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POPULATION POLICY UNDER AN ARBITRARY WELFARE CRITERION: THEORY AND ISSUES

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### Author's Acknowledgements:

This paper was written at the East-West Population Institute, Honolulu, and revised at the Operations Research Center, University of California, Berkeley. While the paper is meant to be self-contained, the reader who is unfamiliar with control theory may find it useful to consult an elementary text such as Intriligator (1971). Throughout the paper, the mathematics are informal; I assume that functions have the properties, such as continuity and differentiability, required for the validity of the argument. Simple examples are chosen in order to emphasize methodology rather than any particular model. Discussions with Paul Demeny, Stuart Dreyfus, Nathan Keyfitz and Geoffrey McNicoll have been helpful. My thanks also to Susan Peterson and to Geoffrey McNicoll for editing the preliminary drafts of this paper.

# INTRODUCTION

#### The Analytical Framework

Population policy-makers, unlike economic planners, have as yet no comprehensive theoretical basis to help them select longterm population policy targets. What complicates the selection of rational population policy is the dynamic nature of the problem. The conditions surrounding the problem are constantly changing: society's preferences change over time; the economy grows because of capital accumulation and technological change; the physical environment changes; and population parameters change. Any static "solution" to the population policy problem would be quickly outdated.

Most of these dynamic difficulties can be eliminated, or can at least be brought under control, if we use dynamic optimization theory as our analytical tool. Dynamic optimization theory is an outgrowth of the classical variational calculus; in its modern form, control theory, it can tell us much about selecting policies in a changing environment. In this paper I shall use control theory to construct an analytical framework for the study of population policy, taking some policy issues as examples of how the framework can be used. I shall try to preserve generality as far as possible so that the framework can be used to examine population policy in an arbitrary context where the population, the economy, and the physical environment are interrelated.

Besides using a dynamic approach and a general setting for the problem, I shall employ a general, arbitrary welfare

criterion. This general criterion is not merely a measure of economic well-being. It may include any aspect of social welfare related to population. The classic study of Coale and Hoover (1958) concentrated the attention of population policy makers on economic effects of policy changes. Although it is useful to deduce the economic effects of population policy changes, when we reverse our point of view and try to derive population policies which maximize some economic effect we run into trouble. For example, under a per-capita consumption criterion the optimal policy would be to run the population down to a low level (under most economic models). To avoid this embarrassment, previous papers on optimal policy have resorted to rather questionable practices. Pitchford's (1968) optimal policy is constrained so that the population reaches a given target level, but the population target level is arbitrary and therefore not "optimal." Das Gupta (1969), discards the per-capita criterion and maximizes the total consumption of the population, ignoring the ethical issues raised by this choice of total welfare and implicitly biasing policy towards higher populations.

Obviously the purpose of population policy is not solely to maximize some purely economic criterion. A broader, more suitable criterion should recognize people's desire for company and for children and the adverse social effects of overcrowding, environmental damage and pollution. By adopting an arbitrary welfare criterion in this paper, we can derive more realistic policy solutions. We can also use the theory to test the effects of different social preferences, different ethical positions, and the importance of initial assumptions.

#### The Issues

In using the theoretical framework to study policy improvement and optimization it is of course possible to extract concrete numerical "solutions" in many applications. But more important than this is the insight that the theory can give on several issues confronting policy makers.

> 1. What is the Shadow Price of a Marginal Birth? To determine a rational population policy we need a fairly precise idea of the value of a marginal birth. It is worth noting that every population policy (including a laissez-faire one) attaches an implicit value to introducing an additional person. The "shadow price" of a birth will be important for determining policy improvement and optimization, and, more practically, for assessing the benefits of a birth control scheme.<sup>1</sup> This problem is complicated by the fact that each birth carries a potential for further births, for resource usage, and for capital accumulation. This "growth potential" is extremely important in determining the value of an additional birth. We should like to know how the "further-growth potential" affects

<sup>&</sup>lt;sup>1</sup>There is a large literature on the topic of valuing prevented births and on the related problem of measuring the benefits of birth control programs or of reduced fertility. See for example: Enke (1966, 1970); Zaidan (1967, 1968); Coale and Hoover (1958); Demeny (1965); for a recent practical approach see King (1971). These papers treat welfare in a purely economic sense and are directed towards family planning in under-developed countries.

the shadow-price and how the shadow-price behaves under changing tastes and conditions and under different ethical positions. The shadow price will be derived assuming that non-optimal policies are operating. Throughout this paper we shall concentrate on issues under <u>non-optimal</u> policies since non-optimality is (alas) more commonly found in the real world.

What Constitutes an Improvement in Policy? In many 2. cases we might be content with determining how present policies can be "improved" rather than "optimized." It is perhaps closer to political reality to assume that policies are changed step by step with a view to increasing society's welfare rather than "optimized" in one sweeping reform. The question of policy improvement merits at least as much study as the question of policy optimization. The problem is this: how should present or proposed policies be modified to increase social welfare? Turned around, the question becomes: if a new policy is recommended, under what circumstances can we say it is an "improvement"? For example, under what combination of social preferences would a policy which tends to lower population growth in the U.S. be an "improvement"?

3. When is a Policy Optimal? Being "optimal" entails striking a balance between social and private costs. In our analysis we can make these tradeoffs between private and social costs explicit and then study how they change as preferences change. There are also intertemporal tradeoffs: to be "optimal" we may have to give up some present utility for future utility. How do these tradeoffs behave and how are they affected by intertemporal discounting?

Answers to these three related policy issues depend critically on the selected criterion for social welfare. Without a suitable welfare objective it is meaningless to talk about "improvement" or "optimality." However, choosing a suitable welfare criterion is by no means easy. Geoffrey McNicoll (1971), in a paper in this series, has pointed out that many ethical problems arise in this area. For example: whose utility should be represented by our welfare criterion? How should we weight the utility of future generations and of unborn people (who are candidates to be born)? These problems are strictly questions of ethics; there are no "correct" answers. We can, however, use the analytical framework to compare the implications of different welfare criteria, reflecting different ethical positions.

#### THEORETICAL FRAMEWORK

The System

Ideally, policies should be analyzed within the context of a complex, interacting and changing system which includes the population, the economy, and the physical environment. To describe the present and future behavior of this system we must select suitable variables. The particular variables to choose will be determined by the problem in question, the required accuracy, and the special feat res of the population and economy under study.

Some of these variables, the "state variables," describe the state of the system (e.g., population, capital stock, etc.). These cannot be directly altered. Other variables, the "control variables," can be directly manipulated (e.g., the savings rate, birth control expenditures, etc.). The values of the system variables change over time, it is useful to think of each variable as having a time-path or trajectory. The control variable trajectories determine the state-variable trajectories and hence the behavior of the system.

We now have a system, S, which is described by an n-dimensional time vector of state variables, x(t). An m-dimensional control vector, y(t), represents the values of the chosen policy instruments at time t.

The system is further described by

$$\dot{x} = f[x(t), y(t), t]$$
, (1)

$$C[x(t), y(t), t] \leq 0$$
 (2)

The differential equations (1), describe the dynamic behavior of the system. These equations might represent population growth, capital accumulation, etc. The inequalities (2) are a set of constraints on the state variables and control policies; examples might be budgetary limitations or political restraints on control. The Objective Criterion

We cannot discuss policy "improvement" or "optimization" without some criterion to improve or optimize. For social policy problems, it seems reasonable to choose as the objective criterion the "amount of welfare" received in the arbitrary time period  $(t_0, t_f)$ . If U is a measure of the per-capita social welfare rate, and w(t) weights welfare at time t , the objective function is:<sup>2</sup>

$$J = \int_{t_0}^{t_f} w(t) U[x(t), y(t), t] dt .$$

<sup>2</sup>Assuming welfare received at different times is comparable, i.e., that we can integrate welfare over a time period. In most of this paper, for convenience, I shall assume w(t) is exp(-pt) where  $\rho$  is zero. (That is,

$$J = \int_{t_0}^{t_f} U[x(t), y(t), t] dt,)$$
 In cases where time discounting is

important, exp(-pt) will appear explicitly. Later, I shall discuss some implications of the total welfare criterion,

$$J = \int_{t_0}^{t_f} L(t) U(x,y,t) dt$$
, where the per-capita welfare rate U is

weighted by the population size.

