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CODED MODULATIONS FOR
MOBILE SATELLITE COMMUNICATION CHANNELS

A DISSERTATION SUBMITTED TO THE GRADUATE DIVISION OF THE UNIVERSITY OF HAWAII IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ELECTRICAL ENGINEERING MAY 1995

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Abstract

The mobile satellite (MSAT) channel is subject to multipath fading, shadowing, Doppler frequency shift, and adjacent channel interference (ACI). Therefore, transmitted signals face severe amplitude and phase distortions. This dissertation investigates various high performance and low decoding complexity coded modulation schemes for reliable voice and data transmissions over the shadowed mobile satellite channel and the Rayleigh fading channel.

The dissertation consists of four parts. The first part presents a systematic technique for constructing MPSK trellis coded modulation (TCM) codes for voice transmission over the MSAT channel. The multilevel coding method is used for constructing TCM codes using convolutional codes with good free branch distances as the component codes or using both convolutional and block codes as the component codes. Since there is a satellite delay constraint for voice transmission, a receiver is subject to the limitation of interleaver size and decoding depth. Optimum choices of the decoding depth and the interleaving depth of TCM codes are investigated. Multi-stage decoding algorithm is proposed to rescue decoding complexity. Simulation results showed that these codes achieve good coding gains over the uncoded reference system for the Rayleigh fading and shadowed MSAT channels.

In the second part, using the multilevel coding method, multilevel block coded modulation (BCM) codes are constructed for voice transmission over the MSAT channel. Even though BCM is generally less power efficient than TCM for AWGN channels, BCM has a great potential to compete with TCM in the MSAT channel because of its shorter decoding depth and hence more effective interleaving. Binary Reed-Muller(RM) codes of length up to 32 are used as component codes. Decoding
complexity of 4-section trellis diagrams of RM codes is presented. Simulation results show that these codes achieve good coding gains over the uncoded reference system and outperform TCM codes with the same decoding complexity for the Rayleigh fading and various shadowed MSAT channels.

In the third part, a simple and systematic technique for constructing multi-level concatenated BCM schemes for data transmission over the shadowed MSAT channel and the Rayleigh fading channel is presented. These schemes are designed to achieve high-performance or large coding gain with reduced decoding complexity. Construction is based on a multilevel concatenation approach in which long powerful (binary or non-binary) codes are used as the outer codes and coset codes constructed from a linear BCM code and its subcodes are used as the inner codes. Sub-optimum multilevel closest coset decoding is proposed for the multilevel concatenated coded modulation scheme. Two-, Three- and 6-level concatenated BCM schemes with inner coset codes constructed from a short (or long) linear BCM code and its subcodes are constructed for the shadowed MSAT and the Rayleigh fading channels. Simulation results show that these codes achieve large coding gains over the uncoded reference system and outperform single-level concatenated BCM codes with the same inner codes. Especially, some multilevel concatenated BCM codes achieve an error-floor free communication at BER of $10^{-5}$.

In the final part of the dissertation, product coded modulation schemes are constructed by using multilevel concatenating approach. In product coded modulation, the product coding technique and coded modulation are combined to achieve high performance with reduced decoding complexity. Good product 8-PSK modulation codes are constructed for various shadowed MSAT channels. Suboptimum multi-stage decoding is proposed. Simulation results show that these codes achieve good
coding gains over the uncoded reference system and outperform single-level concatenated BCM codes with the same inner codes.
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Chapter 1

Introduction

1.1 Coded Modulation and Its Applications

One of the dramatic developments in bandwidth-efficient communications over the past eighteen years is the introduction and rapid application of combined coding and bandwidth efficient modulation, known as coded modulation, for reliable data transmission. Using coded modulation, reliable data transmission can be obtained without compromising bandwidth efficiency. In 1974, Massey [1] showed that the error performance of a coded digital communication system can be improved by combining coding and modulation. The first coded modulation scheme was introduced by Ungerboeck in 1976 [2], and later published in 1982 [3]. In this scheme, trellis (or convolutional) codes were combined with various types of modulation signal sets to form modulation codes by proper bits-to-signal mapping through signal set partitioning. This scheme is now known as the trellis-coded modulation (TCM).

Since the publication of Ungerboeck's prize winning paper, there has been a great deal of research on construction of TCM codes and many good TCM codes have been constructed [4] - [23].

Combining block coding and modulation, now known as block-coded modulation (BCM), originated from Imai and Hirakawa's paper published in 1977 [24], in which
they proposed a multilevel method of combining binary block component codes with a channel signal set to form a block modulation code. This multilevel method was later extended and used by others for constructing block modulation codes [25]-[40].

Coded modulation schemes were originally designed for the AWGN channel [2] - [40]. Recently they have been applied to various fading channels [41]-[65]. In the early research for fading channels, traditional error correction codes were used with binary modulation techniques [66]-[73]. Later, Butman [74] applied the multilevel FSK modulation technique and Johnston and Jones [75] presented schemes with combining block coding and multi-level DPSK modulation for the fading channel in 1981. The primary advantage of the coded modulation scheme over modulation schemes employing traditional error correction codes is its ability to achieve increased power efficiency without bandwidth expansion which is introduced by the coding process. Therefore, coded modulation schemes are ideally suited to power and bandwidth-limited channels. Such as, cellular land mobile radio, aeronautical or maritime satellite, and land mobile satellite channels. It is well known that [79, 80] the cellular land mobile radio channel can be modeled as the Rayleigh channel with its local mean following a lognormal statistics and its phase normally distributed. For an aeronautical or maritime or land mobile satellite channel, there is a line-of-sight component as well as multipath. This channel is usually modeled as Rician, i.e. a constant component plus multipath [81]. Especially, the mobile satellite (MSAT) channel is subject to multipath fading, shadowing, Doppler frequency shift, and adjacent channel interference (ACI). For areas with a lower angle (15° - 20°) between a mobile user and a suitably located geosynchronous satellite, the line-of-sight (LOS) component of the Rician model is subject to a lognormal transformation. This transformation represents the effect of foliage attenuation or blockage, also referred as shadowing. From now on, we call this MSAT channel a “shadowed MSAT” channel
There are three kinds of shadowing, i.e. light, average, and heavy, which correspond to the shadowings faced in the area of farms, suburban, and inner city, respectively.

Therefore, to design good coded modulation schemes for those fading channels, the designer should consider how to overcome the following limitations, constraints and impairments:

1. Bandwidth constraint,

2. Power constraint,

3. Doppler frequency shift due to vehicle motion,

4. Constraints on total allowable delay time,

5. Nonlinearity due to the saturated nonlinear amplifier in the transmitter,

6. Adjacent channel interference, and

7. Multipath fading and shadowing.

Bandwidth and power limitations can be overcome by using properly designed coded modulation schemes. Effects of Doppler frequency shift can be reduced by using an interleaver, however the constraint on allowable delay time will limit the size of the interleaver. Therefore, items 3 and 4 must be optimized to achieve best performance for voice transmission. Constant envelope modulation techniques must be used to limit the impairment by item 5. Since constant envelope modulations can be transmitted by saturated nonlinear amplifier without spectral growth, we can limit the adjacent channel interference. To overcome impairments by item 7, diversity techniques [76, 77], coding, and coded modulation schemes must be designed properly. Due to items 3 and 7, it is very difficult to use coherent detection methods.
Therefore, it is desirable to use differential detection or other noncoherent methods over a mobile satellite channel [82].

The objective of this dissertation is to design coded modulation schemes for various fading channels to achieve high performance with reduced decoding complexity. Differential M-ary phase shift keying (PSK) and coherently detected M-ary PSK are used as modulation techniques. Block interleaver and de-interleaver pairs of finite size are designed to combat burst errors due to fast fading. Interleaving is also used to prevent the error propagation due to differential detection.

1.2 Historical Overview

In this section, previous research on coded modulation for various fading channels are briefly summarized.

Divsalar and Simon [41] investigated the performance of trellis coded MPSK over a fading mobile satellite channel, i.e. the Rician channel. Results were obtained by using a combination of analysis and simulation. They assumed that the effect of the fading on the phase of the received signal is fully compensated for either by tracking it with some form of phase-locked loop or with pilot tone calibration techniques. They derived the bit error probability bound for the MPSK TCM codes for the Rician fading channel. In [46], Wilson and Leung analyzed the behavior of 8-PSK trellis codes transmission on a slow-fading Rayleigh channel, with or without channel interleaving. In particular, they pointed out that the effective diversity order achieved by virtue of coding is related to the minimum Hamming distance (in PSK symbols), and performance is only weakly dependent on the free Euclidean distance. Also they pointed out that Euclidean distance as well as non-dominant error events are important at low SNR. In [42, 43], Divsalar and Simon presented
the design criteria for trellis coded MPSK modulation to achieve minimum error probability performance on the Rician fading channels and also designed multiple trellis coded modulation schemes based on their design criteria. In [44], Simon and Divsalar presented the performance of a trellis coded multilevel differential PSK (MDPSK) modulation over a mobile satellite channel characterized by AWGN and slow Rician fading. The analysis was carried out under the assumption of ideal coherent detection. Therefore, their numerical results derived from this analysis only reflected the degradation due to the effect of the fading on the amplitude of the received signal. In [45], Simon and Divsalar studied the performance of trellis coded MDPSK over the fast fading channels characteristic of a mobile environment with high vehicle speeds. In the fast fading channel, the phase of the fading process varies rapidly over the symbol interval. In this paper, they derived the upper bounds on the bit error probability of the trellis coded MDPSK on fast fading channels. The pair-wise error probability bounds were represented by the function of autocorrelation of the fading process. This autocorrelation function is a function of the vehicle's speed. Infinite interleaving was assumed in the analytical analysis. Because of practical limitations, the interleaver size in the simulation is limited, i.e. 32 x 16 bits. Edbauer [56] investigated the performance of trellis-coded differential 8-PSK over the Rayleigh and Rician channels for various Doppler spreads and interleaver sizes with differentially coherent detection and soft decision Viterbi decoding. Schlegel and Costello [55] designed an 8-PSK TCM codes based on the Divsalar and Simon's design criteria. Their codes showed better error performance than codes of the same complexity designed for Gaussian channels. Cavers and Ho [57] presented the expression for the pair-wise error event probability of TCM signals transmitted over Rayleigh fading channel. However, their pair-wise error event probability is not quite accurate at low signal-to-noise (SNR). Ho and Fung
studied error performance of interleaved trellis coded PSK modulations in correlated Rayleigh fading channels. Both coherent as well as differential detection were considered. They discovered that, in the case of DPSK, full interleaving does not necessarily provide the best error performance. This is especially true when the fading is relatively fast and when the autocorrelation function of the channel fading process exhibits oscillatory behavior, like that of a Bessel function. Vucetic presented several concatenated coded modulation schemes for a slow-fading Rayleigh fading channel, in which the NASA standard (255,223,33) Reed-Solomon code is used as the outer code and various 8-PSK multilevel block modulation codes are used as inner codes. The rates of these single-level concatenated coded modulation schemes are less than 2 bits per 8-PSK symbol. Wu and Lin designed several multilevel trellis MPSK modulation schemes for the slow-fading Rayleigh channel. Important distance parameters for both the AWGN and Rayleigh fading channels were expressed in terms of minimum Hamming distances of the component codes using multilevel coded modulation scheme. Their codes outperform the Divsalar and Simon’s TCM codes and the rate-2/3 8-state Ungerboeck’s TCM code with the same decoding complexity.

In [65], Wei presented the bit error performance of the some block and trellis MDPSK codes for the fast-fading land mobile communication channel (i.e, the Rayleigh fading channel). Rhee et. al. and Lin et. al. presented the bit error performance of several block and trellis coded modulation schemes for the slow-fading Rayleigh fading channel. Their schemes are designed based on the multilevel coding method and multilevel concatenated coding scheme. At the high SNR, their coded modulation schemes outperform all existing codes for the Rayleigh fading channel.

For the fading mobile satellite channel with shadowing, communication scientists
in Canada have presented results based on the Loo’s mobile satellite communication channel (i.e., shadowed Rician channel) [86]-[89]. In [47]-[48], MacLane et. al. have studied the performance of 8-PSK and 8-DPSK trellis codes for a class of fast fading, land mobile satellite communication channels. The fading model is Rician but, in addition, the line-of-sight path is subjected to a fast log-normal attenuation that represents tree shadowing. Sensitivity of the bit error probability to amplitude fading, amplitude and phase fading, decoding delay and interleaving has been presented. Ungerboeck’s rate 2/3 TCM codes were used for simulation. Their simulation results showed that bit-error performances of 8-PSK TCM codes improves as the speed of vehicle was increased. For the case of 8-DPSK TCM codes, the results were the opposite. In [49], Lee and McLane investigated the performance of convolutionally interleaved PSK and DPSK trellis codes for digital speech transmission over a shadowed mobile satellite communication channel by using computer simulation. A test for channel burst error behavior was presented. Based on that test, a periodic convolutional interleaver/de-interleaver was designed to be used with trellis coding to combat the slow fading digital, shadowed mobile satellite channel. In [50], McKay et. al. presented the analytical performance bounds for trellis-coded MPSK over a satellite based land mobile channel. A new tighter upper bound was derived based on a simple modification to the standard transfer function bound, and results were presented for the Ungerboeck’s 4- and 8-state trellis codes in Rician and Rayleigh fading. In [51], Jamali and Le-Ngoc devised a new 4-state TCM code for fast fading, shadowed mobile radio channels. This 4-state TCM code outperforms other available 4-state TCM codes at bit error rate $10^{-3}$. They also presented a BCM scheme using Reed-Solomon (RS) codes with different decoding strategies [52] for the same channel.

Although there have been many analytical studies of TCM codes for the mobile
satellite communication channels and Rician channels, there is very little study of block coded modulation, concatenated coded modulation, and product coded modulation codes.

This dissertation investigates various high performance, and low decoding complexity, coded modulation schemes for reliable voice and data transmission over the shadowed mobile satellite channel and the Rayleigh fading channel. The proposed research includes three area: (1) multilevel TCM and BCM schemes for voice transmission; (2) multilevel concatenated coded modulation; and (3) product coded modulation schemes for data transmission.

1.3 Proposed Research

Even though there have been many studies of TCM schemes for various fading channels, multilevel block modulation codes and combination of TCM and BCM have not been received as much attention. Most TCM schemes in the literature are constructed by computer search. The search for high performance TCM schemes becomes extremely time consuming and a more systematic and practical technique of modulation code construction is required. In the first research area, it is proposed to devise a systematic technique for constructing multilevel coded modulation schemes using the multilevel coding method. The multilevel modulation codes constructed by this method allow the use of multi-stage decoding procedures that provide a good trade-off between performance and decoding complexity. The multilevel coding method will be used for constructing TCM codes using convolutional codes with good free branch distances as the component codes or using both convolutional and block codes as the component codes. Since there is a satellite delay constraint for the voice transmission, a receiver is subject to the limitation of interleaver size and
decoding depth. It is proposed to investigate optimum decoding and interleaving depths of TCM codes for the MSAT channel.

Even though block coded modulation (BCM) is generally less power efficient than TCM for AWGN channels, BCM has a great potential to compete with TCM in mobile satellite channel because of its shorter decoding depth and hence more effective interleaving. For the second part of the first research area, good 3-level BCM codes will be devised for the MSAT channel. Binary Reed-Muller (RM) codes of length up to 32 will be used as component codes. The decoding complexity of 4-section trellises of these RM codes will be investigated. The error performance of these codes based on multi-stage soft-decision decoding will be evaluated for the MSAT channel with different vehicle speeds. Also, effects of interleaving on the error performance will be investigated.

Large coding gain TCM and BCM schemes require large decoding complexity. The large decoding complexity of these codes makes them impractical for applications where high reliability and high data rates are required. To achieve large coding gain without large decoding complexity over the AWGN channel, Deng and Costello [20, 21], and Kasami et al. [36] introduced concatenated coded modulation schemes. For the Rayleigh fading channel, Vucetic [60] presented the error performance of single-level concatenated modulation codes. In their schemes, Reed-Solomon (RS) outer codes were concatenated with TCM (or BCM) inner codes in a single-level manner. But using a single-level concatenated block coded modulation scheme, it is desirable to keep the inner modulation codes short to reduce the decoding complexity. However, using a short inner modulation code in a single-level concatenated block coded modulation scheme limits the improvement of bandwidth efficiency. This shortcoming can be overcome by using multilevel concatenation and coset inner codes derived from a long block modulation code. Since multi-level con-
catenation allows us to use long multi-level modulation inner codes with multi-stage suboptimum decoding [40], it is possible to achieve very high reliability, large coding gain, and high bandwidth efficiency with reduced decoding complexity. In a single-level concatenated coded modulation scheme, the outer code corrects all the output bits of the inner code decoder to the same degree. However, all the output bits of the inner code decoder do not have the same error probability. This shortcoming can be overcome by using multilevel concatenation and coset inner codes derived from a block modulation code. Since modulation schemes using the multilevel concatenation approach allow the freedom of choosing error-correcting capabilities of outer codes, it is easy to provide proper error protection to each level.

In the second and third proposed research area, it is proposed to devise multilevel concatenated coded modulation and product coded modulation schemes for data transmission over the shadowed MSAT channel. These schemes will be designed to achieve high-performance or large coding gain with reduced decoding complexity. Construction is based on a multilevel concatenation approach in which long powerful codes are used as the outer codes and coset codes constructed from a linear BCM code and its subcodes are used as the inner codes. Sub-optimum multilevel closest coset decoding is proposed for the multilevel concatenated coded modulation scheme. Good two-level concatenated 8-PSK modulation codes will be devised for the Rayleigh fading channel. Error performance of the two-level concatenated coded modulation scheme will be evaluated for the MSAT channel with different vehicle speeds. Efficient three- and six-level concatenated block coded modulation schemes with inner coset codes constructed from a long linear block modulation and its subcodes will be devised for the shadowed MSAT channel. In product coded modulation, the product coding technique and coded modulation are combined to achieve high performance with reduced decoding complexity. Good product 8-PSK
modulation codes are to be constructed for various shadowed MSAT channels. Sub-optimum multi-stage decoding will be investigated.

### 1.4 Outlines of the Dissertation

In chapter 2, a brief explanation of multipath fading is given and a MDPSK coded modulation system model for the Rician fading channel is discussed. An upper bound on pair-wise error probabilities of the MDPSK coded modulation scheme over the fast (or slow)-fading Rician and Rayleigh fading channels is discussed.

In chapter 3, a systematic technique for constructing the multilevel coded modulation schemes using the multilevel coding method is presented. Distance parameters, such as the minimum squared Euclidean distance, minimum symbol distance, and minimum product distance, which determine the error performance of a multilevel modulation code are expressed in terms of the minimum Hamming distances of the component codes. Guidelines for constructing good multilevel modulation codes for either the AWGN channel or the Rayleigh fading channel are presented. A multi-stage decoding algorithm is presented.

In chapter 4, MPSK trellis code modulation (TCM) codes using convolutional codes with good free branch distances as the component codes or using both convolutional and block codes, such as Reed-Muller codes, as the component codes are constructed by using the multilevel coding method. The constructed modulation codes are compared to the codes available in the literature. Optimum size decoding depth and interleaving depth for TCM codes are investigated.

Even though block coded modulation (BCM) is generally less power efficient than TCM for AWGN channels, BCM has the potential to compete with TCM in mobile satellite channel because of its shorter decoding depth and hence more effective interleaving. In chapter 5, the multilevel coding method is used to constructing
3-level 8-PSK BCM codes for the Rayleigh fading and shadowed MSAT channels. Binary RM codes with length up to 32 are used as component codes. The decoding complexity of 4-section trellises of these RM codes is presented. Error performances of these codes based on multi-stage soft-decision decoding are presented for the MSAT channel with different vehicle speeds.

In chapter 6, a simple and systematic technique for constructing multilevel concatenated block coded modulation schemes for data transmission over the shadowed MSAT channel and the Rayleigh fading channel is presented. These schemes are designed to achieve high-performance or large coding gain with reduced decoding complexity. Construction is based on a multilevel concatenation approach in which long powerful (binary or non-binary) codes are used as the outer codes and coset codes constructed from a linear BCM code and its subcodes are used as the inner codes. Sub-optimum multilevel closest coset decoding is proposed for the multilevel concatenated coded modulation scheme. Two-, Three- and 6-level concatenated block coded modulation schemes with inner coset codes constructed from a short (or long) linear block modulation and its subcodes are proposed for the shadowed MSAT and the Rayleigh fading channels.

In chapter 7, product coded modulation schemes are constructed by using multilevel concatenating approach. In product coded modulation, product coding technique and coded modulation are combined to achieve high performance with reduced decoding complexity. Good product 8-PSK modulation codes are constructed for various shadowed MSAT channels. Suboptimum multi-stage decoding is investigated.

Chapter 8 concludes the thesis, and discusses future research topics.
Chapter 2

Multipath Fading and
A Mobile Satellite
Communication Channel Model

Considerable literature [76, 77] has been devoted to phenomenological and statistical characterization of the rapid wave interference fading associated with time-varying propagation multipath, i.e., multipath fading. There are two kinds of multipath fading, i.e., short term and long term fading. Long term signal fading is mainly caused by terrain configuration and the built-in environment, i.e., buildings and mountains, between the base station and the mobile unit with the time scales of minutes, hours, or seasonal. Short term fading is rapid fading and mainly caused by multipath reflections of a transmitted wave by local scatterers such as houses, buildings, and other structures. For most fading channels, only the short-term fading variations affect the details of waveform structure and the interrelationships of errors within a message; while the longer term variations determine in effect the availability of the channel. Mathematically, the statistical model of the short term fading usually can be regarded to be conditional upon the instantaneous values of parameters described by the longer term statistics. For reliable communication, the system must be able to combat short fades and recover quickly from long fades.
2.1 Statistical Characterization of Short-term Multipath Fading Channels

A *multipath* channel is one where energy arrives via several paths, usually as a result of *reflections*, or of inhomogeneities in the physical medium that produce *ray-splitting* or *scattering effects*. Continuous physical changes in the channel cause small changes in the individual path lengths, but these may result in large phase changes at high radio frequencies. The variations between constructive and destructive interference resulting from the random phase changes comprise the effect called *multipath fading*.

One characteristic of a multipath fading channel is the time spread introduced in the transmitted signal. The other characteristic is due to the time variations in the structure of the physical medium. Therefore the nature of the multipath varies with time. Moreover the time variations appear to be unpredictable to the user of the channel. Therefore, it is reasonable to characterize the time-variant multipath channel statistically.

2.1.1 Frequency Non-Selective Rayleigh Fading Channel

Using the complex envelope notation, a transmitted signal over the multipath fading channel at some carrier frequency $f_c$, can be represented as

$$s(t) = \text{Re}[u_o(t)e^{j2\pi f_c t}]$$

(2.1)

where $u_o(t)$ has an equivalent low-pass spectrum $U_o(f)$,

$$u_o(t) = \int_{-\infty}^{\infty} U_o(f)e^{j2\pi f t} df.$$  

(2.2)

We assume that there are $N$ independent propagation paths. Both the propagation delays and the attenuation factors for propagation paths are time-variant as a result
of differences in the structure of the medium. The received signal is the superposition of $N$ independently arrived signals as following

$$r(t) = \text{Re}[u(t)e^{j2\pi ft}]$$  \hspace{1cm} (2.3)

where

$$u(t) = \sum_n A_n(t)u_o(t - \tau_n(t))e^{-j2\pi fc\tau_n(t)}$$ \hspace{1cm} (2.4)

where $A_n(t)$ is the attenuation factor for the signal received on the $n$th path and $\tau_n(t)$ is the propagation delay for the $n$th path. With (2.2), it can readily rewritten as

$$u(t) = \int_{-\infty}^{\infty} U_o(f)e^{j2\pi ft}\{\sum_n A_n(t)e^{-j2\pi(f+fc)\tau_n(t)}\}df.$$ \hspace{1cm} (2.5)

Therefore, the *equivalent low-pass transfer function* of the channel is

$$H(f, t) = \sum_n A_n(t)e^{-j2\pi(f+fc)\tau_n(t)}$$ \hspace{1cm} (2.6)

in term of which

$$u(t) = \int_{-\infty}^{\infty} U_o(f)H(f, t)e^{j2\pi ft}df.$$ \hspace{1cm} (2.7)

Suppose $s(t)$ is a unit amplitude continuous wave (CW) signal, then

$$u_o(t) = 1 \quad \text{for all } t.$$ \hspace{1cm} (2.8)

Then the equivalent lowpass received signal is

$$r(t) = \sum_n A_n(t)e^{-j2\pi fc\tau_n(t)}.$$ \hspace{1cm} (2.9)

Let $\theta_n(t) = 2\pi fc\tau_n(t)$. Then (2.9) becomes

$$r(t) = \sum_n A_n(t)e^{-j2\pi fc\tau_n(t)}$$

$$= \sum_n A_n(t)e^{-j\theta_n(t)}.$$ \hspace{1cm} (2.10)
Therefore the received signal is the sum of a time-variant phasors having amplitude $A_n(t)$ and phase $\theta_n(t)$. Dynamic changes in the medium will cause significant changes in $A_n(t)$. These changes in $A_n(t)$ will change the received signal significantly. On the other hand, $\theta_n(t)$ will change by $2\pi$ radian whenever $\tau_n(t)$ changes by $1/f_c$. Since $1/f_c$ is very small number, $\theta_n(t)$ will change by $2\pi$ radian with relatively small motions of the medium. Also we expect that the delays $\theta_n(t)$ associated with different signal paths will change at different rates and in a random manner. This implies that the received signal $r(t)$ can be modeled as a random process. If there are a large number of independent paths, the central limit theorem can be applied. Therefore, $r(t)$ can be modeled as a complex-valued gaussian random process. This means that the time-variant impulse response $h(\tau; t)$ is a complex-valued gaussian random process in the $t$ variable. The received signal $r(t)$ in (2.10) will result in signal fading. In fact, result of the time variations in the phases $\theta_n(t)$ is a primary source of fading phenomenon. Sometimes, the vectors $\{A_ne^{-j\theta_n}\}$ in (2.10) add destructively. The resulting received signal $r(t)$ is very weak. At other times the vectors $\{A_ne^{-j\theta_n}\}$ add constructively, so that received signal is strong. Therefore, the amplitude variations in the received signal, termed signal fading, are due to the time-variant multipath characteristics of the channel. When the impulse response of $h(\tau; t)$ is modeled as a zero mean complex-valued gaussian process, the envelope $|h(\tau; t)|$ at any instant $t$ is Rayleigh distributed. In this case, the channel is said to be a Rayleigh fading channel. Let $h(\tau; t)$ be a zero mean complex-valued gaussian process. Then we may divide $h(\tau; t)$ into its in-phase $h_c(\tau; t)$ and quadrature parts $h_s(\tau; t)$;

$$
\begin{align*}
    h_c(\tau; t) &= \text{Re}[h(\tau; t)] \\
    h_s(\tau; t) &= \text{Im}[h(\tau, t)].
\end{align*}
$$

(2.11)

Let $h_c, h_s$, and $h$ denote the random variables corresponding to $h_c(\tau; t), h_s(\tau; t)$, and $h(\tau; t)$ for fixed $t$, respectively. Then the first-order probability density function
(pdf) of a stationary complex zero-mean Gaussian process (complex envelope)

\[ h = h_c + j h_s \]  \hspace{1cm} (2.12)

is described by the circular Gaussian distribution

\[ p(h_c, h_s) = \frac{1}{2\pi S} \exp\left(-\frac{h_c^2 + h_s^2}{2S}\right) \]  \hspace{1cm} (2.13)

where the expected value

\[ S = E[\frac{1}{2}|h|^2] \]  \hspace{1cm} (2.14)

is the mean power in the random waveform. Its envelope and phase are

\[ R = \sqrt{h_c^2 + h_s^2} \hspace{1cm} \theta = \tan^{-1}\frac{h_s}{h_c}. \hspace{1cm} (2.15) \]

Also, \( R \) and \( \theta \) are statistically independent. Therefore the joint first-order pdf can be decomposed into independent pdf's:

\[ p(\theta) = \frac{1}{2\pi} \]  \hspace{1cm} \( 0 \leq \theta \leq 2\pi \)  \hspace{1cm} uniform distribution  \hspace{1cm} (2.16)

\[ p(R) = \frac{R}{S} \exp\left(-\frac{R^2}{2S}\right) \]  \hspace{1cm} \( 0 \leq R < \infty \)  \hspace{1cm} Rayleigh distribution.  \hspace{1cm} (2.17)

It is important to note that the central limit characterization is strongest around the value zero for each of the quadrature components, namely, around the region where the value of the envelope is zero. Therefore, at large values of instantaneous envelope, the mathematical model of the channel as a complex Gaussian process is less credible. However, large values of signal are not harmful to performance. It is known that [77, 79], the cellular land mobile radio channel can be modeled as Rayleigh with its local mean following a lognormal statistic and its phase distributed uniformly.
Covariance Functions and Power Spectrum

Since $h(\tau; t)$ is a complex-valued Gaussian process in the $t$ variable, it follows that $H(f; t)$ also has the same statistics. Given that $H(f; t)$ is a complex Gaussian process, a spaced-tone complex covariance [78] can be defined to characterize the relationships of values of the transfer function at different frequencies and at different times:

\[ R_F(f_1, f_2; \Delta t) = E\left[ \frac{1}{2} H^*(f_1; t) H(f_2; t + \Delta t) \right]. \]  

(2.18)

We assumed that the short-term fading is a wide-sense stationary process. Then we can write this covariance as a function of only the time difference $\Delta t$. Also the spaced-tone relationship will depend only on the frequency difference $\Delta f = f_2 - f_1$, and not on the actual value of frequency. Therefore the channel is homogeneous in frequency and time. Assuming that the spaced-tone covariance depends only on $\Delta f$, we can write

\[ R_F(\Delta f; \Delta t) = E\left[ \frac{1}{2} H^*(f; t) H(f + \Delta f; t + \Delta t) \right]. \]  

(2.19)

Its normalized value is

\[ \rho_F(\Delta f; \Delta t) = \frac{R_F(\Delta f; \Delta t)}{R_F(0; 0)}. \]  

(2.20)

The selectivity of fading is well characterized by $\rho_F(\Delta f; \Delta t)$. If $\rho_F(\Delta f; 0)$ is near unity for $|\Delta f| < B$, then all frequency components of signal faces same fading. Therefore

\[ \rho_F(B; 0) = 1 \implies H(f_1; t) = H(f_2; t) = \alpha(t) \]  

for all $f_1, f_2$ such that $|f_1 - f_2| \leq B$  

(2.21)

where $\alpha(t)$ is a complex Gaussian process. This kind of channel is called frequency flat-fading or nonselective fading channel. In nonselective fading channel, signal
does not receive frequency selective distortion by the channel. Suppose a complex envelope \( u(t) \) is transmitted over a nonselective fading channel, then the received complex envelope is
\[
r(t) = \alpha(t)u(t).
\] (2.22)

This channel is also called a **multiplicative fading** channel [77]. It is shown in [76] that the wide-sense frequency stationarity of \( H(f; t) \) is equivalent to a statement that the impulse response changes independently at each different time delay. Then the autocorrelation function of \( h(\tau; t) \) is
\[
R_T(\tau_1, \tau_2) = E\left[ \frac{1}{2} h^*(\tau_1; t) h(\tau_2; t + \Delta t) \right]
\] (2.23)
\[
= R_T(\tau_1; \Delta t) \delta(\tau_1 - \tau_2)
\]

where
\[
R_T(\tau; \Delta t) = \int \exp(-j2\pi\tau\Delta f) R_F(\Delta f; \Delta t) d(\Delta f).
\] (2.24)

The value \( R_T(\tau; 0) = R_T(\tau) \) is the mean strength (intensity) of the channel versus delay time and is called a **multipath intensity profile** or the **delay power spectrum** of the channel. The range of values of \( \tau \) over which \( R_T(\tau) \) is essentially nonzero is called the **multipath spread** of the channel and is denoted by \( T_m \). In most radio transmission media, the attenuation and phase shift of the channel associated with path delay \( \tau_1 \) is uncorrelated with the attenuation and phase shift associated with path delay \( \tau_2 \). This is usually called **uncorrelated scattering**. From (2.19) and (2.24), the spaced-tone covariance \( R_F(f_1, f_2; \Delta t) \) is given as follows:
\[
R_F(f_1, f_2; \Delta t) = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E[h^*(\tau_1; t) h(\tau_2; t + \Delta t)] e^{j2\pi(f_1\tau_1 - f_2\tau_2)} d\tau_1 d\tau_2
\] (2.25)
\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_T(\tau_1; \Delta t) \delta(\tau_1 - \tau_2) e^{j2\pi(f_1\tau_1 - f_2\tau_2)} d\tau_1 d\tau_2
\] (2.25)
\[
= \int_{-\infty}^{\infty} R_T(\tau_1; \Delta t) e^{j2\pi(f_1 - f_2)\tau_1} d\tau_1
\] (2.25)
\[
= \int_{-\infty}^{\infty} R_T(\tau_1; \Delta t) e^{-j2\pi\Delta f\tau_1} d\tau_1 \equiv R_F(\Delta f; \Delta t)
\] (2.25)
where $\Delta f = f_2 - f_1$. From (2.25) we observe that $R_F(\Delta f; \Delta t)$ is the Fourier transform of the multipath intensity profile. Under the assumption of uncorrelated scattering, the autocorrelation function of $H(f; t)$ in frequency is a function of only the frequency difference $\Delta f$. That is why $R_F(\Delta f; \Delta t)$ is called spaced-frequency spaced-time correlation function of the channel. Transform relationships between $R_F$ and $R_T$ in (2.24) and (2.25) show that the selectivity of the channel as measured by the width of $R_F$ versus $\Delta f$ is inversely reciprocal to the width of the channel response, the value described by the multipath spread $T_M$. In other words, the reciprocal of the multipath spread is a measure of the coherence bandwidth of the channel. That is,

$$
(\Delta f)_c \approx \frac{1}{T_M}
$$

(2.26)

where $(\Delta f)_c$ denotes the coherence bandwidth. Therefore, if two sinusoids are separated by $(\Delta f)_c$ in frequency, then they are affected differently by the channel.

Next we consider the time variations of the channel as measured by changing the parameter $\Delta t$ in $R_F(\Delta f; \Delta t)$. The time variations in the channel are characterized as a Doppler shift of spectral line. To relate the Doppler effects to the time variations of the channel, we define the Fourier transform of $R_F(\Delta f; \Delta t)$ with respect to the variable $\Delta t$ in following:

$$
S_F(\Delta f; \lambda_D) = \int_{-\infty}^{\infty} R_F(\Delta f; \Delta t) e^{-j2\pi\lambda_D \Delta t} d\Delta t
$$

(2.27)

where $\lambda_D$ is the Doppler frequency. Suppose $\Delta f = 0$, then $S_F(0; \lambda_D) \equiv S_F(\lambda_D)$. And (2.27) becomes

$$
S_F(\lambda_D) = \int_{-\infty}^{\infty} R_F(\Delta t) e^{-j2\pi\lambda_D \Delta t} d\Delta t.
$$

(2.28)

The function $S_F(\lambda_D)$ is a power spectrum that gives the signal intensity as a function of the Doppler frequency $\lambda_D$. Therefore, $S_F(\lambda_D)$ is called the Doppler spectrum of
the channel. If the channel is time-invariant, i.e. \( R_F(\Delta t) = 1 \), \( S_F(\lambda_D) \) becomes equal to the delta function \( \delta(\lambda_D) \). Therefore, if the channel is time-invariant, there is no spectral broadening observed in the transmission of a pure frequency tone. The range of values of \( \lambda_D \) over which \( S_F(\lambda_D) \) is essentially nonzero is called the **Doppler spread** \( B_d \) of the channel. Since \( S_F(\lambda_D) \) is the Fourier transform of the \( R_F(\Delta t) \), the reciprocal of \( B_d \) is a measure of the coherence time of the channel. That is

\[
(\Delta t)_c \approx \frac{1}{B_d}
\]

(2.29)

where \( (\Delta t)_c \) denotes the **coherence time**. Therefore, a slowly changing channel has a large coherence time, or a small Doppler spread and a rapidly changing channel has a large Doppler spread. The rate of Rayleigh envelope fading across the median envelope level is proportional to a specific definition of **rms bandwidth** \( f_N \) of \( S_F(\lambda_D) \):

\[
f_N = \left[ \int \frac{\lambda_D^2 S_F(\lambda_D) d\lambda_D}{\int S_F(\lambda_D) d\lambda_D} \right]^{1/2}
\]

(2.30)

and fade rate \( f_s \) is 1.475 \( f_N \). The fade rate \( f_s \) is the average rate of downgoing crossings of median envelope level.

### 2.1.2 Rician Fading Channel

When there is a single dominant, nonfading component in the received signal along with a fading process, the envelope statistics are Rician, having pdf

\[
p(R) = \frac{R}{S_f} \exp\left(-\frac{S_o}{S_f}\right) \exp\left(-\frac{R^2}{2S_f}\right) I_0\left(\frac{R\sqrt{2S_o}}{S_f}\right) \quad 0 \leq R < \infty
\]

(2.31)

where \( S_o \) and \( S_f \) are mean power in fading and nonfading component, respectively. Due to nonfading component, the phase is no longer uniformly distributed and is more concentrated around that of the nonfading component. For an **aeronautical** or **maritime satellite**, there is generally a line-of-sight component as well as multipath. This channel is usually modeled as **Rician**, i.e., a constant component plus multipath [81].
2.1.3 Shadowed Rician Fading Channel

Research on the mobile satellite communication in both Europe and North America received major stimulus from the 1979 WARC decision which permits a satellite mobile service in the 806-890 MHz band. From 1984, NASA has been conducting the Mobile Satellite Experiment (MSAT-X) to determine the feasibility of a communications network in which private and commercial users send linear predictive coded voice and low-speed data via satellite, using low-cost terminals mounted in vehicles [82]. The MSAT-X channel is subject to multipath fading, shadowing, Doppler frequency shift, and adjacent channel interference (ACI). Shadowing is defined as the effect of foliage attenuation or blockage by buildings. Shadowing is not expected to be a problem except in urban areas, since the elevation angle to the satellite is expected to be more than 25° for the continental U.S. This channel is characterized as the Rician fading channel.

However, shadowing is more severe in Canada than in the continental U.S. due to a lower angle of elevation (15° – 20°) between a mobile user and a suitably located geosynchronous satellite. Recently, a statistical model [86]-[89] has been developed for application in the Canadian Mobile Satellite Communications (MSAT) Program. In this model, the line-of-sight (LOS) component of the Rician model is subjected to lognormal transformation. This transformation represents the effect of foliage attenuation or blockage, also referred as shadowing. From now on, we call this MSAT channel as the shadowed MSAT. There are three kinds of shadowing, i.e. light, average, and heavy shadowing, which correspond to the shadowing faced in the area of farms, suburban, and city, respectively.

The pdf of the received signal envelope $R$ caused by the combined effect of
Table 2.1. System parameters for the shadowed Rician channel

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Light</th>
<th>Average</th>
<th>Heavy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_0$</td>
<td>0.158</td>
<td>0.126</td>
<td>0.0631</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>0.115</td>
<td>-0.115</td>
<td>-3.91</td>
</tr>
<tr>
<td>$\sqrt{d_0}$</td>
<td>0.115</td>
<td>0.161</td>
<td>0.806</td>
</tr>
</tbody>
</table>

Multipath fading and shadowing is given by

$$p(R) = \frac{R}{b_0 \sqrt{2\pi d_0}} \int_0^\infty \frac{1}{z} \exp\left(-\frac{(\ln z - \mu_0)^2}{2d_0} - \frac{(R^2 + z^2)}{2b_0}\right) I_0(Rz/b_0) dz$$  \hspace{1cm} (2.32)

where $\mu_0$ is the mean value due to shadowing, $\sqrt{d_0}$ is the standard deviation due to shadowing, and $b_0$ is the multipath power. Values of $\mu_0$, $b_0$ and $d_0$ for Loo's model [86]-[89] are given in Table 1. Three cases are listed: light, average, and heavy shadowing, which represent an increasing effect of the lognormal process or the degree of shadowing. All parameters are normalized to a signal amplitude of unity.

In [48], they showed that the shadowed Rician model has an envelope distribution that can be approximated by a Rician model (without shadowing).

### 2.2 Analysis Model

The basic analysis model is shown in Figure 2.1. We denote a coded symbol sequence of length $N$ by

$$x = (x_1, x_2, \cdots, x_N)$$  \hspace{1cm} (2.33)

where $x_k$ represents the transmitted MPSK symbol in the $k$th transmission interval. Before transmission over the fading channel, the sequence $x$ is differentially encoded into the sequence $v = (v_1, v_2, \cdots, v_N)$. In phasor notation, $v_k$ and $v_{k+1}$ can be written as

$$v_k = \sqrt{2E_s} e^{j\phi_k} \quad v_{k+1} = v_k \cdot x_{k+1} = \sqrt{2E_s} e^{j(\phi_k + \Delta \phi_{k+1})} = \sqrt{2E_s} e^{j\phi_{k+1}}$$  \hspace{1cm} (2.34)
Figure 2.1. MDPsk coded modulation system model for fast-fading Rician channel

\[ W_k = \alpha_k V_k + N_k \]

\[ Y_{k+1} = W_k \ast W_{k+1} \]

\[ W_k = \alpha_k V_k + N_k \]

\[ Y_{k+1} = W_k \ast W_{k+1} \]
where $E_s$ is the energy per MDPSK symbol and

$$x_k = e^{j\Delta\phi_k}$$

(2.35)

is the phasor representation of the MPSK symbol $\Delta\phi_k$ assigned by the mapper in the $k$th transmission interval.

Corresponding to $x$ the channel output the sequence

$$y = (y_1, y_2, \cdots, y_N)$$

(2.36)

where the $k+1$st element $y_{k+1}$, representing the output in the $k+1$st transmission interval is given by

$$y_{k+1} = w_k^* w_{k+1}$$

$$= (\alpha_k^* e^{-j\phi_k} + N_k^*)(\alpha_{k+1} e^{j(\phi_k + \Delta\phi_{k+1})} + N_{k+1})$$

$$= \alpha_k^* \alpha_{k+1} e^{j\Delta\phi_{k+1}} + \text{noiseterm}s.$$  

(2.37)

where $*$ represents the conjugate of complex r.v. and $\alpha_k, \alpha_{k+1}$ are samples of a normalized stationary, circularly complex Gaussian noise process $\alpha(t)$ (independent of $N(t)$) which represents the fading characteristic of the channel.

The first two moments of the random variables $\alpha_k$ and $N_k$ are given by

$$E\{\alpha_k\} = \eta; \quad E\{|\alpha_k|^2\} - |\eta|^2 = \sigma^2_\alpha; \quad \sigma^2_\alpha + |\eta|^2 = 1;$$

$$E\{\alpha_k - \eta\}^* (\alpha_{k+1} - \eta)\} = \tau_{\alpha(t)}(T_s) = |\eta|^2 = \xi \sigma^2_\alpha, \quad 0 \leq |\xi| \leq 1$$

$$R_{\alpha(t)}(T_s) = E\{\alpha^*(t)\alpha(t + T_s)\} = E\{\alpha_k^* \alpha_{k+1}\}$$

$$E\{N_k\} = 0; \quad E\{N_k^* N_m\} = 2N_0 \delta_{km}; \quad \delta_{km} = \begin{cases} 0; & k \neq m \\ 1; & k = m \end{cases}$$

(2.38)

where $R_{\alpha(t)}(\tau)$ is the autocorrelation function of the complex fading process $\alpha(t)$. When $\xi = 1$, channel becomes slow fading channel.

25
In the fast-fading Rician channel, \( \alpha_k \) can be written in phasor form as following:

\[
\alpha_k = \rho_k e^{j\theta_k}
\]  

(2.39)

where \( \rho_k \) is the normalized (unit mean-squared value) random variable with a Rician probability density function given by

\[
p(\rho_k) = \begin{cases} 
2\rho_k(1 + K)\exp\{-K - \rho_k^2(1 + K)\}I_0(2\rho_k\sqrt{K(1 + K)}); & \rho_k \geq 0 \\
0; & \text{otherwise}
\end{cases}
\]  

(2.40)

where the Rician factor is expressed by \( K = |\eta|^2/\sigma_0^2 \) or

\[
\sigma_0^2 = (1 + K)^{-1}.
\]  

(2.41)

For simplicity of analysis, we shall assume infinite depth interleaving and deinterleaving such that the coding channel is memoryless. Under this assumption, the \( \rho_k \)'s are independent r.v.'s and hence the joint channel probabilities satisfy

\[
p_N(y|x) = \prod_{n=1}^{N} p(y_n|x_n).
\]  

(2.42)

For any coded communication system, the decoding process uses a metric of the form \( m(y, x) \) (assuming channel side information is not available). For the simplicity for the decoding process, it is desirable such that metric has an additive property. Then the total metric for a sequence is the sum of the metrics for each channel input and output pair as following:

\[
m(y, x) = \sum_{n=1}^{N} m(y_n, x_n).
\]  

(2.43)

And the maximum likelihood metric is

\[
m(y, x) = \log p_N(y|x).
\]  

(2.44)

The pairwise error probability \( P(x \rightarrow \hat{x}) \) represents the probability of choosing the coded sequence \( \hat{x} = (\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_N) \) when indeed \( x = (x_1, x_2, \cdots, x_N) \) was
transmitted. Then $P(x \rightarrow \hat{x})$ is given by

$$P(x \rightarrow \hat{x}) = \Pr\{m(y, \hat{x} \geq m(y, x) | x_n, A \}.$$  \hspace{1cm} (2.45)

Using (2.44) and (2.45) and applying a Chernoff bound, we have

$$P(x \rightarrow \hat{x}) \leq \prod_{n \in \eta} E\{\exp(\lambda[m(y_n, \hat{x}_n) - m(y_n, x_n)] | x_n) \}$$ \hspace{1cm} (2.46)

where $\eta$ is the set of all $n$ such that $x_n \neq \hat{x}_n$ and we should choose $\lambda$ such that the right hand side is minimum.

