Order Number 9018960

# Optimal resource allocation, pricing, investment and market structure under a spatial externality: The case of irrigation

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University of Hawaii, 1989

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# OPTIMAL RESOURCE ALLOCATION, PRICING, INVESTMENT AND MARKET STRUCTURE UNDER A SPATIAL EXTERNALITY: THE CASE OF IRRIGATION

A DISSERTATION SUBMITTED TO THE GRADUATE DIVISION OF
THE UNIVERSITY OF HAWAII IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

IN AGRICULTURAL AND RESOURCE ECONOMICS

DECEMBER 1989

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## ACKNOWLEDGEMENTS

This dissertation could not have been written without the assistance I received from a variety of sources. Several professors, from within my dissertation committee as well as from outside, provided critical help. My advisor, Peter Garrod gave valuable comments and strong institutional backing. In addition, he gave me complete freedom to pursue various ideas. John Dixon kept my analysis of irrigation systems honest, and as my advisor at the East-West Environment & Policy Institute, made sure that I received all possible research and funding support. Sumner La Croix provided incredibly detailed comments on my drafts which substantially improved the quality of the dissertation. PingSun Leung gave me freely of his time and ideas. James Roumasset suggested the topic for the dissertation, and his influence in its conceptual development has been profound. Gerard Russo's comments and guidance in the final stages of the dissertation were extremely helpful. I-Pai Wu asked penetrating questions about the engineering aspects of irrigation.

I remain infinitely grateful to Eithan Hochman and David Zilberman for their ideas, encouragement and friendship. Working with them made the task of dissertation writing seem effortless.

I am grateful to the East-West Environment & Policy Institute for a generous dissertation scholarship. The Department of Agricultural & Resource Economics, University of California at Berkeley and the Monaster Center for Economic Research at Ben-Gurion University, Israel provided partial research funding and good working environments. I would also like to thank June Kuramoto and Evelyn Klinckman for providing administrative support.

## ABSTRACT

This dissertation develops the economic principles for spatial water allocation, pricing and investment in water conveyance and on-farm technology for large-scale irrigation systems. Although irrigation is emphasized, the results obtained are applicable to other public utilities such as electricity, natural gas and transportation. The effect of marginal cost pricing on the welfare of farmers located at various distances from the water source is examined, and alternative taxation mechanisms are proposed. The study examines the differential impact of private and public irrigation investments on resource rents accruing to beneficiaries, as well as on irrigated acreage and aggregate output. Performance of purely decentralized irrigation systems are compared to socially optimal and monopolistic regimes, under a range of plausible assumptions of demand elasticity. The theoretical results are illustrated with secondary data from California and Pakistan.

## TABLE OF CONTENTS

TABLE OF CONTENTS (CONTINUED)

## LIST OF TABLES

# LIST OF GRAPHS

## LIST OF SYMBOLS

### Latin Letters

| | |
|---|---|
| a | conveyance efficiency |
| A | irrigated area |
| b | parameter of the $VMP_r$ function |
| B | net benefit function |
| C | cost of water generation |
| CS | consumer surplus |
| d | width of the irrigation system |
| D | demand function parameter |
| e | effective water, exponential symbol |
| f | production function |
| g | conveyance loss function |
| h | on–farm investment function |
| $h_0$ | on–farm efficiency when no on–farm investments are made |
| subscript h | variable measured at the system head |
| H | Hamiltonian functions |
| i | generic index |
| I | on–farm investment |
| j | generic index |
| l | conveyance loss function |
| $l_0$ | conveyance loss under no canal investment |
| ln | natural logarithm |
| L | Lagrangian functions |
| k | conveyance investment |
| m | reduction in conveyance losses due to canal investments |
| MC | marginal cost |
| MR | marginal revenue |
| N | aggregate |
| p | output price |
| $p_o$ | competitive price |
| $p_M$ | monopoly price |
| p** | price in the optimal model |
| P | output price |
| PS | producer surplus |
| $P_W$ | efficiency price of water |
| q | quantity of water applied |
| $q_r$ | water received |
| $q_s$ | water sent from source |
| Q | canal discharge |
| $R_L$ | quasi–rents to land |
| $R_W$ | quasi–rents to water |
| s | distance function |
| subscript t | variable measured at the system tail |
| TC | total cost |
| TNB | total net benefit |
| v | parameter of the $VMP_s$ function |
| $VMP_r$ | value of marginal product for water received |

| | |
|---|---|
| $VMP_s$ | value of marginal product for source water |
| w | cost of on–farm investment |
| WC | water charges |
| x | distance from the water source |
| $X_1$ | distance at which conveyance investments are equal |
| X* | length of irrigation system |
| y | yield per unit area |
| $y_t$ | distance of tail farmer from source |
| Y | output |
| $Y_c$ | competitive output |
| $Y_M$ | monopoly output |
| Y* | aggregate output |
| Y** | output in the optimal model |
| z | quantity of water flowing through the canal |
| $z_0$ | total stock of water |

**Greek Letters**

| | |
|---|---|
| $\mu$ | shadow price of water |
| $\mu_0$ | shadow price of water at source |
| $\varepsilon$ | price elasticity of demand |
| $\varepsilon_r$ | elasticity of marginal product of water in crop production |
| $\varepsilon_M$ | elasticity of conveyance investments |
| $\varepsilon_h$ | elasticity of on–farm investments |
| $\varepsilon_{h'}$ | elasticity of marginal returns to on–farm investment |
| $\alpha$ | proportion of quasi–rents taxed |
| $\beta$ | jump in shadow price |
| $\sigma$ | shadow price of output |
| $\tau$ | jump in shadow price |
| $\pi$ | profit function |

# PREFACE

Most of the research work for this dissertation was done in the form of self-contained, jointly-authored papers intended for publication. Parts of the dissertation that are in manuscript form are identified below, along with their current status and the nature of contribution of each of the co-authors.

Chapter 2 is a revised version of a manuscript entitled "Spatial Efficiency and User Prices: The Case of Irrigation" by Ujjayant Chakravorty and James Roumasset which is currently being considered for publication by the American Journal of Agricultural Economics. Candidate's contribution: model formulation, simulations, research and writing. Co-author's contribution: research direction, elaboration of concepts, editing, and rewriting of sections 2.1 and 2.5.

Chapter 3 was written as "Spatial Externalities and Optimal Investment in Irrigation" by Ujjayant Chakravorty, Eithan Hochman and David Zilberman which has been submitted to the American Economic Review. Candidate's contribution: Solving the model, simulations, research and writing. Co-authors' contribution: specification of the model, research guidance, and detailed comments at various stages.

Chapter 4 is presently a manuscript entitled "Market Structure Under a Spatial Externality: The Case of Irrigation" by Ujjayant Chakravorty, David Zilberman and Eithan Hochman, which is being prepared for submission. Candidate's contribution: Solving the model, simulations, and writing. Co-authors' contribution: problem formulation, research direction and comments.

# CHAPTER 1

# INTRODUCTION

## 1.1 The Problem

Many countries, both in the developed and developing world, face a serious water crisis. While fresh water stocks have remained finite, demand for water has steadily expanded not only in agriculture, but in the urban and industrial sectors as well (Rogers 1985). Over the last three to four decades, large investments have gone into exploiting new water resources, especially in building dams and irrigation projects. However, evaluations of these water projects have revealed considerable inefficiencies in planning, system design, operation and maintenance. They have suffered from inefficiencies in on–farm water use, poor maintenance and low cost recovery, skewed distributions of benefits, a large gap between ex–ante and ex–post benefits, and have often contributed to negative environmental effects such as salinity and waterlogging (Repetto 1986).

The problem of irrigation system inefficiency is partly caused by the continued use of design and management principles which implicitly assume that water supplies are plentiful. The traditional engineering, hydrological and agronomic principles that have dominated irrigation planning, aimed to maximize physical efficiency in irrigation systems. They were appropriate in an era when water shortages did not inhibit expansion or intensification of irrigated agriculture. This situation has changed in recent years, mainly because of the increased demand for water from competing urban and industrial uses, where the marginal value product of water is usually higher than in agriculture (e.g., see Dixon 1986).

Traditional design principles do not adequately incorporate the scarcity value of water or the spatial nature of irrigation, and their use has frequently led to sub-optimal system design. These design principles need to be supplemented by economic criteria that take into account the full opportunity cost of the resource resulting from losses in water transmission (Walker and Skogerboe 1987 p.4; James 1988 p.97).

Although this need to integrate principles of economic efficiency into engineering design has been expressed frequently, a workable theoretical framework has yet to be developed (Merriam 1987; Plusquellec and Wickham 1985). Most economic studies of irrigation have tended to focus on either a qualitative classification of economic concepts associated with irrigation practices (Carruthers and Clark 1981) or benefit-cost analyses and project appraisal of irrigation projects in a with or without (irrigation) framework (Olivares and Wieland 1987; Bergmann and Boussard 1976). Some other studies perform farm-level optimization exercises that determine optimal input mixes under assured water supplies. While these studies have facilitated the use of economic concepts in irrigation planning, they do not provide a methodology for economically efficient irrigation design.

In section 1.2, we discuss the technical approach to irrigation system planning and design and highlight some of its limitations. Section 1.3 introduces the general approach developed in this dissertation. Section 1.4 connects the ideas developed in the individual chapters.

## 1.2 Limitations of the Technical Approach

Design of irrigation facilities have frequently been based on exogenous specifications of "crop water requirements". Historically, the concept of efficient irrigation management entailed periodic refilling of the soil root zone from the "management allowed deficit (MAD)" level to "field capacity". The modern concept of irrigation that has evolved in the last two decades involves providing frequent and controlled water applications that prevent occurence of moisture stress in the soil (Hillel 1987).

Aggregating the amounts of water required to replenish the root zone of crops over a given geographical area gives the Design Daily Irrigation Requirement (DDIR) which then provides the basis for computation of design flow, canal size, and system capacity. This method might be termed the "intensive" method of water development. Alternatively, in the "extensive" method, flow data from an already existing irrigation system provides the design flows per unit area for newly planned systems. The old data can be adjusted for anticipated improvements in the new system (Withers and Vipond 1974, p.195). The size of the irrigated area is then determined by extending the irrigation system until the head net of gradient necessary for water delivery is just equal to the height of the canal at the boundary of the system.

From an economic efficiency point of view, these irrigation scheduling strategies that provide the entire irrigation requirement of the crop ("full irrigation") are sub-optimal because they implicitly assume that the opportunity cost of supplying additional amounts of water is zero. Irrigation techniques such as "deficit irrigation", which allows planned plant stress during the growing season, are nearer the economic optimum (Hargreaves and Samani 1984; Martin, Watts and Giley 1984; Ayer, Hoyt and Cotner 1980). Experimental studies have shown that deficit irrigation of crops can produce greater economic benefits (English and Nuss 1980), even where the assumed cost of water used in the calculations of private profitability did not reflect the full long run social opportunity cost of providing irrigation.

Thus, standard irrigation techniques that provide for the full evapotranspiration requirements of the crop supply more irrigation water to the farm then is socially profitable. As a consequence, this "intensive" method tends to prescribe irrigated area sizes that are smaller than optimal. On the other hand, the "extensive" method of development, which has been popular in many developing countries, is attractive to politicians who wish to maximize the number of beneficiaries subject to constraints. Unfortunately, designating a large number of farmers as target beneficiaries sets up inevitable conflict in water allocation. If water requirements are followed at the head, nothing will be left at the tail. In this case the size of the irrigated area tends to be larger than optimal. Thus political pressures to extend irrigated acreage have often led to extreme disparities in head-tail water allocations, causing tension within the system and demands for more "equitable" water distribution (Repetto; Ali 1985). Therefore economic principles are needed to determine the optimum command area under different conditions.

The choice of irrigation technology at the source is, again, a complex task given the number of physical, engineering and socio-economic parameters involved, and the ultimate decision often seems to be a matter of informed judgement (James; Hillel, op. cit.). Traditionally, comparisons of aggregate capital and operating costs of alternative technological options seems to be the most popular method employed (Holzapfel, Marino and Chavez-Morales 1985). A few studies have compared irrigation methods by incorporating additional criteria such as water conservation, farm profits, and by qualitative ranking of options based on various technical and economic criteria (Holzapfel et. al., op. cit.; Hill and Keller 1980; Keller 1965). However, comparisons of discrete cost and engineering data tend to be <u>ad hoc</u>, and do not lend themselves to the evaluation of trade-offs between the choice parameters available to the irrigation planner, such as, the choice between designing for one or multiple sources of irrigation water, providing intensive vs. extensive area coverage, or evaluating the effect of reallocating water from head to tail. Thus guidelines based on economic efficiency are needed for choosing between irrigation techniques, and to provide a basis for performing sensitivity analyses. As we shall see in chapter 3, our formulation of a marginal cost function at source implicitly takes into account the opportunity cost of water, or its value in alternative uses such as in the industrial or urban sector.

Efficiency rules are also required to determine the optimal amount of conveyance for water delivery. From an engineering viewpoint, delivery systems are designed so that the capacity is sufficient to deliver the required amount of water to any section of the irrigated area. However, the choice between lined and unlined canals, and the quality and length of lining, if any, is frequently made on the basis of comparative costs of alternative lining technologies value adjusted for the associated water savings resulting from reduced seepage losses (James; Flynn and Marino 1987). However, economically efficient design of conveyance systems should equate marginal benefits and costs associated with conveyance investments at each location, since shadow prices of water as well as the volume of flow, changes with location.

## 1.3 Approach

This dissertation develops methodology that could be used to address some of the questions posed in section 1.2. By means of a spatial equilibrium model, we develop a conceptual framework that determines rules for optimal water allocation and pricing, investment in canals and in on-farm conservation technology, and for the endogenous determination of water generating capacity and irrigated acreage. We also examine the effect of market structure (i.e., socially optimal, monopolistic and purely decentralized irrigation institutions) and different demand elasticities on system performance.

The approach we have defined emphasizes the spatial characteristics of irrigation, especially losses that occur in transmission of water over distance. By isolating the problem of conveyance losses, we have chosen to ignore or abstract from several other variables that might be equally critical in irrigation. For instance, we have ignored problems relating to intertemporal optimization and weather and price uncertainty. Similarly, partly for reasons of analytical tractability, we do not consider heterogeneity within an irrigation system, arising from topographical variations, and non-uniform land quality and farm size distributions. Our models also abstract from transactions costs and choice of alternative institutions. Some of these questions, such as optimization over time, have been studied in the literature, and the relevant literature has been cited wherever appropriate. Other issues such as water supply uncertainty could be analysed by extending the basic model we have developed.

### 1.4 Outline of Chapters

Chapter 2 develops a model of irrigation that incorporates conveyance losses in water transfer. Farmers are taken to be located along a straight line from the source, and assuming exogenously given conveyance losses, we obtain results that determine water allocations to each farmer, and the efficiency price of water at each location along the canal. We observe that water allocated to each farmer decreases with distance from the source, while the marginal cost–based price of water increases from head to tail. Rules for the endogenous determination of the total water used at source and irrigated area are determined.

Since marginal cost pricing leads to spatial 'inequity' in the sense that tail farmers pay a higher price than head farmers, a set of second–best pricing rules based on specific distributional objectives are outlined. The theoretical results are illustrated using published data from irrigation systems in Pakistan and California.

Chapter 3 extends the basic spatial model formulated in chapter 2 to incorporate endogenous investments in canal lining, and also in on–farm irrigation technology such as drip or sprinkler systems. Using a control theoretic framework, we derive results that determine optimal spatial investments in conveyance and in on–farm technology. We find that investment in canals (on–farm technology) decreases (increases) with distance from the source.

These results are then compared to stylized real–world scenarios where (i) canal lining investments are spatially uniform, and (ii) on–farm technology is constrained to be furrow irrigation. The analytical results are illustrated by simulating data for California cotton. We show that significant gains in net economic benefits, agricultural output and irrigated acreage could be achieved through optimal investments in canals and on–farm technology.

Chapter 4 relaxes the assumption of a constant output price of the crop and examines the effect of output elasticity on irrigation system parameters such as water use, irrigated area, output price and quantity. Three different organizational structures are considered (i) socially optimal (ii) monopoly in the output market and (iii) a decentralized system where the spatial externality (i.e, conveyance) is not internalized. Again, simulations are run using California data to obtain quantitative results which indicate that at high (low) demand elasticities, the monopoly will produce higher (lower) output and charge a lower (higher) output price than the decentralized model.

Chapter 5 concludes the dissertation by summarizing the main results, noting the major policy implications of the study and recognizing its limitations.

# CHAPTER 2

# A MODEL FOR SPATIAL ALLOCATION AND PRICING

## 2.1 Introduction

Despite massive public investments in irrigation infrastructure, ex-post evaluations of irrigation projects both in developed and developing countries indicate that actual benefits are drastically below projected levels. There is considerable evidence suggesting that these low benefits are largely due to poor on-farm water use efficiencies which are partly related to low water charges (Chaudhry 1985; Repetto 1986; Bowen and Young 1986a). Low water charges have resulted in the creation of large rents in public irrigation systems and water allocation based on the relative political and financial influence of farmers rather than on grounds of economic efficiency (Wade 1982). Location in the system is also an important determinant of water allocation and surplus benefits above water charges. Farmers near the system headworks consume a disproportionate share of irrigation water while tail farmers are left to do with residual supplies that are often scanty and unreliable (Reidinger 1971).

Low water charges have necessitated large government subsidies for irrigation projects thus attracting rent-seeking activity. When neither water charges nor rationing rules effectively limit water use, environmental damages such as waterlogging and salinity are also induced, and groundwater resources are mined too quickly.

In order to improve irrigation performance and promote sustainable use of agricultural water, an analytical framework for allocating water and levying water charges is needed (Repetto, op. cit.). Although water pricing systems vary widely, traditional concepts of irrigation management dictate that farmers be charged uniform prices for equal amounts of water delivered[1] (Plusquellec 1983; Bishop and Long 1983; Yoo and Busch 1985; Burness and Quirk 1980). Similarly, evaluations of irrigation systems implicitly assume that equal water allocation is desirable and that head vs. tail disparities in water allocation are prima facie evidence of both inefficiency and inequity. As shown below, however, irrigation policies that prescribe equal allocations of water to farmers, or uniform pricing of water at its marginal cost at system source may be inefficient since conveyance costs of water increase with distance from the source.[2]

This chapter develops a methodology for efficient spatial allocation of irrigation water that takes into account system conveyance losses caused by seepage, percolation and evaporation.[3,4] It derives optimal rules for allocation of water supplied to farmers at various distances from the water source. When there are losses in water conveyance, it is observed that efficient spatial allocation of water in an irrigation system implies that allocations of source water increase with distance from the head of the system. Towards the tail of the system, efficient allocation reaches a maximum and then declines with further increases in distance from the head. In terms of water received, i.e., net of conveyance losses, the efficient quantity of water per hectare decreases monotonically from head to tail. Under marginal cost pricing, tail farmers must pay more per unit of water received than head farmers. If farms are uniform in size, then total water charges are lowest at the head and highest at the point of maximum source water. Quasi–rents from irrigation are maximized at the head and decline steadily until they equal zero at the tail.

Thus, up to the point of maximum source water, farmers further away from the water source pay more for less, in the sense of higher water charges per hectare for less water received. Since the marginal cost–based price increases and the rents decrease continuously from head to tail, this defines a sense in which marginal cost pricing, although efficient, creates an appearance of spatial inequity. By the use of water entitlements, however, water charges can be made more equitable without decreasing efficiency. As shown in Section 3.4, spatial disparities in rents can be offset by choosing from a menu of alternative pricing schemes that range from equalizing total water charges to taxation according to total irrigation benefits.

Section 2.2 outlines a theoretical model that formulates rules for spatial efficiency and examines its implications for the allocation of irrigation water. Conditions for the determination of the optimal system boundary are developed. Section 2.3 analyses the effect of marginal cost pricing on spatial prices and rents. In order to address specific distributional objectives, an alternate set of pricing mechanisms is discussed. Section 2.4 illustrates the theoretical results in relation to empirically derived demand functions for irrigation water. Section 2.5 concludes by outlining the implications of these efficiency rules for irrigation system planning and management, and for welfare of beneficiaries, especially with respect to their spatial location.

## 2.2 Efficiency Rules

Abstracting from the costs of information and enforcement, efficient spatial allocation requires equal marginal value products of water measured at a common source point (O'Mara 1984). The purpose of this section is to derive the equal (gross) marginal product rule and demonstrate its implications for (net) marginal value products and water allocations at the farm level and for determination of optimal system size.

Consider a simplified one–period (one season) model of an irrigation system with water being supplied from a point source to a canal. We thus abstract from seasonal variabilities in water supply and storage. Farmers draw water at various points along the canal located at a variable distance 'x' from the source, x=0 representing the system head, and x increasing towards the system tail (Fig.1). Let '$q_s$' be the 'gross' volume of water sent from source for any farmer in the system and let '$q_r$' be the 'net' water received. Then, conveyance losses from source to farmer due to seepage, percolation and evaporation would depend upon a loss factor 'a' (which is a function of investments made in canal lining and maintenance) and distance 'x' of farmer from source.[5] Conveyance losses from the farm gate to the root zone of the crop can be regarded as given such that the yield–water relationship is well–defined in terms of water delivered at the farm gate. The model also abstracts from the externality effects of conveyance losses on downstream water quality or on irrigation return flows (Willardson 1985). The relationship between $q_r$ and $q_s$ can thus be stated as:

(1) $q_r = q_s \cdot g(a) \cdot r(x);\ q_r, q_s \geq 0;\ 0 \leq g(a), r(x) \leq 1$

$g(0)=1;\ r(0)=1;\ g'(a), r'(x) < 0$

where g(a) and r(x) are functions that denote water conveyance efficiencies as a function of canal losses and distance of farmer from source, respectively. When the conveyance loss factor 'a' increases, the conveyance efficiency g(a) decreases. In this model, we specify 'a' as a constant parameter exogenous to the system, thereby abstracting from different conveyance loss rates at different locations and from the choice of conveyance technology.[5]

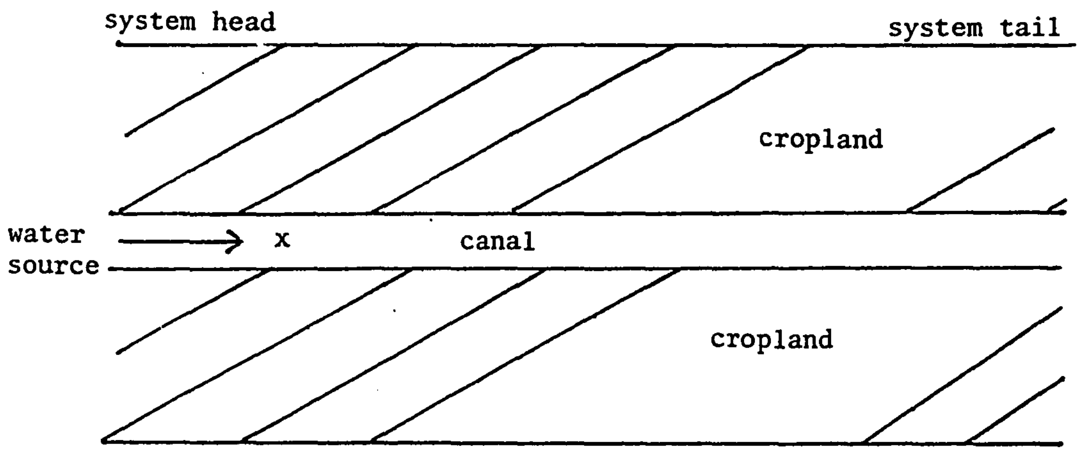



FIGURE 1. SPATIAL SCHEME OF THE MODEL

If we assume that the same crop is grown in the entire command area, then we can formulate a one–input, one–output production function for net (or received) water that holds for each farmer in the system (the monocropping assumption is retained for model simplicity but can be easily relaxed by specifying a unique production function for each farmer). The production function has the usual concavity properties:

(2) $y = f(q_r)$ ; $f > 0$; $f' > 0$; $f'' < 0$

where 'y' is crop output per unit area. The value of marginal product function for received water on land of a uniform quality can be written as

(3) $VMP_r(q_r) = P \cdot f'(q_r)$

where P is a constant crop price in a perfectly competitive market. From (2) and (3),

(4) $\delta VMP_r / \delta q_r < 0$

The value of marginal product for source water ($VMP_S$) can be defined (by using the chain rule and assuming the necessary continuity and differentiability properties) as

(5) $VMP_S(q_S) = P \cdot f'(q_S) = P \cdot f'(q_r) \cdot \delta q_r / \delta q_s = VMP_r(q_r) \cdot g(a) \cdot r(x)$

from (1) and (3). When a=x=0, $VMP_S(q_S) = VMP_r(q_r)$ from (1). Differentiating (5) with respect to x, we get

(6) $\delta VMP_S(q_S)/\delta x = P \cdot f'(q_r) \cdot g(a) \cdot r'(x) < 0$

using (1). Equations (5) and (6) imply that when canal losses are zero, the value of marginal product at source and at the farm are equal. The value of marginal product of water at source decreases with distance of farm from source.

