. The CBDC economy consistently achieves higher welfare than the cash economy across the entire range of inflation rates considered. However, the welfare gap narrows as inflation increases. This implies that while CBDC is welfare-enhancing compared to cash, its relative benefit diminishes in higher inflation environments. The positive welfare difference indicates that CBDC outperforms cash in terms of welfare across all inflation rates. However, the welfare difference declines as inflation rises. This suggests that a CBDC becomes less desirable as the economy experiences higher inflation.

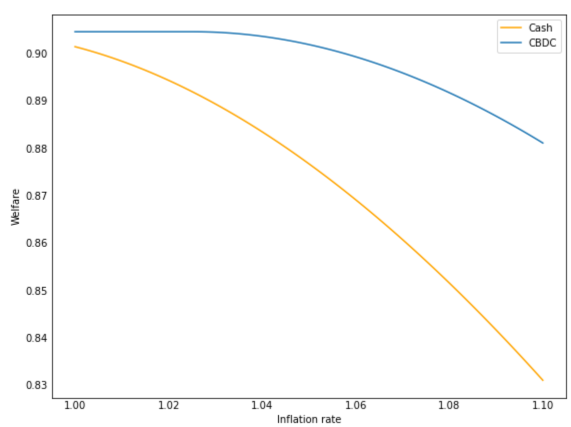

Figure 1.2: Welfare comparisons between cash and CBDC

We also examine how welfare in the CBDC economy varies with the nominal interest rate (Figure 1.3). Welfare monotonically increases with the nominal interest rate, implying that higher interest rates on CBDC balances are associated with improved welfare outcomes. This highlights the potential for interest-bearing CBDC to enhance welfare by providing households with a more attractive saving vehicle.

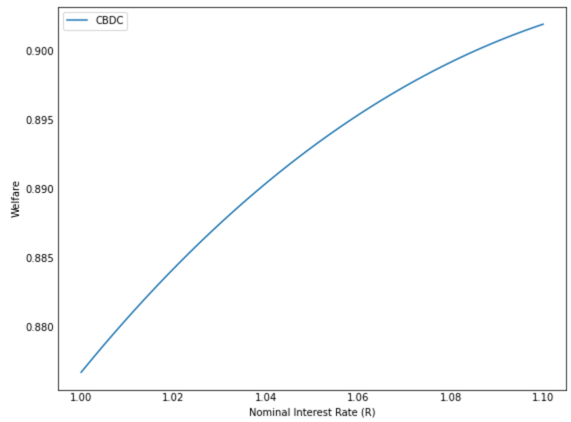

Figure 1.3: Welfare in a CBDC economy vs nominal interest rate

Lastly, we examine how the welfare difference between CBDC and cash economies varies with changes in key parameters. As the discount factor increases, the welfare difference becomes more negative, indicating that the welfare gain from CBDC diminishes relative to cash at higher discount factors. The preference shock parameter η has a relatively small impact, with the welfare difference staying negative but close to zero across the range of η values. Finally, as the production function parameter ω increases, the welfare difference becomes less negative, suggesting that CBDC performs relatively better compared to cash when production is more sensitive to labor input.

1.6 Conclusion

We study how the issuance of CBDC might impact welfare in an economy where tax evasion through the usage of cash is possible. Our model incorporates private information to introduce heterogeneity among agents. A crucial finding is that CBDC can benefit those individuals who have an immediate need for consumption when their debt-constraints are binding. The labor tax, which only applies when CBDC is used

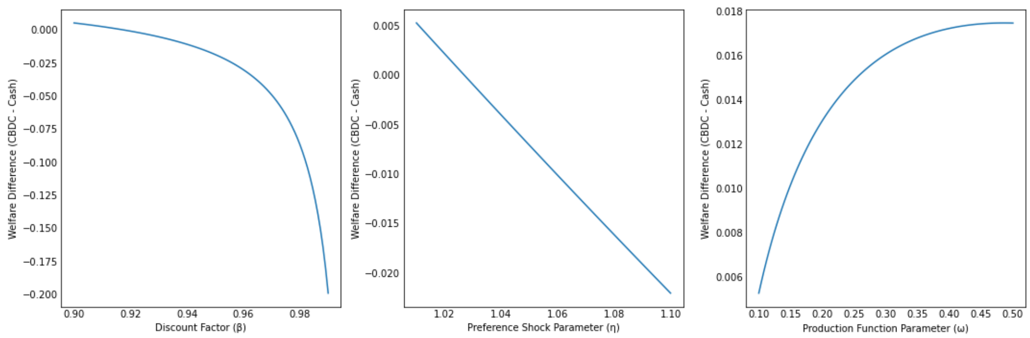

Figure 1.4: Welfare gains from CBDC as a function of key parameters

as a payment instrument, poses no distortion, as individuals can conceal their cash balances. While the government might face challenges financing the cost of CBDC if those with a lower marginal utility are unwilling to pay the CBDC fee, the labor tax itself does not introduce any distortion.

A critical component of our model is the requirement for all exchanges to be voluntary, which naturally limits policies to those respecting individual rationality. Low inflation policies can be achieved if individuals with a lower marginal utility of consumption are incentivized to contribute to the government's CBDC expenditure. Given that inflation with cash is relatively higher than inflation with CBDC, leveraging the labor tax to finance CBDC expenditure presents a valuable opportunity for the government. In other words, the introduction of CBDC can enhance welfare and potentially lead to a more efficient allocation of resources by deterring tax evasion.

CHAPTER 2
INTEREST-BEARING MONEY, ILLIQUID BONDS, AND BANKING IN A NEWS ECONOMY

2.1 Introduction

Private currencies facilitated economic exchange long before the advent of central bank-issued fiat money. From ancient Mesopotamian banks to the Free Banking era in the United States (1837-1863), private institutions issued their own currencies that circulated as media of exchange (Davies (2010); Champ (2007)). While these systems proved successful in some regions like Canada and Scotland, they often exhibited instability, particularly in the United States.[3] Recent technological advances in private money systems, such as gift cards and cryptocurrency, mean that these systems are now more important than ever. This raises the fundamental question: how can private money circulate as a medium of exchange to facilitate intertemporal trade amidst such instability? In this paper, I model news shocks or flows of information as a potential source of this instability, which could lead to excessive volatility in privately-issued bank money, hindering its efficiency as a payment instrument.

The seminal work of Hirshleifer (1971) pioneered the exploration into the role of information in financial markets, emphasizing that public information is not invariably socially beneficial due to its potential to eliminate insurance possibilities. While much of the recent literature focuses on the influence of information disclosure or monetary policy announcements on markets (e.g., Andolfatto and Martin (2013), Andolfatto et al. (2014), Dang et al. (2017), Gu et al. (2020), and Choi and Liang (2021)), there has been relatively limited attention given to the impact of technological uncertainty on economies relying on exchange media when banks carry out the process of asset transformation and are deemed essential. To address this gap, I develop a general equilibrium model where banks issue their own private currencies in the form of bank deposits.

The analysis makes three main contributions to our understanding of private money, banking, and monetary policy. First, I show how banking intermediaries

[3] This was particularly evident during the early stages of development across different states, although the Suffolk system in New England during the Free Banking era stands as an exception.

transmit and amplify information-driven liquidity shocks. Unlike Andolfatto and Martin (2013), where agents hold productive assets directly, I model banks as essential intermediaries that issue deposits backed by firm output. This creates a realistic channel through which productivity shocks affect currency values and introduces bank decision-making as crucial for liquidity provision.

Second, I demonstrate that interest-bearing money provides an effective tool for addressing liquidity shortages caused by private currency instability. Beyond traditional money growth instruments, interest-bearing money allows central banks to influence asset prices through an investment channel: banks use interest-bearing reserves as insurance against productivity shocks. Under observable preferences, type-contingent transfers with interest-bearing money can implement the efficient allocation while requiring a positive inflation rate and strictly positive nominal interest rate—a departure from the traditional Friedman rule.

Third, I examine how government and private currencies can coexist under private information about consumption needs. Moving beyond the unrealistic assumption of observable preferences (Andolfatto (2011); Kocherlakota (2003)), I introduce private information and show how illiquid bonds enable sorting between consumer types. The reason is that consumers with high marginal utility of consumption will sell their illiquid bonds to consumers with lower marginal utility of consumption, which transfers liquidity between households while preserving their anonymity. This makes government debt essential even when type-contingent transfers are infeasible, requiring positive inflation to achieve efficiency in a news economy.

For my formal analysis, I extend the Andolfatto and Martin (2013) framework by incorporating banks and interest-bearing money while accounting for information frictions. To do this, I borrow some elements from Chiu et al. (2019)'s framework, which extends the unified framework of Lagos and Wright (2005) by introducing an imperfect banking sector and inside money creation. In my model, households consisting of consumers and producers use exchange media to realize intertemporal gains to trade in the absence of commitment. Banks act as financial intermediaries between entrepreneurs and households by creating deposits and issuing loans. Bank deposits function as exchange media that facilitate intertemporal exchange, making banks essential—otherwise, there would be no gains from trade.

To illustrate the role of liquidity in determining asset prices, I include agent heterogeneity in the form of consumer preference heterogeneity with two types of consumers. Including consumer preference heterogeneity enables liquidity to be determined endogenously. In particular, I explicitly show how bank deposits are priced at a premium when individuals with a higher marginal utility of consumption have a pressing need to consume (Proposition 2.1). I also find some counterintuitive results. Most notably, imposing cash-in-advance constraints on bank deposits—rendering them illiquid—can improve welfare. This paradox arises because restricting deposit liquidity prevents destabilizing arbitrage that would otherwise amplify news-driven volatility.

The private sector actively seeks ways to improve liquidity with high-quality payment instruments in financial markets, employing techniques such as tranching assets or nondisclosure of information, as highlighted in Andolfatto and Martin (2013). However, when the private sector faces constraints in providing sufficient liquidity through such instruments, central bank intervention becomes necessary to address liquidity shortages and mitigate potential crises by providing liquid liabilities.

While lump-sum tax instruments are typically assumed in monetary theory literature, I relax this assumption. Instead of issuing zero-interest-bearing fiat money, the central bank issues interest-bearing money that can be used by households as a payment instrument (see Andolfatto (2010)). The absence of a lump-sum tax instrument implies that the Friedman rule is not implementable. With this in mind, interest-bearing money in my paper can also be thought of as a weak form of central bank digital currency (CBDC), which has generated considerable interest among policymakers recently.

My paper contributes to the New Monetarist literature with financial intermediation by modeling news shocks with an active banking sector along with interest-bearing assets. Berentsen et al. (2007) was the first to incorporate money and banking into the Lagos and Wright (2005) framework with a perfectly competitive banking sector. In my model, the banking sector is also perfectly competitive but with news shocks and includes consumer heterogeneity. In Keister and Sanches (2019), the banking sector is also perfectly competitive and CBDC—similar to interest-bearing money in this paper—competes with bank deposits, but there are no news shocks and consumer heterogeneity. In a related work, Hu (2021) takes a mechanism-design approach to study the implementation of optimal policy through the interest rate on excess reserves by including a pledgeability constraint on banks. Gu et al. (2019) show that financial intermediation with banking is inherently unstable.

Since interest-bearing money in my paper is very similar to CBDC, other papers that study CBDC with banking include Andolfatto (2021), Williamson (2019), Williamson (2021), Monnet et al. (2019), and Brunnermeier and Niepelt (2019). This paper is also related to transmission channels of monetary policy through the banking system; see the seminal works of Bernanke and Blinder (1992) and Bernanke et al. (1999). Other relevant papers on banking and liquidity include Drechsler et al. (2017), Goodfriend and McCallum (2007), Christiano et al. (2014), and Kiyotaki and Moore (2019). Since I consider three different types of assets, the recent paper by Amendola et al. (2021)—where money, bonds, and equity are included but without informational asymmetries—is also pertinent.

The remainder of the paper proceeds as follows. Section 2 presents the environment. Sections 3 and 4 explore two regimes: one where bank deposits solely function as exchange media in a private economy, and another where bank deposits and interest-bearing money coexist as means of payment. Section 5 introduces an illiquid bond, which becomes essential when the added friction of private information over consumption patterns of households is considered. The model is further extended in this section by imposing a cash-in-advance constraint on bank deposits, rendering them illiquid and establishing the conditions for the coexistence of money, bonds, and bank deposits in equilibrium. The final section concludes.

2.2 Environment

Time is discrete and continues forever. Each time-period t is divided into two subperiods: day and night. There are four types of agents in the economy: a unit measure of infinitely-lived households comprised of consumers and producers (divided evenly), a continuum of entrepreneurs with measure 1, and a continuum of bankers with measure 1. All agents reside in centralized locations in both subperiods (there are no search frictions).

Households belong to one of two permanent groups: Group 1 and Group 2. Each group is of equal measure. Denote by A and B the set of Group 1 and Group

2, respectively. All households have common preferences and have the ability to produce and consume the day output. Let $x_t(i) \in \mathbb{R}$ denote household consumption (production, if negative) of output (or good) during the day by household $i \in A \cup B$ at date t. Preferences are linear in $x_t(i)$, which implies that utility is transferable.

At the beginning of the night, households experience an idiosyncratic shock that determines whether they are consumers or producers with equal probability. The shock is *i.i.d.* across households and time. Consumer heterogeneity is realized at the beginning of each night after another shock, which occurs with equal probability. Let $\omega_t(i)$ denote the shock on consumer type, where $\omega_t(i) \in \{\omega_l = 1, \omega_h = \delta\}$ and $\delta > 1$.[4] This shock is *i.i.d.* across consumers within each group and across time. I will consider both public and private information structures by imposing assumptions on whether or not $\omega_t(i)$ is observable.

Denote by $\{c_t(i), y_t(i)\} \in \mathbb{R}^2_+$ the consumption and production, respectively, of the night good by household $i \in A \cup B$ at date t. The utility associated from consumption at night is given by $\omega_t(i)u(c_t(i))$, where $u'' < 0 < u'$, $u'(0) = \infty$ and $u(0) = 0$. The utility associated from production at night is given by $v(y_t(i))$, where $v' > 0$ for $y > 0$, $v'' \geq 0$ and $v(0) = 0$. Households discount utility payoffs across periods with the discount factor $\beta \in (0, 1)$; so that the utility function for household i can be represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ x_t(i) + \omega_t(i)u(c_t(i)) - v(y_t(i)) \right\}. \tag{2.1}$$

In the spirit of Kocherlakota (2003), a spatial structure is imposed at night with two spatially separated locations: location 1 and location 2. Subsequent to the realization of consumer types, consumers of group 1(2) households move to location 1(2) for consumption of the night output, while producers of group 2(1) households move to location 1(2) for production of the night output. This ensures that the two locations are symmetric in terms of the composition of preference types at night. Moreover, households cannot consume their own output at night as a consequence of this spatial structure.

Entrepreneurs live for two periods and can only participate in the day subperiod.

[4] Note that ω_l represents the marginal utility of type l consumers and ω_h represents the marginal utility of type h consumers, after the shock is realized.

Each day, a generation of young entrepreneurs is born who can consume only in old age and then die in the following day. A young entrepreneur is endowed with an investment opportunity that transforms x_t units of day output at date t to $z_t f(x_t)$ units of day output in date $t+1$, where $0 < z_t < \infty$ denotes a productivity parameter and $f'' < 0 < f'$, $f'(0) = \infty$, and $f'(\infty) = 0$. The entrepreneur then consumes x_t in date $t+1$ when he becomes old.

Productivity evolves stochastically over time and follows a Markov process, $Pr[z_{t+1} \leq z^+ | \eta_t = \eta] = G(z^+|\eta)$; where G is a cumulative distribution function conditional on information η_t (news) at the beginning of each night. Following Andolfatto and Martin (2013), I assume that news can be classified into two categories: good news and bad news; so that $\eta_t \in \{b, g\}$ and denote $\pi \equiv Pr[\eta_t = b]$. Define $z(\eta) = \int z^+ dG(z^+|\eta)$ and assume that $G(z^+|g) \leq G(z^+|b)$ which implies $z(b) \leq z(g)$. In particular, news η_t received at the beginning of the night is a short-term conditional forecast of next day's productivity. Moreover, good news first-order stochastically dominates bad news. In contrast, $z^e \equiv \pi z(b) + (1-\pi)z(g)$, is a long-term forecast of productivity that extends to infinite horizons, where $E_t z_{t+1} = z^e$ for all t.

Given the perishability of the day and the night output along with the spatial structure, the resource constraints are as follows

$$\int_{A \cup B} x_t(i)di + x_{t+1} \leq z_t f(x_t), \tag{2.2}$$

$$\textstyle\int_A c_t(i)di \leq \int_B y_t(i)di \quad \text{and} \quad \int_B c_t(i)di \leq \int_A y_t(i)di. \tag{2.3}$$

The planner weights all agents equally and maximizes the aggregate welfare,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ x_t(i) + \omega_t(i)u(c_t(i)) - v(y_t(i)) \right\}, \tag{2.4}$$

subject to the resource constraints (2.2) and (2.3). Note that the symmetry in location means that the night resource constraint (2.3) can also be expressed as $0.25c_l + 0.25c_h = 0.5y$, with measure 0.25 of type l consumers, measure 0.25 of type h consumers, and measure 0.5 of producers. Because of linear utility in x, the first-best allocation must be consistent with any lottery scheme in $\{x_t(i)\}$ satisfying the expected value $z_t f'(x_t) - x_{t+1}$.[5] Assume, without loss of generality, that for the

[5]Given the environment, I mean by first-best allocation is what allocation is best if there is

solution of the first-best allocation, the planner may assign $x_{t+1} = x^*$ for all t; where

$$\beta z^e f'(x^*) = 1. \tag{2.5}$$

Given the strict concavity of u and strict convexity of v, the first-best allocation is characterized by

$$\begin{aligned} u'(c_l^*) &= \delta u'(c_h^*), \\ u'(c_l^*) &= v'(y^*), \\ c_l^* + c_h^* &= 2y^*. \end{aligned} \tag{2.6}$$

Note that the first-best allocation is independent of news by construction; see Proposition 1 in Andolfatto and Martin (2013). Moreover, given the preferences and endowment, there are gains from trade between households and entrepreneurs. That is, consumers want to consume the output of the producers, and entrepreneurs want to borrow from households to invest in their investment opportunities. I place further restrictions on this environment that will render trade by credit infeasible, which will provide a role for the bankers to facilitate intertemporal exchange. Bank deposits and interest-bearing money are the possible media of exchange that are essential for trade. In what follows, I assume that both entrepreneurs and households lack commitment. They are also anonymous, which together with lack of commitment rules out enforcement of debt repayment by households. This implies that all trade must be *quid pro quo*. Furthermore, I restrict trade between agents to occur in competitive spot markets.

2.3 Private economy with banking

I refer to private economy as a competitive equilibrium free of central bank intervention in which bank deposits can be used to facilitate intertemporal exchange. Similar to the entrepreneurs, bankers live for two periods and can only participate during the day. A generation of young bankers is born in the day, but die in the next day after becoming old. Unlike entrepreneurs and households, bankers can commit and are able to enforce repayment of debt at no cost. This allows the banks (owned by bankers) to act as financial intermediaries between the entrepreneurs and households.

perfect monitoring and agents can commit to future actions.

As in Chiu et al. (2019), banks want to fund investment projects by issuing liquid deposits which can be used as a means of payment by households in the day market. In this private economy, suppose for now that banks do not receive anything in exchange from households for the deposits issued. Money is thus created *ex nihilo* in the private economy when banks issue deposits to the households. Banks also make loans to entrepreneurs in the form of deposits, which the entrepreneurs use to purchase output x from households for investment. The investment of the entrepreneurs is subject to productivity shocks mentioned earlier. In the night market, households use deposits to trade goods. In the next day, entrepreneurs and households settle their debt by repaying loans and deposits, respectively. After selling some of their investment for deposits to settle bank loans, entrepreneurs can retain the leftover output for their own consumption. Bankers collect loan repayments and redeem deposits held by households. The banking sector is assumed to be perfectly competitive with free entry; so that banks make zero profit. Figure 2.1 illustrates the timeline of the model.

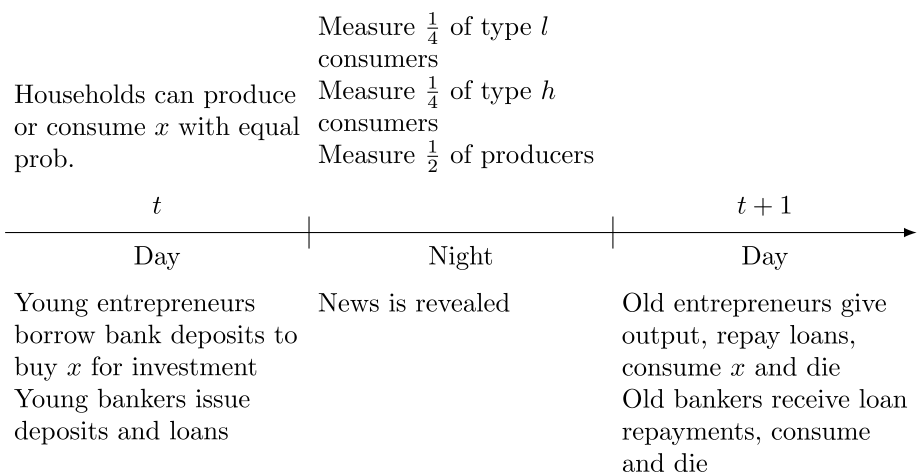

Figure 2.1: Timeline

In what follows, I will characterize a competitive equilibrium in which bank deposits are circulated as exchange media.

2.3.1 Decision-making of banks and entrepreneurs

I examine the optimization problems faced by banks and entrepreneurs, respectively. Their respective optimization problems will determine the demand and supply of loans and deposits in the equilibrium.

2.3.1.1 Entrepreneurs

Consider the entrepreneurs who take the loan rate $R^L(z)$ as given to maximize consumption, $zf(p)$, in the second period of life (that is, day consumption when old); where p denotes loans. When borrowing from the bank, he/she faces a pledgeability constraint, $p \leq zf(p)$; see Rocheteau et al. (2018). That is, the entrepreneur pledges the entirety of their output from investment to obtain loans from the bank.[6] Since banks can enforce repayments, I am assuming that the debt owed by the entrepreneurs can be recovered fully in the event of default.

Formally, the entrepreneur solves the following maximization problem:

$$\begin{aligned} \max_{p} \quad & \left\{ zf(p) - R^L(z)p \right\} \\ \text{s.t.} \quad & p \leq zf(p). \end{aligned} \tag{2.7}$$

There are two cases to consider. First if the pledgeability constraint is slack, then $p = p^*$, where $zf'(p) = R^L(z)$. Second, if the pledgeability constraint binds then $p < p^*$ where $zf'(p) < R^L(z)$. An entrepreneur borrows up to the point where the marginal cost of obtaining the loan equals its marginal benefit. When the marginal cost of obtaining the loan exceeds the marginal benefit, the entrepreneur would not be willing to obtain the loan due to the banks charging a higher $R^L(z)$. The higher $R^L(z)$ stems from the increased risk of default generated by the productivity shock that results in a binding constraint. It follows that the demand for loans decreases with the loan rate. Next, I examine the bank's problem.

