# LEARNING TO FACILITATE MATHEMATICAL DISCOURSE:

# A SIXTH-GRADE TEACHER'S JOURNEY OF SELF-DISCOVERY

A THESIS SUBMITTED TO THE GRADUATE DIVISION OF THE
UNIVERSITY OF HAWAI'I IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF EDUCATION

IN

CURRICULUM STUDIES

AUGUST 2006

By
Andrea M. Aiona

Thesis Committee:

Anne Freese
Neil Pateman
Joseph Zilliox, Chairperson

We certify that we have read this thesis and that, in our opinion, it is satisfactory in scope and quality as a thesis for the degree of Master of Education in Curriculum Studies.

THESIS COMMITTEE

Chairperson

Table of Contents

## Abstract

This paper describes the evolution of my self-study over a two-year period. The introduction summarizes two pilot studies called Phases 1 and 2. Four questions are initially addressed. What was the nature and focus of questions asked during mathematics instruction? How effective was my questioning? Why do I do and say the things I say when I teach and talk with my students? What does this say about the kind of teacher I am? The pilot studies led to Phase 3, a more thorough study about mathematical discourse addressing two additional questions. What did I do and say to facilitate mathematical discourse? How effective was I in facilitating mathematical discourse? Conclusions and implications are given that are unique to my sixth grade classroom and me, but may provide other researchers and teaching professionals useful insights when studying and facilitating mathematical discourse.

## List of Tables

## Introduction

As an elementary school teacher, my journey of self-discovery has been a continuing process. From the first year of teaching until the current one, my ninth, I faced new challenges each year, both imposed upon me and chosen by me. I struggled with day-to-day classroom management along with getting to know and understand the various requirements for each elementary content area. Once I developed some daily routines that were somewhat effective, I was able to experiment with my curriculum.

In order to improve my teaching, I knew that I needed to focus on a limited number of content areas at a time. Because I feel literacy is at the foundation of all learning, I focused on the literacy of reading and writing for several school years. Later, in the summer of 2002, I took a course that focused on the mathematics process standards of problem solving, reasoning and proof, communication, connections, and representation. The following school year, I attempted to teach mathematics with the process standards in mind, although social studies was my primary curricular focus. I also needed to improve my teaching of science, and therefore took three courses involving discovery-based learning in science in the summer of 2003. Although I had planned on implementing new science curriculum, mathematics became my primary interest.

In January of 2003, I was accepted into the Mathematics Teacher Endorsement Program of the University of Hawaiʻi and the Department of Education, beginning the coursework that could be applied to a master degree that February. By August, I applied for entry in the M.Ed. program and was accepted in spring of 2004. That was when my formal study of mathematics education began.

I provide this history because my journey of discovering how to facilitate classroom discourse began when I began teaching and has evolved throughout my experience as a teacher and as a student in various settings, from being the student of my students, of my colleagues and administrators, and in my university coursework. Facilitating general classroom discourse presents any teacher with set of challenges, while facilitating mathematical discourse presents many more. By studying mathematics education, I have learned many methods, systems, processes, and strategies that have helped me better teach all elementary curricular areas.

This paper focuses particularly on the last two school years, from 2004 to 2006 and tells about my journey of attempting to learn about and facilitate effective mathematics discourse. I began by studying the questions and prompts I presented to my students as described in Phases 1 and 2 in this introduction. These pilot studies led to Phase 3, a more thorough study of the conversations that occurred during my mathematics instruction.

### *Phase 1: What do You Mean by That?*<br>*A Self-Study of Questioning During Mathematics Instruction*

I strive to be a teacher who attempts different teaching styles and who takes advantage of multiple professional development opportunities with one of my major goals to apply the new ideas I learn in my classroom. I believe that I do this, but I've wondered to what extent has my teaching actually changed? Has what I've learned in my professional development actually changed the way that I teach for the better? Are my students becoming better learners of mathematics as a result of the changes I attempt to make? In my quest to shed light on these questions, I conducted three studies. First, in the fall of 2004, I studied the types of questions I asked students during mathematics instruction and interactions.

During the 2003 – 2004 school year, I chose once again to be a mentor teacher by accepting a student teacher. As my student teacher began a lesson, he asked my 6th grade students a series of questions as prompts to get the students thinking and to set up what was to come. His questions got me thinking as well. I remembered learning that questioning is an extremely important part of teaching, and that good questions were essential in getting students engaged in a way that provides opportunities for them to communicate their knowledge, skills, and metacognition. I wondered a few things. To what degree do I question my students as precursors to lessons? Are my questions effective? How did students respond to my questions? Upon reflection, I felt that I was spending too much time 'telling' the students what to do rather than discovering where they were, where we were going, and where we could go through questioning.

I felt uncomfortable partly because I had taken a professional development course in the scientific method and statistics pedagogy in the summer of 2003, wherein Socratic questioning was a primary teaching method. We had a crash course on Socratic questioning and practiced it in class. I found that questioning in this manner was extremely difficult as well as uncomfortable, partly because I did not have a deep understanding of the content and therefore found it difficult to formulate questions. I didn't necessarily know what my questions were intended to do or where I was heading with them. Therefore, how could I design questions and anticipate responses that continued a line of questioning to arrive at a predetermined conclusion while simultaneously allowing the students to discover what I, myself, did not know? I not only need broad, deep, and flexible knowledge of content, but also of pedagogical alternatives (Silver & Smith, 1996). I felt I was experiencing some success with regard to content, but I believed there had been little change in my questioning style and, although I was becoming more confident in process-oriented mathematics instruction, I was still uncomfortable in the facilitation of more student-centered, student-guided mathematical situations.

In the summer of 2004, I became more aware of how prevalent questioning was in my teaching. I was co-teaching a university mathematics course to teachers in a summer institute designed to help teachers understand students' difficulties in learning mathematics. Usually, when teachers asked me questions, I didn't provide answers. Instead, I responded to their questions with other questions or supplied them with hypothetical situations to examine. I wanted the teachers to realize that they were the experts along with me. I did not want them to look to me for the answers but learn to depend on themselves and, in turn, on their students. The teachers seemed extremely uncomfortable with my style of teaching, especially at the beginning of the course. Schoen, Bean, and Ziebarth (1996) discuss how students react to participating in new forms of communication in a classroom. They conclude that there may be resistance from students when students are challenged with newly embedded communication in mathematics instruction because previous experiences dictate what they think mathematics should look like. These teachers, like my elementary students, initially resisted when they were confronted with my unfamiliar teaching style.

While in the middle of a lesson, one of my colleagues, a co-instructor of the course, explained to the teachers that I was teaching in the manner that I teach my elementary students. I realized, while teaching this course, that I had an unusual style of teaching. I see more clearly now that my questioning of students (or other teachers) in response to their questions is an integral part of my instruction as well as my formative assessment. I wanted to further develop my own skills of questioning and help others make meaning for themselves rather than depend on prefabricated solutions given to them by outside 'experts.' While some good questions and problems have only one solution, there may be many possible paths to the solution. Other good questions and problems may have several distinctly different solutions. These elements of problem solving are difficult for many to integrate when teaching. I believe that becoming more comfortable with using questions to stimulate mathematical thinking, as well as with not immediately knowing the answers, is a beginning.

Subsequent events extended my self-reflection. While observing other teachers teach, I noticed that many administer directions step-by-step rather than use questioning as a way of encouraging students to think. I believe one reason for this is that elementary teachers are not usually mathematics specialists. Because they often do not grasp fully the underlying concepts and principles, they are guided by the textbook or curriculum, and, many times unfortunately, by their own experiences as students of mathematics. I have been and probably will be, from time to time, exactly this kind of teacher. Previously, I had taken my students through step-by-step algorithms and had them copy down notes while I taught, just as I had done when I was a student. After instruction, many students could not repeat or use the algorithms. I didn't know that understanding the underlying concepts and ideas were necessary for a student to genuinely understand, rather than to merely use, an algorithm.

I try to create a learning environment in which students were free to experience varied activities to guide them to deeper understandings rather than simply following the textbook. The textbook became a support tool rather than the primary source of my curriculum. I found that it did not involve the students in exploration through open-ended activities (even when it claims to) and, in turn, students do not have a chance to generate ideas that elicit discussion.

Rather than having a teacher-centered classroom, I strive for a more student-centered environment. After some time with me, my students tend to think before they ask me questions because they know that I am not likely to give them the 'answer.' I attempt to guide them to think for themselves.

In my quest better understand the nature of the student-centered, process-oriented classroom I was working to create, I sought to study, in detail, my mathematics curriculum. I decided to begin by closely examining the questions I asked during mathematics instruction. I hoped to gain insight into what kinds of questions I asked and the purposes for which I presented them. I wanted to expand my repertoire of questions to better decide what types of questions to use and when to use them. Further, I wanted to know whether the questions I asked were successful in getting students to think mathematically.

*Data Collection and Analysis*

My goal was to examine the questions that I asked during mathematics instruction. My research questions were: (1) What was the nature and focus of my questions? and (2) How effective was my questioning? I found the latter question more difficult to answer, although, through detailed comments within the transcripts, I did shed light on it.

I selected two audio taped episodes from those recorded from October 29 through November 10, 2004. I transcribed and analyzed only my own dialog and conversation, focusing only on questions directed to the students. (See Appendix A for transcripts.) Once the audiotapes were transcribed, I sought to find patterns and categorize the questioning techniques that surfaced. As I examined the transcripts, the idea of a "question" became blurred. Therefore, I use the word prompt from this point on instead of question.

There were a total of 75 prompts in the excerpts. I defined a prompt as an instance where a student or the students were expected to respond to a comment or a gesture that I made. After grouping the questions into categories, I organized the prompts in six ways.

First, there were prompts that solicited student ideas (SOLCT) such as: "What does that mean?" (Episode 1, end line 2); "OK. And what do you think…?" (Episode 1, line 5); "I want you to explain your thinking." (Episode 1, line 12).

Second, I prompted students to apply facts and skills (APPLY) particularly with fractions and percents included: "I want to go back to A-----'s question. She said, D----, that before we were using fractions, but now we're using percents. Does anybody have anything to say about that? Why is it the same thing?" (Episode 2, line 69); "And can you change that?…Into anything else?…What else can you change it to?…Yes?…What other fractions can you change it to?… What else? (Episode 2, lines 83-88).

Third, prompts requested student approval that ideas were clarified for them (CLARI). Fourth, prompts requested students to repeat or confirm what they said (CONFIRM). Fifth, I prompted students to pay attention, focus, and listen (ATTN). And, finally sixth, prompts were used to check for students' correct answers (CHECK).

And, finally sixth, prompts were used to check for students' correct answers (CHECK).

It was my belief that the questions that solicited students' ideas (SOLCT) and those that ask students to apply facts and skills (APPLY) were of a different level than the other kinds of prompts because they required students to generate ideas. Therefore, I did not give specific examples for the other types of questions.

Although clarification type prompts (CLARI) occurred more often, 24% of the prompts presented, as compared to APPLY questions at 17%, the responses from the students were more lengthy and detailed when given APPLY prompts. CONFIRM, ATTN, and CHECK prompts were more closed types of prompts that initiated shorter responses from students. They were generally behavior-management or clerically oriented.

Table 1. Types of Prompts Presented

| Types of Prompts | Number of Prompts | Percent of Total Prompts |
|---|---|---|
| SOLCT | 29 | 39.7 |
| APPLY | 13 | 17.3 |
| CLARI | 18 | 24.0 |
| CONFIRM | 7 | 9.3 |
| CHECK | 5 | 6.7 |
| ATTN | 3 | 4.00 |

As I was listening and transcribing the audiotapes, working hypotheses evolved. While my initial plan was to categorize questions according to their levels and types and attempt to examine their effectiveness, as the subject researcher, I discovered that I needed to know the context in which the question was asked, the purpose and goals of my lessons, the subsequent questions posed as follow-ups, as well as how students responded. In this way, the data I collected was inadequate to answer my original questions.

*Discussion and Lessons Learned*

Merely looking at my dialog was limiting because many issues arose during my transcription and evaluation. I felt that I did not always facilitate shifts in students' ideas. For example, I wanted the students to discuss the difference in the types of probability games that we had played. The previous games we had played were purely chance games while the one that I was introducing was one with a set of probabilities not entirely decided by chance. There were assumptions requiring the integration of new concepts including compound events. My questioning and prompting seemed to confuse some students. I realized the importance of probing questions being used effectively rather than repeating questions over and over again.

I noticed that some students contributed more than others either because I called on them more or they chose to participate more readily. I also noticed that wait time varied depending on the question asked and which student(s) responded or were called on. I gained an increased awareness of my individual students, who I called on, and the behavior of the less confident students. I gained insight into my students' understandings and misunderstandings of concepts as I carefully analyzed and focused in on my questioning. I wondered how I could get more students involved in sharing during the discussion by using questions and prompts more effectively and by modifying instructional strategies.

My values were apparent in the classroom climate of open inquiry and discussion and most students seemed comfortable in participating even though many of the same students participated repeatedly while others did not. Students were expected to be actively involved in discussion, thinking, and reflection about the task at hand and the diversity of ideas was appreciated.

All of these factors are extremely complex and contribute to a positive classroom environment. My interaction with students and use of particular teaching strategies is dependent upon the students, their levels of ability and motivation at the time of instruction, as well as the particular mathematics content and process areas being taught. All in all, it is my belief that varied modes of instruction and a teacher's willingness to be fun and learn with the students is paramount in a successful classroom, and further, in any learning environment. Through this study, I realized, even more, how complex teaching and learning is.

*Conclusions*

Through examination of my transcripts, I felt that I spoke more often and for more time than was necessary. Types of prompts varied based on the lessons as well as in response to the reactions of students. I could have chosen more effective prompts from time to time, although, in general, I presented prompts that solicited student ideas and asked for them to think and apply previously learned facts and skills to new situations. Probing and follow-up prompts were useful in allowing students who did not actively participate to hear other students' ideas numerous times, perhaps allowing them to internalize some of the confusing concepts for themselves.

The purposes of my questioning were diverse. I wanted students to actively participate, to share their ideas, to make connections between concepts and skills, and to use higher level thinking to justify their thinking. It is difficult to see whether this was upon listening to the transcripts, the types of responses from the students indicated to me that many of them were achieving what my goals were for them. Therefore, in general, I believe that my questioning style was effective. However, as any reflective teaching professional would acknowledge, I have much more work to do with learning to facilitate optimum classroom discourse with questioning being just one important aspect.

An important part of my learning was that I could better plan my prompting ahead of time with the purpose of my lesson and outcome more clearly in the design of the sequencing of questions. It also became apparent that I could attempt more to predict student responses in order to have more effective responses and probing questions rather than repeating some of the same questions more than necessary.

### *Phase 2: Reformulating the Self-Study*
### *Further Inquiry to Better Understand Discourse*

I continued to clarify my beliefs and values as I continued my coursework and reflecting on my classroom interactions. The Hawaii State Department of Education (HIDOE) uses the terms traditional and reform to describe different mathematics curriculum. Traditional curriculum has become synonymous with using a textbook that provides particular algorithms in a lesson format followed by problems and assessment items for students to do as practice. Teachers teach in a lecture format and students are expected to regurgitate information presented by the teacher and the textbook. Students generally work alone and the teacher is the evaluator of the student's work. A reform curriculum provides units that integrate many different skills and concepts rather than teaching them in isolation. Students work in peer groups while they are involved in problem solving experiences, inventing algorithms and communicating using multiple problem solving experiences, inventing algorithms and communicating using multiple representations and media. The teacher acts as a facilitator and students reflect on and assess their own work.

I have become uncomfortable with the term 'reform' because there is an implication that all that was done in 'traditional' mathematics instruction was negative or poor pedagogy. Rather, I find a curriculum that travels along a continuum between traditional and reform to be more useful for me as a teacher. I have heard many complaints from teachers on both extreme ends. Most I've spoken with who use reform programs say that students do not know or need more practice with basic skills, but "see the bigger picture" and vice versa for those using traditional textbooks. Rather than calling for a need to reform all that is traditional, I'd rather posit that teaching professionals and teaching 'experts' should be able to take what is good about traditional curriculum and instruction and that which people consider reform to create an educational environment in which the two can be used to create a more enlightened curriculum that best suits *all* students and individual students.

Routine as well as novel tasks and pedagogy have their places. Gopnik (2005) coins the term "routinized learning" where a student learns to perform a skill for mastery while in guided discovery, students learn to solve new problems by figuring out how the world works. There are instances when more routinized learning is appropriate and when guided discovery is appropriate. Knuth and Peressini (2001) suggest that discourse can also be thought of as a continuum. At one end, discourse is primarily univocal wherein a speaker tries to transfer an exact meaning, and, at the other end is dialogic discourse wherein a speaker intends on generating new meaning. They may both be used effectively depending on instructional goals.

Heuser (2005) argues that many teaching methods used to teach basic computation hinder conceptual understanding of mathematics including teaching standard algorithms or "parrot mathematics" and the explicit teaching of alternative procedures. "Because invented procedures stem from children's existing understanding, students are more likely to construct knowledge that is personal and robust, rather than memorize what they may not understand" (p. 405). O'Brien (1999) discusses that some continue to call for the return to recitative instruction although it has been the dominate form of mathematics instruction and has failed the United States in helping students become mathematical thinkers. Finding a balance between the telling of procedures and giving directions to complete a task in the time that is available and allowing students to create their own meaning through open experimentation is a challenge in the environment of high-stakes education that exists today.

There are negative aspects of the mere transmission of algorithms, but I suggest that, as students move toward more complex mathematics, it becomes inevitable that some algorithms will need to be "taught" directly to some students in order for other, more complex concepts and procedures to be undertaken. Lobato, Clarke, and Ellis (2005) discuss that, with the constructivist approach and reform-based mathematics, "telling" has taken on a negative connotation. They rethink the idea of telling as "initiating" by looking at telling in three ways: what the function is of the communication rather than the form, whether the communication seeks to develop conceptual understanding rather than merely providing a list of procedures, and how it is related to other actions that occur in the learning environment. The decision, then, rests in the hands of the teacher in whether "telling" is also "initiating."

I believe that one of the things that makes my classroom a special place is that the climate is welcoming and open with regards to sharing ideas. I do wonder, though, to what extent I impose my own ideas on the students. Do I ask leading questions so that I can get the predetermined answers and conclusions that I have decided are valuable? Do I truly teach in an inquiry-based style wherein students are free to form their own conclusions? And, by achieving the latter, will the students really understand the mathematics that I am required to teach them?

Subsequent to my study on questioning, I wanted to look more closely at my motivations for decision-making during my mathematics instruction and assessment by conducting a second pilot study in the spring of 2004. Because I merely looked at my questioning in the previous study, the data was limiting. I discovered that I needed to know the context in which the questions were being asked, the purpose and goals of my lessons, the subsequent questions posed as follow-ups, and how students responded.

The guiding questions were: (1) Why do I say the things that I say when I teach and talk with my students? (2) What does this say about the kind of teacher I am? (3) And, what can I do better? I decided to examine the discourse that happened in my mathematics instruction and between students while they are engaged in tasks during classroom episodes. I sought to analyze the discourse that occurred in my class by observing my interaction with the class and individual students, and the interaction of my students with each other.

*Data Collection and Analysis*

I videotaped several mathematics episodes while simultaneously having tape recorders at each small group of students. I considered one mathematics period in one day an episode. Only two episodes were transcribed (See Appendix B for transcripts.) due to time constraints. The transcripts included descriptions of the episodes as they occurred and paraphrasing of the conversations in order to glean big ideas from the videotapes. Questions and prompts that I presented to students are italicized and were not always direct quotes.

I noticed I took physical and verbal actions in the attempt that we, as a class, would participate in:

> recalling previous knowledge; applying previous knowledge in old and new contexts; asking for justification; asking for additional input; defining vocabulary and giving information; using absurdity for humor; giving directions; reasoning, forming generalizations, making conclusions, and coming to agreements; using correct mathematical language; giving students choices; checking for understanding and making corrections; applying technology; modeling; making observations; simulate situations to extend thinking; and giving examples.

These were the general themes that emerged regarding why I said what I did in interacting with the class.

Depending on the type of activity, particular types of actions and prompts were used more or less. For example, in Episode 1, Review of Measurement Concepts, there was a lot of application of previous knowledge. In Episode 2, Follow-up on Episode 1, there were more prompts given to facilitate forming generalizations and conclusions. And in Episode 2, Lead into the Lesson, there was a lot of direction giving, modeling, making observations, and providing definitions.

*Discussion and Lessons Learned*

Episode 1 was a repetition of a lesson that my University of Hawai'i observation/participation student had attempted to teach, but had not gone well because she did not understand the big ideas in the lesson. I did the lesson again to model for her. I explained to the students that we were going to revisit the lesson because it had good ideas that we needed to understand. I believe it is important that the students buy in to the idea of doing something over again and value the time we were spending on it.

In the beginning of Episode 1, I asked students to clear their desks as much as possible. I also moved a few students around in the room so that they were in a good place to work with other students and to fill empty desks due to absentees. Being flexible and taking into consideration the use of space in the classroom is important for each teaching activity. However, using manipulatives can be tricky. I always try to anticipate the best times to pass out supplies and how to do so. In Episode 1, I had one person from each table group pick up the rulers. I had previously filled containers with linguini for each table group and passed them out while they were getting the rulers. In hindsight, I would have had them get the rulers and linguini after my demonstration on the overhead. Some students began as I was explaining and one played with the rulers for a couple of minutes. I had them separate the pieces of linguini they had already broken helped alleviate confusion and record inaccurate data. I passed out calculators and the second data sheet while they were writing data on the charts. I asked those that were done to put back the rulers and clean up the linguini. In Episode 2, I had one person on each table collect rulers, protractors, and compasses and I went around to get the supplies. These actions showed that I anticipated times and materials that may cause student distraction.

I try to have the students create meaning from and make connections with their previous experiences. In Episode 1, I had them show with their hands about how big each of the following were: centimeter, inches, feet, yards, meters. All students could simultaneously participate and check around the room if they were not sure. This ensured every student's success. I solicited student ideas when I suggested we think of objects we can measure using something other than in feet, or yards, or meters. When students gave ideas, I tried to build upon them, such as when "C" suggested we could measure the nametags in centimeters. In reviewing episode 2, I wondered how I could better communicate the importance of parallel and perpendicular lines. Perhaps, I could have used real life examples and help build students make connections.

Students need many varied methods of instruction and I try to design different types of activities to not only hold the students' attention, but also my own. I experiment with new strategies often. I had the most attention when I was modeling with the overhead projector in Episode 1. Using see-through rulers and the linguini on the overhead projector was effective. I had the least attention when I gave the students directions orally. The students were most involved when they were actually using the manipulatives and recording and analyzing their data. I had complete attention of all students in Episode 2 when I had them write observations about what they saw me doing on the board.

Giving students a choice and allowing them to have some control over what they are doing is important. In Episode 1, the students got to choose the units they wanted to use. After I had them switch rulers, they got to choose any one that they wanted to use if they didn't like the one that they had. The students had little to no choice in Episode 2. I wondered how I could have created opportunities for students to make choices with parallel and perpendicular lines.

It's important that the students think flexibly. Students were expected to know and use appropriate vocabulary and mathematics language, use different measurement tools accurately, and collect data in a consistent manner. In Episode 1, I asked them to switch rulers because I wanted them to be able to measure using different rulers. Collecting consistent data is an important element of data collection. Having them sketch their triangles and label them consistently reinforced the ideas of labeling units and collecting consistent data while having them estimate before actually measuring the pieces of linguini helped to build understanding of the size of units. Having another student or teacher check the measurements before they were recorded reinforced the idea of accuracy and that measurement is not exact. In Episode 2, the students had to think flexibly because perpendicular lines are intersecting lines that have a special property, 90 degree angles. Defining parallel and perpendicular lines in different ways also helped the students think flexibly.

It's important that students understand directions. Many students needed multiple ways of explanation to understand a task. After modeling in Episode 1, I went over steps again on the board. I instructed students to look at their paper and use the appropriate sides for the appropriate data. I asked if they have questions often and tried to provide wait time. I still had to help students.

Time is a big constraint. In Episode 1, I spent too much time giving directions – about 25 minutes. I also changed directions a few times as I was giving them. I had little time to prepare for the lesson and had to stop and change directions a few time. I gave them time reminders when they were working independently and did not finish all that I intended to. Episode 1 would have been better as two, hour-long sessions. I would like to have spent more time on estimation before actually measuring. Episode 2 was also cut short. Many students did not understand how to construct the parallel and perpendicular line although they could recognize and describe them.

Seeing large amounts of data helped the students make generalizations. In Episode 1, the students recorded all of their data on the charts even though they didn't have time to check with someone. A student recognized inaccurate data right away because it didn't match the other data. Some data was hard to read and it would have helped if I numbered the list.

*Conclusions*

In both episodes, I feel that I could have planned my introductory questions better to facilitate the discussion about choosing appropriate units of measurement and about the attributes of parallel and perpendicular lines. I could have predetermined questions and objects to discuss so that the concepts would be more clearly presented. Some students did not appear to be very engaged during the discussion. What strategies could I use to get students who don't usually participate to do so more effectively?

I felt that I spent too much time giving directions. I would have rather had students involved in the activity instead of watching me model. I could have presented a series of written instructions and just let the students attempt the tasks on their own. I could have used the overhead and modeled the steps, allowing the students to tell me what I was doing rather than me telling them what to do and what I was doing. Further, although I spent so much time giving directions, a few students still did not follow directions. I wanted the students to be better problem solvers and work more independently, yet I was at the center of instruction for at least half the time of each episode.

Many big ideas emerged, but I felt as though I was explaining them to the students. How can I get the students to recognize and learn them for themselves rather than me telling them? If I were more careful in listening to student responses, could I have capitalized on their ideas better? I did facilitate some shifts in thinking well and use students ideas as springboards in many instances. Yet, I had ideas about what I wanted to happen. The desire to have certain predetermined ideas to emerge seemed to have limited the discussions. One example of this was when "J" answered that she would use inches in Episode 1. I preferred a different response. Therefore, I asked the same question again and got an answer from another student. "J" volunteered and I ignored her input. I could have found another way to deal with "J's" response that valued her participation.

Since time is such a constraint, I wonder how can I incorporate more ideas into the lessons to make them richer and be able to spend more time with them. There is always so much content to cover. I often feel as though I need to move on quickly and just expose the students to some ideas so that they will at least recognize the concepts when they are faced with them. I knew the Hawaii State Assessment was coming up and there were many concepts I needed to touch on or revisit so that the students would be prepared for testing.

Another way in which time was a constraint was that I did not have time to look at all of my data. I wanted to focus more on the individual interactions I had with students and that the students had with each other. Instead, I became engrossed in the details of the video and the large group interactions.

## Phase 3: Continuing the Journey
## A Better Understanding of Mathematical Discourse

My previous two studies led me not only to continue to ponder my previous questions, but also to ask new ones. I wanted to study the mathematical discourse that occurred and try to affect students' participation and sharing. I also wondered if and how mathematical discourse affected student perceptions about math. I decided to survey my students at the beginning and end of the fall 2004 semester. I had a range of questions regarding student perceptions about mathematics, but later decided to study only those that I believed related to students' participation in behaviors necessary for mathematical discourse.

Because I wanted to know how to improve the discourse in my lessons, I planned to video and audiotape lessons throughout the fall semester. I wanted to video and audiotape lessons in the beginning, middle, and end of fall 2005 and look for evidence of changes over time with any specific strategies I applied and used regularly. Since one of the implications that arose in both studies is that planning was essential, I wanted to decide on specific strategies that I would apply regularly and plan for them. I also wanted to look at student work samples including journals and math assignments to see if their communication about math changed as a result of my planning and using new strategies to facilitate mathematical discourse. I planned on keeping a journal that highlighted planning for discourse before lessons and reflections on discourse after lessons.

All of these were grand ideas, but were thwarted by new challenges that exhausted my time. One of the simpler problems, that of data collection was easily solved. Rather than recording episodes from the beginning of the semester in July, I began recording again in October because, it was at this point that I began using tasks and strategies in instruction that I thought could stimulate better mathematical discourse.

The more complex challenges were those of the class itself and new state initiatives. In the previous years, I had an enrollment of around 20 students. The first challenge was that the class was much larger, with 27 students. There was less room for student movement and students were in seated in groups of 5 or 6 instead of 3 or 4. I also had fewer math manipulatives per student and per group of students. Because of enrollment, I had less time to spend with each student and I found that this group of students included some who were extremely needy with respect to demanding attention and needing extra help. The range of abilities along with the range of motivation with respect to completing work and actively participating was especially difficult with this larger group of students. I had difficulty creating the classroom community necessary for my usual teaching techniques and for facilitating discourse.

One new state initiative was a change from the Hawaii Content and Performance Standards (HCPS) II to the implementation of the new HCPS III. I had to first understand the new standards and second, realign my curriculum, instruction, and assessment to these new, unfamiliar standards. Another new initiative that consumed much of my energy was the implementation of the new "Standards-Based Report Card." I spent many hours weekly trying to figure out how to collect and record data to support the new kinds of marks being given to the students for reporting purposes. Finally, Act 51 was implemented by the legislature calling for the new School Community Council and the subsequent Strategic Actions Plans that needed to be created including the Academic and Financial Plans. I was intimately involved with this process and it demanded much time and energy. These challenges are not unique to me. However, they made it necessary for me to rethink and redesign my study as the school year progressed.

## Methodology

In this investigation, I sought to examine how and why I chose particular tasks in an attempt to facilitate mathematical discourse and what I did and said in the implementation of these tasks that may or may not have facilitated effective mathematical discourse. The research questions were: (1) What did I do and say to facilitate mathematical discourse, and (2) How effective was I in facilitating mathematical discourse?

### *Participants*

As this is a self-study designed to continuing my evaluation of my own mathematics curriculum, instruction, and assessment, I am the sole researcher and subject. I teach at in a small (approximately 620 students with 28 K-6 teachers), suburban school located in Pearl City, Hawaii. I conducted this study in the first semester of my ninth year teaching there. I have taught one year of special education, two years of fifth grade, and over six years of sixth grade.

### *Setting*

The setting was a fully self-contained, heterogeneous sixth-grade class. Mathematics instruction was usually scheduled daily between 8:30 and 9:30. It sometimes continued later and was integrated with other content areas. The class consisted of 27 students of varied mathematical ability including four special education students, and six other students who received after school tutoring to address academic concerns. All students spoke English as a first language and were representative of a larger population of primarily mixed Asian and Pacific Islanders. An educational assistant was present during most mathematics instruction and was assigned to assist the three special education students that were integrated during mathematics instruction.

### *General and Specific Curriculum*

My mathematics curriculum is personally designed and rarely stagnant. The Harcourt textbook and the state standards are the primary curriculum guide for most teachers at my school. However, teachers have the freedom to create their own curriculum as long as it is consistent with the school's vision and mission, state standards, and the needs of the students. Therefore, I do not use the textbook as the primary teaching tool. Instead, I design lessons, activities, and units by using the textbook as a resource and supplement it with other resources including trade books and many National Council of Teachers of Mathematics (NCTM) publications.

My curriculum is primarily small group and large group discussion based. I emphasize small group and large group sharing but also use individual and paired practice and review. The students sit in table groups of between four and seven students, but the movement of students and grouping is fluid depending on the lessons and activities. In order to have regular review of previously taught concepts and skills, I do not necessarily use any resource in sequence, but instead find common threads from many sources in an attempt to keep students' attention and offer a wealth of different types of activities in my lessons. Pimm (1996) argues "for diverse mathematical activity and communication within a single classroom, one where mathematics is the focus [and] for the importance of maintaining a rich diversity of styles and sources of classroom discourse within any individual teacher's repertoire" (p. 11).

In the 2004-2005 school year, I devised a schedule (Table 2) so that I could engage students in several different content areas at once rather than treat them as isolated concepts taught as a chapter or unit, as is normally how traditional textbooks are designed. I sought to help students retain skills and concepts taught in previous lessons as they are reviewed and integrated on an ongoing basis.

Table 2. Sample First Quarter Curricular Plan

| Day | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| Title | "Measurement Monday" | Other Content Instruction | "What's My Chance? Wednesday" | Other Content Instruction | Other Content Instruction |
| Content | Geometry and Measurement | Whole Numbers and Decimals | Probability and Statistics | Number Theory and Procedures | Review, Connect, Integrate and Debrief |
| Main Topics | Rectangular Prisms: Attributes and Properties; Volume and Capacity; Nets; Perimeter, Area, Surface Area | Operations; Ordering and Comparing; Fraction Concept Building; Conversions Among Fractions, Decimals, Percents; Fraction Operations | Games that Build Understanding; Sample Space and Outcomes; Large Number Principle; Single and Compound Events | Exponents; Prime Factorization; Rules of Divisibility; Order of Operations | Social Studies: Empires Spreadsheets; Science: Force and Motion |

"Other Content Instruction" days focused on the mathematics that support the other concepts as well as skills acquisition, application, and review in order for students to apply the wealth of information gained in classroom experiences and apply them to new situations. I believe that covering many related topics at once that build upon each other while revisiting them often work well to help reinforce concepts for students.

I wanted to execute the same plan for the 2005-2006 school year, but found myself faced with additional challenges. Many students arrived with very little understanding of basic whole number concepts including place value, estimation, and operation. I decided, that for the first quarter, I would focus on these areas.

We spent much of July through August on whole number place value and estimation skills. In the beginning of the year, I gave the students "Find A Place," a game that allowed me to do some formative assessment of how well they understood and could use place value. I observed that many of the students struggled with whole number place value and also had little estimation skills. I believe using estimation effectively is a powerful tool in thinking flexibly and accurately. I used our Harcourt textbook as a resource because it outlines different estimation strategies. We used these strategies with different whole number operations. I wanted to give them multiple experiences to help reinforce place value while also providing opportunities for communication and articulation of concepts that were accessible to the students.

Using the different forms of estimation also helped me begin to design an environment where the students had many correct answers and they could discuss and justify which answers were the best or most reasonable. Since more than one answer was often acceptable, we, as a class moved away from 'right answers' as students seem to like to have. I'm not sure if the students subsequently had more number sense, but some the foundation for discourse was laid down as the students learned about how the class would work together as a community.

Through our time spent on estimation, which was formally assessed on August 18, 2005, I was able to see that much of the class didn't understand the concept of division. I had, in the past begun the year with fractions, decimals, and percents. However, I spent a large amount of time attempting to create opportunities for students to understand division of whole numbers. I felt that students needed to understand wholes and the partitioning of them to make an easier transition to fractions, decimals, and percents. The summative assessment division was given on September 1.

Along with the concepts and operations, the students were involved in a social studies unit on Ancient Civilizations in the Mesopotamian area. Each student was part of a "clan" that progressed in hope of becoming an "empire." The students had to keep an accounting sheet that included income and expenditures. While some students grasped and were able to complete the accounting sheet, others were not able to do so, even after continuous practice from July to October. This is similar to my experience with most classes and with the estimation and division instruction.