Assumed that $|x_n|^2 = |\hat{x}_n|^2 = 1$ which is independent of $n$. Since it is very difficult to evaluate $p(y_n|x_n)$ for the fading channel, we shall use the metric which is optimum for the AWGN channel. Then metric has a following form;

$$m(y_n, x_n) = -|y_n - x_n|^2.$$ \hspace{1cm} (2.47)

From (2.37) and (2.47), the difference of the metric requires in (2.47) becomes

$$m(y_n - \hat{x}_n) - m(y_n - x_n) = -|w_{n-1}^* w_n - \hat{x}_n|^2 + |w_{n-1}^* w_n - x_n|^2$$

$$= w_{n-1}^* w_n (\hat{x}_n - x_n) + w_{n-1}^* w_n (x_n - \hat{x}_n)$$ \hspace{1cm} (2.48)

which can be conveniently written in the matrix form

$$m(y_n, \hat{x}_n) - m(y_n, x_n) = W_n^{* -T}(\hat{A}_n - A_n)W_n$$ \hspace{1cm} (2.49)

where

$$W_n = \begin{bmatrix} w_{n-1} & w_n \\ \end{bmatrix}, \quad \hat{A}_n = \begin{bmatrix} 0 & \hat{x}_n^* \\ \hat{x}_n & 0 \end{bmatrix}, \quad A_n = \begin{bmatrix} 0 & x_n^* \\ x_n & 0 \end{bmatrix}$$ \hspace{1cm} (2.50)

and $T$ denotes the transpose operation.

Assuming the amplitude fading is slowly varying enough that $\rho_{n-1} = \rho_n$, then substituting (2.49) into (2.46) gives

$$P(x \rightarrow \hat{x}) \leq \prod_{n \in \eta} E\{\exp(\lambda W_n^{* -T}(\hat{A}_n - A_n)W_n) | x_n, \rho_n \}$$ \hspace{1cm} (2.51)
where the expectation is over the AWGN and the fading process.

The expectation required in (2.51) was evaluated originally by Stein [76] and later by Johnston [75] in connection with the analysis of a block-coded MDPSK system. In particular, for any \( n \in \eta \),

\[
E\{\exp(\lambda \mathbf{W}_n^T(\hat{\mathbf{A}}_n - \mathbf{A}_n)\mathbf{W}_n)|x_n, \rho_n\} = \frac{\exp\{-\lambda \mu_n^T F_n(I + 2\lambda R_n^* F_n)^{-1} \mu_n\}}{\det(I + 2\lambda R_n^* F_n)} \tag{2.52}
\]

where \( \mathbf{I} \) is the identity matrix and

\[
\mathbf{F}_n \triangleq \hat{\mathbf{A}}_n - \mathbf{A}_n = \begin{bmatrix}
0 \\
(\hat{x}_n - x_n) \\
0
\end{bmatrix} \\
(\hat{x}_n - x_n)^*
\]

\[
\mu_n = E\{\mathbf{W}_n|x_n\} = \begin{bmatrix}
E\{w_{n-1}|x_n, \rho_n\} \\
E\{w_{n}|x_n, \rho_n\}
\end{bmatrix} \triangleq \begin{bmatrix}
\mu_{n-1} \\
\mu_n
\end{bmatrix}
\]

\[
\mathbf{R}_n = \frac{1}{2} E\{(\mathbf{W}_n - \mu_n)^*(\mathbf{W}_n - \mu_n)^T|x_n\} = \frac{1}{2} \begin{bmatrix}
E\{|w_{n-1} - \mu_{n-1}|^2\} & E\{(w_{n-1} - \mu_{n-1})^*(w_n - \mu_n)\} \\
E\{|w_{n-1} - \mu_{n-1}\}(w_n - \mu_n)^* & E\{|w_n - \mu_n|^2\}
\end{bmatrix} \tag{2.53}
\]

Note that (2.52) is valid only when \( \det(I + 2\lambda R_n^* F_n) > 0 \). For the fast-fading Rician channel, Eq. (2.52) evaluates to

\[
E\{\exp(\lambda \mathbf{W}_n^*(\hat{\mathbf{A}}_n - \mathbf{A}_n)\mathbf{W}_n)\} = \frac{1}{\Delta} \exp\{-\lambda E_s \frac{1}{N_0} (\hat{x}_n - x_n)^2 K \left[1 - 2\lambda \left(\frac{1}{1 + K} \frac{E_s}{N_0} (1 - \xi) + 1\right)\right]\}
\]

where

\[
\Delta = 1 + |\hat{x}_n - x_n|^2 \{ \lambda \frac{E_s}{N_0} \left[ 1 + K \right] (\xi - 2\lambda) - \lambda^2 \left[ 1 + (1 - \xi)^2 \right] \} \tag{2.55}
\]

For the Rayleigh fading case \((K=0)\), (2.54) simplifies to

\[
E\{\exp(\lambda \mathbf{W}_n^*(\hat{\mathbf{A}}_n - \mathbf{A}_n)\mathbf{W}_n)\} = \frac{1}{1 + |\hat{x}_n - x_n|^2} \{ \lambda \frac{E_s}{N_0} (\xi - 2\lambda) - \lambda^2 \left[ 1 + (1 - \xi)^2 \right] \} \tag{2.56}
\]
For the Rician case, we must first compute the pairwise error probability by substituting (2.54) into (2.51) and then optimize over the Chernoff parameter. For the Rayleigh fading case, (2.56) can be optimized over the Chernoff parameter independent of the index \( n \). Then we have

\[
\lambda_{\text{opt}} = \frac{\xi \frac{E_s}{N_0}}{4 \frac{E_s}{N_0} + 2[1 + (\frac{E_s}{N_0})^2(1 - \xi^2)]}.
\]  

(2.57)

Substituting (2.57) into (2.56) and then in (2.51) yields the pairwise error probability

\[
P(x \rightarrow \hat{x}) \leq \prod_{n \in \mathbb{N}} \frac{1}{1 + |\hat{x}_n - x_n|^2 \{\frac{\xi \frac{E_s}{N_0}}{4 \frac{E_s}{N_0} + 2[1 + (\frac{E_s}{N_0})^2(1 - \xi^2)]}\}}.
\]  

(2.58)

For the slow-fading Rician channel, i.e. \( \xi = 1 \), Eq. (2.52) evaluates to

\[
E\{\exp(\lambda W_n^* (\hat{A}_n - A_n) W_n)\} = \frac{1}{\Delta} \exp\left\{-\frac{\lambda E_s}{\Delta N_0} \frac{1}{1 + K} (1 - 2\lambda) - \lambda^2\right\}.
\]  

(2.59)

where

\[
\Delta = 1 + |\hat{x}_n - x_n|^2 \{\frac{\lambda E_s}{N_0} \frac{1}{1 + K} (1 - 2\lambda) - \lambda^2\}.
\]  

(2.60)

For the Rayleigh fading case (\( K = 0 \)), (2.58) simplifies to

\[
E\{\exp(\lambda W_n^* (\hat{A}_n - A_n) W_n)\} = \frac{1}{1 + |\hat{x}_n - x_n|^2 \{\frac{\lambda E_s}{N_0} (1 - 2\lambda) - \lambda^2\}}.
\]  

(2.61)

For the Rician case, we must first compute the pairwise error probability by substituting (2.59) into (2.51) and then optimize over the Chernoff parameter. For the Rayleigh fading case, (2.61) can be optimized over the Chernoff parameter independent of the index \( n \). Then we have

\[
\lambda_{\text{opt}} = \frac{\frac{E_s}{2N_0}}{1 + \frac{E_s}{N_0}}.
\]  

(2.62)

Substituting (2.62) into (2.61) and then in (2.51) yields the pairwise error probability

\[
P(x \rightarrow \hat{x}) \leq \prod_{n \in \mathbb{N}} \frac{1 - \nu^2}{1 - \nu^2(|\hat{x}_n - x_n|^2)}.
\]  

(2.63)
where

\[ \nu = \frac{E_s}{N_0} \left(1 + \frac{E_s}{N_0}\right)^{-1} \]  \hspace{1cm} (2.64)

The asymptotic behavior of (2.58) and (2.63) is of significant interest. For \( \xi \neq 1 \),

\[ P(x \to \hat{x}) \leq \prod_{n \in \mathbb{N}} \frac{1}{1 + \frac{\|x_n - x_n^0\|^2}{4} \frac{\xi^2}{1-\xi^2}}. \]  \hspace{1cm} (2.65)

For \( \xi = 1 \),

\[ P(x \to \hat{x}) \leq \prod_{n \in \mathbb{N}} \frac{1}{1 + \frac{\|x_n - x_n^0\|^2}{8} \frac{E_s}{N_0}}. \]  \hspace{1cm} (2.66)

Note that (2.65) is independent of signal-to-noise ratio. This means that regardless of how large we increase the signal-to-noise ratio, we get a nonzero error probability which is referred to as an error floor.

2.3 Digital Computer Simulation Model

For the Rayleigh fading, coherently detected 8-PSK modulation codes were used. Infinite interleaving and perfect phase tracking was assumed.

For the shadowed MSAT channel, both differentially detected and coherently detected 8-PSK modulation codes were used in simulations. Effects of the Doppler frequency shift and random variation of phase are considered in simulations. In the shadowed MSAT channel, carrier frequency \( f_c \) is 870 MHz and symbol rate \( T^{-1} \) is 2400 8-PSK symbols/sec. The Doppler shift \( f \) in frequency from the carrier frequency \( f_c \), for a signal arriving at an angle \( \alpha \) relative to vehicle motion, is

\[ f = f_m \cos \alpha \]  \hspace{1cm} (2.67)

where

\[ f_m = \frac{V}{\lambda \omega} \]  \hspace{1cm} (2.68)
is the maximum Doppler shift at the speed \( V \) and carrier wavelength \( \lambda = \frac{c}{f_c} \) where \( C \) is the speed of the light. In the multipath fading channel, the received signal is a result of many plane waves, each shifted in frequency by the Doppler shift appropriate to the vehicle motion relative to the direction of the plane waves. Then the received signal consists of a large number of sinusoids of comparative amplitude and random phase, whose frequencies are confined to the Doppler spread \( B_d \) around the transmitted frequency. The envelope of the received multipath signal confirms to Rice's model of narrow-band Gaussian noise [84]. Rice has computed the statistical behavior of narrow-band Gaussian noise in terms of its power spectrum. Gans [83] showed that power spectrums of received signal with different antennas have different shapes. However, all power spectrums are confined within the maximum Doppler frequency. For the shadowed MSAT communication channel, Loo [86]-[89] showed that the fading process which was created by using 3-rd order Butterworth shaping filter of 3 dB bandwidth \( B \) has almost the same statistical properties as experimental data from the shadowed MSAT channel. The shadowed Rician fading model is shown in Figure 2.2. Using the phasor form, the fading envelope \( A_k \) is defined as

\[
A_k = \mathcal{L}[Z] + \mathcal{L}[N_s]
\]

(2.69)

where \( \mathcal{L}[] \) is the effect of a third-order Butterworth linear digital filter. In (2.69), \( Z = e^{N_d} \) where \( N_d \) is Gaussian r.v. with mean \( \mu_0 \) and variance \( \sigma_0 \), is a lognormal r.v. representing the effect of shadowing in the fading model and the multipath component \( N_s = N_q + jN_i \) where \( N_q \) and \( N_i \) are Gaussian r.v., with zero mean and variance \( \sigma_0 \), and are independent. In a mobile fading channel, \( B \) is equal to the maximum Doppler frequency \( f_m \). Let \( BT \) be defined as the normalized fading bandwidth. When \( T^{-1} = 2400 \) symbols/sec and \( f_c = 870 \) MHz, for \( V = 37.152 \) miles/hr = 59.581 Km/hr, we have \( BT = 0.02 \). For \( V=92.88 \) miles/hr = 148.95
Km/hr, we have $BT = 0.05$. It was shown in [86]-[89] that a third-order Butterworth of the 3 dB bandwidth $B$ fading spectrum yields the best results when comparing with the experimental results in [87]. The normalized fading bandwidth is $BT$ where $T^{-1}$ is the symbol rate. The normalized fading bandwidth is directly related to the velocity differences of a vehicle and the symbol rate. As the speed of a vehicle increases, the normalized fading bandwidth also increases. Conversely, as the symbol rate increases, the normalized fading bandwidth decreases. In computer simulation, three different shadowed MSAT channels will be considered, i.e., light, average, and heavy shadowed MSAT channel with a carrier frequency 870 MHz, the symbol rate 2400 symbols/s (4800 bit/s/s) and different vehicle speeds. The mobile satellite channel has a constraint on the total allowable delay time which is 310 ms where 250 ms for the propagation delay and 60 ms for deinterleaving and decoding. It limits the size of the interleaver. In simulations, the interleaver size for uncoded
4-DPSK and coded 8-DPSK is 128 symbols and the interleaver size for uncoded binary DPSK and coded 4-DPSK is 256 symbols. In a mobile satellite channel, multipath fading, which is modeled by a Rician distribution, not only introduces an *error floor* into the system but also makes the problem of carrier recovery more difficult. Depending on the ratio of direct and specular to diffuse signal power, one might even be required to employ differentially coherent or noncoherent detection techniques. Also, system installed in the vehicle must be built with a *lower cost and complexity* than that of a telephone channel modem.
Chapter 3
Multilevel Coded Modulation Schemes

3.1 Introduction

One of the dramatic developments in bandwidth-efficient communications over the past eighteen years is the introduction and rapid application of combined coding and bandwidth efficient modulation, known as coded modulation, for reliable data transmission. Using coded modulation, reliable data transmission can be obtained without compromising bandwidth efficiency. The first coded modulation scheme was introduced by Ungerboeck in 1976 [2], and later published in 1982 [3]. In this scheme, trellis (or convolutional) codes were combined with various types of modulation signal sets to form modulation codes by the proper bits-to-signal mapping through signal set partitioning. This scheme is now known as trellis-coded modulation (TCM). Since the publication of Ungerboeck's prize winning paper, there has been a great deal of research on construction of TCM codes and many good TCM codes for both the AWGN and fading channels have been constructed [4] - [23].

Combined block coding and modulation, now known as block-coded modulation (BCM), originated from Imai and Hirakawa's paper published in 1977 [24], in which
they proposed a multilevel method of combining binary block component codes with a channel signal set to form a block modulation code. This multilevel method was later extended and used by others for constructing block modulation codes [25]-[40].

The multilevel method is a very powerful technique for constructing bandwidth efficient modulation codes systematically with arbitrarily large distance parameters from Hamming distance component (block or convolutional) codes in conjunction with the proper bits-to-signal mapping through signal set partitioning. Particularly, it provides the flexibility to coordinate the distance parameters of a code such that the best performance for a given channel can be attained. Furthermore, the multilevel modulation codes constructed by this method allow the use of multi-stage decoding procedures that provide good trade-off between error performance and decoding complexity.

In this chapter, construction of multilevel bandwidth efficient modulation codes for various fading channels is presented. Distance parameters, such as the minimum squared Euclidean distance, minimum symbol distance, and minimum product distance, which determine the error performance of a multilevel modulation code are expressed in terms of the minimum Hamming distances of the component codes. Guidelines for constructing good multilevel modulation codes for either the AWGN channel or the Rayleigh fading channel are presented.

The organization of this chapter is as follows. In section 2, we briefly discuss the evolution of coded modulation from conventional coding, the basic concept of coded modulation, and structural properties of coded modulation such as distance parameters and spectral efficiency. In Section 3, the multilevel method for constructing general multilevel modulation codes is given. Distance parameters of a multilevel modulation code are defined and expressed in terms of the minimum Hamming
distances of the component codes. Guidelines for constructing good modulation codes for either the AWGN or the Rayleigh fading channel are presented in Section 3.

3.2 Evolution of Coded Modulation

In conventional digital communications, channel coding is designed and performed separately from modulation. Error control is provided by transmitting additional well structured redundant bits. This added redundancy results in lowering the information bit rate per channel bandwidth. Hence bandwidth efficiency is traded for increased power efficiency. Therefore coding gain is achieved at the expense of bandwidth expansion. In the case of bandlimited channels, bandwidth expansion is either not desirable or not possible. Therefore, in the past, error control coding has never been popular in bandlimited channels such as telephone, land mobile communication, and mobile satellite channels.

In 1974, Massey [1] suggested that coding and modulation must be treated as a single entity to achieve a more effective utilization of the available bandwidth and power. That is, he suggested that coding must be designed in conjunction with modulation. The design rules associated with designing combined coding and modulation schemes were proposed by Ungerboeck in 1976 [2] and later published in 1982 [3]. Ungerboeck showed that, by combining coding and modulation properly, significant coding gains over conventional uncoded modulation can be achieved without bandwidth expansion. This combination of coding and modulation is now known as coded modulation. For ease of explanation, we will restrict our initial discussion of the design of combined coding and modulation schemes for the AWGN channel. Details of the design of modulation codes for the Rayleigh fading channel will be given towards to the end of the chapter.
3.2.1 Basic Concepts

The basic concept of coded modulation is to encode information symbols onto an expanded signal set, relative to that needed for uncoded modulation. Suppose we want to transmit at the rate of 2 bits/symbol. Then we choose QPSK (Quadrature Phase Shift Keying) for uncoded modulation to transmit at the rate of 2 bits/symbol. This would satisfy the rate criterion, however the performance in most cases would be below the required level. To overcome this, we introduce channel coding. Say a rate-2/3 convolutional code is chosen for the channel coding scheme. To maintain the rate of 2 bits/symbol, we would choose the 8PSK signal constellation at the modulator. At each time instant, 2 uncoded bits would be encoded by the convolutional code into 3 coded bits and these 3 bits are mapped onto an appropriate signal point in the 8PSK signal constellation (since the 8PSK signal constellation has 8 signals, each signal is uniquely represented by a label of 3 bits) by proper bits-to-signal mapping through signal set partitioning. The signal set expansion from QPSK to 8PSK represents the first and most important step associated with coded modulation. The channel signal set expansion provides the needed redundancy for error control without increasing the bandwidth requirement. The channel coding scheme (the rate-2/3 convolutional code in the above example) is selected such that the structured signal sequences being transmitted over the channel have the largest possible minimum squared Euclidean distance for a given decoding complexity. Therefore, by using coded modulation, coding gain is achieved at the expense of decoding complexity.

3.2.2 Types of Coded Modulation

Based on code structure, coded modulation can be classified into two categories:
(1) **Trellis Coded Modulation (TCM)** in which convolutional (or trellis) codes are used, and

(2) **Block Coded Modulation (BCM)** in which block codes are used.

Based on modulation signaling, coded modulation is divided into three types:

(1) *Constant-envelope-type* coded modulation in which the modulation signals have constant envelope, e.g., MPSK, and

(2) *Lattice-type* coded modulation in which the signal constellation has lattice structure,

(3) *Hybrid-type* coded modulation in which the modulation signals have constant envelop and the signal constellation also has lattice structure.

### 3.2.3 Modulation Codes and Distance Parameters

In a coded modulation system, information sequences are encoded into signal sequences over a certain modulation signal set. These signal sequences form a modulation code. In the following, we first define some important distance parameters of a modulation code.

Let \( C \) be a modulation code of length \( n \) with signals from a certain modulation signal space \( S \). The error performance of \( C \) depends on several distance parameters.

Let \( d^2(x, y) \) denote the *squared Euclidean distance* between two code sequences, \( x \) and \( y \), in \( C \). The *minimum squared Euclidean distance* of \( C \), denoted \( d^2_{E}[C] \), is defined as follows:

\[
d^2_{E}[C] \triangleq \min\{d^2(x, y) : x, y \in C \text{ and } x \neq y\}.
\]  

(3.1)

The *symbol (or Hamming) distance* between two code sequences \( x \) and \( y \), denoted \( \delta_{H}(x, y) \), is the number of different symbols between the two sequences. The min-
imum symbol distance of $C$, denoted $\delta_H[C]$, is defined as the minimum symbol distance between any two code sequences in the code. The product distance between $x$ and $y$ denoted by $\Delta_p^2(x, y)$ is defined as follows:

$$
\Delta_p^2(x, y) = \prod_{k=1, x_k \neq y_k}^{n} d^2(x_k, y_k)
$$

(3.2)

where $d^2(x_k, y_k)$ is the squared Euclidean distance between $k$-th signals, $x_k$ and $y_k$, of $x$ and $y$. The minimum product distance of $C$, denoted $\Delta_p^2[C]$, is the minimum product distance between any two code sequences with symbol distance $\delta_H[C]$ in the code.

For the AWGN channel, the error performance of a code depends primarily on its minimum squared Euclidean distance and path multiplicity [3]. For the Rayleigh fading channel, the error performance of a code depends primarily on its minimum symbol distance, minimum product distance, and path multiplicity [42, 43]. It depends on the minimum squared Euclidean distance to a lesser degree.

### 3.2.4 Spectral Efficiency

Let $C$ be a modulation code with signals from a two-dimensional modulation signal set $S$. Suppose $k$ information bits are encoded into a sequence of $n$ signals based on $C$. The spectral efficiency of $C$, denoted $\eta[C]$, is defined as:

$$
\eta[C] \triangleq \frac{k}{n}
$$

(3.3)

which is simply the average number of information bits transmitted per symbol. For a given decoding complexity, the idea is to construct a modulation code with as high a spectral efficiency as possible. High spectral efficiencies would lead to high data rates.
3.2.5 Soft Decision Decoding

Assume that the channel is AWGN and all the code sequences of a modulation code are equally likely to be transmitted. Let \( r = (x_1, y_1, x_2, y_2, \cdots) \) be the output sequence of the demodulator. Then the squared Euclidean distance between \( r \) and a code sequence \( v = (s_1, s_2, \cdots) \) in \( C \) is:

\[
|r - v|^2 = \sum_{j} (x_j - X(s_j))^2 + (y_j - Y(s_j))^2.
\]

(3.4)

The modulation code is decoded using the MLD (Maximum Likelihood Decoding) principle, i.e., the received sequence \( r \) is decoded into the code sequence \( v_i \) for which the following condition holds:

\[
|r - v_i| < |r - v_i|,
\]

for \( i \neq l \). The above corresponds to finding the code sequence which is closest to the transmitted sequence in terms of Euclidean distance. To perform a soft-decision MLD, it is desirable for a modulation code to have a trellis structure so that the Viterbi decoding algorithm can be applied. In the absence of a trellis structure, the decoding complexity associated with the decoding of the modulation code is in most cases very large.

3.2.6 Major Steps in Coded Modulation System Design

The basic method for constructing modulation codes consists of five steps: (1) selection of a modulation signal set \( S \); (2) labeling of signal points by strings of labeling symbols through signal set partitioning; (3) selection of component codes; (4) combining component codes into a code over a signal label set; and (5) label-to-signal mapping to form a multilevel modulation code.
3.2.7 Set Partitioning and Signal Labeling

In this dissertation, we only consider the construction of modulation codes over the MPSK signal constellation. Generalization of the construction to QAM signal constellation is straightforward. Let \( S \) denote the two-dimensional \( 2^\ell \)-PSK signal constellation with unit energy. Label each of the \( 2^\ell \) signal point with a unique string of \( \ell \) bits, \((a_1, a_2, \cdots, a_\ell)\), where \( a_1 \) is the first labeling bit and \( a_\ell \) is the last labeling bit.

Let \( L \) be a set of labeling strings which is of the form

\[
L = \{(a_1, a_2, \cdots, a_\ell) : \text{where } a_i \in \{0, 1\} \text{ for } 1 \leq i \leq \ell\}.
\]

The labeling \( L \) is said to have \( \ell \) levels or length \( \ell \).

The signal labeling is achieved by using the following partition chain, \( 2^\ell \)-PSK/\( 2^{\ell-1} \)-PSK/\( \cdots \)/QPSK/BPSK. At the first partition level, the \( 2^\ell \)-PSK signal constellation is partitioned into two \( 2^{\ell-1} \)-PSK signal sets, one is labeled with "0" and other is labeled with "1". At the second partition level, each \( 2^{\ell-1} \)-PSK signal set is partitioned into two \( 2^{\ell-2} \)-PSK sets, one is labeled with "0" and the other is labeled with "1". The partition process continues until the \( \ell \)-th level is reached where each subset contains a single signal point which is labeled with a unique sequence of \( \ell \) bits. This partitioning and labeling process for an 8-PSK signal constellation is shown in Figure 3.1. For \( 0 < i \leq \ell \), let \( d_i \) be the intra-set distance [3] (the minimum squared Euclidean distance among signal points) of the \( 2^{\ell-i+1} \)-PSK signal set. It is clear that the intra-set distance of a set at the \((i-1)\)-th partition level is \( d_i \), and the intra-set distance increases as the partition level increases. This monotonically increasing property of the intra-set distances, \( d_1, d_2, \cdots, d_\ell \), is a key to the construction of bandwidth efficient modulation codes. The labeling strings formed from the above partitioning process have the following important property: for \( 0 < i \leq \ell \),
Figure 3.1. The partitioning and labeling process of a 8-PSK constellation
two signal points with labels identical at the first $i - 1$ bit positions but different at the $i$-th bit position are at least at a squared Euclidean distance $d_i$ apart. The intra-set distances of the partition chain 8-PSK/QPSK/BPSK for an 8-PSK signal set are: $d_1 = 0.586$, $d_2 = 2.0$, and $d_3 = 4.0$ respectively. The above partitioning process is called the *binary partition* which was first devised by Ungerboeck in his construction of TCM codes [3].

### 3.3 Generalized Multilevel Coded Modulation

In this section, we consider the $m$-level modulation code over the $2^l$-PSK signal constellation $S$. To construct a $m$-level modulation code over $S$, we must segment the labeling into sublabelings and choose the starting symbol position of each sublabeling. Let $m$ be a positive integer not greater than $l$ and let $j_1, j_2, \cdots, j_m, j_{m+1}$ be $m + 1$ integers such that

$$1 = j_1 < j_2 < \cdots < j_m < j_{m+1} = \ell + 1.$$ 

For $1 \leq i \leq m$, let $q_i$ be defined as

$$q_i \triangleq j_{i+1} - j_i,$$

and let $L^{(i)}$, called the $i$-th sublabeling, denote the set of substrings from $j_i$ th symbols to the $(j_{i+1} - 1)$ th symbol of strings in $L$, i.e.,

$$L^{(i)} \triangleq \{ a_{j_i}a_{j_{i+1}}a_{j_{i+1} - 1} : a_h \in \{0,1\} \text{ for } j_i \leq h < j_{i+1} \}$$

Concatenating $L^{(1)}$ to $L^{(m)}$, we obtain

$$L = L^{(1)}L^{(2)} \cdots L^{(m)}.$$ 

Consider an $n$-tuple $v = (v_1, v_2, \cdots, v_n)$ over $L$. For $1 \leq j \leq n$, the $j$ th component $v_j$ of $v$ can be expressed as the following concatenation of substrings in $L^{(1)}$ to $L^{(m)}$
\[ v_j = (v_j^{(1)} v_j^{(2)} \cdots v_j^{(m)}) , \]

where \( v_j^{(i)} \in L^{(i)} \) for \( 1 \leq i \leq m \).

Since \( q_i = j_{i+1} - j_i \) and \( L^{(i)} = \{a_{j_i}a_{j_{i+1}} \cdots a_{j_{i+1} - 1} : a_h \in \{0,1\} \text{ for } j_i \leq h < j_{i+1}\} \) for \( 1 \leq i \leq m \), \( L^{(i)} \) is a set of all elements in \( GF(2^{q_i}) \). For \( 1 \leq i \leq m \), we form the following \( n \)-tuple over \( L^{(i)} \) (or \( GF(2^{q_i}) \)):

\[ \mathbf{v}^{(i)} = (v_1^{(i)}, v_2^{(i)}, \ldots, v_n^{(i)}) \]

This \( n \)-tuple \( \mathbf{v}^{(i)} \) over \( GF(2^{q_i}) \) is called the \( i \)th component \( n \)-tuple of \( \mathbf{v} \), and \( \mathbf{v} \) is denoted as follows:

\[ \mathbf{v} = \mathbf{v}^{(1)} \mathbf{v}^{(2)} \cdots \mathbf{v}^{(m)}. \]

Suppose block component codes are used for the code construction. For \( 1 \leq i \leq m \), let \( A_i \) be a \( (n, k_i, \delta) \) linear block code of length \( n \) over \( GF(2^{q_i}) \), dimension \( k_i \), and minimum Hamming distance \( \delta \). Let

\[ \mathbf{v}^{(1)} = (v_1^{(1)}, v_2^{(1)}, \ldots, v_j^{(1)}, \ldots, v_n^{(1)}) \]

\[ \mathbf{v}^{(2)} = (v_1^{(2)}, v_2^{(2)}, \ldots, v_j^{(2)}, \ldots, v_n^{(2)}) \]

\[ \vdots \]

\[ \mathbf{v}^{(m)} = (v_1^{(m)}, v_2^{(m)}, \ldots, v_j^{(m)}, \ldots, v_n^{(m)}) \]

(3.6)

be \( m \) codewords in \( A_1, A_2, \ldots, A_m \) respectively. We form the following \( n \)-tuple:

\[ \mathbf{A} = \mathbf{v}^{(1)} \ast \mathbf{v}^{(2)} \ast \cdots \ast \mathbf{v}^{(m)} = (v_1^{(1)} v_1^{(2)} \cdots v_1^{(m)}, v_2^{(1)} v_2^{(2)} \cdots v_2^{(m)}, \ldots, v_j^{(1)} v_j^{(2)} \cdots v_j^{(m)}, \ldots, v_n^{(1)} v_n^{(2)} \cdots v_n^{(m)}). \]

(3.7)

Such a code \( \mathbf{A} \) is called an \( m \)-level code with \( m \) component codes, and \( A_i \) called the \( i \)th component code of \( \mathbf{A} \). For \( 1 \leq j \leq n \), we take \( v_j^{(1)} v_j^{(2)} \cdots v_j^{(m)} \) as the
label for a signal point in the $2^\ell$-PSK signal constellation. Let $\lambda(\cdot)$ be the mapping which maps the label $v_j^{(1)}v_j^{(2)}\cdots v_j^{(m)}$ into its corresponding signal point $s_j$, i.e.,

$$\lambda(v_j^{(1)}v_j^{(2)}\cdots v_j^{(m)}) = s_j.$$  

Then

$$\lambda(v^{(1)}v^{(2)}\cdots v^{(m)}) \triangleq (\lambda(v_1^{(1)}v_1^{(2)}\cdots v_1^{(m)}), \lambda(v_2^{(1)}v_2^{(2)}\cdots v_2^{(m)}), \ldots, \lambda(v_n^{(1)}v_n^{(2)}\cdots v_n^{(m)}))$$

is a sequence of $n$ $2^\ell$-PSK signals. Let

$$C \triangleq \lambda[A_1 \ast A_2 \ast \cdots \ast A_m]$$

$$= \{\lambda(v^{(1)}v^{(2)}\cdots v^{(m)}): v^{(1)} \in A_1, v^{(2)} \in A_2, \ldots, v^{(m)} \in A_m\}.$$  

Then $C$ is an $m$-level $2^\ell$-PSK block modulation code of length $n$ and dimension $k = \sum_{i=1}^{m} q_i \cdot k_i$. Since $k$ information bits are encoded into a code sequence of $n$ $2^\ell$-PSK signals, the spectral efficiency is $\eta[C] = \sum_{i=1}^{m} q_i k_i / n$ bits/symbol. In the above construction, $m$ component codes are used and each component code contributes a level of labeling.

For $1 \leq i \leq m$, let

$$d^{(i)}(v^{(i)}, v'^{(i)}) = \min\{d(v^{(1)}\cdots v^{(i-1)}v^{(i)}v^{(i+1)}\cdots v^{(m)}), v^{(1)}\cdots v^{(i-1)}v'^{(i)}v^{(i+1)}\cdots v^{(m)});$$

$$v^{(j)} \in L^{(j)} \text{ with } j = 1, 2, \ldots, i \ldots, i - 1, i + 1, \ldots, m \text{ and}$$

$$v'^{(i)} \in L^{(i)} \text{ with } j = i + 1, \ldots, m\}$$

Since $j_i$ is the starting position of $i$-th sublabeling $L^{(i)}$ and

$$d_{j_1} < d_{j_2} < \cdots < d_{j_m}$$

for $2^\ell$-PSK signal constellation,

$$d^{(i)}(v^{(i)}, v'^{(i)}) = d_{j_i}.$$  

45
The distance parameters of an \( m \)-level \( 2^k \)-PSK modulation code can be expressed in terms of the minimum Hamming distances of its component codes.

**Distance Theorem:** Let \( d_E^2[C] \), \( \delta_H[C] \) and \( \Delta_p^2[C] \) denote the minimum squared Euclidean distance, minimum symbol distance and minimum product distance of an \( m \)-level \( 2^k \)-PSK code, \( C = \lambda[A_1 \ast A_2 \ast \cdots \ast A_m] \), respectively. Then

\[
\begin{align*}
(1) & & d_E^2[C] = \min\{\delta_i d_{ji} : 1 \leq i \leq m\}, \\
(2) & & \delta_H[C] = \min\{\delta_i : 1 \leq i \leq m\}, \\
(3) & & \Delta_p^2[C] = (d_{jk})^{\ell_k}
\end{align*}
\]

where \( d_{j1}, d_{j2}, \ldots, d_{jm} \) are the intra-set distances of the partition chain

\[2^k\text{-PSK}/2^{k-q_1}\text{-PSK}/\cdots/2^{q_{m-1}+q_m}\text{-PSK}/2^{q_m}\text{-PSK},\]

since \( \ell = \sum_{i=1}^{m} q_i \).

The proof of (3.10) can be found in [24, 26, 27] The results of (3.11) and (3.12) are simply generalization of the results for 3-level 8-PSK modulation codes given in Lemma 1 of [62].

From the expressions for the minimum squared Euclidean distance and product distance of a multilevel modulation code given by (3.10) and (3.12), it is clear why the intra-set distances should be kept as large as possible during the signal set partitioning and labeling process. The distance theorem provides general guidelines for constructing good multilevel modulation codes for both the AWGN and fading channels.
3.4 Channel Coding Design Criteria for the AWGN and Multipath Fading Channels

For the AWGN channel, the error performance of a code depends primarily on its minimum squared Euclidean distance and path multiplicity [3]. For the Rayleigh fading channel, the error performance of a code depends primarily on its minimum symbol distance, minimum product distance and path multiplicity [42]. It depends on the minimum squared Euclidean distance to a lesser extent for high SNR. However, for low SNR, minimum squared Euclidean distance and path multiplicity are still very important [46].

Let $x$ be a code sequence in $C$. If $x$ is transmitted over the AWGN channel, the pairwise error probability that represents the probability of choosing the code sequence $\hat{x}$ instead of $x$ is upper bounded by the following expression:

$$P(x \rightarrow \hat{x}) \leq Q(\frac{d_{E}^{2}(x, \hat{x})}{2\sigma})$$ \hspace{1cm} (3.13)

where $\sigma^2$ is the noise variance and $Q(\cdot)$ denotes the Q function which is defined as follows:

$$Q(x) \triangleq \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-y^2/2)dy.$$ \hspace{1cm} (3.14)

If $d_{E}^{2}[C]$ denotes the minimum squared Euclidean distance of the code, and $N[C]$ is the average number of codewords at a squared Euclidean distance from any transmitted codeword, then using (3.13) and the union bound, the error event probability for large signal-to-noise ratios can be approximately upper bounded by:

$$P(\text{error}) \approx N[C] \cdot Q(\sqrt{d_{E}^{2}[C]/2\sigma}).$$ \hspace{1cm} (3.15)

For the frequency non-selective slowly varying Rician fading channel, upper bounds on pairwise error probability of coherently detected M-PSK and differentially detected M-DPSK are summarized in [59] where interleaving depth is assumed.
to be infinite. This assumption provides a memoryless channel. In the following, upper bounds on pairwise error probability of coherently detected and differentially detected $M - PSK$ for the frequency non-selective slow Rician fading channel are summarized.

(1) For the case of coherently detection of $M$-PSK with no channel state information (CSI), the upper bound on pairwise error probability is given by [59]

$$P(x \rightarrow \hat{x}) \leq \left( \frac{2e}{L_n \delta H(x, \hat{x})} \right)^{-2} \left( \frac{\prod_{n \in \eta} |x_n - \hat{x}_n|^2}{\prod_{n \in \eta} \left( \sum_{n \in \eta} |x_n - \hat{x}_n|^2 \right)^{1/2}} \right)^{2(1 + K) e^{-K \delta H(x, \hat{x})}}. \quad (3.16)$$

where $\eta$ is the set of all $n$ such that $x_n \neq \hat{x}_n$ and $L_{eta}$ is the length of the error event path corresponding to $\hat{x}$ (i.e., the number of elements in the set $\eta$). Therefore, $L_\eta$ is equal to the symbol distance $\delta_H(x, \hat{x})$ between two code sequence $x$ and $\hat{x}$. Then, we have

$$P(x \rightarrow \hat{x}) \leq \left( \frac{2e}{L_n \delta H(x, \hat{x})} \right)^{-2} \left( \frac{\prod_{n \in \eta} |x_n - \hat{x}_n|^2}{\prod_{n \in \eta} \left( \sum_{n \in \eta} |x_n - \hat{x}_n|^2 \right)^{1/2}} \right)^{2(1 + K) e^{-K \delta H(x, \hat{x})}}. \quad (3.17)$$

(2) For the differentially coherent detection of $M$-PSK with no CSI (channel state information), for sufficiently large SNR, an upper bound of pairwise error probability is given by [59]

$$P(x \rightarrow \hat{x}) \leq \prod_{n \in \eta} \frac{S_{H(x, \hat{x})}^\delta((1 + K) e^{-K \delta H(x, \hat{x})})}{\left( \frac{E_x}{N_0} \right)^\delta H(x, \hat{x}) \prod_{n \in \eta} |x_n - \hat{x}_n|^2}. \quad (3.18)$$

From (3.2), we have

$$\Delta_k^2(x_k, \hat{x}_k) = \prod_{n \in \eta} |x_k - \hat{x}_k|^2 \quad (3.19)$$

and substitute in (3.18). Then we have

$$P(x \rightarrow \hat{x}) \leq \prod_{n \in \eta} \frac{S_{H(x, \hat{x})}^\delta((1 + K) e^{-K \delta H(x, \hat{x})})}{\left( \frac{E_x}{N_0} \right)^\delta H(x, \hat{x}) \Delta_k^2(x, \hat{x})}. \quad (3.20)$$
Channel state refers to the amount of fading the channel is experiencing at a certain instant. In most cases, knowledge of the channel state is difficult, and as such assuming that the channel state information is unknown is a very valid and practical assumption. All the results that have been derived in this dissertation assume no channel state information. All the simulations for the frequency non-selective slow Rayleigh fading channel, assume infinite interleaving, no channel state information and no frequency spreading.

For the frequency non-selective fast Rayleigh fading channel, an upper bound on pairwise error probability of differentially detected M-DPSK modulation code is summarized in the following:

\[
P(x \rightarrow \hat{x}) \leq \prod_{n \in \eta} \frac{1}{1 + |x_n - \hat{x}_n|^2 \{\frac{\xi^2}{4} \left(\frac{E_s}{N_0}\right)^2 [2\frac{E_s}{N_0} + 1 + (1 - \xi^2)(\frac{E_s}{N_0})^2]\}}
\]  

(3.21)

where \(\xi\) is the covariance coefficient of the complex-valued Gaussian process which represents a characteristic of the fast Rician fading channel. Mason [85] has tabulated \(\xi\) for various types of fading process of practical interest such as exponential, Gaussian, land mobile, and 1st and 2nd order butterworth fading spectrums. For sufficiently large \(E_s/N_0\), (3.21) becomes

\[
P(x \rightarrow \hat{x}) \leq \prod_{n \in \eta} \frac{1}{1 + \frac{|x_n - \hat{x}_n|^2 \xi^2}{4 \left(1 - \xi^2\right)}}.
\]

(3.22)

As can be seen from expressions (3.13) and (3.17-3.21) the parameters of interest for code design for the AWGN and the Rayleigh fading channels are different. In either case, the parameters of the modulation code need to be chosen such that the appropriate error event probability is minimized.

For the AWGN channel, the error performance of a modulation code depends mainly on its minimum squared Euclidean distance. In this case, expression (3.10) should be used as a guideline for code construction. For a given minimum squared
Euclidean distance, the component codes should be chosen to maximize the spectral efficiency and minimize the decoding complexity and path multiplicity. The above design rule is applied to the Rician channel when $K \to \infty$. When $K \to 0$, in the slow- and fast Rician fading channels, the error performance of a modulation code depends strongly on its minimum symbol and product distances for high SNR. Both these distances should be as large as possible. For low SNR, the error performance of a modulation code depends on its minimum squared Euclidean distance and path multiplicity. In designing modulation codes for the Rician fading channel with $K \to 0$, expressions of (3.11) and (3.12) should be used as the design guidelines.

In the fast Rician fading channel, if $E_s/N_0$ is sufficiently large, the pairwise error probability is independent of the signal-to-noise ratio as shown in (3.22). That is, the BER (bit error rate) does not decrease significantly after a certain SNR. This BER is called the error floor. This error floor is dependent on the symbol distance between coded sequences which is shown in (3.22). Therefore, as we increase the minimum symbol distance of code, we can achieve better BER at the same SNR. Since coded modulation does not perform better than the uncoded reference system without interleaving technique [47], interleaving must be used with coded modulation. It has been shown [47] that interleaved TCM achieves a good real coding gain over the uncoded reference system.
Chapter 4

Multilevel TCM Codes

4.1 Introduction

The multilevel coding method presented in the previous Chapter can be used for constructing TCM codes using convolutional codes as the component codes or using both convolutional and block codes as the component codes. In this chapter, we consider the construction of multilevel TCM codes using convolutional codes with good free branch distance [64] as the component codes. Codes constructed in this chapter outperform existing codes with the same decoding complexity. Bit error performances of these codes for the Rayleigh and shadowed MSAT channels are presented and comparisons are summarized in Tables.

4.2 Optimum Free Branch Distance Convolutional Codes

Let $C$ be a rate-$k/n$ binary convolutional code. In particular, the encoder contains $k$ shift registers, not all of which must have the same length. If $K_i$ is the length of the $i$th shift register, then the encoder memory order $\gamma$ is defined as

$$\gamma \triangleq \max_{1 \leq i \leq k} K_i \quad (4.1)$$
i.e., the maximum length of all $k$ shift registers. By using trellis representation, a code sequence in $C$ is a path in the code trellis diagram consisting of a sequence of branches, each branch consists of $n$ coded bits. For two coded sequences $u$ and $v$ in $C$, the branch distance between them, denoted $d_b(u, v)$, is defined as the number of branches for which $u$ and $v$ differ. The minimum free branch distance of $C$, denoted $d_{\text{B-free}}$, is defined as the minimum branch distance between any two code sequences in $C$, i.e.,

$$d_{\text{B-free}} \triangleq \min\{d_b(u, v) : u, v \in C \text{ and } u \neq v\}. \quad (4.2)$$

For a rate-$k/n$ feedforward binary convolutional code of total encoder memory $\gamma$, its minimum free branch distance $d_{\text{B-free}}$ is upper bounded by $1 + \lceil \gamma/k \rceil$ [23]. A search has been performed on rate-1/2, -2/3 and -3/4 codes to find the best codes in terms of minimum free branch distance and the minimum nearest neighbors. Some best codes are given in Tables 4.1, 4.2, and 4.3. Most of the codes meet the upper bound on the minimum free branch distance.

A search [63] was also performed to find the optimal rate-1/2 and rate-2/3 branch distance convolutional codes for coherently detected 4-PSK and 8-PSK transmission respectively, for the frequency non-selective slow Rayleigh fading channel. The optimal codes obtained by the search are listed in Tables 4.4 and 4.5.

Convolutional codes of good minimum free branch distances are quite suitable for constructing TCM codes with good minimum symbol distances. Let $S$ be a modulation signal set with $2^\ell$ signal points and $C$ a rate-$\frac{\ell-1}{\ell}$ convolutional code of minimum free branch distance $d_{\text{B-free}}$. If each group of $\ell$ coded bits is one-to-one mapped into a signal point in $S$, we obtain a TCM code over $S$ with minimum symbol distance equal to the minimum free branch distance of the convolutional
Table 4.1. Rate-1/2 Optimum Branch Distance Convolutional Codes

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$G$</th>
<th>$d_{B\text{-free}}$</th>
<th>$N_{B\text{-free}}$</th>
<th>$d_{H\text{-free}}$</th>
<th>$N_{H\text{-free}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\left( \begin{array}{c} 4 \ 2 \end{array} \right)$</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>$\left( \begin{array}{c} 5 \ 2 \end{array} \right)$</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>$\left( \begin{array}{c} 5 \ 64 \end{array} \right)$</td>
<td>4</td>
<td>1</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>$\left( \begin{array}{c} 44 \ 32 \end{array} \right)$</td>
<td>5</td>
<td>2</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>$\left( \begin{array}{c} 62 \ 35 \end{array} \right)$</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>$\left( \begin{array}{c} 51 \ 664 \end{array} \right)$</td>
<td>7</td>
<td>4</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>$\left( \begin{array}{c} 344 \ 532 \end{array} \right)$</td>
<td>8</td>
<td>6</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>$\left( \begin{array}{c} 622 \ 575 \end{array} \right)$</td>
<td>8</td>
<td>1</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>$\left( \begin{array}{c} 355 \ 6244 \end{array} \right)$</td>
<td>9</td>
<td>1</td>
<td>11</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>$\left( \begin{array}{c} 3576 \ 6322 \end{array} \right)$</td>
<td>10</td>
<td>3</td>
<td>12</td>
<td>2</td>
</tr>
</tbody>
</table>

$\gamma$ : Total encoder memory  
$d_{B\text{-free}}$ : Minimum free branch distance  
$N_{B\text{-free}}$ : Number of codewords with branch distance $d_{B\text{-free}}$  
$d_{H\text{-free}}$ : Free Hamming distance  
$N_{H\text{-free}}$ : Number of codewords with Hamming distance $d_{H\text{-free}}$
### Table 4.2. Rate-2/3 Optimum Branch Distance Convolutional Codes

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>G</th>
<th>$d_{B\text{-}free}$</th>
<th>$N_{B\text{-}free}$</th>
<th>$d_{H\text{-}free}$</th>
<th>$N_{H\text{-}free}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$\begin{pmatrix} 6 &amp; 2 &amp; 6 \ 2 &amp; 4 &amp; 4 \end{pmatrix}_8$</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>$\begin{pmatrix} 0 &amp; 4 &amp; 3 \ 7 &amp; 5 &amp; 0 \end{pmatrix}_8$</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>$\begin{pmatrix} 0 &amp; 54 &amp; 64 \ 54 &amp; 74 &amp; 14 \end{pmatrix}_8$</td>
<td>4</td>
<td>7</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>$\begin{pmatrix} 76 &amp; 26 &amp; 46 \ 64 &amp; 0 &amp; 36 \end{pmatrix}_8$</td>
<td>5</td>
<td>14</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>$\begin{pmatrix} 75 &amp; 57 &amp; 0 \ 66 &amp; 64 &amp; 55 \end{pmatrix}_8$</td>
<td>6</td>
<td>30</td>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table 4.3. Rate-3/4 Optimum Branch Distance Convolutional Codes

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>G</th>
<th>$d_{B\text{-}free}$</th>
<th>$N_{B\text{-}free}$</th>
<th>$d_{H\text{-}free}$</th>
<th>$N_{H\text{-}free}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$\begin{pmatrix} 0 &amp; 6 &amp; 6 &amp; 2 \ 6 &amp; 6 &amp; 2 &amp; 4 \ 6 &amp; 2 &amp; 2 &amp; 2 \end{pmatrix}_8$</td>
<td>2</td>
<td>11</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>$\begin{pmatrix} 7 &amp; 1 &amp; 0 &amp; 4 \ 5 &amp; 7 &amp; 1 &amp; 7 \ 0 &amp; 5 &amp; 6 &amp; 7 \end{pmatrix}_8$</td>
<td>3</td>
<td>16</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>$\begin{pmatrix} 74 &amp; 2 &amp; 34 &amp; 0 \ 44 &amp; 7 &amp; 74 &amp; 74 \ 54 &amp; 0 &amp; 4 &amp; 74 \end{pmatrix}_8$</td>
<td>4</td>
<td>30</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 4.4. Rate-1/2 Optimum Branch Distance Convolutional Codes for the Rayleigh Fading Channel

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$G$</th>
<th>$d_{\text{B-free}}$</th>
<th>$\Delta_p^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\left( \begin{array}{c} 4 \ 2 \end{array} \right)$ &amp; 2 &amp; 8.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$\left( \begin{array}{c} 5 \ 2 \end{array} \right)$ &amp; 3 &amp; 32.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$\left( \begin{array}{c} 7 \ 24 \end{array} \right)$ &amp; 4 &amp; 64.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$\left( \begin{array}{c} 64 \ 52 \end{array} \right)$ &amp; 5 &amp; 128.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$\left( \begin{array}{c} 66 \ 37 \end{array} \right)$ &amp; 6 &amp; 128.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$\left( \begin{array}{c} 77 \ 224 \end{array} \right)$ &amp; 7 &amp; 256.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$\left( \begin{array}{c} 552 \ 364 \end{array} \right)$ &amp; 8 &amp; 1024.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$\left( \begin{array}{c} 706 \ 251 \end{array} \right)$ &amp; 8 &amp; 4096.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.5. Rate-2/3 Optimum Branch Distance Convolutional Codes for the Rayleigh fading channel

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$G$</th>
<th>$d_{\text{B-free}}$</th>
<th>$\Delta_p^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$\left( \begin{array}{c} 142 \ 432 \end{array} \right)$ &amp; 3 &amp; 5.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$\left( \begin{array}{c} 7345 \ 64140 \end{array} \right)$ &amp; 4 &amp; 7.99</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
code $C$. If the resultant TCM code also has good minimum product distance, the code would perform well for the Rayleigh fading channel.