In this criterion there are no end-targets, for we can only choose an end-target statically which, in a dynamic problem, is a non-optimal procedure. For a qualitative discussion of this point see Myrdal (1968), pp. 2063-2066. So far the social welfare function U, is abstract. Behind social welfare, however, lies a wide range of factors which are important to the "quality of life." In post-industrial countries, these factors might include the standard of living, the state of the environment, health standards, ability to realize family-size goals, etc.

We should like to be able to include such factors explicitly in the analysis. This would enable us to study the policy implications of the relative preferences between the welfare factors. For example, if tastes changed so that people wanted a cleaner environment, how should population policy change?<sup>3</sup> I shall now outline how relative preferences can be included in the theoretical framework. Broadening the Welfare Function

For population policy problems we need only select welfare factors which will be influenced by changes in population policy. Such factors might include the standard of living, familial preferences, the state of the environment, perception of crowding, and so on. Denoting these factors by  $F_i$ , we have

$$U = U(F_1, F_2, \dots, F_k)$$
, (3)

where there are k such factors. Since these welfare factors are rather vague entities, we choose an index function,  $A_i(x,y,t)$ , to

<sup>&</sup>lt;sup>3</sup>There is an added advantage in including welfare factors explicitly. Constructing a welfare function in terms of quality-oflife factors such as consumption, leisure, etc., is easier in practice than attempting to construct one directly from the system variables (e.g., population, capital stock, etc.).

represent each factor, F<sub>i</sub>. For example, to represent the factor standard of living we might choose the index per-capita consumption, to represent the state of the environment we might choose a pollution index, and so on.

The index functions  $A_i(x,y,t)$  are known functions of the system variables. For example, per-capita consumption and the chosen pollution index can be defined in terms of the system variables, population, capital stock, etc. Thus every point (x,y,t)in system space implies a corresponding point  $(A_1,A_2,\ldots,A_k)$  in index space. Since I shall be working partly in system space and partly in index space, I shall call this a "two-space approach."

Our welfare function is now  $U = U(F_1, F_2, \dots, F_k)$  which can be represented by:

$$U = U(A_1, A_2, \dots, A_k)$$
 (4)

Assuming for convenience that the chosen indices are independent, a small change in welfare is given by

$$dU = \frac{\partial U}{\partial A_1} dA_1 + \frac{\partial U}{\partial A_2} dA_2 + \dots + \frac{\partial U}{\partial A_k} dA_k .$$
 (5)

The small welfare change is due to changes in the welfare indices,  $dA_1$ ,  $dA_2$ , ...,  $dA_k$ , the index changes being weighted respectively by their relative importance,  $\frac{\partial U}{\partial A_1}$ ,  $\frac{\partial U}{\partial A_2}$ , ...,  $\frac{\partial U}{\partial A_k}$ . The weights  $\frac{\partial U}{\partial A_1}$ ,  $\frac{\partial U}{\partial A_2}$ , ...,  $\frac{\partial U}{\partial A_k}$  can be interpreted as the relative preferences between the factors  $F_1,F_2,\ldots,F_k$  (as measured by the indices  $A_1,A_2,\ldots,A_k)\,.^4$ 

The welfare changes due to a small change in a state variable,  $x_i(t)$ , or in a control variable,  $y_j(t)$ , are given respectively by

$$\frac{\partial U}{\partial x_{i}} = \frac{\partial U}{\partial A_{1}} \frac{\partial A_{1}}{\partial x_{i}} + \frac{\partial U}{\partial A_{2}} \frac{\partial A_{2}}{\partial x_{i}} + \cdots + \frac{\partial U}{\partial A_{k}} \frac{\partial A_{k}}{\partial x_{i}}$$
(6)

$$\frac{\partial U}{\partial y_{j}} = \frac{\partial U}{\partial A} \frac{\partial A_{1}}{\partial y_{j}} + \frac{\partial U}{\partial A} \frac{\partial A_{2}}{\partial y_{j}} + \cdots + \frac{\partial U}{\partial A} \frac{\partial A_{k}}{\partial y_{j}}$$
(7)

That is, a marginal additional unit of a system variable changes each welfare index. This in turn changes the welfare function, U. To illustrate this two-space approach let us take a concrete example. Example 1

Suppose we choose the indices: per capita consumption, C ; population density, D ; a pollution index, E ; and average family-size, B . These represent the more vague factors of standard of living, "crowding," adverse effects on the environment, and family-size preference.

<sup>4</sup>The preferences change over time and also depend on the value of the indices. Hence, we should write  $\frac{\partial U}{\partial A_i}[(A_1,\ldots,A_k),t]$ . For notational simplicity I shall simply write  $\frac{\partial U}{\partial A_i}$ , and assume the argument is understood. Throughout this paper I drop time arguments, unless they are necessary for clarity. E.g., I write U(x,y,t) for U(x(t),y(t),t). Now,

$$U = U(C, D, E, B) , \qquad (8)$$

and

$$dU = \frac{\partial U}{\partial C} dC + \frac{\partial U}{\partial D} dD + \frac{\partial U}{\partial E} dE + \frac{\partial U}{\partial B} dB .$$
 (9)

The above equation describes how social welfare will change for small changes in per capita consumption, density, etc.

The partials  $\frac{\partial U}{\partial C}$ ,  $\frac{\partial U}{\partial D}$ ,  $\frac{\partial U}{\partial E}$  and  $\frac{\partial U}{\partial B}$  can be thought of as the relative preferences between marginal consumption, population density, pollution and family size. <u>Measurement of Relative Preferences</u>.<sup>5</sup> In most cases, by assuming

these preferences are arbitrary we can study the effect of different preference weightings. However, if we wished, we could measure the individual's tradeoffs between marginal units of C, D, E and B.<sup>6</sup>

<sup>&</sup>lt;sup>5</sup>In dealing with dynamic optimization problems, we have to use a cardinal criterion. An ordinal criterion carries less information and is usually too weak to reduce the set of optimal policies to a finite number. For example, under an ordinal criterion, policy  $y_A(t)$  is "better" than policy  $y_B(t)$  only if  $y_A$  can deliver preferred quantities of all welfare factors at all times. Since there are nearly always tradeoffs over time, it is unlikely that such a  $y_A$  exists for  $y_B$ . Hence, any policy  $y_B$ , unless extremely poor, is "optimal" under an ordinal criterion and we have gained little insight into what constitutes a good policy.

<sup>&</sup>lt;sup>6</sup>The question of <u>whose</u> preferences should be represented is discussed in McNicoll (1971). Under certain axioms, it is impossible to find preferences that represent all of society--see Arrow (1951). We are interested in the policy implications of a <u>given</u> set of preferences. Hence, we shall avoid the measurement problem, and consider the arbitrary preference values as given.

