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Lava flow dynamics: Clues from fractal analysis

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LAVA FLOW DYNAMICS: 
CLUES FROM FRACTAL ANALYSIS

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By

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Xerxes Tata
Para mamá
**Instantes**

Si pudiera vivir nuevamente mi vida
en la próxima trataría de cometer más errores.
No intentaría ser tan perfecto, me relajaría más.
Sería más tonto de lo que he sido, de hecho
tomaría muy pocas cosas con seriedad.
Sería menos higiénico.
Correría más riesgos, haría más viajes, contemplaría
más atardeceres, subiría más montañas, nadaría más ríos.
Iría a más lugares adonde nunca he ido, comería
más helados y menos habas, tendría más problemas
reales y menos imaginarios.

Yo fui una de esas personas que vivió sensata y prolíficamente
cada minuto de su vida; claro que tuve momentos de alegría.
Pero si pudiera volver atrás trataría de tener
solamente buenos momentos.
Por si no lo saben, de eso está hecha la vida, sólo de momentos
no te pierdas el ahora.
Yo era uno de esos que nunca iban a ninguna parte sin un
termómetro, una bolsa de agua caliente, un paraguas y un paracaídas.

Si pudiera volver a vivir, viajaría más liviano.
Si pudiera volver a vivir comenzaría a andar descalzo a
principios de la primavera y seguiría así hasta concluir el
otoño.

Daria más vueltas en callesitas, contemplaría más amaneceres
y jugaría con más niños, si tuviera otra vez la vida por delante.
Pero ya ves, tengo 85 años y sé que me estoy muriendo.

- Jorge Luis Borges
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ABSTRACT

This research aims at developing a better understanding of the rheological and fluid dynamic processes that govern lava flow emplacement, both for terrestrial and extraterrestrial lavas. Two techniques are developed: fractal analysis and fluid dynamic modeling, both of which exploit the final shape of a flow as a source of information regarding flow rheology and dynamics. Analysis of terrestrial flow margins reveals systematic differences in the fractal properties of a’a and pahoehoe basaltic lava flows, as well as between basalts and more silicic lavas. These systematic differences have been developed into a remote sensing tool and applied to selected lava flows on Mars, Venus and the Moon. The vast majority of these extraterrestrial lavas have fractal properties indicating basalt, and have been further categorized into a’a, pahoehoe and transitional morphologies. For Venusian flows, this identification has been independently confirmed by radar measurements of surface roughness. Fractal analysis of extraterrestrial flood basalt margins suggests that these vast flow fields were emplaced at both high (a’a-like) and low (pahoehoe-like) eruption rates. Four flow margins on Mars were found to have fractal properties consistent with a rheology more viscous than typical basalt, possibly a more silicic composition. These results are in agreement with distal lobe width measurements, which indicate an andesitic composition.
Based on fluid dynamic modeling of lava flow emplacement, I generate an equation to describe gravity-driven flows on an inclined plane and solve it analytically. This solution models changes in flow depth and width with distance from the vent, based on different rheological characteristics. Consequently, by comparing known depth and width profiles with the model’s output, these rheological characteristics can be determined. This approach resulted in the following conclusions: (1) The basaltic flows studied were generally non-Newtonian; and (2) Downstream viscosity increases for the flows studied were 2 to 4 orders of magnitude. These results are consistent with the results of several independent studies, attesting to the validity of the model.
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CHAPTER 1

INTRODUCTION

This dissertation is aimed at developing a better understanding of the rheological and fluid dynamic processes that govern lava flow emplacement, both for terrestrial and extraterrestrial lavas. The perspective taken is one of technique development. In this work, I develop two different techniques, both of which exploit the final shape of a flow as a source of information regarding flow rheology and dynamics. The advantage of these techniques is they can be applied with a minimum of data; the only required information is contained in the plan-view shape of the flow.

In Chapter 2, I describe a technique that uses fractal parameters to distinguish various types of lava flows. It has long been known that different flow types have different plan-view shapes. Based on measurements taken of lava flow margins of known flow types (a‘a and pahoehoe basalts, basaltic andesites, andesites, dacites and rhyolites), I quantify these differences using fractal parameters. I have found that different flow types have systematically different fractal properties. Thus, if the only information available of a given lava flow is an image of its plan-view shape (as is often the case for planetary lavas), fractal parameters can be used as a remote sensing tool to
identify flow type, which in turn provides insights into rheology, as well as eruption and emplacement conditions.

In Chapter 3, I lay the groundwork for a second remote sensing technique, also aimed at “working backwards” from the information contained in the final shape of a flow to gain insights into flow rheology and dynamics. By examining how gravitational transport and magmastic pressure combine to drive flow movement, I develop a fluid dynamic model of emplacement of unconfined flows. I generate a differential equation and solve it analytically. The solution models changes in flow depth and width with distance from the source of the flow for different rheological characteristics. Thus, I can use downstream width and/or depth profiles to infer the rheology of lavas or other fluids.

In Chapter 4, I evaluate the potential of the fluid dynamic model developed in Chapter 3 as a remote sensing tool. My approach is to compare model predictions against measurements of lava flows, taken both in the field and from remote sensing images. The model predicts downstream changes in flow depth and width for different rheological characteristics. By comparing these predictions to measurements taken of lava flows, I evaluate the model and show it to be reasonable.

In Chapter 5, I extend the fractal analysis presented in Chapter 2 to lavas on Mars, Venus and the Moon. Included in the database are measurements of extraterrestrial flow fields considered analogous to terrestrial flood basalts. Using fractal parameters, I first categorize these flows into flow types. Then, I evaluate fractal analysis as a remote sensing technique for extraterrestrial flows by comparing this categorization with two other independent remote sensing techniques also aimed at remotely identifying flow type. One of these techniques is based on systematic
differences in the radar-backscatter signals of a’a and pahoehoe basalts; the other uses
distal lobe widths to distinguish basalts from more silicic flows.

Chapter 6 briefly summarizes the conclusions of this research and outlines future
research endeavors.
CHAPTER 2

QUANTIFYING THE EFFECT OF RHEOLOGY ON LAVA FLOW MARGINS USING FRACTAL GEOMETRY

In Press:

2.1 Abstract

This chapter quantifies the effect of rheology on plan-view shapes of lava flows using fractal geometry. Plan-view shapes of lava flows are important because they reflect the processes governing flow emplacement and may provide insight into lava flow rheology and dynamics. The flow margins of basalts are fractal, having a characteristic shape over a wider range of scale (0.125 m - 2.4 km). Furthermore, the fractal dimension (D, a parameter which quantifies flow margin convolution) can distinguish between the two endmember types of basalts: 'a'a (D: 1.05-1.09) and pahoehoe (D: 1.13-1.23). Additionally, we planimetrically analyze ten silicic flows (SiO₂: 52-74%) over a similar scale range (10 m to 4.5 km) and note that silicic flows...
tend to exhibit non-fractal, or scale-dependent, behavior. We attribute this breakdown of fractal behavior at increased lava silica contents to the suppression of small-scale features in the flow margin, due to the higher viscosities and yield strengths of silicic flows. The results suggest we can use the fractal properties of flow margins as a remote sensing tool to distinguish flow types. Evaluation of the nonlinear aspects of flow dynamics indicates a tendency towards fractal behavior for basaltic lavas provided that flow is controlled by internal fluid dynamic processes. For silicic flows, or basaltic flows where flow is controlled by steep slopes, our evaluation indicates non-fractal behavior, consistent with our observations.

2.2 Introduction

Plan-view shapes of lava flows reflect the processes governing flow emplacement; they are frozen snapshots of the final moments of flow (e.g., Wadge and Lopes, 1991). As such, they provide insight into the final stages of lava flow dynamics and rheological state. Plan view shapes and other morphological characteristics have been studied extensively and important quantitative parameters have been developed to extract rheological properties and eruption and emplacement processes of lava flows. Useful parameters include flow length and width as indicators of eruption rate and duration (Walker, 1973; Hulme and Fielder, 1977); widths and thicknesses of flows to estimate yield strengths (Hulme, 1974); widths of distal lobes to deduce rheological properties and SiO$_2$ content (Wadge and Lopes, 1991); flow length and width coupled with levee and channel width to yield effusion rate (Crisp and
Baloga, 1990); average thickness and the ratio of maximum width to maximum length to calculate eruption duration (Lopes and Kilburn, 1990); and ridge heights and spacings to estimate viscosity of flow interiors (Fink and Fletcher, 1978; Fink, 1980). Use of these measurements has led to improved insight into lava flow dynamics and planetary volcanism, but many questions about their quantitative use remain.

A new approach to quantitatively characterize lava flow morphology is the fractal properties of flow margins. Fractals are objects (real or mathematical) that look the same at all scales (Mandelbrot, 1967, 1983). Many geologic features exhibit such "self-similar" behavior (e.g., rocky coastlines, topography, river networks). A qualitative example of self-similar behavior of a lava flow margin appears in Fig. 2.1.

The key parameter is the fractal dimension. Fractal dimension (D) is based on a similar concept as topological dimensions (Dₜ), which are integer measures of the amount of space occupied. For example, a line can be contained in a plane; thus a line (Dₜ=1) has a lower topological dimension than a plane (Dₜ=2). Similarly, a plane can be contained in a volume; thus a volume has a greater topological dimension (Dₜ=3) than a plane. Fractal dimensions are also measures of the amount of space occupied, but they do not have integer values. The following example illustrates the difference between D and Dₜ. Any curve, such as those shown in Fig. 2.2, can be contained in a plane; thus the curve has Dₜ=1. However, in some geometric sense, the convoluted curves (Fig. 2.2b, 2.2c) take up more space than do simple curves (Fig. 2.2a); they are longer. Thus, these convoluted curves have D > 1. As curves becomes increasingly convoluted in a self-similar fashion, D continues to increase, approaching an upper limit at the topological dimension of a plane (since no plan-view curve can take up more space than a plane). Thus, the fractal dimensions of all plan-view shapes of self-similar
Figure 2.1. Margin of a typical pahoehoe flow from the 1972 eruption of Mauna Ulu, Kilauea Volcano with small section enlarged to show self-similarity. The similar shapes of the entire flow margin and the enlarged section at different scales suggests fractal behavior.
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objects are in the range: $1 < D < 2$. The method by which fractal dimension is calculated is described below.

Bruno et al. (1992) showed that the flow margins of both endmember types of basaltic lavas (a'a and pahoehoe) are fractals, with the scale of self-similarity extending from about 0.5 m to over 2 km. This suggests that the processes that control the shapes of basaltic flows at a small (1-meter) scale are dynamically similar to the processes that control flow shapes at a 10-meter or 100-meter scale. For pahoehoe flows, this implies that the same factors that control the outbreak of a small toe control the outbreak of a larger eruptive unit. For a'a flows, which have crenulations superimposed upon larger flow lobes, self-similarity implies that the same factors that cause these crenulations to form (presumably related to differential shear stress) are also responsible for forming the lobes themselves; i.e., the lobes are large-scale crenulations. Kilburn (1990) made a similar point in describing the fractal properties of the surfaces of a'a flows. Bruno et al. (1992) discovered that the margins of a'a and pahoehoe flows have different fractal dimensions. Pahoehoe margins have higher D than do a'a flows. This is consistent with the observation that pahoehoe margins are qualitatively different from a'a margins. Pahoehoe margins have numerous embayments and protrusions (e.g., toes) and are generally quite convoluted. A'a margins tend to be more "linear", with undulatory lobes and, superposed on these lobes, crenulations; see Fig. 2.3.

The differences in lava flow geometry do not reflect differences in composition, but rather differences in rheology and emplacement processes. Whether an erupting basalt becomes a'a or pahoehoe depends on a critical relationship between volumetric flow rate (largely controlled by effusion rate and ground slope), effective viscosity and
Figure 2.3. Flow margins of typical a’a and pahoehoe flows from the 1935 eruption of Mauna Loa Volcano. The pahoehoe margin is more convoluted than the a’a margin, corresponding to a higher D.
shear strength (Shaw et al., 1968; Shaw, 1969; Peterson and Tilling, 1980; Kilburn, 1981; Rowland and Walker, 1990). Pahoehoe flows are associated with low volumetric flow rates (typically < 5 - 10 m$^3$/s for Hawaiian eruptions) and/or very fluid lavas. Upon emplacement, lobes tend to be thin (< 1-2 m) and advance with a smooth rolling motion (e.g., Cas and Wright, 1987). Pahoehoe flows frequently form compound flow fields composed of numerous thin overlapping units. In contrast, a'a flows are generally associated with higher effusion rates (typically 10 - 1000 m$^3$/s for Hawaiian eruptions) and/or more viscous lavas (Rowland and Walker, 1990). A'a flows are generally thicker (typically several meters), and have massive interiors and clinkery exteriors. Unlike most pahoehoe flows, they are erupted -- and flow -- as a single unit, with no part of the flow transported very far from the central channel. This results in a fairly linear a'a margin (low fractal dimension). Pahoehoe flows are fed by a branching tube system, enabling flow over long distances with small radiative heat losses from tubes (Mattox et al., 1993; Realmuto et al., 1992). The smaller tubes transport pahoehoe lavas in various directions, potentially far from the center of the flow field. This results in a more convoluted margin for pahoehoe flows, corresponding to a higher fractal dimension.

One of the objectives of investigations of flow morphology is to determine, without recourse to detailed analysis and field measurements of active flows, rheological properties and perhaps lava flow composition, particularly SiO$_2$ content. So, in addition to basalts, we have studied more silicic flows ranging from SiO$_2$ = 52 to 74 wt. %. Silicic flows tend to erupt as single flow units characterized by a high aspect ratio and a blocky surface morphology, and are also often associated with channel
formation. Thus, in terms of both morphology and emplacement mechanism, high-silica flows are more similar to a'a flows than pahoehoe flows. We have found that higher silica contents and the accompanying increase in viscosity and presumed yield strength lead to qualitative as well as quantitative differences in plan-view shapes. Figure 2.4a shows a basaltic a'a flow characterized by fairly linear margins, superimposed upon which are small-scale features that resemble crenulations. Basaltic andesite (Fig. 2.4b) has finger-like lobes, hundreds of meters in diameter, and appears less "linear". Like basaltic a'a, this basaltic andesite has a crenulated appearance. Figure 2.4c, an example of an andesite lava, also has multiple lobes and they appear shorter, stubbier and wider (approaching 1 km), and the crenulations appear to be absent. Figure 2.4d (dacite) has the highest silica content of these lavas. The dacitic lobes are wider still (> 1 km) and protrude less from the main mass of the lava flow, causing the flow to assume a more bulbous appearance. We note that silica content is just one controlling factor on plan-view shape; there are many other controlling factors (e.g., overall volume, volatile content, eruption rate). Nevertheless, each range of silica content (basalts, basaltic andesites, andesites, and dacites/rhyolites) appears to show qualitative differences in plan-view shape. In this paper, we quantify these differences using fractal analysis.

2.3 Methodology

The fractal analysis employed in this study uses three quantitative parameters: correlation coefficient ($R^2$), fractal dimension ($D$), and quadratic coefficient ($a$). These
Figure 2.4. Plan-view shapes of lava flows of various compositions (in order of increasing silica content). As silica content increases, flow lobes tend to widen, thicken and protrude less from the main mass of the lava flow, and the smaller-scale features become suppressed. (a) Basalt (Galapagos Islands). Note the small-scale crenulation-like features in the flow margin. (SPOT image courtesy of Duncan Munro.)
Figure 2.4b. Basaltic andesite (Hekla, Iceland). Note the finger-like lobes characterizing the flow margin. Like the basaltic margin shown in (a), this margin has a crenulated appearance. (Air photograph courtesy of Thorvaldur Thordarson.)
Figure 2.4c. Andesite (Mount Shasta, U.S.). Compared to (a) and (b), these flow lobes are shorter, stubbier and wider, and the crenulations appear absent. (AIRSAR image courtesy of Peter Mouginis-Mark.)
Figure 2.4d. Dacite (Chao, Chile). Note the bulbous appearance, characterized by wide lobes that do not significantly protrude from the main mass of the lava flow. (Landsat TM image by Shanaka de Silva, courtesy of Stephen Self.)
parameters are all calculated in accordance with the "structured-walk" method (Richardson, 1961). Alternative methods of calculating fractal dimension include "equipaced polygon", "hybrid walk" and "cell-count" methods; these are discussed in detail in Longley and Batty (1989). We selected the structured-walk method because it can be readily applied, both in field measurements as well as on remotely-sensed images. According to the structured-walk method, the apparent length of a lava flow margin is measured by walking rods of different lengths along the margin. For each rod length \( r \), we can determine flow margin length \( L \) according to the number of rod lengths \( N \) needed to approximate the margin; that is, \( L = Nr \). By plotting \( \log L \) vs. \( \log r \) (called a "Richardson plot", after Richardson, 1961), fractal behavior can be determined.

**Calculating Correlation Coefficient \((R)\)**

A linear trend on a Richardson plot indicates the data form a fractal set, indicating self-similarity over the range of rod lengths used. Our criterion for linearity (i.e., fractal behavior) is an \( R^2 \) value exceeding 0.95, where \( R \) is the correlation coefficient of the linear least squares fit. This criterion was chosen somewhat arbitrarily, but follows that used by Mueller (1987). Care was taken to ensure the data array did not artificially flatten out at long rod lengths as a result of choosing rod lengths that are so large that they approach the length of the object. One can avoid this problem altogether by letting \( r \) approach the length of the object (that is, letting \( N \) approach 0) and plotting all the data on a Richardson plot. One can then visually select the linear portion of the curve and fit
a least squares line to the selected segment. Although we have found this technique suitable in measurements of lava flows taken from images, it is quite impractical in field, as it would involve a large number of time-consuming measurements. We have found that choosing our longest rod length such that it can be placed at least five times along a flow margin (i.e., \( N=5 \) is a minimum value) is sufficient to prevent this artifact from compromising our results.

Calculating Fractal Dimension (D)

The fractal dimension of a curve (such as a lava flow margin) is a measure of the curve's convolution, or deviation from a straight line. The fractal dimension (D) can be calculated as:

\[
D = 1 - m,\]

where \( m \) is the slope of the linear least squares fit to the data on the Richardson plot. In other words, D is defined as follows (e.g., Turcotte, 1991):

\[
N = Cr^D \quad (2.2)
\]
\[
L = Nr = Cr^{1-D} \quad (2.3)
\]
\[
\log L = C_1 + (1-D) \log r, \quad (2.4)
\]

where \( C \) is a constant and \( C_1 = \log C \) is the \( y \)-intercept of the Richardson plot.

Consider the theoretical case in which a lava flow margin is a straight line of apparent length \( L \) when measured with a rod of length \( r \). If we again measure its length using a rod of half the original length \( (r/2) \), we count twice as many lengths \( (2N) \).
Similarly, dividing the rod length by four \((r/4)\) results in four times as many lengths \((4N)\). In each case, \(r\) and \(N\) are inversely proportional; \(L\) remains constant. Thus, the Richardson plot is a straight line with slope \(m=0\) and \(D=1\) (Fig. 2.5). However, lava flow margins are not straight lines; instead, they are convoluted. Since smaller rods traverse more of the smaller-scale features in the flow margin, \(L\) increases as \(r\) decreases. Thus the Richardson plot has a negative slope \((m < 0)\) and \(D > 1\).

**Calculating Quadratic Coefficient \((a)\)**

In the above discussions of calculating fractal dimensions and correlation coefficients, the data on the Richardson plot are fitted to a least squares line of the form \(y=mx+b\). Alternately, the data can be approximated by a second-order least squares fit, having the form \(y=ax^2+mx+b\). The quadratic coefficient \((a)\) can provide insight into fractal tendency. An ideal fractal would be expected to have \(a=0\). A negative value of \(a\) on a Richardson plot (concave-downward) translates to a decrease in slope with decreasing rod length, indicating a relative lack of small-scale features. A positive value of \(a\) (concave-upward) correlates with an increase in slope with decreasing rod length, or a relative abundance of small-scale features.

**Testing the Methodology on a Synthetic Fractal**

In order to test this methodology, we consider the case of a computer-generated or
Figure 2.5. Richardson plot of a straight line. The length of a straight line (L) is a constant function of the length of the measuring rod (r), thus the Richardson plot (i.e., plot of Log L vs. Log r) has slope $m=0$, and fractal dimension ($D=1-m$) of 1.
"synthetic" fractal. Such ideal fractals have fractal dimensions that can be calculated mathematically. For example, the Koch triad (discussed below) has a fractal dimension:

\[
D = \frac{\ln 4}{\ln 3} \tag{2.5}
\]

or \( D \sim 1.26 \) (Turcotte, 1991). Applying this methodology to the Koch triad and constructing a Richardson plot reveals fractal behavior \((R^2 > 0.95)\) and \( D = 1.26 \). If we instead fit the data with a second-order least-squares curve, we get a quadratic coefficient \( a = 0.002 \), which is reasonably close to zero. Thus, the results obtained by the structured-walk method closely match the theoretical results.

**Field measurement technique**

We applied the structured-walk method to lava flow margins both in the field and on images. The field technique requires two people, a tape measure, and measuring rods of various lengths. We used wooden dowels for the smaller rod lengths (1/8, 1/4, 1/2 and 1m) and light-weight chains for the longer rod lengths (2, 4, 8 and 16m). First, we isolate a section of flow margin to be measured and, somewhat arbitrarily, choose a point along the margin as the starting point. If the selected section of flow margin is sufficiently long to permit, the measurement begins with one person holding one end of the 16-m chain at the starting point (a). A second person walks along the boundary until the other end of the taut chain exactly intersects the outline. This new point (b) becomes the next starting point. Now, as the second person holds the end of the 16-m chain fixed over point (b), the first person walks along the boundary until the
next intersection point (c) is found. This process continues until a given number of lengths (N) are measured, and the ending point is marked. To maximize accuracy, the measurement is replicated using the same chain length, but this time the persons walk in the opposite direction (from the ending point to the starting point). We have found that the N values from both directions match well (Table 2.1). The results (N) are averaged and L (in meters) is calculated as \( L = N \cdot r \). Ideally, this first length calculation \( L_1 \) will be based on 5 lengths of a 16-m chain, so \( L_1 = 80 \) meters.

We then recalculate the length of the same segment \( L_2 \), using a chain half of the original length \( (r = 8 \text{ m}) \). Since the 8-m chain will likely intersect some undulations in the flow margin that were not encountered by the 16-m chain, \( L_2 > L_1 \), implying \( N_2 > 10 \). Note that it is possible (and likely) that \( N_2 \) will be a fraction. To illustrate this, consider the case when \( 11 > N_2 > 10 \), that is, the case in which the 11th rod length extends beyond the ending point of the given margin segment. In this case, the distance between the last (here, 10th) rod length and the ending point is measured, and expressed as a fraction of \( r \). This fraction is added to the whole-number rod lengths (here, 10) measured, resulting in a fractional \( N \) and a calculated \( L_2 \). We continue dividing the chain length by two and repeating the procedure until at least 5 measurements of \( L \) have been made using 5 different rod lengths, i.e., the Richardson plots have a minimum of five data points.

For sufficiently long flow margin segments, these data points generally correspond to chain lengths of 1, 2, 4, 8, and 16 m. In some cases, we included an additional rod length of 0.5 m. For shorter flow margin segments that cannot accommodate five lengths of a 16-m chain, the first (longest) chain length we chose is the longest chain length that can be walked along the flow margin at least five times;
Table 2.1. Directional Analysis of Field Data

<table>
<thead>
<tr>
<th>r (meters)</th>
<th>$N_1$</th>
<th>$N_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>5.00</td>
<td>5.02</td>
</tr>
<tr>
<td>8</td>
<td>11.52</td>
<td>10.81</td>
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<tr>
<td>4</td>
<td>26.73</td>
<td>25.55</td>
</tr>
<tr>
<td>2</td>
<td>63.61</td>
<td>63.63</td>
</tr>
<tr>
<td>1</td>
<td>140.10</td>
<td>140.19</td>
</tr>
</tbody>
</table>

Values of $N$ obtained by replicating field measurement in opposite direction. Data corresponds to a segment of the 1972 Mauna Ulu pahoehoe flow, Kilauea Volcano.
this may be 8, 4, or 2 meters. In these cases, rod lengths smaller than one meter are necessarily used to meet the minimum requirement of five measuring rods/chains, separated by a factor of two in length. The smallest rod lengths used were 0.25 m for a'a flows and 0.125 m for pahoehoe flows.

