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Golden rules and second best shadow prices for sustainable development

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GOLDEN RULES AND SECOND BEST SHADOW PRICES
FOR SUSTAINABLE DEVELOPMENT

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ABSTRACT

This study puts the popular but vague concept of sustainable development on a firmer theoretical foundation. Drawing on neoclassical growth theory and the field of public economics, the study concentrates on two themes central to a theoretical consideration of sustainability: golden rules for resource management and capital accumulation, and second best shadow prices for ecological capital in distorted economies.

Current research on sustainability focuses primarily on the case of nonrenewable resources with constant extraction cost and no backstop technology for providing a substitute resource. Sustainability is typically viewed in terms of imposing constants on growth, such as requiring aggregate consumption to be nondecreasing over time, or maintaining the value of the total capital stock, including both man-made and ecological capital, at a nondecreasing or even constant level. Another limitation of the literature is its reliance on first best, rather than second best, shadow prices for cost-benefit analysis and national income accounting in distorted economies.

In contrast, this study takes a unified approach to resource modeling that admits renewable and nonrenewable resources as special cases. Extraction cost is assumed to rise as the resource stock is drawn down, but is bounded above by the unit cost of a backstop resource that serves as a substitute. This framework is used to drive modified golden rules and golden rules for capital accumulation and resource management that maximize utilitarian welfare, but also meet society's obligation to the future. In the case
of golden rules, this is accomplished by setting the rate of social time preference equal to zero. The unified model of natural resources is also used to derive second best shadow prices of man-made and ecological capital in imperfect economies, so that net national product can serve as a true measure of social welfare. The study concludes that sustainability constraints on growth are likely to be redundant, infeasible, or dominated. Sustainable growth is best achieved by designing policies, based on golden rules and appropriate shadow prices, that are compatible with both economic efficiency and stewardship for the future.
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CHAPTER 1

INTRODUCTION AND BACKGROUND

This study puts the popular but vague concept of sustainable development on a firmer theoretical foundation. Drawing on neoclassical growth theory and the field of public economics, the study concentrates on two themes central to a theoretical consideration of sustainable development: golden rules for resource management and capital accumulation, and second best shadow pricing of ecological capital in distorted, or imperfect economies.

1.1. Motivation for the Study

With the end of the Cold War, concern about economic security has emerged as a serious policy issue in many countries. It has taken on added force in recent policy debates in the United States and has achieved a leading position on both domestic and foreign policy agendas of the Clinton Administration. As expressed by Secretary of State, Warren Christopher, during his Senate confirmation hearings, the first pillar of U.S. foreign policy will be "to elevate America's economic security as a primary goal." And according to the comments of scholars and policy makers from the dynamic economies of the Asia-Pacific region, continued economic development is viewed as a national security issue of top priority.
Economic security has not been precisely defined, but nonetheless is a meaningful concept with significant political appeal. One description that has influenced both domestic and foreign policy makers in the United States has been put forward by Romm (1993): "Economic security measures a nation's ability to improve the living standards of its citizens in an equitable and sustainable fashion." In this formulation, sustainability is viewed as central to economic security.

A more fundamental approach to economic security in the United States would entail going back to first principles of the U.S. Constitution, in particular, those advanced in the Preamble: justice, welfare, and the blessings of liberty to ourselves and our posterity. Within this framework, economic security may be defined as the enhancement of aggregate welfare for this and future generations consistent with individual liberty. The phrase "to ourselves and our posterity," may be interpreted as a mandate for stewardship and sustainability.

The notion of sustainable development has been especially high on the agendas of both academic researchers and government policy makers internationally since publication of the Brundtland Commission report in 1987. Despite its popularity, however, sustainable development has eluded any consensus regarding a precise definition. As Solow (1993) has stated about sustainability, "the less you know about it, the better it sounds." Solow goes on to note that sustainable development is a vague concept that conveys the idea of obligation to the future. This is captured by the definition offered in the Brundtland Report: "Sustainable development is development that meets the needs of the present without compromising the ability of future generations
to meet their own needs." Unfortunately, this definition has not served to resolve the controversy between the pro-growth and anti-growth schools of development.

In a recent article addressing the "perils" of free trade, Daly (1993) states: "Development without growth is sustainable development." In contrast, Pearce and Warford (1993) open their recent book, 'World without End,' with the following assertion: "If poverty is to be reduced and the standard of living of the average person improved, economic growth must remain a legitimate objective of national governments and the world community."

This study aligns with pro-growth point of view, consistent with the goal of economic security, and adopts the Pearce and Warford (1993) premise that the fundamental question is not whether to grow, but how to grow. Specifically, the study addresses the question of 'how to grow' from two standpoints. First, the study extends the results of neoclassical growth theory to an economy dependent on ecological capital and derives golden rules of resource management and capital accumulation. These golden rules serve not only to guide growth of the economy, but to contribute greater precision to the meaning of sustainability. Second, the study presents methods for computing second best shadow prices of man-made and ecological capital in dynamic economies with dynamic market distortions. These second best shadow prices have direct application to project evaluation and national income accounting.
1.2. Relationship of the Study to Present State of Knowledge

1.2.1. Sustainability as a Constraint on Growth

Traditional growth theory considers trajectories for output, consumption and capital accumulation that reflect both dynamic efficiency and maximization of welfare; that is, growth theory attempts to identify trajectories consistent with a utilitarian optimum. Recently, several papers have explored sustainability in the context of neoclassical growth theory (see especially Toman, et. al., 1993 and the references cited therein). However, current research on sustainability focuses primarily on the case of nonrenewable resources with constant extraction cost and no backup technology for providing a substitute resource. The elasticity of substitution between capital and the nonrenewable resource is typically assumed to be unity. Under these conditions, with no technology change and the pure rate of social time preference greater than zero, optimal consumption eventually declines to zero, a condition clearly not compatible with sustainability.

Notions of sustainability reflect concern for future generations and are often framed in terms of constraints on growth or on the use of ecological stocks. Examples of proposed sustainability constraints include: (1) the stock of natural capital, $K_n$, remains constant (Pearce, Barbier and Markandya, 1990); (2) per-capita consumption, $c$, always remains above some minimum level $c_{\text{min}}$, which might represent subsistence (e.g. Pezzey, 1989); and (3) negative growth must be precluded so that growth today does not come at the expense of consumption in the future (Siebert, 1992). One formulation of this constraint is that $\dot{c} \geq 0, \forall t \geq 0$. 

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The constraint on natural capital advanced by Pearce et. al. has been sharply criticized by Dasgupta and Mäler (1991). A decline in a resource stock on its own is not a reason for concern. In particular, the criterion makes no sense in the context of exhaustible resources: to not reduce their stocks is to not use them at all. But even in the case of renewable resources maintaining stock levels may not be prudent. "..... there is nothing sacrosanct about the stock levels we have inherited from the past." Similarly, Solow (1986) observes, "The current generation does not especially owe to its successors a share of this or that particular resource. If it owes anything, it owes generalized productive capacity or, even more generally, access to a certain standard of living or level of consumption."

Interest in maintaining stocks of natural or ecological capital is evidently motivated by concern about such factors as the degree to which natural and man-made capital are substitutes, scientific uncertainty about the role environmental systems play in the economy, and potential irreversibility of the effects of ecological degradation. These may be legitimate concerns, but the sustainability criterion proposed by Pearce, et. al. is ad hoc and without foundation. A more illuminating approach is to incorporate these concerns in a model and pursue their implications analytically.

More broadly, sustainability could be interpreted to mean that the value of the total capital stock, including both man-made and ecological capital is always nondecreasing. Under this constraint, any decrements to ecological capital must be offset by additions to produced capital; i.e., capital accumulation minus the decrease in natural capital, evaluated at its relative price, must always be nonnegative. If, for example, the
elasticity of substitution between man-made capital and natural capital is low, as proponents of natural capital preservation fear, then decrements to natural capital will be evaluated at a high and rising relative price, thus making it increasingly difficult to offset by increases in man-made capital. Again, simply asserting capital preservation as the appropriate criterion of sustainability does not appear to be well-grounded in terms of specified welfare objectives. By relating sustainability to welfare or consumption levels, one can explore under what conditions preservation of the total capital stock may be appropriate or not.

The second sustainability constraint, \( c_t \geq c_{\min}, \forall t \geq 0, \) might appear reasonable except for the fact that specification of \( c_{\min} \) is arbitrary. One possible candidate for \( c_{\min} \), which has some specificity, is maximin consumption, i.e., the maximum level of constant per-capita consumption. We discuss maximin welfare and its weaknesses as a basis for sustainability in the development of the simple model below.

The third constraint, \( c_t \geq 0, \forall t \geq 0, \) is the most widely accepted criterion for sustainability. A variation of this approach is to specify the constraint in terms of nondecreasing utility (Pezzey, 1994). A consumption path which maximizes utilitarian welfare subject to a nondecreasing utility constraint is called an "opsustimal path" by Pezzey (1994) and Toman et. al. (1993). Upon initial reflection, this criterion for sustainability has considerable appeal. However, in the context of modified golden rules and golden rules, as developed in Chapter 2, the sustainability constraint of nondecreasing consumption or nondecreasing utility becomes either dominated or redundant. "Opsustimality" is discussed further in Section 1.2.5.
1.2.2. A Basic Model of Natural Resources

The approach to sustainable development pursued in this study is organized around a basic model of natural resource use and its extensions. The model has the advantage of offering great generality in that it integrates consideration of renewable and nonrenewable resources into a common framework.

Consider an economy that uses three inputs, capital (K), Labor (L) and a natural resource (R) to produce a single homogeneous good. Assume that production technology is constant returns to scale, so that the production function Q(K, R, L) is homogeneous of degree one. For simplicity, and following Dasgupta and Heal (1979), we abstract from population growth and technological change and take L = 1. We then set F(K, R) = Q(K, R, 1). Following the standard approach, output of production is divided among consumption, gross investment, and the cost of providing the resource as an input to the production process.

Let $\theta$ be the unit cost of extracting the natural resource and providing it as an input to production. Capital depreciation occurs at the rate $\delta K$. The basic dynamic equation for the simple economy becomes

$$\dot{K} = F(K, R) - \delta K - \theta R - C.$$  \hspace{1cm} (1)

The resource stock $X$ is drawn down at the rate $R$. The case of a renewable resource is typically addressed by modeling growth of the resource as a function of the stock level $X$. Representing the growth function as $G(X)$, the dynamic equation governing the resource stock becomes
\[ \dot{X} = G(X) - R. \quad (2) \]

For exhaustible resources, \( G = 0. \)

We exploit the convenience of continuous time, and express the social welfare function as

\[ W = \int_0^\infty U(C) e^{-\rho t} \, dt, \quad (3) \]

where \( e^{-\rho t} \) is the utility discount factor and \( \rho \) is the utility discount rate or the pure rate of social time preference.

As in Dasgupta and Heal (1979), we assume \( \rho \geq 0, \, U'(C) \geq 0, \, U''(C) < 0, \)

\[ \lim_{C \to 0} U'(C) = \infty \text{ and } \lim_{C \to \infty} U(C) = 0. \]

Consistent with these conditions, we will find it useful to employ the iso-elastic utility function

\[ U(C) = -C^{-\eta}, \quad \eta > 1. \quad (4) \]

A utilitarian optimum trajectory for consumption and capital accumulation can then be derived as a solution to the following problem, given that such a solution exists:

\[ \text{Max } W = \int_0^\infty U(C) e^{-\rho t} \, dt \quad (5) \]

s.t. \( \dot{K} = F(K, R) - \delta K - \theta R - C, \quad K(0) = K_0 \)

\[ \dot{X} = G(X) - R, \quad X(0) = X_0. \]

Application of the maximum principle to this optimal control problem yields the
following efficiency conditions:

\[
[F_R - \theta] = \frac{1}{[F_R - \delta]} (\dot{F}_R^+ [F_R - \theta] G'(X) - \theta'(X)G(X)) \tag{6}
\]

\[
\text{and } \eta \frac{\dot{C}}{C} = F_K^- (\delta + \rho). \tag{7}
\]

Condition (6) is the generalized Hotelling's Rule for natural resources. For the case of an exhaustible resource, \(G(X) = 0\) and equation (6) can be written in the more familiar Hotelling form

\[
\frac{\dot{F}_R}{[F_R - \theta]} = [F_K^- - \delta]. \tag{6a}
\]

This is analogous to the equilibrium condition

\[
\dot{P} = r\pi \tag{6b}
\]

in partial equilibrium models of exhaustible resources. In such models, \(P\) is the market price of the resource, \(r\) is the exogenous interest rate, and \(\pi\) is producer royalty, given as price minus extraction cost.

For a renewable resource in the steady state, \(\dot{F}_R = 0\) and equation (6) assumes a form similar to that developed in the economics of fisheries:

\[
[F_K^- - \delta] = G'(X) - \frac{G(X) \theta'(X)}{[F_R - \theta]}. \tag{6c}
\]

Condition (7) is usually referred to as the Ramsey condition. If a steady state exists, \(\dot{C} = 0\) and condition (7) becomes
1.2.3. The Case of Exhaustible Resources

Of particular concern to resource economists is the degree to which an exhaustible resource is an essential input to the production process. A straightforward approach to addressing this concern is to consider substitution possibilities between capital, \( K \), and the natural resource, \( R \). In general terms, the greater the degree to which \( K \) can substitute for \( R \) in production, the less essential \( R \) is as a production input in the economy. The elasticity of substitution, \( \sigma \), between \( K \) and \( R \) is, of course, the appropriate index for examining substitutability. Dasgupta and Heal (1979) make the following observations with respect to the class of CES production functions for the case \( \delta = \theta = 0 \). If \( \sigma > 1 \), the natural resource, \( R \), is not necessary for production, so that sustainability is not an issue. If \( \sigma < 1 \), consumption necessarily declines to zero in the absence of technical progress; sustainable consumption is not possible. The interesting case is \( \sigma = 1 \), which corresponds to the Cobb-Douglas form.