2.3.1.2 Banks

Banks issue deposits d to households and invest in loans p offered to the entrepreneurs. Let $\psi_1(z)$ denote the price of deposits at the end of each day. Competitive markets imply that banks take the deposit price $\psi_1(z)$ and the lending rate $R^L(z)$ as given. The pledgeability constraint of the entrepreneurs now translates to

[6] There may be restrictions on the amount of output that can be pledged. These restrictions can not only arise from institutions including the legal system, but also from information and commitment frictions. I abstract from such restrictions on pledgeability here.

a lending constraint for the bankers: $p \leq zf(p)$. Banks also face a balance sheet constraint, $p = d$, where the right-hand side is the liability and the left-hand side is the asset. In this case, the loans represented by p constitute the bank's assets, while the issued bank deposits, denoted by d, serve as the liabilities.. The constraint is the balance sheet identity of the bank. The bank's maximization problem can then be written as

$$\begin{aligned} \max_{p,d} \quad & \left\{R^L(z)p - \psi_1(z)d\right\} \\ \text{s.t.} \quad & p = d, \\ & p \leq zf(p). \end{aligned} \tag{2.8}$$

Substitute out d using the balance sheet identity and rewrite the bank's maximization problem as

$$\begin{aligned} \max_{p} \quad & \left\{R^L(z)p - \psi_1(z)p\right\} \\ \text{s.t.} \quad & p \leq zf(p). \end{aligned} \tag{2.9}$$

Once again there are two cases to consider. First, if the lending and pledgeability constraints are slack, then $p = p^*$ where $\psi_1(z) = R^L(z) = zf'(p)$. Second, if the lending and plegeability constraints bind then $p < p^*$ where $\psi_1(z) > R^L(z) > zf'(p)$. The higher risk of loan default—as a result of the productivity shock—implies that the banks will cut back on their lending to firms. This is because the marginal benefit of issuing an extra unit of loan is below its marginal cost. The banks will increase their lending rates, which will make it more expensive for entrepreneurs to borrow, leading to a reduction in loan demand.

2.3.2 Decision-making of households

I now examine the household maximization problem for the day market, and then describe the producer's problem and the consumer's problem for the night market.

2.3.2.1 The day market

At the beginning of the day, each household enters with d real deposits priced at $\psi_1(z)$. Let $s \geq 0$ denote the real deposits carried forward into the night market. The day-market budget constraint can then be written as

$$x = \psi_1(z)d - \psi_1(z)s. \tag{2.10}$$

Let $W(d, z)$ denote the utility value of a household beginning the day with d real deposits when the productivity shock is z; let $V(s, \eta)$ denote the utility value associated with entering the night market with s real deposits conditional on news η. These two value functions must satisfy the following recursive relationship:

$$W(d, z) \equiv \max_{s \geq 0} \{\psi_1(z)d - \psi_1(z)s + E_\eta V(s, \eta)\}, \tag{2.11}$$

where V satisfies $\frac{\partial^2 V}{\partial s^2} \leq 0 < \frac{\partial V}{\partial s}$. The demand for real deposits can then be characterized by the first-order condition:

$$\psi_1(z) = E_\eta \frac{\partial V(s, \eta)}{\partial s}. \tag{2.12}$$

Notice that the optimal choice of s is identical across all households entering the night market. This is because the demand for real deposits is independent of initial deposit holdings d. Furthermore, the envelope condition yields

$$\psi_1(z) = \frac{\partial W(d, z)}{\partial d}. \tag{2.13}$$

The above condition implies that W is quasilinear in d and given stochastic productivity, the deposit price is time-invariant; that is, $\psi_1(z) = \psi_1^+(z)$.

2.3.2.2 The night market

At the beginning of the night, households realize whether they are consumers or producers. Consumer preference shock is also realized at the beginning of the night. Then the consumers of type $j \in \{l, h\}$ and the producers in households separate to travel to different locations, as in Xiang (2013). A household makes the consumption and production decisions *ex ante* on behalf of the type j consumer and the producer; instructions of each household are simply carried out by the producers and consumers. News about the entrepreneurs' productivity is also revealed at the beginning of the night.

Let $c_j = c_j^d$ represent the total purchases of output by a type j consumer using bank deposits, and $y_j = y_j^d$ denote the total amount of output produced where deposits are accepted as payments. Because of limited commitment and lack of record keeping, each consumer with realized type j of a household faces a deposit constraint[7]

$$c_j \leq \psi_2(\eta)s. \tag{2.14}$$

The price of deposits at night is influenced by news. Denote by $\psi_2(\eta)$ the price of deposits at night paid by a household with realized consumer type j. The price is paid to purchase output y_j for consumption c_j. Accordingly, the future deposit balances are given by

$$d_j^+ = \frac{1}{\psi_2(\eta)} \left(\psi_2(\eta)s + y_j - c_j\right).$$

The choice problem for a household with realized consumer type $j \in \{l, h\}$ can be expressed as

$$V_j(s, z) \equiv \max_{c_j, y_j} \left\{ \omega_j u(c_j) - v(y_j) + \beta \mathbb{E}\left[W\left(\frac{1}{\psi_2(\eta)} \left(\psi_2(\eta)s + y_j - c_j\right), z^+ \right) |\eta \right] \right\}. \tag{2.15}$$

Since a producer has the desire to accumulate deposit balances for future consumption, the deposit constraint $d_j^+ \geq 0$ will not bind. Independent of household types, all producers produce output y; so that the supply of output y at night is characterized by

$$v'(y(\eta)) = \beta \frac{\psi_1(z(\eta))}{\psi_2(\eta)}. \tag{2.16}$$

The consumption of output c_j at night will depend on whether or not the deposit constraint for type j binds. By applying (2.13), the desired consumption c_j at night

[7]The deposit constraint can also be interpreted as a debt-constraint that has been used in many Lagos-Wright type models, but more specifically I am referring to the environments in Andolfatto and Martin (2013) and Andolfatto (2013).

is characterized by

$$\begin{aligned}
\omega_j u'(c_j(\eta)) &= \beta\tfrac{\psi_1(z(\eta))}{\psi_2(\eta)} \qquad \text{if } \psi_2(\eta)s \geq c_j(\eta) \\
c_j(\eta) &= \psi_2(\eta)s \qquad\qquad \text{otherwise.}
\end{aligned} \tag{2.17}$$

Moreover, the envelope condition in either case is

$$\frac{\partial V_j(s,z)}{\partial s} = \psi_2(\eta)\omega_j u'(c_j(\eta)). \tag{2.18}$$

2.3.3 Equilibrium

Denoting the total supply of deposits by S, the loan demand and loan supply by p^d and p^s, respectively, market-clearing conditions imply

$$\begin{gathered}
s = S, \\
0.25c_l(\eta) + 0.25c_h(\eta) = 0.5y(\eta), \\
p^s = p^d.
\end{gathered} \tag{2.19}$$

Condition (2.19) states that in each period, the deposit and the loan markets must clear along with the competitive spot markets in the day and night.

To solve for the equilibrium allocation, four cases must be considered.

Case 1. *Both the deposit constraints for type h and type l consumers remain slack, that is, $\psi_2(\eta)s \geq c_h(\eta)$ and $\psi_2(\eta)s \geq c_l(\eta)$.*

Case 2. *The deposit constraints for type h consumers remains slack while it binds for type l, that is, $\psi_2(\eta)s \geq c_h(\eta)$ and $\psi_2(\eta)s = c_l(\eta)$.*

Case 3. *The deposit constraints for type l consumers remains slack while it binds for type h, that is, $\psi_2(\eta)s \geq c_l(\eta)$ and $\psi_2(\eta)s = c_h(\eta)$.*

Case 4. *Both the deposit constraints for type h and type l consumers bind, that is, $\psi_2(\eta)s = c_h(\eta)$ and $\psi_2(\eta)s = c_l(\eta)$.*

Considering Case 1, by (2.17) one obtains

$$u'(c_l(\eta)) = \beta\frac{\psi_1(z(\eta))}{\psi_2(\eta)} = \delta u'(c_h(\eta)). \tag{2.20}$$

By applying the market clearing conditions, both Case 2 and Case 3 imply

$$\begin{aligned} u'(c_h(\eta)) &= \tfrac{\beta}{\delta}\tfrac{\psi_1(z(\eta))}{\psi_2(\eta)} \quad \text{and} \quad c_l(\eta) = \psi_2(\eta)S, \\ u'(c_l(\eta)) &= \beta\tfrac{\psi_1(z(\eta))}{\psi_2(\eta)} \quad \text{and} \quad c_h(\eta) = \psi_2(\eta)S, \end{aligned} \tag{2.21}$$

respectively. Applying the market-clearing conditions, Case 4 leads to

$$\psi_2(\eta)S = y(\eta) < y^*. \tag{2.22}$$

On the other hand, applying the market-clearing conditions for Case 1 results in

$$\psi_2(\eta)S \geq y(\eta) = y^*. \tag{2.23}$$

For cases in which the deposit constraint for type l consumers does not bind, applying (2.16) yields

$$u'(c_l(\eta)) = \beta\frac{\psi_1(z(\eta))}{\psi_2(\eta)} = v'(y(\eta)). \tag{2.24}$$

Similarly, a slack deposit constraint for type h implies $u'(c_h(\eta)) = {}^{v'(y(\eta))}/_{\delta}$. Note that ${}^{\partial V(s,z)}/_{\partial s} = 0.5{}^{\partial V_l(s,z)}/_{\partial s} + 0.5{}^{\partial V_h(s,z)}/_{\partial s}$. Appealing to (2.13), the following equilibrium restriction must be true,

$$\psi_1(z^e) = \frac{\pi\psi_2(b)}{2}\left[u'(c_l(b)) + \delta u'(c_h(b))\right] + \frac{(1-\pi)\psi_2(g)}{2}\left[u'(c_l(g)) + \delta u'(c_h(g))\right]. \tag{2.25}$$

Assuming that the deposit constraint for type l consumers is slack, condition (2.25) may be rewritten as

$$\psi_1(z^e) = \pi\psi_2(b)v'(y(b))A(y(b)) + (1-\pi)\psi_2(g)v'(y(g))A(y(g)), \tag{2.26}$$

where

$$A(y) \equiv 0.5\left[\frac{\delta u'(y)}{v'(y)} + 1\right]. \tag{2.27}$$

Notice that $A(y^*) = 1$ and $A'(y) < 0$.

Next, from the respective maximization problems of the entrepreneurs and banks, one can derive the deposit-price function

$$\psi_2(\eta) = \beta\frac{z(\eta)f'(p)}{v'(y(\eta))} = \beta\frac{R^L(z^e)}{v'(y(\eta))}, \tag{2.28}$$

by assuming that the plegeability and lending constraints of the entrepreneurs and banks, respectively, are slack. The equilibrium allocation $(c_l(\eta), c_h(\eta), y(\eta))$ at night in which only bank deposits is used as a medium of exchange is characterized by the conditions (2.22), (2.23), (2.26) and (2.28). Next, I consider cases in which news may or may not be of importance in this private economy.

2.3.3.1 Equilibrium with no news

In this section, I examine the competitive equilibrium where news is of no importance. This means $z(\eta) = z^e$ for $\eta \in \{b, g\}$ with the implication that $y(\eta) = y$ and $\psi_2(\eta) = \psi_2$.

I now seek to solve for ψ_1. Notice that with no news,

$$\psi_2 = \beta\frac{z^e f'(p)}{v'(y)} = \beta\frac{R^L}{v'(y)}. \tag{2.29}$$

Combining (2.26) with (2.28) yields the following expression for the deposit day-price:

$$\psi_1 = \beta z^e f'(p)A(y) = \beta R^L A(y) > 0. \tag{2.30}$$

Condition (2.30) states the rate of return on bank deposits must compensate for discounting across time by the loan rate or by the expected marginal product of the day output. Following Andolfatto and Martin (2013), I refer to this condition as the "fundamental" price of deposits, as this reflects the average price relative to extreme price fluctuations. Given the strict concavity of u, from an *ex ante* perspective, society prefers average prices to extremes to smooth out consumption over time. Note that the deposit price also depends on the preference parameter δ. We have the following proposition.

Proposition 2.1. *ψ_1 is strictly increasing in δ.*

Proof. Rewriting the deposit day-price by using the definition of $A(y)$ from (2.27) gives us

$$\psi_1 = \frac{\beta z^e f'(p)\left[\delta u'(y) + v'(y)\right]}{2v'(y)} = \frac{\beta R^L \left[\delta u'(y) + v'(y)\right]}{2v'(y)} > 0.$$

Differentiating the expression above w.r.t. δ gives rise to

$$\frac{\partial \psi_1}{\partial \delta} = \frac{\beta z^e f'(p) u'(y)}{2v'(y)} = \frac{\beta R^L u'(y)}{2v'(y)} > 0.$$

■

The interpretation of Proposition 2.1 is as follows. Since all consumption in the night market must be purchased by using deposits, the consumer preference shock can be interpreted as a liquidity shock, where δ measures the magnitude of this shock. An increase in the magnitude of this liquidity shock is reflected in a higher deposit price during the day in the form of a liquidity premium.[8] This is due to the high demand from type h consumers for the night output as δ gets larger and also because of the usefulness of bank deposits as a means of payment. Moreover, given $A(y^*) = 1$ and using the restriction $\beta z^e f'(p^*) = 1$ (from the solution in the planner's problem in (2.5)) implies $\psi_1^* = 1$, $R^{L^*} > 0$ and $p = p^*$.

Next, I verify the conditions under which the deposit constraint for either type of consumers will not bind, that is, $\psi_2 S \geq y^*$. First, suppose that both the pledgeability constraint and the lending constraint from the entrepreneur's and the bank's optimization problems are slack, that is, $p \leq z f(p)$. This implies $p = p^*$. Using (2.29), condition $\psi_2 S \geq y^*$ can be expressed in terms of parameters,

$$\beta \geq \frac{y^* v'(y^*)}{S z^e f'(p^*)} = \frac{y^* v'(y^*)}{S R^{L^*}}. \tag{2.31}$$

[8] If an asset is used as a medium of exchange, then the asset will be traded at a premium relative to other illiquid assets. The fact that financial assets are valued for their liquidity when they are used as exchange media has been highlighted in Lagos (2010).

Defining

$$\beta^*(z^e, R^L) = \frac{y^* v'(y^*)}{S z^e f'(p^*)} = \frac{y^* v'(y^*)}{S R^{L^*}} \tag{2.32}$$

as the equilibrium object corresponding to the efficient level of production y^*, we get the following result.

Proposition 2.2. *If $\beta \in \big[\beta^*(z^e, R^L), 1\big)$ and $p \leq zf(p)$, then $p = p^*$ and where R^{L^*} corresponds to $zf'(p) = R^L(z)$. A competitive equilibrium corresponds to the efficient allocation y^*. Moreover, $\beta^*(z^e, R^L)$ is independent of δ.*

Liquidity shock has no influence on the parameters for which the efficient allocation can be implemented. Observe that $\beta^*(z^e, R^L)$ is strictly decreasing in z^e and R^L. The higher the risk of loan default is, the lower is the bank lending to firms; so that $R^L < \psi_1(z^e)$. This is up to the point where the pledgeability constraint and the lending constraint for both the firms and banks may bind, that is, $p = zf(p)$. The efficient allocation is only implementable for patient economies—that is, for economies with sufficiently high β—and up to the point on which the set of economies can be expanded. Beyond this point, an efficient allocation is no longer feasible.

A liquidity shortage arises for impatient economies—that is, for the case when $\beta \in \big(0, \beta^*(z^e, R^L)\big]$ and $p = zf(p)$—in the sense of Caballero (2006); see also Proposition 2 in Andolfatto and Martin (2013) for an analogous result. Since entrepreneurs pledge their future output as collateral to the banks, limited commitment means that liquidity is in short supply. Owing to a lack of commitment from the entrepreneurs, there is a liquidity shortage when the pledgeable future output is subject to binding constraints as a result of technological uncertainty. Hence, deposits are in short supply, creating a liquidity shortage. This makes the deposit constraints for both type l and type h consumers bind tightly; so that $\psi_2(\eta)S = y(\eta) < y^*$.

When deposit constraints bind for both consumer types, combined with the restrictive lending constraints of the banks and the pledgeability constraints of the entrepreneurs, the deposit-price function (2.30) indicates an overvaluation of the bank deposit compared to its fundamental value. Though household members wish to borrow money from banks overnight, restrictions imposed by both the banks and entrepreneurs from the previous day hinder this possibility. Entrepreneurs, eager to secure loans from banks, find themselves constrained, leading to banks' reluctance to lend. Since $A(y) > 1$, the expected rate of return on deposit is

$$\frac{\psi_1}{\beta} > \psi_1 > R^L > z^e f'(p) > 0,$$

which suggests that the effect of a liquidity shortage is to confer a liquidity premium on the price of deposit when bank deposit is used as a medium of exchange. The implication of this liquidity premium is that the deposits will earn a lower expected rate of return, as originally highlighted in Lagos and Rocheteau (2008).

2.3.3.2 Equilibrium with news

I now consider the case when news is of importance. This means that $z(b) < z^e < z(g)$. I present the results that are similar to Andolfatto and Martin (2013), but when the consumer deposit constraints for both types, along with the banks and the entrepreneurs' respective constraints are considered.

Lemma 2.1. *The deposit constraints for both type l and type h consumers cannot remain slack in both news states along with slack lending and pledgeability constraints for the banks and the entrepreneurs, respectively.*

Proof. Suppose that Case 1 holds in both news states, that is, the deposit constraints for both type l and type h are slack in both news states. Also, assume that the lending and the pledgeability constraints of the banks and the entrepreneurs, respectively, are slack. Then $y(b) = y(g) = y^*$ and $p = p^*$. In this case, $A(y^*) = 1$ and so by condition (2.30), $\psi_1 = \beta z^e f'(p^*) = \beta R^{L^*}$. Moreover, condition (2.29) implies $\psi_2(\eta) = \beta \frac{z(\eta) f'(p^*)}{v'(y^*)}$. Using (2.23), $\beta \frac{z(\eta) f'(p^*)}{v'(y^*)} \geq {}^{y^*}/s$ for $\eta \in \{b, g\}$. This implies $\beta z(b) f'(p^*) \geq {}^{y^* v'(y^*)}/s$. Since $z(b) < z^e < z(g)$, it follows that $\psi_1 = \beta z^e f'(p^*) > \beta z(b) f'(p^*) \geq {}^{y^* v'(y^*)}/s = S\beta R^{L^*} = S\psi_1$, by using condition (2.32). But this is a contradiction; as $S > 0$. ■

Lemma 2.2. *The deposit constraints for both type l and type h consumers cannot bind tightly in both news states along with binding pledgeability and lending constraints.*

Proof. Assume that Case 4 holds in both news states, that is, the deposit constraints for both type l and type h bind tightly in both news states. Also, assume that the constraints for the banks and the entrepreneurs bind. Then (2.26) and (2.28) imply $\psi_1(z^e) = \pi\beta z(b) f'(p) A(y(b)) + (1-\pi)\beta z(g) f'(p) A(y(g))$. If the debt constraints for all the agents bind, then (2.22) implies $y(\eta) < y^*$ for $\eta \in \{b, g\}$. This means that $A(y(\eta)) > 1$ for $\eta \in \{b, g\}$ and $p < p^*$ given binding pledgeability and lending constraints. Then by the inequality in (2.31), $\psi_1 > \beta z^e f'(p) = \beta R^L > \frac{y^* v'(y^*)}{S}$. Moreover, condition (2.22) implies $\psi_2(\eta) S = y(\eta) < y^*$ for $\eta \in \{b, g\}$. Then (2.28) implies $y(b)v'(y(b)) = \beta z(b) f'(p)$, $y(g)v'(y(g)) = \beta z(g) f'(p)$. Since $z^e = \pi z(b) + (1 - \pi) z(g)$, the two equalities above imply that $\pi y_2(b) v'(y(b)) + (1-\pi) y_2(g) v'(y(g)) = \beta z^e f'(p) = \beta R^L$. Therefore, $\pi y_2(b) v'(y(b)) + (1-\pi) y_2(g) v'(y(g)) > \frac{y^* v'(y^*)}{S}$. However, this is impossible; as $y' v'(y)$ is strictly increasing in y, and as $y(\eta) < y^*$. ■

Lemma 2.3. *The deposit constraints for both type l and type h consumers cannot bind tightly in the good-news state along with binding pledgeability and lending constraints. The deposit constraints for both types cannot remain slack in the bad-news state along with slack pledgeability and lending constraints.*

Proof. Suppose that the deposit constraints for both types of consumers bind tightly in the good-news state and remain slack in the bad-news state. Then (2.22) and (2.23) imply $\psi_2(b) S > y^*$ and $\psi_2(g) S = y(g) < y^*$. Also, by condition (2.28) $\psi_2(b) v'(y^*) = \beta z(b) f'(p^*) = \beta R^{L^*}$, $\frac{y(g) v'(y(g))}{S} = \beta z(g) f'(p^*) = \beta R^{L^*}$. Observe that these two latter restrictions together with $z(g) > z(b)$ imply that $\frac{y(g) v'(y(g))}{S} > \psi_2(b) v'(y^*) > \frac{y^* v'(y^*)}{S}$, which is a contradiction; as $y v'(y)$ is strictly increasing in y and as $y(g) < y^*$. ■

The three lemmas above effectively rule out Case 2 and 3. The remaining possibility is presented below, which corresponds to Proposition 3 in Andolfatto and Martin (2013).

Proposition 2.3. *If $z(b) < z^e < z(g)$ and $\beta = \beta^*(z^e, R^L)$, then the consumer deposit constraints for both type l and type h bind tightly in the bad-news state when $p = z f(p)$ and $p < p^*$, along with binding lending and pledgeability constraints for the banks and the entrepreneurs, respectively. The deposit constraints for both types become slack in the good-news state when $p \leq z f(p)$ and $p = p^*$, along with slack lending and pledgeability constraints.*

Proposition 2.3 fixes a pair $\beta(z^e, R^L)$ so that the competitive equilibrium barely implements the efficient allocation when there is no news. Following Andolfatto and Martin (2013), I perform a mean-preserving spread over the short-run conditional forecast of future productivity. Specifically, I keep z^e fixed and increase the variance of the short-run forecast around this mean. The analysis follows from their paper, namely that good news slackens a weakly binding constraint while bad news induces the deposit constraints of both type l and type h consumers to bind tightly. Despite potential fluctuations in deposit prices during short-term news events related to entrepreneurs' productivity, bank deposits can still function as exchange media, as long as efficiency can be maintained. The implementation of an efficient allocation is possible if consumers of both types are not debt-constrained in either news state, which will correspond to the case when the entrepreneurs' pledgeability constraint and the bank's lending constraint are not binding.