For example, holding E and B constant, we can question the individual as to how much consumption he would trade for less (or more) density. In a tradeoff, where there is no change in utility,

$$dU = 0 = \frac{\partial U}{\partial C} dC + \frac{\partial U}{\partial D} dD$$
(10)

Let us call  $\frac{\partial U}{\partial C}$ , the utility increase for a marginal unit of per capita consumption (at a certain standard (C,D,E,B)), one unit--a "vibe." We can now use the tradeoff information, dD and dC, and equation (10) to derive  $\frac{\partial U}{\partial D}$  in "vibes." In this way, we can determine the relative preferences for each factor, over the entire index space.<sup>7</sup>

Welfare Effect of an Additional Person. The welfare effect of a marginal change in population, L(t), is given by:

$$\frac{\partial U}{\partial L} = \frac{\partial U}{\partial C} \frac{\partial C}{\partial L} + \frac{\partial U}{\partial D} \frac{\partial D}{\partial L} + \frac{\partial U}{\partial E} \frac{\partial E}{\partial L} + \frac{\partial U}{\partial B} \frac{\partial B}{\partial L}$$
(11)

The impact on the welfare rate of a marginal person is the sum of his effect on per capita consumption, density, pollution, and family size; each index being weighted by its "relative preference." The partials of the welfare indices  $\frac{\partial C}{\partial L}$ ,  $\frac{\partial D}{\partial L}$ , etc., are known from the consumption and density functions; the preference partials  $\frac{\partial U}{\partial C}$  and  $\frac{\partial U}{\partial D}$  are arbitrary or can be measured if necessary.

This two-space approach enables us to include arbitrarily broad welfare functions in our analysis, with arbitrary preferences

<sup>&</sup>lt;sup>7</sup>For an alternative method and discussion of some of the experimental difficulties in measuring multiattribute ordinal utility functions, see McCrimmon and Toda (1969).

between the indices. It also facilitates interpretation of the partials  $\frac{\partial U}{\partial x}$  and  $\frac{\partial U}{\partial y}$ , which often arise in the theory.

We now have a suitable format in which to embed population policy problems and a means of introducing an arbitrary welfare function explicitly into the analysis. Before we turn to the issues outlined in the introduction, I shall develop one or two theoretical results from control theory. These will be fundamental to the later analysis of policies and shadow prices.

# The Effect of Marginal Policy Changes Over Time

Let us start with a nominal time curve of the policy vector, y(t). We should like to determine the effect of small changes in y(t) on the objective function, J. Knowledge of the effect of marginal control changes will give us the theoretical results basic to the discussion of the policy issues in the following sections.

It might help the reader to have a concrete example in mind. Suppose we can control only government expenditures on birth control. Then the schedule of expenditures on birth control is the control variable, y(t). It is reasonable to ask: what is the total welfare effect of marginal changes in the birth control expenditure trajectory?

Suppose we perturb y(t) slightly, so that the new control function is slightly different at each point in time.<sup>9</sup> The

<sup>8</sup>The reader who finds difficulty in following this section is referred to Intriligator (1971) or to Bryson and Ho (1969). Either of these texts provide a good background to this material.

<sup>9</sup>This analysis is valid whether y(t) is a single variable or a vector of control variables.

difference between the nominal and the perturbed functions is called  $\delta y(t)$ , the variation in y(t). (See Figure 1.)



Figure 1

The arbitrary control variation  $\delta y(t)$ , produces a corresponding variation in the state variables  $\delta x(t)$ , and a change  $\delta J$  in the objective function J. To determine the change in  $\delta J$  due to the variation  $\delta y(t)$  we first adjoin the system differential equations  $\dot{x} = f(x,y,t)$  to the objective function, using a vector of arbitrary multiplier functions p(t):

$$\overline{J} = \int_{t_0}^{t_f} \left( U(x,y,t) + \sum_{i} p_i(t) [f_i(x,y,t) - \dot{x}_i] \right) dt .$$
 (12)

The new objective function,  $\overline{J}$ , is the same as the old one, J, because the expression in square brackets is zero as long as the dynamic equations hold. The arbitrary multipliers  $p_i(t)$  are introduced to bring the constraints  $\dot{x} = f(x,y,t)$  into the objective function. (This device is merely a mathematical trick which will simplify the solution procedure. It is similar to the use of Lagrange multipliers to handle constraints.)

The expression  $U(x,y,t) + p^{T}(t)f(x,y,t)$  occurs frequently, and for notational convenience we shall denote it

$$H(x,y,t) = U(x,y,t) + p^{T}(t)f(x,y,t)$$
 (13)

(In the literature, H(x,y,t) is called the Hamiltonian of the system.)

Now, 
$$\overline{J} = \int_{t_0}^{t} f[H(x,y,t) - p^{T}(t)\dot{x}] dt . \qquad (14)$$

Integrating the last term on the right by parts,

$$\overline{J} = -p^{T}(t_{f})x(t_{f}) + p^{T}(t_{0})x(t_{0}) + \int_{t_{0}}^{t_{f}} [H(x,y,t) + \dot{p}^{T}(t)x(t)]dt .$$
(15)

We can now determine the effect on J of a control variation  $\delta y(t)$  which produces the state variable variation  $\delta x(t)$ .

<sup>10</sup> For convenience, I shall write  $\sum_{i} p_i f_i(x,y,t)$  in the vectornotation  $p^{T}(t)f(x,y,t)$  where  $p^{T}(t)$  is the transpose of p(t).

$$\delta \overline{J} = -p^{T}(t_{f})\delta x(t_{f}) + p^{T}(t_{0})\delta x(t_{0})$$

$$+ \int_{t_{0}}^{t} \int_{0}^{t} \left( \left[ \frac{\partial H}{\partial x} + \dot{p}^{T}(t) \right] \delta x(t) + \frac{\partial H}{\partial y} \delta y(t) \right) dt , \quad (16)$$

It would be tedious to determine the variations  $\delta x(t)$  produced by a given control variation  $\delta y(t)$ , so we <u>choose</u> the multiplier functions p(t) to cause the coefficients of  $\delta x(t)$  in (16) to vanish. That is, we set

$$\dot{\mathbf{p}}^{\mathrm{T}}(\mathbf{t}) = -\frac{\partial \mathbf{H}}{\partial \mathbf{x}} = -\frac{\partial \mathbf{U}}{\partial \mathbf{x}} - \mathbf{p}^{\mathrm{T}}(\mathbf{t})\frac{\partial \mathbf{f}}{\partial \mathbf{x}}, \qquad (17)$$

and

$$p^{T}(t_{f}) = 0$$
 (18)

Then, provided we define p(t) using equations (17) and (18), the change in the objective function for an arbitrary control variation  $\delta y(t)$  is given by

$$\delta \overline{J} = p^{T}(t_{0}) \delta \mathbf{x}(t_{0}) + \int_{t_{0}}^{t_{f}} \frac{\partial H}{\partial y} \delta y(t) dt .$$
 (19)

Since  $x(t_0)$ , the starting vector for the state variables, is given, the control variation must be chosen so that  $x(t_0)$  stays fixed. Hence,  $\delta x(t_0)$  is zero and we have:

$$\delta \overline{J} = \int_{t_0}^{t_f} \frac{\partial H}{\partial y} \, \delta y(t) \, dt \quad .$$
<sup>11</sup> (20)

Equation (20) now shows the effect on the objective function of a small change (variation) in the nominal control trajectory y(t).<sup>12</sup>

# Interpretation of the Multiplier Functions

From equation (19), for  $\delta y(t) = 0$ , a unit increase in the starting value of the state variable,  $x_i$ , adds  $p_i(t_0)$  to the objective function. Thus, we can interpret  $p_i(t_0)$  as the <u>value</u> of a marginal unit of variable  $x_i$  at time  $t_0$ . Since the starting time,  $t_0$ , is arbitrary, we could as easily start at time t. It follows that  $p_i(t)$  is also the value of a marginal unit of variable  $x_i$  at time t.

We now have the theoretical framework and the results necessary to examine the three policy issues raised above, namely the value of a marginal birth, policy improvement, and policy optimization.