Consider a simple economy without capital depreciation or resource extraction cost, so that the dynamic equations of the economy become

\[
\dot{K} = F(K, R) - C, \quad K(0) = K_0
\]

\[
\text{and } \dot{X} = -R, \quad X(0) = X_0.
\]

Let the production function be specified as

\[
F_K - \delta = \rho. \quad (7a)
\]
with \( a, b > 0 \) and \( a+b < 1 \). Then the following is well known. If \( a > b \), (i.e., output elasticity of capital exceeds that of the natural resource), then the economy is capable of providing a sustained level of per-capita consumption. This result does not hold when capital is allowed to depreciate, i.e., when \( \delta > 0 \) (Solow 1974). However, if \( b \geq a \), consumption per head declines to zero in the long run in the absence of technical progress.

Given the simple case of a production function with Cobb-Douglas form and the iso-elastic utility function, the following results are known with respect a utilitarian optimum (i.e., the solution to problem (5), with dynamic constraints given by (8a) and (8b)). If the utility discount rate, \( \rho \), is positive and the elasticity of marginal utility, \( \eta \), is greater than 1, then a utilitarian optimum exists. However, along the optimum trajectory, output and consumption eventually decline to zero. A small value of \( \rho \) or large value of \( \eta \) will only delay the inevitable decline in output and consumption.

For the case \( \rho = 0 \), a utilitarian optimum exists iff \( \eta > (1-b)/(a-b) \). Along the optimum trajectory, both output and consumption increase without bound, although \( \dot{C} \to 0 \). The larger the value of \( \eta \), the flatter the consumption path. If \( \rho = 0 \) and \( \eta \leq (1-b)/(a-b) \), no path exists which meets required efficiency conditions (6a) and (7).

Readily apparent in this consideration of a utilitarian optimum are the highly restrictive conditions necessary to guarantee the existence of an optimum program. And even in the case where the optimum program does exist, it may be subject to criticism on grounds that it fails to be consistent with intergenerational equity. For example, in
the case $\rho > 0$ and $\eta > 1$, current consumption is enjoyed at the expense of future
generations, who eventually see output and consumption decline to zero. Alternatively,
for the case $\rho = 0$ and $\eta < \infty$, the existence of an optimum dictates that the current
generation must sacrifice for future generations, whose welfare continues to increase.

This brief overview for the case of exhaustible resources applies in the absence
of both population growth and technological change. For a consideration of consumption
possibilities with population growth and technological change, see Stiglitz (1974) and the
survey by Toman et. al. (1993). Assuming a production function of the Cobb-Douglas
form, Stiglitz (1974) proves the following proposition: a constant level of per-capita
consumption can be sustained iff the ratio of the rate of technological change, $\lambda$, to the
rate of population growth, $n$ (assumed positive), is greater than or equal to the share of
the natural resource. That is, if $Q(K, R, L, t) = K^\alpha R^\beta L^\delta e^{\lambda t}$, where $\dot{L}/L = n$, then a
necessary and sufficient condition that a constant level of per-capita consumption be
sustainable is that $\lambda/n \geq b$.

1.2.4. Maximin Welfare

Since Rawl's (1971) path-breaking work on the theory of justice, maximin welfare
has received much attention as an alternative to utilitarianism that provides for
intergenerational equity as well as dynamic efficiency. At least three motivations for
maximin may be put forward. While distinct in outline, they are conceptually equivalent.

The motivation closest to Rawl's framework is that which appeals to the idea of
choice behind the veil of ignorance: "Since no one knows to which generation he
belongs, the question [of choice] is viewed from the standpoint of each." (Rawls, 1971,
p. 287). From the so-called 'original position of equal ignorance,' individuals are induced to choose consumption programs that maximize the welfare of the least well-off generation. This idea can be expressed analytically in the following way. Let $E$ be the set of consumption trajectories $(C_t)$ that are consistent with dynamic efficiency (i.e., conditions (6) and (7)). Within the set $E$, one then seeks a consumption trajectory $(C^m_t)$ that satisfies:

$$\inf_{t \geq 0} (C^m_t) = \sup_{(C_t) \in E} \{ \inf_{t \geq 0} (C_t) \}$$

(10)

As discussed in Dasgupta (1974), a direct implication of such a choice process is that if the desired trajectory, $(C^m_t)$, exists, it must satisfy $C^m_t = \bar{C}$ for all $t \geq 0$.

Alternatively, one may view maximin welfare as a process of lexicographic ordering whereby consumption trajectories are ranked according to the following scheme.

Define an operator, $L$, on the set of dynamically efficient consumption trajectories, $E$. To each $(C_t) \in E$, the operator, $L$, assigns an ordered pair $(I, W)$. Specifically,

$$L[(C_t)] = (I[(C_t)], W[(C_t)])$$

(11)

where $I[(C_t)] = \inf_{t \geq 0} (C_t)$,

and $W[(C_t)] = \int_0^\infty U(C_t)e^{-pt} dt$.

Ranking occurs lexicographically. Given two trajectories, $(C_1)$ and $(C_2)$, $(C_1)$ is said to dominate $(C_2)$, written $(C_1) > (C_2)$, iff $I[(C_1)] > I[(C_2)]$. If a tie occurs at the first stage, relative ranking then proceeds to the second stage. Now $(C_1)$ dominates $(C_2)$ iff $I[(C_1)] = I[(C_2)]$ and $W[(C_1)] > W[(C_2)]$. This ranking process ultimately identifies the maximin trajectory $(C^m_t) = \bar{C}_{max}$ as the maximal element of the set $E$. 

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Finally, maximin welfare may be discussed in the context of the parameter $\eta$, the elasticity of marginal utility. A large elasticity $\eta$ implies a high degree of curvature on the utility function, which, in turn, renders a high level of 'egalitarianism' in the distribution of utility across generations. In the limit as $\eta \to \infty$, the utilitarian optimum consumption trajectory is flattened. Alternatively, $\eta$ may be taken as the index of relative risk aversion. In this context, the maximin solution would naturally arise if $\eta = \infty$, signifying infinite risk aversion on the part of individuals confronted with a uniform probability distribution of being a member of any generation. The result may be seen analytically by rearranging equation (7) for the case $\delta = 0$ to give

$$\frac{\dot{C}}{C} = \frac{F_k - \rho}{\eta}. \quad (12)$$

As $\eta \to \infty$, $\dot{C}/C \to 0$, generating constant consumption for all $t \geq 0$.

The popularity of the maximin framework for achieving intergenerational equity has no doubt been boosted by the identification of a simple investment rule, that under certain restrictive conditions is both necessary and sufficient to achieve a maximum level of constant per-capita consumption. That rule, due to Hartwick (1977), is to follow Hotelling's Rule for resource extraction, equation (6), and invest the profits from the flow of depletion into capital accumulation. More generally, as developed by Dixit et al. (1980) and Solow (1986), the Hartwick rule may be viewed as keeping the total value of net investment equal to zero. With consumption $C$ as numeraire, let $\rho$ and $\psi$ be the unit prices of capital, $K$, and the national resource, $R$, respectively. The Hartwick condition can then be written as
\[ \lambda \dot{X} + \psi \dot{X} = 0. \quad (13) \]

The Hartwick investment rule and the maximin welfare that it renders point to the existence of at least one sustainable consumption path. However, the moral framework of maximin has serious weaknesses. First and foremost are its implications for development. As discussed by Dasgupta and Heal (1979), the maximin welfare criterion leaves countries at the mercy of their initial capital stock. Countries that are capital poor are forever constrained to have lower levels of per capita consumption than more advanced countries that are already capital rich. Secondly, the assumptions needed to yield a maximin solution are quite stringent. They include zero population growth in the absence of technological change, no capital depreciation, and output elasticity of capital greater than that of the resource. On these two counts, the maximum criterion fails as an appropriate basis for sustainability and should be rejected on both moral and practical grounds.

1.2.5. Constrained Utilitarian Optimization

Recognizing the theoretical and ethical weakness of the maximin approach to welfare, Asheim (1988, 1991) and Pezzey (1994) offer a compromise between maximin welfare and utilitarian optimization in the form of "opsustimality." In this approach, nondecreasing utility is added as a sustainability constraint to the problem of utilitarian optimization.

Pezzey describes the two-phase nature of the opsustimal path, a result established by Asheim (1988). The opsustimal path is continuous and exhibits one of two possible profiles: (1) constant consumption (and hence utility) for all time \( t \); or (2) rising
consumption and utility for $0 \leq t \leq T$, where $T$ is endogenous, and then constant consumption and utility for $t > T$. See Figure 1.1 for an illustration of an opsustimal path. Pezzey also shows that along the opsustimal path, the nondecreasing consumption constraint is equivalent to the constraint that total capital be nondecreasing. In the context of our basic model, this implies that $\dot{C} \geq 0$ iff $\lambda \dot{K} + \psi \dot{X} \geq 0$. Moreover, consumption and the value of the total capital stock both rise together and then are both constant together.

There are several drawbacks to the opsustimal paradigm. In general, constrained optimization models are likely to be in conflict with generally accepted axioms of rational choice. That turns out to be the case here. Constrained optimization cannot provide a full ranking of alternatives, because alternatives that violate the constraint cannot be compared. In the case of opsustimal growth with a finite stock of nonrenewable resources, if either the elasticity of substitution between natural capital and produced capital is less than one, or the output elasticity of natural capital is greater than that of produced capital (with elasticity of substitution equal to one), then the sustainability constraint renders the opsustimal problem infeasible. Clearly, among all the infeasible consumption trajectories some trajectories are preferable to others. Constrained welfare maximization does not provide such a ranking.

The present study takes an alternative approach to sustainability, one that retains the utilitarian framework, but avoids sustainability constraints and the restrictive assumptions needed to guarantee the existence of an optimum consumption trajectory. Key to this approach is augmenting the basic model by incorporating a superabundant
Figure 1.1
Modified Golden Rule Path vs. Opsustimal and Maximin Paths
resource that flows from a backstop technology. Within the context of this augmented model, one can derive modified golden rules and golden rules for resource management and capital accumulation. An overview of this approach will be presented in Section 1.3 below.

The typical profile of an optimal consumption path with a positive, but modest utility discount rate, for an economy with either a backstop or a renewable resource is single-peaked. It then asymptotically approaches some positive lower bound consumption level defined by the modified golden rule. If that lower bound is above the maximum consumption level for opsustimal growth, then the latter is strictly dominated under the axiom that more is better than less. This situation is illustrated in Figure 1.1. If the utility discount rate is zero, the optimal consumption trajectory is strictly increasing up to the golden rule steady state level of consumption. In that case, the sustainability constraint is redundant.

1.2.6. Economic Depreciation of Ecological Capital

The alarming level of resource depletion and environmental degradation now occurring in developing countries has increased the sense of urgency among public officials and development economists. As a consequence, the research agenda has moved beyond environmental impact assessments to comprehensive economic planning and policy design for effective management of natural resources and the environment (see, e.g., Schramm and Warford, 1989). Two aspects of this recent literature are especially notable. The first is the emphasis on the resource systems approach to analysis, which applies systems engineering philosophy and methodology to investigate the complex
interactions between government policy and large scale physical resource systems. This approach is being used to good effect in the study of problems related to forestry, water systems and soil management. The second is the attempt to incorporate measures of natural resource depletion in the national income accounts. The motivation here is to help policy makers to take better account of resources and the environment in constructing policy for economic development.

Significant progress has been made in both of these areas of investigation, but a rigorous conceptual foundation for measuring the economic depreciation of ecological capital is wanting. A number of approaches to environmental and resource accounting have been suggested in the literature (see e.g. Landefeld and Hines, 1985; El Seraphy and Lutz, 1989; Repetto, 1989; Hartwick, 1990; and Mäler, 1991). While these approaches are generally informative, they are limited in applicability. Properly interpreted, they either provide the basis for pragmatic approximation of resource depletion or they apply only in special cases. In particular, most approaches are restricted to the case of exhaustible resources and are only correct when such resources are efficiently mined. Hartwick (1990) and Mäler (1991) do consider renewable resources, but still assume that the economy follows or nearly follows the optimal trajectory for growth.

In traditional national income accounts, depreciation of produced capital is deducted from Gross National Product in estimating income. In this context, income can be defined as "the maximum amount a recipient can consume in a given period without reducing possible consumption in a future period (El Seraphy and Lutz, 1989, based on
Hicks, 1946). From this definition, it follows that true income is sustainable income (Daly, 1989; El Seraphy and Lutz, 1989). If ecological capital as well as man-made capital is essential to production, depreciation of ecological capital should be treated analogously to man-made capital in the calculation of national income.

At the heart of a suitable methodology for measuring resource and environmental depreciation is the derivation of appropriate shadow prices. However, the literature on the "greening" of income accounts treats depreciation in the context of optimal control trajectories for undistorted economies. Both Hartwick (1990) and Mäler (1991), for example, apply linear approximation to terms in the current value Hamiltonian expression associated with the optimal control model of the economy. This approach can be illustrated using the basic model presented in Section 1.2.2 above.

Consider the case of exhaustible resources (i.e., the resource growth function $G = 0$). Suppose utilitarian welfare $W$ is maximized subject to the two dynamic equations

$$\dot{K} = F(K, R) - \delta K - \theta R - C$$

and

$$\dot{X} = -R.$$

The current value Hamilton for this optimal control problem is given by

$$H = U(C) + \lambda(t)[F(K, R) - \delta K - \theta R - C] + \psi(t)[-R],$$

(14)

where $\lambda(t)$ and $\psi(t)$ are costate variables. Hartwick then applies the linear approximation $U(C) \equiv U_c C$. Using the first order conditions $\lambda(t) = U_c$ and $\psi(t) = (F_K - \theta) \lambda(t)$, the Hamiltonian expression becomes:
Dividing equation (15) through by $U_c$ and substituting $\dot{K}$ and $\dot{X}$ in the dynamic equations of the economy yield the ‘dollar value’ NNP function

$$H = U_c C + U_c [F(K, R) - \delta K - \theta R - C] + (F_R - \theta) U_c [-R].$$

(15)

This says that current ‘Hotelling rents’ on stock depletion should be netted out of GNP to yield NNP. In this optimality framework, depreciation of the resource is taken as the first best shadow price, which is seen from equation (16) to be equal to royalty from resource extraction, i.e., $(F_R - \theta)$. This result is not at all surprising in that a necessary condition for optimality is that the resource be extracted up to the point that royalty equals user cost, or depreciation, as specified by the Hotelling Rule.