### 2.2.1 Derivation of $VMP_s$ Schedule

We can now derive $VMP_s$ curves as a function of source water for farmers at different locations along the canal. From (5), for the head farmer, a=x=0 implies $VMP_s^h(q_s) = P \cdot f'(q_r)$ where $VMP_s^h$ represents the $VMP_s$ function for the head farmer. Similarly for a farmer located at any distance $x_t$ from the source (referred to as the 'tail' farmer), $VMP_s^t$ is given by $VMP_s^t(q_s) = P \cdot f'(q_r) \cdot g(a) \cdot r(x_t)$. Since $g(a) \cdot r(x_t) < 1 \ \forall \ a, x_t > 0$, this implies

$$(7) \quad VMP_s^t(q_s) < VMP_s^h(q_s) \ \forall \ q_s > 0.$$

Also at head, $VMP_r^h = VMP_s^h = 0$ implies $q_{rmax} = q_{s\,max}^h$ using (1) and (4), where $q_{rmax}$ and $q_{s\,max}^h$ represent values at which the relevant value of the marginal product is zero. Since the same $VMP_r$ function is faced by each farmer (by the monocropping assumption), for the tail farmer, $VMP_r^t = 0$ and $q_{rmax} = q_{s\,max}^t \cdot g(a) \cdot r(x_t)$ implies

$$(8) \quad q_{s\,max}^h < q_{s\,max}^t.$$

Differentiating (5) with respect to $q_s$, we get

$$(9) \quad \delta VMP_s(q_s)/\delta q_s = P \cdot f''(q_r) \cdot [\delta q_r/\delta q_s]^2 = P \cdot f''(q_r) \cdot [g(a) \cdot r(x)]^2 < 0.$$

From (7), (8) and (9) we can draw $VMP_S$ curves at different distances from the head as shown in Fig.2. Since there are no conveyance losses at the head, $VMP_S = VMP_f$ at $x_0 = 0$. Curves $VMP_S(x_1)$ and $VMP_S(x_2)$ located at increasing distances from the source flatten out towards the x–axis as shown.

### 2.2.2 The Optimal Allocation Rule

We can now derive the rule for optimal spatial allocation of water by applying the usual welfare criterion of maximising consumers' plus producers' surplus subject to a capacity constraint as follows:

$$(10) \quad \text{Maximise} \quad \sum_{i=1}^{n} \int_{0}^{q_s} VMP_S^{i}(\beta)\, d\beta - \int_{0}^{z_0} C'(\Theta)\, d\Theta$$

subject to

$$z_0 = \sum_{i=1}^{n} q_s$$

where 'i' represents the $i^{th}$ farmer and $i=1,2,..,n$, and $z_0$ is the system capacity at source, $C'(z_0)$ is the total long run marginal cost of water and $\beta$ and $\Theta$ are variables of integration. The first term in (10) represents the aggregate willingness to pay schedule and is obtained by the horizontal summation of the $VMP_S$ curves for each of the 'n' farmers in the system.[7]

$C'(z_0)$ is the sum of the marginal costs of supply and distribution. The marginal cost of supply includes the per period equivalent of capital construction costs[8], and costs of operation and maintenance of the head works. Empirical studies have found it to decrease with system capacity (Koenig 1966; Orlob and Lindorf 1958). The marginal cost of distribution includes construction, operation and maintenance of the canal as well as pumping and metering costs. It would increase with system capacity if we

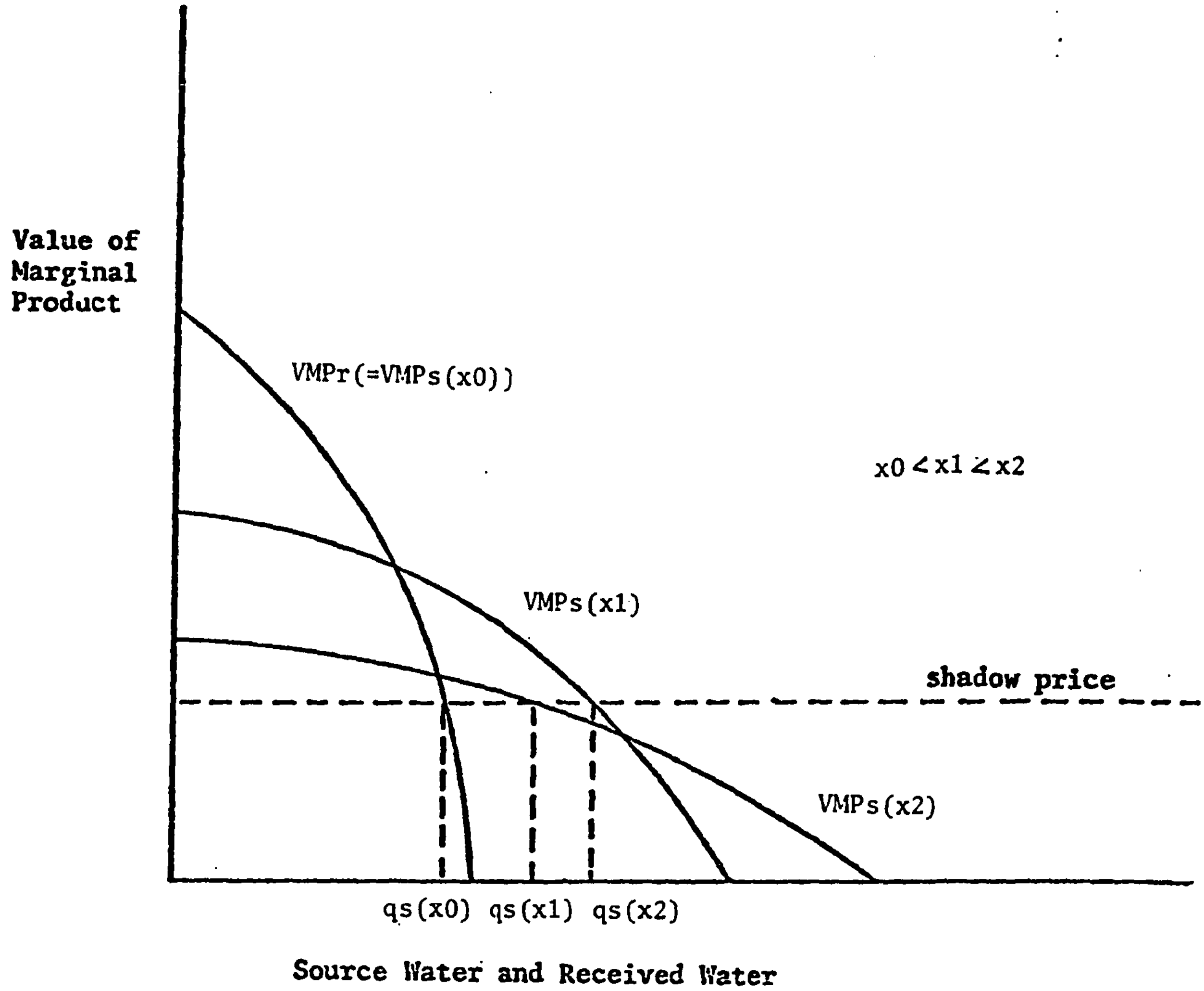



FIGURE 2. POSITION OF VMPr, VMPs CURVES AT VARIOUS DISTANCES FROM THE WATER SOURCE

assume that further increases in the supply of water necessitates extending the distribution system into new command areas (Yoo and Busch, op. cit.). Thus the resultant total marginal cost function could be rising or falling with $z_0$, and our model results are robust for either of the two cases. For illustrative purposes, however, we have assumed that costs of distribution dominate costs of supply, giving us a rising marginal cost curve.

From (10), we can maximize the following Lagrangian

$$(11)\quad L=\sum_{i=1}^{n} \int_{0}^{q_s} VMP_s^i \, d\beta - \int_{0}^{z_0} C'(\Theta)\, d\Theta - \mu(z_0 - \sum_{i=1}^{n} q_s)$$

with respect to the decision variables $q_s$ and $z_0$. $\mu$ is the usual Lagrange multiplier. We obtain the following first order conditions:

(12a) $VMP_s^i = \mu$ and

(12b) $C'(z_0) = \mu$ giving

(12c) $VMP_s^i = C'(z_0*)$

where $z_0*$ is the optimal system capacity, and $\mu$ represents the system shadow price of water. Equations (12) equate the shadow price at system source, $\mu$, to the value of marginal product at source for each farmer and to the long run marginal cost at optimal system capacity. It gives the equilibrium conditions for optimal allocation of water under spatial efficiency.

The above discussion can be summarised as follows:

Proposition 1: Optimal allocation implies that the marginal product of water at the source is equal across farmers. It also means setting the value of marginal product at source equal to long run marginal cost at optimal system capacity.

Fig.3 shows the determination of the system shadow price $\mu$, at the intersection of the aggregate marginal benefit and marginal cost curves. $\mu$ is equated to individual $VMP_s$ curves to give the optimal source water allocation $q_h*$ at the head, and $q_s*$ at the tail. Water received at the head is the same as water sent $q_h*$, while an amount $q_r*=q_s*\cdot g(a)\cdot r(x)$ is received at the tail after conveyance losses.

2.2.3 **Spatial Allocation of Water: Comparative Statics**

We can now derive comparative statics propositions for the spatial variation of received water and source water in the system. From (5) and (12a),

(13) $\mu = P\cdot f'(q_r)\cdot g(a)\cdot r(x)$. By total differentiation,

$$0 = P\cdot f''(q_r)dq_r\cdot g(a)\cdot r(x) + P\cdot f'(q_r)\cdot g(a)\cdot r'(x)dx$$

Rearranging terms, we get

(14) $dq_r/dx = -f'(q_r)\cdot r'(x)/f''(q_r)\cdot r(x) < 0$ from (1) and (2).

Thus farmers further from the source receive less net (or received) water. To examine the sensitivity of head–tail quantities to changes in the conveyance loss coefficient, from (12a) and (13), for any two farmers with net quantities $q_{r1}$, $q_{r2}$ $\varepsilon$ $[0,q_{rmax}]$, $f'(q_{r1})/f'(q_{r2}) = g(a_2)/g(a_1)$. Thus $a_2 > a_1$ gives $f'(q_{r1}) < f'(q_{r2})$ since $g'(a) < 0$ from (1). Since $f''(q_r) < 0$, $q_{r1} > q_{r2}$. Thus the difference in net quantities between any two farmers increases with a higher conveyance loss coefficient resulting from lower investments in canal lining.

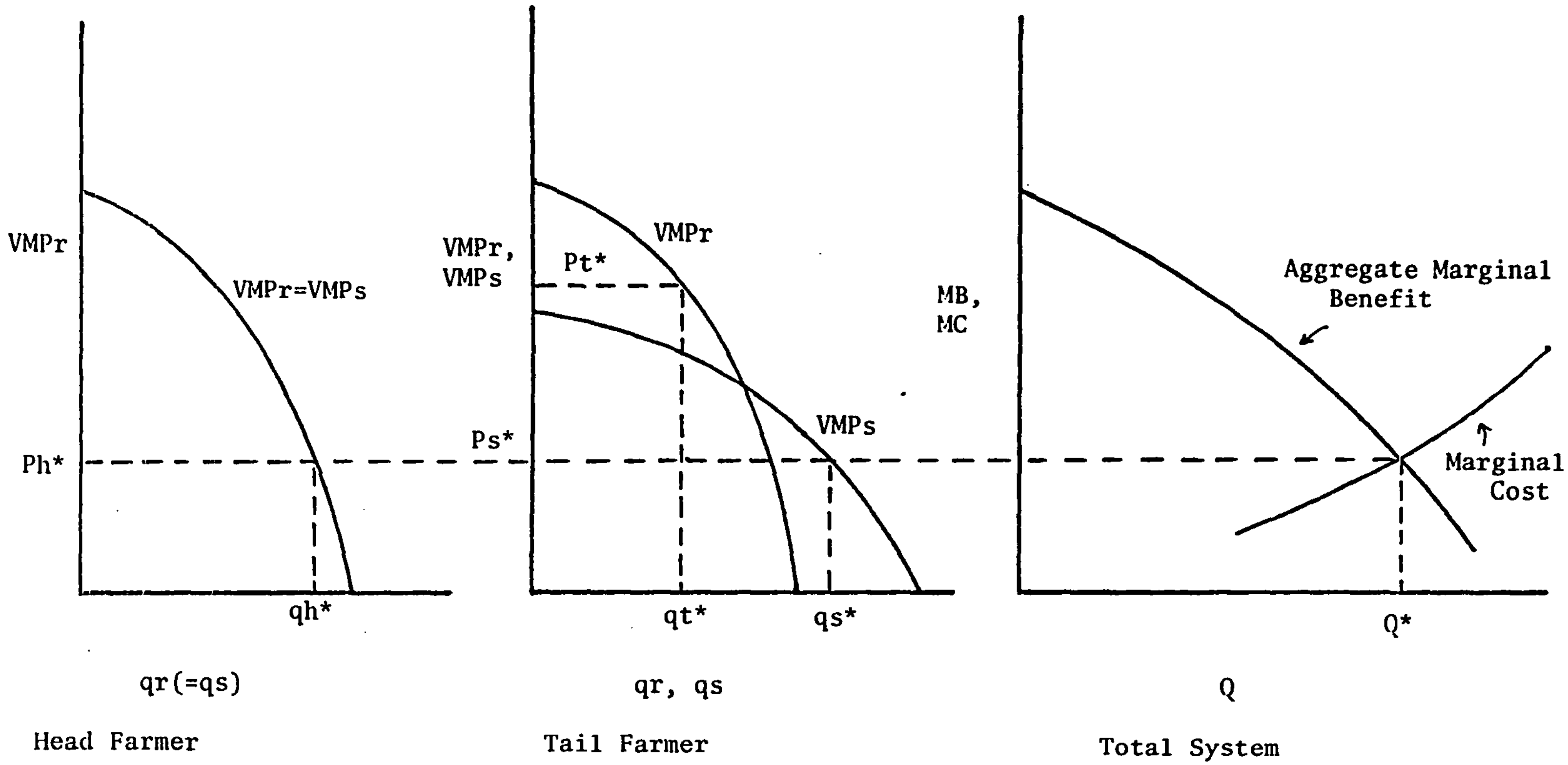



FIGURE 3. DETERMINATION OF OPTIMAL SYSTEM CAPACITY, ALLOCATION AND PRICING

In order to examine the variation of source water across space, we differentiate (1) and substitute (14) to get

$$(dq_s/dx)\cdot g(a)\cdot r(x) = - f'(q_r)\cdot r'(x)/f''(q_r)\cdot r(x) - q_s\cdot g(a)\cdot r'(x)$$

Thus the sign of $dq_s/dx$ depends on the relative magnitudes of the two expressions on the right hand side. Simplifying, $dq_s/dx > 0$ implies

$$|f'(q_r)/f''(q_r)\cdot h(x)| < |q_s\cdot g(a)| \quad \text{or}$$

$$|f'(q_r)/f''(q_r)\cdot q_r| < 1 \text{ implies } |\varepsilon_r(q_r)| > 1$$

where $\varepsilon_r(q_r)$ is the elasticity of marginal product of received water in crop production. Thus $|\varepsilon_r(q_r)|=1$ implies $dq_s/dx=0$. It can be seen that the $q_s(x)$ function has a local maximum at $|\varepsilon_r(q_r)| =1$ since when $|\varepsilon_r(q_r)|<1$, $dq_s/dx<0$. Differentiating with respect to 'x' and substituting from (14),

$$d(\varepsilon_r(q_r))/dx \quad = f''/f'\cdot dq_r/dx = f''/f'\cdot(-f'\cdot r'/f''\cdot r)$$

$$= -r'(x)/r(x) > 0$$

Since $\varepsilon_r(q_r) < 0$, the above equation indicates that its absolute value decreases with distance from source. Thus for $x < x_{qSmax}$, $dq_s/dx > 0$ and for $x > x_{qSmax}$, $dq_s/dx < 0$, where $x_{qSmax}$ represents the location which receives maximum source water.

The above results can be combined as follows:

Proposition 2: Spatial efficiency implies that farmers at the tail receive less net water. Allocation of source water increases with distance from source until the absolute value of elasticity of the marginal product function for received water equals unity. Water sent decreases with distance beyond that point.

Fig.2 provides a diagrammatic explanation of the variation of source water with distance for a hypothetical shadow price $\mu$. From $x_0$ to $x_1$, source water allocations increase from $q_s(x_0)$ to $q_s(x_1)$. However, from $x_1$ to $x_2$, source water sent actually decreases from $q_s(x_1)$ to $q_s(x_2)$. Received water falls monotonically from $x_0$ to $x_2$ and falls dramatically from $x_1$ to $x_2$.

### 2.2.4 The Boundary Condition

Beyond the system boundary, farmers do not receive any water allocation. Rules for its endogenous determination based on the optimality conditions derived in (12) are as follows:

Proposition 3.(i) If the production function is strictly concave, the marginal farmer receives no irrigation water. (ii) If the value of marginal product is constant over some intial range of water application, then the production function is not strictly concave and the marginal farmer receives a positive quantity of water bounded by the parameters of the production function.

Proof 3(i): We need to prove that

$$(15)\quad \mu = VMP_S\,(a,q_r*,X* \mid q_r* = 0)$$

where X* denotes the boundary and 'a' is assumed to be constant for the system. To prove by contradiction, let us assume that $q_r* \neq 0$ where $q_r*$ is the amount of net water received by the marginal farmer located at a distance X* from the system source. Then since $q_r* > 0$ from condition (1), and $\delta VMP_S/\delta q_r < 0$ and $\delta VMP_S/\delta x < 0$

from (4) and (6), this implies that there exist $q_r{**}$ and $X{**}$, such that $0 < q_r{**} < q_r{*}$ and

$$\mu = VMP_s\ (a, q_r{**}, X{**})\ |\ X{**} > X{*})$$

Thus (15) holds for farmers at distance $X{**}$ larger than $X{*}$. Therefore the marginal farmer is not located at $X{*}$, thus contradicting the initial assumption. Hence at any distance $x \geq X{*}$, farmers do not receive any water at all.

<u>Proof 3(ii)</u>: The boundary condition in this case is given by

$$(16)\quad \mu = VMP_s\ (a, q_r{*}, X{*}\ |\ 0 \leq q_r{*} \leq q_{rmin})$$

where $VMP_r = VMP_{rmax}$ for $0 \leq q_r \leq q_{rmin}$ is given by the production function, $VMP_{rmax}$ being a constant value. We need to prove that $q_r{*} \leq q_{rmin}$, since $q_r{*} \geq 0$ by (1). Let $q_r{*} > q_{rmin}$. Then there exists $q_{rmin} < q_r{**} < q_r{*}$ such that (4), (5) and (6) imply that (15) holds for some $X{**} > X{*}$, which again, contradicts the fact that $X{*}$ is the system boundary.

Thus (16) holds for all $0 \leq q_r{*} \leq q_{rmin}$ since $VMP_S$ is constant in that range by definition or $\delta VMP_S/\delta q_r = 0\ \forall\ 0 \leq q_r \leq q_{rmin}$. However, (6) implies $\delta VMP_S/\delta x < 0$, so for any $x > X{*}$, (16) implies that farmers do not receive any water and hence are outside the system boundary.

Conditions leading to the existence of the system boundary can be stated by the following lemma:

<u>Lemma (i) The existence of the system boundary is guaranteed by a downward–sloping and concave $q_r(x)$ function. (ii). The $q_r(x)$ function is concave if either h(x) is concave or</u>

$|\varepsilon_{r'}(x)| \leq |\varepsilon_{r}(x)|$.

where $\varepsilon_{r'}(x)$ and $\varepsilon_{r}(x)$ are the respective first and second elasticities of the conveyance loss function with respect to distance of farm from source.

<u>Proof (i)</u>: A downward–sloping and concave $q_r(x)$ function implies

$dq_r/dx < 0$ ; $d^2q_r/dx^2 \leq 0 \;\forall\, q_r > 0$.

Let $q_r(x_1) = b$ and $q_r'(x_1) = k$. Then $k < 0 < b$ and $q_r'(x) \leq k$ for $x > x_1$.

To prove by contradiction, we assume that $q_r$ approaches a lower bound x(l) as x goes to some upper bound l. Then

$$q_r(l) - q_r(x_1) = \int_{x_1}^{l} q_r'(x)\, dx$$

or

$$x(l) - b \leq \int_{x_1}^{l} k\, dx$$

implies $x(l) \leq b + k(l - x_1)$.

Thus since $l > x_1$, $k(l - x_1)$ is negative and we can make RHS as small as possible by increasing l. Hence there is no lower limit to x(l), which contradicts our initial assumption. Thus there exists $x \in [0,l]$ such that $q_r(x) = 0$.

<u>Proof (ii)</u>: Concavity implies $d^2q_r/dx^2 \leq 0$.

Differentiating (14) using the quotient rule and cancelling terms, we get

(17) $d^2q_r/dx^2 = [-f''(q_r)\cdot r(x)\cdot f'(q_r)\cdot r''(x) + f'(q_r)\cdot r'(x)\cdot f''(q_r)\cdot r'(x)]/[f''(q_r)\cdot r(x)]^2$

Thus $d^2q_r/dx^2 \leq 0$ if either

i) $r''(x) \leq 0$ meaning r(x) is concave or

ii) even if $r''(x) \geq 0$,

$|\varepsilon_{r'}(x)| \leq |\varepsilon_r(x)|$

i.e. $|x \cdot r''(y)/r'(x)| \leq |x \cdot r'(x)/r(x)|$ or

$|r(x) \cdot r''(x)| \leq |[r'(x)]^2|$

Substituting the above condition in (17) ensures that $d^2q_r/dx^2 \leq 0$.

## 2.3 Efficiency Pricing Rules and Quasi–Rents

In this section we analyse the effect of marginal cost pricing on spot prices and quasi–rents, and propose a set of alternative taxation schemes that are efficient yet achieve different degrees of equity.

### 2.3.1 Marginal Cost Pricing

One mechanism for achieving optimal spatial allocation is marginal cost pricing, which is the same as marginal product pricing (see condition 12c). It implies setting the system price of water equal to long run marginal cost at optimal system capacity. Since farmers at each location will pay off their demand curves for net water, from (5) and (12a), we get

(18) $\mu = P_w(x) \cdot g(a) \cdot r(x)$

where $P_w(x)$ is the efficiency price of water at any point 'x' in the system. Totally differentiating and simplifying, we obtain

(19) $dP_w/dx = - P_w(x) \cdot r'(x)/r(x) > 0 \;\; \forall \; P_w > 0.$

Thus farmers further away from the source pay a higher price per unit of net water. Also $\mu \cdot q_s = P_w \cdot g(a) \cdot r(x) \cdot q_s = P_w \cdot q_r$ from (1) and (18) which implies that farmers at any location can pay either the shadow price of water per unit of source water or off their demand curves per unit of net water.

Intuitively, both these pricing solutions are equivalent because the total products from source water and received water are equal. In other words, the area under the corresponding value of marginal product curves for source water and net water are equal or

$$(20) \quad \int_0^{q_r} VMP_r \, d\beta_r = \int_0^{q_s} VMP_s \, d\phi_s$$

where $\beta_r$ and $\phi_s$ are variables of integration. This can be proved as follows:

$$LHS = \int_0^{q_s} P \cdot f'(q_r) \cdot g(a) \cdot r(x) \, d\phi_s = \int_0^{q_r} P \cdot f'(q_r) \, d\beta_r = RHS$$

by changing the variable of integration and using (1), (3) and (5).

At marginal cost prices, rents from land accruing to a farmer $R_L(x)$ at any location x are given by

$$(21) \quad R_L(x) = \int_0^{q_r*} [VMP_r(\sigma_r) - P_w] \, d\sigma_r = [P \cdot f(q_r) - P_w \cdot q_r] \Big|_0^{q_r*}$$

where $\sigma_r$ is a variable of integration and $q_r*$ is the water received by a farmer at distance 'x'. In order to analyse the effect of farmer location on spot rents, we can differentiate (21) with respect to 'x', to get

$$R_L'(x) = [P \cdot f'(q_r) \cdot \delta q_r/\delta x - P_w \cdot \delta q_r/\delta x] \Big|_0^{q_r*} < 0 \text{ for all } P \cdot f'(q_r) > P_w$$

and $R_L'(x) = 0$ for $P \cdot f'(q_r) = P_w$. Or, spot rents decrease over 'x' till the slope of R'(x) becomes zero at the system boundary. Thus rents are maximum at the head and zero at the tail. By differentiating (18) and (21) with respect to 'a' we can see that price and rent differences over space increase with higher conveyance losses. Now we can state the following:

Proposition 4. Under marginal cost pricing, spot prices per unit of received water increase and rents decrease with distance of farm from system source. Head farmers get maximum rents while rents at the boundary are zero. Both price and rent differentials increase with increased conveyance losses.