<sup>11</sup>Equations (17) and (18) are expressed in vector notation. For each  $x_i$  there is a corresponding  $p_i$ . Hence,  $\dot{p}^T(t)$  and  $p^T(t_f)$  are n-dimensional vectors,  $\frac{\partial f}{\partial x}$  in (17) is an n x n matrix. Equation (20), written more fully is  $\delta J = \int_{t_0}^{t_f} \left[\frac{\partial U}{\partial y} + p^T(t)\frac{\partial f}{\partial y}\right] \delta y(t) dt$ . Here  $\delta y(t)$  is an m-dimensional vector,  $\frac{\partial f}{\partial y}$  is an m x n matrix and  $\delta J$  is a scalar.

<sup>12</sup>Provided that no constraints are violated, the nominal policy curve y(t) can be quite arbitrary. Note that result (20) holds whether y(t) is optimal or not. We are also assuming that the variation  $\delta y(t)$  is permissible, that is, that it does not violate any constraints.

#### THE VALUE OF A MARGINAL BIRTH

The first issue is that of determining the "value" or "shadow price" of a marginal birth.<sup>13</sup> What is the net benefit (or cost) of introducing an extra person into the system? I shall define this shadow price as the contribution the person would make to the objective criterion, i.e., to "welfare," in the time period  $(t_0, t_f)$ .

Recall from the previous section that if population, L(t), is one of the state variables of the system, then  $p_L(t)$  is the "value" of introducing a marginal person at time t. From (17) and (18), we chose  $p_T(t)$  so that

$$-\dot{p}_{L} = \frac{\partial U}{\partial L} + p^{T}(t) \frac{\partial f}{\partial L} , \qquad (21)$$

$$p_{1}(t_{f}) = 0$$
 (22)

#### Changes in the Shadow Price Over Time

The above equations, determining  $p_L(t)$ , at first sight bear little relation to our intuitive idea of the value of introducing another person into the system. But we can interpret them in the following way. According to our objective function, the world

<sup>&</sup>lt;sup>13</sup>In this theory we are ignoring time lags. The term "marginal birth" will be used interchangeably with the term "marginal person." Here, the newly born can work and breed immediately. Provided the system changes slowly, this is not a serious error. The author hopes at a later date to improve the analysis by taking account of time lags.

ceases to exist at time  $t_f$ . (Later we shall take  $t_f$  at infinity.) A birth just before  $t_f$  would have little opportunity to contribute to the total welfare in the time interval and its value would be zero. This explains equation (22). The other equation, (21), describes how the shadow price  $p_L(t)$  changes over time. As time runs out there is progressively less opportunity for a potential birth to contribute to welfare in the period  $(t_0, t_f)$ . Equation (21) states that the shadow price loses value at the rate at which "opportunity" to contribute to the objective function is lost. The opportunity in the next time unit is made up of two components: the instantaneous welfare change due to introducing the person, and the value of the new person's instantaneous contribution to further growth. (These components are represented by  $\frac{\partial U}{\partial L}$  and  $\sum_i p_i(t) \frac{\partial f}{\partial L}$ , respectively.)

Example 2. A system contains two growth equations,  $\dot{L} = f_{L}$  and  $\dot{K} = f_{K}$ , to describe population growth and capital growth respectively.<sup>14</sup> From (21) and (22),

$$-\dot{p}_{L}(t) = \frac{\partial U}{\partial L} + p_{L}(t)\frac{\partial f_{L}}{\partial L} + p_{K}(t)\frac{\partial f_{K}}{\partial L} , \qquad (23)$$
$$p_{L}(t_{f}) = 0 .$$

By waiting an instant to introduce the marginal person, opportunity is lost. The opportunity here is the marginal person's potential

 $<sup>^{14}\</sup>rm Notation:$  The subscripts L and K on  $\rm f_L$  and  $\rm f_K$  are used to identify the growth equations--they do not denote partial derivatives.

impact on welfare plus his potential contribution to further births and to accumulating new capital. Equation (23) tells us that the shadow price falls at the rate this opportunity is lost.

### The Shadow Price under an Infinite Time Horizon

Choosing the welfare criterion on a finite interval enables us to derive the equation for  $\dot{p}(t)$  , but unfortunately the shadow price p<sub>I</sub>(t) depends on the choice of t<sub>f</sub>, the time horizon. Since a model where everything stops at  $t_f$  is somewhat questionable, we should like to take the time horizon at infinity. In this case we might guess that  $p_{I}(0)$ , the present value of introducing a person, is infinite or else converges to an asymptotic value, changing slowly as preferences and system parameters change. Let us derive  $p_{L}(0)$  , the shadow price at time zero, in the context of a simple model. Example 3. We shall take a simplified situation where capital formation and other non-population growth processes are determined exogenously with respect to population. Further, let us assume that the impact of an additional person on welfare,  $\frac{\partial U}{\partial L}$ , is constant (or changes slowly relative to the discount rate). Lastly, in this model the control policy is laissez-faire, and crude population growth remains steady, at the rate n , in the indefinite future. We wish to determine the present value of a marginal birth, p(0), discounting welfare at a rate o over time.

In this model,  $\frac{\partial U}{\partial L}$  is constant and population growth is given by

$$\dot{L} = n L(t)$$
 . (24)

Adding time discounting to (21),

$$\dot{p}_{L}(t) = -\frac{\partial U}{\partial L} e^{-\rho t} - p_{L}(t) \frac{\partial f_{L}}{\partial L} ; \quad p_{L}(t_{f}) = 0 , \qquad (25)$$

which, from (24), becomes

$$-\dot{p}_{L}(t) = \frac{\partial U}{\partial L} e^{-\rho t} + p_{L}(t)n ; \quad p_{L}(t_{f}) = 0 \quad .$$
 (26)

The solution to this differential equation is easily derived as

$$p_{L}(t) = \frac{e^{-nt} \frac{\partial U}{\partial L}}{n - \rho} \left[ e^{-(\rho - n)t} f - e^{-(\rho - n)t} \right].$$
(27)

From (27), the present shadow price,  $p_L(0)$  , is given by

$$p_{L}(0) = \frac{\partial U/\partial L}{n - \rho} \begin{bmatrix} e & -1 \end{bmatrix}, \qquad (28)$$

And in particular, the infinite time horizon present value is

lim  $p_L(0)$  , which gives the result:  $t_f \rightarrow \infty$ 

$$p_{L}(0) = \begin{cases} \frac{1}{\rho - n} \frac{\partial U}{\partial L} &, \rho \geq n \\ \\ \pm^{\infty}, & \rho \leq n \end{cases}$$
(29)

Let us examine the two cases, where  $\rho \leq n$  and  $\rho > n$ .

<u>Case 1:</u> Infinite Shadow Price,  $\rho \leq n$ . If the discount rate,  $\rho$ , is less than or equal to the population growth rate, n, the present value of an additional person, from (29), becomes infinite.<sup>15</sup> Why is this? The marginal person, in Example 2, derives his value from being able to contribute to present utility and to further population growth. Under a low discount rate, his potential for introducing an infinite number of descendants makes his "value" infinite. The value is positive or negative depending on the sign of  $\partial U/\partial L$ .

In many countries it is reasonable to assume that positive growth rates will persist for some time--at least until the advent of a contraceptive revolution. In that case the assumptions of the above, crude model might be reasonably valid. If a marginal birth had an infinite (or extremely large) value, population policy should logically consist of super-encouragement or super-discouragement of births (encouragement or discouragement depending on the sign of  $\partial U/\partial L$ ).

<u>Case 2: Finite Shadow Price</u>,  $\rho > n$ . If we are willing to accept a discount rate  $\rho$  larger than n, then we discount the population growth potential of the new person heavily enough that his shadow price converges. The shadow price, although finite, can still be large under high population growth rates or a low discount rate. For the discussions of policy in the following sections, we shall assume  $p_1(0)$  is finite.

 $^{15}\text{Even}$  if n were less than  $\rho$  eventually,  $\text{p}_{L}(0)$  would still be very large.