While teaching division, I attempted to build conceptual understanding, but still fell short with many students. Place value was still difficult for some students and I wandered away from the idea of estimation although I wanted to keep it in the forefront. I felt as though I wandered in to 'how to do division' rather than providing opportunities for students to build number sense. I wished I had the time to explore ideas such as: What happens when the divisor gets smaller? Or the dividend gets bigger? as concepts to further stretch the idea of number sense. I wondered and still wonder if the remediation with division actually made a difference when we started looking at concepts associated with probability. (See Appendix C for 1st Quarter Curricular Artifacts.)

Along with these concepts, we played many games. I wanted to create some common experiences that I could pull on later as we applied probability concepts. The games created situations where we could collect data in order to do operations with fractions, decimals, and percents. I provided calculators much of the time because many students still could not do division and it allowed us to do more as a class without the time constraints. In additional most people would use calculators when dealing with these types of operations.

By October, I was a couple of months behind where I usually like to be in the school year. I was also behind with respect to the HCPSIII expectations because the students lacked fluency with some basic concepts and skills. I knew, however, that I did not want to continue in the same fashion. I hadn't felt that I provided the wealth of experiences that I had in the previous years and I wanted to provide more rich experiences rather than 'skills in isolation.' With ideas involved with whole number place value and computation and probability introduced through various games, the students had common experiences to call upon. This allowed me to move into more applications of and problem solving and data analysis with fractions, decimals, and percents. (See Appendix D for 2nd Quarter Curricular Artifacts.)

### *Data Collection, Considerations, and Analysis*

#### *Recorded Data*

I audio-recorded most, but not all, of my mathematics episodes from October 14 through December 13, 2005 in my regular classroom and during after school tutoring I held from November 29, 2005 through February 28, 2006. I had intended to begin recording earlier, but, due to the challenges discussed earlier, I began formal data collection on October 14. The audio recordings were done at one source, an iPod with iTalk rather than with the groups of students as had been done in the Phase 1 and 2 studies. This allowed for much more flexibility when I had previously recorded episodes on tape recorders. It was still difficult, however, to hear all of what the students said.

I intended on including video-recordings of the episodes in the data analysis. However, the video proved to be cumbersome and ineffective. Besides having no space to put the video camera where I could clearly see my interactions with students and my complete movements, purchasing tapes became too costly and changing the tape several times during a math episode detracted from my teaching and interaction with students. Besides these logistical problems, when viewing the recordings, the picture did not show details I believed would effectively add to the data such as what was recorded on the whiteboard during discussions. In addition, the student dialogue was generally inaudible. I stopped recording with video on October 21.

For this paper, I consider one continuously recorded math experience in one day an episode. Episodes ranged from approximately 12 minutes to 1 hour, 40 minutes during the school day and between 1 hour, 8 minutes and 1 hour, 44 minutes during after school tutoring.

*Student Data*

The students were given a Likert survey twice, once toward the beginning of the semester, on July 28, 2005 and once toward end of the semester, on November 29, 2005. I also gave the students an open-ended questionnaire twice, once on July 28, 2005 and once on December 16, 2005.

Various work samples were collected including individual student work in the form of reflections, notes, logs, and students work. Others were generated during discussions involving the whole class.

Students were given an identification number so that work and responses could be kept anonymous. This number was used consistently for recording in the survey, questionnaire, and on student work.

*Personal Reflections*

Finally, I kept reflections at various times in writing and in audio form. Some reflections included two other teachers that team taught with me during after school tutoring sessions.

*Analyzed Data*

Due to time constraints and other considerations, only some of the data described above was formally used for this self-study. Audio-recordings from October 14 – November 9 were transcribed. Only excerpts of the transcriptions are included in this paper due to the length of the full transcripts (approximately 180 pages). Details or big ideas I could gleam from viewing the video recordings from October 15 – October 19 were added to the transcripts. My personal reflections have been used in the text of this paper and have been added to the transcripts where appropriate.

I also chose to study the Student Math Questionnaire and the Student Math Survey. (See Appendix D.) Some questions and responses were not related to the research questions and therefore, I used only those parts that I believed were related to or gave insight into mathematical discourse.

Finally, some instructional artifacts, copies of student work, and teacher resources are shared in the appendices to  provide context.

*Analysis of Transcripts*

I listened to the audio recording of an episode and transcribed it. If video was available, I watched the video and reviewed personal reflections, adding notes to the transcript. I listened again, checking the transcript for accuracy, and added more notes including personal motivations for saying particular things, acting in particular ways, and student responses.

After repeating this process for all of the episodes, I examined Episode 1, in detail. I found that lines of transcript could be grouped together based on one or more themes. As I designated new themes, I found that many were similar enough to be grouped together. The larger list of themes became smaller as themes became sub-themes under a new name. Once I had a manageable list of recurring themes from Episode 1, I examined the other Episodes to see if I could continue to apply the designated themes and if any others occurred.

I chose excerpts from the lengthy complete transcripts to present in this paper. These excerpts were chosen because they showed or told me something about mathematical discourse as I understood, or did not understand, it. I examined the excerpts first so that I could apply the themes to particular lines and groups of lines. Second, I wrote notes and commentary about the excerpts. Finally, I incorporated outside research to help shed light on the nature of discourse community as exhibited in the excerpts.

## Findings

The findings are given in three parts. First, I present analysis and conclusions regarding the complete transcripts followed by excerpts of the transcripts with detailed examination and connections to outside research. Second, I summarize my findings regarding the student surveys. Finally, I provide analysis of the student questionnaires.

### *Transcripts*

The *Professional Standards for Teaching Mathematics* (NCTM, 1991) states that:

> The discourse of a classroom – the ways of representing, thinking, talking, agreeing and disagreeing – is central to what students learn about mathematics as a domain of human inquiry with characteristics of knowing. Discourse includes both the way ideas are exchanged and what the ideas entail: Who talks? About what? In what ways? What do people write, what do they record and why? What questions are important? How do ideas change? Whose ideas and ways of thinking are important? Who determines when to end a discussion? The discourse is shaped by the tasks in which students engage and the nature of the learning environment; it also influences them.

In order to answer the question "Who talks?" I counted the number of lines of transcripts to find that approximately 2499 out of 6808, or 36.7% of the transcribed lines were those of students. Episodes 5 and 9 have the least amount of student lines of dialogue. On November 2 (Episode 5), I had lectured the students about being unprepared for class by not completing work and studying for announced assessments. My interactions with the students were more short and commanding. Evidence of my frustration with the students is that I had the students who did not complete their work stand while we went over the homework. I had reached my wit's end on this day, possibly resulting in less student dialogue.

Table 3. Percentages of Student Lines of Dialogue in the Full Transcript

| Episode | Number of Student Lines | Number of Total Lines | Percent of Lines Students Spoke |
|---|---|---|---|
| 1: October 14 | 489 | 1219 | 40.0 |
| 2: October 18 | 252 | 676 | 37.3 |
| 3: October 19 | 452 | 1263 | 35.8 |
| 4: October 21 | 271 | 617 | 43.9 |
| 5: November 2 | 310 | 972 | 31.9 |
| 6: November 3 | 125 | 237 | 52.7 |
| 7: November 4 | 315 | 791 | 39.8 |
| 8: November 8 | 116 | 310 | 37.4 |
| 9: November 9 | 169 | 723 | 23.4 |
| Total | 2499 | 6808 | 36.7 |

Table 4. Highest Percentages of Student Lines on Pages of Transcripts

| Percentage of Student Lines | Episode, Page | Context of Discussion |
|---|---|---|
| 60.0 | 1, 11 | Where to put the probability that something will occur on a scale of 0 to 1 |
| 50.0 | 1, 23 | Comparing the theoretical and experimental probability that a number will occur when rolling 6 sided dice |
| 56.1 | 1, 26 | Estimating and confirming the percent equivalent of a fraction using calculators and mathematical procedures |
| 53.1 | 1, 29 | (same as 1, 26) |
| 50.0 | 3, 21 | Assisting a group who is trying to play "The Card Game" |
| 52.5 | 4, 2 | Finding errors in a compilation of class data being used to find the probability that the sums of two dice will occur |
| 54.8 | 4, 4 | Finding the GCF of the numerator and denominator of a fraction |
| 59.0 | 4, 5 | Discussing and modeling: after student question regarding an alternate way to find the simplest form of fractions with large numerators and denominators, procedure to divide a fraction to get the equivalent percent |
| 62.2 | 4, 12 | Finding the GCF of a fraction with a large numerator and denominator, modeling finding an error |
| 55.6 | 4, 13 | (same as 4, 12) |
| 50.0 | 6, 1 | Discussing student's solution to NCTM Assessment Problem 7 |
| 58.7 | 6, 3 | (same as 6, 1) |
| 52.3 | 6, 5 | Discussing students' solutions to NCTM Assessment Problems 8 and 9 |
| 70.0 | 6, 6 | Discussing student's solution to NCTM Assessment Problem 9 |
| 56.5 | 7, 20 | Discussing possible choices in solving NCTM Assessment Problem 14 |
| 58.7 | 7, 21 | Discussing the solution of NCTM Assessment Problem 15 |

On November 9 (Episode 9), I tried a new strategy of giving the students transparencies of some problems in the *Mathematics Assessment Sampler, Grades 3-5*

(NCTM, 2005) at their desks to write their solution on before coming up to the overhead projector to share. I was trying to find ways to discuss more problems because it was taking so long in the previous episodes. I didn't realize at the time that this caused me to do more of the talking. I tended to read the problems and the solutions the students had written. After, I asked the students to tell what they thought of the solution. I did more talking through the problems than the students did. In order to use this strategy more effectively, the students have to be versed at giving opinions and reasons more independently and readily.

I noticed that throughout the transcripts, teacher lines were generally longer than student lines. Asking open-ended questions rather than "yes/no" questions is generally preferred because closed-questions are lower-order questions and usually elicit one-word answers, while open-ended questions have many different responses and require students to think rather than to recall facts. Probing questions or timely questions can challenge the learner to advance their understanding in several ways through student reexamination of solutions and reflections that stimulate students making reflections and connections (Martino & Maher, 1999).

Although many of my prompts were open-ended, many of the student lines were answers of yes or no in response. I found it to be difficult to get the students to express their thinking although the question "What do you think and why do you think that?" consistently appeared throughout the episodes. I hoped that the prompts could eventually be removed. Mason (2000) found that questions that become more general and more indirect as prompts until they disappear can have an effect of transferring initiative from teacher to student, becoming part of each student's inner monitor. However, Nathan and

Knuth (2003), posit that when teachers "step-out" and allow for student led discourse, a decline in teacher mathematical participation and analytic scaffolding occurs and, although student-to-student talk increases, there may not be dramatic improvement in the quality of conversation regarding the mathematics. Therefore, it is difficult to decide when to step-out and when to stay in. Partly because of my discomfort, I chose to stay in.

To answer the questions "About what? In what ways? What do people write, What do they record and why? What questions are important? How do ideas change? Whose ideas and ways of thinking are important?" I reviewed the transcripts to find 'themes' that occurred. Along with reviewing all of the transcripts to generate themes, I coded Episode 1 with the themes in detail to find specific purposes I had for the conversations I led through my choice of tasks, questions, prompts, and responses to students.

I found it extremely difficult to code using the different themes because of the complexity involved in questioning, prompting and responding. For example, in Excerpt 1-2, six themes are listed for thirteen lines of transcript. Many themes occur at the same time such as in Excerpt 1-3, lines 1-20, where questioning for clarification of misconception, prompting for estimation and questioning for understanding all occur simultaneously. These themes are listed, but I also repeat student comments and check for understanding in my prompts or questions while I am prompting for student participation. In many instances, the complexity of the conversation was immense with many purposes for prompts and questions receiving different responses from different students. Some of the themes are listed with the excerpts of the transcript following this section.

Mathematical reasoning and evidence is the basis for discourse (NCTM, 1991). Throughout most of the excerpts, I model my thinking and reasoning and prompt students to get at their reasoning. The students estimated the probability that events may occur and were asked to give the reasoning for their estimations. I guided the class in determining the theoretical probability that outcomes would occur and, as a class, we validated the estimates with experiments and through calculations regarding what occurred during and after playing games. Students were continually prompted to explain why they thought something was true and to provide any evidence.

In *Principles and Standards for School Mathematics* (2000), NCTM calls for instructional programs that

> enable students to – organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teachers, and others; analyze and evaluate the mathematical thinking and strategies of others; use the language of mathematics to express mathematical ideas precisely.

Beginning in episode 5, I attempted to get students to communicate more in these ways by giving them opportunities to solve problems together. I asked students to share their solutions with the class on the overhead projector and for other students to come up to add to or change the work as they thought necessary. This allowed for more communication about problems solving as well as for discussion about issues that arose naturally out of the context of problem solving. Therefore, the problem solving standard wherein NCTM calls for instructional programs that "enable students to: build new mathematical problem solving; solve problems that arise in mathematics and other contexts; apply and adapt a variety of appropriate strategies to solve problems; and monitor and reflect on the process of mathematical problem solving" was also being emphasized.

Many of the aspects of a teacher's role in discourse that are identified in the *Professional Teaching Standards* (NCTM, 1991) are exhibited and grappled with throughout the following excerpts. These include:

> posing questions and tasks that elicit, engage, and challenge each student's thinking; listening carefully to students' ideas; asking students to clarify and justify their ideas orally and in writing; deciding when and how to attach mathematical notation and language to students' ideas; deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let the student struggle with a difficulty; and, monitoring students' participation in discussions and deciding when and how to encourage each student to participate.

I chose the following excerpts to examine most closely because I felt they brought up issues that are related but not limited to the discourse, communication, and problem solving standards as outlined by NCTM.

*Excerpts of Class Transcripts*

Episode 1: October 14, 2005

I began the episode by referring to circle charts I had made on the board representing landmark fractions such as 1, 3/4, 1/2, 1/4, and 1/8 along with their decimal and percent forms. I did this to introduce a basis for communicating about these three forms as well as review some basic facts. Following this teacher led discussion, I asked the students to go the board to record data they had considered the night before such as:

What are the chances a tornado will hit today? What are the chances it will never snow here? And, What are the chances you will get homework tomorrow? We examined the data the students recorded as a class.

Excerpt 1-1 (90)

> OK. Thank you, everybody. K. Look at the list on the left. Somebody read the first category and example and then tell us what you think about it and then they will call on the next person who has something to say about what they said.

[Themes: showing appreciation, transition to new task, "what do you think about that?" prompting for student to call on another student.]

I wanted the students to respond to the class data. In an attempt to remove myself from a leadership role, I asked the students to call on the next student who volunteered. That next student should then paraphrase what the previous student said. My purposes were to get the students to actively listen to each other, speak loudly enough for others to hear, as well as to continue to remove myself as the leader by allowing myself to step out of repeating what students said.

Excerpt 1-2 (118-137)

1 Come on, somebody, raise you hand and read the next one. Thank you, N----.

2 *Student:* *There is a 25% that I will have Chinese food for dinner.*

3 What do you think of that?

4 *Student:* *Yeah.*

5 So you think that person has, eats Chinese food 1 out of every 4 days?

6 *Many students respond negatively.*

7 Who wrote that one?

8 *Student responds.*

9 M----, do you eat Chinese food 1 out of every 4 days?

10 Did you write, there's a 25% chance that I will have Chinese food for dinner?

11 Did you have another reason why you had 25% chance of having Chinese food for dinner?

*12 Student: Left overs*

13 Ah! Does that make sense then?

[Themes: prompt for student participation (1), appreciating student participation (2), "what do you think of that?" (4), restating student idea in a different way (5), questioning for reasoning and understanding (6-13), prompting for agreement (14).]

I deviated from my strategy when I called on a student instead of allowing another student to call on somebody. Unless I am consistent, students will not get used to choosing another person to speak and I will never be able to remove the prompts. Since it was a new strategy, it was difficult for me to be consistent. In line 4, I ask an open-ended question, but the student responds in a yes/no format rather than telling what they think. Students consistently responded this way.

It was important that I sought different reasons why the student said the probability he would eat Chinese food was 25%, because he was able to validate his answer. The line of questioning was an example of how students could justify their reasoning. I could have directly discussed that this was justification. Through the questions I pose in lines 9-11 above, the student is asked to clarify and justify his answer. I could have been more explicit so that the other students knew that I was modeling my thinking with these questions and that I expected them to be asking questions as well. Below, a student shares that he has a 50-50 chance of watching TV that day.

Excerpt 1-3 (153-915)

1 So you watch TV every other day?

*2 Student: I watch TV everyday.*

3 If you watch TV everyday, is that a 50-50 chance that you would watch TV?

*4 Students: 100*

*No.*

*100*

*5 Student: I'm not sure if I watch it-*

6 Do you watch it almost every day?

*7 Student: No.*

8 You said you watch it every day.

*9 Student: Uh. Yeah. Sometimes I watch TV, sometimes I don't.*

10 So about-

*11 Student: Mostly I do.*

12 Mostly you do. If you mostly watch TV, is it a 50-50 chance that you watch TV?

*13 Students: No.*

*75*

14 That means you're not sure, you could or could not, but if you mostly do, it's closer to-

*15 Students: 90%*

100%.

16 Do you watch TV everyday?

*17 Student: Yes.*

18 Then it's gonna be much higher.

19 *Student: I watch TV everyday.*

20 Do you understand why? Why it is more than 50-50?

*21 Student: I'm not sure-*

22 OK. Help him out. Explain to him why, if he watches almost everyday, why it's not, why it's more than 50%. Go ahead M----.

23 *Student: -(inaudible)-usually watch TV.*

24 OK, but the question, I'm asking you, if he says he mostly watches TV, almost everyday he watches TV, why is that not 50%? Why is it more than 50%?

*25 Students:* *Because you usually watch TV.*

*Because mostly is more than half and mostly is like every time so it's closer to more times than not watching it.*

26 95% of the time you watch TV? When you go home?

*27 Student:* *I do my homework first.*

28 But then you watch TV? So it's not very often that you don't watch TV. When was the last time you had a day that you didn't watch any TV?

*29 Student:* *When I got grounded.*

30 When was that? This year?

*31 Student responds (inaudible).*

32 OK. So he's watched TV everyday this school year. So, does that, that's not 50-50 then, yeah? That's more like 100. So you'll watch TV pretty much 100% of the days unless you're grounded. OK. Thanks.

[Themes: questioning for clarification of misconception (1-20), prompting for estimation (14), questioning to check for understanding (20), prompting for reasoning (22), restate the question (24), questioning for understanding (26-31), summarizing (33)]

In line 20, I addressed the student directly as I moved closer to him. It became a more lengthy class discussion since he didn't seem to understand. I wonder if the student felt uncomfortable since he was at the center of the class discussion.

In line 22, I wanted the students to share in their words since I may have been unclear. The student seems to be confused when he says, "I do my homework first." After following up with lines 28-33, the student seemed to understand. Excerpt 1-3 exhibits the difficulty experienced when a student doesn't grasp an idea such as 50% and 100% probable. I struggled with clarifying the issue and allowing the student to struggle and decided to pursue this by asking other students to share their ideas. The next discussion involves food. One thing to note is that all of these discussions are related to students' real life experiences.

Excerpt 1-4 (234-268)

1 There's 100% chance-

*2 Student* *I won't eat dog food.*

3 That's great!

*4 Students laugh*

5 I'm glad for you.

6 *Student:* *I ate dog food before.*

7 What does the next one say?

*8 Student:* *Breakfast. There is a 100% chance that I'll eat eggs for breakfast.*

9 So this person eats eggs every single day.

*10 Student responds (inaudible).*

11 Almost every day. Then is it 100%?

*12 Students:* *No*

*90*

13 100% means that it's absolutely going to happen, no questions asked, no exceptions.

*14 Students:* *They might run out.*

*They could have eggs.*

15 Then it can't be 100%.

*16 There is a lot of student discussion.*

17 But do you do it all the time, every day, no matter what?

*18 Student responds (inaudible).*

19 Some of it is not 100%. All is 100%. So you gotta change it. Do you eat eggs most days?

*20 Student:* *Yeah*

21 9 out of 10 days?

22 *Student:* *No*

23 8 out of 10 days

24 *Student:* *More like 7*

25 7 out of 10 days. What percent is that?

26 *Student:* *75*

27 7 out of 10 days

28 *Student:* *70%*

29 70%. Thank you N-------.

30 7 out of 10, 70%.

31 OK. So take a look back at your categories and examples and make sure, that if you have something down for 100%, that it is absolutely gonna happen, no possible way it could not. Otherwise, you should change your percentage.

[Themes: prompt for student participation (1, 5), providing humor (3, 5), restating [student] ideas in different ways (9, 13), providing correction to student error (19), prompting for improvement to work (19, 31), prompting for fact (25), questioning for clarification of misconception (19-27), appreciating student participation (29)]

At the end of line 1, I paused to solicit student participation. I did this consistently throughout all of the episodes. I ignored the student response in line 6. I think it was appropriate to do so as a way of redirecting student attention. The questioning in lines 19-27 seemed effective because a student refines his thinking. During line 30 above, I modeled that 7 out of 10 is also 7/10 and 70/100 to show how it is also 70%. Here, I decided to attach mathematical notation and language to students' ideas. Students were consistently prompted to change and improve their work subsequent to our discussions.

Below, the excerpt involves the students being given ideas labeled with capital letters. The students were to put the letter on a line to represent a prediction that something could happen along a probability scale from 0 to 1.

Excerpt 1-5 (286-328)

1 Where did you put your B and why?

*2 Many students.*

3 Raise your hand. Raise your hand. The next president of the United States will be from our state, Hawaii. And, C—.

*4 Student: Um, Impossible*

5 Why?

*6 Student: Because every single president was never from this state and it says the next president so, yeah.*

7 OK. So G—, repeat what he said and tell us what you think about what he said and what you think. Ha ha.

*8 Student: He thinks absolutely no because all the other presidents have not been from our state and it says the next state, the next president. Um. I think a little closer to possible but not all the way and not close to 50-50 chance because we're one of fifty states and there's a chance that there could be a president from our state.*

9 Anybody else have something to say about that, about what G— said or about what they put?

10 So where did you put the B? Tell me where.

11 Here?

*12 Student: No.*

13 Here?

*14 Student: No.*

15 Here?

*16 Student: Yes.*

17 OK. And some people have it at zero. Is that OK to have it at zero?

*18 Students: Yeah.*

19 Cause this is your opinion.

[Themes: "what do you think and why?" (1), redirect class attention (3), prompt for student to repeat what another student said (7), "what do you think of that?" (7), accepting multiple answers (17-19)]

The student who shares in line 6 has a good idea and I try to have students build on it in line 7. Ideally, I wouldn't have to prompt others to say what they think of what others have shared. I believe that the questions and prompts are a beginning to facilitating students' spontaneous reflection and participation. I would need to step out of the role of leader and stay silent after explaining to the students why I do so and what I expect in classroom conversations more explicitly. In line 8, the student makes a good attempt at explaining her thinking. I wonder how I could get the students to communicate more clearly. Perhaps students could be taught to ask, "What do you mean?" when they don't understand. First, however, the students need to understand that participation starts with their active listening. Whether students are actually listening is not always apparent.

Next, I asked the students to discuss some items in their groups. The prompt was as follows:

Excerpt 1-6 (334)

K. Talk in your group. See what you put. See if you agree. Come up with, Sh. Come up with one number for your table. So you guys have to come to an agreement. Explain to each other why you think it should be [the number you agree to as a group].

[Themes: redirect class attention, prompting students to work collaboratively in their groups and come to an agreement]

I asked the students to work in their groups to come to an agreement because I became aware, through the previous lines in the conversation, that many students either did not complete that part because they didn't know how or they weren't sure of their conclusions. I wanted the students to share their ideas while I circulated throughout the room. In this way, the students could help each other and I could also help the ones who needed it. It also extended their thinking because they had to come to a probability that their entire group would do something, not just themselves as individuals.

In David Pimm's essay "Diverse Communications" (NCTM, 1996), he argues "Mathematics involves focusing on relationships between parts and wholes, exploring change and constancy, stressing this and ignoring that. Mathematical activity is the means to an end, to encountering some idea or isolating some property, to seeing or realizing that something *must* happen or *cannot* happen" (p. 13). As students discussed in their groups, I circulated in the room and joined a student group. The following excerpt shows Pimm's idea in action.

Excerpt 1-7 (367-392)

1 I got a question for you though. Now looking at the TV, what are the chances that your entire group will watch TV tonight?

2 *Student:* *OK. What did you get?*

3 *Student:* *25%*

4 *Student:* *K. That's one 25, one 0.0, one 1.0...K. The chances that we will watch TV tonight absolutely is 3 out of 5.*

5 But, the question is, what are the chances that your entire group will watch TV tonight.

6 Tonight. Your whole group.

*7 Students: fifty...*

*45%*

*More than 50-50.*

8 Who's not gonna watch?

9 If he's not gonna watch, what are the chances that your whole group will watch? If he's not going to.

*10 Student: 4 out of 5?*

11 Your whole group. The chances your entire group is gonna watch if he's not gonna watch.

*12 Student: 50-50 chance, I guess.*

13 Is he gonna watch tonight?

*14 Student: No.*

15 Is he part of your group?

*16 Student: Yes.*

17 So is your whole group gonna watch tonight?

*18 Student: No.*

19 So what are the chances your whole group is gonna watch tonight?

*20 Student: Zero*

*21 Student: Oh! 0%*

22 Cause he's not, so then your whole group will not be. Some of you might be. But, if you were gonna put it up there for your whole group, you're gonna have to put close to zero.

[Themes: questioning for understanding (1-9), restating the question (5), questioning for reasoning (8-19), summarizing (22).]

The students discussed together, with me as a facilitator in lines 2-8. In line 2, a student takes the leadership for the group discussion. I notice that in line 5, the student is looking at the part of the whole instead of the whole group. In response, I question and prompt to get them to reason in that way instead. This ends with the students understanding the concept as evidenced in lines 20-21. I wonder if I could have got a student to summarize instead of me doing so in line 22. Later, that group shares with the whole class. I follow up that I noticed one student said that the chances he would watch TV was 0.0003.

Excerpt 1-8 (428-453)

1 OK. Call on somebody else, N-------.

*2 Student: Uh. G----.*

*3 Student: Our group-*

4 I'm sorry, some people are talking and I'm gonna ask them to leave because I already warned them. So see you later. Bye. Go talk outside. Come back when you're ready to stop talking while other people are talking.

5 Thanks G----. Go ahead.

*6 Student: Um. There was a 0% chance that our whole group was gonna watch TV.*

7 Why?

*8 Student: Because one person put that they weren't gonna watch TV tonight.*

9 OK. And actually one person had this number. How many zeros are there?

*10 Student: 0 point 3-*

11 These are the tenths, hundredths, thousandths, ten-thousandths. Three out of ten thousand times, they watch TV. How many times a year do they watch TV?

*12 Students: Hardly*

*once*

13 Once out of-

*14 Students: a thousand*

*three-*

15 Have they…I wonder if they've been alive 10,000 days.

*16 Students laugh.*

17 3 out of 10,000 days they watch TV. How many days are in a year?

*18 Students respond*

19 365 times10 is 3,065. This person has to be 20 years old to only watch TV this much.

20 Maybe you watch TV a little bit more than that? I think you might. So you gotta be kinda careful about this.

[Themes: prompting student to call on anther student (1), addressing behavior problem (4), "why do you think that?" (7), building on student idea (9-18), questioning for reasoning (9-20), modeling appropriate math language (11)]

In line 4, I quickly and effectively dealt with students who were distracting each other and the class. In line 19, I made a mistake, but the meaning was not hindered. I wonder if I could have had the students think through the calculating instead of me doing so. Through this conversation, we built on a real life experience and checked for accuracy in the context of the problem and through our discussion.

In lines 11-20 as well as in the next two excerpts, I tried to give the students experiences that allowed them to determine whether something was mathematically correct, to reason mathematically, and to connect mathematics, its ideas, and its applications (NCTM, 1991). In Excerpt 1-9, we began by using fractions to compute theoretical probability in percent.

Excerpt 1-9 (640-683)

1 2 out of 8. K? And, if you think about 2 out of 8, well, 1 out of 8 is 12 1/2%. So 2 out of 8 would be double that, which is this. And you can also, if you remember how to reduce fractions, know that 2/8 is the same thing as 1/4. And 1/4 is 25%. So 25% of the time, you would get red...And 100% of the time would be all the time. 25% of the time you get red. So what's left over?

2 *Students:* *Wait.*

*Oh!*

*75*

3 75% of the time, you should get white. That's what should happen. That's the theoretical probability.

4 OK. When we did the dice, you told me that you have a 1 out of 6 chance that you would get each number...1 out of 6 chance. Is that more or less than 1 out of 4?

*5 Students: Less*

6 Less cause it's even smaller. K? So what would you guess, 1 out of 6, what percent of the time do you get a number on a dice?

*7 Student: So...50*

8 Not 50, that would be 1 out of 2 times. It's less than that. It's less than 25%.

*9 Student: 10%*

10 10% of the time? Do you agree?...It's 1/10?

*11 Students: No.*

*15?*

12 1/10 is too small. So what's 1/6?

*13 Student: 14%*

14 So let's take a look at these numbers. 1/6. 1/10 is less than 1/6, right? 1/2 is more than 1/6. This is 50%. This is 10%. Where's 1/8 fit?...In between here or in between here?

*15 Students: In between.*

16 Here?

*17 Students: Yeah*

18 That's 12 1/2 %. So what do you think 1/6 might be?

*19 Students: 14 1/2*

*17 1/2*

20 14 1/2? 17 1/2?

*21 Student: No, just regular 17.*

22 17? So we have between 14 and 17%...Well, we can figure this out by dividing. This means 1 divided by 6. You're taking 1 and you're dividing it up into 6 equal pieces. K. 1 divided by 6. I can add a decimal and zeros. Like we did before. 6 goes into 1?

23 Oh. And look at that. It's gonna go on, and on, and on like that. Very good estimate. 17%. Because here it is. 16.6%. So it is, if you round it up, about 17%. 16 and it keeps going forever. Very close.

[Themes: modeling thinking/reasoning (1), introducing vocabulary in context (3), questioning for reasoning (4-21), modeling mathematical procedure (22-23)]

In addition to introducing the concept of theoretical probability, I could have prompted students to examine the root of the word "theoretical" and discussed the idea of a theory after line 3. In line 12, I noticed that students were not grasping the reasoning; I provided a number line on the whiteboard as another representation to stimulate understanding. It worked well because it was another visual representation that becomes a subject in later episodes. With the circle graphs and line representations, there is more chance that students will be able to make meaning of the fractions and their equivalent percentages. I could have also used the number line representation to review the notation of probability on a scale of 0 to 1 since it was discussed previously (Excerpt 1-5). In lines 22-23, I modeled the division, a review for the students, to show how to use the textbook instructed algorithm to find the value of 1/6 and validated the students' estimations of between 15-17%.

We related the theoretical probability to the experimental probability that resulted from a game we had played together. Students used calculators to convert 17/110 and 14/110 into decimals and percents. I then asked them to do some thinking in Excerpt 1-10.

Excerpt 1-10 (773-815)

1 13%. 22 out of 110. Put your calculators down, don't touch it. 22 out of 110. Is it more or less than how many times we chose the 1s or 2s?

2 *Students respond differently.*

3 We picked it 22 times. Is that more or less than 17 or 14 times?

4 *Students:* *more*

5 So take a guess. What do you think, what percent will 22 out of 110 be?

6 *Student:* *23%*

7 *Student:* *I say 20 because 5 times 20 is 100 plus the other 2, plus 5 is gonna be 10.*

8 Cause you know that 20 out of 100 is 20%. 22 out of 110...[is] probably close to 20%. K. Try it, 22 divided by 110. N----, I'm coming over.

9 *Student:* *I think that it's 22% because 20%, um, 20 out of 100 is 20%, and you just have to add 2%.*

10 Yep. So it's a little bit more than 20, but, so, you think it could be 22. Good guess.

[Themes: prompting reasoning and estimation (1-10), restating student idea in different way (7)]

In line 1, I wanted the students to think ahead and estimate instead of using calculators. I restated my question in another way and gave additional context by highlighting previous data in line 3 because I noticed that students were giving me different answers, and therefore, may not have understood the question. The students responded and showed reasoning in the responses on lines 7 and 9. This is what I'd like all of the students to be able to do. In excerpt 1-10, the discourse is dialogic rather than univocal because I am not attempting to convey a particular message or approach for the students to follow. Instead I intend on understanding my students' thinking and using the students' statements as thinking devices as well as expect the students to use each others' statements as thinking devices (Knuth and Peressini, 2001). To extend this idea, I could have recorded the two students' ideas on the board and had the students discuss what they meant in order to stimulate listening to, thinking about, and reasoning through other students' ideas. Sometimes, during class, the discussions go so quickly that it is difficult to record and capitalize on student ideas. Taking more time to listen to and discuss becomes more evident as a need for a discourse community.

Excerpt 1-11 (859-912)

1 OK. S--- said he noticed a pattern. What pattern did you notice? Don't touch the calculators, put 'em down.

2 *Student:* *Um, all the percents are two below the top number ---*

3 below the numerator?...OK. If you look at these, all of these numbers are around the same. There's not that big of a difference. Yeah? So you're saying that this is close to this? Cause this is almost 100. Yeah?...This is about 100 and remember a percent means, whatever it is out of 100. So that's why...this is a little bit more than 100, so it makes this amount a little bit less. I'm glad you noticed that.

4 Anything else you notice?

5 *Student:* *Um. Sort of like what S--- said...the percents, they look the same, too, about.*

6 OK. And why should all the percents be the same?...Why should all of the percents be about the same?...What are the chances you're gonna get a 1?...What are the chances, if you get a 1, how many times out of, cause you have 1, 2, 3, 4, 5, 6. What are the chances you're gonna get a 1?

7 *Student responds (inandible).*

8 1 out of

9 *Student:* *6*

10 What about 2?

11 *Student responds (inaudible).*

12 1 out of 6, 1 out of 6, 1 out of 6. All of them should be about 1 out of 6. And what was 1/6, if you change it to a percent?

*13 Students:* *16*

*0 point 16*

14 Should be about 16%. All of our numbers should be around 16%. We did roll a lot of 3s. And we noticed that. For some reason we rolled more 3s than we should have. But, other than that-

*15 Student:* *6*

16 Oh and more 6s. And we noticed that, oh, it felt like we rolled more 1s, but that was because we were losing whenever we got a 1. But, we actually rolled more 6s and more 3s, than we did 1s. But in a perfect world, every thing, should be 16%, because you had a 1 out of 6 chance that you would get each of those numbers. But, that's not how it really works when you really play the game. Yes?

[Themes: building on student idea (1-3), appreciating student participation (3), modeling thinking/reasoning (3, 6, 12, 14, 16), questioning to build understanding (6-12)]

In lines 3 and 6, I think I missed an opportunity to get the students to explain and understand this students reasoning. I, instead, take over the idea and explain it. If the students who shared had the opportunity to repeat or restate their ideas, they may have clarified their own thinking. In addition, I would have allowed the students to take responsibility for the discussion instead of me. In lines 6-16, I bring to light the idea of theoretical probability in relation to experimental probability. I could have made more explicit this relationship. Again, in line 16, I could have asked the student why they thought we felt like we rolled more ones instead of giving my opinion. I think it was good, however, that in line 15, a student corrects me and I accept and build on that correction, modeling that everyone makes mistakes.