Proposition 2.3 implies $\psi_2(b)S = y(b) < \psi_2(g)S = y(g) = y^*$. Combining (2.26) with (2.28), we have the price of deposits in the day

$$\psi_1 = \beta R^L \left[\pi A(y(b)) + (1-\pi)A(y(g))\right]$$

Note that the above equation reduces to (2.30) when $y(b) = y^*$. Since $\psi_1 > 0$ and $\psi_1 = \beta R^L < 1$ when $y(b) = y(g) = y^*$, we have the following restriction that characterizes the equilibrium

$$\frac{\psi_1}{R^L} = \beta \left[\pi A(y(b)) + (1-\pi)A(y(g))\right]. \tag{2.33}$$

The implication of (2.33) is that information itself will carry a premium in the day-deposit price. Due to uncertainty, banks will raise their deposit rate, so that $\psi_1 > \psi_1^* > R^L$ as opposed to the no-news case. Here, information itself carries a premium in how the deposit prices will be set by the banks. The high deposit price means that bank lending in the news economy may be suboptimal, that is, $p^* > p$.

As for the equilibrium price of deposits at night, once again recalling condition (2.28)

$$\psi_2(b) = \beta \frac{z(b)f'(p)}{v'(y(b))} = \beta \frac{R^L}{v'(y(b))},$$

and

$$\psi_2(g) = \beta \frac{z(g)f'(p^*)}{v'(y^*)} = \beta \frac{R^{L^*}}{v'(y^*)}.$$

We have $\psi_2(g) > \psi_2(b)$; an implication of Proposition 3 due to the deposit constraints for both types becoming slack in the good-news state and binding in the bad-news state, and also when v is linear (a special case).

Next, I examine how central bank intervention with interest-bearing money can coexist with bank deposits. More specifically, I explore whether interest-bearing money can help overcome the liquidity shortage in the news economy when $\beta \in \big(0, \beta^*(z^e, R^L)\big]$.[9]

2.4 Bank deposits and interest-bearing money

The central bank intervenes at the night market after the realization of agents' types. I assume that $\omega_t(i)$ is publicly observable upon realization. This assumption implies that the central bank can operate on a type-contingent transfer policy by observing household types. The type-contingent transfer policy is introduced along the lines of Andolfatto (2011). Lump-sum taxation in the day market is ruled out and assume money as a divisible object. Let $\{c, p\}$ denote the household types, which are categorized into consumers and producers. The central bank's policy rule is to make lump-sum transfers of money $T_j^\iota \geq 0$ at each night after observing these household types, where $\iota \in \{c, p\}$. The central bank also pays a positive nominal interest rate $R^M \geq 1$ on money balances. Banks are required by the central bank to hold a fraction ρ of their deposits as currency reserves.

Banks now issue deposits to households in exchange for interest-bearing money which can be retained as bank reserves. Banks still make loans to entrepreneurs in the form of deposits. Loans and deposits are settled in the following day. Entrepreneurs use either deposits or interest-bearing money to purchase the day good from households. Households use a combination of bank deposits and interest-bearing money to trade goods in the night market. The process is the same as described earlier other than interest-bearing money now coexisting with deposits.

Denote by (ϕ_1, ϕ_2) the value of money in the day and night markets, respectively. Let M denote the total stock of money at the beginning of the day; with M^+ denoting

[9] The implicit assumption made in this context is that a nondisclosure of news is infeasible, that is, society does not have the power to hide bad news from the individuals. Hiding bad news and revealing good news is only time-consistent for sufficiently patient economies. This is the main motivation for government or central bank intervention for this class of models; see Andolfatto and Martin (2013) for more details.

the "next" period's money supply. Assume that this stock evolves at the constant gross rate $\mu \geq 1$, so that $M^+ = \mu M$. Since the central bank makes lump-sum transfers and also pays a nominal interest rate, the central bank budget constraint must satisfy $\left(R^M - 1\right) M = M^+ - M + 0.25T_l^c + 0.25T_h^c + 0.5T^p$, where $(R^M - 1)$ is the central bank's aggregate interest obligation. Suppose $T_l^c = T^p = 0$. Then, $T_h^c = 4\left(\frac{R^M}{\mu} - 1\right) M^+$, or in real terms can be expressed by

$$\tau_h^c = 4\left(\frac{R^M}{\mu} - 1\right)\phi_1 M^+, \tag{2.34}$$

where $\tau_j^c \equiv \phi_1 T_j^c$. Note that linearity restricts the transfers to be proportional.

2.4.1 Decision-making of banks and entrepreneurs

Note that the optimization problem of the entrepreneurs stays the same as before. I now examine the optimization problem of banks when they have the option of investing in government-issued interest-bearing reserves.

2.4.1.1 Banks

Banks now issue deposits d to households and invest in loans p and interest-bearing money m_1 issued by the central bank, where $m_1 \geq 0$ is the nominal money balances during the day. Banks acquire the real quantity of outside interest-bearing money, $a \equiv \phi_1 m_1$, and earn interest R^M. Both the deposit market and the loan market are competitive as before. Banks also face a reserve requirement. At the end of each date, the bank's beginning-of-the-day real money balances, a, must be at least ρ fraction of the total deposits, that is, $\rho d \leq a$, where ρ is a policy parameter set by the central bank. The bank solves the following maximization problem:

$$\begin{aligned} \max_{p,d,a} \quad & \left\{R^L(z)p + R^M a - \psi_1(z)d\right\} \\ \text{s.t.} \quad & p + a = d, \\ & \rho d \leq a, \\ & p \leq zf(p). \end{aligned} \tag{2.35}$$

Once again, substitute out d using the balance sheet identity and rewrite the bank's maximization problem as

$$\begin{aligned} \max_{p,d} \quad & \left\{ \left(R^L(z) - R^M\right) p + \left(R^M - \psi_1(z)\right) d \right\} \\ \text{s.t.} \quad & p \leq (1-\rho)d, \\ & p \leq zf(p). \end{aligned} \tag{2.36}$$

There are several cases to consider. For the first case, suppose that the reserve requirement, the lending and the pledgeability constraints are all slack. Then $p = p^*$ when $zf'(p) = R^L(z) = R^M = \psi_1(z)$. When the marginal benefit of investing in loans is equal to the marginal benefit of investing in interest-bearing money, the bank is indifferent between investing in loans and investing in interest-bearing money. This is because both interest-bearing reserves and loans have the same rate of return. For the second case, suppose that the reserve requirement is slack but the lending and pledgeability constraints bind. Then $zf'(p) < R^L(z) < R^M = \psi_1(z)$ and $p < p^*$. Interest-bearing reserves have a higher return than loans; so that banks will reduce their lending and hold more cash reserves (earns interest) due to a higher risk of loan default from the entrepreneurs. In this way, banks invest in interest-bearing reserves as insurance against the limited commitment friction of the entrepreneurs. If the reserve requirement binds along with binding pledgeability and lending constraints, then $zf'(p) < R^L(z) < R^M < \psi_1(z)$. In words: if the reserve requirement binds, the bank will need to charge a higher deposit rate than the interest it earns from reserves given the productivity shock z, which the bank is exposed to from the entrepreneur's collateral.

2.4.2 Decision-making of households

In this section, I examine the household maximization problem for the day market, and then describe the producer's problem and the consumer's problem for the night market when interest-bearing money and bank deposits may coexist as exchange media.

2.4.2.1 The day market

A household enters the day with m_1 nominal money balance. Let m_2 denote the nominal money balance taken by this household into the night market. Recall that we already defined the real money balance at the beginning of the day as $a \equiv \phi_1 m_1$ in the bank's maximization problem. Define the real money balance carried forward into the night $q \equiv \phi_1 m_2$. The day budget constraint of a household is now given by

$$x = \psi_1(z)d - \psi_1(z)s + R^M a - q. \tag{2.37}$$

Analogous to (2.11), the choice problem in the day is

$$W(d, a, z) \equiv \max_{s \geq 0, q \geq 0} \left\{ \psi_1(z)d - \psi_1(z)s + R^M a - q + E_\eta V(s, q, \eta) \right\}. \tag{2.38}$$

The demand for real deposits and real money, respectively, must satisfy

$$\psi_1(z) = E_\eta \frac{\partial V(s, q, \eta)}{\partial s}, \tag{2.39}$$

$$1 = E_\eta \frac{\partial V(s, q, \eta)}{\partial q}. \tag{2.40}$$

The envelope conditions are

$$\psi_1(z) = \frac{\partial W(d, a, z)}{\partial d}, \tag{2.41}$$

$$R^M = \frac{\partial W(d, a, z)}{\partial a}. \tag{2.42}$$

Note that the conditions above imply that both $\psi_1(z)$ and ϕ_1 are invariant over time in a stationary equilibrium.

2.4.2.2 The night market

Households take portfolio (s, q) into the night market, when the news is η. Consumers and producers separate and move to their respective locations. A type j consumer receives a lump-sum transfer of money T_j^c, and travels to another location with real money balances $q + \tau_j^c$. Denote by $c_j = c_j^d + c_j^m$ the total real purchases of output of a type j consumer with deposits and cash, where c_j^d is the output purchased by using deposits and c_j^m is the output purchased by using interest-bearing fiat money. Also denote by $y_j = y_j^d + y_j^m$ the amount of the output produced where a combination of deposits and money is accepted for purchase, with y_j^d as the amount of output that can be purchased by using deposits and y_j^m as the amount of output that can be purchased by using money. In addition to the deposit constraint (2.14), each consumer type j now faces a cash constraint

$$c_j^m \leq \frac{\phi_2(\eta)}{\phi_1}\left(q + \tau_j^c\right), \tag{2.43}$$

respectively. The combined deposit and cash constraints can be viewed as a single consumer debt-constraint, defined as

$$c_j \leq \psi_2(\eta)s + \frac{\phi_2(\eta)}{\phi_1}\left(q + \tau_j^c\right).$$

The nominal money balances brought forward by a household into the next day are $m_1^+(j) = m_2 + T_j^c + {}^1/_{\phi_2(\eta)}\left(y_j^m - c_j^m\right)$, which can be expressed in real terms,[10]

$$a_j^+ = \frac{\phi_1^+}{\phi_1}\left(q + \tau_j^c + \frac{\phi_1}{\phi_2(\eta)}\left(y_j^m - c_j^m\right)\right).$$

The choice problem for a household with realized consumer type $j \in \{l, h\}$ can be stated as

$$\begin{aligned} V_j(s,q,z) \equiv \max_{c_j^d, c_j^m, y_j^d, y_j^m} \Bigg\{ & \omega_j u(c_j) - v(y_j) \\ & + \beta \mathbb{E}\Bigg[W\Bigg(\frac{1}{\psi_2(\eta)}\left(\psi_2(\eta)s + y_j^d - c_j^d\right), \\ & \frac{\phi_1^+}{\phi_1}\left(q + \tau_j^c + \frac{\phi_1}{\phi_2(\eta)}\left(y_j^m - c_j^m\right)\right), z^+ \Bigg) \Bigg| \, \eta \Bigg] \Bigg\}. \end{aligned} \tag{2.44}$$

I want to restrict attention to equilibria in which bank deposits and interest-bearing money coexist. For both of these two assets to be accepted as payment, their expected

[10] Since $a_j^+ \equiv \phi_1^+ m_1^+(j)$, multiplying by ϕ_1^+ gives $a_j^+ = \phi_1^+ m_2 + \phi_1^+ T_j^c + {}^{\phi_1^+}/_{\phi_2(\eta)}\left(y_j^m - c_j^m\right)$. Again, multiplying by ϕ_1, the evolution of real money balances may be stated, alternatively, as $a_j^+ = {}^{\phi_1^+}/_{\phi_1} q + {}^{\phi_1^+}/_{\phi_1} \tau_j^c + {}^{\phi_1^+}/_{\phi_2(\eta)}\left(y_j^m - c_j^m\right)$.

rate of return from the night to the next day (conditional on news η) must be equal. That is, the following no-arbitrage condition must hold:

$$\frac{\psi_1(z(\eta))}{\psi_2(\eta)} = \frac{R^M \phi_1^+}{\phi_2(\eta)}. \tag{2.45}$$

Following similar steps as before, the total supply of output y at night is characterized by

$$v'(y(\eta)) = \frac{\beta R^M \phi_1^+}{\phi_2(\eta)}. \tag{2.46}$$

Applying (2.39) and (2.40), the total consumption of output c_j at night is characterized by

$$\begin{aligned} \omega_j u'(c_j(\eta)) &= \tfrac{\beta R^M \phi_1^+}{\phi_2(\eta)} && \text{if } \psi_2(\eta)s + \tfrac{\phi_2(\eta)}{\phi_1}\left(q + \tau_j^c\right) \geq c_j(\eta) \\ c_j(\eta) &= \psi_2(\eta)s + \tfrac{\phi_2(\eta)}{\phi_1}\left(q + \tau_j^c\right) && \text{otherwise.} \end{aligned} \tag{2.47}$$

In either case, the envelope conditions are

$$\frac{\partial V_j(s,q,z)}{\partial s} = \psi_2(\eta)\omega_j u'(c_j(\eta)), \tag{2.48}$$

$$\frac{\partial V_j(s,q,z)}{\partial q} = \frac{\phi_2(\eta)}{\phi_1}\omega_j u'(c_j(\eta)). \tag{2.49}$$

2.4.3 Equilibrium

Denoting the supply of money by Q and defining $\phi_1 M^+ \equiv Q$, the market-clearing conditions in a monetary equilibrium now imply

$$\begin{aligned} s &= S, \\ q &= Q, \\ 0.25c_l(\eta) + 0.25c_h(\eta) &= 0.5y(\eta), \\ p^s &= p^d. \end{aligned} \tag{2.50}$$

Note that (2.34) and the money-market clearing condition, $q = Q$, together can be used to express the lump-sum transfers received by type h consumers as shown below

$$\tau_h^c = 4\left(\frac{R^M}{\mu} - 1\right)Q. \tag{2.51}$$

In a stationary equilibrium all the real variables are constant over time, so that $S = S^+$ and $Q = Q^+$. It follows that ${}^{\phi_1^+}/_{\phi_1} = {}^1/_\mu$.

As before, the four cases will still apply. That is, with market-clearing, conditions (2.20), (2.21), (2.22), and (2.23) can be restated as

$$u'(c_l(\eta)) = \frac{\beta R^M \phi_1}{\phi_2(\eta)} = \delta u'(c_h(\eta)), \tag{2.52}$$

$$\begin{aligned} u'(c_h(\eta)) = \tfrac{\beta R^M \phi_1}{\phi_2(\eta)} \quad &\text{and} \quad c_l(\eta) = \psi_2(\eta)S + \tfrac{\phi_2(\eta)}{\phi_1}Q, \\ u'(c_l(\eta)) = \tfrac{\beta R^M \phi_1}{\phi_2(\eta)} \quad &\text{and} \quad c_h(\eta) = \psi_2(\eta)S + \tfrac{\phi_2(\eta)}{\phi_1}\left(Q + \tau_h^c\right), \end{aligned} \tag{2.53}$$

$$\psi_2(\eta)S + \frac{\phi_2(\eta)}{\phi_1}Q + 0.5\tau_h^c = y(\eta) < y^*, \tag{2.54}$$

$$\psi_2(\eta)S + \frac{\phi_2(\eta)}{\phi_1}Q + 0.5\tau_h^c \geq y(\eta) = y^*. \tag{2.55}$$

Once again, invoking the envelope conditions allows us to get an equivalent condition to (2.26) that may characterize the monetary equilibrium. Following similar steps, condition (2.40) may be stated as

$$\phi_1 = \frac{\pi\phi_2(b)}{2}\left[u'(c_l(b)) + \delta u'(c_h(b))\right] + \frac{(1-\pi)\phi_2(g)}{2}\left[u'(c_l(g)) + \delta u'(c_h(g))\right], \tag{2.56}$$

which by assuming that the type l deposit constraint is slack can be rewritten as

$$\phi_1 = \pi\phi_2(b)v'(y(b))A(y(b)) + (1-\pi)\phi_2(g)v'(y(g))A(y(g)). \tag{2.57}$$

Note that deposit price is still characterized by condition (2.28). In fact, in a monetary economy, the deposit price can also be expressed as a function of nominal interest rate,

$$\psi_2(\eta) = \frac{\beta R^M}{v'(y(\eta))}, \tag{2.58}$$

when considering slack reserve requirement and lending constraints.

Condition (2.58) shows a channel through which a policy of paying interest on money can influence asset prices. We want to derive an analogous condition for the value of money at night. From condition (2.46) describing the optimal behavior of the household, we can derive the expression

$$\phi_2(\eta) = \frac{\beta R^M \phi_1}{\mu v'(y(\eta))}. \tag{2.59}$$

Conditions (2.54), (2.55), (2.57) and (2.59), together with conditions (2.22), (2.23), (2.26) and (2.28) derived earlier, characterize the competitive equilibrium allocation at night in which both bank deposits and interest-bearing money are valued. Furthermore, after some manipulation, condition (2.56) may be rewritten as

$$\frac{\mu}{R^M} = \beta \left[\pi A(y(b)) + (1-\pi)A(y(g))\right]. \tag{2.60}$$

Note that a stationary monetary equilibrium will concurrently require the equilibrium price of deposit $0 < \psi_1(z) < \infty$ to satisfy the restriction in (2.33). This gives rise to the following proposition.

Proposition 2.4. *i) In a news economy with bank deposits and interest-bearing money, the type-contingent transfer policy $R^{M^*} = \beta^{-1} > \mu^* = 1$, $\tau_h^{c*} = 4\left(\frac{1-\beta}{\beta}\right) Q > 0$, and $\tau_l^{c*} = \tau^{p*} = 0$ implements the efficient allocation y^*. The lending market may be suboptimal with $p < p^*$.*

ii) In a no-news economy, there does not exist a monetary equilibrium when $R^{M^*} = \mu^* \geq 1$ and $p = p^*$.

Proof. Since $A(y^*) = 1$, condition (2.60) is satisfied with $R^{M^*} = \beta^{-1} > \mu^* = 1$. We also need to check the condition that can guarantee positive money balances along with $\psi_1(z) > 0$ by also satisfying condition (2.33). Substituting τ_h^{c*} into (2.55) means that we require

$$\phi_1 \geq \frac{\mu^*[y^*v'(y^*) - \beta z(b)f'(p)S] - 2(R^{M^*} - \mu^*)v'(y^*)Q}{\beta R^{M^*}Q}$$

$$\text{or, } \phi_1 \geq \frac{\mu^*[y^*v'(y^*) - \beta R^L S] - 2(R^{M^*} - \mu^*)v'(y^*)Q}{\beta R^{M^*}Q},$$

where $R^{L^*} < R^L$ and $p < p^*$ by assuming that the pledgeability constraint, the lending constraint and the reserve requirement are all binding. This is because from (2.32), $y^*v'(y^*) = \beta z^e f'(p^*)S > \beta z(b)f'(p) = \beta R^L S$. It follows that any value $\phi_1 < \infty$ satisfying the above inequalities is a competitive monetary equilibrium that will guarantee positive money balances. To see this, note that we can rearrange the above inequalities to write down

$$q = \frac{\mu^*[y^*v'(y^*) - \beta z(b)f'(p)S]}{\beta R^{M^*}\phi_1 + (R^{M^*} - \mu^*)2v'(y^*)} > 0.$$

If $y^*v'(y^*) = \beta z^e f'(p^*)S = \beta R^{L^*}S$ then $q = 0$, which implies that interest-bearing money cannot coexist with bank deposits in the no-news case. ■

Note that with interest-bearing money, the implementation of an efficient allocation is independent of parameters β and z^e. Since I have assumed that a lump-sum tax instrument is not available to the central bank, the standard Friedman rule of setting $(R^M, \mu) = (1, \beta)$ is not feasible. Hence, deflation is not optimal. Taxes cannot be collected by the central bank to finance a deflationary policy. Instead, running an inflationary policy can help overcome a liquidity shortage with a positive nominal interest rate on money. This is because β is strictly decreasing in R^M; so that a higher nominal interest rate and positive inflation expands the set of economies for which the efficient allocation is achievable. This result is in contrast to Andolfatto and Martin (2013), where a stationary monetary equilibrium does not coexist with another asset when there is a constant supply of fiat money, namely, $\mu \geq 1$ (see Proposition 5 in Andolfatto and Martin (2013)). Here, paying nominal interest rates and positive inflation rate makes up for the lack of power to lump-sum tax, which in turn may prevent the liquidity shortage that arises as a result of the technological uncertainty faced by the entrepreneurs with limited commitment.

The central bank possesses the capacity to generate assets from the day good x through the issuance of an interest-bearing debt instrument. By investing in interest-bearing reserves, banks can insure against the volatility associated with technological risks in entrepreneurial ventures. However, the lending market may still be suboptimal due to the uncertain nature of information and also due to the short supply of commitment from the entrepreneurs. The crucial assumption used here is that the central bank can observe household preferences, which allows for the type-contingent transfers conditional on household types. The main goal of these type-contingent transfers is to redistribute the purchasing power of households in a manner that is socially desirable.

Not surprisingly, money introduced in this manner cannot coexist with bank deposits in a no-news economy. This is because in the no-news case, bank deposits operating as the sole medium of exchange can achieve the first-best solution. Since there is no benefit from introducing an asset that is dominated in the rate of return, interest-bearing money is not valued and is therefore redundant. After all, if the economy is functioning to its best capacity with private money then why would there be any reason for the central bank or the government to intervene? Outside money in this case is not essential, as money creation does not improve ex ante welfare relative to what can be achieved with private money.[11] The idea here is quite similar to Proposition 3 in Andolfatto and Martin (2009).

One advantage of including interest-bearing money in this manner is that it does not require us to artificially impose a cash-in-advance constraint on bank deposits to essentially evade the price volatility in deposits, as was highlighted in Andolfatto and Martin (2013). That is, imposing the constraint of $s = 0$, so that individuals can only use money to settle their debt in the night market does not confer any substantial welfare gains. Even though money is affected by news, the nominal interest rate R^M is an additional policy tool (apart from the money growth rate μ) that makes money less sensitive to news. This is considering that the economy experiences the adverse effects of excessive price sensitivity of bank deposits due to information frictions, and particularly when commitment in financial markets is limited.

[11]See Wallace (2014) for an exposition on the sufficient conditions that can guarantee the essentiality of money.

To see how an additional policy tool R^M can be beneficial even in the deposit market, referring to (2.58) results in the following proposition.

Proposition 2.5. *In a news economy, $\psi_2(\eta)$ is increasing in R^M.*

The proposition above asserts that interest-bearing money can provide an additional policy tool to alleviate depressed asset prices. Since $\psi_2(b) < \psi_2(g) = {\beta R^{M*}}/{v'(y^*)}$, raising the interest rate on bank reserves during tougher economic times may provide relief and help the economy recover by relaxing the debt constraints of the banks and consumers. With the intended policy design of promoting financial intermediation, undervalued real deposit prices can reach their long-run fundamental value. In practical terms, this could also be a motivation for introducing interest-bearing CBDC, although this paper only studies a weak-form of CBDC.[12]

To see if there is some empirical evidence to support this proposition, I obtain time series on interest rate on excess reserves (IOER) and interest rates on transaction deposits. The IOER data were obtained from FRED, and the rate on transaction deposits was sourced from Wharton Research Data Services (WRDS) using the Stata code by Chiu et al. (2019).[13] Figure 2.2 lends some credence to Proposition 2.5, namely, that there is a positive association between the deposit rate and the IOER.