The problem, of course, is that in imperfect economies, distortions drive a wedge between marginal benefits and marginal user cost, so that royalty no longer equals depreciation. To address this and other conceptual deficiencies in the calculation of economic depreciation, the present study proposes alternative approaches that reflect market distortions and consider the opportunity cost of resource depletion.

1.3. Overview of the Study

This section provides a guide to the organization of the study and outlines the conceptual approaches considered in the subsequent chapters.

1.3.1. Golden Rules and Sustainability

The conceptual framework for this research centers on the notion of sustainable
resource management as derived from models of aggregate growth with reproducible capital, natural resources, and environmental stocks. In this framework, sustainable resource management is tied directly to the idea of sustainable development. Chapter 2, adapted from Endress and Roumasset (1994), considers the basic model presented in Section 1.2 for the case of exhaustible resources and augments it by incorporating a superabundant resource that flows from a backstop technology. The augmented model also allows for capital depreciation ($\delta > 0$) and assumes that unit cost $\theta$ is a decreasing function of the resource stock $X$.

Take the case of oil as an example. Oil stock is drawn down as the economy grows, until unit cost, $\theta$, has risen sufficiently to warrant the switch to a superabundant, but high cost, alternative energy source with unit cost, $\theta_b$ (e.g., coal, solar energy). Once the switch has been made, capital and labor costs alone determine the price of the energy resource; i.e., scarcity rents are no longer significant.

Description of the augmented model can be facilitated by letting $R_1$ denote the flow of the primary resource and $R_b$ the flow of the backstop resource. Assuming the two natural resources are perfect substitutes, the optimization problem may be written

$$\begin{align*}
\text{Max} & \int_0^\infty U(C) e^{-\rho t} dt \\
\text{s.t.} & \dot{K} = F(K, R_1 + R_b) - \delta K - \theta R_1 - \theta_b R_b - C \\
 & \dot{X} = -R_1, \quad \theta \geq \theta_b.
\end{align*}$$

(17)

At some endogenous time $T$, the unit cost, $\theta$, of the primary resource, $R_1$, reaches the backstop cost, $\theta_b$, and a transition is made to the substitute resource, $R_b$. Standard
application of the maximum principle from optimal control theory yields the following efficiency conditions:

\[
\frac{\dot{F}_R}{[F_R - \theta]} = [F_K - \delta], \text{ for } 0 < t < T,
\]

\[
F_R = \theta_b', \text{ for } t \geq T,
\]

\[
\eta \frac{\dot{C}}{C} = F_K^-(\delta + \rho).
\]  

Along the steady state path, \( \dot{C}/C = 0 \) and \( F_K = \delta + \rho \). Two conditions now define the modified golden rule for growth and capital accumulation when a backstop natural resource, essential to production is available in infinite supply:

\[
F_K = (\delta + \rho) \text{ and } F_R = \theta_b'.
\]  

Efficient evolution of the economy toward the modified golden rule steady state growth path is governed by equations (18) and (19). In particular, resource extraction should be governed by Hotelling's rule. Overuse of the resource, counter to his rule, would be inefficient in both the short run and the long run. But, just as important, under use would be inefficient as well.

Chapter 2 specifies conditions under which the modified golden rule is consistent with the notion of sustainable development. The approach proposed relies on comparison of consumption trajectories rather than capital stocks. For the case of constant population, we may define a consumption trajectory \( C_t \) to be sustainable relative to \( C_{\min} \) iff there exists some \( T \) such that \( C_t \geq C_{\min} \forall t \geq T \). In particular, it is interesting to consider the case where \( C_{\min} = \bar{C} \), the maximin constant level of consumption that could
be attained in the absence of a backstop, since $\tilde{C}$ has been previously singled out as a possible benchmark for intergenerational equity. Other benchmark levels of $C_{\min}$ are possible, such as some agreed upon subsistence level. However, the specification of subsistence level may be somewhat arbitrary.

A central critique of ecologists and ecological economists regarding the maximization of aggregate discounted welfare is that discounting necessarily prejudices the case against future generations. Some authors have suggested (see e.g., Pearce and Turner, 1990) that the present generation is properly viewed as a steward for the future. Setting $\rho = 0$ in the model introduced above is one way of representing these concerns. Setting the pure rate of time preference equal to zero transforms the modified golden rule into a golden rule and provides an alternative criterion for intergenerational equity. Instead of requiring consumption in all periods to be equal, this approach simply removes any \textit{a priori} discrimination between generations.

1.3.2. Economic Depreciation and Resource Shadow Price

A major portion of the present study is devoted to consideration of market distortions and their effect on resource use and shadow prices. Calculation of shadow prices for natural resources and environmental stocks has direct application to project evaluation, leasing policy and the national income accounts.

The previous literature, however, is restricted to examination of resource depletion along the optimal trajectory of an evolving economy. Such an approach yields first best shadow prices, which do not correctly value resources when market distortions are present.
The approach in this study is to consider efficiency condition (6) for the case of a renewable resource. Independent of the condition of equality in equation (6), we note that each side can be regarded as a schedule or function of the resource flow rate, \( R \). The resulting resource rent, \( \pi \), will be

\[
\pi(R) = F_R - \theta. \tag{21}
\]

Similarly, the right hand side of (b) represents the schedule of marginal user cost, \( MUC \), so that

\[
MUC(R) = \frac{1}{(F_K - \delta)}(\dot{F}_R + [F_R - \theta]G'(X) - \theta'(X)G(X)). \tag{22}
\]

Marginal net benefit, \( MNB \), can then be expressed as

\[
MNB(R) = \pi(R) - MUC(R).
\]

As usual, efficiency is achieved if, during each time interval \( dt \), the natural resource is used up to the point that net marginal benefit is zero, i.e., until \( \pi(R) = MUC(R) \).

A primary concern of the present research, however, is computation of resource depreciation when efficiency conditions are not met and the economy is off the optimal trajectory.

1.3.2.1. A Partial Equilibrium Approach

To overcome some of the deficiencies outlined above, Chapter 3 presents a method for constructing second best shadow prices in a partial equilibrium framework. Start with the analogue of efficiency condition (6) in a model with resource price \( P \) and interest rate \( r \):
\( P - \theta = \left( \frac{1}{r} \right) \{ \dot{P} + (P - \theta)G'(X) - G(X)\theta'(X) \} \). \hspace{1cm} (23)

Again, the left-side represents producer royalty, and the right side marginal user cost.

Tax distortions are then included in the model to alter royalty and user cost schedules from the vantage point of the resource concessionaire. For example, imposition of a tax at rate \( \tau \) on both profits and interest earnings would alter the concessionaire's profit maximizing condition to

\[
(P - \theta) = \left( \frac{1}{r(1-\tau)} \right) \{ \dot{P} + (P - \theta)G'(X) - G(X)\theta'(X) \}.
\hspace{1cm} (24)
\]

The effective discount rate as perceived by the concessionaire declines from \( r \) to \( r(1-\tau) \). The imposition of the tax produces a parametric shift in the marginal user cost schedule. The concessionaire is induced to reduce his harvest rate; he underproduces and overprices relative to the social optimum. In the case at a production subsidy, \( \tau \) is negative, increasing the discount rate as perceived by the concessionaire, and decreasing marginal user cost. This induces the concessionaire to overproduce and underprice relative to the optimum.

The heart of Chapter 3 is the construction of the shadow price of a natural resource using a weighted average formula similar to that developed by Harberger (1969) for static cost-benefit analysis. Using the nonoptimal trajectories for resource extraction and stock depletion that would be used by a resource concessionaire in the distorted economy, Chapter 3 presents the derivation of closed form expressions for second best shadow prices. Also presented are numerical approximations to make the shadow price
expressions operational in practical applications. The model is applied to the case of fisheries economics in order to illustrate the shadow price calculation for the open access situation.

1.3.2.2. A General Equilibrium Approach

Chapter 4 tackles the problem of second best shadow prices in the context of the basic model of a production economy with renewable resources. Using this model as a framework for analysis, Section 4.2 reviews recent literature on national income accounting (e.g., Hartwick, 1990 and Máler, 1991) and makes the case that second best shadow prices are needed to enable net national product to serve as a welfare measure in distorted economies. This argument is pursued through the development of two models of economic distortion is Section 4.3. One model considers a direct distortion in the natural resource market, while the other model examines the distortionary effects of an economy-wide generalized profits tax. The first example of distortion is assumed to be removable through correction of market incentives. In the second example, the distortion is regarded as permanent because of political or institutional constraints on economic policy.

The second best shadow prices derived in connection with the two models of distorted economies are used to construct constrained first best net national product in Section 4.4. Though graphical representation, these alternatives to net national product based on first best shadow prices are shown to be better measures of society’s aggregate welfare in the presence of market distortion. Central to the theme of this study is the conclusion in Section 4.5 that sustainable growth in distorted economies is best pursued
by using second best, rather than first best, shadow prices as a basis for economic policy, cost-benefit analysis, and national income accounting.

1.3.3. Extensions and Conclusions

Finally, Chapter 5 extends the approach and findings of earlier chapters to more general models. Section 5.2 builds on Chapter 1 and develops both a modified golden rule and then a golden rule for the case of a production economy with renewable resources. As in Chapter 2, we allow for population growth and express major variables in the economy in per-capita units. In the context of optimal growth in an economy with renewable resources, Section 5.3 examines the distinction between personal discount rates and generational discount rates. An overlapping generations model, based on an approach in Burton (1993), is used to support the case for setting the generational discount rate equal to zero in aggregate growth models. The case for not discounting the utility of future generations lends weight to the legitimacy of the golden rule as a basis for sustainable growth. This linkage is developed in Section 5.4 though consideration of a golden rule net national product. Drawing on the analysis in Chapter 4, Section 5.5 derives second best golden rules for application in economies with market distortions. Section 5.6 suggests directions for future research and summarizes the major findings of this study. Paramount among these findings is that sustainable growth is best achieved, not by imposing artificial constraints on economic growth, which are likely to be redundant, infeasible or dominated, but by designing policies, based on golden rules and appropriate shadow prices, that are compatible with both economic efficiency and stewardship for the future.
2.1. Introduction

In a recent review, Ruttan (1992) observes that academic discourse is experiencing a “third wave” of heightened post-war sensitivity about the limits to growth inherent in the finite stocks of natural and environmental resources. In these discussions, the concepts of “sustainable growth” and “sustainable development” have been suggested, reminiscent of “growth with equity,” as an objective that gives appropriate weights to possibly conflicting goals. Despite its popularity, however, sustainability has largely eluded any consensus regarding a precise definition. We suggest that greater precision could be achieved by drawing from and extending the standard golden rules from the growth theory literature.

The common concern shared by sustainability proponents is that present consumption is at the expense of future generations because of the resulting depletion of natural and environmental resource stocks. Solow (1986) has proposed that this problem may be appropriately addressed by the criterion of *intergenerational equity*, defined as the maximin level of consumption that can be sustained over all time. Under special assumptions, maximin consumption can be attained by extracting resources according to the Hotelling principle and investing the resource royalties thus derived into capital
accumulation (Hartwick, 1978). This rule provides a solution to the maximin consumption problem and simultaneously shows that, under the assumptions specified, at least one sustainable consumption path exists. As discussed by Dasgupta and Heal (1979), however, the maximin welfare criterion leaves countries at the mercy of their initial capital stock. Countries that are capital poor are forever constrained to have lower levels of per-capita consumption than more advanced countries that are already capital rich. In addition, the assumptions used in the derivation are quite stringent, including zero population growth in the absence of technological change, no capital depreciation, and output elasticity of capital greater than that of the resource.

Solow (1974) examines intergenerational equity and exhaustible resources under conditions of zero extraction costs. In Hartwick (1978), the rule of investing rents from exhaustible resources is derived in the case of constant, but nonzero, extraction cost. Cairns (1986) extends the Hartwick investment rule to the case where Ricardian differential rents are generated through exploitation of a nonhomogeneous resource. In the Cairns model, mineral production costs increase as the quality of ore exploited decreases. Cairns shows that the basic model can be modified to incorporate the possibility that extraction costs increase with cumulative extraction.

In the face of exponential population growth and limited resources, Solow (1974) shows that "... no positive constant consumption per worker is maintainable forever." With respect to the problem of capital depreciation, Hartwick (1978) notes, "... (the) savings investment rule will not provide for the maintaining of per-capita consumption constant over time. The current decline in per-capita consumption is simply the amount
of the produced commodity required to offset the current amount of depreciation in reproducible capital.” Solow (1974), Stiglitz (1974), Dasgupta and Heal (1979), and Wan (1989) all present demonstrations that a necessary condition for maximin consumption is that output elasticity of capital be greater than that of the finite resource.

An alternative representation of intergenerational equity involves suppressing to zero the social rate of time preference in intertemporal welfare maximization. In particular, Ramsey (1928) held that it is “ethically indefensible for society to discount future utilities” (paraphrased by Solow, 1974).2 “We ought to act as if the social rate of time preference were zero (though we would simultaneously discount future consumption if we expect the future to be richer than the present)” (Ramsey, 1928).

In the Solow model, the optimal trajectory of per-capita consumption with a zero social rate of time preference increases indefinitely, and there is no steady state. Both consumption and growth are sustainable. In the present study, we explore an alternative model. Extraction costs are permitted to rise as a function of the cumulative amount of the nonrenewable resource extracted, in contrast to the constant extraction cost assumption in the models of Hartwick and Solow, and the output elasticity of capital is not required to exceed that of the resource. Instead, the extraction cost function is assumed to be bounded from above by a backstop technology. The backstop assumption may be thought of as an approximation which may be made arbitrarily close to the actual extraction cost function. This assumption permits the restoration of steady state results for which both golden and modified golden rules can be derived. Also, in contrast to
additional restrictive assumptions needed to support maximin consumption, the model in this study allows for both population growth and capital depreciation.