Episode 2: October 18, 2005

This episode begins with asking the students some questions to get the class thinking about fractions.

Excerpt 2-1 (1-39)

1 OK. Umm. Some notes for you to take in your math log. Fractions. Decimals. [writing on the board] Percents…K. Somebody give me an example of a fraction.

*2 Student: 7 / 10*

3 Somebody tell me what you call the different parts of the fraction.

*4 Student: Numerator, denominator.*

5 Can anybody tell me what the numerator and denominator mean? What does the numerator mean? What does the denominator mean?

*6 Student: The numerator is the number that you divided by the denominator.*

7 OK. But if I have a picture like this, what is the fraction?

*8 Students: 1/ 4*

*yeah (different students answer)*

9 What does the 1 mean?…By looking at the picture, what does the 1 mean?…

10 *[A-----] struggles…long pause]*

11 Do you have the whole thing?…You have just one part of it. K?…So the numerator is…

*12 Students: The part of one*

*the part of four*

*the part of the whole*

13 The part of the whole that…that's actually there. Yeah?…So the numerator is what part… you actually have …

[Themes: reviewing concepts and vocabulary (3-9), prompting student ideas, summarize (11), questioning to clarify misconception (9-13)]

The students were taking notes based on my prompts and their ideas. This is much different than a lecture format while it is still guided by me as the teacher. In line 9, I responded to a student who is showing difficulty explaining a concept. Since I am not sure that he understands, I question and prompt in another way in lines 9 and 11 and the students respond in different and appropriate ways.

We later reviewed changing fractions into decimals and percents using calculators. The concepts of repeating and terminating decimals appeared naturally. After the class has time for individual practice with me circulating around the room, we came back together to review decimal place value and reading and writing decimal numbers.

Excerpt 2-2 (503-514)

1 Make up your own number, with a decimal in it, and you're gonna have to read it to us. Write it in your book. [N---- is raising her hand while I am erasing the board and walk away.] OK. N---- is gonna give us her number, and you have to try to write it down in your book. So, N----, please say your number really loud and really carefully, and we might ask you to repeat it several times.

*2 Student: Four hundred and seven tenths.*

3 Say it again.

*4 Student: Four hundred and seven tenths.*

*5 Student: Four hundred and seven tenths.*

6 Four hundred seven tenths. [I heard it differently].

7 This is tricky. You need tenths. That's all you can have. And you need four hundred seven of them. Four hundred seven tenths. Otherwise known as forty and seven tenths. I don't think you wrote that did you?

*8 Student: No.*

9 What did you write in your book?

*10 Student: Um. Four zero zero point seven.*

11 Let me see.

12 Good. This is what she wrote. This says four hundred <u>and</u> seven tenths. When she said four hundred seven tenths, I knew I was going to have tenths here and everything else was going to be

on this side. You see why it's four hundred seven tenths? Four hundred seven tenths. This is very tricky. We don't call it like that cause it's too hard to, to see. Four hundred and seven tenths. Whenever you see a decimal point, you say...And!

[Themes: using appropriate math language (2-12), building on student idea (6-12)]

In Excerpt 2-2, I heard what the student inaccurately, an instance where a teacher misinterprets what a student says. It ends up being a teachable moment, however, because of the ideas in line 12. I could have had the students grapple with how to write four hundred and seven tenths and four hundred seven tenths instead of explaining is myself.

Listening is key in facilitating discourse. "Mathematical communication can take place effectively only if *all* participants are prepared to adopt *both* roles [listener and speaker], to listen actively as well as to talk" (Pirie, 1996, p. 105). There is the "need to attend to the subtle power of the individual words that students use; mathematics is about precision of thought, and this is best expressed through precision of language, be it verbal or symbolic" (Pirie, 1996, p. 115). In the excerpt above, I hear the student incorrectly. I needed to listen more carefully. However, the resulting discussion became an important teachable moment because it touched on the importance of the precision of mathematical language.

Episode 3: October 19, 2005

I quickly went over changing fractions to their simplest form. I expected and confirmed that many students had forgotten how, but decided to address the need at a later time. Using the context of a game with different color tiles in a bag, we used calculators to see that equivalent fractions are the same when turned into their decimal form. I introduced a probability game, "The Card Game" (Griffin, 1998). After going over the directions I asked the students to record predictions they then shared with the class.

Excerpt 3-1 (462-511)

1 Make a prediction. Does this card game seem like it's a fair game?...Write it down. If you don't think it's fair, who do you think has the advantage?...Write it down, who do you think has a better chance of getting what they want? And why do you think so?...Raise your hand if you want to share you prediction.

*2 Student: I don't think this is a fair game. And I think that player A seems to have a better chance of, better chances of winning because there are more aces than any other number, [other? than are odd]*

3 Do you agree with her or do you think something different?

*4 Student: I think I'm different...I think the game's fair because there's always a --- card, and you can either get something or not.*

5 Can you say that one more time?

*6 Student: There's four of each kind of card, like four aces, four kings, and four -----, so it's a fair game.*

7 Are there four kings?

*8 Students: No [Many students respond.]*

9 Look at your, look at the cards that you're gonna have. It says right here. So, how many aces do you have?

*10 Students: Four [Many students respond.]*

11 You have one jack, one queen, and one king. Do you still think so?

*12 Student: Oh. Nevermind, I changed my, I changed my-*

13 Um. R------, what do you think?

*14 Student: I think it's a fair game.*

15 Why?

*16 Student: Because there's four aces for player A, but with player B, you can get an ace, a jack, a queen, and a king.*

17 So your saying, you have four chances to get two aces, and you have four chances cause you could get an ace and a jack, an ace and a queen, and an ace and a king. That's only three combinations. Ace and jack, ace and queen, king and ace.

*18 Student: Or a queen and a king.*

19 Still think so?

*20 Student: Could get queen and king.*

21 Still think so.

22 K. Anybody else? N---?

*23 Student: No, I don't think this game is fair because player B has the advantage because it's hard to get the same, um, get the same cards.*

24 He thinks it's going to be difficult to get two aces, why?

*25 Student: Because it's hard to get the same cards.*

26 It's hard to get two aces because-

*27 Student: You might not be able to get the right cards.*

28 You never know and you might pick one of the other ones too? [I prompt for more sharing]

*29 Student: Uh, what it it's too hard to explain.*

30 Just try.

*31 Student: ---It's not fair because, even though, that player A has has six chances ---, player B, there's no way that player B can get a pair of a jack, queen, or king, because there's only one of them each and there's the fact that it's easier to get an ace with another one of the um, jacks, queens, or kings. And there's more chance of getting not that.*

32 So there's a lot of chances, your saying, that there will be an ace and a jack, cause there's a lot of aces, but then you have the jack. There's a lot chances there could be an ace and a queen cause there's a lot of aces and there's a queen. There's a lot of chances there could be an ace and a king cause there's a lot of aces and there's a king. OK. So you're saying could be ace – jack, ace –

jack, ace – jack, ace – jack, ace – queen, ace – queen, ace – queen, ace – queen, ace – king, ace – king, ace- king, ace- king. Cause there's four aces. Thank you. K. We're gonna play.

[Themes: encouraging student participation (1, 3, 13, 30), prompting reasoning (2-28), restating student idea for clarification (17, 32), modeling reasoning (32)]

I used a strategy of students reflecting individually about their predictions before they shared with the class. I wanted them to consider the fairness of a game before playing it. They could also revise their thinking while hearing other peoples' ideas. In lines 6, it becomes evident that the student does not understand the game. Therefore, by presenting predictions, rules of play could be clarified. A student was hesitant to share in line 29, but I encouraged him by saying "Just try" resulting in him sharing reasoning in line 31 that I could build on. In line 32, I modeled a strategy of making a list that would be more formally presented at a later time. I set up experiences to scaffold learning about how to represent theoretical probability in visual forms. After playing the game for a while, a student asked if she could change her prediction.

Excerpt 3-2 (676-694)

*1 Student: What if we changed [our prediction from] before we played the game.*

2 You can erase it…Yeah, yeah, you can change it.

*3 Student: OK. Cause I think that player, there's a chance that player B might win because player B can have these two,*

4 Um hm.

*5 Student: these two,*

6 Um hm.

*7 Student: these two,*

8 Um hm.

*9 Student: and these two,*

10 Um hm.

*11 Student: and these two,*

12 Um hm.

*13 Student: and these...so much*

14 Um hm.

*15 Student: because um, because um, their cards are all different*

16 Um hm.

*17 Student: so, but player A's cards are all the same.*

18 Um hm.

[Theme: actively listening to student idea and thinking]

In this interaction with a student, I allowed her to show her reasoning after she asked if she can change her prediction. It shows me actively listening to her idea while she explains with non-verbal information as well as me allowing her to revise her ideas based on new thinking. Perhaps, I could have asked to her express her new prediction and an explanation so that she could record it clearly in writing.

We later compiled the results of "The Card Game" as a class and used calculators to find the experimental probability that Players A and B would win.

Excerpt 3-3 (795-831)

1 OK. Calculators down. Look at your paper. You should have everything filled out. All of the scores, the totals points, the grand total, and look at number 4. It says compute the percent for each outcome using the class totals. I did that up here. 75 out of 261, when I divided it, was zero point two eight seven, so I rounded it to 29%. So 29% for player A. For player B, 186 divided by 261 was that decimal so I rounded to 71%. I do a quick check. Is 29% plus 71%, 100%?

*2 Students: Yes.*

3 OK. Number 5. According to your class results, which player appears to have the advantage?

4 *Students:* *Player B*

5 It looks like player B has the advantage, because, give me a couple different reasons. G—, what's one reason?...There's lot of, there's a few reasons you could put because you need to explain your thinking.

6 *Student:* *OK. Because, um, player B has more points.*

7 Player B got more points. Another reason.

8 *Student:* *Um. They got more points because the, you could make more groups.*

9 Just looking at the data...Who won every single game?

10 *Students answer.*

11 Player B won every game. Player B has the most points. And player B had 71%. Player B won 71% of the time.

[Themes: transition to new task (1), modeling thinking (1), questioning for reasoning (1-11), modeling providing evidence for conclusions (11)]

I attempted to get students to give pieces of evidence from looking at the class data to support the conclusion that player B had the advantage. In the complete transcript, three students have a difficult time limiting their evidence to the data that was recorded. Instead of allowing them time to grapple with is, I take the leadership role throughout the discussion and give three pieces of data in line 11. I think I should have recorded all of the students' ideas on the whiteboard and led a discussion about what could be concluded directly from the data.

At the end of the episode, I gave the students two ideas for trying the figure out the theoretical probability of "The Card Game," a listing method and a tree method. They were given the choice to use one of the methods I introduced, or use their own, to explain why player B had the advantage. Shield and Swinson (1996) argue that "The idea of connecting procedures and concepts with different representations and prior knowledge has been described as *elaboration*" and "that students who generate elaborations, such as visual images…display a deeper understanding and…are better able to apply the ideas in problem situations" (p. 35). I hoped that the common experiences we had, along with the guidance I gave in showing the students some possible ways of representing theoretical probability would help them understand and apply visual representations of theoretical probability.

Excerpt 3-4 (877-919)

1 Explain why player B won and will win. And I'm gonna give you some hints because I'd like to have specific evidence why you are thinking this. My thinking is. My thinking is, I have an ace of spades. Ace of spades. I'm just gonna have an S there because it's easier for me to draw. If I pull an ace of spades, I could pull an ace of diamonds, with it. If I have an ace of spades, I could pick and ace of hearts. If I have an ace of spades, I could pick an ace of clubs. If I have an ace of spades, I could pick a jack with it. If I have an ace of spades, I could pull an ace of spades, I could pull an ace of diamonds, with it. If I have an ace of spades, I could pick and ace of hearts. If I have an ace of spades, I could pick an ace of clubs. If I have an ace of spades, I could pick a jack with it. If I have an ace of spades, I could pick a queen with it. If I have an ace of spades, I could pick a king with it…Right now, it looks even, but it might not be. This is an organized list that might help you. You'll see later, if you continue this list. Another way to think about this. You might want to write this down, in case you want to continue this pattern. This is an organized list. Organized lists can help you solve problems….Another way to think about this is by making a tree. If I have an ace of spades, all of my different combinations are: an ace of diamonds, an ace of hearts, an ace of clubs, a jack, a queen, and a king. What if I pick a king? What are my choices?

*2 Student: Ace of hearts*

3 ace of spades, which I already used.

*4 Student: Ace of diamonds.*

5 I could pick an ace of diamonds.

6 *Student:* *Ace of hearts.*

7 An ace of hearts.

8 *Student:* *Ace of clubs.*

9 Ace of clubs.

10 *Student:* *Ace of, jack.*

11 Ace of clubs, a jack and a queen. Who wins a king with an ace?

12 *Students:* *Player B*

13 Who wins a king with an ace?

14 *Students:* *Player B*

15 Who wins a king with an ace?

16 *Students:* *Player B*

17 Who wins a king with a jack?

18 *Students:* *Player A*

*Player B*

*Oh.*

19 A king with a jack?

20 *Students:* *Player B.*

21 A king with a queen?

22 *Students:* *Player B*

23 This one, one, two, three, four, five, times, player B would win. Yeah?...In this one, one, two, three times player A and three times player B. If you keep doing something like this, you're gonna see how many times player A could win and how many times player B could win. If you continue this pattern. So you have a couple of different ways. Or you could do it your own way. You could make trees and count who could win or you could make an organized list and count who would win. Again, three A, times A could win and three times and three times B could win. So if you continue either a list or a tree, you should be able to figure that out.

[Themes: questioning for reasoning (1-21), modeling reasoning (1, 23), prompting for student participation (1-21), providing for students choice (23)]

Episode 4: October 21, 2005

Students had recorded, on chart paper, their explanations for why one player had an advantage over another player. We spent some time trying to figure out why different groups had different results. Of note is line 37 where the word addend is used in context of the discussion. Pirie (NCTM, 1996) discusses that in order to help students use mathematical language appropriately, teachers can "use that language in ways that help the mathematical words become part of the common language in the classroom" (p. 105). Using mathematical language appropriately is a theme that occurs often in the episodes.

Excerpt 4-1 (7-102)

1 OK. Great. You can have a seat. Everybody, look at what M----, G----, and C----- wrote and see if you have something similar or something different. Did you get 16 combinations?...If not, what's different about yours'? [time to reflect]

2 Why did they get way more, do you think, B------?

*3 Student: Because they did like 3 + 1 and 1 + 3.*

4 Yeah, why do you think they have two, they have that two times?

*5 Student: ---different combinations.*

6 Yeah. So if you have two dice, let's say you have a red and an orange dice, yeah? Here's a 1 plus a 5, yeah? A, I'm sorry, a 5 in the yellow and a 1 in the red. But you could roll it again, and this time, this could be-

*7 Student: -a 5*

8 -a 5, and this could be-

*9 Student: --- 1*

10 -a 1. So you actually have two different combinations cause this could be the 5 or this could be the 1, yeah?

11 *Student:* *Can we play this game again?*

12 *K.* It looks like player B get's 18 combinations-

*13 Student:* *No.*

14 No, there's more?

*15 Student:* *Yes. Wait, um, player A wins 1.*

16 OK. Which one?

*17 Student responds (inaudible).*

18 They have 3 + 6 and 6 + 3. Do you see it up there?...Yeah, look at A, I see it.

*19 Student:* *Player B, I think it's 21.*

K. Grace says 21 for player B. Anybody else get 21.

*20 Student:* *I got 2-.*

21 Anybody else get 21 for player B?

22 M——— did. So what is your extra combination that they don't have. Do you know, S———?

23 *Student:* *I don't know. No. I got 22 combinations.*

24 OK. So which ones do you have that they don't have?....Maybe you can organize it like this: Did you have four combinations for 5? Four combinations, no five combinations for-

*25 Student* *Something's wrong. They put 21 up there.*

26 OK.

*27 Student:* *I counted wrong?*

28 Let's count. 1, 2, 3, 4, 5, 6, 7, 8, 9....20...Let me try that again. 1, 2, 3...They have 20. Is there any other way to get a sum of 5 besides 1 + 4, 2 + 3, and 3 + 3. Any other way?

*29 Students:* *No.*

*3+3?*

30 How about, uh, oh yeah, I'm sorry. How about the sixes. Any other way to get 6 as a sum?

*31 Students:* *No.*

*Is it..?*

*No. There's not other way.*

32 Any other way to get a sum of 7?

*33 Students: Nope.*

*No.*

*Wait.*

*Yes.*

34 What?

*35 Student responds (inaudible).*

36 You can't have 7 on a dice. There's only the number 6 on a dice. Remember the highest you can go is a 6. I think that might be why we have some extras. Did anybody write a 7 as one of the addends?

*37 Students: No.*

*I did.*

38 You did. K. Cross out anything that has 7s or 8 or 9 as an addend.

39 Now do you have 20, if you cross off those extra ones?

*40 Student: Yes.*

41 You have 21 still?

*42 Student: Um hm.*

43 So what is it G----?

*44 Student: -for 5, I put, 1+4, 4+1, 2+3, 3+2*

45 Um hm.

*46 Student: For 6, I put 1+5, 5+1, 2+4, 4+2, 3+3*

47 Um hm.

*48 Student: For 7, 1+6, 6+1, ---2, 3+4, 4+3.*

49 Um hm.

*50 Student: For 8, 2+6, 6+2, 3+5, 5+3, and 4+ 4.*

51 You have the exact same thing that's up there.

*52 Student: Did you have 4 + 4 twice?*

53 No. I just heard her say it once…OK. So the total different number of combinations, all together-

*54 Student: 36*

55 -36. So you have, player A, you've got 16 out of 36. And for player B, you have 20 out of 26. And from now on, we won't need a calculator. Somebody tell me how you reduce a fraction to lowest terms. What do you need to do to reduce your fraction to lowest terms?

[Themes: questioning and prompting for reasoning (1-10), modeling and questioning to find errors (12-52), reviewing vocabulary in context (37)]

In Excerpt 4-1, we discussed a list of possible outcomes for "the Dice Game." I had my list on the whiteboard while the students crosschecked theirs' with mine. We found that we had different amounts of combinations and tried to find out why. I modeled checking the data over in line 27. Line 30 shows that students were generating their own conclusions. Lines 38-53 shows a student becoming very active in the discussion by calling out her combinations while I checked mine on the whiteboard. Students become more responsible for finding mistakes as we did so as a class. Also, in this excerpt, I seemed to be doing less talking than usual, allowing the students to take a more active role. Further, the pace of the episode was much slower than the previous ones.

Later, I used the fractions that were produced from the game to review changing fractions to simplest form and then dividing without using calculators to find the equivalent decimals and percents. We discussed using the GCF and a second method that was brought up by a student.

Excerpt 4-2 (139-155)

1 I think we're done. Probably, the greatest common factor is 4. So you divide the top and the bottom by 4, and you get 4 over -

*2 Student:* *9*

3 -9. Now that's in simplest form.

*4 Student* *Oh.*

*5 Student:* *Is it-?*

6 Some people call it reduced.

*7 Student:* *Is there any, is there numbers really big, like 100 something out of like 1000 or something, then could you, is it possible that you could divide it 2 times?*

8 Yeah. If you don't get the greatest factor the first time, you can always divide by another number later. So let's say in this one, we divided by 2. What do we get? 8 over

*9 Students:* *13...21...16*

*18*

10 And you look at it, you go hey, I can divide that again by 2. Right?

*11 Student:* *Um hm.*

12 Both of those numbers are even.

*13 Students:* *4/16*

*Oh.*

14 Yeah. That's what we had, because we divided by 4. That was the biggest common factor. Same things as dividing by 2 and then dividing by 2 again. K? So the chances A will win? 4/9. And we write the probability that A would win is 4 out of 9 times.

[Themes: building on student idea (8-14), modeling thinking/reasoning (8-14), reviewing different methods (14)]

We went over how to find the greatest common factor to change fractions into simplest form. Although it is a skill they are supposed to know, many students did not remember how to do it. I wanted to use simplest form in context to show students that it's easier to calculate the probability, but did not make it explicit. In line 7, the student asked a question providing another way to change fractions to their simplest form if the students didn't know the GCF. It also helped with fractions that have large numerators and denominators as well as stimulating reasoning since the numerators and denominators can be divided multiple times by different factors. I told the students about a future quiz and assigned homework wherein the numerators and denominators were very large and, therefore, I used both of the methods we discussed as tools for completing the homework, along with rules for divisibility that we had used earlier in the year that they had forgot.

Episode 5: November 2, 2005

This day was interesting because I had the students who did not complete their work stand while we go over the review homework that was given. I had told the students that we couldn't wait for those that did not complete their work anymore. I had more of a short, commanding tone throughout the episode. After going over the homework, I had the students to reflect on errors they may have made.

Excerpt 5-1 (29-57)

1 What are the mistakes that you could make on this stuff? What did you do wrong? What are some things? [I record students' shared ideas.] What are some things? G—. [Great prompt that could be used more often.]

2 *Student:* *You could divide wrong.*

3 If you divided wrong, and you make mistakes with your division, then you'll have an incorrect answer. What else could you do wrong?

4 *Student:* *Um, multiply wrong.*

5 You could multiply wrong. What else could go wrong?

6 *Student:* *You could mix up your fractions.*

7 What do you mean?

8 *Student:* *Like adding the wrong numerator ---*

9 OK. So mixing up the steps. Yeah? So like adding the wrong thing.

10 What else. Yes?

11 *Student:* *-wrong numbers.*

12 What do you mean?...

13 *Student responds (inaudible).*Oh. You just wrote the wrong number to begin with? You wrote the wrong problem? What else?

14 *Student:* *You might reduce incorrectly.*

15 OK. Reduce wrong, or you didn't reduce, or put in simplest form, yeah? What else could go wrong?

16 *Student:* *For numbers 24 and 23, put the wrong numbers on the top and on the bottom.*

17 OK. So switching numbers. Putting numbers in the wrong place.

18 What else can go wrong?

19 *Student:* *You can forget to multiply or divide.*

20 Forget a step? Anything else you did wrong?...Get a brand new piece of paper...Please write about how you did...and, if you made mistakes, why did you make those mistakes?

[Themes: prompting for student ideas (1-20), reflecting on work (1-20)]

After students practiced fraction concepts such as GCF, LCM, simplest form, mixed numbers and improper fractions, as well as converting fractions, decimals, and percents, I asked the students to give mistakes they could have made and record them on a list. The students then used that list to independently reflect on their work in writing. Because, in many instances, students have difficulty finding where mistakes occur in algorithms, I could have given the students some problems with mistakes in the work and/or solution as modeling and practice for finding and naming the errors. The next excerpt shows one student asking for help while trying to reflect on her work.

Excerpt 5-2 (59-67)

*1 Student: I'm confused about this part...I got this as an answer, but then I put it in simplest form, and I got this, but then the correct answer was that.*

2 So that means that there wasn't, um, something's wrong when you did, cause, oh, I see. 3 times 15 is 45. You should have a remainder of 10 here.

*3 Student: Oh.*

4 So it should be 3 10/15. So you had a subtraction error. Do you see why? 55 divided by 15 is 3. Which gives you 45 and then you had 10 left over. So you had a subtraction error.

*5 Student: OK.*

6 Or a multiplication error. I could've had Nikki go over how she did the problem so that she could try to figure out, on her own, what her mistakes were.

*7 Student: OK. Thank you.*

[Themes: responding to student request for help, modeling finding errors]

While assisting this student, I took the leadership role instead of questioning and prompting her so that she could find the error herself. I think I should have allowed her to attempt to find the error instead of essentially showing her. I was doing the thinking rather than she was.

I assigned homework, which was additional practice for those who had more than 3 wrong. Students then took notes about how to use the Least Common Multiple for comparing and ordering fractions as a review. While the students have time to practice, I went around to see if they need assistance.

Excerpt 5-3 (407-437)

*1 Student: We have to reduce, yeah?*

2 Reduce it?

3 *Student:* *Yeah, 4/12.*

4 You're finding equivalent fractions so that you can order your original fractions.

5 *Student:* *You don't have to reduce-*

6 This is the answer. So there's nothing to reduce, cause this is what your answer look like.

7 *Ms. Aiona. Do you have to show your work?*

8 Yes.

9 *No but like-*

10 You're not finding simplest form-

11 *I know but-*

12 You're finding if it's greater than, equal to, or less than.

13 *Student:* *(inaudible)-after-*

14 You're ordering fractions.

15 *Never mind.*

16 *-Cause this is already 4/12 and your answer came out to be 4/12, it's already---*

17 When you order the fractions, you order these fractions, yeah?

18 *OK. Never mind.*

19 These are already in simplest form.

20 *OK.*

21 So what are you doing now?

22 Number ---

23 OK. So 5/6 is equal to

24 *---*

25 10/18? No, 15/18. K. Hey 15/18 is the same as 15/18, so you see why you don't have to anything with simplest form in this.

26 *Oh, OK.*

27 Cause you're just putting the answer right there. These are the ones that you're comparing.

28 *Yeah.*

29 The only time you put it in simplest form is if you're told to, yeah, and you have an answer that's a fraction. Write now your answer is these.

[Themes: responding to student request for help, questioning and prompting for understanding]

The student didn't seem to understand what the answer to the problem is supposed to look like, several fractions in order. He asked whether or not the answer had to be in lowest terms. What may have confused him is that I had told the students that, if they're working with fractions, their answers should be in lowest terms from this point on. He seemed to generalize the idea to different contexts showing some misunderstanding of the task at hand. After much questioning and prompting, he finally understood what the solution should look like. After some thought, he may also have been asking if he needed to order the original fractions or the fractions with the common denominators. The previous interpretation seems more likely, although both are possible. Slowing down to really listen to the student would have given me the most information. Time was an issue, however, since many students were asking for assistance at the same time. Later, the students were presented with another way of representing fractions.

Excerpt 5-4 (471-495)

1 Did you find the least common multiple, least common denominator, for all those fractions?...And then change them all into equivalent fractions?...That's what you have to do. You have to follow the same steps that we did for the other one.

2 *Student:* *All right.*

3 So you have 9/12, 1/2 and 2/6. So you have to look at the 12 and the 2 and the 6 and see what is the least common multiple for each of those.

4 *Students:* *Then what is the number line for?*

*Yeah.*

5 Oh! You use the number line. So do you have to find the multiple?

*6 Student: No*

7 You could just look at the number line.

*8 Student: It'll help though.*

9 It, you could double check. But, do you see 9/12 on the number line?

*10 Student: No*

11 Why not?

*12 Student? Because it's in between.*

13 What is 9/12? Is that in lowest terms, in simplest form?

*14 Student: 3/4*

15 So where's the, do you see 3/4 on that line?

*16 Student: Yeah.*

17 So that could help you.

18 *Students: Oh. I get it.*

*So do you have to show your work?*

19 Um. If you want to show that 9/12 was the same thing as 3/4.

*20 Student: No. I mean do you have to show your work like you did up there?*

21 If you think so. You decide.

[Themes: reviewing procedures (1-2), responding to student request for help (1, 4), questioning for understanding (9-17)]

In line 1, I told the students that they should have been using the GCF to solve fraction problems without reading the directions myself. This causes some confusion as seen beginning in line 4. However, since I listened to the student's ideas, I was able to help them understand the thinking necessary to complete the task. I had told the students that they had to show their work, which became a problem when there was no work to be shown. Again, I could have planned better by presenting examples so that this didn't become an issue that took so much time to address in this excerpt as well as earlier in the episode.

Students had been given problems from *Mathematics Assessment Sampler, Grades 3-5* (NCTM, 2005) to solve independently. I made overhead transparencies of the problems and asked students to come up and share their solutions. The first problem we discuss has fractions on a number line.

Excerpt 5-5 (579-639)

1 A volunteer who will go up there who will go up there and justify their answer and right it on that paper. N----.

2 *Student:* *What is justify?*

3 Justify means explain and prove what you're saying with evidence.

4 There's a pen right there and you can write right on that paper and I want you to talk out loud to explain what you're thinking.

5 *Student:* *I think that 1 1/8 is wrong because, if you count, this is only a forth away from the number 2 so this right here would be a forth and right here would be in the middle which would be another forth so I think 1 1/18 should be 1/14.*

6 OK. Go ahead and write down your thoughts. What you just said. How could you explain it clearly? [student records]

7 Go ahead and cap that pen and I need another person to go up and make any changes they think is necessary.

8 You all agree that 1 1/8 is actually 1 1/4? [long pause]

9 Cause I disagree. I disagree with what she said. Anybody disagree as well?

10 K. S-----. Go ahead and get up there and explain to us why you disagree. Go up there. Point to what you see on the number line.

11 *Student:* *Um. I think the 1 1/8 should be where – the - 1 2/8 so- (inaudible).*

12 Show us where 1 2/8 is on the number line. K. Write it down. What is 1 2/8 equal to in lowest terms?

13 *Student:* *1 1/4*

14 So write that right underneath. That's where 1 1/4 is. Do you see why?

15 No? Do you see why? Look at her dots. This is from 1 to 2, this is 1/4, 2/4, what would that be?

16 *Student:* *3/4*

17 So what do you think, S——? What should 1 1/8 be?

18 *Student:* *Um, 1 3/4, that's simplest form, or 1 6/8.*

19 Show us why you think 1 6/8, by looking at the number line.

20 *Student:* *This right here is the…area, so since that's 1 1/4, that's 1/2 that's 2/4, and that's 3/4.*

21 So underneath the 1 1/8 [I mean to say 1 6/8], write 1 3/4. Thanks S-----.

22 Thanks, S——. Can somebody go up and show why that's 1 6/8? Show why 1 3/4 is 1 6/8. Come on up. And you can draw on the number line if you want. The pens right there.

23 *Student:* *This is 1 6/8 because-*

24 Show us. Look B------.

25 *Student:* *This is the first one. This is the second one. This is the 3rd. This is the 4th---*

26 Do you see why?

27 So we can also show it. This is, you start from here. This is 1/8, 2/8, 3/8, 4/8, 5/8, 6/8. 1 and 6/8.

28 K. Wait. So S---, can you change her answer so that it's more correct by crossing things out in her answer? [long pause]

29 *Student:* *Can I just erase it?*

30 Yeah. Or, No. Just cross it out. Anything else you need to cross out?

31 *Student responds (inaudible).*

32 Anybody disagree now? Anybody another reason, Thanks S---, you can have a seat now, why, another justification why 1 1/8 is not correct on that number line? Does anybody have a different reason why? You did M——.

33 *Student responds (inaudible).*

34 Um hm. You did. Do you remember what you said?

35 *Student responds (inaudible).*

36 Michael wrote that 1/8 is less that 1/2. Do you agree?

37 *Students: Yeah.*

38 Is 1/8 less than 1/2?

39 *Students: Yeah.*

40 On the number line is 1 1/8 less than 1 1/2?

41 *Students: No.*

42 Look at the number line, the original one.

43 *Students respond (inaudible).*

44 It's on the other side,

45 *Students: Yes*

46 1 1/8 is bigger than 1 1/2 on that number line, when you look at it. Do you see where the numbers are on the number line?

47 *Student: Uh huh.*

48 Can you see, right away, 1 1/8 should not be after 1 1/2?

49 *Student: Yeah*

50 1 1/8 should come first, because it's smaller. K?

51 *Ms. Aiona. I got three.*

52 Three what?

53 *Student responds (inaudible).*

54 How can you get a 3 when it say, which point is incorrectly labeled on the number line shown below?

55 *Student: I put a 3 because there's a ---*

56 Is there anything in there that says 3? Yeah, up here, but is that a point?

57 *Students: No*

58 The points are labeled with a dot. Here's a point, here's a point, here's a point. And it was labeled with the numbers. Points look like points and they're labeled, but these were the numbers to show you that it starts at 1 and then it goes to 3. But, I can see what you meant. But, the number 3 is not wrong, because 1 comes first, then 2, then 3. And they're all in order and they have the right number, the right space. So 3 is in the right place, S-----. Do you see why?

59 *Student:* *Yeah.*

60 1 comes first, then 2, then 3. So the only thing that could be wrong when you look at all of the points is 1 1/8.

61 Volunteer who hasn't gone up to explain what they were thinking with this problem. Remember you can draw pictures, explain in words, or numbers.

[Themes throughout: encouraging student participation, questioning and prompting for reasoning, building on student ideas, assessing student solutions]

This was the first episode in which I had the students come to the overhead projector to show their solutions to problems. After one student shows his work, I asked other students to go up to add or change anything based on their solutions. In this way, I was hoping they would build on each others' thinking and become more responsible for assessing others' work. The students in the audience were very attentive. It seemed that students like to see each others' work, as I found earlier in Phase 2. Also, while students are sharing, I get to step back and watch all of the students instead of leading them through tasks. I continued to have a difficult time allowing the student discussion to take over partly because of the brief nature of their comments and also because they tended not to ask questions of each other. I need to be more clear and explicit about what good conversations look like. The students are too used to me questioning them and commenting on their work. One positive aspect of this way of sharing solutions and thinking is that it requires longer wait time as students ponder the task and other students' solutions.

In lines 53-60, a student's misunderstanding of the task allows for clarification of what a point is and how they are labeled. I validated his idea of having the idea that answer of 3 although it is not what the problem is asking for and it is a misconception.

Sherin, Louis, and Mendez (2000) discuss one aspect of discourse they call building. Building involves two related goals: "(1) that student respond to other's comments rather than just state their own ideas and (2) that student use one another's ideas as the basis for thinking and learning about mathematics" (p. 186). In parts of the episodes, I prompted the students to repeat what other students said and to agree or disagree, which is consistent with the researchers' concept of building. To get students to "provide new evidence, or insight into, someone else's idea" (p. 187), I asked students to come to the overhead projector to change or add their ideas to student work.

One area the students had a more difficult time with was in relating their ideas to other students' ideas. I could have asked the students to share how the answer the student gave in line 5 was similar or different to the answer the student gave during the presentation during line 23. In line 9 above, my prompt is evidence of me modeling in an attempt to create a context for argument by creating opportunities for students to learn to participate in disagreement and argument (Wood, 1999). In lines 51-60 the idea of really listening to what students say by paying attention to the context in addition to the words (Pirie, 1996) allow me to understand why the student gets an answer of 3 when it would otherwise make little sense. The problem presented next involved fractions of a whole and ended the episode.

Excerpt 5-6 (642-673)

1 *Student:* *K. It says they both ate half of a pizza, or their own so*

2 How do you know it's their own pizza?

*3 Student:* *Cause it say another.*

4 Ah! A pizza and then *another* pizza. [pause for student working]

5 *Student:* So. *It could be a bigger pizza and they both ate half of them. For an example....(he draws). So they had a bigger pizza---bigger number ---*

6 Really?

*7 Student:* *So half of their own is ---fraction will be bigger—*

8 OK. Pull you example down a little bit. So they could have a bigger pizza. Then he's saying that 8/8 is bigger than 4/4. Is 8/8 bigger than 4/4? What is 8/8?

*9 Student responds (inaudible.)*

*10 Studens:* *One whole.*

*11* One whole thing.

12 What is 4/4?

*13 Student:* *One whole thing.*

14 One whole thing. Hey. Do you still want to add that?

*15 Student:* *I'll erase it.*

16 This is really good thinking. He was thinking OK , if I have more pieces, the pizza's bigger. Not always. Let's pretend these are the same size. Actually they are. Right now if they're broken into fourths, see it?