Error and Variation Analyses of Field Measurement Technique

We conducted analyses, based on field measurements, to confirm both the precision of this technique ("error analysis") as well as its applicability to the entire flow margin ("variation analysis"). To assess the precision, we conducted 5 replicate measurements of a Hawaiian pahoehoe margin (a portion of the 1972 Mauna Ulu pahoehoe flow). We began each measurement at the same starting point, and measured off 5 lengths of a 16-m chain. Therefore, the ending points of each measurement did not necessarily coincide, but instead were chosen such that \( L_i = 80 \) m in each case. Each measurement consisted of five data points, corresponding to chain lengths of 1, 2, 4, 8, and 16 m. The results of this error analysis are summarized in Table 2.2; note the negligible variance of \( D: \sigma = 0.008 \). Although this error analysis implies that the technique is precise, it does not suggest that the calculated \( D \) of a given flow margin segment is representative of the entire flow. Different segments of a flow margin may have different fractal dimensions, and the above-described error analysis does not measure this segment-to-segment variation. Therefore, we performed an additional analysis on the 1972 Mauna Ulu pahoehoe flow to rigorously study variation along a flow margin. We measured \( D \) of seven adjacent segments of a flow margin in the field.
Table 2.2. Error Analysis of Field Data

<table>
<thead>
<tr>
<th>Trial number</th>
<th>D</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.163</td>
<td>.980</td>
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<tr>
<td>2</td>
<td>1.173</td>
<td>.977</td>
</tr>
<tr>
<td>3</td>
<td>1.177</td>
<td>.988</td>
</tr>
<tr>
<td>4</td>
<td>1.182</td>
<td>.980</td>
</tr>
<tr>
<td>5</td>
<td>1.182</td>
<td>.990</td>
</tr>
</tbody>
</table>

Mean of D: 1.175
Standard Deviation ($\sigma$): 0.008

Error analysis of field measurement technique. Data corresponds to a segment of the 1972 Mauna Ulu pahoehoe flow, Kilauea Volcano.
with each segment defined as five lengths of a 16-m chain \( L_1 = 80 \text{m} \). These results, summarized in Table 2.3, show a significantly larger variation, with \( \sigma = 0.05 \). However, note that all measurements are within the pahoehoe range \( \geq 1.15 \).

*Image measurement technique*

A form of the same "structured-walk method" was utilized to determine fractal dimensions of lava flows from aerial photographs and other images, at scales ranging from 1:6,000 to 1:70,000. We tried to use flow margins in the centers of the images to avoid distortion. Flow margins were digitized using the software package GRASS (Geographical Resources Analysis Support System). These digital data were converted to an ascii file in GRASS, and then read into IDL (Interactive Data Language). In IDL, data were checked for spikes (digitizing errors), which, if found, were smoothed by averaging the neighboring values. Spikes occurred in less than 0.1\% of the data values. We then ran a computer program to calculate fractal dimension using the EXACT algorithm (Hayward *et al.*, 1989). Fractional N-values were handled in the same manner as in the field technique (see above discussion). Computerization facilitates changing the rod lengths in small increments, improving the precision of the calculated \( D \). The computer program was executed using 30 rod lengths, equally spaced on a log scale. (Using more than 30 rod lengths did not significantly improve the calculated \( D \).) Consistent with the field methodology, the minimum flow margin segment included in the image data set corresponds to \( N=5 \) for the longest rod length. The actual length of this longest rod depends on the scale of the image, and ranges up
Table 2.3. Variation Analysis of Field Data

<table>
<thead>
<tr>
<th>Segment Number</th>
<th>D</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (avg.)</td>
<td>1.175</td>
<td>.987</td>
</tr>
<tr>
<td>2</td>
<td>1.207</td>
<td>.958</td>
</tr>
<tr>
<td>3</td>
<td>1.315</td>
<td>.960</td>
</tr>
<tr>
<td>4</td>
<td>1.186</td>
<td>.997</td>
</tr>
<tr>
<td>5</td>
<td>1.183</td>
<td>.984</td>
</tr>
<tr>
<td>6</td>
<td>1.161</td>
<td>.980</td>
</tr>
<tr>
<td>7</td>
<td>1.185</td>
<td>.956</td>
</tr>
</tbody>
</table>

Mean of D: 1.202
Standard Deviation ($\sigma$): 0.052

Variation analysis of field measurement technique. Data corresponds to a segment of the 1972 Mauna Ulu pahoehoe flow, Kilauea Volcano.
to 2.4 km. The minimum rod length was chosen to be sufficiently large as to exceed both the noise inherent in the digitization process as well as the spatial resolution of the images.

*Error and Variation Analyses of Image Measurement Technique*

Analogous with our analyses of the field technique, we conducted error and variation analyses to confirm the image measurement technique. Since this technique is computerized, it is perfectly reproducible; every measurement taken from a given starting point will, after a certain number of rod lengths are measured, result in the exact same ending point. Thus, any error analysis of fractal dimension would necessarily yield $\sigma = 0$. In order to assess variation of fractal dimension among different segments of flow margin, we selected the longest lava flow margin in the image database (Hell's Half Acre, a pahoehoe flow). We divided this margin, which contains over 8000 data points, into 7 overlapping flow margin segments. Each of these segments contains 2000 points and overlaps adjacent segments by 1000 points. Thus segments 1, 3, 5, and 7 are non-overlapping, as are segments 2, 4 and 6. To be consistent with our field variation analysis, we would ideally like to have seven non-overlapping flow segments. However, data limitations prevent this. The results of this analysis, summarized in Table 2.4, show a comparable variation, with $\sigma = 0.04$. As with our field measurements, the minimum D is within the pahoehoe range ($\geq 1.15$).
Table 2.4. Variation Analysis of Photographic Data

<table>
<thead>
<tr>
<th>Segment Number</th>
<th>D</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.204</td>
<td>.970</td>
</tr>
<tr>
<td>2</td>
<td>1.263</td>
<td>.953</td>
</tr>
<tr>
<td>3</td>
<td>1.243</td>
<td>.936</td>
</tr>
<tr>
<td>4</td>
<td>1.188</td>
<td>.954</td>
</tr>
<tr>
<td>5</td>
<td>1.177</td>
<td>.969</td>
</tr>
<tr>
<td>6</td>
<td>1.218</td>
<td>.960</td>
</tr>
<tr>
<td>7</td>
<td>1.270</td>
<td>.953</td>
</tr>
</tbody>
</table>

Mean of D: 1.223
Standard Deviation (σ): 0.036

Variation analysis of photographic measurement technique. Data corresponds to a segment of the Hell's Half Acre pahoehoe flow, Snake River Plain, Idaho.
2.4 Data

The database of field and photographic measurements consists of 55 lava flow margins (or segments thereof). In choosing suitable candidates for measurement, we used the following criteria: (1) The margin is continuous, well-preserved and unambiguous (e.g., not obscured by forest or younger flows). (2) It is unaffected by external controls, such as steep slopes or preexisting topography (e.g., channels). (3) The segment appears to be representative of the entire margin. We categorize the analyzed flows based on composition, separating the basalts from the more silicic flows. We further divide the more silicic flows into two categories: intermediate silica (SiO₂: 52 - 58%) and high silica (SiO₂: 61 - 74%). This database is an extension of that considered by Bruno et al. (1992), which included 28 basaltic lava flows.

Basaltic Lava Flows

This analysis of basaltic lava flows is based on two types of data (Table 2.5a): (1) Field studies of 28 flows on Kilauea, Mauna Loa and Hualalai volcanoes on Hawaii. These flows have different morphologies, and include 7 a'a, 16 pahoehoe and 4 "transitional" flows, i.e., flows with morphologies intermediate between a'a and pahoehoe. (2) Aerial photographs of 18 lava flows in Hawaii, the western United States, Iceland, and the Galapagos Islands. These flows include 8 pahoehoe and 10 a'a flows. Scales of photographs range from 1:6,000 to 1:60,000, which determine the range of equivalent rod lengths to be 12 m to 2.4 km.
Table 2.5a. Database of Basaltic Flows

<table>
<thead>
<tr>
<th>Flow description</th>
<th>Flow Type</th>
<th>D</th>
<th>R²</th>
<th>Data Type</th>
<th>Substrate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kilauea Volcano, Hawaii</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1971 Mauna Ulu</td>
<td>pahoehoe</td>
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<td>.962</td>
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<tr>
<td>1972 Mauna Ulu</td>
<td>pahoehoe</td>
<td>1.20</td>
<td>.994</td>
<td>field</td>
<td>pahoehoe</td>
</tr>
<tr>
<td>1972 Mauna Ulu</td>
<td>pahoehoe</td>
<td>1.18</td>
<td>.973</td>
<td>field</td>
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</tr>
<tr>
<td>1972 Mauna Ulu</td>
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<td>1.21</td>
<td>.987</td>
<td>field</td>
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<tr>
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<tr>
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<td>pahoehoe</td>
</tr>
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<td>1972 Mauna Ulu</td>
<td>a'a</td>
<td>1.06</td>
<td>.988</td>
<td>field</td>
<td>pahoehoe</td>
</tr>
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<td>1974 Mauna Ulu</td>
<td>pahoehoe</td>
<td>1.15</td>
<td>.963</td>
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</tr>
<tr>
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<td>pahoehoe</td>
</tr>
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<td>a'a</td>
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</tr>
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<td>.989</td>
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<td>.995</td>
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</tr>
</tbody>
</table>

Mauna Loa Volcano, Hawaii

<table>
<thead>
<tr>
<th>Flow description</th>
<th>Flow Type</th>
<th>D</th>
<th>R²</th>
<th>Data Type</th>
<th>Substrate</th>
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</thead>
<tbody>
<tr>
<td>prehistoric, Saddle Rd.</td>
<td>pahoehoe</td>
<td>1.23</td>
<td>.988</td>
<td>field</td>
<td>a'a</td>
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<td>a'a</td>
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<td>.969</td>
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<td>pahoehoe</td>
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<tr>
<td>1852 Mauna Loa</td>
<td>pahoehoe</td>
<td>1.13</td>
<td>.992</td>
<td>photo</td>
<td></td>
</tr>
</tbody>
</table>
Table 2.5a. (continued) Database of Basaltic Flows

<table>
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<tr>
<th>Year</th>
<th>Volcano</th>
<th>Type</th>
<th>$T_a$</th>
<th>$T_v$</th>
<th>$F_a$</th>
<th>Notes</th>
</tr>
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<tbody>
<tr>
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<td>a'a</td>
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<tr>
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<tr>
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<tr>
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<td>pahoehoe</td>
<td>1.15</td>
<td>.988</td>
<td>field</td>
<td>a'a</td>
</tr>
<tr>
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<td>Mauna Loa</td>
<td>a'a</td>
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**Hualalai Volcano, Hawaii**

<table>
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<th>Type</th>
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<th>$T_v$</th>
<th>$F_a$</th>
<th>Notes</th>
</tr>
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<tr>
<td>1800</td>
<td>Hualalai</td>
<td>a'a</td>
<td>1.09</td>
<td>.967</td>
<td>field</td>
<td>pahoehoe</td>
</tr>
<tr>
<td>1800</td>
<td>Hualalai</td>
<td>a'a</td>
<td>1.08</td>
<td>.995</td>
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<td>pahoehoe</td>
</tr>
</tbody>
</table>

**Other volcanoes**

<table>
<thead>
<tr>
<th>Volcano</th>
<th>Type</th>
<th>$T_a$</th>
<th>$T_v$</th>
<th>$F_a$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hell's Half Acre, Idaho</td>
<td>pahoehoe</td>
<td>1.21</td>
<td>.981</td>
<td>photo</td>
<td></td>
</tr>
<tr>
<td>Volcano Peak, California</td>
<td>pahoehoe</td>
<td>1.23</td>
<td>.963</td>
<td>photo</td>
<td></td>
</tr>
<tr>
<td>Fernandina, Galapagos</td>
<td>a'a</td>
<td>1.07</td>
<td>.972</td>
<td>photo</td>
<td></td>
</tr>
<tr>
<td>Fernandina, Galapagos</td>
<td>a'a</td>
<td>1.09</td>
<td>.952</td>
<td>photo</td>
<td></td>
</tr>
<tr>
<td>Fernandina, Galapagos</td>
<td>a'a</td>
<td>1.05</td>
<td>.985</td>
<td>photo</td>
<td></td>
</tr>
<tr>
<td>Krafla, Iceland</td>
<td>pahoehoe</td>
<td>1.16</td>
<td>.971</td>
<td>photo</td>
<td></td>
</tr>
</tbody>
</table>
Silicic Lava Flows

This analysis of silicic lava flows is based exclusively on data obtained from aerial photographs and radar images; no field data have been taken to date. The database, summarized in Table 2.5b, consists of ten flows with silica contents ranging from 52 - 74%. We divide these flows into two categories based on silica content: basaltic andesites (SiO$_2$: 52-58%) and more silicic flows (SiO$_2$: 61-74%), the latter being primarily dacites and rhyolites. The images have scales ranging from 1:8,250 to 1:70,000, which determine the lengths of rods used (equivalently, 10 m - 4.5 km).

2.5 Results and Discussion: Basaltic Lava Flows

Basaltic lava flow margins are fractals

Our preliminary results (Bruno et al., 1992) indicated that both a'a and pahoehoe flow margins are fractals within the range of scale studied (r: 0.5 m to 2.4 km). Richardson plots are linear (Fig. 2.6), demonstrating self-similarity. Based on a larger database (45 flows) and over a wider range of scale (r: 0.125 m to 2.4 km), the present analysis confirms that conclusion. Furthermore, transitional flows have also been shown to be fractal. The only cases where the margins of basaltic flows are not fractal are on steep slopes. In these cases, when the margin is externally controlled by a steep ground slope, it becomes more linear, with fewer convolutions.
Table 2.5b. Database of Silicic Flows

<table>
<thead>
<tr>
<th>Flow description</th>
<th>SiO₂ (%)</th>
<th>Flow Type</th>
<th>Scale</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andes Mountains</td>
<td>52</td>
<td>Bas. Andesite</td>
<td>1: 27,000</td>
<td>Thorpe et al. (1984)</td>
</tr>
<tr>
<td>Andes Mountains</td>
<td>52</td>
<td>Bas. Andesite</td>
<td>1: 13,500</td>
<td>P. Francis (pers. comm.)</td>
</tr>
<tr>
<td>1980 Hekla, Iceland</td>
<td>55</td>
<td>Bas. Andesite</td>
<td>1: 8,250</td>
<td>Gudmundsson et al. (1991)</td>
</tr>
<tr>
<td>SP Flow, AZ</td>
<td>57</td>
<td>Bas. Andesite</td>
<td>1: 36,000</td>
<td>Ulrich and Bailey (1987)</td>
</tr>
<tr>
<td>Lava Park Flow, CA</td>
<td>61</td>
<td>Andesite</td>
<td>1: 30,000</td>
<td>Smith and Carmichael (1968)</td>
</tr>
<tr>
<td>Ludent, Iceland</td>
<td>65</td>
<td>Dacite</td>
<td>1: 8,250</td>
<td>Nicholson (pers. comm.)</td>
</tr>
<tr>
<td>1100 Hekla, Iceland</td>
<td>65</td>
<td>Dacite</td>
<td>1: 8,250</td>
<td>Sigmarssson (pers. comm.)</td>
</tr>
<tr>
<td>Chao, Chile</td>
<td>66</td>
<td>Dacite</td>
<td>1: 70,000</td>
<td>Guest and Sanchez (1969)</td>
</tr>
<tr>
<td>Glass Mountain, CA</td>
<td>74</td>
<td>Rhyolite</td>
<td>1: 12,000</td>
<td>Eichelberger (1975)</td>
</tr>
</tbody>
</table>

Database of silicic flows is based entirely on photographs and other images; scales of images are listed above. We have subdivided this database into two, separating the first five flows (basaltic andesites) from the remaining, more silicic flows (primarily dacites and rhyolites).
Figure 2.6. Richardson plots of typical a'a and pahoehoe flows, in meters, based on field data. High $R^2$ values (> 0.95) indicate fractal behavior. The more convoluted margins of pahoehoe flows translate to higher $D$. 
The fractal behavior of pahoehoe and a'a flows might be predicted by their basaltic composition. Low viscosities on the order of 1000 Pa-sec for typical eruption temperatures of 1150° C, coupled with a negligible yield strength for most basalts, offers no obstacle to prevent self-similar features from being formed on a wide range of scales. We note at some small scale below the detection limit of this study, fractal behavior will eventually break down due to material properties, such as surface tension.

*Pahoehoe and A'a have different D*

We find that over a wide range of geographic locations (Hawaii, Iceland, western U.S., Galapagos Islands), basaltic lavas divide into two populations with regard to their fractal dimensions. A’a flows generally have $D$ ranging between 1.05 and 1.09 whereas pahoehoe flows typically have $D$ ranging between 1.15 and 1.23. Figure 2.7a summarizes the results for basaltic flows. Most (12 of 14) of the Hawaiian a’a flows have $D$ between 1.05 and 1.09; all have $D$ between 1.05 and 1.13. Most Hawaiian pahoehoe flows (18 of 21) have $D$ between 1.15 and 1.23; all have $D$ between 1.12 and 1.23. The two pahoehoe flows in the western U.S. yield measurements of 1.21 and 1.22, consistent with the range of Hawaiian pahoehoe flows. Similarly, a Krafla, Iceland basalt (pahoehoe) falls into the Hawaiian pahoehoe range, with a fractal dimension of 1.16. The three Galapagos flows measured, all a’a, yield $D$ values of 1.05, 1.07 and 1.09, in agreement with the range of Hawaiian a’a flows. This suggests that regardless of geographic location, the pahoehoe flows consistently have
higher D than a'a flows. This is important because it suggests that eruption and emplacement mechanisms in greatly varying locations are similar.

By definition, fractals should have constant ranges of fractal dimensions, regardless of the rod lengths used to measure D. Thus, if lava flows are fractals over the range of scale studied, the fractal dimensions obtained at the field scale (0.125 - 16 m) should be similar to the range of fractal dimensions obtained at the image scale (12 m - 2.4 km) for a'a as well as pahoehoe. This is confirmed by our results. All seven a'a flows measured in the field have D between 1.05 and 1.09 (Fig. 2.7b), the same range we find for image data of a'a flows (Fig. 2.7c). All sixteen pahoehoe field measurements have D between 1.12 and 1.23, compared to a range of 1.13 - 1.23 for image data of pahoehoe flows.

For three pahoehoe flows, we measured margins of the same flow in the field and from aerial photographs. The fractal dimensions as measured from aerial photographs are 1.19 (1855 flow, Mauna Loa), 1.14 (1859 flow, Mauna Loa) and 1.20 (1935 flow, Mauna Loa). Field measurements yielded corresponding D of 1.17, 1.16 and 1.15, respectively. These variations in D are within the overall variation (Table 2.3), and indicate fractal behavior.

Flows that we have determined to be transitional between a'a and pahoehoe based on field observations tend to have intermediate fractal dimensions, as might be expected (Fig. 2.7d). Of the four field measurements of transitional flows, three have D between 1.09 and 1.12; the fourth has a slightly higher D of 1.15.

One might expect the fractal dimensions of flow margins to be affected by the nature of the underlying substrate. For example, a pahoehoe margin that formed on a preexisting a'a flow might be different than one that formed on a preexisting pahoehoe
Figure 2.7. Histograms of D values of a‘a and pahoehoe flows. (a) a‘a and pahoehoe flows based on all (field and photographic) data. Pahoehoe flows tend to have higher D than a‘a flows.
Figure 2.7b. Histogram of D values of a'a and pahoehoe flows based on field data. For these field data, pahoehoe flows tend to have higher D than a'a flows. This tendency also emerges from the photographic data; see (c).
Figure 2.7c. Histogram of D values of a'a and pahoehoe flows based on photographic data. For these photographic data, pahoehoe flows tend to have higher D than a'a flows. This tendency also emerges from the field data; see (b).
Figure 2.7d. Histogram of D values of transitional flows (i.e., flows with morphologies transitional between a‘a and pahoehoe). This histogram comprises field data only, as no photographic measurements were taken of transitional flows. Transitional flows tend to have values of D intermediate between those of a‘a and pahoehoe.
flow. However, a detailed analysis shows that D values are unaffected by differences in substrate. We took 16 field measurements of Hawaiian pahoehoe flows. Some of these lavas flowed upon preexisting a'a lava flows (5), others upon preexisting pahoehoe flows (9), and still others atop ash deposits (2). Figure 2.8 shows the lack of correlation between D and the substrate for these 16 flows. In one case (1855 Mauna Loa pahoehoe), we performed a controlled experiment on the effect of substrate on D. We measured D in one location where this pahoehoe flowed over a preexisting pahoehoe, and again nearby (within 100m), where the same flow covered an a'a substrate. The D values obtained for the pahoehoe and a'a substrates (1.17 and 1.19, respectively) are well within the observed variation of D along a flow margin with a constant substrate (see Table 2.3).

Clearly, a pattern emerges for the fractal dimensions of terrestrial basaltic lava flows. Regardless of geographic location, lengths of rods used, or substrate material, pahoehoe flow margins consistently have higher D than a'a flow margins. This is consistent both with the preliminary results of Bruno et al. (1992) and also observational evidence that the outlines of pahoehoe and a'a flows are qualitatively different.

A note about topographically-controlled flows

Topographically-controlled flows have been excluded from this analysis because such external controls can have a significant effect on D. Positive topography (e.g., hills) may deflect or bifurcate flows, increasing the degree of flow margin convolution and therefore increasing D. Negative topography (e.g., channels) serves to confine or
Figure 2.8. Histogram of D values for pahoehoe flows on three different substrates -- a'a, pahoehoe, or ash -- based on field data. There is no apparent correlation between D and substrate type.
channelize flows, causing the margin to become more linear and thus decreasing D. In many cases, these external controls interfere with the development of self-similar features, and prevent fractal behavior. Similarly, we have found fractal behavior to break down, with an accompanying decrease in D, at steep (>15 - 28°) slopes; see Table 2.6. This tendency toward nonfractal behavior as the gravity-driven force on the flow increases is discussed further below.

**Implications for Flow Dynamics**

The fractal properties of lava flows may offer insight into the dynamics of flow emplacement because fractals reflect nonlinear processes (e.g., Campbell, 1987). We have made a preliminary evaluation of the nonlinear aspects of flow dynamics to obtain a qualitative indication of the tendency toward fractal behavior. Following earlier fluid dynamic models (e.g., Baloga and Pieri, 1986; Baloga, 1987), we depict variations in the free surface of a lava flow as due to both a gravitational transport term and the fluid dynamic ("magmastatic") pressure gradient (Baloga et al., 1992). On steep slopes, the gravitational term dominates; on nearly flat slopes, the gravitational term is negligible. Here, we examine how these two forces combine to drive lava flow movement by considering the volumetric flow rate \( Q \) of a lava flow. We define a control volume of a lava flow as having width \( w \), length \( dx \) and thickness \( h=h(x,t) \), as shown in Fig. 2.9.

\[
V = whdx
\]  

(2.6a)
Data obtained from field measurements of the 1972 Mauna Ulu a‘a flow on various slopes, Kilauea Volcano.
Figure 2.9. Flow model. The x and y directions are downstream and cross-stream respectively, and z is vertical. Lava flow has width w, thickness h, and flows on a plane inclined to the horizontal at an angle \( \theta \). Control volume \( V = whdx \).
Volumetric flow rate:
\[
\frac{\partial V}{\partial t} = wdx \left( \frac{\partial h}{\partial t} \right)
\]  
(2.6b)

At first, for ease of illustration, let us restrict volumetric flow to the x-direction \(Q = Q_x\). Thus, all volumetric flow is occurring through the front and back faces of the control volume. Volume-conservation considerations require that the flow satisfies:
\[
\frac{\partial V}{\partial t} = Q_x(x) - Q_x(x + dx) = -\left( \frac{\partial Q_x}{\partial x} \right) dx
\]  
(2.7)

Equating eqs. (2.6b) and (2.7),
\[
w\left( \frac{\partial h}{\partial t} \right) + \left( \frac{\partial Q_x}{\partial x} \right) = 0
\]  
(2.8a)

If we also account for volumetric flow in the y-direction, eq. (2.8a) becomes:
\[
w\left( \frac{\partial h}{\partial t} \right) + \left( \frac{\partial Q_x}{\partial x} \right) + \left( \frac{\partial Q_y}{\partial y} \right) = 0
\]  
(2.8b)

where \(Q_l\) = volumetric flow rate through lateral faces of the control volume.

There are two components of \(Q_x\): a gravitational term and a magmastatic pressure gradient term. \(Q_y\) has only one component (a magmastatic pressure gradient term); the y-direction was chosen perpendicular to slope. For Newtonian fluids,
\[
Q_x = \frac{gw}{3} \left[ \sin \theta h^3 - \cos \theta h^3 \left( \frac{\partial h}{\partial x} \right) \right]
\]  
(2.9a)
where \( \theta = \) slope, \( \nu = \) kinematic viscosity and \( g = \) gravitational acceleration. Discussion of flow rates for non-Newtonian fluids is reserved for Chapter 3.

Substituting eqs. (2.9a) and (2.9b) into (2.8b) and non-dimensionalizing using the following equations:

\[
\begin{align*}
    y &\Rightarrow wy^* \\
    x &\Rightarrow Lx^* \\
    y &\Rightarrow wy^* \\
    t &\Rightarrow Tt^*
\end{align*}
\]

for some thickness scale \( h_o^* \), length scale \( L \), width scale \( w \) and time scale \( T \), yields:

\[
\frac{\partial h^*}{\partial t^*} + qh^* \left( \frac{\partial h^*}{\partial x^*} \right) = pq \left[ \left( \frac{w}{L} \right)^2 \left( \frac{\partial}{\partial x^*} \left( h^* \frac{\partial h^*}{\partial x^*} \right) \right) + \frac{\partial}{\partial y^*} \left( h^* \frac{\partial h^*}{\partial y^*} \right) \right]
\]

where:

\[
q = \frac{g \sin \theta h_o^2 T}{\nu L}
\]

\[
p = \frac{\cot \theta h_o L}{3w^2}
\]

We define \( q \) as the coefficient of the gravitational term and \( pq \) as the coefficient of
the magmastatic pressure gradient term in eq. (2.13). Therefore, $p$ is proportional to
the ratio of the pressure gradient to the gravitational terms. A small value of $p$ ($p \ll 1$)
reflects a large gravitational term, i.e., a steep slope. A large value of $p$ ($p \gg 1$)
indicates the gravitational force is negligible compared to the pressure gradient.