The efficiency pricing solutions are shown in Figure 3. At the head, the efficiency price corresponding to the optimal quantity received $q_h*$ is read off the $VMP_r$ curve, $P_h*$. At the tail ('tail' here indicates a farmer in the system interior, not the marginal farmer at the boundary in which case the diagram will be slightly different), the efficiency price corresponding to $q_s*$ units of source water is $P_s*$. The unit price for $q_t*$ units of received water is read off the $VMP_r$ curve, $P_t*$. Thus the tail farmer could pay either $P_s* \cdot q_s*$ or $P_t* \cdot q_t*$.

### 2.3.2 Alternative Pricing Rules

While efficient, marginal cost pricing may be objectionable on equity grounds, given that rents at the system head are much higher than those at the tail. If rationing mechanisms are available, then efficient allocation can be achieved without resorting to marginal cost pricing. This affords greater flexibility in meeting equity and sustainability objectives and in reducing the incentives for rent–seeking activities and redistributive pork–barrel expenditures (Repetto 1986). We can thus propose a mix of pricing and rationing devices that attain efficiency objectives while preserving a sense of equity.[9] The preferred taxation or rationing scheme could be chosen based on the cost of the associated institutional mechanisms needed to implement them.

**Rule 1. Proportional Benefit Taxation.** Charge farmers an equal proportion of individual rents. This rule conforms to the Wicksellian notion of levying beneficiary charges that are proportional to total project benefits (Wicksell 1950; Lindahl 1950). If a constant proportion of benefits were taxed, head farmers would pay more per unit of water as compared to tail farmers since they get larger rents. Proportional benefit taxation would reduce inequities between have (irrigation) and have–not farmers as well as within a given irrigation system. Total water charges $WC_i$ for the $i^{th}$ farmer is given by

$$(22)\ WC_i = \alpha \cdot R_{Li} \text{ such that } \sum_{i=1}^{n} WC_i \leq TC \quad \text{for } i=1,..,n$$

where $R_{Li}$ is given by (18) when the $i^{th}$ farmer is located at a distance '$x_i$', $\alpha$ is a constant fraction that is obtained by the above relation, and TC is the total cost of water supplied (equal to area under the marginal cost curve), net of government subsidies and possible irrigation externalities. Since $R_{Li}$ decreases with increasing x as shown earlier, water charges also decrease from head to tail, or $dWC_i/dx_i<0$.

**Rule 2. Equal Charges.** Charge farmers a fraction of rents such that overall water charges per unit of water are equated across the system. This can be represented as

$$(23)\quad WC = \alpha_i \cdot R_{Li} \text{ such that } n \cdot WC \leq TC \text{ and } \alpha_i \cdot R_{Li} = \alpha_j \cdot R_{Lj}\ \forall\ i \neq j \text{ and } i,j=1,..,n$$

where WC is a uniform water charge and $\alpha$ is a fraction that is an increasing function of distance, $d\alpha_i/dx_i>0$. This rule does not discriminate between farmers at different locations except in rationing optimal quantities, and has the added advantage of lower administrative costs, being immune from bureaucratic problems of differential pricing across an irrigation system.

**Rule 3. Equal Rents.** Charge farmers a fraction of rents such that net rents for each farmer are equated across space, as follows:

$$(24)\quad WC_i = \alpha_i \cdot R_u \text{ such that } \sum_{i=1}^{n} WC_i \leq TC \quad \text{and}$$

$$(1-\alpha_i)R_u = (1-\alpha_j)R_{uj} \quad \forall \; i \neq j,\; i,j=1,\ldots,n$$

where $\alpha$ is a fraction given by the above relation. Obviously, $\alpha_i$ will decrease from head to tail, $d\alpha_i/dx_i<0$.

The billing and collection agency can use the above equations for computing water rations and charges for each farmer along the system. The relevant parameters for the irrigation system (e.g., conveyance loss coefficients, demand equations, cost functions for water supply) can be inputed into a suitably designed software program in order to streamline administrative procedures and reduce transactions costs.[10]

In the absence of administration and other transactions costs, marginal cost pricing and other pricing institutions discussed are equally efficient. The determination of an optimal pricing scheme depends on the nature of transactions costs in the particular irrigation system. For instance, rationing different parcels of water to farmers along the head–tail spectrum might involve high measurement costs and might be inappropriate in the absence of low-cost metering technologies (Bowen and Young, 1986b). It is possible that in certain systems, administrative (or political) costs of billing and collecting differential water charges are so high that equal charges are preferred. Local management networks like water-users associations can reduce the costs of procuring information regarding farmers' usage and demand for water and can undertake low-cost monitoring of rationing and taxation schemes.

All of the rules reviewed above for setting user fees must be anchored in explicit cost recovery targets. When some of the project benefits accrue to indirect beneficiaries, benefit taxation implies that the direct beneficiaries (farmers) be responsible for the proportion of costs equal to the proportion of total benefits that they receive. Indirect benefits will be especially large where imperfect competition and marketing costs result in inelastic demand schedules facing the agricultural producers (see also Ghate 1987; Roumasset 1987).

2.4 Low vs. High Conveyance Losses: An Application

The above model is illustrated by assuming demand and conveyance functions that closely represent the physical and engineering characteristics of irrigation water. We obtain representative patterns for the price, quantity and rent functions under spatial efficiency for conditions typical to actual irrigation systems in developed and developing countries.

We arrive at a stylized approximation of the $VMP_r$ function from a survey of functional water-use yield relationships (Hillel 1987) and from linear programming studies of cash crops in California and Pakistan (Dawson 1957; Gotsch 1968, cited in Carruthers and Clark, 1981). Hillel approximates a yield-water use curve with two segments — a flat portion with constant marginal product and a downward sloping portion with decreasing marginal product at higher quantities of water.[11] In order to compare typical water–use scenarios in developing and developed countries, we adapt this functional relationship to data obtained from the California and Pakistan studies as follows:

$$(25)\quad \text{Pakistan: } VMP_r = \begin{cases} 1.75 & \forall\ 0 \le q_r \le 0.3 \\ 2.66 - 3.0\ q_r & \forall\ 0.3 \le q_r \le 0.875 \end{cases}$$

$$(26)\quad \text{California: } VMP_r = \begin{cases} 5.0 & \forall\ 0 \le q_r \le 0.6 \\ 8.33 - 5.55 q_r & \forall\ 0.6 \le q_r \le 1.5 \end{cases}$$

where $VMP_r$ and $q_r$ are in cents and m/sq.m. respectively (Fig. 4). The curve for California demonstrates a higher value of marginal product per unit of net water and a greater demand per unit price, as compared to the one for Pakistan. This difference in slope and intercept is to some extent due to better on–farm practices and application of modern technologies in California that raises the marginal product of net water, and a higher willingness to pay that stems from a greater degree of commercialization of irrigated agriculture in developed countries.

To arrive at a functional form for conveyance losses, we assume that the loss of water from seepage and evaporation is proportional to the volume of water carried in the canal, an assumption standard in the irrigation literature (Tolley and Hastings, op. cit.; Goel and Sharma 1985). Thus (1) becomes

$$(27)\ q_r = q_s \cdot e^{-ax}$$

and the quantity of water received at any location 'x' is given by

$$(28)\ q_r = q_{min} \cdot e^{ax} + e^{ax}(q_{max} - q_{min})(b \cdot e^{-ax} - v)/b \cdot e^{-ax}) \ \forall \qquad q_{min} \le q \le q_{max}$$

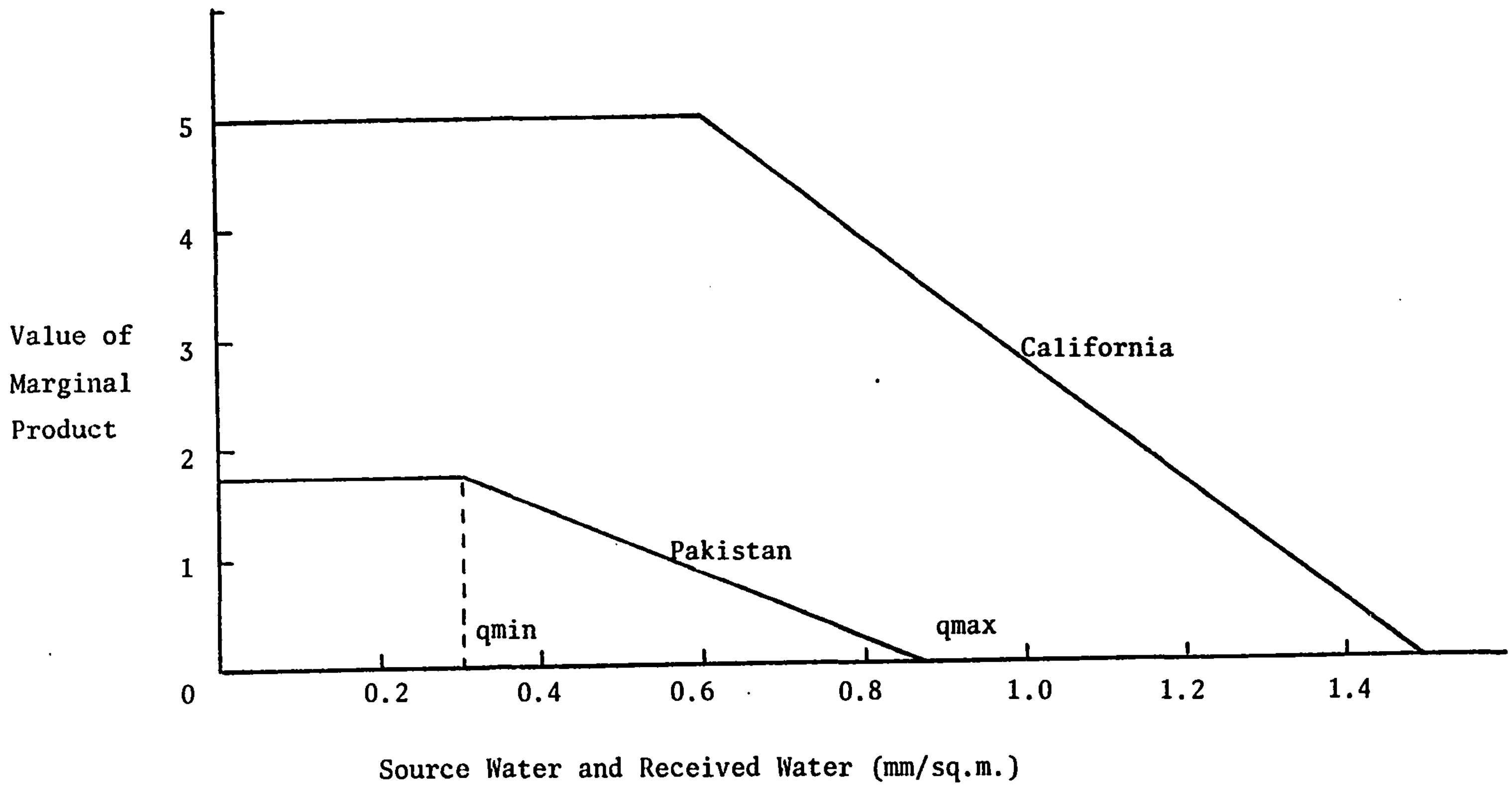



FIGURE 4. VALUE OF MARGINAL PRODUCT CURVES FOR PAKISTAN AND CALIFORNIA

where v stands for the value of marginal product at source, and b, $q_{min}$ and $q_{max}$, are given parameters of the $VMP_f$ function (Fig.4). Substituting these demand parameters into the boundary condition, we get an expression for the limits of the irrigation system as follows:

$$\mu = VMP_s(a,q_r,X* \mid q_r=q_{min}) = b \cdot e^{-aX*} \quad \text{or}$$

(29) $X* = -1/a \cdot \ln(\mu/b)$

We assume a hypothetical long run marginal cost function

(30) $C'(z_0) = z_0 * 10^{-6}/30$

where $C'(\cdot)$ is in cents and $z_0$ is in cu.m.. Although the marginal cost function for water supply can vary with specific irrigation technologies, projects and geographical region (Ghate 1986), a linear form is assumed for simplicity.[12]

We assume the total width of the command area is assumed to be 1000 m (500 m on each side of the canal) with uniform farm sizes of 5 ha each[13], which means that canal outlets to successive farms are located every 50 m (Fig.1). We use typical conveyance loss coefficients ('a') of 0.02 for Pakistan and 0.01 for California. Values of 0.015–0.02 are commonly found in developing country irrigation systems where canals are often unlined or have linings of low quality, maintenance is poor, and evapotranspiration is high because of higher ambient temperatures. Conveyance loss coefficients of 0.01 are more representative of surface irrigation systems in developed countries, although more sophisticated technologies like drip and sprinkler systems approach negligible loss coefficients (Bos and Nugteren 1974; Garbrecht 1975; Hillel

1987). A lower 'a' value of 0.01 has also been considered for Pakistan to facilitate additional comparisons.

Integrating (28) from 0 to $X*$ gives the total system capacity in terms of the shadow price. Using (12b), we can substitute (29) and (30) for $X*$ and $z_0$ to solve for $\mu$ and $z_0*$. Conditions (27) and (28) then give received water and source water allocations at each location in the system and (29) gives the location of the marginal farmer, or the efficient boundary of the system. Efficiency prices and rents can be obtained from (21), (25) and (26); (22), (23) and (24) can then be used to compute water charges under the alternative pricing schemes. Tables 1 and 2 show the water allocations and charges under alternative conveyance and pricing systems.

Fig.5 shows that if conveyance losses are low, the aggregate marginal benefit curve shifts up, leading to a larger system size and a higher shadow price of water at source. Lower conveyance losses thus expand irrigated acreage, as shown in Fig.6 where the efficient length[14] of the system increases from 37.2 kms. to 54.5 kms. Thus reducing the conveyance loss coefficient by half leads to expansion of the command area by almost 50 per cent. We can also observe that under both high and low conveyance loss regimes, source water allocations increase away from the head, and are maximised at about 25 km from the source. Farmers beyond that point get decreasing amounts of source water.

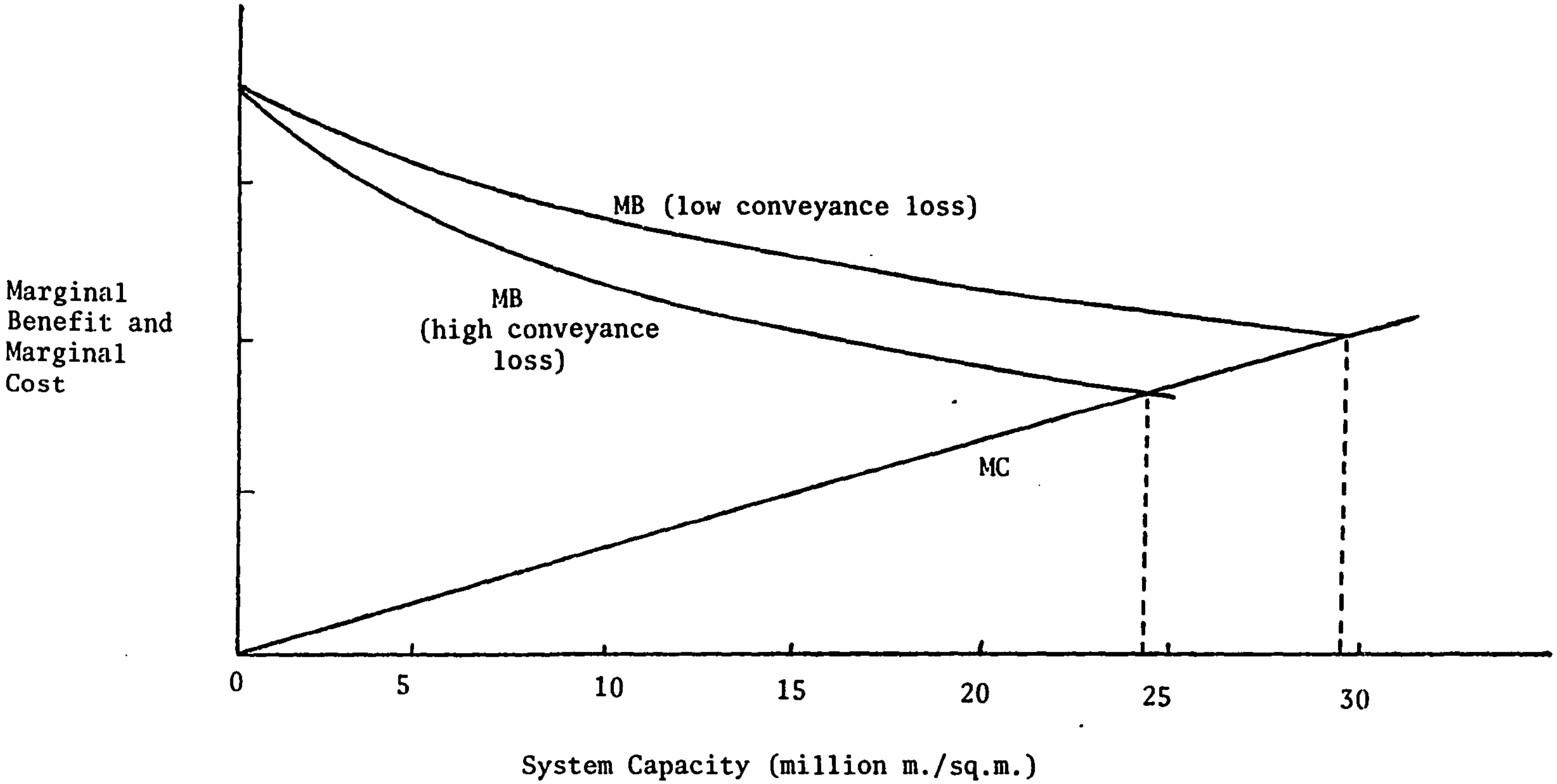
Fig.6 also shows that received water decreases monotonically with distance from source. With lower conveyance losses, head farmers receive less water while the tail (marginal) farmer receives the same amount in each case (check point A and B in figure; since the tail allocation is constrained by the demand curve for received

TABLE 1. SPATIAL ALLOCATIONS OF WATER, EFFICIENCY PRICES, WATER CHARGES AND RENTS (PER FARM) IN THREE DIFFERENT IRRIGATION REGIMES

| | California(low loss) | | | Pakistan(high loss) | | | Pakistan(low loss) | | |
|---|---|---|---|---|---|---|---|---|---|
| parameter | head | $q_s$max | tail | head | $q_s$max | tail | head | $q_s$max | tail |
| location 'y' (km) | 0 | 50.0 | 66.4 | 0 | 25.0 | 37.2 | 0 | 25.0 | 54.5 |
| $q_s$ ($10^3$m) | 51.8 | 60.7 | 58.3 | 30.1 | 35.0 | 31.6 | 27.1 | 28.7 | 25.8 |
| $q_r$ ($10^3$m) | 51.8 | 36.8 | 30.0 | 30.1 | 21.2 | 15.0 | 27.1 | 22.3 | 15.0 |
| $P_w$ (c/m$^3$) | 2.6 | 4.2 | 5.0 | 0.83 | 1.4 | 1.75 | 1.0 | 1.3 | 1.75 |
| water charges ($ per farm) | 1333 | 1561 | 1500 | 250 | 291 | 262 | 275 | 291 | 262 |
| R ($ per farm) | 993 | 253 | 0 | 207 | 68.9 | 0 | 154 | 83.4 | 0 |

TABLE 2. EFFECT OF ALTERNATIVE PRICING SCHEMES ON WATER CHARGES AND RENTS FOR PAKISTAN (WITH HIGH CONVEYANCE LOSS)

| | Water Charges per farm ($) | | | Rents per farm ($) | | |
|---|---|---|---|---|---|---|
| | head | $q_{smax}$ | tail | head | $q_{smax}$ | tail |
| Marginal Cost Pricing | 250.1 | 291.0 | 262.5 | 207.2 | 68.9 | 0 |
| Proportional Benefit Taxation | 247.0 | 194.3 | 141.8 | 210.4 | 165.5 | 120.8 |
| Equal Charges | 207.8 | 207.8 | 207.8 | 249.6 | 152.1 | 54.8 |
| Equal Rents | 282.2 | 183.6 | 86.6 | 175.0 | 175.0 | 175.0 |




FIFURE 5. OPTIMAL SYSTEM CAPACITY, BOUNDARY AND SHADOW PRICE FOR PAKISTAN

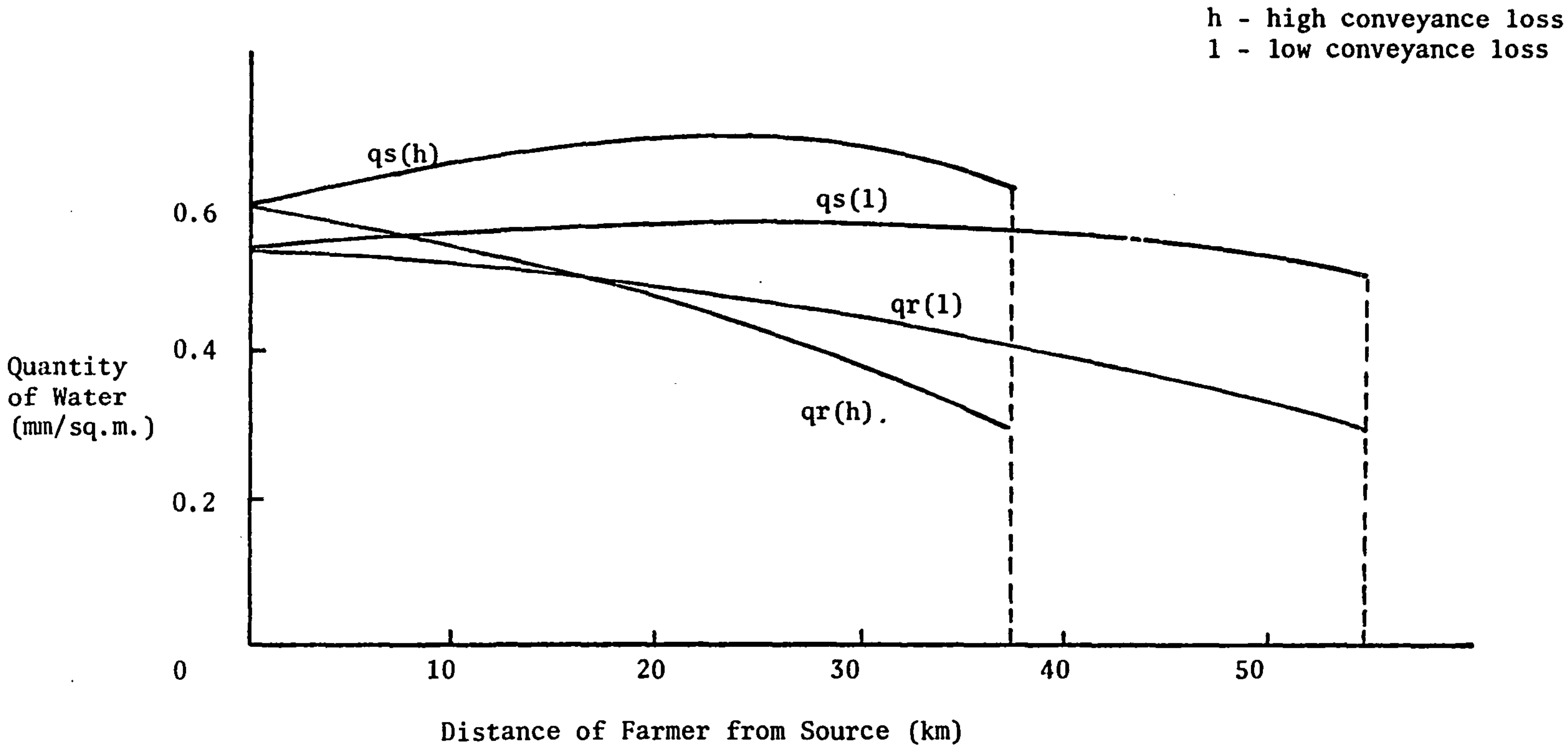



FIGURE 6. SPATIAL VARIATION OF SOURCE WATER AND RECEIVED WATER FOR PAKISTAN

water, see section 2.2.1) which is in a sense, more 'equitable'. With an increase in conveyance efficiency, the gradient of the curve for received water becomes flatter. Under high conveyance losses, efficient allocations at head are approximately double that of tail.

Price and rent differentials, too, increase when conveyance losses are higher (Fig.7). In a characteristic developing country situation (a=0.02), efficient tail prices are more than double of head prices. With improved conveyance, tail-head price ratios are still high – about 175 per cent. Head rents are higher when conveyance losses are high, while tail rents are zero in each case.