*17 Yeah*

18 Now, it's broken into eighths. It's still the same size pizza.

*19 Oh.*

20 It has nothing to do with the number of pieces. It has everything to do with how big the whole is.

*21 Yeah.*

22 The whole pizza. You have to start with a bigger whole pizza. See the difference? So this is enough, right here cause right here, "You may use drawings, words, and numbers to explain your answer. Be sure to show all you work." Jose ate half of a pizza, here it is. Here's Ella's half of another pizza. Jose said that he ate more pizza than Ella, but Ella said that they were both the same amount. Use words and pictures to show that Jose could be right. Jose is right, if his pizza is bigger.

23 When is Jose wrong?

*24 Common sense.*

25 If the pizza was the same size or Ella's one is bigger.

*26 Common sense.*

27 If the pizzas are the same or Ella's is bigger. So be really careful, just cause the denominator is bigger doesn't mean it's a bigger pizza. It just means that the same pizza has more slices. So this right here, is 1/2. It's also 2/4, it's also 4/8, but it's still the same size pizza, and they ate the same amount.

28 OK. If you're having a hard time with your math homework, you should get help at recess. Otherwise, have a great recess. Those of you who didn't do your math homework. Oh. Where are you going? Those of you who didn't have homework last night should be doing it right now. The page H39 Set B.

[Themes throughout: encouraging student participation, questioning and prompting for reasoning, building on student ideas, assessing student solutions]

In line 5 above, a student shows a misconception when he discusses that a pizza is bigger if it has more slices because the denominator is larger. Through questioning and modeling, in lines 8-16, this idea is highlighted. It becomes a meaningful and important discussion since it gives the opportunity for clarification. At the end, I tell students to come for additional help if needed as I found I did at the end of most math episodes.

Episode 6: November 3, 2005

This entire episode involves students coming up to the overhead projector to show and explain their answers to the assessment problems. The first problems involved fractions on a number line, building on the problems from the previous episode.

Excerpt 6-1 (5-90)

1 Somebody read the problem for us.

2 *Student:* *Jennifer is making a number line for fractions from 0 and 1. At which point should she put 1/2? At what, I mean which point should she put 7/8?*

3 K. So Angelo wrote the fraction then. Do you agree with where he wrote those fractions?

*4 Students:* *Yes*

*Uh*

*Yes*

5 Who says no?

6 Who says yes?

*7* OK. Uh, M----. Go up and explain why you agree that he has 1/2 and 7/8 in the right place. A----, you can stay up there. If you need to draw on the number line, that's fine, while you talk.

*8 Student:* *K. It's because, evenly spread, there are three lines on this side and also three lines on this side. So, in the middle, it should be half, which is, that's where he put it. And all together, the number line is separated into 7 lines, including this one is 8. And the one before this one, which is 8, should be 7, which is what he put here, 7/8.*

9 So how would you label each of those little lines?

*10 Student:* *In simplest form or?*

11 Just do it how you would label it.

12 [student showing work]

13 So 0, then 1/8, then 1/4, then 3/8, then 1/2, 5/8, 3/4, and 7/8. Yes?

14 Would, did anybody label it a different way? Thanks, M----, you can have a seat. Did anybody label it a different way with different fractions?

*15 Student:* *You could - one more.*

16 Well, I'm talking about just the lines up there, the fractions that are there, but with the fractions looking different.

17 G—-, go ahead. If you need to write a whole nother number line, go ahead.

18 *Student:* *Do I have to change all of them?*

19 You can just write, make a whole nother number line right underneath it and label it the way you would label it.

20 I hope you're watching. I see some people playing with things at their desk. I would hope that—with your eyes and ears up there.

21 [student is writing]

*22 Student: You have two 6/8ths. You have two.*

*23 Student: What?*

*24 Student: You have two 6/8.*

*25 Student: I don't have. This is a 5.*

*26 Student: Oh.*

27 What would be the 1, what would 1 be in 8ths?

*28 Student: We can not see the whole thing.*

29 Don't worry, she'll put it back.

30 What would the 1 be in 8ths? You guys?

*31 Student: 8*

32 8/8. You could also write 8/8 underneath the 1.

33 See that? 1/8, 2/8, 3/8, 4,/8, 5/8, 6/8, 7/8, yes?

*34 Student: Yes.*

35 But you could reduce those to lowest terms or simplest form and that's why M—- has 1/8 and then 1/4. 2/8 is the same thing as 1/4. OK? Thanks, G—-.

36 OK. A—- go back to the original —. Bring it down a little bit so we can see-

*37 Student: OK*

38 -the original section. Did A—- write the answer to the original question?

39 *Students:* *Yes.*

*No*

*Yes*

*In a way.*

40 Did he tell, at which point, she should put 1/2?

41 *Students:* *No*

*Yeah*

*He didn't exactly tell.*

42 He put the answer on the number line. Could he give the answer in another way?

43 *Student:* *Yes.*

44 How, P------? [long pause]

45 A------. So she can tell you what to write while you write it underneath the number line. She's gonna tell you how---

46 *Student:* *I am?*

47 Yep. [long pause]

48 Come on, A------. At which point should Jennifer put 1/2 and at which point should she put 7/8.

49 *Student:* *In the middle, she could put 1/2.*

50 That's not a point. How do you know a point?

51 *Student responds (inaudible).*

52 It's labeled with-

53 *Students:* *numbers*

*Letters*

54 the letters.

55 *Student:* *K. Jennifer should put 1/2 at D and at F and---*

56 Slow down so he can write it down.

57 *Student:* *What did she say?*

58 *Student:* *Jennifer should put 1/2*

59 *Student:* *at point D?*

60 *Student:* *Huh?*

61 *Student:* *At point D.*

62 *Student:* *OK. And-*

63 K. Somebody finish the answer.

64 S---.

65 *Student:* *She should put 7/8-*

66 *Student:* *Huh?*

67 *Student:* *She should put 7/8 [there is a pause as A----- records]*

68 *Student:* *on F.*

69 *Student:* *On F?*

70 *Student responds (inaudible).*

71 OK. So if you read the question again, go ahead, Angelo, read your question and read your answer.

72 *Student:* *OK. Jennifer is making a number line for fractions between 0 and 1. At which point should she put 1/2? At which point should she put 7/8? Jennifer should put 1/2 at point D. She should put 7/8 at point F.*

73 Did he answer the question? Did he show his thinking when he wrote the numbers on the number line? Thanks, A-----.

74 *Student:* *You're welcome.*

[Themes throughout: encouraging student participation, prompting students to verbalize, questioning and prompting for reasoning, building on student ideas, assessing student solutions]

After hearing different responses from students, I took an informal assessment of the class in lines 5 and 6. Later, in line 14, the question I presented seemed to cause some confusion with the students. They weren't sure what I was asking for. I clarified what I wanted in line 16. In lines 19, I tell the student how to record her answer, where instead, I think I should have allowed her the freedom to decide what do write. However, it was good because she made a number line labeled completely differently that modeled different equivalent fractions. In lines 19-28, a student is making sense of what the other student was presenting. While the student was presenting, I asked the class an extending question in line 30 that could have been added to the work being presented. In lines 36-73, we discussed whether the original question was answered completely. One motivation for this is that I wanted the students to answer state assessment questions in the most complete and appropriate way. The next two problems involved geometrical shapes and fractions and ended the episode.

Excerpt 6-2 (96-186)

*1 Student:* *Um. If triangle is 1/2 of a unit, then draw one unit.*

2 Talk it out while you're doing it.

*3 Student:* *This is a triangle. I added two triangles together and I forget what the name of the-*

4 Can somebody help her?

*5 Student:* *I don't know the name of it, but, um, never mind.*

6 S---- knows the name of it.

*7 Student:* *A diamond and a square?*

8 Hm?

*9 Student:* *A diamond and a square.*

10 Not square. Oh, wait, I see a square. It could be a square or a it could be a diamond. Anybody know another name for a diamond?

*11 Student:* *Quadrilateral.*

12 It's a quadrilateral because I has-

*13 Students:* *four sides.*

14 And another name for a four sided figure where all the sides are equal, but they don't have 90 degree angles.

*15 Students: [many answers] rhombus*

16 Thank you! Who said rhombus?

*17 Students: All of us did.*

18 Good job.

19 OK. Do you think her answer is complete?

*20 Students: No.*

*21* Read the question and tell me why you think it is or isn't.

22 *Student: It's not complete because she only drew it, but she didn't really explain it.*

23 Does it say to explain it?

*24 Student: No*

25 What does the question say?

*26 Student: To draw one unit. [many students answer]*

27 Did she do what the question told her to do?

28 *Students respond positively.*

29 She drew it. Now, if you wanted to explain, two triangles can make a square or a rhombus, you could also add that. But, she really did everything that she was asked to. But, yeah! She's adding one unit, 1/2, 1/2. That helps. The labels are good cause now it's really clear.

30 Did you want to add anything else?

*31 Student: Um. No.*

32 Would anybody like to add anything else? If you want to you can. No? I'll add something. Just what we were saying before. Two triangles equals one

*33 Students: square*

*quadrilateral*

*34* square or rhombus and, this is a special kind of triangle. Do you, can you tell what kind it is?

35 *Student: I forget the name of it.*

36 When every side is the same length.

37 *Students:* *Equil*

*Equilateral*

38 Yeah!

39 Next victim.

40 G----

41 *Student:* *I want to be the victim*

42 *Student:* *Me?*

43 Go ahead G----.

44 Talk it out while you're doing it.

45 *Student:* *Since this is one unit, and their asking what that is, then that would be like that so you can divide that-*

46 Instead of saying "that" could you use other words?

47 *Student:* *OK. Yeah, um, you can change the rhombus into 2 triangles. And so in that shape, there would be 3 triangles. And since the rhombus is shaded, that would be 2 out of 3 because you can change this figure into 3 equal parts. So the answer is 2 out of 3. Yeah.*

48 *Student:* *It doesn't answer the question. [long pause]*

49 What do you folks think?

50 G----. Can you explain it one more time?

51 *Student:* *OK. Um. OK. This is a trapezoid, and that's a rhombus, and in the trapezoid, you can make a rhombus right here. And you can divide the rhombus into 2 equal parts to make 2 triangles. And so there is gonna be 3 triangles in the trapezoid and 2 of the triangles are shaded to make a rhombus, and so 2/3.*

52 So the rhombus is 2/3 of the-

53 *Student:* *trapezoid.*

54 I see a lot of people nodding.

55 Did she answer the question?

*56 Students: Yeah.*

*Nope.*

*57 [The student sharing adds to her answer in response.]*

58 Ah! Now she answered the question.

*59 Student: Ah.*

60 Could she answer the question in another way?

*61 Student: Yes.*

62 How? Go ahead.

*63 Student: A rhombus is 2/3 of a trapezoid.*

64 Make sense?

*65 Students: Yes.*

66 Good job. Thanks everybody. I think it's recess.

*67 Student: Thanks.*

68 Thank you!

[Themes throughout: encouraging student participation, prompting students to verbalize, questioning and prompting for reasoning, building on student ideas, assessing student solutions, vocabulary in context]

Throughout the episodes where students are sharing their solutions on the overhead projector, I asked the students to verbalize their thinking. This proves difficult for most of them. To build on this strategy, I asked the students to talk about what they saw happening (lines 2, 4, 44). I validated what a student saw although he saw it differently than I did in line 10 and tried to extend the vocabulary in lines 11-15 as well as stimulate providing definitions in lines 13, 15, and 34-38. It's important that in line 29, I emphasized that labeling helps make a better answer. In line 32, I modeled providing the best answer by adding some of what students had contributed earlier. Lines 39-41 showed that I had used the work "victim" to ask students to participate, as I had done with other content areas. It added humor, makes participating less threatening, and as evidenced in line 41, students want to be the "victim." Lines 46-54 are effective because the student who is sharing restated her idea and used appropriate vocabulary. This provided reinforcement for others to hear her thinking and helped build everyone's understanding. In line 51, the student used the word trapezoid and I sought to get her to repeat it in line 52 so that the students could hear it again. She restated her answer succinctly in line 63.

In studying adolescent pairs completing math problems, Kieran (2001) found that "bridging the individual and the social in mathematical problem solving can be extremely difficult to practice, especially when it involves novel problem situations. Making one's emergent thinking available to one's partner in such as way that the interaction be highly mathematically productive for both may be more of a challenge to learner than is suggested by the current mathematics education research literature" (p. 220). In lines 45, 47, 51, and 63, the student clarifies her answer as a result of my prompting. By eliciting this clarification from the student, I built on the mathematical culture of our class (van Oers, 2001) where appropriate vocabulary is used effectively and ideas are presented concisely.

Episode 7: November 4, 2005

Episode 7 began with students at the overhead with problems involving fractions including using the LCM and GCF to put fractions in simplest form and comparing and ordering them. In line 62, notice that the student restates her ideas again, lending to the notion that mathematics is about precision of thought (Pirie, 1996). Through the discussion, the student refined her statement.

Excerpt 7-1 (1-56)

1 —here and talk us through this problem. Go ahead.

2 What do you have to do to solve that problem?...Help him out. The first thing I notice is-

3 *Student:* *-(inaudible)-times is 25.*

4 Why are you doing that though?

5 *Student:* *Um. To find the LCM.*

6 OK. Sienna, repeat what Michael said and then say what you would do next.

7 *Student:* *I could here part of it but-*

8 -M------.

9 *Student:* *Um. 5-*

10 Wait, why are you doing that?

11 *Student:* *To find the LCM.*

12 The what?

13 *Student:* *the LCM.*

14 So what does he say he needs to do?

15 *Student:* *Um. You have to find the LCM of 5 and 25.*

16 OK. Did you hear her? Could you here her? Say it louder.

17 *Student:* *Um. M------ said that you have to find the LCM of 5 and 25.*

18 OK. You're not speaking clearly enough yet. One more time.

19 *Student:* *M------ said you have to find the LCM of 5 and 25.*

20 You have a hint how to get started.

21 So, S----- what is she doing?

22 *Student:* *Um, she's writing, um, the multiples? Is that how you say it?*

23 *Student:* *5, 10, 15, 20*

24 Oh he's gonna keep going. What do you think he's gonna do next?

25 *Student:* *Underneath the 5 -circle the-(inaudible).*

26 Oh. Did you hear what he said? Say it again S-----.

27 *Student: Underneath the 5, he's gonna put 25 and ---*

28 **Why is he circling both of the 25s?**

29 *Student: Because it's the lowest one.*

30 **The lowest what?**

31 *Student: Multiple*

32 **Lowest multiple that they both have in common. What does he do next? -**

33 *Student: He ---changed---5---25---*

34 **Why is S---- saying he changed the 5 to 25 in the fraction 3 over 25? What is he doing S---?**

35 *Student responds (inaudible).*

36 *Student: Making---*

37 **Making them equal. Making what equal?**

38 *Student: the denominators.*

39 **Why is he making the denominators equal? G----.**

40 *Student: So that he can compare.*

41 **Compare what?**

42 *Student: The fractions.*

43 **What fractions?**

44 *Student: 7/25 and 3/5.*

45 **7/25 and what?**

46 *Student: 3/5*

47 **So what did he have to do to the 3/5?**

48 *Student: Change it, make the denominator changed to 25.*

49 **What do you call that, when you have two fractions that have different denominators, but they're worth the same amount?**

50 *Student: Equivalent.*

51 **Equivalent fractions. Yeah? So we need to make equivalent fractions? Go for it.**

[Themes: "what do you think and why?" (2-4), encouraging student participation, prompting students to verbalize, questioning and prompting for reasoning, building on student ideas, assessing student solutions, vocabulary in context (49)]

This excerpt begins with me giving review problems regarding fractions in preparation for a summative assessment. The students were excited to share on the overhead projector. I asked or prompted the students to share what they saw happening in hopes that I would get a complete ideas such as, "She is finding the LCM so that she can make equivalent fractions to be able to compare them." It proved difficult as shown in lines 33-50. Consistent with Kieran's findings (2001), the utterances of the interlocutors tend to be fragmented and under elaborated.

I continued to review by going over a math textbook assignment and problems I presented on the overhead projector. Once we finished reviewing, I gave students time to go over their solutions with each other, at their table groups, to double check their problems and see if they agreed with each other. I then asked the students who hadn't yet shared to go up and share.

Excerpt 7-2 (445-564)

1 *Student:* *I put C because 4/5 is really closer to 1 than 2/3 because 2/3 is beyond 1.*

2 [Other students go up to share.] Ha ha! What is she doing?

*3 Student:* *Number line.*

4 What is she doing first?

*5 Student:* *Drawing a number line [and breaking it up].*

6 into what?

*7 Student:* *equal*

8 equal pieces.

9 How many are in there?

*10 Students:* *4*

no

*5*

*6*

11 Look at what she's doing.

*12 Student:* *5*

13 How many equal parts?

*14 Student:* *6*

15 Look at her fractions.

*16 Student:* *5*

17 If there's 5ths, there's 5 equal parts. Do you see it?

*18 Student:* *No*

19 You don't see it? It says 0/5, 1/5, 2/5, 3/5, 4/5

*20 Student:* *Yep, there's 5.*

21 List it up a little bit, Grace. Push your paper up. 0/5, then 1/5, then

*22 Student:* *Yeah*

23 2/5, then, 3/5, then 4/5, then 5/5. [long pause]

24 1/3. She has another number line broken up into 3rds. 2/3, 3/3. Now what is she doing? Hey, look how close 4/5 is. Look how close 2/3 is. Which one's closer?

*25 Student responds (inaudible).*

26 Which one is closer to 1 whole?

*27 Student:* *4/5*

28 Do you see it? Can you see it?

*29 Students:* *Yes*

30 Is there anything you would change about anybody else's work up there, G----? [pause]

31 What does it mean if you say something is beyond 1?

*32 Students:* *farther*

*above*

33 Above! More than 1. 2/3 is not more than 1. How could she change that, I put C because 4/5 is really closer to 1/3-

*34 Student: 2/3 [ Student corrects me.]*

35 Uh, 2/3. [pause]

36 Couldn't you just put a period after the 2/3 and that would be, the first 2/3, and that would be the end of the sentence. Yeah?

37 But there's not proof up there. So you'd have to either have B——'s method or your method to prove that sentence. You can just say "4/5 is closer to 1 than 2/3." It doesn't really prove it. But now it's proved with the other work. Yeah?

38 Good job. Give them all a hand. [applause] Somebody who hasn't come up yet. [pause]

39 Remember you should be fixing your answers to show that you understand better now, after they talk about them.

40 [long pause while student works] K. So read your answer to us.

*41 Student: 0 point 6 is the closest to 1/2, because, if you put 1/2 in a decimal, you get 0 point 5 and 0 point 6, is close to 0 point 5.*

42 Good job. [applause]

43 Does anybody want to add anything?

44 And actually, we do need to you add something, because she has proof that 5/6, uh, 5/10 and 6/10 are close, but she didn't say anything about 1/4 and 1. So it doesn't quite have enough information yet. N——, you want to go up and add some stuff.

*45 Student: Never mind.*

46 The other thing about this problem is it says "on a number line." So if it says on a number line, probably a really good thing to do is-

*47 Student: make a number line*

48 -to make a number line.

49 Yes, go ahead, N——. Get up there. Or you could also explain 1/4 and 1— [student working]

50 Push the paper up a little. [student working]

51 Ah! What's that one?

52 *Students:* *1/2*

*0 point 5*

53 Ah! [student working] Stop. What else should she put on her number line? Move your hand. You see where 1 is, you see where 5/10 is. What else should she put up there to answer the question completely?

54 *Students:* *1/4*

*1/2*

55 1/4 would be a really good thing to put up there cause that's in the question. Where are you gonna put 1/4?

56 *Student:* *Here*

57 Is that 1/4? You're not looking.

58 *Student:* *We don't know, cause then there's only one line.*

59 That's not 1/4 right there? Is it broken up into 4 pieces? Where's 1/4?

60 *Student:* *there*

61 Right there. That's 1/4. N----, 1/4. Push your paper up. [pause] K. Now where would 6/10 be? [pause] Somewhere there?

62 Do you need 3/4?

63 *Student:* *No.*

64 You don't even need 3/4 cause all you need to show is how that is close to-

65 Anybody want to add anything? Thank you N----. [applause]

[Themes: encouraging student participation, questioning and prompting for reasoning, building on student ideas, assessing student solutions, vocabulary in context, talking through problems (2-24)]

I continued to try to get students to verbalize how students they were solving problems when they shared in front of the class by questioning students in lines 1-22 and ended modeling my thinking aloud on line 23. With line 31, I brought the students back to the comment made in line 1 because it was incorrect or misstated. In lines 37-64 we discussed what could be added to the solution to make it the best answer. It's evident in lines 61-64 that I am still very much in control of the discussion. I feel like I am leading instead of allowing the students to take ownership of their thinking. Much of the interactions in the class were nonverbal. In the next excerpts, I was at the overhead to lead the discussions about the students' solutions in an because we were running out of time.

Excerpt 7-3 (633-662)

1 OK. That would mean that I'm breaking this up into 22 pieces. So you want 23 over 22.

*2 Student:* *Yeah. So it would be 1 over-*

3 Is that a little bit more than 1?

*4 Student:* *Yeah.*

5 Does anybody have another answer?

*6 Students:* *No*

*It could be like-*

*-23 over 24*

7 Is 23 over 24, more than 1?

*8 Student:* *No*

9 Anybody have another answer?

*10 Students:* *respond (inaudible)*

Yes

*I got -*

*- little bit more.*

11 You want a little bit more than 1 and this has to be- So can it be anything else? M----, what did you say?

*12 Student responds (inaudible)*

13 over 21, is that a little bit more than 1?

*14 Student: Yes.*

15 That's a little bit more than 1.

*16 Student: 20*

17 Is 23 over 20 a little bit more than 1?

*18 Student: Yeah.*

19 All of those are a little more than 1. So there's lots of different answers. But, what would be the best answer.

*20 Students: 22*

*over 22.*

21 This one would probably be the best...But there are lots of different answers.

[Themes throughout: encouraging student participation, prompting for reasoning and different answer]

This problem could have been used much more effectively if I had planned by making different pictorial representations. I asked students to give a numerator to go with the denominator of 22 to make the fraction a little more than 1. As it stood, I led them to know that there were many solutions while they could have otherwise noticed it themselves. The problem could lead to a more rich discussion. To facilitate building, I could have asked questions such as, "How is your answer, 23/22 similar and different to my idea of 23/20?" The next discussion I led involved a problem asking students to compare shaded regions of different shapes by estimating the percentages of the shaded regions.

Excerpt 7-4 (672-725)

1 10 out of 18. Anybody have something else they would do to help them with this?

*2 Student: Um, change it down, I mean, half of 18 is 9*

3 Why do you want to find half of 18?

*4 Students: Cause then the little, um, the little, circle, they can be by 3*

*Because the circles are divided in half and 10 out of 18 is a little bit more than half which is 9 out of 18-*

*Yeah*

*So it would be, in the shaded, a little bit more, but not a lot.*

*Yeah.*

5 So somebody tell me what you want to do to this fraction.

*6 Student: Reduce it*

7 Reduce it to what?

*8 Student: 5/9*

9 5/9? And what does that tell you?

*10 Students: Uh, nuttin'*

*A little more than half.*

11 What?

*12 Student: 5 shaded out of 9*

13 You said a little more than half. How do you know it's a little more than half?

*14 Student: Because there's 10-*

15 Yes?

*16 Student: Because half of 9 is 4 1/2.*

17 Half of 9 is 4 1/2 so this is a little bit more than 1/2. I can see some people weren't listening so we might not have recess at all, even if we do finish, because you're being rude.

18 This is a little more-

*19 Student: than half.*

20 So would A be the answer?

*21 Student: No*

22 Why not?

*23 Student: 1/2*

24 A is 1/2. That's not it.

*25 Student:* *B is*

26 B is

*27 Students:* *more than*

*it's about like*

*it's more than half, way more*

*it's about 7/9*

28 It's 7/9?

*29 Student:* *No*

30 Maybe. It's bigger than,

*31 Student:* *6/9*

32 it's pretty big. Too big. This is-

*33 Student:* *too small.*

34 Too small. That's about 1/4. That's about 3/4. This one?

*35 Student:* *Perfect.*

36 Oh, this is a little more than half. D is a little more than half. What's E?

*37 Students:* *It's less than*

*too little*

38 A little bit less than 1/2. So it has to be

*39 Student:* *C*

*You wrote the side wrong.*

40 10/18 is a little more than 1/2 because 9/18 would

*41 Student:* *Half*

42 be a half.

*43 Student:* *Yeah*

[Themes throughout: encouraging student participation, prompting and questioning for reasoning, encouraging estimation]

In the excerpt above, a student attempted to explain that 10/18 was a little more than half to help him compare the shaded region of a rectangle to the shaded regions in five circles. This problem and the ones that followed gave students another visual representation extending the work we had already done with fractions and percents by applying knowledge about percentages to different shaded regions of different shapes.

Episode 8: November 8, 2005

Excerpt 8-1 (1-63)

1 Need somebody who hasn't come up here yet to come and show us what you did for number 16. Thanks, N----. Go on up. The pens are right there.

2 [long pause while student is writing] K. By looking at her picture, somebody explain what she did.

3 Uh, B-----.

*4 Student: She circled the groups, I mean, divided the rectangle into groups.*

5 What kinds of groups?

*6 Student: Equal groups?*

7 Exactly. Cause you have to have equal groups. And three equal groups in each picture. And then-

*8 Student: She told how many times that...that each rectangle has two equal sections that are shaded in.*

9 K. What is the 8/12 for? Anybody have an idea? A-----.

*10 Student: It's the exact fraction for figuring-*

11 OK. So how does 8/12 help you?

*12 Student: Um, that, you could put that in lowest terms -*

13 Could you add that up there, N----? Show the equivalent fractions and what you call that little 4.

*14 Students: The least common multiple.*

*LCM*

15 The LCM - the greatest common factor. Could she have another equivalent fraction [for that] picture?

*16 Student: Yeah.*

17 What is it, G----?

*18 Student: 4/8*

19 - why? 4 what?

*20 Student: 8ths*

21 4/8, for the first picture? Somebody says no. C----- says 6.

*22 Students: Yeah, 6.*

*4/6*

23 Could you add that up there also? Can anybody think of another equivalent fraction that is there? With those pictures? [long pause] Does anybody see another way they could break up one or both of those pictures? See one S-----?...See one B------?

*24 Student: Maybe 1/6 --- count the non-shaded.*

25 OK. So you're breaking it up into 6ths? Figure 1 or figure 2?

*26 Students: Figure 1.*

*[long pause] I lost it.*

27 You lost it. S-----, do you know what he's talking about?

*28 Student: Um. The non-shaded one --- bigger --- would be 4/12.*

29 My question is, though, can you break up the pictures in any other way? So, I think N---- knows now. Is there another way you can break up those pictures? You have 3rds or 12ths. Is there another way you can break up figure 2?

*30 Student: By doing it like, this is one, this is one, this is one.*

31 So how many pieces would there be then, altogether?

*32 Student: 4/6*

33 Will you use a different color to show — the other one? [long pause] See it now?

*34 Student: Yeah.*

35 So how much is shaded now?

*36 Student: 4*

37 4 what?

*38 Student: 6.*

39 4/6. So that's another equivalent fraction you could put with 8/12 and 2/3. Can you break it up in any other way?

*40 Student: ---break it up in 1---*

41 1? They're both whole things but not the whole thing is shaded. I think that's it. So your other fraction is 4/6? Is that right?

*42 Student responds (inaudible).*

43 Cool. Thanks, N----. I'm gonna add that here.

44 Now remember, I'm gonna look for improvements on your papers. So you need to make improvements.

45 So really, you can see that this 4/6 is here from Figure 1 and this 4/6, they're the same. And you can also see that 2/3 is the same as 2/3 here. So there's lots of reasons why those are, the same amount is shaded in. It doesn't mean you actually have the same amount, cause these ones are different. This is way bigger than this. But, you have the same fraction of the whole thing that's colored in.

[Themes: encouraging student participation, questioning for understanding]

I attempted to get students who didn't regularly volunteer to participate in line 1. It doesn't prove very effective as shown in the lines of the complete transcript. In lines 13-23, I wanted the students to see all of the equivalent fractions in the pictures to reinforce the concept. My questioning does not seem to be very effective and I had to repeat the question a few times, although the questioning in line 29 seems to give enough information about what I am asking for. In line 45, I touch on an important concept, that the whole may be different, but the fraction is the same. Below, a student finds his mistake while showing his solution to a problem and later, students disagreed because of a confusion of the meaning of numerators and denominators.

Excerpt 8-2 (74-102)

1 K. So show us how you got 9/13 by looking at the picture. [Long pause while student examines the problem.]

*2 Student: I counted wrong.*

3 Oh! So now you agree with them? You agree with 9/12 then?

*4 Student: Yeah.*

5 OK. So do you think 9/12 is the same thing as 3/4?

*6 Student: Yeah.*

7 Is there any way you could show us on the picture that --- 3/4?

*8 Student: Yeah.*

9 How?

*10 Student: -put it in groups of 4.*

11 Groups of 4?

*12 Student: -3*

13 Groups of what?

*14 Students: 3*

*3*

*4 [there is disagreement]*

15 And how many groups are there altogether?

*16 Students: 4 groups ---*

*3 [more disagreement]*

17 So that there's a total of 4 groups...OK. Sometimes we get something in our heads and stick to it, instead of opening our minds to other ideas.

18 I'm convinced. Are you?

*19 Student: Yeah.*

20 Two different ways, both using the GCF, and, showing on the picture. Thanks. Good job.

[Themes: finding errors, dealing with disagreement]

A student discovered his own mistake while sharing with the class in line 2. If he had read the question completely he would have known there were 12ths instead of 13ths. This is powerful since this student exhibited difficulty following directions and working independently to solve problems. Through sharing, he found his own error. Because of my prompting beginning in line 9, the students disagreed about whether there were 3 groups or 4 groups partly because I was unclear about whether I was asking for the total number of groups, or the number of groups that are shaded in lines 7-16. The students end up coming to a common conclusion and I made an important statement in line 19 that I should remember myself.

Excerpt 8-2 shows a time when I was not listening sensitively to the students' responses. They did not understand the difference between the number of groups shaded as opposed to the total number of groups. Pirie (NCTM, 1996) described a danger to classroom communication that "until the talk ceases to be compatible with the thinking of one of the participants, each participant will be assuming that all have a common understanding of the words being used" (p. 114). I could have made more clear what was meant by the word groups in the phrases 'putting in groups' and 'groups all together.'

Episode 9: November 9, 2005

In Episode 9, I tried a new strategy of giving the students the problems on overhead transparencies in their table groups. They were to write their solutions to the problems and then give them back to me. I then put their solutions up to guide the discussion. I hoped that since the students had worked together to write their solutions on the overhead, more students who did not regularly share would have to since all students should have participating in discussing and generating the solutions to share. The first problem we discussed as a class involved using multiples in a different context.

Excerpt 9-1 (112-148)

1 What about C? Could Jose design the same game using multiples of 4? Explain your answer. She says no. Jose can't design the same game using multiples of 4 because you can only get even numbers. Do you agree?

*2 Students:* *Um hm*

*No*

*It's not really-*

3 Who says no?

4 OK. Why do you say no?

*5 Students:* *Because it's possible that you can get odd numbers too.*

*No.*

6 OK. So you have two different cubes The green cube has odd multiples of 3. The red cube has even multiples of 3. And their asking you, could he do it with odd multiples of 4 and even multiples of ? And your saying no because there could be odd multiples of 4.

*7 Students:* *No there can't.*

*Because –*

8 So tell me, S-----, what are the multiples of 4?

*9 Student responds (inaudible).*

10 Uh huh.

*11 Student:* *---24---[can't hear S-----'s numbers, but I am writing them on the overhead]*

12 Are you noticing anything?

*13 Students:* *Yes.*

14 What is it? What are all multiples of 4?

*15 Student: They're evens.*

16 They're all evens. Do you know why?

*17 Student: Because, when it goes up, the tens place change, but the ones stays the same, like 4, 8-*

18 The tens place change, but the ones stays the same?

*19 Student: Like, um-*

20 Give me an example.

*21 Student: for 24, there's a 4, and the 4 and then the 8 and the 8, so-*

22 Why is there a 4 and an 8? What are you doing every time that you're multiplying by 4?

*23 Student: Your multiplying by 4-*

24 Uh huh.

*25 Student: And that means that you would, um, never mind. You're adding 4 to it and every time, it's gonna be a multiple of 4.*

26 Do you agree?

*27 Student: Yeah.*

28 Every time you add 4, you're adding an even number. So you're gonna keep getting even numbers. What were you gonna say, S——?

*29 Student: Um, 2 + 2 is 4 ---2 is – (inaudible)*

30 2 + 4 is gonna be even, 4 + 4 is gonna be even, 6 + 4 is gonna be even, 0 + 4 is gonna be even. You're always adding an even number, and you started with an even number. So now do you agree with her?

31 So a lot of times, it helps me to make list, especially if there's numbers, cause then I can look at the list, and I can decide. If I didn't make the list, I might not know.

[Themes: prompting for agreement/disagreement (1, 30), "why do you think that?" (17), questioning to clarify misconception (1-16), prompting for examples (8), questioning for reasoning (16-30)]

In lines 1-4, students shared their ideas about whether a game would be possible given specifications. One student's idea became a focus of this conversation because she assumed that multiples of 4 could be odd. In lines 12-15, I got her to list the possible multiples of 4 as evidence that the game is not possible. I also asked students to explain why the multiples of 4 are even. Their reasoning is given on lines 17-29. On line 31, I solicit agreement from students and on line 31, I hint that a list is a strategy that can assist students in problem solving.

In line 3, I had an opportunity to prompt students to align themselves with a claim and provide mathematical evidence to support that claim (Stein, 2001). I prompted the students to explain why multiples of 4 were even numbers and while students were explaining, I feel as though I stepped in too much. I don't know if the students were given ample opportunity to be heard and understood by the other students. The cognitive conflict that resulted from the discussion allowed the student to understand that the game as designed would not be possible. Allowing students to see evidence that contradicts their thinking can be an effective teaching strategy.

Excerpt 9-2 (164-200)

*1* *[Student reads problem aloud.] Patrick had only quarters, dines, and nickels to buy snacks. He spent all his money and received no change. Could he have spent 99 cents? Justify your answer.*

2 So what kind of answer are you gonna have? Are you gonna have a number answer? Are you gonna have a yes, no answer? Are you gonna have an explanation answer? How will you answer that question?...The question is, could he have spent 99 cents?

*3* *Student:* *No.*

4 That's a yes, no answer.

5 *Student:* *Yeah.*

6 Justify your answer. That's an explanation with numbers. But you couldn't answer, could he have spent 99 cents? The answer is 99 cents. You can't do that. Sometimes people do that. They answer questions that don't make sense. [laughing] Happens all the time.

7 He couldn't have spent 99 cents because 25, 10, and 5 cents all end in a 5 or a 0. So all multiples of 5 and 0 end up as 5 and 0. 5 + 5 is 10, 10 + 5 is 15, 35 + 5 is 50, 50 + 25. They gave examples, very nice examples. Do you agree?