In the following examples, the error performance of modulation codes is evaluated for the frequency non-selective slow Rayleigh fading channel and the fast-fading shadowed MSAT channel. For the slow Rayleigh fading channel, coherently detected $M$-PSK modulation codes are used under conditions that perfect phase synchronization and infinite interleaving are available. In the shadowed MSAT channel for the voice transmission, we have limitation on buffer size in the receiver due to constraint on satellite delay time. Therefore, we have to use a finite size of interleaver and de-interleaver. The interleaver size is 256 QPSK symbols or 128 8-PSK symbols.

4.3 Design of an Interleaver with Finite Size

Let $\alpha$, and $\beta$ be non-zero positive integers and let $M = \alpha \beta$. A coded MPSK sequence $c = (c_1, c_2, \cdots, c_k, \cdots, c_M)$ will fill in the interleaver buffer row by row, while the output sequence $c' = (c'_1, c'_2, \cdots, c'_k, \cdots, c'_M)$ from the interleaver is formed by reading out column by column as shown in Figure 4.1. Therefore, the number of row $\alpha$ is related to the interleaving depth and the size of column $\beta$ is related to the decision depth of modulation codes. This implies that symbols in the two code sequences $c$ and $c'$ have the following relationship:

$$ c_{\beta a + b + 1} = c'_{\alpha a + b + 1} \quad (4.3) $$

where $0 \leq a \leq \beta - 1$ and $0 \leq b \leq \alpha - 1$. Since our interleaver store only 128 8-PSK symbols (or 256 QPSK symbols), the interleaving depth $\alpha$ is finite. In the following, we examine the bit-error performance of a rate-2/3, 16-state, 8-PSK TCM code with different numbers of rows and columns. For fixed $\alpha = 20$, bit error rates (BERs) of the rate-2/3, 16-state, 8-DPSK TCM code $C(1)$ which is the 1st code in Table

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4.5 with three different values of $\beta$ are shown in Figure 4.2. The three different values of $\beta$ are 16, 20, and 30. When $\beta$ is 20 and 30, the decoding depth $d_1$ of a code is 20. And when $\beta$ is 16, $d_1$ is 16. It shows that if the interleaving depth is fixed, then the BER of the code is strongly dependent on the decision depth of the TCM code. Figure 4.3 shows BERs of the 8-PSK TCM code $C(1)$ when the size of column is fixed at 30 and the number of rows are 10, 20, and 30. In this case, the decoding depth $d_1$ is 20 for all three cases. The bit error performance of the code with $\alpha = 20$ is almost the same as that of the code with $\alpha = 30$. For this code, if $\beta = 30$, then the size of interleaving depth 20 is enough to achieve the best possible error performance.

When the size of interleaver is only 128 8-PSK symbols and the memory size of a code is 4, we examine the effect of the size of decision depth on the error performance of a code. Figure 4.4 shows the BER of the 8-PSK TCM code $C(1)$

Figure 4.1. A block interleaver with $\alpha$ rows and $\beta$ columns.
Figure 4.2. Bit error performance of the R=2/3 16-state TCM code C(1) as the number of columns varies

Figure 4.3. Bit error performance of the R=2/3 16-state TCM code C(1) as the number of rows varies
Figure 4.4. Bit error performance of the R=2/3 16-state TCM code C(1) with an interleaver size 128 8-PSK symbols

when the decoding depth is 20 and 16, respectively. When the size of the decoding depth is 20, our choice is $\alpha = 4$ and $\beta = 32$. When the decoding depth is 16, our choice is $\alpha = 8$ and $\beta = 16$. As shown in Figure 4.4, decreasing the decoding depth and increasing the interleaving depth do not improve the error performance for this code. In the following, important points to design an interleaver with finite size are summarized:

(1) First, the number of column $\beta$ must be greater than the decoding depth of a code. When the decoding depth $d_1$ of a TCM code is about five times the constraint length of a TCM code, a code achieves the optimum bit error performance for the AWGN channel [92]. However, we can not use 5 times the constraint length of TCM code as the decoding depth of a code on a MSAT channel because of the limitation on decoding delay time. The main reason for using the decoding depth of a code which is smaller than five times constraint
length of a code in the simulation is following. Let $d_1$ be the decoding depth of a $2^\ell$-PSK TCM code $C = (c_1, c_2, \cdots)$ where $c_i$ is a point in a $2^\ell$-PSK constellation. For a $2^\ell$-PSK TCM code, we will use a rate-$\frac{\ell-1}{\ell}$ convolutional code with the memory size $\mu$. Let $R = (r_1, r_2, \cdots)$ be the received signal sequence. To recover a code sequence $C$ from a received sequence $R$, we will use the Viterbi algorithm with a finite size of decoding depth $d_1$, i.e., $d_1$ is normally 5 times the constraint length of code $C$. The Viterbi algorithm is applied to the trellis of a code $C$. In the trellis of a code $C$, there are $2^\mu$ states and $2^{\mu+\ell-1}$ branches in each section of the trellis. In the Viterbi algorithm, informations about the past $d_1$ sections are used to decode $c_k$ at a time instant $t_k$. Therefore, we always have to trace back $d_1$ sections of the trellis to decode a $2^\ell$-PSK signal point. That is why sometimes we can not use 5 time the constraint length of a code as the decoding depth of a code. In simulations, 5 or 4 times of memory size of TCM code is used as decoding depth to reduce decoding delay.

(2) Since the interleaver is finite and the number of column must be bigger than the decoding depth, the size of interleaving depth $\alpha$ is determined after deciding the size of column.

For BCM codes, the number of column (i.e., the size of decoding depth) is equal to the code length. Therefore, short BCM codes can have better interleaving depth than long BCM codes with the same interleaver.

### 4.4 Single-level TCM Codes

In this section, we construct 8-PSK and QPSK TCM codes for the frequency non-selective Rayleigh fading and shadowed MSAT channels with finite interleaver size.
4.4.1 8-PSK TCM Code

Suppose the first code given in Table 4.5 is used for constructing an 8-PSK TCM code. This code is a rate-2/3 feedforward convolutional code with total encoder memory 4, minimum free branch distance 3, and generator matrix

$$G(D) = \begin{pmatrix} D^2 & 1 & D \\ 1 & D + D^2 & D \end{pmatrix}$$

Suppose the natural mapping as shown in Figure 3.1 is used and every three coded bits are mapped into an 8-PSK signal point. The resultant modulation code $C(1)$ is a 16-state 8-PSK TCM code with spectral efficiency 2 bits per symbol. The minimum symbol distance of this TCM code is 3. We also find that the minimum product and squared Euclidean distances of this code are 4.6864 and 5.17 respectively. Furthermore, this code has only two nearest neighbors in terms of both minimum symbol and product distances. It turns out that this code has the same distance param-
eters as those of the 16-state Ungerboeck code (or the 16-state Schlegel-Costello code) [3, 55] which has been regarded as optimal for the Rayleigh fading channel. The 16-state Ungerboeck code was actually designed for the AWGN channel and the convolutional code used in the construction is a rate-2/3 feedback convolutional code. The error performances of the TCM code constructed in this example and the 16-state Ungerboeck code over the Rayleigh channel are shown in Figure 4.5. The code achieves a 13 dB coding gain over the uncoded QPSK at the bit-error-rate of $10^{-3}$ and performs slightly better than the 16-state Ungerboeck code for SNR greater than 11.5 dB, but the Ungerboeck code gives slightly better error performance for SNR less than 11.5 dB. For the AWGN channel, the Ungerboeck code performs slightly better than the code in this example for SNR less than 7 dB, and their error performance curves overlap with each other for SNR greater than 7 dB. This is due to the fact that the Ungerboeck code has slightly better squared Euclidean distance profile.

**Coherently detected 8-PSK TCM code over the shadowed MSAT channel**

In the shadowed MSAT channel, we have limitations on the buffer size in the receiver due to constraints on delay time, Therefore, we have to use a finite interleaver and deinterleaver. Since the size of an interleaver is 128 8-PSK symbols and the decoding depth is 20, let the number of column be 32 and the number of row be 4. The error performance of the coherently detected 8-PSK modulation codes $C(1)$ for the light-, average-, and heavy-shadowed MSAT channel is shown in Figure 4.6, 4.7, and 4.8, respectively. In this case, perfect phase synchronization is assumed. The Rician factors $K$ of light-, average-, and heavy-shadowed MSAT channel are 6.16, 5.46, and -19.33 dB, respectively. As $K \to 0$, the characteristic of the channel becomes close to the Rayleigh fading channel. And as $K \to \infty$, the characteristic
of the channel becomes close to the AWGN channel. Therefore, the light-shadowed MSAT channel is closer to the AWGN channel than the Rayleigh fading channel. And the heavy-shadowed MSAT channel is closer to the Rayleigh fading channel.

As shown in these Figures, code \( C(1) \) achieves an impressive real coding gain over the coherently detected uncoded QPSK system. If perfect phase synchronization is not available, there will be a severe degradation in error performance of a code and furthermore the BER will not decrease even though we increase the SNR. Therefore, the BER will be constant for high SNR. This phenomenon is called the **error floor**.

Since we assume perfect phase synchronization, we do not observe the **error floor** in the bit error performance curves in Figures 4.6, 4.7, and 4.8. It has been shown [47] that if the speed of a moving object is increasing from slow speed to fast speed, the channel changes from the bursty error channel to random error channel. As shown in figures 4.6, 4.7, and 4.8, the code \( C(1) \) performs better as the speed of a moving object is increasing from 37.152 mile/hr (or \( BT = 0.02 \)) to 92.88 miles/hr (or \( BT = 0.05 \)). In the above, \( BT \) is the normalized Doppler frequency [47], \( T^{-1} \) is the symbol rate, and \( B \) is the 3 dB bandwidth of a third-order Butterworth filter or is the maximum Doppler frequency shift [47]. The bit error performance of the rate-2/3 16-state Ungerboeck code \( C(U) \) for various shadowed MSAT channels is also shown in Figures 4.6, 4.7, and 4.8. At BER \( 10^{-3} \) or less, code \( C(1) \) outperforms the Ungerboeck code in light-, average, and heavy shadowed MSAT channels. Table 4.6 summarized error performance of the coherently detected 8-PSK modulation code \( C(1) \) for various shadowed MSAT channels. Real coding gains over the uncoded QPSK and the Ungerboeck rate-2/3, 16-state, TCM code are also summarized in the Table 4.6. In the table, \( C(U) \) represents the rate-2/3, 16-state, Ungerboeck TCM code. As shown in Table 4.6, the coherently detected 8-PSK modulation code \( C(1) \) outperforms the Ungerboeck code \( C(U) \) at BER \( 10^{-4} \) for light-, average-, and heavy-
shadowed MSAT channel. It has been known [55] that the rate-2/3, Ungerboeck 16-state, TCM code is optimum for both AWGN channel and Rayleigh fading channel.

Table 4.6. Bit error performance of the coherently detected 8-PSK TCM code $C(1)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_0$ @ $10^{-3}$ BER</th>
<th>$E_b/N_0$ @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(U)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(U)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02</td>
<td>14.39</td>
<td>20.44</td>
<td>1.522</td>
<td>4.594</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>11.88</td>
<td>17.04</td>
<td>4.662</td>
<td>9.011</td>
<td>0.531</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>15.80</td>
<td>22.01</td>
<td>2.501</td>
<td>6.159</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>13.22</td>
<td>18.724</td>
<td>5.808</td>
<td>9.91</td>
<td>0.39</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>26.35</td>
<td>30.465</td>
<td>3.029</td>
<td>7.99</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>19.248</td>
<td>24.244</td>
<td>12.26</td>
<td>16.806</td>
<td>1.217</td>
</tr>
</tbody>
</table>
Figure 4.6. BER of the 8PSK TCM code $C(1)$ over a light-shadowed MSAT channel.

Figure 4.7. BER of the 8PSK TCM code $C(1)$ over an average-shadowed MSAT channel.
Figure 4.8. BER of the 8PSK TCM code $C(1)$ over a heavy-shadowed MSAT channel.

**Differentially detected 8-PSK TCM code over the shadowed MSAT channel**

If perfect phase synchronization is not possible, the bit error performance of a coherently detected 8-PSK TCM code without perfect phase synchronization shows the error floor before BER $10^{-3}$ over the light-shadowed MSAT channel. Therefore coherently detected 8-PSK modulation is hardly used in multipath fading channels. However, coherently detected M-PSK modulation codes over the this kind of severe fading channel are worthy of study. In this dissertation, the bit error performance of both coherently and differentially detected M-PSK modulation codes are evaluated for the slow Rayleigh fading channel and shadowed MSAT (fast Rician) channel.

The bit error performance of the differentially detected 8-PSK modulation code $C(1)$ for the light-, average-, and heavy-shadowed MSAT channel is shown in Figures
4.9, 4.10, and 4.11, respectively. Since we do not assume perfect phase synchronization, we observe error floors in the bit error performances because of randomly time-varying phase and Doppler frequency shift. When a normalized fading bandwidth $BT = 0.05$ (i.e., the speed of a moving object is 99.88 miles/hr), bit error performance of uncoded QPSK reaches an error floor faster than when the speed of a moving object is 37.152 miles/hr (i.e., $BT = 0.02$). When the vehicle is slow, the channel is a burst error channel. And when the vehicle is fast, the channel behaves like a random error channel. In the coherently detected 8-PSK modulation code, we do not observe the error floor and a code performs better as the object moves faster because we assume perfect phase synchronization. Unlike a coherently detected 8-PSK code bit error performance of a differentially detected code increases as the speed of vehicle increases because of the randomly varying phase and Doppler frequency shift. However, if a code has a large minimum symbol distance (e.g., 4) and has a large minimum product distance (e.g., 256), regardless of random phase and Doppler frequency shift, the performance of the code improves as the speed of an object increases. For differentially detected QPSK TCM codes and BCM codes with a minimum symbol distance of 4 or more, these codes perform better as the speed of object increases.

Sliding-window Viterbi soft-decision decoding is used to decode $C(1)$. Window size (or decision depth) of code $C(1)$ is 20. These values are 5 times the memory size of code $C(1)$ and $C(U)$. Since the size of the buffer is only 128 8-PSK symbols and the number of column must be greater than 20, we let the number of column be 32 and the interleaving depth (the number of row) be 4. Table 4.7 summarizes the error performance of differentially detected 8-PSK modulation code $C(1)$ for various shadowed MSAT channels. Table 4.7 also summarizes the coding gain over the differentially detected 8-PSK modulation code $C(U)$ for various shadowed MSAT
channels. In contrast to the case of coherently detected 8-PSK TCM codes, the bit error performance does not improve even though we increase the signal power. As shown in the case of the coherently detected, 8-PSK, TCM codes, codes perform better as the speed of object increases because a burst error channel becomes a random error channel. However, in the case of differentially detected 8-PSK TCM, if the minimum symbol and product distance of a modulation code are not large enough (i.e., large diversity), a modulation code can not utilize the random error channel characteristic and reaches the error floor before a bit error rate of $10^{-4}$. If a code has the minimum symbol distance 4 and the minimum product distance 256, it will perform better as the speed of a object increases in the light- and average-shadowed MSAT channels. For heavy shadowing, the Rician factor $K$ is about -20 dB and the error floor is present for almost all the modulation codes. These results will be presented in the following section and chapters.

![Figure 4.9. BER of the 8-DPSK TCM code C(1) over a light-shadowed MSAT channel.](image)

Figure 4.9. BER of the 8-DPSK TCM code $C(1)$ over a light-shadowed MSAT channel.
Figure 4.10. BER of the 8-DPSK TCM code $C(1)$ over an average-shadowed MSAT channel.

Figure 4.11. BER of the 8-DPSK TCM code $C(1)$ over a heavy-shadowed MSAT channel.
Table 4.7. Bit error performance of the differentially detected 8-PSK TCM code $C(1)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$ @ $10^{-3}$ BER</th>
<th>$E_b/N_o$ @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over 4-DPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over 4-DPSK</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(U)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(U)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02</td>
<td>18.663</td>
<td>27.13</td>
<td>0.464</td>
<td>1.82</td>
<td>-0.495</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>18.831</td>
<td>34.63</td>
<td>1.869</td>
<td>0.0001973*</td>
<td>0.0</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>20.614</td>
<td>29.69</td>
<td>0.786</td>
<td>4.764</td>
<td>-0.315</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>21.65</td>
<td>0.00018*</td>
<td>3.447</td>
<td>0.0004744*</td>
<td>0.255</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>36.74</td>
<td>0.0004993*</td>
<td>0.00144*</td>
<td>0.00142*</td>
<td>1.62</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.0063*</td>
<td>0.00803*</td>
<td>0.00477*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*: bit-error-rate at the error floor

4.4.2 QPSK TCM Codes

In this section, we consider the construction of QPSK TCM codes for the Rayleigh fading and various shadowed MSAT channels using codes in Table 4.4. Let $C(2)$ denote the fifth code in Table 4.4. The code $C(2)$ is a rate-1/2 convolutional code of constraint length 6 with generator matrix

$$G(D) = [1 + D + D^2 + D^4, D + D^2 + D^3 + D^4 + D^5]. \quad (4.4)$$

This code has minimum free branch distance 6, minimum free distance 8, and a 32-state trellis diagram. Let $C(3)$ denote the fourth code in Table 4.4. The code $C(3)$ is a rate-1/2 convolutional code of constraint length 5 with generator matrix

$$G(D) = [1 + D + D^2, 1 + D^2 + D^4]. \quad (4.5)$$

This code has minimum free branch distance 5, minimum free distance 7, and a 16-state trellis diagram. At each time unit, the two code bits at the output
of the rate-1/2 convolutional code encoder form the two label bits for a QPSK signal point. The resultant QPSK code is a TCM code $C(2)$ with the following parameters: $\eta[C(2)] = 1.0$ bits/symbol, minimum symbol distance $\delta_H[C(2)] = 6$, minimum product distance $\Delta_p^2[C(2)] = 128$, and minimum free squared Euclidean distance $d_{\text{free}}^2 = 14.0$ And code $C(3)$ has the following parameters: $\eta[C(3)] = 1.0$ bits/symbol, minimum symbol distance $\delta_H[C(3)] = 5$, minimum product distance $\Delta_p^2[C(3)] = 128$, and minimum free squared Euclidean distance $d_{\text{free}}^2 = 12.0$. The error performances of these codes over the Rayleigh fading channel are shown in Figure 4.12. $C(2)$ and $C(3)$ achieves an impressive 17.8 and 16.8 dB real coding gain over the uncoded BPSK at the BER of $10^{-3}$, respectively. These coding gains are achieved without bandwidth expansion. And $C(2)$ and $C(3)$ are compared with the optimum free distance rate-1/2 32- and 16-state convolutional codes $C(H2)$ and $C(H3)$ [92] in Figure 4.12. The optimum distance code is a convolutional code with the largest minimum Hamming distance among codes with the same memory size. As shown in Figure 4.12, our optimum branch distance codes outperform optimum free distance codes.

Coherently-detected QPSK modulation codes over the shadowed MSAT channel

Due to the limitation on the size of interleaver, the size of the interleaver is 256 QPSK symbols for the data rate 2400 bits/sec. Since the constraint length of the code $C(2)$ is 6, the decoding depth $d_1$ of the code is chosen to be $5 \times 6 = 30$. Then the resultant interleaver is 8 by 32. The constraint length of code $C(3)$ is 5, but we choose the decoding depth to be $4 \times 4 = 16$. Reasons to choose those interleaver sizes are as follows:
(1) For code $C(2)$, when we choose $d_1 = 5 \times 5 = 25$, the number of columns is still 32 because the total size of the interleaver is 256 QPSK symbols. Therefore, either $d_1 = 25$ or $d_1 = 30$, we have the same 8 by 32 interleaver. If the interleaving depth is fixed, then a code with longer decoding depth performs better than a code with shorter decoding depth. Therefore, in the above case, we choose the decoding depth $d_1$ of 30.

(2) For code $C(3)$, when we choose $d_1 = 25$, the number of column is 32. When we choose $d_1 = 16$, the number of column can be 16. To find out the effect of the row (or interleaving depth) on the performance of code, we evaluate bit error performances of differentially detected QPSK TCM code $C(3)$ with different row which are 4 and 8, respectively. Figure 4.13 shows bit error performances of code $C(3)$ with different sizes of interleaving depth (or row). Case I, we choose $\alpha = 16, \beta = 16$, and the decoding depth 16. For case II, we choose $\alpha = 8, \beta = 32$, and the decoding depth $d_1 = 25$ which is five times of the constraint length of code $C(3)$. The bit error performance of case I is better than that of case II for high SNR because case I has larger interleaving depth than case II. Therefore, in this case, we trade the interleaving depth with the decoding depth $d_1$. And the resultant interleaver is a 16 by 16 interleaver.

The error performance of the coherently detected QPSK modulation code $C(2)$ for the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 4.14, 4.15, and 4.16, respectively. The error performance of coherently detected QPSK modulation codes $C(3)$ for the light-, average-, and heavy-shadowed MSAT channel is shown in Figures 4.17, 4.18, and 4.19, respectively. In the coherently detected QPSK case, perfect phase synchronization is assumed. As shown in these Figures, codes $C(2)$ and $C(3)$ achieve impressive real coding gains over the coher-
Figure 4.12. BER of QPSK TCM codes $C(2)$ and $C(3)$ over a frequency non-selective slow Rayleigh fading channel.

Figure 4.13. BER of the QPSK TCM code $C(3)$ over a light-shadowed MSAT channel with different interleaving depths.
ently detected uncoded BPSK system and outperform optimum free distance codes [92] with the same decoding complexity. The error performance of coherently detected QPSK modulation codes $C(2)$ and $C(3)$ for various shadowed MSAT channels are summarized in Table 4.8 and Table 4.9. Since the channel is close to Rayleigh fading, the optimum branch codes outperform codes which are designed for maximizing Hamming distance [92]. As the speed of vehicle increases, the bit error performances of codes improve for codes $C(2)$ and $C(3)$ up to BER $10^{-4}$ because these codes have minimum symbol distances 5 and 6. However, for high SNR and for $BT = 0.05$, large Doppler frequency causes an error floor as shown in Figure 4.19.

Table 4.8. Bit error performance of the coherently detected QPSK TCM code $C(2)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>Shadowing</th>
<th>$E_b/N_0$ @ $10^{-3}$ BER</th>
<th>$E_b/N_0$ @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over BPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over BPSK</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(H2)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(H2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 8.377 12.75 6.923 11.351 1.0 2.115</td>
<td>0.05 6.15 10.389 9.317 14.293 2.046 2.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.02 9.401 13.897 8.01 13.103 1.168 2.69</td>
<td>0.05 6.706 11.989 11.021 15.5 1.855 1.707</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 14.323 18.491 12.77 17.041 1.07 1.403</td>
<td>0.05 9.571 13.64 18.729 23.556 1.39 2.32</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.14. BER of the QPSK TCM code $C(2)$ over a light-shadowed MSAT channel.

Figure 4.15. BER of the QPSK TCM code $C(2)$ over an average-shadowed MSAT channel.
Figure 4.16. BER of the QPSK TCM code $C(2)$ over a heavy-shadowed MSAT channel.

Figure 4.17. BER of the QPSK TCM code $C(3)$ over a light-shadowed MSAT channel.
Figure 4.18. BER of the QPSK TCM code $C(3)$ over an average-shadowed MSAT channel.

Figure 4.19. BER of the QPSK TCM code $C(3)$ over a heavy-shadowed MSAT channel.
Table 4.9. Bit error performance of the coherently detected QPSK TCM code $C(3)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>Shadowing</th>
<th>$E_b/N_o$ @ $10^{-3}$ BER</th>
<th>$E_b/N_o$ @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over BPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(H3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 7.754</td>
<td>12.04</td>
<td>7.546 12.171</td>
<td>0.068 0.0</td>
</tr>
<tr>
<td></td>
<td>0.05 6.368</td>
<td>10.0</td>
<td>9.099 14.682</td>
<td>0.159 0.161</td>
</tr>
<tr>
<td>Average</td>
<td>0.02 8.66</td>
<td>13.09</td>
<td>8.751 13.91</td>
<td>0.09 0.14</td>
</tr>
<tr>
<td></td>
<td>0.05 6.84</td>
<td>10.73</td>
<td>10.887 16.76</td>
<td>0.207 0.18</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 12.537</td>
<td>16.55</td>
<td>14.563 18.982</td>
<td>0.43 0.382</td>
</tr>
<tr>
<td></td>
<td>0.05 9.603</td>
<td>13.619</td>
<td>18.697 23.577</td>
<td>0.797 0.92</td>
</tr>
</tbody>
</table>

Differentially detected QPSK TCM code over the shadowed MSAT channel

The bit error performance of the differentially detected QPSK modulation code $C(2)$ for the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 4.20, 4.21, and 4.22, respectively. The error performance of the differentially detected QPSK modulation code $C(3)$ for the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 4.23, 4.24, and 4.25, respectively. As shown in these Figures, codes $C(2)$ and $C(3)$ achieve impressive real coding gains over the uncoded differentially detected BPSK system and outperform the optimum free distance codes with the same decoding complexity. The number of row is 8 and the number of column is 32 for code $C(2)$, and the number of row is 16 and the number of column is 16 for code $C(3)$. Sliding-window Viterbi soft-decision decoding is used to decode $C(2)$ and $C(3)$. Window size of code $C(2)$ is 30 and that of code $C(3)$ is
16. The error performance of the differentially detected QPSK modulation codes, $C(2)$ and $C(3)$, for various shadowed MSAT channels are summarized in Table 4.10 and Table 4.11. Table 4.10 and Table 4.11 show very interesting results.

(1) For a normalized Doppler frequency of $BT = 0.02$ (i.e., the channel is the bursty error channel), the 16-state code $C(3)$ outperforms the 32-state code $C(2)$ because an interleaving depth of code $C(3)$ is 16 and that of code $C(2)$ is only 4.

(2) For a normalized Doppler frequency of $BT = 0.05$ (i.e., the channel is the random error channel), the 32-state code $C(2)$ outperforms the 16-state code $C(3)$ because the minimum symbol distance of code $C(2)$ is 6 and that of code $C(3)$ is 5. For the heavy-shadowed channel, the code $C(2)$ achieves a BER of $10^{-4}$ at SNR 25.0 dB, however the code $C(3)$ reaches the error floor at BER of $1.068 \times 10^{-4}$. Code (1) in the previous subsection also shows an error floor at BER of $6.3 \times 10^{-3}$. The minimum symbol distance of a code $C(1)$ is 3.

For reliable communications between a high speed train or an airplane and satellites, a code is required to have large symbol distance. And for reliable communications between ground transportation vehicles and satellites, a code is required to have a short decoding decision depth.

### 4.5 Two-level 8-PSK TCM Codes

#### 4.5.1 Two-level 8-PSK TCM Scheme I

In this section, we consider the construction of 2-level 8-PSK TCM codes for the Rayleigh fading channel using two component codes, a convolutional code and a
Figure 4.20. BER of the 4-DPSK TCM code $C(2)$ over a light-shadowed MSAT channel.

Figure 4.21. BER of the 4-DPSK TCM code $C(2)$ over an average-shadowed MSAT channel.
Figure 4.22. BER of the 4-DPSK TCM code $C(2)$ over a heavy-shadowed MSAT channel.

Figure 4.23. BER of the 4-DPSK TCM code $C(3)$ over a light-shadowed MSAT channel.
Figure 4.24. BER of the 4-DPSK TCM code $C(3)$ over an average-shadowed MSAT channel.

Figure 4.25. BER of the 4-DPSK TCM code $C(3)$ over a heavy-shadowed MSAT channel.
Table 4.10. Bit error performance of the differentially detected QPSK TCM code $C(2)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>Shadowing</th>
<th>$E_b/N_0$ @ $10^{-3}$ BER</th>
<th>$E_b/N_0$ @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over DBPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(H2)$</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(H2)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(H2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 13.10 18.251 2.21 5.939 1.538 2.588</td>
<td>0.05 11.159 15.01 4.309 9.675 1.675 3.331</td>
<td>0.02 14.521 19.509 2.975 7.484 1.718 3.239</td>
<td>0.05 12.226 16.76 5.663 10.724 2.05 4.466</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.02 21.729 26.524 14.471 0.000529* 2.262 3.456</td>
<td>0.05 17.647 24.989 0.00326* 0.00326* 2.853 0.000249*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 12.86 17.442 2.45 6.748 0.303 1.291</td>
<td>0.05 11.098 15.352 4.37 9.333 0.475 0.809</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.02 14.065 19.053 3.4 7.94 0.557 0.942</td>
<td>0.05 12.221 17.27 5.668 10.214 0.442 0.912</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* : bit-error-rate at the error floor

Table 4.11. Bit error performance of the differentially detected QPSK TCM code $C(3)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>Shadowing</th>
<th>$E_b/N_0$ @ $10^{-3}$ BER</th>
<th>$E_b/N_0$ @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over DBPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(H3)$</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(H3)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(H3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 12.86 17.442 2.45 6.748 0.303 1.291</td>
<td>0.05 11.098 15.352 4.37 9.333 0.475 0.809</td>
<td>0.02 14.065 19.053 3.4 7.94 0.557 0.942</td>
<td>0.05 12.221 17.27 5.668 10.214 0.442 0.912</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.02 20.704 25.893 15.496 0.000529* 1.22 1.533</td>
<td>0.05 18.89 0.0001068* 0.00326* 0.00326* 1.33 0.000142*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 21.729 26.524 14.471 0.000529* 2.262 3.456</td>
<td>0.05 17.647 24.989 0.00326* 0.00326* 2.853 0.000249*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* : bit-error-rate at the error floor

83
block code. The convolutional component code, denoted $A_1$, is a rate-1/2 code of constraint length 6 with generator matrix (the fifth code in Table 4.5)

$$G(D) = [1 + D + D^3 + D^4, D + D^2 + D^3 + D^4 + D^5].$$ (4.6)

This code has minimum free branch distance 6, minimum free distance 7, and a 32-state trellis diagram. The block component code, denoted $A_2$, is the $(32, 26, 4)$ RM code which has a 4-section 16-state trellis diagram consisting of two parallel and structurally identical 8-state sub-trellis diagrams without cross connections between them. At each time unit, the two code bits at the output of the convolutional code encoder form the first two label bits for an 8-PSK signal point, the block component code contributes the third label bit as shown in Figure 4.26. The resultant two-level 8-PSK code, denoted $C(4) = \lambda[A_1 * A_2]$, is a TCM code with the following parameters: $\eta(C(4)) = 1.8125$ bits/symbol, minimum symbol distance $\delta_H[C(4)] = 4$, minimum product distance $\Delta_p^2[C(4)] = 256$, and minimum free squared Euclidean distance $d_{free}^2 = 4.93$. Another choice of the block component code, denoted $A_3$, is the $(32, 31, 2)$ RM code which has a simple 2-state trellis. The resultant two-level 8-PSK code, denoted $C(5) = \lambda[A_1 * A_3]$, is a TCM code with the following parameters: $\eta(C(5)) = 1$ bits/symbol, minimum symbol distance $\delta_H[C(5)] = 2$, minimum product distance $\Delta_p^2[C(5)] = 16$, and minimum free squared Euclidean distance $d_{free}^2 = 4.93$. The error performances of these codes over the Rayleigh fading channel with two-stage decoding is shown in Figure 4.27. $C(4)$ achieves an impressive 15.2 dB real coding gain over the uncoded QPSK at the BER of $10^{-3}$. This coding gain is achieved at the expense of a 9.375% bandwidth expansion. $C(5)$ achieves an impressive 14.2 dB real coding gain over the uncoded QPSK at the BER of $10^{-3}$. This coding gain is achieved at the expense of a 0.0312% bandwidth expansion. The decoding complexities are reasonably simple.
Figure 4.26. Two-level 8-PSK TCM code $C = \lambda[A_1, A_2]$.

Figure 4.27. BER of 8-PSK TCM codes $C(4)$ and $C(5)$ over a Rayleigh fading channel.
Coherently detected 2-level 8-PSK TCM codes for the shadow MSAT channels

The bit error performance of the coherently detected 8-PSK modulation code $C(4)$ for the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 4.28, 4.29, and 4.30, respectively. The error performance of the coherently detected 8-PSK modulation codes $C(5)$ for the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 4.31, 4.32, and 4.33, respectively. As shown in these Figures, codes $C(4)$ and $C(5)$ achieve impressive real coding gains over the uncoded coherently detected QPSK system. Let $C(H4)$ denote a two-level TCM code with a rate-1/2 optimum free distance convolutional code of constraint length 6 as a first-level component code and the (32,26,4) RM code as a second-level component code. Let $C(H5)$ denote a two-level TCM code with a rate-1/2 optimum free distance convolutional code of the constraint length 6 as the first-level component code and a (32,31,2) RM code as the second-level component code. Code $C(H4)$ is compared with code $C(4)$ and code $C(H5)$ is compared with code $C(5)$. For the interleaver, the number of row is 4 and the number of column is 32 for both codes $C(4)$ and $C(5)$ because the decoding depth of the block code is 32. Sliding-window Viterbi soft-decision decoding is used to decode the 1st level convolutional code of TCM codes $C(4)$ and $C(5)$. The decoding depth $d_1$ of this convolutional code is 30. The error performance of the coherently detected 8-PSK modulation codes $C(4)$ and $C(5)$ for various shadowed MSAT channels are summarized in Table 4.12 and Table 4.13. Code $C(4)$ achieves more coding gain than code $C(5)$ over the uncoded QPSK modulation with bandwidth expansion. Code $C(5)$ also achieves an impressive real coding gain over the uncoded QPSK modulation with less decoding complexity than code $C(4)$. Codes $C(4)$ and $C(5)$ outperform codes $C(H4)$ and $C(H5)$, respectively.
Figure 4.28. BER of the 8-PSK TCM code $C(4)$ over a light-shadowed MSAT channel.

Figure 4.29. BER of the 8-PSK TCM code $C(4)$ over an average-shadowed MSAT channel.
Figure 4.30. BER of the 8-PSK TCM code $C(4)$ over a heavy-shadowed MSAT channel.

Figure 4.31. BER of the 8-PSK TCM code $C(5)$ over a light-shadowed MSAT channel.
Figure 4.32. BER of the 8-PSK TCM code $C(5)$ over an average-shadowed MSAT channel.

Figure 4.33. BER of the 8-PSK TCM code $C(5)$ over a heavy-shadowed MSAT channel.
Table 4.12. Bit error performance of the coherently detected 2-level 8-PSK TCM code $C(4)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$</th>
<th>$E_b/N_o$</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$10^{-3}$</td>
<td>$10^{-4}$</td>
<td>BER over QPSK</td>
<td>BER over QPSK</td>
<td>BER over $C(H4)$</td>
<td>BER over $C(H4)$</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>13.163</td>
<td>19.597</td>
<td>2.749</td>
<td>5.437</td>
<td>1.472</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>10.355</td>
<td>14.63</td>
<td>6.187</td>
<td>11.421</td>
<td>1.338</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>14.71</td>
<td>20.413</td>
<td>3.651</td>
<td>7.756</td>
<td>1.203</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>11.46</td>
<td>15.668</td>
<td>7.568</td>
<td>12.966</td>
<td>1.288</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>23.263</td>
<td>27.4</td>
<td>6.116</td>
<td>10.57</td>
<td>0.361</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>15.472</td>
<td>19.57</td>
<td>16.119</td>
<td>21.48</td>
<td>1.548</td>
</tr>
</tbody>
</table>

Table 4.13. Bit error performance of the coherently detected 2-level 8-PSK TCM code $C(5)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$</th>
<th>$E_b/N_o$</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$10^{-3}$</td>
<td>$10^{-4}$</td>
<td>BER over QPSK</td>
<td>BER over QPSK</td>
<td>BER over $C(H5)$</td>
<td>BER over $C(H5)$</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>13.54</td>
<td>19.43</td>
<td>2.372</td>
<td>5.604</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>11.09</td>
<td>16.08</td>
<td>5.452</td>
<td>9.971</td>
<td>1.1</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>14.8</td>
<td>20.764</td>
<td>3.561</td>
<td>7.405</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>12.28</td>
<td>17.80</td>
<td>6.748</td>
<td>10.834</td>
<td>1.21</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>25.873</td>
<td>30.09</td>
<td>3.506</td>
<td>8.367</td>
<td>0.157</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>20.6</td>
<td>26.116</td>
<td>10.991</td>
<td>14.934</td>
<td>0.299</td>
</tr>
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</table>
Differentially detected 2-level 8-PSK TCM codes over the shadowed MSAT channel

The error performance of the differentially detected 8-PSK modulation code $C(4)$ for the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 4.34, 4.35, and 4.36, respectively. The error performance of the differentially detected 8-PSK modulation code $C(5)$ for the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 4.37, 4.38, and 4.39, respectively. As shown in these Figures, codes $C(4)$ and $C(5)$ achieve impressive real coding gains over uncoded 4-DPSK system and outperform codes $C(H4)$ and $C(H5)$ with same decoding complexities. The error performances of 8-DPSK modulation codes $C(4)$ and $C(5)$, for various shadowed MSAT channels are summarized in Table 4.14 and Table 4.15. For the heavy shadowed MSAT channel with $BT = 0.05$, codes $C(4)$ and $C(5)$ show an error floor at lower BER than codes $C(H4)$ and $C(H5)$, respectively.

![Figure 4.34. BER of the 8-DPSK TCM code $C(4)$ over a light-shadowed MSAT channel.](image)

- Uncoded 4-DPSK (Light, $BT=0.02$)
- Uncoded 4-DPSK (Light, $BT=0.05$)
- $C(4)$ Light, $BT=0.02$
- $C(H4)$ Light, $BT=0.02$
- $C(4)$ Light, $BT=0.05$
- $C(H4)$ Light, $BT=0.05$
Figure 4.35. BER of the 8-DPSK TCM code $C(4)$ over an average-shadowed MSAT channel.

Figure 4.36. BER of the 8-DPSK TCM code $C(4)$ over a heavy-shadowed MSAT channel.
Table 4.14. Bit error performance of the differentially detected 2-level 8-PSK TCM code $C(4)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>Coding</th>
<th>$E_b/N_0$</th>
<th>$E_b/N_0$</th>
<th>Coding</th>
<th>$E_b/N_0$</th>
<th>$E_b/N_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain</td>
<td>$10^{-3}$</td>
<td>$10^{-4}$</td>
<td>Gain</td>
<td>$10^{-3}$</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>@</td>
<td>BER</td>
<td>BER</td>
<td>@</td>
<td>BER</td>
<td>BER</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>18.03</td>
<td>10^{-3}</td>
<td>1.097</td>
<td>3.44</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>16.65</td>
<td>10^{-4}</td>
<td>4.05</td>
<td>0.0001973*</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>20.226</td>
<td>$10^{-3}$</td>
<td>1.741</td>
<td>8.414</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>18.3</td>
<td>$10^{-4}$</td>
<td>6.802</td>
<td>0.0004744*</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>30.84</td>
<td>$10^{-3}$</td>
<td>0.00144</td>
<td>2.15</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>35.0</td>
<td>$10^{-4}$</td>
<td>0.000851*</td>
<td>0.0023*</td>
</tr>
</tbody>
</table>

*: bit-error-rate at the error floor

Figure 4.37. BER of the 8-DPSK TCM code $C(5)$ over a light-shadowed MSAT channel.
Figure 4.38. BER of the 8-DPSK TCM code $C(5)$ over an average-shadowed MSAT channel.

Figure 4.39. BER of the 8-DPSK TCM code $C(5)$ over a heavy-shadowed MSAT channel.
Table 4.15. Bit error performance of the differentially detected 2-level 8-PSK TCM code $C(5)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$ @ $10^{-3}$ BER</th>
<th>$E_b/N_o$ @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over 4-DPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over 4-DPSK</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(H5)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(H5)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02</td>
<td>17.85</td>
<td>24.546</td>
<td>1.277</td>
<td>4.404</td>
<td>1.45</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>16.68</td>
<td>23.83</td>
<td>4.02</td>
<td>0.001973*</td>
<td>1.565</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>19.83</td>
<td>27.173</td>
<td>1.57</td>
<td>1.777</td>
<td>1.36</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>18.657</td>
<td>28.04</td>
<td>6.445</td>
<td>0.0004744*</td>
<td>1.767</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>31.929</td>
<td>0.00018*</td>
<td>4.271</td>
<td>0.000144*</td>
<td>2.111</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.0018*</td>
<td>0.00803*</td>
<td>0.0018*</td>
<td>0.00803*</td>
<td>0.0018*</td>
</tr>
</tbody>
</table>

* : bit-error-rate at the error floor

4.5.2 Two-level 8-PSK TCM Scheme II

In this subsection, we consider the construction of 2-level, 8-PSK, TCM codes for the Rayleigh fading channel using two component codes, a convolutional code and a block code. The convolutional component code, denoted $A_1$, is a rate-1/2 code of constraint length 5 with generator matrix (the fourth code in Table 4.4)

$$G(D) = [1 + D + D^2, 1 + D^2 + D^4].$$

(4.7)

This code has minimum free branch distance 5, minimum free distance 6, and a 32-state trellis diagram. The block component code, denoted $A_2$, is the (32, 26, 4) RM code which has a 4-section 16-state trellis diagram consisting of two parallel and structurally identical 8-state sub-trellis diagrams without cross connections between them. At each time unit, the two code bits at the output of the convolutional code encoder form the first two label bits for an 8-PSK signal point, the block component
code contributes the third label bit as shown in Figure 4.26. The resultant 3-level 8-PSK code, denoted $C(6) = \lambda[A_1 \ast A_2]$, is a TCM code with the following parameters: $\eta[C(6)] = 1.8125$ bits/symbol, minimum symbol distance $\delta_H[C(6)] = 4$, minimum product distance $\Delta_p^2[C(6)] = 256$, and minimum free squared Euclidean distance $d^2_{\text{free}} = 5.758$. Another choice of the block component code, denoted $A_3$, is the $(32, 31, 2)$ RM code which has a simple 2-state trellis. The resultant 3-level 8-PSK code, denoted $C(7) = \lambda[A_1 \ast A_3]$, is a TCM code with the following parameters: $\eta[C(7)] = 1$ bits/symbol, minimum symbol distance $\delta_H[C(7)] = 2$, minimum product distance $\Delta_p^2[C(7)] = 16$, and minimum free squared Euclidean distance $d^2_{\text{free}} = 5.758$. The error performances of these codes over the Rayleigh fading channel with two-stage decoding is shown in Figure 4.40. $C(6)$ achieves an impressive $12.7$ dB real coding gain over the uncoded QPSK at the BER $10^{-3}$. This coding gain is achieved at the expense of a $9.375\%$ bandwidth expansion. $C(7)$ achieves an impressive $10.43$ dB real coding gain over the uncoded QPSK at the BER $10^{-3}$. This coding gain is achieved at the expense of a $0.0312\%$ bandwidth expansion. The decoding complexities of these codes are reasonably simple.

**Coherently detected 2-level 8-PSK TCM codes for the shadow MSAT channels**

The bit error performance of the coherently detected 8-PSK modulation code $C(6)$ over the light-, average-, and heavy-shadowed MSAT channel are shown in Figures 4.41, 4.42, and 4.43, respectively. The error performance of the coherently detected 8-PSK modulation code $C(7)$ for the light-, average-, and heavy-shadowed MSAT channel is shown in Figures 4.44, 4.45, and 4.46, respectively. As shown in these Figures, codes $C(6)$ and $C(7)$ achieve impressive real coding gains over the uncoded, coherently detected QPSK system. Also codes, $C(6)$ and $C(7)$, outperform two-level
TCM codes which use optimum free distance codes as first-level component code and use the same block codes as second-level component code. Let $C(H6)$ denote the 2-level TCM code with a 16-state, rate-1/2, optimum free distance, convolutional code [92] as the 1st level component code and the $(32,26,4)$ RM code as the second level component codes. Let $C(H7)$ denote the 2-level TCM code with a 16-state, rate-1/2, optimum free distance, convolutional code [92] as the 1st level component code and the $(32,31,2)$ even-parity check code as the second level component codes. Let the number of row be 4 and the number of column be 32 for codes $C(6)$ and $C(7)$ because the decoding depth of the second-level component code is 32. Sliding-window Viterbi soft-decision decoding is used to decode the 1st level component convolutional code of $C(6)$ and $C(7)$. The Decoding depth is 20 for both $C(6)$ and $C(7)$. This value is 4 times the constraint length of each convolutional code. The error performance of the coherently detected 8-PSK modulation codes, $C(6)$ and
$C(7)$, for various shadowed MSAT channels are summarized in Table 4.16 and Table 4.17. Both first-level convolutional codes in codes $C(6)$ and $C(H6)$ have the same minimum symbol distance 5. Therefore, they show almost the same performance, but a code with the optimum branch convolutional code always shows a better performance than a code with the optimum free distance convolutional code.