The chapter is organized as follows. The model is described in Section 2.2. The modified golden rule is derived and conditions are stated for whether that rule violates a sustainability condition that constrains consumption not to fall below the maximin level. Comparative statics results are also derived showing that steady state consumption decreases as the backstop cost and rate of time preference increase. In Section 2.3, a special case is considered, where the concern for the future is manifested by setting the rate of time preference at zero and solving for the golden rule. This solution does not violate the sustainability constraint. These and additional concluding remarks are summarized in Section 2.4.

2.2. A Modified Golden Rule

Consider an economy that uses three inputs, capital (K), labor (L), and a natural resource (R) to produce a single homogeneous good. Assume that the production technology is constant returns to scale, so that the production function, $F(K, R, L)$, is homogeneous of degree one. Following the standard approach (see e.g., Wan, 1989), output of production is divided among consumption, gross investment, and the cost of providing the resource as input to the production process. Let $\theta$ be the unit cost of the natural resource, which we take to be a decreasing function of the resource stock, $X(t)$. Capital depreciation occurs at the rate $\delta K$. Then,
\[ F(K, R, L) = \dot{K} + \delta K + \theta R + C. \]  \hspace{1cm} (1)

In the case of oil, for example, oil stocks are drawn down as the economy grows, until the unit cost, \( \theta \), of providing oil as an input to production has risen sufficiently to warrant the switch to a superabundant, but high cost, alternative energy source with unit cost, \( \theta_b \) (e.g., coal, solar energy, geothermal, or nuclear fusion). Following Nordhaus (1973, 1979) and Heal (1976), we consider such a superabundant energy source to flow from a backstop technology with constant unit cost. Capital and labor costs alone determine the price of the energy resource; i.e., scarcity rents are no longer significant.

In their supply-side study of oil prices, Roumasset, Isaak, and Fesharaki (1983) conducted a sensitivity analysis showing that the assumption of an unlimited backstop technology does not introduce a substantial inaccuracy in the estimation of efficiency prices, if the backstop price is set sufficiently high and if total resources available up to the backstop price are abundant. The test of this condition is that the efficiency prices in the present and future period of interest (e.g., 50-100 years) are not sensitive to changes in the backstop price. On this basis, we take the incorporation of the backstop technology as an empirically sound approach to resource modeling. Moreover, the point at which the extraction cost reaches its ultimate backstop plateau may be made arbitrarily remote. For example, if we take the exhaustible resource to be a composite, all-purpose energy resource, measured e.g., in oil-barrel equivalents, intermediate plateaus can be taken to represent various grades of oil, coal, nuclear fission, nuclear fusion, etc. In such an application, extraction cost would include conversion costs of the oil-alternative technology as well as the extraction and production costs of the energy source (see, e.g.,
Nordhaus, 1979). In this way, the quantity of aggregate resource use, beyond which the backstop substitute comes in, can be arbitrarily high, restricted only by limits on information and estimating capability.

As a related matter, we briefly comment on the issue of optimal sequencing of resource extraction with rising unit extraction costs. Solow and Wan (1976) considered a simple two-period, two-deposit model of an exhaustible resource and showed that it is preferable to fully exploit the low cost deposit first. Subsequently, Kemp and Long (1980) developed a more general model, wherein it may be preferable to exploit high and low resource deposits simultaneously for the purpose of smoothing consumption over time. Commenting on the work by Kemp and Long, Lewis (1982) derived sufficient conditions under which strict sequencing of extraction (from low cost to high cost) becomes optimal, consistent with the model of Solow and Wan: extracted resources can be converted into productive capital.

A different problem arises in the case of physical mining constraints which prevent optimal sequencing of resource extraction per Solow and Wan (1976). Hartwick (1978) and Cairns (1986) examined situations where sources of varying quality must be exploited at a single time. While yielding important insights related to the generation of Ricardian rents, the Hartwick/Cairns model would add little to our consideration of the steady state golden rules made possible by the existence of a backstop technology, beyond the modifications of trajectories leading to the steady state. We therefore adhere to the simpler model of extraction costs rising with cumulative extraction.

The basic model in the present paper, as represented by dynamic equation (1), is
consistent with the sufficient condition established by Lewis (1982). We therefore assume that the backstop resource will not be used until unit extraction cost, $\theta$, rises to the backstop cost, $\theta_b$.

Until the transition to the backstop resource, there is a finite constraint on the resource stock:

$$\int_0^\infty R(t) \, dt \leq X_0 < \infty.$$  \hspace{1cm} (2)

Assume that the labor force grows at rate $n$ from an initial level of $L(0) = L_0$. Given the homogeneity of $F(K, R, L)$, labor can be factored out to yield a production function of the form, $f(k, r)$, where $k$ is the capital to labor ratio, and $r$ is the resource to labor ratio. The dynamic equation of growth now becomes

$$k = f(k, r) - \mu k - \theta r - c, \quad \theta \leq \theta_b.$$  \hspace{1cm} (3)

Here, $c$ is per-capita consumption, $\mu = (n + \delta)$, and $\theta_b$ is the constant unit cost of supplying the backstop resource.

A modified golden rule can be obtained when we include time preference in the model and maximize the conventional measure of social welfare. With $\rho$ as the rate of time preference and $U(c)$ as the utility of consumption of the representative agent, the planner solves

$$\text{Max} W = \int_0^\infty U(c) e^{-\rho t} \, dt$$  \hspace{1cm} (4)

s.t.  \hspace{1cm} \begin{align*}
\dot{k} &= f(k, r) - \mu k - \theta r - c, \quad k(0) = k_0 \\
\dot{X} &= -rL, \quad X(0) = X_0 \hspace{1cm} (4a) \\
\theta &\leq \theta_b. \hspace{1cm} (4b)
\end{align*}$$  \hspace{1cm} (4c)

At some endogenous time, $T$, the unit cost, $\theta$, of the exhaustible resource reaches the
the backstop cost, \( \theta_b \), and a transition is made to the substitute resource.\(^3\) Because of the inequality constraint on \( \theta \), the Hamiltonian \( H \) must be augmented to form the Lagrangean function, \( \mathcal{L} \):

\[
\mathcal{L} = H + \gamma [\theta_b - \theta]
\]  

where \( H = U(c)e^{pt} + \lambda [f(k, r) - \mu k - \theta r - c] - \psi[rL]. \) (5)

The complementary slackness condition associated with the inequality constraint is

\[
\gamma \frac{\partial \mathcal{L}}{\partial \gamma} = \gamma [\theta_b - \theta] = 0.
\]

Standard application of the maximum principle yields the following efficiency conditions:

\[
\frac{f_r}{f_r - \theta} = [f_r - \delta], \text{ for } 0 < t < T, \tag{6}
\]

\[
f_r = \theta_b, \text{ for } t \geq T, \tag{6'}
\]

\[
- \frac{U''(c)}{U'(c)} = f_k - (\mu + \rho). \tag{7}
\]

Equation (6) is essentially Hotelling's rule in a general equilibrium context.\(^4\) Equation (7), the Ramsey condition, can be simplified by introducing the consumption elasticity of marginal utility,

\[
\eta(c) = -c \frac{U''(c)}{U'(c)}. \tag{8}
\]

Using this definition, equation (7) becomes
\[ \eta(c) \frac{\dot{c}}{c} = f_k - (\mu + \rho). \]  

(9)

For a production technology, \( F(K, R, L) \), that is constant returns to scale, the only possible steady state growth rate is zero (see e.g., Sala-i-Martin, 1990). Therefore, along the steady state path, \( \dot{c}/c = 0 \), and

\[ f_k = (\mu + \rho). \]

Two conditions now define the modified golden rule for growth and capital accumulation when a backstop natural resource, essential to production is available in infinite supply:

\[ f_k = (\mu + \rho) \text{ and } f_r = \theta_b, \quad t \geq T. \]  

(10)

There is no presumption in this formulation that the backstop and the steady state are reached simultaneously.

Efficient evolution of the economy toward the modified golden rule steady state growth path is governed by equations (6) and (7). In particular, resource extraction should be governed by Hotelling’s rule. Overuse of the resource, counter to this rule, would be inefficient in both the short run and the long run. But, just as important, underuse would be inefficient as well.

We now consider conditions under which the modified golden rule is consistent with the notion of sustainable development. The approach we propose relies on comparison of consumption trajectories rather than capital stocks. If one accepts the premise that consumption is the ultimate goal of economic activity, then this approach
goes to the heart of the matter. We may define a consumption trajectory to be sustainable relative to \( c_{\text{min}} \) iff there exists some \( T \) such that \( c(t) \geq c_{\text{min}}, \forall t \geq T \). In particular, it is interesting to consider the case where \( c_{\text{min}} = \bar{c} \), the maximin constant level of consumption that could be attained in the absence of a backstop, since \( \bar{c} \) has been previously singled out as a possible benchmark for intergenerational equity (Hartwick, 1977; Solow, 1986). Other benchmark levels of \( c_{\text{min}} \) are possible, such as some agreed upon subsistence level. However, the specification of subsistence level may be somewhat arbitrary.

The computation of \( \bar{c} \) can be formulated as a standard optimization problem (see Wan, 1989):

\[
\bar{c} = \max_{c_0} c_0 \quad \text{(11)}
\]

\[\text{s.t.} \quad \dot{c} = 0 \text{ and } (4a), (4b), (4c).\]

The solution to this problem, if it exists, yields

\[
\bar{c} = \bar{c}(k_0, X_0). \quad \text{(12)}
\]

The dependence of \( \bar{c} \) on \( k_0 \) and \( X_0 \) reinforces the idea that economies constrained to maximin per-capita consumption are at the mercy of initial conditions. Solow (1974), Dasgupta and Heal (1979), and Wan (1989) derive solutions to versions of problem (11), showing the specific dependence of per capita consumption, \( \bar{c} \), on the initial capital stock, \( k_0 \), and resource stock, \( X_0 \). In contrast, we advance the following proposition concerning the modified golden rule steady state.
Proposition 1: Given an economy governed by dynamic equation (3), let $c^*$ be the steady state modified golden rule level of per-capita consumption associated with condition (10). Then $c^*$ is independent of the initial per-capita capital stock, $k_0$, and resource stock, $X_0$.

Proof: The modified golden rule (10) defines the steady state marginal products $f_k$ and $f_r$, which in turn determine the well-defined steady state values $k^*$ and $r^*$. Consumption level $c^*$ can then be computed as $c^* = f(k^*, r^*) - \mu k^* - \theta_b r^*$.

As noted by Wan (1989), there is no guarantee that a maximin solution exists for production functions other than those with Cobb-Douglas form, $f(k, r) = Ak^r$. Even in this case, a necessary condition for existence of a maximin solution is that $a > b$. Moreover, both population growth and capital depreciation must be zero, so that $\mu = 0$. The behavior of extraction cost, as a function of the resource stock, $X$, may also affect the existence of a maximin solution.

The relationship between $c^*$ and $\bar{c}$ (assuming $\bar{c}$ exists) will depend on the rate of time preference, $\rho$, and on the backstop cost, $\theta_b$. The following proposition verifies that, as expected, the steady state level of per capita consumption, $c^*$, declines with an increase in $\rho$ and with an increase in $\theta_b$.

Proposition 2: Under the assumptions of Proposition 1, let $\rho$ be the rate of time preference and $\theta_b$ be the unit cost of the backstop resource. Then,
i) \( \frac{\partial c^*}{\partial \rho} < 0 \) and ii) \( \frac{\partial c^*}{\partial \theta_b} < 0 \).

Proof: In the steady state \( \dot{k} = 0 \), so that

\[
  c^* = f(k^*, r^*) - \mu k^* - \theta_b r^* .
\]  

(13)

i) Differentiating equation (13) with respect to \( \rho \) we obtain

\[
  \frac{\partial c^*}{\partial \rho} = f_k \frac{\partial k^*}{\partial \rho} + f_r \frac{\partial r^*}{\partial \rho} - \mu \frac{\partial k^*}{\partial \rho} - \theta_b \frac{\partial r^*}{\partial \rho} .
\]

(14)

Using the modified golden rule, condition (10), we can rewrite equation (14) as

\[
  \frac{\partial c^*}{\partial \rho} = \rho \frac{\partial k^*}{\partial \rho} .
\]

(15)

The partial derivative, \( \partial k^*/\partial \rho \), can be signed by differentiating the two modified golden rule conditions with respect to \( \rho \):

\[
  f_k \frac{\partial k^*}{\partial \rho} + f_r \frac{\partial r^*}{\partial \rho} = 1 ,
\]

(16)

\[
  f_k \frac{\partial k^*}{\partial \rho} + f_r \frac{\partial r^*}{\partial \rho} = 0 .
\]

Application of Cramer's rule yields

\[
  \frac{\partial k^*}{\partial \rho} = \frac{f_{rr}}{f_{kr} f_{rr} - (f_{rk})^2} .
\]

(17)
By assumption, $f(k, r)$ is concave (diminishing returns to scale), so that $f_{rr} < 0$ and $f_{kr} f_{rr} - (f_{rr})^2 > 0$. Hence for $\rho > 0$,

$$\frac{\partial c^*}{\partial \rho} = \rho \frac{\partial k^*}{\partial \rho} < 0.$$  

ii) Differentiating equation (13) with respect to $\theta_b$, we obtain:

$$\frac{\partial c^*}{\partial \theta_b} = f_k \frac{\partial k^*}{\partial \theta_b} + f_r \frac{\partial r^*}{\partial \theta_b} - \mu \frac{\partial k^*}{\partial \theta_b} - \theta_b \frac{\partial r^*}{\partial \theta_b} - r^*.$$  

With the modified golden rule, equation (18) can be written

$$\frac{\partial c^*}{\partial \theta_b} = \rho \frac{\partial k^*}{\partial \theta_b} - r^*.$$  

To sign $\partial k^*/\partial \theta_b$, we differentiate the two modified golden rule conditions with respect to $\theta_b$:

$$f_k \frac{\partial k^*}{\partial \theta_b} + f_r \frac{\partial r^*}{\partial \theta_b} = 0,$$

$$f_r \frac{\partial k^*}{\partial \theta_b} + f_r \frac{\partial r^*}{\partial \theta_b} = 1.$$  

By Cramer's rule and concavity of $f(k, r)$,

$$\frac{\partial k^*}{\partial \theta_b} = \frac{-f_r}{f_{kr} f_{rr} - (f_{rr})^2} < 0.$$  

Therefore,
Assuming the maximin solution, \( \bar{c} \), exists, it is reasonable to suspect that it will depend on backstop unit cost, \( \theta_b \), as well as \( k_0 \) and \( X_0 \), if \( \theta_b \) serves as an upper bound to rising unit cost, \( \theta \). We conjecture that when \( \bar{c} \) does depend on \( \theta_b \), \( \frac{\partial \bar{c}}{\partial \theta_b} < 0 \). If the upper bound on rising unit resource cost increases, resource royalties eventually decrease, so that, under the Hartwick savings rule, investment in capital accumulation is not as great as it otherwise would have been. The constant level of per-capita consumption, \( \bar{c} \), that can be sustained at the reduced rate of capital accumulative must decline.