*8 Student responds (inaudible).*

9 Do you think that's a good answer M------?

*10 Student: Um hm.*

11 Um hm? Why?

*12 Student: Because –(inaudible)*

13 -look at that answer. Do you agree? Do you think it's a good one? Why is it a good one?

14 Does it answer the question?

*15 Student responds (inaudible).*

16 OK. Say that louder so I can write it up here.

*17 Student: 95 is the lowest you could go, 99 and-*

18 Slow down. And the next part.

*19 Student responds (inaudible).*

20 Ha ha! Very nice thinking. G----?

*21 Student: 99 cents is 1 less, 1 cent less than a dollar and a nickel is 5 cents less than a dollar and a quarter is 25 cents less than a dollar.*

22 So what kinds of coins would you need in order to have 99 cents?

*23 Student: Pennies.*

24 You would definitely need pennies and we don't have any pennies. He would need pennies. How many pennies would he need-

*25 Student: Four.*

26 -to have 99 cents?

27 *Students:* *Four.*

28 Four cause he would have 95 cents, plus 4 pennies. That would give him 99 cents. Excellent thinking.

29 Hopefully your answer says yes or no or says "He could not have" cause that would answer "Could he have." Could he have 99 cents? No. He couldn't have 99 cents. And then some sort a reason, either in words or in numbers.

[Themes: appropriate answers to problems, "what do you think and why?" (9-14), encouraging student participation]

Throughout the excerpt above, I focused on an appropriate answer to a problem. I did this in others parts of the episode, but less effectively. In order to be successful in answering questions, students need to understand the question and respond appropriately. In addition, I thought that focusing on whether an solution appropriately answers a question was important because students don't always check their solutions with the original question and make unnecessary mistakes on work and therefore, most likely on state assessments.

Consistent with Inagaki, Morita, and Hatano's findings (1999), there seemed to be a pattern in public discourse called Inquiry-Response-Feedback. Although I attempted to encourage the students to offer their own arguments and evaluate other students' arguments, I also provided feedback that directly or indirectly evaluated student responses, elaborated on student responses, and gave my own opinion of the mathematics and validity of argumentation (lines 18-20, 22-24, 26-28). I would rather that the students responded to each other more often, instead of me being the "sole evaluator of student thinking and reasoning in the classroom" (Stein, 2001, p. 112).

Excerpt 9-3 (218-248)

1 **And we did this in the beginning of the year. Remember all of the different ways we divided.**

2 ***Students:*** *Yeah.*

3 **And we showed lots of different ways. So hopefully you remember your past experiences when you do this problem. [I read problem aloud]**

4 **And how nice this is because they say, both are correct. They answer the question. Who is correct?**

5 ***Students:*** *Both*

6 **Both are correct because you can subtract 12. 13 times from 156 and you can divide 156 by 12. So what's so nice is this sentence is really, really clear. And then in addition, the work is shown. And you can see that both ways end up being the same. So that is really nice work. Yes?**

7 ***Student:*** *Um, when I did it, at the end, I got 11 subtracted by 12, not 12 subtracted by 12.*

8 **Why do you think she would have 11 subtracted by 12 instead of 12 subtracted by 12? B------.**

9 ***Student:*** *She subtracted wrong.*

10 **Somewhere in your work, in all of your subtraction, you subtracted 1 too many. So what you would have to do is go back and see where that was. Somewhere, here, or here, or here, or here, or here, or here, or here, there's an extra 1 that you subtracted. Does that make sense N----?**

11 **Did you list all the subtraction?**

12 **So did you have 156?**

13 ***Student:*** *Yeah*

14 **and then 144?**

15 ***Student:*** *Yeah*

16 **and then 132?**

17 ***Student:*** *Yeah*

18 **120?**

19 ***Student:*** *Yeah*

20 **108?**

21 *Student:* *Yeah*

22 96?

23 *Student:* *Yeah*

24 84?

25 *Student:* *No.*

26 Oh! That's where it was. You had 83?

27 Yeah. That's where that 1 went.

28 *Student:* *Oh. She thought as in multiplication. 3 times 2 equals 6.*

29 Somehow, when you did 6 minus 2, you got 3. Oh. She saying you did 6 divided by 2, that's why you got a 3. Maybe. Could've been. Or you were just doing it and somehow you put the wrong number. That's why I tell you to check your work, P-------, before you turn in a test, because there's little things like that that happen. Like when S------ was counting the dots yesterday. He counted 13, yeah? So you always go back and look at the problem again.

[Themes: recalling previous common experiences (1-3, 29), multiple solutions (4), questioning and modeling to understand error (7-29), recalling previous common experiences (1, 29)]

A student expressed that she had a different answer in line 7. I followed up with soliciting student ideas in line 8 and in lines 10-29, and I modeled reviewing student work to check for error, with a student giving her opinion of what the error was in line 28.

Excerpt 9-4 (278-300)

1 This person said they didn't finish. Which [I read the problem] equals 18? They circled B [I read that choice]. Did they finish?

2 *Student:* *Yeah*

3 Does that look good?

4 *Student:* *Yeah*

5 Yeah, it actually looks good. So looks like they are done.

6 *Student:* *Almost.*

7 But I'm not convinced about the other one. For A, what could be a number sentence to show that, if Steve had 6 baseball cards, and bought 3 more cards, how many would he have? What would be a good number sentence?

8 *Students:* *6 + 3*

9 is what?

10 *Student:* *9*

11 So this says 6 + 3 is 9. That is not what I'm looking for. B, we see he already wrote it here is 6 times 3 is 18. That worked really well. What about C. What is the number sentence for C?

12 *Student:* *6 – 3*

13 Yeah. Because he had 6 and he gave 3 away. And to finish this sentence, I'd put an equal sign and a 3. And the last one, he had 6 baseball cards and-

14 *Student:* *Divide*

15 -put them in three equal stacks. What's my number sentence?

16 *Student:* *6 divided by 3*

17 That's an expression. If I add the equal and the rest of the sentence-

18 *Student:* *2*

19 It has an answer to it. Now I have proof. The other ones are not it. It has to be B.

[Themes throughout: encouraging student participation, building on student ideas, modeling thinking, exhausting all possible solutions, vocabulary in context]

The problem discussed in this excerpt involved four multiple-choice answers. I modeled exhausting all possible answers by going through all of the options and prompting students to give their number expressions and equations. Through the questioning and prompting with subsequent recording of the number sentences, I reinforced the definitions of numerical expressions and numerical sentences. This is consistent with Pimm's (1996) conclusion that

In mathematics, there is always movement back and forth between the potential (the possible) and the actual. The question of alternative possibilities can be partially explored by looking at particular cases, but one important mathematical challenge lies in identifying all possibilities, eliminating possibilities, and convincing other that all cases have been considered" (p. 18).

I model this way of thinking in Excerpt 9-5 while trying to make sense of a student's method for solving a problem. In sharing about the last problem, a student provides her solution. It is a challenge because it seems to work as a solution at first, but later doesn't.

Excerpt 9-5 (339-429)

1 Did anybody do it a different way? You did S-----? Can you explain to me how you did it?

*2 Student: I counted by 2s.*

3 Why'd you count by 2s?

*4 Student: Because I thought- [long inaudible response]*

5 OK. So you counted by 2. K. So you start with 2. Why would you start with 2?

*6 Student: Because the teacher called the first person. That person called 2 people.*

7 OK. So these two people are the people who got the calls from the first student. So what do you do about the first student?

*8 Student: After - I got 30 and I added then I added that student.*

9 You added 1 after you got to 15.

*10 Student: No. I added 1 after I got to 30 students.*

11 K. So 1 person called 2 students. Those 2 students called 2 students each. But you counted by 2s. Oh. So you counted, this is 2 students calling, now 4 students are calling. And then, 8 students are calling.

*12 Student: That's not what I ---- adding---2 +2.*

13 OK. I'm not sure how that works. 2+ 2, this first 2 is the first 2 students that calls 2 students.

*14 Student responds (inaudible).*

15 So what does this plus 2 mean?

*16 Student: The other 2 students.*

17 But these 2 students actually called 4 students. Do you see what I'm saying?

*18 Student: Yes.*

19 If you start with 2 students, they would call 2 students each. So how-

*20 Student: That's 4*

21 -many should you actually add?

*22 Student: 4*

23 You should actually add 4.

*24 Student: Times it by 2.*

25 And when those 4 students call 2 students each, how many are they calling?

*26 Student: 8*

27 They're calling 8. And when those 8 students call 2 students each-

*28 Student: 16*

29 -They're calling 16. 16 + 8 + 4 + 2, 16 + 8 + 4 + 2. What is it?

*30 Students 31*

*30*

31 That's 30. What are we missing?

*32 Student: 1*

33 That first caller. So S-----, I don't know how you would just add 2 each time.

*34 Student responds (inaudible).*

35 K. So you add 2, then 4, then 6, then 8, then 10, then 12, and when did you stop?

*36 Student: Until I got to 30.*

37 So 14, 16, 18, 20, 22, 24, 26, 28, 30. And that shows 15 students. No it has, 15 multiples of 2. How does that show the chain of calls? How does it show the chain of calls?

*38 Students: Student responds (inaudible).*

39 What are they doing so far? Does it show the chain of calls?

40 *Students:* *No.*

*Do we have to make a chain?*

41 Can anybody see how it shows who called who? Maybe I'm missing something. Yeah, bring up your paper, S——.

42 *Students:* *Are you multiplying?*

*No*

*--multiplying- (inaudible)*

*No*

43 You get the same answer, but I don't see how that is the chain.

44 OK. I can see why it works. She's got this. [writing student's work]

45 *Student:* *Ho. That's long.*

46 K. You can see it this way. If 2 students called 4 student, who called 8 students, who called these many students. And this person called that, now I can see it. But with just the 2s. So really, the 15 times that you added 2 doesn't show the chain, but if you circled it this way, you would see that these 2 called these 4, these 4 called these 8, these 8 called these 16, and this 1 called them.

47 *Student responds (inaudible).*

48 What?

49 So basically you had 1 person calling. Then another person calling. Then 2 people calling. Then 4 people calling. Then-

50 *Students:* *16*

51 Wait-

52 *Student:* *Isn't that 1 person calling –*

53 people calling. So 7, 8, 9, 10, 11, 12, 13, 14, 15.

54 *Student:* *Oh.*

55 Now you can see. These are the people that actually made calls. [long pause]

56 *Students:* *Oh.*

*Why is that other-?*

57 Huh?

58 *Student responds (inaudibly).*

59 No this is part of my other answer. Good thinking. Do you see the difference?

60 *Student:* *Yeah.*

61 K. Somebody has a question.

62 *Students:* *How'd you get 30-*

*There are 31 students*

*Cause 7 times 2 is 14 plus the 1*

63 2, 4, 6, 8, 10, 12, 14, 16, 18, 10, 22, 24, 26, 28. We're missing a 2 somewhere.

64 *Student responds (inaudible).*

65 and then 1 is here.

66 *Student:* *Oh.*

67 Yeah, sorry. 7, 8, yeah we're missing-

68 *Student:* *That should be 8.*

69 [counting to 16]. It doesn't work. I don't know about his. I think you have to make a chain.

70 So the key word is go back to your problem and look at the clues, cause it'll help you.

71 On the bottom of the last page. Bottom of the last page. OK. Here we go. I wanna call, copy this down. All 6$^{th}$ grade students. I will call 1 student. Who will call 4 students. Each student will then call 4 students. There are 95 students altogether. How many students will make phone calls?

72 Your job over the weekend is to make corrections. On all the ones that we talked about. Double check. And do this one. That's your homework.

73 Do that problem using the chain or trying to use S-----'s method so we can see if S-----'s method works. I have a feeling it doesn't, but I'm not sure. Correct the ones we just went over so we can see if S-----'s method works. I have a feeling it doesn't, but I'm not sure. Correct the ones we just went over. Make any changes you want to make to make them better. And just double-check the rest that you did that we didn't have time to look at yet.

[Themes: questioning for reasoning and understanding, attempting to understand student thinking, modeling and engaging in problem solving, building on student idea]

A student shared her solution to a problem that stumped me. I engaged in trying to make sense of her method and provided a follow up problem for the class to solve with a desire to create an opportunity for her, the other students, and myself to understand her reasoning. At the conclusion of the episode, the educational assistant and I conversed [not in the excerpt] about her method and could not grasp how it worked. In lines 71-73, I created a problem similar to the one we discussed in an attempt to see other students' methods for solving a similar problem, but also to get the student who presented her solution to attempt the same method again in hopes that it would clarify her thinking.

In the last excerpt, I was not so focused on where the students' attention was, but rather, my attention was solely focused on me making sense of the method the student had presented. "One thing that teachers need to do continually is attend to the mathematics in the situation and communicate where their own attention is – and not be too concerned at directing the student's attention there as well" (Pimm, 1996, p. 13). The other students participated in trying to understand the solution method presented by the student. In this way, students participated in collective reflection because they reflected on and objectified the mathematical contribution as it was shared (Cobb, 1997). In order to better facilitate students mathematizing, my questions and prompts could have initiated shifts in thinking if I had stepped back and reorganized what had been done in an episode regularly.

*Student Math Survey*

Although there were 21 questions on the survey, I analyzed the results from only the prompts I thought could be related to classroom discourse. The prompts, as they were numbered on the survey, were:

3. I listen to other students talk about math.
4. I talk about how I solve math problems.
5. I ask my teacher questions when I do math.
6. I ask my classmates questions when I do math.
10. I work with other people when I am doing math.]
16. It is important to talk about math.
18. I have good ideas to share about math.
19. When I share my ideas about math, people listen.

The students rated the prompts by checking off always, usually, often, sometimes, rarely and never. I assigned the numbers 5, 4, 3, 2, and 1, with 5 being the 'highest' and 0 being the 'lowest' ratings.

Table 5 shows that there was an increase in raw total points and in the mean rating per student response for questions 3, 4, 6, 10, 16, and 18, while the points and mean for questions 5 and 19 were lower in November as compared with the July survey results.

Table 5. Total Raw Points and Mean Ratings

| Question Number | 3 | | 4 | | 5 | | 6 | | 10 | | 16 | | 18 | | 19 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Survey Month | Jul | Nov | Jul | Nov | Jul | Nov | Jul | Nov | Jul | Nov | Jul | Nov | Jul | Nov | Jul | Nov |
| Total | 97 | 100 | 71 | 84 | 84 | 73 | 81 | 84 | 68 | 79 | 98 | 104 | 56 | 68 | 76 | 71 |
| Mean | 3.73 | 3.85 | 2.73 | 3.23 | 3.36 | 2.92 | 3.12 | 3.23 | 2.72 | 3.16 | 3.80 | 4 | 2.15 | 2.62 | 2.92 | 2.73 |

Table 6. Direction of Movement along the Scale of Single Student Ratings

| Question | 3 | 4 | 5 | 6 | 10 | 16 | 18 | 19 | Totals |
|---|---|---|---|---|---|---|---|---|---|
| Up | 8 | 13 | 7 | 9 | 10 | 10 | 10 | 6 | 73 |
| Down | 8 | 3 | 11 | 8 | 4 | 6 | 2 | 8 | 50 |
| No Change | 10 | 10 | 7 | 9 | 11 | 10 | 14 | 12 | 83 |

Table 6 shows that there was more movement of ratings upward (toward always) in questions 4, 6, 10, 16, and 18 while there was more movement of ratings downward (toward rarely) for questions 5 and 19. The number of ratings that went up and down was the same for question 3. However, when looking at the total number of ratings, 73 moved up the scale and 50 moved down, while 83 remained the same. This shows that students rated more prompts higher than they did lower.

Table 7 gives the number of occurrences of each rating for each question in the two surveys given and Table 8 shows that the total number of occurrences of the rating of 5 increased by 1, the total number of occurrences of the rating of 4 increased by 7, and the total number of occurrences of the rating of 3 increased by 8, while the total number of occurrences of the ratings of 2,1, and 9 decreased in the November survey as compared with the July survey results.

Table 7. Number of Occurrences of Each Rating

| Question | 3 | | 4 | | 5 | | 6 | | 10 | | 16 | | 18 | | 19 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ratings | Jul | Nov | Jul | Nov | Jul | Nov | Jul | Nov | Jul | Nov | Jul | Nov | Jul | Nov | Jul | Nov |
| 5 | 8 | 6 | 5 | 4 | 3 | 4 | 5 | 4 | 2 | 4 | 9 | 11 | 1 | 3 | 3 | 1 |
| 4 | 9 | 13 | 2 | 7 | 9 | 8 | 7 | 7 | 8 | 8 | 8 | 6 | 3 | 4 | 7 | 7 |
| 3 | 3 | 4 | 6 | 8 | 7 | 1 | 3 | 7 | 1 | 4 | 3 | 7 | 5 | 4 | 5 | 6 |
| 2 | 6 | 3 | 8 | 5 | 6 | 7 | 8 | 7 | 9 | 6 | 6 | 2 | 9 | 10 | 8 | 8 |
| 1 | 0 | 0 | 4 | 2 | 0 | 4 | 3 | 1 | 5 | 3 | 0 | 0 | 6 | 5 | 2 | 4 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 1 | 0 |

Table 8. Total Number of Occurrences of Ratings

| Rating | 5 | 4 | 3 | 2 | 1 | 0 |
|---|---|---|---|---|---|---|
| July | 36 | 53 | 33 | 60 | 20 | 4 |
| November | 37 | 60 | 41 | 48 | 19 | 1 |

*Student Math Questionnaire*

There were some difficulties with the questionnaire. The responses on the questionnaire seem to support the idea that there are different views of what mathematics is (van Oers, 2001). Students may have perceived that a teacher is teaching mathematics if s/he is actively showing the students how to solve a problem. I, however, in the design of the questionnaire considered teaching mathematics as inclusive of when a teacher is observing students solving problems together, joining a problem solving situation, or discussing students' solutions with the group. These views of teaching were probably not shared by the students. Therefore, most of the responses were not appropriate for this study. However, I recorded some responses in Table 9 that I found to be interesting. For the complete student responses from the questionnaires, see Appendix F. Spelling and grammatical errors have not been corrected.

Students 13 and 22 did not respond to "What is math?" in July but responded in ways that corresponded to the types of activities and discussions during mathematics instruction. More students responded that mathematics was something in life and involved problems in December than in July. Student 12 wrote that mathematics is understanding his or herself as well as others. Of note is that student 21 wrote that mathematics was boring and later wrote that mathematics is when you use your brain to solve problems. In question 3, students 13 adds that s/he would be talking in the later questionnaire and student 14 initial wrote that s/he would be playing, yelling, cheating, and hiding during mathematics in July and later wrote that s/he would be studying or doing homework. This seems like an improvement in classroom participation. In answering question 4, both students below wrote that they would be figuring out problems in different ways which was a goal I had while having students share their solutions with the class.

Table 9. Student Responses to Questionnaire Showing Possible Change in Perceptions

| Question 1: What is math? | | |
|---|---|---|
| Student Number | July | December |
| 6 | Numbers Digits Varibals | Math is something in everyday life |
| 8 | Math is work you see people doing every day around you | Math is a problem |
| 12 | Math is a subject that will show up in every day life. Its numbers that are put together. It is also another way of communication. | Math to me is a way to express a different part of you. It is a way for me to understand different parts of other people too. |
| 13 | | Math is something that is involved in your whole life. |
| 15 | I think math is supposed to be something to help us when we get older. | Math is where you learn how to add, subtract, multiply, and divide. |
| 18 | I think math is a bunch of numbers and equations, and alot of problem solving skills. | Math is like a language with numbers. Math uses strategies to solve problems. |
| 19 | Math is fun sometimes but not most of the time. | I think math is lots of numbers that are added subtracted, multiplied, and divided. |
| 21 | Math is boreing | Math is when you use your brain to slove problems |
| 22 | | math to me is divission, multiplying, adding, and subtracting. math is needed to get jobs. |
| 23 | Math is addition, subtraction, multiplication, division, geometry, and fractions. | Math is something that you do in eveyday life like in question 5. [Math is used in the kitchen, at work, and shopping.] |
| 24 | Math is numbers and sequances | Math is something we use every day to figure things out |
| 26 | | math is using numbers and solving question with numbers. |
| 27 | Math is calculating numbers, measurement, and other neat stuff. Stuff meaning multiplying, subtracting, and using fractions. | I think that math is learning and preparing for other things in life. |

| Question 3: If I were to walk into your _____ classroom while your teacher was teaching math, what would I see you doing? | | |
|---|---|---|
| 7 | working , writeing in are composition book, raiseing are hand for the answer | You would se me doing the probles and asking my firend for help or see that I would be asking qesutions about math. |
| 8 | 1. listening 2. not talking 3. solving a problem 4. writing or erasing | trying to solve it |
| 13 | You would see me looking at my own paper, listening, taking notes | You would see me listening or sometimes talking. |
| 14 | playing yelling hiding cheating | Studing or doing some homeowkr i didint finish |
| 18 | - rasing my hand - talking to the teacher - asking questions if I don't understand something - writing down answers - looking at my mathbook - listening | You would see me working on math problems and listening to the teacher. |
| 23 | You would see me taking notes, asking for help, or reading. | You would see me working or asking for help. |

| Question 4: If I were to watch you do your math homework, what would I see you doing? | | |
|---|---|---|
| 19 | You would see me working silently on the table. | You would see me answering problems in diffrent ways. |
| 25 | Maybe asking some questions and trying to figure out the answer. If don't, then I'll skip it and go to the next question. | Trying to figure out a problem, trying different ways and double checking my work. |

## Discussion and Lessons Learned

I found it extremely difficult to study the discourse that exists in my classroom due to the complexity involved partly because "…discourses are dynamic and ever-changing entities, and thus, determining their exact identities and mapping their boundaries is not as straightforward a task as a researcher would hope "(Sfard, 2000, pp. 160-161). Sfard (2000) concludes that mathematical discourse can be learned if teachers demonstrate ways of doing mathematics while teachers also respect students' individual thinking. Although I was mostly in a leadership role, I believe that I began to set expectations for the students in their discussions. I attempted to look at themes that arose in the transcripts of the episodes. One thing I could do in the future is look, in a more detailed way, at my patterns of questioning once an initial question is posed. The quality of 'whole class interactive teaching' varies depending on how teachers scaffold, the opportunities created for reflection and degree of student ownership along with teachers abilities to anticipate responses and errors that may arise and the ability of teachers to go with students while continuing to fulfill lesson objectives (Jones & Tanner, 2002). Sherin (2000) describes a format that seemed to be successful in allowing student ideas to be the basis of discussion while ensuring that these discussions were mathematically productive including collecting students' ideas, comparing and evaluating ideas, and focusing the range of ideas. This is termed funneling and allows the teacher to direct the content while emphasizing student ideas. Herbel-Eisenmann and Breyfogle (2005) suggest changing patterns of discourse from Initiation-Response-Feedback patterns to funneling and focusing patterns to allow for more student-centered discourse.

Strategies to get the students to take a more active discourse role were attempted including getting students to call on other students and prompting students to try to explain the thinking exhibited while students were showing their solutions to the class. By having the students share their solutions on the overhead projector, I was more able to step out of the traditional teacher role and become a spectator. I still found that I was continually questioning, prompting, and responding to students. I would like to remove myself from the analytic center so that student-led discussion would increase (Nathan & Knuth, 2003) and I would like to implement strategies such as a "filtering approach" so that the conversations could continue to have mathematical precision and significant mathematical content (Sherin, 2002).

Some of the elements of discourse such as gestures, tone during conversations, and written communication that were presented on the whiteboard are missing from the transcripts. These add additional dimensions to discourse. Webb (2004) found that student behavior largely mirrored the discourse modeled by and the expectations communicated by teachers. Throughout the episodes, I modeled behaviors I would like to see the students participate in – answering questions completely, labeling answers, adding information that clarifies solutions, making mistakes and finding errors.

Hufferd-Ackels, Fuson, and Sherin (2004) describe a math-talk learning community as "a community in which individuals assist one another's learning of mathematics by engaging in meaningful mathematical discourse" (p. 81). The researchers delineate levels 0 to 3 of the math-talk learning community and four developmental trajectories: questioning, explaining mathematical thinking, source of mathematical ideas, and responsibility for learning. In examining the excerpts, my questioning seemed to shift between a level 0 and 1 because I was generally the only questioner and I focused on student thinking by following up on student methods and answers. The student-to-student talk was not student initiated. The learning community was generally at a level 2 for explaining mathematical ideas because I probed student thinking but sometimes filled in explanations myself. The "students give information about their math thinking usually as it is probed by the teacher (minimal volunteering of thoughts). They provide *brief descriptions* of their thinking" (p. 89). I was the main source of ideas (level 1) although I followed up on explanations and used student errors as mathematical ideas (level 2). Student ideas were raised, but not explored in detail (level 1). For responsibility for learning, the students had become more engaged by repeating what other students said at my request (level 1) while I asked students to comment on other students' work by asking whether they agreed or not and why (level 2).

Challenges I faced were similar to other researchers. Both process and content issues need to be included in mathematics instruction, teachers need to promote learning as students engage with appropriate activities, and especially, teachers need to elicit comments from students and decide how to react to and with these comments. Depending on the needs of the students, the beliefs of the teacher, and the nature of the content, a teacher may chose to what degree classroom discourse is oriented around content or processes (Sherin, 2002; Sherin, Mendez, & Louis, 2004).

Sfard (2001) stresses that communicational approaches to teaching should complement rather than replace more traditional outlooks such as learning to acquire knowledge or skills. Nathan and Knuth (2003) found that, as a teacher continued to implement reform-based instructional practices, horizontal discourse increased. Vertical discourse was prevalent in my classroom in that most of the conversation was between the students and me. In the future, I would like to see more horizontal discourse, students talking to each other, during whole class discourse.

The episodes show that the lessons built on each other allowing for practice and reinforcement of skills and concepts in order for the students to build understanding. Basic skills and concepts of what a fraction means and how it can be changed into decimals and percents were taught directly as well as through games. The placement of the teaching and reinforcement of mathematical procedures were dispersed throughout the episodes.

In the episodes where we reviewed homework, I was more traditional since I gave students correct answers. Still, within this context, the students gave answers most of the time, sometimes chorally. I did not have to tell the students all of the correct answer. When the students were sharing their solutions with the class, the answers came from them although I did lead them by showing my own agreement and disagreement. In every episode, no matter what type of activity or task we were engaged in, I solicited student ideas. I noticed that I rarely called on particular students, but instead gave prompts to the whole class to participate. Students readily volunteered most of the time. Many of the same students volunteered more regularly and I used different strategies to try to get students who had not shared their solutions to come up to share. It was somewhat effective since more students who don't normally volunteer did so. A couple of students, however, still did not respond despite encouragement from their classmates and me.

In the future, I would like to implement more inquiry-based tasks to allow students to explore and build on each other's mathematical ideas (Manouchehri & Enderson, 1999). I tended to give problems or tasks that were short term and generally dichotomous in that they resulted in particular answers. Instead, I would like to implement a style of questioning and intense group work as demonstrated in "The Art of Posing Problems and Guiding Investigations" where the teacher "proceeds through defining a problem, focusing students' attention, and guiding the dialogue" (p. 147) so that students' ownership of the problem is established and the discussion follows students' thinking rather than mine.

Throughout my research, I have discovered, even more so, that teaching is an extremely dynamic venture. In an attempt to summarize the different layers of a more enlightened teaching, Table 10 shows different continuums that may be traveled along in order to create a student-centered learning environment while also teaching the skills and concepts that the world expects students to learn.

Table 10. Enlightened Teaching: Multi-Level Continuum

| Traditional | Reform-Based |
|---|---|
| Routine Task | Novel Task |
| Parrot Math | Invented Procedure |
| Telling | Initiating |
| Skill for Mastery | Guided Discovery |
| Whole Class Lecture | Peer Collaboration |
| Open-Ended Response | Dichotomous Response |
| Univocal Discourse | Dialogic Discourse |
| Vertical Discourse | Horizontal Discourse |
| Sort Term Problems | Long Term Problems |

While the Student Questionnaire gave data that was generally inappropriate to the study of discourse, I attempted to show some of the responses that may have shown students responding to the prompts in different ways. The student survey seemed to show that more students responded with higher ratings in most of the questions that related to behaviors needed for or perceptions associated with participation in mathematical discourse. In attempting to use questionnaires and surveys, I learned the importance of having a definite focus for these data collection methods and piloting them before using them in research.

## Conclusions

Throughout all of the studies presented in this paper, I found that student participation is essential. Although I attempted to solicit more students to participate by trying new strategies, many of the same students volunteer more readily and more consistently. I found that, in the episodes where the students were sharing their solutions with the class, the wait time increased. I watched the students and waited for responses more often and for more extended lengths of time.

In all of the studies, there was a climate of open sharing and disagreement. The students were comfortable in asking for help as well as in questioning me about my ideas and correcting me when they found that I made mistakes. I model respect and interest in student ideas and they seemed to respond well to that. One difficulty I found was that this did not always translate into students showing respect for and interest in learning from each other when working together in small groups. In the episodes, I placed the iPod at student groups and found that they did not communicate effectively when I was not present. I noticed that their attention was not always to the task at hand and that they did not necessarily know how to listen to each other and build on each others' ideas. Connected to this is the idea that students generally discuss what the right answer is or what they got as an answer. I would like the discussion to move toward the process rather than the product. I would like also like to focus on establishing a better discourse community with respect to collaborative students groups.

Throughout all of the studies, I continued to speak much more than I needed to. Instead of continually prompting and questioning, I would like to experiment with being more of a silent spectator. Although students were engaged in all discussions and student solutions were the topic of conversation, the direction of the dialogue was generally between the students and me and I controlled the dialogue rather than having peer discussion that seemed more autonomous. I would like the content of the discourse to be shaped by refinements the students make in their thinking rather than closure on presented tasks or problems. I would like the product of discourse shared understanding and sophisticated mathematical relationships and connections. (Manouchehri & St. John, 2006)

One area I continue to struggle with is planning. It's difficult to find time to choose tasks carefully, plan questions and prompts ahead with attention to anticipating student responses, and to plan for formative and summative assessment that acts to inform and guide instruction. However, because of my research, I have found numerous resources that provide worthwhile tasks and ideas for implementing them. I plan on following up by using these resources in my future teaching.

I found that throughout the last study, I interrupted students while they were verbalizing their thoughts. I could better listen to the ideas of my students and take the time to think about how to respond or to allow other students to respond. While I was trying to predict and guide instruction, the way I interacted with students sometimes hindered my understanding of what they were trying to communicate. It is usually a time issue, but I should slow down instead of attempting to expedite learning. Speed and learning are contradictory concepts. By planning and taking my time to listen to students, I can facilitate learning because I will spend less time giving, repeating, and clarifying directions and rules.

Many different models and representations were presented at different times providing opportunities to apply skills in new situations. These also acted as reviews for the students. The problems presented to students seemed to be accessible while also challenging for students. I wanted to give the students opportunities to be clear and flexible in their thinking as well as use mathematical language and vocabulary appropriately. Even though the tasks were generally below grade level, I found that the solution sharing method took a lot more time than when I was in an instructor and assessor mode. It took time for the students to present their solutions, for the class to share what they thought about the solutions, and to decide if the solution was it's best at the time.

One area that I seemed to do well with was in providing experiences that the students could build on and connect to. All of the tasks could be connected with each other and provided opportunities for students to review and practice skills in new situations. There were varied types of tasks and methods of instruction, continuous opportunities for students to get additional help, and many opportunities for students to improve their work. Most of the discourse provided opportunities for students to share their ideas, justify their thinking and form conclusions and generalizations. In the end, this self-study has shed light on how I can get "...students [to] engage in authentic mathematical inquiry, act like mathematicians as they explore ideas and concepts; and negotiate the meaning of, and the connections among those ideas with others in class." (Manouchehri, 1999, p. 216).

## Implications

I found that doing a self-study is extremely personal. It caused me to reevaluate my beliefs and values and therefore my curriculum. Although my findings and conclusions are limited because they may not be generalizable to others, I identify some implications for my own teaching and for the educational community in general.

### *My Teaching*

I have consistently found that certain students participate in large group discussions more readily and regularly while certain other students rarely participate. One area that I would like to research in the future is participation. What does participation mean? What does participation look like? Do students need to be participating verbally? What methods or strategies can be employed so that quieter students have opportunities to share their ideas? What are some strategies for knowing that students are actively listening?

One way I believe I can increase student participation is to set discourse expectations in the beginning of the school year. I recorded student conversations and found that many students didn't show active listening and thinking in their dialog. They interrupted each other, spoke of unrelated topics that distracted other students, and failed to question and follow up on other students' ideas. In the future, I will use recordings and play them to the class. I will have the students identify what is and is not beneficial while they are working in peer groups and when in whole class discussions. Checklists and rubrics can be created by the class and used for reflection and assessment.

Even if students do not understand concepts and skills that I expect them to when entering 6$^{th}$ grade, I will not teach skills in isolation. I will use a curricular plan similar to the one presented in the methodology and rich problem solving experiences, finding other opportunities to remediate as needed. The time that was lost in the 1st quarter to teaching basic skills could have been better spent with experiences that lead to deeper thinking and the establishment of expectations for a discourse community.

I hope to provide more explicit instruction regarding how people can show they are thinking and how to build on others' ideas. One thing I can do is build a list of questions that students can choose from when they are confronted with a problem or solution they do not understand. I can teach and model using these questions and have expectations that they students will also use them. I plan on remaining more quiet once I give the students some ideas about how they can show that they are considering others' ideas. Carefully listening to students' ideas and recording them for discussion will help.

I will work for more horizontal discourse. One way to achieve this may be to require the students to look at each other when sharing ideas and asking questions instead of looking at me. Although I have done this, I could be more consistent with my expectations. Another strategy may be to break the class into smaller groups. A small group could sit in a circle facing each other while the rest of the class, including me, is on the outside of the circle. The expectation is that they students in the inner circle will hold an effective discourse circle. If they have a difficult time sharing, questioning each other, and building on each others' ideas, the audience on the outside could whisper ideas into ears or write ideas for the inner circle to share. The inner circle would not be allowed to ask the audience questions, but may share the audience's ideas and ask their peers in the inner circle what they think the ideas could mean and how it may help them. I plan on interrupting students less and on having longer wait time more consistently. I would also like to research more strategies for increasing horizontal discourse.

I would like to plan more effectively for mathematics instruction by spending time choosing tasks, completing them myself, and planning initial and follow up questions and prompts. I need to develop and use formative and summative assessments more efficiently in everyday teaching. I need to research and develop strategies that address these needs.

Opportunities for students to share their work, solutions to problems, and reflections are extremely useful. I will continue to use this strategy and along with it, I will do less of the talking about the work and solutions to empower the students as problem solvers. Giving the students time to reflect on work should be a daily occurrence. Students should also be given opportunities to share their reflections about other students' work in more routine ways.

*Beyond Me*

My study began small, with examining the types of questions I asked. It then became broader, so broad, that it became difficult to manage. It was useful to look at the entire semester informally, but for more formal self-studies, I think it could be more informative to start smaller, with particular lessons chosen for examination rather than larger periods of time. The amount of data was daunting. Also, had I chosen particular lessons, I may have been better able to study the planning that was or was not effective in facilitating discourse and creating a discourse community.

