Mathematics

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Mathematics is one of the fastest growing fields of learning today; there has never been a time when changes have occurred with such rapidity. These changes include the extensive development of topics such as measure theory, linear programming, theory of games, and topology. They also include the application of mathematics to areas such as communication, traffic, and the biological and physical sciences. More significant and basic than all of these changes, however, is the development of a new and broader view of the nature of mathematics itself. This change in the conception of mathematics has permitted greater freedom which in turn has stimulated new discoveries.

The evolution of mathematical theories and innovations in the applications of new mathematical processes and techniques occur with such breathtaking rapidity that any attempt at a comprehensive definition would be like clutching the wind. Closer observation of what is happening, however, reveals certain characteristics: emphasis on the abstract and deductive nature of mathematics, concern for its structure, interest in logical foundations, clarity and precision in expression of ideas, and recognition of the role of intuition and imagination in mathematics.

The development of mathematical research and the needs of science have brought about an awareness of the power of mathematics to create new entities. These adaptations, dependent not upon an "intuition of the senses directly inspired by the material world," but rather on abstract manipulations, make possible the creation of new numbers, new spaces of many dimensions, and relations impossible to visualize in our space (1:51).

To say that mathematics is deductive is to make reference to that aspect of mathematics which starts with undefined words, other words defined in terms of these, and assumptions accepted in the particular system, and which builds through successive statements proved by logical reasoning. To stop at this point, however, would be to disregard the important human elements in mathematics. The discovery of new ideas and new theorems is a highly intuitive, creative process requiring imagination and involving aesthetic enjoyment. According to A. E. Meder, Jr., imagination, conception, and generalization are the characteristics of the mathematician and "reasoning is, in the words of a philosopher of a bygone generation, but the smooth pavement on which the chariot of the mathematician rolls" (2:584).

When structure is of importance, mathematics cannot be presented as a collection of manipulative tricks or rules telling students what to do or how to get the answer in so many steps. Unifying themes replace unrelated topics. A study of fundamental ideas which characterize mathematics permits one to recognize the interrelationships which exist among seemingly distinct entities and equips him with the power to attack new problems.

Set theory, or a way of thinking of several things or ideas as being grouped together and combining and comparing several such collections to determine relationships, has found extensive use in almost every new treatment of mathematics. Ideas may be expressed more precisely, basic concepts may be clarified, and extensions of the known or new ways of looking at things are made possible through the set-theoretic approach. Logically speaking, it would be possible to derive almost all of mathematics from the theory of sets. Lucienne Felix says that "modern mathematics rests on the substructure of mathematical logic and the theory of sets" (1:91).

NEW SECONDARY MATHEMATICS CURRICULA

With the rapid development of new ideas, new vocabulary, and new points of view, it is evident that the old mathematics curriculum is woefully inadequate. Mathematics courses in the schools must change too if students are to be able to understand mathematics and appreciate the important place which it occupies in today's culture.

During the past ten years, many different groups have been working with changes in the secondary school program. The Summer 1960 issue of *Studies in Mathematics Education,* published by Scott, Foresman and Company, described 28 improvement programs for school mathematics. A U.S. Office of Education publication in 1960 listed 73 projects concerned with improving mathematics education across the nation. Most of them represent the cooperative effort of teachers of mathematics and contemporary mathematicians in research and industry. Many have made recommendations for changes in the content and emphases in high school courses. Many have developed curriculum materials for use with secondary school students.

Among the groups which have made important contributions to the improvement of mathematics teaching are: The College Entrance Examination Board Advanced Placement Program in organizing college level courses in high schools and obtaining credit and placement in colleges for completion of courses and satisfactory examination scores; the College Entrance Examination Board Commission on Mathematics which has made far-reaching recommendations on the improvement of the curriculum; the University of Maryland Mathematics Project in its preparation of seventh and eighth grade textbooks and teachers' guides; the School Mathematics Study Group and its development of materials for grades 4 - 12; the National Council of Teachers of Mathematics Secondary School Curriculum Committee and its recommendations for instruction and proposals for strengthening mathematics education; and the University of Illinois Committee on School Mathematics.

Of all of these, the School Mathematics Study Group, composed of high school teachers, university mathematicians, school administrators, and representatives of science and industry, is the largest united effort to improve school mathematics. As many as 100 mathematicians and 100 teachers have served on writing teams since 1958 to produce mathematically sound material for high school classes. Revisions were made after trial use of the materials by hundreds of teachers and thousands of students all over the United States. Textbooks, supplementary pamphlets, and teachers' manuals are now available for grades 7 - 12. A sample text for the upper elementary grades is also available.

Along with this increasing involvement of university scholars in the development of new curricula has been another cooperative effort of professional educators, mathematicians, scientists, and psychologists to re-examine the nature of the learning process and investigate new approaches to the teaching of various courses.

No program, however, seems to have captured the spirit of modern mathematics, provided for the development of "enthusiastic students who understand mathematics," and utilized sound pedagogical principles better than the University of Illinois Committee on School Mathematics (UICSM). For this reason I am taking illustrations of what the newer programs in mathematics are attempting to do from the UICSM program.

The UICSM, a committee composed of members of the College of Education, the College of Engineering, and the College of Liberal Arts and Sciences of the University of Illinois, has guided over a ten-year period the work of the project staff in developing instructional materials and training teachers for a new secondary school mathematics curriculum.

To develop a curriculum which would stimulate students' interest in mathematics and promote understanding of mathematical concepts, the language of the text is made as unambiguous as possible and discovery of generalizations by students is encouraged.

In developing precision in language, a distinction is made between things and names of things, hence between numbers and numerals. Developing the understanding that each number has many names allows greater flexibility and simplifies working with numbers, since one can then select the particular numeral which is most helpful in each situation. For example, in finding the sum of 1,2,3,4,5,6,7,8, and 9, recognizing '1+9', '2+8', '3+7', and '4+6' as names for the number 10 makes it easy to arrive at 45. More important, this distinction clears the way for a meaningful treatment of variables and serves as preparation for the important job of stating generalizations about numbers.

These two emphases of discovery of principles and procedures on the part of students, and of precision in language are closely connected and seem to complement each other well, for new discoveries are easier to make once earlier discoveries have been precisely stated, and skill in precise use of language enables students to express discoveries with clarity. For example, students are not given a sequence of steps to follow in solving equations. Instead they are helped to understand what an equation is and what it means to solve equations. If the student realizes that in solving an equation such as '$2x + 5 = 15$', he is simply looking for a replacement for 'x' which will convert the sentence into a true one; he can then devise his own rules accordingly.

Contrast the approach to a basic principle in the following illustra-

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