Is it always reasonable to assume that the central bank can observe household preferences? Probably not. The next section explores this limitation.

2.5 Bank deposits, interest-bearing money, and an illiquid bond market

In this section, I assume that the shock on consumer type, $\omega_t(i)$, is not publicly observable upon realization. In other words, household types are private information. Since there is no record-keeping, a welfare-improving transfer policy is infeasible, as type l consumers will misrepresent themselves as type h consumers. The optimal

[12]A weak of form of CBDC because money in this model does not solve the problem of private information. The technology here is not superior to prevent individuals from hiding their money balances if they have private information about their types.

[13]Chiu et al. (2019) obtained the data on interest rates on transaction deposits from the WRDS by using the SAS codes by Drechsler et al. (2017). They obtained the rates on transaction deposits by first dividing interest expenses on transaction accounts (item code: RAID4508) by total transaction deposits (RCON2215) to obtain the quarterly rates for each bank. They then obtained a quarterly industry average by taking a weighted average across banks by taking into account their transaction deposits. See their paper for more details on the data methodology.

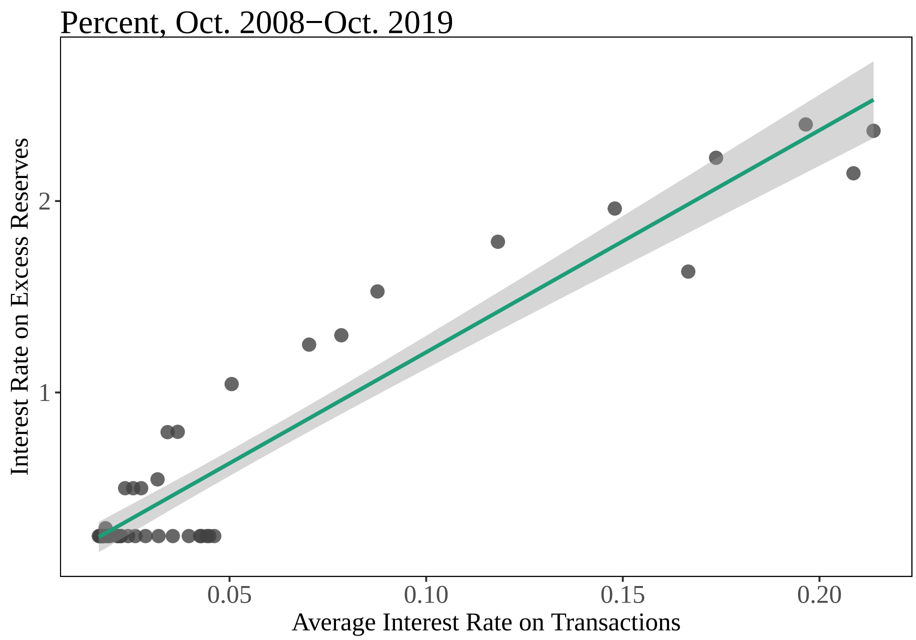

Figure 2.2: Interest Rates

Source: Federal Reserve Board and Chiu et al. (2019).

transfer policy would then essentially be a zero transfer (Andolfatto (2011)). In what follows, I introduce an illiquid bond in the monetary economy, that is subject to news shocks.

The central bank now issues two intrinsically worthless tokens, money and bonds, denoted by M and O, respectively. During the day, new bonds are issued at the discount price $0 < \alpha \leq 1$. Bonds are redeemed at par for money on the following day, and hence represent risk-free claims to future money. Since bonds are illiquid, they cannot be used to make payments. Instead, they can be exchanged for money in a secondary market at a competitive price α_2. I assume that this secondary market opens and closes before the news shock is realized, so that α_2 is independent of η. I also assume that this market opens right after the shock on consumer preferences is realized and closes before households travel to their respective locations.[14]

Money supply now evolves according to the central bank budget constraint, $M^+ -$

[14]This restriction on bond liquidity is what makes bonds essential in improving welfare when the added friction of private information is integrated into the environment.

$R^M M = O - \alpha O^+$. By assuming a constant bond-money ratio $\chi \equiv O/M > 0$, the budget constraint can be expressed as

$$\mu = \frac{R^M + \chi}{1 + \alpha\chi}. \tag{2.61}$$

Clearly, a zero discount policy ($\alpha = 1$) and zero nominal interest rate on money ($R^M = 1$) imply $\mu = 1$. In what follows, I will describe the respective optimization problems of the household for the day and the night.

2.5.1 Decision-making of households

As before, the entrepreneur's problem is unaffected. Since all bonds issued during the day will be redeemed into money at par, the composition of a money-bond portfolio during the day is irrelevant. This implies that the bank's problem remains unaffected, as total real money balances is all that really matters.

2.5.1.1 The day market

Let o denote the real bond holdings purchased by a household during the day. The household's choice problem is given by:

$$W(d, a, z) \equiv \max_{s \geq 0, q \geq 0, o \geq 0} \left\{ \psi_1(z)d - \psi_1(z)s + R^M a - (q + \alpha o) + E_\eta V(s, q, o, \eta) \right\}. \tag{2.62}$$

The demand for real deposits s, real money demand q, and real bond demand o are characterized by:

$$\psi_1(z) = E_\eta \frac{\partial V(s, q, \eta)}{\partial s} \tag{2.63}$$

$$1 = E_\eta \frac{\partial V(s, q, \eta)}{\partial q} \tag{2.64}$$

$$\alpha = E_\eta \frac{\partial V(s, q, \eta)}{\partial o} \tag{2.65}$$

Note that the same envelope conditions (2.41) and (2.42) apply.

2.5.1.2 The night market

Households enter the night market with a portfolio (d, q, o) when news is η. The bond market opens right after the consumer preference shock is realized. Subsequently, there is a news shock on the entrepreneurs' productivity right after the bond market closes. Let o_j denote the quantity of real bonds sold (where $o_j < 0$ denotes a purchase of real bonds) by a type j household in the bond market. Because the quantity of bonds sold cannot exceed the quantity available, there is a trading restriction on bond sales; in particular,

$$o_j \leq o. \tag{2.66}$$

Consumers with liquidity needs, may only purchase output at night by using either money or deposits. The deposit constraint (2.14) remains unaffected, but each consumer now faces the following cash constraint:

$$c_j^m \leq \frac{\phi_2(\eta)}{\phi_1}\left(q + \alpha_2 o_j\right). \tag{2.67}$$

The consolidated consumer debt-constraint now becomes

$$c_j \leq \psi_2(\eta)s + \frac{\phi_2(\eta)}{\phi_1}\left(q + \alpha_2 o_j\right), \tag{2.68}$$

with the evolution of real balances,

$$a_j^+ = \frac{\phi_1^+}{\phi_1}\left(q + \alpha_2 o_j + \frac{\phi_1}{\phi_2(\eta)}\left(y_j^m - c_j^m + o - o_j\right)\right).$$

For a household with realized consumer type $j \in \{l, h\}$, the choice problem is given by

$$\begin{aligned} V_j(s,q,o,z) \equiv \max_{c_j^d, c_j^m, y_j^d, y_j^m} \Bigg\{ & \omega_j u(c_j) - v(y_j) \\ & + \beta \mathbb{E}\Bigg[W\Bigg(\frac{1}{\psi_2(\eta)}\left(\psi_2(\eta)s + y_j^d - c_j^d\right), \\ & \qquad \frac{\phi_1^+}{\phi_1}\left(q + \alpha_2 o_j + \frac{\phi_1}{\phi_2(\eta)}\left(y_j^m - c_j^m + o - o_j\right)\right), z^+ \Bigg) \Bigg| \, \eta \Bigg] \\ & \qquad + \varepsilon_j(o - o_j) \\ & \qquad + \lambda_j(\eta)\left[\frac{\phi_2(\eta)}{\phi_1}(q + \alpha_2 o_j) + \psi_2(\eta)s - (c_j^d + c_j^m)\right] \Bigg\}, \end{aligned} \tag{2.69}$$

where $\varepsilon_j \geq 0$ is a Lagrange multiplier associated with constraint (2.66), and $\lambda_j(\eta) \geq 0$ is a Lagrange multiplier associated with the consolidated consumer debt-constraint (2.68).

Once again, assuming the no-arbitrage condition, the total supply of output at night is characterized by (2.46). The total consumption at night is characterized by

$$\begin{array}{ll} \omega_j u'(c_j(\eta)) = \frac{\beta R^M \phi_1^+}{\phi_2(\eta)} + \lambda_j(\eta) & \text{if } \psi_2(\eta)s + \frac{\phi_2(\eta)}{\phi_1}(q + \alpha_2 o_j) \geq c_j(\eta) \\ c_j(\eta) = \psi_2(\eta)s + \frac{\phi_2(\eta)}{\phi_1}(q + \alpha_2 o_j) & \text{otherwise.} \end{array} \tag{2.70}$$

The first-order condition with respect to unsold bond holdings yields

$$\beta R^M \left[\frac{\phi_1^+}{\phi_1}\alpha_2 - \frac{\phi_1^+}{\phi_2(\eta)} \right] + \alpha_2 \frac{\phi_2(\eta)}{\phi_1}\lambda_j(\eta) = \varepsilon_j. \tag{2.71}$$

Combining the latter two expressions gives

$$\varepsilon_j = \frac{\phi_2(\eta)}{\phi_1}\alpha_2 \omega_j u'(c_j(\eta)) - \beta R^M \frac{\phi_1^+}{\phi_2(\eta)}. \tag{2.72}$$

Another envelope condition in addition to (2.48) and (2.49) is

$$\frac{\partial V_j(s,q,o,z)}{\partial o} = \frac{\phi_2(\eta)}{\phi_1}\alpha_2 \omega_j u'(c_j(\eta)). \tag{2.73}$$

2.5.2 Equilibrium

In addition to the market-clearing conditions in (2.50), the bond market-clearing conditions are

$$\begin{array}{c} o = \chi q \\ o_l + o_h = 0. \end{array} \tag{2.74}$$

Note that $\partial V(s,q,o,z)/\partial o = 0.5 \partial V_l(s,q,o,z)/\partial o + 0.5 \partial V_h(s,q,o,z)/\partial o$. The latter expression combined with condition (2.65) imply that $\alpha_2 = \alpha$. Moreover, using the envelope condition (2.73), once again we have the same restriction as (2.56). It can be easily shown that type l households will use their money to buy bonds and type h households will sell their bonds for money; that is, $o_l < 0 < o_h$. If the consolidated debt-constraint for type l consumers is slack (so that $\lambda_l(\eta) = 0$), then $u'(c_l(\eta)) = v'(y(\eta))$. This together with a slack bond-sales constraint for type l consumers ($\varepsilon_l = 0$) yields

$$\frac{\phi_2(\eta)}{\phi_1}\alpha u'(c_l(\eta)) = v'(y(\eta)) = \beta R^M \frac{\phi_1^+}{\phi_2(\eta)}. \tag{2.75}$$

Assuming stationarity, combining the latter expression with (2.56) leads to

$$\frac{\alpha\mu}{R^M} = \beta\left[\pi A(y(b)) + (1-\pi)A(y(g))\right]. \tag{2.76}$$

Note the similarity between the above expression and condition (2.60).

Condition (2.76) and (2.26) derived earlier, characterize the competitive equilibrium in which bank deposits, interest-bearing money, and illiquid bond are valued. Also, from condition (2.76), implementation of a first-best allocation will require a policy that satisfies $\alpha\mu = \beta R^M$. This is a case when the bond market supplies the agents with sufficient liquidity, as the ability of the government to repay its debt means that bonds may generally be accepted in exchange for money to meet the different liquidity needs of agents. Agents adjust their asset portfolio and liquidity is channeled from bond buyers (type l consumers) to bond sellers (type h consumers). Assuming that the policy $\alpha\mu = \beta R^M$ is satisfied, then together with the central bank budget constraint (2.61), implies

$$\mu = R^M + (1-\beta R^M)\chi. \tag{2.77}$$

Observe that the implied inflation rate is strictly positive for any $\chi > 0$ and $R^M \geq 1$. Clearly, this policy restriction requires the discount rate $\alpha = {}^{\beta R^M}/_{\mu} < 1$. Furthermore, an increase in the bond-money ratio is associated with a higher nominal interest rate.

I will now check if the bond-sales constraint for type h consumers will bind or not.

Contrary to the literature, since the consumer debt-constraint is influenced by news, the question of whether this constraint binds or not is not entirely determined by the bond discount price α or the nominal interest rate. Suppose the debt-constraint for type h consumers is slack, then $\delta u'(c_h(\eta)) = {}^{\beta R^M \phi_1^+}/_{\phi_2(\eta)}$. If type-$h$ bond-sales constraint (2.66) is also slack then this together requires $\phi_1 \le \phi_2(\eta)$ for $\alpha \le 1$.

First, consider the case $o_h = o$ and suppose the debt-constraint for type h binds, that is, $\lambda_h > 0$. Invoking (2.74) yields

$$c_h(\eta) = \psi_2(\eta)s + \frac{\phi_2(\eta)}{\phi_1}(q + \alpha\chi q).$$

Assuming a binding debt-constraint for type l and once again using the market-clearing conditions, we can solve for q and rewrite the following restriction on policy variables α and χ:

$$c_h(\eta) = (1 + \alpha\chi)y(\eta) - \alpha\chi\psi_2(\eta)S. \tag{2.78}$$

Note that the binding bond-sales constraint for type h consumers implies $\varepsilon_h > 0$. At the same time, if both the bond-sales constraint and the debt-constraint for type l consumers bind, then it is impossible for type h to have a binding bond-sales constraint.

Lemma 2.4. *The bond-sales constraint for type h consumers cannot bind tightly in a news economy.*

Proof. Since $\varepsilon_h > \varepsilon_l = 0$, (2.72) reveals that $\delta u'(c_h(\eta)) > u'(c_l(\eta))$. Also, if the cash constraint for type l consumers is slack then $\delta u'(c_h(\eta)) > v'(y(\eta))$. This implies $A(y) > 1$ and using (2.76), $\alpha > {}^{\beta R^M}/_{\mu} > 1$. But this is a contradiction; as $\alpha \le 1$. ■

Now, consider the case $o_h < o$. Then the slack bond-sales constraint for type h consumers means $\varepsilon_h = 0$. If $\varepsilon_h = \varepsilon_l = 0$, then $u'(c_l(\eta)) = \delta u'(c_h(\eta))$. Owing to $\lambda_l = 0$ yields $\delta u'(c_h(\eta)) = v'(y(\eta))$. Clearly, $A(y^*) = 1$ entails a policy that satisfies $\alpha\mu = \beta R^M$. Substituting $\alpha = {}^{\beta R^M}/_{\mu}$ into (2.78), where μ is given by (2.77), leads to

$$c_h(\eta) = \frac{(R^M + \chi)y(\eta) - \beta R^M \chi\psi_2(\eta)S}{R^M + (1 - \beta R^M)\chi}. \tag{2.79}$$

The expression above implies that there exists a $\chi^* > 0$, $R^{M^*} > 1$, and $\mu^* > 1$ that can implement the first-best allocation y^*, so that the bond-sales constraint of type h consumers remains slack. We have the following proposition.

Proposition 2.6. *i) In a news economy with bank deposits, interest-bearing money, and illiquid bond, if $\phi_1 < \phi_2(\eta)$ then the efficient allocation is implementable for any* *bond-money ratio* $0 < \chi^* \leq \chi < \infty$ *and money growth rate* $\mu^* = R^M + (1 - \beta R^M)\chi > 1$, *with an associated nominal interest rate* $R^{M^*} = \beta^{-1} > 1$ *and a discount rate* $\alpha^* < 1$. *The lending market may remain suboptimal with* $p < p^*$.

ii) In a no-news economy, there is no monetary equilibrium when policy is restricted to $0 < \chi^* \leq \chi < \infty$, $\mu^* = R^M + (1 - \beta R^M)\chi > 1$, *with an associated* $R^{M^*} = \beta^{-1} > 1$ *and* $\alpha^* < 1$; *most importantly, when* $p = p^*$.

Proof. Since $A(y^*) = 1$, condition (2.76) is satisfied with $\alpha^*\mu^* = \beta R^{M^*}$. Now we will need to check the condition that can guarantee positive bond balances and concurrently satisfy $\psi_1(z) > 0$. Substituting $c_l(\eta)$ and $c_h(\eta)$ into (2.70) and then using the market-clearing conditions (2.50) and (2.74) imply

$$o \geq \frac{\chi^*\mu^*}{\beta R^{M^*}} \left[y^* v'(y^*) - \beta z(b) f'(p) S\right] \tag{2.80}$$

$$\text{or, } o \geq \frac{\chi^*\mu^*}{\beta R^{M^*}} \left[y^* v'(y^*) - \beta R^L S\right], \tag{2.81}$$

where $R^{L^*} < R^L$ and $p < p^*$ by assuming binding constraints for the entrepreneurs and the banks. As long as $y^* v'(y^*) > \beta R^L S = \beta z(b) f'(p) S, o > o_h > 0$ and the bond-sales constraint for type h consumers will remain slack (following Lemma 2.4). Consequently, any value $o < \infty$ satisfying the latter inequalities is an equilibrium. Similar to the proof in Proposition 2.4, if $y^* v'(y^*) = \beta z^e f'(p^*) S = \beta R^{L^*} S$ then $o = 0$, which violates our assumption of $o_l < 0 < o_h < o$. ■

Having a limit on bond holdings for type h consumers means that type h cannot get sufficient liquidity on bond sales. An optimal allocation of liquidity between the two types of consumers requires the bond market to generate sufficient liquidity against news shocks. An insufficient liquidity from bond sales is infeasible, especially with news shocks creating an additional liquidity shortage. As a consequence, the coexistence of government debt instruments with other private assets requires the sufficiency of liquidity provision. With the exception of strictly positive inflation with illiquid bonds, Proposition 2.6, by and large, replicates the result achieved by the optimal type-contingent transfer policy described in Proposition 2.4. The illiquid bond helps achieve socially desirable allocations, even though household preferences are unknown to the central bank. The lending market may still remain suboptimal for the same aforementioned reasons. However, illiquid bond and interest-bearing money cannot coexist with bank deposits in the no-news case. The reasoning is the same as what was stated earlier; namely, that there is no need for government intervention if the first-best solution can be achieved with private money. In the no-news case, both interest-bearing money and illiquid bond are not essential.

In the next subsection, I investigate the possibility of rendering the short-run rate of return on interest-bearing money insensitive to news by restricting the use of bank deposits as private money.

2.5.3 Illiquid bank deposits

I assume now that bank deposits are illiquid, that is, they cannot be used to make payments at night. In this case, only interest-bearing money can be used to make payments at night. This type of a cash-in-advance constraint is frequently imposed in the literature. I will show how a cash-in-advance constraint of this form is welfare-enhancing.[15]

The choice problem of the agents in the day is unaffected, but the decisions on consumption and production at night will clearly change. The supply of night output y is still characterized by condition (2.46). However, the no-arbitrage condition (2.45) is not relevant in this case, as fiat money can only be used for the purchase of goods in the night. Adding the constraint $s = 0$ means that the desired consumption at night is characterized by

$$\begin{aligned} \omega_j u'(c_j(\eta)) &= \tfrac{\beta R^M \phi_1^+}{\phi_2(\eta)} + \lambda_j(\eta) \qquad \text{if } \tfrac{\phi_2(\eta)}{\phi_1}(q + \alpha_2 o_j) \geq c_j(\eta) \\ c_j(\eta) &= \tfrac{\phi_2(\eta)}{\phi_1}(q + \alpha_2 o_j) \qquad\qquad \text{otherwise.} \end{aligned} \tag{2.82}$$

I anticipate that the cash-in-advance constraint will bind for a growing supply of money ($\mu \geq 1$), a positive interest rate $R^M \geq 1$, a bond-money ratio $\chi > 0$, and a bond discount price $\alpha \leq 1$. Applying the market-clearing conditions and combining

---

[15] A result that has also been pointed out in Lagos and Rocheteau (2008), where placing an exogenous restriction to render capital less liquid generates a demand for outside money, so that such a restriction is indeed welfare improving.

(2.50) and (2.70), the equilibrium value of money in the night is expressed as

$$\phi_2(\eta) = \frac{\phi_1 y(\eta)}{Q} - 2\phi_1 \left(\frac{R^M}{\mu} - 1\right).$$

Together, these restrictions lead to

$$v'(y(\eta)) = \frac{\beta R^M \phi_1}{\mu \left[\frac{y(\eta)}{M^+} - 2\phi_1 \left(\frac{R^M}{\mu} - 1\right)\right]}.$$

Clearly, this implies that the equilibrium level of night output is independent of news, that is, $y = y(\eta)$. This is because payments at night are now solely made with a risk-free asset. Then solving for ϕ_1 results in the equilibrium restriction

$$\phi_1 = \frac{\mu v'(y) y / M^+}{\beta R^M + 2\mu v'(y) \left(\frac{R^M}{\mu} - 1\right)}. \tag{2.83}$$

Substituting the value of money in the day and night into (2.56), we have the equilibrium restriction

$$\frac{\alpha\mu}{R^M} = \beta A(y). \tag{2.84}$$

Conditions (2.83) and (2.84) characterize the equilibrium pair (ϕ_1, y). Furthermore, we can achieve a first-best allocation with the given policy below.

Proposition 2.7. *In a no-news economy, rendering bank deposits illiquid by imposing a cash-in-advance constraint at night improves welfare with the given policy* $\alpha\mu = \beta R^M$. *Interest-bearing money and an illiquid bond can coexist with bank deposits when* $p = p^*$.

Proof. Since $A(y^*) = 1$, the restriction (2.84) is satisfied when $\alpha\mu = \beta R^M$. ■

In contrast to Andolfatto and Martin (2013), this paper finds that imposing a cash-in-advance constraint to make bank deposits less liquid clearly enhances social welfare in the absence of news. In their study, such a constraint actually diminishes welfare when there is no news. This discrepancy might arise from their omission of private information regarding consumer types. Conversely, in my model, factoring in private information about consumer liquidity shocks necessitates an illiquid bond to broaden the scope of trades that improve welfare. Here, a cash-in-advance is a trading restriction on bank deposits that is designed to improve welfare. This type of result has been highlighted in Andolfatto (2011) and Kocherlakota (2003), where restricting the liquidity properties of bonds improves allocative efficiency. Without the friction of private information on consumer preferences, a cash-in-advance may restrict trading opportunities. This could explain the discrepancies between my result and the result in Andolfatto and Martin (2013). And as consequence, a cash-in-advance constraint in a no-news economy eliminates the suboptimality in the lending market that may exist with news; so that $p = p^*$. The restriction of using bank deposits as a means of payment reduces currency competition and fiat money is no longer linked by an arbitrage condition to the price of private money. The value of interest-bearing money becomes insensitive to news or information and its average rate of return is independent of news shocks. Most importantly, interest-bearing money and an illiquid bond can coexist with bank deposits when there are no news shocks.