In the macro-economic models of Coale-Hoover (1958) and Demeny (1965), the above condition that the shadow price is infinite if  $\rho < n$  is equivalent to the case where the two income streams (with and without reduced fertility) diverge indefinitely at a rate greater than the discount rate. In that case on an infinite time horizon the benefits of reducing fertility would be infinite.

#### Choice of Discount Rate

Obviously, the value of a marginal birth in example 2 is extremely sensitive to the choice of discount rate, as indicated in Figure 2. More complicated models would show much the same behavior.<sup>16</sup>





In fact, we are led to a dilemma. Discount rates high enough to converge  $p_L(0)$  may put very little weight on the utility of succeeding generations. On the other hand low discount rates such as Ramsey (1928) advocated, would imply infinite (or under more realistic models, extremely high) shadow prices. The present would be almost completely subservient to the future. Thus, if high discount rates are unethical and low discount rates are impractical we are left with the problem of choosing a suitable rate in the

<sup>&</sup>lt;sup>16</sup>Probably in more complex models an even higher value of  $\rho$  would be necessary to converge  $p_L(0)$ . In a model with an economic growth sector, I would conjecture that  $\rho$  must be at least as large as the economic growth rate for convergence. For a numerical example of sensitivities of policies to the discount rate in a more complicated model see Arthur and McNicoll (1972).

region where the shadow price is most sensitive.

This discount rate problem arises for all attempts to calculate the benefits of preventing a birth. For example, to converge the stream of benefits caused by a proposed family planning scheme in Jamaica (in a paper by King (1971))we would need a discount rate of 10 per cent. The traditional escape route of using a cut-off time horizon is highly questionable.<sup>17</sup>

# Effect of Preferences

Let us use the "two-space theory" to determine whether the present value of a marginal birth is positive or negative. Using the welfare indices C, D, E and B(per capita consumption, density, pollution, and family-size),

 $P_{L}(0) = \frac{1}{\rho - n} \frac{\partial U}{\partial L} = \frac{1}{\rho - n} \left[ \frac{\partial U}{\partial C} \frac{\partial C}{\partial L} + \frac{\partial U}{\partial D} \frac{\partial D}{\partial L} + \frac{\partial U}{\partial E} \frac{\partial E}{\partial L} + \frac{\partial U}{\partial B} \frac{\partial B}{\partial L} \right] .$ (30)

(Here  $\partial B/\partial L$  is zero, because the average number of children per family is not significantly altered by the addition of one person to the system.)

 $^{17}$ Ethically, we could argue that for any finite time horizon, T, we should be interested in the welfare of generations beyond T and thus that a finite time horizon should not be used. Analytically, we can illustrate the time horizon effect using the simple model. Define the time horizon error as  $[p_L^\infty(0) - p_L^T(0)] / p_L^\infty(0)$ , where the superscript denotes the time horizon. Then from (28) and (29) the error is  $e^{-(\rho-n)T}$  when  $\rho > n$ , and infinite when  $\rho \leq n$ . End-error is most severe at low time horizons and where the discount rate is close to n. The effect of a finite time horizon is to make the shadow prices smaller in an absolute sense, since growth effects become less important. Thus, a finite time horizon is equivalent to choosing a higher discount rate whose actual value rises as we approach the time horizon. When we are basing our policies on shadow-prices which contain terminal error, there will be a corresponding "policy error" which may build up systematically.

Now in most cases, introducing an extra person means lower per-capita consumption (unless there are significant positive returns to population), higher density, and higher pollution. If we favor more consumption and less pollution,  $p_L(0)$  is likely to be negative, unless a strong preference for company overcomes these effects (i.e., where  $\partial U/\partial D$ , the preference for density, is strongly positive). From equation (30), we can see the effect on the shadow price of changes in taste in this simple model.

Because of the broad welfare function, we can evaluate the contribution to the shadow price of non-economic welfare factors such as "crowding" and the "state of the environment." This facilitates policy analysis in post-industrial countries.

#### The Benthamite Shadow Price

So far, I have assumed that U is a per-capita welfare function--the welfare of the average individual. Babies or immigrants are valued only in terms of their welfare impact on the average individual already present in the system. A wider notion of welfare would also consider the welfare of the potential entrant, in valuing the marginal person. The total welfare (or Benthamite) criterion does this; per-capita welfare is weighted by the number of people in the system. The total welfare ethic naturally puts a different valuation on the marginal birth.<sup>18</sup> In the simple model, equations (29) and (30) are replaced by:

<sup>18</sup> In our notation, the total welfare rate is L(t)U(t). The choice between per-capita or total welfare is an ethical problem. For a more complete discussion see McNicoll (1971).

$$p_{L}(0) = \frac{1}{\rho - n} \frac{\partial (LU)}{\partial L} , \qquad (31)$$

$$P_{L}(0) = \frac{1}{\rho - n} \left[ U + L \left( \frac{\partial U}{\partial C} \frac{\partial C}{\partial L} + \frac{\partial U}{\partial D} \frac{\partial D}{\partial L} + \frac{\partial U}{\partial E} \frac{\partial E}{\partial L} + \frac{\partial U}{\partial B} \frac{\partial B}{\partial L} \right].$$
(32)

The sign of  $P_L(0)$  is determined by the sign of the terms in the square brackets, which show the change in the welfare rate due to an additional person. This is the newly-found utility of the entering person plus the effect of his presence on the total population. From (32),  $P_L(0)$  will be positive if the new person's welfare gain, U, outweighs society's net welfare loss. Presuming happiness is still to be found in this society, U will be positive and the total-welfare or Benthamite shadow price will be positive in many situations where the per capita shadow price is negative. In most cases then, a Benthamite ethic will encourage births relative to a per capita ethic.

#### Summary

For a realistic numerical solution to the shadow-price problem, we would naturally use a much more complex model than the one in example 3 above. However, the simple model does give us some insight into valuing marginal births. It was seen that the value is extremely sensitive to the choice of discount rate. Under low intertemporal discount rates the "further-growth-potential" of the marginal birth may mean that the shadow price is infinite or at least extremely large. Again, in valuing a marginal birth, different ethical positions yield widely different results. The per capita valuation may be negative while the more generous Benthamite valuation is positive. The simple model also showed how the shadow price would behave under changes in preference.

#### POLICY IMPROVEMENT

Let us now examine the problem of improving population policy. I shall say that policy A, with the time vector of policyinstrument variables,  $y_A(t)$ , is <u>better</u> than policy B (with  $y_B(t)$ ) if

$$\int_{t_0}^{t_f} w(t) U[x(t), y_A(t), t] dt > \int_{t_0}^{t_f} w(t) U[x(t), y_B(t), t] dt .$$
(33)

That is, policy A is <u>better</u> than policy B if it produces more welfare in the time period  $(t_0, t_f)$ .<sup>19</sup> This approach is useful for comparing two potential policies. The Coale-Hoover method is a practical case which is similar to this approach. Coale and Hoover (1958) compare the per capita income resulting from two potential fertility-rate trajectories to estimate how much "better" one trajectory is than the other.

Although in practice we would want to study how the system responds to different control trajectories, in this paper we are interested in gaining insight into what constitutes policy <u>improve-</u> <u>ment</u>. For our purpose it is better to concentrate on the effect of <u>marginal</u> changes in the control trajectory (i.e., variations in y(t)).

 $<sup>^{19}</sup>$  For simplicity I assume a per capita welfare ethic. The arguments in the case of a total welfare ethic are similar.