Figure 2.1 provides a schematic of the possible relationships between \( c^* \) and \( \bar{c} \) for different values and \( \rho \) and \( \theta_b \). For a fixed value of \( \theta_b \), \( c^* = c^*(\rho, \theta_b) \) is a declining function of \( \rho \). The maximin level of consumption, \( \bar{c}(\theta_b) \), then determines the maximum rate of time preference, \( \bar{\rho}(\theta_b) \), consistent with sustainable consumption. As illustrated in Figure 2.1, the defining relationship is

\[
\frac{\partial c^*}{\partial \theta_b} = \rho \frac{\partial k^*}{\partial \theta_b} - r^* < 0.
\]

If \( \rho \leq \bar{\rho} \) for a given backstop unit cost, \( \theta_b \), then the modified golden rule path of per capita consumption, \( c^* \), will satisfy sustainability relative to \( \bar{c} \). The case \( \rho > \bar{\rho} \) (i.e., sustainability is not satisfied), leaves the planner with at least two options. The planner could reduce the social rate of time preference to \( \bar{\rho} \) or less. (Section 2.3 considers the case where \( \rho = 0 \), yielding a golden rule). Alternatively, the planner could
Figure 2.1
Steady State Consumption and the Rate of Time Preference
incorporate directly into problem (4) the constraint $c(t) \geq \bar{c}$ for all $t \geq t_0$, while retaining the prevailing rate of time preference, $\rho$. This forces the consumption trajectory to eventually dominate the maximin consumption path. The planner may choose $t_0 > 0$ so as initially to allow $c < \bar{c}$, thereby building up the capital stock to sustain greater consumption in the future.

In Figure 2.2, we sketch plausible trajectories of per-capita consumption leading to modified golden rule growth paths. These trajectories are analogous to those depicted in Diagram 10.3 of Dasgupta and Heal (1979), with the addition of a backstop substitute. For the case of an exhaustible resource with a production function of the Cobb-Douglas form, Dasgupta and Heal (1974, 1979) showed that the consumption trajectory will have at most one peak. Moreover, the lower the rate of time preference, $\rho$, the further in the future will be the peak. Trajectory 2 in Figure 2.2, satisfies the condition of sustainability relative to maximin consumption, while trajectory 1 does not.

2.3. Intergenerational Equity and Time Preference: A Golden Rule

A central critique of ecologists and ecological economists to maximizing aggregate discounted welfare is that discounting necessarily prejudices the case against future generations. Some authors have suggested (see e.g., Pearce and Turner, 1990) that the present generation is properly viewed as a steward for the future. Setting $\rho = 0$ in the model introduced above is one way of representing these concerns. Setting the social rate of time preference equal to zero also provides an alternative criterion for
Figure 2.2
Plausible Consumption Trajectories Leading to the Modified Golden Rule
intergenerational equity. Instead of requiring consumption in all periods to be equal, this approach simply removes any a priori discrimination between generations.

In general, however, maximizing undiscounted aggregate welfare presents a technical problem that was recognized by Ramsey in his classic paper on optimal savings (see Ramsey, 1928). With $\rho = 0$, the integral in equation (4) becomes infinite, so that there is no way to discriminate among alternative paths of sustainable consumption; all such paths produce infinite social welfare. Ramsey cleverly tackled this problem by incorporating into the integrand a postulated bliss point. Koopmans (1965) refined this approach by showing that the golden rule path for capital accumulation could be taken as the Ramsey bliss point, i.e., the comparison path against which to measure utility as the economy evolves over an infinite time horizon.

In the resource context, an analogous bliss point can be derived from the dynamic equation of growth, equation (3). Once the shift to the backstop has been made, the strict concavity of $f(k, r)$, and the conditions,

$$f_k(0, 0) = \infty, \quad f_r(0, 0) = \infty,$$

$$\lim_{k \to 0} = 0, \quad \lim_{r \to 0} = 0,$$

will be sufficient to guarantee the existence of a steady state for which $\dot{k} = 0$. Equation (3) then becomes

$$c = f(k, r) - \mu k - \theta r.$$

The golden rule is now obtained as the set of first order conditions for maximizing steady state per-capita consumption. The symbol "\$\$" designates golden rule levels:
Golden rule levels of per-capita consumption and utility are then given by

\[ f_k(\hat{k}, \hat{r}) = \mu, \quad f_r(\hat{k}, \hat{r}) = \theta_b. \] (25)

The welfare criterion for this problem can be written as

\[ J = \int_0^\infty \left[ U(c) - \hat{U} \right] dt, \] (27)

which is bounded above when both the production technology and the utility function are concave (see Burmeister and Dobell, 1970). The criterion, \( J \), therefore, has a maximum, and the first order conditions show that this maximum will be attained along the growth path of per-capita consumption governed by the equation,

\[ \eta(c) \frac{\dot{c}}{c} = f_k - \mu. \] (28)

Moreover, the dynamic equations of the marginal product, \( f_c \), for this problem are identical to equation (6) and (6'). Burmeister and Dobell (1970) show that, in general, maximizing the welfare criterion, \( J \), is equivalent to maximizing discounted social welfare with time preference, \( \rho \) (such as solving the problem represented by equation (4)), and then letting \( \rho \) tend to zero. The golden rule path for capital accumulation and resource management can therefore be defined by the conditions

\[ f_k = \mu \quad \text{and} \quad f_r = \theta_b. \]

We may now compare the golden rule to the maximin rule as alternative standards of intergenerational equity. One plausible scenario is illustrated in Figure 2.3. By
Figure 2.3
Golden Rule Turnpike vs. Maximin Path as Alternative Trajectories of Sustainable Consumption
definition, the golden rule path yields the maximum possible level of steady state consumption per capita. Hence, maximin justice necessarily implies a level of constant per-capita consumption less than or equal to that rendered by the golden rule steady state. As the figure shows, this may result in large and sustained (and therefore infinite) losses in the future in order to raise consumption in the present and near future by small increments.

2.4. Concluding Remarks

We believe that the results above help to illuminate the nature of sustainable consumption. Relative sustainability is linked to the idea of eventually meeting or exceeding the maximin level of per-capita consumption, \( \bar{c} \), or some other level of per capita consumption, \( \bar{c} < \bar{c} \), chosen by the planner.

The existence of a superabundant backstop natural resource admits the possibility of deriving modified golden rules and golden rules that govern efficiency in both capital accumulation and resource management. These golden rules can be conveniently related to the issue of sustainability through comparative analysis of per capita consumption trajectories. While the notion of a backstop technology as a basis for resource management remains controversial, we submit that the existence per se of a backstop is not the critical issue. Virtually limitless and renewable resource substitutes do exist (e.g., solar energy). The critical issue is at what unit costs can they be made effective substitutes for nonrenewable resources.

An important implication of the analysis presented here is that, given a backstop
technology, the conditions for existence of the modified golden rule are less stringent than conditions for the existence of a maximin consumption path. In particular, existence of the modified golden rule does not require that the elasticity of output with respect to man-made capital exceed the elasticity with respect to natural resources. Moreover, population growth and capital depreciation are both allowable.

The modified golden rule yields a steady state level of per-capita consumption, \( c^* \), that depends on the prevailing social rate of time preference, \( \rho \), and the unit cost of the backstop resource, \( \theta_b \), but not on initial capital and resource stocks. This result is in stark contrast with maximin constant per-capita consumption (when it exists), which is at the mercy of initial conditions. The consumption level, \( c^* \), declines with an increase in \( \rho \). For a given backstop unit cost, \( \theta_b \), there is a maximum social rate of time preference below which the modified golden rule yields a level of per-capita consumption that exceeds the maximin level. Thus for \( \rho \leq \bar{\rho} \), the modified golden rule satisfies sustainability relative to maximin consumption. The golden rule is a special case of the modified golden rule where the social rate of time preference is set to zero. The golden rule also solves the problem of finding the consumption trajectory which is sustainable relative to the highest possible \( c_{\text{min}} \). That is, the golden rule gives the eventually sustainable maximin.

An objection might be raised to the strong dependence of the modified golden rule and golden rule levels of consumption on the backstop unit cost, \( \theta_b \). A high backstop cost might threaten the dominance of \( c^* \) (modified golden rule) or \( \bar{c} \) (golden rule) over the maximin level \( \bar{c} \). We suggest, however, that the backstop unit cost, \( \theta_b \), serves as an
upper bound on rising extraction costs for the primary resource, so that a high $\theta_{C}$ will depress $\bar{c}$ as well as $c^*$ or $\dot{c}$.

Moreover, the model could be readily extended to allow for a composite resource along the lines of Nordhaus (1979). This would require specifying the extraction costs of oil, coal, natural gas, uranium and other resources and the conversion costs of each and of solar radiation into usable energy. In this way a minimum cost schedule could be calculated that extends thousands of years into the future. Even though this extraction cost function may also be unbounded (e.g., even photovoltaic cells would face a rising rental cost for the space they occupy), it can always be approximated by a rising extraction/conversion cost function and an arbitrarily high backstop price. Thus, the critical issue is not whether a backstop technology exists, but how high and how far into the future the analyst calculates the rising function.

As a final remark, we recommend that policy discussions not put undue emphasis on results arising from long run steady state conditions. Despite the existence of golden rules under the assumptions set forth in this analysis, the concern of most relevance to policy is what happens in the interim on the way to the steady state. It is the trajectory to the steady state, rather than long run sustainability, that captures concern for the future.
Notes

1. The first such episode was in the late 1940's and early 1950's when the manufacturing boom turned the United States from a net resource exporter to a net importer. The second wave followed the boom in resource prices, which followed the apparently prophetic publication of Meadows, et al., *Limits to Growth*. See Goeller and Wienberg (1976), for a debunking of second wave mythology.

2. As cited in the same paper (1974), Solow notes his ambivalence toward the maximin rule.

3. Equivalent formulations of the problem expressed by (4) are possible. Heal (1976) tackles a similar problem by solving two separate problems and piecing the resulting solution together. In the notation of the present paper, the two problems are:

   \[ \text{i) } \max \int_0^\infty U(c) e^{-pt} \, dt \]
   \[ \text{s.t. } \dot{k} = f(k, r) - uk - \theta r - c \]
   \[ \dot{X} = -rL \]
   \[ \theta \leq \theta_b, \]

   \[ \text{and ii) } \max \int_0^\infty U(c) e^{-pt} \, dt \]
   \[ \text{s.t. } \dot{k} = f(k, r) - uk - \theta r - c. \]

Heal shows that any optimal path must link the two solutions together.
Alternatively, let \( r_1 \) denote per capita use (at time \( t \)) of the primary natural resource, and let \( r_b \) denote per capita use of the backstop resource. Then assuming that the two natural resources are perfect substitutes, the problem may be written,

\[
\text{Max} \int_0^\infty U(c)e^{-\rho t} \, dt
\]

s.t. \( \dot{k} = f(k, r_1 + r_b) - \theta k - \theta r_1 - \theta_b r_b - c \)

\( \dot{X} = -r_1 L, \)

\( \theta \leq \theta_b. \)

4. Given that the unit cost, \( \theta \), is a function of the resource stock \( X \), one might expect that the derivative of \( \theta \) with respect to \( X \) would appear in necessary condition (6). The appearance of such a derivative would, in fact, occur in the case of a more general total cost function \( \bar{\theta}(r,X) \) depending on both the resource extraction rate, \( r \), and resource stock, \( X \). In this more general case, covered by Fisher (1981), pp. 28-33, the necessary condition becomes,

\[
\dot{r} = (f_k - \delta)(f_r - \bar{\theta}_r) + \bar{\theta}_X - r \bar{\theta}_{Xr}.
\]

When cost function can be written less generally as

\( \bar{\theta}(r, X) = r \theta(X), \)

the last two terms on the right hand side cancel each other; i.e., \( \bar{\theta}_X = r \bar{\theta}_{Xr} \). The remaining expression is equivalent to (6).
3.1. Introduction

Sustainable development is now at the forefront of research in the overlapping fields of resource and environmental economics and development economics. As a broad concept, sustainable development promises to serve as the foundation for a win-win approach to the combined objectives of economic growth and the management of ecological capital (World Development Report, 1992). The basic sustainability issue, as noted by Pearce and Warford (1993), is how to grow, not whether to grow. Panayotou (1993) suggests that sustainable development is attainable through the functioning of undistorted, competitive and all-encompassing markets that promote efficiency and get the incentives right. Unfortunately, market distortions are pervasive, especially in the case of natural resources. Making a win-win approach operational, therefore, requires constructing shadow prices of ecological capital in a way that is consistent with dynamic efficiency, even in cases where market distortions are known to exist. The main contribution of this paper is to offer a method for constructing these "second best" shadow prices that can serve as market signals for getting the incentives right.

In general, a shadow price refers to an accounting price of an input or output
which reflects its social value. This accounting price will differ from the market price in the presence of market distortions and will substitute for a market price when markets are missing. Because of its generality, the term "shadow price" has been used in different ways in the literature, giving rise to some degree of ambiguity.