2.5.3.1 Numerical Example

To see how the model works with a cash-in-advance constraint on bank deposits in a news economy with interest-bearing money and an illiquid bond, I provide some numerical examples. For functional forms, I assume $u(c) = {(c^{1-\sigma}-1)}/{(1-\sigma)}$, $v(y) = y$, and $f(p) = p^{\theta}$, which implies $c_l^* = 1$, $c_h^* = \delta^{1/\sigma}$, and $y^* = {(1+\delta^{1/\sigma})}/2$, and $x^* = {1}/{\{(\beta\theta)[\pi z(b)+(1-\pi)z(b)]\}^{1/(\theta-1)}}$. Given linear preferences in the day, I consider the average consumption in the day. Period expected utlity or welfare in the cash-in-advance economy is

$$W^{\text{CIA}} = f(p) - p + \frac{1}{2}u(c_l) + \frac{1}{2}\delta u(c_h) - v(y).$$

Similarly, welfare in the news economy is

$$\begin{aligned} W^{\text{News}} = f(p) - p &+ \frac{\pi}{2}\left[u(c_l(b)) + \delta u(c_h(b))\right] \\ &+ \frac{(1-\pi)}{2}\left[u(c_l(g)) + \delta u(c_h(g))\right] \\ &- \left[\pi v(y(b)) + (1-\pi)v(y(g))\right]. \end{aligned}$$

Using the functional forms and condition (2.84), the equilibrium allocation in the cash-in-advance economy is as follows

$$y^{\text{CIA}} = \left[\frac{\beta\delta R^M}{2\alpha\mu - \beta R^M}\right]^{\frac{1}{\sigma}},$$

$$c_h^{\text{CIA}} = \left[\frac{\left(R^M + \chi\right)}{R^M + \left(1 - \beta R^M\right)\chi}\right]\left[\frac{\beta\delta R^M}{2\alpha\mu - \beta R^M}\right]^{\frac{1}{\sigma}},$$

$$c_l^{\text{CIA}} = \left[\frac{\beta\delta R^M}{2\alpha\mu - \beta R^M}\right]^{\frac{1}{\sigma}}\left[\frac{R^M + \chi - 2\beta R^M\chi}{R^M + \left(1 - \beta R^M\right)\chi}\right].$$

For the good state, fix $c_l(g) = c_l^*$, $c_h(g) = c_h^*$, and $y(g) = y^*$. Then using (2.76), the equilibrium allocation in the bad state is characterized by

$$y(b) = \frac{\delta^{\frac{1}{\sigma}}}{\left[\frac{2\alpha\mu}{\beta\pi R^M} - \frac{1-\pi}{\pi}\frac{2^\sigma\delta}{(1+\delta^{\frac{1}{\sigma}})^\sigma} - \frac{1-2\pi}{\pi}\right]^{\frac{1}{\sigma}}},$$

$$c_h(b) = \frac{\delta^{\frac{1}{\sigma}}\left(R^M + \chi\right)}{\left[R^M + \left(1 - \beta R^M\right)\chi\right]\left[\frac{2\alpha\mu}{\beta\pi R^M} - \frac{1-\pi}{\pi}\frac{2^\sigma\delta}{(1+\delta^{\frac{1}{\sigma}})^\sigma} - \frac{1-2\pi}{\pi}\right]^{\frac{1}{\sigma}}},$$

$$c_l(b) = \frac{\delta^{\frac{1}{\sigma}}\left(R^M + \chi - 2\beta R^M\chi\right)}{\left[R^M + \left(1 - \beta R^M\right)\chi\right]\left[\frac{2\alpha\mu}{\beta\pi R^M} - \frac{1-\pi}{\pi}\frac{2^\sigma\delta}{(1+\delta^{\frac{1}{\sigma}})^\sigma} - \frac{1-2\pi}{\pi}\right]^{\frac{1}{\sigma}}}.$$

For parameters, I assume $\beta = 0.95$, $\delta = 100$, $\sigma = 10$ and $\theta = 0.02$. With these parameters, the first-best allocation is $c_l^* = 1$, $c_h^* = 1.58$ and $y^* = 1.29$. First, I fix $\alpha = 0.48$, $O = 2$, $M = S = 10$, $R^M = 1.01$, $\pi = 0.25$, $z(b) = 0$ and illustrate how changing $z(g)$ affects the allocation in the competitive equilibrium of the cash-in-advance economy and the news economy, respectively.[16] Then, I compute and compare the welfare in these respective economies. Pick $z(g) = 0.0001$ so that condition (2.31) just about satisfied.[17] The equilibrium allocation in the cash-in-advance

[16]Note that the money growth rate μ is determined by the budget constraint (2.61) once the nominal interest rate, the bond discount price, and the bond-money ratio are given.

[17]In fact, this is the minimum value of $z(g)$ so that the deposit constraint in the good state is

economy is $p^{CIA} = p^* = 1.08$, $c_l^{\text{CIA}} = 1.61$, $c_h^{\text{CIA}} = 2.36$ and $y^{\text{CIA}} = 1.99$. Welfare in the cash-in-advance economy is 4.38. On the other hand, the equilibrium allocation in the news economy is $p^{News} = p^* = 1.08$, $c_l(b) = 0.95$, $c_h(b) = 1.39$, $c_l(g) = c_l^* = 1$, $c_h(g) = c_h^* = 1.58$ and $y(b) = 1.17$, $y(g) = y^* = 1.29$. Welfare in the news economy is 4.91. For this parameterization, the difference in welfare between the cash-in-advance economy and the news economy is -0.53. Now consider $z(g) = 0.5$. Then the equilibrium allocation in the cash-in-advance economy is $p^{CIA} = p^* = 0.006$, with the night-allocation mostly unchanged. Welfare in the cash-in-advance economy now increases slightly. Similar result applies for the equilibrium quantities and welfare in the news economy. Most importantly, the difference in welfare between these two respective economies now increases by a small margin. As expected, an increase in the magnitude of the good state, represented by $z(g)$, implies that the news economy outperforms the cash-in-advance economy in terms of welfare, although only marginally. This has been tested by creating a grid of $z(g)$ that takes values between 0.0001 and 0.5. Overall, 1000 grid points of $z(g)$ had been generated between the first and last elements of the grid.

I now fix $z(g) = 0.0001$ and vary the nominal interest rate, R^M, to better understand the welfare effects of restricting bank deposits as a means of payment. Let $\Delta(R^M)$ represent the welfare difference between the cash-in-advance and news economies as a function of the nominal interest rate. I create a range of 1000 R^M values between 1 and 10. For each R^M, I compute the welfare in both economies and calculate the welfare difference, $\Delta(R^M)$, for the overlapping values. This calculation is repeated 1000 times for each R^M value in the range. Moreover, for a fixed nominal interest rate, $R^M > 1$, money supply, bond supply, and $z(b)$, I determine how many parameterizations of $\{\alpha, \beta, \delta, \sigma, \theta, \pi, z(g)\}$ satisfy the condition $\Delta(R^M) > 0$. After a total of $10,000,000$ iterations, there are $11,404,000$ parameter combinations for which $\Delta(R^M)$ is strictly positive. This suggests that we can construct policies such that restricting the liquidity of bank deposits may improve welfare, although this is not universally the case (see figure 2.3 below).

barely slack.

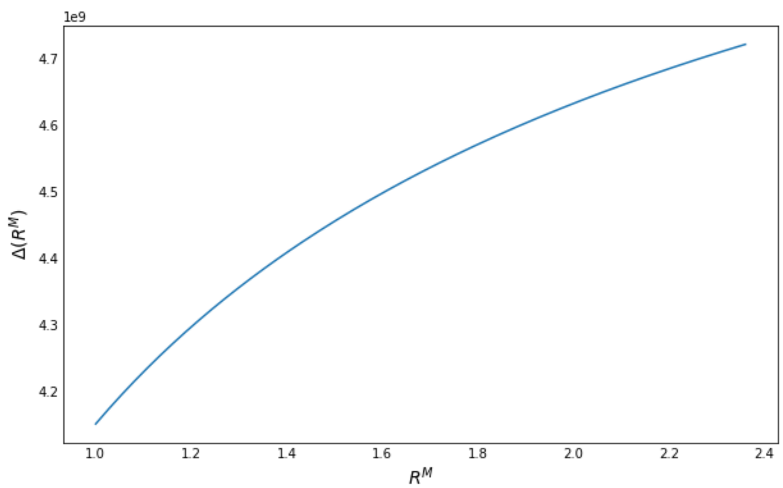

Figure 2.3: Welfare gain from restricting bank deposits as a function of R^M

2.6 Conclusion

If private currencies were not subject to information and limited commitment frictions, there would be no reason why they shouldn't be used as payment instruments. Historically, they were widely used before the dominance of government-issued fiat money. The problem lies when these private currencies are backed by other productive assets. In the model, banks issue private currencies in the form of bank deposits, which circulate as exchange media. The price of bank deposits may fluctuate excessively in response to news events surrounding technological innovations of the firms' future output, which is pledged as collateral by the firms to obtain loans from the banks. The price volatility of bank deposits does not inherently inhibit their use as exchange media, as long as the supply of deposits is not scarce. Individuals with higher liquidity needs are still willing to pay a premium for intertemporal gains to trade. The problem emerges when bad news (although socially irrelevant) results in binding debt-constraints, which can lead to a liquidity shortage. If bank deposits were not used as a medium of exchange, their price fluctuations might be benign. However, when they play a role in the payments system, any extraneous information can lead to excessive volatility in deposit prices, particularly when there is a liquidity shortage due to limited commitment.

To a degree, the adverse impact of private currencies might explain the widespread use of government-issued fiat money. While private banks might use asset tranching from an existing asset pool to create informationally insensitive exchange media, this approach faces significant limitations if there is an initial asset scarcity. Non-disclosure practices might help mitigate short-term asset price fluctuations. But when tranching or non-disclosure is not viable, central banks can provide high-quality debt instruments to overcome the shortcomings of the highly price sensitive nature of private currencies.

If a lump-tax instrument is not available, then the central bank can conduct a type-contingent transfer policy to restore efficiency. This is assuming if individual preferences over desired consumption needs are public information. Interest-bearing money provides the central bank with an additional policy tool that can positively impact depressed asset prices. Banks can invest in interest-bearing reserves as a hedge against risk shocks, enabling the central bank to influence asset prices through an investment channel. In particular, a welfare-improving policy entails a positive inflation rate and a strictly positive nominal interest rate. The fact that interest-bearing money is not backed by any productive asset allows the central bank to effectively create assets by issuing debt.

When the added friction of private information over desired consumption needs is incorporated into the model, an illiquid bond becomes essential. As before, first-best implementation is also feasible through a well-designed policy that permits some inflation and a strictly positive nominal interest rate. By imposing a cash-in-advance constraint, government debt instruments can become insensitive to news and coexist with private currencies under certain conditions. Of course, this only works to the extent that the central bank is willing to maintain low levels of inflation.

The model framework is sufficiently simple to allow for many interesting extensions. One such extension could involve allowing entrepreneurs to default on their debt obligations and banks to fail, with the banks then paying premiums to a deposit insurance agency. Additionally, introducing a meaningful role for equity finance, as detailed in Dermine (1986), could be explored. Another concept is to include scenarios where banks invest in riskier projects to gain access to cheaper funding. Examining the impact of banks possessing some degree of market power, especially in the context of information frictions, is also a realistic and worthwhile endeavor.

CHAPTER 3
INTEREST-BEARING CENTRAL BANK DIGITAL CURRENCY AND BANKING REDUX

3.1 Introduction

Central bank digital currencies (CBDCs) have emerged as a topic of significant interest among policy makers and economists in recent years. The potential benefits and risks of CBDCs have been widely discussed, with many central banks actively exploring the possibility of issuing their own digital currencies.[18] Major economies worldwide are actively researching CBDCs, which would represent a third form of currency accessible to the public, akin to cash, and also accessible to many financial institutions, similar to central bank reserves.[19] Several central banks have conducted or are in the process of planning pilot programs, and operational CBDCs already exist in Caribbean-island countries, such as DCash in the Eastern Caribbean Currency Union (ECCU), the Sand Dollar in the Bahamas, and Jam-Dex in Jamaica. China stands out as a key player among populous nations, aiming to extend financial services across extensive sectors of its economy. Additionally, India and Indonesia are currently conducting trials for digital versions of their respective currencies, the rupee and the rupiah.[20]

A wide range of technological designs for CBDCs have been proposed, but a fundamental characteristic of a CBDC is that it must be universally accessible, meaning it can be held by anyone for any purpose. A second feature relates to whether CBDCs are interest-bearing. The interest rate can serve as an additional policy tool, expanding upon the existing monetary toolkit to stabilize inflation and output. One frequent policy concern surrounding CBDCs is their potential impact on the banking system and financial intermediation. Specifically, many economists and policymakers

[18] According to Boar and Wehrli (2021), a survey conducted by the Bank for International Settlements involved 65 central banks. The survey revealed that 86% of these banks are actively engaged in initiatives related to CBDCs, with 60% having initiated experiments or proofs-of-concept for CBDCs. Additionally, 14% of the banks have progressed to the stage of developing and piloting CBDC arrangements.

[19] See Auer et al. (2020) for an overview of the policy discussion surrounding CBDCs and their different designs across a diverse mix of countries.

[20] Updated lists of countries investigating or issuing CBDCs are reported by https://www.atlanticcouncil.org/cbdctracker/.

have expressed concerns about whether CBDCs could lead to a crowding out of bank deposits, as households substitute traditional bank accounts for holding CBDCs. This disintermediation effect could have significant implications for bank lending, investment, and overall financial stability.[21]

In this paper, I develop a general equilibrium model to study the conditions under which the introduction of a CBDC could crowd out bank deposits and lead to a reduction in bank-financed investment. I build on the framework of Andolfatto et al. (2016) and the subsequent New Monetarist literature, by introducing a CBDC that competes with bank deposits as a means of payment. In the model, households can voluntarily choose their portfolio allocation between CBDC and bank deposits based on their relative returns and liquidity properties. Banks, in turn, make investment decisions based on the level of deposits they attract. I distinguish between the liquidity properties of bank deposits by introducing checkable deposits (more liquid) and time deposits (less liquid). A key feature of the model is that I explicitly incorporate the design choices around CBDC, such as the interest rate it pays and any fees associated with its use. This allows us to study how these policy parameters interact with household portfolio choices and banks' investment decisions. Importantly, the government is assumed to lack a lump-sum tax instrument to implement its monetary policy rule. This implies that households' currency choices must adhere to sequential rationality constraints, as there is no coercion or forced participation. Monetary policy, in this sense, must be incentive-feasible.

I focus my attention on implementing first-best allocations. The main results characterize the government policies required to implement the first-best allocation in an economy where CBDC and bank deposits coexist as payment instruments. In sufficiently patient economies, a passive policy with constant money supply is enough to achieve the first-best outcome. No inflation or taxes are needed, and CBDC does not crowd out bank deposits if CBDC is non-interest bearing. Banks are willing to issue deposits and pay interest rates to households at a sufficiently high level, as households are willing to sacrifice enough of their consumption for labor in the production of the output. Moreover, asset prices are priced at their fundamental level. There is no liquidity shortage as the consumer debt-constraints are slack.

[21] See Meaning et al. (2018) for a discussion on the potential impact of CBDC on monetary policy transmission and the risks CBDC poses to the banking sector.

In impatient economies, active policies with positive inflation and nominal interest rates are required to implement a first-best allocation with competing payment instruments. Binding debt-constraints lead to a shortage of liquidity. Banks issue fewer deposits to households and offer excessively high interest rates to motivate households to work hard and produce enough output. Liquidity premium arises due to asset scarcity as a result of the binding debt-constraints. In the absence of lump-sum taxes, the government must use some combination of seigniorage revenue, labor income taxes, and CBDC fees to incentivize efficient production by households and deposit issuance by banks. In this case, interest-bearing CBDC can potentially crowd out bank deposits if the interest rate on CBDC is too high relative to deposit rates. Distortionary taxes and CBDC fees are necessary to relax the debt-constraints of households.

To quantify these theoretical predictions, I calibrate the model to match key features of the US economy using data from 1987 to 2008. Consistent with the theoretical prediction that interest-bearing CBDC can crowd out bank deposits, the quantitative analysis shows that raising the nominal CBDC interest rate from 0% to 5% leads to an approximately 5% decline in bank deposits. Despite this disintermediation effect, PM market production improves by approximately 0.5%, and overall welfare increases by 1.2% in consumption-equivalent terms. These findings validate the theoretical prediction that while CBDC introduction can lead to some bank disintermediation, the efficiency gains through improved payment mechanisms can outweigh these costs when CBDC is properly designed.

3.2 Literature Review

There has been a burgeoning number of CBDC papers recently that is impractical to review here, but my paper complements the CBDC papers in the New Monetarist literature. Keister and Sanches (2023) study the potential effects of introducing a CBDC on the banking system and monetary policy in perfectly competitive markets. In their model, banks face a pledgeability constraint. They find that a CBDC can lead to a disintermediation effect, where households substitute private bank liabilities for CBDC holdings, which can lead to a reduction in productive investment and social welfare. In contrast, my paper focuses on the conditions under which a CBDC could crowd out bank deposits and the policies required to implement the first-best allocation in an economy where CBDC and bank deposits coexist.

Chiu and Davoodalhosseini (2023) investigate the macroeconomic benefits of a cash-like CBDC design. They find that a CBDC can improve welfare by reducing the cost of holding and using money, as well as by promoting financial inclusion. However, they also note that a CBDC may lead to a reduction in bank deposits and a decline in bank lending. In my paper, I explicitly incorporate the design choices around CBDC, such as the interest rate it pays and any associated fees, and studying how these policy parameters interact with household portfolio choices and banks' investment decisions. Household portfolio choices are also voluntary, meaning that there is no coercion or forced participation, so that sequential rationality is respected.

There are some influential papers that study the effect of CBDC issuance in economies with imperfect competition among banks. Andolfatto (2021) examines the impact of a CBDC on the banking system and monetary policy transmission when there is monopoly power in the banking system. He argues that a CBDC could discipline the banks by compelling them to increase their deposit rate, leading to an increase in bank deposits and financial inclusion. In Chiu et al. (2023), banks also have market power in the deposit market. They find that CBDC issuance could expand bank intermediation if the interest rate on CBDC lies within an intermediate range and causes disintermediation only if the interest rate is too high.

Williamson (2022) focuses on efficiency where the central bank competes with the private sector for safe assets. Welfare is increased through households substituting CBDC for private bank liabilities and a CBDC may disintermediate banks when there is an overaccumulation of capital. This implies that disintermediation comes at the expense of improving economic efficiency. In my paper, the financial frictions in the banking sector themselves lead to an overproduction of goods but investment is too low to satisfy the demand. Lower investment in my model then reduces economic efficiency and bank deposits are priced at a premium.

Many studies have also investigated various aspects of CBDCs, such as their optimal design, their impact on monetary policy transmission, and their potential risks to financial stability. Barrdear and Kumhof (2022) examine the macroeconomic effects of CBDC issuance in a DSGE model, while Davoodalhosseini (2022) examines the coexistence of cash and CBDC with balance-contingent transfers. Fernández-Villaverde et al. (2021) explore the effects of a CBDC on financial stability and bank runs within the framework established by Diamond and Dybvig (1983). Similar papers that also study CBDC and financial stability include Keister and Monnet (2022), Rahman (2024), and Williamson (2022). Brunnermeier and Niepelt (2019) and Niepelt (2023) show that a CBDC might not impact macroeconomic outcomes, including bank intermediation. Jiang and Zhu (2021) delve into the interactions between CBDC and reserves as tools for monetary policy. Various papers, including those by Agur et al. (2022), Davoodalhosseini and Rivadeneyra (2020), Wang (2023), and Kumhof and Noone (2018) contribute to understanding CBDC motivations and designs. However, none of these papers assume the absence of a government lump-sum transfer and consider individual rationality behind the interaction between CBDC and bank deposits. This is how my paper differs from this literature, and I also study the coexistence issues and which conditions are necessary for first-best allocations.

3.3 The Physical Environment

The physical environment is based on Andolfatto et al. (2016) and Chiu and Davoodalhosseini (2023). Time is discrete and the horizon is infinite. As is typical in models like those in Lagos and Wright (2005), each period is divided into two subperiods. In this context, I refer to these subperiods as AM and PM, respectively. Search friction is abstracted away and agents meet in centralized locations in both subperiods. Two distinct perishable goods (or outputs) are produced and consumed in each subperiod called the AM good and the PM good.[22]

There is a continuum of infinitely-lived households, distributed uniformly on the unit interval $[0, 1]$. In each AM subperiod, a unit measure of new competitive bankers enters the economy, and they exit in the subsequent AM. Households are identical *ex ante*, but may differ *ex post*. Let $\{c_t(i), y_t(i)\} \in \mathbb{R}^2_+$ denote consumption and production of the PM good, respectively, at date t by agent i. Households discount utility payoffs across periods with the discount factor $0 < \beta < 1$; so that the preferences for household i are given by

[22]They can also be thought of *day good* and *night good*, respectively, as described in Andolfatto (2010). Provided that the two goods are unique and pertain to separate subperiods, the specific terminology used to label them is not critical to the analysis.

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(x_t(i)) - Ah_t(i) + \pi \left[u(c_t(i)) - g(y_t(i)) \right] \right\}. \tag{3.1}$$

At the beginning of the PM, each member of a household experiences an idiosyncratic shock that determines their types. Let the types be classified as *consumers*, *producers*, and *idlers*.[23] A member of a household can become a consumer or a producer with equal probability π, so that the probability of becoming an idler is $1 - 2\pi$. A consumer derives flow utility $u(c_t(i)) \in \mathbb{R}_+$ from consuming the PM good, where $u'' < 0 < u'$, and $u(0) = 0$, $u'(0) = \infty$. A producer derives flow utility $-g(y_t(i)) \in \mathbb{R}_+$ from producing the PM good, where $g(0) = g'(0) = 0$, $g' > 0$ for $y > 0$, and $g'' \geq 0$. The PM flow utility for idlers is normalized to zero. Since there is an equal measure of consumers and producers, feasibility and efficiency imply $c = y$.

In the AM subperiod, all households share identical preferences and opportunities. Their utility flow in the AM is given by $U(x_t(i)) - Ah_t(i)$, where $x_t(i) \in \mathbb{R}$ denotes the consumption of the AM good by individual i at date t, and $h_t(i)$ denotes their labor at date t. Assume that $U'' < 0 < U'$ with $U(0) = -\infty$ and $U'(0) = \infty$. The parameter A represents the relative emphasis households place on consumption versus labor in their utility preferences, a key factor that significantly influences the outcomes analyzed in this paper.