To simplify notation, we can drop the asterisks from eq. (2.11), with the
understanding that all variables are dimensionless, to arrive at:

$$\frac{\partial h}{\partial t} + q h^2 \left( \frac{\partial h}{\partial x} \right) = p g \left[ \left( \frac{w}{L} \right)^2 \frac{\partial}{\partial x} \left( h^3 \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( h^3 \frac{\partial h}{\partial y} \right) \right]$$

(2.13)

We now consider the case where:

$$\left( \frac{w}{L} \right)^2 \frac{\partial}{\partial x} \left( h^3 \frac{\partial h}{\partial x} \right) \sim 0$$

(2.14)

Eq. (2.14) is satisfied when either the width of a flow is substantially less than its
length (a reasonable assumption for typical a’a flows), i.e., when:

$$\left( \frac{w}{L} \right)^2 \ll 1$$

(2.15a)

and/or when the downstream magmastatic pressure gradient is negligible, i.e., when:

$$\frac{\partial}{\partial x} \left( h^3 \frac{\partial h}{\partial x} \right) \ll 1$$

(2.15b)

When either eq. (2.15a) or (2.15b) is satisfied, eq. (2.14) is satisfied, reducing eq.
By assuming $\partial h/\partial t$ is on the order of 1, we can evaluate eq. (2.16) for selected values of $p$ and $q$. These special cases are shown in the matrix of Fig. 2.10. Some of the equations in this matrix are linear; others are nonlinear. The linear equations would not be expected to produce fractals, whereas the nonlinear equations are candidates for producing fractals, provided that they are physically plausible.

High $p$ values (right column of matrix) indicate the magmastic pressure gradient term is important relative to the gravitational term. Low $q$ values (top row of matrix) indicate a weak gravitational term. Thus, in Case 1c (large $p$, small $q$), the gravitational term is the least important, both relatively (to the pressure gradient) and absolutely, and the magmastic pressure gradient dominates. Thus, since the lava flow is being largely driven by internal fluid dynamic forces in Case 1c, we predict that this combination of $p$ and $q$ is likely to produce fractal behavior. As expected, the resulting equation is explicitly nonlinear.

For the same $q$ ($q<<1$), consider the cases corresponding to $p$ values that are low (Case 1a) and moderate (Case 1b). Both of these equations are linear, and would therefore not be expected to produce fractals. Since $p$ is proportional to the ratio of magmastic pressure gradient to gravitational driving force, this has important implications for the effect of gravity on fractal behavior. When gravity plays a non-negligible role (small or moderate $p$), the matrix predicts the flow margin would not be
<table>
<thead>
<tr>
<th></th>
<th>(p \ll 1)</th>
<th>(p = 1)</th>
<th>(p \gg 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1a</td>
<td>Case 1b</td>
<td>Case 1c</td>
</tr>
</tbody>
</table>
| \(q \ll 1\) | \(\partial h/\partial t = 0\) | \(\partial h/\partial t = 0\) | Assume \(p_k = 0(1)\)  
\(\partial h/\partial t = \partial/\partial y[h^3(\partial h/\partial y)]\) |
| \(q = 1\)  | Case 2a    | Case 2b  | Case 2c     |
|       | \(\partial h/\partial t + h^2(\partial h/\partial x) = 0\) | \(\partial h/\partial t + h^2(\partial h/\partial x) = \partial/\partial y[h^3(\partial h/\partial y)]\) |  
\(0 = \partial/\partial y[h^3(\partial h/\partial y)]\) |
| \(q \gg 1\) | Case 3a    | Case 3b  | Case 3c     |
|       | Assume \(p_k = 0(1)\)  
\(h^2(\partial h/\partial x) = 0\) | \(h^2(\partial h/\partial x) = \partial/\partial y[h^3(\partial h/\partial y)]\) |  
\(0 = \partial/\partial y[h^3(\partial h/\partial y)]\) |

Figure 2.10. Matrix of special cases of eq. (2.16) for select values of \(p\) and \(q\), obtained by assuming \(\partial h/\partial t\) is of order 1. Some of the equations in this matrix are linear; others are nonlinear. The linear equations would not be expected to produce fractals, whereas the nonlinear equations could be expected to produce fractals (see text for details).
fractal. This is consistent with our field observations on Hawaii that flow outlines are not fractals when slopes are steep.

Case 2b is nonlinear diffusion with a kinematic transport term. Case 3b is steady-state nonlinear diffusion equation. These are also likely candidates for producing fractals. Cases 2c and 3c are both nonlinear and are dominated by the pressure gradient term ($p>>1$). These cases may be expected to produce fractal behavior, but are difficult to interpret physically.

This analysis suggests that nonlinear processes are common in lava flows, consistent with the production of fractal outlines. It is likely that the margins are produced when nonlinear instabilities arise inside a flow and then grow to various sizes, producing a set of self-similar crenulations and protrusions. Once an instability begins to grow, feedback mechanisms allow further growth. But what exactly are the nonlinear processes that give rise to instabilities? The equations in our matrix give some clues. In Case 1c, the nonlinear diffusion equation is of the same form as studied by Nagatani (1991) for random particle deposition on flat surfaces. Although derived for understanding surface deposition processes, many aspects of the governing physics are similar. The processes of momentum and volume transport in lavas are similar to random deposition except that the instabilities in lava flows form inside the bounding surface, rather than onto it. Nagatani's (1991) studies indicate formation of fractal surfaces, suggesting that the lobate shapes of many lava flows result from instabilities in a nonlinear diffusive process associated with the conservation of lava volume. This is discussed further in Chapter 3.

Alternatively, lava flow outlines may be fractal due to processes involving viscous
fingering. In systems in which a low-viscosity fluid is injected into one of higher viscosity, instabilities develop (Saffman and Taylor, 1958); the instabilities lead to the development of fingers of the low-viscosity fluid in the more viscous one. Lava flows are like this in that they have a cool, viscous outer layer surrounding a hot, fluid interior. Instabilities result in formation of lobes and toes, analogous to viscous fingers. The process of viscous fingering has been modeled by diffusion-limited aggregation (e.g., Witten and Sander, 1981, 1983; Feder, 1988) in which random walking particles finally attach to a growing cluster. In lava flows, the random walkers can be viewed as carrying little chunks of melt with them, thus moving the boundary. The equations in cases 2c and 3c would be of the same form as equations producing viscous fingers if we relax the assumption $w << L$ (S. Baloga, pers. comm.). This might be most applicable to pahoehoe flows, where emplacement involves complex lava tube systems. Plan-view maps of tube systems are dendritic, fractal patterns much like those produced by diffusion-limited aggregation. (See Taylor, 1992 for a discussion of the Mauna Ulu lava tube system).

2.6 Results and Discussion: Silicic Lava Flows

Silicic lava flows are generally not fractals.

Silicic lava flows are generally not fractals within the range of scale studied (r: 10 m to 4.5 km). Typical Richardson plots for basalt, basaltic andesite, and dacite are shown in Fig. 2.11. Unlike the basaltic case, the Richardson plots for basaltic andesite
Figure 2.11. Richardson plots of flows of various compositions based on image data. (a) 'a'a basalt (Galapagos Islands). Note that (a) is closely approximated by a straight line, indicating fractal behavior.
Figure 2.11b. Richardson plot of a basaltic andesite (SiO$_2$ = 55%) from Hekla Volcano, Iceland based on photographic data. Note that, unlike (a), these data are not closely approximated by a straight line, indicating non-fractal behavior.
Figure 2.11c. Richardson plot of a dacite (SiO$_2$ = 66%) from Northern Chile based on image data. Note that, unlike (a), these data are not linear, indicating non-fractal behavior.
and dacite are non-linear, characterized by relatively low $R^2$ values. Instead of fractal behavior, these Richardson plots exhibit scale-dependent behavior: longer rod lengths have steeper slopes, most notably in the case of dacite. Thus, $D$ tends to increase as $r$ increases, contradicting the definition of $D$ as a scale-independent parameter. This breakdown of fractal behavior at increased silica content is presumably related to the higher viscosities and yield strengths, which suppress smaller-scale features and thus prevent self-similarity over a wide range of scales.

**Quantifying the Effect of Silica Content on $D$**

We seek to develop parameters that can be used remotely to quantify the effect of increasing silica content on fractal properties by comparing basalts, basaltic andesites, and dacites/rhyolites for two main purposes: (1) To gain insight into yield strength and rheological processes, and (2) to develop a remote sensing tool that can differentiate flow type based solely on plan-view shape. Our approach is to use the study of basaltic flows as a benchmark for comparison with the more silicic flows. However, we restrict our basaltic "benchmark" to a'a flows, which are similar to silicic flows in terms of both morphology and emplacement mechanism.

Ideally, we would like to compare $D$ of silicic flows to basaltic flows, to see if and how $D$ changes with silica content. However, this approach is tricky because, as noted above, silicic flows are generally not fractals; instead $D$ tends to increase with $r$ for the majority of the silicic flows. Hence, the concept of a scale-independent fractal dimension for silicic flows is not valid. However, small regions of Richardson plots
for silicic flows locally fit a line. Here we introduce the concept of a "local fractal dimension". This does not imply the data set is fractal, nor that the local fractal dimension is scale-independent. It simply exploits our observation that select regions of the data can be fit by a line and we can estimate locally the degree of convolution for a selected range of rod lengths. Here we describe two methods used to compare silicic and basaltic flows.

**Method 1: Disjoint subsets of log r**

This method dissects the abscissa of the Richardson plot into disjoint subsets of log r. The specific choice of subsets (summarized in Table 2.7a) is constrained by the data. Each of these subsets is fit locally by a least squares line; that is, the Richardson plot is fit by a series of lines. For each line, the slope \( m \) is calculated, and a local fractal dimension is calculated as \( D = 1 - m \), consistent with our methodology for basaltic flows. Since this method can be used to describe fractals as well as non-fractals, it can be employed to compare basaltic and silicic lava flows.

Figure 2.12 shows sample Richardson plots of basalt, basaltic andesite and dacite, with the abscissa dissected according to the methodology described above. The data on these plots are the same as shown in Fig. 2.11; the only difference is the number of lines used to fit the data. Note that for the basalt, the three segments have essentially the same slope. This is consistent with our conclusion that basalts are fractals. Unlike the basalts, the basaltic andesite and dacite show noticeable differences in slope among the various subsets.
Table 2.7a. Ranges of Log \( r \) (meters) for Method 1

<table>
<thead>
<tr>
<th>Log ( r ) (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range 1:</td>
</tr>
<tr>
<td>Range 2:</td>
</tr>
<tr>
<td>Range 3:</td>
</tr>
<tr>
<td>Range 4:</td>
</tr>
</tbody>
</table>

Table 2.7b. Ranges of Log \( r \) (meters) for Method 2

<table>
<thead>
<tr>
<th>Log ( r ) (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range 1:</td>
</tr>
<tr>
<td>Range 2:</td>
</tr>
<tr>
<td>Range 3:</td>
</tr>
</tbody>
</table>
Figure 2.12. Dissected Richardson plots of sample flows based on image data (same data as Fig. 2.11). Here, the abscissa is dissected into ranges of log r, and each range is locally fit with a straight line. See text regarding Method 1. (a) a'a basalt (Galapagos Islands). Note that the slopes of these lines are relatively constant, indicating fractal behavior.
Figure 2.12b. Dissected Richardson plot of a basaltic andesite (SiO₂ = 55%) from Hekla Volcano, Iceland based on photographic data (same data as Fig. 2.11b). Note that, unlike (a), the slopes of these best-fit lines are not constant, indicating non-fractal behavior.
Figure 2.12c. Dissected Richardson plot of a dacite (SiO$_2$ = 66%) from Northern Chile based on image data (same data as Fig. 2.11c). Note that, unlike (a), the slopes of these best-fit lines are not constant, indicating non-fractal behavior.
By plotting D of these segments vs. log r for the entire database of silicic flows, patterns begin to emerge among basaltic andesites and dacites/rhyolites (Fig. 2.13). Basaltic andesites have roughly the same D values for the first 2 subsets. At rod lengths of about 100 m (log r = 2 m), D tends to increase, and the values also have a greater range. For the first three subsets of log r, dacites/rhyolites have D plotting in a rather compact area, showing only negligible differences among the various ranges. At log r ~ 2.5 m, D begins to increase. We suggest that this technique can be used to distinguish basaltic andesites from dacites. Both have a general increase in D with longer r, but the basalt andesites tend to have greater D for each of these categories. Furthermore, dacites/rhyolites show negligible changes within the first three subsets (log r < 2.5 m), whereas the basaltic andesites only remain relatively constant for the first two subsets (log r < 2.0 m). Figure 2.13 also emphasizes that D is not a constant function of log r for both basaltic andesites and dacites/rhyolites, indicating scale-dependent (non-fractal) behavior. Fractals would be expected to have relatively constant fractal dimensions across the various subsets, as shown for basalts in Fig. 2.14. However, a sufficiently large range of log r is needed to discern fractal and non-fractal behavior. Note that a'a basalts and dacites have a similar range of fractal dimensions for the first three categories. Since data limitations often prevent obtaining a large range of log r, we invoke a second method to differentiate a'a basalts from dacites, described below.

Method 2: Overlapping subsets of log r

Like Method 1, this method dissects the abscissa of the Richardson plot into
Figure 2.13. Summary diagram of "local fractal dimension" (D) based on image data of basaltic andesites (open circles) and dacites and rhyolites (solid triangles). D is calculated as $D = 1 - m$, where $m$ is the local slope as calculated according to Method 1. Note that $D$ is not a constant function of log $r$, indicating non-fractal behavior.
Figure 2.14. Summary diagram of "local fractal dimension" (D) based on field and photographic data of a'a basalts. D is calculated as $D=1-m$, where $m$ is the local slope as calculated according to Method 1. Note that D is a constant function of log r, consistent with fractal behavior.
distinct regions of log r. However, it is different from the previous method in two respects. First, the selected ranges (as summarized in Table 2.7b) of log r are overlapping. Although the exact choice of ranges is again constrained by the data, they were intentionally chosen to overlap. This is to explicitly show the effect of a restricted range of rod lengths on local fractal dimension. For example, by comparing Region 1 (log r: 1.7 - 2.8 m) and Region 2 (log r: 1.7 - 2.5 m), we can explicitly see the effect of the restricted range (2.5 - 2.8 m) on fractal parameters. Additionally, we calculate the value of the corresponding correlation coefficient (R²). Second, as these regions span a greater range of log r than those in Method 1, we have sufficient data points to fit a second-order least squares curve to the data, in addition to the standard first-order least squares line. In this method, we fit a curve of the form \( y = ax^2 + mx + b \), and note the value of the leading (or quadratic) coefficient \( a \). In summary, this method compares three quantitative parameters (D, R², a) for three overlapping ranges of log r.

Figure 2.15 shows the results of Method 2 applied to dacites and rhyolites. In Fig. 2.15a, note that fractal dimension varies systematically with range of rod length. In 4 out of 5 cases, the longest rod lengths have the highest D and the shortest rod lengths have the lowest D. Figure 2.15b shows the corresponding R² values. The shortest rod lengths have the highest R² values, whereas the longest rod lengths have lowest R². Fitting a quadratic curve (Fig. 2.15c), the longest rod lengths for each flow all have negative \( a \). For the shortest rods, \( a \) can be either positive or negative. The D, R² and \( a \) for the dacites/rhyolites provide remarkably consistent results: all suggest scale-dependent (or non-fractal) behavior, characterized by an increase in D with increasing r. We attribute this to the suppression of small-scale features, due to the
Figure 2.15. Plots (a) through (c) show the results of Method 2 applied to dacites and rhyolites. In each plot, the x-axis is individual flows. Each flow has 3 values, corresponding to the three ranges of rod lengths summarized in Table 2.7b: triangles (shortest rods), circles (intermediate rods) and squares (longest rods). (a) Fractal dimension (D). For each flow (each column), the shortest rods tend to have the lowest D and the longest rods have the highest D, indicating non-fractal behavior (see text for discussion).
Figure 2.15b. $R^2$ values. Shortest rods (triangles) have the highest values of $R^2$, whereas the longest rods (squares) have lowest $R^2$. This indicates non-fractal behavior (see text for discussion).
Figure 2.15c. Quadratic coefficient ($a$). The longest rods (squares) all have negative $a$, or an increase in D with longer r. For the shortest rods (triangles), $a$ can be either positive or negative. Like (a) and (b), this plot indicates non-fractal behavior (see text for discussion).
higher viscosities and yield strengths of silicic flows. We suggest the margin appears "linear" to a certain range of small rod lengths because the scale of features they would otherwise detect are suppressed. This explains a fractal dimension close to 1 for the shortest range of rod lengths and, as expected, the corresponding $R^2$ values are quite high. We interpret the results of Fig. 2.15 to suggest that the shortest range of rod lengths ($\log r: 1.3 - 2.0$) measures the flow margin at scales below the limit of self-similarity.

Figure 2.16 shows the results of Method 2 applied to basalts. For consistency, we chose the same ranges of $\log r$ as for dacites/rhyolites; thus only a small number of basalt flows could be included (e.g., all field data had to be excluded). In Fig. 2.16a, note that fractal dimension shows no systematic variation with range of rod length. Figure 2.16b, which shows the corresponding $R^2$ values, again shows no systematic variation among the ranges. Note that the $R^2$ values are high, generally exceeding 0.95 and that $a$ can be positive or negative, and is generally close to zero (Fig. 2.16c). Again, there is no systematic pattern among the various ranges. These results for $D$, $R^2$ and $a$ for basalts all suggest fractal behavior.

We believe we can use these fractal parameters to remotely differentiate flow types. Basaltic a'a and basaltic andesites can be distinguished primarily by their $D$ values; basaltic andesites generally have higher $D$ ($\geq 1.15$) than basaltic a'a ($D: 1.05 - 1.09$) and are less likely to exhibit fractal behavior. Although dacites/rhyolites and basalts have similar fractal dimensions for extensive ranges of $\log r$ (1.05 - 1.10), dacites and rhyolites distinctly show non-fractal behavior. Systematic evaluation of $D$, $R^2$, and $a$ at different ranges of rod lengths (as done in Method 2) could be used to remotely distinguish dacites and rhyolites from basalts.
Figure 2.16. Plots (a) through (c) show the results of Method 2 applied to basalts. In each plot, the x-axis is individual flows. Each flow has 3 values, corresponding to the three ranges of rod lengths summarized in Table 2.7b: triangles (shortest rods), circles (intermediate rods) and squares (longest rods). (a) Fractal dimension (D). There is no systematic variation of D among the various ranges, indicating fractal behavior (see text for discussion).
Figure 2.16b. $R^2$ values. $R^2$ are generally high, and show no systematic variation among the various ranges. This is consistent with fractal behavior (see text for discussion).
Figure 2.16c. Quadratic coefficient \(a\). \(a\) can be positive or negative and is generally close to zero for all ranges of log \(r\). Like (a) and (b), this plot indicates fractal behavior (see text for discussion).
There may be a critical value of $r$, related to silica content, which serves as a boundary for self-similar behavior (i.e., a value of $r$ above which the flow appears fractal). This critical value may be related to lobe dimensions and/or the degree of suppression of smaller-scale features. Note that dacites/rhyolites show a marked increase in $D$ after about log $r$ of 2.5 m ($r = 300$ m); see Fig. 2.13. This may be related to the lobe width of dacites, typically hundreds of meters. If so, we would expect the apparent $D$ of basaltic andesites to increase at shorter rod lengths. This may be suggested by Fig. 2.13 but our database is too small to be conclusive. We believe that a larger database of silicic flows would reveal a critical value of breakdown of fractal behavior related to silica content. The fact that basaltic andesites appear to have relatively constant fractal dimensions up to log $r = 2$ m while dacites/rhyolites appear to have relatively constant fractal dimensions up to log $r = 2.5$ m suggests an effect of yield strength which is related to silica content. Our field observations show that fractal behavior for basalts also breaks down, but at $r < 10$ cm.

We investigate this hypothesis by simulating suppression of smaller-scale features on a synthetic fractal called the Koch Triad. This curve is computer-generated by systematically applying a generator function to an equilateral triangle (Fig. 2.17a). Applying the generator an infinite number of times results in a true mathematical fractal. We apply the generator a finite ($n$) number of times ($n=4$ and $n=2$) to create triads (see Figs. 2.17b and 2.17c). Note that a triad constructed at the $n=4$ approximates fractal behavior (for all but the tiniest rod lengths), while an $n=2$ triad is clearly not fractal. Theoretically, this is identical to starting with an ideal fractal and filtering out the smaller-scale features, causing it to no longer be fractal. Applying the "structured-walk" methodology described above, we generate Richardson plots (Fig. 2.18). For
Figure 2.17a. Generator function for Koch triad, applied to a straight line N number of times (Feder, 1988). Replacing each side of a triangle with this generator function applied an infinite number of times results in an ideal fractal. Alternately, we can apply the generator function a finite number of times, to generate the modified Koch triads shown in (b) and (c).
Figure 2.17. Modified Koch triad at the (b) N=4 level (i.e., generator is applied four times) and (c) N=2 level (i.e., generator is applied twice). We compare (b) with basalt, as both are self-similar over a wide range of scales. We compare (c) with more silicic flows (e.g., dacite), where high viscosities and yield strengths suppress smaller-scale features.
Figure 2.18a. Richardson plot of Fig. 2.17b (N=4 level). Axes are in data numbers. Note that (a) is fractal, resembling the Richardson plot for basalt (Fig. 2.12a).
Figure 2.18b. Richardson plot of Fig. 2.17c (N=2 level). Axes are in data numbers. Unlike (a), this Richardson plot is non-linear, indicating non-fractal behavior. Note this plot resembles the Richardson plot for dacite (Fig. 2.12c).
the range of rod length selected (selected sufficiently large such that Fig. 2.17b appears fractal), the Richardson plot for the Koch triad at the n=4 level is fractal (D= 1.26, R² = .95); see Fig. 2.18a. However, for the same range of rod length, the Koch triad at the n=2 level has a Richardson plot that is distinctly non-linear (not fractal; Fig. 2.18b), with a breakdown of fractal behavior at some critical value of r. This critical value is related to the size of the small-scale features suppressed. We liken the n=4 case to basaltic flows, and the n=2 case to silicic flows; note the similarity with Figs. 2.11a and 2.11c. Silicic flows may also have critical values, and may be remotely distinguished by these values.

The suppression of smaller-scale features in silicic flows implies that nonlinear instabilities are also suppressed inside the flows. Either the sluggish rheology prevents their formation, or it prevents their growth by rapidly damping out feedback mechanisms. The generally nonfractal nature of the margins of silicic flows is consistent with our simplified flow model (Fig. 2.10). Viscosities of silicic flows can be very large: \( \nu > 10^6 \text{ Pa s} \) for basaltic andesites and \( \nu > 10^8 \text{ Pa s} \) for dacites and rhyolites, thus \( q << 1 \). Thus, unless the flows have very large initial pressures, it is likely that their behavior would tend to be linear.

2.7 Conclusions

_Basaltic Lava Flows Are Fractals_

Bruno _et al._ (1992) suggested that basaltic lava flows are fractals, with pahoehoe
flow margins having higher fractal dimension (1.13 - 1.23) than a'a flow margins (1.05-1.09). This study, based on a larger database (45 flows) and over a wider range of scale (0.125 m - 2.4 km), confirms that earlier conclusion. Richardson plots are consistently linear, characterized by high R² values. Furthermore, we have shown that basaltic lava flows having transitional morphologies also exhibit fractal behavior, and tend to have dimensions intermediate between a'a and pahoehoe. This indicates that basaltic lavas, regardless of the emplacement mechanism, exhibit self-similar behavior. We interpret this to suggest that basalts are sufficiently fluid and lack a sizable yield strength, offering no obstacle to deter the formation of small-scale self-similar features.

Silicic Flows Are Generally Not Fractals

Unlike basalts, silicic lava flows tend to exhibit scale-dependent (non-fractal) behavior within the range of scale studied (r: 10 m - 4.5 km). Typical Richardson plots for basaltic andesites and (especially) the more silicic dacites and rhyolites are non-linear. This breakdown of fractal behavior at increased silica content is presumably related to the higher viscosities and yield strengths, which suppress smaller-scale features.

Flow dynamics are nonlinear

Our observations that basaltic lava flows have fractal outlines when they are internally controlled yet have non-fractal outlines when they are controlled by
gravitational forces are consistent with our theoretical model. An assessment of flow
dynamics suggests that nonlinear processes operate for lava flow emplacement on
relatively flat slopes. These nonlinear mechanisms are damped out in silicic flows,
leading to nonfractal margins, especially at small rod lengths.

Quantifying the Effect of Rheology

One of the primary objectives of this study is to remotely distinguish flow types. We suggest that fractal dimension (or local fractal dimension), correlation coefficient, and quadratic coefficient can be used, in combination, to attain this objective. We define "local fractal dimensions" for select ranges of log r, and find that D tends to increase with increasing r after certain critical rod lengths are exceeded. We can use local fractal dimension to differentiate basaltic andesites from dacites, provided we are sensitive to the range of rod lengths used in constructing Richardson plots. Although basaltic a'a and dacites have similar fractal dimensions over a wide range of r, the parameters R^2 and a can be used to remotely differentiate between these flow types.
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CHAPTER 3

AN EXACT ANALYTIC SOLUTION
FOR GRAVITY-DRIVEN FLOWS ON AN INCLINED PLANE

In preparation for publication (jointly with Chapter 4)

With Co-authors: S.M. Baloga and G.J. Taylor.