Table 4.16. Bit error performance of the coherently detected 2-level 8-PSK TCM code $C(6)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>Shadowing</th>
<th>$E_b/N_o$ @ 10^{-3} BER</th>
<th>$E_b/N_o$ @ 10^{-4} BER</th>
<th>Coding Gain @ 10^{-3} BER over QPSK</th>
<th>Coding Gain @ 10^{-4} BER over QPSK</th>
<th>Coding Gain @ 10^{-3} BER over $C(H3)$</th>
<th>Coding Gain @ 10^{-4} BER over $C(H3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 14.5 21.27 1.412 3.764 0.0 1.29</td>
<td>0.05 11.59 15.95 4.952 10.1 0.1437 0.68</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.02 16.04 20.88 2.321 7.39 0.0 0.98</td>
<td>0.05 12.67 17.22 6.358 11.414 0.199 0.539</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 24.90 28.854 4.479 9.603 1.11 0.757</td>
<td>0.05 16.768 21.554 14.823 19.496 0.606 0.646</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.41. BER of the 8-PSK TCM code $C(6)$ over a light-shadowed MSAT channel.

Figure 4.42. BER of the 8-PSK TCM code $C(6)$ over an average-shadowed MSAT channel.
Figure 4.43. BER of the 8-PSK TCM code $C(6)$ over a heavy-shadowed MSAT channel.

Figure 4.44. BER of the 8-PSK TCM code $C(7)$ over a light-shadowed MSAT channel.
Figure 4.45. BER of the 8-PSK TCM code $C(7)$ over an average-shadowed MSAT channel.

Figure 4.46. BER of the 8-PSK TCM code $C(7)$ over a heavy-shadowed MSAT channel.
Table 4.17. Bit error performance of the coherently detected 2-level 8-PSK TCM code $C(7)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$ @ $10^{-3}$ BER</th>
<th>$E_b/N_o$ @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(H7)$</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(H7)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(H7)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 14.45</td>
<td>20.54</td>
<td>1.462</td>
<td>4.494</td>
<td>0.0</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>0.05 12.09</td>
<td>17.13</td>
<td>4.452</td>
<td>8.921</td>
<td>0.0</td>
<td>0.18</td>
</tr>
<tr>
<td>Average</td>
<td>0.02 15.76</td>
<td>22.26</td>
<td>2.6</td>
<td>5.909</td>
<td>0.25</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>0.05 13.28</td>
<td>19.09</td>
<td>5.748</td>
<td>9.544</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 26.61</td>
<td>30.74</td>
<td>2.769</td>
<td>7.717</td>
<td>0.08</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>0.05 20.9</td>
<td>26.03</td>
<td>10.691</td>
<td>15.02</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Differentially detected 2-level 8-PSK TCM codes over the shadowed MSAT channel

The error performance of the differentially detected 8-PSK modulation code $C(6)$ for the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 4.47, 4.48, and 4.49, respectively. The error performance of the differentially detected 8-PSK modulation code $C(7)$ for the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 4.50, 4.51, and 4.52, respectively. As shown in these Figures, codes $C(6)$ and $C(7)$ achieve impressive real coding gains over uncoded differentially detected QPSK system and outperform codes, $C(H6)$ and $C(H7)$, with the same decoding complexity. The number of row is 4 and the number of column is 32 for codes $C(6)$ and $C(7)$. Sliding-window Viterbi soft-decision decoding is used to decode $C(6)$ and $C(7)$. Window size for both code $C(6)$ and code $C(7)$ is 20. This value is 5 times the constraint length of each code. Error performances of dif-
ferentially detected 8-PSK modulation codes, $C(6)$ and $C(7)$, for various shadowed MSAT channels are summarized in Table 4.18 and Table 4.19. When the speed of an object is fast and the multipath fading is more severe, a code with the optimum branch distance convolutional code as the first-level component code outperforms a code with optimum free distance convolutional code as the first-level component code.

Table 4.18. Bit error performance of the differentially detected 2-level 8-PSK TCM code $C(6)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_0$ @ $10^{-3}$ BER</th>
<th>$E_b/N_0$ @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over 4-DPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over 4-DPSK</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(H6)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(H6)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02</td>
<td>18.699</td>
<td>26.699</td>
<td>0.428</td>
<td>2.251</td>
<td>0.425</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>16.9</td>
<td>24.969</td>
<td>3.8</td>
<td>0.0001973*</td>
<td>1.23</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>20.71</td>
<td>26.578</td>
<td>0.69</td>
<td>7.876</td>
<td>0.439</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>19.88</td>
<td>26.895</td>
<td>5.105</td>
<td>0.0004744*</td>
<td>0.12</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>31.813</td>
<td>38.88</td>
<td>0.00144*</td>
<td>1.72</td>
<td>0.0001389*</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>44.69</td>
<td>0.000854*</td>
<td>0.00803*</td>
<td>0.00188*</td>
<td></td>
</tr>
</tbody>
</table>

*: bit-error-rate at the error floor
Figure 4.47. BER of the 8-DPSK TCM code $C(6)$ over a light-shadowed MSAT channel.

Figure 4.48. BER of the 8-DPSK TCM code $C(6)$ over an average-shadowed MSAT channel.
Figure 4.49. BER of the 8-DPSK TCM code $C(6)$ over a heavy-shadowed MSAT channel.

Figure 4.50. BER of the 8-DPSK TCM code $C(7)$ over a light-shadowed MSAT channel.
Figure 4.51. BER of the 8-DPSK TCM code $C(7)$ over an average-shadowed MSAT channel.

Figure 4.52. BER of the 8-DPSK TCM code $C(7)$ over a heavy-shadowed MSAT channel.
Table 4.19. Bit error performance of the differentially detected 2-level 8-PSK TCM code $C(7)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_0$</th>
<th>$E_b/N_0$</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$10^{-3}$</td>
<td>$10^{-4}$</td>
<td>BER over 4-DPSK</td>
<td>BER over 4-DPSK</td>
<td>BER over $C(H7)$</td>
<td>BER over $C(H7)$</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>18.846</td>
<td>26.51</td>
<td>0.281</td>
<td>2.44</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>18.323</td>
<td>25.963</td>
<td>2.377</td>
<td>0.0001973*</td>
<td>0.139</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>20.859</td>
<td>28.549</td>
<td>0.541</td>
<td>5.905</td>
<td>0.211</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>20.183</td>
<td>30.727</td>
<td>4.919</td>
<td>0.0004744*</td>
<td>0.679</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>33.713</td>
<td>0.0001329*</td>
<td>0.00144*</td>
<td>0.9</td>
<td>0.0001794*</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.0018*</td>
<td>0.00863*</td>
<td>0.0028*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* : bit-error-rate at the error floor

4.5.3 2-level 16-PSK TCM Codes

In this example, we construct a two-level 16-PSK TCM code. The first component code $A_1$ is the first code given in Table 2 which is a rate-2/3 convolutional code with minimum free branch distance 3 and a 16-state trellis. The generator matrix for this code is

$$G(D) = \begin{pmatrix} D^2 & 1 & D \\ 1 & D + D^2 & D \end{pmatrix}$$

The second component code $A_2$ is the (32, 31, 2) RM code. At each time unit, the first three code bits at the output of the convolutional code encoder form the first three label bits for a 16-PSK signal point and the block component code contributes the fourth label bit. The resultant two-level 16-PSK TCM code, denoted, $C(8) = \lambda [A_1 * A_2]$, has spectral efficiency $\eta[C(8)] = 2.96875$, minimum symbol distance $\delta_H[C(8)] = 2$ and minimum product distance $\Delta_p^2[C(8)] = 16$.  

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The bit error performance of $C(8)$ over the Rayleigh fading channel with two-stage decoding is shown in Figure 4.53. The $(32, 31, 2)$ code has a very simple two state trellis. The entire decoding complexity is dominated by the convolutional component code. Recently, Du, Vucetic and Zhang have found some new 16-PSK trellis codes for the Rayleigh fading channel by computer search [61]. They showed that their 16-PSK trellis codes outperform all the other known 16-PSK trellis codes over the Rayleigh fading channel [4, 46, 55]. For comparison, we consider one of their codes, that is a 16-state code $C(V)$. The minimum symbol distance of $C(V)$ is 2, the minimum product distance is 2.47, and the minimum squared Euclidean distance is 0.89 respectively. The bit error performance of the code $C(V)$ is also shown in Figure 4.53. We see that $C(8)$ outperforms the 16-state Du-Vucetic-Zhang’s code $C(V)$. At the BER of $10^{-3}$, $C(8)$ has a 1.5 dB coding gain over the Du-Vucetic-Zhang’s 32-state code and a 3 dB coding gain over the Du-Vucetic-Zhang’s 16-state code. The
decoding complexity of \( C(8) \) is about the same as that of the Du-Vucetic-Zhang’s 16-state code \( C(V) \).

**Coherently detected 2-level 16-PSK TCM codes over the shadowed MSAT channel**

The error performance of the coherently detected 16-PSK modulation code \( C(8) \) for the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 4.54, 4.55, and 4.56, respectively. As shown in these Figures, code \( C(8) \) achieves an impressive real coding gain over the uncoded QPSK system and outperforms \( C(V) \) with less decoding complexity. The number of row is 4 and the number of column is 32 for code \( C(8) \). Sliding-window Viterbi soft-decision decoding is used to decode \( C(8) \). The window size of code \( C(8) \) is 20. The error performance of the coherently detected 16-PSK modulation code \( C(8) \) for various shadowed MSAT channels is summarized in Table 4.20. As shown in Table 4.20, code \( C(8) \) outperforms code \( C(V) \) with almost the same spectral efficiency. And the decoding complexity of code \( C(8) \) is almost half that of code \( C(V) \) because a trellis of code \( C(V) \) has twice as many branches as a trellis of the first-level component code of code \( C(8) \).

**Differentially detected 2-level 16-PSK TCM codes over the shadowed MSAT channel**

The error performance of the differentially detected 16-PSK modulation code \( C(8) \) for the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 4.57, 4.58, and 4.59, respectively. As shown in these Figures, the code \( C(8) \) achieves an impressive real coding gain over the uncoded, differentially detected QPSK system and outperforms \( C(V) \) with less decoding complexity. The number of row is 4 and the number of column is 32 for the code \( C(8) \). Sliding-window Viterbi soft-
decision decoding is used to decode $C(8)$. The window size of the code $C(8)$ is 20. The error performance of the differentially detected 16-PSK modulation code $C(8)$ over various shadowed MSAT channels is summarized in Table 4.21. Other multi-level TCM codes can be constructed in the same manner. Except heavy-shadowed channel with $BT = 0.05$, the code $C(8)$ outperforms the code $C(V)$ or reaches an error floor at much lower BER than the code $C(V)$.

Table 4.20. Bit error performance of the coherently detected 2-level 16-PSK TCM code $C(8)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$BT$</th>
<th>$E_b/N_o$ @ $10^{-3}$ BER</th>
<th>$E_b/N_o$ @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over 8-PSK</th>
<th>Coding Gain @ $10^{-4}$ BER over 8-PSK</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(V)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(V)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02</td>
<td>17.31</td>
<td>23.967</td>
<td>2.332</td>
<td>4.333</td>
<td>0.368</td>
<td>1.677</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>15.50</td>
<td>20.388</td>
<td>4.414</td>
<td>8.16</td>
<td>1.708</td>
<td>3.783</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>19.16</td>
<td>22.0</td>
<td>2.78</td>
<td>6.64</td>
<td>0.689</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>16.71</td>
<td>22.4</td>
<td>5.28</td>
<td>9.2319</td>
<td>2.18</td>
<td>3.158</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>30.03</td>
<td>33.758</td>
<td>2.44</td>
<td>7.5058</td>
<td>2.98</td>
<td>5.154</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>23.39</td>
<td>27.918</td>
<td>10.94</td>
<td>15.65</td>
<td>3.44</td>
<td>5.382</td>
</tr>
</tbody>
</table>

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Figure 4.54. BER of the 16-PSK TCM code $C(8)$ over a light-shadowed MSAT channel.

Figure 4.55. BER of the 16-PSK TCM code $C(8)$ over an average-shadowed MSAT channel.
Figure 4.56. BER of the 16-PSK TCM code $C(8)$ over a heavy-shadowed MSAT channel.

Figure 4.57. BER of a 16-DPSK TCM codes $C(8)$ over a light-shadowed MSAT channel.
Figure 4.58. BER of a 16-DPSK TCM codes $C(8)$ over an average-shadowed MSAT channel.

Figure 4.59. BER of a 16-DPSK TCM codes $C(8)$ over a heavy-shadowed MSAT channel.
Table 4.21. Bit error performance of differentially detected 2-level 16-PSK TCM code $C(8)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_0$ @ $10^{-3}$ BER</th>
<th>$E_b/N_0$ @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over 8-DPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(V)$</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(V)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(V)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 22.196</td>
<td>33.173</td>
<td>1.17</td>
<td>3.687</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.05 24.8</td>
<td>0.000415</td>
<td>3.51</td>
<td>0.000659*</td>
<td>2.956</td>
<td>0.000409*</td>
</tr>
<tr>
<td>Average</td>
<td>0.02 24.355</td>
<td>42.08</td>
<td>0.56</td>
<td>0.000184*</td>
<td>1.58</td>
<td>0.00018*</td>
</tr>
<tr>
<td></td>
<td>0.05 0.001</td>
<td>0.00151*</td>
<td></td>
<td>0.001359*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 0.00398*</td>
<td>0.00398*</td>
<td></td>
<td>0.00453*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05 0.05006*</td>
<td>0.03127*</td>
<td></td>
<td>0.023*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*: bit-error-rate at the error floor

4.6 Summary

In this chapter, various multilevel TCM codes have been constructed by using the multilevel coding method for voice transmission over various shadowed MSAT channels. In constructing multilevel TCM codes, convolutional codes with good free branch distance are used as the component codes or both convolutional and block codes are used as the component codes.

Due to the multipath fading and the Doppler frequency shift, the MSAT channel is a bursty error channel. Interleaving must be used in a bursty error channel. However the shadowed MSAT channel has a constraint on the total allowable delay time for voice transmission, this constraint limits the size of the interleaver and the decoding depth of the Viterbi decoder for TCM codes. Therefore, we must consider the size of an interleaver and the decoding depth of the decoder to design a good
Table 4.22. Multilevel TCM codes

<table>
<thead>
<tr>
<th>Code</th>
<th>Code Description</th>
<th>Minimum Symbol Distance</th>
<th>Minimum Product Distance</th>
<th>Minimum Squared Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>QPSK TCM</td>
<td>C(2)</td>
<td>32-state</td>
<td>6</td>
<td>128</td>
</tr>
<tr>
<td></td>
<td>C(3)</td>
<td>16-state</td>
<td>5</td>
<td>128</td>
</tr>
<tr>
<td>8-PSK TCM</td>
<td>C(1)</td>
<td>16-state</td>
<td>3</td>
<td>4.68</td>
</tr>
<tr>
<td></td>
<td>C(4)</td>
<td>A1: 32-state</td>
<td>6</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A2: (32,26,4)</td>
<td>4</td>
<td>256</td>
</tr>
<tr>
<td></td>
<td>C(5)</td>
<td>A1: 32-state</td>
<td>6</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A2: (32,31,2)</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>C(6)</td>
<td>A1: 16-state</td>
<td>5</td>
<td>0.8047</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A2: (32,26,4)</td>
<td>4</td>
<td>256</td>
</tr>
<tr>
<td></td>
<td>C(7)</td>
<td>A1: 16-state</td>
<td>5</td>
<td>0.8047</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A2: (32,31,2)</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>16-PSK TCM</td>
<td>C(8)</td>
<td>A1: 16-state</td>
<td>3</td>
<td>0.178</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A2: (32,31,2)</td>
<td>2</td>
<td>16</td>
</tr>
</tbody>
</table>

Proposed multilevel TCM codes are listed in the Table 4.22.

**QPSK TCM codes**

The bit error performance of proposed multilevel TCM codes for various shadowed MSAT channels have been evaluated for two different vehicle speeds, 37.152 and 92.88 mile/hour and summarized in Table 4.23. The normalized fading bandwidth $BT$s for vehicle speed of 37.152 and 92.88 mile/hour are 0.02 and 0.05, respectively. The decoding depths of the codes $C(2)$ and $C(3)$ are 20 and 16, respectively. The size of an interleaver is 256 QPSK symbols. Since the number of column of interleaver...
Table 4.23. Error performances of QPSK TCM codes

<table>
<thead>
<tr>
<th>Detection Method</th>
<th>Shadowing</th>
<th>BT</th>
<th>C(2)</th>
<th>C(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(E_b/N_o) @ (10^{-3})</td>
<td>(E_b/N_o) @ (10^{-4})</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>BER</td>
<td>BER</td>
</tr>
<tr>
<td>Coherent Detection</td>
<td>Light</td>
<td>0.02</td>
<td>8.377</td>
<td>12.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.05</td>
<td>6.15</td>
<td>10.389</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>0.02</td>
<td>9.401</td>
<td>13.897</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.05</td>
<td>6.706</td>
<td>11.989</td>
</tr>
<tr>
<td></td>
<td>Heavy</td>
<td>0.02</td>
<td>14.323</td>
<td>18.491</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.05</td>
<td>9.571</td>
<td>13.64</td>
</tr>
<tr>
<td>Differential Detection</td>
<td>Light</td>
<td>0.02</td>
<td>13.10</td>
<td>18.251</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.05</td>
<td>11.159</td>
<td>15.01</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>0.02</td>
<td>14.521</td>
<td>19.509</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.05</td>
<td>12.226</td>
<td>16.76</td>
</tr>
<tr>
<td></td>
<td>Heavy</td>
<td>0.02</td>
<td>21.729</td>
<td>26.524</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.05</td>
<td>17.647</td>
<td>24.989</td>
</tr>
</tbody>
</table>

* : bit-error-rate at the error floor

must be larger than the decoding depth of a TCM code, the interleaving depth of the code \(C(2)\) is 8 and that of the code \(C(3)\) is 16. Therefore, the code \(C(3)\) has an interleaving depth twice that of the code \(C(2)\). Even though the 32-state QPSK TCM code \(C(2)\) has a better distance profiles than the 16-state QPSK TCM code \(C(3)\), QPSK TCM code \(C(3)\) outperforms the QPSK TCM code \(C(2)\) when a channel is the bursty error channel because \(C(3)\) has twice the interleaving depth of \(C(2)\). For the coherently detected case, a 16-state code \(C(3)\) outperforms the 32-state code \(C(2)\) for \(BT = 0.02\). However, for \(BT = 0.05\), the 4-DPSK TCM code \(C(2)\) outperforms the 4-DPSK TCM code \(C(3)\) because the channel becomes a random error channel.
Table 4.24. Error performances of coherently detected 8-PSK TCM codes

<table>
<thead>
<tr>
<th>Shadowing</th>
<th>BT</th>
<th>C(1)</th>
<th>C(5)</th>
<th>C(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>@ $E_b/N_o$ @ BER</td>
<td>@ $E_b/N_o$ @ BER</td>
<td>@ $E_b/N_o$ @ BER</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>14.39</td>
<td>20.44</td>
<td>13.54</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>11.88</td>
<td>17.04</td>
<td>11.09</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>15.8</td>
<td>22.01</td>
<td>14.8</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>13.22</td>
<td>18.724</td>
<td>12.28</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>26.35</td>
<td>30.465</td>
<td>25.873</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>19.248</td>
<td>24.244</td>
<td>20.6</td>
</tr>
</tbody>
</table>

8-PSK TCM codes

The bit error performance of the coherently detected multilevel 8-PSK TCM codes for various shadowed MSAT channels is summarized in Table 4.24. The bit error performance of differentially detected multilevel 8-PSK TCM codes for various shadowed MSAT channels is summarized in Table 4.25. The size of an interleaver is 128 8-PSK symbols. All three codes C(1), C(5), and C(7) use the same interleaver where the interleaving depth is 4. In the coherently detected case, code C(1) performs slightly better than code C(7) and code C(5) performs slightly better than code C(1), except for the heavy shadowed channel with BT=0.05. In the differentially detected case, code C(7) outperforms code C(1). Even though the minimum symbol distance of code C(1) is larger than that of code C(7), the product distance of code C(7) is much larger than that of code C(1).

As shown in this chapter, the error performance of a TCM code for voice transmission for various shadowed MSAT channels heavily depends on the minimum symbol distance, the minimum product distance, and the interleaving depth.

The bit error performance of the proposed multilevel TCM codes for various
Table 4.25. Error performances of differentially detected 8-PSK TCM codes

<table>
<thead>
<tr>
<th>Shadowing</th>
<th>BT</th>
<th>$E_b/N_o$</th>
<th>$E_b/N_o$</th>
<th>$E_b/N_o$</th>
<th>$E_b/N_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$10^{-3}$</td>
<td>$10^{-4}$</td>
<td>$10^{-3}$</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>18.663</td>
<td>27.13</td>
<td>17.85</td>
<td>24.546</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>18.831</td>
<td>34.63</td>
<td>16.68</td>
<td>23.83</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>20.614</td>
<td>29.69</td>
<td>19.83</td>
<td>27.173</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>21.65</td>
<td>0.00018</td>
<td>18.657</td>
<td>28.04</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>36.74</td>
<td>0.000529*</td>
<td>31.929</td>
<td>0.00018*</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.0063*</td>
<td>0.0018*</td>
<td>0.0018*</td>
<td></td>
</tr>
</tbody>
</table>

* : bit-error-rate at the error floor

Shadowed MSAT channels have been compared with existing codes or codes with the same decoding complexity. All codes constructed in this chapter outperform existing codes with the same decoding complexity.
Chapter 5

Multilevel Block Coded Modulation

5.1 Introduction

Even though block coded modulation (BCM) is generally less power efficient than trellis coded modulation (TCM) with the same decoding complexity for the AWGN channels, BCM has the potential to compete with TCM in mobile satellite channel because of its shorter decoding depth and hence more effective interleaving. Even though computer searches are mandatory for finding good TCM codes for the AWGN and the Rayleigh fading channels, BCM codes can be constructed systematically. BCM codes can achieve better spectral efficiencies than TCM codes with the same, or less, decoding complexity. Furthermore, the multilevel structure of BCM codes allows multi-stage soft-decision decoding which reduces decoding complexity and enables us to investigate the behavior of BCM codes in depth. As shown in the previous chapter 4, computer searches are mandatory for finding good TCM codes for specific communication channels. However, BCM codes can be constructed algebraically based on design rules to provide the expected performance. In the code construction, the component codes are chosen to have simple trellis structure so that the Viterbi decoding algorithm can be used at each decoding stage. In the following,
several 3-level 8-PSK (or 8-DPSK) block modulation codes for the Rayleigh fading and various shadowed MSAT channels are constructed using the distance theorem in chapter 3 as the general guideline. The bit error performance of these codes based on multi-stage soft-decision decoding [40] are given.

Even though there has been a great deal of research effort on TCM codes over the Rayleigh and shadowed MSAT channels, there is little research on BCM codes for those channels. In this chapter, many BCM codes are constructed and compared with TCM codes. Results show that BCM codes achieve very impressive real coding gains over the uncoded QPSK modulation. Also, BCM codes achieve better coding gain than TCM codes for the shadowed MSAT channel where the size of the interleaver is finite. Furthermore, some BCM codes achieve impressive coding gains over TCM codes with higher spectral efficiency.

5.2 Design Criteria for Multilevel BCM Codes

To design good multilevel BCM codes over the AWGN channel, we must maximize the minimum squared Euclidean distance and minimize the path multiplicity.

For the fading channel, we must maximize the minimum symbol distance and minimum product distance for high SNR and we must maximize the minimum squared Euclidean distance for low SNR. In constructing codes to achieve the required error performance, the overall decoding complexity, spectral efficiency, phase symmetry, and other factors must also be taken into consideration.

5.3 Multi-Stage Soft-Decision Decoding

Let \((n, k, \delta)\) denote a linear block code of length \(n\), dimension \(k\) and minimum Hamming distance \(\delta\). From section 3.3, let \(C = \lambda[A_1 * A_2 * A_3]\) be a 3-level 8-PSK
block modulation code where the $i$-th component code $A_i$ is a linear block code $(n, k, \delta_i)$ for $1 \leq i \leq 3$. Suppose a modulation code $C$ is decoded with the multi-stage decoding method. Let $P_C$ denote the total bit-error-rate of a modulation code $C$ and $P_{A_i}$ denote the BER of a $i$-th level decoding for $1 \leq i \leq 3$. Then

$$P_C = P_{A_1} + P_{A_2} + P_{A_3}.$$  

By knowing the BER of each level decoding, we can understand more about effects of distance parameters, (i.e., the minimum symbol distance (MSD), the minimum product distance (MPD), and the minimum squared Euclidean distance (MSED)), for the error performance of a modulation code $C$ for various communication channels. Therefore, Multi-stage decoding not only reduces the decoding complexity but also enables us to understand the behavior of multilevel modulation codes for various communication systems.

### 5.4 Decoding Complexity of Reed-Muller Codes

In block modulation code construction, the component codes are chosen to have a simple trellis structure so that the Viterbi decoding algorithm can be used at each decoding stage. It is well known [19] that RM codes have a very simple trellis structure. Therefore, RM codes can be decoded with the soft-decision Viterbi decoding algorithm. Let $RM(r, n)$ be a $r$-th order RM code of length $2^n$. Table 5.1 shows complexities of 4-section trellises of RM codes up to length 32. As shown in Table 5.1, all $RM(r, n)$ codes have structurally identical parallel subtrellises. Therefore, all parallel subtrellises can be decoded at the same time with identical decoders. In this dissertation, the decoding complexity of the $RM(r, n)$ code is considered as the decoding complexity of a subtrellis. The decoding complexities of 4-section trellises of $RM(r, n)$ codes are derived in Appendix 1.
Table 5.1. Complexity of 4-section trellises of $RM(r,n)$ codes

<table>
<thead>
<tr>
<th>Code</th>
<th>No. of Parallel Sub-trellises</th>
<th>No. of States in each Sub-trellis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RM(1,3)$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$RM(1,4)$</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>$RM(2,4)$</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>$RM(1,5)$</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>$RM(2,5)$</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>$RM(3,5)$</td>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

5.5 Multilevel BCM Schemes

In the following examples, we construct various BCM codes and simulate their performance for the Rayleigh and various shadowed MSAT channels. For the Rayleigh fading channel, coherent 8-PSK is used for detection. Infinite interleaving and perfect phase tracking are assumed. For the shadowed MSAT channel, differentially detected 8-DPSK as well as coherently detected 8-PSK are used as modulation techniques. In choosing the component codes to achieve the required error performance, the overall decoding complexity, spectral efficiency, phase symmetry, and other factors must also be taken into consideration.

For each shadowed MSAT channel, bit-error performances of BCM codes are evaluated for two different vehicle speeds, 37.152 and 92.88 mile/hour. Normalized fading bandwidth $BT$ for vehicle speeds of 37.152 and 92.88 mile/hour are 0.02 and 0.05, respectively. Since the Ungerboeck rate-2/3 16-state TCM code $C(U)$ has been known as an optimum code for both the AWGN and Rayleigh fading channels. BER performances of BCM codes is compared with those of the code $C(U)$. Code $C(U)$ has the spectral efficiency 2.0 bits/symbol, the minimum symbol distance 3,
and the minimum product distance 4.68. For the code \( C(U) \), the decision depth of the Viterbi decoder is 20. Since the size of the interleaver is 128 8-PSK symbols and the number of columns must be larger than the decoding depth, the number of column is 32 and the interleaving depth is 4. For the light shadowed MSAT channel with \( BT = 0.05 \), the bit error performance of the 8-DPSK code \( C(U) \) with different the decoding depths are shown in Figure 5.1.

### 5.5.1 Sixteen Dimensional 8-PSK Block Modulation Codes

#### Example 1

One possible choice of the component codes is: (1) \( A_1 = (8, 4, 4) \), the 2nd-order RM code of length 8; (2) \( A_2 = A_3 = (8, 7, 2) \), the even-parity check code of length 8. With this choice, the resultant 3-level 8-PSK code of length 8, \( C(9) = \lambda[A_1 * A_2 * A_3] \), has the following parameters: \( \eta[C] = 18/8 = 2.25 \) bits/symbol, \( d_E[C] = 2.344, \delta_H[C] = \)
2, and $\Delta_p^2[C] = 4$. This code has a higher spectral efficiency than the 2 bits/symbol of the uncoded QPSK. This modulation code is suitable for use as the inner code in the multilevel concatenated modulation schemes because it has high spectral efficiency and simple decoding complexity. Decoding complexity of $A_1$ is a 4-state trellis and that of $A_2$ and $A_3$ is a simple 2-state trellis. The BER performance of the 8-PSK modulation code $C(9)$ over a frequency non-selective slow Rayleigh fading channel is shown in Figure 5.2. The BER of one-stage optimum decoding is also shown in Figure 5.2. As shown in Figure 5.2, one-stage decoding is slightly better than three-stage decoding. Three-stage decoding achieves 9.0 dB real coding gain over the uncoded QPSK modulation while one-stage decoding achieves a 9.2 dB real coding gain. The decoding complexity of one-stage decoding is a 16-state trellis. The BER of each level decoding in 3-level decoding is also shown in Figure 5.2. Since the first level component code has the largest MSD, it achieves best BER at high SNR. Even though $A_2$ and $A_3$ have the same MSD, for the same SNR, $A_3$ achieves a lower BER than $A_2$ because $A_3$ has better minimum product distance than $A_2$.

Coherently detected 8-PSK modulation code over shadowed MSAT channels

The error performance of the coherently detected 8-PSK modulation code $C(9)$ for the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 5.3, 5.4, and 5.5 respectively. Perfect phase synchronization is assumed. The number of row is 32 and the number of column is 8 for an interleaver. The error performance of the coherently detected 8-PSK modulation code $C(9)$ for various shadowed MSAT channels is summarized in Table 5.2. The coherently detected modulation code $C(9)$ achieves an impressive real coding gain over the uncoded QPSK modulation.
Figure 5.2. BER of the 8-PSK modulation code $C(9)$ over a frequency non-selective slow Rayleigh fading channel.

Comparison with the Ungerboeck's code $C(U)$ are summarized below:

(1) For the light-shadowed MSAT channel, $C(9)$ achieves more coding gain over $C(U)$ at a BER for $10^{-4}$ than at a BER of $10^{-3}$.

(2) For the average-shadowed MSAT channel, $C(9)$ achieves less coding gain over $C(U)$ at a BER for $10^{-4}$ than at BER of $10^{-3}$.

(3) For the heavy-shadowed MSAT channel, $C(9)$ achieves coding gain over $C(U)$ at a BER of $10^{-3}$. However, $C(U)$ achieves more coding gain over $C(9)$ at a BER of $10^{-4}$ because $C(U)$ has large minimum symbol distance than $C(9)$.

For light and average shadowed MSAT channels, the BCM code $C(9)$ achieves better coding gain than the TCM code $C(U)$ because $C(9)$ has better interleaving depth. However, in the heavy shadowed MSAT channel, $C(U)$ outperforms $C(9)$ at a BER of $10^{-4}$ because the minimum symbol and product distances are more important.
than the minimum squared Euclidean distance and the interleaving depth. However, $C(9)$ has much better spectral efficiency than $C(U)$. 

Table 5.2. Bit error performance of the coherently detected 8-PSK BCM code $C(9)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$ @ $10^{-3}$ BER</th>
<th>$E_b/N_o$ @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(U)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(U)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02</td>
<td>12.80</td>
<td>17.22</td>
<td>3.112</td>
<td>5.467</td>
<td>1.579</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>11.075</td>
<td>13.911</td>
<td>7.814</td>
<td>12.14</td>
<td>1.336</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>14.122</td>
<td>18.76</td>
<td>4.239</td>
<td>6.998</td>
<td>1.598</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>12.03</td>
<td>15.89</td>
<td>9.409</td>
<td>12.744</td>
<td>1.59</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>24.89</td>
<td>29.477</td>
<td>4.489</td>
<td>10.881</td>
<td>2.81</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>20.71</td>
<td>26.188</td>
<td>8.98</td>
<td>20.34</td>
<td>-0.262</td>
</tr>
</tbody>
</table>
Figure 5.3. BER of the 8-PSK modulation code $C(9)$ over a light shadowed MSAT channel.

Figure 5.4. BER of the 8-PSK modulation code $C(9)$ over an average shadowed MSAT channel.
Figure 5.5. BER of the 8-PSK modulation code $C(9)$ over a heavy shadowed MSAT channel.

Differentially detected 8-PSK modulation code

The error performance of the differentially detected 8-PSK modulation code $C(9)$ over shadowed MSAT channels is shown in Figures 5.6, 5.7, and 5.8. The number of row is 16 and the number of column is 8 for an interleaver. The error performance of the differentially detected 8-PSK modulation code $C(9)$ for various shadowed MSAT channels is summarized in Table 5.3. The code $C(9)$ compares favorably with the Ungerboeck code $C(U)$. The minimum symbol distance of $C(U)$ is 3 and that of $C(9)$ is 2. However, the decoding depth of $C(9)$ is 8 and that of $C(U)$ is 20. Therefore, the number of row is 4 and the number of column is 32 for the code $C(U)$. And the size of row is 16 and the size of column is 8 for an interleaver. Since $C(9)$ has larger interleaving depth than $C(U)$, $C(9)$ outperforms $C(U)$ with less decoding complexity and a higher spectral efficiency for the case of differentially detected
8-PSK modulation. Even though $C(9)$ has worse distance profile than $C(U)$, $C(9)$ outperforms $C(U)$ because $C(9)$ has larger interleaving depth than $C(U)$. That is also why $C(9)$ reaches an error floor at lower BER than $C(U)$ when $BT = 0.05$.

Table 5.3. Bit error performance of the differentially detected 8-PSK BCM code $C(9)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$ @ $10^{-3}$ BER</th>
<th>$E_b/N_o$ @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over 4-DPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over 4-DPSK</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(U)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(U)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 17.52</td>
<td>23.52</td>
<td>1.607</td>
<td>4.18</td>
<td>0.648</td>
<td>1.731</td>
</tr>
<tr>
<td></td>
<td>0.05 16.52</td>
<td>23.12</td>
<td>5.43</td>
<td>0.0001973</td>
<td>3.03</td>
<td>11.51</td>
</tr>
<tr>
<td>Average</td>
<td>0.02 19.07</td>
<td>25.52</td>
<td>2.33</td>
<td>6.742</td>
<td>1.229</td>
<td>3.545</td>
</tr>
<tr>
<td></td>
<td>0.05 18.36</td>
<td>28.89</td>
<td>5.564</td>
<td>0.0004744*</td>
<td>0.8</td>
<td>0.00018*</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 33.254</td>
<td>0.000334*</td>
<td>0.00144*</td>
<td>5.106</td>
<td>0.0063*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05 0.004744*</td>
<td>0.00803*</td>
<td></td>
<td></td>
<td>0.00477*</td>
<td></td>
</tr>
</tbody>
</table>

*: bit-error-rate at the error floor
Figure 5.6. BER of the 8-DPSK modulation code $C(9)$ over a light shadowed MSAT channel.

Figure 5.7. BER of the 8-DPSK modulation code $C(9)$ over an average shadowed MSAT channel.
5.5.2 32-Dimensional 8-PSK Block Modulation Codes

In this section, RM codes of length 16 are used as component codes in constructing 3-level block modulation codes.

Example 2

The first choice of the component codes of length 16 is: (1) $A_1 = (16, 5, 8)$, the 1st-order RM code of length 16; (2) $A_2 = A_3 = (16, 11, 4)$, the 2nd-order RM code of length 16. With this choice, the resultant 3-level 8-PSK code of length 16, $C(10) = \lambda[A_1 * A_2 * A_3]$, has the following parameters: $\eta[C(10)] = 27/16 = 1.6875$ bits/symbol, $d_H^2[C(10)] = 4.688$, $\delta_H[C(10)] = 4$, and $\Delta_H^2[C(10)] = 16$. This code has a spectral efficiency less than the 2 bits/symbol of the uncoded QPSK. $A_1$ has a 4-state trellis and $A_2$ and $A_3$ have 2 identical 4-state subtrellises. The BER of the...
Figure 5.9. BER of the 8-PSK modulation code $C(10)$ over a frequency non-selective slow Rayleigh fading channel.

8-PSK modulation code $C(10)$ over a frequency non-selective slow Rayleigh fading channel is shown in Figure 5.9. Three-stage decoding achieves 13.0 dB real coding gain over the uncoded QPSK. BER of each level decoding in three-level decoding is also shown in Figure 5.9. Since the first level component code has the largest MSD, it achieves the lowest BER at high SNR. Even though $A_2$ and $A_3$ have the same MSD, $A_3$ achieves lower BER than $A_2$ because $A_3$ has better MPD than $A_2$.

Coherently detected 8-PSK modulation code over shadowed MSAT channels

The bit-error performance of the coherently detected 8-PSK modulation code $C(10)$ is evaluated for the three different shadowed MSAT channels, i.e., light, average, and heavy.

The BER performance of the coherently detected 8-PSK BCM code $C(10)$ for
the light-, average-, and heavy-shadowed MSAT channels is shown in Figure 5.10, Figure 5.11, and Figure 5.12, respectively. The error performance of the coherently detected 8-PSK modulation code $C(10)$ for various shadowed MSAT channels is summarized in Table 5.4. In Table 5.4, the code $C(10)$ is compared with the two-level TCM code $C(4)$ in the chapter 4. Code $C(4)$ has the spectral efficiency 1.8125 bits/symbol, minimum symbol distance 4, and the minimum product distance 256. $C(10)$ has the spectral efficiency 1.6875 bits/symbol. Therefore, the code $C(10)$ transmit 0.125 bits/symbol less than the code $C(4)$. However, $C(10)$ outperforms $C(4)$ even though $C(4)$ has larger minimum product distance than $C(10)$. The interleaving depth of the code $C(10)$ is 8 and that of the code $C(4)$ is 4 because the decoding depth of $C(10)$ is 16 and that of $C(4)$ is 32. Therefore, the code $C(10)$ has better interleaving depth than the 2-level TCM code $C(4)$. For decoding complexity of the code $C(10)$, $A_1$ has a 4-state trellis and $A_2$ and $A_3$ have 2 identical 4-state subtrellises. For decoding complexity of the code $C(4)$, the first level rate-1/2 convolutional code has a 32-state heavily connected trellis and the 2nd level block code (32,26,4) have 2 identical 8-state subtrellises. Therefore, the decoding complexity of $C(4)$ is much bigger than that of $C(10)$.

**Differentially detected 8-PSK BCM code over shadowed MSAT channels**

The BER of the differentially detected 8-PSK BCM code $C(10)$ for the light-, average-, and heavy-shadowed MSAT channels is shown in Figure 5.13, Figure 5.14, and Figure 5.15, respectively. Due to the randomly changing phase and the Doppler frequency shift, the code $C(9)$ displays the error floor phenomenon. However, bit-error performances of the code $C(10)$ show that $C(10)$ does not display the error floor phenomenon because of the large minimum symbol distance, i.e., 4. The error performance of the differentially detected 8-PSK modulation code $C(10)$ for various
Figure 5.10. BER of the 8-PSK modulation code $C(10)$ over a light-shadowed MSAT channel.

Figure 5.11. BER of the 8-PSK modulation code $C(10)$ over an average-shadowed MSAT channel.
Figure 5.12. BER of the 8-PSK modulation code $C(10)$ over a heavy-shadowed MSAT channel.

Table 5.4. Bit error performance of the coherently detected 8-PSK BCM code $C(10)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$</th>
<th>$E_b/N_o$</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$BT$</td>
<td>$10^{-3}$ BER</td>
<td>$10^{-4}$ BER</td>
<td>@ BER over QPSK</td>
<td>@ BER over QPSK</td>
<td>@ BER over $C(4)$</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>10.266</td>
<td>12.85</td>
<td>5.646</td>
<td>12.184</td>
<td>2.897</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>9.79</td>
<td>11.62</td>
<td>6.752</td>
<td>14.43</td>
<td>0.565</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>12.64</td>
<td>16.48</td>
<td>5.721</td>
<td>11.689</td>
<td>2.07</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>10.4</td>
<td>12.49</td>
<td>8.628</td>
<td>16.144</td>
<td>1.06</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>20.28</td>
<td>24.44</td>
<td>9.099</td>
<td>14.017</td>
<td>2.983</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>14.295</td>
<td>17.3</td>
<td>11.756</td>
<td>23.75</td>
<td>1.177</td>
</tr>
</tbody>
</table>
shadowed MSAT channels is summarized in Table 5.5. For $BT = 0.05$, uncoded 4-DPSK modulation faces the error floor. However, the code $C(10)$ does not show the error floor. Therefore, code $C(10)$ provides the error floor free communication at a BER of $10^{-4}$ with the randomly changing phase and the Doppler frequency shift. Comparing with the code $C(4)$, the code $C(10)$ achieves better coding gain than the code $C(4)$ over the uncoded 4-DPSK modulation because $C(10)$ has better interleaving depth than $C(4)$. Therefore, the code $C(10)$ displays an error floor at a lower BER than code $C(4)$ with smaller decoding complexity and more coding gain. However, the spectral efficiency of code $C(4)$ is 0.125 bits/symbol higher than that of code $C(10)$.

Table 5.5. Bit error performance of the differentially detected 8-PSK BCM code $C(10)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$ Gain @ $10^{-3}$ BER</th>
<th>$E_b/N_o$ Gain @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over 4-DPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over 4-DPSK</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(4)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 16.57 20.9 2.557 13.554 1.46 2.21</td>
<td>0.05 14.44 17.31 6.26 0.0001973 5.14 7.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.02 17.64 22.17 3.76 12.284 2.586 3.87</td>
<td>0.05 15.51 18.89 9.592 0.0004744 2.79 5.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 23.135 29.05 0.00144* 7.746 11.012</td>
<td>0.05 23.02 0.0002404* 0.00803* 11.51 0.000826*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*: bit-error-rate at the error floor
Figure 5.13. BER of the 8-DPSK modulation code $C(10)$ over a light-shadowed MSAT channel.

Figure 5.14. BER of the 8-DPSK modulation code $C(10)$ over an average-shadowed MSAT channel.
Example 3

Another possible choice is component codes of length 16: (1) \( A_1 = A_3 = (16, 11, 4) \), the 2nd-order RM code of length 16; (2) \( A_3 = (16, 15, 2) \), the even-parity check RM code of length 16, With this choice, the resultant 3-level 8-PSK code of length 16, \( C(11) = \lambda[A_1 * A_2 * A_3] \), has the following parameters: \( \eta[C(11)] = 37/16 = 2.312 \) bits/symbol, \( d^2_E[C(11)] = 2.344, \delta_H[C(11)] = 2 \), and \( \Delta^2_p[C(11)] = 16 \). This code has a higher spectral efficiency than that of uncoded QPSK, 2 bits/symbol. \( A_1 \) and \( A_2 \) have 2 identical 4-state subtrellises and \( A_3 \) has a simple 2-state trellis. Since code \( C(11) \) has a high spectral efficiency, this code is suitable for use as inner code construction in the multilevel concatenated modulation code and product modulation code. The BER of the 8-PSK modulation code \( C(11) \) over a frequency non-selective slow Rayleigh fading channel is shown in Figure 5.16. Three-stage
decoding achieves 9.237 dB real coding gain over the uncoded QPSK. The BER of each level decoding in 3-level decoding is also shown in Figure 5.16, respectively. Since the first level component code has the largest MSD, it achieves the best BER at high SNR. Even though $A_1$ and $A_2$ has the same MSD, $A_2$ achieves a lower BER than $A_1$ because $A_2$ has better a MPD than $A_1$.

Coherent detection 8-PSK modulation code over the shadowed MSAT channel

The bit-error performance of the coherently detected 8-PSK modulation code $C(11)$ is evaluated for the three different shadowed MSAT channels, i.e., light, average, and heavy, respectively. The BER of the coherently detected 8-PSK BCM code $C(11)$ for light-, average-, and heavy-shadowed MSAT channels are shown in Figure 5.17, Figure 5.18, and Figure 5.19, respectively. The error performance of the coherently
detected 8-PSK modulation code $C'(11)$ for various shadowed MSAT channels is summarized in Table 5.6. In Table 5.6, the code $C'(11)$ is compared with the Ungerboeck TCM code $C(U)$. Code $C'(11)$ has a spectral efficiency of 2.3125 bits/symbol, minimum symbol distance 2, and minimum product distance 16. Therefore, the code $C'(11)$ transmits 0.3125 bits/symbol more than the code $C(U)$. Also, the code $C'(11)$ outperforms code $C(U)$ even though $C(U)$ has larger minimum symbol distance than $C'(11)$. Both codes use an interleaver with size 128 8-PSK symbols. Code $C'(11)$ has an interleaving depth of 8 and the code $C(U)$ has an interleaving depth of 4 because the number of column of the code $C'(11)$ and $C(U)$ is 16 and 32, respectively. For decoding complexity of the code $C'(11)$, $A_1$ and $A_2$ have 2 identical 4-state subtrellises and $A_3$ has a simple 2-state trellis. The code $C(U)$ has a closely connected 16-state trellis. The decoding complexity of $C(U)$ is larger than that of $C'(11)$.

Table 5.6. Bit error performance of the coherently detected 8-PSK BCM code $C'(11)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$ @ $10^{-3}$</th>
<th>$E_b/N_o$ @ $10^{-4}$</th>
<th>Coding Gain @ $10^{-3}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(U)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(U)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 14.43 19.44</td>
<td>1.469 5.594 0.0</td>
<td>1.737</td>
<td>2.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05 12.36 15.13</td>
<td>4.182 10.921 0.051</td>
<td>2.28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.02 15.797 20.784</td>
<td>2.564 7.385 0.003</td>
<td>1.716</td>
<td>2.908</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05 13.325 16.245</td>
<td>5.703 12.389 0.285</td>
<td>2.908</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 25.18 29.4</td>
<td>4.199 9.059 2.52</td>
<td>2.151</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05 18.786 23.18</td>
<td>12.805 17.83 1.697</td>
<td>2.151</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.17. BER of the 8-PSK modulation code $C(11)$ over a light-shadowed MSAT channel.