Bankers live for two periods, participate only in the AM, and consume only in old age. They are endowed with an investment technology. Households can consume the AM good, but they can also transfer these goods to young bankers. Young bankers can transform k units of the AM good into $f(k)$ units of the AM good in the next AM. The banker then consumes k in date $t+1$ when he becomes old. The aggregate resource constraint in the AM is given by

$$X_t + k_{t+1} \leq H_t + f(k_t), \tag{3.2}$$

where $X \equiv \int x_t(i)di$ and $H \equiv \int h_t(i)di$.

As the PM good is perishable, another aggregate resource constraint requires

$$\int c_t(i)di \leq \int y_t(i)di \tag{3.3}$$

[23] The idlers are inactive agents or nonparticipants who are intended to mimic the unmatched agents in Lagos and Wright (2005).

for all $t \geq 0$.

Consider a planner who weights all the agents equally and maximizes the *ex ante* utility of the agents with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(X_t) - AH_t + \pi \left[u(c_t(i)) - g(y_t(i)) \right] \right\} \tag{3.4}$$

subject to the aggregate resource constraints (3.2) and (3.3). The steady-state first-best allocation constitutes a set of numbers (X^*, k^*, y^*) satisfying:

$$U'(X^*) = A, \tag{3.5}$$

$$\beta f'(k^*) = 1, \tag{3.6}$$

$$u'(y^*) = g'(y^*). \tag{3.7}$$

Lemma 3.1 is directly derived from the results presented in equations (3.5) through (3.7).

Lemma 3.1. *X^* is strictly decreasing in A. k^* and y^* are determined independently of A.*

3.4 Agent Decision-making

I will impose restrictions on the environment that will render trade by credit to become infeasible, so that a medium of exchange is essential. A medium of exchange is essential in the sense that it will allow society to achieve desirable outcomes that could not be achieved in its absence. Firstly, I assume limited commitment among household members, contrasting with banks' ability to commit and enforce debt repayment. Limited commitment implies that all trade is voluntary, respecting sequential rationality. This leads to the absence of a lump-sum tax instrument, a point I will discuss later. Secondly, I assume household anonymity, which, combined with the first assumption, rule out private debt between households and makes a medium of exchange essential. Thirdly, I assume that households engage in a sequence of competitive spot market trades, exchanging bank deposits and interest-bearing CBDC for goods in both subperiods. In the following discussion, I will explore how bank deposits and CBDC, as voluntary payment instruments chosen by individual household members, possess distinct properties crucial in determining the welfare consequences of monetary policy.

3.4.1 Banker decision-making

I first consider the decision-making of bankers who derive utility from consuming the AM good in old age. All markets are assumed to be competitive. Each of the bankers possesses an investment technology that allows them to invest in k units of the AM good at date t. The banker then produces $f(k)$ units of the AM good at date $t+1$. Assume $f'' < 0 < f'$, $f'(0) = \infty$, and $f'(\infty) = 0$. The banker finances the investment by issuing deposits d and pays a gross real interest rate R^D to each member of a household who is using bank deposits for payment. Figure 3.1 presents the timeline for all agents.

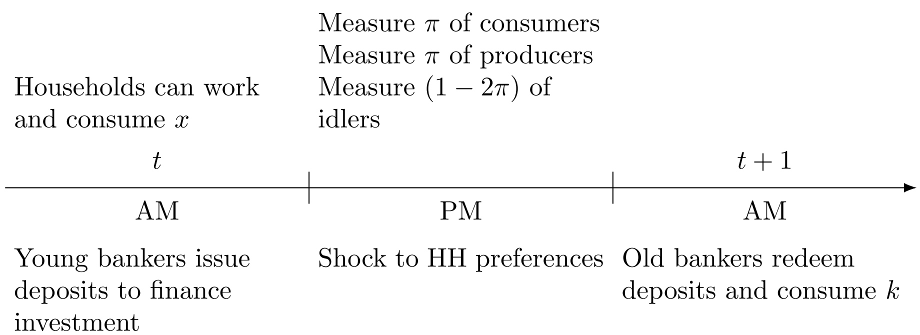

Figure 3.1: Timeline

Each banker takes the deposit price ϕ_1 in the AM as given and maximizes their profit

$$\max_{k} \quad \left\{ \phi_1 f(k) - R^D k \right\}. \tag{3.8}$$

The first-order condition is then given by

$$f'(k) = \frac{R^D}{\phi_1}. \tag{3.9}$$

Given that f is an increasing and strictly concave function, condition (3.9) implies that the demand for investment k is decreasing in R^D.

Lemma 3.2. *The bankers' demand for investment spending $k(R^D)$ is decreasing in the deposit rate R^D.*

3.4.2 Household-member decision-making

Members of a household use CBDC and bank deposits as payment instruments. Denote by $\{(v_1, v_2), (\phi_1, \phi_2)\}$ the price of CBDC and bank deposits in the AM and PM markets, respectively.

Government policy pertinent to household decision-making will be described in detail below. Here, I outline key policy elements that impact the choices of individual household members. The government's policy rule operates before the start of the AM-market trading. A household member enters the AM with money balances in the form of CBDC and bank deposits, denoted by z and a respectively. The individual then has the option to approach either a government or a bank counter to transform these balances into $R^M z - \tau$ or $R^D a + f(a)$ units of money, respectively. Household members also have to pay a labor income tax $\tau_h \in [0, 1)$ on their labor income wh, where h is an individual's labor supply and and w is the market price for leisure. If $R^M > 1$ and $\tau > 0$, then CBDC here is akin to an interest-bearing bond subject to a fixed fee, as in Andolfatto (2010).

If an individual member of a household decides not to use CBDC, he simply uses bank deposits. Subsequently he enters the AM-market with $R^D a + \phi_1 f(a)$ units of money in the form of bank deposits. In contrast to CBDC, bank deposits are partially illiquid financial instruments not issued by the government. The $f(a)$ component captures the illiquid aspect of bank deposits. I model this illiquidity aspect to capture the real-world diversity of bank deposits, differentiating between more liquid forms like checkable deposits and less liquid forms such as time deposits.[24]

After activities in the AM market, household members carry their CBDC and bank deposit balances into the PM market, where their roles as producers, consumers, or idlers are realized. Subsequently, following the PM market transactions, individuals retain their remaining CBDC and deposit balances, moving into the next AM. There, they are again faced with the choice between using CBDC or bank deposits, each

[24]For a more detailed exploration of the dynamics of various forms of exchange media, consider the insights offered in the studies by Chiu et al. (2023) and Wright (2010), which delve into the complexities and implications of different exchange mechanisms.

offering different interest rates.

3.4.2.1 The AM market

Denote by $(z, a) \geq 0$ a household member's CBDC and bank deposit balance, respectively, in the AM at date t; and denote by $(m, d) \geq 0$ the CBDC and bank deposit balance, respectively, that the individual household member carries forward into the PM market. Let $\sigma \in [0, 1]$ denote the probability of an individual household member exercising the CBDC interest vehicle option. This σ also represents the probability of paying the fixed CBDC fee. Depending on the decision to pay the fee or not, the individual household member can then purchase or sell output x at either the market price v_1 if selecting CBDC as currency, or the market price ϕ_1 if selecting bank deposits as the preferred currency of use for transactions. The AM-market budget constraint is then given by

$$x = (1 - \tau_h) wh + \sigma \left(R^M v_1 z - \tau\right) + (1 - \sigma)\phi_1 \left(R^D a + f(a)\right) - v_1 m - \phi_1 d. \quad (3.10)$$

A recursive representation of a household member's optimal choice problem is as follows. Let $W(z, a)$ denote the value function in the AM with CBDC and bank deposit balances, $(z, a) \geq 0$, respectively; and let $V(m, d)$ denote the value function in the PM before the household member realizes his type. The value functions $W(z, a)$ and $V(m, d)$ must satisfy the following recursive relationship:

$$\begin{aligned} W(z, a) \equiv \max_{x,h,\sigma,m,d} \quad & \{U(x) - Ah + V(m, d)\} \\ \text{s.t.} \quad & x = (1 - \tau_h) wh + \sigma \left(R^M v_1 z - \tau\right) \\ & + (1 - \sigma)\phi_1 \left(R^D a + f(a)\right) - v_1 m - \phi_1 d. \end{aligned} \quad (3.11)$$

Assuming that $V(m, d)$ is strictly concave, substituting out for h yields the following first-order conditions

$$U'(x) = \frac{A}{(1 - \tau_h) w}, \quad (3.12)$$

$$v_1 = \frac{(1 - \tau_h) w V_1(m, d)}{A}, \quad (3.13)$$

$$\phi_1 = \frac{(1-\tau_h)\, w V_2(m,d)}{A}. \tag{3.14}$$

The demand for both CBDC and bank deposits, respectively, is independent of a household member's initial CBDC and deposit holdings, z and a, respectively. This implies that all household members enter the PM market with identical CBDC and deposit holdings. By comparing conditions (3.5) and (3.12), it becomes evident that labor taxes introduce distortions that lead to overconsumption in the equilibrium compared to the optimum, as indicated by $x > x^*$. The optimal CBDC interest vehicle choice must satisfy

$$\sigma = \begin{cases} 1 \\ [0,1] \\ 0 \end{cases} \quad \text{if } R^M v_1 z - \tau \begin{cases} > \\ = \phi_1 \left(R^D a + f(a)\right), \\ < \end{cases} \tag{3.15}$$

so that the act of CBDC fee payment is sequentially rational if, and only if,

$$R^M v_1 z - \tau \geq \phi_1 \left(R^D a + f(a)\right). \tag{3.16}$$

For a given CBDC interest vehicle choice σ, by the envelope theorem:

$$W_1(z,a) = \frac{A \sigma R^M v_1}{(1-\tau_h)\, w}, \tag{3.17}$$

$$W_2(z,a) = \frac{A(1-\sigma)\, \phi_1 \left(R^D + f'(a)\right) \phi_1}{(1-\tau_h)\, w}. \tag{3.18}$$

Given the assumptions that $R^M > 1$ and $\tau > 0$, the function $W(z,a)$ is characterized as piece-wise linear and convex in z and a. Furthermore, $W(z,a)$ is non-differentiable at the point $R^M v_1 z - \tau = \phi_1 \left(R^D a + f(a)\right)$.

3.4.2.2 The PM market

After AM-market activity, the household member carries CBDC and deposit balances with him into the PM market. Just before entering the PM market, the individual experiences a stochastic shock, where he realizes he is a consumer, a producer, or an idler. Following PM-market activity, the individual carries any remaining CBDC and deposit balances forward to the next AM, where he once again decides whether to exercise the CBDC interest vehicle option. Let $V^C(m,d)$, $V^P(m,d)$, and $V^I(m,d)$ denote the utility value associated with being a consumer, a producer, and an idler, respectively. The ex ante value function associated with entering the PM market is given by

$$V(m,d) = \pi V^C(m,d) + \pi V^P(m,d) + (1-2\pi)\, V^I(m,d). \tag{3.19}$$

A consumer who enters the PM with a wealth portfolio (m,d) faces the budget constraint $c = v_2\,(m - z^+) + \phi_2\,(d - a^+)$. Substituting out for c, the choice problem can be stated as

$$V^C(m,d) \equiv \max_{z^+\geq 0, a^+ \geq 0} \left\{ u\left(v_2\left(m - z^+\right) + \phi_2\left(d - a^+\right)\right) + \beta W\left(z^+, a^+\right)\right\}. \tag{3.20}$$

In what follows, the consumer's debt-constraint $\{(z^+, a^+) \geq (0,0)\}$ will play an important role in the results below. It is also important to note that if $z^+ = 0$, then $a^+ = 0$, and conversely. The implication of this assumption is that a consumer returning to the AM-market will likely find it optimal to refrain from exercising the CBDC interest vehicle option, meaning they will not pay the CBDC fee. By making use of equations (3.17) and (3.18), the PM consumption is characterized by

$$\begin{aligned} v_2 u'(c) &= \tfrac{\beta A R^M v_1^+}{(1-\tau_h)w} \\ \phi_2 u'(c) &= \tfrac{\beta A (R^D + f'(a^+))\phi_1^+}{(1-\tau_h)w} \end{aligned} \qquad \text{if} \qquad v_2 m + \phi_2 d \geq c$$
$$c = v_2 m + \phi_2 d \qquad \text{otherwise} \tag{3.21}$$

By the envelope theorem:

$$V_1^C(m,d) = v_2 u'(c) \tag{3.22}$$

$$V_2^C(m,d) = \phi_2 u'(c) \tag{3.23}$$

A producer who enters the PM with a wealth portfolio (m,d) faces the budget constraint $y = v_2\,(z^+ - m) + \phi_2\,(a^+ - d)$. Substituting out for y, the choice problem can be stated as

$$V^P(m,d) \equiv \max_{z^+ \geq 0, a^+ \geq 0} \left\{ -g\left(v_2\left(z^+ - m\right) + \phi_2\left(a^+ - d\right)\right) + \beta W\left(z^+, a^+\right)\right\}. \quad (3.24)$$

Since a producer has no desire to consume, his debt-constraint is necessarily slack. Therefore, a producer must strictly prefer to exercise his CBDC interest vehicle option, meaning he will pay the CBDC fee the next AM. Utilizing equations (3.17) and (3.18), the PM production is characterized by

$$v_2 g'(y) = \frac{\beta A R^M v_1^+}{(1-\tau_h)w} \quad (3.25)$$

$$\phi_2 g'(y) = \frac{\beta A(R^D + f'(a^+))\phi_1^+}{(1-\tau_h)w} \quad (3.26)$$

Idle household members entering the PM market with a wealth portfolio (m, d) simply carry their CBDC and bank deposit balances forward to the next AM. Consequently, we have $V^I(m,d) \equiv \beta W(m,d)$. The envelope theorem yields the following equations, applicable to both idlers and producers:

$$V_1^P(m,d) = V_1^I(m,d) = v_2 g'(y) \quad (3.27)$$

$$V_2^P(m,d) = V_2^I(m,d) = \phi_2 g'(y) \quad (3.28)$$

As the choice of a preferred payment instrument also comes into question, I want to restrict attention to equilibria where both bank deposits and CBDC coexist. For this to occur, the following rate-of-return equality condition must be satisfied:

$$\frac{R^M v_1^+}{v_2} = \frac{\left(R^D + f'(a^+)\right)\phi_1^+}{\phi_2} \quad (3.29)$$

That is, for both assets to be accepted as payment, the expected rate of return on assets from the PM to the next AM must be the same. Consequently, at the individual level, portfolio composition becomes indeterminate in equilibrium.

3.5 Government Policy

I will now outline the government's policy. As a reminder, the government's operational approach involves intervening before AM-market trading begins. The policy entails offering a nominal interest rate of R^M on CBDC balances to household members willing to pay the fixed CBDC fee τ. Additionally, the government has the authority to impose labor income taxes, denoted as τ_h, on the labor earnings (wh) of individual household members, irrespective of their choice of currency.[25]

Let M^- denote the supply of outside money in the form of CBDC at the beginning of the AM-market (prior to any injection or withdrawal). Assume that this digital money supply grows at the constant (gross) rate $M = \mu M^-$, where M denotes the supply of digital money in the "next" period. Based on the assumptions, the initial CBDC supply M^- is entirely held by producers and idlers at the beginning of the AM. This is because both producers and idlers find it optimal to pay the CBDC fee τ. Consequently, the government bears an aggregate interest obligation of $(R^M - 1)M^-$, along with revenue from labor income tax $\tau_h wH$, and revenue from CBDC fee payments, $(1 - \pi)\tau$.

The government can also earn seigniorage revenue by printing new digital money $M - M^-$. Thus, a feasible government policy will have to satisfy the government budget constraint:

$$\underbrace{\left(R^M - 1\right) M^-}_{\text{Government spending}} = \underbrace{\tau_h wH}_{\text{Labor income tax revenue}} + \underbrace{(1 - \pi)\,\tau}_{\text{CBDC fee revenue}} + \underbrace{M - M^-}_{\text{Seigniorage revenue}} . \tag{3.30}$$

By defining $\delta \equiv {}^{R^M}\!/_{\mu}$ and rearranging the equation above, the government budget constraint may alternatively be expressed as:

$$\tau = \frac{(\delta - 1)M - \tau_h wH}{1 - \pi}. \tag{3.31}$$

Invoking the results derived from the aforementioned assumptions, which are established to be valid within this class of quasilinear models, the joint equilibrium distribution of CBDC and bank deposit holdings (z, a) will be massed over points:

[25] In this context, the approach to implementing a labor tax differs significantly from that presented in Rahman and Wang (2023). In their paper, τ_h can also be viewed as a sales tax.

$\{(0,0),(M,D),(2M,2D)\}$. This means that the mass π of PM consumers enter the AM with zero units of CBDC and deposit holdings, the mass $(1-2\pi)$ of PM idlers enter with (M,D) units of wealth, and the mass π of PM producers enter with $(2M,2D)$ units of wealth. Hence, an incentive-feasible government policy is one designed to ensure that both the fraction $(1-2\pi)$ of idlers and the fraction π of producers voluntarily pay the CBDC fee τ, while also satisfying (3.31) with the policy parameters (δ,τ,τ_h). It is worth noting that unlike the CBDC fee τ, the labor income tax τ_h will be voluntarily paid by all household members.

I define a *passive policy* as a government policy with the property $(\delta,\tau,\tau_h)=(1,0,0)$. In a passive policy, CBDC is non-interest bearing. Any policy that does not meet this criterion is referred to below as an *active policy*, where CBDC bears interest.

3.6 Stationary Monetary Equilibrium

In this section, I will examine the characteristics of stationary monetary equilibria under different currency regimes. These regimes include economies where only bank deposits serve as exchange media, economies where only CBDC is used, and economies where both bank deposits and CBDC coexist and compete as exchange media. My primary focus is on analyzing the properties of a stationary equilibrium where both CBDC and bank deposits coexist, given an incentive-feasible government policy. Briefly outlined, such equilibria must meet the following requirements: (i) Household and banker decisions are optimal; (ii) decisions are symmetric across all producers and consumers; (iii) markets clear at every date; and (iv) all real quantities remain constant over time.

3.6.1 A CBDC economy

Suppose that outside money, specifically CBDC, can only be used for payment in the PM. In particular, the rate of return on CBDC is higher than that on bank deposits. That is

$$\frac{R^M v_1^+}{v_2} > \frac{\left(R^D + f'(a^+)\right)\phi_1^+}{\phi_2}.$$

The market-clearing conditions for the money market are given by $m=M$ and $c=y$, as well as $v_2 = {}^{y}/_{M}$.

Gathering restrictions implied by individual behavior, I combine (3.13), (3.22), (3.27) to form

$$\frac{Av_1}{(1-\tau_h)w} = v_2 \left[\pi u'(c) + (1-\pi)g'(y)\right]. \tag{3.32}$$

Updating the latter expression by one period and combining with (3.25) yields

$$g'(y) = \beta R^M \left(\frac{v_2^+}{v_2}\right) \left[\pi u'(y^+) + (1-\pi)g'(y^+)\right]. \tag{3.33}$$

Restricting our attention to steady-state ($y = y^+ > 0$) it follows that $v_2^+/v_2 = v_1^+/v_1 = 1/\mu$. This then together with the market-clearing conditions yields

$$\beta\delta L(y) = 1, \tag{3.34}$$

where

$$L(y) = \frac{\pi u'(y) + (1-\pi)g'(y)}{g'(y)}. \tag{3.35}$$

Market clearing implies

$$v_2 M \geq y^* \text{ or } v_2 M = y < y^*. \tag{3.36}$$

Conditions (3.16), (3.34), and (3.36) characterize the monetary equilibrium as a function of parameters, contingent upon an incentive-feasible policy $\delta \equiv R^M/\mu$ in an economy with CBDC as the only medium of exchange. Note that $L'(y)$ can either increase or decrease with respect to y, and $L(y^*) = 1$. Furthermore, it should be emphasized that the "standard" Friedman rule, where $(R^M, \mu) = (1, \beta)$ or $\delta = 1/\beta$, is not incentive-feasible, as the CBDC fee is not voluntarily paid by every individual.

3.6.2 A bank credit economy

Now, let us consider the case where inside money, specifically bank deposits, can only be used for payment in the PM. Conversely, the condition below holds:

$$\frac{R^M v_1^+}{v_2} < \frac{\left(R^D + f'(a^+)\right)\phi_1^+}{\phi_2}.$$

This implies that the return on bank deposits is higher than that on CBDC. The market-clearing conditions for the deposit market are given by $d = D$ and $c = y$.

Gathering restrictions implied by individual behavior, I combine (3.14), (3.23), (3.28) to form

$$\frac{Av_1}{(1-\tau_h)w} = \phi_2\left[\pi u'(c) + (1-\pi)g'(y)\right]. \tag{3.37}$$

Updating the latter expression by one period and combining with (3.26) yields

$$g'(y) = \beta\left(R^D + f'(a^+)\right)\left(\frac{\phi_2^+}{\phi_2}\right)\left[\pi u'(y^+) + (1-\pi)g'(y^+)\right]. \tag{3.38}$$

Restricting our attention to steady-state ($y = y^+ > 0$, $a = a^+ = 0$) it follows that $\phi_2^+ = \phi_2 > 0$ and $\phi_1^+ = \phi_1 > 0$. This then together with the market-clearing conditions yields

$$\beta\left(R^D + f'(a)\right)L(y) = 1. \tag{3.39}$$

To solve for the AM price of bank deposits assume that $L(y^*) = 1$, so that there is zero-liquidity premium at the first-best allocation and that assets are efficiently priced at their "fundamental level". By combining the banker's first-order condition (3.9) with (3.39), we obtain:

$$\phi_1^* = \frac{\beta R^D}{1-\beta R^D} > 0, \tag{3.40}$$

which bears resemblance to the standard asset-pricing formula derived for risk-neutral agents. Note that ϕ_1 is increasing in the bank interest rate R^D. Furthermore, we need $1 < R^D < 1/\beta$ to satisfy $0 < \phi_1 < \infty$ for a bank credit equilibrium to exist.