It seemed that the more I was speaking and modeling in class, the more I was doing the thinking instead of the students. By not allowing students to do the thinking, I felt I took away their sharing and questioning voices. More is less and less is more. If I listen more carefully, I will do less talking. If I model less, the students do more thinking. If I teach more explicitly, I will have to teach less. If I slow down, the students may learn to think deeper, and may not learn more content, but will learn to think more about the content.

I found it extremely useful to look closely at and listen to my teaching. It was very uncomfortable at times because it brought things to the surface that I otherwise did not notice such as how often and how much I interrupted the students during their sharing and questioning. I was anticipating too much in order to save time, but was also removing their power through removing their opportunity to share complete thoughts and ideas.

Finally, conducting research and communicating with others about teaching is important. Finding and grappling with other researcher's studies is invaluable in learning to teach better. Sharing research with other teachers and discussing the issues it brings up allowed me to broaden my understanding of mathematics and of teaching. Openly communicating about my teaching with others and learning about other peoples' ideas and instructional strategies has helped remove boundaries that could limit my learning and my students' learning.

## Appendix A
## Phase 1 Data

Episode 1: October 29, 2004

1 ...before...it tells you the rules of the game in the box.

2 So...It says that a dart player has entered a contest in which he throws two darts at a dart board. If both darts hits the bull's eye, he wins $10. If one dart hits the bull's eye, he wins a key chain. If no darts hit the bull's eye, he gets no prize. After much practice, this dart player has found that he's very accurate when he throws the first dart, hitting the bull's eye 80% of the time. Unfortunately, he's less accurate with the second dart. He hits the bull's eye 50% of the time with his second shot. So think about that. If he hits the bulls eye 80% of the time. What does that mean?

*Upon reflection, students could have been given time to read on their own and discuss together. rather than me reading to them.*

3 Sorry, what?

4 So you're saying, he'll mostly hit it, because 80% is most of the time?

5 OK. And what do you think, I----?

6 K. So he's most likely to hit the bull's eye and then miss. So if he threw the darts 10 times, how many times would he hit it? If it was just his first time?

This question asks for application of a skill.

7 8, yeah, because 8 out of 10 is 80%.

8 OK. But then the second time he throws it, he hits it 50% of the time. What does that mean?

*This is an open-ended question that leads to student internalization and application of the problem presented.*

9 Yeah. Threw 10, only hit 5.

10 What were you going to say?

11 About half the time, He'll hit the bull's eye.

*Many different students are given the opportunity to participate and give varied types of responses. This allows students that are not yet able to participate, hear others' ideas.*

12 OK. It says, make a prediction. What seems the most likely outcome for this dart player? Is he most likely going to win $10? Is he most likely gonna win a key chain? Is he going to get no prize? I want you to explain your thinking. So you have to decide, [pause] is he going to win $10 [pause] is he not gonna win $10, [pause] is he gonna win the key chain? So you need to come up with some sort of way to figure this out and you might get some clues if you look at the next page. Look at number 6. [pause] It starts off a tree of possibilities of what might happen [pause] and you might be able to figure it out from there. So you're only doing number 1, you're only making a prediction, but you have to explain in detail why. So it might help you to look at number 6 and try to add on to that if you need to. Does that make sense?

*Wherever there's a pause, the students are thinking and agreeing with responses like "yeah" or "yes".*

13 You're just making a prediction. You're not doing anything else. Only number 1. You might circle it so that you remember or make a note to yourself "only do number 1" but will you underline explain your thinking your thinking cause that's what's most important, that you explain. Are there questions about what "Playing Darts" is?

*These questions are asking to check for student understanding of the directions of the task and to get them to explain thinking. It emphasizes that explanations are the most important things.*

14 No? You all understand? You're just writing...prediction and explanation.

Episode 2: October 30, 2004

15 The shaded squares in the 10 by 10 grid on the right represent 54 hundredths, written as a decimal, 54 hundredths, everybody with me?

16 or 54 percent of the whole square, because percent means, per?

*This is a recall question.*

17 Hundred, K. So percent can also represent a value and they show a dollar there. A dollar is basically a hundred pennies so one penny would be 1% of a dollar. So, a dime would be what percent of a dollar?

18 Thanks I----,

19 yeah and a quarter would be what percent of a dollar, Kanani?

20 25%, yeah, good. And a nickel would be...

21 5%. So you add up cents, you're just adding up percents. So you have some money there and it says 43 cents represents 43 hundredths or 43 percent of a dollar. Easy, yeah?

These questions ask students to apply the skill. Many students answer, showing their understanding in order to move on to the next tasks.

22 70 cents represent 70 hundredths which is 70% of a dollar. So for number 1, it says write the decimal and percent for the amount of money as part of one dollar. So you have 10 cents plus 5 cents plus 3 cents which is 13 cents, right?

*This questions tests if the students are listening because I make an error on purpose.*

23 18 cents. I'm glad you're listening. Which looks like this? Right? Yeah? Which is 18%. [pause] 25 plus 25 plus 25 plus is 75 plus 5 is 80 plus 2, is that right?

24 82 hundredths of a dollar. Which is 82%. And then we got 50, 60 cents. Now this is where a lot of people were messing up. If they saw that, they told me, sometimes, that that is 6%. That is NOT 6%, that is 60% percent because this is in the tenths place, and you need hundredths in order to have percent. So I need you to be really careful with that. Some people aren't looking up here. And it's those people that sometimes make mistakes like that. K So for Number 4, 75 percent, looks like this as a decimal. Did you have that for number 4?

*Anticipating possible misconceptions that students will make is the point of this discussion. Students are getting a little distracted and their responses indicate that they seem bored or would rather be doing something else.*

25 K. Number 5. Should look like this. This is where people mess up. You're looking at this paper right now. Because this is a review. I'd like you to remember this, forever. Remember we looked through the newspapers in the beginning of the year? We found all kids of percents in there. We talked a little bit about how you put money in a bank and you get an interest rate. And if you don't know how to use percent, you may get jipped out of some money. Chances are you want to keep as much money as you can, right? And make as much money as you can. So you gotta know what interests rates are. If somebody tells you this and you think that that looks like

*Using real life examples and previous discussions to reinforce concepts and show them the importance of the skill and concept.*

26 Yeah. This. That's very very different. You've gotta be really really careful with that. Number 6. Did you have this?

27 Number 7. Did you have this?

28 K. If you didn't have these, then you should probably come and see because you may need help. Um. Number 8. Should have this.

29 You have that?

*The students respond in a more positive way after explaining the importance of the skill.*

30 K. Number 9. You should have this.

31 Number 10. You should have this.

32 Number 11.

*A student asks for clarification showing that she has interest in the topic.*

33 Yes. Always put the zero before the decimal point. That way we know that it's a decimal. If you just have a decimal, sometimes we can't see it or we're not sure what it means. Everybody in the math world, anywhere in the world, will know that that's a decimal cause it has a zero and then a little point, and then some numbers. If you just have a point some people might not know what you're talking about. And the last one, number 11, should look like that.

34 Yes?

*All of these questions were to check for correct answers that students gave in an in class exercise.*

35 You should have all of these correct. If you don't, what you need to do, is you need to make a note to yourself that you need to get help. Maybe write at the top of this paper "I need help" because I got some of these wrong and I should know this already. This was not difficult because there were not any operations, you just had to put the decimal in the right place. So remind yourself "I need some help" and make a plan to get it. Put that inside your Weekly Report Folder in the left hand pocket so as soon as you open it, you'll see it. That way you'll remember every time you open that folder that you need help, if you need help.

*I give these types of directions to put the responsibility on the students to care for their own learning.*

36 K. Everyone got a Playing Darts sheet and I told you yesterday that, if you lost it, you would write everything out by hand. So you should have it, or you're gonna have to copy it from somebody. Your job last night was to read through the directions of the game and to make a prediction and explain why you think so. So who would like to share their prediction? Anybody, please, share what they think will happen and why. K. C——.

37 Did you all hear her?

38 Do you agree with her?

This questions is asking students to discuss what another student said.

39 You thought that? That he would probably get a key chain cause it's really likely that he'll get the 80 out of 100, the 80 %, but it's only half-half that he'll get 50. Looks like T—— disagrees. What do you think, T----?

40 Why?

41 D----. Right now you're being rude, because T—— is talking.

*Tiffany gives a very detailed explanation that disagrees with a previous student's idea.*

42 So she thinks that the percentage is high enough because of the 80 and them half-half.

43 K. Keku. What do you think?

*K---, again, disagrees and feels free to share this disagreement showing an open discussion.*

44 Any other ideas?

45 This one's not as clear maybe as the other ones that we played, the Card Game and the Dice Game? What makes this one a little different?

46 OK.

47 And, what about darts makes it a little bit different.

*This repeated questions is trying to get the students to notice similarities and differences to make connections with activities previously done.*

48 K. So how does that make it different than the Card Game or the Dice Game?

*One of my special education students was sharing ideas that did not fit the question. I had to redirect and answer in a way that did not address his question.*

49 OK. So, in this case, you've got different degrees of winning. You could win all the way, or you could win part way, or you could not win at all. In the other games, you either got a point, or you didn't. So that makes it a little more complicated.

*I would have rather had the students come up with their own reasons, but the conversation was not going as I had anticipated it or wanted it to.*

50 OK. Was the Dice game probability?

*This questions was in response to a students detailed description that we were dealing with percent and percent is just probability.*

51 So it was all probability, but this is a little different, because, you mentioned, percent of the time, yeah? Whereas before, we were talking about the role of a dice. So you had maybe a one out of a six chance of getting a number, so M——, what were you going to say?

52 What makes it different than the Dice Game and the Card Game? Did you forget what you were going to say?

53 I'm sorry. I'll come back to you. K----?

*K---- brings up the idea of skill versus chance games.*

54 OK. So, if you practice at playing darts, you usually will get better. That's kind of different than what's happening here, though, because he's doing better the first time, and then he get's only 50% on the second. So this isn't following the normal skill that you're talking about. But normally, when you do play, the more you play, the better you get, so like when I play darts, the first dart may be a little bit off and then

my second darts is usually closer. But this is kind of backwards. And maybe that makes it a little more complicated too.

*If I were able to do this again, I would not explain, but instead let the students discuss the ideas of skill and chance.*

55 OK. Using a spinner is a little bit different, like C—— said. Now, we're not actually throwing darts, we're using a spinner. So the skill is taken out, that you had mentioned. When you actually play darts, you can use your skill, the more that you play, probably, the better you'd get. Other ideas? Do you remember yours' M——?

56 No? Any other ideas?

57 Yes?

58 Yeah. You don't have player A or player B. Yeah. Any other ideas?

59 OK. So she says this one we're using percents and the other one we were using fractions. What do you folks think of that? M——, you remembered?

*I think this line of questioning was good because I did not tell the students that fractions were percents, but instead let them discuss it and remember previously taught skills with fraction, decimal, and percent conversion.*

60 You're saying that this one has less evidence? What do you mean?

61 So you're saying that it's easy to look at a number of cards? And it's easy to know that there are only 6 chances when you role dice, but in this one, it feels a little different because you're unsure of what's going to happen. Is that what you mean? Because you're dealing with two different percents?

62 And so there's lots of different places that this could land. Yeah? It's not just going to land on a 1 or a 2 or a 3 or a 4 or a 5. But you can land anywhere in this circle. So it's a little bit confusing. Is that what you're trying to say? It's hard to make a choice. Because you don't know what evidence to use.

*Here, I am answering the student's question. I don't think I should have done this.*

63 OK. What were you guys talking about back there?

64 Do you have an idea?

65 Talking about something else?

66 What were you thinking? So that we can hear your thoughts?

67 Yeah. That's what Keola was saying. That while it's saying that it's a dart game. This is actually a spinner game. So there are different things going on.

68 So in this case, they're giving you some things that you have to accept. They're telling you that every time he shoots the dart the first time, he's gonna get a bulls eye 80% of the time. They're telling you that that's what's going to happen. So, when you look at this first spinner, it's either gonna land here, or it's going to land here. And there's more chances that it's gonna land in this bigger spot because this is covering 80% of the circle. So we're not actually throwing the darts, but they're giving you something that you have to accept. He will hit 80% of the time the first shot and 50% of the time the second shot. So you're kind of pretending that you're throwing the darts and this is his first shot and this is his second shot. So it makes it a little bit confusing. But, it'll make more sense as we practice, as we play it.

69 I want to go back to A——'s question, she said, D——, that before we were using fractions, but now we're using percents. Does anybody have anything to say about that? Why is it the same thing?

*Here, I went back to the problem the students had with percents, decimals, and fractions so that we can clarify the issue more.*

70 OK. So give me an example of what you mean. How are these percents like fractions?

*An open-ended question that could have many responses helps continue the discussion.*

71 OK.

72 Do you want to finish or do you want me to ask someone else?

73 Sorry I can't hear you.

74 OK.

75 Continue what he's saying, L——.

*Asks another student to internalize someone else's idea and make it her own.*

76 OK.

77 So you had your hand up after that, A——, what were you going to say?

78 So A——'s asking "can you turn a fraction into a percent?"

79 Can you turn a percent into a fraction?

*I put A-----'s question back on the class to answer.*

80 So. If this is 80%, what does that mean? 80?

81 Hundreths.

82 Which means?

83 K. And can you change that?

84 Into anything else?

85 What else can you change it to?

86 Yes?

87 What other fractions can you change it to?

88 What else?

89 All of that means 80%.

90 4 out or 5. If you think about that.

91 8 out of 10, 80 out of 100...So are these fractions?

92 They're just looking at the circle as being the whole thing and in this case they changed this into 20 out of a hundred of the circle and 80 out of 100 of the circle. For this one 5 tenths or one-half. One-half, one-half. Yes?

*The previous line of questioning challenges students to make meaning and come to conclusions for themselves.*

93 Yep. Then we would be breaking up the circle into really really tiny pieces. It could go on forever.

94 Infinity.

95 OK. Um..It's time to play the game. So if f you look at number 2, it says use spinners to model this player's dart throws. Spin the 80% spinner first and then the 50% spinner. So if you get, if you land in the 80% here you get a hit. Then you would spin on this one and if you get the hit here, you get the $10. Right?

96 But, if you land here at the 80% and then you miss here. What do you get?

97 K. What if you miss here and you win here?

98 And if you miss both.

99 Yep. I don't have enough money to pay you all $10.

100 This is a simulation. A pretend game.

101 OK. So thanks, D---- for speaking up, but that's OK. Record the prize outcome for each set of spins in the chart below using tally marks. Do this at least 30 times. The more data you collect, the more reliable your results will be. What do we call that when we have more data?

102 Yeah?

103 Beautiful. Say that one more time.

*A student makes a generalization that was learned before, the Large Number Principle.*

104 Remember the Large Number principle? The more you play something, the more chances all of the different things will come up and it'll become closer to what should actually happened, I mean in pretendland. Like the difference between ideal mechanical advantage and actual mechanical advantage, what really happens in a machine in real life and what the machine was made to do? Different things. So the more you play, the more it'll be like the theoretical probability. So I want you to get into partners. Wait. How many people do we have? We're gonna to have partner and then one group of three. And take turns spinning and it tells you how to do it. You just put the paper on the table, one person can hold the paper down to make sure it doesn't move. Use your pencil, get a paper clip, and kind-of just straighten one end out and spin using the paper clip. Very simple. Take turns. You don't each have to do it 30 times, yeah? You can go back and forth.

105 Don't mess with the paper clip. And. Cause with the spinner sometimes you can make it do something by hitting it with a certain amount of strength or starting it in one place and just pushing it a little bit. Don't do that so that your working with chance and not skill. So don't try to cheat is what I'm saying.

*I think I would have rather let them "cheat" while playing the game. It would be a good way to introduce the idea of spinners rather than me giving them the rules. It may have led to more heated discussions at the conclusion of the game.*

106 Can you make your own partners or should I choose for you?

107 OK. Some people aren't getting up. I guess that means you're working with someone sitting next to you.
108 S------, who's your partner?
109 Yep. You know where they are.
110 Yes.
111 So that it doesn't roll off?

# Appendix B
# Phase 2 Data

### Episode 1: The Triangle Inequality Rule (March 16, 2005)

1. Review of Measurement Concepts

One of the major ideas I wanted to review with the students is the size of units of measurements and even, further, how to choose an appropriate unit of measurement. I did this through discussion.

> *Show me with your hands about how big a centimeter is. How big an inch is. How big a foot is. About how big a meter or yard is.*
>
> *Tell me something in this room that you would measure with a meter stick. J, Do you agree? M, Do you agree. Someone else, tell me if you would agree? Why not? So why wouldn't that be good for you? Can anyone add on to what K said?*

I was hoping to get the students to talk about accuracy and realize that it would be good to measure a table in the class in feet or inches.

> *What would be the best unit to measure the table? What would be the best most accurate way of measuring?*

Students responded that feet would be most appropriate.

> *Tell me something else you would measure in the room and how you would measure it. Anyone agree or disagree and tell why?*
>
> *What else could we measure, but not in feet or yards, or meters? What could you measure and with what other units?*

A student responded that they could measure their name tags in inches. I discussed how, when I bought them, they were described in inches on the box.

> *Does anyone know why I'm thinking that it would be OK to measure the name tags in centimeters instead of inches?*
>
> *The smaller unit you have, the more accurate your measurement is. So if you have big objects, you would want to use bigger units, but if you have smaller objects, you would want smaller units. Measurement is never exact, never. You could always use smaller units. Objects could be microscopic, when you would need to have a microscope to see it. Then you would need really small units. But measurements is never exact, it can just be more or less accurate depending on the size of the units.*

2. Transition

> *You have noodles at your desks. Raise your hand if you would use yards to measure this?*
>
> Students laugh as I hold up a piece of linguini and respond no.
>
> *Good! How about feet? No? Good. Well you could use these, but you would have fractions of a yard, or feet.*

*I'd like you to take a piece of linguini and break it into three pieces.* I was modeling on the overhead projector. *Try to make a triangle. After you do that, I want you to measure it. But tell me, what would be the best unit to use to measure these three pieces?*

*What is the best unit, to use to get the most accurate measurement. OK let's try it with centimeters. Look at that. Where is it? It's between 7 and 8. So I would have to choose 7 or 8. So are centimeters the most accurate unit? What would be the best unit? Yes, millimeters.*

*How many millimeters are in a centimeter? 10 millimeters in each centimeters and how 75 millimeters is the same at 7 and 5 tenths centimeters. That's why you used decimals yesterday. You can do it either way. If you have 75 then you should have mm next to it or 7.5 with cm next to it. What would be easier for you? I'm going to take a vote. Do you want to use decimals or whole numbers?*

The students voted for using centimeters with tenths. I measured the three pieces of my linguini on the overhead and reminded the that measurement is never exact, but that the data should be consistent. Eight centimeters should be written as 8.0 instead of 8 and when talking about data to say it mathematically correctly such as "ten and 3 three-tenths centimeters."

*Which side is the smallest...the medium...and the long side? You are going to record that data and sketch it. I want to see the sketch labeled with the measurements including the unit label. Record data on the charts as you create your non-triangle and triangle.*

I distributed the rulers and first data collection sheet.

*Does everyone understand how to use the rulers they have? Yes. Good. Now you have a new ruler. Switch rulers.*

Students are confused.

*Do you understand how to read that ruler? If you don't like that ruler, you can get up and get another one.*

I went over the directions and wrote steps on the board with student assistance.

*What is your first step? Get a piece of linguini. Break into three pieces. Try to make a triangle. You may or may not have one. If you don't have a triangle then, it's a non-triangle. You need to use the appropriate side of the paper. One is for non-triangles and one is for triangles. Sketch it. Label the sides with the units. If you make a triangle, then next you will make a non-triangle. Measure the sides. Have a friend check it. Sketch it. Label it. Do the opposite. Continue to ask kids to repeat directions. Asked for questions. Are you sure? OK. Now you may start.*

3. Students Measure, Collect, and Record Data
4. Whole Class Data Collection and Discussion

M notices that some data does not make sense. We move it to the other chart.

*We can do a few things to make sense of this data. All of the linguini started at about the same length. Then, when we break it into three pieces, all of the pieces have to be about the same length. You're not going to have a small piece that's going to be a foot long. You're not going to have a long piece that is about a mile long. We can find the average lengths. Around how big is the small side of a non-triangle. Around how big is the side the long side of a triangle? Using the pieces of linguni. We're looking at measures of central tendency. Mean, media, and mode. These tell us around how big all of the pieces should.*

I asked students for their definitions of the three measures of central tendency. They gave their definitions and how they would find the mean, median, and mode. I went over how to use the calculators.

> *You're job is to choose one of the pieces of data. Could just do the small side of the small triangle, or medium side of the non-triangle, etc. Circle the top of the table to show which piece of data you do. I suggest that you do the mean in school because you have 5 minutes and a calculator in school. You can do the median and mode at home.*

While the students were calculating their means, I was doing it on the chart for all of the data. I had to remind them that we were rounding to the nearest tenth since we were using tenths of centimeters. I circulated around the room while students were working with their data. K had a mean that was way to high. J wasn't sure how to read his data. I asked him questions so that he could decide for himself. The video-tape ran out when we had about 4 more minutes of class.

The next day, I followed up with their data and used some from our chart to discuss outliers and how to determine what the most appropriate measures of central tendency are given certain data.

Episode 2: Parallel and Perpendicular Lines (March 17, 2005)

1. Follow-up on Episode 1

> *Did anyone have a hard time finding their mean, median or their mode? Since nobody had a problem finding their mean, median, and mode, please take that paper, make sure your name is on it and make sure I can tell what your mean, median, and mode are. Everyone should have them labeled in centimeters. Some people will not have a mode because a number may not have occurred more than once. If that is true, put "mode" and then "none" so that I know.*
>
> *Did anyone have a question before they turn that in? Other questions? If you finished it, you can give it to one person at your table and that person bring it to me.*
>
> *Yesterday I mentioned that sometimes you have a piece of data that doesn't fit. What did I call that?*

K answered outlier. I spelled it on the board and explained that an outlier can mess up the data.

> *Let's think about the small side of a non-triangle. What could it be around?*

Students answered two-tenths, three and two-tenths, and four and two-tenths.....

> *I saw a lot [of measurements] like this....and some more like this....*

I was writing on the board.

> *Right now you should have this paper, and using this data (on the board), I want you to find the mean, median, and mode. You don't need a calculator. These numbers are easy. Yep, all of these, on the board. So here we go. I'd like to do to start is by putting them in order. That helps me. What's the smallest one?*

Students and I discuss list the numbers: 0.2, 0.4, 0.8, 1.2, 1.5, 1.8. 2, 2 again, 2.5, 3.2, 3.3, 4.2

> *and I'm going to put an outlier in here. I'm going to say somebody had 25.1. Yep, something's off. Let's pretend somebody got that. OK?*
>
> *So you add it all up right? Right? To find the mean?*

I proceed to add the data on the board using compatible numbers and mental math, while most of the students are doing it as well on their own.

*That's what I get. So somebody else, I hope that you get that. See if you got the same thing that I got. Yes, you should be adding it up too. You got 48? Hmm. OK.*

And I double-check my work on the board as I talk through my mental math.

> *You got 48? I got that too. OK. Looks like 48.2 might be it. Several people got that. And if you were a scientist, you'd actually use a calculator. We're going to use 48.2. With me? What do I divide by. 13 goes into 48....3...yeah...13 goes into 92? 7? K.*

We discuss what we do with the remainder...

> *OK. Take a look at our data. Most of our numbers are less than 3. Our mean is 3.7 centimeters. What's our mode?* 2 centimeters. *What's our median?*

Students answer 2.0

> *Notice...What's up over there? Stop already. There's a big difference here. When you're talking about almost 2 centimeters difference, that's a lot when you're talking about a small piece of a noodle. So in this case, what is my best measure of central tendency? What really tells us about how big the smallest pieces are. These do. Remember yesterday, we talked about what outliers do? This 25 centimeters really threw us off. That's a big difference when you're looking at a little noodle. That's what important...sometimes certain measures of central tendency are better than others. Got it? Always look at the mean, median, and mode and then look back at your original data and decide "what is giving me the best information about this data?"*

We continued to discuss that the mean, median, and mode worked well yesterday. But today, the best would be the mode and median today because of the outliers. I collected their papers.

2. Lead into the Lesson

Students were asked to get a piece of paper. I asked them to observe what I was doing on the board and write down what they observed.

> *Please write your name and date in the top right. And at the top on the first line, I would like you to write Parallel and Perpendicular Lines. I want you to take observations about what I am doing. You are not copying what I'm doing, you're describing what I'm doing. You're going to describe what I'm doing on the board. No talking, just describing.*

3. Lesson

I use a compass and protractor to make parallel lines and stop in between the steps that I take so that they students can describe what I did.

> *Somebody tell me what I did first. Then what did I do next? Yes, they intersect there. Those are intersecting lines at that point.*

Students continue to describe, in writing, what they observe me doing.

> *OK. Somebody tell me what I did.*

M: You made a straight line. And drew one intersecting line....

> *Yes. I measured my angle and found out that it was 75 degrees. And in order to make a parallel line is by making the same angle measurement at the intersection of the lines. But, there's another way to tell that they are parallel.*

A student volunteers and touches on the equidistance between the two lines. I continue to discuss the idea of equidistance between each line at every point.

> *They will never touch each other. The opposite of parallel lines are skew lines. Somewhere, skew lines will intersect. These two do not intersect, but if they kept going on, they will eventually touch. They are not intersecting yet. Some lines are parallel, some are intersecting, and some are skew. We have a special kind of intersecting line which is called perpendicular. Now turn your paper over and observe what I do now.*

I make perpendicular lines on the board, taking small breaks, so that they can write their observations. We discuss that one of the angles is 90 degrees, then all of the angles are 90 degrees and how they are complementary and supplementary.

> *These are intersecting lines that are special because they intersect at 90 degrees and they are also called right angles. Any questions? With me? OK. Now....*

(the tape ended) I told them that they have to make parallel and perpendicular lines following the steps they observed. I went around the room helping students as needed. A few minutes later, I changed the video-tape.

> *I'm giving you two more minutes and that's all the time we have.*

Some students were struggling and I hade to continue to help them. While I was helping some students, others were playing around. How could I get them to have good discussions during this time? I realized they didn't know what they were doing and I was running out of time.

> *OK. Our time is up. We will try to make parallel and perpendicular lines tomorrow so save your paper. Give one person each of your supplies. To hold.*

> *I'm running out of time. I'm just going to collect the unit test. Make sure your names are on them.*

I went around collecting supplies. The next day was the last day in the quarter and we were unable to go back to revisit parallel and perpendicular lines until two weeks later. I assigned homework because we were running out of time.

4. The Assignment

> *Take out your math book. Turn to page 326-327. Your homework is page 327, numbers 6-13 and page 328, numbers 1-21. So right now, look over the problems and let me know if you have any questions.*

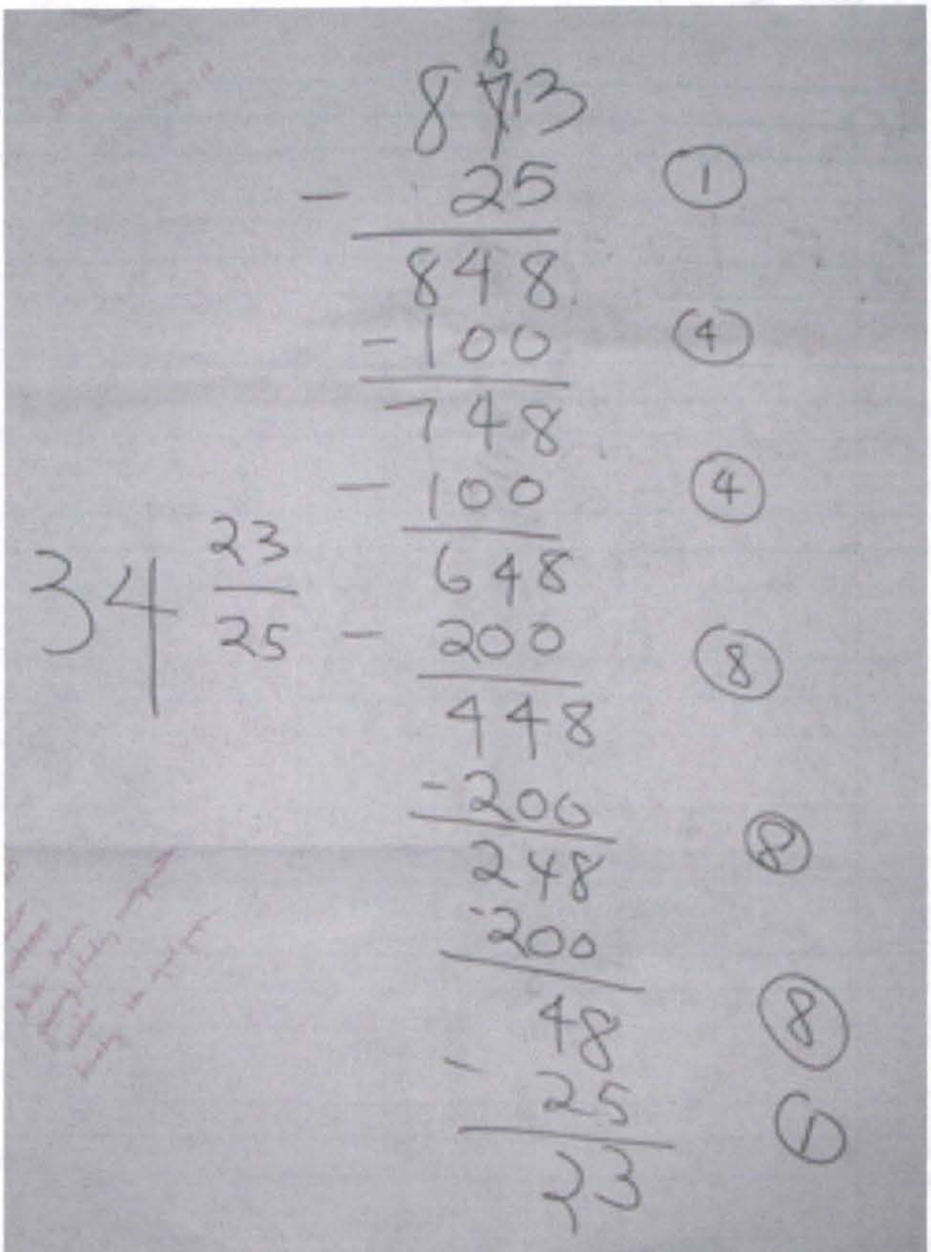
> *Classify means name. Like we were classifying triangles as isoscleles, scalene, equilnteral...So if you're classifying lines, you would call them perpendicular, parallel, intersecting...Any other questions?*

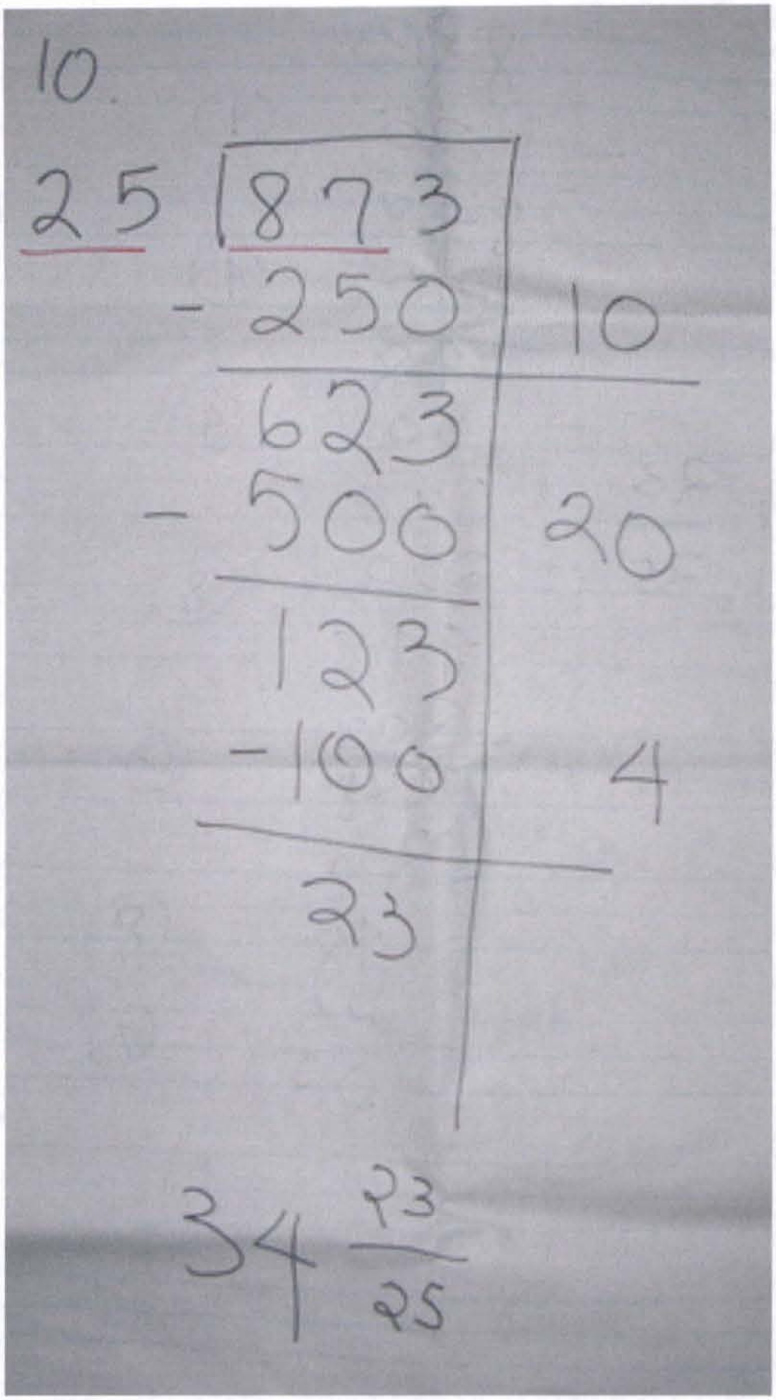
> *OK. Everybody look at numbers 10-12.*

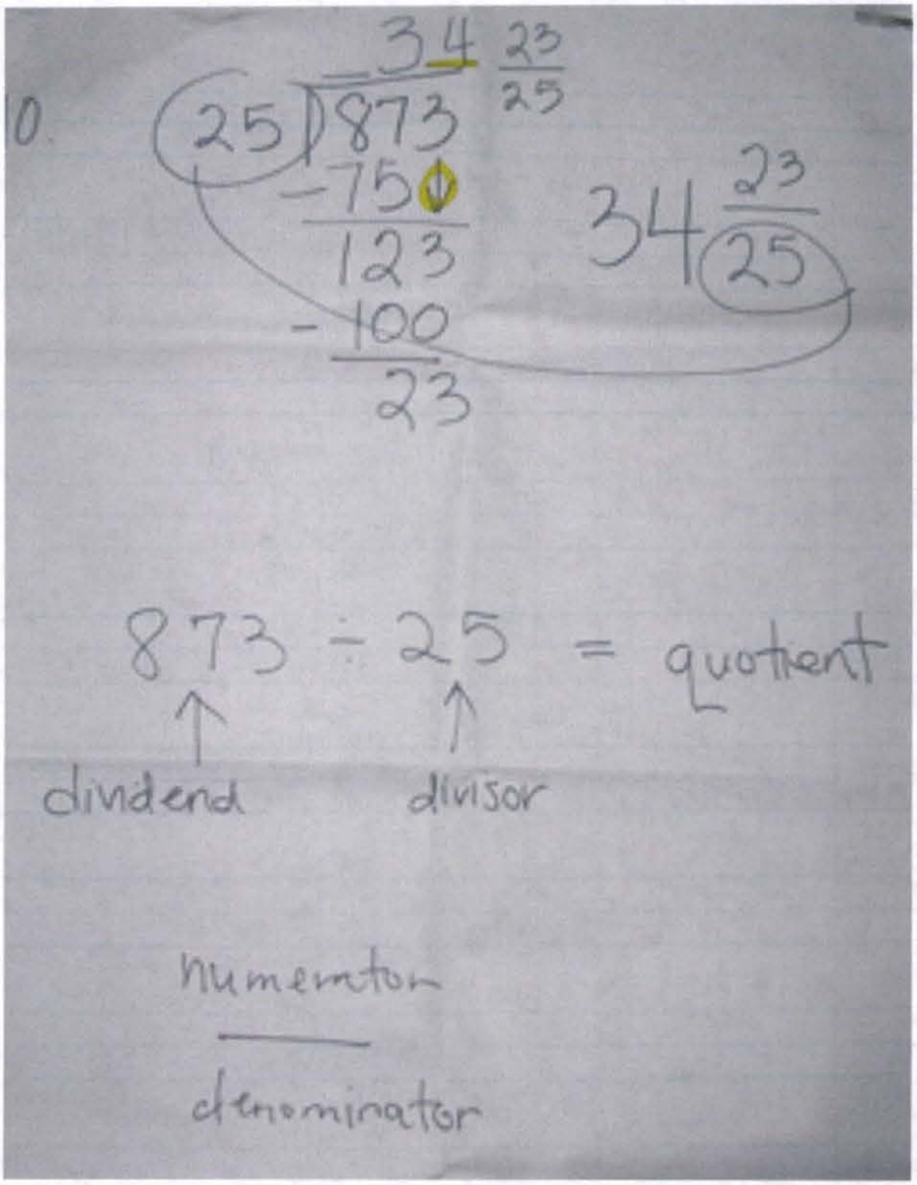
I read aloud the directions and restate them in another way. I do an example and ask for volunteers to check for understanding.