Figure 5.18. BER of the 8-PSK modulation code $C(11)$ over an average-shadowed MSAT channel.
Figure 5.19. BER of the 8-PSK modulation code $C(11)$ over a heavy-shadowed MSAT channel.

Differentially detected 8-DPSK modulation code over shadowed MSAT channels

Simulations are done for 3 different shadowed MSAT channels, i.e., light, average, and heavy. The BER for light-, average-, and heavy-shadowed MSAT channels are shown in Figure 5.20, Figure 5.21, and Figure 5.22, respectively. The error performance of the differentially detected 8-DPSK modulation code $C(11)$ for various shadowed MSAT channels is summarized in Table 5.7. For a $BT = 0.02$ and BER of $10^{-3}$, the code $C(U)$ achieves better coding gain than of $C(11)$ in the light- and average-shadowed MSAT channel. However, at a BER of $10^{-4}$, $C(11)$ outperforms $C(U)$ because the interleaving depth of $C(11)$ is larger than that of $C(U)$. For $BT = 0.05$, the code $C(11)$ outperforms the code $C(U)$ in the light- and average-shadowed channels. Especially, the code $C(U)$ faces the error floor at a BER of
$1.8 \times 10^{-4}$, but the code $C(11)$ achieves the BER $10^{-4}$ at SNR 26.02 dB. However, for the heavy shadowed channel with $BT^{-1} = 0.05$, the code $C(11)$ reaches the error floor at BER $7.9 \times 10^{-3}$ but the code $C(U)$ reaches the error floor at the BER of $6.3 \times 10^{-3}$ and the code $C(11)$ almost always achieves an impressive coding gain over $C(U)$ with higher spectral efficiency and lower decoding complexity.

Table 5.7. Bit error performance of the differentially detected 8-PSK BCM code $C(11)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$EB/No$</th>
<th>$EB/No$</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$@$ $10^{-3}$</td>
<td>$@$ $10^{-4}$</td>
<td>over 4-DPSK</td>
<td>over 4-DPSK</td>
<td>over C(U)</td>
<td>over C(U)</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>19.1</td>
<td>25.06</td>
<td>0.027</td>
<td>3.89</td>
<td>-0.932</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>18.0</td>
<td>22.63</td>
<td>2.7</td>
<td>0.0001973*</td>
<td>0.831</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>20.64</td>
<td>26.6</td>
<td>0.76</td>
<td>7.854</td>
<td>-0.341</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>19.66</td>
<td>25.02</td>
<td>5.442</td>
<td>0.0004744*</td>
<td>2.245</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>32.86</td>
<td>48.0</td>
<td>0.00144*</td>
<td>5.5</td>
<td>0.000529*</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.0079*</td>
<td>0.00803*</td>
<td>0.0063*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*: bit-error-rate at the error floor
Figure 5.20. BER of the 8-DPSK modulation code $C(11)$ over a light-shadowed MSAT channel.

Figure 5.21. BER of the 8-DPSK modulation code $C(11)$ over an average-shadowed MSAT channel.
Figure 5.22. BER of the 8-DPSK modulation code $C(11)$ over a heavy-shadowed MSAT channel.

**Example 4**

The third choice of component codes of length 16 is: (1) $A_1 = (16, 11, 4)$, the 2nd-order RM code of length 16; (2) $A_2 = A_3 = (16, 15, 2)$, the even-parity check RM code of length 16, With this choice, the resultant 3-level 8-PSK code of length 16, $C(12) = \lambda[A_1 \ast A_2 \ast A_3]$, has the following parameters: $\eta[C(12)] = 41/16 = 2.5625$ bits/symbol, $d_S^2[C(12)] = 2.344$, $\delta_H[C(12)] = 2$, and $\Delta_d^2[C(12)] = 4$. This code has a larger spectral efficiency than code $C(11)$. $A_1$ has 2 identical 4-state subtrellises and $A_2$ and $A_3$ have a simple 2-state trellis. Since code $C(12)$ has high spectral efficiency, this code is suitable for use as an inner code construction in the multilevel concatenated modulation code and product modulation code. Since the code $C(12)$ has higher spectral efficiency than code $C(11)$, the multilevel concatenated modulation code with $C(12)$ as the inner code can have more freedom to choose outer codes
Figure 5.23. BER of the 8-PSK modulation code $C(12)$ over a frequency non-selective slow Rayleigh fading channel.

than that with $C(11)$ as the inner code. Multilevel concatenated modulation codes and product modulation codes using codes $C(11)$ and $C(12)$ as inner codes will be presented in the following chapters. The BER of the 8-PSK modulation code $C(12)$ over a frequency non-selective slow Rayleigh fading channel is shown in Figure 5.23. Three-stage decoding achieves 7.67 dB real coding gain over the uncoded QPSK.

The BER of each level decoding in 3-level decoding is also shown in Figure 5.23. Since the first level component code has the largest MSD, it achieves the lowest BER at high SNR. Even though $A_2$ and $A_3$ have the same MSD, $A_3$ achieves lower BER than $A_2$ because $A_3$ has better MPD than $A_2$. 
Coherently detected 8-PSK modulation code over shadowed MSAT channels

The bit-error performance of the coherently detected 8-PSK modulation code $C(12)$ is evaluated for the three different shadowed MSAT channels, i.e., light, average, and heavy, respectively. The BERs of the coherently detected 8-PSK BCM code $C(12)$ for light-, average-, and heavy-shadowed MSAT channels are shown in Figure 5.24, Figure 5.25, and Figure 5.26, respectively. The error performance of the coherently detected 8-PSK modulation code $C(12)$ for various shadowed MSAT channels is summarized in Table 5.8. In Table 5.8, the code $C(12)$ is compared with the Ungerboeck TCM code $C(U)$. Code $C(12)$ has the spectral efficiency 2.5625 bits/symbol, minimum symbol distance 2, and the minimum product distance 4. Therefore, the code $C(11)$ transmits 0.5625 bits/symbol more than the code $C(U)$. The code $C(U)$ outperforms the code $C(12)$ at a BER of $10^{-3}$. However, at a BER of $10^{-4}$, the code $C(12)$ outperforms the code $C(U)$ even though $C(U)$ has larger minimum symbol distance and minimum product distance than $C(12)$. Both codes use an interleaver with size 128 8-PSK symbols. The code $C(12)$ has the interleaving depth 8 because the decoding depth of $C(12)$ is 16. For decoding complexity of the code $C(12)$, $A_1$ has 2 identical 4-state subtrellises and $A_2$ and $A_3$ have a simple 2-state trellis. The code $C(U)$ has a closely connected 16-state trellis. Therefore, the decoding of the code $C(U)$ is much more complex than that of code $C(12)$. However, in the heavy shadowed MSAT channel with $BT = 0.05$, the code $C(U)$ outperforms the code $C(12)$ because $C(U)$ has the bigger minimum symbol and product distance than $C(12)$.
Figure 5.24. BER of the 8-PSK modulation code \( C(12) \) over a light-shadowed MSAT channel.

Figure 5.25. BER of the 8-PSK modulation code \( C(12) \) over an average-shadowed MSAT channel.
Figure 5.26. BER of the 8-PSK modulation code $C(12)$ over a heavy-shadowed MSAT channel.

Table 5.8. Bit error performance of the coherently detected 8-PSK BCM code $C(12)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$ @ $10^{-3}$ BER</th>
<th>$E_b/N_o$ @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(U)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(U)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02</td>
<td>14.82</td>
<td>19.77</td>
<td>1.092</td>
<td>0.67</td>
<td>-0.44</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>12.56</td>
<td>16.565</td>
<td>3.982</td>
<td>2.159</td>
<td>-0.149</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>16.344</td>
<td>21.87</td>
<td>2.017</td>
<td>6.299</td>
<td>-0.544</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>13.745</td>
<td>18.319</td>
<td>5.283</td>
<td>10.315</td>
<td>-0.135</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>27.594</td>
<td>31.5</td>
<td>1.785</td>
<td>6.89</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>22.59</td>
<td>27.57</td>
<td>9.0</td>
<td>13.44</td>
<td>-2.125</td>
</tr>
</tbody>
</table>
Differentially detected 8-DPSK modulation code over shadowed MSAT channels

The BERs of the 8-DPSK modulation code $C(12)$ for light-, average-, and heavy-shadowed MSAT channels are shown in Figure 5.27, Figure 5.28, and Figure 5.29, respectively. The error performance of the differentially detected 8-PSK modulation code $C(12)$ for shadowed MSAT channels is summarized in Table 5.9. In Table 5.9, the code $C(12)$ is compared with the Ungerboeck's code $C(U)$. Code $C(12)$ has a spectral efficiency of 2.344 bits/symbol. Even though the code $C(U)$ has larger minimum symbol and product distances than the code $C(12)$, $C(12)$ outperforms $C(U)$ at BER of $10^{-4}$ because $C(12)$ has larger interleaving depth than $C(U)$ the exception is the heavy shadowed case with $BT = 0.05$. With such high spectral efficiency and small decoding complexity, the code $C(12)$ shows an impressive error performance for the shadowed MSAT channels.

Table 5.9. Bit error performance of the differentially detected 8-PSK BCM code $C(12)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$</th>
<th>$E_b/N_o$</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>@ $10^{-3}$</td>
<td>@ $10^{-4}$</td>
<td>@ $10^{-3}$</td>
<td>@ $10^{-4}$</td>
<td>BER over 4-DPSK</td>
<td>BER over 4-DPSK</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>19.10</td>
<td>25.60</td>
<td>0.027</td>
<td>2.32</td>
<td>-0.932</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>18.495</td>
<td>25.6</td>
<td>2.205</td>
<td>0.0001973$^*$</td>
<td>0.336</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>20.85</td>
<td>27.65</td>
<td>0.235</td>
<td>6.804</td>
<td>-0.55</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>20.66</td>
<td>0.000123$^*$</td>
<td>4.442</td>
<td>0.0004744$^*$</td>
<td>0.99</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>37.25</td>
<td>0.000495$^*$</td>
<td>0.00144$^*$</td>
<td>1.11</td>
<td>0.000529$^*$</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.0102$^*$</td>
<td>0.00803$^*$</td>
<td>0.0063$^*$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^*$: bit-error-rate at the error floor
Figure 5.27. BER of the 8-DPSK modulation code $C(12)$ over a light-shadowed MSAT channel.

Figure 5.28. BER of the 8-DPSK modulation code $C(12)$ over an average-shadowed MSAT channel.
5.5.3 64-Dimensional 8-PSK Block Modulation Codes

In this section, RM codes of length 32 are used as component codes in constructing a 3-level block modulation code. By using longer component codes, we can construct more powerful modulation codes, i.e., large minimum symbol and product distances.

Example 5

The first choice of the component codes of length 32 is: (1) $A_1 = A_2 = (32, 16, 8)$, the 2nd-order RM code of length 32; (2) $A_3 = (32, 26, 4)$, the 3rd-order RM code of length 32. With this choice, the resultant 3-level 8-PSK code of length 32, $C(13) = \lambda[A_1 * A_2 * A_3]$, has the following parameters: $\eta[C(13)] = 58/32 = 1.8125$ bits/symbol, $d_E^2[C(13)] = 4.688$, $\delta_H[C(13)] = 4$, and $\Delta_p^2[C(13)] = 256$. This code has a spectral efficiency less than the uncoded QPSK, 2 bits/symbol. $A_1$ and $A_2$
Figure 5.30. BER of the 8-PSK modulation code $C(13)$ over a frequency non-selective slow Rayleigh fading channel.

have 8 identical 4-section 8-state subtrellises and $A_3$ has 2 identical 4-section 8-state subtrellises. The BER of the 8-PSK modulation code $C(13)$ over a frequency non-selective slow Rayleigh fading channel is shown in Figure 5.30. Three-stage decoding achieves a 12.655 dB real coding gain over the uncoded QPSK. The BER of each level decoding in 3-level decoding is also shown in Figure 5.30. Even though $A_1$ and $A_2$ have the same MSD, $8$, $A_2$ achieves a lower BER than $A_1$ because $A_2$ has a better MPD than $A_1$.

Coherently detected 8-PSK modulation code for shadowed MSAT channels

The bit-error performance of the coherently detected 8-PSK modulation code $C(13)$ is evaluated for three different shadowed MSAT channels, i.e., light, average, and heavy. The BERs of the coherently detected 8-PSK BCM code $C(13)$ for light-
, average-, and heavy-shadowed MSAT channels are shown in Figure 5.31, Figure 5.32, and Figure 5.33, respectively. The error performance of the coherently detected 8-PSK modulation code $C(13)$ for various shadowed MSAT channels is summarized in Table 5.10. In Table 5.10, the code $C(13)$ is compared with the two-level TCM code $C(4)$. Code $C(13)$ has a spectral efficiency 1.8125 bits/symbol, minimum symbol distance 4, and the minimum product distance 256. Code $C(4)$ has a spectral efficiency 1.8125 bits/symbol, the minimum symbol distance 4, and the minimum product distance 256. Therefore, the code $C(13)$ has the same distance parameters as the code $C(4)$. Both codes use an interleaver of size 128 8-PSK symbols. The code $C(13)$ has interleaving depth 4 and the code $C(4)$ has interleaving depth 4 because the column size of interleaver for codes $C(13)$ and $C(4)$ is 32. For decoding complexity of the code $C(13)$, $A_1$ and $A_2$ have 8 identical 4-section 8-state subtrellises and $A_3$ has 2 identical 4-section 8-state subtrellises. The first-level component code of code $C(4)$ has a heavily connected 32-state trellis and the second-level component code of code $C(4)$ has 2 identical 4-section 8-state subtrellises. Therefore, code $C(4)$ is more complex than than code $C(13)$. For light and average shadowed channels, the code $C(4)$ outperforms the code $C(13)$ at the BER of $10^{-3}$. However, at the BER of $10^{-4}$, the code $C(13)$ outperforms the code $C(4)$. Even though both codes have the same minimum symbol 4, the second smallest minimum symbol distance of the code $C(13)$ is 8 and that of the code $C(4)$ is only 6. Therefore, for the high SBR or at low BER, code $C(13)$ outperforms code $C(4)$ even though both codes have the same interleaving depth 4.
Figure 5.31. BER of the 8-PSK modulation code \( C(13) \) over a light-shadowed MSAT channel.

Figure 5.32. BER of the 8-PSK modulation code \( C(13) \) over an average-shadowed MSAT channel.
Figure 5.33. BER of the 8-PSK modulation code $C(13)$ over a heavy-shadowed MSAT channel.

Table 5.10. Bit error performance of the coherently detected 8-PSK BCM code $C(13)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_0$ @ $10^{-3}$ BER</th>
<th>$E_b/N_0$ @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(4)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>13.721</td>
<td>18.04</td>
<td>2.149</td>
<td>6.98</td>
<td>-0.558</td>
<td>1.557</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>11.09</td>
<td>13.4</td>
<td>5.43</td>
<td>12.62</td>
<td>-0.735</td>
</tr>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>15.06</td>
<td>19.10</td>
<td>3.3</td>
<td>9.06</td>
<td>-0.35</td>
<td>1.313</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>0.05</td>
<td>11.885</td>
<td>14.614</td>
<td>7.125</td>
<td>14.0</td>
<td>-0.425</td>
</tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>Heavy</td>
<td>22.73</td>
<td>26.68</td>
<td>6.63</td>
<td>11.72</td>
<td>0.533</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>15.741</td>
<td>19.51</td>
<td>15.959</td>
<td>21.35</td>
<td>-0.269</td>
</tr>
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<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Differentially detected 8-DPSK modulation code over shadowed MSAT channels

The BER of the differentially detected 8-DPSK modulation code \( C(13) \) for light-, average-, and heavy-shadowed MSAT channels is shown in Figure 5.34, Figure 5.35, and Figure 5.36, respectively. The error performance of the differentially detected 8-PSK modulation code \( C(13) \) for shadowed MSAT channels is summarized in Table 5.11. Except for the case of light shadowing at a BER of \( 10^{-3} \), the code \( C(13) \) outperforms the code \( C(4) \). For the light shadowed MSAT channel at low SNR, code \( C(4) \) performs better than code \( C(13) \) because code \( C(4) \) has better minimum squared Euclidean distance than code \( C(13) \).

Table 5.11. Bit error performance of the differentially detected 8-PSK BCM code \( C(13) \) over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>( E_b/N_0 ) @ ( 10^{-3} ) BER</th>
<th>( E_b/N_0 ) @ ( 10^{-4} ) BER</th>
<th>Coding Gain @ ( 10^{-3} ) BER over 4-DPSK</th>
<th>Coding Gain @ ( 10^{-4} ) BER over 4-DPSK</th>
<th>Coding Gain @ ( 10^{-3} ) BER over ( C(4) )</th>
<th>Coding Gain @ ( 10^{-4} ) BER over ( C(4) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 18.48</td>
<td>22.64</td>
<td>0.62</td>
<td>6.25</td>
<td>-0.2297</td>
<td>2.87</td>
</tr>
<tr>
<td></td>
<td>0.05 15.6</td>
<td>19.64</td>
<td>5.43</td>
<td>0.0001973*</td>
<td>1.39</td>
<td>2.86</td>
</tr>
<tr>
<td>Average</td>
<td>0.02 18.91</td>
<td>23.9</td>
<td>2.6</td>
<td>10.47</td>
<td>1.316</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>0.05 16.74</td>
<td>19.78</td>
<td>4.6</td>
<td>0.0004744*</td>
<td>1.56</td>
<td>5.22</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 29.04</td>
<td>33.1</td>
<td>0.00144*</td>
<td>1.8</td>
<td>4.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05 30.15</td>
<td>0.0002*</td>
<td>0.00803*</td>
<td>4.85</td>
<td>0.000863</td>
<td></td>
</tr>
</tbody>
</table>

* : bit-error-rate at the error floor
Figure 5.34. BER of the 8-DPSK modulation code \( C(13) \) over a light-shadowed MSAT channel.

Figure 5.35. BER of the 8-DPSK modulation code \( C(13) \) over an average-shadowed MSAT channel.
Example 6

The second choice for the component codes of length 32 is: (1) $A_1 = (32, 16, 8)$, the 2nd-order RM code of length 32; (2) $A_2 = A_3 = (32, 26, 4)$, the 3rd-order RM code of length 32, With this choice, the resultant 3-level 8-PSK code of length 32, $C(14) = \lambda[A_1 * A_2 * A_3]$, has the following parameters: $\eta[C(14)] = 68/32 = 2.125$ bits/symbol, $d[k][C(14)] = 4.688$, $\delta_H[C(14)] = 4$, and $\Delta_2^2[C(14)] = 16$. This code has a higher spectral efficiency than 2 bits/symbol of the uncoded QPSK. $A_1$ has 8 identical 4-section 8-state subtrellises and $A_2$ and $A_3$ have 2 identical 4-section 8-state subtrellises. The BER of the 8-PSK modulation code $C(14)$ over a frequency non-selective slow Rayleigh fading channel is shown in Figure 5.37. Three-stage decoding achieves a 12.3 dB real coding gain over the uncoded QPSK. The BERs of each level decoding in 3-level decoding are also shown in Figure 5.37. Even though

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Figure 5.36. BER of the 8-DPSK modulation code $C(13)$ over a heavy-shadowed MSAT channel.
Figure 5.37. BER of the 8-PSK modulation code $C(14)$ over a frequency non-selective slow Rayleigh fading channel.

$A_2$ and $A_3$ have the same MSD, 4, $A_3$ achieves lower BER than $A_2$ because $A_3$ has stronger MPD than $A_2$. $A_1$ achieves the best error performance because it has the largest MSD 8.

**Coherently detected 8-PSK modulation code for shadowed MSAT channels**

The bit-error performance of the coherently detected 8-PSK modulation code $C(14)$ is evaluated for the three different shadowed MSAT channels, i.e., light, average, and heavy. These error performances are shown in Figure 5.38, Figure 5.39, and Figure 5.40, respectively and are summarized in Table 5.12. In Table 5.12, the code $C(14)$ is compared with the Ungerboeck TCM code $C(U)$. Code $C(14)$ has a spectral efficiency of 2.125 bits/symbol, minimum symbol distance 4, and the minimum product distance 256. Therefore, the code $C(14)$ has better distance parameters

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than code $C(U)$. Both codes use the interleaver with size 128 8-PSK symbols. The code $C(14)$ has interleaving depth 4 and the code $C(U)$ has interleaving depth 4 because the decoding depth of the code $C(13)$ and $C(U)$ are 32 and 20, respectively.

For the code $C(14)$, $A_1$ has 8 identical 4-section 8-state subtrellises and $A_2$ and $A_3$ have 2 identical 4-section 8-state subtrellises. The code $C(U)$ has a closely connected 16-state trellis. Therefore, code $C(14)$ is slightly more complex to decode than code $C(U)$. However, code $C(14)$ outperforms code $C(U)$ in all three different shadowed MSAT channels.

Table 5.12. Bit error performance of the coherently detected 8-PSK BCM code $C(14)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_0$ @ $10^{-3}$ BER</th>
<th>$E_b/N_0$ @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(U)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(U)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 13.969 18.81 1.758 3.14 0.693 2.239</td>
<td>0.05 10.279 13.963 6.241 12.037 2.11 3.396</td>
<td>0.02 14.655 19.077 3.676 9.149 1.5759 3.463</td>
<td>0.05 11.76 15.46 7.268 13.174 1.85 3.693</td>
<td>0.02 24.467 28.36 4.912 10.099 3.233 3.115</td>
<td>0.05 17.479 21.02 14.112 19.99 2.986 3.94</td>
</tr>
<tr>
<td>Average</td>
<td>0.02 13.969 18.81 1.758 3.14 0.693 2.239</td>
<td>0.05 10.279 13.963 6.241 12.037 2.11 3.396</td>
<td>0.02 14.655 19.077 3.676 9.149 1.5759 3.463</td>
<td>0.05 11.76 15.46 7.268 13.174 1.85 3.693</td>
<td>0.02 24.467 28.36 4.912 10.099 3.233 3.115</td>
<td>0.05 17.479 21.02 14.112 19.99 2.986 3.94</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 13.969 18.81 1.758 3.14 0.693 2.239</td>
<td>0.05 10.279 13.963 6.241 12.037 2.11 3.396</td>
<td>0.02 14.655 19.077 3.676 9.149 1.5759 3.463</td>
<td>0.05 11.76 15.46 7.268 13.174 1.85 3.693</td>
<td>0.02 24.467 28.36 4.912 10.099 3.233 3.115</td>
<td>0.05 17.479 21.02 14.112 19.99 2.986 3.94</td>
</tr>
</tbody>
</table>
Figure 5.38. BER of the 8-PSK modulation code \( C(14) \) over a light-shadowed MSAT channel.

Figure 5.39. BER of the 8-PSK modulation code \( C(14) \) over an average-shadowed MSAT channel.
Differentially detected 8-DPSK modulation code over shadowed MSAT channels

The BER the differentially detected 8-DPSK modulation code $C(14)$ is evaluated for various shadowed MSAT channels. Simulations are done for 3 different shadowed MSAT channels, i.e., light, average, and heavy. The BERs of the 8-DPSK modulation code $C(14)$ for light-, average-, and heavy-shadowed MSAT channels are shown in Figure 5.41, Figure 5.42, and Figure 5.43, respectively. The error performance of the differentially detected 8-PSK modulation code $C(14)$ for various shadowed MSAT channels is summarized in Table 5.13. The code $C(14)$ outperforms the code $C(U)$, except for the case of the light shadowed channel at the BER of $10^{-3}$. For the heavy shadowed channel, the code $C(14)$ reaches the error floor at a lower BER than the code $C(U)$.
Figure 5.41. BER of the 8-DPSK modulation code $C(14)$ over a light-shadowed MSAT channel.

Figure 5.42. BER of the 8-DPSK modulation code $C(14)$ over an average-shadowed MSAT channel.
Figure 5.43. BER of the 8-DPSK modulation code C(14) over a heavy-shadowed MSAT channel.

Table 5.13. Bit error performance of the differentially detected 8-PSK BCM code C(14) over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_0$</th>
<th>$E_b/N_0$</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$10^{-3}$</td>
<td>$10^{-4}$</td>
<td>BER @ 4-DPSK</td>
<td>BER over C(U)</td>
<td>BER @ 4-DPSK</td>
<td>BER over C(U)</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>18.21</td>
<td>25.08</td>
<td>0.917</td>
<td>3.87</td>
<td>-0.042</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>16.0</td>
<td>20.5</td>
<td>4.7</td>
<td>0.0001973*</td>
<td>2.831</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>19.61</td>
<td>24.83</td>
<td>1.79</td>
<td>9.624</td>
<td>0.689</td>
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<td></td>
<td>0.05</td>
<td>17.54</td>
<td>22.11</td>
<td>7.562</td>
<td>0.0004744*</td>
<td>4.365</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>30.67</td>
<td>35.54</td>
<td>0.00144*</td>
<td>7.69</td>
<td>0.00529*</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.00128*</td>
<td>0.00803*</td>
<td></td>
<td>0.0063*</td>
<td></td>
</tr>
</tbody>
</table>

*: bit-error-rate at the error floor
Example 7

The third choice for the component codes of length 32 is: (1) \( A_1 = (32, 16, 8) \), the 2nd-order RM code of length 32; (2) \( A_2 = (32, 26, 4) \), the 3rd-order RM code of length 32; and (3) \( A_3 = (32, 31, 2) \), the parity-check RM code of length 32. With this choice, the resultant 3-level 8-PSK code of length 32, \( C(15) = A_1 * A_2 * A_3 \), has the following parameters: \( \eta[C(15)] = 73/32 = 2.281 \) bits/symbol, \( d_{HC}[C(15)] = 4.688 \), \( \delta_H[C(15)] = 2 \), and \( \Delta_\tau^2[C(15)] = 16 \). This code has a spectral efficiency higher than the 2 bits/symbol of the uncoded QPSK. \( A_1 \) has 8 identical 4-section 8-state subtrellises, \( A_2 \) has 2 identical 4-section 8-state subtrellises and \( A_3 \) has a simple 2-state trellis. The BER of the 8-PSK modulation code \( C(15) \) over a frequency non-selective slow Rayleigh fading channel is shown in Figure 5.44. Three-stage decoding achieves an 11.258 dB real coding gain over the uncoded QPSK. The BERs of each
level decoding in 3-level decoding are also shown in Figure 5.44. The BER of $A_3$ dominates the BER of the code $C(15)$ because the minimum symbol distance of $A_3$ is the smallest among three component codes. $A_1$ achieves best error performance because it has the largest MSD 8. These results agree with our modulation design criteria.

Coherently detected 8-PSK modulation code over shadowed MSAT channels

The bit-error performance of the coherently detected 8-PSK modulation code $C(15)$ is evaluated for three different shadowed MSAT channels, i.e., light, average, and heavy. The BER of the coherently detected 8-PSK BCM code $C(15)$ for light-, average-, and heavy-shadowed MSAT channels are shown in Figure 5.45, Figure 5.46, and Figure 5.47, respectively. The error performance of the coherently detected 8-PSK modulation code $C(15)$ for various shadowed MSAT channels is summarized in Table 5.14. In Table 5.14, the code $C(15)$ is compared with the Ungerboeck TCM code $C(U)$. Two information bits in a 8-PSK signal in $C(U)$ are protected with the minimum symbol distance 3 and the minimum product distance is 4.68 in the worst case. Code $C(15)$ has a spectral efficiency 2.28125 bits/symbol, minimum symbol distance 2, and a minimum product distance of 16. Among 2.28125 bits/symbol, 0.96875 information bits of the $A_3$ is protected with the minimum symbol distance 4 and the minimum product distance 16 in the worst case, 0.625 information bits of $A_2$ is protected with the minimum symbol distance 4 and the minimum symbol distance 16, and 0.5 information bits of $A_1$ are protected with minimum symbol distance 8. Therefore, code $C(15)$ outperforms code $C(U)$ for various shadowed MSAT channels. Both codes use an interleaver with size 128 8-PSK symbols. Code $C(15)$ has an interleaving depth 4 and code $C(U)$ also has an interleaving depth
4 because the decoding depth of $C(15)$ and $C(U)$ are 32 and 20, respectively. For $C(15)$, $A_1$ has 8 identical 4-section 8-state subtrellises and $A_2$ has 2 identical 4-section 8-state subtrellises. And $A_3$ has a simple 2-state trellis. The code $C(U)$ has a closely connected 16-state trellis. Therefore, $C(15)$ is more simpler to decode than $C(U)$.

**Differentially detected 8-DPSK modulation code over shadowed MSAT channels**

Simulations are done for 3 different shadowed MSAT channels, i.e., light, average, and heavy. BERs of the 8-DPSK modulation code $C(15)$ for light-, average-, and heavy-shadowed MSAT channels are shown in Figure 5.48, Figure 5.49, and Figure 5.50, respectively. The error performance of the differentially detected 8-PSK modulation code $C(15)$ for various shadowed MSAT channels is summarized in Table 5.15. Even though the code $C(15)$ has smaller minimum symbol distance than the code $C(U)$, $C(15)$ has better overall distance parameters than $C(U)$. Also, for an given interleaver with size 128 8-PSK symbols, each interleaver contains 4 independent code $C(15)$. In decoding, these 4 codes are decoded independently. Therefore, there is no error propagation between these four independent codes $C(15)$. However, for the code $C(U)$, there exists an error propagation through entire 128 8-PSK symbol sequence because of the nature of the Viterbi algorithm. In the Viterbi algorithm, a decoder must use past history of trellis to decide present information. Therefore, the code $C(15)$ outperforms the code $C(U)$ for all three shadowed MSAT channels. Even though the code $C(U)$ faces an error floor phenomenon before achieving the BER of $10^{-4}$, the code $C(15)$ achieves the BER of $10^{-4}$. 

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Figure 5.45. BER of the 8-PSK modulation code $C(15)$ over a light-shadowed MSAT channel.

Figure 5.46. BER of the 8-PSK modulation code $C(15)$ over an average-shadowed MSAT channel.
Figure 5.47. BER of the 8-PSK modulation code $C(15)$ over a heavy-shadowed MSAT channel.

Table 5.14. Bit error performance of the coherently detected 8-PSK BCM code $C(15)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$ @ $10^{-3}$ BER</th>
<th>$E_b/N_o$ @ $10^{-4}$ BER</th>
<th>Coding Gain @ $10^{-3}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-3}$ BER over $C(U)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(U)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 13.63 18.91 2.26 6.124 0.76 2.17</td>
<td>0.05 11.1 15.4 5.442 10.651 1.311 2.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.02 14.75 20.22 3.611 7.949 1.05 2.28</td>
<td>0.05 12.3 16.03 6.728 12.604 1.31 3.123</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 25.61 30.03 3.769 8.429 2.09 1.477</td>
<td>0.05 19.586 24.545 12.0 16.465 0.879 0.385</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.48. BER of the 8-DPSK modulation code $C(15)$ over a light-shadowed MSAT channel.

Figure 5.49. BER of the 8-DPSK modulation code $C(15)$ over an average-shadowed MSAT channel.
Figure 5.50. BER of the 8-DPSK modulation code $C(15)$ over a heavy-shadowed MSAT channel.

Table 5.15. Bit error performance of the differentially detected 8-PSK BCM code $C(15)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$ @ (10^{-3}) BER</th>
<th>$E_b/N_o$ @ (10^{-4}) BER</th>
<th>Coding Gain @ (10^{-3}) BER over 4-DPSK</th>
<th>Coding Gain @ (10^{-4}) BER over 4-DPSK</th>
<th>Coding Gain @ (10^{-3}) BER over $C(U)$</th>
<th>Coding Gain @ (10^{-4}) BER over $C(U)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.05</td>
<td>16.26</td>
<td>21.79</td>
<td>4.44</td>
<td>0.0001973*</td>
<td>2.571</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.05</td>
<td>17.647</td>
<td>24.521</td>
<td>7.455</td>
<td>0.000495*</td>
<td>4.258</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heavy</td>
<td>0.05</td>
<td>31.1</td>
<td>42.25</td>
<td>0.00132*</td>
<td>7.26</td>
<td>0.000529*</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* : bit-error-rate at the error floor

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5.6 Summary

In this chapter, various multilevel BCM codes have been constructed by using the multilevel coding method for voice transmission for various shadowed MSAT channels. Since the shadowed MSAT channel has a constraint on the total allowable delay time for voice transmission, this constraint limits the size of the interleaver. Therefore, short BCM codes have the potential to compete with TCM codes in the shadowed MSAT channel because of their shorter decoding depth and hence more effective interleaving. The proposed multilevel BCM codes are listed in the Table 5.16. The bit error performances of proposed multilevel BCM codes for various shadowed MSAT channels are summarized in Table 5.17, 5.18, 5.19, 5.20, and 5.21. For 8-PSK BCM codes, an interleaver contains 128 8-PSK symbols. The interleaving depth of the code \( C(9) \) is 16. The thirty-two dimensional BCM codes, \( C(10), C(11), \) and \( C(12) \), have the same interleaving depth 8. The 64-dimensional BCM codes \( C(13), C(14), \) and \( C(15) \) have the same interleaving depth of 4 because the number of interleaver column must be larger than the length (or decoding decision depth) of a BCM code.

<table>
<thead>
<tr>
<th>Code</th>
<th>Rate</th>
<th>Minimum Symbol Distance</th>
<th>Minimum Product Distance</th>
<th>Minimum Squared Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C(9) )</td>
<td>2.25</td>
<td>2</td>
<td>4</td>
<td>2.344</td>
</tr>
<tr>
<td>( C(10) )</td>
<td>1.6875</td>
<td>4</td>
<td>16</td>
<td>4.688</td>
</tr>
<tr>
<td>( C(11) )</td>
<td>2.3125</td>
<td>2</td>
<td>16</td>
<td>2.344</td>
</tr>
<tr>
<td>( C(12) )</td>
<td>2.5625</td>
<td>2</td>
<td>4</td>
<td>2.344</td>
</tr>
<tr>
<td>( C(13) )</td>
<td>1.8125</td>
<td>4</td>
<td>256</td>
<td>4.688</td>
</tr>
<tr>
<td>( C(14) )</td>
<td>2.125</td>
<td>4</td>
<td>16</td>
<td>4.688</td>
</tr>
<tr>
<td>( C(15) )</td>
<td>2.28125</td>
<td>2</td>
<td>16</td>
<td>4.688</td>
</tr>
</tbody>
</table>
If two BCM codes with different lengths have the same minimum symbol distance and minimum product distance, then the shorter code outperforms the longer code because the shorter code has a larger interleaving depth than the longer code for a given finite size of an interleaver. If two BCM codes have the same minimum symbol distance, then the code with the large minimum product distance faces an error floor phenomenon at a lower BER than the code with the small minimum product distance.

As shown in this chapter, BCM codes achieve impressive coding gains over TCM codes with almost the same decoding complexity. Since short BCM codes have larger interleaving depths than TCM codes with same decoding complexity, BCM codes are highly recommended for voice transmission for various shadowed MSAT channels.

Table 5.17. Error performances of 8-PSK BCM code $C(9)$

<table>
<thead>
<tr>
<th>Shadowing</th>
<th>$BT$</th>
<th>Coherent @ $E_b/N_o$</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BER</td>
<td>$E_b/N_o$</td>
<td>BER</td>
<td>$E_b/N_o$</td>
<td>BER</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>12.8</td>
<td>17.22</td>
<td>17.52</td>
<td>23.52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>11.075</td>
<td>13.911</td>
<td>16.57</td>
<td>20.9</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>14.122</td>
<td>18.76</td>
<td>19.07</td>
<td>25.52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>12.03</td>
<td>15.89</td>
<td>18.36</td>
<td>28.89</td>
<td></td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>24.89</td>
<td>29.477</td>
<td>33.254</td>
<td>0.000334*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>20.71</td>
<td>26.188</td>
<td>0.00474*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* : bit-error-rate at the error floor
Table 5.18. Error performances of coherently detected 32-dimensional BCM codes $C(10), C(11),$ and $C(12)$

<table>
<thead>
<tr>
<th>Shadowing</th>
<th>$B\ell T$</th>
<th>$C(10)$</th>
<th>$C(11)$</th>
<th>$C(12)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_b/N_o$ @ BER</td>
<td>$E_b/N_o$ @ BER</td>
<td>$E_b/N_o$ @ BER</td>
<td>$E_b/N_o$ @ BER</td>
</tr>
<tr>
<td></td>
<td>$10^{-3}$</td>
<td>$10^{-4}$</td>
<td>$10^{-3}$</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>10.266</td>
<td>12.85</td>
<td>14.43</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19.44</td>
<td>14.82</td>
<td>19.77</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>9.79</td>
<td>11.62</td>
<td>12.36</td>
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<tr>
<td></td>
<td></td>
<td>15.13</td>
<td>12.56</td>
<td>16.565</td>
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<tr>
<td>Average</td>
<td>0.02</td>
<td>12.64</td>
<td>16.48</td>
<td>15.797</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20.784</td>
<td>16.344</td>
<td>21.87</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>10.4</td>
<td>12.49</td>
<td>13.325</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16.245</td>
<td>13.745</td>
<td>18.319</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>20.28</td>
<td>24.44</td>
<td>25.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>29.4</td>
<td>27.594</td>
<td>31.5</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>14.295</td>
<td>17.3</td>
<td>18.786</td>
</tr>
<tr>
<td></td>
<td></td>
<td>23.18</td>
<td>22.59</td>
<td>27.57</td>
</tr>
</tbody>
</table>

Table 5.19. Error performances of differently detected 32-dimensional BCM codes $C(10), C(11),$ and $C(12)$

<table>
<thead>
<tr>
<th>Shadowing</th>
<th>$B\ell T$</th>
<th>$C(10)$</th>
<th>$C(11)$</th>
<th>$C(12)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_b/N_o$ @ BER</td>
<td>$E_b/N_o$ @ BER</td>
<td>$E_b/N_o$ @ BER</td>
<td>$E_b/N_o$ @ BER</td>
</tr>
<tr>
<td></td>
<td>$10^{-3}$</td>
<td>$10^{-4}$</td>
<td>$10^{-3}$</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>16.57</td>
<td>20.9</td>
<td>19.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>25.06</td>
<td>19.1</td>
<td>25.6</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>14.44</td>
<td>17.31</td>
<td>18.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>22.63</td>
<td>18.495</td>
<td>25.6</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>17.64</td>
<td>22.17</td>
<td>20.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td>26.6</td>
<td>20.85</td>
<td>27.65</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>15.51</td>
<td>18.89</td>
<td>19.66</td>
</tr>
<tr>
<td></td>
<td></td>
<td>25.02</td>
<td>20.66</td>
<td>0.000123*</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>23.135</td>
<td>29.05</td>
<td>32.86</td>
</tr>
<tr>
<td></td>
<td></td>
<td>48.0</td>
<td>37.25</td>
<td>0.000495*</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>23.02</td>
<td>0.000221*</td>
<td>0.0079*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0102*</td>
<td></td>
</tr>
</tbody>
</table>

* : bit-error-rate at the error floor
Table 5.20. Error performances of coherently detected 64-dimensional BCM codes $C(13), C(14),$ and $C(15)$

<table>
<thead>
<tr>
<th>Shadowing</th>
<th>$BT$</th>
<th>$C(13)$</th>
<th>$C(14)$</th>
<th>$C(15)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$E_b/N_o$ @ BER</td>
<td>$E_b/N_o$ @ BER</td>
<td>$E_b/N_o$ @ BER</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$10^{-3}$</td>
<td>$10^{-4}$</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>13.721</td>
<td>18.04</td>
<td>13.969</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>11.09</td>
<td>13.4</td>
<td>10.279</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>15.06</td>
<td>19.10</td>
<td>14.655</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>11.885</td>
<td>14.614</td>
<td>11.76</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>22.73</td>
<td>26.68</td>
<td>24.467</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>15.741</td>
<td>19.51</td>
<td>17.479</td>
</tr>
</tbody>
</table>

Table 5.21. Error performances of differently detected 8-PSK TCM codes $C(13), C(14),$ and $C(15)$

<table>
<thead>
<tr>
<th>Shadowing</th>
<th>$BT$</th>
<th>$C(13)$</th>
<th>$C(14)$</th>
<th>$C(15)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$E_b/N_o$ @ BER</td>
<td>$E_b/N_o$ @ BER</td>
<td>$E_b/N_o$ @ BER</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$10^{-3}$</td>
<td>$10^{-4}$</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>18.48</td>
<td>22.64</td>
<td>18.21</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>15.6</td>
<td>19.64</td>
<td>16.0</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>18.91</td>
<td>23.79</td>
<td>19.61</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>16.74</td>
<td>19.78</td>
<td>17.54</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>20.94</td>
<td>33.1</td>
<td>30.67</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>30.15</td>
<td>0.00128*</td>
<td>0.00158*</td>
</tr>
</tbody>
</table>

* : bit-error-rate at the error floor
Chapter 6

Multi-level Concatenated Block Coded Modulation Schemes

In the previous two chapters, it was shown that when the received coded sequence is distorted by severe multipath fading and large Doppler frequency shift, multilevel coded modulation schemes face an error floor phenomenon before achieving the BER of $10^{-4}$. For reliable data communication, the desirable BER is at least $10^{-4}$. This chapter investigates multi-level concatenated coded modulation schemes for reliable data communications over various fading channels. Our objective is to design coded modulation schemes to remove the error floor phenomenon or to lower the BER of the error floor. In a multi-level concatenated coded modulation scheme, the inner codes are block modulation codes and the outer codes are generally block codes with code symbols from some Galois fields. If the inner codes, outer codes, and the level of concatenation are properly chosen, very good performance can be achieved with high spectral efficiency and large coding gain. Several multi-level concatenated coded modulation schemes are proposed. Simulation studies show that these schemes perform very well and achieve significant real coding gains over uncoded reference modulation systems.
6.1 Introduction

Coded modulation in conjunction with concatenation is a powerful technique for achieving high reliability, large coding gain, and high spectral efficiency with reduced decoding complexity. This combination of coded modulation and concatenation is known as concatenated coded modulation. Deng and Costello [20, 21] studied the single-level concatenated trellis coded modulation (TCM) scheme, Kasami et al [36] studied the single-level concatenated block coded modulation (BCM) scheme, and Vucetic [60] presented error performance for the Rayleigh fading channel using single-level concatenated BCM and TCM scheme in 1993. It was shown that by properly choosing the inner codes and outer codes, large coding gains and high spectral efficiency could be achieved with reduced decoding complexity. Suppose a multilevel block modulation code is used as the inner code. As shown in Chapter 5, each component code of the multilevel modulation code has a different distance profile, it is therefore desirable to provide different error protection to each component code for better error performance. A major shortcoming of single-level concatenation is that the outer code corrects all output bits of the inner code decoder to the same degree. This shortcoming can be overcome by using multilevel concatenation and coset inner codes derived from the BCM code. Since modulation schemes using the multilevel concatenation approach give us the freedom to choose error-correcting capabilities of outer codes, it is easy to provide proper error protection to each level. Using a single-level concatenated block coded modulation scheme, it is desirable to keep the inner modulation codes short (or to keep the number of states of the trellis diagrams for the inner modulation codes small) to reduce the decoding complexity. However, using a short inner modulation code in a single-level concatenated block coded modulation scheme limits the improvement of bandwidth efficiency. This
shortcoming can be overcome by using multilevel concatenation and coset inner codes derived from a block modulation code with high spectral efficiency. In fact, multi-level concatenation allows us to use large block modulation inner codes with multi-stage suboptimum decoding [40] to achieve very high reliability, large coding gain, and high bandwidth efficiency with reduced decoding complexity. In this chapter, several multi-level concatenated block coded modulation schemes are proposed for data transmission over the Rayleigh fading and various shadowed MSAT channels.

6.2 Multilevel Concatenated Block Coded Modulation Scheme

The proposed multi-level concatenated coded modulation schemes are constructed using a multi-level concatenation approach [90]. In a $q$-level concatenated coded modulation system, $q$ pairs of outer and inner codes are used. In block coding, Reed-Solomon (RS) codes are used as the outer codes, and coset codes constructed from a linear block modulation code and its linear subcodes are used as the inner codes.

The encoding and decoding are each accomplished with $q$ levels. The decoding at each level consists of an inner code decoding and an outer code decoding. Closest coset decoding is performed at the first stage inner code decoder. Estimated sequence of the coset representatives is obtained at the first stage of the inner code decoder. This estimated coset representative is converted to RS code symbols from some Galois field. Outer codes are hard-decision decoded based on the RS outer codes. If the inner codes have a trellis structure, then these codes can be decoded with a soft decision Viterbi decoding algorithm. By using decoded RS code symbols, we form an estimated coset representative and the estimates are passed to the second...
stage inner code decoder where the process is repeated. Successive applications of closest coset decoding at the each of the individual stages give estimates of the coset representatives at all the q-stages.

Since the estimates from the previous stage decoding are passed to the next stage decoding, severe error propagation is expected in the fading channel. However, the multilevel concatenation modulation scheme allows us to choose error-correcting capabilities of outer codes to reduce the effect of the error propagation to the next stage decoding. Methods to determine error correcting capabilities of outer codes in multilevel concatenated modulation codes is discussed in the following section.

By using high spectral efficiency block modulation codes as inner codes and powerful RS codes as outer codes, proposed 8-PSK (or 8-DPSK) multilevel concatenated BCM schemes achieve impressive real coding gains over uncoded QPSK for the various fading channels. Even though these multilevel concatenated BCM schemes require more decoding complexity than the multilevel coded modulation schemes presented in chapter 4 and 5, they provide high performance data transmission for the various fading channels. They achieve low BER's at low SNR's and do not exhibit an error floor. Section 6.2.1 presents a general formulation of the multi-level concatenated coded modulation scheme. In Section 6.2.2, methods to determine outer codes of the multilevel concatenated modulation codes are discussed. Encoding is discussed in the section 6.2.3. In Section 6.3, multilevel closest coset decoding is proposed. In Section 6.4, several 3-level concatenated coded modulation schemes are proposed and their error performance is simulated over Rayleigh fading and various shadowed MSAT channels. In Section 6.5, 6-level concatenated coded modulation scheme is proposed and its error performance is simulated for the Rayleigh fading and various shadowed MSAT channels. In Section 6.6, a 2-level concatenated coded modulation scheme is proposed and its error performance is simulated over
the Rayleigh fading and various shadowed MSAT channels.