If farm-size is uniform, total water charges under marginal cost pricing are maximised at the point of maximum source water, and do not vary substantially over space[15], while delivered water is almost double at head relative to tail (Fig.8). Thus pricing rules that charge spatially uniform prices but ration efficient water allocations might be an attractive policy option.

A comparison of irrigation systems in Pakistan and California reveals that a higher marginal product per unit of on-farm water and higher intensity of water use in California contribute to larger optimal system capacities and generally higher efficiency prices (Fig.9). The absolute value of rents, as well as head-tail disparities in prices and rents are much larger in magnitude in California than in Pakistan (see also Table 1).

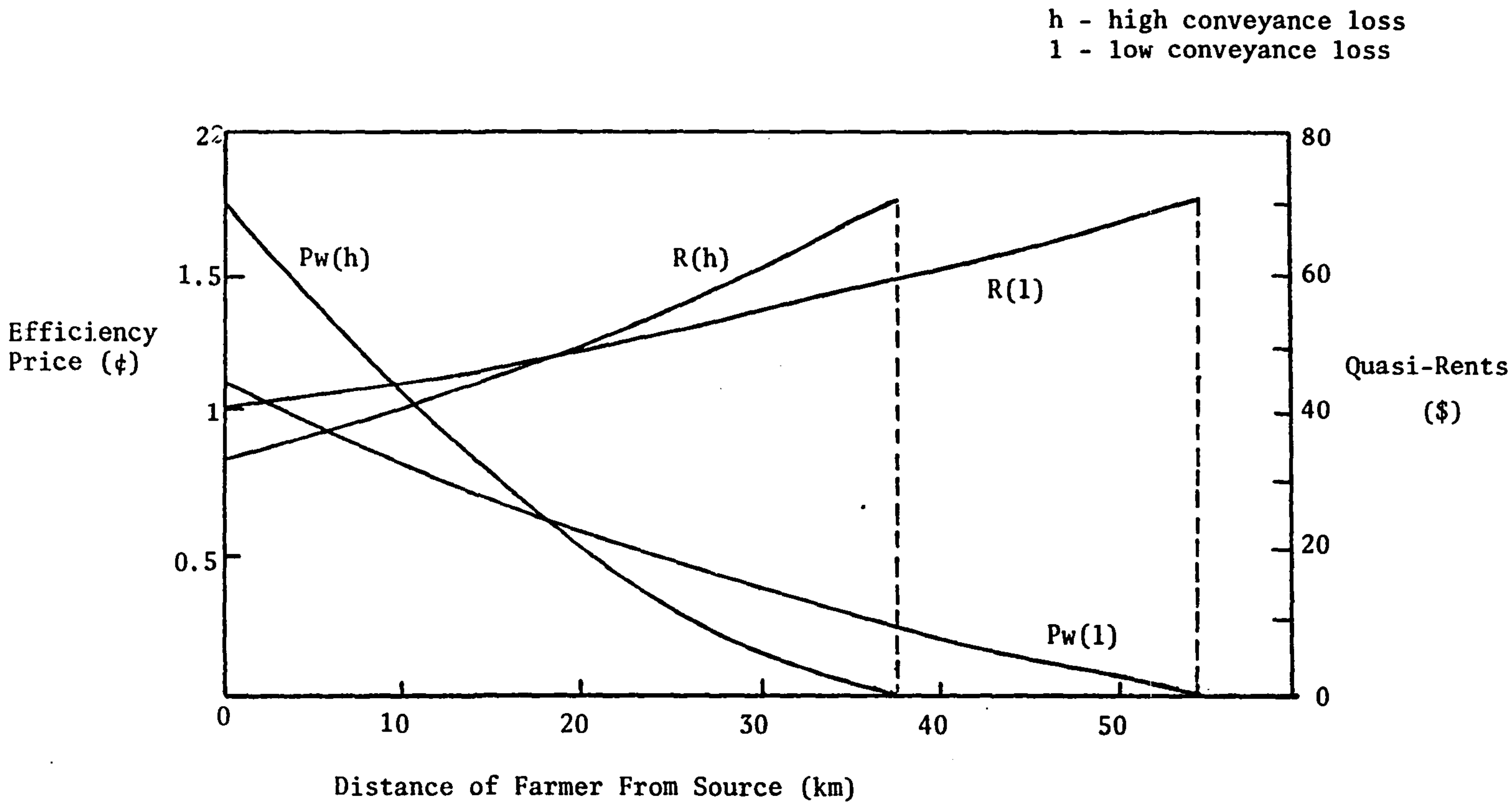



FIGURE 7. VARIATION OF EFFICIENCY PRICES AND QUASI-RENTS OVER SPACE FOR PAKISTAN

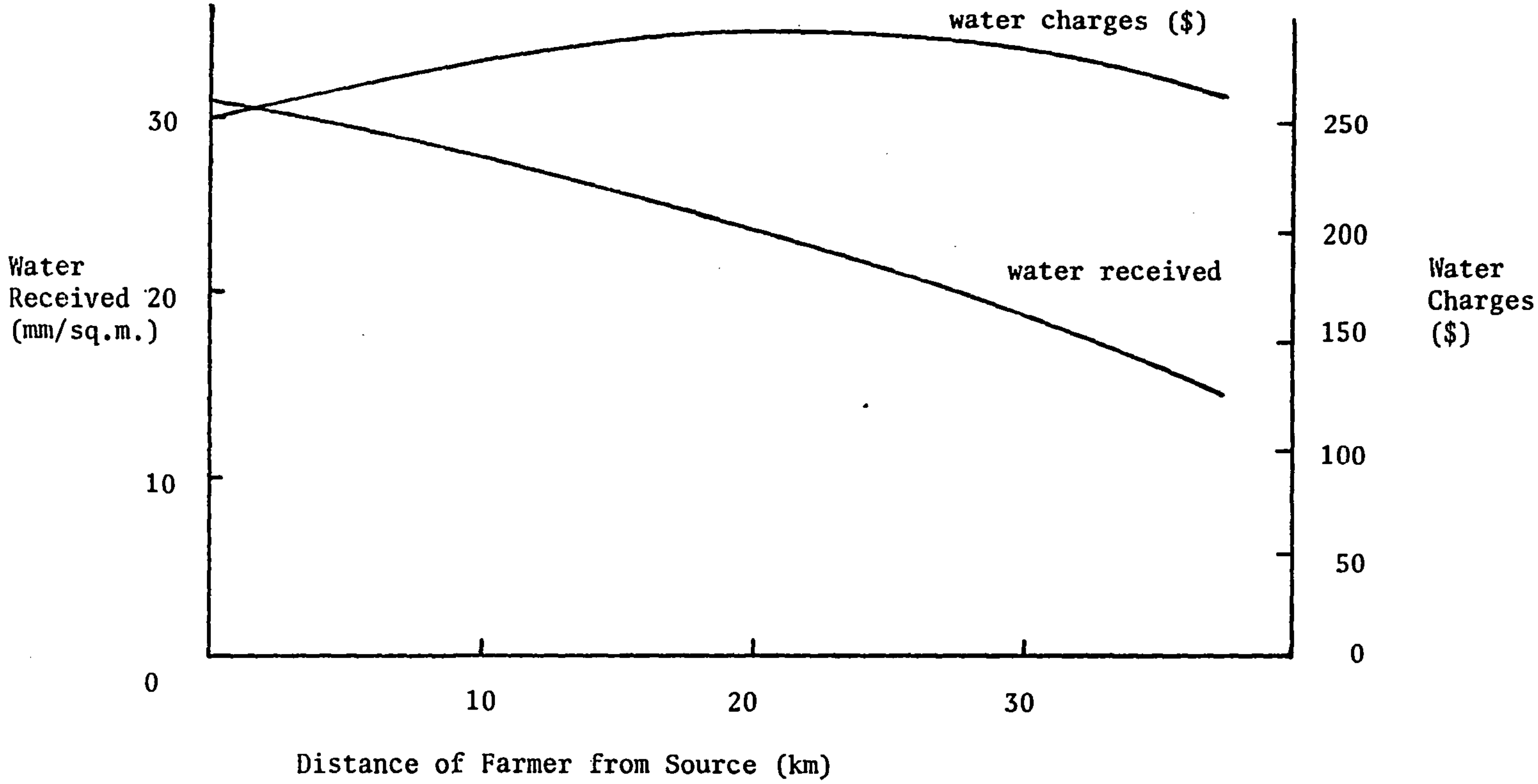



FIGURE 8. TOTAL PER FARM WATER CHARGES AND ALLOCATIONS IN PAKISTAN

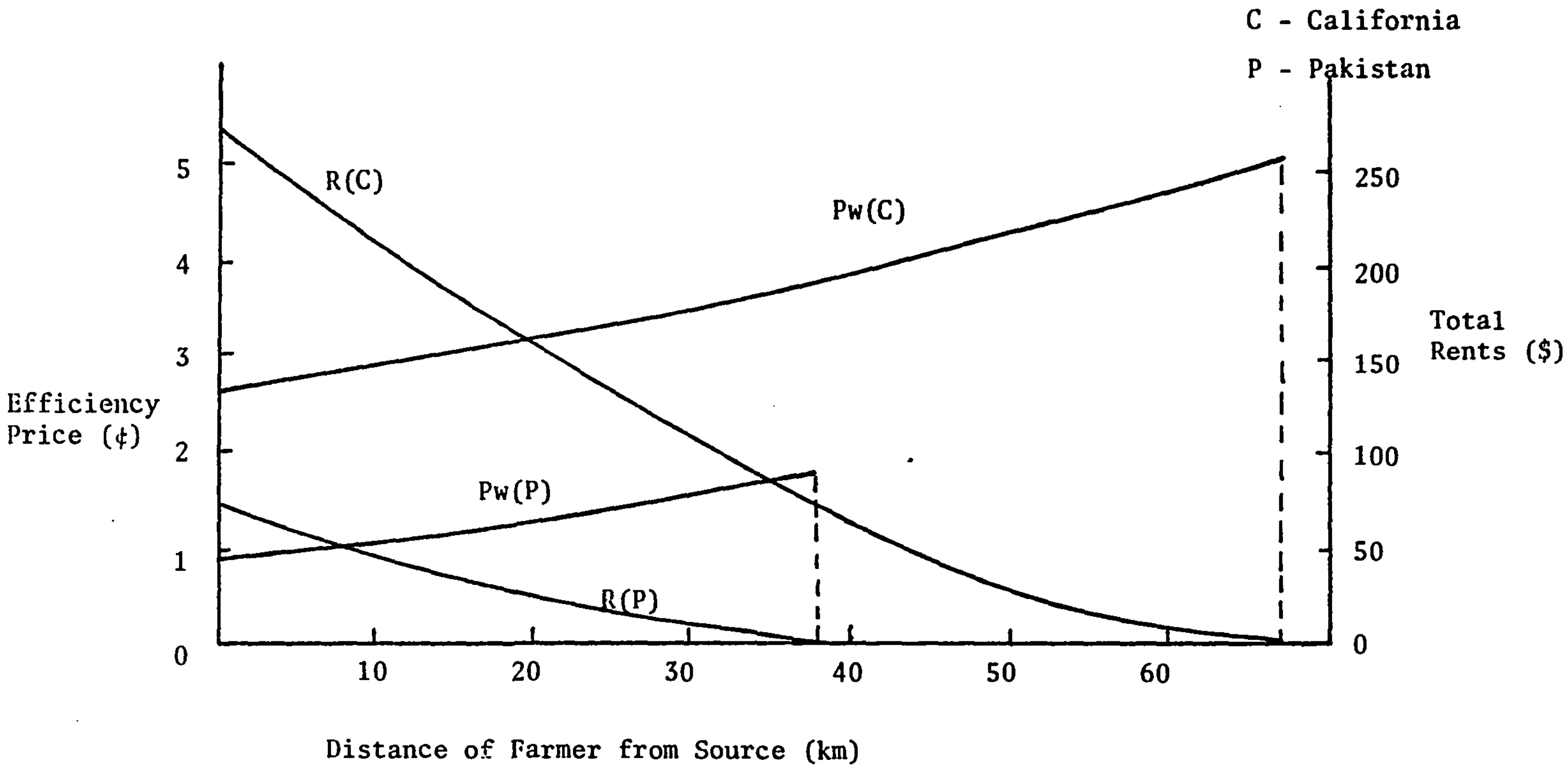



FIGURE 9. COMPARISON OF SPATIAL PRICES AND QUASI-RENTS FOR PAKISTAN AND CALIFORNIA

When compared to the different pricing schemes discussed, marginal cost pricing appears to be most inequitable while proportional benefit taxation permits all farmers to collect a higher degree of rents than if there was no taxation. The scheme that equalizes net rents across locations benefits tail farmers the most (Table 2), while equalising water charges has the advantage of administrative simplicity. Under very high conveyance losses, equalising net rents could result in tail farmers being subsidised for consumption of irrigation water.

## 2.5 Conclusions

We have shown that under conveyance loss regimes, efficient spatial allocation of irrigation water implies a continuous reduction in water delivered as the conveyance distance increases. In terms of water produced at the source (before subtracting conveyance losses), efficiency requires that greater amounts of water be allocated to further reaches of the system so long as the absolute value of elasticity of water demand is greater than unity. If the absolute elasticity of water demand is less than unity, the opposite is the case. Under the plausible assumption that the elasticity increases with a decrease in water use, this implies that the efficient allocation of source water increases with distance from the source, reaches a maximum where the elasticity of demand is minus one, and then decreases up to the system boundary.

We have also seen that efficiency prices increase and quasi-rents decrease away from the source. A variety of second-best pricing schemes can be adopted to make the spatial distribution of rents more uniform, the choice of which depends upon the welfare objectives of the irrigation agency and the nature of transactions costs in the system.

Schemes that tax proportional benefits or equalise net benefits seem to be most equitable, since they allow for moderate rents to accrue at the system tail. This could, however, lead to political pressures for expansion of irrigated area beyond its efficient size. Equalising water charges provides an attractive compromise by limiting rents to be gained from command area expansion, avoiding the additional administrative problems of differential pricing, and responding to the appearance of unfairness associated with paying more for less.

This model also provides a rigorous and operational basis for estimating irrigation benefits - by explicitly relating the area under the demand curve to the allocation rule. Existing methods of estimating irrigation project benefits have largely been based on ad hoc estimates of expected increases in revenues or profits, not on theoretical foundations that take into account the specific characteristics of irrigation systems. This helps to explain the gross overestimation of project benefits experienced worldwide.

The normative principles of allocation and water charges developed here should be thought of as a point of departure, not as a final solution. Actual choice of management and cost recovery systems will depend on the administrative costs and relative effectiveness of alternative arrangements. Additional research is needed to determine the effectiveness of alternative institutional arrangements. But in order to compare alternative institutional systems, one requires a metric of efficiency losses due to departing from the "first-best" allocation against which to compare the organizational costs of moving closer to that ideal.

In order to expand the scope of the model, the efficiency framework could also be extended to incorporate trade-offs between water allocation, reduced conveyance losses, expansion (or contraction) of the command area and supplementing tail water from additional sources. To facilitate practical applications, methods for classifying land quality can also be incorporated (see e.g., Caswell and Zilberman 1986).

Drainage can be included in the optimization exercise by including a cost function that removes the excess water from the fields, and waterlogging could be considered as a negative effect on crop yield (Reddy and Clyma 1981; Peri, Hart and Norum 1979). Water supply and conveyance systems can be optimized on the basis of rate, duration and frequency of water supply rather than the simple volumetric analysis performed here (Merriam 1987; Clemmens and Dedrick 1984).

Since the shape of the irrigation water demand schedule, especially the underlying marginal product function, is important for determining efficient water allocation, total system benefits, and horizontally equitable user fees, empirical work is needed to determine the shape of the marginal product schedule under varying environmental conditions. The framework suggested above can also be extended to estimate expected benefits where irrigation supplements stochastic rainfall.

## Notes to Chapter 2

1. The traditional warabandi system of irrigation prevalent in India and Pakistan charges equal price by crop type where each holding is allocated water by time. However, due to conveyance losses farmers at the tail pay the same price for less quantities of water delivered. Although this system is superior to systems that allocate equal quantities and charge equal prices, it is sub–optimal as compared to the allocation and pricing rules proposed here, and does not provide a theoretical basis for computing water charges and other system parameters.

2. Spatial pricing models have also been developed in the transportation and regional development literature (Takayama and Judge 1971; Judge and Takayama 1973; Guise and Flinn 1973). Most of these studies use programming algorithms to generate shadow prices of resources based on optimization of transportation costs over space. These shadow prices are then used as spatially constant parameters or are modified by a constant freight charge based on distance. As shown in this chapter, spatial efficiency implies resource allocation and pricing rules quite different than those proposed in the transportation literature.

3. The basic principles are somewhat similar to spot pricing models developed for electrical utilities (Vickrey 1971; Uri 1976; Bohn 1982; Caramanis 1982; Caramanis, Bohn and Schweppe 1982). The physical and engineering characteristics of irrigation systems require some modification of spot pricing models, however. For instance, since irrigation systems are usually smaller in size and conveyance efficiencies in water transfer are of a lower order of magnitude than in electrical power, policy questions such as the determination of the extensive margin, spatial equity among

farmers from head to tail, rent-seeking and cost recovery become important in the context of irrigation. On the other hand, energy and maximum safe capacity constraints in transmission lines and market regulation are more important considerations in electricity planning (Bohn, Caramanis and Schweppe 1984). Also the feasibility of basing water entitlements on farm characteristics makes it possible to separate the problems of efficient water allocation and appropriate water charges.

4. Tolley and Hastings (1960) incorporate conveyance losses to investigate the gains from reallocation of water from the North Platte river. Their model, however, does not yield general results. The model proposed here is not only more general, but considers the implications of conveyance efficiency on irrigation system design, farmer welfare, and proposes a set of pricing rules for attaining efficiency and equity objectives.

5. Evaporation losses are often considered unavoidable and ignored, since even if the water is stored and conveyed, it will evaporate (although at a different rate). Thus the major sources of conveyance loss are seepage and percolation.

6. Although, in contrast to electrical models, conveyance losses are usually not spatially constant, and depend on soil quality which can vary within the same irrigation scheme.

7. However, all the 'n' farmers need not be within the efficient 'boundary' of the system, since the allocation rule derived from the above optimization exercise gives an efficient boundary beyond which farmers will not receive any water allocation (discussed later in this section). Choosing a value of 'n' large enough will ensure the optimal solution (the allocation to some farmers may be zero).

8. For the purposes of allocation of capital costs among consumers, we assume homogeneity in demand. The marginal cost–based pricing rules, however, will, generally not be optimal if users are heterogenous, or there are decreasing costs or jointness of supply. Other pricing rules, such as those based on incremental pricing, might be more relevant (see Loehman and Whinston 1971; also Garrod 1978 for an application).

9. Note that these pricing rules will meet the desired objectives only if the actors have no prior knowledge of (i) the rents their land might accrue, and (ii) the proposed taxation scheme. This might be the case when new irrigation projects are being built. If the above conditions do not hold, our proposed taxation scheme will not work. For instance, if the rents from land are already capitalized in the value of the land, which might be true for irrigation projects already in place, the taxation scheme might produce perverse results. On the other hand, if farmers have prior information on both the spatial distribution of rents and the taxation scheme, the pricing rules we propose will have no effect on spatial equity.

10. Such a taxation program, however, might have to be made analytically simple for it to be understood by farmers.

11. The two-part function seems to be more accurate than the uniformly downward sloping curve usually assumed since at low quantities of water, on-farm losses through deep percolation and evaporation are proportionately higher, justifying a relatively high but constant marginal product over some range of water use. It can be treated as a special case of the more general concave, downward-sloping functions analysed in the previous sections.

12. Given the capital intensive orientation in California and the labor intensive pattern of agriculture in developing countries, long run marginal cost curves might be falling in California and rising in Pakistan. In that case, optimal system capacity in California might be larger than that computed in this section.

13. Although the average farm size in California might be larger than in Pakistan, restricting the farm size to 5 ha facilitates comparison, without affecting the central results.

14. Since width is assumed constant, irrigated area is proportional to canal length.

15. In a low conveyance regime, the spatial variation in water charges would be still lower, strengthening the case for uniform water charges over space.

# CHAPTER 3

# OPTIMAL INVESTMENT IN CONVEYANCE AND ON–FARM TECHNOLOGY

## 3.1. Introduction

In recent years, increased food production and expanding urban and industrial needs have led to a rising demand for water. Thus, governments the world over are making large–scale investments towards developing new water resources. For instance, in irrigation alone, developing countries will have invested more than 350 billion dollars by the turn of the century (Repetto 1986).

Policies for water management have traditionally emphasized the creation of new capacity through financing of newer and bigger projects (Rogers 1985). On the other hand, systems already completed suffer from inadequate investments in operation and maintenance. Many public irrigation projects, for example, 'deteriorate' within a few years of commissioning and experience a steady decline in performance. Distribution channels get clogged, control structures are altered or damaged, and farmers rarely make investments that would increase the on–farm efficiency of the water supplied (Repetto).

This 'bias' towards creation of new capacity and away from operation and maintenance is aggravated by the lack of a conceptual framework that determines optimal investments in irrigation system capacity, in maintenance and repair of conveyance structures, and in the choice of supplemental on–farm technology (Young and Haveman 1985). Given the spatial nature of public irrigation projects and the externalities associated with investing in water transfer, competitive market mechanisms do not ensure optimal investments in irrigation (Burness and Quirk 1979).

This chapter takes into account the specific technological characteristics of irrigation systems, and develops a model that incorporates the above externality in a spatial equilibrium framework.[1] The approach used is similar to that of dynamic control theoretic models with the parameters moving over space instead of time (Hochman, Pines and Zilberman 1977).[2] We determine optimal investments in water conveyance and in on-farm technology, and then compare the optimal outcome with "traditional" models that do not account for these externalities in irrigation design.

Through an empirical example, it is shown that significant economic gains could be achieved through investment policies that emphasize maintenance of existing facilities, and use of more efficient on-farm technology. Agricultural output and irrigated acreage could increase four fold, while net benefits can multiply almost three times.

These investments are found to have differential effects on farmer groups and on system efficiency and "equity". Conveyance improvements actually decrease the value of land and increase rents to water substantially. Private investment on the farm, however, has a positive effect on both land and water rents. While on-farm investments raise efficiency by increasing agricultural output, public investment in canals "saves" water and thus permits expansion of irrigated acreage. Improved conveyance is also more "equitable" since it reduces head-tail rent disparities. In contrast, irrigation structures that are allowed to 'deteriorate' through time allow head farmers to accrue a larger share of project benefits.

Section 3.2 develops a theoretical framework that incorporates both conveyance and on–farm investments, and derives results for optimal water use, pricing, output, and spatial allocation of investments. In Section 3.3 two stylized irrigation models are analysed — when conveyance investments are constrained to be spatially uniform, and when there is no on–farm technology choice. Section 3.4 compares the above models through an empirical example. Section 3.5 discusses implications for irrigation reform and concludes the chapter.

### 3.2. The Theoretical Framework

Consider a simple one–period (i.e., one irrigation season) model of an irrigation system that abstracts from considerations of time and uncertainty. Water is supplied from an in situ point source (e.g., a dam or a diversion structure) into a canal. Farms of homogenous land quality are located over a continuum on either side of the canal[3]. Farmers draw water at various locations 'x' along the canal, where 'x' is measured from the source. The spatial layout is as shown in Fig.1.

Let the spatial distribution of farm size at any 'x' be s(x). For simplicity we assume a uniform distribution $s(x)=d$ where 'd' is a positive constant[4]. Here 'd' can be interpreted to be the width of the system.

#### 3.2.1 Water Generation and Conveyance

Let the amount of water available at the source be denoted by $z_0$. The cost of generating water at source $C(z_0)$ is assumed to be an increasing, twice differentiable, convex function with $C'(z_0)>0$, $C''(z_0)>0$. It represents per period capital construction costs, and costs of operation (e.g., pumping) and maintenance of the head works.

Let $q(x)$ be the quantity of water applied on the farm at any location 'x' with $q(x) \geq 0$, and $l(x)$ be the conveyance loss per unit length of canal at any 'x', where $l(x) \geq 0$. Let '$z(x)$' be the cumulative quantity of water flowing in the canal through any location 'x', $z(x) \geq 0$. Then

$$(1) \quad z(x) = z_0 - \int_0^x [q(\tau)s(\tau) + l(\tau)z(\tau)]\, d\tau$$

where '$\tau$' is a variable of integration, and the first and second terms under the integral sign in (1) indicate, respectively, water applied on the farm and water lost in conveyance at each location 'x'. In addition, $l(x)$ can be written as

$$(2) \quad l(x) = l_0 - m(k(x))$$

where $k(x)$ is the investment in conveyance at any 'x', '$l_0$' represents the base loss — the proportion of water lost if there were no investments in conveyance (for instance, if the canals were unlined), $l_0 \varepsilon [0,1]$, and $m(k)$ is the fraction of water saved by conveyance investments $k(x)$.[5] Assume $m(k)$ to be an increasing, strictly concave, twice differentiable function, $m(k(x)) \varepsilon [0,l_0]$, $m'(k)>0$, $m'(0)=\infty$, $m''(k)<0$ which implies decreasing returns to scale in conveyance investments. Thus $l(x) \varepsilon [0,l_0]$ from (2).