3.1 Abstract

In this chapter, we find an exact analytic solution for unconfined flows having an arbitrary power-law rheology and advancing on an inclined plane. We consider time-dependent flow movement to be the result of competition between two forces: gravitational transport and magmastatic pressure. We examine how these forces combine to drive flow movement in the downstream and cross-stream directions by adopting a volume conservation approach. Simplifying assumptions reduce the governing equation to the dimensionless form:

$$\frac{\partial}{\partial x}(\omega h^n) = \frac{\partial}{\partial y} \left( \omega h^n \frac{\partial h}{\partial y} \right)$$

(3.1)
where \( x \) and \( y \) are the spatial coordinates in the downstream and cross-stream directions, respectively; \( h \) is the flow depth (thickness); and \( a = a(x) \) and \( m \), a positive constant, are both prescribed by the rheology of the fluid. Smith (1973) solved this equation analytically for a Newtonian fluid \( (m=3) \) with a constant rheology \( (a = 1/3v) \) where \( v \) is viscosity) using a similarity transformation. This method involves replacing the variables appearing in the free-boundary problem for eq. (3.1) with other variables that preserve the invariance of the governing equation and reduce it to a nonlinear ordinary differential equation. By invoking additional transformations of the dependent and independent variables, we use this same procedure to obtain an analytic solution for flows of arbitrary rheology. Using the same boundary conditions, our solution reduces to that of Smith (1973) for the Newtonian case with a constant rheology.

Our solution determines changes in flow depth (thickness) and width with distance from the source of the flow for different rheological characteristics. Consequently, these results may be used to infer the rheology of unconfined geologic deposits, such as lava flows, lahars, or mud flows, from the geometric dimensions of the deposits. Chapter 2 showed that basaltic lava flow margins are fractal, i.e., the features that comprise the margin have a power-law distribution. This work also explores the relationship between the fractal properties of the flow margins of fluids with rheologies consistent with eq. (3.1) and the underlying fluid dynamic processes. We suggest that the fluid dynamic processes and physical principles used in developing the volume conservation equation (eq. 3.1) which determine the power-law dependence of our solution may be the same as those that govern the fractal nature of the lava flow margins. The analysis investigates the quantitative details of
this conjecture, including sensitivities to the parameters of the rheological variation along the flow path and the effects of different magma compositions.

3.2 Introduction

In Chapter 2, we developed a technique to glean information regarding flow emplacement and rheology from the fractal properties of lava flow margins. We found that, as frozen snapshots of the final moments of flow, plan-view shapes hold important information regarding lava flow dynamics and rheology. In this chapter and the following one, we again exploit the final shape of a flow as a source of rheological information, using an altogether different method. This method is based on the geometric dimensions of the lava flow (or other static eruptive product): flow depth and width. Our approach is to first develop a differential equation and then solve it. Our solution models changes in flow depth and width with distance from the source of the flow for different rheological characteristics. In Chapter 4, we show how this model can be applied to lava flow data, both in the field and from remote sensing images, to infer or constrain the flow rheology.

As derived in Chapter 2, the conservation of volume for an unconfined flow of depth $h = h(x, y, t)$ on an inclined plane is described by:

$$ w \frac{\partial h}{\partial t} + \bar{\nabla} \cdot \bar{Q} = 0 $$

(3.2a)

where

$$ \bar{\nabla} = \frac{\partial}{\partial x} \bar{i} + \frac{\partial}{\partial y} \bar{j} $$

(3.2b)
and where $w$ is half-width; $x$ and $y$ are spatial coordinates in the downstream and cross-stream directions, respectively; $t$ is time; and $Q$ is the volumetric flow rate. This is equivalent to the expression:

$$\frac{\partial h}{\partial t} + \nabla \vec{q} = 0$$  \hspace{1cm} (3.3a)

where the volumetric flow rate per unit width is given by:

$$\vec{q} = \frac{\vec{Q}}{w}$$  \hspace{1cm} (3.3b)

Eqs. (3.2a) and (3.3a) are forms of the first-order conservation equation relating flow of a given variable (i.e., $h$) to its time rate of change. Solutions to these equations are called kinematic waves (e.g., Lighthill and Whitham, 1955; Baloga, 1987). The reader is directed to these references for a discussion of the conservation equation and its relation to kinematic waves.

In this work, we consider volumetric flow driven by both gravitational transport and the magmastatic pressure gradient. Gravity drives flow movement only in the downstream ($x$) direction, as the $y$-direction is chosen perpendicular to slope. The magmastatic pressure gradient, which is due to the curved upper surface of the flow, operates in both the downstream and cross-stream directions. For Newtonian fluids, the downstream, cross-stream and net volumetric flow rates (per unit width) are respectively given by:

$$q_x = \frac{g}{3v} \left( \sin \theta h^3 - \cos \theta h^3 \frac{\partial h}{\partial x} \right)$$  \hspace{1cm} (3.4a)
\[ q_y = -\frac{g}{3\nu} \left( \cos \theta h^j \frac{\partial h}{\partial y} \right) \]

\[ \bar{q} = -\frac{g}{3\nu} \cos \theta h^j \bar{v} h + \frac{g}{3\nu} \sin \theta h^j \bar{l} \]

where \( g \) = gravitational acceleration; \( \nu \) = viscosity; and \( \theta \) = groundslope.

We generalize the volumetric flow rate per unit width given for Newtonian flows in eq. (3.4c) to include arbitrary, non-Newtonian rheologies. We assume this generalized volumetric flow rate per unit width has the form:

\[ \bar{q} = -\alpha \cos \theta h^m \bar{v} h + \alpha g \sin \theta h^m \bar{l} \]

where \( \alpha = \alpha(x) \) and \( m \) are prescribed by the rheology of the fluid.

Equation (3.5) assumes a power-law dependence on the depth of the flow and an arbitrary spatially dependent change in the flow rheology. This power-law dependence may be related to the formation of fractal outlines, and will be discussed further below. Like eq. (3.4c), eq. (3.5) expresses the influence of gravity in the downstream \( (x) \) direction and the influence of magmatic pressure as a function of the gradient of the depth of the flow. For Newtonian fluids of constant rheology \( (m=3 \text{ and } \alpha=1/3\nu, \text{ where } \nu \text{ is viscosity}) \), eq. (3.5) reduces to eq. (3.4c). Smith (1973) found an analytical solution for this special case. For non-Newtonian flows, the parameter \( m \) may be greater than or less than 3, but it is required to be a positive constant. For flows of non-constant rheology, the parameter \( \alpha \) is free to vary.
spatially. However, this spatial dependence is permitted only in the downstream direction; any cross-stream variations in rheology are assumed negligible. Like \( m \), \( \alpha(0) \) is a positive constant.

Many different types of transport processes have been modeled with a flow rate of the form shown in eq. (3.5) featuring a power-law dependence on flow depth \((i.e., q = h^m)\), a spatially dependent prefactor \( \beta \) (which in eq. (3.5) depends on \( \alpha(x) \) and the surface slope), and a positive constant \( m \). Rainfall runoff has been modeled in this manner with \( m > 1 \) and \( \beta \) dependent on the surface roughness (Sherman, 1978). Sherman (1981) similarly modeled channel flow with infiltration, with \( \beta \) dependent on the underlying slope. As noted by Weir (1982), \( m \) values of 3/2 and 5/3 give the Chezy and Manning laws, respectively, for rivers with \( \beta \) dependent on the square root of the slope. The Shamov law is given by \( m=1/6 \) with \( \beta \) independent of slope, whereas the Sribniy law is given by \( m=2/3 \) with \( \beta \) dependent on the fourth root of slope. Both the Shamov and Sribniy laws have been used to describe transport of mud flows (Gol’din and Lyubashevskiy, 1966). Sediment transport in alluvial streams has been modeled with \( m=9/4 \) and a nearly linear dependence of \( \beta \) on slope (Engelhund and Hansen, 1972; chapter 4). In quantifying the risk from volcanic lahars, Weir (1982) obtained \( m \) values in the range 1.2 to 2.0 with a \( \beta \) dependence on the underlying slope similar to that of empirical river laws. Baloga and Pieri (1986) and Baloga (1987) have used \( m=2 \) for the emplacement of lava flows with \( \beta \) spatially dependent on the viscosity of the lava and the local topographic slope of the flow bed. Laminar flow of glaciers has been similarly modeled: experimentally-obtained \( m \) values range from 1.5 to 3.9 with a mean of about 2.5 and \( \alpha \) is dependent on the local
temperature, density, and slope (Paterson, 1969; chapter 6). In modeling solute transport in soils, a power-law dependence between \( q \) and water content in the flow paths has been used, with \( m = 2.5 \) and \( \beta \) dependent on the local characteristics of the soil macropores (Beven and Germann, 1981; Germann et al., 1986; Levy and Germann, 1988). Other diverse applications based on such a form for flow rate include surface irrigation (Sherman and Singh, 1978) and dam-bursting (Hunt, 1982); see Weir (1983) for a thorough review of the literature. The classic treatment of such flow rate forms derives from the problem of river flooding (Lighthill and Whitham, 1955) with other notable early papers addressing the response of glaciers to various environmental factors (e.g., Nye, 1960, 1963). These \( m \) values are summarized in Table 3.1.

Smith (1973) showed that substituting the flow rate for a Newtonian fluid (eq. 3.4c) into the volume conservation law (eq. 3.3a) generates a differential equation having the form of the nonlinear diffusion equation under certain restrictions and simplifications with free boundaries on the surface of the flow. Obtaining this differential equation requires that gravity is the predominant influence on the downstream motion (i.e., the influence of the magmastatic pressure gradient in the downstream direction is relatively small) while the concomitant lateral expansion of the flow is due solely to magmastatic pressure. In the steady-state, Smith (1973) found a similarity solution that matches the free-boundary condition for the lateral expansion of the flow as a function of distance from the source. A critical assumption of the formulation of the problem is that the volumetric flow rate in the downstream direction represents a conserved quantity.

In this paper, we generalize the volumetric flow rate used by Smith (1973) to
Table 3.1. Summary of Transport Processes and m values

<table>
<thead>
<tr>
<th>Transport Process</th>
<th>m value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transport models of volcanic lahars</td>
<td>1.2 - 2</td>
<td>(1,2)</td>
</tr>
<tr>
<td>Transport models of mud flows</td>
<td>1/6 - 2/3</td>
<td>(3)</td>
</tr>
<tr>
<td>Chezy and Manning laws (rivers)</td>
<td>3/2 and 5/3</td>
<td>(1,2)</td>
</tr>
<tr>
<td>Solute transport in soils</td>
<td>2.5</td>
<td>(4)</td>
</tr>
<tr>
<td>Solute transport in alluvial streams</td>
<td>9/4</td>
<td>(5)</td>
</tr>
<tr>
<td>Emplacement of lava flows</td>
<td>2</td>
<td>(6,7)</td>
</tr>
<tr>
<td>Emplacement of Newtonian fluids</td>
<td>3</td>
<td>(8)</td>
</tr>
<tr>
<td>Laminar flow of glaciers</td>
<td>1.5 - 3.9</td>
<td>(9)</td>
</tr>
</tbody>
</table>

References: (1) Weir (1982); (2) Weir (1983); (3) Gol'din & Lyubashevskiy (1966); (4) Levy and Germann (1955); (5) Engelhund and Hansen (1972); (6) Baloga (1987); (7) Baloga and Pieri (1986); (8) Smith (1973); (9) Paterson (1969).
embrace an arbitrary power-law dependence on the depth of the flow and an arbitrary spatially dependent change in the flow rheology. To solve the resulting nonlinear diffusion equation for volume conservation, we show that there exists a transformation of the differential equation into the original form of Smith (1973) and obtain corresponding solutions for different types of rheologies. As required by the case of a Newtonian fluid of constant rheology (Smith, 1973), the generalized case requires both that gravity is the predominant influence on the downstream motion and the accompanying widening of the flow downstream is due solely to magmastatic pressure.

3.3 The Steady State Similarity Solution

The solution described in this section applies to the steady-state, i.e.,

\[ \frac{\partial h}{\partial t} = 0 \]

(3.6a)

The steady-state (or time-independent) solution is presumably the asymptotic form of physically reasonable time-dependent solutions. In the steady-state, eq. (3.3a) reduces to:

\[ \bar{V}\bar{q} = 0 \]

(3.6b)

where the volumetric flow rate per unit width is assumed to be of the form described in eq. (3.5).

At the source of the flow, we have the boundary conditions:
\[ h(x = 0, y = 0) = h_0 \]  \hspace{1cm} (3.7a)

\[ h(x = 0, y = \pm w_0) = 0 \]  \hspace{1cm} (3.7b)

where \( h_0 \) and \( w_0 \) are prescribed positive constants which represent the depth and extent of the flow, respectively, at the source. Because the flow margin is a free boundary and can expand according to the local dynamics, we must also find the function \( w = w(x, t) \) such that, for all \( t \geq 0 \),

\[ h(x, y = \pm w) = 0 \]  \hspace{1cm} (3.7c)

that is, the flow has an edge and is therefore of finite width. We do not require a comparable boundary condition be satisfied in the downstream direction, thus allowing the flow to have infinite length.

To obtain an exact analytic solution in the steady-state, we convert the steady-state partial differential equation and supplementary conditions to dimensionless form using the following substitutions:

\[ h \Rightarrow h_0 h^* \]  \hspace{1cm} (3.8a)

\[ x \Rightarrow L x^* \]  \hspace{1cm} (3.8b)

\[ y \Rightarrow w_0 y^* \]  \hspace{1cm} (3.8c)

\[ w \Rightarrow w_0 w^* \]  \hspace{1cm} (3.8d)

\[ \alpha \Rightarrow \alpha_0 \alpha^* \]  \hspace{1cm} (3.8e)
where asterisked parameters are dimensionless and the downstream length scale \((L)\) is yet to be determined. Unlike the other scaling factors \((h_0, w_0, \alpha_0)\), the parameter \(L\) does not represent a condition at the source, nor is it constrained by flow dimensions. Thus, it is a free parameter which can be defined in any manner to simplify the problem.

Combining eqs. (3.5), (3.6) and (3.8), the resulting dimensionless partial differential equation is:

\[
\frac{\partial}{\partial x} \left( \alpha^* h^m \frac{\partial h^*}{\partial x} \right) + \left( \frac{w_0}{L} \right)^2 \frac{\partial}{\partial x} \left( \alpha^* h^m \frac{\partial h^*}{\partial x} \right) = \left( \frac{\sin \theta w_0}{\cos \theta L h_0} \right) \frac{\partial}{\partial x} \left( \alpha^* h^m \right) \tag{3.9a}
\]

or, dropping asterisks to simplify notation (but remembering that all terms are dimensionless!),

\[
\frac{\partial}{\partial x} \left( \alpha h^m \frac{\partial h}{\partial x} \right) + \left( \frac{w_0}{L} \right)^2 \frac{\partial}{\partial x} \left( \alpha h^m \frac{\partial h}{\partial x} \right) = \left( \frac{\sin \theta w_0}{\cos \theta L h_0} \right) \frac{\partial}{\partial x} \left( \alpha h^m \right) \tag{3.9b}
\]

The corresponding dimensionless form of the supplementary conditions is given by:

\[
h(x = 0, y = 0) = 1 \quad (3.10a)
\]
\[
h(x = 0, y = \pm 1) = 0 \quad (3.10b)
\]
\[
h(x, y = \pm w(x)) = 0 \quad (3.10c)
\]
\[
\frac{\partial h}{\partial y}(x, y = 0) = 0 \quad (3.10d)
\]
\[
w(0) = 1 \quad (3.10e)
\]
\[
\alpha(0) = 1 \quad (3.10f)
\]
In the interest of simplifying equation (3.9b), we now define the length scale:

\[ L = \frac{w_0^2}{h_0} \tan \theta \]  

(3.11a)

Furthermore, we now consider flow regimes such that:

\[ \left( \frac{w_0}{L} \right)^2 \frac{\partial}{\partial x} \left( \alpha h^m \frac{\partial h}{\partial x} \right) \ll 1 \]  

(3.11b)

This allows us to ignore the second term in eq. (3.9b) and brings the partial differential equation (3.9b) into the form:

\[ \frac{\partial}{\partial x} \left( \alpha h^m \right) = \frac{\partial}{\partial y} \left( \alpha h^m \frac{\partial h}{\partial y} \right) \]  

(3.12)

Specifically, eq. (3.11b) requires that the influence of the magmastatic pressure in the downstream direction is small compared to the direct gravitational transport of fluid elements and the cross-stream influence of magmastatic pressure, and can be disregarded. When the geometric considerations for a flow indicate that eq. (3.11b) is satisfied, solutions to eq. (3.12) describe how flow depth changes, both downstream and laterally. We can then solve for the unknown function \( w = w(x) \), which describes how flow width changes downstream.

Equation (3.12) describes the non-linear character of the magmastatic pressure term as the flow evolves according to the gravitational transport term. We call attention to the similarity between eq. (3.12) and the steady-state form of the non-linear diffusion equation. Building on earlier work on random particle deposition models (Family, 1986; Nagatani, 1991) introduced non-linear surface diffusion into
the random deposition model. Assuming the diffusivity $D(h)$ behaves as a power law $h^k$, the growth of an interface from a substrate can be described by the right-hand side of eq. (3.12), where $h(x,t)$ is the height of the interface at position $x$ and time $t$. As this equation was found to produce self-affine fractals (Nagatani, 1991), we suggest this nonlinear "diffusive" character of the magmastic pressure may be responsible for forming the fractal outlines discussed in Chapter 2.

Smith (1973) solved eq. (3.12) analytically for a Newtonian fluid with a constant rheology using the method of similarity transformation. This is a transformation of the independent variables of a partial differential equation to new independent variables (called similarity variables) in such a manner that the number of independent variables is reduced. If the given equation has only two independent variables (e.g., eq. (3.12)), the transformed equation is an ordinary differential equation, having only one similarity variable. Under similarity transformations, the given equation is required to remain absolutely invariant (Ozisik, 1980). Similarity transformations are invoked with the intention of finding physically meaningful analytic solutions to difficult nonlinear equations.

Smith (1973) showed that for Newtonian fluids (i.e., $\alpha(x)=1/3v$, where $v=$viscosity and $m=3$), a remarkable similarity solution does exist, which gives the explicit form of $w(x)$ as a natural by-product. Our approach requires transformation of eq. (3.12) to the form obtained by Smith, but our subsequent analysis highlights some important differences in the dynamics of the problem.

We begin our approach by transforming away the dependence on the arbitrary function $\alpha(x)$ in the governing equation (3.12). This function characterizes the
spatially-dependent resistance to flow caused by viscosity, surface friction and any other forces. Here, we rid our equation of \( \alpha(x) \), leaving a transformed equation that can be more readily solved by similarity methods, even though the arbitrary function remains embedded in the dependent and independent variables. Instead of a differential equation for \( h \), we transform the partial differential equation to one that applies to a related variable \( \xi \) defined by:

\[
h(x, y) = \alpha(x)^{-\nu/m} \xi(x, y)
\]

(3.13)

Instead of the independent variable \( x \), we use a related variable \( z \):

\[
z = \int_{0}^{x} \frac{dx'}{\alpha(x')^{1/m}}
\]

(3.14)

With these new variables, eq. (3.12) becomes:

\[
\frac{\partial}{\partial z} (\xi^m) = \frac{\partial}{\partial y} \left( \xi^m \frac{\partial \xi}{\partial y} \right)
\]

(3.15a)

Explicitly differentiating (3.15a) yields the equivalent expression:

\[
\frac{\partial \xi}{\partial z} = \xi \frac{\partial^2 \xi}{\partial y^2} + \left( \frac{\partial \xi}{\partial y} \right)^2
\]

(3.15b)

Equation (3.15) is a partial differential equation with one dependent variable (\( \xi \)) and two independent variables (\( y, z \)). It is a transformed version of eq. (3.12) which also has one dependent variable (\( h \)) and two independent variables (\( x, y \)). We now apply a similarity construct for the purpose of transforming eq. (3.15) into an ordinary
differential equation with one dependent variable ($G$) and one independent variable ($\eta$). Following Smith (1973), we introduce the similarity construct:

$$\xi(\eta,z) = (1 + az) G(\eta)$$  \hspace{1cm} (3.16)

with

$$\eta = y(1 + az)^r$$  \hspace{1cm} (3.17)

where $a$, $r$ and $q$ are positive constants to be determined.

This construct will constitute a similarity solution provided: (1) the constants $a$, $r$ and $q$ can be determined; (2) the partial differential equation (3.15b) reduces to an ordinary differential equation; and (3) the resulting ordinary differential equation has a solution that satisfies boundary conditions corresponding to those appearing in eq. (3.10).

With the transformations shown in eqs. (3.16) and (3.17), the supplementary conditions shown in eq. (3.10) translate to:

G(0) = 1 \hspace{1cm} (3.18a)

G'(0) = 0 \hspace{1cm} (3.18b)

G(\pm 1) = 0 \hspace{1cm} (3.18c)

The reader may check, by explicitly differentiating eq. (3.16), that:

$$\frac{\partial \xi}{\partial z} = a(1 + az)^{r-1} \left[ rG + q\eta G' \right]$$  \hspace{1cm} (3.19a)
Substituting the expressions of eq. (3.19) into eq. (3.15b) yields:

$$a(1+az)^{-1}[rG + q\eta G'] = (1+az)^{2r+2q}\left[\frac{GG''}{m} + G'^2\right]$$  

Note that the factor \((1+az)\) appears on each side of eq. (3.20) with different exponents. By definition, similarity transformations are absolutely invariant; they necessarily preserve power relationships. Thus, we require the constants \(r\) and \(q\) satisfy:

$$2r + 2q = r - 1$$  

The governing equation becomes:

$$\left[\frac{GG''}{m} + G'^2\right] - a[rG + q\eta G'] = 0$$  

and we look for a solution of the form:

$$G = c_1 + c_2\eta + c_3\eta^2$$
noting that there may be other solutions to eq. (3.22) that are not of the form of eq. (3.23). We now find a solution to eq. (3.23) that satisfies the boundary conditions listed in eq. (3.18):

From eq. (3.18a):
\[ c_1 = 1 \]  \hspace{1cm} (3.24a)

From eq. (3.18c):
\[ c_2 = 0, \quad c_j = 1 \]  \hspace{1cm} (3.24b)

Thus, equation (3.23) becomes:

\[ G = 1 - \eta^2 \]  \hspace{1cm} (3.25)

The reader may check that the solution shown in eq. (3.25) also satisfies eq. (3.18b), as required. Substituting eq. (3.25) into eq. (3.22):

\[ \frac{-2}{m}(1 - \eta^2) + 4\eta^2 - a(r + \eta^2) (-r - 2q) = 0 \]  \hspace{1cm} (3.26a)

and, from eq. (3.21),

\[ \frac{-2}{m}(1 - \eta^2) + 4\eta^2 - a[r + \eta^2] = 0 \]  \hspace{1cm} (3.26b)

We now wish to solve for the three unknown constants \((a, r, q)\). All three constants appear in the similarity construct and can be related explicitly to \(m\), the only parameter of the transformed differential equation (3.12). By equating coefficients of powers of \(\eta\) in eq. (3.26b), we obtain two conditions for the constants \(r\) and \(a\), required for a solution of the governing equation (3.22).
Equating coefficients of $\eta^2$:

$$\frac{2}{m} + 4 - a = 0$$

(3.27a)

Equating coefficients of $\eta^0$:

$$\frac{-2}{m} - ar = 0$$

(3.27b)

With eq. (3.21), we thus have three equations for three unknowns $(a, r, q)$. The result is:

$$a = \frac{2(1 + 2m)}{m}$$

(3.28a)

$$r = \frac{-1}{1 + 2m}$$

(3.28b)

$$q = \frac{-m}{1 + 2m}$$

(3.28c)

This completes the solution of the problem in the sense that all constants required by the similarity construct are given in terms of the parameter $m$ and, once some form of $\alpha(x)$ has been chosen, eqs. (3.14), (3.16), (3.17), and (3.25) can be used to relate the integral in eq. (3.13) back to the $h$ variable. The solution is:

$$h(x, y) = \alpha^{1/m} \left[ 1 + a \int_0^x \frac{d\alpha'}{\alpha'^{1/m}} \right] \left[ 1 - y^2 \left( 1 + a \int_0^x \frac{dx'}{\alpha'^{1/m}} \right)^2 \right]$$

(3.29)

where $a, r, q$ are given in terms of $m$ in eq. (3.28) and $\alpha = \alpha(x)$ is arbitrary. We
note that, for Newtonian fluids \((m=3)\), our solution reduces to Smith's (1973), with 
\(\alpha=14/3\), \(r=-1/7\) and \(q=-3/7\).