The project shadow price of a commodity is its social value as an input to or output of some public project. Shadow pricing for project evaluation is especially important when input and output commodities associated with the project are traded in distorted markets. When market distortions exist, prices will not reflect the true social value of commodities. In cost-benefit analysis of public projects, for example, the project shadow price of an input would be computed as the social opportunity cost of drawing one unit of the commodity out of the private sector and into the public project.

As typically applied, project shadow pricing relates to static analysis in the context of a distorted market equilibrium. The objective of this chapter is to extend the concept of project shadow pricing to dynamic analysis. In particular, we develop closed form expressions for shadow prices of natural resources when dynamic inefficiencies exist. We also suggest how these expressions can be made operational for practical application by numerical approximation. These shadow prices form the basis for proposed policy to induce dynamic efficiency on the part of resource managers. In addition, they can serve as a means of adjusting the national income accounts for inefficiencies in the use of ecological capital.

In the context of dynamic optimization, we show how the programming shadow price of a state variable can be considered a special case of the project shadow price.
The distinction between "project" and "programming" shadow prices follows Starrett (1988). Programming shadow prices are sometimes refined to as "first-best" shadow prices. Specifically, the programming shadow price of a state variable is its project shadow price in the absence of market distortions. While previous discussions of the marginal valuation of natural resources have implicitly focused on programming shadow prices, the more general project shadow price is more germane. Indeed, it is the concern that natural resources are being overused that often motivates a heightened concern for their valuation. We consider distortionary tax policy as a possible source of dynamic inefficiency in resource management. As an illustration of this approach to dynamic shadow pricing, we contrast the open access solution with the efficient depletion of a renewable resource using the Schaefer (1957) model of fisheries economies. Finally, we consider implications for the national income accounting.

### 3.2. Shadow Pricing of Renewable Resources

#### 3.2.1. An Undistorted Dynamic Equilibrium

Under conditions of perfect competition and full definition of property rights, the decentralized solution to the problem of resource management will be equivalent to a centralized solution (see e.g., Blanchard and Fisher, 1989). Thus, an independent resource concessionaire, who seeks to maximize profits, will manage the resource consistent with the following basic model of renewable resources. The key variables in the model are:
X(t): The stock of the resource

G(X): The natural growth function of the stock

h(t): The harvest rate

C(X): The unit cost of harvesting as a function of stock level X

P(h): The demand schedule for the resource

The concessionaire establishes the efficient harvesting program by maximizing W, the net present value of the resource:

\[ \text{Max } W = \int_0^\infty e^{-rt} \left( \int_0^{h(t)} P(q) \ dq - C[X(t)]h(t) \right) \ dt \]  
\[ \text{s.t. } \dot{X} = G(X) - h(t), \ X(0) = X_0. \]  

In the absence of distortion, (1) is also the problem from the perspective of a central planner attempting to maximize the net present value of the resource to society. Letting \( \pi = P - C(X) \), the net price or royalty, the necessary first order conditions for an interior solution can be expressed as the no-arbitrage condition

\[ r\pi = \dot{\pi} + \pi G'(X) - G(X)C'(X). \]  

The left hand side of (2) is the interest the concessionaire earns by harvesting a unit of the resource now and investing the resulting royalty at interest rate \( r \). The right hand side of (2) represents future returns foregone by harvesting the marginal unit now. At the optimum, interest earnings now equate to future earnings foregone. The equality implies that arbitrage opportunities between the present and the future have been eliminated. Alternatively, the first order conditions may be written
In this form, the left hand side is producer royalty or net price, while the right hand side is the marginal user cost. The efficiency condition indicates that along the optimum trajectory, royalty equals marginal user cost. At the optimum, both royalty and marginal user cost are equal to the current value programming shadow price of the state variable, i.e., the resource stock. In the natural resource literature, this programming shadow price is typically called the scarcity rent.

By following the trajectory defined by (3), the resource may be drawn down to the steady state level, after which time $\dot{P} = 0$. This yields the classic steady state solution of fisheries economics (see Munroe and Scott, 1985):

$$\pi = \frac{1}{r} (\dot{P} + \pi G'(X) - G(X)C'(X)).$$

(3)

The case of a nonrenewable resource can also be addressed. Setting $G(X) = 0$ in equations (2) and (4) leads to the classic Hotelling rule, modified to allow for changing marginal cost of extraction:

$$rr\pi = \dot{P} \quad \text{or} \quad \pi = \frac{1}{r} \dot{P}.$$  

(5)

Equation (3) provides a synthesis and generalization of the apparently disparate models of renewable and nonrenewable resource economics. One implication of particular interest is that during the transitional phase of renewable resource use, before
A key to the subsequent development is to observe that the left and right hand sides of equation (3) may be considered as independent functions of the stock level X and the harvest rate h. That is, we have $\pi = \pi(X, h)$ and $D = D(r, X, h)$, where $D$, the marginal user cost, represents the expression on the right hand side of (3). The dependence of $\pi$ and $D$ on both $X$ and $h$ can easily be seen by writing $\pi = P(h) - C(X)$. It is only by choice of the optimal harvest rate $h'(t)$, i.e., the optimal control, that first order conditions (3) are satisfied, resulting in the optimal trajectory for the resource stock, $X'(t)$.

3.2.2. Tax Policy and Dynamic Distortions

Government tax policy often produces distortion in the market for natural resources. Dasgupta and Heal (1979) note that of the many forms of taxation that affect resource depletion, one of the most prevalent is a profits tax. It is essentially a tax on royalty. If $\tau$ is the rate of the tax on the rent going to the concessionaire, the after-tax royalty, $\pi_a$, can be expressed as

$$\pi_a = \pi(1 - \tau) = P(1 - \tau) - C(X)(1 - \tau).$$

(6)

The first order conditions for the decentralized solution in the presence of a profits tax will be

$$\pi_a = \frac{1}{r}(\dot{P}_a + \pi_a G'(X) - G(X)C_a(X)),$$

(7)

where $P_a = P(1 - \tau)$ and $C_a = C(1 - \tau)$. 

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Noting that \( \dot{P} = \dot{P}(1-\tau) \), we observe that equation (7) may be divided through by the factor \((1-\tau)\), rendering (7) equivalent to equation (3). This result is consistent with that discussed by Dasgupta and Heal (1979) for the case of a profits tax on an exhaustible resource. The tax produces no distortion in the management of the resource, because royalty and marginal user cost are reduced in the same proportion. The concessionaire loses surplus to the government, but he does not change his harvest rate.

However, in the typical case, governments impose taxes on interest earnings as well as on profits. For a constant tax rate, \( \tau \), on both profits and interest earnings, the concessionaire’s profit maximizing condition becomes

\[
\pi = \frac{1}{r(1-\tau)} (\dot{P} + \pi G'(X) + G(X)C'(X)).
\]

The effective discount rate, as perceived by the concessionaire, declines from \( r \) to \( r(1-\tau) \) and hence, the concessionaire’s marginal user cost increases for any given stock level \( X \) and harvest rate \( h \). The imposition of the tax on interest earnings has produced a parametric shift in the marginal user cost schedule from \( D \) to new schedule \( \tilde{D} \), represented by the right hand side of equation (8). The effect of the shift is to induce the concessionaire to reduce his harvest rate; he now underproduces and overprices relative to the social optimum.

The effect of a sales tax will differ from that of a profits tax. Suppose the government levies a constant sales tax, \( s \), so that the after-tax producer royalty becomes \( \pi_s = P-C(X)-s = \pi-s \). Then the decentralized efficiency condition becomes
\[ \pi - s = \frac{1}{r} \left( \dot{P} + (\pi - s) G'(X) - G(X) C'(X) \right). \] (9)

The impact of the sales tax on resource management is ambiguous. The royalty schedule is shifted down by amount \( s \), while the marginal user cost schedule is shifted by amount \( (-sG'(X)/r) \). But the direction of the latter shift depends on the sign of \( G'(X) \). If the stock level is greater than maximum sustainable yield (MSY), for which \( G'(X) = 0 \), it will be the case that \( G'(X) < 0 \). The marginal user cost schedule will then shift up. The combination of the downward shift in royalty and the upward shift in marginal user cost will induce the concessionaire to reduce his harvest rate. On the other hand, if the stock level \( X \) is less than MSY, then \( G'(X) > 0 \). Now the marginal user cost schedule will shift in the downward direction. The overall effect on stock level and harvest rate will depend on the relative magnitudes of the two shifts, and on the elasticities of the royalty and marginal user cost schedules as functions of \( X \) and \( h \).

Other tax structures may also be considered, including capital gains taxes, excess profits taxes, and depletion allowances. In addition, a more elaborate analysis can be performed by allowing taxes to vary with time. Thus, we might consider a profits tax \( \tau = \tau(t) \) or a sales tax \( s = s(t) \) as time dependent policy variables. Elements of these extensions, in the context of exhaustible resources, are discussed in Dasgupta and Heal (1979).

3.2.3. Construction of the Shadow Price

The previous section described dynamic market distortions arising from tax policies that will generate decentralized solutions to profit maximization that differ from
the centralized, socially optimum solution. When such market distortions exist, we will call the resulting decentralized solution a distorted equilibrium. Such a distorted equilibrium is achieved by the concessionaire's choice of harvest rate h(t) to equate his private marginal benefit \( \bar{\pi} \) and his private marginal user cost \( \bar{D} \) at each time t. Because private benefits and costs in the distorted equilibrium will generally differ from social benefits and costs, the harvest rate h(t) will not be the same as the socially optimum rate, h*(t). As a result, the trajectory of the resource stock X(t) will differ from the optimum trajectory, X*(t).

For purposes of the subsequent development, however, we assume that distorted equilibrium, whatever its origin, generates well defined trajectories h(t) and X(t) that are twice differentiable as functions of t. We take as a measure of distortion at time t the difference \( \pi(X(t), h(t)) - D(r, X(t), h(t)) \). When \( \pi - D > 0 \), marginal social benefit exceeds marginal social cost and we have underharvesting of the resource. Alternatively, \( \pi - D < 0 \) implies that marginal social cost exceeds marginal social benefits; the concessionaire overharvests. When \( h = h^* \) and \( X = X^* \), \( \pi = D \) and distortion is zero.

Key to construction of second best shadow prices is recognition that tax distortion can generate parametric shifts in both royalty and user cost schedules as viewed by the concessionaire, say from \( \pi \) to \( \bar{\pi} \) and D to \( \bar{D} \). In the face of such distortion, the concessionaire's profit maximizing condition becomes \( \bar{\pi} = \bar{D} \), not \( \pi = D \), so that socially optimum resource management is not achieved.

For given trajectories X(t) and h(t), let \( W_t \) be the welfare function at time t, defined by
$W_t = W(X(t), h(t), t)$

$$= \int_{t}^{\infty} e^{-\tau} \left( \int_{0}^{h(\tau)} P(q) \ dq - C[X(\tau)] h(\tau) \right) d\tau,$$

where $X$ and $h$ must satisfy the dynamic equation $\dot{X} = G(X) - h(t)$. If no distortions exist, the concessionaire will choose the optimal harvest rate $h^*$, thereby generating the optimal path for the resource stock, $X^*$. Social welfare will be maximized; in particular, for the case $t = 0$,

$$W^* = W(X^*, h^*) = \text{MAX } W_0.$$

We now consider the change in social welfare that results when the resource stock is depleted by one unit, which is taken out of the private sector. That is, we want to compute $dW/dX$.

In the case of no market distortion, the quantity $dW^*/dX$ is just the current value programming shadow price, as is well established in the optimal control literature. For the example under consideration, this programming shadow price is given by the common value of $\pi$ or $D$, since they are equal along the optimum trajectory.

For the case of market distortion, where $\pi$ and $D$ are not equal, we invoke the small project assumption and equate $dW/dX$ with the shadow price of $X$, $S_X$, as specified in the following proposition:

**Proposition 1:** Let $\pi$ and $D$ be royalty and marginal user cost respectively, as viewed by a central planner. As a result of tax distortion to the economy, let $\pi$ and $\tilde{D}$ be the private royalty and user cost schedules as viewed by an individual resource concessionaire. The shadow price or total opportunity cost of depleting the stock by one
unit can expressed as a weighted average of royalty, \( \pi \), and user cost, \( D \):

\[
S_X = \frac{\pi (d\tilde{D}/dX) - D(d\tilde{\pi}/dX)}{(d\tilde{D}/dX) - (d\tilde{\pi}/dX)}.
\] (12)

A formal derivation of the weighted average expression for the shadow price of a natural resource in a distorted resource market is given in Chapter 4, using a more general model. For the partial equilibrium model of the present chapter, we provide a graphical argument with the aid of Figure 3.1. Producer royalty, \( \pi \), is shown as an increasing function of the stock level, \( X \), while marginal user cost, \( D \), is displayed as a decreasing function. In the absence of market distortion, the optimum solution is represented by point \( E \), where the schedules for \( \pi \) and \( D \) intersect. The introduction of market distortion results in shifting the royalty and marginal user cost schedules from \( \pi \) to \( \tilde{\pi} \) and \( D \) to \( \tilde{D} \) respectively. A distorted equilibrium is achieved at point \( \tilde{E} \), the intersection of the \( \tilde{\pi} \) and \( \tilde{D} \) schedules. Consider the effect, starting from distorted equilibrium \( \tilde{E} \) of depleting the stock level by \( \Delta X \) and drawing the resource away from the private sector. This depletion produces a shift in the royalty schedule \( \tilde{\pi} \) to the left by amount \( \Delta X \).

The social opportunity cost of depleting the stock by amount \( \Delta X \) is realized by both the user cost incurred and the producer royalty foregone. The user cost incurred by society is represented by the area under the undistorted marginal user cost curve, \( D \), from \( X = a \) to \( X = b \). This is the area \( ahkb \), which can be approximated by the rectangle of area \(-D_o\Delta X_i\). Note that the quantity \( \Delta X_i \) is negative. Royalty foregone
Figure 3.1
Shadow Price of a Natural Resource
is represented by area bdec, the area under the undistorted royalty schedule, \( \pi \), from \( X = b \) to \( X = c \). This area can be approximated by the rectangle \( \pi \Delta X_2 \).