Differentiating (1) with respect to 'x', we get

$$(3) \quad z'(x) = -q(x)s(x) - l(x)z(x)$$

where $l(x)$ is given by (2). Thus (3) implies that $z'(x) \leq 0$. If $X*$ is the length of the system (to be determined endogenously), then

$$(4) \quad z_0 = \int_0^{X*} [q(x)s(x) + l(x)z(x)]\, dx$$

which implies that $z(X*)=0$ from (1).

### 3.2.2 On-farm Production and Technology Choice

Farmers have a choice of investing in on-farm technology that increases the effectiveness of water applied, q(x). This is represented by an increasing, twice differentiable, concave function h(I) where 'I' is the amount of on-farm investment and $h(I)>0$; $h'(I)>0$; $h''(I)<0$. The price of 'I' is taken to be unity.[6]

Assuming monocropping[7], specify a production function of crop yield in terms of water applied, q(x) and investments in on-farm technology, h(I(x)) as $f(q \cdot h(I))$ where $f(\cdot)$ is twice differentiable and has the usual neoclassical properties, $f(\cdot)>0$; $f'(\cdot)>0$; $f''(\cdot)<0$.[8] For notational convenience define $e = q \cdot h(I)$ where 'e' can be termed "effective water"[9]. Conveyance losses perpendicular to the length of the canal can be incorporated into the model by specifying a production function that is well-defined in terms of water received at the farm-gate.

### 3.2.3 The Optimization Model

Let 'p' be the constant output price of the crop.[11] Applying the usual welfare criterion of maximizing net producer and consumer surplus, the problem as formally stated, is to find piecewise continuous, non-negative control functions q(x), k(x), I(x), optimal $X*$ and $z_0$ and an associated continuous and piecewise differentiable state function z(x) defined on the interval $[0,X*]$ that will[12,13]

$$\text{(5a)} \quad \underset{q,k,I,X*,z_0}{\text{maximise}} \int_0^{X*} \{[pf(qh(I))-I]s(x) - k\}dx - C(z(0))$$

subject to

(5b) condition (3),

(5c) $q(x),k(x),l(x),X_*\geq 0$

(5d) $z(x)\geq 0$

(5e) $z(0)$ free, $z(X_*)=0$, $X_*\varepsilon(0,\infty)$.

The problem defined in (5) is a standard optimal control problem with an initial salvage value, a pure state constraint and a free terminal condition. Associating auxiliary functions $\mu(x)$ and $\sigma(x)$ for each $x\varepsilon[0,X_*]$ to the differential equation (3) and the state constraint (5d), the Hamiltonian function and the corresponding Lagrangian for the maximization problem in (5a)–(5e) can be written as

(6a) $H(q,k,l,z,\mu) = [pf(qh(l))-l]s(x) - k - \mu[qs(x)+lz]$

(6b) $L(q,k,l,z,\mu,\sigma) = H(q,k,l,z,\mu) + \sigma z$

Formally, if $q,k,l,z,X_*$ comprise an optimal interior solution, then there exists a constant $\mu_o$, continuous and piecewise differentiable functions $\mu(x)$ and $\sigma(x)$ in $x\varepsilon[0,X_*)$, and a number 'ß' such that (5b)–(5e) is satisfied for all $x\varepsilon[0,X_*]$, and the necessary conditions are as follows:[14]

(7) $\delta H/\delta q = [pf'h(l) - \mu]s(x) = 0$

(8) $\delta H/\delta k = -1 + \mu z m'(k) = 0$

(9) $\delta H/\delta l = [pf'qh'(l) - 1]s(x) = 0$

(10) $\mu'(x) = -\delta L/\delta z = \mu l - \sigma$

(11) $\sigma(x) \geq 0$ ($=0$ if $z(x)>0$)

the terminal conditions[15]

(12) $L(x=X*)=0$

(13) $\mu(X*-) - (\mu(X*) = \beta$

and the salvage value condition

(14) $\mu_0 = C'(z(0))$.

Assume that the sufficiency conditions for optimization (as given for e.g., in Seierstad and Sydsaeter (1987), theorem 6.7 pp.377) are met. It is easily observed that from condition (3), $z'(x) \leq 0$ implies that if $z(x)$ were to be zero or negative in $x \varepsilon [0,X*)$, it could not later increase from that value (see Kamien and Schwarz (1978) for a similar argument). Hence from (11), $\sigma(x)=0$ for $x \varepsilon [0,X*)$ since the constraint is not tight in that interval, which implies that $\mu(x)$ has a jump discontinuity at X* given by (13).

3.2.4 **Interpretation of the Necessary Conditions**

It is obvious that $\mu(x)$ is the shadow price of each unit of water at location 'x'. Condition (7) states that at each point along the optimal path the shadow price of water '$\mu$' is equal to its marginal benefit in crop production, which is the marginal value product of a unit of water $pf'(\cdot)$ times the proportion of water 'h' that reaches the plant.

In condition (8) the benefits from water saved by an additional unit of 'k' is $\mu zm'(k)$ (since $zm'(k)$ is the amount of water saved by the marginal dollar of conveyance investment and '$\mu$' is its shadow price). Along the optimal path, this marginal benefit is equated to the marginal cost of investing in each unit of conveyance (assumed to be unity).

Similarly, condition (9) can be interpreted to mean that the marginal cost of each unit of investment in on–farm technology which is unity, equals its marginal benefit, the latter being equal to the value of marginal product of each unit of water ($pf'(\cdot)$) times the marginal efficiency of each additional unit of investment in on–farm technology, $h'(I)$.

Since $\sigma(x)=0$ for $x\varepsilon[0,X*)$, condition (10) implies that $\mu'(x)=\mu l$ for $x\varepsilon[0,X*)$ ($\mu'(x)=\mu-\sigma$ for $x=X*$). It indicates that in the interval $x\varepsilon[0,X*)$, the rate of change of shadow price is equated to the loss rate of water in conveyance. From (2), $l(x)\geq0$ which implies that the shadow price of water increases monotonically with distance from the source for $x\varepsilon[0,X*)$. Integrating (10) and using (14), we get

$$(15)\quad \mu(x) = \mu_0 e^{\int_0^x l(\tau)d\tau} = C'(z0) + C'(z0)[e^{\int_0^x l(\tau)d\tau} - 1] \text{ for } x\varepsilon[0,X*)$$

The shadow price of water increases at an exponential rate '$\int l(\tau)d\tau$' with 'x'.[16] It has a fixed component, the marginal cost of capacity $C'(z_0)$ and a variable component given by the cost of conveyance that is an increasing function of distance 'x'. When $x=0$, conveyance costs are zero, and thus (15) reduces to (14) indicating that the shadow price at source is just the marginal cost of capacity.

Notice that the first–best efficiency price of water will be its shadow price. Rearranging the terms in conditions (7), (8) and (9), we get

$$(16)\quad \mu = k/zm\varepsilon_m = l/q\varepsilon_h$$

where $\varepsilon_m$(=m'(k)k/m ) and $\varepsilon_h$(=h'(I)I/h) are the elasticities of conveyance and on–farm investments, respectively. It indicates that at the margin, the optimal solution equates the returns from investments in conveyance and on–farm technology. For instance, a higher conveyance or on–farm investment elasticity would imply higher investments for the same shadow price of the resource.

### 3.2.5 Behavior Along the Optimal Path

By differentiating the necessary conditions stated above, the following propositions are obtained that characterize the functions the functions q(x), k(x), I(x); output Y(x); and quasi–rents to land $R_L$(x). These results are obtained for the case when the elasticity of marginal product of effective water ($\varepsilon_f$'=f''qh(I)/f') is in the range $-\infty<\varepsilon_f'<-1$. This restriction ensures that production occurs well within the economic region of the production function and that a critical amount of water is needed to sustain production.[17]

Proposition 1. (a) Water applied q(x) decreases, while (b) on–farm investment I(x) increases monotonically with 'x', or q'(x)<0; and I'(x)>0[18].

Proof: (a) Differentiating (9) with respect to 'x' and simplifying, we get

f''h'(I)I'q/f'+f''h(I)q'/f'+q'/q+h''(I)I'/h'(I)=0 or

(17) $I'/I = -q'/q\ [(\varepsilon_f'+1)/(\varepsilon_f'\varepsilon_h+\varepsilon_h']$

where $\varepsilon_h'$=h''(I)I/h'(I) is the elasticity of marginal returns to on–farm investment. Again, differentiating (7) with respect to 'x', we get

$f'qh'I'/f' + f'q'h/f' + h'I'/I = \mu'/\mu = I(x)$ from (10), or $\varepsilon_f'q'/q + [(1+\varepsilon_f')/\varepsilon_h]I'/I = I(x)$.

Simplifying, and substituting for I'/I from (17), we obtain

(18) $q'/q\ [\varepsilon_f'\varepsilon_h' - \varepsilon_h - 2\varepsilon_f'\varepsilon_h] = [\varepsilon_f'\varepsilon_h + \varepsilon_h']I <0$

since $\varepsilon_f'<-1, \varepsilon_h'<0$; and $\varepsilon_h>0, I>0$ by assumption. Thus, on the left hand side of (18), $|\varepsilon_h|<|2\varepsilon_f'\varepsilon_h|$ and the entire bracketed term is positive, giving q'(x)<0.

(b) From Proposition 1(a) and condition (17), $-\infty<\varepsilon_f'<-1$ implies $l'(x)>0$.

The optimal trajectories of q(x) and l(x) are shown in Fig.10. An increase in the shadow price of the resource from head to tail causes a decrease in the amount of water used. On the other hand, an increase in the value of water makes investing in on–farm technology more worthwhile. So tail farmers receive less water, but make higher investments on the farm, relative to those located upstream. This result supports empirical findings that higher water prices lead to an increased adoption of modern irrigation technology.

Proposition 2. (a) Investment in conveyance, k(x), (b) output Y(x) and (c) quasi-rents accruing to land $R_l(x)$ all decrease monotonically with 'x', or $k'(x)<0$; $Y'(x)<0$; $R_l'(x)<0$.

Proof: (a) Differentiating (8) with respect to 'x', we get $\mu'/\mu + z'/z + m''(k)k' = 0$ or $\varepsilon_m'(k)k'/k = -\mu'/\mu - z'/z = q(x)s(x)/z$ using (3) and (10), where $\varepsilon_m'(k)=m''(k)k/m'(k) <0$ by assumption. Thus $k'(x)<0$.

(b) Since $e=h(l)q$, differentiating with respect to 'x', and simplifying, we get

(19) $e'/e = \varepsilon_h l'/l + q'/q$

Substituting for $l'/l$ from (17) and cancelling terms, we get

(20) $e'/e = [(\varepsilon_h' - \varepsilon_h)/(\varepsilon_f'\varepsilon_h + \varepsilon_h)]q'/q <0$

using Proposition 1(b) and $\varepsilon_h>0$, $\varepsilon_f'<-1$ and $\varepsilon_h'<0$.

The output function Y(x) can then be defined as $Y(x)=f(e)s(x)$, differentiating which we obtain $Y'(x)=f'(e)e'(x)s(x)<0$ using $e'(x)<0$, $f'(e)>0$, $s(x)>0$, $s'(x)=0$.

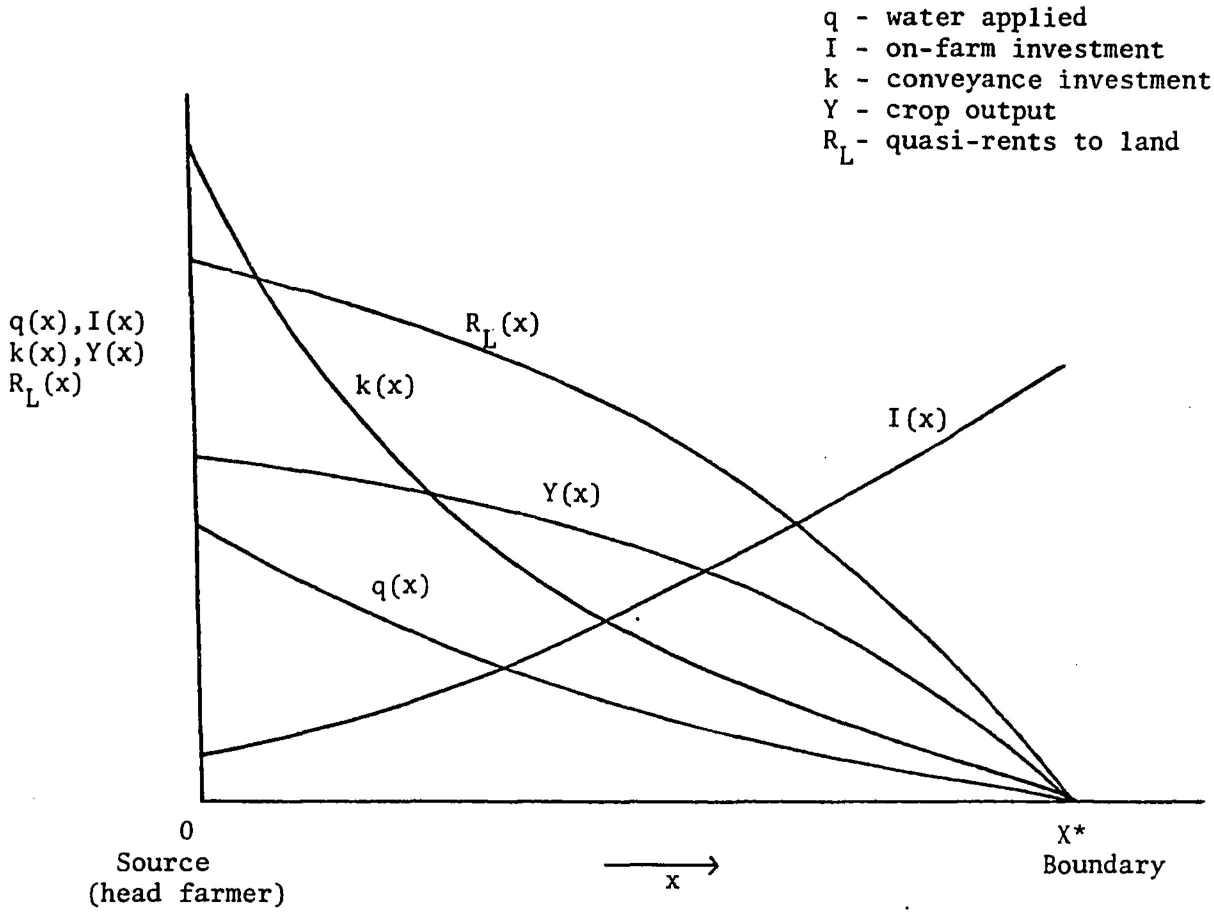



FIGURE 10. MOVEMENT OF VARIABLES IN THE OPTIMAL MODEL

(c) Quasi-rents $R_L(x)$ accruing at each location 'x' is given by

(21) $R_L(x) = [pf(qh(l)) - l]s(x) - \mu q(x)s(x)$

Differentiating (21) with respect to 'x' and using s'(x)=0, we get

$R_L'(x) = (pf'h'l'q + pf'q'h - l')s(x) - \mu'q(x)s(x) - \mu q'(x)s(x)$

Cancelling terms by using (7) and (10), we are left with

(22) $R_L'(x) = -\mu'(x)q(x)s(x) < 0$

using condition (15).

Fig.10 plots the movement of k(x), Y(x) and $R_L(x)$. Conveyance investments are higher nearer the source (k'(x)<0) because a larger volume of flow requires increased cross-section and depth of the canal. This is analogous to the familiar result in urban economics where the optimal width of road is a decreasing function of distance from the center of the city (see e.g., Solow and Vickrey 1971).

Monotone decreasing output and rent-distance functions indicate that the von Thunen results for land use are preserved. Since farmers located downstream benefit from the positive externalities of investing in conveyance at the head reaches, it implies that, left to the market mechanism, these investments will be sub-optimal. Head farmers would have little incentive to invest optimally in conveyance (as we shall see later, they have an incentive not to invest in conveyance, since head rents actually increase when conveyance is sub-optimal, see Proposition 3(c)).

The decrease in output from head to tail (Y'(x)<0) is the result of two opposing effects – a decrease in water use, and an increase in technology use on the farm. The net result is negative because although technology use increases, the marginal returns to investment (measured by the elasticity $\varepsilon_h' = h''l/h'$) decrease. This implies that the contribution of on–farm technology to yields does not increase at the same rate as the decrease in yields due to reduced application of water (see condition (19)). Hence effective water use decreases and so does output, leading to a higher intensity of cropping in the head region.

The monotonically decreasing output result is partly driven by our assumption of a continuum of on–farm technology choices being available (a continuous, monotone–increasing h(l) function). It is conceivable that if given a choice of say, two discrete technologies, one traditional and one modern, output could increase discontinuously in the neighborhood of the switch point. We could then observe decreasing output away from the head, a jump at the switch point, and again, decreasing output towards the tail.

To investigate the use of effective water at each location, we can rewrite condition (7) as $Pf'(e)=\mu/h(l)$. The left hand side then represents the 'price' of a unit of effective water. By differentiating the expression using the quotient rule, we get

$$d(Pf')/dx = [h(l)\mu'(x) - \mu h'(l)l'(x)]/h^2>0$$

using (10), (19), (20) and Proposition 1(a) and some algebraic manipulation. Thus the 'price' of effective water increases from head to tail.

From Fig.10 the negative slope of $R_L(x)$ (sometimes called the 'rent–distance function' after Muth (1961)) indicates that head farmers accrue higher rents than those at the tail. This is obvious given the low shadow price of water nearer the source. Condition (22) indicates that the slope of the rent–distance function moves with the slope of the conveyance loss function, $l(x)$. Thus higher conveyance loss rates between any two locations would increase rent differentials, because higher conveyance losses increase the shadow price (through condition (10)).

The irrigation system is terminated when the net private benefits from supplying an additional unit of water equal the shadow cost of the resource. This can be seen from expanding condition (12) at $x=X*$, the boundary of the system,

$$L(X*) = [pf(qh(l) - l]s(x) - k(x) + \mu z' + \sigma z = 0$$

where all the parameters are evaluated at $x=X*$. Thus, $z(X*)=0$ from (4), and using Lemma 1 (see Appendix to Chapter 3) we get

$$[pf(q(X*)h(l(X*)) - l(X*)]s(X*) = -\mu(X*)z'(X*).$$

The left hand side gives the net benefit of bringing one more unit of land under irrigation, and the right hand side its cost.

### 3.3 Comparisons With Stylized Irrigation Systems

In this section, the optimal spatial model (with conveyance and on–farm investments as choice parameters) developed in section 3.2 is used as a basis for comparison with two situations that approximate real–world irrigation practices, when (a) conveyance investments are spatially uniform, and (b) there is no on–farm technology choice.

#### 3.3.1 Uniform Conveyance

Irrigation practices typically cover a broad spectrum and range from no investment in lining to partial lining of the canal, and at best a classification of canal cross-sections into primary, secondary and tertiary types, with differential investments for each type. Standard irrigation engineering textbooks, however, prescribe a lining of uniform quality that maximizes engineering efficiency (e.g., see Walker and Skogerboe 1987).

In order to determine 'k' endogenously, an additional constraint of the form $k'(x)=0$ is imposed on the optimal model. Thus the optimization procedure can be accomplished in two stages (Hochman and Zilberman 1985). First determine the optimal allocation for each exogenous 'k', then choose the constant 'k' that maximizes net benefit. The first stage is similar to the maximization problem in (5) with 'k' fixed, and control and state variables $q(x)$, $l(x)$, $X*$ and $z(x)$ respectively. The relevant necessary conditions are (7) and (9)–(14).[19]

In the second stage $q*(x)$, $l*(x)$, $X*$, and $z_0*$ are taken to be the optimal allocations corresponding to the first stage, and '$\pi(k)$' is defined to be the objective function that is to be maximized over 'k' as follows:

$$\pi(k) = \int_0^{X*} \{[pf(q*h(l*)) - l*]s(x) - k\}dx - C(z_0*)$$

Differentiating $\pi(k)$ with respect to 'k', using conditions (4) and (2), and equating to zero, we have

$$\delta\pi/\delta k = \int_0^{X*} (-1)\, dx + C'(z_0*) \int_0^{X*} zm'(k)dx = 0 \quad \text{which gives}$$

$$(23) \quad C'(z_0*)m'(k)(1/X*)\int_0^{X*} z(x)\, dx = 1$$

Condition (23) is the equilibrium condition that determines optimal investment in uniform conveyance and is analogous to condition (8). It states that the unit cost of conveyance is equated to the 'average' benefit of the water saved. Notice that the unique value of 'k' in this system is determined by averaging across all values of z(x), while in the optimal model each z(x) determines an optimal 'k(x)' at each location. Moreover, the shadow cost of the water used here is its marginal cost at source $C'(z_0)$, rather than $\mu(x)$ as in the optimal case. This model is thus spatially 'myopic' in the sense that it does not take into account the differential flow of water at each location in the canal, nor the contribution of conveyance losses to the shadow price of the resource.

Conditions (7) and (9) and Propositions (1) and (2) imply that with uniform conveyance investments, water used q(x), output Y(x) and quasi-rents $R_L(x)$ will decrease with 'x', and the amount of on-farm technology I(x) will increase with 'x', as in the optimal model.

The following propositions compare the above model with the optimal model developed in Section 3.2. We make the simplifying assumptions that $I^u(x)=I*(x)$[20] and that the control functions q(x), k(x), and I(x) are continuous in the relevant intervals $[0,X*)$ and $[0,X^u)$, where '*' and 'u' denote the parameters in the optimal and sub-optimal models respectively.[21]

Proposition 3. When conveyance is spatially uniform, (a) aggregate water used by the system is less than the optimal, $z_0^u<z_0*$, and (b) water applied $q^u(x)$ and (c) quasi-rents accruing to land, $R_L^u(x)$, are higher than optimal at the head.

Proof: (a) The maximization problem in (5) can be written in a parameterized form as

(24) Max $B(q*(x), l*(x), z_0*(x), X*, k*(q, l, z_0, X))$ subject to (5b–5e)

where $B(\cdot)$ represents the net benefit function, and $k*(\cdot)$ indicates the choice of an optimal 'k' for corresponding values of the other parameters. Applying the envelope theorem (Varian 1984, pp.327) to (24) when k(x) is a constrained parameter, we obtain the result $z_0*>z_0^u$.

(b) Using (16), Lemma 2(b), and the assumption $l^u(x)=l*(x)$, we get $q^u(x)>q*(x)$ for all $x \in [0,\varepsilon]$.

(c) Evaluating condition (21) at x=0, we get

$R_L^u(0)-R_L*(0)=ps(x)[f(q^u h(l^u))-f(q*h(l*))]-s(x)[\mu_0^u q^u-\mu_0*q*+l^u-l*]$

Using the assumption $l^u(x)=l*(x)$, (16) gives $\mu_0^u q^u=\mu_0^u q*$. Proposition 3(b) and $f'(\cdot)>0$ together imply $R_L^u(0)-R_L*(0)>0$. From (22), differentiability of $R_L(x)$ implies continuity in the interval $[0,x*)$. This implies that we can find an $\varepsilon>0$ such that $R_L^u(x)>R_L*(x)\ \forall\ x \in [0,\varepsilon]$.

Let $X_1=(x \mid k*=k^u)$. Then we can state the following proposition that relates conveyance investments and irrigated acreage in the two models:

Proposition 4: (a)(i) If $X^u \leq X_1$, conveyance investments are lower throughout the system and (ii) if $X^u>X_1$, conveyance investments are lower than optimal at the head and higher than optimal towards the tail. (b) $k^u(x) \leq k*(x)$ is a sufficient condition for irrigated area under uniform lining to be smaller than optimal.

Proof: (a) Proposition 3(a) and Lemma 2(c) (see Appendix) and the assumption $X^u \leq X_1$ give (i). By Lemma 2(c) and Proposition 2(a), $k*(x)>k^u(x)$ for $x \varepsilon [0,\varepsilon]$ and $k*(x)$ is continuous and monotonically decreasing. Since $k^u(x)>0$ (nontrivially), Lemma 1 implies there exists $k*(x)<k^u(x)$ for $x>X_1$, yielding (ii).