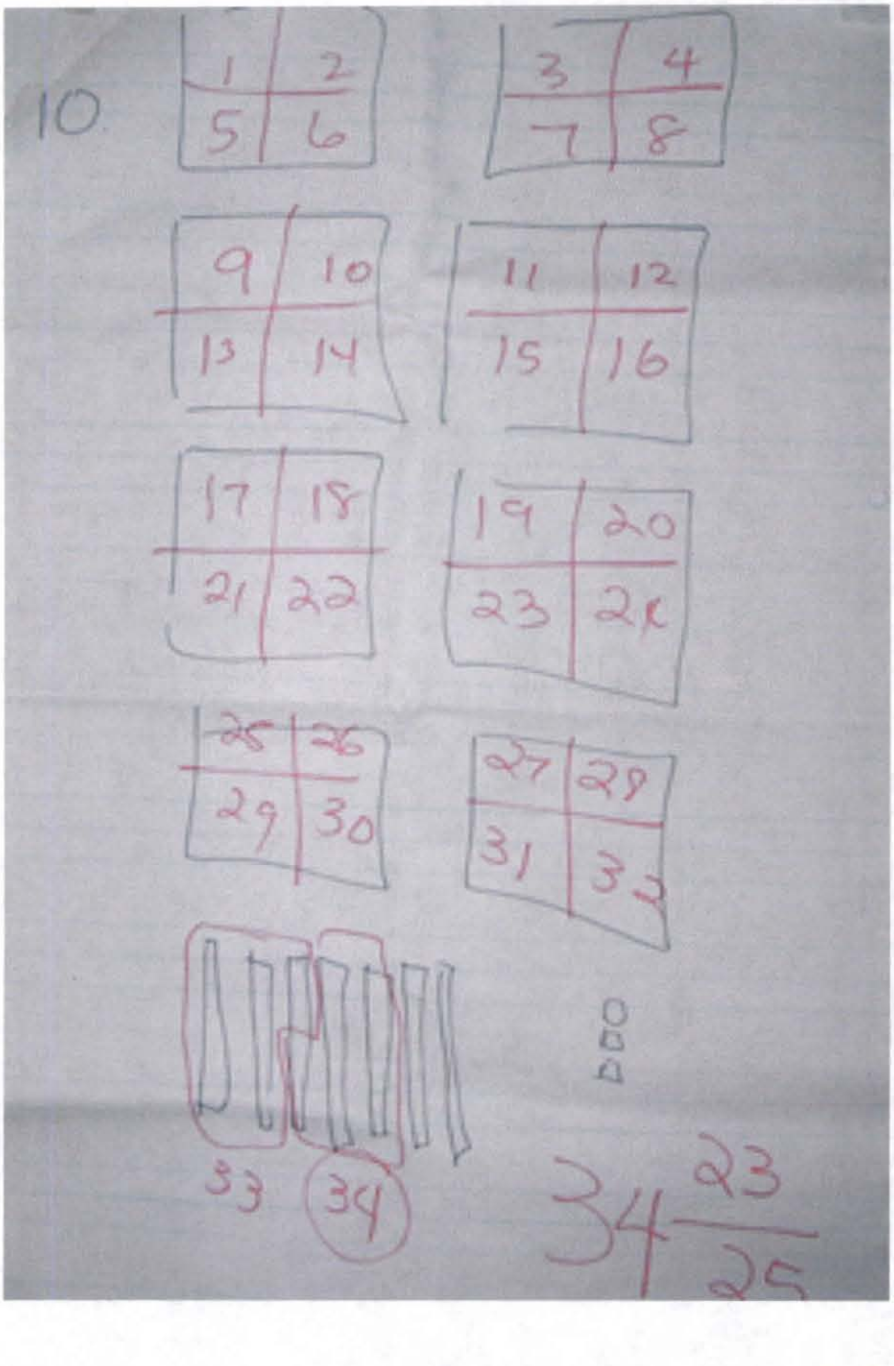
*Any questions from the next page? Remember you do not need a protractor because you have enough information since you know what complementary, supplementary, and vertical angles are. This is your last math assignment of the quarter. You have to turn it in tomorrow. If you don't have time to do it at home, you should do it in school. Does everyone understand that?*

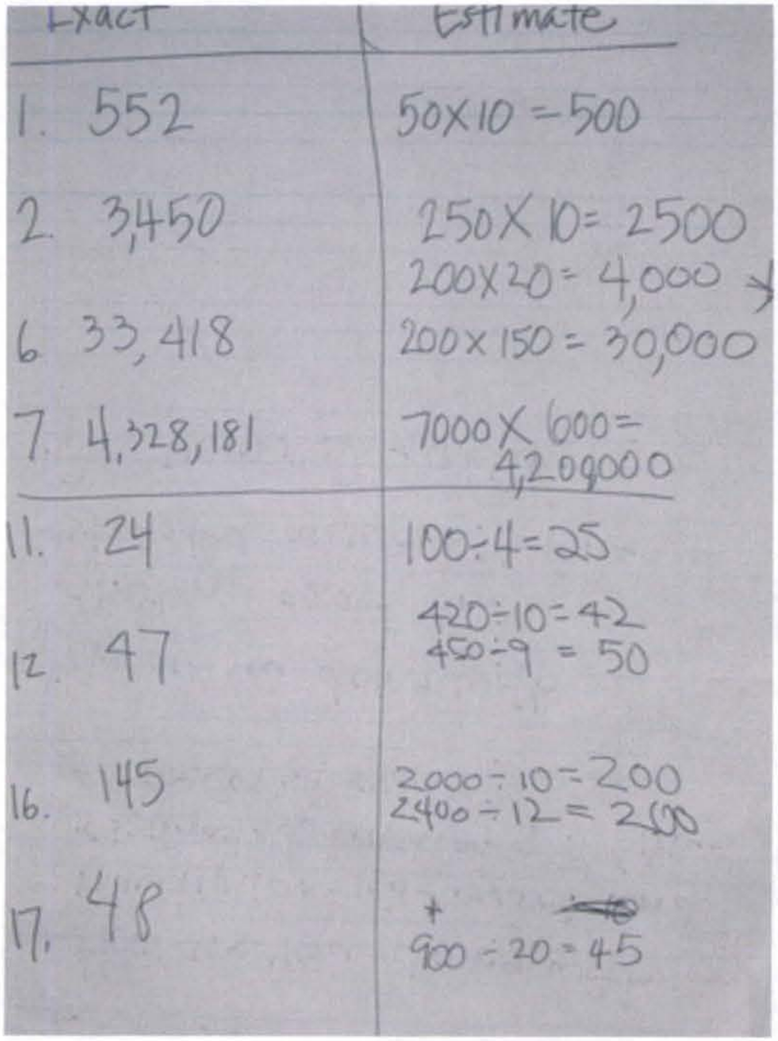
Appendix C
1st Quarter Curricular Aartifacts



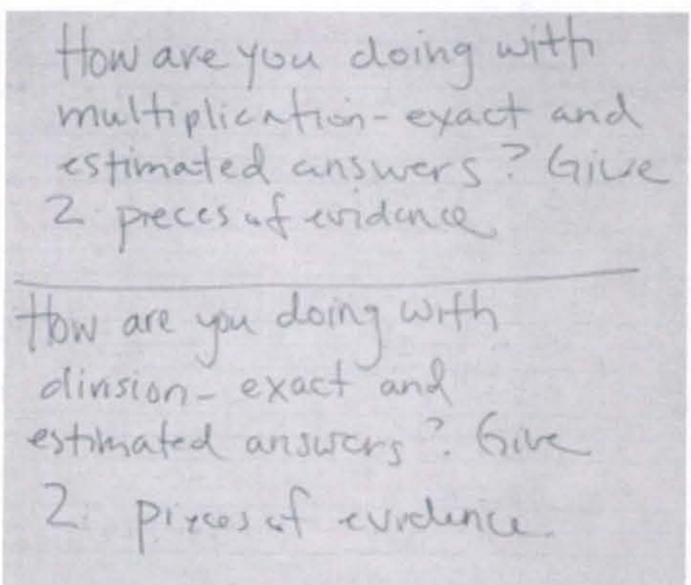

10.
25 | 873
- 250 | 10
623
- 500 | 20
123
- 100 | 4
23
34 23/25


34 23/25
10.
25)873
-750
123
-100
23
34 23/25
873 ÷ 25 = quotient
dividend
divisor
numerator
denominator


10.
1 2 5 6
3 4 7 8
9 10 13 14
11 12 15 16
17 18 21 22
19 20 23 24
25 26 29 30
27 28 31 32
33 34
34 23/25



| Exact | Estimate |
|---|---|
| 1. 552 | 50×10 = 500 |
| 2. 3,450 | 250×10= 2500<br>200×20= 4,000 |
| 6 33,418 | 200×150 = 30,000 |
| 7. 4,328,181 | 7000×600=<br>4,200000 |
| 11. 24 | 100÷4=25 |
| 12. 47 | 420÷10=42<br>450÷9 = 50 |
| 16. 145 | 2000÷10=200<br>2400÷12=200 |
| 17. 48 | 900÷20=45 |



How are you doing with multiplication - exact and estimated answers? Give 2 preces of evidance

How are you doing with division - exact and estimated answers?. Give 2 pieces of evidence

FIND A PLACE

Use a set of 50 cards numbered 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, (five of each). Shuffle the cards and deal 19 to each of two players. Players alternate turning over a card from their stack and recording the number in one of the squares on their side of this record sheet. Target scores are identified in center column. After all squares are filled, each player records the difference between his score and the target score in the lines on the far right or far left. The scores are totaled to provide a final score at the bottom of the page—low score wins.

| Score | PLAYER A | | | Target | PLAYER B | | | Score |
|---|---|---|---|---|---|---|---|---|
| 2 | | | 2 | Closest to 0 | 3 | | | 3 |
| | | | 9 | Closest to 10 | | 5 | | |
| 3 | | 5 | 3 | Closest to 50 | 6 | 5 | | 15 |
| 11 | | 8 | 9 | Closest to 100 | | 2 | | |
| | 1 | | | Closest to 100 | | | 3 | |
| | | | | Closest to 500 | 4 | | 2 | |
| | 8 | | | Closest to 800 | | | | |
| | | | | Closest to 1000 | | | | |

Total score for A

Total score for B

# Appendix D
## Student Survey and Questionnaire

Student Number 1 Date 7-25-05

| | always | usually | often | sometimes | rarely | never | |
|---|---|---|---|---|---|---|---|
| 1. I like math. | | | | X | | | +2 |
| 2. I listen to my teacher during math lessons. | X | | | | | | -2 |
| 3. I listen to other students talk about math. | | X | | | | | 0 |
| 4. I talk about how I solve math problems. | | | | | X | | +1 |
| 5. I ask my teacher questions when I do math. | | | | X | | | 0 |
| 6. I ask my classmates questions when I do math. | | | | X | | | +1 |
| 7. I am good at math. | | | | X | | | +1 |
| 8. My teacher thinks I am good at math. | | | | X | | | +1 |
| 9. My classmates think I am good at math. | | | | X | | | 0 |
| 10. I work with other people when I am doing math. | | | | | X | | 0 |
| 11. I work by myself when I do math in school. | | | X | | | | +1 |
| 12. I work by myself when I do math at home. | | X | | | | | +1 |
| 13. I ask for help from my family when I do math. | | | | X | | | -1 |
| 14. My family helps me when I ask for help with my math. | | X | | | | | +1 |
| 15. My family thinks I am good at math. | | | X | | | | 0 |
| 16. It is important to talk about math. | | X | | | | | -1 |
| 17. It is important to write about math. | | | X | | | | +1 |
| 18. I have good ideas to share about math. | | | | X | | | 0 |
| 19. When I share my ideas about math, people listen. | | X | | | | | 0 |
| 20. Math is important in my life right now. | | X | | | | | +1 |
| 21. Math is important in the lives of adults. | X | | | | | | 0 |
| | 5 | 4 | 3 | 2 | 1 | 0 | |

Student Math Questionnaire

Student Number 1

What is math?

Math is a subject in school that teachers you how to add, subtract, divid and multiply.

If I were to walk into your classroom while your teacher was teaching math, what would I see your teacher doing?

teaching, reading kik and also and she would explain and help kids. Also having fun.

If I were to walk into your classroom while your teacher was teaching math, what would I see you doing?

taking down notes and listing also paying atetion. Also most of of all learning and last but not least having fun.

If I were to watch you do your math homework, what would I see you doing?

me writing on scrach paper and thinking also solving problems.

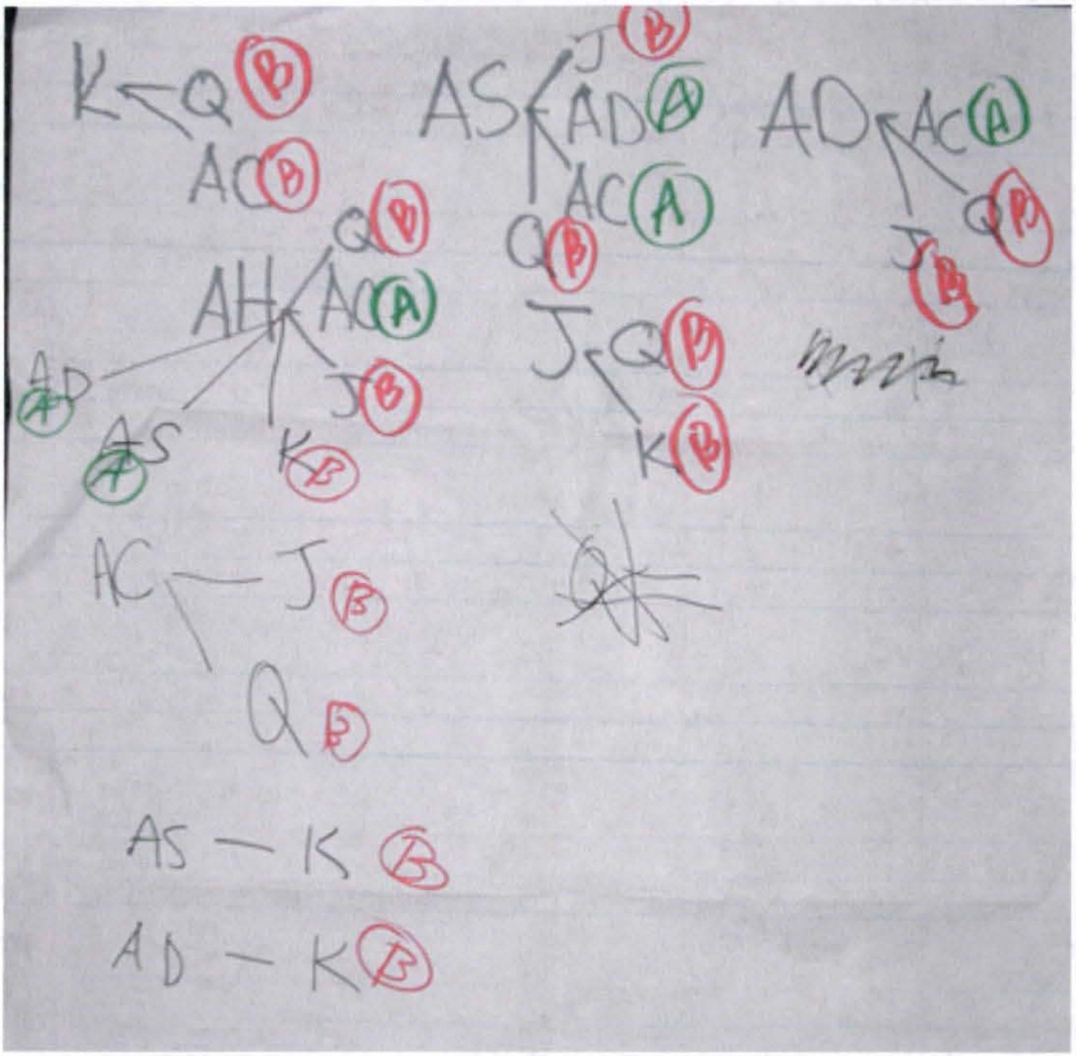
Describe times in everyday life (not in school) where math is used? How is math used? Try to give some examples of when, where, and how math is used.

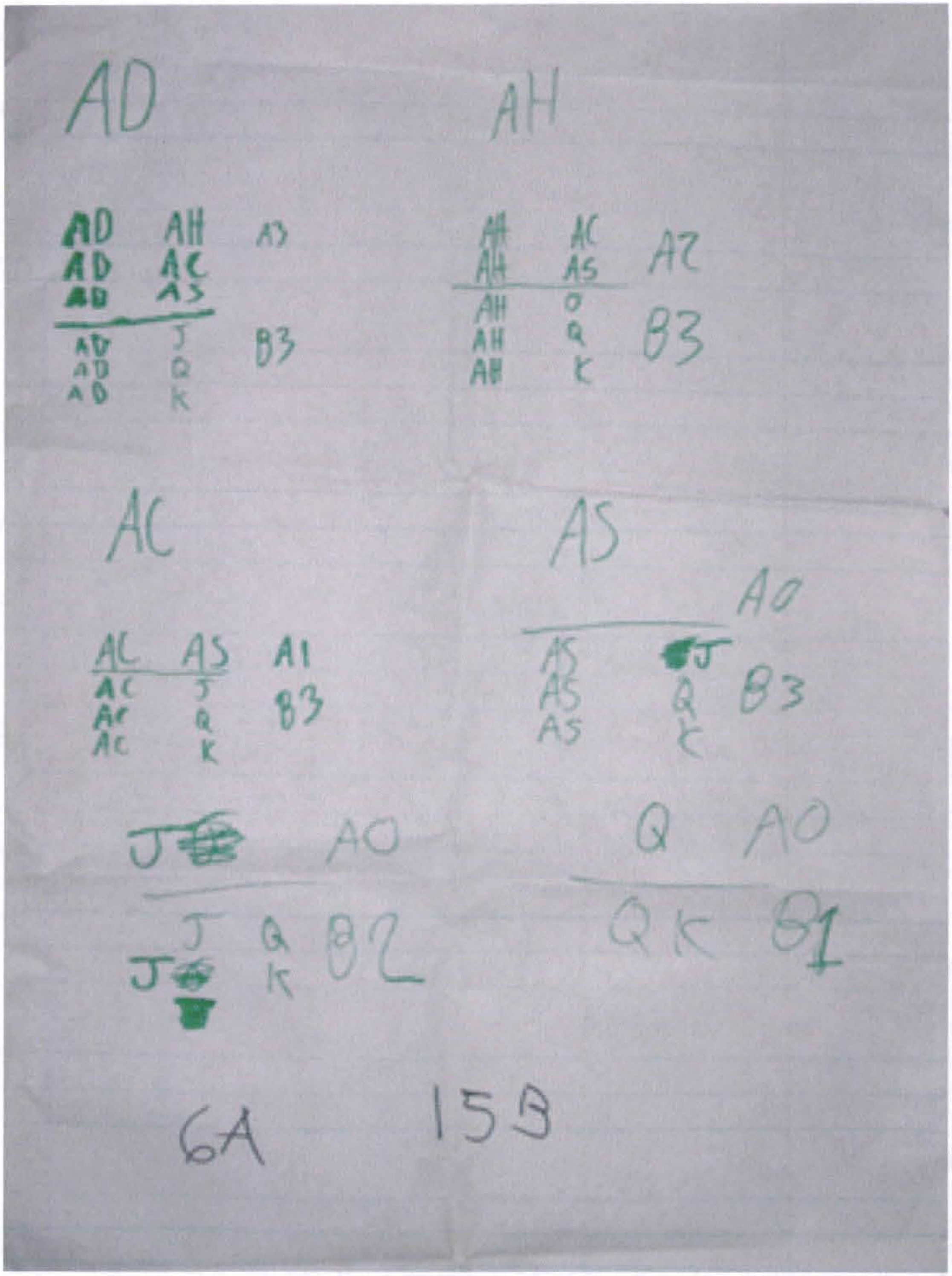
Math is use every were its used to add up prices and also we have to figure how much fence there is.

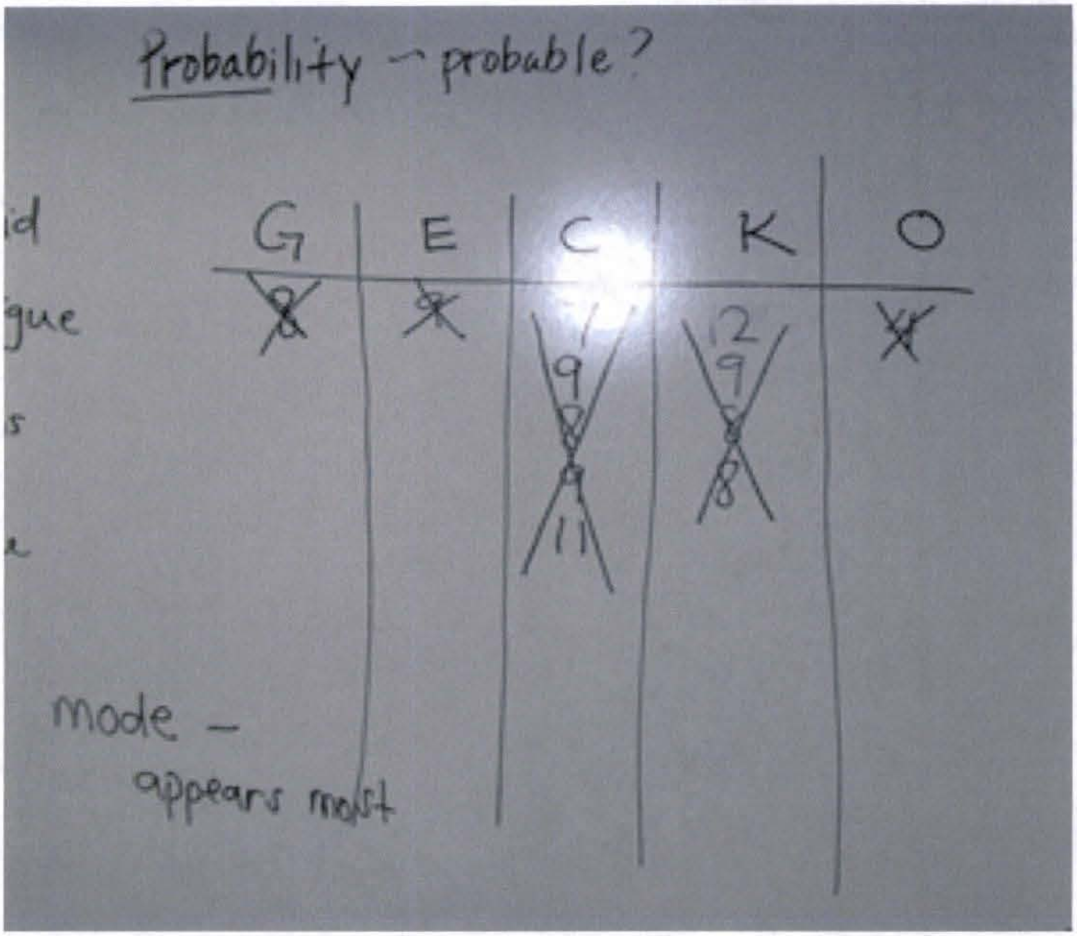
Is there anything else you'd like to say about math?

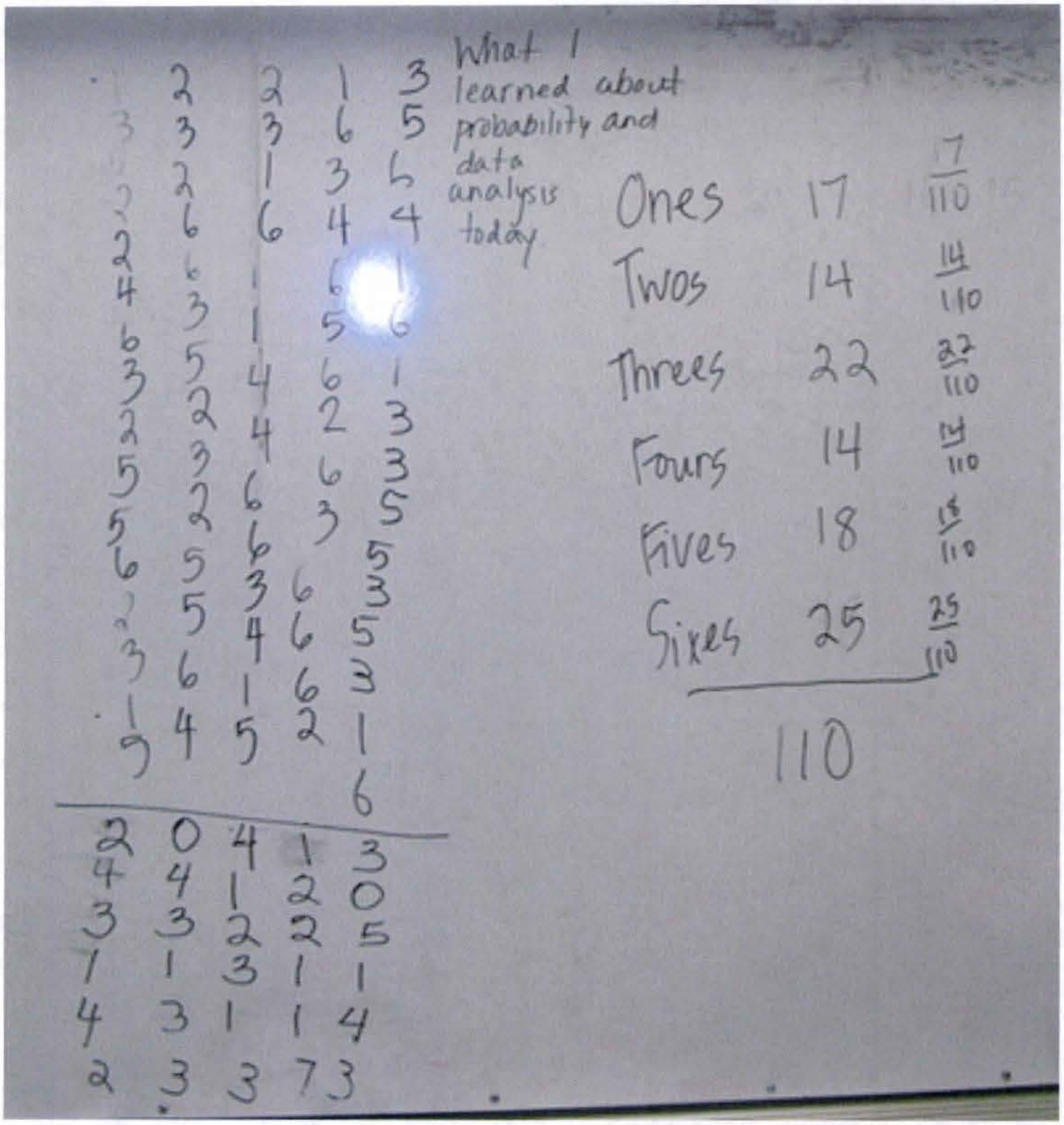
no.

Appendix E
2nd Quarter Curricular Artifacts



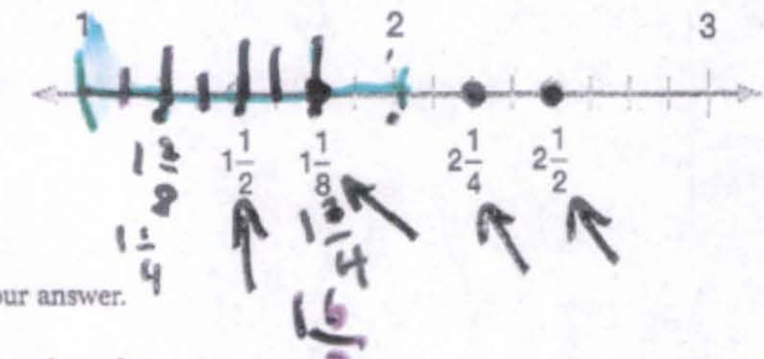

AD
AH
AD AH A3
AD AC
AD AS
AD J B3
AD Q
AD K
AH AC
AH AS A2
AH Q B3
AH Q
AH K
AC
AS
AC AS A1
AC J
AC Q B3
AC K
A0
AS J
AS Q B3
AS K
J AO
J Q B2
J K
Q A0
QK B1
6A 15B


Probability - probable?
G
E
C
K
O
mode -
appears most


What I learned about probability and data analysis today
Ones 17 17/110
Twos 14 14/110
Threes 22 22/110
Fours 14 14/110
Fives 18 18/110
Sixes 25 25/110
110

## 5

Which point is incorrectly labeled on the number line shown below?




Justify your answer.

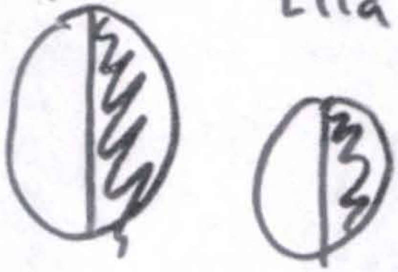
$1\frac{1}{8}$ should be $1\frac{3}{4}$ because if you go from the middle out you will see that $1\frac{1}{8}$ is really $1\frac{3}{4}$.

6 $^{11}/_{2}$

Think carefully about the following question. Write a complete answer. You may use drawings, words, and numbers to explain your answer. Be sure to show all your work.

José ate $^{1}/_{2}$ of a pizza.
Ella ate $^{1}/_{2}$ of another pizza.
José said that he ate more pizza than Ella, but Ella said they both ate the same amount. Use words and pictures to show that Jose could be right.

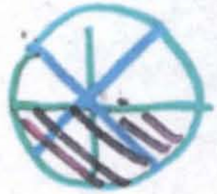
José Ella



José is right if his pizza is bigger. 7

José

$$\frac{8}{6} - \frac{1}{2} = \frac{4}{8}$$



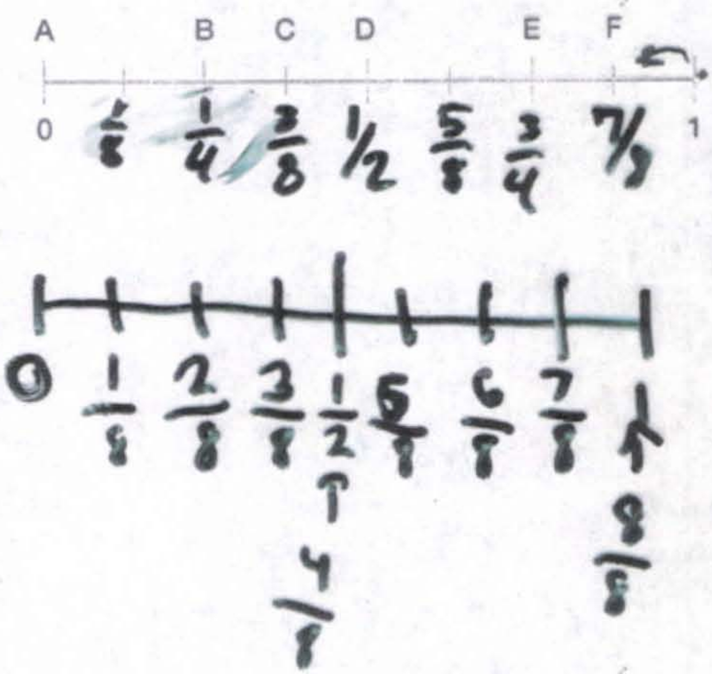
Ella

$$\frac{4}{4} - \frac{1}{2} = \frac{2}{4}$$

7

11/3

Jennifer is making a number line for fractions between 0 and 1. At which point should she put $^1/_2$? At which point should she put $^7/_8$?




Jennifer should put 1/2 at point D. She should put 7/8 at point F.

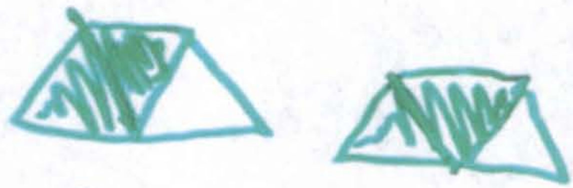
If  is $^1/_2$ of a unit, then draw 1 unit.


1 unit 1 unit

2 Δ = 1 square or rhombus
equilateral

11/3

If  is 1 unit, then  is $\frac{2}{3}$.



$\frac{2}{3}$ A rhombus is $\frac{2}{3}$ of a trapezoid.

10

Students in Mrs. Johnson's class were asked to tell why $^4/_5$ is greater than $^2/_3$. Whose reason is best? Explain your answer.