Recall that the variables in eq. (3.29) are dimensionless; they were non-dimensionalized according to the transformations of eq. (3.8). Here, we undo those transformations, arriving at:

\[
\begin{align*}
 h(x, y) &= h_0 \left( \frac{\alpha(x)}{\alpha_0} \right)^{-1/m} \left( 1 + a\alpha_0^{1/m} \int_0^{x/L} \frac{dx'}{\alpha^{1/m}} \right) \left[ 1 - \frac{y^2}{w_0^2} \left( 1 + a\alpha_0^{1/m} \int_0^{x/L} \frac{dx'}{\alpha^{1/m}} \right)^{-2q} \right] \\
 &= h_0 \left( \frac{\alpha(x)}{\alpha_0} \right)^{-1/m} \left( 1 + a\alpha_0^{1/m} \int_0^{x/L} \frac{dx'}{\alpha^{1/m}} \right) \left[ 1 - \frac{y^2}{w_0^2} \left( 1 + a\alpha_0^{1/m} \int_0^{x/L} \frac{dx'}{\alpha^{1/m}} \right)^{-2q} \right]
\end{align*}
\]

(3.30)

The explicit form of \(w(x)\) naturally follows from eqs. (3.7c) and (3.30). At the flow margin (i.e., \(y=\pm w\)):

\[
1 - \frac{y^2}{w_0^2} \left( 1 + a\alpha_0^{1/m} \int_0^{x/L} \frac{dx'}{\alpha^{1/m}} \right)^{-2q} = 0
\]

(3.31a)

therefore,

\[
w = w_0 \left( 1 + a\alpha_0^{1/m} \int_0^{x/L} \frac{dx'}{\alpha^{1/m}} \right)^{-q}
\]

(3.31b)

### 3.4 Effect of Rheology on the Steady State Similarity Solution

Our steady-state similarity solution \(h(x,y)\) in eq. (3.30) has a rheological dependence; both \(\alpha(x)\) and \(m\) are prescribed by the rheology of the fluid. We note, with the conditions given in eq. (3.7), that regardless of the values of \(\alpha(x)\) and \(m\), eq.
(3.30) reduces to:

\[ h(0, y) = h_0 \left( 1 - \frac{y^2}{w_0^2} \right) \]  

(3.32)

Thus, the similarity method produces a physically-realistic boundary condition: eq. (3.32) indicates a parabolic cross-section at the vent. The downstream evolution of this parabolic form, however, varies with the rheological characteristics. In the following discussion, we show the effect of various values of \( \alpha(x) \) and \( m \) on the form of \( h(x, y) \) both in cross-sectional and longitudinal profiles. In Chapter 4, we compare the resulting profiles with data obtained from field and photographic measurements.

*Models for \( a(x) \)*

During surface flow, a lava (or other geologic material) often experiences a change in resistance to flow. This could be due to changes in the fluid's properties (e.g., a downstream increase in viscosity in a lava due to cooling or crystallization) and/or changes in the underlying topography (e.g., changes in the slope or roughness of the underlying flow bed). Determining the nature of these processes and their effect on \( \alpha(x) \) requires either empirical data from specific applications or an independent physical law, such as a cooling-induced viscosity or crystallinity change. In the absence of such information, we consider three endmember models to approximate the form of changes in \( \alpha(x) \) with distance from the source of the flow. These choices for \( \alpha(x) \) are arbitrary, but based on our knowledge that at least for
Newtonian flows, $\alpha(x)$ is inversely related to viscosity. These three models for $\alpha(x)$ correspond to constant, linearly increasing and exponentially increasing viscosity, and are given below:

**Constant:**

$$\alpha(x) = \alpha_0$$  \hspace{1cm} (3.33a)

**Linear:**

$$\alpha(x) = \frac{\alpha_0}{1 + x / L_\alpha}$$  \hspace{1cm} (3.33b)

**Exponential:**

$$\alpha(x) = \alpha_0 e^{-x/L_\alpha}$$  \hspace{1cm} (3.33c)

where $\alpha_0 = \alpha(0)$ and $L_\alpha$ is a constant scale factor. For laminar flow of Newtonian fluids, $\alpha_0 = 1/3\nu_0$, where $\nu_0$ is the viscosity at the source of the flow. Equation (3.33a) precludes any downstream changes in rheology, requiring $\alpha$ (and thus viscosity) to remain constant. Equations (3.33b) and (3.33c) allow for downstream rheological changes. In formulating these two equations, we assumed viscosity increases downstream. For many geological materials (including silicate lava flows), this is reasonable and consistent with observations that cooling and crystallization induce viscosity increases (e.g., Crisp et al., 1994 and references therein). However, this is not always the case; some materials (e.g., sulfur) show a decrease in viscosity during cooling in certain temperature ranges, and the reader is hereby cautioned. The rate at which viscosity increases (and $\alpha(x)$ decreases) in eqs. (3.3b) and (3.3c) is related to some scale factor $L_\alpha$, which is generally controlled by physical processes (e.g., crystallization), and may be unrelated to the length scale $L$ defined in eq. (3.8b).

In the following section, we substitute the expressions for $\alpha(x)$ given in eq.
(3.33) into our solution (eq. 3.30 and 3.3a,b) along with selected $m$ values to determine the dependence of $h(x,y)$ on these rheological parameters. Also required by these equations are flow depth and width at its source ($h_0$ and $w_0$) and surface slope ($\theta$); $L$ is defined in eq. (3.11a). Note that our solution is not sensitive to $\alpha_0$; once some form of $\alpha(x)$ has been selected and substituted into the solution, the parameter $\alpha_0$ will cancel upon integration. Thus, our solution is insensitive to the initial viscosity, and is only sensitive to downstream rheological changes.

Effect of Rheology on the Solution

In this section, we construct depth and width profiles of a theoretical lava flow for various choices of $\alpha(x)$ and $m$ to determine the effect of these rheological parameters on our solution. This theoretical flow is assumed to have the following values: $w_0 = 10$ m, $h_0 = 1$ m and $\theta = 5.7$ degrees; thus the downstream length scale is calculated from eq. (3.11a) as $L = 10$ m. Longitudinal depth and width profiles are calculated from eqs. (3.30) and (3.31b), respectively, for selected $m$ values (0.5, 1 and 10) for constant, linearly increasing and exponentially increasing viscosities (Figs. 3.1, 3.2 and 3.3, respectively).

For constant $\alpha$, all flows (regardless of $m$ values) thin and widen downstream (fig 3.1). Following an initial marked near-vent thinning which is a consequence of our model (discussed in Chapter 4), the flow depth remains relatively constant, showing a slight downstream thinning. (Note: this near-vent thinning is not shown in fig 3.1a, but follows from $h_0 = 1$ m); Higher $m$ values correlate with thicker, wider
Figure 3.1a. Theoretical longitudinal profiles of centerline flow depth as a function of distance from source, based on constant rheology and assumed initial parameters. These profiles correspond to $m=0.5$ (asterisks), $m=1$ (diamonds) and $m=10$ (squares). For constant rheology, all flows (regardless of $m$ values) show modest thinning. Higher $m$ values correspond to thicker flows.
Figure 3.1b. Theoretical longitudinal profiles of flow half-width as a function of distance from source, based on constant rheology and assumed initial parameters. These profiles correspond to $m=0.5$ (asterisks), $m=1$ (diamonds) and $m=10$ (squares). For constant rheology, all flows (regardless of $m$ values) widen downstream. Higher $m$ values correspond to wider flows.
flows of higher aspect ratios \((h/w)\). Near the source, the flow has the highest aspect ratio; cross-sectional profiles become progressively thinner and wider downstream. All flows show convex longitudinal depth \((h \text{ vs. } x)\) profiles (fig. 3.1a) and concave longitudinal width \((w \text{ vs. } x)\) profiles (fig. 3.1b).

For linear \(\alpha\) (i.e., linearly increasing viscosity), flows tend to both thicken and widen downstream. Unlike the case of constant \(\alpha\), lower \(m\) values generally correlate with wider, thicker flows having higher aspect ratios (figure 3.2). Both the longitudinal depth (fig. 3.2a) and width (fig. 3.2b) profiles are concave. Compared with the case of constant \(\alpha\), these flows are wider and thicker, with the differences becoming more pronounced downstream, that is, as the differences in \(\alpha(x)\) become more significant.

Note that Figs. 3.2a and 3.2b have a strong dependence on the choice of scaling factor \(L_\alpha\). For sufficiently large values of \(L_\alpha\), downstream increases in \(\alpha(x)\) are negligible, approximating constant rheology. Thus the flow would exhibit a marked near-vent thinning, followed by a near-constant, slowly decreasing depth resulting in a convex longitudinal depth profile. Choosing sufficiently small \(L_\alpha\) precludes near-vent thinning, such that the downstream longitudinal profile shows continuous thickening, most noticeably for low \(m\) values. Fig. 3.2 is based on \(L_\alpha = 10\) in eq. (3.33b), which corresponds to a 100-fold viscosity increase over the first kilometer.

For exponential \(\alpha\) (i.e., exponentially increasing viscosity), flows also tend to both thicken and widen downstream. Like fig. 3.2, fig. 3.3 corresponds to a viscosity increase of two orders of magnitude over the first km. This corresponds to \(L_\alpha = 215\) in eq. (3.33c). Flow depth (fig. 3.3a) and width (fig. 3.3b) each increase
Figure 3.2a. Theoretical longitudinal profiles of centerline flow depth as a function of distance from source, based on linearly increasing viscosity and assumed initial parameters including \( \Lambda_{0} = 10 \) (100 fold viscosity increase). These profiles correspond to \( m = 0.5 \) (asterisks), \( m = 1 \) (diamonds) and \( m = 10 \) (squares). For linear rheology, all flows (regardless of \( m \) values) thicken downstream. Lower \( m \) values correspond to thicker flows.
Figure 3.2b. Theoretical longitudinal profiles of flow half-width as a function of distance from source, based on linearly increasing viscosity and assumed initial parameters including $L_o=10$ (100 fold viscosity increase). These profiles correspond to $m=0.5$ (asterisks), $m=1$ (diamonds) and $m=10$ (squares). For linear rheology, all flows (regardless of $m$ values) widen downstream. Lower $m$ values correspond to wider flows.
Figure 3.3a. Theoretical longitudinal profiles of centerline flow depth as a function of distance from source, based on exponentially increasing viscosity and assumed initial parameters including $L_a = 215$ (100 fold viscosity increase). These profiles correspond to $m=0.5$ (asterisks), $m=1$ (diamonds) and $m=10$ (squares). For exponential rheology, all flows (regardless of $m$ values) thicken downstream, but flows characterized by high $m$ values show only modest thickening over the distance shown (1 km).
Figure 3.3b. Theoretical longitudinal profiles of flow half-width as a function of distance from source, based on exponentially increasing viscosity and assumed initial parameters including $L_2=215$ (100 fold viscosity increase). These profiles correspond to $m=0.5$ (asterisks), $m=1$ (diamonds) and $m=10$ (squares). For exponential rheology, all flows (regardless of $m$ values) widen downstream.
exponentially downstream. Longitudinal depth profiles are convex, whereas the width profiles may show a change in concavity from concave near the source to convex further downstream. Flows characterized by low $m$ values show only modest widening and thickening near the source; however, further downstream, these flows eventually become thicker and wider than those characterized by higher $m$ values. Again, there is a strong dependence on the choice of $L_a$. Choosing a smaller value for $L_a$ would result in significantly wider and thicker longitudinal profiles.

### 3.5 Discussion and Conclusions

Our model predicts that flows characterized by an increase in viscosity (either linear or exponential) are typically thicker and wider than flows that show no downstream rheological change (figs. 3.1 - 3.3). This is reasonable and consistent with observations that lava flows tend to thicken as they cool. Both the linear and exponential rheology models predict downstream widening and thickening. Since the model assumes volume conservation, such concomitant widening and thickening necessarily implies a decrease in flow rate.

By comparing the theoretical profiles predicted by the model with known flow dimensions, we can work "backwards" to infer the rheology of geologic deposits. This can be a valuable method of studying lavas whose flow has not been recorded, including prehistoric flows and flows in remote areas of Earth or other planets. However, we note that this model is simplified and must be applied with caution. One key assumption is flow must be unconfined. Once a channel and/or levees have been
formed, flow becomes confined and tends to maintain an equilibrium width. Furthermore, the underlying topography is assumed to be smooth and characterized by constant slope. It is essential that this condition be satisfied because a lava of constant viscosity flowing on irregular topography may form a deposit having dimensions similar to that of a lava flowing on a smooth surface that cools during flow. Finally, our model assumes flow is driven in the downstream direction by gravitational transport only, without a contribution from magmastatic pressure. Since eruptions are generally driven by magmastatic pressure, this assumption is not valid at the vent. The sharp-near vent thinning which characterizes the model's longitudinal profiles is a consequence of pressure-driven flow. If these assumptions are satisfied by a given flow, matching the flow's geometric dimensions to those predicted by the model can be used to infer or constrain the governing rheology (i.e., the rheological parameters $\alpha = \alpha(x)$ and $m$). A quantitative comparison between such theoretical and actual profiles is the focus of Chapter 4.
References


CHAPTER 4

LAVA FLOW RHEOLOGY

A COMPARISON OF DATA AND THEORY

In preparation for publication (jointly with Chapter 3)

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4.1 Abstract

This chapter involves testing the fluid dynamic model of emplacement of unconfined flows described in Chapter 3 by comparing its predictions against measurements obtained from field and remote sensing studies of basaltic lava flows. Specifically, we took field measurements of flow width and depth as a function of distance from the source of the flow. From remote sensing images, we constructed downstream profiles of flow width only. We compare these data to the theoretical profiles generated by our fluid dynamic model. This model predicts changes in flow depth and width with distance from the source of the flow for different rheological characteristics based on known or assumed input parameters (e.g., flow depth and
width at its source, surface slope). Such a comparison allows us to “work backwards” and solve for the two rheological parameters in question: $m$ (a positive constant, characteristic of the fluid rheology) and $\alpha(x)$ (a parameter which records downstream changes in rheology). Our results suggest that $m$ between 1 and 2 is characteristic of the basaltic flows studied. This indicates a non-Newtonian rheology, as Newtonian fluids are characterized by $m=3$. Based on these $m$ values, we ran the model for various choices of $\alpha(x)$. Choosing $\alpha(x)$ such that the corresponding downstream increases in viscosity were 2-4 orders of magnitude generally provided a good approximation to the data. The reasonableness of these values for downstream viscosity increases attests to the validity of this model.

4.2 Introduction

In Chapter 3, we found an exact analytic solution for unconfined flows with an arbitrary power-law rheology advancing on an inclined plane. Following Smith (1973), we used the method of similarity transformation to arrive at the steady-state solution:

$$h(x, y) = h_0 \left( \frac{\alpha}{\alpha_0} \right)^{-1/m} \left( 1 + a_{\alpha_0} \frac{1}{1/m} \int_0^{x/L} \frac{dx'}{\alpha^{1/m}} \right)^q \left( 1 - \frac{y^2}{w^2} \right)$$

(4.1)

where:

$$w(x) = w_0 \left( 1 + a_{\alpha_0} \frac{1}{1/m} \int_0^{x/L} \frac{dx'}{\alpha^{1/m}} \right)^{-q}$$

(4.2)
and where \( h \) and \( w \) are flow depth and half-width, respectively; \( x \) and \( y \) are the downstream and cross-stream directions, respectively; \( \alpha=\alpha(x) \) and \( m \) are rheological parameters; \( h_0, w_0, \) and \( \alpha_0 \) are the values of \( h, w \) and \( \alpha \) at the source (i.e., at \( x = 0 \)), respectively; \( L \) is the downstream length scale, given by:

\[
L = \frac{w_0^2}{h_0} \tan \theta
\]

(4.3)

where \( \theta \) is the surface slope; and \( a, q \) and \( r \) are constants defined in terms of \( m \) below:

\[
a = \frac{2(1+2m)}{m}
\]

(4.4a)

\[
q = \frac{-m}{1+2m}
\]

(4.4b)

\[
r = \frac{-1}{1+2m}
\]

(4.4c)

This solution determines changes in flow depth and width with distance from the source of the flow for different rheological characteristics based on known or assumed initial parameters. Consequently, these results may be used to infer downstream changes in rheology (i.e., \( \alpha(x) \)) of unconfined flows from the width and/or thickness of the deposits. This can be useful way of studying the rheology of lavas whose flow has not been observed, including prehistoric flows and flows in geographically remote areas, whether on Earth or other planets.
The database for this analysis consists of 9 basaltic lava flows (or segments thereof). Each flow included in this study is an individual flow unit, as compound flows or flow fields are not described by our model. In choosing suitable candidates for measurement, we used the following criteria: (1) unconfined, gravity-driven flow; (2) continuous, well-preserved and unobscured flow margin; (3) constant underlying slope; and (4) relatively smooth substrate such that any irregularities in the substrate do not significantly affect flow behavior. There are two data sets: field and image. The field data set comprises 5 tholeiite basalts on Kilauea volcano. These flow units are individual pahoehoe breakouts, with lengths ranging from 0.6 - 5 m from the point of breakout. Three of these pahoehoe flows have ropy morphology; the remaining two are pahoehoe toes. For each flow unit, we measured centerline flow thickness and width as a function of distance from the source. For two ropy pahoehoe flows, we also constructed an additional longitudinal width profile: we measured the end-to-end widths of the ropes at various downstream locations. No such measurement could be made on the third ropy pahoehoe, as it had a double-rope morphology. Table 4.1 summarizes the field data set.

The image data set comprises 4 lavas in aerial photographs and other images: La Poruna (Andes, Chile), Citelli Phase 4 (Mount Etna, Italy), Marcath (Lunar Crater, USA) and SP (San Francisco Volcanic Field, USA) flows. These flows range in composition from alkali basalt to basaltic andesite and have lengths of 2 - 8 km. For these flows, we constructed longitudinal profiles of flow width only; no thickness
Table 4.1. Summary of Field Data Set

<table>
<thead>
<tr>
<th>Flow Location</th>
<th>Flow Type</th>
<th>Slope (deg.)</th>
<th>$w_0$ (cm)</th>
<th>$h_0$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Pu‘u O’o/Kupaianaha</td>
<td>Pahoehoe (toe)</td>
<td>18</td>
<td>35</td>
<td>11</td>
</tr>
<tr>
<td>b) Mauna Ulu</td>
<td>Pahoehoe (toe)</td>
<td>11</td>
<td>21</td>
<td>9</td>
</tr>
<tr>
<td>c) Mauna Ulu</td>
<td>Pahoehoe (ropy)</td>
<td>6</td>
<td>17</td>
<td>21</td>
</tr>
<tr>
<td>d) Mauna Ulu</td>
<td>Pahoehoe (ropy)</td>
<td>6</td>
<td>75</td>
<td>10</td>
</tr>
<tr>
<td>e) Mauna Ulu</td>
<td>Pahoehoe (ropy)</td>
<td>6</td>
<td>113</td>
<td>13</td>
</tr>
</tbody>
</table>

All flows are from Kilauea Volcano, Hawaii and have a tholeiitic basaltic composition. Field data were obtained in December, 1993.
measurements were conducted. Where available, previously published estimates of flow thickness have been noted. Table 4.2 summarizes the remote sensing data set.

4.4 Methodology

We compare the model's theoretical profiles of flow thickness and width against measurements obtained from our field and image studies. To generate these profiles, the model requires a variety of input parameters. Some of these parameters (i.e., \( h_0 \), \( w_0 \) and \( \theta \)) are easily measured; others (i.e., \( m \) and \( \alpha \)) are unknown and must be deduced. Recall from Chapter 3, \( m \) is characteristic of the fluid so its value remains constant during flow (e.g., Newtonian fluids have \( m=3 \)), whereas the parameter \( \alpha \) records changes in rheology during flow. Our model assumes any cross-stream variations in rheology are negligible, that is, \( \alpha=\alpha(x) \). We consider three endmember approximations for \( \alpha(x) \), all based on the assumption that \( \alpha \) is inversely related to viscosity. These three models for \( \alpha(x) \) correspond to constant, linearly increasing and exponentially increasing viscosity, are introduced in Chapter 3 and reviewed below:

- **Constant:** \( \alpha(x) = \alpha_0 \)  
  \( \alpha_0 = \alpha(0) \) and \( L_\alpha \) is a constant scale factor that is generally controlled by

- **Linear:** \( \alpha(x) = \frac{\alpha_0}{1 + x / L_\alpha} \)  
  \( \alpha_0 \) and \( L_\alpha \) are constants.

- **Exponential:** \( \alpha(x) = \alpha_0 e^{-x/L_\alpha} \)  
  \( \alpha_0 \) and \( L_\alpha \) are constants.
Table 4.2. Summary of Remote Sensing Data Set

<table>
<thead>
<tr>
<th>Flow Name and Location</th>
<th>Flow Type</th>
<th>Slope ( w_0 )</th>
<th>( h_0 )</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) SP flow (San Francisco Volcanic Field, USA)</td>
<td>Basaltic Andesite</td>
<td>6 ( \text{deg.} )</td>
<td>40 ( \text{m} )</td>
<td>20 ( \text{m} )</td>
</tr>
<tr>
<td>b) Citelli Phase 4 flow (Mt. Etna, Italy)</td>
<td>Alkali Basalt</td>
<td>13 ( \text{deg.} )</td>
<td>20 ( \text{m} )</td>
<td>2 ( \text{m} )</td>
</tr>
<tr>
<td>c) Marcath flow (Lunar Crater, USA)</td>
<td>Alkali Basalt</td>
<td>3 ( \text{deg.} )</td>
<td>220 ( \text{m} )</td>
<td>- ( \text{m} )</td>
</tr>
<tr>
<td>d) La Poruna (Andes, Chile)</td>
<td>Basaltic Andesite</td>
<td>2 ( \text{deg.} )</td>
<td>170 ( \text{m} )</td>
<td>10's ( \text{m} )</td>
</tr>
</tbody>
</table>

References: (1) Schaber et al. (1980), USGS Topographic Map, Coconino Co. (1989); (2) Booth and Self (1973); (3) Lum et al. (1989), USGS Topographic Maps, Nye Co. (1967); (4) Francis (1993, pers. comm.). Images of SP and Lunar Crater flows were provided by R. Lopes-Gautier.
physical processes (e.g., crystallization) and may be unrelated to the length scale $L$ defined in eq. (4.3). Note that our solution (eq. 4.1 and 4.2) is not sensitive to absolute values of $\alpha_0$, only to the ratio of $\alpha_0/\alpha(x)$. Thus, $\alpha_0$ is a free parameter which we define as $\alpha_0=1$.

We first consider the field data. These flow units are sufficiently short ($< 5m$) such that viscosity (and thus $\alpha$) is assumed to remain constant during emplacement. Thus, the only unknown input parameter is $m$, which can be deduced by a best fit of the constant-$\alpha$ model to the data. We assume this value of $m$ to be characteristic of all basaltic lavas.

We then consider the remote sensing data. Over the length of these flows (2-8 km) significant cooling and/or crystallization may have occurred, resulting in a downstream viscosity increase. Thus, we cannot assume a constant $\alpha$. However, using the $m$ value obtained from the field data above, the form of $\alpha(x)$ can be inferred for each flow by fitting the model to these data. By computing the corresponding predicted viscosity increase, the model can be tested for reasonableness.

4.5 Results

In accordance with the above-described methodology, we begin our analysis by comparing the field data to the theoretical output of our fluid dynamic model assuming constant rheology. Fig. 4.1 and 4.2 show such a comparison, based on $m=1$. For all five flows, the model predicts flows that are thinner (Fig. 4.1) and wider (Fig. 4.2) than the field data suggests. This is not strictly a consequence of the
Figure 4.1. Longitudinal profiles of centerline flow depth as a function of distance from source. Each plot (a through e) shows a comparison between the model’s predicted profile based on constant rheology and $m=1$ (solid line) and the measured profile based on field data (+). The field data are basaltic pahoehoe flows from Kilauea Volcano and are described in Table 4.1. In each case (a through e), the model predicts flows that are thinner than the field data. This discrepancy is explained in the text. (a) Field data is pahoehoe toe from the Puu Oo/Kupaianaha flow field (flow (a) in Table 4.1).
Figure 4.1b. Comparison between the model's predicted depth profile based on constant rheology and $m=1$ (solid line) and the measured profile based on field data (+). Field data is pahoehoe toe from the Mauna Ulu flow field (flow (b) in Table 4.1).
Figure 4.1c. Comparison between the model's predicted depth profile based on constant rheology and $m=1$ (solid line) and the measured profile based on field data (+). Field data is ropy pahoehoe from the Mauna Ulu flow field (flow (c) in Table 4.1).
Figure 4.1d. Comparison between the model’s predicted depth profile based on constant rheology and \( m=1 \) (solid line) and the measured profile based on field data (+). Field data is ropy pahoehoe from the Mauna Ulu flow field (flow (d) in Table 4.1).
Figure 4.1e. Comparison between the model’s predicted depth profile based on constant rheology and $n=1$ (solid line) and the measured profile based on field data (+). Field data is ropy pahoehoe from the Mauna Ulu flow field (flow (e) in Table 4.1).
Figure 4.2. Longitudinal profiles of flow half-width as a function of distance from source. Each plot (a through e) shows a comparison between the model’s predicted profile based on constant rheology and \( m=1 \) (solid line) and the measured profile based on field data (+). The field data are basaltic pahoehoe flows from Kilauea Volcano, and are described in Table 4.1. In each case (a through e), the model predicts flows that are wider than the field data. This discrepancy is explained in the text. (a) Field data is pahoehoe toe from the Puu Oo/Kupaianaha flow field (flow (a) in Table 4.1).
Figure 4.2b. Comparison between the model's predicted width profile based on constant rheology and $m=1$ (solid line) and the measured profile based on field data (+). Field data is pahoehoe toe from the Mauna Ulu flow field (flow (b) in Table 4.1).
Figure 4.2c. Comparison between the model's predicted width profile based on constant rheology and $m=1$ (solid line) and the measured profile based on field data (+). Field data is ropy pahoehoe from the Mauna Ulu flow field (flow (c) in Table 4.1).
Figure 4.2d. Comparison between the model's predicted width profile based on constant rheology and $m=1$ (solid line) and the measured profile based on field data (+). Field data is ropy pahoehoe from the Mauna Ulu flow field (flow (d) in Table 4.1).
Figure 4.2e. Comparison between the model’s predicted width profile based on constant rheology and $m=1$ (solid line) and the measured profile based on field data (+). Field data is ropy pahoehoe from the Mauna Ulu flow field (flow (e) in Table 4.1).
selected \( m \) value; a wide range of \( m \) values (\( m = 0.5 \) to 5) produces similar inconsistencies between the predicted and actual profiles (Fig. 4.3). Instead, the discrepancy in flow thickness reflects an inherent shortcoming of the model in modeling near-vent flow. The lack of agreement in flow width between the data and theory reflects a limitation of the measuring technique. These discrepancies are discussed separately below.