(b) Since s(x) is constant, irrigated area is proportional to the length of the system. Thus we only need to show that $X*>X^u$. The proof is by contradiction. Let the boundary of the uniform lining model, $X^u$ be larger than $X*$, the boundary of the optimal model, that is $X^u=X*+X\sim$ for some $X\sim\geq 0$. Then by condition (4),

$$z_0^u = \int_0^{X^u} (q^u s + l^u z^u)dx = \int_0^{X*} (q^u g + l^u z^u)dx + \int_{X*}^{X^u} (q^u s + l^u z^u)dx$$

Assumption $k*>k^u$ gives $l*<l^u$ from condition (2), and Proposition 3(b) gives $q^u(x)>q*(x)$. Thus

$$z_0* < \int_0^{X*} (q^u s + l^u z^u)dx \Longrightarrow z_0^u > \quad z_0* + \int_0^{X^u} (q^u s + l^u z^u)\, dx$$

which contradicts Proposition 3(a).

The net benefits in the optimal model are higher than in the constrained sub–optimal model, and with a rising marginal cost function, this results in a larger equilibrium value of $z_0*$ (where the marginal benefit of increasing capacity equates its marginal cost).

The two bottom panels of Fig.11 show the path of shadow prices and water use in the two models.[22] These results indicate that uniform conveyance reduces system

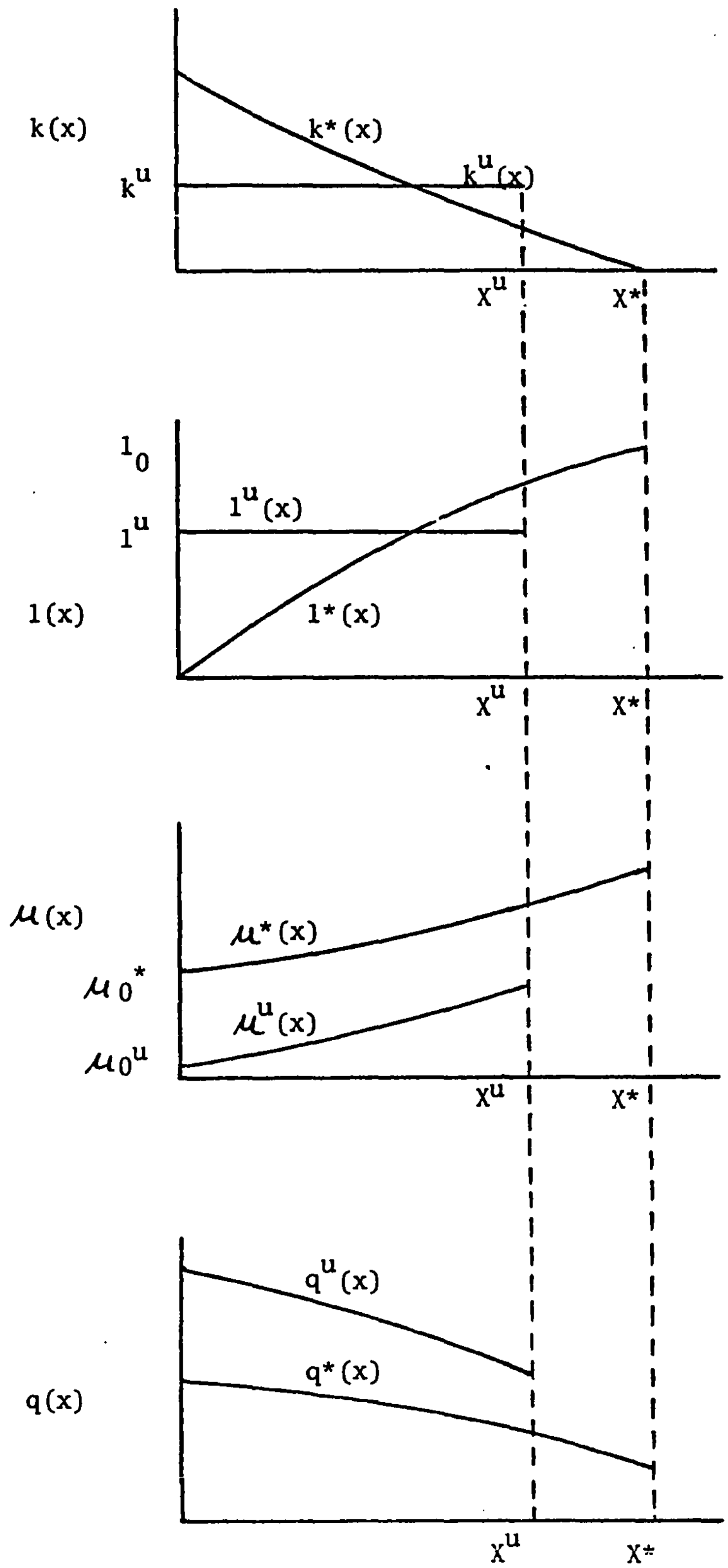



FIGURE 11. COMPARISON OF MODELS WITH OPTIMAL AND SUBOPTIMAL (UNIFORM) LINING WHEN $\mu^*(x) > \mu^u(x)$ EVERYWHERE

benefits and causes less water to be generated at the source. This in turn reduces the shadow price of the resource at source. As shown in the figure, head farmers benefit from the lower price of water by using too much of it and by accruing super–normal rents. Thus the twin effects of a lower water stock and excess withdrawal by head farmers 'shrinks' the system, resulting in a smaller irrigated area.

The two top panels in Fig.11 show the relative movement of k(x) (and hence l(x) from condition (2)) in the two models. The 'averaging' effect of the equilibrium condition (23) causes less than optimal conveyance investments at the head region, while in the tail region, both over and under investment are possible (notice that since two different regimes are being compared, their 'tail' regions do not share the same location).

Note also from Fig.11 that $l''(x)$ is constant in the sub–optimal model causing the shadow price to grow at a constant rate, as compared to an increasing rate in the optimal model.

### 3.3.2 No On–farm investment

Irrigation systems, both in developed and developing countries, are typically built without taking into account the possibility of augmenting water productivity by private on–farm investments (Repetto, op. cit.). In this section we examine a model that provides for optimal canals but does not include on–farm technology choice as a decision parameter.

The basic framework described earlier can be extended to the case $I(x)=0$ by specifying $h(I=0)=h_0$, where '$h_0$' is a constant, and is a measure of on–farm efficiency when no investments are made. It can be conceived as the ratio of the amount of water reaching the plant to the amount delivered at the farm–gate for $l(x)=0$. Thus $0 \leq h_0 \leq 1$ and since $h'(l)>0$, $h_0<h(l)$ for all $l>0$.

The maximization problem in (5) remains exactly the same, except that $h(l)=h_0$, and $l(x)$ is no longer a decision variable. The necessary conditions in this model are conditions (7), (8), and (10)–(14). Following Propositions (1) and (2), it is clear that water used $q(x)$, conveyance investments $k(x)$, output $Y(x)$ and quasi–rents $R_L(x)$ will decrease away from the source.

Using the symbols '$*$' and 'c' to denote the parameters in the optimal and constrained models respectively, we can state the following proposition:

<u>Proposition 5. When there is no on–farm investment, (a) system capacity is smaller than optimal. When $\mu*>\mu^c>h*/h_0$, (b) water applied is higher than optimal, $q^c>q*$,[23,24] (c) conveyance investments are less than optimal, $k*>k^c$,[25] and (d) irrigated area is smaller than optimal.</u>

<u>Proof</u>: (a) The proof parallels that of Proposition 3(a), giving $z_0^c<z_0*$.

(b) From (7), $pf'(qh)h=\mu$ which implies $f'(e*)/f'(e^c)=(\mu*/\mu^c)(h_0/h*)$. When $\mu*/\mu^c>h_0/h*$, $f'(e*)>f'(e^c)$ and $e*<e^c$; $h*(l)>h_0$ gives $q*<q^c$.

(c) Applying Proposition 5(a) and Lemma 2, we get $k*(x)>k^c(x)$ for $x\varepsilon[0,\varepsilon]$. Using Lemma 2 again in the condition $\varepsilon_m'(k)k'/k=q(x)s(x)/z$, $q^c>q*$ (by Proposition 5(b)), and $|\varepsilon_m'(k^c)|<|\varepsilon_m'(k*)|$ since $\varepsilon_m'(k)=m''(k)k/m'(k)$ and $k^c<k*$ and $m'(k^c)>m'(k*)$, we get $|k^{c\prime}/k^c|>|k*'/k*|$. Thus $k^c<k*$ and $k'/k<0$ by Proposition 2(a), implies $k^c(x)<k*(x)$ for all $x$.

(d) The proof is very similar to that of Proposition 4(b) and is not repeated.

Figure 12 plots the variables in the model without technology choice relative to the optimal model. As in the earlier model, a lower marginal cost of water causes excess application of water. A lower shadow price of water (and smaller volume of flow) induces less than optimal investments in conveyance, as shown in the figure. The net effect, again, is a smaller irrigated area.

Unlike in the previous model, the behavior of rents at head is ambiguous, and depends on the opposing effects of higher water use (which increases rents) and an absence of yield–increasing technology (which decreases rents).

### 3.4 An Illustration

The above models are illustrated by assuming supply and demand parameters that closely represent the physical and technological properties of irrigation systems. We perform simulations to determine the effect of decision parameters such as conveyance investments and on-farm technology on spatial water allocations, efficiency prices and quasi-rents accruing to land and water, and on stock of water at source, irrigated area, crop output and aggregate net benefits of the system.

#### 3.4.1 The Production Function

We derive a quadratic crop production function for California cotton in terms of effective water such that a maximum yield of 1,500 lbs. can be obtained with an effective water application (i.e., an evapotranspiration requirement) of 3.0 acre–feet

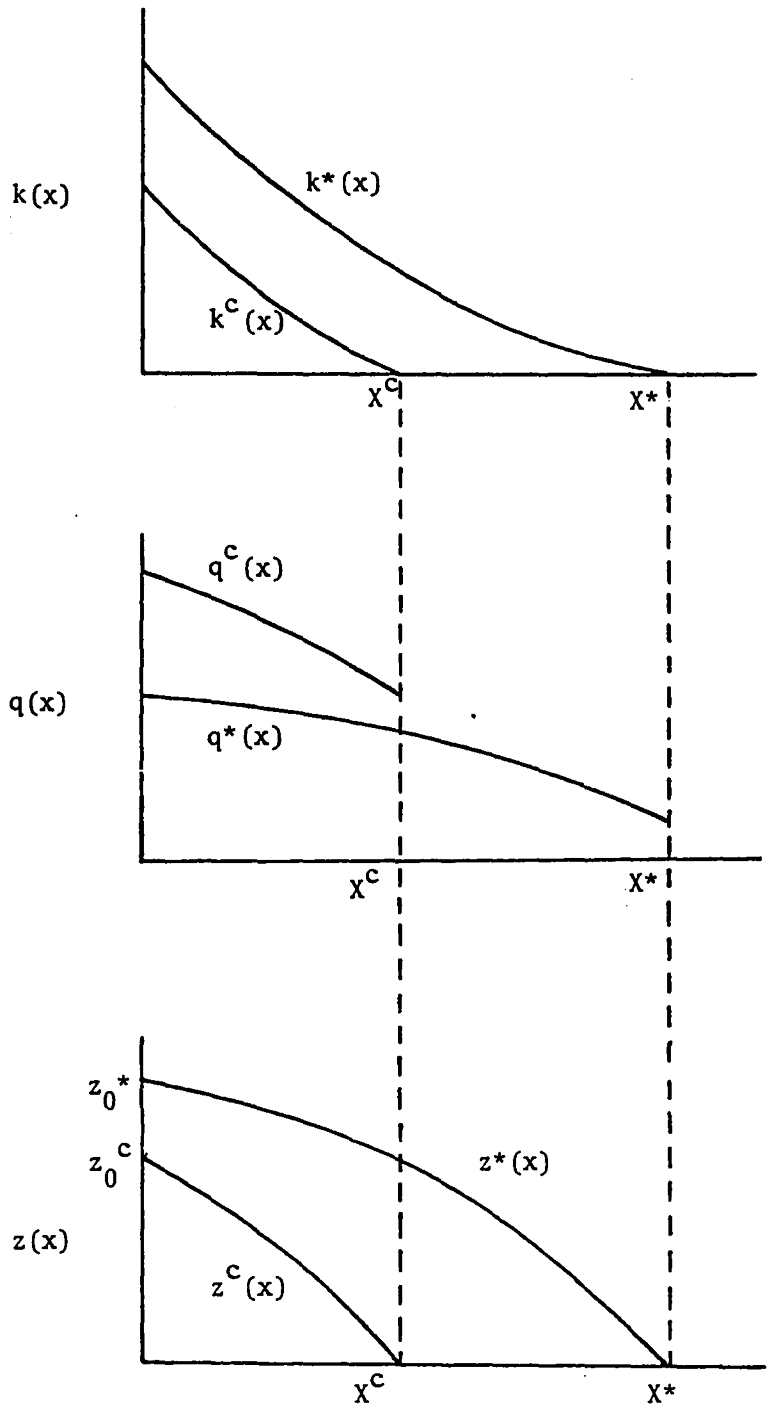



FIGURE 12. COMPARISON OF MODELS WITH AND WITHOUT ON-FARM INVESTMENT

WHEN $\mu^*(x)/\mu^c(x) > h^*/h_0$

and a yield of 1,200 lbs. with 2.0 acre–feet (W.M. Hanemann et al., 1987). Using cotton prices of 0.75 per lb., and converting into metric units, we obtain the revenue function

(25) $Pf(e) = -0.2224 + 1.0944*e - 0.5984*e^2$

where revenue is in US dollars, and 'e' is in $m/m^2$ of water. Differentiating (25) with respect to 'e', we get the value of marginal product function

(26) $Pf'(e) = 1.0944 - 1.1968*e$

### 3.4.2 The On–Farm Investment Function

The on–farm investment function is approximated from per acre cost estimates of investing in irrigation technologies in California (Hanemann et al. 1987). They range from traditional labor-intensive techniques that save water delivered to the field by cutting run-off, to installing modern sprinkler and drip systems. The relevant cost estimates for different levels of technology choice are shown in Table 3. We assume that when furrow irrigation is applied, there is no investment cost to the farmer, and irrigation effectiveness $h(I)(=h_0)$ is 0.6, i.e., only 60 per cent of the water delivered at the farm–gate reaches the plant. The irrigation effectiveness increases at a decreasing rate as more sophisticated technologies such as sprinkler and drip are employed. We assume that h(I) can be approximated by a continuous function of 'I', and write the investment function as

(27) $h(I) = 0.6 + 21.67*I - 333.3*I^2$

where h(I) is in efficiency units per $m^2$ and 'I' is in dollars.[26]

TABLE 3. UNIT COST OF ON-FARM TECHNOLOGY

| Technology | Cost ($/acre) | h(I) |
| --- | --- | --- |
| Furrow Irrigation | 0 | 0.6 |
| Short-Run | 20 | 0.7 |
| Hand-move Sprinkler | 60 | 0.8 |
| Drip | 120 | 0.95 |

Source: Adapted from Hanemann et al. (1987).

3.4.3 **Cost and Conveyance Functions**

Fixed costs for irrigated farming were taken to be $433 per acre or 0.107/$m^2$ (University of California, 1988). A quadratic function for conveyance investments was constructed from average lining and piping costs in 17 states in western United States (U.S. Department of the Interior et al., 1979, Table 15, p.87). An investment of $100/m length of canal in piped systems[27] results in zero conveyance losses in the system. Concrete lining with an investment of $50/m achieves a loss factor of $10^{-5}$/m or a conveyance efficiency of 0.8 over a 20 km length of canal.[28]

When k=0, the loss factor is $4*10^{-5}$/m giving an overall conveyance efficiency of 0.2. We get

(28) $l(k) = 4*10^{-5} - (8*10^{-7}k - 4*10^{-9}k^2)$

where from condition (2), $l_0=4*10^{-5}$, and

(29) $m(k) = 8*10^{-7}k - 4*10^{-9}k^2, \quad 0 \le k \le 100.$

It is obvious that $m'(k) \ge 0$, $m''(k) \le 0$.

A rising long–run marginal cost function for water supply was constructed from average water supply cost data from 18 irrigation projects in the western United States (Wahl 1985 cited in Repetto, op. cit., p.16) as

(30) $C'(z_0) = 0.00757 + (3.785*10^{-11}z_0)$

where marginal cost is in US dollars and $z_0$ is in cu.m.. This function gives a marginal cost of 0.00757\$/$m^3$ (9.34 dollars per acre–foot) when $z_0$=0, and marginal cost values in the range 0.068 to 0.16 dollars/$m^3$ (83.84 to 197.28 dollars per acre–foot) for the various models analyzed (see Table 4). Although the marginal cost function can vary with specific water generation or diversion technologies, projects, or geographical regions, a linear form is assumed to keep the formulation simple.

The cropped area is taken to be rectangular in shape with farms located on either side of the canal. In addition, we assume the system width to be $s(x)=d=10^5$m.

### 3.4.4 The Algorithm

A computer algorithm was written that starts by assuming a value of $z_0$, and computes $\mu_0$ from condition (14). At x=0, condition (8) gives m'(k). By iterating on 'k', we compute k(x) that satisfies (29), and (28) gives l(x). Knowing $\mu_0$, conditions (7) and (9) used simultaneously yield l(x), q(x) and thus e(x), Y(x) and $R_L(x)$ respectively. Next, when x=1, knowing l(x=0) and $\mu_0$, (15) gives $\mu(x)$, and z(x=1) is obtained from (1) by subtracting the water already used up previously. Again, $\mu(x=1)$ and z(x=1) give k(x=1) from (8) and the cycle is repeated to give q,l, etc. at the location x=1. The process is continued until exhaustion of $z_0$ terminates the cycle, and a new value of $z_0$ is assumed. Aggregate land rents are calculated for each $z_0$ by summing over each $R_L(x)$ (using condition (21)) and aggregate rents to water $R_W$ are computed from the relation

$$\int_0^{X*} R_W \, dx = \int_0^{X*} [\mu(x)q(x)s(x) - k(x)]dx - \int_0^{z_0} C(\tau)d\tau.$$

Therefore, the aggregate net benefits NB of the system are given by

$$NB = \int_0^{X*} [(pf(qh(l))-l)s(x) - k(x)]dx \quad - \quad \int_0^{z_0} C(\tau)d\tau$$

It is easily seen that the total net benefits are equal to the sum of the aggregate quasi–rents to land and water. The algorithm selects the value of $z_0$ that maximizes total net benefits.[29]

### 3.4.5 Simulation Results

We performed simulations for a set of models that represent a variety of management options in irrigation (Table 4). Model 1 is the optimal case (analyzed in section 3.2) with conveyance and on–farm investments while model 4 is the 'primitive' case without conveyance or on–farm investments and is a stylized example of irrigation systems that have deteriorated sharply over time (for documentation of such cases, see Repetto, op. cit. and Wade (1982)). Intermediate between these two polar extremes are models 3 and 4, which have on–farm technology (but no conveyance), and conveyance (without on–farm technology), respectively. As we shall see later in this section, these two models reveal the trade–offs between private and public investment. Model 5 examines the case of spatially uniform conveyance investments. Note that models 3 and 5 have been analysed earlier in section 3.3. We have not analysed models 2 and 4 separately in the theoretical sections mainly because they are special cases of models 1 and 3 respectively, and the analysis is straightforward.

Table 4 summarizes the results which can be described as follows:

# TABLE 4. SUMMARY STATISTICS

| Model | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Parameter | Optimal With k,I | Without k With I | With k Without I | 'Primitive' Without k,I | Model 1 with k constant |
| $A(10^3$hectares) | 490 | 140 | 230 | 100 | 430 |
| $z_0(10^6$cu.m) | 41 | 18 | 29 | 17 | 37 |
| Aggr.$Y(10^6\$)$ | 13.28 | 4.09 | 6.31 | 2.97 | 11.77 |
| Aggr.$R_L(10^6\$)$ | .136 | .872 | .309 | .566 | .657 |
| Aggr.$R_w(10^6\$)$ | 3.254 | .658 | 1.711 | .614 | 2.663 |
| Aggr.NB$(10^6\$)$ | 3.39 | 1.53 | 2.02 | 1.18 | 3.32 |
| $NB/z_0$($/cu.m) | .83 | .85 | .70 | .69 | .90 |
| $Y/z_0$($/cu.m) | .32 | .23 | .22 | .17 | .32 |
| $A/z_0(m^2$/cu.m) | 1.20 | .78 | .79 | .58 | 1.16 |
| $k_{head}$($/m) | 98.61 | 0 | 98.41 | 0 | 92.0 |
| $I_{head}(\$/m^2)$ (in $/acre) | .023 (93.1) | .015 (60.7) | 0 (0) | 0 (0) | .022 (89.05) |
| $I_{tail}(\$/m^2)$ ($/acre) | .023 (93.1) | .021 (93.1) | 0 (0) | 0 (0) | .022 (89.05) |
| $q_{head}(m/m^2)$ | .835 | .993 | 1.26 | 1.366 | .858 |
| $q_{tail}(m/m^2)$ | .835 | .879 | 1.26 | 1.288 | .854 |
| $e_{head}(m/m^2)$ | .77 | .84 | .76 | .82 | .79 |
| $e_{tail}(m/m^2)$ | .77 | .80 | .76 | .77 | .78 |
| $\mu_{head}$($/m3) ($/acre-ft) | .159 (196.05) | .072 (88.78) | .113 (139.33) | .068 (83.84) | .14 (172.62) |
| $\mu_{tail}$($/m3) ($/acre-ft) | .159 (196.05) | .126 (155.36) | .113 (139.33) | .102 (125.77) | .16 (197.28) |
| $R_{Lhead}$(106$) | .28 | 8.16 | 1.29 | 7.25 | 1.56 |
| $R_{Ltail}$(106$) | .26 | 3.12 | 1.28 | 2.81 | 1.43 |

Notes:

1. A = area irrigated, Y = crop output, $R_L$ and $R_w$ = quasi-rents accruing

to land and water respectively, $z_0$ = total water used, NB = net benefit.

2. Since area irrigated is different in each model, the location of the tail farmer is unique to each model.

3. '$\mu$' represents the shadow price of water, which is also the efficiency price (see discussion in text).

1. From the 'primitive' (model 4) to the optimal model (1), area irrigated and agricultural output multiply fourfold, while total net benefits increase three times. These results show the economic gains from optimal maintenance of irrigation systems.

2. Aggregate rents to land ($R_L$) decrease when conveyance structures are improved (compare models 1 and 2, and models 3 and 4), a result that runs counter to the general notion that conveyance investments are capitalized in the value of the land. This finding is consistent with the literature following the Tiebout theorem, where it has been shown that public good improvements need not always improve the value of nearby property (see Stiglitz 1977; Hartwick 1980).

3. Land rents accruing to farmers located upstream decrease in the optimal model (check $R_L$ at the head and tail; also shown in Proposition 3) leading to a more 'equitable' distribution of rents from head to tail. Points 2 and 3 together indicate that landowners in general, and head farmers in particular, lose from improved maintenance of irrigation facilities.[30]

4. Quasi–rents accruing to water ($R_W$) increase when public and private investments are made. This suggests that agencies that supply water to farmers, provide canal maintenance, and collect water charges could be made financially viable.

5. The 'intensive' and 'extensive' nature of private and public investments is easily seen. Use of modern on–farm technology without optimal maintenance increases total output and rents but has a smaller impact on irrigated acreage and total water use. Improvements in conveyance, however, expand acreage and use more water.

6. Under conveyance investments, head–tail shadow prices and water allocations are almost equal (check model 2 and 3).[31] All farmers invest in sprinkler irrigation, and cropping intensities are spatially uniform.

7. Model results with constant lining quality (model 5) suggest that conveyance

investments, even if spatially uniform, could markedly improve irrigation performance. For instance, the practice of building primary, secondary and tertiary canals each with a uniform quality might be a near approximation to models with optimal conveyance.[32]

### 3.5 Conclusions and Policy Implications

In summary, we have shown that in irrigation systems with conveyance losses, 'water–saving' investments in conveyance decrease from head to tail, while use of 'yield–increasing' technology increases away from the source. Optimal water use, output, and quasi–rents also decrease with distance, thus preserving the von Thunen results.

We then compared the optimal system to stylized irrigation models that represent the real world. These models were found to have lower aggregate net benefits, smaller irrigated area, higher water use, and lower conveyance investments.

An empirical example revealed the phenomenal output and acreage gains that could be achieved by making optimal investments in reducing conveyance losses and in yield–increasing technology. We observed the complimentary nature of these public (conveyance) and private (on–farm technology) investments, as well as their differential impacts on aggregate land and water rents, irrigated acreage, and head–tail water and rent distribution.

Although we have attempted a static analyses, the problem we have highlighted is essentially dynamic in nature, and our models can be thought of as being part of a dynamic spectrum. It is conceivable that soon after commissioning of an irrigation system, it behaves according to our optimal model, and the need for maintenance investments is low. But with time, if maintenance is poor, the control structures deteriorate, and conveyance losses increase. Effective acreage is reduced, and farmers located towards the system tail get left out as the system 'shrinks'. Land rents at the head rise, but overall output and net benefits decrease. The performance of the system diverges from optimality and moves towards the sub-optimal models presented in Section 3.4.