To solve for the PM price of bank deposits, we can combine (3.26) and (3.40) to find:

$$\phi_2^* = \frac{\beta A R^D}{(1-\beta R^D)(1-\tau_h)wg'(y^*)} > 0. \tag{3.41}$$

An immediate observation from the above equation is the influence of labor income tax τ_h on deposit prices. Once again, market clearing implies

$$\phi_2^* D \geq y^* \text{ or } \phi_2 D = y < y^*. \tag{3.42}$$

Conditions (3.9), (3.39), (3.40), (3.41), and (3.42) constitute the key restrictions that characterize the general equilibrium allocation and price system in this competitive economy, where only bank deposits are used as the medium of exchange. If the debt-constraints are binding, bank deposits may become overvalued ($\phi_1 > \phi_1^* \implies \phi_2 > \phi_2^*$) due to a shortage in their supply. Consequently, this creates a liquidity premium on the price of bank deposits, leading to a lower expected rate of return. The following condition must hold for deposit prices to be overvalued with binding debt-constraints:

$$\frac{1}{\beta} > \frac{R^D(1+\phi_1)}{\phi_1} > 1.$$

3.6.3 A mixed CBDC and bank credit economy

I now consider an economy where CBDC and bank deposits coexist as payment instruments, with condition (3.29) being satisfied. Market clearing now implies

$$v_2 M + \phi_2^* D \geq y^* \text{ or } v_2 M + \phi_2 D = y < y^*. \tag{3.43}$$

To confirm whether the conjecture made regarding (3.43) holds in equilibrium, we can use (3.21) to derive:

$$A \geq \frac{(1-\tau_h) w g'(y^*) y^*}{\beta \left[\delta v_1 M + (R^D + f'(a))\phi_1^* D\right]}. \tag{3.44}$$

Since we can use (3.40) to derive an equilibrium value for $\phi_1 > 0$, then all we require is any value $v_1 < \infty$ satisfying (3.44) to obtain an equilibrium in which both CBDC and bank deposits coexist. In economies with a sufficient level of A, multiple assets can coexist and hold value.

To derive the consumption allocation across household types in each AM, we can combine the joint steady-state distribution of wealth above with the household budget constraint (3.10). For those who were consumers in the previous PM, $\sigma = 0$ and $(z^+, a^+) = (z, a) = (0, 0)$, so:

$$x = (1-\tau_h)\, wh - v_1 M - \phi_1 D. \tag{3.45}$$

For those who were idlers in the previous PM, $\sigma = 1$ and $(z^+, a^+) = (z, a) =$

(M, D), so:

$$x = (1 - \tau_h)\, wh + (R^M - 1)v_1 M - \tau - \phi_1 D. \tag{3.46}$$

For those who were producers in the previous PM, $\sigma = 1$ and $(z^+, a^+) = (z, a) = (2M, 2D)$, so:

$$x = (1 - \tau_h)\, wh + (2R^M - 1)v_1 M - \tau - \phi_1 D. \tag{3.47}$$

We can easily verify that the population-weighted sum of (3.45), (3.46), and (3.47) is nonzero.[26]

Recall that we want to restrict our attention to incentive schemes that satisfy (3.16), ensuring that producers strictly prefer to pay the CBDC fee, while idle household members weakly prefer to do so. Consider that idlers will find it optimal to pay the CBDC fee the next AM. In equilibrium, idlers enter the AM market with wealth $(z, a) = (M, D)$. By appealing to (3.31), the CBDC fee constraint $R^M v_1 z - \tau \geq \phi_1 \left(R^D a + f(a)\right)$ can be written as follows:

$$(1 - \pi)\left[R^M v_1 M - \phi_1(R^D D + f(D))\right] + \tau_h w H \geq (\delta - 1)M. \tag{3.48}$$

In this class of models, it is crucial to consider the sequential participation of a consumer who enters the AM market with zero CBDC and deposit balances. To accumulate wealth, this agent must exert significant effort, sacrificing transferable utility. Let $W(0, 0)$ denote the payoff for rebalancing asset holdings in the AM on the equilibrium path. If the cost or sacrifice of rebalancing is excessively high, an individual household member would forego that opportunity and enter the next PM with zero CBDC and deposit holdings. However, the individual can still consume in the AM and opt to work in the PM market. If he chooses to work in the PM, then he takes his CBDC and deposit holdings and spend them in the AM. Along this alternative path, the individual never consumes in the PM. Let $\widehat{W}(0, 0)$ represent the payoff of this alternate strategy. Then, sequential rationality must satisfy:

$$W(0, 0) \geq \widehat{W}(0, 0). \tag{3.49}$$

[26] Although the process may appear similar, the result differs significantly from that of Andolfatto (2010).

Another constraint that must be met is that producers in the PM market must be willing to produce good y for (z, a) units of money. It is straightforward to verify that this constraint is satisfied in equilibrium.

Condition (3.43) and (3.49), along with conditions (3.9), (3.16), (3.34), (3.39), (3.40), and (3.41) derived earlier, constitute the key restrictions characterizing the general equilibrium allocation and price system in an economy where both CBDC and bank deposits hold value.

In the next section, I will examine the various policies required for first-best implementation in a mixed CBDC and credit economy.

3.7 Optimal Policies

I now study the implementation of first-best allocation in a mixed CBDC and bank credit economy. Depending on parameters, the level of output may or may not be efficient. The goal is to identify the policies that are required for a first-best implementation.

3.7.1 Passive policy

I first determine the conditions under which a passive policy $(\delta, \tau, \tau_h) = (1, 0, 0)$ is optimal. With a passive policy where CBDC is non-interest bearing, $\sigma = 1$ holds trivially for all household members. Hence, the CBDC fee constraint (3.48) can be ignored. The government budget constraint (3.31) will also be satisfied, so the passive policy is trivially an incentive-feasible policy. The question is whether the consumer's debt constraint (3.21) will bind or not. From (3.44), if the consumer debt constraint is slack, then the conjecture that needs to be satisfied is the following:

$$A \geq \frac{wg'(y^*)y^*}{\beta\left[v_1 M + (R^D + f'(a))\phi_1^* D\right]}. \tag{3.50}$$

Proposition 3.1. *Under a passive government policy* $(\delta, \tau, \tau_h) = (1, 0, 0)$ *with non-interest bearing CBDC, there exists a unique* $0 < A_0 < \infty$ *that satisfies* $v_2 M + \phi_2 D = y^*$. *For economies with* $A \geq A_0$*, the competitive monetary equilibrium is efficient. That is,* (3.40) *and* (3.41) *hold, as well as*

$$v_2 M + \phi_2 D \geq y^*. \tag{3.51}$$

For economies with $0 < A < A_0$, the competitive monetary equilibrium is inefficient. That is,

$$v_2 M + \phi_2 D < y^*, \tag{3.52}$$

$$\phi_1 > \phi_1^*, \tag{3.53}$$

$$\phi_2 > \phi_2^*. \tag{3.54}$$

According to Proposition 3.1, if $A \geq A_0$, the household's preference for the AM good is sufficiently high for the AM good to be produced at its efficient level, as there are no distortionary effects from labor income tax. As the real rate of return on money with a non-interest bearing CBDC is high enough, the debt constraint for consumers remains slack. The prices of CBDC and bank deposits are at their fundamental level. Bankers will issue deposits and pay an interest rate R^D that is sufficiently high, as household members are willing to sacrifice enough of their consumption for labor to produce the AM good. Furthermore, producers will be willing to produce the first-best amount of goods in the PM.

On the other hand, if $A < A_0$, the household's preference for the AM good is too low for the AM good to be produced efficiently, even without distortionary effects from labor income tax. The assets are priced inefficiently due to a shortage of liquidity. Bankers issue fewer deposits and offer an excessively high interest rate R^D, which results in a liquidity premium of deposit prices. Consequently, binding consumer debt constraints will lead to the producers delivering an inefficient level of PM goods, and consumption in the PM is too small.

Proposition 3.1 is related to the nonmonetary and monetary equilibria discussed in Lagos and Rocheteau (2008) and Andolfatto et al. (2016). Lagos and Rocheteau (2008) find that if the capital stock is small, there is an overaccumulation of capital in the equilibrium, leading to lower consumption in the PM. Conversely, Andolfatto et al. (2016) assume an efficient capital stock, resulting in an overvaluation of money (backed by the capital stock) in equilibrium and lower consumption in the PM. Here, based on Lemma 3.1, since the AM good X is strictly decreasing in A, if $A < A_0$, this results in an overproduction of the AM good, meaning $X > X^*$. This occurs because with a higher interest rate R^D, there is a lower demand for the AM good from the bankers via the investment channel, as outlined in Lemma 3.2. Consequently, there will be lower consumption of the PM good with $y < y^*$. Finally, the liquidity premium of deposits when the debt constraints are binding, that is, when $A < A_0$, is also similar to that of Keister and Sanches (2023), except for the fact that their result depends on the parameter β instead of A here.

When $A < A_0$, policies need to be necessarily inflationary, that is, $\delta > 1$ ($R^M > 1, \mu > 1$), to restore efficiency. Moreover, the private banks will have to lower R^D to not reduce their investment demand.

3.7.2 Active policies

If first-best implementation under a passive policy is infeasible for economies with low A, then our only recourse is a range of active policies necessitating strictly positive inflation and nominal interest rates. This implies an interest-bearing CBDC is necessary to improve welfare. I now focus solely on economies where $A < A_0$, specifically in regions of the parameter space where a passive policy fails to implement the first-best allocation. This is because the real rate of return is too low to motivate bankers to issue deposits and producers to supply the first-best level of output. The government has three instruments to finance its CBDC interest obligation: seigniorage, labor income tax revenue, and CBDC fee revenue. Below, I restrict attention to policies that satisfy $1 < \delta < 1/\beta$; since otherwise, a monetary equilibrium will fail to exist, except in the limiting case of $\delta \nearrow \beta^{-1}$.

Case 1: $1 < \delta < 1/\beta$, $\tau = 0$, *and* $\tau_h > 0$. In what follows, I ask whether seigniorage and labor income tax revenue help to implement the first-best allocation in the absence of CBDC fees. Note that from (3.44) it is easy to verify that A is decreasing in both δ and τ_h. As $A < A_0$, we will have

$$A_0 \equiv \frac{(1-\tau_h)wg'(y_0)y_0}{\beta\left[\delta v_1 M + (R^D + f'(a))\phi_1 D\right]}. \tag{3.55}$$

We simply need to reduce the right-hand side of condition (3.55) to relax the debt constraint for consumers, ensuring that (3.44) holds. This can be accomplished through a strictly inflationary policy and a strictly positive labor income tax. Since A is decreasing in both δ and τ_h, there exists a unique critical value $0 < A_1 < A_0$ so that the consumer debt constraint is slack. Note that since the labor income tax is distortionary (see condition (3.12)), AM consumption in the equilibrium is lower than in the optima. Unlike Andolfatto et al. (2016) and Andolfatto (2010), money injected in this manner is not superneutral when it is introduced in the form of interest. This is because seigniorage and labor income tax expand the set of economies that can attain the first-best allocation.

Case 2: $1 < \delta < 1/\beta$, $\tau > 0$, *and* $\tau_h = 0$. In the absence of labor income tax with $\tau_h = 0$, I want to now show how CBDC expenditures can be financed by seigniorage and CBDC fees to improve the allocation. For this instance, we have

$$A_0 \equiv \frac{wg'(y_0)y_0}{\beta\left[\delta v_1 M + (R^D + f'(a))\phi_1 D\right]}. \tag{3.56}$$

The following lemma characterizes the optimal CBDC fee necessary to implement the first-best allocation.

Lemma 3.3. *For economies with* $A_1 \leq A < A_0$*, the optimal CBDC fee* τ *that can be attained to implement the first-best allocation is*

$$\tau^*(\delta, R^D, A) = (1-\pi)^{-1}\left\{(\delta-1)M + \frac{\beta AH\left[\delta v_1 M(1-\beta R^D) + R^D D\right]}{(1-\beta R^D)g'(y^*)y^*} - wH\right\}. \tag{3.57}$$

$\tau^*(\delta, R^D, A)$ *is increasing in* A*.* $\tau^*(\delta, R^D, A)$ *is increasing in* δ *and the effect of* R^D *on* $\tau^*(\delta, R^D, A)$ *is ambiguous.*

Proof. To derive $\tau^*(\delta, R^D, A)$, set $A = wg'(y^*)y^*\left\{\beta\left[\delta v_1 M + (R^D + f'(a))\phi_1 D\right]\right\}^{-1}$ in (3.50) to obtain

$$\tau_h = 1 - \frac{A\beta[\delta v_1 M + (R^D + f'(a))\phi_1^* D]}{wg'(y^*)y^*}.$$

Use the government budget constraint (3.31) to substitute out τ_h to obtain

$$\tau = (1-\pi)^{-1}\left\{(\delta-1)M + \frac{\beta AH[\delta v_1 M + (R^D + f'(a)\phi_1^*)D]}{g'(y^*)y^*} - wH\right\}.$$

Use (3.40) to substitute ϕ_1^* and simplify to obtain (3.57). Differentiating (3.57) with respect to A leads to

$$\frac{\partial \tau^*(\delta, R^D, A)}{\partial A} = \frac{\beta H(1-\pi)^{-1}[\delta v_1 M(1-\beta R^D) + R^D D]}{(1-\beta R^D)g'(y^*)y^*} > 0.$$

Now, differentiating (3.57) with respect to δ leads to

$$\frac{\partial \tau^*(\delta, R^D, A)}{\partial \delta} = (1-\pi)^{-1}M + \frac{\beta A H(1-\beta R^D)(1-\pi)^{-1} v_1 M}{(1-\beta R^D)g'(y^*)y^*} > 0.$$

Finally, differentiating (3.57) with respect to R^D gives us

$$\frac{\partial \tau^*(\delta, R^D, A)}{\partial R^D} = \frac{(1-\beta R^D)g'(y^*)y^*[\beta A H(1-\pi)^{-1}D - \beta^2 A(1-\pi)^{-1}\delta v_1 M]}{[(1-\beta R^D)g'(y^*)y^*]^2} + \frac{\beta^2 A H(1-\pi)^{-1}\left[\delta v_1 M(1-\beta R^D) + R^D D\right] g'(y^*)y^*}{[(1-\beta R^D)g'(y^*)y^*]^2} \lesseqgtr 0.$$

■

Unlike in Andolfatto et al. (2016), the CBDC fee $\tau(\delta, R^D, A)$ here is increasing in A. Although the interpretation of A here and in their paper is similar, A enters slightly differently in my model. The CBDC fee is also not independent of inflation μ and interest rates R^M and R^D. As δ (the ratio of nominal interest on CBDC to money growth rate) increases, the CBDC fee also increases.

The positive effect of R^M on τ^* suggests that when the government pays a higher interest rate on CBDC, it can afford to charge a higher CBDC fee without significantly reducing the demand for CBDC. This is because the higher interest rate on CBDC compensates households for higher CBDC fees, maintaining the overall attractiveness of CBDC as a payment instrument. The effect of R^D on τ^* can potentially induce both income and substitution effects. If R^D increases, bank deposits become relatively more attractive compared to CBDC, prompting the government to potentially reduce CBDC fees to incentivize households to use CBDC instead of bank deposits (substitution effect). Conversely, as households earn more interest income from their bank deposits, they may have additional disposable income to spend on transaction fees. Consequently, the government could potentially charge a higher CBDC fee without significantly discouraging CBDC use (income effect). Ultimately, the sign of $\partial \tau^* / \partial R^D$

will depend on which effect dominates. However, for $A \geq A_0$, no CBDC fee income and inflation are necessary to implement the first-best allocation.

Since coercion is ruled out and all trade must be voluntary, it is necessary to determine the conditions under which the CBDC fee τ^* can be collected through voluntary contributions. Specifically, the CBDC fee τ^* must satisfy the CBDC fee constraint given by equation (3.16). This constraint ensures that the fee is set at a level that incentivizes agents to participate in the CBDC system voluntarily, without the need for coercion or forced participation.

It is also crucial to consider how CBDC could potentially displace bank deposits in an environment with strictly positive inflation and positive nominal interest rates. If $R^M > 1$, maintaining the rate-of-return equality condition (3.29) requires an increase in R^D. However, as Lemma 3.2 suggests, an increase in R^D would lead to a decline in investment demand. This is a channel through which bank deposits could be crowded out by CBDC, even though sequential rationality is respected.

Case 3: $1 < \delta < 1/\beta$, $\tau > 0$, *and* $\tau_h > 0$. Suppose now that $A < A_1$, indicating that the government cannot implement the first-best allocation exclusively with either labor income tax (and seigniorage) as in *Case 1*, or with a fixed fee (and seigniorage) as in *Case 2*. With further restrictions in the economy, the government will have to utilize all the available policy instruments at its disposal. This comes at the expense of giving up more degrees of freedom than desired.

Note that with both $\tau_h > 0$ and $\tau > 0$, the CBDC fee constraint (3.48) needs to be satisfied to induce voluntary participation. Rearranging (3.48) further, we can obtain the expression

$$\tau_h \geq \frac{(1-\pi)(R^D + f(D)) - [(1-\pi)R^M v_1 + 1 - \delta]M}{wH}. \tag{3.58}$$

Condition (3.58) is an expression for the minimum labor income tax when the government uses all the policy tools available in a constrained equilibrium. A higher CBDC fee and labor income tax relax the CBDC fee constraint, which is a channel through which households can relax their debt constraint. Since $A < A_1$ and A decreases with both δ and τ_h, there exists a unique critical value $0 < A_2 < A_1 < A_0$ that will relax the consumer debt constraint. The following proposition identifies the regions of the parameter space in which active government policies alone, within a constrained CBDC and credit equilibrium, are sufficient for achieving the first-best allocation.

Proposition 3.2. *If $A_0 > A \geq A_1$, the first-best allocation can be implemented with $1 < \delta < 1/\beta$, $\tau = 0$, and $\tau_h > 0$, or with $1 < \delta < 1/\beta$, $\tau > 0$, and $\tau_h = 0$ where CBDC is interest-bearing. If $A_1 > A \geq A_2$, implementing the first-best allocation requires $1 < \delta < 1/\beta$, $\tau > 0$, and $\tau_h > 0$. Price stability is achieved with $\phi_1 = \phi_1^*$ and $\phi_2 = \phi_2^*$ satisfying* (3.40), (3.41), *and* (3.50).

Propositions 3.1 and 3.2 identify three regions of the parameter space with different rates of returns on CBDCs. For $A \geq A_0$, a passive policy ($(\delta, \tau, \tau_h) = (1, 0, 0)$) is sufficient to deliver a first-best allocation where CBDC bears zero interest. There is adequate liquidity provisioning in the economy, so having a constant supply of CBDC is enough to induce household members to work hard and produce the PM good at the first-best level. Bankers will issue deposits and pay an interest rate R^D high enough so that household members sacrifice enough of their consumption to produce the AM good. Moreover, all trade is voluntary among agents as sequential rationality is respected. CBDC in this case is non-interest bearing with $R^M = 1$ and can be viewed as digital cash with which agents cannot hide their money balances. The unbacked nature of this form of government-issued fiat money over private money (bank deposits) is an advantage. There is no disintermediation in the banking system in this case as the rate-of-return equality condition (3.29) holds.

For $A_0 > A \geq A_1$, the constrained-efficient policy entails positive inflation and non-zero nominal interest rates, as a constant supply of CBDC is insufficient to restore efficiency. In this case, the government will require a combination of seigniorage with labor income tax ($1 < \delta < 1/\beta$, $\tau = 0$, and $\tau_h > 0$) or a combination of seigniorage with fee income ($1 < \delta < 1/\beta$, $\tau > 0$, and $\tau_h = 0$) to induce producers to produce an efficient level of PM goods and for bankers to issue enough deposits. CBDC in this case must be interest-bearing to deliver efficiency. However, CBDC could crowd out bank deposits if R^M is too high relative to R^D and decrease investment. For $A_1 > A \geq A_2$, the constrained-efficient policy requires the government to use both labor income tax and fee income (and seigniorage) to induce producers to produce y^* and for bankers to issue deposits k^*. Any optimal policy in this region must have a CBDC that bears interest. In the equilibrium, however, there will be an overproduction of the AM good X^* due to the distortionary effects of labor income tax. The optimal CBDC fee income and labor income tax through voluntary contributions are incentive-feasible. Strictly positive inflation and nominal interest rates encourage households to use CBDC and pay its associated fees, as it relaxes their CBDC fee constraint so that their debt constraint does not bind. Disintermediation of banks could still occur with this policy as it is strictly inflationary with strictly positive interest rates.

Finally, it is worth pointing out that achieving the first-best allocation is impossible under deflation within any parameter space, in contrast to Andolfatto et al. (2016). The government policy here mirrors that of Andolfatto (2010), where an incentive-feasible policy precludes deflation. Unlike in Andolfatto et al. (2016), there is no dividend fee income from holding assets to encourage voluntary contributions for a small CBDC fee. Additionally, in Andolfatto et al. (2016), there are no competing payment instruments; however, in this model, both CBDC and bank deposits compete as exchange media, and their choice must adhere to individual rationality. Thus, positive inflation and non-zero nominal interest rates are always necessary in more impatient economies when CBDC and bank deposits compete as exchange media, as deflation is infeasible.

3.8 Quantitative Analysis

In this section, I calibrate the theoretical model to the US economy in order to assess the impact of introducing different versions of a CBDC. As noted earlier, the CBDC in my paper can either be non-interest-bearing or interest-bearing.

3.8.1 Calibration

For the quantitative exercise, I consider an annual model with utility functions $U(y) = B\log(y+\epsilon)$ in the AM and $u(c) = ((c+\epsilon)^{1-\chi} - 1)/(1-\chi)$ in the PM, PM disutility $g(y) = y^{1+\gamma}/(1+\gamma)$, and production technology $f(k) = 2\sqrt{k}$, where ϵ is a utility normalization parameter set to 0.001.

The calibration procedure consists of two parts, involving the calibration of ten parameters: $(A,\ B,\ \beta,\ \chi,\ \gamma,\ \mu,\ \pi,\ R^M,\ \tau,\ \tau_h)$. In the first step, some parameters are set directly. These include the discount factor, money growth rate, nominal interest rate, labor income tax, CBDC fee, and preference shock parameter. Following the literature, I set $\beta = 0.97$ and choose a money growth rate of $\mu = 1.02$, which aligns with the historical long-term inflation rate in the United States. The nominal interest rate on CBDC, $R^M = 1.021$, is set to reflect the FRED data on the Interest Rate on Excess Reserves (IOER) in 2019, ensuring that $\delta > 1$. The labor income tax, $\tau_h = 0.145$, reflects the average income tax rate in 2021. Second, I jointly calibrate four parameters (A, B, χ, γ) to match the empirical money demand curve using data from 1987 to 2008. I use the new M1 series from Lucas and Nicolini (2015), excluding the post-crisis period when M1 demand rose sharply due to non-transactional motives. The calibration employs a grid search followed by numerical optimization to minimize a weighted objective function that combines two criteria: the root mean squared error between model-predicted and actual M1/GDP ratios, and the difference between their averages. Table 3.1 summarizes all the parameter values along with their calibration targets. Figure 3.2 shows the predicted money demand curve from the model compared to the actual data from 1987 to 2008.