#### Marginal Policy Improvement

Recall that each control policy, y(t), has a set of corresponding multiplier functions p(t) (value of a marginal unit of capital, shadow price of a birth, etc.).<sup>20</sup> Then, since we know from (20) the effect of a control variation  $\delta y(t)$  on the objective function, we have

$$\delta J = \int_{t_0}^{t_f} \left[\frac{\partial U}{\partial y} + p^T(t)\frac{\partial f}{\partial y}\right] \delta y(t) dt \quad . \tag{34}$$

Let us define  $\delta y(t)$ , the marginal policy change, as an <u>improvement</u> if  $\delta J$  is positive--that is, if  $\delta y(t)$  yields a net increase in the welfare received in  $(t_0, t_f)$ . From (34), at time t, a small increase in the value of y(t) over the next time unit (provided we subsequently stay close to the nominal path), will be an <u>improvement</u> if

$$\frac{\partial \mathbf{U}}{\partial \mathbf{y}} + \mathbf{p}^{\mathrm{T}}(\mathbf{t})\frac{\partial \mathbf{f}}{\partial \mathbf{y}} > 0 \quad . \tag{35}$$

Equation (35) tells us the proper direction in which to make small policy changes.<sup>21</sup> In the case of a two-dimensional policy vector,

<sup>20</sup>For marginally different control trajectories, the shadow prices, p(t), will remain the same.

<sup>&</sup>lt;sup>21</sup>We must, however, be careful. If we want to determine whether a present policy instrument value is "too high" or "too low" relative to the optimal policy value we should use  $p^*(t)$ , the optimal shadow prices, in (35). In the above I use the shadow prices p(t), corresponding to the real future policy (probably non-optimal) since these valuations, not the optimal ones, will be realized in the future. Also note that this analysis is valid only for small policy changes. It tells us the correct <u>direction</u> in which to change policy. Large control changes could cause the shadow-prices to change.

this is illustrated in Figure 3; the arrows indicate the policy improvement direction over time in a hypothetical case.



"Improvement" direction for policy variables  $y_1^{y_2}$ .

Figure 3

Under what Circumstances Should a Country Increase or Decrease its Population Growth Rate?

Suppose the population growth rate, n(t), is one of the control variables.<sup>22</sup> Equation (35) says we should <u>increase</u> the population growth rate if

The population growth rate n(t) can be treated as a control variable if we can eliminate one of the <u>real</u> control variables  $Y_i(t)$ , (e.g., family planning expenditures) and replace it by  $Y_{m+1}(t) = n(t)$ .

$$\frac{\partial \mathbf{U}}{\partial \mathbf{n}} + \mathbf{p}^{\mathrm{T}}(\mathbf{t})\frac{\partial \mathbf{f}}{\partial \mathbf{n}} > 0 .$$
 (36)

Conversely, we should decrease the population growth rate if

$$\frac{\partial \mathbf{U}}{\partial \mathbf{n}} + \mathbf{p}^{\mathrm{T}}(\mathbf{t}) \frac{\partial \mathbf{f}}{\partial \mathbf{n}} < 0 , \qquad (37)$$

In an actual analysis, we would use a fairly complex model, perhaps incorporating several geographical regions, economic sectors, the age-distribution, time lags, and so on. To illustrate the method we shall use a fairly simple model.

Example 4. Let us take a simple model where welfare depends on the indices: C , per capita consumption; D , density; E , the environment; and B , family size. In this model there are two growth processes, for population and capital respectively,

$$L = f_L$$
, (38)

$$K = f_{K}$$
 (39)

The partials  $\partial D/\partial n$  and  $\partial E/\partial n$  are zero, since population density and pollution are not <u>directly</u> functions of population growth. From (37), we should decrease population growth if

$$\frac{\partial U}{\partial C} \frac{\partial C}{\partial n} + \frac{\partial U}{\partial B} \frac{\partial B}{\partial n} + p_{L}(t) \frac{\partial f_{L}}{\partial n} + p_{K}(t) \frac{\partial f_{K}}{\partial n} < 0 .$$
(40)

That is, in example 4 we should <u>decrease</u> the growth rate if having fewer people  $(\partial f_L/\partial n)$  plus the eased dependency burden  $(\partial C/\partial n)$ , more than compensate for giving up desired children ( $\partial B/\partial n$ ) and losing some capital growth ( $\partial f_K/\partial n$ ).<sup>23</sup> Conversely, we should increase the growth rate if parental desires for extra children plus a strong population growth stimulus to investment outweigh the merits of more consumption (due to fewer children) and having fewer potential people.

<u>Effect of Tastes</u>. Whether we should increase or decrease population growth depends on the preferences between the welfare factors. We see this more clearly if we phrase the problem in this way: under what preferences can we say a certain policy change would be an improvement?

Taking example 4 of the previous section, and substituting for  $p_{L}(0)$ , we can say a reduction in growth rate is an <u>improvement</u> if

$$e^{-\rho t} \left(\frac{\partial U}{\partial C} \frac{\partial C}{\partial n} + \frac{\partial U}{\partial B} \frac{\partial B}{\partial n}\right) + \frac{1}{\rho - n} \left(\frac{\partial U}{\partial C} \frac{\partial C}{\partial L} + \frac{\partial U}{\partial D} \frac{\partial D}{\partial L} + \frac{\partial U}{\partial E} \frac{\partial E}{\partial L}\right) \frac{\partial^{2} L}{\partial n} < 0$$
(41)

Whether population growth should be encouraged or discouraged depends on the social preferences  $\partial U/\partial C$ ,  $\partial U/\partial D$ ,  $\partial U/\partial E$ ,  $\partial U/\partial B$ . One combination of social preferences might indicate a higher growth rate, another combination a lower growth rate. Stated more precisely,

The cost of raising or lowering population growth may also be part of  $\partial C/\partial n$  .

 $<sup>^{23}{\</sup>rm In}$  this paper, we are not interested in the specific form of the partials  $\,\partial{\rm C}/\partial{\rm n}$ ,  $\partial{\rm B}/\partial{\rm n}$ ,  $\partial{\rm f}_{\rm L}/\partial{\rm n}$ ,  $\partial{\rm f}_{\rm K}/\partial{\rm n}$ , etc. The reader who wants to explore these partials further should consult the extensive literature in this area. Robinson and Horlacher's (1971) paper is useful as an overview and has a good bibliography.

<sup>&</sup>lt;sup>24</sup>To be consistent with example 3 where we derived  $p_L(0)$ , we must assume that  $\partial f_K / \partial n = 0$  and that there is positive discounting,  $e^{-\rho t}$ .

let there be a "preference space," P , the space of all possible preference combinations:

$$P = \left\{ \frac{\partial U}{\partial C} \times \frac{\partial U}{\partial D} \times \frac{\partial U}{\partial E} \times \frac{\partial U}{\partial B} \right\} .$$
(42)

Let F, the "reduction-favorable space," be the subspace of Pwhere preferences would indicate reduction in growth rate, i.e.,

$$F = \left\{ \frac{\partial U}{\partial C} , \frac{\partial U}{\partial D} , \frac{\partial U}{\partial E} , \frac{\partial U}{\partial B} \right| \text{ Inequality (41) holds} \right\}.$$
(43)

If an individual's preferences lie within F he will favor lowering the population growth rate; if they lie outside he will favor raising it (under this model). This is illustrated in two dimensions in Figure 4 ; the diagram shows the spread of individual tastes, some people favoring the policy change, others not.



Figure 4

<u>Conclusions</u>.<sup>29</sup> The above analysis enables us to lay down some conditions on what can be said about the currently debated problem of whether a country should attempt to increase or decrease its population growth rate. Views on this issue cover a wide spectrum, from those of Paul Ehrlich to those of Colin Clark, with less extreme positions in between. Implicit in each argument is the idea of improving welfare, whether that of members of the biosphere, of underdeveloped countries, of the church or of any other group. Also implicit is some valuation of future versus present welfare and a set of welfare preferences. The arguments usually purport to "prove" that tradeoffs are such that growth or reduction in growth is highly desirable.