These results emphasize the need for 'rehabilitation' of existing irrigation systems as compared to building new capacity. They hold relevance both in the developed and developing world. For instance, 85 per cent of the canals in projects of the U.S. Bureau of Reclamation are unlined. Coupled with high on-farm losses, this results in only about 50 per cent of the diverted water being used for plant growth (U.S. GAO 1977 in Repetto, op. cit.). In the developing world, the situation is much worse, with only about 25-30 per cent of the water actually reaching the plant (Rangeley 1985).

Upgrading these existing systems through lining of canals and adoption of yield-increasing technology would release water that could be used either to expand irrigated acreage and crop output, or for meeting competing urban and industrial needs. Concrete lining of canals in California's Imperial Valley to meet growing urban needs is already a distinct possibility (The New York Times, 1988). Similar plans have been proposed for Pakistan.

The effect of conveyance investments in reducing aggregate land rents leads us to conjecture that to the extent political power lies with the landowners, they will oppose rehabilitation of irrigation projects through better maintenance. These pressures will be stronger if, as is typically the case in most developing countries, the more powerful farmers are located at the head reaches. To some extent, these factors might explain the low level of political interest in allocating maintenance expenditures. On the other hand, conveyance investments might be politically favored if the more influential farmers are located at the tail, which seems to be the case in areas like California.

Our results also suggest a workable strategy for rehabilitation of existing public irrigation systems, a concept increasingly gaining ground among policy makers (Easter 1985). Improving the water distribution system, and providing financing packages and extension and education services for technology adoption on the farm will move the system towards optimal performance. Additional acreage at the tail reaches could be brought under irrigation, more water could be generated at source, and an increasing number of farmers could share in a more uniform distribution of project benefits.

Since improved conveyance results in higher shadow prices of water, which in turn creates substantial water rents, this implies that the task of supplying water, providing optimal maintenance, and collecting water charges could be a profit-making activity. The externalities associated with making canal investments would then be internalized. However, such an arrangement is feasible only when well-defined property rights to water exist, in the sense that owners of the resource can allocate it to its highest valued use. Existing legal regimes, such as the appropriative rights doctrine prevailing in parts of the United States, would restrict the emergence of markets for water (Burness and Quirk, 1979). Examples of water auctions, and trading in private tubewell water among neighboring farmers in many countries of the world, demonstrate the need for institutional reform.

Although our model abstracts from crop choice, we can see that the pattern of water allocation dictates that when land quality is uniform, water-dependent crops could be grown at the head, while tougher, water-efficient varieties would be more suitable towards the tail.

We have also abstracted from considering land quality and topographical variations within the system, as also the possibility of supplementing irrigation water through rainfall or groundwater sources. If private groundwater extraction were allowed, optimal extraction might increase with distance from the source, analogous to our analysis of on-farm technology. However, the absolute value of conveyance investments might decrease because of the positive externality effects of seepage and percolation on groundwater recharge. This analyses could also be extended to incorporate the stochastic nature of water supply, and possibilities of water storage.

## APPENDIX TO CHAPTER 3

The following lemmas have been used in proving results in this chapter:

Lemma 1. $k(X*)=0$.

Proof: Condition (4) gives $z(X*)=0$. Substituting in condition (8) and using $\mu(X*)\geq 0$, we get $m'(k(X*))=\infty$ which gives the result.

Lemma 2. If $z_0*>z_0^u$, then there exists $x\varepsilon[0,\varepsilon]$ such that (a) $z*(x)>z^u(x)$ (b) $\mu*(x)>\mu^u(x)$ and (c) $k*(x)>k^u(x)$.

Proof: $z_0*>z_0^u$ and continuity of $z(x)$ implies that there exists $\varepsilon_1>0$ such that $z*(\varepsilon_1)>z^u(\varepsilon_1)$. Together with $z'(x)<0$, this gives $z*(x)>(1/X^u)\int_0^{X^u} z^u(x)dx$ for $x\varepsilon[0,\varepsilon_1]$. Again, $z_0*>z_0^u$ implies $\mu_0*>\mu_0^u$ from (14). Since $\mu(x)$ is continuous everywhere except at the boundary (from (10)), there exists $\varepsilon_2>0$ such that $\mu*(x)>\mu^u(x)$ for $x\varepsilon[0,\varepsilon_2]$. Taking $\varepsilon=\min(\varepsilon_1,\varepsilon_2)$, we get (a) and (b). Applying (a) and (b) to conditions (8) and (23), we get $m'(k*)<m'(k^u)$. Using $m''(k)<0$, gives (c).

## NOTES TO CHAPTER 3

1. See Caswell and Zilberman (1986) for a non–spatial model of technology choice in irrigation.

2. Models that derive efficient pricing and allocation rules based on location of the consumer have been eveloped in the transportation and electrical utilities literature, and also in the literature on product differentiation and optimal location of firms (see for e.g., Takayama and Judge 1971; Bohn, Caramanis and Schweppe 1984; Salop 1979; Thisse and Zoller 1983). The transportation models generally determine shadow prices of the resource throug cost minimization, and then apply a constant freight charge based on distance. The electrical utility models take transmission losses as fixed. A spatial model for irrigation has been developed elsewhere (Chakravorty and Roumasset 1988), but it treats conveyance and on-farm investments as exogenously given. Our model, in a sense, builds on this framework by determining conveyance losses and technology choice by consumers endogenously.

3. The model could easily be extended to accommodate differential land quality (e.g., see Caswell and Zilberman, op. cit.).

4. Although it is not attempted here, we could incorporate a non-uniform farm size distribution by writing $\int_0^x s(x)dx$ as the integral of the density function of the distribution of farm size at any 'x'. Then farm size in the interval $(x-\delta x/2, x+\delta x/2)$ where $\delta>0$ is small, can be approximated by $s(x)dx$.

5. Ideally, $m(\cdot)$ should be a function of both $k(x)$ and canal discharge, '$Q(x)$' (which is

is equal to z(x) divided by the total duration of irrigation) with $\delta m/\delta Q<0$, indicating that savings per unit investment in conveyance decrease with discharge. However, our formulation already allows for 'k' to be sensitive to 'z' (see Proposition 2 and related discussion; also Section 3.4 of this chapter).

6. These investments could include practices such as land–smoothing for more uniform water application, re–use of tail water that runs off the downstream end of fields, techniques that reduce runoff by cutting back the rate of water inflow into the soil to match the soil's infiltration capacity (e.g., sprinkler and drip systems), or surgeflow irrigation that provides intermittent rather than continuous inflow to furrows (James 1988). Yield-increasing inputs other than irrigation, such as modern seed varieties, fertilizers or other farm machinery can also be incorporated in this framework.

8. Implicitly, we are assuming that the production function is linear homogenous in 'e' and all other production inputs.

9. Multicropping can be easily accomodated by indexing on the production function for each crop type, but is not attempted here in order to keep the formulation simple.

10. Effective water could be taken to be the evapotranspiration requirement of the crop (Hillel 1986).

11. Our analytical results remain unchanged if the constant output price is replaced by an inverse demand function (see chapter 4).

12. To avoid notational clutter, we omit writing the independent variable 'x' whenever convenient.

13. We have written $z_0$ as z(0) to be consistent with control theory notation.

14. To take into account the possibility of corner solutions, we can write conditions (7)–(9) as

$\delta H/\delta q = [pf'h(l)-\mu]s(x) \leq 0 \quad (=0 \text{ if } q(x)>0)$

$\delta H/\delta k = -1 + \mu zm'(k) \quad \leq 0 \quad (=0 \text{ if } k(x)>0)$

$\delta H/\delta l = [pf'qh'(l)-1]s(x) \leq 0 \quad (=0 \text{ if } l(x)>0)$

The inequalities can be eliminated by assuming an indefinitely large first derivative as q,k,l approach zero, as is common in the literature (see, for e.g. Kamien and Schwarz (1978)). Given the nature of our problem, and the simulation results which affirm that the control variables are strictly positive in the interval [0,X*), we only work with the interior solution.

15. After Takayama (1985), pp. 656, remark to theorem 8.C.3. For a discussion of the various necessary and sufficient conditions for optimization with state constraints, see Seierstad and Sydsaeter (1987), theorems 5.1 and 5.2, pp. 317, 332; also Takayama (op. cit.), pp. 646.

16. The behavior of 'μ' in this model is somewhat analogous to the movement of user cost over time in the case of exhaustible resources with a constant marginal cost of extraction, where $\mu'(t)/\mu(t) = r$, where 'r' is the real rate of interest (e.g. Dasgupta and Heal 1979). In our model, the conveyance loss 'l' is the discount rate equivalent, the difference being that 'l' is a function of 'k' and hence endogenous.

17. From Knightian production theory, $\varepsilon_f'$ takes on values in the interval $(-\infty,0)$ as the marginal product increases from zero to its maximum value. Restricitng $\varepsilon_f'$ from taking values in $[-1,0)$ means that we ignore the production phase where marginal product is near its maximum value and increasing with small input levels (Caswell and Zilberman, op. cit.). From Proposition 1, $q'(x)<0$ implies that the above production phase occurs in the neighborhood of the system tail. Since input levels in this region are approaching zero, it might be the case that applied water (and hence effective water) levels are less than the minimum needed to sustain production activity, which makes our assumption more plausible. Cobb–Douglas production functions also require the same restriction on elasticity values (Hexem and Heady 1974). Moreover, values of $\varepsilon_f'$ obtained from our simulations in Section 3.4 of this chapter are all within the assumed range $-\infty<\varepsilon_f'<-1$.

18. Removing the above assumption (letting $\varepsilon_f'$ take on all values in $(-\infty,0)$) would affect Proposition 1 in the following way, as can be verified from conditions (17) and (18):

$q'(x)<0$ for $\varepsilon_f' \varepsilon (-\infty,-1/2]$

$\quad \gtrless 0$ for $\varepsilon_f' \varepsilon (-1/2,0]$ depending on whether

$\quad |\varepsilon_f'| \gtrless |\varepsilon_h|/[2|\varepsilon_h|+|\varepsilon_h'|]$.

$l'(x)>0$ for $\varepsilon_f' \varepsilon (-\infty,-1)$

$\quad \gtrless 0$ depending on $q'(x) \gtrless 0$.

From the above, we can observe that in the neighborhood of $x=X*$, there would exist regions such that both investments in on–farm technology and water use (i) initially decrease and then (ii) increase with 'x'.

19. Note that although the conditions are the same, the parameter values need not be. We use the same notation as in the earlier model for simplicity.

20. The elasticity of on-farm investments is typically small, and $I*(x)$ is not very sensitive to variations in $\mu(x)$. Thus for all practical purposes, variations in shadow price over 'x' are reflected through changes in the amount of water used, and $I(x)$ remains almost constant (from condition (16); also see Table 4).

21. This assumption simplifies the proofs without loss of relevant information, since we have seen that the control functions could only jump at the boundary, a case which is of peripheral interest to our problem.

22. Either of the functions $\mu(x)$ or $q(x)$ could intersect towards the tail. It can be shown that there can exist (at most) three regions and $X_2$ and $X_3$ such that

$\mu*>\mu^u$ and $q*<q^u$ when $x \varepsilon [0,X_2]$

$\mu*<\mu^u$ and $q*>q^u$ when $x \varepsilon (X_2,X_3)$

$\mu*>\mu^u$ and $q*<q^u$ when $x\varepsilon(X_3,\min(X^u,X*)]$

and $\mu*=\mu^u, q*=q^u$ at $x=X_2,X_3$.

Thus, the three regions can degenerate into two or one depending on the location of $X^u$ with respect to $X_2$, $X_3$.

If (a) $X^u\varepsilon[0,X_2)$, shadow prices are lower everywhere in the sub-optimal model, and water use is higher. Intuitively, although higher conveyance losses (due to a lower 'k') cause the shadow price $\mu^u(x)$ to rise faster than $\mu*(x)$, 'i' is a small number (of the order of $10^{-3}$), and the higher marginal cost at source ($\mu_0*>\mu_0^u$) ensures $\mu*>\mu^u$

everywhere. If (b) $X^u \varepsilon (X_2, X_3)$, shadow prices in the optimal model are higher(lower) at the head(tail); and if (c) $X^u \varepsilon (X_3, X*]$, shadow prices are higher at the head and tail, and lower inbetween.

Notice that in Proposition (3b,c), we have not analysed behavior at the tail of the two systems, because the definition of 'tail' depends on the precise magnitude of $X^u$ as specified above.

23. When $\mu*/\mu^o < h_0/h*$, (see proof), $e* > e^o$ and $q*h* > q^o h_0$, and hence $q* > q^o$ depending on the relative magnitude of $h*(l)$ and $h_0$. Thus the relationship $\mu*(x)/\mu^o(x) > h*/h_0$ provides a sufficiency condition, which is met by our simulations.

24. $h*(\cdot)$ is a function of 'l', although the argument is sometimes omitted in writing.

25. When $q^o(x) < q*(x)$ for some $x \varepsilon [0, \min(X^o, X*)]$, it can be shown (following proof of Proposition 3(b)) that $k^o < k*$ at the head while conveyance investments in the tail depend on the relative magnitude of $q^o/q*$.

26. For computational purposes, we assume divisibility.

27. In order to determine the sensitivity of our results to the cost of conveyance, we experimented with a maximum value of 'k' of $320/m that includes the cost of building control structures and consolidation and realignment of canals. However, the order of magnitude of our results did not change appreciably.

28. These conveyance efficiency figures are supported by engineering studies (i.e.,

see Bos and Nugteren 1974); the exact loss coefficient, however, would depend on soil characteristics, ambient temperatures, and other environmental factors. Our results were found to be relatively insensitive to variations in the value of $l_0$.

29. The algorithm was modified suitably when solving models 2–5 (see section 3.4.5).

30. However, if the rights to water are vested in the land, then this conclusion may not hold true.

31. Shadow prices actually increase and water allocations q(x) decrease from head to tail, but this is not captured when rounded off to two decimal places.

32. Another model was investigated (not detailed here) that examined uniform pricing (at marginal cost of capacity) from head to tail. Its performance resembled an 'averaging' of models 2 and 3. In irrigation systems with sub–optimal conveyance and high transactions costs of metering and collecting differential water charges, this model might be a viable option. It has the added political appeal of being 'equitable' since all farmers in the system pay the same price and use equal amounts of water.

# CHAPTER 4

# EFFECT OF OUTPUT ELASTICITY AND MARKET STRUCTURE

## 4.1 Introduction

In this chapter, the basic analytical model presented in chapter 3 is extended to examine the performance of competitive vs. monopolistic market structures under a spatial externality. The theoretical literature on the theory of the second best in general and externalities in particular has dealt with the relationship between the monopoly price and the socially optimal price in the presence of consumption diseconomies (see, for e.g., Diamond and Mirrlees (1973), Edelson (1971), Buchanan (1969)), and the effect of elasticity of the utility function on the above (Luski and Lusky 1975).[1]

In the following section, we have applied the theory of the second best to analyse the effect of spatial externalities in irrigation. We derive relationships between the socially optimal price, and pricing under monopoly and competitive (i.e., when farmers behave competitively and there is no investment in conveyance) systems. The effect of elasticity on output price and quantity, as well as on related parameters such as irrigated area, water use, and rents are examined. Using the parameters values and functional forms given in chapter 3, the quantitative effects of alternate assumptions about elasticity of the demand function are generated.

## 4.2 The Theoretical Framework

Consider a simple one-period (i.e., one irrigation season) model of an irrigation system. A point source (e.g., a dam or diversion structure) supplies water into a canal. Farms of uniform land quality are located on a continuum on either side of the canal. Farmers draw water at various locations, x along the canal, where x is measured from the source.

Let the spatial distribution of farm size at any x be s(x). We assume a uniform distribution s(x)=d where d is a positive constant. In our model, d can be interpreted to be the width of the system.

The amount of water available at the source is taken to be $z_0$. The cost of generating water at source $w(z_0)$ is assumed to be an increasing twice-differentiable convex function with $w'(z_0)>0$, $w''(z_0)>0$. These costs could represent per period capital construction costs and operating costs (e.g., pumping) of the facility.

We define z(x) to be the cumulative quantity of water flowing in the canal through any location x, $z(x)\geq 0$, q(x) is the quantity of water applied on the farm at any location x, $q(x)\geq 0$, and l(x) is the conveyance loss per unit length of canal at any x, $l(x)\geq 0$. Then

(1a) $$z(x) = z_0 - \int_0^x [q(\tau)s(\tau) + l(\tau)z(\tau)]\, d\tau$$

where the first and second terms under the integral sign in (1a) indicate respectively, water applied on the farm and water lost in conveyance at each location x. We can write l(x) as

(2) $$l(x) = l_0 - m(k(x))$$

where k(x) is the investment in conveyance at any x; $l_0$ represents the base loss, or the proportion of water lost per unit length of canal if there were no investments in conveyance (for instance, if the canals were unlined), $l_0 \varepsilon [0,1]$; and m(k) is the fraction of water saved by conveyance investments k(x). We assume m(k) to be an increasing, strictly concave, twice-differentiable function, $m(k(x)) \varepsilon [0,l_0]$, $m'(k)>0$, $m'(0)=\infty$, $m''(k)<0$, which implies decreasing returns to scale in conveyance investments. We thus obtain $l(x) \varepsilon [0,l_0]$ from (2).

Differentiating both sides of (1a) which is an identity, we get

$$(1b) \quad z'(x) = - q(x)s(x) - l(x)z(x)$$

and we obtain $z'(x) \leq 0$. If X* is the length of the system (determined endogenously), then

$$(3) \quad z_0 = \int_0^{X*} [q(x)s(x) + l(x)z(x)]\, dx$$

which gives z(X*)=0 using (1a). Farmers can choose to invest in on–farm technology that increases the effectiveness of the water applied, q(x). If I(x) is the amount of on–farm investment (per unit area) at any x, then effective water 'e' is defined as $e=q \cdot h(I)$ where h(I) is an increasing twice–differentiable, concave function, $h(I)>0$, $h'(I)>0$, $h''(I)<0$. Assuming monocropping, the on–farm production function is $y=f(q \cdot h(I))$ where y is yield per unit area, $f(\cdot)$ is twice differentiable and $f(\cdot)>0$, $f'(\cdot)>0$, $f''(\cdot)<0$. The cumulative output at any x, Y(x) can then be defined as

$$(4a) \quad Y(x) = \int_0^{x} f(q \cdot h(I))s(\alpha)d\alpha$$

differentiating which we obtain

(4b) $Y'(x) = f(\cdot)s(x)$.

### 4.2.1 A Cost Minimization Problem

Let $Y*$ be the total output produced by the irrigation project given by

$$(5)\quad Y* = \int_0^{X*} f(q \cdot h(l))s(x)dx$$

Then we can define a cost function $C(Y*)$ as the total cost of producing an output level $Y*$, written as the sum of the individual costs of water generation, and investments in conveyance and on–farm technology, and thus

$$(6)\quad C(Y*) = w(z_0) + \int_0^{X*} [k(x) + l(x)s(x)]\,dx$$

In formulating the associated cost minimization problem, we need to find piecewise continuous, non–negative control functions $q(x)$, $k(x)$ and $l(x)$, optimal values for $X*$ and $z_0$, and associated continuous and piecewise differentiable state functions $z(x)$ and $Y(x)$ defined on the interval $[0,X*]$ that will

(7a) $\underset{q,k,l,X*,z_0}{\text{minimize}} \quad C(Y*)$

(7b) subject to conditions (1b)

(7c) and (4b),

(7d) $q(x), k(x), l(x), X* \geq 0$

(7e) $z(x) \geq 0$

(7f) $Y(x) \geq 0$

(7g) $z(0)$ free, $z(X*)=0$, $X* \varepsilon\ (0,\infty)$.

The problem in (7) is easily identified as a standard control problem with an initial salvage value, pure state constraints, and a free terminal condition. Associating auxiliary functions $\mu(x)$, $\sigma(x)$ and $\tau(x)$ to differential equations (1b) and (4b) and state constraint (7e) respectively, the Hamiltonian function and the corresponding Lagrangian for the minimization problem in (7a)–(7g) can be written as

(8a) $H(q,k,l,z,\mu,\sigma) = k(x) + l(x)s(x) + \mu[l(x)z(x)+q(x)s(x)] - \sigma f(\cdot)s(x)$

(8b) $L(q,k,l,z,\mu,\sigma,\tau) = H(q,k,l,z,\mu,\sigma) + \tau z$

If q,k,l,z,X* comprise an optimal interior solution, then there exist constants $\mu_0$ and $\sigma*$, continuous and piecewise differentiable functions $\mu(x)$, $\sigma(x)$ and $\tau(x)$ in $x \in [0,X*)$ and a number ß such that (7b)–(7f) is satisfied for all $x \in [0,X*]$ and the necessary conditions are as follows:

(9a) $\delta H/\delta q = [\mu - \sigma f' h(l)]s(x) = 0$

(9b) $\delta H/\delta k = [1 - \mu z m'(k)] = 0$

(9c) $\delta H/\delta l = [\sigma f' q h'(l) - 1]s(x) = 0$

(10a) $\mu'(x) = -\delta L/\delta z = \mu l - \tau$

(10b) $\sigma'(x) = -\delta L/\delta Y = 0$

(11) $\tau(x) \geq 0$ (=0 if $z(x)>0$)

the terminal conditions

(12a) $L(x=X*) = 0$

(12b) $\mu(X*^-) - \mu(X*^+) = ß$

and the salvage value conditions

(13a) $\mu_0 = w'(z_0)$

(13b) $\sigma* = C'(Y*)$.

We assume that the sufficiency conditions for optimization are met (see Seierstad and Sydsaeter 1987, theorem 6.7 p.377). Note that the above program defines a cost, or expenditure function $C*(Y*)$ which gives the minimum cost for any level of output $Y*$.

Obviously, $\mu(x)$ and $\sigma(x)$ are the respective shadow prices of each unit of water and output at location x. Since $z'(x)\leq 0$ from condition (1b), z(x) could not be zero or negative in $x \varepsilon [0,X*)$, or it could not increase from that value. Hence, $\tau(x)=0$ and thus $\mu'(x)>0$ (from (10a)) for $x \varepsilon [0,X*)$. Integrating (10a) and using (13a), we get

$$(14) \quad \mu(x) = \mu_0 e^{\int_0^x l(\Gamma)d\Gamma} = w'(z_0) + w'(z_0)[e^{\int_0^x l(\Gamma)d\Gamma} - 1] \text{ for } x\varepsilon[0,X*)$$

The shadow price of water increases at an exponential rate '$\int_0^x l(\Gamma)d\Gamma$' with x. It has a fixed component – the marginal cost of capacity $w'(z_0)$, and a variable component that is a function of the conveyance loss over a distance x.

Conditions (10b) and (13b) suggest that the shadow price of output is constant over distance, and is equal to the marginal cost of producing a unit of output from the project $C'(Y*)$. Let us now assume that social utility is given by a concave utility function $U(Y*)$ where $U'(\cdot)>0$, $U''(\cdot)<0$. Then output $Y*$ is chosen such that

$$(15a) \quad Y** \; \varepsilon \; \underset{Y*}{\text{argmax}} \; U(Y*) - C*(Y*)$$

Here $Y**$ denotes the output of the commodity in the optimal model and solves the following necessary and sufficient conditions:

(15b) $U'(Y*)=C*'(Y*)$ and

(15c) $U''(Y*) - C*''(Y*) < 0$.

Together, (13b) and (15b) further imply that $\sigma$ gives the optimal output price, which we will denote as p** for notational convenience.

We can now state the following proposition that characterizes the properties of the optimal model:

Proposition 1: (a) Water applied q(x), (b) conveyance investments k(x) (c) output per unit area y(x) and (d) land rents $R_l(x)$ decrease monotonically with x; while (e) investment in technology I(x) increases with x.

Proof: See chapter 3, Propositions 1 and 2, and related discussion.

4.2.2 **Output and Pricing Under Monopoly**

We now examine the case when the irrigation project is operated as a monopoly in the output market.[2] The monopolist faces the same cost function C*(Y*) obtained from the above optimization exercise, but chooses output $Y_m$ that maximizes profits as follows:

(16a) Maximize $\pi = pY* - C*(Y*)$ and $Y_m$ solves

(16b) $MR(Y*) - C*'(Y*) = 0$ and

(16c) $MR'(Y*) - C*''(Y*) < 0$

where p is the price of the agricultural commodity. Let $p_m$ be the output price under monopoly. Then $p_m=U'(Y_m)$.