The total opportunity cost, represented by the shaded area in Figure 3.1, is therefore approximated by \( \pi \Delta X_2 - D_0 \Delta X_1 \). The shadow price of the resource is then the opportunity cost per unit. Taking \( \Delta X = -1 \), the shadow price can be written as

\[
S_x = \pi \Delta X_2 - D_0 \Delta X_1
\]

where \( -\Delta X_1 \) and \( \Delta X_2 \) are the weighting factors. Triangle \( \tilde{E}fg \) in Figure 3.1 generates the approximate relationship

\[
\Delta X_1 \frac{d\tilde{D}}{dX} - \Delta X_2 \frac{d\tilde{\pi}}{dX} = 0.
\]

Combining (14) with the relationship \( \Delta X_1 - \Delta X_2 = -1 \) yields the following result:

\[
\Delta X_1 = \frac{(d\tilde{\pi}/dX)}{(d\tilde{D}/dX) - (d\tilde{\pi}/dX)}, \quad \text{(15a)}
\]
\[
\Delta X_2 = \frac{(d\tilde{D}/dX)}{(d\tilde{D}/dX) - (d\tilde{\pi}/dX)}. \quad \text{(15b)}
\]

Substituting (15a) and (15b) into (13) then gives the shadow price formula, equation (12), which is reminiscent of the Harberger shadow price formula used in cost-benefit analysis (see Harberger, 1969). As suggested by the formula, the distorted schedule with the flatter slope will leave its undistorted relative with the greater weight in the averaging process. As an extreme example, when schedule \( \tilde{D} \) has a flat slope, full weight goes to \( D \). This is the case for the open access situation, which is developed in Section 3.3. In many practical situations, overharvesting of the resource, due to tax distortion or
truncated property rights, approximates the open access solutions. For such situations, marginal user cost, D, may be taken as an estimate of the shadow price of the resource.

The remaining exercise is to compute the derivatives $d\bar{D}/dX$ and $d\bar{\pi}/dX$, given $P(h)$, $G(X)$, $C(X)$, $X(t)$ and $h(t)$. We do this for the example of distortion arising from a tax on profits and interest earnings, where the equilibrium condition is represented by equation (8). In this example, all the distortion and the resulting parametric schedule shift occur on the side of marginal user cost. The royalty schedule remains unaffected so that in this case, $\pi = \bar{\pi}$. The computations are supported by three simple lemmas, which are presented in the appendix. In all cases, we assume that local inverses exist and are differentiable.

From lemma 2, we obtain $d\pi/dX$ directly:

$$\frac{d\pi}{dX} = \frac{dP}{dh} \left[ G'(X) - \frac{\dot{X}}{X} \right] - C'(X).$$

(16)

Lemmas 1 and 3 permit computation of $d\bar{D}/dX$:

$$\frac{d\bar{D}}{dX} = \frac{1}{r(1-\tau)} \left\{ \frac{d\bar{P}}{dX} + \pi G''(X) + G'(X) \frac{d\pi}{dX} - G(X)C''(X) - G'(X)C'(X) \right\}$$

$$= \frac{1}{r(1-\tau)} \left\{ \left[ \frac{d^2P}{dh^2} \right] G'(X) - \frac{\dot{X}}{X} + \pi G''(X) ight\}$$

$$+ G'(X) \frac{d\bar{P}}{dh} \left[ G'(X) - \frac{\dot{X}}{X} \right] - G(X)C''(X) - G'(X)C'(X).$$

(17)

Two special cases can be considered, which reduce the complexity of the calculations.

For the case of a nonrenewable resource, $G(X) = 0$. In this case we have:
Substitution into formula (12) and further manipulation yield the closed form expression:

\[
\dot{S}_X = \frac{\dot{P} \left[ \frac{dP}{dh} \dot{X} + C'(X) \right] - \pi \left[ \frac{d^2P}{dh^2} \dot{X} + \frac{dP}{dh} \frac{dP}{dh} \right] \dot{X}}{\left[ \frac{d^2P}{dh^2} \dot{X} + \frac{dP}{dh} \frac{dP}{dh} \right] \dot{X} - r(1-\tau) \left[ \frac{dP}{dh} \dot{X} + C'(X) \right]}.
\]  

The other case is that of a renewable resource in the steady state, for which \( \dot{P} = 0, \dot{X} = 0 \) and \( \dot{h} = 0 \). These conditions simplify the expressions for \( d\pi/dX \) and \( d\bar{D}/dX \) as follows:

\[
\frac{d\pi}{dX} = \frac{dP}{dh} G'(X) - C'(X),
\]

\[
\frac{d\bar{D}}{dX} = \frac{1}{r(1-\tau)} \left( \frac{dP}{dh} \right)^2 + \pi G''(X) - G(X)C''(X) - G'(X)C'(X).
\]  

Again, substitution into formula (12) yields the corresponding shadow price in closed form.

While actually calculating such shadow prices may be tedious, the closed form solution relieves the user of having to solve a messy system of equations. And, as Robert Solow (1992) has suggested, "If we -- the country, the government, the research community -- are serious about doing the right thing for the resource endowment and the environment, then the proper measurement of stocks and flows ought to be high on the
list of steps toward intelligent and foresighted decisions." The information required for the calculation is contained in the functions $P(h)$, $G(X)$ and $C(X)$, which may be estimated, and in the trajectories $X(t)$ and $h(t)$, which are fully determined in distorted equilibrium. Specifically, in the case of a government tax on interest earnings and profits, $X(t)$ and $h(t)$ are determined according to (8).

Formula (12) and closed form expressions such as (20) can be made operational for practical application by estimation and numerical approximation of component terms. For example, the growth of a renewable resource is typically modeled as a logistic function of the form $G(X) = gX(1-X/k)$, where $g$ and $k$ are constants. Similarly, extraction cost is often formulated as inverse function of the stock level $X$, i.e.,

$$C(X) = a/X$$

for some constant $a$. These formulations render easy computation of $G'(X)$, $G''(X)$, $C'(X)$ and $C''(X)$ for insertion in shadow price formulas. Time derivatives of $X(t)$ and $h(t)$ may be approximated numerically using discrete time intervals (see e.g., Acton, 1990). For example, if we measure time, $t$, in years, taking $\Delta t = 1$, and estimate resource stock levels at half-year intervals, we obtain the following mid-year approximations:

$$\frac{X_t - X_{t-1}}{2} = \frac{X_t - X_{t-1}}{2}, \quad (23)$$

$$\frac{\dot{X}_t - \dot{X}_{t-1}}{2} = \frac{X_t - X_{t-1}}{2} - 2\frac{X_{t-1}}{2} + \frac{X_{t-2}}{2}.$$ 

Similar computations could be done for the resource extraction rate $h(t)$.  

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3.3 Application to Fisheries Economics: The Open Access Solution

As an application of second best shadow pricing, we now contrast the open access solution with the efficient depletion of a renewable resource using the Schaefer (1957) model of fisheries economics.

Models of fisheries typically specify the harvest rate $h$ in terms of fishing effort $E$:

$$ h = qEX, \quad (24) $$

where $q$ is the catchability coefficient. Total cost $TC$ is assumed to be directly proportional to fishing effort:

$$ TC = cE, \quad (25) $$

where $c$ is a constant. Substituting for $E$ from (24), we obtain

$$ TC = h(c/qX). \quad (26) $$

We therefore take unit harvesting cost, $C(X)$, as inversely proportional to the stock, $X$,

$$ C(X) = c/qX. \quad (27) $$

The natural growth function $G(X)$ is of the logistic form,

$$ G(X) = gX(1-X/k), \quad (28) $$

where $g$ is the intrinsic growth rate, and $k$ is the environmental carrying capacity. To simplify the analysis, we consider the steady state condition with $\dot{X} = 0$. Equation (4) then becomes the efficiency condition for optimal resource management. Substituting for $G(X)$ and $C(X)$, and computing associated derivatives, equation (4) becomes
\[
[P - c/qX] = \frac{1}{r} \{ [Pg + gc/qk] - (2Pg/k)X \}. 
\]  
(29)

As independent schedules in their own right, royalty, \( \pi \), and marginal user cost, \( D \), can be written as

\[
\pi = P - c/qX 
\]  
(30)

\[
and \quad D = \frac{1}{r} \{ [Pg + gc/qk] - (2Pg/k)X \}.
\]  
(31)

Computing derivatives, we obtain

\[
d\pi/dX = c/qX^2 > 0, \quad \text{for } X > 0, 
\]  
(32)

\[
dD/dX = -2Pg/rk < 0, \quad \text{for } r < \infty. 
\]  
(33)

This shows that the royalty schedule is upward sloping and marginal user cost downward sloping as functions of stock level \( X \).

The open access solution can be derived by regarding the open access situation as one in which the individual fishing firm or concessionaire realizes no private marginal user cost. In the context of a tax-induced distorted equilibrium, open access is equivalent to the case where the concessionaire's marginal user cost is completely subsidized. As a result, the concessionaire completely discounts the future in favor of the present; that is, his subjective discount rate becomes infinite and his marginal user cost schedule shifts down to \( D = 0 \) for all \( X \). Equating private marginal benefit to private marginal cost, the individual concessionaire will expand his fishing effort, \( E \), until royalty, \( \pi \), declines to zero. Thus, for the open access solution, \( X^o \),
\[ P - \frac{c}{q}X^* = 0 \quad \text{or} \quad X^* = \frac{c}{q} P. \] (34)

This contrasts with the socially optimum solution, \( X^* \), which is obtained by solving efficiency condition (29). As computed by Schaefer (1957),

\[ X^* = \frac{k}{4} \left( \frac{c}{Pqk} + 1 - \frac{r}{g} \right) + \left[ \frac{c}{Pqk} + 1 - \frac{r}{g} \right]^2 + \frac{8cr}{Pqk} \frac{1}{2}. \] (35)

Overharvesting occurs in the open access fishery so that \( X^* > X^\infty \). Since \( \frac{dD}{dX} < 0 \), marginal user cost, \( D \), is a decreasing function of \( X \). Hence we have the result that

\[ D(r,X^*) < D(r,X^\infty). \] (36)

Substituting for \( X^\infty \) in (31), we get the value of social marginal user cost at the open access solution:

\[ D(r,X^\infty) = \frac{qkPg - gc}{rqk}. \] (37)

Since the concessionaire’s private marginal user cost is zero, formula (37) also gives the difference between social marginal cost and private marginal cost. Because \( \bar{D} = 0 \) at all stock levels \( X \), \( \frac{d\bar{D}}{dX} = 0 \). Hence \( D \) gets full weight in the shadow price formula. That is,

\[ s_{x^\infty} = D(r, X^\infty). \] (38)

This result may be applied as an estimate of the resource shadow price for the case of overharvesting which is less extreme than the open access situation.
The principles developed here for fisheries economics can be readily generalized to other renewable resources. In forestry management, for example, growth enhancement might be achieved by accelerated harvesting of slow growing trees to make room for faster growing trees. That is, cutting a tree may increase natural capital, including the inventory of cut timber. In both the under and overharvesting cases, the problem is that resource managers do not correctly incorporate economic depreciation in their harvesting and production decisions.

3.4 Concluding Remarks

In this chapter we have attempted to extend the concept of a project shadow price to dynamic analysis in order to facilitate managing ecological capital consistently with sustainable development. The traditional programming shadow price of dynamic optimization (i.e., the multiplier) is a special case of the extended version of the project shadow price, but is inappropriate in the presence of market distortions; that is, it is a first best shadow price. The principle contribution of this chapter is a partial equilibrium construction of a second best shadow price with a closed form representation.

The development also shows that the second best project shadow price has practical significance for policy design in resource management to control overuse or underuse of a resource. A number of economists and international organizations have advocated government taxation of resource rents as a means of generating needed revenue and encouraging efficient use of natural resources (see e.g., Repetto, 1989b; World Development Report, 1992). In the case of open access and other instances of overuse,
where royalty is less than economic depreciation, taxing the royalty does not provide adequate incentive for resource conservation. For such cases, the development in this chapter suggests that the marginal user cost, rather than royalty, should serve as an estimate of the resource shadow price for setting taxes or user fees. Conversely, in the case of underuse of the resource, royalty will generally exceed economic depreciation. Taxing away the entire royalty, however, penalizes the producer, who now sees no incentive to increase production to the socially efficient level.

A government leasing policy based on second best shadow prices could be devised to accomplish two important policy objectives: (1) provide the proper incentives for resource management consistent with social optimality; and (2) extract the resource rents to which the government is entitled by virtue of public ownership of the resource. An effective government leasing policy that meets these objectives can be achieved by two policy instruments, a user fee and an auction. An incentive compatible user fee would be based on the shadow price of the resource. Consideration of the marginal benefits and marginal costs of harvesting in each time period would induce the resource manager to adhere to an efficient harvesting program, even in the presence of market distortions. The total amount assessed by the government during any time period would depend directly on the amount of resource harvested. Hence, extensive government monitoring of the operation would be required.

The other recommended component of an effective government leasing policy is an auction to allocate production licenses and property rights. The auction functions as the mechanism for capturing the resource rents the public is entitled to, but is serves
other related objectives as well: (1) it reduces potential inequities within the private sector regarding the distribution of surpluses generated by the resource; (2) it eliminates inefficiencies created by rent seeking behavior; and (3) it serves as a means for selecting the most capable resource managers. To enforce compliance with the terms of the lease, the government could also require the resource manager to post a performance bond. The bond would be subject to forfeit if user fees were not properly paid or in the event of incomplete restoration of the production site following shutdown of the enterprise.

Another application of the approach outlined in this paper is national income accounting. As argued forcefully by Repetto (1989) and others, failure to include ecological capital in the national income accounts has resulted in inefficient management of these capital stocks. The approach offered in this paper provides a method for valuing resource depletion at the margin as a basis for netting out depreciation of natural capital in the calculation of net national product. This aspect of second best shadow pricing is developed in Chapter 4.
Appendix

Lemma 1:

i) If \( \dot{X} = 0 \), \( \frac{dh}{dX} = G'(X) \).

ii) If \( \dot{X} \neq 0 \), \( \frac{dh}{dX} = G'(X) - \frac{\ddot{X}}{\dot{X}} \).