Explanation of discrepancies in flow thickness between field data and theory

The calculated curves in Figs. 4.1 (a - e) all show a sharp decrease in flow thickness near the vent. This is an artifact of the fluid dynamic model. Essentially, our model assumes a big pile of lava (or other geologic material) exists at the source \((x=0, y=0)\) and spreads out both downstream and laterally, primarily due to gravity. Clearly, this is not realistic. In actuality, pressure drives near-vent flow. Thus, the field data show no sharp decrease in flow depth near the vent. In an attempt to compensate for the model’s near-vent artifact in flow thickness, we could replace the first data point with an assumed value of \( h_0 \). This value would be artificially selected such that, based on this value, the model predicts a thickness value that closely matches the subsequent thickness value of the data.

Here, we evaluate one such attempt. Figure 4.4a shows the predicted longitudinal thickness profile based on \( h_0 = 1 \) for a wide range of \( m \) values (0.1, 1, 10). Also shown in Fig. 4.4a is a sample field measurement. The profile corresponding to \( h_0 = 1 \) meter and \( m \sim 1 \) provides a reasonable approximation to the field data; however,
Figure 4.3a. Longitudinal profiles of centerline flow depth as a function of distance from source. This figure shows a comparison between predicted profiles based on constant rheology for selected $m$ (solid lines) and a measured profile based on field data (+). The predicted profiles correspond to $m=0.5$ (bottom), $m=1$ (middle) and $m=5$ (top). The field measurement is flow (c), a ropy pahoehoe from Mauna Ulu; it is the same data shown in plot Fig. 4.1c. Note that, for all values of $m$ shown, the model predicts flows that are thinner than the field data. See text for explanation.
Figure 4.3b. Longitudinal profiles of flow half-width as a function of distance from source. This figure shows a comparison between predicted profiles based on constant rheology for selected \( m \) (solid lines) and a measured profile based on field data (+). The predicted profiles correspond to \( m=0.5 \) (bottom), \( m=1 \) (middle) and \( m=5 \) (top). The field measurement is flow (c), a ropy pahoehoe from Mauna Ulu; it is the same data shown in plot Fig. 4.2c. Note that, for all values of \( m \) shown, the model predicts flows that are wider than the field data. See text for explanation.
Figure 4.4. Theoretical longitudinal profiles of centerline flow depth as a function of distance from source based on an assumed value of $h_0$ and the following $m$ values: $m=0.5$ (bottom line), $m=1$ (middle line), $m=10$ (top line). Also shown is a sample field measurement (+) for comparison (flow (c) in Table 4.1). (a) $h_0=1$ meter. Note the field data is best approximated by $m=1$. 

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this pair of $h_0$ and $m$ is not unique. Many combinations of values for $h_0$ and $m$ (e.g., a larger $h_0$ and a smaller $m$, or vice versa) would provide similarly good fits. For example, $h_0=10$ meters and $m=.25$ or $m=.50$ also approximate the data (Fig. 4.4b). Thus, it's next to impossible to deduce $m$ (or extract anything meaningful) based on a comparison between thickness data and the model's predictions. In a very general sense, we note that, regardless of $m$, the theoretical profiles show $h$ remains relatively constant downstream, which is consistent with the field data (fig. 4.1). However, any detailed comparison between the data and theory will be limited to the width data.

*Explanation of discrepancies in flow width between field data and theory*

The longitudinal width profiles shown in Figs. 4.2 (a - e) also show a discrepancy between field data and theory; the model consistently predicts flow widths to be greater than the measured values. We attribute this discrepancy to a fundamental limitation of the measurement technique. We measured $w_0$ of these pahoehoe toes at the point of breakout. Thus, this is not the true source width but instead represents the location where flow width can be first measured. The true $w_0$, which represents the critical mass of basaltic lava required to cause a breakout, cannot be measured: it's hidden from view, located inside the source flow from which the breakout occurred. For basaltic lava at typical eruption temperatures, we assume $w_0$ can be closely approximated by a point source. Furthermore, we assume that the formation of the lava sub-system inside the main pahoehoe flow that eventually
Figure 4.4b. Theoretical longitudinal profiles of centerline flow depth as a function of distance from source based on $h_o=10$ meters and $m=0.25$ (bottom line) and $m=0.5$ (top line). Also shown is a sample field measurement (+) for comparison (flow (e) in Table 4.1). Like the profile corresponding to $h_o=1$ and $m=1$ in (a), these profiles approximate the field data.
formed a breakout is sufficiently close to the point of breakout such that we can treat the point of breakout as \( x=0 \), as opposed to having to rescale the \( x \) values (Fig. 4.5).

Field data: Solving for \( m \)

The longitudinal width profiles shown in Fig. 4.6 are generated in the identical fashion as those shown in Fig. 4.2 except that for all five flows, the parameter \( w_0 \) has been reset to 1 cm to reflect a point source. The parameters \( h_0 \) and \( \theta \) remain unchanged. The best fit of these “corrected” profiles to the field data generally corresponds to \( m \) values of 1 - 2. We assume this value of \( m \) to be characteristic of all basaltic lavas.

As Newtonian fluids are characterized by \( m=3 \), our analysis suggests most basaltic flows are non-Newtonian. (Of the five flows, only one was well-approximated by \( m=3 \); see Fig. 4.2e). This is in agreement with a wide variety of field and laboratory measurements of basaltic lavas which indicate the presence of a yield strength (e.g., Shaw et al., 1968; Shaw, 1969; Pinkerton and Sparks, 1978). This non-Newtonian rheology has been attributed to dispersed crystals and gas bubbles contained in the lava, and possibly to the development of molecular structural units at sub-liquidus temperatures (Cas and Wright, 1987, section 2.4).

As discussed in Section 4.3 above, we also measured the end-to-end widths of surface ropes as a function of downstream location of two pahoehoe flows. These data are shown along with the model’s predicted longitudinal width profiles for various \( m \) (Fig. 4.7). For both flows, the model approximates the data for \( 1/2 < m < \)
Figure 4.5. This figure illustrates the difference between the locations of the measured $w_0$ (i.e., at $x_1$, the breakout from the main mass of the lava flow) and the true source width (i.e., at $x_0$, located inside the main flow). In this work, we assume that the true source width can be modeled as a point source (i.e., $x_0$) and that the distance between $x_0$ and $x_1$ is sufficiently small such that $x_0 - x_1$. (Illustration by Cynthia Wilburn.)
Figure 4.6. Longitudinal profiles of flow half-width as a function of distance from source. Each plot (a through e) shows a comparison between the model's predicted profile based on constant rheology (solid lines) and the measured profile based on field data (+). The calculated profiles correspond to $m=0.5$ (bottom), $m=1$ (lower middle), $m=2$ (upper middle), and $m=3$ (top). The field data are described in Table 4.1, except that the source width has been reset to $w_o=1$ cm to approximate a point source. Compared with Fig. 4.2 (which are based on the measured values of $w_o$), these profiles more closely approximate the field data. (a) Field data is pahoehoe toe from the Puu Oo/Kupaianaha flow field (flow (a) in Table 4.1), and is best approximated by the theoretical profiles corresponding to $m=1-2$. This indicates non-Newtonian rheology.
Figure 4.6b. Comparison between the model’s predicted width profile based on constant rheology and $w_0=1$ cm and the measured profile based on field data (+). Field measurement is pahoehoe toe from the Mauna Ulu flow field (flow (b) in Table 4.1). These data are best approximated by the theoretical profiles corresponding to $m=0.5-1$. Although this “best-fit” $m$ value is slightly lower than those obtained for the remainder of the data, it again indicates non-Newtonian rheology.
Figure 4.6c. Comparison between the model's predicted width profile based on constant rheology and $w_0=1$ cm and the measured profile based on field data (+). Field measurement is ropy pahoehoe from the Mauna Ulu flow field (flow (c) in Table 4.1). These data are best approximated by the theoretical profiles corresponding to $m=1-2$, indicating non-Newtonian rheology.
Figure 4.6d. Comparison between the model's predicted width profile based on constant rheology and \( w_0 = 1 \) cm and the measured profile based on field data (+). Field measurement is ropy pahoehoe from the Mauna Ulu flow field (flow (d) in Table 4.1). These data are best approximated by the theoretical profiles corresponding to \( m = 1 \), indicating non-Newtonian rheology.
Figure 4.6e. Comparison between the model's predicted width profile based on constant rheology and $w_0=1$ cm and the measured profile based on field data (+). Field data is ropey pahoehoe from the Mauna Ulu flow field (flow (e) in Table 4.1). Unlike the remainder of the flows studied, these data are best fit by the theoretical profiles corresponding to $m=3$. This corresponds to Newtonian rheology.
Figure 4.7. Longitudinal profiles of rope half-width as a function of distance from source. Each plot shows a comparison between the model's predicted profile based on constant rheology (solid lines) and the measured profile based on field data (+). The calculated profiles correspond to $m=0.5$ (bottom), $m=1$ (lower middle), $m=2$ (upper middle), and $m=3$ (top). (a) Field data is flow (c) in Table 4.1, and is best approximated by the theoretical profiles corresponding to $m=0.5-1$. 
Figure 4.7b. Longitudinal profiles of rope half-width as a function of distance from source. Field data is flow (d) in Table 4.1, and is best approximated by the theoretical profiles corresponding to $m=0.5-1$. 
1. Recall, our model assumes unconfined flow. We conducted these rope-width measurements because we were interested in seeing how our model would fit data from a confined flow. Like the flow itself, the ropes widen downstream. However, the downstream widening of the ropes is hampered by the confining effect of the previously-established flow margins. Thus, we might expect the ropes to experience less net widening compared to the total flow, corresponding to a smaller $m$ value. This is precisely what Fig. 4.7 shows.

Remote sensing data: Solving for $\alpha$

Using $m$ values of 1 and 2, we run the model for parameters corresponding to the remote sensing data (see Table 4.1), first using constant rheology. Again, we assume a point source ($w_0=1$ cm). Fig 4.8 shows the resulting longitudinal width profiles. For most (i.e., 3 of 4) of the flows shown in Fig. 4.6, the data are inconsistent with the model’s predictions. The measured widths generally exceed those predicted, and these differences become more pronounced downstream. This results in a steeper longitudinal width profile than that predicted by the model. The inconsistency between the majority of the data and the model suggests basaltic flows of these lengths are generally characterized by non-constant rheology.

Using the same $m$ values and initial parameters, we re-run the model using non-constant viscosity. Figures 4.9 and 4.10 show longitudinal width profiles for linear and exponential rheologies, based on eqs. (4.5b) and (4.5c), respectively. In these equations, the parameter $L_a$ quantifies the downstream change in rheology. In the
Figure 4.8. Longitudinal profiles of flow half-width as a function of distance from source. Each plot shows a comparison between the model's predicted profile based on constant rheology (solid lines) and the measured profile based on remote sensing data (+). The calculated profiles correspond to $m=1$ (bottom) and $m=2$ (top). (a) Remote sensing data are of SP flow (see Table 4.2). The data are steeper than the given theoretical profiles.
Figure 4.8b. Longitudinal profiles of flow half-width as a function of distance from source (constant rheology). Remote sensing data are of Citelli Phase 4 flow (see Table 4.2). Unlike the remainder of the remote sensing data, this flow is well approximated by the given theoretical profiles based on constant rheology.
Figure 4.8c. Longitudinal profiles of flow half-width as a function of distance from source (constant rheology). Remote sensing data are of Lunar Crater Marcath flow (see Table 4.2). The data are steeper than the given theoretical profiles based on constant rheology.
Figure 4.8d. Longitudinal profiles of flow half-width as a function of distance from source (constant rheology). Remote sensing data are of La Poruna flow (see Table 4.2). The data are steeper than the given theoretical profiles based on constant rheology.
Figure 4.9. Longitudinal profiles of flow half-width as a function of distance from source. Each plot shows a comparison between the model's predicted profile based on linear rheology (solid lines) and the measured profile based on remote sensing data (+). The calculated profiles correspond to $m=1$ (bottom) and $m=2$ (top). For both profiles, $L_a = 50$, which corresponds to a downstream viscosity increase of approximately two orders of magnitude over 4 km. (a) Remote sensing data are of SP flow (see Table 4.2). These data are better approximated by this linear-rheology model than the constant-rheology model shown in Fig. 4.8a.
Figure 4.9b. Longitudinal profiles of flow half-width as a function of distance from source (linear rheology). Remote sensing data are of Citelli Phase 4 flow (see Table 4.2). Unlike the remainder of the remote sensing data, this flow is better approximated by the constant rheology profiles of Fig. 4.8b.
Figure 4.9c. Longitudinal profiles of flow half-width as a function of distance from source (linear rheology). Remote sensing data are of Lunar Crater Marcath flow (see Table 4.2). These data are better approximated by this linear-rheology model than the constant-rheology model shown in Fig. 4.8c.
Figure 4.9d. Longitudinal profiles of flow half-width as a function of distance from source (constant rheology). Remote sensing data are of La Poruna flow (see Table 4.2). These data are better approximated by this linear-rheology model than the constant-rheology model shown in Fig. 4.8d.
Figure 4.10. Longitudinal profiles of flow half-width as a function of distance from source. Each plot shows a comparison between the model's predicted profile based on exponential rheology (solid lines) and the measured profile based on remote sensing data (+). The calculated profiles correspond to $m=1$ (bottom) and $m=2$ (top). For both profiles, $L_a = 500$, which corresponds to a downstream viscosity increase of 3-4 orders of magnitude over 4 km. (a) Remote sensing data are of SP flow (see Table 4.2). These data are better approximated by this exponential-rheology model than the constant-rheology model shown in Fig. 4.8a.
Figure 4.10b. Longitudinal profiles of flow half-width as a function of distance from source (exponential rheology). Remote sensing data are of Citelli Phase 4 flow (see Table 4.2). Unlike the remainder of the remote sensing data, this flow is better approximated by the constant rheology profiles of Fig. 4.8b.
Figure 4.10c. Longitudinal profiles of flow half-width as a function of distance from source (exponential rheology). Remote sensing data are of Lunar Crater Marcath flow (see Table 4.2). These data are better approximated by this exponential-rheology model than the constant-rheology model shown in Fig. 4.8c.
Figure 4.10d. Longitudinal profiles of flow half-width as a function of distance from source (constant rheology). Remote sensing data are of La Poruna flow (see Table 4.2). These data are better approximated by the linear-rheology model (Fig. 4.9d) than this exponential-rheology model, particularly as distance from source increases.
absence of actual data or an empirical law, a value of \( L_a \) must be assumed. Choosing \( L_a \) sufficiently large has the effect of reducing eqs. (4.5b) and (4.5c) to eq. (4.5a), the case of constant rheology. In this work, our method is to empirically choose values of \( L_a \) to approximate the data, and then test these values for reasonableness by calculating the corresponding changes in viscosity. This approach results in choices of \( L_a = 50 \) for the linear model (Fig. 4.9) and \( L_a = 500 \) for the exponential model (Fig. 4.10).

For the three flows whose with profiles were poorly predicted by the constant-rheology model shown in Fig. 4.8 (Lunar Crater, La Poruna and SP), the linear-rheology model more closely approximates the data (Fig. 4.9). For two of these flows (Lunar Crater and SP), the exponential-rheology model also produces a good fit to the data (see Fig. 4.10a and 4.10c). The downstream viscosity increases corresponding to the model predictions shown in Fig. 4.9 and 4.10 are 2 and 3-4 orders of magnitude, respectively, over a distance of 4 km. These downstream viscosity increases are comparable to those documented for basaltic flows; see Crisp et al. (1994) for a review of the literature. The viscosity of the 1983-1984 Pu‘u O‘o flows has been measured by Fink and Zimbelman (1990) to have increased approximately 2 or 3 orders of magnitude during emplacement. Moore and Ackerman (1989) similarly estimate downstream viscosity increases of Kilauea basalts to be 2 orders of magnitude. For the 1971 Mount Etna flows, Booth and Self (1973) estimate viscosity increases of 2 orders of magnitude over 4 km. Thus, the downstream viscosity increases corresponding to the linear and exponential rheology profiles shown in Fig. 4.9 and 4.10 are reasonable.
4.6 Conclusions

Basaltic lava flows have $m \sim 1 - 2$.

Fig. 4.6 shows a comparison of longitudinal width profiles of sample field data and model predictions, assuming constant rheology. The majority of these field data are well approximated by the constant viscosity model, for $m$ between 1 and 2. This indicates non-Newtonian rheology, as Newtonian fluids are characterized by $m=3$.

Model predicts downstream viscosity increases of 2 - 4 orders of magnitudes.

Using $m$ values of 1 and 2, we ran the model for endmember approximations of $\alpha$. The best fits of the model to sample photographic data are shown for constant $\alpha$ (Fig. 4.8), linear $\alpha$ (Fig. 4.9) and exponential $\alpha$ (Fig. 4.10). In Fig. 4.8, the data are generally inconsistent with the model's predictions, indicating non-constant $\alpha$. Instead in most cases the data are better approximated by linearly or exponentially decreasing $\alpha$. The downstream viscosity increases corresponding to the model predictions shown in Fig. 4.9 and 4.10 are 2 and 3-4 orders of magnitude, respectively, over a distance of 4 km. The reasonableness of these values attests to the validity of this model.
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CHAPTER 5

FRACTAL PROPERTIES OF PLANETARY LAVAS:
IMPLICATIONS FOR ERUPTION RATES
AND EMBLACEMENT STYLES

Preliminary results have been published as:


5.1 Abstract

This work extends the fractal analysis of lava flow margins presented in
Chapter 2 to extraterrestrial lavas. We have measured 44 lava flow margins from
orbital images of Venus, Mars and the Moon. The vast majority (40) of these extra-
terrestrial flows are fractal, indicating a basaltic composition (see Chapter 2).
Furthermore, the ranges of fractal dimensions (D) are consistent with the terrestrial
range of a'a (D: 1.04 - 1.09) and pahoehoe (1.13 - 1.24). This suggests that fractal
analysis can be used to remotely identify extraterrestrial a’a and pahoehoe flows, which can in turn be used to provide insights into eruption and emplacement conditions. Included in this analysis are four flow margins of lavas from Alba Patera, Mars that have fractal properties different from typical basalts. This may suggest a more viscous rheology (possibly a more silicic composition) and/or topographic control on the flow margin. Included in this database are measurements of vast lava plateaus, analogous to terrestrial flood basalts. We identify both a’a and pahoehoe margins, suggesting emplacement of these high-volume lavas at both high (a’a-like) and low (pahoehoe-like) eruption rates. Our identification of flow type has been confirmed for selected Venusian lavas by an independent technique, based on the radar-backscatter properties of the flow surface. Furthermore, our identification of selected Martian lavas are consistent with a second independent technique, which relates the width of distal flow lobes to silica content.

5.2 Introduction

Identifying and distinguishing flow types on other planets have been ongoing goals of planetary volcanologists (e.g., Hulme, 1974, 1976; Schaber et al., 1976, 1978; Moore et al., 1978; Zimbelman, 1985; Theilig and Greeley, 1986; Wadge and Lopes, 1991; Campbell and Campbell, 1992; Roberts et al., 1992). However, with the only available data often being an image of the flow, it is essential to develop remote sensing techniques that fully exploit this limited information. One of the most useful pieces of information contained in an image of a lava flow is its plan-view shape.
In Chapter 2, we describe a technique that uses fractal parameters to distinguish various types of lava flows. Briefly, we have found that fractal behavior characterizes internally-controlled basalts, and the fractal dimension, a parameter which measures flow margin convolution, is useful in distinguishing the two main morphological types of basalts (a’a and pahoehoe). Additionally, we have found that more silicic flows (basaltic andesites through rhyolites) as well as externally (i.e., topographically) controlled basalts are generally associated with non-fractal behavior. The breakdown of fractal behavior at increased silica contents is attributable to the higher viscosities and/or yield strengths that normally characterize silicic flows, and is discussed in detail in Chapter 2.

Identifying a flow as a’a or pahoehoe is important because it reflects the conditions of emplacement. A’a flows are generally associated with high effusion rates (> 5-10 m$^3$/s for Hawaiian lavas) and/or viscous lavas (Rowland and Walker, 1990). Flowing as one main mass in a central channel, no part of the flow is transported far from the main body of the flow unless there is a major breakout (e.g., Lockwood et al., 1987), and this tends to result in a fairly linear flow margin. In Hawaii, pahoehoe flows are associated with low effusion rates (< 5m$^3$/s) and/or fluid lavas. Fed by a branching tube system, pahoehoe lavas get transported in various directions, potentially far from the center of the flow field. This tends to make pahoehoe flow margins more convoluted (i.e., have higher fractal dimensions. These systematic differences in fractal dimension can provide insights into eruption and emplacement conditions via the remote identification of a basaltic flow as a’a or pahoehoe.

Of particular interest to volcanologists and climatologists is the eruption and
emplacement of flood basalts. Flood (or plateau) basalts are defined as vast accumulations of horizontal or subhorizontal lavas of basaltic composition, generally believed to be the product of fissure eruptions (Macdougall, 1988). How these high-volume flows were emplaced has been a subject of recent controversy and debate (Kerr, 1994). The traditional view holds that flood basalts form thick, extensive flows that are erupted from fissure vents at extremely high rates (e.g., Swanson et al., 1975; Greeley, 1976, 1982; Plescia, 1990). According to this view, such eruptions produce vast basaltic provinces in a matter of days. Volcanic aerosols (e.g., sulfuric acid) could potentially be injected high into the stratosphere, blocking sunlight and perhaps causing dramatic changes in global climate (Stothers et al., 1986).

Largely based on field evidence gathered at the Columbia River Plateau, a different theory has recently been proposed: flood basalts may have been emplaced at lower rates (Self et al., 1993). According to this theory, initially-thin (20-30 cm) flow lobes became subsequently inflated by continued injection of lava from within an emplaced flow lobe (Self et al., 1991; Finnemore et al., 1993). This theory was inspired by the ongoing Kilauea eruption, where such endogenous growth has been documented (Walker, 1991; Hon et al., 1994). Accretion of many such inflated flow units could form massive, compound flow fields, consistent with the great thicknesses of flood basalts. According to this theory, emplacement of the Columbia River basalts would require periods of months to years, and this could have a different effect on the global atmosphere. For example, rather than a sudden pulse of aerosols injected into the stratosphere, one might expect smaller, repeated, or constant injections which would be less likely to reach the stratosphere, but would increase aerosol concentrations in the troposphere, especially locally.

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Identification of flood basalt flow margins as either a’a or pahoehoe could provide insight into the emplacement rates of these flows. Although fractal dimension of flow margins is one way to distinguish a’a and pahoehoe, this requires plan-view exposures and such features have not been documented for terrestrial flood basalts due to erosion of flow margins. However, flood basalt margins are exposed on Venus, Mars and the Moon, and we have used these to make inferences about the emplacement of terrestrial examples.

Extraterrestrial lava flows are generally thought to be basaltic; if more silicic flows exist, they are believed to be of limited areal extent (see Basaltic Volcanism on the Terrestrial Planets, 1981 for details.) First, lunar samples returned by the Apollo and Luna missions have basaltic compositions. The lunar maria have also been identified as basaltic through Earth-based spectral reflectance studies. On Mars, several lines of evidence indicate widespread basaltic volcanism. X-ray fluorescence analysis of soil samples collected at the Viking 1 (Chryse Planitia) and Viking 2 (Utopia Planitia) lander sites indicate mafic to ultramafic source materials (Baird and Clark, 1981; Clark et al., 1982). This is confirmed by the results of reflectance spectroscopy studies of the Martian surface (e.g., Singer et al., 1979). Further evidence for basaltic volcanism on Mars comes from the SNC meteorites, a group of eight achondrites believed to be derived from Mars (Wood and Ashwal, 1981). The SNC meteorites are compositionally quite similar to terrestrial basaltic and ultramafic rocks (Banin et al., 1992).

On Venus, the Soviet Venera and Vega missions reported basalts, based on the results of x-ray fluorescence analysis; see Barsukov et al. (1992) for a summary of

Clearly, there is considerable evidence suggesting that the vast majority of planetary lavas are basaltic. If we assume that the range of fractal dimension for planetary basalts is the same for terrestrial flows, then ‘a’ and pahoehoe can be remotely distinguished by the fractal dimension. Furthermore, flows that are found to be non-fractal or to have fractal dimensions different from ‘a’ and pahoehoe could be of a non-basaltic composition. Thus, fractal analysis has the potential of being an important remote sensing technique for planetary volcanology. However, in order to have confidence in fractal analysis as a remote sensing tool, we must confirm its identification of flow types using other, independent techniques.