When we formalize this process, as we have done, it is clear that even under perfect knowledge of all interactions and under perfect data, rigorously speaking, no absolute "conclusion" on the issue is possible. Any conclusion is relative to one's Weltanschauung and one's relative preferences.

For those who would attempt to resolve this issue for any given country, the following caveats apply.

 To determine whether a country should increase or decrease its growth rate, one must know (a) the future parameter values of the system, and (b) the complete trajectory of policy instruments in the future. Without a precise knowledge of the future, one cannot, strictly speaking, properly value growth effects, (or in our

<sup>&</sup>lt;sup>25</sup>The above arguments would equally well apply to the **question:** Should a country increase or decrease its birth control **expenditures**/savings rate/immigration rate (or any other combination of policy variables)?

context, derive the "correct" shadow prices p(t).)<sup>26</sup> For example, a good model, with reasonable assumptions about future behavior, tastes and policies, may show the U.S. growth rate to be "too high." But suppose we expect that a large national disaster is going to eliminate 90 per cent of the population in ten years' time. We would then estimate the shadow prices differently in comparison with the no-disaster future, and perhaps even conclude that the U.S. should increase its growth rate. Uncertainty about the future means uncertainty about present policy modifications.

- 2. There is no "correct" way to view the problem. Different system models may lead to different conclusions. Even very sophisticated models are abstractions of the real world and hence permit some bias in conclusions drawn from them.
- 3. The conclusion depends on the assumed relative preferences in the model. Individuals with different tastes may arrive at different results. Since we cannot, according to Arrow (1951), find a set of preference tradeoffs which represent society as a whole, how can we find a "conclusion" that represents society as a whole?
- 4. The conclusion depends on the ethical position taken in the choice of the welfare function. A Benthamite ethic would probably give a result quite different from the per capita ethic, yet either ethic is perfectly defensible.

 $<sup>^{26}</sup>$ In the case of uncertainty about the future, we could define  $\delta y(t)$  as an <u>improvement</u> if it increased the <u>expected</u> value of J. The growth rate problem is then decidable under uncertainty, provided we know the relevant probability distributions.

5. The choice of intertemporal discount rate also affects the conclusion. Yet, there is no "correct" way to value the welfare of future generations.

Conclusions on this issue are possible of course; the polemicist need not give up. However, the above conditions show that a conclusion on this issue is a highly relative entity. A mathematical analysis has the advantage that it forces one to state explicitly one's assumptions and ethical position at the outset.

#### OPTIMALITY

In the previous section, we assumed an <u>arbitrary</u> nominal policy trajectory, y(t), and asked: What marginal changes in this policy constitute improvement? When the nominal trajectory is already optimal, the objective function, J, cannot be increased by policy changes.<sup>27</sup> That is, for any policy variation,  $\delta y(t)$ ,

$$\delta \overline{J} = \int_{t_0}^{t_{f}} \frac{\partial H}{\partial y} \, \delta y(t) dt \leq 0 , \qquad (44)$$

For a  $\delta y(t)$  of arbitrary sign, this condition can only hold if along the optimal trajectory

$$\frac{\partial H(t)}{\partial y(t)} = 0 , \text{ for all } t .$$
(45)

Written out more fully, the necessary condition for optimality is that

$$\frac{\partial U}{\partial y} + p^{*T}(t)\frac{\partial f}{\partial y} = 0$$
, for all t.<sup>28</sup> (46)

Along with this condition, the earlier conditions must also obtain: the system differential equations must hold and p(t) must satisfy the multiplier equations (17) and (18).

<sup>27</sup>Under uncertainty about the future there is no optimal <u>trajectory</u>. A policy which optimizes Expected(J) does exist, however, at each point in the state-space. See Dreyfus (1965), Chapter VII.

<sup>28</sup>When the control vector is optimal, the corresponding multiplier functions p(t) are written  $p^{*}(t)$ .

The notation here is concise. Recall that x(t) is n-dimensional and y(t) is m-dimensional; therefore, we have m equations (46), one for each  $y_i$ . There are n p-values and  $\partial f/\partial y$ is an m x n dimensional matrix.

#### The Optimality Equation

How do we interpret (46) in the context of population policy? Optimality in population policy is a matter of balancing welfare tradeoffs. Suppose we adjust the value of the policy instrument at time t ; as we approach the optimal value the <u>costs</u> of increasing control become as large as the <u>benefits</u> of increasing control.<sup>29</sup> We should stop increasing control when the welfare costs <u>equal</u> the benefits of a further unit of control. The marginal unit of control has an immediate impact on welfare  $\partial U/\partial y$ , plus a longerterm effect on the system growth  $\partial f/\partial y$  (with present value p(t)). Equation (46), states that for optimality the net contribution to the objective function of any marginal change in control, must be zero.

# Optimality Tradeoffs

To illustrate the tradeoffs in maintaining optimality, let us take example 4 again, but this time with  $\rho = 0$ . In this model, there are two dynamic processes:  $\dot{L} = f_L(t)$  for population growth and  $\ddot{K} = f_K(t)$  for capital growth. Assuming we control the population growth rate, n(t), the necessary condition for optimality is:

$$\frac{\partial U}{\partial C}\frac{\partial C}{\partial n} + \frac{\partial U}{\partial B}\frac{\partial B}{\partial n} + p_{L}^{*}(t)\frac{\partial f_{L}}{\partial n} + p_{K}^{*}(t)\frac{\partial f_{K}}{\partial n} = 0 \quad .$$
 (47)

Along the optimal trajectory, if we make an arbitrary marginal decrease in the growth rate, there will be <u>private</u> welfare effects. For optimality, the net impact on welfare received, J,

 $^{29}_{\mbox{ Assuming we maintain an optimal policy after time t .$ 

must be zero (else we could improve the trajectory, and n(t) would not be optimal). Thus, in this model, for a marginal decrease in growth rate at time t, the private welfare effects of the reduced dependency burden plus the smaller family size must balance the social effects of fewer people and a (possibly) lower capital growth rate.

There are also tradeoffs between receiving welfare in the present and in the future. For changes in control there will be an immediate impact on U , the welfare rate, but although the growth equations also are changed now, the welfare effect of growth changes is spread over the future (p(t) gives the present value of these longer-term welfare effects). Optimal control theory sets policy levels so that these tradeoffs between present and future will balance along the optimal trajectory.

# Particular Population Control Schemes

These social/private and intertemporal tradeoffs will vary from one population control scheme to the next. For a scheme <u>coercing</u> a lower growth rate, the chief tradeoffs are: foregoing children versus achieving a less crowded environment; for a <u>bribery/tax</u> scheme: expense of the transfer payments versus achieving a less crowded environment; for <u>family planning clinics</u> in under-developed countries: expense of the scheme versus achieving a reduced dependency burden and higher per capita income.

It is not the purpose of this paper to examine particular policy schemes. $^{30}$  In constructing a theoretical framework for

<sup>&</sup>lt;sup>30</sup>For a more detailed analysis of optimal policies under different control schemes see Arthur and McNicoll (1972).

examining these schemes, however, we provide a means whereby the particular tradeoffs in each scheme can be brought into the open and analyzed. The study of specific programs in this manner is an area where further research would be useful.

#### Optimal Policy and Overpopulation

Let us define a society to be <u>overpopulated</u> if at time t, the per-capita welfare rate could be increased if the population were marginally reduced; that is, if <u>3U/3L</u> is negative.

This is not the only possible definition of overpopulation; it is a time static one, but it has intuitive appeal. A corresponding Benthamite definition could be proposed.

A reduction in population will have associated social and private benefits and costs. Although the net costs may indicate that society is overpopulated in the static sense defined above, the optimal policy may well be to increase the population level.

The optimal policy must consider not only present welfare but future welfare; working through the dynamics of the system, present policies affect the future and this effect must be costed into the decision. We see this clearly in the optimality equation (46).