### 6.2.1 Construction

In a $q$-level concatenated coded modulation system, $q$ pairs of outer and inner codes are used as shown in Figure 6.1. In block coding, Reed-Solomon (RS) codes are used as the outer codes, and coset codes constructed from a block modulation code and its subcodes are used as the inner codes. The encoding and decoding are accomplished in $q$ levels respectively.

**Outer Code Construction**

For $1 \leq i \leq q$ and $1 \leq j \leq m_i$, let $B_{ij}$ be an $(N, K_i)$ RS (or shortened RS) code over $GF(2^p)$ with minimum Hamming distance $D_i = N - K_i + 1$. In the $i$-th level outer code encoder, a $(N, K_i, D_i)$ RS code is interleaved to depth $m_i$. For $1 \leq j \leq m_i$, let $B_{ij}$ represent the $j$-th code among $m_i$ interleaved RS codes. After $i$-th level outer code encoding, symbols from $GF(2^p)$ in each RS code are converted into $p$-bits binary representation. After conversion, $m_i N p$-bits are stored in the $m_i$ by $N p$ array such that every column has $m_i$ bits and each bit in this column is selected from each RS code $B_{ij}$ for $1 \leq j \leq m_i$. Since $B_{ij} = B_{ik}$ for $1 \leq j, k \leq m_i$, let $B_i = \{B_{i1}, B_{i2}, \cdots, B_{im_i}\}$ represents an $i$-th level outer code for $1 \leq i \leq q$. The $i$-th level outer code encoder is shown in Figure 6.2. Later, these $q$-sets of RS codes will be used as the outer codes in $q$ levels of concatenation.

**Inner Coset Code Construction**

Let $\Lambda_0$ be a block modulation code over a certain elementary signal set $S$ with length $n$, dimension $k_0$ and minimum squared Euclidean distance $\Delta_0$. We require that

$$k_0 = m_1 + m_2 + \cdots + m_q. \tag{6.1}$$

From $\Lambda_0$, we form a sequence of subcodes, $\Lambda_0$, $\Lambda_1$, $\Lambda_2$, $\cdots$, $\Lambda_q$, where $\Lambda_q$ consists
Figure 6.1. Multilevel Concatenated Coded Modulation Encoder
$K_i$ p - bits

$(N, K_i, D_i)$

RS Code Encoder

$B_{i,1}$

$K_i$ p - bits

$(N, K_i, D_i)$

RS Code Encoder

$B_{i,2}$

$m_i$-bits from each column

$m_i \times N_p$ Array

$\Gamma_i$

$m_i$
of the all-zero codeword, i.e. \( \Lambda_q = \{0\} \). The dimension of these subcodes satisfy the following conditions: For \( 1 \leq i \leq q \), \( \Lambda_i \) is a linear subcode of \( \Lambda_{i-1} \) with dimension
\[
k_i = k_{i-1} - m_i
\]
and minimum squared Euclidean distance \( \Delta_i \). From 6.1 and 6.2, we have
\[
\begin{align*}
k_1 &= m_2 + m_3 + \cdots + m_{q-1} + m_q \\
k_2 &= m_3 + m_4 + \cdots + m_q \\
&\vdots \\
k_{q-1} &= m_q \\
k_q &= 0
\end{align*}
\]
We also note that \( \Delta_0 \leq \Delta_1 \leq \cdots \leq \Delta_q \) and that \( \Lambda_q \) consists of only the all-zero codeword with \( \Delta_q = \infty \).

Now we are going to construct \( q \) coset codes from \( \Lambda_0, \Lambda_1, \ldots, \Lambda_q \). These \( q \) coset codes will be used as the inner codes in the proposed \( q \)-level concatenated coded modulation scheme. First we partition \( \Lambda_0 \) into \( 2^{m_1} \) cosets modulo \( \Lambda_1 \). Let \( \Lambda_0/\Lambda_1 \) denote the set of cosets of \( \Lambda_0 \) modulo \( \Lambda_1 \). The minimum squared Euclidean distance of each coset in \( \Lambda_0/\Lambda_1 \) is \( \Delta_1 \). The minimum squared distance between two cosets in \( \Lambda_0/\Lambda_1 \) is \( \Delta_0 \). \( \Lambda_0/\Lambda_1 \) is called the coset code of \( \Lambda_0 \) modulo \( \Lambda_1 \). Next we partition each coset in \( \Lambda_0/\Lambda_1 \) into \( 2^{m_2} \) cosets modulo \( \Lambda_2 \). Let \( \Lambda_0/\Lambda_1/\Lambda_2 \) denote the set of cosets of a coset in \( \Lambda_0/\Lambda_1 \) modulo \( \Lambda_2 \). It is clear that the minimum squared Euclidean distance of a coset in \( \Lambda_0/\Lambda_1/\Lambda_2 \) is \( \Delta_2 \), and the minimum squared distance among the cosets of a coset in \( \Lambda_0/\Lambda_1/\Lambda_2 \) is \( \Delta_1 \). We call \( \Lambda_0/\Lambda_1/\Lambda_2 \) the coset code of \( \Lambda_0/\Lambda_1 \) modulo \( \Lambda_2 \). We continue the above partition process to form coset codes. For \( 1 \leq i \leq q \), let \( \Lambda_0/\Lambda_1/\cdots/\Lambda_{i-1} \) be the coset code of \( \Lambda_0/\Lambda_1/\cdots/\Lambda_{i-2} \) modulo \( \Lambda_{i-1} \). We partition each coset in \( \Lambda_0/\Lambda_1/\cdots/\Lambda_{i-1} \) into \( 2^{m_i} \) cosets modulo \( \Lambda_i \). Then \( \Lambda_0/\Lambda_1/\cdots/\Lambda_i \) is the
coset code of $\Lambda_0/\Lambda_1/\cdots/\Lambda_{i-1}$ modulo $\Lambda_i$. The minimum squared Euclidean distance of a coset in $\Lambda_0/\Lambda_1/\cdots/\Lambda_i$ is $\Delta_i$, and the minimum squared distance among the cosets of a coset in $\Lambda_0/\Lambda_1/\cdots/\Lambda_{i-1}$ modulo $\Lambda_i$ is $\Delta_{i-1}$. Note that each coset in $\Lambda_0/\Lambda_1/\cdots/\Lambda_{q-1}$ consists of $2^{m_q}$ codewords in $\Lambda_0$. Since $\Delta_q = \{0\}$, each coset in $\Lambda_0/\Lambda_1/\cdots/\Lambda_q$ consists of only one codeword in $\Lambda_0$. Hence the minimum squared Euclidean distance of each coset is $\Delta_q = \infty$. The minimum squared distance among the cosets of a coset in $\Lambda_0/\Lambda_1/\cdots/\Lambda_{q-1}$ modulo $\Lambda_q$ is $\Delta_{q-1}$. The above partition process results in a sequence of $q$ coset codes,

$$A_1 = \Lambda_0/\Lambda_1$$

$$A_2 = \Lambda_0/\Lambda_1/\Lambda_2$$

$$\vdots$$

$$A_q = \Lambda_0/\Lambda_1/\cdots/\Lambda_q.$$  \hspace{1cm} (6.4)

These $q$ coset codes are used as inner codes in the proposed $q$-level concatenated coded modulation scheme. This $q$-level concatenated modulation code $C$ is denoted as follows:

$$C \triangleq \{B_1, B_2, \cdots, B_q\} \ast \{A_1, A_2, \cdots, A_q\}. \hspace{1cm} (6.5)$$

If $\Lambda_0, \Lambda_1, \cdots, \Lambda_{q-1}$ have simple trellis diagrams, the coset inner codes, $A_1, A_2, \cdots, A_q$, also have a simple trellis diagrams. If the coset inner codes have simple trellis structure, then we can use Viterbi decoding to decode the coset inner codes. This will decrease the decoding complexity of the inner codes drastically.

Table 6.1 gives a list of various BCM inner codes for this multilevel concatenation coded modulation scheme. Codes in Table 6.1 do have phase invariance. Kasami et al. in [38] derived a condition on phase invariance of basic $l$-level block modulation codes over $2^l$-PSK signal constellation.
### Table 6.1: Block Modulation Inner Codes

<table>
<thead>
<tr>
<th>Code</th>
<th>MSED</th>
<th>MSD</th>
<th>MPD</th>
<th>Phase Invariance</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8,4)<em>(8,7)</em>(8,7)</td>
<td>2.0</td>
<td>4.0</td>
<td></td>
<td>90°</td>
</tr>
<tr>
<td>(16,11)<em>(16,11)</em>(16,15)</td>
<td>2.0</td>
<td>4.0</td>
<td></td>
<td>90°</td>
</tr>
<tr>
<td>(16,11)<em>(16,15)</em>(16,15)</td>
<td>2.0</td>
<td>4.0</td>
<td></td>
<td>90°</td>
</tr>
</tbody>
</table>

#### 6.2.2 Determination of Outer Codes

In a multilevel concatenated coded modulation scheme, multilevel closest coset decoding is used. In the multilevel closest coset decoding process, decoded estimates from the present level are passed to the next stage of decoding. Therefore, in the severe fading channel, error propagation is expected with multilevel decoding. However, the multilevel concatenation modulation scheme allows us the freedom to choose error-correcting capabilities of outer codes to reduce the effects of error propagation to the next stage of decoding. In the following, a method to determine error correcting capabilities of outer codes in the multilevel concatenated modulation scheme is discussed.

Suppose RS codes from $GF(2^p)$ are used as the outer codes. To find the best combination of outer codes for the multilevel concatenated BCM code, we must find the conditional $p$-bit symbol error probability from the inner code decoder for the $i$-th level outer code $B_i = \{B_{i,1}, B_{i,2}, \ldots, B_{i,m_i}\}$ for $1 \leq i \leq q$. In $i$-th level inner closest coset decoding, we assume that all inner and outer code decodings of the previous $i-1$ levels are correct. Based on these assumptions, we can find the conditional $p$-bit symbol error probabilities after $i$-th level inner code decoding. Using these conditional symbol error probabilities, we can find error correcting capabilities for $q$-sets of outer codes. Error correcting capabilities of outer codes are chosen to achieve bit error rate $10^{-6} - 10^{-7}$. Since decodings of each level achieve bit error
rate $10^{-6} - 10^{-7}$, error propagation becomes negligible. Therefore, we can achieve the best possible error performance and communication at the BER $10^{-4}$ before encountering the error floor. By finding the best combination of outer codes for the multilevel concatenated BCM scheme, we can achieve reliable communication with high performance, high spectral efficiency, reduced decoding complexity, and without the *error floor effect* due to the multipath fading and the Doppler frequency shift.

### 6.2.3 Encoding

An organization of the overall encoder for a $q$-level concatenated coded modulation system is shown in Figure 6.1. Every inner code encoder, except the first-level, has two inputs, one from the output of an outer code encoder and one from the output of the inner code encoder of the *preceding* level. For $1 \leq i \leq q$, the $i$-th level encoding is accomplished in two steps:

1. **Outer code encoding** - For $1 \leq i \leq q$ and $1 \leq j \leq m_i$, a message of $K_ip$ bits (or $K_i$ $p$-bit bytes) is encoded into a codeword of $N$ $p$-bit bytes in the $i$-th level outer code $B_{i,j}$. After encoding of $m_i (N,K_i,D_i)$ RS codes, we form a array of $m_i$ by $N_p$ such that each element of the array is a symbol from $GF(2^p)$. After converting all symbols over $GF(2^p)$ in the array into $p$-bits binary representation, $m_i N_p$ bits are put into $m_i$ by $Np$ array $\Gamma_i$. From now on, we refer to this array $\Gamma_i$ as the $i$-th level outer code array. Each column of the array $\Gamma_i$ contains $m_i$-bits where each bit came from $m_i$ outer codes $B_{i,1}, B_{i,2}, \cdots, B_{i,m_i}$, respectively.

2. **Inner code encoding** - The input coset from the $(i-1)$-th level inner encoder is partitioned into $2^{m_i}$ cosets modulo $\Lambda_i$. Each $m_i$-bit byte input from the $i$-th
level outer code array $\Gamma_i$ is encoded into a coset (or coset leader) in the coset code $A_i = A_0 / A_1 / \cdots / A_i$. Therefore the output of the $i$-th level inner code encoder is a sequence of cosets from $A_i = A_0 / A_1 / \cdots / A_i$. The output of the $q$-th level inner code encoder is a sequence of codewords from the base inner code $A_0$.

The above multilevel concatenated coded modulation scheme is easy to implement because outer codes at each level have same length, say $N$. Therefore, the overall concatenated code $\hat{C}$ is a modulation code over the signal set $S$ with length $nNp$, dimension $\hat{K} = (m_1K_1+m_2K_2+\cdots+m_qK_q)p$ and minimum squared Euclidean distance

$$D[\hat{C}] \geq \min_{1 \leq i \leq q} \{ D_i \Delta_{i-1} \}. \quad (6.6)$$

The spectral efficiency and effective rate of $\hat{C}$ are:

$$\eta[\hat{C}] \triangleq \frac{\hat{K}}{nNp} \text{ bits/signal} \quad (6.7)$$

and

$$R[\hat{C}] \triangleq \frac{\hat{K}}{2nNp} \text{ bits/dimension} \quad (6.8)$$

respectively.

### 6.3 Multilevel Closest Coset Decoding

A multilevel closest coset decoding for the proposed scheme is presented in this section. Suppose that all of the outer codes have the same length, say $N = 2^p - 1$. Then the overall concatenated code $\hat{C}$ is a modulation code of length $nNp$. Let

$$\mathbf{V} = (\mathbf{v}_0, \mathbf{v}_1, \cdots, \mathbf{v}_t, \cdots, \mathbf{v}_{N(p-1)}) \quad (6.9)$$
be the transmitted code sequence, where \( v_\ell \) is a codeword in one of the cosets of the coset code \( A_1 = \Lambda_0/\Lambda_1 \) with length \( n \). Let

\[
R = (r_0, r_1, \ldots, r_\ell, \ldots, r_{N(p-1)})
\]  

(6.10)

be the received sequence. Decoding is carried out in \( q \) steps, from the first level to the \( q \)-th level.

At the first-level of decoding, we decode \( r_\ell \) into one of the cosets in \( A_1 = \Lambda_0/\Lambda_1 \). Based on the decoded coset, we identify an \( m_1 \)-bit byte and form an \( m_1 \) by \( Np \) array by a reversing the order of the first level outer code encoding. Later we form \( m_1 \) by \( N \) array with symbols from \( GF(2^p) \). Then we identify the \( p \)-bit byte \( \tilde{e}_\ell^{(1,j)} \) which is the estimate of the output byte of the first-level RS outer code \( B_{1,j} \) encoder at the time \( \ell \) for \( 1 \leq j \leq m_1 \). Then, decode the sequence

\[
\tilde{E}^{(1,j)} = (\tilde{e}_0^{(1,j)}, \tilde{e}_1^{(1,j)}, \ldots, \tilde{e}_\ell^{(1,j)}, \ldots, \tilde{e}_{N-1}^{(1,j)})
\]  

(6.11)

based on the RS outer code \( B_{1,j} \). Let

\[
\tilde{B}^{(1,j)} = (\tilde{b}_0^{(1,j)}, \tilde{b}_1^{(1,j)}, \ldots, \tilde{b}_\ell^{(1,j)}, \ldots, \tilde{b}_{N-1}^{(1,j)})
\]  

(6.12)

be the decoded codeword in \( B_{1,j} \) for \( 1 \leq j \leq m_1 \). Then the input message sequence is retrieved from this decoded codeword. Furthermore, by using \( m_1 \) set of decoded codeword, we reproduce a coset sequence

\[
\Omega^{(1)} = (\Omega_0^{(1)}, \Omega_1^{(1)}, \ldots, \Omega_\ell^{(1)}, \ldots, \Omega_{N(p-1)}^{(1)})
\]  

(6.13)

at the output of the first-level decoder, where \( \Omega_\ell^{(1)} \in \Lambda_0/\Lambda_1 \). The reproducing process is exactly the same as the 1st level outer and inner code encodings. This coset sequence is then applied to the second-level inner code decoder.

Now we perform the second-level decoding. For \( 0 \leq \ell \leq N(p-1) \), we used the input information \( \Omega_\ell^{(1)} \) to decode \( r_\ell \) into one of the cosets in \( \Omega_\ell^{(1)}/\Lambda_2 \). We use the
decoded coset to identify the p-bit byte $\tilde{e}_t^{(2,j)}$ which is the estimate of the output byte of the second-level RS outer code $B_{2,j}$ encoder at time $t$ for $1 \leq j \leq m_2$. Next, decode

$$\mathbf{\tilde{E}}^{(2,j)} = (\tilde{e}_0^{(2,j)}, \tilde{e}_1^{(2,j)}, \ldots, \tilde{e}_t^{(2,j)}, \ldots, \tilde{e}_{N-1}^{(2,j)})$$  \quad (6.14)$$

based on the RS outer code $B_{2,j}$. Let

$$\mathbf{\tilde{B}}^{(2,j)} = (\tilde{b}_0^{(2,j)}, \tilde{b}_1^{(2,j)}, \ldots, \tilde{b}_t^{(2,j)}, \ldots, \tilde{b}_{N-1}^{(2,j)})$$  \quad (6.15)$$

be the decoded codeword in $B_{2,j}$. Retrieve the second-level input message sequence from this decoded codeword. For $1 \leq j \leq m_2$ and we use $(\tilde{b}_0^{(2,j)}, \tilde{b}_1^{(2,j)}, \ldots, \tilde{b}_t^{(2,j)}, \ldots, \tilde{b}_{N-1}^{(2,j)})$, to reproduce a coset sequence

$$\Omega^{(2)} = (\Omega_0^{(2)}, \Omega_1^{(2)}, \ldots, \Omega_t^{(2)}, \ldots, \Omega_{N(p-1)}^{(2)})$$  \quad (6.16)$$

at the output of the second-level decoder, where $\Omega_t^{(2)} \in \Lambda_0 / \Lambda_1 / \Lambda_2$. The reproducing process is exactly same as the 2nd level outer code and inner code encodings. This coset sequence is then applied to the third-level decoder. Other levels of decoding are carried out in the same manner. The overall multilevel decoder is shown in Figure 6.3.

Since decoded information at each level is passed to the next level, decoding at each level depends on decoded information from the preceding level. Therefore, error propagation may occur. To reduce the probability of error propagation, outer codes must be selected by considering the specific channel characteristic. In the following sections, multilevel concatenated codes are constructed by the following rules. In the AWGN channel, strong outer codes must be used for levels where the inner codes have small minimum squared Euclidean distances. In the Rayleigh fading channel, strong outer codes must used for levels where inner codes have small minimum symbol and product distances.
Figure 6.3. Multilevel Concatenated Modulation Code Decoder
In the following section, 3-level 8-PSK concatenated BCM schemes are presented. In the 3-level concatenated schemes, long and short BCM codes are used to construct coset inner codes. Later 6-level and 2-level concatenated BCM schemes are also constructed using short inner coset codes.

6.4 Three-level Concatenated BCM Schemes

In the following, several 3-level 8-PSK (or 8-DPSK) concatenated BCM schemes are constructed for the Rayleigh fading and various shadowed MSAT channels. The RS code over $GF(2^8)$ is used as the outer code. The error correcting capability of the outer code for each level is determined by the conditional symbol error probability from the inner code decoder. To minimize error propagation to the next level decoding, the error correcting capability of the outer code at the present level is chosen to achieve a bit error rate less than $10^{-6}$.

6.4.1 Scheme 1:

Inner Coset Codes Construction:

$\Lambda_0 = \lambda[ C_1 * C_2 * C_3 ] = \lambda[(8,4) * (8,7) * (8,7)]$ is a BCM code of length 8, dimension 18, minimum squared Euclidean distance 2.344, minimum symbol distance 2, and minimum product distance 4. From $\Lambda_0$, we construct a sequence of subcodes as following;

$\Lambda_1 = \lambda[0 * C_2 * C_3] = \lambda[(0) * (8,7) * (8,7)]$

$\Lambda_2 = \lambda[0 * 0 * C_3] = \lambda[(0) * (0) * (8,7)]$

$\Lambda_3 = \lambda[0 * 0 * 0] = \lambda[(0) * (0), (0)]$
Then we have the following 3 coset codes

\[ A_1 = \Lambda_0/\Lambda_1 \]
\[ A_2 = \Lambda_0/\Lambda_1/\Lambda_2 \]
\[ A_3 = \Lambda_0/\Lambda_1/\Lambda_2/\Lambda_3 \]

The minimum squared Euclidean distances of \( A_1, A_2, \) and \( A_3 \) are 2.344, 4, and 8, respectively. The minimum symbol distances of \( A_1, A_2, \) and \( A_3 \) are 4, 2, and 2, respectively. The minimum product distances of \( A_1, A_2, \) and \( A_3 \) are 0.117, 4, and 16, respectively. Outer codes are chosen based on conditional symbol error probabilities of inner coset codes for a given communication channel.

**Single-level concatenated BCM code:**

For comparison purpose, we construct a single-level concatenated code with \( \Lambda_0 \) as the inner code. The outer code is the (255,227,29) RS code which is interleaved to a depth of 18. Let \( C(S16) \) denote this single-level concatenated code. Each block of the code \( C(S16) \) contains \( 227 \times 18 \times 8 \) bits. In decoding the code \( C(S16) \), the inner code \( \Lambda_0 \) is decoded using three stage decoding and decoded estimates from the inner decoder are passed to the outer code decoder at the same time. Therefore, all information bits of the inner code are protected to the same degree. The spectral efficiency of the resulting single-level 8-PSK concatenated modulation code \( C(S16) \) is 2.00294 bits/dimension. The bit-error performances of \( C(S16) \) over the Rayleigh and shadowed MSAT channels are compared with the following 3-level concatenated modulation code with the same inner code.
Frequency non-selective Rayleigh fading channel

In the frequency non-selective Rayleigh fading channel, the error performance of modulation codes are heavily dependent on minimum symbol and minimum product distances for high SNR. Therefore, the bit error rate of the BCM code \( A_0 \) will be dominated by the 2nd component code \( C_2 \) at high SNR because \( C_2 \) has the smallest minimum symbol distance. Therefore, the bit error rate of a 3-level BCM code will be dominated by that of 2nd level code \( C_2 \). Even though the 1st level code \( C_1 \) has the smallest MSED among the three codes, \( C_1 \) achieves the best error performance at high SNR because it has the largest minimum symbol distance.

For the frequency non-selective Rayleigh fading channel, selected outer codes are as follows:

\[
\begin{align*}
B_1 &= (255, 217, 39) \text{ Reed Solomon code} \\
B_2 &= (255, 221, 35) \text{ Reed Solomon code} \\
B_3 &= (255, 239, 17) \text{ Reed Solomon code}
\end{align*}
\]

Then we have the following 3-level concatenated BCM codes \( C(16) \)

\[
C(16) = \{B_1, B_2, B_3\} \ast \{A_1, A_2, A_3\}. \tag{6.17}
\]

The spectral efficiency of the code \( C(16) \) is 2.003921569 bits/symbol. A codeword in \( C(16) \) contains \( (217 \times 4 + 221 \times 7 + 239 \times 7) \times 8 = 32704 \) information bits and a codeword in \( C(S16) \) contains \( 227 \times 18 \times 8 = 32688 \) information bits. Therefore, a codeword in \( C(16) \) can transmit 160 information bits more than a codeword in \( C(S16) \).

Three-level closest coset decoding is used to decode the code \( C(16) \). Each decoding level consists of inner closest coset decoding and outer code decoding. At
$i$-th level of decoding, the inner coset code $A_i$ is decoded by using the decoded estimates from the first level decoder to the $(i - 1)$ level decoder. In the multilevel concatenated scheme, the $(i - 1)$ level outer code is designed to reduce the error propagation from the $(i - 1)$ level decoder to the $i$ level decoder. The inner code decoding complexity of the single-level code $C(S16)$ is the same as that of the code $C(16)$. For the code $C(S16)$, the inner code $A_0$ is decoded first using 3-stage decoding and then followed by the outer decoding. To decode the code $C(S16)$, we decode $A_1$ first and then decode $A_2$ and then decode $A_3$ and finally decode the outer code. To decode the code $C(16)$, we decode $A_1$ and $B_1$, $A_2$ and $B_2$, and $A_3$ and $B_3$. Therefore, for the code $C(S16)$, all 18 information bits in the inner code $A_0$ are decoded by the same Reed-Solomon code $(255,227,29)$ and hence are protected with the same degree. This means that the error performance of the code $C(S16)$ heavily depends on the worst error performance among the three-stage inner code decodings. However, the BER of 4 bits in the first level component code $(8,4,4)$, the BER of 7-bits in the 2nd level component code $(8,7,2)$, and the BER of 7-bits in the 3rd level component code $(8,7,2)$ are all different. Therefore, they need different levels of protection. For example, over the AWGN channel, the 1st level component code $(8,4,4)$ dominates the error performance because the minimum squared Euclidean distance is the most important factor in designing a code. Over the Rayleigh fading channel, the 2nd level component code $(8,7,2)$ dominates the error performance because the minimum symbol and product distance are the most important factors in designing a code. That is why the second level inner code is more heavily protected than the third level inner code of the code $C(16)$. By choosing the outer codes as such in $C(16)$, we can reduce the error propagation to the next stage decoding in the multi-stage closest coset decoding and we can achieve an impressive coding gain over the uncoded QPSK modulation.
Infinite interleaving and perfect phase tracking are assumed in simulation. The error performance of the coherently detected 8-PSK modulation code $C(16)$ over the Rayleigh fading channel is shown in Figure 6.4. Error performance of the coherently detected 8-PSK single-level concatenated modulation code $C(S16)$ over the Rayleigh fading channel is also shown in Figure 6.4. At the BER $10^{-5}$, the code $C(16)$ achieves a 30.757 dB real coding gain over the uncoded QPSK modulation and a 0.798 dB coding gain over the code $C(S16)$. Both concatenated codes have about the same spectral efficiency.

Coherently detected 8-PSK modulation code over the shadowed MSAT Channels

The error performance of the coherently detected 8-PSK modulation code $C(16)$ are evaluated for various shadowed MSAT channels. The error performance of the
coherently detected 8-PSK modulation code $C(16)$ over the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 6.5, 6.6, and 6.7, respectively. Table 6.2 summarizes the error performance of the coherently detected 8-PSK modulation code $C(16)$ over various shadowed MSAT channels. The code $C(16)$ achieves an impressive coding gain over the code $C(S16)$ and over uncoded QPSK modulation over all shadowed MSAT channels.

**Differentially detected 8-PSK modulation code over the shadowed MSAT channels**

The error performance of the 8-DPSK modulation code $C(16)$ over the light-, average-, heavy-shadowed MSAT channels is shown in Figures 6.8, 6.9 and 6.10, respectively. Table 6.3 summarizes the error performance of the 8-DPSK modulation code $C(16)$ for various shadowed MSAT channels. In the case of the differentially detected 8-PSK modulation code, we assume that the phase of the received 8-DPSK signal is randomly changing due to the multipath fading. Over a heavy shadowed MSAT channel with $BT = 0.05$, the single-level concatenated BCM code $C(S16)$ shows an error floor phenomenon. The outer code of $C(S16)$ can correct 14 symbols. To achieve a BER of $10^{-6}$ with these outer codes, the 8-bit symbol error probability of the inner codes must be at most 0.015. However, when the inner code of $C(S16)$ encounters an error floor, the 8-bit symbol error probability of the 1st level component code (8,4,4) of the inner code $\Lambda_0$ is bigger than 0.015. Therefore, $C(S16)$ faces an error floor at $5.169 \times 10^{-4}$. However, the first level outer code of $C(16)$ can correct 19 symbols and the second level $C(16)$ can correct 17 symbols. Therefore, the code $C(16)$ can provide better protection to the 1st and 2nd level component codes (8,4,4) and (8,7,2) of the inner code. As a result, the code $C(16)$ encounters an error floor at $2.35 \times 10^{-5}$ which is much lower than $5.169 \times 10^{-4}$. 

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Figure 6.5. BER of the 3-level 8-PSK concatenated modulation code $C(16)$ over a light-shadowed MSAT channel.

Figure 6.6. BER of the 3-level 8-PSK concatenated modulation code $C(16)$ over an average-shadowed MSAT channel.
Table 6.2. Bit error performance of the coherently detected 8-PSK 3-level concatenated modulation code $C(16)$ over the shadowed MSAT channel.

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_0$</th>
<th>$E_b/N_0$</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>@ $10^{-4}$ BER</td>
<td>@ $10^{-5}$ BER</td>
<td>over QPSK</td>
<td>over QPSK</td>
<td>over $C(16)$</td>
<td>over $C(S16)$</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>9.959</td>
<td>10.321</td>
<td>15.157</td>
<td>25.104</td>
<td>0.065</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>9.791</td>
<td>10.016</td>
<td>16.409</td>
<td>25.751</td>
<td>0.558</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>10.554</td>
<td>11.03</td>
<td>17.823</td>
<td>27.604</td>
<td>0.343</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>10.228</td>
<td>10.506</td>
<td>18.372</td>
<td>28.128</td>
<td>0.41</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>13.4</td>
<td>14.247</td>
<td>27.546</td>
<td>37.03</td>
<td>0.776</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>13.4</td>
<td>14.144</td>
<td>25.04</td>
<td>33.853</td>
<td>0.865</td>
</tr>
</tbody>
</table>

Figure 6.7. BER of the 3-level 8-PSK concatenated modulation code $C(16)$ over a heavy-shadowed MSAT channel.
Figure 6.8. BER of the 3-level 8-DPSK concatenated modulation code $C(16)$ over a light-shadowed MSAT channel.

Figure 6.9. BER of the 3-level 8-DPSK concatenated modulation code $C(16)$ over an average-shadowed MSAT channel.
Figure 6.10. BER of the 3-level 8-DPSK concatenated modulation code $C(16)$ over a heavy-shadowed MSAT channel.

Table 6.3. Bit error performance of the differentially detected 8-PSK 3-level concatenated modulation code $C(16)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_0$</th>
<th>$E_b/N_0$</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$@10^{-4}$</td>
<td>$@10^{-5}$</td>
<td>@10^{-4} BER</td>
<td>@10^{-5} BER</td>
<td>over 4-DPSK</td>
<td>over $C(S16)$</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>14.341</td>
<td>14.923</td>
<td>14.614</td>
<td>0.0000235*</td>
<td>0.559</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>14.3</td>
<td>14.67</td>
<td>0.0001973*</td>
<td></td>
<td>0.434</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>15.261</td>
<td>15.959</td>
<td>19.477</td>
<td>0.0000566*</td>
<td>0.466</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>15.172</td>
<td>15.598</td>
<td>0.0004744*</td>
<td></td>
<td>0.868</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>19.477</td>
<td>20.602</td>
<td>0.00144*</td>
<td></td>
<td>0.964</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>29.437</td>
<td>0.0000235*</td>
<td>0.00803*</td>
<td></td>
<td>0.0005169*</td>
</tr>
</tbody>
</table>

* : bit-error-rate at the error floor

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6.4.2 Scheme 2:

In this scheme, RM codes of length 16 are used as the inner codes of the three-level concatenated BCM code.

**Inner Coset Codes Construction:**

\[ \Lambda_0 = \lambda[C_1 * C_2 * C_3] = \lambda[(16,11) * (16,11) * (16,15)] \]

is a BCM code of length 16, dimension 37, minimum squared Euclidean distance 2.344, minimum symbol distance 2, and minimum product distance 16. From \( \Lambda_0 \), we construct a sequence of subcodes as following;

\[ \Lambda_1 = \lambda[0 * C_2 * C_3] = \lambda[(0) * (16,11) * (16,15)] \]
\[ \Lambda_2 = \lambda[0 * 0 * C_3] = \lambda[(0) * (0) * (16,15)] \]
\[ \Lambda_3 = \lambda[0 * 0 * 0] = \lambda[(0) * (0) * (0)]. \]

Then we have the following 3 coset codes

\[ A_1 = \Lambda_0/\Lambda_1 \]
\[ A_2 = \Lambda_0/\Lambda_1/\Lambda_2 \]
\[ A_3 = \Lambda_0/\Lambda_1/\Lambda_2/\Lambda_3. \]

The minimum squared Euclidean distances of \( A_1, A_2, \) and \( A_3 \) are 2.344, 8, and 8, respectively. The minimum symbol distances of \( A_1, A_2, \) and \( A_3 \) are 4, 4, and 2, respectively. The minimum product distances of \( A_1, A_2, \) and \( A_3 \) are 0.117, 16, and 16, respectively. The selection of the outer codes is based on the conditional symbol error probabilities of the inner coset codes for a given communication channel.

For the Rayleigh fading channel, the error performance of modulation codes heavily depends on the minimum symbol and minimum product distances for high SNR's. Therefore, the bit error rate of a BCM code \( \Lambda_0 \) is dominated by the that
of \( C_3 \) at high SNR's. The code \( C_3 \) needs more protection than the code \( C_2 \) even though they have the same minimum squared Euclidean distance.

For the AWGN channel, the error performance of modulation codes depends heavily on the minimum squared Euclidean distance for high SNR's. Therefore, the bit error rate of \( \Lambda_0 \) will be dominated by the that of the 1st level component code, \( C_1 \) at high SNR's. The error performance of the inner code \( \Lambda_0 \) over the Rayleigh fading and various shadowed MSAT channels is shown in the chapter 5.

**Outer Codes Construction:**

For comparison purposes, we construct a single-level concatenated code \( C(S17) \) with \( \Lambda_0 \) as the inner code. As the outer code, a \((255,221,35)\) RS code is interleaved to a depth of 37. A codeword in \( C(S17) \) contains \( 221 \times 37 \times 8 = 65416 \) information bits. The spectral efficiency of the resulting single-level 8-PSK concatenated modulation code \( C(S17) \) is 2.004166 bits/signal. The bit-error performance of \( C(S17) \) for the Rayleigh and various shadowed MSAT channels is compared with the 3-level 8-PSK concatenated modulation codes using the same BCM code \( \Lambda_0 \).

**Frequency Non-selective Rayleigh Fading Channel**

In the frequency non-selective Rayleigh fading channel, the error performance of the inner code heavily depends on the minimum symbol distance and the minimum product distance of the inner code. Therefore, we provide a powerful outer code to the inner code with the smallest minimum symbol and product distances. The selected outer codes are as follows:

\[
B_1 = (255,185,71)\text{Reed Solomon code} \\
B_2 = (255,239,17)\text{Reed Solomon code}
\]
$$B_3 = (255, 235, 21)$$ Reed Solomon code.

Then we have a 3-level concatenated BCM code $C(17)$

$$C(17) = \{B_1, B_2, B_3\} \ast \{A_1, A_2, A_3\}. \quad (6.18)$$

A codeword in $C(17)$ contains $(185 \times 11 + 239 \times 11 + 235 \times 15) \times 8 = 65512$ information bits. The spectral efficiency of $C(17)$ is 2.0071. A codeword in $C(17)$ transmits 96 bits more than that of $C(S17)$. For the frequency non-selective Rayleigh fading channel, infinite interleaving and perfect phase tracking are assumed in simulation. The error performance of the coherently detected 8PSK modulation code $C(17)$ for the Rayleigh fading channel is shown in Figure 6.11. The error performance of the coherently detected 8PSK modulation code $C(S17)$ for the Rayleigh fading channel is also shown in Figure 6.11. $C(17)$ achieves a coding gain of 1.081 dB over $C(S17)$ and 30.852 dB over the uncoded QPSK modulation at the BR of $10^{-5}$. 

Coherently detected modulation codes over the shadowed MSAT channels

The error performance of the coherently detected 8PSK modulation code $C(17)$ over various shadowed MSAT channels is shown in Figures 6.12, 6.13 and 6.14 respectively. The code $C(17)$ achieves impressive coding gains over the uncoded QPSK modulation and $C(S17)$. Table 6.4 summarizes the error performance of the coherently detected 8-PSK modulation code $C(17)$ over various shadowed MSAT channels.
Figure 6.11. BER of the 3-level 8-PSK concatenated modulation code $C(17)$ over the Rayleigh fading channel.

Figure 6.12. BER of the 3-level 8-PSK concatenated modulation code $C(17)$ over a light-shadowed MSAT channel.
Figure 6.13. BER of the 3-level 8-PSK concatenated modulation code $C(17)$ over an average-shadowed MSAT channel.

Figure 6.14. BER of the 3-level 8-PSK concatenated modulation code $C(17)$ over a heavy-shadowed MSAT channel.
Table 6.4. Bit error performance of the coherently detected 8-PSK 3-level concatenated modulation code C(17) over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_0$ Coding Gain @</th>
<th>$E_b/N_0$ Coding Gain @</th>
<th>@</th>
<th>@</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$10^{-4}$ BER</td>
<td>$10^{-5}$ BER</td>
<td>$10^{-4}$ BER</td>
<td>$10^{-5}$ BER</td>
</tr>
<tr>
<td>Light</td>
<td>0.02 10.891 11.092 14.225 24.333 0.602 0.823</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05 10.73 10.951 15.47 24.816 0.691 0.715</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.02 11.465 11.538 16.912 27.096 0.727 1.046</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05 11.208 11.317 17.392 27.317 0.852 0.966</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 13.879 14.2 27.067 37.08 0.982 0.996</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05 13.799 14.08 24.641 33.92 1.08 1.14</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Differentially detected 8-PSK modulation code over various shadowed MSAT channels

The error performance of the 8-DPSK modulation code $C(17)$ over various shadowed MSAT channels is shown in Figures 6.15, 6.16 and 6.17, respectively. For the heavy shadowed MSAT channel and $BT = 0.05$, the code $C(S17)$ exhibits an error floor at a BER of $3.53 \times 10^{-3}$. However, $C(17)$ achieves a BER of $10^{-4}$ at SNR of 39.718 dB and exhibits an error floor phenomenon at a BER of $7.65 \times 10^{-5}$. As shown in Chapter 5, the inner code of $C(16)$ and $C(17)$ exhibits an error floor at a BER of $4.744 \times 10^{-3}$ and $7.9 \times 10^{-3}$, respectively. Therefore, compared with the code $C(16)$, the code $C(17)$ faces the error floor phenomenon at the BER which is higher than the code $C(16)$, i.e., $2.35 \times 10^{-5}$. Table 6.5 summarizes the error performance of the 8-DPSK modulation code $C(17)$ for various shadowed MSAT channels.
Figure 6.15. BER of the 3-level 8-DPSK concatenated modulation code $C(17)$ over a light-shadowed MSAT channel.

Figure 6.16. BER of the 3-level 8-DPSK concatenated modulation code $C(17)$ over an average-shadowed MSAT channel.
Figure 6.17. BER of the 3-level 8-DPSK concatenated modulation code $C(17)$ over a heavy-shadowed MSAT channel.

Table 6.5. Bit error performance of the differentially detected 8-PSK 3-level concatenated modulation code $C(17)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$ @ $10^{-4}$ BER</th>
<th>$E_b/N_o$ @ $10^{-5}$ BER</th>
<th>Coding Gain @ $10^{-4}$ BER over 4-DPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(S)$</th>
<th>Coding Gain @ $10^{-5}$ BER over $C(S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 15.319</td>
<td>0.05 15.253</td>
<td>0.0000235*</td>
<td>0.705</td>
<td>0.783</td>
</tr>
<tr>
<td></td>
<td>0.02 15.542</td>
<td>0.05 15.562</td>
<td>0.0001973*</td>
<td>0.783</td>
<td>0.783</td>
</tr>
<tr>
<td>Average</td>
<td>0.02 15.947</td>
<td>0.05 16.244</td>
<td>0.0000566*</td>
<td>1.047</td>
<td>1.082</td>
</tr>
<tr>
<td></td>
<td>0.02 16.317</td>
<td>0.05 16.55</td>
<td>0.0004744*</td>
<td>1.237</td>
<td>1.542</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 20.04</td>
<td>0.05 20.441</td>
<td>0.00144*</td>
<td>1.606</td>
<td>1.928</td>
</tr>
<tr>
<td></td>
<td>0.02 20.441</td>
<td>0.05 20.441</td>
<td>0.00144*</td>
<td>1.606</td>
<td>1.928</td>
</tr>
</tbody>
</table>

*: bit-error-rate at the error floor
6.4.3 Scheme 3:

In this scheme, an inner code with a higher spectral efficiency than the inner code, \( C(17) \), is used to construct a 3-level concatenated BCM code. Therefore, in this construction, we can provide much stronger outer code than the code \( C(17) \) and maintain the same spectral efficiency.

**Inner Coset Codes Construction**:

\[ \Lambda_0 = \lambda[C_1 \times C_2 \times C_3] = \lambda[(16,11) \times (16,15) \times (16,15)] \]

is a BCM code of length 16, dimension 41, minimum squared Euclidean distance 2.344, minimum symbol distance 2, and minimum product distance 4. The spectral efficiency of \( \Lambda_0 \) is 2.5625 bits/symbol. We construct a sequence of subcodes of \( \Lambda_0 \) as following:

\[ \Lambda_1 = \lambda[0 \times C_2 \times C_3] = \lambda[(0) \times (16,15) \times (16,15)] \]

\[ \Lambda_2 = \lambda[0 \times 0 \times C_3] = \lambda[(0) \times (0) \times (16,15)] \]

\[ \Lambda_3 = \lambda[0 \times 0 \times 0] = \lambda[(0) \times (0), (0)]. \]

Then we have the following 3 coset codes

\[ A_1 = \Lambda_0/\Lambda_1 \]
\[ A_2 = \Lambda_0/\Lambda_1/\Lambda_2 \]
\[ A_3 = \Lambda_0/\Lambda_1/\Lambda_2/\Lambda_3. \]

The minimum squared Euclidean distances of \( A_1, A_2, \) and \( A_3 \) are 2.344, 4, and 8, respectively. The minimum symbol distances of \( A_1, A_2, \) and \( A_3 \) are 4, 2, and 2, respectively. The minimum product distances of \( A_1, A_2, \) and \( A_3 \) are 0.117, 4, and 16, respectively. For a given communication channel, the outer code of each level is selected based on conditional symbol error probabilities of inner coset codes. In the frequency non-selective Rayleigh fading channel, the error performance of a
modulation code depends heavily on the minimum symbol and minimum product distances for high SNR. Therefore, the bit error rate of the BCM code $A_0$ will be dominated by the that of the 2nd component code $C_2$ at high SNR.

**Outer Codes Construction:**

For comparison purpose, we construct a single-level concatenated code $C(S18)$ with $A_0$ as the inner code. As the outer code, a $(255,199,57)$ RS code is interleaved to depth 41. A codeword in $C(S18)$ contains $199 \times 41 \times 8 = 65272$ information bits. The spectral efficiency of the resulting single-level 8-PSK concatenated modulation code $C(S18)$ is 1.99975 bits/signal. The bit-error performance of $C(S18)$ the Rayleigh fading and various MSAT channels is compared with the 3-level 8-PSK concatenated modulation code using the same BCM code $A_0$.

**Frequency Non-selective Rayleigh Fading Channel**

For frequency non-selective Rayleigh fading channel, selected outer codes are as follows:

\[\begin{align*}
B_1 &= (255, 165, 91) \text{ Reed Solomon code} \\
B_2 &= (255, 195, 61) \text{ Reed Solomon code} \\
B_3 &= (255, 235, 21) \text{ Reed Solomon code}
\end{align*}\]

We provide more protection for the 2nd level than the 3rd level because the minimum product distance of the 2nd level inner coset code is 4 and that of the 3rd level inner coset code $A_3$ is 16 even though they have the same minimum symbol distance. Then we have the following 3-level concatenated BCM codes $C(18)$

\[C(18) = \{B_1, B_2, B_3\} \ast \{A_1, A_2, A_3\}.\]  
(6.19)
Figure 6.18. BER of the 3-level 8-PSK concatenated modulation code \( C(18) \) over the Rayleigh fading channel.

A codeword in \( C(18) \) contains \((165 \times 11 + 195 \times 15 + 235 \times 15) \times 8 = 66120\) information bits. A codeword in \( C(18) \) transmit 848 information bits more than a codeword in \( C(S18) \). The spectral efficiency of \( C(18) \) is 2.0257 bits/symbol. The outer codes of \( C(18) \) have better error correcting capabilities than those of \( C(17) \) because the inner code of \( C(18) \) has better spectral efficiency than that of \( C(17) \). For the non-selective Rayleigh fading channel, perfect phase tracking is assumed in simulation. The error performance of the coherently detected 8-PSK modulation code \( C(18) \) over the Rayleigh fading channel is shown in Figure 6.18. The error performance of the coherently detected 8-PSK modulation code \( C(S18) \) over the Rayleigh fading channel is also shown in Figure 6.18. The code \( C(18) \) achieves \( 0.865 \) dB coding gain over the code \( C(S18) \) at the BER of \( 10^{-5} \) without bandwidth expansion. Also, the code \( C(18) \) achieves real coding gain of \( 31.198 \) dB of uncoded QPSK modulation at the BER of \( 10^{-5} \).
Coherently detected 8-PSK modulation code over the shadowed MSAT channel

The error performance of the coherently detected 8-PSK modulation code $C(18)$ over various shadowed MSAT channels is shown in Figures 6.19, 6.20, and 6.21, respectively. The code $C(18)$ outperforms $C(S18)$ and achieves impressive coding gain over uncoded QPSK modulation over all three shadowed MSAT channels without bandwidth expansion. Table 6.6 summarizes error performance for the coherently detected 8-PSK modulation code $C(18)$ over various shadowed MSAT channels. As shown in Table 6.4 and Table 6.6, the $C(18)$ outperforms $C(17)$ over light, average, and heavy shadowed MSAT channels with the same spectral efficiency.

Differentially detected 8-PSK modulation code over shadowed MSAT channels

The error performance of the differentially detected 8-PSK modulation code $C(18)$ over various shadowed MSAT channels is shown in Figures 6.22, 6.23, and 6.24, respectively. The code $C(18)$ achieves a BER of $10^{-5}$ without an error floor over the heavy shadowed MSAT channel with $BT = 0.05$. However, the single level concatenated BCM code $C(S18)$ exhibits an error floor at a BER of $3.489 \times 10^{-4}$ under the same conditions. Codes $C(16)$ and $C(17)$ also exhibit an error floor at a BER of $2.35 \times 10^{-5}$ and $7.65 \times 10^{-5}$, respectively. As shown above, it is desirable to use a block modulation code with large spectral efficiency as the inner code of the multilevel concatenated BCM code because we can use a more powerful RS code as the outer code. Table 6.7 summarizes the error performance of the differentially detected 8-PSK modulation code $C(18)$ for various shadowed MSAT channels.
Figure 6.19. BER of the 3-level 8-PSK concatenated modulation code $C(18)$ over a light-shadowed MSAT channel.