As part of this chapter, we compare the results of this fractal analysis to those of two independent remote sensing techniques, also aimed at distinguishing flow types. The first technique identifies flows as ‘a’, pahoehoe, or transitional based on systematic differences in their radar-backscatter signals (Campbell and Campbell, 1992). The second technique, based on the observation that distal lobe width is directly correlated to SiO₂ content, remotely distinguishes basalts from more silicic flows (Wadge and Lopes, 1991). We have applied these techniques to selected flows on Venus (radar technique) and Mars (lobe width technique), and compare the results with those of the fractal technique.
5.3 Methodology

This analysis invokes a three-part methodology. First, we perform a fractal analysis of selected planetary lavas and, based on this analysis categorize these lavas into four flow types. Then, we evaluate this categorization of flow type by comparing the results to those of two independent, remote sensing techniques. For selected Venusian flows, we compare the fractal results to those of a radar technique (Campbell and Campbell, 1992). For selected Martian flows, we compare the fractal results to those of the distal lobe width technique (Wadge and Lopes, 1991).

1. Fractal Analysis of Planetary Lavas

Fractal properties of selected planetary lavas are determined in the same way as for terrestrial flows (see Section 2.3). The apparent length (L) of a flow margin is measured by walking rods of different lengths (r) along the margin. Since shorter rods traverse more smaller-scale embayments and protrusions in the flow margin, L increases as r decreases. A linear trend ($R^2 > .95$) on a log L vs. log r plot indicates the data are fractal. Fractal dimension (D) can then be calculated as $D=1-m$, where $m$ is the slope of the linear least squares fit to the data.

As all flows are measured from images, only the computerized measurement technique was employed. The range of equivalent rod lengths used depends on both flow margin length and the scale of the image, and ranges from 140 m to 58 km. For terrestrial flows, the equivalent range was .125 m - 2.4 km. We note there is no gap in
scale between the terrestrial and extraterrestrial measurements: in fact, there is a significant overlap in rod lengths.

Based on the results of this fractal analysis, and assuming terrestrial analogy, the planetary lavas are categorized into four flow types. Flows that are fractal and have fractal dimensions consistent with terrestrial basalts are interpreted to be internally-controlled basalts (i.e., basalts with flow margin shapes controlled by internal, fluid dynamic processes, as opposed to topography). Such lavas are further categorized into a'a, pahoehoe, and transitional flow types. Flows that are not fractal and/or have fractal dimensions inconsistent with terrestrial basalts are placed into a fourth category: "different from internally-controlled basalts". This category could comprise topographically-controlled basalts (e.g., basalts flowing in preexisting channels) as well as flows of more evolved compositions.

2. Radar Analysis of Venusian Lavas

For selected Venusian lavas, we use a second remote sensing tool to independently evaluate our identification of flow type based on fractal analysis. Campbell and Campbell (1992) have developed a technique, sensitive to incidence angle, that identifies Venusian flows as a'a, pahoehoe or transitional based on radar-backscatter signal at the S band (12.6 cm) wavelength. This technique is based on the observation that a'a and pahoehoe flow surfaces have different radar-backscatter properties (Gaddis et al., 1990). Surface roughness is a major contributor to the backscatter signal: rougher surfaces tend to produce brighter images. A'a flows, with
their jagged, clinkery surfaces, are systematically rougher than pahoehoe flows at small (≤ 10’s cm) scales, and thus tend to produce higher backscatter signal at these short wavelengths for a given incidence angle. Thus, all other factors being equal, a’a flows would be expected to appear brighter on radar-backscatter images (e.g., Magellan images) than pahoehoe flows.

This technique, ground-truthed with AirSAR data of Hawaiian basalts, effectively discriminates between a’a and pahoehoe when the short-wavelength topography dominates the radar-backscatter return; this occurs at incidence angles exceeding about 30° (Campbell and Campbell, 1992). At lower incidence angles, the long-wavelength topography significantly contributes to the radar backscatter signal and this technique cannot be applied. Fortunately, Magellan’s Cycle I left-looking coverage used sufficiently high incidence angles (≥ 30°) over most of the Venusian surface: at latitudes from 54°N to 34°S. Of the 30 Venusian flows measured as part of this fractal analysis, 21 fall within this latitudinal range. The remaining 9 are located at latitudes south of 34°S or north of 54°N, and thus cannot be independently checked by the radar technique.

3. Lobe Width Analysis of Martian Lavas

For selected Martian lavas, we use the technique of Wadge and Lopes (1991) to evaluate our identification of flow type based on fractal analysis. This technique is based on the observation that more silicic flows are characterized by wider distal lobes than are basaltic flows. Wadge and Lopes (1991) showed a direct correlation between
distal flow lobe width and SiO₂ content for terrestrial flows, suggesting that distal lobe width can be used to remotely distinguish basalts from more silicic flows (Fig. 5.1). Before applying this technique to remotely identify planetary lavas, distal lobe width values must first be normalized to account for the differing planetary gravities (Wadge and Lopes, 1991).

There is substantial overlap in Fig. 5.1 among the ranges of distal lobe widths comprising each flow type. For example, distal lobe widths of 10 - 100 m could be classified as either basalts or andesites. This overlap is due to significant scatter in the data, which is in turn attributable to the fact that silica content is only one of many factors influencing viscosity. Other significant factors include the eruption temperature, crystallinity, and the concentrations of H₂O and alkalis. If the abscissa of Fig. 5.1 were a more accurate viscosity index, less scatter would be anticipated, as it is viscosity -- not just silica content -- that ultimately controls distal lobe widths. Therefore, in using distal lobe widths to classify flows, we use only the middle portion of the distal lobe width range, as this likely represents the distal lobe widths of basalts and andesites of typical viscosities. This approach narrows the range of distal lobe widths to approximately 20 - 40 m for basalts and 40 - 200 m for andesites.

5.4 Data

We measured 44 extraterrestrial lava flow margins. In selecting suitable lavas for measurement, we used the same criteria as for terrestrial lavas (see Section 2.4). On Mars, we measured the fractal properties of 13 flow margins from Viking Orbiter.
Figure 5.1. Plot of distal flow lobe width vs. SiO₂ content for terrestrial flows. General increase in lobe width with increasing SiO₂ content suggests that lobe widths, after normalization to account for differing planetary gravities, can be used to infer silica content of extraterrestrial lavas. (After Wadge and Lopes, 1991.)
photographic images using equivalent rod lengths ranging from 240 m to 18 km (Table 5.1). The general locations of these flows are shown in Fig. 5.2. Three of these margins are of lavas from Elysium Mons Volcano (Figs. 5.3 and 5.4); these lavas have been studied in depth by Mouginis-Mark et al. (1984; also see Mouginis-Mark, 1985, 1990). We also measured 8 lava flow margins (2 margins each of 4 lava flows) in Alba Patera, which have been studied in detail by Lopes-Gautier and co-workers (e.g., Lopes and Kilburn, 1990; Lopes-Gautier et al., 1993). Two of these lava flows are located north of the summit caldera of Alba Patera (Flow margins M4-7); the other two are located to the southeast of the caldera (Flow margins M8-11); see Figs. 5.5 and 5.6. The remaining two (M12 and M13) are margins of the Eastern Cerberus Formation in Western Amazonis Planitia, which has been identified as a flood basalt (Plescia, 1990); see Figs. 5.7 and 5.8.

Also given in Table 5.1 is a single measurement of a lunar flow (L1): the Phase III flow margin in Mare Imbrium, measured from Apollo 15 photographic images (Figs. 5.9 and 5.10). This was the only lunar lava flow margin that we found to satisfy the criteria outlined in Section 2.4; see Schaber et al. (1976) for a discussion on the scarcity of mappable flow margins in the lunar maria. We measured the Phase-III Mare Imbrium flow with equivalent rod lengths ranging from 3 to 58 km. Like the Martian Cerberus Formation, these young Imbrium flows form vast lava plateaus, having areal extents comparable to terrestrial flood basalts (Plescia, 1990; Schaber, 1973).

On Venus, we measured the fractal properties of 30 flow margins from Magellan radar images, using equivalent rod lengths of 140 m - 36 km. These data are summarized in Table 5.2. Three of the measured flow margins are of Mylitta Fluctus,
Table 5.1. Database of Martian and Lunar Flows

<table>
<thead>
<tr>
<th>Flow location (Image Number)</th>
<th>D</th>
<th>R²</th>
<th>Interpretation</th>
<th>Scale (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mars (Viking Orbiter Image):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1. Elysium Mons (651A07-12)</td>
<td>1.04</td>
<td>.95</td>
<td>a'a</td>
<td>0.8 - 13.0</td>
</tr>
<tr>
<td>M2. Elysium Mons (651A07-12)</td>
<td>1.08</td>
<td>.97</td>
<td>a'a</td>
<td>0.8 - 13.0</td>
</tr>
<tr>
<td>M3. Elysium Mons (651A07-12)</td>
<td>1.19</td>
<td>.98</td>
<td>phh</td>
<td>0.3 - 10.3</td>
</tr>
<tr>
<td>M4. N. of Alba Patera (253S07)</td>
<td>1.06</td>
<td>.98</td>
<td>a'a</td>
<td>0.2 - 7.4</td>
</tr>
<tr>
<td>M5. N. of Alba Patera (253S07)</td>
<td>1.05</td>
<td>.95</td>
<td>a'a</td>
<td>0.2 - 7.4</td>
</tr>
<tr>
<td>M6. N. of Alba Patera (253S07)</td>
<td>1.06</td>
<td>.98</td>
<td>a'a</td>
<td>0.2 - 7.4</td>
</tr>
<tr>
<td>M7. N. of Alba Patera (253S07)</td>
<td>1.07</td>
<td>.97</td>
<td>a'a</td>
<td>0.2 - 7.4</td>
</tr>
<tr>
<td>M8. SE. of Alba Patera (254S48)</td>
<td>1.02</td>
<td>.98</td>
<td>different</td>
<td>0.3 - 4.7</td>
</tr>
<tr>
<td>M9. SE. of Alba Patera (254S48)</td>
<td>1.03</td>
<td>.94</td>
<td>different</td>
<td>0.3 - 5.3</td>
</tr>
<tr>
<td>M10. SE. of Alba Patera (254S48)</td>
<td>1.03</td>
<td>.97</td>
<td>different</td>
<td>0.3 - 5.3</td>
</tr>
<tr>
<td>M11. SE. of Alba Patera (254S48)</td>
<td>1.03</td>
<td>.96</td>
<td>different</td>
<td>0.3 - 6.7</td>
</tr>
<tr>
<td>M12. SE Cerberus Plains (583A76-77)</td>
<td>1.06</td>
<td>.96</td>
<td>a'a</td>
<td>0.4 - 17.5</td>
</tr>
<tr>
<td>M13. NE Cerberus Plains (583A77)</td>
<td>1.06</td>
<td>.97</td>
<td>a'a</td>
<td>0.4 - 11.0</td>
</tr>
<tr>
<td><strong>The Moon (Apollo 15 Image):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L1. Mare Imbrium Phase III (1553, 1556, 1557)</td>
<td>1.20</td>
<td>.96</td>
<td>phh</td>
<td>2.9 - 57.6</td>
</tr>
</tbody>
</table>

Flows are interpreted according to fractal dimension (D) and R² as one of four flow types: a'a, pahoehoe ("phh"), transitional ("trans.") or different from internally-controlled basalts ("different"). Scale refers to range of rod lengths used to measure flow margin. Flow numbers beginning with M and L are Martian and Lunar flows, respectively.
Figure 5.2. Regional map of Mars, showing locations of three study areas: (a) Elysium Planitia, (b) Alba Patera, and (c) Cerberus Plains. These areas are shown in detail in Figs. 5.3, 5.5 and 5.7, respectively. (After Baker, 1982.)
Figure 5.3. Morphologic map of Elysium Planitia, showing distribution of surface units. Detail of box (a) of Fig. 5.2. Flows measured with fractal technique (shown in Fig. 5.4) are located at 33N, 215W, just west of Hecates Tholus. (From Mouginis-Mark et al., 1984.)
Figure 5.4. Photomosaic of Viking Orbiter frames 651A07-12 in Elysium Planitia, west of Hecates Tholus. Image center 33°N, 215°W; resolution 52m/pixel. Arrows denote lava flow margins measured with fractal technique: flow margin numbers correspond to Table 5.1. (After Mouginis-Mark, 1985.)
Figure 5.5. Map of Alba Patera, showing distribution of lava flows. Detail of box (b) of Fig. 5.2. Flow margins measured with fractal technique correspond to flows 2 (M6-M7), 3 (M4-M5), 16 (M8-M9) and 17 (M10-M11). S-type (or single) flow fields are composed of one major flow, whereas M-type (or multiple) flow fields consist of several major flows. All flows measured by fractal analysis are S-type. C designates summit caldera. (From Lopes and Kilburn, 1990.)
Figure 5.6a. Lava flows of Alba Patera, north of summit caldera. Flow margins measured with fractal technique are numbered, and correspond to flows 2 (M6-M7) and 3 (M4-M5) of Fig. 5.5. Viking frame 253507. Image center 44°N, 108°W; resolution 72 m/pixel. (Image courtesy of R. Lopes-Gautier.)
Figure 5.6b. Lava flows of Alba Patera, southeast of summit caldera. Arrows denote lava flow margins measured with fractal technique, and correspond to flows 16 (M8-M9) and 17 (M10-M11) of Fig. 5.5. Viking frame 254S48. Image center 37°N, 101°W; resolution 87 m/pixel. (Image courtesy of R. Lopes-Gautier.)
Figure 5.7. Geologic map of southern Elysium Planitia, showing distribution of surface units including the Cerberus Plains. Detail of box (c) of Fig. 5.2. (From Plescia, 1990.)
Figure 5.8. Eastern flow margin of Cerberus Formation in western Amazonis Planitia. Arrows denote lava flow margins measured with fractal technique; flow margin numbers correspond to Table 5.1. Viking frame 583A77. Image center 27°N, 168°W; resolution 110 m/pixel. (After Plescia, 1990.)
Figure 5.9. Map of Imbrium basin, showing distribution of the earliest (phase-I) to most recent (phase-III) lavas. Areal extent of the Phase-III mare is shown in black. Arrows indicate flow direction; hachures indicate flow scarp limits of each phase. Dashed lines represent basin ring structures. (After Schaber, 1973).
Figure 5.10. Oblique photograph of a portion of the Phase-III lava flow field in Mare Imbrium. As image is not vertical, scale is approximate. Arrow denotes lava flow margin measured with fractal technique; flow margin number corresponds to Table 5.1. Apollo 15 metric photograph 1556. Image center 28°N, 27°W.
<table>
<thead>
<tr>
<th>Flow location (Magellan image)</th>
<th>D</th>
<th>$R^2$</th>
<th>Interpretation</th>
<th>Scale (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1. W. of Aphrodite Terra (C100N043)</td>
<td>1.12</td>
<td>.99</td>
<td>trans.</td>
<td>0.6 - 17.9</td>
</tr>
<tr>
<td>V2. E. of Atla Regio (C100N215)</td>
<td>1.15</td>
<td>.90</td>
<td>different</td>
<td>3.6 - 35.7</td>
</tr>
<tr>
<td>V3. E. of Atla Regio (C100N215)</td>
<td>1.09</td>
<td>.90</td>
<td>different</td>
<td>2.3 - 28.3</td>
</tr>
<tr>
<td>V4. E. of Atla Regio (C100N215)</td>
<td>1.14</td>
<td>.98</td>
<td>phh</td>
<td>0.5 - 22.5</td>
</tr>
<tr>
<td>V5. N. of Phoebe Regio (C100N283)</td>
<td>1.06</td>
<td>.95</td>
<td>a’a</td>
<td>0.6 - 7.1</td>
</tr>
<tr>
<td>V6. N. of Phoebe Regio (C100N283)</td>
<td>1.07</td>
<td>.90</td>
<td>different</td>
<td>0.6 - 7.1</td>
</tr>
<tr>
<td>V7. N. of Phoebe Regio (C100N283)</td>
<td>1.06</td>
<td>.97</td>
<td>a’a</td>
<td>0.6 - 9.0</td>
</tr>
<tr>
<td>V8. N. of Phoebe Regio (C100N283)</td>
<td>1.10</td>
<td>.97</td>
<td>trans.</td>
<td>0.6 - 9.0</td>
</tr>
<tr>
<td>V9. N. of Phoebe Regio (C100N283)</td>
<td>1.09</td>
<td>.95</td>
<td>a’a</td>
<td>0.4 - 10.0</td>
</tr>
<tr>
<td>V10. N. of Phoebe Regio (C100N283)</td>
<td>1.09</td>
<td>.95</td>
<td>a’a</td>
<td>0.5 - 14.0</td>
</tr>
<tr>
<td>V11. E. of Phoebe Regio (C100N317)</td>
<td>1.13</td>
<td>.98</td>
<td>phh</td>
<td>0.6 - 17.9</td>
</tr>
<tr>
<td>V12. N. of Sif Mons (F25N351)</td>
<td>1.21</td>
<td>.98</td>
<td>phh</td>
<td>0.3 - 11.1</td>
</tr>
<tr>
<td>V13. N. of Sif Mons (F25N351)</td>
<td>1.15</td>
<td>.95</td>
<td>phh</td>
<td>0.3 - 5.6</td>
</tr>
<tr>
<td>V14. N. of Sif Mons (F25N351)</td>
<td>1.20</td>
<td>.96</td>
<td>phh</td>
<td>1.8 - 17.9</td>
</tr>
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<td>V15. N. of Sif Mons (F25N351)</td>
<td>1.17</td>
<td>.96</td>
<td>phh</td>
<td>1.4 - 14.2</td>
</tr>
<tr>
<td>V16. N. of Sif Mons (F25N351)</td>
<td>1.09</td>
<td>.97</td>
<td>a’a</td>
<td>2.3 - 28.3</td>
</tr>
<tr>
<td>V17. N. of Sif Mons (F25N351)</td>
<td>1.16</td>
<td>.96</td>
<td>phh</td>
<td>0.3 - 11.1</td>
</tr>
<tr>
<td>V18. N. of Gula Mons (F25N357)</td>
<td>1.22</td>
<td>.96</td>
<td>phh</td>
<td>1.1 - 28.3</td>
</tr>
<tr>
<td>V19. N. of Gula Mons (F25N357)</td>
<td>1.16</td>
<td>.98</td>
<td>phh</td>
<td>0.3 - 8.9</td>
</tr>
<tr>
<td>V20. N. of Gula Mons (F25N357)</td>
<td>1.24</td>
<td>.96</td>
<td>phh</td>
<td>1.8 - 35.7</td>
</tr>
<tr>
<td>V21. N. of Gula Mons (F25N357)</td>
<td>1.23</td>
<td>.99</td>
<td>phh</td>
<td>0.1 - 7.9</td>
</tr>
<tr>
<td>V22. N. Sedna Planitia (F55N346)</td>
<td>1.20</td>
<td>.96</td>
<td>phh</td>
<td>0.3 - 8.9</td>
</tr>
<tr>
<td>V23. Mylitta Fluctus (F55S355)</td>
<td>1.09</td>
<td>.99</td>
<td>a’a</td>
<td>0.4 - 8.9</td>
</tr>
<tr>
<td>V24. Mylitta Fluctus (F55S355)</td>
<td>1.09</td>
<td>.97</td>
<td>a’a</td>
<td>0.3 - 11.2</td>
</tr>
<tr>
<td>V25. Mylitta Fluctus (F55S355)</td>
<td>1.04</td>
<td>.98</td>
<td>a’a</td>
<td>0.4 - 5.6</td>
</tr>
<tr>
<td>V26. SE Lavinia Planitia (F50S356)</td>
<td>1.20</td>
<td>.98</td>
<td>phh</td>
<td>0.3 - 22.2</td>
</tr>
<tr>
<td>V27. SE Lavinia Planitia (F50S356)</td>
<td>1.21</td>
<td>.97</td>
<td>phh</td>
<td>0.3 - 7.0</td>
</tr>
<tr>
<td>V28. SE Lavinia Planitia (F50S356)</td>
<td>1.13</td>
<td>.96</td>
<td>phh</td>
<td>0.3 - 8.9</td>
</tr>
<tr>
<td>V29. SE Lavinia Planitia (F50S356)</td>
<td>1.18</td>
<td>.98</td>
<td>phh</td>
<td>0.3 - 11.1</td>
</tr>
<tr>
<td>V30. SE Lavinia Planitia (F50S356)</td>
<td>1.18</td>
<td>.98</td>
<td>phh</td>
<td>0.3 - 11.1</td>
</tr>
</tbody>
</table>
also considered to be a flood basalt (Roberts et al., 1992); see Figs. 5.11 and 5.12. A compilation of area, volume and thickness estimates for the flood basalts included in this analysis is given in Table 5.3. Also shown in Table 5.3 are these same parameters for the Columbia River Plateau, a terrestrial flood basalt. Note these flood basalts on Mars, Venus, Moon and Earth are of comparable areal and volumetric extents, as are the individual flow fields which comprise these vast plateaus.

5.5 Results

Results of Fractal Analysis of Planetary Lavas

Of the 44 flow margins measured, most (40, or over 90%) are fractal over the range of scale measured (equivalent rod lengths of 140 m - 58 km); see Tables 5.1 and 5.2. Based on the terrestrial analogy, fractal behavior indicates basaltic composition. Fractal dimension (D) can thus be used to distinguish among basaltic flow types. Recall from Chapter 2 that terrestrial basalts show a bimodal distribution in fractal dimension, with a'a flows generally having lower D (1.04-1.09) than pahoehoe flows (1.13-1.24). Terrestrial basalts with morphologies transitional between a'a and pahoehoe tend to have intermediate fractal dimensions. A histogram of the fractal dimensions of extraterrestrial flows is given in Fig. 5.13. Assuming the terrestrial ranges of D also apply to planetary lavas, we make the following interpretations, as summarized in Tables 5.1 and 5.2:
Figure 5.11. Map of Mylitta Fluctus flow field, showing the relative positions of individual phases. Phases (oldest to youngest) are shown in green, light blue, dark blue, red, orange, yellow. (From Roberts et al., 1992.)
Figure 5.12. Mylitta Fluctus flow field (part of Phase IV, shown in red in Fig. 5.9). Arrows denote lava flow margins measured with fractal technique; flow margin numbers correspond to Table 5.2. Magellan image F55S355. Image center 55°S, 357°E; resolution 75 m/pixel.
Table 5.3. Comparison of Selected Flood Basalt Provinces on the Terrestrial Planets

<table>
<thead>
<tr>
<th>Flow Field (Planet)</th>
<th>Area (km²)</th>
<th>Volume (km³)</th>
<th>Thickness (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Columbia River (Earth)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entire Flow Field</td>
<td>1-2 x 10⁵</td>
<td>1-2 x 10⁵</td>
<td>10³</td>
</tr>
<tr>
<td>Individual Flows</td>
<td>1-2 x 10⁵</td>
<td>10² - 10³</td>
<td>10 - 10² (avg=30)</td>
</tr>
<tr>
<td>References: Tolan et al., 1989; Reidel et al., 1989.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cerberus Formation (Mars)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entire Flow Field</td>
<td>1-2 x 10⁵</td>
<td>?</td>
<td>≥ 400</td>
</tr>
<tr>
<td>Individual Flows</td>
<td>?</td>
<td>?</td>
<td>≥ 10</td>
</tr>
<tr>
<td>Mylitta Fluctus (Venus)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entire Flow Field</td>
<td>3 x 10⁵</td>
<td>2 x 10⁴</td>
<td>250 - 400 (?)</td>
</tr>
<tr>
<td>Individual Flows</td>
<td>≤ 1 x 10⁵</td>
<td>≤ 2 x 10⁴</td>
<td>10 - 30 (?)</td>
</tr>
<tr>
<td>Mare Imbrium (Moon)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entire Flow Field</td>
<td>2 x 10⁵</td>
<td>4 x 10⁴</td>
<td>2 - 4 x 10³</td>
</tr>
<tr>
<td>Individual Flows</td>
<td>≤ 1 x 10⁵</td>
<td>?</td>
<td>10 - 60 (avg=30)</td>
</tr>
</tbody>
</table>
Figure 5.13. Histogram of the fractal dimensions (D) of extraterrestrial flows. Based on terrestrial analogy, fractal dimension is used to categorize these flows into a’a (D: 1.04-1.09), pahoehoe (D: 1.13-1.24) and transitional (D: 1.10-1.12) flow types. (See Tables 5.1 and 5.2.)
(1) **The Moon.** The single lunar measurement (Mare Imbrium Phase III, a flood basalt) is fractal with $D = 1.20$, suggesting basaltic pahoehoe.

(2) **Mars.** Eight of the 13 Martian flow margins measured have fractal properties consistent with basaltic a'a ($D: 1.04 - 1.08$); only one resembles basaltic pahoehoe ($D: 1.19$). The remaining four flow margins (three of which are fractal) all have $D$ of 1.02 or 1.03; these are Flows M8-11 of Table 5.1, located in the Alba Patera area. These fractal dimensions are unusually low, even for a'a flows. We categorize these flows as "different from internally-controlled basalts", leaving open their interpretation as either topographically-controlled basalts or flows of a non-basaltic composition. These flows are discussed further below.

(3) **Venus.** Of the 30 Venusian flow margins measured, 27 are fractal. Eight of these fall into the field of a'a ($D: 1.04 - 1.09$), 17 as pahoehoe ($D: 1.13 - 1.24$), and 2 as transitional ($D: 1.10 - 1.12$).