# Optimal Policy Paths and Social Adjustment

It is interesting to ask the question: To what extent do societies follow an optimal population path? In a frontier society, there are significant increasing returns to population for economic

(and possibly military) reasons. The social value of a marginal birth would be positive. According to the optimality condition in the above example, we would expect an optimizing frontier society to produce children until: a) people already had more than they privately desired or they could properly afford, or b) the limits of the childbearing capacity were reached. On the other hand, in the postindustrial society, high population density plus a deteriorating environment combine to make the shadow price of a marginal person negative. We would expect this type of society to discourage population growth, at least until the "cost of control" (expense of social disruption, unfulfilled parental desires, etc.) began to outweight the benefits.

Of course, the decision to have children rests with parents rather than with society as a whole. The socially optimal policy will only be realized if society can influence parents to act in its interest. Here, we are perhaps unnecessarily imputing a teleological meaning to the word "society." Society is merely a collection of individuals and these individuals do collectively possess welfare preferences and some judgement as to what is in their collective interest. The question to what extent society perceives welfare tradeoffs and readjusts its institutions to come closer to optimality may be an interesting area for further research.<sup>31</sup>

<sup>&</sup>lt;sup>31</sup>This question is decidable only under assumed social welfare preferences, ethics and time preferences. Or it may be possible to <u>assume</u> society's time path optimal and to try to deduce the corresponding preferences and ethics.

#### SUMMARY AND CONCLUSIONS

In this paper I have attempted to provide a theoretical framework for the study of macro-population policy. Since population policy impinges on the economy, the environment, and naturally the population, I have selected a broad systems approach, so that policy can be analyzed in the context of a changing, interrelated demographiceconomic-environmental system. The systems approach enables us to preserve generality in the formulation of policy problems. The user of this framework is free to choose the model and the system variables which best suit his particular problem. In the development of the theory we have relied on the mathematics of control theory. The advantage here is that control theory can recognize the dynamic nature of the changing system--we are not confined to a time-static model.

The framework also allows the possibility of an arbitrarily broad social welfare function. In the literature, most normative policy studies have rested on a very simple welfare criterion, such as per capita consumption or the economic growth rate.<sup>32</sup> Simple criteria, such as these, have usually produced untenable policy recommendations. Hence, we must resort to an approach which can handle a wide range of welfare factors which might be affected by population policy (for example, crowding, family size, the environment, leisure, etc.). By extending the welfare criterion to include an

<sup>&</sup>lt;sup>32</sup>This paper differs from those of the so-called Pontryagin school in another respect. Here, control theory is used to investigate several issues under non-optimal policies. The sections on the value of a marginal birth, policy improvement and on whether a country's growth rate is "too high" assume the continuance of policies which are not necessarily optimal.

arbitrary number of such factors, we can analyze the role of relative preferences among factors in determining rational population policy. This is especially useful in the study of population policy in developed countries, where non-economic welfare factors are important.

An important feature of this type of analysis is that precise functional relationships need not be specified. Problems can be analyzed in general terms; if numerical results are required, the implications of assuming a given functional relationship stand out clearly at the solution stage and are not obscured by prior analysis.

By means of simple examples, plus a little variational calculus theory, we have examined three policy issues:

# 1. The Shadow Price of a Marginal Birth

Most studies of the value of preventing a birth have obscured the role of the time-discount factor, and the potential contribution of the marginal birth to further population and economic growth. Also, the role of non-economic factors in determining the appropriate shadow price has been ignored. Our analysis allows us to make the contribution of each of these factors explicit. One result of the analysis is that, under reasonable assumptions, the shadow price of a marginal birth may be <u>intinite</u> unless the discount rate is greater than the population-growth rate. A marginal birth carries the potential for introducing infinite descendants with infinite impact on the future economy and future welfare. Thus, the shadow price is extremely sensitive to the choice of the intertemporal discount rate, raising again the problem of selecting a suitable value. Also,

different welfare ethics cause major changes in the shadow price. For example, a per capita welfare criterion may indicate a negative value of a marginal birth, a total-welfare (Benthamite) ethic may indicate a positive value.

#### 2. Policy Improvement

The second issue we examined was the question of improvement. Marginal policy changes (in any policy time curve, optimal or nonoptimal) cause short and long-term welfare tradeoffs. If the net benefit is positive, then we define the marginal policy change as an <u>improvement</u>. Which policy changes then qualify as improvements? It is also interesting to turn the problem around: under what range of social preferences is a given marginal policy change an improvement? Under one individual's preferences a certain policy change may be an improvement, under the preferences of another it may not.

The study of policy improvements gave us a means of deciding when a country should attempt to increase or decrease its population growth rate. The conclusion is affected by the assumed social preferences, the ethics of the welfare criterion, the discount rate, and the assumed parameters and dynamics of the model. However, in a particular study, we might examine the "robustness" of the conclusion, that is, determine whether the solution holds over a wide range of plausible assumptions.

# 3. Optimality

Finally, we examined the problem of optimality in population policy. Because of the changing conditions surrounding the policy

problem there is no such thing as a population optimum or a single optimal population growth rate. Optimality in population policy is a matter of adjusting population policy so that at the margin it does not pay to adjust policy further. This implies the existence of welfare tradeoffs to maintaining optimal policy. Our analysis can show explicitly these tradeoffs between social and private welfare as well as tradeoffs between present and future welfare.

The theoretical framework presented in this paper is quite general. We are not restricted to simple models containing four welfare factors and two growth equations. Our goal, in this paper, was to obtain some insight into the structure of the policy issues and I have therefore avoided complex models which would have added little and merely cluttered the argument. Naturally, to analyze actual population problems, planners would use fairly complicated models. The main use of this type of analysis would be not so much to determine actual policies but to bring to the surface the key issues which must be faced by planners in a given situation.

Study of the simple models enabled us to add some insight and new rigor to such problems as what constitutes "optimal" policy and under what circumstances a suggested policy is an "improvement." Rigorously, we can talk about "optimal population policy," but we should remember that the concept exists relative to the chosen model, the selected welfare ethic and discount rate, and the assumed present and future relative social preferences. In addition, to talk about the "value of preventing a birth" and "policy improvement" we need

precise assumptions (or at least probability distributions) on the value of future policy instruments.

The existence of a theoretical framework does not "solve" the population policy problem. The fundamental ethical problems remain: What is a suitable welfare ethic? Whose relative preferences should be considered? And how should we weight the utility of succeeding generations? However, given a particular set of answers to these questions, the theoretical framework allows the planner to examine the logical policy implications of his ethical position.

#### ABSTRACT

Many well-defined policy problems can be formally described as follows: Let x(t) be an n-dimensional vector describing the state of a system at time t , and y(t) be an m-dimensional vector of policy instrument variables at time t . Let U = U[x(t), y(t), t] be a measure of the condition or "welfare" of a system in state x and adopting policies y at time t . Then it is possible to evaluate any specified policy  $\{y(t), 0 \le t \le T\}$ by its impact on  $J = \int_0^T U dt$ . In particular, an optimal policy is one that maximizes J subject to whatever constraints on x and y are applicable.

Analyzing this formulation by means of the calculus of variations enables one to relate a variation  $\delta y(t)$  in policy to the variation  $\delta J$  in J that it induces. This relation involves also a set of multiplier functions (analogous to Lagrange multipliers) that can be interpreted as the "shadow prices" of the state variables.

In this paper, population policy is analyzed in the above format, with stress on the demographic insights that follow from the variational approach. Particular problems investigated in terms of simple but fairly general models are: the value of a marginal birth (the shadow price of the state variable, population); the conditions under which  $\delta y$  is a policy improvement, i.e.,  $\delta J$ is positive (enabling, inter alia, a rigorous definition of "overpopulation"); and the characteristics of an optimal policy.

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