Parameters	Notation	Value	Calibration Targets
Calibrated externally			
Discount factor	β	0.97	Standard in literature
Money growth rate	μ	1.02	2% inflation
Nominal interest rate	R^M	1.021	2.1% IOER rate
Labor income tax	τ_h	0.145	14.5% average income tax rate
CBDC fee	τ	0.01	Set directly
Preference shock	π	0.55	Set directly
Calibrated internally			
PM utility curvature	χ	0.30	Money demand curve
AM utility parameter	B	5.34	Money demand curve
Production curvature	γ	10.00	Money demand curve
AM preference	A	1.003	Money demand curve

Table 3.1: Calibration Results

The model captures the downward-sloping relationship between the M1 to GDP ratio and the 3-month T-bill rate. The model reasonably approximates the data, with the average money demand broadly consistent with empirical observations. The efficiency measures indicate that the calibrated economy exhibits slight overconsumption in both markets while showing some underinvestment relative to first-best. Using

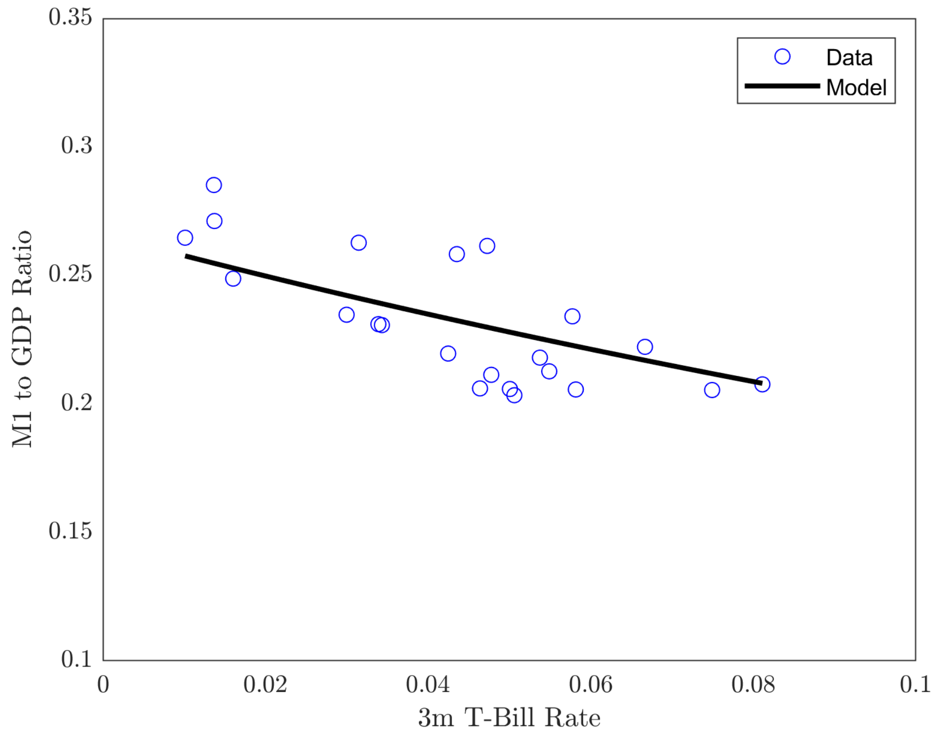

Figure 3.2: Money Demand Curve: Model vs Data

these calibrated parameters, I analyze the quantitative effects of introducing different versions of CBDC policy.

3.8.2 Effects of an interest-bearing and non-interest-bearing CBDCs

Taking the economy without interest-bearing CBDC ($\delta = 1$) as my benchmark, I find several key effects when introducing CBDC policy. Figure 3.3 shows the results. First, AM consumption remains essentially unchanged, consistent with the theoretical prediction that AM consumption is determined by labor-leisure tradeoffs and is largely insulated from monetary policy.

PM production shows meaningful responses to CBDC policy, increasing by approximately 0.50% (from 0.99 to about 1.0) , as δ rises to 1.03. This aligns with the theoretical model where monetary policy affects PM market efficiency through the interest rate channel. More specifically, interest-bearing CBDC improves the returns on money holdings, which affects agents' PM consumption-production decisions.

My model generates significant bank disintermediation effects, with bank deposits declining by approximately 3% (from 1.01 to 0.98) as δ increases to 1.03. This result is replicated when examining the effects through R^M, where deposits decline by approximately 5%, (from 1.03 to 0.98), as the interest rate on CBDC, R^M, rises from 0% to 5%. This supports the theoretical prediction that as CBDC becomes more attractive, agents substitute away from traditional bank deposits. However, despite this disintermediation effect, I find that overall welfare improves from 2.48 to 2.51, representing about a 1.2% increase. This result suggests that in sufficiently impatient economies, active CBDC policies with positive interest rates can enhance welfare even though they may reduce bank intermediation.

These findings quantify an important policy tradeoff in my theoretical model: while interest-bearing CBDC can improve welfare through enhanced payment efficiency in the PM market, it comes at the cost of reduced bank deposits. My calibrated results suggest that the efficiency gains outweigh the costs of disintermediation within the policy range considered, whether we examine the effects through δ or R^M.

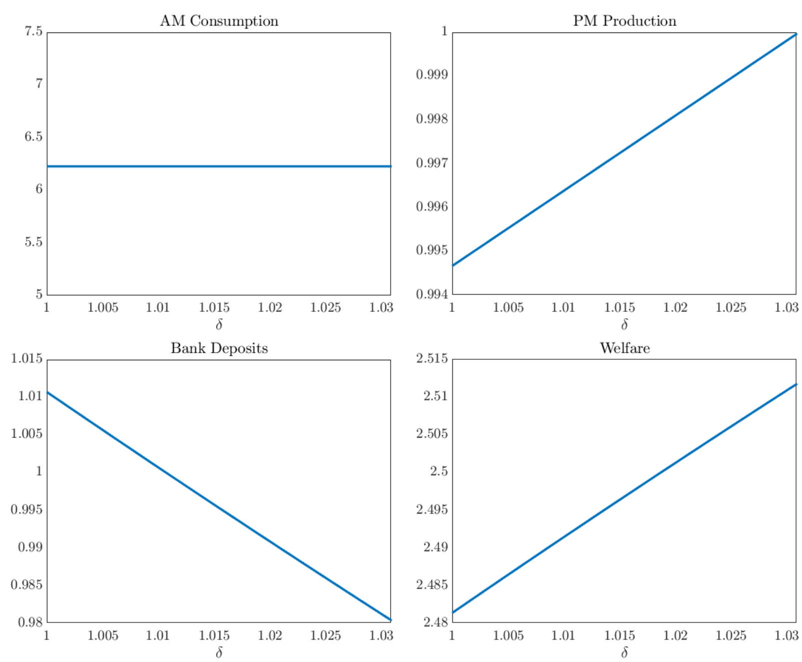

Figure 3.3: Effects of Interest-bearing CBDC

3.8.3 Welfare costs of active and passive policies

To quantify the welfare implications of moving from passive to active policy, I compare the welfare under the benchmark passive policy ($\delta = 1$) with active policies where $\delta > 1$. Figure 3.4 shows the results. I find that increasing δ from 1 to 1.05 leads to a welfare gain of 2% (as welfare increases from 2.48 to 2.53).

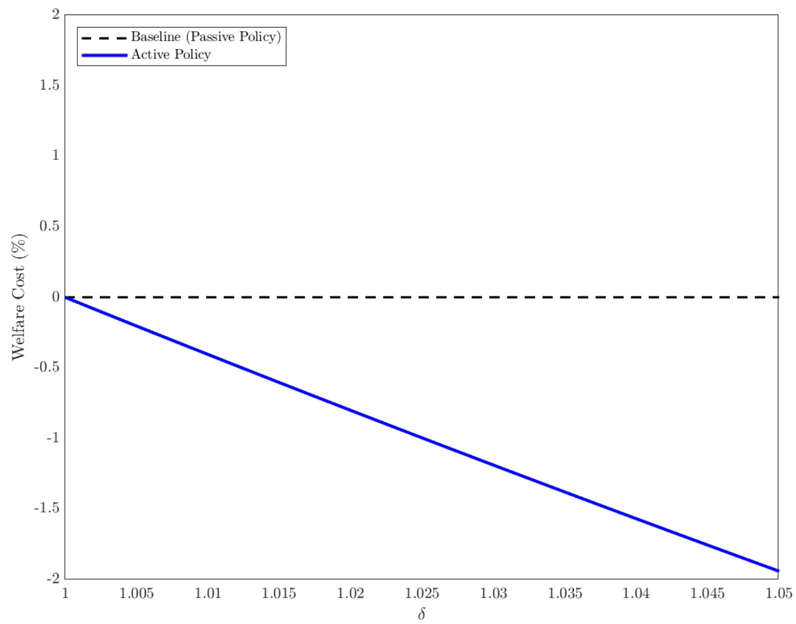

Figure 3.4: The Welfare Cost of CBDC

This welfare improvement can be decomposed into several effects. First, under the active policy, AM consumption remains stable. Second, PM production increases by a small margin, indicating improved production efficiency. However, bank deposits decrease from 1.01 to 0.96, reflecting a significant crowding out of bank intermediation of approximately 5%.

The overall welfare gain despite lower bank deposits suggests that the benefits from improved PM production efficiency outweigh the costs of reduced bank intermediation. This finding quantitatively supports the theoretical prediction that active policy can be welfare-improving even when it leads to some disintermediation of the banking sector. The magnitude of these effects suggests that the tradeoff between efficiency gains and banking sector stability should be a key consideration in CBDC policy design.

3.9 Conclusion

This paper examines how information frictions in asset-backed private currencies create liquidity shortages and investigates the role of interest-bearing central bank money in addressing these challenges. The analysis reveals that while private currencies can facilitate intertemporal exchange, they become vulnerable to excessive volatility when backed by productive assets subject to news shocks. In the model, banks issue deposits backed by firm output as collateral. Although news about firm productivity carries no social value, adverse information can trigger binding debt constraints that generate liquidity shortages and depress economic activity.

Three key results emerge from this analysis. First, banking intermediation amplifies the transmission of information-driven liquidity shocks compared to economies where agents hold productive assets directly. Banks' dual role as currency issuers and asset holders creates a channel through which productivity news affects deposit values and overall liquidity provision. Second, interest-bearing money provides an effective policy tool for central banks to address private currency instability. Unlike traditional monetary policy that operates through money growth, interest-bearing money influences asset prices through an investment channel, allowing banks to use reserves as insurance against productivity shocks. This mechanism enables welfare-improving policies that require positive inflation and positive nominal interest rates—departing from the conventional Friedman rule.

Third, the coexistence of government and private currencies is possible even under private information about household consumption needs. Illiquid bonds serve as a sorting mechanism that enables efficient allocation when type-contingent transfers are infeasible. My analysis shows how in a news economy, illiquid bonds may play a crucial role in reallocating money towards consumers with different marginal utility of consumption. Remarkably, imposing cash-in-advance constraints on bank deposits can improve welfare by preventing destabilizing arbitrage that amplifies news-driven volatility.

These findings carry important implications for contemporary monetary policy, particularly as central banks worldwide explore central bank digital currencies (CBDCs). The analysis suggests that interest-bearing government money can effectively compete with private bank deposits while providing additional monetary policy flexibility. However, the success of such policies depends critically on maintaining credible low inflation rates and carefully designed institutional frameworks.

To a degree, the adverse impact of private currencies documented here might explain the historical shift toward government-issued fiat money. While private banks can employ techniques like asset tranching or information nondisclosure to mitigate volatility, these approaches face limitations when asset scarcity exists. Central banks can overcome these constraints by providing high-quality debt instruments that are insensitive to productivity shocks.

The framework developed here opens several avenues for future research. One extension could examine bank default and deposit insurance, where banks pay premiums to protect depositors against institutional failure. Incorporating equity finance following Dermine (1986) would add another dimension to firm financing and bank portfolio choices. Another promising direction involves analyzing scenarios where banks strategically choose riskier projects to access cheaper funding, potentially creating systemic risks.

Examining the implications of market power in banking, particularly in the presence of information frictions, represents another important research frontier. Additionally, empirical work testing the model's predictions about the relationship between news shocks, banking sector stability, and monetary policy effectiveness would provide valuable insights for policymakers designing digital currency frameworks.

Ultimately, this paper demonstrates that the design of monetary systems must carefully balance the efficiency gains from private sector innovation against the stability concerns that arise when currencies are backed by volatile productive assets. As financial technology continues to evolve, understanding these trade-offs becomes increasingly crucial for maintaining stable and efficient payment systems.

BIBLIOGRAPHY

Agur, I., Ari, A., and Dell'Ariccia, G. (2022). Designing central bank digital currencies. *Journal of Monetary Economics*, 125:62–79.

Amendola, N., Carbonari, L., Ferraris, L., et al. (2021). Three liquid assets. Technical report, Tor Vergata University, CEIS.

Andolfatto, D. (2010). Essential interest-bearing money. *Journal of Economic theory*, 145(4):1495–1507.

Andolfatto, D. (2011). A note on the societal benefits of illiquid bonds. *Canadian Journal of Economics/Revue canadienne d'économique*, 44(1):133–147.

Andolfatto, D. (2013). Incentive-feasible deflation. *Journal of Monetary Economics*, 60(4):383–390.

Andolfatto, D. (2021). Assessing the impact of central bank digital currency on private banks. *The Economic Journal*, 131(634):525–540.

Andolfatto, D., Berentsen, A., and Waller, C. (2014). Optimal disclosure policy and undue diligence. *Journal of Economic Theory*, 149:128–152.

Andolfatto, D., Berentsen, A., and Waller, C. (2016). Monetary policy with asset-backed money. *Journal of Economic Theory*, 164:166–186.

Andolfatto, D. and Martin, F. M. (2009). Money and capital as competing media of exchange in a news economy. *Federal Reserve Bank of St. Louis Working Paper*.

Andolfatto, D. and Martin, F. M. (2013). Information disclosure and exchange media. *Review of Economic Dynamics*, 16(3):527–539.

Aruoba, S. B. (2021). Institutions, tax evasion, and optimal policy. *Journal of Monetary Economics*, 118:212–229.

Auer, R., Cornelli, G., and Frost, J. (2020). Rise of the central bank digital currencies: drivers, approaches and technologies.

Bajaj, A. and Damodaran, N. (2022). Consumer payment choice and the heterogeneous impact of india's demonetization. *Journal of Economic Dynamics and Control*, page 104329.

Barrdear, J. and Kumhof, M. (2021). The macroeconomics of central bank digital currencies. *Journal of Economic Dynamics and Control*, page 104148.

Barrdear, J. and Kumhof, M. (2022). The macroeconomics of central bank digital currencies. *Journal of Economic Dynamics and Control*, 142:104148.

Berentsen, A., Camera, G., and Waller, C. (2007). Money, credit and banking. *Journal of Economic theory*, 135(1):171–195.

Bernanke, B. S. and Blinder, A. S. (1992). The federal funds rate and the channels of monetary transmission. *American economic review*, 82(4):901–21.

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, 1:1341–1393.

Boar, C., Holden, H., and Wadsworth, A. (2020). Impending arrival–a sequel to the survey on central bank digital currency. *BIS paper*, (107).

Boar, C. and Wehrli, A. (2021). Ready, steady, go?-results of the third bis survey on central bank digital currency. *BIS papers*.

Brunnermeier, M. K. and Niepelt, D. (2019). On the equivalence of private and public money. *Journal of Monetary Economics*, 106:27–41.

Caballero, R. J. (2006). On the macroeconomics of asset shortages.

Champ, B. A. (2007). The national banking system: A brief history.

Chiu, J., Davoodalhosseini, M., Jiang, J. H., and Zhu, Y. (2019). Bank market power and central bank digital currency: Theory and quantitative assessment. Technical report, Bank of Canada.

Chiu, J. and Davoodalhosseini, S. M. (2023). Central bank digital currency and banking: Macroeconomic benefits of a cash-like design. *Management Science*, 69(11):6708–6730.

Chiu, J., Davoodalhosseini, S. M., Jiang, J., and Zhu, Y. (2023). Bank market power and central bank digital currency: Theory and quantitative assessment. *Journal of Political Economy*, 131(5):1213–1248.

Chiu, J. and Wong, T.-N. (2015). On the essentiality of e-money. Technical report, Bank of Canada.

Choi, M. and Liang, F. (2021). Learning and money adoption. *Available at SSRN 3632489*.

Christiano, L. J., Motto, R., and Rostagno, M. (2014). Risk shocks. *American Economic Review*, 104(1):27–65.

Dang, T. V., Gorton, G., Holmström, B., and Ordonez, G. (2017). Banks as secret keepers. *American Economic Review*, 107(4):1005–29.

Davies, G. (2010). *History of money*. University of Wales Press.

Davoodalhosseini, S. M. (2021). Central bank digital currency and monetary policy. *Journal of Economic Dynamics and Control*, page 104150.

Davoodalhosseini, S. M. (2022). Central bank digital currency and monetary policy. *Journal of Economic Dynamics and Control*, 142:104150.

Davoodalhosseini, S. M. and Rivadeneyra, F. (2020). A policy framework for e-money. *Canadian Public Policy*, 46(1):94–106.

Dermine, J. (1986). Deposit rates, credit rates and bank capital: the klein-monti model revisited. *Journal of Banking & Finance*, 10(1):99–114.

Diamond, D. W. and Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of political economy*, 91(3):401–419.

Dong, M. and Jiang, J. H. (2010). One or two monies? *Journal of Monetary Economics*, 57(4):439–450.

Drechsler, I., Savov, A., and Schnabl, P. (2017). The deposits channel of monetary policy. *The Quarterly Journal of Economics*, 132(4):1819–1876.

Fernández-Villaverde, J., Sanches, D., Schilling, L., and Uhlig, H. (2021). Central bank digital currency: Central banking for all? *Review of Economic Dynamics*, 41:225–242.

Ferrari, M., Mehl, A., and Stracca, L. (2022). Central bank digital currency in an open economy. *Journal of Monetary Economics*.

Gomis-Porqueras, P., Peralta-Alva, A., and Waller, C. (2014). The shadow economy as an equilibrium outcome. *Journal of Economic Dynamics and Control*, 41:1–19.

Goodfriend, M. and McCallum, B. T. (2007). Banking and interest rates in monetary policy analysis: A quantitative exploration. *Journal of Monetary Economics*, 54(5):1480–1507.

Gu, C., Han, H., and Wright, R. (2020). The effects of news when liquidity matters. *International Economic Review*, 61(4):1411–1435.

Gu, C., Monnet, C., Nosal, E., and Wright, R. (2019). On the instability of banking and financial intermediation. Technical report, Department of Economics, University of Missouri.

Hirshleifer, J. (1971). The private and social value of information and the reward to inventive activity. *American Economic Review*, 61(4):561–574.

Hu, T.-W. (2021). Optimal monetary policy with interest on reserves and capital over-accumulation. *Journal of Economic Theory*, 196:105319.

Internal Revenue Service (2019). Federal tax compliance research: Tax gap estimates for years 2011–2013. Technical report.

Jiang, J. H. and Zhu, Y. (2021). Monetary policy pass-through with central bank digital currency. Technical report, Bank of Canada Staff Working Paper.

Keister, T. and Monnet, C. (2020). Central bank digital currency: Stability and information. *Unpublished, Rutgers University and University of Bern*.

Keister, T. and Monnet, C. (2022). Central bank digital currency: Stability and information. *Journal of Economic Dynamics and Control*, 142:104501.

Keister, T. and Sanches, D. (2023). Should central banks issue digital currency? *The Review of Economic Studies*, 90(1):404–431.

Keister, T. and Sanches, D. R. (2019). Should central banks issue digital currency?

Keister, T. and Sanches, D. R. (2021). Should central banks issue digital currency? *Available at SSRN 3966817*.

Kim, Y. S. and Kwon, O. (2019). Central bank digital currency and financial stability. *Bank of Korea WP*, 6.

Kiyotaki, N. and Moore, J. (2019). Liquidity, business cycles, and monetary policy. *Journal of Political Economy*, 127(6):2926–2966.

Kocherlakota, N. R. (1998). Money is memory. *journal of economic theory*, 81(2):232–251.

Kocherlakota, N. R. (2003). Societal benefits of illiquid bonds. *Journal of Economic Theory*, 108(2):179–193.

Kumhof, M. and Noone, C. (2018). Central bank digital currencies-design principles and balance sheet implications.

Kwon, O., Lee, S., and Park, J. (2020). Central bank digital currency, inflation tax, and central bank independence. *Inflation Tax, and Central Bank Independence (February 13, 2020)*.

Lagos, R. (2010). Asset prices and liquidity in an exchange economy. *Journal of Monetary Economics*, 57(8):913–930.

Lagos, R. and Rocheteau, G. (2008). Money and capital as competing media of exchange. *Journal of Economic theory*, 142(1):247–258.

Lagos, R. and Wright, R. (2005). A unified framework for monetary theory and policy analysis. *Journal of political Economy*, 113(3):463–484.

Lahcen, M. A. (2020). Informality, frictional markets and monetary policy. *Working Paper*.

Lucas, R. E. and Nicolini, J. P. (2015). On the stability of money demand. *Journal of Monetary Economics*, 73:48–65.

Meaning, J., Dyson, B., Barker, J., and Clayton, E. (2018). Broadening narrow money: monetary policy with a central bank digital currency.

Monnet, C., Petursdottir, A., and Rojas-Breu, M. (2019). Central bank account for all: Efficiency and stability. Technical report, Mimeo.

Niepelt, D. (2022). Monetary policy with reserves and cbdc. Technical report, Swiss National Bank.

Niepelt, D. (2023). Money and banking with reserves and cbdc. *Journal of Finance*. Forthcoming.

Rahman, A. (2024). Interest-bearing money, illiquid bonds, and banking in a news economy. Mimeo.

Rahman, A. and Wang, L. (2023). Central bank digital currency (cbdc) and monetary policy with heterogeneous agents. Mimeo.

Rocheteau, G. and Wright, R. (2005). Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium. *Econometrica*, 73(1):175–202.

Rocheteau, G., Wright, R., and Zhang, C. (2018). Corporate finance and monetary policy. *American Economic Review*, 108(4-5):1147–86.

Wallace, N. (2014). Optimal money creation in "pure currency" economies: a conjecture. *The Quarterly Journal of Economics*, 129(1):259–274.

Wang, Z. (2020). Tax compliance, payment choice, and central bank digital currency. *Payment Choice, and Central Bank Digital Currency (December 15, 2020)*.

Wang, Z. (2023). Money laundering and the privacy design of central bank digital currency. *Review of Economic Dynamics*, 51:604–632.

Whited, T. M., Wu, Y., and Xiao, K. (2022). Central bank digital currency and banks. *Available at SSRN 4112644*.

Williamson, S. (2019). Central bank digital currency: Welfare and policy implications. Technical report, Society for Economic Dynamics.

Williamson, S. (2022). Central bank digital currency: Welfare and policy implications. *Journal of Political Economy*, 130(11):2829–2861.

Williamson, S. D. (2021). Central bank digital currency and flight to safety. *Journal of Economic Dynamics and Control*, page 104146.

Wright, R. (2010). A uniqueness proof for monetary steady state. *Journal of Economic Theory*, 145(1):382–391.

Xiang, H. (2013). Optimal monetary policy: distribution efficiency versus production efficiency. *Canadian Journal of Economics/Revue canadienne d'économique*, 46(3):836–864.

Zhu, Y. and Hendry, S. (2019). A framework for analyzing monetary policy in an economy with e-money. Technical report, Bank of Canada.