4.2.3 **Output and Pricing Under Competition**

Competitive (or decentralized) behavior will result when the spatial externalities arising from conveyance losses are not internalized.[3] In our framework, farmers would invest in private on-farm technology, but not in conveyance. The cost function $C(Y*)$ is now defined for $k=0$ while all the other parameters are allowed to vary as given in conditions (7a–g). Let us denote the new 'constrained' cost function as $C_o(Y*)$. Output $Y_o$ and price $p_o$ in the competitive model will be obtained as follows:

(17a) $Y_o \in \underset{Y*}{\operatorname{argmax}}\ U(Y*) - C_o(Y*)$ and solves

(17b) $U'(Y*) - C_o'(Y*) = 0$ and

(17c) $U''(Y*) - C_o''(Y*) < 0$.

The following propositions contrast output and prices of the commodity, as well as water use and irrigated acreage in the three models. We assume that investment in on-farm technology does not vary across models, or $I*(x)=I_m(x)=I_o(x)$ and that the marginal cost of producing output is increasing.

<u>Lemma 1: (i) $C*(Y*) \leq C_o(Y*)$ (ii) $C*'(Y*)>0$; (iii) $C*''(Y)>0$ (iv) $C_o'(Y*) \geq C*'(Y*)$ and (v) $C_o''(Y*) \geq C*''(Y*)$ for all $Y*$, the equality holding if and only if $k*(x)=0$ for all $x \in [0,X*]$.</u>

<u>Proof</u>: (i), (ii) and (v) are straightforward applications of the Le Chatelier Principle (Silberberg 1971).

(iii) By assumption.

(iv) See Silberberg (1978) p.298, problem 3, i.e., marginal cost rises faster when a factor is held fixed than when all factors are variable.

The above lemma states that the cost of producing a unit of output under the competitive system where conveyance investments are fixed to be zero is always higher than in the optimal system. If the marginal cost of output is rising, then the cost function in the competitive case (with a fixed factor of production) is more convex than in the optimal model (see Silberberg (1978) for a detailed discussion of cost functions with variable and fixed factors of production).

Fig.13 shows the marginal cost functions in the optimal and competitive case, $C^{*\prime}(Y^*)$ and $C_o'(Y*)$. Both the socially optimal irrigation system and the monopolist operate with the marginal cost function $C*'(Y)$. The socially optimal price $P**$ and output $Y**$ are given by the intersection of the demand function D and $C*'(Y*)$. The competitive price $P_o$ and quantity $Y_o$ are given by intersecting demand with $C_o'(Y*)$. The monopolist equates marginal revenue $MR(Y*)$ with $C*'(Y*)$ to give price $P_m$ and quantity $Y_m$. The figure has been drawn such that the monopolist produces more and charges a lower price than the competitive case. However, it is easy to see that the reverse could also happen.

<u>Proposition 2: If $P_m=P_o$ then (i) $Y_m=Y_o$ (ii) $z_{0m}<z_{0o}$ (iii) $X_m<X_o$ (iv) $\delta C'(Y*)/P=-\varepsilon$</u> where $\varepsilon(=U''Y*/U')$ is the elasticity of marginal utility or equivalently, the price elasticity of demand of the commodity $Y*$.

<u>Proof</u>: (i) By definition, $P_m=U'(Y_m)$ and $P_o=U'(Y_o)$ which gives $Y_m=Y_o$.

(ii) By Lemma 1(i), $C_o(Y)\geq C*(Y)$ which implies

$$\int_0^{X_o} l_o(x)s(x)dx + w(z_{0o}) > \int_0^{X_m} [l_m(x)s(x) + k(x)]dx + w(z_0m)$$

which implies $w(z_{0o}) > w(z_0)$, or $z_{0o}>z_0m$.

(iii) From Proposition 2(iii), $z_{0o}>z_{0m}$ implies $\mu_{0o}>\mu_{0m}$ by condition (13a). Since $k_c(x)=0$, $k_m(x)>k_c(x)$, which gives $l_m(x)<l_o(x)$ by condition (2). From (10a), $\mu_o(x)>\mu_m(x)$ $\forall x$. From

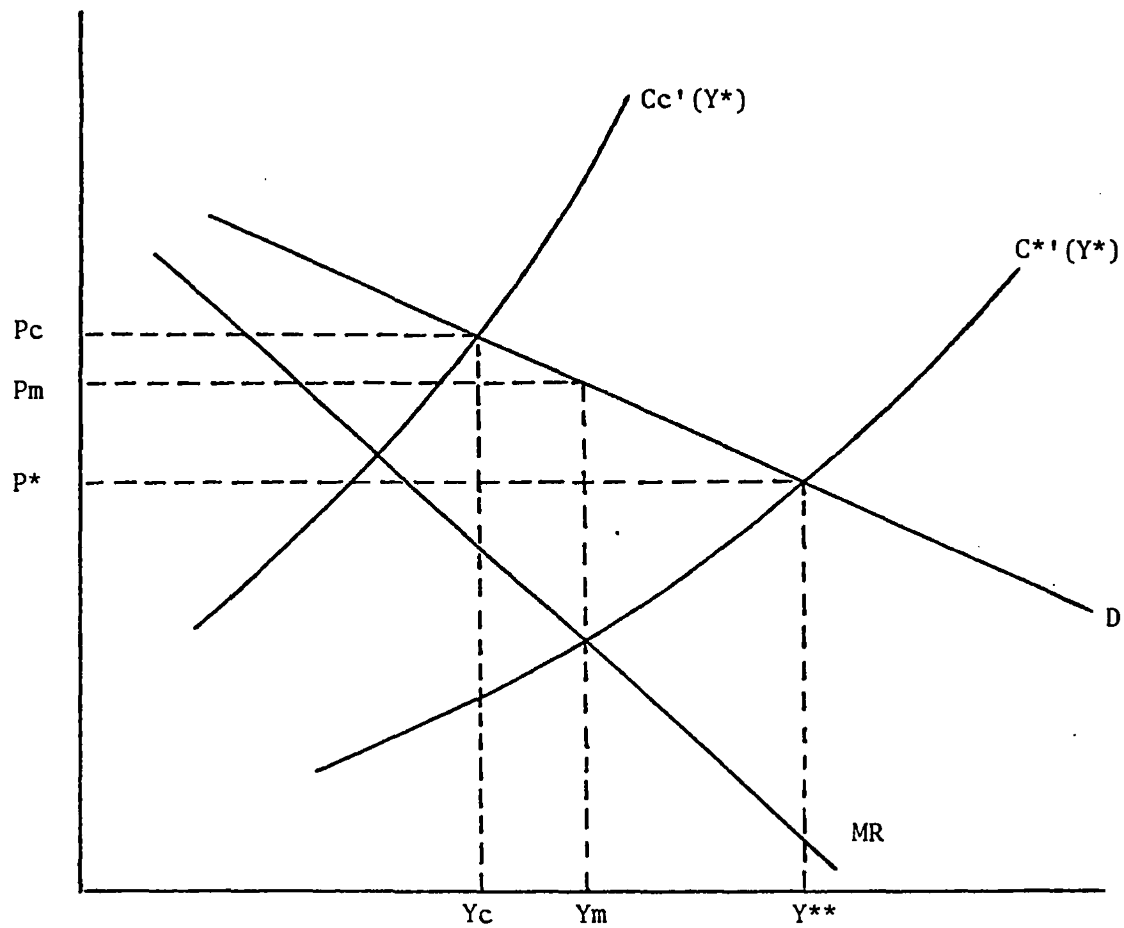



FIGURE 13. MONOPOLY, COMPETITIVE AND SOCIALLY OPTIMAL PRICES AND OUTPUT UNDER AN EXTERNALITY

(9a), and the assumptions $l_m(x)=l_o(x)$, $P_m=P_o=\sigma_o=\sigma_m$, we get $q_o(x)<q_m(x)\ \forall x$. But from Proposition 2(i), we know that

$$Y_o = \int_0^{X_o} f(q_o h(l_o)s(x)dx = Y_m = \int_0^{X_m} f(q_m h(l_m)s(x)dx$$

$f'(\cdot)>0$ and $q_o(x)<q_m(x)$ together imply $X_o>X_m$.

(iv) The price charged by a monopolist $P_m$ can be written as

$P_m(1 + 1/\varepsilon)=C*'(Y)$ where $Y=Y_m=Y_o$ (Varian 1978). Substituting $P_m=P_o=C_o'(Y)$, we get $C_o'(Y)-C*'(Y)=-P/\varepsilon$ or $\delta MC/P=|1/\varepsilon|$.

<u>Proposition 3. If $P_m>P_o$ then (i) $Y_m<Y_o$ (ii) $z_{om}<z_{oo}$ (iii) $X_m<X_o$.</u>

<u>Proof</u>: Similar to proof of Proposition 3 and hence is not repeated.

The above propositions indicate that if the monopoly price is greater than (or equal to) the competitive price, the monopoly output is lower than (or equal to) the competitive output, an intuitively obvious result. In both these cases, total water used and area irrigated by the monopoly is less than that under competition. However, when $P_m<P_o$, $Y_m>Y_o$, but the relative sizes of the water stock and acreage are unclear.

<u>Proposition 4.(i) $P_m>P**$ (ii) $Y_m<Y**$ (iii) $z_{om}<z_o**$ (iv) $X_m<X**$, where '**' denotes the parameters of the socially optimal model.</u>

<u>Proof</u>: We prove 3(ii) first.

(ii) From (16b) monopoly output $Y_m$ is given by $MR(Y)=C*'(Y_m)$, and the optimal output $Y**$ equates $P=C*'(Y**)$. Since $P>MR(Y)$, $C*'(Y**)>C*'(Y_m)$. Using Lemma 1(iii), $Y**>Y_m$.

(i) Output prices under monopoly and competition are given by $P_m=U'(Y_m)$ and

$P_o=U'(Y^{**})$. Using Proposition 3(i) and the assumption $U''(\cdot)<0$ gives $P_m>P_o$.

(iii) Using the chain rule, $\delta z_o/\delta Y^*=(\delta z_o/\delta x)/(\delta Y^*/\delta x)$. Substituting from conditions (3) and (4a), we obtain $\delta z_o/\delta Y^*=[q(x)s(x)+l(x)z(x)]/[f(qh(l))s(x)]>0$. Therefore, using Proposition 3(ii), we get $z_o^{**}>z0_m$.

(iv) Differentiating condition (3) by using Leibnitz rule, we obtain

$$\delta z_o/\delta X^* = [q(x)s(x) + l(x)z(x)]\Big|_{x=X^*} >0$$

which, used with Proposition 3(iii) gives the result.

The monopoly price (output) is always higher (lower) than optimal. Therefore, an irrigation system under monopoly uses less water and irrigates a smaller area, as compared to a system that maximizes net social benefits.

Proposition 5.(i) $P_c>P^{**}$ (ii) $Y_c<Y^{**}$.

Proof: (ii) The proof is by contradiction. Let $Y_c>Y^{**}$. Then $U'(Y_c)<U'(Y^{**})$. From (15b) and (17b), we have $U'(Y_c)=C_c'(Y_c)$ and $U'(Y^{**})=C^{*\prime}(Y^{**})$, which gives $C_c'(Y_c)<C^{*\prime}(Y^{**})$. By Lemma 1(iv), this gives $Y_c<Y^{**}$ which is a contradiction.

(i) $Y_c<Y^{**}$ implies $U'(Y_c)>U'(Y^{**})$, which gives $P_c>P^{**}$.

The competitive price (quantity) is always higher (lower) than optimal. However, the relative magnitude of water use and acreage in the two models is indeterminate.

Proposition 6.(i) $dP_m/d|\varepsilon|<0$ (ii) $dY_m/d|\varepsilon|>0$ (iii) $dz_{0m}/d|\varepsilon|>0$ (iv) $dX_m/d|\varepsilon|>0$ (v) $dP_c/d|\varepsilon|<0$ (vi) $dY_c/d|\varepsilon|<0$ (vii) $dz_{0c}/d|\varepsilon|<0$ (viii) $dX_c/d|\varepsilon|<0$.

Proof: The proofs of (iii), (iv), (vi), (vii) and (viii) are omitted because they are similar to those given in Propositions 4,5,and 6(i),(ii) and (v).

(i) The pricing rule for a monopolist is given by

$P_m(1 + 1/\varepsilon) = C*'$ which gives $P_m = C*'\varepsilon/(1+\varepsilon)$. Differentiating with respect to $\varepsilon$ by using the quotient rule, we obtain

$dP_m/d\varepsilon = C*'/(1+\varepsilon)^2 > 0$. Since $\varepsilon<0$, we get the desired result.

(ii) The monopolist sets the output price off the consumer's demand function, or $U'(Y_m) = P_m$. Differentiating totally, we get $U''(Y_m)dY_m/dP_m = 1$ or $dY_m/dP_m < 0$. By the chain rule, using Proposition 6(i), we get $dY_m/d|\varepsilon| > 0$.

(v) The competitive price is set by the condition $P_c = U'(Y_c)$, or $P_c = U''(Y_c)Y_c/\varepsilon$. Differentiating with respect to $\varepsilon$, we get $dP_c/d\varepsilon = -U''(Y_c)Y/\varepsilon^2 > 0$, which gives the result.

The above proposition suggests that as the absolute value of demand elasticity increases, output price under both the monopolistic and the competitive systems decreases. However, the output under monopoly increases while the competitive output decreases. With increase in absolute elasticity, the monopolist produces more output by using more water and expanding irrigated acreage, while the competitive system shrinks in acreage, and uses a smaller water stock.

4.3 <u>An Illustration</u>

The above models are illustrated by using the functional forms given in section 3.4. An iso–elastic demand function for the commodity (California cotton) is constructed for elasticity values ranging from –1 to –3 (with intervals of –0.5) such that at the price of \$0.75, the quantity produced is $17.7*10^8$ lbs. The demand function is of the form $Y=DP^{\varepsilon}$ where D is a constant and $\varepsilon<0$. The results are shown in Table 5 and can be summarised as follows:

1. With increase in the elasticity of demand, monopoly output increases while prices decrease, while both price and output under competition decrease. Therefore, at high

## Table 5. Summary Statistics

| $\epsilon$ | -1 | | -1.5 | | -2 | | -2.5 | | -3 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Opt | Comp | Mon | Comp | Mon | Comp | Mon | Comp | Mon | Comp |
| P($) | .75 | 1.38 | 1.5 | 1.19 | 1.11 | 1.09 | .93 | 1.03 | .88 | .99 |
| Y($10^8$ lbs.) | 17.7 | 9.6 | 6.29 | 8.91 | 8.1 | 8.22 | 10.24 | 7.88 | 10.95 | 7.55 |
| A($10^3$ ha) | 490 | 260 | 160 | 240 | 210 | 220 | 270 | 210 | 290 | 200 |
| $z_0$($10^8$ cu.m.) | 41 | 40 | 17 | 35 | 21 | 32 | 26 | 30 | 27 | 28 |
| PS($10^8$ $) | 3.39 | 6.46 | 6.73 | 4.84 | 5.33 | 3.88 | 4.59 | 3.42 | 4.38 | 3.15 |
| $R_L$($10^8$ $) | 0.14 | 3.59 | 6.15 | 2.54 | 4.46 | 2.06 | 3.31 | 1.80 | 2.89 | 1.70 |
| $R_W$($10^8$ $) | 3.26 | 2.87 | 0.57 | 2.30 | 0.87 | 1.82 | 1.28 | 1.62 | 1.48 | 1.45 |
| CS($10^8$ $) | 41.77 | 33.91 | 9.20 | 11.40 | 5.91 | 6.07 | 4.77 | 3.91 | 3.74 | 2.79 |
| TNB($10^8$ $) | 45.16 | 40.37 | 15.93 | 16.24 | 11.24 | 9.95 | 9.36 | 7.33 | 8.12 | 5.94 |
| $R_h$($10^6$ $) | .28 | 23.4 | 36.2 | 18.17 | 20.27 | 15.57 | 11.83 | 14.07 | 9.65 | 13.3 |
| $R_t$($10^6$ $) | .27 | 1.36 | 36.2 | .98 | 20.25 | 1.6 | 11.82 | 1.78 | 9.64 | 2.5 |
| $Y_h$($10^8$ lbs.) | .35 | .37 | .37 | .37 | .37 | .37 | .37 | .37 | .36 | .37 |
| $Y_t$1($10^8$ lbs.) | .35 | .34 | .37 | .34 | .37 | .35 | .37 | .35 | .36 | .35 |
| $q_h$(m/sq.m.) | .84 | .91 | 1.04 | .91 | .99 | .92 | .94 | .92 | .93 | .93 |
| $q_t$(m/sq.m.) | .84 | .74 | 1.04 | .76 | .99 | .78 | .94 | .79 | .93 | .80 |
| $I_h$($/sq.m.) | .023 | .023 | .015 | .022 | .017 | .021 | .019 | .021 | .019 | .020 |
| $I_t$($/sq.m.) | .023 | .029 | .015 | .028 | .017 | .027 | .019 | .026 | .019 | .026 |
| $e_h$(m/sq.m.) | .77 | .84 | .88 | .84 | .86 | .84 | .84 | .84 | .83 | .84 |
| $e_t$(m/sq.m.) | .77 | .70 | .88 | .72 | .86 | .73 | .84 | .74 | .83 | .75 |
| $K_h$($/m.) | 99.8 | 0 | 98.9 | 0 | 99.3 | 0 | 99.5 | 0 | 99.6 | 0 |
| $\mu_h$($/cu.m.) | .159 | .155 | .068 | .136 | .080 | .125 | .102 | .117 | .106 | .110 |
| $\mu_t$($/cu.m.) | .159 | .440 | .068 | .356 | .080 | .300 | .102 | .272 | .110 | .240 |

Notes:

1. Note that when elasticity is minus one, monopoly price and quantity are the same as socially optimal.

2. $\epsilon$ =output elasticity, P=output price, Y=output, A=area irrigated, $z_0$=total water used, PS=producer surplus, $R_L$=rents to land, $R_w$=rents to water, CS=consumer surplus, TNB=total net benefit, $R_h$ and $R_t$=rents from land at head and tail, q=water applied, e=effective water, I=on-farm investment, k=canal investment, $\mu$=shadow price of water.

demand elasticities, monopoly produces more output and charges a lower output price.

2. Irrigated acreage and water use also increases with elasticity under monopoly, while they decrease under a competitive system. Also note that when demand elasticity is unity, both the competitive and the optimal models use roughly the same water, but the latter produces double the output.

3. Land rents at the head reaches decrease with increasing elasticity in both monopoly and competitive systems. However, rents accruing to farmers under monopoly and competition are many times higher than in the optimal model, mainly because of the combined effect of higher output prices and lower water charges (due to a smaller water stock) in the former models.

4. Welfare effects are highest under the optimal model, closely followed by the competitive model when demand elasticity is unity. Producer surplus decreases with increasing elasticity under both monopoly and competition, since output price decreases, while consumer surplus goes down because of shifts in the demand curve resulting from increasing elasticity. Thus total welfare decreases with increasing output elasticity. In our example, the welfare gains from monopoly always exceed those from competition.

5. Aggregate rents to water are highest in the optimal model. They decrease with elasticity in the competitive case, but increase with increased output elasticity under monopoly. This is because, as elasticity increases, the monopolist produces more output and uses more water, leading to rising shadow prices of water. The reverse is true for the competitive model.[4]

## 4.4 Conclusions and Policy Implications

In this chapter we have examined the effect of market structure on irrigation system performance. The behavior of optimal, purely competitive and monopolistic systems were examined under a range of assumptions regarding elasticity of the demand function. The analytical results show that under low elasticity conditions, competitive (without conveyance investments) behavior will result in a higher output and lower output price than monopoly. However, when the output elasticities are high, monopoly conditions will produce more output at a lower price. A system that permits intervention in the form of conveyance investments but also maximises net social benefit would dominate both of the above.

This indicates that for crops that are characterised by high demand elasticities, e.g., high–valued crops, or export–oriented agricultural products, a monopolist might be preferred to a competitive system. However, for low–elasticity crops such as those grown in subsistence farming or for domestic consumption, a competitive system is likely to result in lower prices, more output and larger net economic benefits. Farmers' coperatives, or marketing boards that engage in monopoly behavior in the output market could be viable policy options in the high–elasticity regime. This is consistent with observed behavior, e.g., in the case of plantation crops such as rubber or cocoa, where producers often operate under a marketing cooperative or cartel.

The fact that rents to water increase with absolute value of the elasticity in the case of the monopoly point to the viability of promoting institutions that combine the maintenance and marketing functions in irrigation. Organizations that supply water, and buy the produce from individual farmers might be an economically attractive proposition, especially for high–elasticity crops. Such an arrangement would also reduce administrative costs, since the task of collecting water charges and paying farmers their output price could be integrated into one.

## NOTES TO CHAPTER 4

1. See also Crew (1969) for an application of the theory of second best to the provision of medical insurance coverage.

2. Where the irrigation project already exists, the monopoly buys the water at marginal cost from the irrigation authority. In a new project, the monopoly might be engaged in building water generation capacity at source. The solutions will be equivalent given transferable water rights. In both cases, the monopoly invests optimally in conveyance, chooses the profit maximising output, and tells the farmers how much to produce.

3. The competitive model proposed here assumes that irrigation water is being supplied and priced efficiently by a water authority, but conveyance investments are not being made. Farmers, however, make optimal investments on the farm. This scenario corresponds to real–world cases where irrigation systems have been built, but canal structures have deteriorated sufficiently over time. The social planner has to choose between maintaining the status quo (the competitive model) and contracting the management of the system to a monopoly. As shown later in this chapter, this choice would depend on the elasticity of the demand function.

4. As in chapter 3, we assume that rights to land and water are well–defined and separable, and also abstract from capitalization of rents into land.

# CHAPTER 5

# CONCLUSIONS

This dissertation studies problems relating to the planning and operation of spatial irrigation systems. It develops the economic principles that determine parameters such as water allocations, pricing, distribution of system benefits, and examines the sensitivity of these results under alternate institutional and market structures. Although irrigation is emphasized, the results obtained here are relevant to other types of public utilities, such as electricity, natural gas, and transportation.

We argue that the traditional approach to irrigation based on exogenous specification of water requirements does not incorporate the opportunity cost of water, and is thus economically inefficient. By means of a static optimization model where water is supplied to farmers located along a canal, we propose a methodology to derive optimal rules for water allocation and pricing to farmers located at various distances from the water source. We find that given losses in water conveyance, efficient spatial allocation requires equating the net marginal value product of water measured at the source. This rule implies that allocations of source water initially increase and then decrease with distance from the system head. However, water received net of conveyance losses decreases monotonically from head to tail. Under marginal cost pricing, tail farmers pay more than those located closer to the source, and therefore quasi-rents from irrigation decline steadily from head to tail. We identify a range of alternative taxation schemes that are more 'equitable', ranging from equalizing total water charges to proportional benefit taxation. The analytical results are illustrated by using stylized data from California and Pakistan.

The basic model is then extended to determine optimal investment in canals and in conservation technologies (e.g., drip or sprinkler irrigation) on the farm. Again, since the shadow price of water increases from head to tail (it is more 'costly' to send water to increasing distances). We find that canal investments decrease (because of a lower volume of flow) while on-farm investments increase (a higher shadow price makes conservation more attractive) from head to tail. These results are then compared to irrigation systems where (i) lining is spatially uniform, and (ii) the on-farm technology is constrained to be furrow irrigation. These later models are found to have lower aggregate net benefits, smaller irrigated area, and higher water use on the farm. An empirical example with California cotton revealed that by making optimal canal and on-farm investments, agricultural output and irrigated acreage could increase fourfold, while net benefits can multiply almost three times. We also found that conveyance investments decrease the value of land and increase rents to water substantially, while private on-farm investments have a positive effect on both. Improved conveyance was revealed to be more 'equitable' since it reduces head-tail rent differentials. In contrast, irrigation structures that deteriorate through time might allow head farmers to accrue a larger share of project benefits.

The model was then expanded to allow for competitive (without conveyance investments) and monopolistic behavior, under assumptions of varying output elasticities. Under low (high) elasticities, competitive behavior was found to produce a higher (lower) output and charge a lower (higher) price when compared to a monopoly. With increasing output elasticities, irrigated area and water use by the monopoly increases, while that under competition decreases. Intuitively, higher output elasticities imply lower market power for the monopolist, and beyond a critical elasticity value, a monopoly might be preferred to a purely competitive (or decentralized) structure. Rents to water under monopoly increased with elasticity, suggesting that organizations that provide maintenance of irrigation structures and market the output might be economically viable.

Our analysis has many limitations that could alter some of our analytical results, or modify the orders of magnitude obtained in the simulations. We have abstracted from considerations of time and uncertainty. Land quality is assumed homogenous. Long–run marginal costs of water at source could be falling in which case, marginal cost pricing will not result in cost recovery. The administrative costs of implementing our proposals could be sizeable, which could affect our numerical results. The model could be extended to incorporate heterogenous land quality and multicropping, or discrete choice of on–farm technology. Conjunctive use of ground and surface water could be modelled by extending the proposed framework. Game–theoretic models that include rent–seeking and collusive behavior by large farmers (who are often located at the head reaches) and irrigation managers might provide additional insights into irrigation performance.

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