Figure 6.20. BER of the 3-level 8-PSK concatenated modulation code $C(18)$ over an average-shadowed MSAT channel.
Figure 6.21. BER of the 3-level 8-PSK concatenated modulation code \( C(18) \) over a heavy-shadowed MSAT channel.

Table 6.6. Bit error performance of the coherently detected 8-PSK 3-level concatenated modulation codes \( C(18) \) over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>( BT )</th>
<th>( E_b/N_o ) ( @ ) ( 10^{-4} ) BER</th>
<th>( E_b/N_o ) ( @ ) ( 10^{-5} ) BER</th>
<th>Coding Gain @ ( 10^{-4} ) BER over QPSK</th>
<th>Coding Gain @ ( 10^{-5} ) BER over QPSK</th>
<th>Coding Gain @ ( 10^{-4} ) BER over ( C(S18) )</th>
<th>Coding Gain @ ( 10^{-5} ) BER over ( C(S18) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02</td>
<td>10.55</td>
<td>10.77</td>
<td>14.566</td>
<td>24.655</td>
<td>0.57</td>
<td>0.527</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>10.4</td>
<td>10.574</td>
<td>15.8</td>
<td>25.193</td>
<td>0.562</td>
<td>0.646</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>11.01</td>
<td>11.257</td>
<td>17.367</td>
<td>27.37</td>
<td>0.5</td>
<td>0.614</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>10.987</td>
<td>11.1</td>
<td>17.613</td>
<td>27.534</td>
<td>0.442</td>
<td>0.614</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>13.475</td>
<td>13.836</td>
<td>28.20</td>
<td>37.45</td>
<td>4.951</td>
<td>5.164</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>13.426</td>
<td>13.557</td>
<td>24.976</td>
<td>34.443</td>
<td>4.541</td>
<td>4.902</td>
</tr>
</tbody>
</table>

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Figure 6.22. BER of the 3-level 8-DPSK concatenated modulation code $C(18)$ over a light-shadowed MSAT channel.

Figure 6.23. BER of the 3-level 8-DPSK concatenated modulation code $C(18)$ over an average-shadowed MSAT channel.
Figure 6.24. BER of the 3-level 8-DPSK concatenated modulation code $C(18)$ over a heavy-shadowed MSAT channel.

Table 6.7. Bit error performance of the differentially detected 8-PSK 3-level concatenated modulation code $C(18)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_0$ @ $10^{-4}$ BER</th>
<th>$E_b/N_0$ @ $10^{-5}$ BER</th>
<th>Coding Gain @ $10^{-4}$ BER over 4-DPSK</th>
<th>Coding Gain @ $10^{-5}$ BER over 4-DPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(S18)$</th>
<th>Coding Gain @ $10^{-5}$ BER over $C(S18)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 14.698 15.048</td>
<td>14.257</td>
<td>0.0000235*</td>
<td>0.627</td>
<td>0.777</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05 14.819 15.024</td>
<td>0.0001973*</td>
<td></td>
<td>0.735</td>
<td>0.825</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.02 15.574 15.895</td>
<td>0.0000566*</td>
<td></td>
<td>0.675</td>
<td>0.763</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05 15.783 16.024</td>
<td>0.0004744*</td>
<td></td>
<td>0.771</td>
<td>0.923</td>
<td></td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 19.317 19.799</td>
<td>0.00144*</td>
<td></td>
<td>1.044</td>
<td>0.964</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05 29.116 33.775</td>
<td>0.00803*</td>
<td></td>
<td>0.0003489*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* : bit-error-rate at the error floor
Table 6.8. Decoding complexities of the outer codes of single- and 3-level concatenated BCM schemes

<table>
<thead>
<tr>
<th></th>
<th>$C(16)$</th>
<th>$C(S16)$</th>
<th>$C(17)$</th>
<th>$C(S17)$</th>
<th>$C(18)$</th>
<th>$C(S18)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication</td>
<td>7830</td>
<td>7056</td>
<td>31358</td>
<td>21386</td>
<td>74550</td>
<td>64288</td>
</tr>
<tr>
<td>Addition</td>
<td>7830</td>
<td>7056</td>
<td>31358</td>
<td>21386</td>
<td>74550</td>
<td>64288</td>
</tr>
</tbody>
</table>

Remarks

As shown in the examples, three level concatenated BCM codes outperform single level concatenated BCM codes using the same inner BCM codes over the shadowed MSAT channels. Since 3-level concatenated BCM codes can provide different level of protection to each level, we prevent error propagation and achieve error floor free communication over a channel with the Rician factor $K = -19.9$ dB. In contrast to the single-level concatenated BCM codes, 3-level concatenated BCM codes have extra outer code decoding complexity and decoding delay due to the multilevel multi-stage decoding. However, the decoding complexity of the outer codes is usually very small compared with those of the inner coset codes. The decoding complexity [93] of the RS code over $GF(2^t)$ with error-correcting capability $t$ is typically $2 \times t^2$ multiplications and $2 \times t^2$ additions over $GF(2^t)$ (assuming the Berlekamp-Massey decoding algorithm). The decoding complexity of the outer codes of single- and 3-level concatenated BCM schemes is summarized in the Table 6.8. As shown in Table 6.8, the multilevel concatenated scheme needs slightly more decoding complexity than the single-level concatenated scheme with the same inner code. However, the multilevel scheme outperforms the single-level scheme with the same, or more, spectral efficiency.
6.5 Six-Level Concatenated BCM Scheme

In the previous section, a 3-level BCM code is used in the 3-level concatenated BCM scheme. In that scheme, the 3-level BCM code is partitioned into 3-levels and used to form a sequence of inner coset codes. Outer codes are chosen to prevent error propagation to the next stage of decoding. As shown in the examples of the previous section, 3-level concatenated BCM schemes achieve impressive real coding gains over the uncoded reference system for various shadowed MSAT channels. However, it is possible to construct a scheme which achieves a larger coding gain than the 3-level concatenated BCM scheme with slightly more decoding complexity. In the following, we construct a 6-level concatenated BCM code. The 3-level concatenated BCM code provide different level of protection to an inner coset code at each level. However, the 6-level concatenated BCM code provides bit-level error protection to achieve better error performance than the 3-level concatenated BCM code.

6.5.1 Construction

Let \( A_0 = A[C_1 \ast C_2 \ast C_3] \) be the 3-level BCM code where \( C_i \) is a linear binary block code of length \( n \), dimension \( k_i \). Suppose \( C_{i,t} \) is a linear binary code of dimension \( k_{i,t} \) and subcode of \( C_i \) for \( 1 \leq i \leq 3 \). If \( C_{1,t} \) is subcode of \( C_1 \), then we have the following code decomposition:

\[
C_1 = C_{1,1} + \lceil C_1/C_{1,1} \rceil.
\]  

(6.20)

The minimum symbol distance of \( [C_1/C_{1,1}] \) is the same as that of \( C_1 \) but less than that of \( C_{1,1} \). Therefore the error performance of \( C_1 \) will be dominated by that of \( [C_1/C_{1,1}] \). However, in the 3-level concatenated BCM code, the same outer code is used to protect \( k_{1,1} \) bits for \( C_{1,1} \) and \( k_1 - k_{1,1} \) bits for \( [C_1/C_{1,1}] \). If we choose the error correcting capability of the 1st level outer code based on the error performance of
we waste spectral efficiency because we over protect \( k_{1,1} \). To attain better error performance and higher spectral efficiency, we decode \([C_1/C_{1,1}]\) first and then use a proper outer code to produce better estimates of \([C_1/C_{1,1}]\). Then we use these estimates to decode \( C_{1,1} \) and proper outer codes are used to prevent error propagation to the next level of decoding. By doing this at each level, we can achieve better error performance and higher spectral efficiency with little additional decoding complexity.

### 6.5.2 Scheme 4:

**Inner Code Construction:**

Let \( \Lambda_0 = \lambda[C_1 * C_2 * C_3] = \lambda[(8, 4) * (8, 7) * (8, 7)] \) be a 3-level BCM code and form a sequence of subcodes as following:

\[
\begin{align*}
\Lambda_0 &= \lambda[C_1 * C_2 * C_3] = \lambda[(8, 4) * (8, 7) * (8, 7)] \\
\Lambda_1 &= \lambda[C_{1,1} * C_2 * C_3] = \lambda[(8, 1) * (8, 7) * (8, 7)] \\
\Lambda_2 &= \lambda[0 * C_2 * C_3] = \lambda[(0) * (8, 7) * (8, 7)] \\
\Lambda_3 &= \lambda[0 * C_{2,1} * C_3] = \lambda[(0) * (8, 4) * (8, 7)] \\
\Lambda_4 &= \lambda[0 * 0 * C_3] = \lambda[(0) * (0) * (8, 7)] \\
\Lambda_5 &= \lambda[0 * 0 * C_{3,1}] = \lambda[(0) * (0) * (8, 4)] \\
\Lambda_6 &= \lambda[0 * 0 * 0] = \lambda[(0) * (0) * (0)]
\end{align*}
\]

Then we have following 6 coset codes

\[
\begin{align*}
A_1 &= \Lambda_0/\Lambda_1 \\
A_2 &= \Lambda_0/\Lambda_1/\Lambda_2 \\
A_3 &= \Lambda_0/\Lambda_1/\Lambda_2/\Lambda_3
\end{align*}
\]
\[ A_4 = \Lambda_0 / \Lambda_1 / \Lambda_2 / \Lambda_3 / \Lambda_4 \]
\[ A_5 = \Lambda_0 / \Lambda_1 / \Lambda_2 / \Lambda_3 / \Lambda_4 / \Lambda_5 \]
\[ A_6 = \Lambda_0 / \Lambda_1 / \cdots / \Lambda_5 / \Lambda_6 \]

The distance parameters of coset codes are given in Table 6.9.

Table 6.9. Distance parameters for inner coset codes for 6-level concatenated modulation code

<table>
<thead>
<tr>
<th>Coset Code</th>
<th>MSD</th>
<th>MPD</th>
<th>MSED</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>2</td>
<td>4</td>
<td>2.344</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>( A_4 )</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>( A_5 )</td>
<td>2</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>( A_6 )</td>
<td>2</td>
<td>16</td>
<td>16</td>
</tr>
</tbody>
</table>

**Outer Codes Construction:**

For comparison purpose, we consider the single level concatenated BCM code \( C(S16) \) which uses the same inner code \( \Lambda_0 \). The spectral efficiency of the resulting single-level 8-PSK concatenated modulation code \( C(S16) \) is 2.00294 bits/signal. The bit-error performance of the code \( C(S16) \) over the Rayleigh fading and various shadowed MSAT channels is compared with that of the 6-level 8-PSK concatenated modulation code using the same inner code \( \Lambda_0 \).

**Frequency Non-selective Rayleigh Fading Channel**

For this 6-level 8-PSK concatenated BCM codes for the frequency non-selective Rayleigh fading channel, selected outer codes are as follows:

\[ B_1 = (255,207,49) \text{ Reed Solomon code} \]
\[
B_2 = (255, 251, 5) \text{ Reed Solomon code}
\]
\[
B_3 = (255, 181, 75) \text{ Reed Solomon code}
\]
\[
B_4 = (255, 243, 13) \text{ Reed Solomon code}
\]
\[
B_5 = (255, 233, 23) \text{ Reed Solomon code}
\]
\[
B_6 = (255, 251, 5) \text{ Reed Solomon code}
\]

Then we have the following 6-level concatenated BCM codes \( C(19) \)

\[
C(19) = \{B_1, B_2, B_3, B_4, B_5, B_6\} \ast \{A_1, A_2, A_3, A_4, A_5, A_6\}. \quad (6.21)
\]

For the non-selective Rayleigh fading channel, infinite interleaving and perfect phase tracking are assumed in simulation. The error performance of the coherently detected 8PSK modulation code \( C(19) \) over the Rayleigh fading channel is shown in Figure 6.25. The error performance of the coherently detected single-level concatenated 8PSK modulation code \( C(S16) \) over the Rayleigh fading channel is also shown in Figure 6.25. The code \( C(19) \) achieves a 1.31 dB coding gain over the code \( C(S16) \) at a BER of \( 10^{-5} \). Also, the code \( C(19) \) achieves a 31.23 dB coding gain over the uncoded QPSK modulation at a BER of \( 10^{-5} \).

**Coherently detected 8-PSK modulation code over shadowed MSAT channels**

The error performance of the coherently detected 8PSK modulation code \( C(19) \) over the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 6.26, 6.27, and 6.28 respectively. Table 6.10 summarizes the error performance of the coherently detected 8-PSK modulation code \( C(19) \) over various shadowed MSAT channels. The 3-level concatenated BCM code \( C(16) \) and \( C(19) \) use the same BCM code as the inner code. The code \( C(19) \) outperforms the 3-level code \( C(16) \) because inner coset codes are more precisely protected in \( C(19) \) than in \( C(16) \).
Figure 6.25. BER of the 6-level 8-PSK concatenated modulation code $C(19)$ over the Rayleigh fading channel.

**Differentially detected 8-PSK modulation code over shadowed MSAT channels**

The error performance of the differentially detected 8PSK modulation code $C(19)$ over the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 6.29, 6.30, and 6.31, respectively. Table 6.11 summarizes the error performance of the differentially detected 8-PSK modulation code $C(19)$ for various shadowed MSAT channels. For the heavy shadowed MSAT channel with $BT = 0.05$, the code $C(19)$ achieves a BER of $10^{-6}$ without exhibiting an error floor. The code $C(19)$ outperforms codes $C(16)$, $C(17)$, and $C(18)$ over all three different shadowed channels.
Figure 6.26. BER of the 6-level 8-PSK concatenated modulation code $C(19)$ over a light-shadowed MSAT channel.

Figure 6.27. BER of the 6-level 8-PSK concatenated modulation code $C(19)$ over an average-shadowed MSAT channel.
Figure 6.28. BER of the 6-level 8-PSK concatenated modulation code $C(19)$ over a heavy-shadowed MSAT channel.

Table 6.10. Bit error performance of the coherently detected 8-PSK 6-level concatenated modulation codes $C(19)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_0$</th>
<th>$E_b/N_0$</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>@ 10^{-4}</td>
<td>@ 10^{-5}</td>
<td>BER over QPSK</td>
<td>BER over QPSK</td>
<td>BER over $C(S16)$</td>
<td>BER over $C(S16)$</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>0.02</td>
<td>9.686</td>
<td>9.885</td>
<td>15.44</td>
<td>25.54</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.05</td>
<td>9.469</td>
<td>9.783</td>
<td>16.731</td>
<td>25.984</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>0.02</td>
<td>10.186</td>
<td>10.463</td>
<td>18.191</td>
<td>28.171</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.05</td>
<td>9.915</td>
<td>10.2</td>
<td>18.685</td>
<td>28.434</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>0.02</td>
<td>13.04</td>
<td>13.526</td>
<td>27.906</td>
<td>37.754</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.05</td>
<td>13.116</td>
<td>13.518</td>
<td>25.324</td>
<td>34.482</td>
</tr>
</tbody>
</table>
Figure 6.29. BER of the 6-level 8-DPSK concatenated modulation code $C(19)$ over a light-shadowed MSAT channel.

Figure 6.30. BER of the 6-level 8-DPSK concatenated modulation code $C(19)$ over an average-shadowed MSAT channel.
Figure 6.31. BER of the 6-level 8-DPSK concatenated modulation code $C(19)$ over a heavy-shadowed MSAT channel.

Table 6.11. Bit error performance of the differentially detected 8-PSK 6-level concatenated modulation codes $C(19)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_0$</th>
<th>$E_b/N_0$</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>@ 10^{-4}</td>
<td>@ 10^{-5}</td>
<td>@ 10^{-4}</td>
<td>@ 10^{-5}</td>
<td>@ 10^{-4}</td>
<td>@ 10^{-5}</td>
</tr>
<tr>
<td></td>
<td>BER</td>
<td>BER</td>
<td>BER</td>
<td>BER</td>
<td>BER</td>
<td>BER</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>13.923</td>
<td>14.421</td>
<td>15.44</td>
<td>25.54</td>
<td>0.338</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>13.967</td>
<td>14.216</td>
<td>0.0001973*</td>
<td>0.338</td>
<td>0.759</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>14.722</td>
<td>15.124</td>
<td>20.016</td>
<td>0.0000566*</td>
<td>1.318</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>14.779</td>
<td>15.068</td>
<td>0.0004744*</td>
<td>1.261</td>
<td>1.638</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>18.995</td>
<td>19.638</td>
<td>0.00144*</td>
<td>1.446</td>
<td>2.088</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>26.787</td>
<td>31.0</td>
<td>0.00803*</td>
<td>0.000509*</td>
<td></td>
</tr>
</tbody>
</table>

*: bit-error-rate at the error floor
6.6 Two-Level Concatenated BCM Scheme

6.6.1 Construction

For the heavy-shadowed MSAT channel with large Doppler frequency shift (e.g., $BT = 0.05$), the 3-level and 6-level concatenated BCM schemes with Reed-Solomon codes over $GF(2^8)$ did not show the water-fall like bit error performance. To achieve better error performance, we need to use longer Reed-Solomon codes as outer codes. If the total number of information bits of the inner coset codes are big, the decoding complexities of 3-level and 6-level concatenated BCM schemes with longer outer codes will grow exponentially.

To achieve better error performance with less decoding complexity than 3- and 6-level concatenated BCM schemes, a two-level concatenated BCM scheme is devised.

6.6.2 Scheme 5:

In this scheme, only a single Reed-Solomon code over $GF(2^9)$ is used at each level. And a BCM code $\Lambda_0 = \lambda[(8,4,4)\ast(8,7,2)\ast(8,7,2)]$ and its subcode $\Lambda_1 = \lambda[(8,1,8)\ast(8,4,4)\ast(8,4,4)]$ are used to construct inner coset codes $A_1$ and $A_2$. Then we have the following 2 coset codes:

$$A_1 = \Lambda_0/\Lambda_1$$
$$A_2 = \Lambda_0/\Lambda_1/\{0\}.$$

The minimum squared Euclidean distances of $A_1$ and $A_2$ are 2.344 and 4.688, respectively. The minimum symbol distances of $A_1$ and $A_2$ are 2 and 4, respectively. The minimum product distances of $A_1$ and $A_2$ are 0.1179 and 16, respectively. Since we are using Reed-Solomon codes over $GF(2^9)$, we can provide more error protection at the first level of outer decoding. Therefore, we can provide almost error free decoded...
estimates to the second level inner coset code decoding. Therefore, the resultant
two-level code can achieve better error performance than the 3- and 6-level codes.

Closest coset decoding of inner coset codes

First, we decode the 1st level inner coset code $A_1 = \Lambda_0/\Lambda_1$. To decode $A_1$, we use
multi-stage decoding. The decoding steps are as follows:

1. Decoding of the first-level inner coset code $A_1$

   (1) Decode the 1st-stage coset representative $[(8,4,4)/(8,1,8)]$ based on the
       $(8,1,8)$ code. At the same time, temporarily decoded estimates of $(8,1,8)$
       code are obtained.

   (2) Decode the 2nd-stage coset representative $[(8,7,2)/(8,4,4)]$ based on the
       $(8,4,4)$ code by using the decoded estimates from the 1st stage of decoding.

   (3) Decode the 3rd-stage coset representative $[(8,7,2)/(8,4,4)]$ based on the
       $(8,4,4)$ code, the 1st-stage decoded estimates, and the 2nd-stage decoded
       estimates.

   (4) After all three-stage decodings, decode the 1st level outer code and re-
       produce the 1st, 2nd and 3rd stage decoded estimates of $A_1$. Use these
       decoded estimates to decode the 2nd-level inner coset code $A_2$.

2. Decoding of the second-level inner coset code $A_2$

   (1) Decode the 1st-level component code $(8,1,8)$ of $A_2$ using the 1st stage
       decoded estimates of $A_1$.

   (2) Decode the 2nd-level component code $(8,4,4)$ of $A_2$ using the 1st and
       2nd stage decoded estimates of $A_1$ and decoded estimates of 1st-level
(3) Decode the 3rd-level component code \((8,4,4)\) of \(A_2\) using the 1st, 2nd, and 3rd stage decoded estimates of \(A_1\) and decoded estimates of 1st- and 2nd-level component codes of \(A_2\).

(4) After all three-stage decodings, decode the 2nd level outer code.

**Outer code Selection**

For comparison purpose, we construct a single-level concatenated code. In the single-level concatenated block modulation code, we use \(A_0\) as the inner code and the \((511,455,57)\) RS code as the outer code. Let \(C(S20)\) denote this single-level concatenated BCM code. Decoding of the inner codes follows the same steps as the two-level code \(C(20)\). However, for \(C(S20)\), inner codes are decoded and before the outer codes. The spectral efficiency of the code \(C(S20)\) is 2.0034 bits/symbol.

For the frequency non-selective Rayleigh fading and various shadowed MSAT channel, the same set of outer codes are used. Selected outer codes are as follows:

\[
\begin{align*}
B_1 & = (511,411,101) \text{ Reed Solomon code} \\
B_2 & = (511,499,13) \text{ Reed Solomon code}
\end{align*}
\]

Then we have following 2-level concatenated BCM codes \(C(20)\)

\[
C(20) = \{B_1, B_2\} \ast \{A_1, A_2\}.
\]

(6.22)

**Coherently detected 8-PSK modulation code over the frequency non-selective Rayleigh fading channel**

For the non-selective Rayleigh fading channel, perfect phase tracking is assumed in simulation. Each codeword in \(C(20)\) contains 8190 bits and has a spectral efficiency
of 2.003424 bits/symbol. The error performance of the coherently detected 8-PSK modulation code $C(20)$ over the Rayleigh fading channel is shown in Figure 6.32. The two-level code $C(20)$ achieves a 1.58 dB real coding gain over the single-level code $C(S20)$ at a BER $10^{-5}$ with the same spectral efficiency. Also, $C(20)$ achieves a 33.347 dB real coding gain over the uncoded QPSK modulation without bandwidth expansion. Compared with the 3-level codes $C(16)$, $C(17)$, and $C(18)$, the two-level code $C(20)$ achieves an impressive coding gain with the same spectral efficiency.

Coherently detected 8-PSK modulation code over the shadowed MSAT Channels

The error performance of the coherently detected 8-PSK modulation codes $C(20)$ over the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 6.33, 6.34 and 6.35 respectively. Table 6.12 summarizes the error performance of the
coherently detected 8-PSK modulation code $C(20)$ over shadowed MSAT channels. The two-level code $C(20)$ achieves an impressive coding gain over the single-level code $C(S20)$ and the uncoded QPSK modulation at a BER of $10^{-5}$ with the same spectral efficiency. Compared with the 3-level codes $C(16)$, $C(17)$, and $C(18)$, the two-level code $C(20)$ achieves an impressive coding gain with the same spectral efficiency.

**Differentially detected 8-PSK modulation code over the shadowed MSAT channels**

The error performance of the differentially detected 8-PSK modulation code $C(20)$ over the light-, average- and heavy-shadowed MSAT channels is shown in Figures 6.36, 6.37 and 6.38 respectively. Table 6.13 summarizes the error performance of the differentially detected 8-PSK modulation code $C(20)$ over various shadowed MSAT channels. In the case of the differentially detected 8-PSK modulation code, we assume that phase of the received 8-PSK signal is randomly changing due to the multipath fading and the Doppler frequency shift. At a BER of $10^{-5}$, simulation results show that the code $C(20)$ achieves more than 1.0 dB real coding over the single-level code $C(S20)$ over all three shadowed MSAT channels. Especially, in the heavy shadowed MSAT channel, $C(20)$ achieves an very impressive coding gain over $C(S20)$. Compared with codes $C(16)$, $C(17)$, $C(18)$, and $C(19)$, the code $C(20)$ outperforms those codes with same spectral efficiency.
Figure 6.33. BER of the 2-level 8-PSK concatenated modulation code $C(20)$ over a light-shadowed MSAT channel.

Figure 6.34. BER of the 2-level 8-PSK concatenated modulation code $C(20)$ over a average-shadowed MSAT channel.
Figure 6.35. BER of the 2-level 8-PSK concatenated modulation code $C(20)$ over a heavy-shadowed MSAT channel.

Table 6.12. Bit error performance of the coherently detected 8-PSK 2-level concatenated modulation code $C(20)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>Shadowing</th>
<th>$E_b/N_0$ BER</th>
<th>$E_b/N_0$ BER</th>
<th>Coding Gain @ $10^{-4}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-5}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(S20)$</th>
<th>Coding Gain @ $10^{-5}$ BER over $C(S20)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 9.357</td>
<td>0.05 8.779</td>
<td>15.579 25.8</td>
<td>0.94 1.533</td>
<td>1.51 1.635</td>
<td>1.52 1.52</td>
</tr>
<tr>
<td>Average</td>
<td>0.02 10.228</td>
<td>0.05 9.277</td>
<td>18.09 26.852</td>
<td>10.642 17.42</td>
<td>27.992 1.065</td>
<td>1.374 1.374</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 13.124</td>
<td>0.05 12.116</td>
<td>27.976 37.426</td>
<td>13.923 19.32</td>
<td>37.426 1.51</td>
<td>1.635 1.635</td>
</tr>
</tbody>
</table>
Figure 6.36. BER of the 2-level 8-DPSK concatenated modulation code $C(20)$ over a light-shadowed MSAT channel.

Figure 6.37. BER of the 2-level 8-DPSK concatenated modulation code $C(20)$ over an average-shadowed MSAT channel.
Figure 6.38. BER of the 2-level 8-DPSK concatenated modulation code $C(20)$ over a heavy-shadowed MSAT channel.

Table 6.13. Bit error performance of the differentially detected 8-PSK 2-level concatenated modulation code $C(20)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_0$</th>
<th>$E_b/N_0$</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$@ 10^{-4}$</td>
<td>$@ 10^{-5}$</td>
<td>BER</td>
<td>BER</td>
<td>over 4-DPSK</td>
<td>over 4-DPSK</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>14.224</td>
<td>14.883</td>
<td>14.731</td>
<td>0.0000235*</td>
<td>1.157</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>13.341</td>
<td>13.678</td>
<td>0.0001973*</td>
<td>1.341</td>
<td>1.06</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>15.14</td>
<td>15.883</td>
<td>19.598</td>
<td>0.0000566*</td>
<td>1.847</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>14.341</td>
<td>14.955</td>
<td>0.0004744*</td>
<td>1.679</td>
<td>1.711</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>18.963</td>
<td>19.542</td>
<td>0.00144*</td>
<td>1.837</td>
<td>2.015</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>21.01</td>
<td>21.863</td>
<td>0.00803*</td>
<td>7.052</td>
<td>11.671</td>
</tr>
</tbody>
</table>

*: bit-error-rate at the error floor
6.7 Application: All-digital High-definition TV and Advanced TV

All-digital high-definition television (HDTV) systems for terrestrial broadcasting have been proposed in the United States, as well as an all-digital advanced television (ATV) system for satellite broadcasting. All-digital HDTV systems have also been proposed in Europe for terrestrial and satellite broadcasting. In [94], Sablatash described data rates, channels and channel transmission encoder-decoder and modulator-demodulator pairs for the four all-digital HDTV systems proposed in each of the United States and in Europe for terrestrial broadcasting as well as for advanced all-digital television systems for satellite broadcasting. All of these HDTV or ATV systems employ fairly modern methods for error control in the system to minimize the effects of interference and noise in the system and to minimize errors in other systems, caused by that system. The four all-digital HDTV systems proposed and tested for terrestrial broadcasting in the United States are: (1) Digicipher, (2) Digital Spectrum Compatible (DSC-HDTV), (3) Advanced Digital Television (ADTV), and (4) Channel Compatible Digicipher (CCDC) HDTV System. These systems compress a raw digitized source image data rate of about 1 Gbps to between 12.59 and 18.88 Mbps. After digitized audio and control data are added, as well as a considerable number of bits for error control because of the sensitivity to errors of the highly compressed data, the transmission data rates lie between 19.2 and 26.43 Mbps. The Digicipher, ADTV and CCDC systems all employ a choice of 16, or 32, quadrature amplitude modulation (QAM), and a concatenation of an outer RS code with an inner TCM code. The Digicipher and CCDC RS codes have \( t = 5 \), and the ADTV system uses a RS code with \( t = 10 \). The DSC-HDTV system selects 2-vestigial sideband (VSB) or 4-VSB and uses an RS code with \( t = 10 \) and
erasure correcting. The goals of recent research directions and trends are to: (1) provide sufficient rejection of co-channel interference, (2) achieve low BER, high data rate and reasonable implementation complexity and cost, (3) create a signal which minimizes degradation due to interferences and noise, and minimize interference into other systems, (4) provide bandwidth efficiency, and (5) provide graceful degradation, coverage range and robustness.

As shown in this chapter, proposed multilevel concatenated BCM codes achieve low BER, high data rate with low decoding complexity and no bandwidth expansion. Since, all four proposed HDTV systems achieve low BER with bandwidth expansions. Therefore, the proposed multilevel concatenated BCM system is more suitable for error control for the all-digital HDTV systems and the advanced ADTV systems than four proposed systems.

6.8 Summary

In this chapter, several multilevel concatenated BCM codes have been constructed for the Rayleigh fading and various shadowed MSAT channels. Simulation results showed that if the inner codes, outer codes, and the level of concatenation are properly chosen, very good performance can be achieved with high spectral efficiency and large coding gain. For the channel with severe multipath fading and large Doppler frequency shift, the proposed multilevel concatenated BCM schemes achieve very impressive real coding gains over the corresponding single-level concatenated BCM schemes with the same inner code. Even though single-level concatenated BCM codes exhibit an error floor phenomenon before reaching the BER of $10^{-5}$, some of the proposed multilevel schemes achieve error floor phenomenon free communication at the BER of $10^{-4}$. The two-level scheme achieves especially an impressive error performance over various fading channels.
Chapter 7

Product Modulation Codes

In [91], Rajpal constructed product coded modulation schemes by combining multilevel coded modulation and product coding technique to achieve high performance with reduced decoding complexity. He constructed many good product modulation codes for the AWGN and the frequency non-selective slow Rayleigh fading channels. His simulation results show that product coded modulation schemes achieve impressive coding gains over the uncoded reference systems without bandwidth expansion. In this chapter, various product modulation scheme is constructed for various shadowed MSAT channels. Their error performances are compared with the uncoded reference systems as well as with the single-level concatenated BCM schemes. Simulation results show that the proposed product modulation codes achieve impressive coding gains over the uncoded reference systems and outperform single-level concatenated BCM codes with a little extra decoding complexity.

7.1 Introduction

For reliable data communication over various shadowed MSAT channels, the BER of the coded system must be lower than $10^{-4}$. As shown in chapter 6, the multilevel concatenated BCH scheme achieve the BER of $10^{-4}$ at low SNR without exhibiting
the error floor phenomenon. In this chapter, the product coded modulation schemes
for the data transmission over various shadowed MSAT channels are constructed.

In product coded modulation, product coding technique and coded modulation
are combined to achieve high performance with reduced decoding complexity. Binary
(or non-binary) block codes are used as the horizontal codes, and a short block
modulation code is used as the vertical code in product coded modulation. The
product coded modulation scheme can provide different level of error protection for
each level of vertical component codes just as with multilevel concatenated coded
modulation scheme. If the vertical codes and horizontal codes are properly chosen,
large coding gains and high spectral efficiency can be achieved with reduced decoding
complexity.

In section 7.2, the construction of two-dimensional product codes is briefly re­
viewed. In section 7.3, the construction of product modulation codes is discussed and
the decoding is discussed in section 7.4. In section 7.5, various product modulation
codes are proposed for data transmission over various shadowed MSAT channels.
The bit error performance of proposed product modulation codes is evaluated and
compared with those of single-level concatenated BCM codes. Simulation results
show that proposed product modulation codes achieve more coding gain than the
single-level concatenated BCM codes.

7.2 Two-dimensional Product Codes

In this section, we give a brief review of the construction of two-dimensional product
codes. Let $C_1$ be an ($N_1, K_1$) binary block code and $C_2$ be an ($N_2, K_2$) binary block
code. The product of $C_1$ and $C_2$, denoted $C_1 \times C_2$, is formed in three steps. A
message of $K_1K_2$ bits is first arranged in a $K_2 \times K_1$ array of $K_2$ rows and $K_1$
columns as shown in Figure 7.1. Each row of this array is encoded into an $N_1$-bit
codeword in $C_1$. This row encoding results in a $K_2 \times N_1$ array of $K_2$ rows and $N_1$ columns. Then each column of this second array is encoded into an $N_2$-bit codeword in $C_2$. As a result of the two-step encoding, we obtain an $N_2 \times N_1$ code array. This code array is then transmitted. The collection of all the distinct code arrays form a two-dimensional product code [92]. If the minimum Hamming distances of $C_1$ and $C_2$ are $\delta_1$ and $\delta_2$ respectively, then the minimum Hamming distance of their product $C_1 \times C_2$ is $\delta_1 \times \delta_2$. $C_1$ and $C_2$ are called the horizontal and vertical component codes of the product code respectively.

![Diagram of two-dimensional product code](image)

Figure 7.1. Two-dimensional product code
7.3 Construction of Product Modulation Codes

In this section, the construction of an 8-PSK product modulation codes is discussed. Generalization of the construction technique to other constellation is straightforward.

The overall encoder for the product modulation code is shown in Figure 7.2. For $1 \leq i \leq 3$, let $C_{i,1} = (N, k_{i,1}, \delta_{i,1})$ be a binary block code with minimum Hamming distance $\delta_{i,1}$ and $C_{i,2} = (n, k_{i,2}, \delta_{i,2})$ be a binary block code with minimum Hamming distance $\delta_{i,2}$. Now we form three product codes, $P_1 = C_{1,1} \times C_{1,2}$, $P_2 = C_{2,1} \times C_{2,2}$ and $P_3 = C_{3,1} \times C_{3,2}$ as shown in Figure 7.3. Let $A$, $B$, and $C$ be three code arrays from $P_1$, $P_2$ and $P_3$ respectively. Let

\[ a_j = (a_{j,1}, a_{j,2}, \cdots, a_{j,n}) \]
\[ b_j = (b_{j,1}, b_{j,2}, \cdots, b_{j,n}) \]  \hspace{1cm} (7.1)
\[ c_j = (c_{j,1}, c_{j,2}, \cdots, c_{j,n}) \]

be the $j$-th columns of the code arrays, $A$, $B$, and $C$ respectively. We form the following sequence:

\[ a_j \ast b_j \ast c_j = (a_{j,1} b_{j,1} c_{j,1}, a_{j,2} b_{j,2} c_{j,2}, \cdots, a_{j,n} b_{j,n} c_{j,n}). \]  \hspace{1cm} (7.2)

For $1 \leq \ell \leq n$, we take $a_{j,\ell} b_{j,\ell} c_{j,\ell}$ as the label for a signal point in the 8-PSK signal constellation. Let $\lambda(\cdot)$ be the mapping which maps the label $a_{j,\ell} b_{j,\ell} c_{j,\ell}$ into its corresponding signal point $s_{j,\ell}$, i.e., $\lambda(a_{j,\ell} b_{j,\ell} c_{j,\ell}) = s_{j,\ell}$. Then

\[ \lambda(a_j \ast b_j \ast c_j) = (\lambda(a_{j,1} b_{j,1} c_{j,1}), \lambda(a_{j,2} b_{j,2} c_{j,2}), \cdots, \lambda(a_{j,n} b_{j,n} c_{j,n})) \]
\[ = (s_{j,1}, s_{j,2}, \cdots, s_{j,n}) \]  \hspace{1cm} (7.3)

is a sequence of $n$ 8-PSK signals. For $1 \leq j \leq N$, combining the corresponding columns $a_j$, $b_j$, and $c_j$ of code arrays $A$, $B$ and $C$ by the above bits-to-signal mapping,
Figure 7.2. Product Modulation Code Encoder
\[ P_1 = C_{1,1} \times C_{1,2} \]
\[ P_2 = C_{2,1} \times C_{2,2} \]
\[ P_3 = C_{3,1} \times C_{3,2} \]

\[ A \in P_1 \]
\[ B \in P_2 \]
\[ C \in P_3 \]

\[ (a_1 \times b_1 \times c_1) \quad \lambda (a_1 \times b_1 \times c_1) \]

8-PSK signal column

\[ \Lambda = \lambda (C_{1,2} \times C_{2,2} \times C_{3,2}) \]

\[ \lambda (a_1 \times b_1 \times c_1) \in \Lambda \]

\[ \Omega \]

\[ n \quad \lambda (a_1 \times b_1 \times c_1) \]

\[ \Lambda \]

\[ n \]

Figure 7.3. Product Modulation Code
we obtain an \( n \times N \) array of 8-PSK signals, denoted \( \lambda(A * B * C) \). This array is then transmitted column by column.

Note that, for \( 1 \leq j \leq N \), the columns \( a_j, b_j, \) and \( c_j \) are codewords from the vertical codes \( C_{1,2}, C_{2,2} \) and \( C_{3,2} \) respectively. Then, for \( 1 \leq j \leq N \),

\[
\Lambda \triangleq \lambda[C_{1,2} * C_{2,2} * C_{3,2}]
\]

\[
= \{ \lambda(a_j * b_j * c_j) : a_j \in C_{1,2}, b_j \in C_{2,2} \text{ and } c_j \in C_{3,2} \} \quad (7.4)
\]

forms a 3-level 8-PSK modulation code of length \( n \), dimension \( k = k_{1,2} + k_{2,2} + k_{3,2} \), and minimum squared Euclidean distance \( d_E[\Lambda] = \min\{0.586 \times \delta_{1,2}, 2 \times \delta_{2,2}, 4 \times \delta_{3,2}\} \). Consequently, the following collection of distinct arrays of 8-PSK signals,

\[
\Omega = \{ \lambda(A * B * C) : A \in P_1, B \in P_2 \text{ and } C \in P_3 \} \quad (7.5)
\]

form a product 8-PSK modulation code of length \( nN \), dimension \( k_{1,1} \times k_{1,2} + k_{2,1} \times k_{2,2} + k_{3,1} \times k_{3,2} \), and minimum squared Euclidean distance

\[
d_E^2[\Omega] = \min\{0.586 \times \delta_{1,1} \times \delta_{1,2}, 2 \times \delta_{2,1} \times \delta_{2,2}, 4 \times \delta_{3,1} \times \delta_{3,2}\}. \quad (7.6)
\]

From Eq. (7.6), we see that we can construct product 8-PSK codes with arbitrarily large minimum squared Euclidean distance by choosing the horizontal and vertical component codes properly. If the vertical component codes are chosen properly for the fading channel, good codes with large minimum symbol and product distances can be constructed.

In the above construction of a product modulation code, the binary linear block codes are used as the horizontal codes for forming the product codes. In fact, we can use nonbinary codes of length \( N \) with symbols from \( GF(2^m) \) as the horizontal codes for the product codes. In this case, the horizontal encoding must be done first followed by the column encoding. This is to ensure that the columns of the
$n \times N$ signal array are codewords in $\Lambda$. Note that in product modulation code construction, horizontal component codes are normally long powerful codes for hard-decision decoding.

### 7.4 Decoding

One obvious way (though impractical) of decoding the proposed product modulation code is to compare each received code sequence with all the possible code sequences and find the closest one in terms of minimum squared Euclidean distance. The decoding complexity associated with this technique is enormous. We will focus on a suboptimal decoding procedure which allows decoding of the codes with reduced decoding complexity while maintaining good performance.

Consider the decoding of an 8-PSK product modulation code $\Omega$. Recall that each codeword in $\Omega$ can be written in the form

$$V = (v_1, v_2, \cdots, v_N)$$

with $v_i \in \Lambda$ for $1 \leq i \leq N$. Let $R = (r_1, r_2, \cdots, r_N)$ be the received sequence. The overall decoder is shown in Figure 7.4. The decoding is performed in 3 stages with $P_1$, $P_2$ and $P_3$ decoded in sequence. At the first stage of decoding, each column $a_j$ of $P_1$ is decoded based on the received sequence $r_j$, using a multi-stage soft-decision decoding algorithm for $\Lambda$ [40]. After the column decoding of $P_1$, the horizontal code $C_{1,1}$ of $P_1$ then uses the decoded estimates of columns to decode the rows of $P_1$ using hard-decision decoding. The horizontal row decoding is advantageous since it corrects additional errors in the columns and thus helps reduce the error propagation into the next stage of decoding. In fact, if the horizontal codes are chosen powerful enough, the error propagation effect is negligible, and thereby eliminates the major problem of multi-stage decoding and improving error performance. Thus, after the
horizontal row decoding of $P_1$ we have new estimates $\Omega^{(1)}$ of the columns (the corrected estimates). These new estimates along with the received sequence are passed to the second decoding stage and are used to decode the columns of $P_2$. The decoding of $P_2$ and $P_3$ follows the same procedure as that of $P_1$.

### 7.5 Product Modulation Codes Schemes

#### 7.5.1 Scheme 1:

In this scheme, we construct a product modulation code for various shadowed MSAT channels. In the construction, vertical codes of length 8 are used.

**Vertical Codes Construction**

Choose $C_{1,2}$ to be the $(8,4,4)$ RM code with minimum distance 4, $C_{2,2}$ and $C_{3,2}$ to be the $(8,7,2)$ even parity code with minimum distance 2. These three vertical codes are used to form $\Lambda$. Note, that $\Lambda$ is designed specifically for the multipath fading channel. The code has minimum symbol distance $\delta_H[\Lambda] = 2$ and the minimum product distance $\Delta_p[\Lambda] = 4$. The error performance of $\Lambda$ over various fading channels is shown in Chapter 5.

**Horizontal Codes Construction**

Let $C_{1,1}$ be the $(255,231,7)$ BCH code with error correcting capability $t_1 = 3$. $C_{2,1}$ is chosen to be the $(255,215,11)$ BCH code with error correcting capability $t_2 = 5$. $C_{3,1}$ is chosen to be the $(255,239,5)$ BCH code with error correcting capability $t_3 = 2$.

The overall spectral efficiency of the product modulation code $\Omega$ is $\eta = 2.01$ bits/symbol and the phase invariance is $90^\circ$. The $(8,4,4)$ RM code associated with
Figure 7.4. Product Modulation Code Decoder
the first labeling bit of $A$ has a 4-state trellis which is used to decode the columns of $P_1$ at the first stage of decoding with the Viterbi algorithm. Also, the $(8, 7, 2)$ even parity code associated with the second level of $A$ has a very simple 2-state trellis which is used to form the decoded estimates of the columns of $P_2$ (i.e. the second stage of decoding). The decoding complexity associated with the third stage of decoding is comparable to that of the second stage. The total decoding complexity of $A$ due to the soft-decision multi-stage decoding is therefore $4 + 2 + 2 = 8$ states. The above construction of product modulation codes can be generalized to other signal constellations in a straightforward manner.

Let $C(21)$ denote this product modulation code. We construct a single-level concatenated BCM code with $A$ as the inner code for comparison purposes. The outer code is the $(255, 223, 9)$ BCH code with error correcting capability $t = 4$ which is interleaved to a depth of 18. Let $C(S21)$ denote this single-level concatenated BCM code. Each codeword of the code $C(S21)$ contains $239 \times 18$ bits. In decoding the code $C(S21)$, the inner code $A$ is decoded using three stage decoding and decoded estimates from the inner decoder are passed to the outer code decoder at the same time. Therefore, all information bits of the inner code are protected by the same degree and the error performance of the code $C(S21)$ heavily depend on the worst error performance among three-stage inner code decoding. The bit error performance of the code $C(S21)$ over various shadowed MSAT channels is compared with the product modulation code with the same $A$.

For each shadowed MSAT channel, the bit-error performance of product modulation code $C(21)$ is evaluated for two different vehicle speeds, 37.152 and 92.88 mile/hour. Normalized fading bandwidth $BT$ for vehicle speeds of 37.152 and 92.88 mile/hour are 0.02 and 0.05, respectively.
Coherently detected 8-PSK product modulation code over shadowed MSAT channel

The error performance of the coherently detected 8-PSK product modulation code \( C(21) \) over the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 7.5, 7.6, and 7.7 respectively. Table 7.1 summarizes the error performance of the coherently detected 8-PSK modulation code \( C(21) \) for various shadowed MSAT channels. At a BER of \( 10^{-5} \), the product modulation code \( C(21) \) outperforms the single-level concatenated BCM code \( C(S21) \) over light, average, and heavy shadowed MSAT channels. Also, \( C(21) \) achieves an impressive coding gain over uncoded QPSK modulation for various shadowed MSAT channels.

Differentially detected 8-PSK modulation code over the shadowed MSAT channels

The error performance of the differentially detected 8-PSK modulation code \( C(21) \) over the light-, average-, heavy-shadowed MSAT channels is shown in Figures 7.8, 7.9 and 7.10 respectively. Table 7.2 summarizes the error performance of the differentially detected 8-PSK modulation code \( C(21) \) over various shadowed MSAT channels. The product modulation code \( C(21) \) outperforms the single-level concatenated BCM code \( C(S21) \) over light, average, and heavy shadowed MSAT channels at a BER of \( 10^{-5} \). Also, \( C(21) \) achieves an impressive coding gain over the uncoded 4-DPSK modulation system. For the differentially detected 8-PSK modulation code, we assumed that phase of the received 8-DPSK signal is randomly changed due to the multipath fading.
Figure 7.5. BER of the 8-PSK product modulation code $C(21)$ over a light-shadowed MSAT channel.

Figure 7.6. BER of the 8-PSK product modulation code $C(21)$ over an average-shadowed MSAT channel.
Figure 7.7. BER of the 8-PSK product modulation code $C(21)$ over a heavily shadowed MSAT channel.

Table 7.1. Bit error performance of the coherently detected 8-PSK product modulation code $C(21)$ over the shadowed MSAT channel.

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$ @ $10^{-4}$ BER</th>
<th>$E_b/N_o$ @ $10^{-5}$ BER</th>
<th>Coding Gain @ $10^{-4}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-5}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(S21)$</th>
<th>Coding Gain @ $10^{-5}$ BER over $C(S21)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 12.069 14.713 13.047 20.712 0.0 0.549</td>
<td>0.05 10.686 12.028 15.514 27.739 0.0 0.028</td>
<td>0.02 13.385 16.545 14.992 22.089 0.11 0.483</td>
<td>0.05 11.54 13.348 17.06 25.254 0.234 0.529</td>
<td>0.02 16.95 19.459 24.15 31.89 0.0 0.0</td>
<td>0.05 14.946 16.618 23.503 31.372 0.234 0.504</td>
</tr>
</tbody>
</table>
Figure 7.8. BER of the 8-DPSK product modulation code $C(21)$ over a lights shadowed MSAT channel.