**Results of Radar Analysis of Venusian Lavas**

The above categorization of flow types is highly dependent on the assumption that the ranges of fractal dimension of planetary a’a, pahoehoe and transitional basalts are the same as their terrestrial counterparts. In this section, we evaluate this assumption by comparing the results of this fractal analysis of Venusian flows to those of a radar technique, also aimed at distinguishing basaltic flow types.

Using the methodology of Campbell and Campbell (1992), we use the radar-backscatter signal of the 18 Venusian lavas that are both fractal and located between
Based on the level of this signal, we categorize these lavas into a'a (high), transitional (intermediate), and pahoehoe (low) basalts. This interpretation is shown visually in Fig. 5.14: a'a flows are shown in red, transitional flows are shown in yellow, and pahoehoe flows are shown in shades of grey. A greyscale is used to show the wide range of surface smoothness interpreted to be pahoehoe textures. Table 5.4 compares this radar categorization of Venusian lavas with the results of the fractal technique. Of these 18 flows, 4 are identified by both techniques as a'a and 10 are identified by both techniques as pahoehoe. Three flows are identified by the radar technique as having both transitional and pahoehoe morphologies. The corresponding fractal dimensions (1.10, 1.12, and 1.14) are in the transitional or low end of the pahoehoe range.

In only one case (Flow V16; see Table 5.4) is there an obvious disagreement between the two techniques. Thus, in 17 of the 18 cases, there is a good match between these two, independent techniques. This is excellent evidence that Venusian basalts have the same ranges of fractal dimensions as their terrestrial counterparts. Thus, we can confidently use D to remotely distinguish a'a, pahoehoe and transitional flows on other planets.

Results of Lobe Width Analysis of Martian Lavas

Using the technique of Wadge and Lopes (1991), Rosaly Lopes-Gautier measured the distal flow lobes of three of the four Alba Patera flows (i.e., corresponding to 6 of the 8 margins in Table 5.1: Flows M4, M5, and M8-M11)
Figure 5.14. Venusian lava flows, colored according to surface roughness at the 12.6 cm wavelength (Campbell and Campbell, 1992). Based on surface roughness, flows are interpreted as a‘a (shown in red), transitional (shown in yellow) and pahoehoe flows (shown in shades of grey). Color bar shows surface roughness increasing toward the right. Arrows denote lava flow margins measured with fractal technique; flow margin numbers correspond to Tables 5.2 and 5.4. Interpretation of flow type based on fractal analysis and surface roughness are summarized in Table 5.4. (a) Magellan image C100N043, showing flow margin V1. This flow margin, interpreted as transitional by fractal analysis, appears mostly pahoehoe in this radar image.
Figure 5.14b. Magellan image C100N215, showing flow margin V4. This flow margin, interpreted as pahoehoe by fractal analysis, appears mostly transitional in this radar image.
Figure 5.14c. Magellan image C100N283, showing flow margins V5 and V6. V5, interpreted as a’a by fractal analysis, also appears a’a in this radar image. (V6 was not fractal).
Figure 5.14d. Magellan image C100N283, showing flow margins V7 and V8. V7, interpreted as a'a by fractal analysis, appears a'a in this radar image. V8, interpreted as transitional by fractal analysis, appears mostly transitional (with some pahoehoe) in this radar image.
Figure 5.14e. Magellan image C100N283, showing flow margins V9 and V10. Both flow margins, interpreted as a`a by fractal analysis, also appear a`a in this radar image.
Figure 5.14f. Magellan image C100N317, showing flow margin V11. This flow margin, interpreted as pahoehoe by fractal analysis, also appears pahoehoe in this radar image.
Figure 5.14g. Magellan image F25N351, showing flow margins V12, V14 and V15. These flow margins, interpreted as pahoehoe by fractal analysis, also appear pahoehoe in this radar image.
Figure 5.14h. Magellan image F25N351, showing flow margins V13, V16 and V17. These flow margins each appear pahoehoe in this radar image. Flow margins V13 and V17 are also interpreted as pahoehoe by fractal analysis; flow margin V17, however, is interpreted as a'a by fractal analysis.
Figure 5.14i. Magellan image F25N357, showing flow margins V18 and V19. These flow margins, interpreted as pahoehoe by fractal analysis, also appear pahoehoe in this radar image.
Figure 5.14j. Magellan image F25N357, showing flow margins V20 and V21. These flow margins, interpreted as pahoehoe by fractal analysis, also appear pahoehoe in this radar image.
Table 5.4. Comparison of Fractal and Radar Techniques of Venusian Flows

<table>
<thead>
<tr>
<th>Flow location (Magellan image)</th>
<th>Fractal Interpretation</th>
<th>Radar Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1. W. of Aphrodite Terra (C100N043)</td>
<td>trans.</td>
<td>mostly phh/trans.</td>
</tr>
<tr>
<td>V4. E. of Atla Regio (C100N215)</td>
<td>phh</td>
<td>trans./ phh</td>
</tr>
<tr>
<td>V5. N. of Phoebe Regio (C100N283)</td>
<td>a'a</td>
<td>a'a</td>
</tr>
<tr>
<td>V7. N. of Phoebe Regio (C100N283)</td>
<td>a'a</td>
<td>a'a</td>
</tr>
<tr>
<td>V8. N. of Phoebe Regio (C100N283)</td>
<td>trans.</td>
<td>trans./ phh</td>
</tr>
<tr>
<td>V9. N. of Phoebe Regio (C100N283)</td>
<td>a'a</td>
<td>a'a</td>
</tr>
<tr>
<td>V10. N. of Phoebe Regio (C100N283)</td>
<td>a'a</td>
<td>a'a</td>
</tr>
<tr>
<td>V11. E. of Phoebe Regio (C100N317)</td>
<td>phh</td>
<td>phh</td>
</tr>
<tr>
<td>V12. N. of Sif Mons (F25N351)</td>
<td>phh</td>
<td>phh</td>
</tr>
<tr>
<td>V13. N. of Sif Mons (F25N351)</td>
<td>phh</td>
<td>phh</td>
</tr>
<tr>
<td>V14. N. of Sif Mons (F25N351)</td>
<td>phh</td>
<td>phh</td>
</tr>
<tr>
<td>V15. N. of Sif Mons (F25N351)</td>
<td>phh</td>
<td>phh</td>
</tr>
<tr>
<td>V16. N. of Sif Mons (F25N351)</td>
<td>a'a</td>
<td>phh</td>
</tr>
<tr>
<td>V17. N. of Sif Mons (F25N351)</td>
<td>phh</td>
<td>phh</td>
</tr>
<tr>
<td>V18. N. of Gula Mons (F25N357)</td>
<td>phh</td>
<td>phh</td>
</tr>
<tr>
<td>V19. N. of Gula Mons (F25N357)</td>
<td>phh</td>
<td>phh</td>
</tr>
<tr>
<td>V20. N. of Gula Mons (F25N357)</td>
<td>phh</td>
<td>phh</td>
</tr>
<tr>
<td>V21. N. of Gula Mons (F25N357)</td>
<td>phh</td>
<td>phh</td>
</tr>
</tbody>
</table>

Flows V22-30 of Table 5.2 are not included in this table because, as they are located outside the latitudinal band 54°N - 34°S, no radar data are available for comparison. Flows V2, V3 and V6 are not included because they are not fractal.
included in this fractal analysis. The distal lobes corresponding to flow margins M6 and M7 were not suitable for distal lobe width measurements (Rosaly Lopes-Gautier, pers. comm.). As discussed in Section 5.3, typical basalts of average viscosities have distal lobe widths of approximately 20-40 m, whereas typical andesites have wider distal lobes: approximately 40 - 200 m.

R. Lopes-Gautier (unpublished data) found that the normalized distal lobe widths of the northern flows (M4 and M5) are in the range 24 to 35m. Although these values are consistent with either basalts or andesites using the Wadge and Lopes (1991) classification, they likely indicate basalts, as they are less than 40m. The distal flow lobes of the southeastern flows (M8-M11) are wider, ranging from 37 to 55m. According to the Wadge and Lopes (1991) classification, these lobe widths exceed those of typical basalts, and, while still in the basaltic range, are more typical of basaltic andesites or andesites.

We now compare these distal lobe width results with those of our fractal analysis. Fractal analysis of M4 and M5 revealed fractal behavior (R^2 > 0.95), with fractal dimensions ranging from 1.05 - 1.06. These results are consistent with those obtained for terrestrial a’a basalts (1.04 - 1.09; see Chapter 2). Measurements of Flows M8 through M11 again yielded high R^2 values (all but one had R^2 >.95), but with significantly lower fractal dimensions: all four margins had D values between 1.02 and 1.03 and were therefore classified as “different from internally-controlled basalts”.

The low D values of flow margins M8 through M11 indicate that these flow margins appear fairly linear at the range of rod lengths used, possibly due to the
suppression of features in this scale range. Consider a flow margin that has a relative lack of small-scale features. A fractal analysis on such a flow over a wide range of scales would reveal non-fractal behavior (i.e., low $R^2$ values). A fractal analysis over only the scale range of features that have been suppressed, however, would generally reveal fractal behavior with $D \approx 1$; in this scale range, the margin appears to be a straight line. (A fractal analysis on a perfectly straight line would theoretically yield $R^2 = 1.00$ and $D = 1.00$.)

Thus, one explanation for the low $D$ values of M8-M11 is that relative to typical basalts, these flows have higher viscosities and/or yield strengths which cause suppression of small-scale features in the flow margin (discussed in Section 2.6). Such rheologies are normally associated with flows of more silicic compositions as well as cooler, more crystalline basalts. An alternative explanation for the low $D$ values of flows 8 through 11 is that these margins are topographically controlled.

Interestingly, the flow margins we have interpreted as a‘a basalts based on this fractal analysis (M4 and M5) were also classified as basalts by Wadge and Lopes (1991), whereas the flow margins we have classified as different from internally-controlled basalts (M8 - M11) have distal flow lobe widths typically associated with andesites or more viscous basalts. The results of these two independent remote sensing techniques are consistent and complementary, and suggests that these two techniques can be used in combination to better understand planetary lavas (Lopes-Gautier et al., 1993).
5.6 Discussion

Planetary lavas are basalts

Most of the flow margins included in this analysis are fractal, with fractal dimensions indicating a basaltic composition. This is consistent with an abundance of evidence suggesting planetary lavas are basalts, as reviewed in Section 5.2. On Earth, small volumes of silicic rocks can be produced by differentiation of basaltic magma in a magma chamber, but large silicic deposits tend to be concentrated in areas having a thick granitic crust: subduction zones, continental rift zones and isolated mid-plate continental hot spots (Coulon and Thorpe, 1981; Francis and Wood, 1982). Francis and Wood (1982) interpret the apparent lack of large-scale silicic volcanism on Mars to indicate that Mars lacks such a crust. Potential small-scale silicic volcanism on Mars is discussed below in the context of lava flows from Alba Patera.

Eruption rates and emplacement processes

Whether a basaltic lava assumes an a’a or pahoehoe morphology depends on the conditions or emplacement. In Hawaii, a’a flows are generally associated with high effusion rates and/or viscous lavas; pahoehoe flows tend to be associated with low effusion rates and/or fluid lavas (Peterson and Tilling, 1980; Kilburn, 1981; Rowland and Walker, 1990). Low effusion of a viscous basalt in Hawaii results in toothpaste morphology, transitional between a’a and pahoehoe (Rowland and Walker, 1987).
Of the 13 Martian flow margins studied, only one has a fractal dimension indicative of pahoehoe-style emplacement (M3; see Table 5.1). This could reflect a sampling bias. Relative to a’a flows, pahoehoe flows tend to have lower near-margin thicknesses and could more easily be obscured by dust or eroded. If, however, this is not a sampling bias and the flows studied in this analysis are representative of the global populations of a’a and pahoehoe flows on Mars -- or at least for these volcanoes on Mars -- then this would indicate that a’a formation is favored. This could suggest (1) higher eruption rates and/or (2) more viscous lavas for these volcanoes in particular, or for Martian volcanoes in general. We evaluate these possibilities using the theoretical model of Wilson and Head (1981, 1983):

Over a wide range of conditions, magma rise velocity ($u$) is given by:

$$u = w\left(\frac{wg\Delta \rho - 2\gamma}{12\eta}\right)$$

and the corresponding mass eruption rate ($M$) is given by:

$$M = w^2 L \rho \left(\frac{wg\Delta \rho - 2\gamma}{12\eta}\right)$$

where $w$ is fissure width, $g$ is planetary gravity, $\Delta \rho$ is the effective density contrast between the rising magma and the surrounding country rock, $\gamma$ is yield strength, $\eta$ is viscosity, $L$ is fissure length.

The Martian planetary gravity is (3.7 m/s$^2$) is lower than the terrestrial value (9.8 m/s$^2$). Thus, if all other parameters on the right hand sides of Eqs. (5.1) and (5.2)
are held constant, lower magma rise velocities and eruption rates would be expected, not the higher effusion rates normally associated with a’ā flows.

An alternate explanation for the observed proportional abundance of Martian a’ā flows is that, relative to terrestrial lavas, Martian lavas are more viscous. However, according to Eqs. (5.1) and (5.2), more viscous magmas would serve to decrease the magma rise velocity and mass eruption rates. As noted above, low effusion of a viscous basalt gives rise to toothpaste lava, not a’ā (Rowland and Walker, 1987).

If, on the other hand, typical Martian lavas were less viscous than terrestrial lavas, this would give rise to higher mass eruption rates, and possibly a’ā lavas (despite these lower viscosities). From Eqs. (5.1) and (5.2), this would be particularly likely if Martian fissures were typically wider. One possible explanation for relatively fluid Martian lavas follows from the large size of Martian volcanoes. Bigger volcanoes tend to have bigger magma chambers, and would likely produce hotter, less viscous magmas.

In contrast, the majority of the Venusian flows measured have fractal dimensions corresponding to pahoehoe flows (Table 5.2). Similarly, the Magellan radar data suggest that volcanic surfaces in the areas studied by Campbell and Campbell (1992) are generally smooth, analogous to terrestrial pahoehoe flows.

A priori, it’s theoretically difficult to explain why pahoehoe formation would be favored on Venus. However, if more detailed studies using larger databases show that pahoehoe flows are common on Venus -- or on certain volcanoes on Venus -- this has important implications for the sub-volcanic plumbing systems. As summarized by Wilson and Head (1981) and Rowland (1987), an eruption will continue only if (1) magma pressure is sufficient to maintain eruptive conduits, and (2) magma flux is
sufficient to replenish heat lost through the cooling of country rocks. Otherwise, the conduit closes and/or the dike freezes, and the eruption stops. The low magma pressures associated with eruptions of pahoehoe lava are insufficient to keep eruptive conduits open; the presence of pahoehoe lavas indicates a preestablished connection between the magma chamber and the planetary surface. Perhaps, the hotter temperatures on Venus retard magma freezing, thereby keeping dikes fluid longer and allowing them to become mechanically-stable conduits. If the dikes remain fluid even after the eruption stops, they can be later re-used by subsequent low-magma-flux eruptions.

Flood basalts

As part of this fractal analysis, we studied three extraterrestrial flood basalt provinces: the Cerberus Formation (Mars), Mare Imbrium, Phase III (Moon) and Mylitta Fluctus (Venus). In terms of areal extent, volume and thickness, these flow fields are roughly comparable to each other, as well as to terrestrial flood basalt provinces such as the Columbia River Plateau (Table 5.3).

Fractal analysis reveals a'a-like fractal dimensions for the margins of Mylitta Fluctus and the Cerberus Formation, suggesting high effusion rates. This is consistent with the traditional view that flood basalts are erupted at high eruption rates (e.g., Swanson et al., 1975) in short periods of time (c. 10 days). However, the measurement of Mare Imbrium (L1; see Table 5.1) yielded a significantly higher fractal dimension, consistent with pahoehoe-style emplacement. This suggests that at least
some flood basalts may be emplaced more slowly at lower effusion rates, as proposed by Self et al. (1993). Clearly, many more flood basalts need to be measured with this technique before any firm conclusions are made.

Discussion of Alba Patera lavas

Both the fractal and distal lobe width analyses identified the flows to the north of Alba Patera as typical a’a basalts and the southeastern flows (M8-M11) as different from typical basalts. One explanation for the wide distal flow lobes and low fractal dimensions of the southeastern flows is a more viscous rheology, suggesting a more silicic and/or cooler lava.

Additional evidence that Flows M8-M11 ("southeastern flows") may be more viscous and/or silicic than Flows M4-M7 ("northern flows") comes from flow geometry and estimated effusion rates. Lopes and Kilburn (1990) performed a detailed study of eighteen Alba Patera lavas; four of these flows correspond to the eight margins measured in this fractal analysis. They measured lengths, maximum widths and thicknesses of these flows; these data are summarized in Table 5.5. Note the northern flows are longer and have lower aspect ratios than the southeastern flows, indicating a more fluid rheology. Also shown in Table 5.5 are Lopes and Kilburn's (1990) estimates for average effusion rates for these four flows. Note the northern flows have higher estimated effusion rates (13 - 74 x 10^3 m^3/s) than the southeastern (3 - 9 x 10^3 m^3/s). On Earth, viscous lavas tend to erupt at lower effusion rates. Thus, there is a
Table 5.5. Data of Alba Patera Lavas

<table>
<thead>
<tr>
<th>Flow Reference Numbers</th>
<th>$L_M$ (m)</th>
<th>$W_M$ (m)</th>
<th>$H$ (m)</th>
<th>A.R.</th>
<th>$Q \times 10^3$ m$^2$/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M4-M5), [3]</td>
<td>206</td>
<td>7,300</td>
<td>35</td>
<td>.0048</td>
<td>18 - 74</td>
</tr>
<tr>
<td>(M6-M7), [2]</td>
<td>181</td>
<td>11,300</td>
<td>50</td>
<td>.0044</td>
<td>13 - 30</td>
</tr>
<tr>
<td>(M8-M9), [16]</td>
<td>87</td>
<td>7,500</td>
<td>46</td>
<td>.0061</td>
<td>3</td>
</tr>
<tr>
<td>(M10-M11), [17]</td>
<td>99</td>
<td>5,000</td>
<td>28</td>
<td>.0056</td>
<td>8 - 10</td>
</tr>
</tbody>
</table>

For flow reference numbers, flow margin numbers in Table 5.1 are shown in round brackets and the flow numbers given in Lopes and Kilburn (1990) are shown in square brackets; maximum length ($L_M$); maximum width ($W_M$); average thickness from photoclinometry ($H$); Aspect Ratio (A.R. = $H/W_M$); estimated average effusion rates ($Q$). All data from Lopes and Kilburn (1990). Data have been rounded.
broad base of evidence suggesting that the southeastern flows are more viscous, and possibly more silicic, than the northern flows.

As noted by Francis and Wood (1982) and many others, small-scale silicic volcanism is often the product of magma differentiation in a basaltic magma chamber, and is generally associated with explosive volcanism. Thus, if these lavas are indeed silicic, the likelihood of explosive volcanism in Alba Patera is strengthened, as suggested by Mouginis-Mark et al. (1988).

5.7 Conclusions

Planetary lavas are generally fractal, indicating a basaltic composition, with fractal dimensions similar to the terrestrial range. Combining the terrestrial and extraterrestrial data, the fractal nature of lava flow outlines holds over five orders of magnitude in scale: from 10 cm to 60 km. Using fractal dimension, we have remotely identified flows on Venus, Mars and the Moon as a'a, pahoehoe and transitional basalts, and, for Venusian flows, this identification has been independently confirmed by a radar technique. Such flow type identification can lead to a better understanding of planetary eruption rates and eruption styles. Our fractal analysis of flood basalts suggests that these high-volume flows have been emplaced at both high (a'a-like) and low (pahoehoe-like) eruption rates, but this is based on limited data.

Additionally, we have identified four flow margins in the Alba Patera area of Mars with fractal properties different from internally-controlled basalts. These flow margins may have been controlled by preexisting topography. Alternately, these
unusual fractal properties may indicate a more viscous rheology, typically associated with cooler basalts and more silicic flows.
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CHAPTER 6

CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

Fractal properties of lava flows

Based on measurements of terrestrial flows, I have found systematic differences in fractal properties between a’α and pahoehoe basalts, as well as between basalts and more silicic lavas. Basaltic lava flow margins are generally fractal within the range of scale measured (rod lengths: 10 cm - 2 km), with pahoehoe flow margins having higher fractal dimensions than a’α flow margins; transitional flows tend to have intermediate fractal dimensions. Unlike basalts, more silicic lava flows tend to exhibit non-fractal behavior within the range of scale studied (rod lengths: 10 m - 4.5 km). This non-fractal behavior is interpreted to reflect the suppression of small-scale features in the flow margin by the higher viscosities and yield strengths associated with more silicic flows.
Fractal analysis as a remote sensing tool for planetary lavas

Based on these systematic differences in the fractal properties of terrestrial lava flow margins, I used fractal analysis as a remote sensing tool to study lava flows on Mars, Venus and the Moon. I found that the vast majority of these extraterrestrial lavas are fractal, indicating a basaltic composition. Combining the terrestrial and extraterrestrial data, the fractal nature of lava flow margins holds for over five orders of magnitude: from 10 cm to 60 km. Using the fractal dimension, I have identified these extraterrestrial lavas as internally-controlled a’a, pahoehoe and transitional basalts, or different from internally-controlled basalts. For Venusian flows, this identification has been independently confirmed by a radar technique, also aimed at remotely identifying flow type. Identifying basaltic flows as a’a and pahoehoe is significant, as these different morphologies reflect different eruption and emplacement styles. Fractal analysis of extraterrestrial flood basalt margins suggests that these vast flow fields have been emplaced at both high (a’a-like) and low (pahoehoe-like) eruption rates. I also note four flow margins in Alba Patera, Mars have fractal properties different from internally-controlled basalts. This may indicate topographic control of flow margin shape, or may indicate a more viscous rheology -- possibly a more silicic composition.

Fluid dynamic model of lava flow emplacement

I have developed a fluid dynamic model of lava flow emplacement which, based on given rheological parameters (i.e., m and α), predicts changes in flow width
and depth with distance from the source of the flow. By comparing these predicted flow geometries with field data, I work backwards to solve for $m$. I found basalts are characterized by $m$ values of 1 to 2; this indicates non-Newtonian flow, as Newtonian flows have $m=3$. The other rheological parameter ($\alpha$), which is a measure of the downstream change in rheology, is solved for by comparing the model’s predictions with remote sensing data. Such comparisons yield $\alpha$ values corresponding to downstream viscosity increases of 2 to 4 orders of magnitude. These values are reasonable and consistent with the results of several independent studies, attesting to the validity of this model.

6.2 Future Work

Future work involves (1) further developing both fractal analysis and the fluid dynamic model as remote sensing techniques and (2) applying these techniques to gain insights into rheology and dynamics of lava flows as well as other naturally occurring geologic flows (e.g., pyroclastic flows).

Fractal Analysis

In order to apply fractal analysis as a remote sensing tool for planetary lavas, it is essential to first have a solid database of terrestrial lava flows for comparison. Although my database of terrestrial basalts is fairly large, my database of silicic flows is limited to only ten flows. Clearly, it is virtually impossible to characterize the fractal
properties of silicic flows based on such a small amount of data. Future work must certainly include expanding this database of silicic flows.

Future work will largely involve applying this technique to planetary lavas. Of particular interest are:

(1) Measuring the fractal properties of more planetary flood basalt margins to gain insights into eruption rates and emplacement styles. This would in turn provide a better understanding of terrestrial flood basalts.

(2) Systematically searching for extraterrestrial lavas of non-basaltic composition. Certain lava flows on Mars and Venus have been identified as possibly silicic by other authors, and fractal analysis can test these suggestions. Also, there has recently been speculation that lava flows on Io may be sulfuric. Just as fractal analysis can be used to distinguish basalts from more silicic flows, it may be similarly developed to distinguish basalts from sulfur flows. In order to develop the fractal properties of sulfur for comparison, either terrestrial sulfur lava flows (e.g., from Poas Volcano, Costa Rica) and/or industrial sulfur flows could be measured.

(3) Measuring fractal properties of flows other than lavas. Candidate flows include rampart ejecta deposits, rock and debris flows, lahars and pyroclastic flows.

Fluid Dynamic Model

Unlike fractal analysis, which is ready to be used as a remote sensing tool, much work must still be done to make the fluid dynamic model widely applicable to
real-life lava flows. Several of the model's assumptions, which were made to simplify the mathematics, are overly restrictive and physically unrealistic. Particularly objectionable are the assumptions that underlying topography is smooth, ground slope is constant, and rheology has no cross-stream variation. Future work includes relaxing these assumptions, and solving the resulting equations.

Once the model is further developed, it must be thoroughly tested by comparing its predictions to data. Such a comparison would likely involve an experimental approach, as changes in topography and rheology can more easily be modeled in a laboratory than in nature. But even in its current simplified form, the model must be further tested. This involves collecting more data, both in the field and from images, of flow width and depth profiles.

Finally, as is the case with fractal analysis, I would like to continue this work by modeling fluids other than lavas. Specifically, I'm interested in developing this work to be relevant to volcanic hazard assessment. This model relates flow rate \( q \) to flow thickness \( h \) and a rheological parameter \( m \) using the power-law relation: \( q \sim h^m \). Thus, applying this model to volcanic lahars and pyroclastic flows, and solving for \( m \), can provide insights into the emplacement (e.g., flow rates) of these potentially hazardous flows.
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