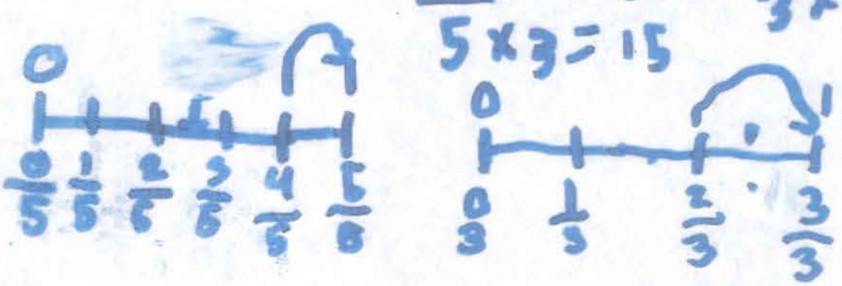
a. Kelly said, "Because 4 is greater than 2."
b. Keri said, "Because 5 is larger than 3."
c. Kim said, "Because $^4/_5$ is closer to 1 than $^2/_3$."
d. Kevin said, "Because 4 + 5 is more than 2 + 3."

I put c because 4/5 is really closer to 1 than 2/3. ~~because~~ ~~[illegible]~~

3: 3 6 9 12 15

5: 5 10 15

$\frac{4 \times 3 = 12}{5 \times 3 = 15}$ $\frac{2 \times 5}{3 \times 5} = \frac{10}{15}$




E6

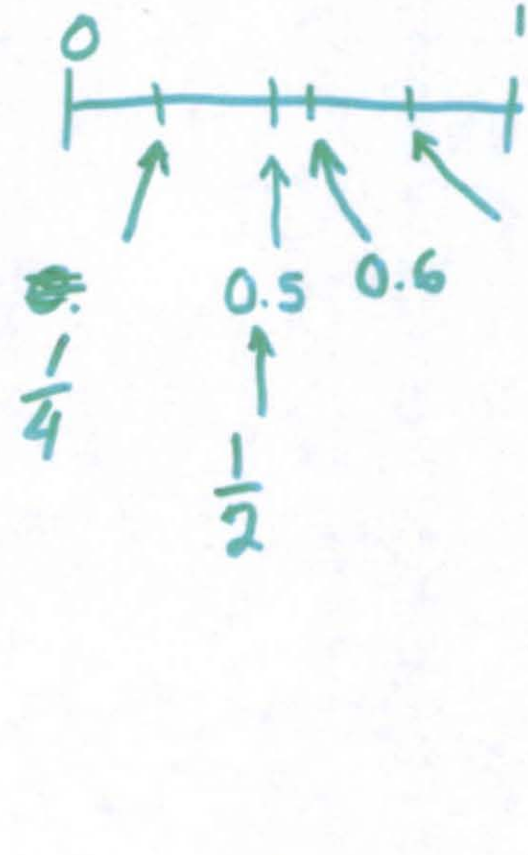
On a number line, 0.6 is closest to which of the following:

$^1/_4$, $^1/_2$, or 1?

Justify your answer.

11

0.6 is ~~the~~ closest to $\frac{1}{2}$ because if you put $\frac{1}{2}$ in a decimale you get 0.5 and 0.6 is close to 0.5.



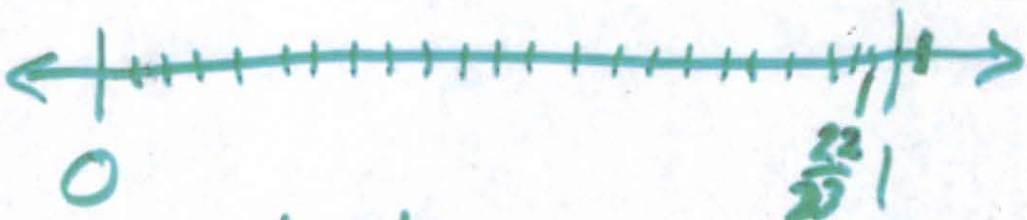
## 13

Answer the following question about this fraction:

$$\frac{23}{\boxed{?}}.$$

If this fraction is just a little more than 1, what would $\boxed{?}$ be?

1 would be $\frac{23}{23}$




$\frac{23}{23}$

best

$\boxed{\frac{23}{22}}$ $\frac{23}{21}$

$\cancel{\frac{23}{24}}$ $\frac{23}{20}$

Which circle has approximately the same fraction shaded as that of the rectangle?


$\boxed{\frac{10 \div 2}{18 \div 2}} = \frac{5}{9}$ is a little more than $\frac{1}{2}$.

D is a little more than $\frac{1}{2}$

THIS one

## 16

Which shaded region(s) show fractions equivalent to $^2/_3$?

shaded

Figure 1 Figure 2

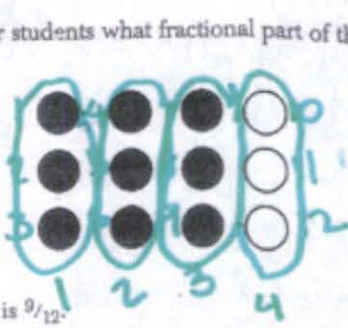
$\boxed{\frac{2}{3}}$ $\boxed{\frac{4}{6}}$

shaded

a. Figure 1 only
b. Figure 2 only
c. Both figure 1 and figure 2
d. Neither figure 1 nor figure 2

$\boxed{\frac{4}{6}} = \frac{8 \div 4}{12 \div 4} = \boxed{\frac{2}{3}}$

Mrs. Washington asked her students what fractional part of these 12 circles is shaded.




3 Group shaded
4 groups all together

Odessa thinks the answer is $^9/_{12}$.

Bob thinks the answer is $^3/_4$.

a. Who is correct—Odessa, Bob, or both or neither?
b. Write an explanation to Odessa and Bob about how you figured out your answer. Draw your own pictures to go with your explanation.

$\frac{9}{12}$ 9: 1, 3, 9 $\frac{9 \div 3}{12 \div 3} = \frac{3}{4}$

12: 1, 2, 3, 4, 6, 12

I think it's not either of theyre answers I got 9/[illegible]

José created a game using two number cubes of different colors. The green cube had ODD multiples of 3, and the red cube had EVEN multiples of 3.

a. What was the color of the cube that had the number 6?
b. List SIX numbers that could be on the OTHER cube.
c. Could José design the same game using multiples of 4? Explain your answer.

a. The color of the cube that had the number six was red. 6 is a multiple of 3 and it's even.

b. 3, 9, 15, 21, 27, 33, 39, 45

c. No. José can't design the same game using multiples of 4 because you can only get even numbers.

4, 8, 12, 16, 20, 24, 28, ...

22

Patrick had only quarters, dimes, and nickels to buy snacks. He spent all his money and received no change. Could he have spent 99¢? Justify your answer.

He couldn't have spent 99¢ because 25, 10, and 5¢ all end with a 5 or a 0. So all multiples of 5 and 0 end up as 5 and 0.

95¢ is the lowest you could get to $1 and 100¢ is the highest. We would need pennies.

$$\begin{array}{r} 5 \\ +5 \\ \hline 10 \\ +5 \\ \hline 15 \end{array} \qquad \begin{array}{r} 25 \\ +25 \\ \hline 50 \\ +25 \\ \hline 75 \end{array} \qquad \begin{array}{r} 10 \\ +10 \\ \hline 20 \\ +10 \\ \hline 30 \end{array}$$

Sharon and Jessica have a bag of 156 candies. They want to find the greatest number of candies to put into 12 bags so that each bag will have the same amount.

Sharon says they should see how many times they can subtract 12 from 156.
Jessica says they should divide 156 by 12.
Who is correct? Sharon, Jessica, both, or neither?

How do you know?

Both are correct because you can subtract 12 13 times from 156 and you can divide 156 by 12. 21

1) 156 − 12 = 144
2) 144 − 12 = 132
3) 132 − 12 = 120
4) 120 − 12 = 108
5) 108 − 12 = 96
6) 96 − 12 = 84
7) 84 − 12 = 72
8) 72 − 12 = 60
9) 60 − 12 = 48
10) 48 − 12 = 36
11) 36 − 12 = 24
12) 24 − 12 = 12
13) 12 − 12 = 00

$$\begin{array}{r} 13 \\ 12\,\overline{)\,156} \\ -12 \\ \hline 36 \\ -36 \\ \hline \end{array}$$

E13
B14

## 26

Which word problem below could be represented by the number sentence $6 \times 3 = 18$?

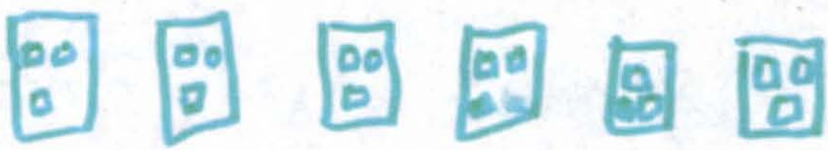
a. Steve had 6 baseball cards. He bought 3 more cards. How many cards does he now have? $6+3=9$

b. Steve bought 6 baseball card packages with 3 cards in each package. How many cards did he buy? $6 \times 3 = 18$

c. Steve had 6 baseball cards. He gave away 3 of them. How many cards did he have left? $6-3=3$

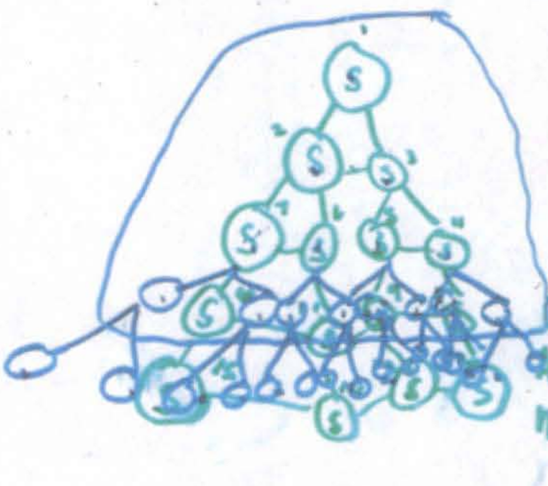
d. Steve had 6 baseball cards. He put the cards in 3 equal stacks. How many baseball cards were in each stack? $6 \div 3 = 2$

$6 \times 3 = 18$



Mrs. Forest wanted to plan how to contact her students by phone in case the field trip they were going on the next day needed to be canceled. She decided to call 1 student, who would then call 2 other students. Each of these students would then call 2 other students. This telephone chain would continue until all students had been called. Mrs. Forest has 31 students. How many students will need to make phone calls if Mrs. Forest calls the first student?

How do you know? Show your work.



15
−31
16

So 15 people called. And 16 people did not call.

~~2, 4, 8~~ 1+2+4+8+16

~~2+2~~ 2, 4, 6, 8, 10, 12, 14, 16

18, 20, 22, 24, 26, 28, 30

15 multiples of 2.

$\boxed{1}$ · $\boxed{2}$ · $\boxed{4}$ · $\boxed{8}$

+2 + 2+2 + 2+2+2+2 + 2+2+2+2+2+2+2+2

EG

## Appendix F
## Complete Responses to Questionnaire

Student Math Questionnaire
Responses

---- no survey administered

Question 1: What is math?

| Student Number | July 29, 2005 | December 16, 2005 |
|---|---|---|
| 1 | | Math is a subject in school that teachers you how to add, subtract, divid and multiply. |
| 2 | | ---- |
| 3 | | I think math is a learning subject that is taught in school. |
| 4 | Math is calculating numbers and problem solving. | ---- |
| 5 | Math is a matter of academic learning and an everyday need. | Math is the skill used to organize and configure numbers. |
| 6 | Numbers Digits Varibals | Math is something in everyday life |
| 7 | | Math is adding subacrting multpling and divdeding things |
| 8 | Math is work you see people doing every day around you | Math is a problem |
| 9 | | Math is something that we need to and learn. For example: like adding, subtracting, multiply and divide |
| 10 | Math is when you deal with weighing, buying things, building things, and other stuff. | ---- |
| 11 | Math is something educational that uses numbers, math symbols, and helps you a lot during life. | ---- |
| 12 | Math is a subject that will show up in every day life. Its numbers that are put together. It is also another way of communication. | Math to me is a way to express a different part of you. It is a way for me to understand different parts of other people too. |
| 13 | | Math is something that is involved in your whole life. |
| 14 | a bunch of numbers tha mean the same thing | |
| 15 | I think math is supposed to be something to help us when we get older. | Math is where you learn how to add, subtract, multiply, and divide. |
| 16 | Math is basicly addition, multiplication, division, and subtraction | ---- |
| 17 | When you yous these + - X [division symbol]. | ---- |
| 18 | I think math is a bunch of numbers and equations, and alot of problem solving skills. | Math is like a language with numbers. Math uses strategies to solve problems. |
| 19 | Math is fun sometimes but not most of the time. | I think math is lots of numbers that are added subtracted, multiplied, and divided. |
| 20 | Math you yous in every day | Math is wath I do at school |
| 21 | Math is boreing | Math is when you use your brain to slove problems |
| 22 | | math to me is divission, multiplying, adding, and subtracting. math is needed to get jobs. |
| 23 | Math is addition, subtraction, multiplication, division, geometry, and fractions. | Math is something that you do in eveyday life like in question 5. [Math is used in the kitchen, at work, and shopping.] |
| 24 | Math is numbers and sequances | Math is something we use every day to figure things out |
| 25 | I think math is a way to make counting faster. | Math is a easier way to count and it makes life easier. |
| 26 | | math is using numbers and solving question with numbers. |
| 27 | Math is calculating numbers, measurement, and other neat stuff. Stuff meaning multiplying, subtracting, and using fractions. | I think that math is learning and preparing for other things in life. |

Question 2: If I were to walk into your _____ classroom while your teacher was teaching math, what would I see you teacher doing?

| | | |
|---|---|---|
| 1 | My teacher would be…1. writing on the white bord 2. looking in her book so she can ask us questions 3. telling us the definition of the word. | Teaching, reading kik and also and she would explain and help kids. Also having fun. |
| 2 | When you talk into our 5th grade class I would be writing, taking notes, or working on all kinds of math problems. | ---- |
| 3 | writtin on the white board expaining how to solve this problem | Shy would be yelling a Tyler, Michael I, and I when we walk around to much or shy will be writing on the board. |
| 4 | My teacher would be explaining to me how to do everything and would be asking me problems. | ---- |
| 5 | You'de see here giving us many, many examples | My teacher would be deeply explaining the math subject |

| | | |
|---|---|---|
| 6 | Talking to the students to see if they understand math problemes. | You would see her explaining what we were suppose to do. |
| 7 | you would see her wrieting on the boared t see her talk t ouse about the problem | You would see her/him reading there math book to the class and wirting problems on the bord or helping people with work |
| 8 | 1. Writing on the board 2. Giving us problems. Passing pout worksheets. 4. Reading | Writing a problem on the board or talking about the problem. |
| 9 | My teacher would be teaching us what were gonna do for Homework on that day. | Is doing math on the borad that we need to learn. |
| 10 | Writing on the board. | — |
| 11 | If you were to walk into my 5th grade classroom while my teacher was teaching math, you would see my teacher on a chair facing the white board. On the board would be all different kinds of numbers and words and stuff like that. He would be explaining math to us. | ---- |
| 12 | You would see my teacher answering question and helping us learn new things. | You would see that me teacher would be on the overhead or the board. You would see that Ms. Aiona would be working hard with us. |
| 13 | You would see her talking to us and writing on the board. | You would see her teaching math |
| 14 | you would see the teacher teaching the class how to do math | teaching us math that we dont know |
| 15 | You would see her maybe yelling –writing down problems – answering questions – calling us up to do a problem | You would see her writing on the whiteboard about how to do it. |
| 16 | you would see my teacher teaching us how to figure out the problem. | — |
| 17 | Writing on the bord. | ---- |
| 18 | -writing things on the board – exaplaining problems - asking questions - looking at her math book. | I would see my teacher at the projecter explaining problems. |
| 19 | The teacher is showing you how to solve problems with numbers in them. | You would see my teacher explaining things with numbers in it. |
| 20 | showin uas what to do and how to do it | she would be ask people qastion |
| 21 | The teacher is taching math, write ing. Looking at his books | teaching math |
| 22 | She would be writeing on the white board explaining what our math homework would be. | You would see her explaining problems to us and giving examples. |
| 23 | My 5th grade teacher would be writing on the board or teaching math. | You would see me teacher teaching & helping. |
| 24 | Teaching on the white bord | Helping use learn and go over things good |
| 25 | You would see Mrs. Omura writing on the whiteboard and making math easyer. | Explaining things to use and writing things on the board |
| 26 | You would see my teacher talking to us about math and answering our question | ower teacher teaching us. |
| 27 | She would be writing on the white board, explaining how to do things and giving us problems to do. | My teacher would be explaining about a certain math method and giving us problems. |

Question 3: If I were to walk into your _____ classroom while your teacher was teaching math, what would I see you doing?

| | | |
|---|---|---|
| 1 | I would be… 1. Taking down notes. 2. listening 3. not playing around 4. looking at her 5. siting down nicely | taking down notes and listing also paying atntion. Also most of of all learning and last but not least having fun. |
| 2 | I would be listening, or taking some notes while the teacher is talking and giving us some hints. | listening for what she has to say |
| 3 | I would be [we walk around to much] (or) I would be listening to Mrs. Aiona | ---- |
| 4 | I would not be playing and I would be paying attention and listening. | ---- |
| 5 | Listing and looking at the white board. | I would probly be focusing on my mat book. |
| 6 | listening looking at the teacher writting notes | You would see me paying attention to what she was saying. |
| 7 | working , writeing in are composition book, raiseing are hand for the answer | You would se me doing the probles and asking my firend for help or see that I would be asking qesutions about math. |
| 8 | 1. listening 2. not talking 3. solving a problem 4. writing or erasing | trying to solve it |
| 9 | If you walk in the class I would be listening to the teacher and writing notes. | I wouledsee me paying attention and writing things that is on the borad. |
| 10 | Doing my work. | ---- |
| 11 | If you were to walk in my 5th grade classroom while my teacher was teaching math, you would see me staring at my teacher trying to figure out what he was teaching us. | — |
| 12 | I would be sitting quietly in my seat so myself and other | You would see me paying attention to what Ms. Aiona |

| | | |
|---|---|---|
| | could hear, be raising my hand if I have any questions and taking notes. | would be saying. I would be taking notes or I would be asking questions so that I can understand so that I can do my homework good. |
| 13 | You would see me looking at my own paper, listening, taking notes | You would see me listening or sometimes talking. |
| 14 | playing yelling hiding cheating | Studing or doing some homeowkr i didint finish |
| 15 | - taking notes - solving a problem - listening - maybe asking a question | — |
| 16 | You would see me paying attention to what my teacher is explaining to me. | You would see me paying attention to what my teacher is trying to explain to the class. |
| 17 | Looking on the bord. | — |
| 18 | - rasing my hand - talking to the teacher - asking questions if I don't understand something - writing down answers - looking at my mathbook - listening | You would see me working on math problems and listening to the teacher. |
| 19 | I will be listening to the teacher. And taking notes so I understand math. | You would see me writing numbers on a paper |
| 20 | Lisning to what he sed wright nots or reding my math book | lisning s plsyin with cetres |
| 21 | listing. playing writeing, helping people with math | |
| 22 | - I would be paying attention - writing notes - Asking questions – or doing math problems | You would see me listening and looking at the board. |
| 23 | You would see me taking notes, asking for help, or reading. | You would see me working or asking for help. |
| 24 | Listen and speaking about math promblems | I would be listening and doing my work |
| 25 | You would see me paying attention to Mrs. Omura and writing down notes. | Wrting stuff down and looking at the board. |
| 26 | My 5th grade class would eather be sitting and listening or we would be walking around working. | listening |
| 27 | I would be paying attention and being quiet. I would be raising my hand to answer questions. | You would see me taking notes and paying attention. |

Question 4: If I were to watch you do your math homework, what would I see you doing?

| | | |
|---|---|---|
| 1 | I would be doing 1. Siting in my room. 2. have my math book open. 3. writing down the answer in foulder paper. 4. not using a cowculater 5. not fowling around. | me writing on scrach paper and thinking also solving problems. |
| 2 | Writing some math work problems or just be doing some other math problems that I didn't finish yet. | — |
| 3 | doing my home work, look in my math book for what page I am doing | doing my math one at a time. |
| 4 | You would see me working by myself and not playing. | — |
| 5 | At home I would be working alone./ At class I would be workin with a friend or two. | I would want to finish as fast an carfully as posible. |
| 6 | Doing my own work Solving the probleme Not playing around Writting | You would see me working out the problem to show my work. |
| 7 | it would be qiet no goofing off no cheating me geting help form my parents | doing my work on scrapaper and righting it out my problems and asking if it was right or not |
| 8 | 1. thinking 2. not cheating with the built in caculater eraser. 3. not writing any thing on top | trying solve it. |
| 9 | You would see me doing my math homework and figuring out how to do my math. | You would see me doing my homework. |
| 10 | Asking for some help from my dad or doing it by myself. | — |
| 11 | If you were to watch me do my math homework, you would either see me looking at my math paper doing my homework or up at the ceiling trying to figure out the problem in my head. | — |
| 12 | You would see me sitting quietly in my seat focusing on my work + looking only at my work and not somebody elses. | You would see me paying attention and working as hard as I could and not paying attention to me sister or cats. It sometimes gets hard. |
| 13 | You would see me consentrating, workin hard, saying "ow my brain hurts | You would see me writing or thinking. |
| 14 | consentrating doing our work and not drawing on your work | thinking or getting cratable |
| 15 | - working quietly -asking for help - asking questions - solving a problem -trying to concentrate -skiping a question | — |
| 16 | You would see me trying to figure out how to do it correctly. | You would see me doing the best I can to get everything correct. |
| 17 | Writing. | — |

| | | |
|---|---|---|
| 18 | - answers -writing -reading -sitting down -asking me dad questions -thinking | You would see me thinking and writing problems out on paper and trying to figure out equations. |
| 19 | You would see me working silently on the table. | You would see me answering problems in diffrent ways. |
| 20 | Thinkinfg of what the answer is and woking with my friends. | Staying foces with my work |
| 21 | writeing, sleeping, being bored | |
| 22 | -consentrating on my math homework -asking questions to my parents about math | You would see me trying hard to do math problems. |
| 23 | You would see me working out a math problem. | Yo9u would see me working on the work. |
| 24 | Thinking and fustratien | Trying to figure out the problems |
| 25 | Maybe asking some questions and trying to figure out the answer. If don't, then I'll skip it and go to the next question. | Trying to figure out a problem, trying different ways and double checking my work. |
| 26 | You would either see me hand on my head, that means I need help, or you would see me working my hardest. | usally doing mento math |
| 27 | I would be doing my homework and reading the directions on my paper. Also wouldn't be cheating myself. | You would see me showing my work on another paper. |

Question 5: Describe times in everyday life (not in school) where math is used. How is math used? Try to give some examples of when, where, and how math is used.

| | | |
|---|---|---|
| 1 | Math is used to figuer out how much lunch cost. Also figureing outhow much something cost and how much tax is. If my sandwich cost $3.00 and my drink cost 50 cents and tax was $1 what would it cosst: #3.00 + $1.00 + 50 cents = $4.50 | Math is use everywere its used to add up prices and also we have to figure how much fence there is. |
| 2 | homes places were I get bord sometimes like when I have to what for my sister in the dentist. Math is used to think hard and strait. like for example Math is mostly used in school because Math is good for people that don't think strait. | ---- |
| 3 | In you house bill, By adding/subtracting/multiplying ect. adding: bills subtract the money when you go to the math what your buening(?) like 4 come so $3.00 300X 4 = $12.00 | at work, to pay bills, when at home –paying there bills –solving problems for homework and other stuff |
| 4 | Math is used at Nasa because they need to calculate and now where to put everything in a space shuttle. | ---- |
| 5 | Math is used when purchasing things. We use math at store, markets,mall, etc. to count the money givien to buy the item(s). | Math is used in purchasing items at a store. It is used to count the needed money. |
| 6 | At the store Paying bills | Math is used at the mall because you need to pay for things which cost money which is math. |
| 7 | say you are at the store and you get m+m and chip the chip are 1.25 and the m+m is 50 cents how much is it togeter it is 1.75 | I use it when I go shoping like if I have $10 then the thing I'm buying is $5 I don't want to buy it but if it wa good en I would proble buy it |
| 8 | Trying to figure out how many people are coming to the party. or trying to figure out how many cans of paint you need to paint the house | math is used everywhere math is used with numbers, words, and operation signs |
| 9 | Math would be used by adding and mulilyplying | Math is use when you have a cupcake for example: when you need to cut the cupcake into four piece's. |
| 10 | Math is used everyday in your life. Math is used when you want to buy something. | ---- |
| 11 | Math is used in stores when you have to pay for stuff you're going to buy. Math is used when you have to pay for your bills. Math is used so that you can find the total amount of something. | ---- |
| 12 | Math is used when my uncle is on the computer at work so that he can figure out how many people came into get a certine medication. | Math is use everyday at my house when ever my brother is home and my mom is home. My mom is always doing math to figure out the buject for work and my brother because he's in Kamehameha. |
| 13 | Math is used at sea life park when you want to figure out how many dolphins there are in the dolphin observatory. | Math is used at work, at home, when you want to give equal amount of stuff to you friends. |
| 14 | Mc donals has a caculater to count money and money is math | ? I don't know |
| 15 | -figuring out their taxes -figuring out how many square foot we need for wood -for their homework -for the house | ---- |
| 16 | Math is used every where. Like at a store when someone is doing the cashier Its used when you're trying to figure out how much your bill is at the store its used when you don't have a calculator and you add, subtract, multiply and divide. | You use math at the store when you go shopping. You can use it when you add all you things up. When I go shopping I always try to figure out an estimate. |
| 17 | When we go to sams clud by coming the foods. | ---- |
| 18 | -math is used all the time, in stores, home, offices, everywhere -math is used, probably, all the time -math is used to count money and do taxes some ex: my dad uses math to check my homework. My mom uses math to see how many people walk into her store. | Math is used at the store or in a business ot in factories. If you're at the store, you pay at the cash register with money, you're using decimals. |

| | | |
|---|---|---|
| 19 | Math is used everday at stores. to add prices of items there. When you go to the bank you need to know how much money you need. | At the store people use math to find out how much items cost. |
| 20 | You use it at the bank in the store or look on the clock | at the store to add up tax %5.65 + 10 cents = 570 |
| 21 | Paying cash. Driving. Sports. Working | |
| 22 | -Math is used when you need to count how much pencils you have fir school supplies -Math is also used when I get a new bag and I count how much pockets I have. | math is used in jobs. if you were working on a airplane you might need math to count how many people are aboard. |
| 23 | Math is used at home when were baking brownies by measuring how much oil and water you need. | math is used in the kitchen, at work, ands shopping |
| 24 | 1. Restruants the cost of money 2. Bilders use math forleng. wi. | In the store when I'm buying something at pearlridge |
| 25 | Math is used in supermarkets or gardening. | If you need to figure out how much money you need, how much things you have. If I'm shopping and I need to figure out how much money I need to pay. I round up and give the amount of cash I need to pay. |
| 26 | When I was at home I used math because I had to do math homework after school when I got home. It was used by multiplying. | I forgette |
| 27 | Math is used at the bank because the people in the back have to subtract and add to get the correct amount of money. | Math is used in cooking. It's used by measureing ingredient in fractions. |

Question 6: Is there anything else you'd like to say about math?

| | | |
|---|---|---|
| 1 | That math is challaging and some times confusing. | no |
| 2 | NO thank you. | — |
| 3 | very chalenging | math is one of my favorite subject. |
| 4 | Math is fun to me because I get to usee my brin moe than my hands. | — |
| 5 | nope. | I think math is important in life because it sort of guides us thourough our daily activitys. |
| 6 | | No. |
| 7 | | if we didn't have math we would not have a lot to do we would not have tv or playgrounds or even have rooms math is like everything |
| 8 | I like this math sheet | I need help in division |
| 9 | | Yes, because theres any kind of thing we can learn from math. |
| 10 | Math is good for you but I hate math. | — |
| 11 | | — |
| 12 | | Yes,......but I can not think of it right now. |
| 13 | It is fun because it is challenging and I like challenging things. | NO |
| 14 | that math is very important to the world. | math is very help full but sometimes its hard |
| 15 | Sometimes I don't really like math But sometimes I do like math. | — |
| 16 | Math is sometimes fun if you're dong it correctly. | Sometimes math is fun except the hard kind of math that I don't understand. |
| 17 | That math is not easy. | |
| 18 | Well...math is used 24-7, and everybody uses it. | I love math!!! |
| 19 | No. | |
| 20 | Math is fun and you will always need it for when yo becom an adult. | lisning a playin with cetres |
| 21 | No | |
| 22 | In math sometimes I have hard times. | I'd like to say that I like math most of the time. |
| 23 | In math I have a hard time doing fractions. | no |
| 24 | Is fdun and somtimes not. | Its sometimes fun |
| 25 | | Math is sometimes hrd, but useful |
| 26 | no | |
| 27 | Math is really irritating but it becomes very important in an adults life | There is nothing else I'd like to say. |

References


Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal for Research in Mathematics Education, 28, 3,* 258-277.

Dugdale, S., Matthews, J.I., & Guerrero, S. (2004), The art of posing problems and guiding investigations. *Mathematics Teaching in the Middle School, 10, 3,* 140-147.

Gopnik, A. (January 16, 2005). How we learn. *The New York Times.* Retrieved January 17, 2005 from http://www.nytimes.com/2005/01/16/education/edlife/EDSCIENCE

Griffin, L. & DeMoss, G. (1998). *What are my chances? exploring the laws of probability.* Grand Rapids, Michigan: Instructional Fair TS Denison.

Herbel-Eisenmann, B.A., & Breyfogle, M.L. (2005). Questioning our patterns on questioning. *Mathematics Teaching in the Middle School, 10, 9,* 484-489.

Heuser, D. (2005). Teaching without telling: computational fluency and understanding through invention. *Teaching Children Mathematics, 11,8,* 404-412.

Hufferd-Ackles, K., Fuson, K.C., & Sherin, M.G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35, 2,* 81-116.

Inagaki, K., Morita, E., & Hatano, G. (1999). Teaching-learning of evaluative criteria for mathematical arguments through classroom discourse: a cross-national study. *Mathematical Thinking and Learning 1(2),* 93-111.

Jones, S., & Tanner, H. (2002). Teacher interpretations of effective whole-class interactive teaching in secondary mathematics classrooms. *Educational Studies, 28, 3,* 265-274.

Kieran, C. (2001). The mathematical discourse of 13-year-old partnered problem solving and its relation to the mathematics that emerges. *Educational Studies in Mathematics, 46*, 187-228.

Knuth, E., & Peressini, D. (2001). Unpacking the nature of discourse in mathematics classrooms. *Mathematics Teaching in the Middle School, 6, 5*, 320-325.

Lobato, J., Clarke, D., & Ellis, A.B. (2005). Initiating and eliciting in teaching: a reformulation of telling. *Journal for Research in Mathematics Education, 36, 2,* 101-136.

Manouchehri, A., & Enderson M.C. (1999). M.C. Promoting mathematical discourse: learning from classroom examples. *Mathematics Teaching in the Middle School, 4, 4,* 216-222.Manouchehri, A., & St. John, D. (2006). From classroom discussions to group discourse. *Mathematics Teacher, 99, 8*, 544-551.

Martino, A.M., & Maher, C.A. (1999). Teacher questioning to promote justification and generalization in mathematics: what research practice has taught us. *The Journal of Mathematical Behavior, 18, 1,* 53-78.

Mason, J. (2000). Asking questions mathematically. *International Journal of Mathematics Education in Science and Technology, 31, 1,* 97-111.

Nathan, M.J., & Knuth, E.J. (2003). A study of whole classroom mathematical discourse and teacher change. *Cognition and Instruction, 21(2),* 175-207.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics* [Electronic Version], Reston, VA.

National Council of Teachers of Mathematics. (2000). *Principles and standards for teaching school mathematics* [Electronic Version], Reston, VA.

National Council of Teachers of Mathematics. (2005) *Mathematics assessment sampler, grades 3-5*, Reston, VA.

O'Brien, T.C. (1999). Parrot Math. [http://www.pdkintl.org/kappan/kobr9902.htm]

Pimm, D. (1996). Diverse communications. In P.C. Elliott, (Ed.), *1996 Yearbook: Communication in mathematics, K-12 and beyond* (pp. 11-19). Reston, VA: NCTM.

Pirie, S.E.B. (1996). Is anybody listening? In P.C. Elliott, (Ed.), *1996 Yearbook: Communication in mathematics, K-12 and beyond* (pp.105-115). Reston, VA: NCTM.

Schoen, H.L., Bean, D.L., & Ziebarth, S.W. (1996). Embedding communication throughout the curriculum. In P.C. Elliott, (Ed.), *1996 Yearbook: Communication in mathematics, K-12 and beyond* (pp. 170-179). Reston, VA: NCTM.

Sfard, A. (2000). On reform movement and the limits of mathematical discourse. *Mathematical Thinking and Learning, 2(3),* 157-189.

Sfard, A. (2001). There is more to discourse than meets the ears: looking at thinking as communicating to learn more about mathematical learning. *Educational Studies in Mathematics, 26*, 13-57.

Sherin, M.G. (2000). Facilitating discussion meaningful of mathematics. *Mathematics Teaching in the Middle School, 6, 2*, 122-125.

Sherin, M.G. (2002). A balancing act: developing a discourse community in a mathematics classroom. *Journal of Mathematics Teacher Education, 5*, 205-233.

Sherin, M.G., Louis, D., & Mendez, E.P. (2000). Students' building on one another's mathematical ideas. *Mathematics Teaching in the Middle School, 6, 3,* 186-190.

Sherin, M.G., Mendez, E.P., & Louis, D.A. (2004). A discipline apart: the challenges of 'fostering a community of learners' in a mathematics classroom. *Journal of Curriculum Studies, 36, 2*, 207-232.

Shield, M., & Swinson, K. (1996). The link sheet: a communication aid for clarifying and developing mathematical ideas and processes. In P.C. Elliott, (Ed.), *1996 Yearbook: Communication in mathematics, K-12 and beyond* (pp. 35-39). Reston, VA: NCTM.

Silver, E.A., & Smith, M.S. (1996). Building discourse communities in mathematics classrooms: a worthwhile but challenging journey. In P.C. Elliott, (Ed.), *1996 Yearbook: Communication in mathematics, K-12 and beyond* (pp. 20-28). Reston, VA: NCTM.

Stein, M.K., & Strutchens, M.E. (2001). Mathematical argumentation: putting unmph into classroom discussions. *Mathematics Teaching in the Middle School, 7, 2*, 110-112.

van Oers, B. (2001). Educational forms of initiation in mathematical culture. *Educational Studies in Mathematics, 46*, 59-85.

Webb, N., Nemer, K.M., Kerstin, N., Ing, M., & Forrest, J. (2004). The effects of teacher discourse in student behavior and learning in peer-directed groups. CSE Report 627, University of California, Los Angeles.

Wood, T. (1999). Creating a context for argument in mathematics class. *Journal for Research in Mathematics Education, 30, 2*, 171-191.