Figure 7.9. BER of the 8-DPSK product modulation code $C(21)$ over an average shadowed MSAT channel.
Figure 7.10. BER of the 8-DPSK product modulation code $C(21)$ over a heavy-shadowed MSAT channel.

Table 7.2. Bit error performance of the differentially detected 8-DPSK product modulation code $C(21)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$ @ $10^{-4}$ BER</th>
<th>$E_b/N_o$ @ $10^{-5}$ BER</th>
<th>Coding Gain @ $10^{-4}$ BER over 4-DPSK</th>
<th>Coding Gain @ $10^{-5}$ BER over 4-DPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(S21)$</th>
<th>Coding Gain @ $10^{-5}$ BER over $C(S21)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 17.372 20.372 11.583 0.0000235* 0.394 0.608</td>
<td>0.05 15.946 18.024 0.0001973* 0.283 0.701</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.02 18.95 22.391 15.788 0.0000566* 0.35 1.66</td>
<td>0.05 17.389 19.768 0.00004744* 0.504 0.441</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 23.35 26.475 0.00144* 0.289 0.441</td>
<td>0.05 0.000210* 0.00803* 0.000274*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*: bit-error-rate at the error floor
Comparison with the 3-level concatenated BCM code \(C(16)\)

The 3-level concatenated BCM code \(C(16)\) in the chapter 6 uses the inner code \(A\) which is the vertical code of the product modulation code \(C(21)\), \(A\) is decoded by three-stage decoding. However, \(C(16)\) uses RS codes over \(GF(2^8)\) as the horizontal codes and \(C(21)\) uses binary BCH codes of length 255 as the outer codes. Since RS codes are maximum distance separable codes, they have the largest possible minimum distance. Also, RS codes are useful for correcting bursts of errors and the rate of a RS code is higher than that of a BCH code with the same error correcting capability for most cases. Let the length of a RS code and a BCH code be 255 and their error correcting capability 4. Then, we have \((255,247,t=4)\) RS code and \((255,223,t=4)\) BCH code. The rate of \((255,247)\) RS code is 0.9686 and the rate of \((255,199)\) is 0.7803. Table 7.3 summarizes the bit error performances of the coherently detected 8-PSK modulation code \(C(21)\) and the code \(C(16)\) over various shadowed MSAT channels. For various shadowed MSAT channels, at BER of \(10^{-5}\), the 3-level concatenated BCM code \(C(16)\) achieves at least 2.46 real coding gain over the product modulation code \(C(21)\) with a little extra decoding complexity. Table 7.4 summarizes the error performance of the differentially detected 8-PSK modulation code \(C(21)\) and the code \(C(16)\) for various shadowed MSAT channels. For the heavy shadowed MSAT channel with \(BT = 0.05\), the code \(C(21)\) and the code \(C(16)\) exhibits an error floor phenomenon at BER \(2.1 \times 10^{-4}\) and \(2.35 \times 10^{-5}\), respectively. Therefore, the code \(C(16)\) faces an error floor phenomenon at much lower BER than the code \(C(21)\). Also, for the light and average shadowed MSAT channels, the code \(C(16)\) achieves impressive coding gains over the code \(C(21)\) over various shadowed MSAT channels.

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Table 7.3. Bit error performances of the coherently detected 8-PSK product modulation code \( C(21) \) and the 3-level concatenated BCM code \( C(16) \) over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>( BT )</th>
<th>Code ( C(21) )</th>
<th>Code ( C(16) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( E_b/N_o ) @ BER ( 10^{-4} )</td>
<td>( E_b/N_o ) @ BER ( 10^{-5} )</td>
<td>( E_b/N_o ) @ BER ( 10^{-4} )</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>12.069</td>
<td>14.713</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>10.686</td>
<td>12.028</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>13.385</td>
<td>16.545</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>11.54</td>
<td>13.348</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>16.96</td>
<td>19.459</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>14.946</td>
<td>16.618</td>
</tr>
</tbody>
</table>

Table 7.4. Bit error performances of the differentially detected 8-PSK product modulation code \( C(21) \) and the 3-level concatenated BCM code \( C(16) \) over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>( BT )</th>
<th>Code ( C(21) )</th>
<th>Code ( C(16) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( E_b/N_o ) @ BER ( 10^{-4} )</td>
<td>( E_b/N_o ) @ BER ( 10^{-5} )</td>
<td>( E_b/N_o ) @ BER ( 10^{-4} )</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>17.372</td>
<td>20.372</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>15.946</td>
<td>18.024</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>18.95</td>
<td>22.391</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>17.389</td>
<td>19.768</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>23.35</td>
<td>26.475</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.00021*</td>
<td>0.0000235*</td>
</tr>
</tbody>
</table>

\* : bit-error-rate at the error floor
7.5.2 Scheme 2:

In this scheme, non-binary block codes as well as binary block codes are used as the horizontal codes.

**Vertical code construction**

The same vertical component codes of the code $C(21)$ are used as vertical codes. These vertical codes are used to form $\Lambda$. $\Lambda$ has minimum symbol distance $\delta_H[\Lambda] = 2$ and the minimum product distance $\Delta_p^2[\Lambda] = 4$.

**Horizontal code construction**

Let $C_{1,1}$ be the $(127,106,7)$ BCH code with error correcting capability $t = 3$. $C_{2,1}$ is chosen to be the 9-error-correcting $(127,109,19)$ RS code over $GF(2^7)$. $C_{3,1}$ is chosen to be the four-error-correcting $(127,119,9)$ RS code over $GF(2^7)$. The overall spectral efficiency of $\Omega$ is $\eta = 1.988$ bits/symbol and the phase invariance is $90^\circ$. The $(8,4,4)$ RM code associated with the first labeling bit of $\Lambda$ has a 4-state trellis which is used to decode the columns of $P_1$ at the first stage of decoding with the Viterbi algorithm. The $(8,7,2)$ even parity code associated with the second level of $\Lambda$ has a very simple 2-state trellis which is used to form the decoded estimates of the columns of $P_2$ (i.e. the second stage of decoding). The decoding complexity associated with the third stage of decoding is comparable to that of the second stage. The total decoding complexity of $\Lambda$ due to the soft-decision multi-stage decoding is therefore $4 + 2 + 2 = 8$ states.

Let $C(22)$ denote this product modulation code. For each shadowed MSAT channel, the bit-error performance of the product modulation code $C(22)$ is evaluated for two different vehicle speeds, 37.152 and 92.88 mile/hour.

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Since the single-level concatenated BCM code $C(S21)$ uses the same inner (or vertical) code $\Lambda$ with the code $C(22)$, bit error performances of the code $C(S21)$ over various shadowed MSAT channels are compared with those of the code $C(22)$.

**Coherently detected 8-PSK product modulation code over shadowed MSAT channel**

The error performances of the coherently detected 8-PSK product modulation code $C(22)$ for the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 7.11, 7.12, and 7.13 respectively. Table 7.5 summarizes the error performance of the coherently detected 8-PSK modulation code $C(22)$ for various shadowed MSAT channels. Even though the horizontal codes of $C(22)$ are shorter than those of the single-level concatenated BCM code $C(S21)$, $C(22)$ outperforms $C(S21)$ over various shadowed MSAT channels.

**Differentially detected 8-PSK modulation code over the shadowed MSAT channels**

The error performance of the differentially detected 8-PSK modulation code $C(22)$ over the light-, average-, heavy-shadowed MSAT channels is shown in Figures 7.14, 7.15 and 7.16, respectively. Table 7.6 summarizes the error performance of the differentially detected 8-PSK modulation code $C(22)$ over various shadowed MSAT channels. The code $C(22)$ is compared with the single-level concatenated BCM code $C(S21)$. Simulation results show that $C(22)$ outperforms $C(S21)$ over various shadowed MSAT channels with shorter horizontal codes. $C(22)$ also achieves impressive coding gains over the uncoded 4-DPSK modulation for various shadowed MSAT channels.

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Figure 7.11. BER of the 8-PSK product modulation code \( C(22) \) over a light-shadowed MSAT channel.

Figure 7.12. BER of the 8-PSK product modulation code \( C(22) \) over an average-shadowed MSAT channel.
Figure 7.13. BER of the 8-PSK product modulation code $C(22)$ over a heavily shadowed MSAT channel.

Table 7.5. Bit error performance of the coherently detected 8-PSK product modulation code $C(22)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$</th>
<th>$E_b/N_o$</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$BT$</td>
<td>$10^{-4}$</td>
<td>$10^{-5}$</td>
<td>@ $10^{-4}$</td>
<td>@ $10^{-5}$</td>
<td>@ $10^{-4}$</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>11.364</td>
<td>13.18</td>
<td>13.802</td>
<td>22.245</td>
<td>0.755</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>10.389</td>
<td>11.5</td>
<td>15.811</td>
<td>24.267</td>
<td>0.297</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>12.479</td>
<td>15.04</td>
<td>15.898</td>
<td>23.594</td>
<td>1.011</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>11.155</td>
<td>12.491</td>
<td>17.445</td>
<td>26.143</td>
<td>0.422</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>16.864</td>
<td>19.33</td>
<td>24.236</td>
<td>32.019</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>14.528</td>
<td>15.934</td>
<td>23.921</td>
<td>32.056</td>
<td>0.623</td>
</tr>
</tbody>
</table>
Figure 7.14. BER of the 8-DPSK product modulation code \( C(22) \) over a light-shadowed MSAT channel.

Figure 7.15. BER of the 8-DPSK product modulation code \( C(22) \) over an average-shadowed MSAT channel.
Figure 7.16. BER of the 8-DPSK product modulation code $C(22)$ over a heavy-shadowed MSAT channel.

Table 7.6. Bit error performance of the differentially detected 8-DPSK product modulation code $C(22)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$</th>
<th>$E_b/N_o$</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>@ $10^{-4}$</td>
<td>@ $10^{-5}$</td>
<td>BER over 4-DPSK</td>
<td>BER over 4-DPSK</td>
<td>BER over $C(S21)$</td>
<td>BER over $C(S21)$</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>16.72</td>
<td>19.01</td>
<td>12.235</td>
<td>0.0000235*</td>
<td>1.046</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>13.93</td>
<td>15.245</td>
<td>0.0001973*</td>
<td>2.299</td>
<td>3.48</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>18.03</td>
<td>20.737</td>
<td>16.708</td>
<td>0.0000566*</td>
<td>1.27</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>15.245</td>
<td>16.885</td>
<td>0.0004744*</td>
<td>2.648</td>
<td>3.36</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>23.196</td>
<td>26.311</td>
<td>0.00144*</td>
<td>0.443</td>
<td>0.092</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>31.311</td>
<td>0.0000565*</td>
<td>0.000274*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* : bit-error-rate at the error floor
Comparison with the 3-level concatenated BCM code $C(16)$

The 3-level concatenated BCM code $C(16)$ designed in chapter 6 uses the inner code $A$ which is the vertical code of the product modulation code $C(22)$ and $A$ is decoded in the three-stage decoding. Table 7.7 summarizes the bit error performance of the coherently detected 8-PSK modulation code $C(22)$ and the code $C(16)$ for various shadowed MSAT channels. For various shadowed MSAT channels, at a BER of $10^{-5}$, the 3-level concatenated BCM code $C(16)$ achieves an impressive coding gain over $C(21)$ with little extra decoding complexity. Table 7.8 summarizes the bit error performance of the differentially detected 8-PSK modulation code $C(22)$ and the code $C(16)$ for various shadowed MSAT channels. For the heavy shadowed MSAT channel with $BT = 0.05$, the code $C(22)$ and the code $C(16)$ exhibit an error floor phenomenon at BER $6.17 \times 10^{-5}$ and $2.35 \times 10^{-5}$, respectively. Therefore, $C(16)$ faces an error floor phenomenon at a much lower BER than $C(22)$. Also, for the light and average shadowed MSAT channels, $C(16)$ achieves an impressive coding gain over $C(22)$ for various shadowed MSAT channels.

7.5.3 Scheme 3:

In this scheme, we construct a product modulation code with vertical codes of length 16.

Vertical codes construction

Choose $C_{1,2}$ and $C_{2,2}$ to be the $(16,11,4)$ RM code with minimum distance 4 and $C_{3,2}$ to be the even parity $(16,15,2)$ code with minimum distance 2. These three vertical codes are used to form $A$. Note, that $A$ is designed specifically for the Rayleigh fading channel. The code has minimum symbol distance $\delta_H[A] = 2$ and the minimum product distance $\Delta_p^2[A] = 16$. 

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Table 7.7. Bit error performances of the coherently detected 8-PSK product modulation code $C(22)$ and the 3-level concatenated BCM code $C(16)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>Shadowing</th>
<th>$BT$</th>
<th>Code $C(22)$</th>
<th>Code $C(16)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$E_b/N_o$ @ $10^{-4}$ BER</td>
<td>$E_b/N_o$ @ $10^{-5}$ BER</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>11.364</td>
<td>13.18</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>10.389</td>
<td>11.5</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>12.479</td>
<td>15.04</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>11.155</td>
<td>12.491</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>16.864</td>
<td>19.33</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>14.528</td>
<td>15.934</td>
</tr>
</tbody>
</table>

Table 7.8. Bit error performances of the differentially detected 8-PSK product modulation code $C(22)$ and the 3-level concatenated BCM code $C(16)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>Shadowing</th>
<th>$BT$</th>
<th>Code $C(22)$</th>
<th>Code $C(16)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$E_b/N_o$ @ $10^{-4}$ BER</td>
<td>$E_b/N_o$ @ $10^{-5}$ BER</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>16.72</td>
<td>19.01</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>13.93</td>
<td>15.245</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>18.03</td>
<td>20.737</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>15.245</td>
<td>16.885</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>23.196</td>
<td>26.311</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>31.311</td>
<td>0.0000565*</td>
</tr>
</tbody>
</table>

* : bit-error-rate at the error floor
Horizontal codes construction

Let $C_{1,1}$ be the $(255,191,17)$ BCH code with error correcting capability $t_1 = 8$. $C_{2,1}$ is chosen to be the $(255,239,5)$ BCH code with error correcting capability $t_2 = 2$. $C_{3,1}$ is chosen to be the $(255,231,7)$ BCH code with error correcting capability $t_3 = 3$.

The overall spectral efficiency of $\Omega$ is $\eta = 2.00857$ bits/symbol and the phase invariance is 90°. The $(16,11,4)$ RM code associated with the first labeling bit of $\Lambda$ has two parallel 4-state sub-trellises which are used to decode the columns of $P_1$ and $P_2$ with the Viterbi algorithm at the first and second stage of decoding. The $(16,15,2)$ even parity code associated with third level of $\Lambda$ has a very simple 2-state trellis which is used to form the decoded estimates of the columns of $P_3$. The total decoding complexity of $\Lambda$ due to the soft-decision multi-stage decoding is therefore $4 + 4 + 2 = 10$ states.

Let $C(23)$ denote this product modulation code. For comparison purpose, we construct a single-level concatenated BCM code with $\Lambda$ as the inner code. The outer code is the $(255,223,t=4)$ binary BCH code which is interleaved to a depth of 37. Let $C(S23)$ denote this single-level concatenated BCM code. Each code block in the code $C(S23)$ contains $223 \times 37$ bits. In decoding the code $C(S23)$, the inner code $\Lambda$ is decoded using three stage decoding and the decoded estimates from the inner decoder are passed to the outer code decoder at the same time. Therefore, all information bits of the inner code $\Lambda$ are protected to the same degree. The spectral efficiency of the resulting single-level 8-PSK concatenated modulation code $C(S23)$ is 2.0223 bits/dimension.
Coherently detected 8-PSK product modulation code over shadowed MSAT channel

The error performance of the coherently detected 8-PSK product modulation code $C(23)$ for the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 7.17, 7.18, and 7.19, respectively. Table 7.9 summarizes the error performance of the coherently detected 8-PSK modulation code $C(23)$ over various shadowed MSAT channels. The code $C(23)$ achieves an impressive coding gain over the code $C(S23)$ for various shadowed MSAT channels.

Table 7.9. Bit error performance of the coherently detected 8-PSK product modulation code $C(23)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_0$</th>
<th>$E_b/N_o$</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
<th>Coding Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$@ 10^{-4}$</td>
<td>$@ 10^{-5}$</td>
<td>@ $10^{-4}$</td>
<td>@ $10^{-5}$</td>
<td>@ $10^{-4}$</td>
<td>@ $10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>BER</td>
<td>BER</td>
<td>BER</td>
<td>BER</td>
<td>BER</td>
<td>BER</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>12.4</td>
<td>13.524</td>
<td>12.716</td>
<td>21.901</td>
<td>-0.13</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>16.8</td>
<td>11.36</td>
<td>15.4</td>
<td>24.403</td>
<td>0.91</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>12.479</td>
<td>13.73</td>
<td>15.898</td>
<td>24.904</td>
<td>0.557</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>11.504</td>
<td>12.479</td>
<td>17.096</td>
<td>26.155</td>
<td>0.975</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>15.245</td>
<td>16.491</td>
<td>25.855</td>
<td>34.858</td>
<td>0.664</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>14.409</td>
<td>15.549</td>
<td>24.04</td>
<td>32.441</td>
<td>0.968</td>
</tr>
</tbody>
</table>
Figure 7.17. BER of the 8-PSK product modulation code $C(23)$ over a light-shadowed MSAT channel.

Figure 7.18. BER of the 8-PSK product modulation code $C(23)$ over an average-shadowed MSAT channel.
Figure 7.19. BER of the 8-PSK product modulation code $C(23)$ over a heavy-shadowed MSAT channel.

Differentially detected 8-PSK modulation code over the shadowed MSAT channels

Error performances of the differentially detected 8-PSK modulation code $C(23)$ over the light-, average-, heavy-shadowed MSAT channels are shown in Figures 7.20, 7.21 and 7.22 respectively. Table 7.10 summarizes error performances of the differentially detected 8-PSK modulation code $C(23)$ over various shadowed MSAT channels. The code $C(23)$ achieves an impressive coding gain over the uncoded QPSK modulation and the code $C(S23)$ for various shadowed MSAT channels.
Figure 7.20. BER of the 8-DPSK product modulation code $C(23)$ over a light-shadowed MSAT channel.

Figure 7.21. BER of the 8-DPSK product modulation code $C(23)$ over an average-shadowed MSAT channel.
Figure 7.22. BER of the 8-DPSK product modulation code $C(23)$ over a heavy-shadowed MSAT channel.

Table 7.10. Bit error performance of the differentially detected 8-DPSK product modulation code $C(23)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$ @ $10^{-4}$ BER</th>
<th>$E_b/N_o$ @ $10^{-5}$ BER</th>
<th>Coding Gain @ $10^{-4}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-5}$ BER over $C(S23)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(S23)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 16.5</td>
<td>18.614</td>
<td>12.455</td>
<td>0.0000235*</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>0.05 15.959</td>
<td>16.778</td>
<td>0.0001973*</td>
<td>0.71</td>
<td>1.336</td>
</tr>
<tr>
<td>Average</td>
<td>0.02 17.7</td>
<td>19.745</td>
<td>17.038</td>
<td>0.00000566*</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>0.05 17.073</td>
<td>18.27</td>
<td>0.004744*</td>
<td>1.32</td>
<td>1.385</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 21.639</td>
<td>23.5</td>
<td>0.00144*</td>
<td>1.475</td>
<td>1.622</td>
</tr>
<tr>
<td></td>
<td>0.05 0.000484*</td>
<td>0.00803*</td>
<td>0.00215*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*: bit-error-rate at the error floor
Comparison with the 3-level concatenated BCM code \( C(17) \)

The 3-level concatenated BCM code \( C(17) \) of chapter 6 uses the inner (or vertical) code \( \Lambda \) which is the vertical code of the product modulation code \( C(22) \) and \( \Lambda \) is decoded with three-stage decoding. Table 7.11 summarizes bit error performances of the coherently detected 8-PSK modulation code \( C(23) \) and the code \( C(17) \) over various shadowed MSAT channels. For various shadowed MSAT channels, at a BER of \( 10^{-5} \), the 3-level concatenated BCM code \( C(17) \) achieves an impressive coding gain over the product modulation code \( C(23) \) with little extra decoding complexity. Table 7.12 summarizes the bit error performance of the differentially detected 8-PSK modulation code \( C(23) \) and the code \( C(17) \) for various shadowed MSAT channels. For the heavy shadowed MSAT channel with \( BT = 0.05 \), \( C(23) \) and \( C(17) \) exhibit an error floor phenomenon at a BER of \( 4.84 \times 10^{-4} \) and \( 7.65 \times 10^{-5} \), respectively. For light and average shadowed MSAT channels, \( C(17) \) also achieves impressive coding gains \( C(23) \) for various shadowed MSAT channels.

### 7.5.4 Scheme 4:

In this scheme, we construct another product modulation code which has the vertical code with length 16.

**Vertical codes construction**

Choose \( C_{1,2} \) to be the (16,11,4) RM code with minimum distance 4 and \( C_{2,2} \) and \( C_{3,2} \) to be the even parity (16,15,2) code with minimum distance 2. These three vertical codes are used to form \( \Lambda \). Note, that \( \Lambda \) is designed specifically for the Rayleigh fading channel. The code has minimum symbol distance \( \delta_{H}[\Lambda] = 2 \) and the minimum product distance \( \Delta_{p}^{2}[\Lambda] = 4 \).
Table 7.11. Bit error performances of the coherently detected 8-PSK product modulation code $C(23)$ and the 3-level concatenated BCM code $C(17)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$BT$</th>
<th>Code $C(23)$</th>
<th>Code $C(17)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_b/N_o$ @</td>
<td>$E_b/N_o$ @</td>
<td>$E_b/N_o$ @</td>
</tr>
<tr>
<td></td>
<td>$BER$ 10^{-4}$</td>
<td>$BER$ 10^{-5}$</td>
<td>$BER$ 10^{-4}$</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>12.4</td>
<td>13.524</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>10.8</td>
<td>11.36</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>12.479</td>
<td>13.73</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>11.504</td>
<td>12.479</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>15.245</td>
<td>16.491</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>14.409</td>
<td>15.549</td>
</tr>
</tbody>
</table>

Table 7.12. Bit error performances of the differentially detected 8-PSK product modulation code $C(23)$ and the 3-level concatenated BCM code $C(17)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$BT$</th>
<th>Code $C(23)$</th>
<th>Code $C(17)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_b/N_o$ @</td>
<td>$E_b/N_o$ @</td>
<td>$E_b/N_o$ @</td>
</tr>
<tr>
<td></td>
<td>$BER$ 10^{-4}$</td>
<td>$BER$ 10^{-5}$</td>
<td>$BER$ 10^{-4}$</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>16.5</td>
<td>18.614</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>15.959</td>
<td>16.778</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>17.7</td>
<td>19.745</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>17.073</td>
<td>18.27</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>21.639</td>
<td>23.524</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.000484*</td>
<td>0.0000765*</td>
</tr>
</tbody>
</table>

*: bit-error-rate at the error floor
Horizontal codes construction

Let $C_{1,1}$ be the $(255, 199, 15)$ BCH code with error correcting capability $t_1 = 7$. $C_{2,1}$ is chosen to be the $(255, 179, 21)$ BCH code with error correcting capability $t_2 = 10$. $C_{3,1}$ is chosen to be the $(255, 223, 9)$ BCH code with error correcting capability $t_3 = 4$.

The overall spectral efficiency of $\Omega$ is $\eta = 2.01446$ bits/symbol and the phase invariance is $90^\circ$. The $(16, 11, 4)$ RM code associated with the first labeling bit of $\Lambda$ has two parallel 4-state sub-trellises which are used to decode the columns of $P_1$ with the Viterbi algorithm at the first stage of decoding. The $(16, 15, 2)$ even parity code associated with second and third level of $\Lambda$ has a very simple 2-state trellis which is used to form the decoded estimates of the columns of $P_2$ and $P_3$. The total decoding complexity of $\Lambda$ due to the soft-decision multi-stage decoding is therefore $4 + 2 + 2 = 8$ states. Let $C(24)$ denote this product modulation code.

We construct a single-level concatenated BCM code with $\Lambda$ as the inner code for comparison purposes. The outer code is the $(255, 199, t=7)$ binary BCH code which is interleaved with a depth of 41. Let $C(S24)$ denote this single-level concatenated BCM code. Each block of the code $C(S24)$ contains $199 \times 41$ bits. In decoding for the code $C(S24)$, the inner code $\Lambda$ is decoded using three stage decoding and decoded estimates from inner decoder are passed to the outer code decoder at the same time. Therefore, all information bits of the inner code are protected by the same degree. The spectral efficiency of the resulting single-level 8-PSK concatenated modulation code $C(S24)$ is $1.99975$ bits/dimension.
Coherently detected 8-PSK product modulation code over shadowed MSAT channel

The error performance of the coherently detected 8-PSK product modulation code $C(24)$ for the light-, average-, and heavy-shadowed MSAT channels is shown in Figures 7.23, 7.24, and 7.25, respectively. Table 7.13 summarizes the error performance of the coherently detected 8-PSK modulation code $C(24)$ over various shadowed MSAT channels. The code $C(24)$ achieves an impressive coding gain over the uncoded QPSK modulation and the single-level concatenated BCM code $C(S24)$ for various shadowed MSAT channels.

Table 7.13. Bit error performance of the coherently detected 8-PSK product modulation code $C(24)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$ @ $10^{-4}$ BER</th>
<th>$E_b/N_o$ @ $10^{-5}$ BER</th>
<th>Coding Gain @ $10^{-4}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-5}$ BER over QPSK</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(S24)$</th>
<th>Coding Gain @ $10^{-5}$ BER over $C(S24)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 11.762</td>
<td>13.37</td>
<td>13.354</td>
<td>22.055</td>
<td>0.065</td>
<td>0.124</td>
</tr>
<tr>
<td></td>
<td>0.05 10.963</td>
<td>11.67</td>
<td>15.237</td>
<td>24.097</td>
<td>0.184</td>
<td>0.35</td>
</tr>
<tr>
<td>Average</td>
<td>0.02 12.885</td>
<td>14.596</td>
<td>15.492</td>
<td>24.038</td>
<td>0.748</td>
<td>0.942</td>
</tr>
<tr>
<td></td>
<td>0.05 11.665</td>
<td>12.29</td>
<td>16.935</td>
<td>26.344</td>
<td>0.308</td>
<td>0.759</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 15.741</td>
<td>17.35</td>
<td>25.39</td>
<td>33.999</td>
<td>0.871</td>
<td>1.127</td>
</tr>
<tr>
<td></td>
<td>0.05 14.522</td>
<td>15.762</td>
<td>23.927</td>
<td>32.228</td>
<td>0.646</td>
<td>1.0383</td>
</tr>
</tbody>
</table>
Figure 7.23. BER of the 8-PSK product modulation code $C(24)$ over a light-shadowed MSAT channel.

Figure 7.24. BER of the 8-PSK product modulation code $C(24)$ over an average-shadowed MSAT channel.
Differentially detected 8-PSK modulation code over the shadowed MSAT channels

The error performance of the differentially detected 8-PSK modulation code $C(24)$ for the light-, average-, heavy-shadowed MSAT channels is shown in Figures 7.26, 7.27 and 7.28, respectively. Table 7.14 summarizes the error performance of the differentially detected 8-PSK modulation code $C(24)$ for various shadowed MSAT channels. The code $C(24)$ achieves impressive coding gains over the uncoded QPSK modulation and the code $C(S24)$ for various shadowed MSAT channels.
Figure 7.26. BER of the 8-DPSK product modulation code $C(24)$ over a light-shadowed MSAT channel.

Figure 7.27. BER of the 8-DPSK product modulation code $C(24)$ over an average-shadowed MSAT channel.
**Figure 7.28.** BER of the 8-DPSK product modulation code $C(24)$ over a heavy-shadowed MSAT channel.

**Table 7.14.** Bit error performance of the differentially detected 8-DPSK product modulation code $C(24)$ over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>$E_b/N_o$ @ $10^{-4}$ BER</th>
<th>$E_b/N_o$ @ $10^{-5}$ BER</th>
<th>Coding Gain @ $10^{-4}$ BER over 4-DPSK</th>
<th>Coding Gain @ $10^{-5}$ BER over $C(S24)$</th>
<th>Coding Gain @ $10^{-4}$ BER over $C(S24)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.02 16.485 18.534 12.47 0.0000235* 0.084 1.466</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05 15.881 16.844 0.0001973* 0.209 0.625</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.02 17.9 19.59 16.838 0.0000566* 0.95 1.71</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05 17.09 18.491 0.0004744* 0.229 0.539</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02 21.885 23.688 0.00144* 0.983 1.392</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05 0.000356* 0.00803* 0.000474*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*: bit-error-rate at the error floor
Comparison with the 3-level concatenated BCM code $C(18)$

The 3-level concatenated BCM code $C(18)$ of chapter 6 uses the inner (or vertical) code $\Lambda$ which is the vertical code of the product modulation code $C(24)$ and $\Lambda$ is decoded with three-stage decoding. Table 7.15 summarizes the bit error performance of the coherently detected 8-PSK modulation code $C(24)$ and $C(18)$ for various shadowed MSAT channels. For various shadowed MSAT channels, at a BER of $10^{-5}$, the 3-level concatenated BCM code $C(18)$ achieves an impressive coding gain over the product modulation code $C(24)$ with little extra decoding complexity. Table 7.16 summarizes the bit error performance of the differentially detected 8-PSK modulation code $C(24)$ and the code $C(18)$ for various shadowed MSAT channels. For the heavy shadowed MSAT channel with $BT = 0.05$, the code $C(24)$ face an error floor phenomenon at a BER of $4.74 \times 10^{-4}$. However, the code $C(18)$ achieves the BER $10^{-5}$ without facing the error floor phenomenon. For light and average shadowed MSAT channels, the code $C(18)$ also achieves impressive coding gains over the code $C(24)$ for various shadowed MSAT channels.

7.6 Summary

In this chapter, we have shown that product modulation is a powerful technique for constructing modulation codes for various shadowed MSAT channels. Proposed product modulation schemes have been compared with single-level concatenated BCM schemes with the same inner (or vertical) and outer (or horizontal) codes. Simulation results showed that the proposed schemes achieve impressive coding gains over the uncoded QPSK (or 4-DPSK) modulation and single-level concatenated BCM schemes. Also, the proposed product modulation schemes are compared with multilevel concatenated BCM schemes with the same vertical (or inner) codes. Since
Table 7.15. Bit error performances of the coherently detected 8-PSK product modulation code \( C(24) \) and the 3-level concatenated BCM code \( C(18) \) over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>( BT )</th>
<th>Code ( C(24) )</th>
<th>Code ( C(18) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( E_{b}/N_{o} ) @ BER ( 10^{-4} )</td>
<td>( E_{b}/N_{o} ) @ BER ( 10^{-5} )</td>
<td>( E_{b}/N_{o} ) @ BER ( 10^{-4} )</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>11.762</td>
<td>13.37</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>10.963</td>
<td>11.67</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>12.885</td>
<td>14.596</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>11.665</td>
<td>12.29</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>15.741</td>
<td>17.35</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>14.522</td>
<td>15.762</td>
</tr>
</tbody>
</table>

Table 7.16. Bit error performances of the differentially detected 8-PSK product modulation code \( C(24) \) and the 3-level concatenated BCM code \( C(18) \) over the shadowed MSAT channel

<table>
<thead>
<tr>
<th>shadowing</th>
<th>( BT )</th>
<th>Code ( C(24) )</th>
<th>Code ( C(18) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( E_{b}/N_{o} ) @ BER ( 10^{-4} )</td>
<td>( E_{b}/N_{o} ) @ BER ( 10^{-5} )</td>
<td>( E_{b}/N_{o} ) @ BER ( 10^{-4} )</td>
</tr>
<tr>
<td>Light</td>
<td>0.02</td>
<td>16.485</td>
<td>18.534</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>15.881</td>
<td>16.884</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>17.9</td>
<td>19.59</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>17.09</td>
<td>18.491</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.02</td>
<td>21.885</td>
<td>23.688</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.000356*</td>
<td>29.116</td>
</tr>
</tbody>
</table>

\* : bit-error-rate at the error floor
multilevel concatenated BCM codes use RS codes as the outer codes and product modulation codes use binary BCH codes as the horizontal codes, a multilevel concatenated BCM code outperforms a product modulation code with the same inner (or vertical) codes. However, the multilevel concatenated BCM code needs more decoding complexity than the product modulation code. Also, proposed product modulation codes achieve impressive coding gains over multilevel modulation codes in chapter 4 and 5.
Chapter 8

Conclusion

8.1 Conclusion

The main objective of this dissertation is to construct low decoding complexity and high performance modulation codes for the Rayleigh fading and various shadowed mobile satellite (MSAT) channels. Both voice and data communications over various shadowed MSAT channels have been considered in this dissertation. For voice communication over various shadowed MSAT channels, multi-level TCM and BCM codes have been constructed by using the multilevel coding method. For data communication over various shadowed MSAT channels, multi-level concatenated BCM schemes and product modulation schemes have been constructed by using the multilevel concatenated coding method.

In chapter 3, a systematic technique for constructing the multilevel coded modulation scheme using the multilevel coding method was presented. Distance parameters, such as the minimum squared Euclidean distance, minimum symbol distance, and minimum product distance, which determine the error performance of a multi-level modulation code were expressed in terms of the minimum Hamming distances of the component codes. Guidelines for constructing good multilevel modulation
codes for either the AWGN channel or the Rayleigh fading channel were presented. A multi-stage decoding algorithm was presented.

In chapter 4, MPSK trellis code modulation (TCM) codes using convolutional codes with good free branch distances as the component codes or using both convolutional and block codes, such as Reed-Muller codes, as the component codes were constructed by using the multilevel coding method. These multilevel TCM codes were specifically designed for voice communication over shadowed MSAT channels. The constructed modulation codes were compared to the available in the literatures. Simulation results showed that these codes achieved good coding gains over the uncoded reference system for Rayleigh fading and shadowed MSAT channels with reduced decoding complexity. Single- and two-level TCM codes using convolutional codes with good free branch distances as component codes outperform TCM codes using convolutional codes with good minimum Hamming distance as component codes. The optimum size of the decoding depth and the interleaving depth of TCM codes was investigated.

Even though block coded modulation (BCM) was generally less power efficient than TCM for AWGN channels, BCM has the potential to compete with TCM for the voice communication over shadowed MSAT channel because of its shorter decoding depth and hence more effective interleaving. In chapter 5, the multilevel coding method was used for constructing 8-PSK, 3-level, BCM codes for Rayleigh fading and shadowed MSAT channels. Binary RM codes of lengths up to 32 were used as component codes. The decoding complexity of 4-section trellis of RM codes was presented. The error performance of these codes for various shadowed MSAT channels was evaluated. Multi-stage soft-decision decoding was used to decode BCM codes. Simulation results showed that BCM codes outperform TCM codes for high SNR with the same (or less) decoding complexity.
Based on our study, we may summarize the design guidelines as follows. To design good multilevel modulation codes for voice transmission over the shadowed MSAT channels with an interleaver of finite size, codes must be designed based on following facts:

1. For a bursty error channel with small Doppler frequency shift (BT=0.02), it is better to use a code with short decoding decision depth because a code can use an interleaver with a larger interleaving depth.

2. For a random error channel with a large Doppler frequency shift (BT=0.05), it is better to use a code with the large minimum symbol and product distances.

3. If two codes have the same minimum symbol distance, then the code with larger minimum product distance faces an error floor phenomenon at lower BER's than the code with the smaller minimum product distance.

Multilevel BCM codes have following advantages over TCM codes.

1. In decoding TCM codes, error propagation is avoidable. However, in decoding BCM codes, there is no error propagation between blocks because each block is decoded independently.

2. With the same decoding complexity, we can find a BCM code with a shorter decoding depth than a TCM code. Even though the resultant BCM code has a smaller minimum symbol distance than the TCM code, the BCM has a larger interleaving depth than the TCM code. Therefore, the resultant BCM code will outperform the TCM code over a bursty error channel.

3. It is easy to construct a BCM code with high spectral efficiency, large minimum symbol distance, large minimum product distance, and small decoding
complexity. However, it is not easy to construct TCM codes with high spectral efficiency. A TCM code with a large minimum symbol distance and minimum product distance usually needs a large decoding depth to achieve a desirable BER. However, with an interleaver with finite size, good error performance is not expected with a powerful TCM code over the bursty error channel.

4. BCM codes with high spectral efficiency are suitable for use as the inner codes of a multilevel concatenated modulation code. It is easy to construct a concatenated BCM code without bandwidth expansion and this code achieves a desirable BER. However, it is not easy to construct a concatenated TCM code without bandwidth expansion over the uncoded system.

In chapter 6, a simple and systematic technique for constructing multilevel concatenated block coded modulation schemes for data transmission over the shadowed MSAT channel and the Rayleigh fading channel was presented. Various schemes were designed to achieve high-performance or large coding gain with reduced decoding complexity. Construction was based on a multilevel concatenation approach in which long powerful binary (or non-binary) codes were used as the outer codes and coset codes constructed from a linear BCM code and its subcodes were used as the inner codes. Sub-optimum multilevel closest coset decoding was proposed for the multilevel concatenated coded modulation scheme.

Since each component code of the multilevel BCM inner code has a different distance profile, it is desirable to provide different error protection to each component code for better error performance. A major shortcoming of single-level concatenation is that the outer code corrects all output bits of the inner code decoder to the same degree. This shortcoming can be overcome by using multilevel concatenation and coset inner codes derived from block modulation code. Since modulation
schemes using the multilevel concatenation approach give us the freedom to choose the error-correcting capabilities of the outer codes, it is easy to provide proper error protection to each level. Two-, three- and six-level concatenated block coded modulation schemes with inner coset codes constructed from a short (or long) linear block modulation and its subcodes were devised for the shadowed MSAT and the Rayleigh fading channels. Simulation results showed that these schemes achieve very high coding over the uncoded reference system without bandwidth expansion. Even though single-level concatenated BCM codes exhibits an error floor phenomenon before reaching the BER of $10^{-5}$, some of the proposed multilevel schemes do not exhibit an error floor at the BER of $10^{-5}$.

In chapter 7, product coded modulation schemes were constructed by using a multilevel concatenating approach. In product coded modulation, the product coding technique and coded modulation were combined to achieve high performance with reduced decoding complexity. Since modulation schemes using the multilevel concatenation approach give us the freedom to choose error-correcting capabilities of outer codes, it is easy to provide proper error protection to each level in the product modulation scheme. Good product 8-PSK modulation codes were constructed for various shadowed MSAT channels. Simulation results showed that the proposed schemes achieve impressive coding gains over the uncoded QPSK (or 4-DPSK) modulation and single-level concatenated BCM schemes. Suboptimum multi-stage decoding was used to decode the proposed product modulation codes.

8.2 Proposed Researches for Future Work

Possible future research in coded modulation includes:
(A) Research on applications of coded modulation techniques to code division multiple access (CDMA) for digital cellular systems is booming. CDMA uses the direct sequence spread spectrum (DS/SS) technique to support more users per base station per MHz of spectrum. As shown in this dissertation, proposed modulation codes perform very well over various shadowed MSAT channels. The digital cellular communication channels exhibit rapid fading and are bandwidth limited. The error performance of proposed modulation codes with DS/SS technique for digital cellular communication channel will be evaluated.

(B) Digital cordless systems require low-complexity equipment and high-quality speech with respect to user mobility in contrast with digital cellular system. Proposed multilevel TCM and BCM code are constructed for voice transmission over various shadowed MSAT channels. Results show that the proposed multilevel TCM and BCM code performs very well with reduced decoding complexity. Good low-complexity multilevel TCM and BCM codes will be devised for digital cordless systems and their error performances will be evaluated.

(C) Research on the global mobile satellite services (MSS). MSS systems are categorized by the orbital altitude of the satellites. High altitude satellites can cover more range. However, there is a long propagation delay. Low altitude satellites have a small propagation delay. However, they have limited lifetime. Therefore, replacement satellite will have to be launched frequently. In the future, the error performance of modulation codes for MSS systems will be evaluated with different propagation delays, i.e., satellites with different altitudes.

(D) Recently several single-level concatenated TCM codes were proposed for all-digital high-definition television (HDTV) system for terrestrial broadcasting as
wel as an all-digital advanced television (ATV) system for satellite broadcasting. In this dissertation, we showed that the proposed multilevel concatenated BCM codes perform very well over various shadowed MSAT channels. Application of multilevel concatenated BCM codes for all-digital HDTV and ATV systems is a good research problem.

(E) For deep space satellite communications, powerful concatenated coding techniques have been proposed to NASA. Further research on power-efficient error correcting codes for deep space satellite communications will be conducted.
Appendix A

Decoding Complexity of the 4-section $r$-th order Reed-Muller code, $RM(r,m)$

The $r$-th order Reed-Muller code $RM(r,m)$ is a linear binary code with the length $2^m$, the dimension $K = \sum_{i=0}^{r} \binom{m}{i}$, and the minimum Hamming distance $D = 2^{m-r}$.

Forney [19] constructed 4-section trellis for the $r$-th order Reed-Muller code $RM(r,m)$ by using the iterated squaring construction. In the following, decoding complexities of the Viterbi decoding on 4-section $RM_{m,r}$ code trellis are discussed.

Kasami, et. al. [95] showed that the 4-section trellis diagram of the $RM(r,m)$ code is loosely connected and has $2^{\binom{m-2}{r}}$ parallel and structurally identical sub-trellis diagrams without cross connections between them. As a result, $2^{\binom{m-2}{r-1}}$ identical Viterbi decoders of $2^{\binom{m-2}{r-1}}$ states can be built to process the decoding in parallel. This not only simplifies the decoding complexity but also speeds up the decoding process.

In the following, the decoding complexity of one of $2^{\binom{m-2}{r}}$ parallel $2^{\binom{m-2}{r-1}}$ states sub-trellis are computed. There are three categories of computations: (1) compute the branch metric (C1) (2) find a survivor at each state (comparison) (C2) (3) add the new branch metric (C3) to corresponding survivor’s metric (addition)
In each section, there are only \(2^{\left(\begin{array}{c}
m-2 \\
r-1
\end{array}\right)}\) branches with distinct label. Each branch corresponds to the coset codes of \(RM(r-2, m-2)\). Therefore, each branch contains \(2^\sum_{i=0}^{r-2} \left(\begin{array}{c}
m-2 \\
-i
\end{array}\right)\) parallel transitions. However, if \(r < 2\), there is a only one codeword in \(RM(r-2, m-2)\) code, i.e., all-zero codeword. Suppose the branch metric computation and additions of those parallel transitions in each branch can be done in parallel. For branch metric calculations and additions, we only consider \(2^{\left(\begin{array}{c}
m-2 \\
r-1
\end{array}\right)}\) branches for \(1 \leq r \leq m-2\). For \(r = m-1\), a \(RM(m-1, m)\) code is an even parity check code which has simple 2-state trellis with \(2^m\) sections.

Each branch consists of \(2^{m-2}\) M-PSK signals.

For the first section, we have

1. \(C_1 = 2^{m-2} 2^{\left(\begin{array}{c}
m-2 \\
r-1
\end{array}\right)},\)

2. \(C_2 = (2^\sum_{i=0}^{r-2} \left(\begin{array}{c}
m-2 \\
-i
\end{array}\right) - 1)(2^{\left(\begin{array}{c}
m-2 \\
r-1
\end{array}\right)} - 1),\)

3. \(C_3 = (2^{m-2} - 1)(2^{\left(\begin{array}{c}
m-2 \\
r-1
\end{array}\right)} - 1).\)

For the second and third section, we have

1. \(C_1 = 2^{m-2} 2^{\left(\begin{array}{c}
m-2 \\
r-1
\end{array}\right)},\)

2. \(C_2 = (2^\sum_{i=0}^{r-2} \left(\begin{array}{c}
m-2 \\
-i
\end{array}\right) - 1)(2^{\left(\begin{array}{c}
m-2 \\
r-1
\end{array}\right)} - 1) + (2^{\left(\begin{array}{c}
m-2 \\
r-1
\end{array}\right)} - 1),\)

3. \(C_3 = (2^{m-2} - 1)(2^{\left(\begin{array}{c}
m-2 \\
r-1
\end{array}\right)} + (2^{\left(\begin{array}{c}
m-2 \\
r-1
\end{array}\right)} - 1).\)

For the fourth section, we have

1. \(C_1 = 2^{m-2} 2^{\left(\begin{array}{c}
m-2 \\
r-1
\end{array}\right)},\)

2. \(C_2 = (2^\sum_{i=0}^{r-2} \left(\begin{array}{c}
m-2 \\
-i
\end{array}\right) - 1)(2^{\left(\begin{array}{c}
m-2 \\
r-1
\end{array}\right)} - 1) + (2^{\left(\begin{array}{c}
m-2 \\
r-1
\end{array}\right)} - 1)\)

3. \(C_3 = (2^{m-2} - 1)(2^{\left(\begin{array}{c}
m-2 \\
r-1
\end{array}\right)} + (2^{\left(\begin{array}{c}
m-2 \\
r-1
\end{array}\right)} - 1).\)

Table A.1 shows decoding complexities of \(r\)-th order Reed-Muller codes.
Table A.1. Decoding complexities of each sub-trellis of the 4-section $r$-th order Reed-Muller code trellis

<table>
<thead>
<tr>
<th>Code $(N, K, D)$</th>
<th>$(r, m)$</th>
<th>total No. of states</th>
<th>total No. of parallel subtrellis</th>
<th>Metric computation</th>
<th>Comparison</th>
<th>Addition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(8,4,4)$</td>
<td>$(1,3)$</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>5</td>
<td>14</td>
</tr>
<tr>
<td>$(16,5,8)$</td>
<td>$(1,4)$</td>
<td>8</td>
<td>2</td>
<td>24</td>
<td>5</td>
<td>36</td>
</tr>
<tr>
<td>$(16,11,4)$</td>
<td>$(2,4)$</td>
<td>8</td>
<td>4</td>
<td>64</td>
<td>31</td>
<td>60</td>
</tr>
<tr>
<td>$(32,6,16)$</td>
<td>$(1,5)$</td>
<td>16</td>
<td>2</td>
<td>64</td>
<td>5</td>
<td>62</td>
</tr>
<tr>
<td>$(32,16,8)$</td>
<td>$(2,5)$</td>
<td>64</td>
<td>8</td>
<td>256</td>
<td>95</td>
<td>248</td>
</tr>
<tr>
<td>$(32,26,4)$</td>
<td>$(3,5)$</td>
<td>16</td>
<td>2</td>
<td>256</td>
<td>543</td>
<td>248</td>
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</tbody>
</table>
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