Proof: i) If \( \dot{X} = 0 \), the resource constraint takes the form

\[ G(X) = h. \]

Totally differentiating, we get

\[ G'(X)dX = dh, \]

and the result follows.

ii) Differentiate the constraint,

\[ \dot{X} = G(X) - h(t), \]

with respect to t:

\[ \ddot{X} = G'(X)\dot{X} - \dot{h}(t). \]

Since \( \dot{X} \neq 0 \), we can divide to get

\[ \frac{\dot{h}}{\dot{X}} = \frac{dh}{dX} = G'(X) - \frac{\ddot{X}}{\dot{X}}. \]

Lemma 2:

\[ \frac{d\pi}{dX} = \frac{dP}{dh} \left[ F'(X) - \frac{\ddot{X}}{\dot{X}} \right] - C'(X). \]
Proof: Write $\pi = P(h) - C(X)$. 

Then $\frac{d\pi}{dX} = \frac{dP}{dh} \frac{dh}{dX} - C'(X)$.  

Now substitute for $\frac{dh}{dX}$ using Lemma 1.

Lemma 3:

$$\frac{d\dot{P}}{dX} = \frac{d^2P}{dh^2} \dot{h} + \frac{dP}{dh} \ddot{h} \left[ G'(X) - \frac{\ddot{X}}{X} \right].$$  

(A7)

Proof: Write $\dot{h} = \frac{dP}{dh} \frac{dh}{dt}$.

We assume $\frac{dh}{dt} \neq 0$, so that the function $h(t)$ has a differentiable inverse. Now use the chain rule:

$$\frac{d\dot{P}}{dh} = \frac{d}{dh} \left[ \frac{dP}{dh} \frac{dh}{dt} \right]$$

or

$$\frac{d\dot{P}}{dh} = \frac{dh}{dt} \frac{d^2P}{dh^2} \frac{dh}{dt} + \frac{dP}{dh} \frac{d}{dt} \left[ \frac{1}{dt/dh} \right].$$  

(A8)

Using the quotient rule,

$$\frac{d}{dh} \left[ \frac{1}{dt/dh} \right] = -\frac{d^2t}{dh^2} (\frac{dt}{dh})^2$$

(A9)

$$= -\frac{d^2t}{dh^2} (\frac{dh}{dt})^2.$$
Invoking the formula for the second order derivative of an inverse function, we have

\[
\frac{d^2 t}{dh^2} = -\frac{d^2 h}{dt^2} / (\frac{dh}{dt})^3. \tag{A10}
\]

Substituting (A10) into (A9) gives

\[
\frac{d}{dh} \left[ \frac{1}{dt/dh} \right] = \frac{d^2 h}{dt^2} / \frac{dh}{dt} = \frac{\dot{h}}{\ddot{h}}. \tag{A11}
\]

Therefore

\[
\frac{d\dot{P}}{dh} = \frac{d^2 P}{dh^2} \frac{\ddot{h}}{\dot{h}} + \frac{dP}{dh} \frac{\ddot{h}}{\dot{h}}. \tag{A12}
\]

Writing \( \frac{d\dot{P}}{dX} = \frac{d\dot{P}}{dh} \frac{dh}{dX} \) and using Lemma 1 gives the result.
4.1. Introduction

The goal of sustainable development may be interpreted as the pursuit of economic growth compatible with environmental stewardship and obligation to the future. This does not imply, as Daly (1993) has suggested, that, "development without growth is sustainable development." Rather, sustainable development entails implementing policies that provide incentives for efficient use of ecological capital and satisfy society's concern for the welfare of future generations. Moreover, the efficiency in the use of environmental resources need not be achieved without regard for the amenity value of such resources. Designing policies appropriate to these objectives requires the ability to monitor the depreciation of ecological capital and its effect on national income.

A fundamental presumption of researchers and policy makers is that sustainability involves the management of "social capital," which consists not only of physical (man-made) capital, but also human and natural, or ecological capital. The Economic Report of the President (1994) states, "it will be necessary to measure social capital and the value its use brings in order to understand whether a growth path for the economy is sustainable. This is the province of "green accounting." Acting in accordance with this
perspective, President Clinton, in his 1993 Earth Day speech, directed that the national income accounts be augmented to include natural resources.

The recent literature on "green accounting" has moved beyond pragmatic approximation of resource depletion to a deeper consideration of the welfare significance of net national product. Drawing on the foundations established by Samuelson (1961), Weitzman (1976), and Usher (1980), researchers such as Hartwick (1990) and Mäler (1991) have cast the computation of national income in a growth theoretic framework, using the technology of optimal control. Within this framework, net national product, based on the current value Hamiltonian expression arising from constrained optimization of a utilitarian welfare function, serves as the appropriate measure of aggregate welfare. As Weitzman (1976) proves, the maximum level of aggregate welfare attainable along a competitive trajectory is the same as what would be obtained from a hypothetical constant level of consumption equivalent to current net national product. The Hamiltonian is thus a measure of sustainable income. Building on this idea, Mäler (1991) says of the Hamiltonian, "along the optimal trajectory [it] is the national welfare measure in utility terms we are looking for."

A fundamental limitation of recent analysis of net national product and its welfare significance is the restriction to "first best" situations. The formulations of Weitzman (1976), Hartwick (1990) and Mäler (1991) are valid for economies without distortion. Hartwick (1990) comments on market imperfections such as property rights failure and cites Mirrlees (1969) as noting that if distorted market prices are used in the national income accounts, NNP ceases to reflect the production and consumption capacity of the
economy. In place of observed prices that are likely to be distorted, Hartwick advocates the use of first best shadow prices. Mäler (1991) justifies this approach by claiming that if the production feasibility set in the economy is convex, optimal, or first best, prices are the only prices that, in all circumstances, will render net national product a true measure of aggregate welfare.

This claim appears inconsistent with the vast body of literature on cost-benefit analysis and second best shadow pricing. The challenge in cost-benefit analysis in distorted economies is to determine the appropriate accounting prices of inputs and outputs of public projects so that projects yielding positive profits at these shadow prices increase social welfare. As pointed out by Dasgupta (1993), a project may be viewed as a perturbation of the current trajectory followed by the economy. From a general equilibrium standpoint, cost-benefit analysis is equivalent to determining the effect of the perturbation on net national product. If net national product at appropriate prices increases as a result of the perturbation, the project meets the cost-benefit test and should be accepted. But for the case of distorted economies, second best shadow prices seem called for as a basis for a general equilibrium approach to cost benefit analysis. This suggests the need for the computation of a second best net national product. While our concern here is motivated primarily by interest in "green accounts," the applicability of second best shadow prices naturally extends to all consumption and capital entries in the national income accounts.

The plan of the chapter is as follows. Section 4.2 establishes the basic framework in the context of an optimal control model of an economy with renewable resources. To
facilitate the exposition, this section reviews some results already known in the resource economics literature. In Section 4.3, we present two simple models of distorted economies as examples of the effects of distortion on relative resource prices. The first model considers a direct distortion in the resource market that might arise due to misguided policies or incomplete information, which render market incentives inadequate to achieve efficiency. These distortions are presumed to be correctable. The second model involves a generalized profits tax on private production as a source of revenue for public production. In contrast to the case of the first model, this distortion is taken as irremovable because of political imperatives or institutional rigidities in the economy. Section 4.4 uses the models of Section 4.3 to compute second best net national product for comparison with the first best case. Concluding remarks are offered in Section 4.5.

4.2 Framework and Basic Model

We take as our basic model for comparing first best and second best approaches to shadow pricing an optimal control model of aggregate growth in closed economy with a renewable resource. Key variables in the model are as follows:

- \( C(t) \): Aggregate Consumption
- \( K(t) \): Stock of Man-made or Physical Capital
- \( X(t) \): Stock of the Renewable Resource
- \( R(t) \): Resource Harvest Rate
- \( \theta(X) \): Unit Cost of Harvesting the Resource
- \( F(K, R) \): Aggregate Production Function
G(X): Natural Growth Function of the Resource

U(C): Utility of Consumption

The standard optimization problem is to maximize the discounted utility of aggregate consumption over an infinite time horizon, subject to dynamic constraints on capital accumulation and changes in the resource stock. We designate this first problem P1:

$$\text{Max } W = \int_0^\infty U(C) e^{-pt} \, dt$$

s.t. \( \dot{K} = F(K, R) - \delta K - \theta R - C, \quad K(0) = K_0 \)

\( \dot{X} = G(X) - R, \quad X(0) = X_0 \)

Capital depreciation occurs at the rate \( \delta K \) and the resource extraction cost is assumed to increase as the stock is depleted, that is, \( \theta'(X) < 0 \).

The present value Hamiltonian expression is given by:

$$H = U(C)e^{-pt} + \lambda [F(K, R) - \delta K - \theta R - C] + \psi [G(X) - R],$$

where \( \lambda \) and \( \psi \) are costate variables. Application of the maximum principle yields the following necessary conditions for an optimum solution:

$$\frac{\partial H}{\partial C} = U'(C)e^{-pt} - \lambda = 0,$$  \hspace{1cm} (3a)

$$\frac{\partial H}{\partial R} = \lambda [F_R - \theta] - \psi = 0,$$  \hspace{1cm} (3b)
Standard manipulation of equations (3a) through (3d) to eliminate costate variables $\lambda$ and $\psi$ yields the two fundamental arbitrage conditions which govern the optimal trajectory of the economy,

$$\frac{U''(C)}{U'(C)} \dot{C} = \rho - [F_K - \delta]$$  \hspace{1cm} (4)

and

$$[F_R - \theta] = \frac{1}{[F_K - \delta]}(\dot{F}_R + [F_R - \theta]G'(X) - G(X)\theta'(X)).$$ \hspace{1cm} (5)

Equation (4) is typically referred to as the Ramsey condition, while equation (5) is a generalized form of Hotelling’s Rule for renewable resources. The expression \{\rho - U''(C)\dot{C}/U'(C)\} is often referred to as the consumption rate of interest or the social rate of discount, with parameter $\rho$ as the rate of pure time preference. The Ramsey condition specifies that along the optimum trajectory, the consumption rate of interest equals the return to capital. The Hotelling Rule states that along the optimum trajectory, the resource is harvested at each time $t$ up to the point that the resource royalty, or return to the resource, is equal to marginal resource depreciation, or resource user cost.

Following Hartwick (1990), the current value Hamiltonian $\tilde{H}$ serves as the basis for determining net national product. Designating $\tilde{\lambda}$ and $\tilde{\psi}$ as the current value costate
variables and using the dynamic equations for the economy along with first order conditions (3a) and (3b) yield:

\[ \tilde{H}^* = U(C^*) + \bar{\lambda}^* \dot{K}^* + \bar{\psi}^* \dot{X}^* \]

(6)

\[ = U(C^*) + U'(C^*) \dot{K}^* + U'(C^*) [F_R - \theta] \dot{X}^*. \]

The costate variables \( \bar{\lambda} \) and \( \bar{\psi} \) have the standard interpretations as first best shadow prices. Specifically, if we set

\[ W_t^* = \text{Max} \int_t^\infty U(C) e^{-\rho^* \tau} d\tau, \]

(7)

where maximization is performed subject to the dynamic constraints on \( K \) and \( X \), then,

\[ \bar{\lambda}^* = \frac{\partial W_t^*}{\partial K_t}, \quad \bar{\psi}^* = \frac{\partial W_t^*}{\partial X_t}. \]

(8)

The symbol (*) in equations (6) through (8) is used to designate values of variables along the optimum trajectory.

Dividing equation (6) by \( U'(C) \) gives net national product as the current value Hamiltonian expressed in consumption units rather than utils. Net national product is then taken as this "dollar value" Hamiltonian:

\[ NNP = \frac{\tilde{H}^*}{U'(C^*)} = \frac{U(C^*)}{U'(C^*)} + \dot{K}^* + [F_R - \theta] \dot{X}^*. \]

(9)

Hartwick (1990) makes a further refinement through the approximation

\[ U(C^*) \approx U'(C^*)C^*, \]

so that NNP has the more familiar appearance:
Unfortunately, the validity of the approximation is weak for many basic functional forms of utility. Consider, for example, \( U(C) = -C^{(\eta-1)}, \eta > 1 \). Then,

\[
\frac{U(C)}{U'(C)} = -\frac{C}{\eta - 1}.
\]

Mäler (1991) and Dasgupta (1993) use a more elaborate approach and consider small perturbations about the optimum. Although they work with a more complex model, the approach can be readily applied to the basic case. Consider a point \( 0 = (C, K, X) \), close to, but away from \( 0^* = (C^*, K^*, X^*) \), which lies on the optimum trajectory at time \( t \). Applying a first order Taylor expansion around \( 0^* \) gives an approximation to the current value Hamiltonian at point \( 0 \):

\[
\tilde{H}(0) = \tilde{H}(0^*) + U'(C^*) (C - C^*) + \tilde{\lambda}^*(K - K^*) + \tilde{\psi}^*(\dot{X} - \dot{X}^*).
\]

Mäler (1991) argues that constant terms related to \( \tilde{H}(0^*) \) will be unaffected by changes in current economic activity. Therefore they may be neglected in representing net national product. Accordingly, net national product is taken to be what Mäler calls the linear support of \( \tilde{H} \), or

\[
NNP = U'(C^*) C + \tilde{\lambda}^* K + \tilde{\psi}^* \dot{X}.
\]

Again, we may divide by \( U'(C) \) to express net national product in terms of the aggregate consumption numeraire. A critical feature of equation (12) is that even though the economy has departed from the optimal trajectory due to the perturbation, net national product is still reckoned using first best shadow prices.

\[
NNP = C^* + \dot{K}^* + [F_R - \theta] \dot{X}^*.
\]
While the linerization technique and use of first best shadow prices seem justified for small perturbations to the economy, their applicability in the case of larger projects or major economic distortions is questionable. This leads us to consider the construction of a "second best" net national product as a basis for welfare measurement in distorted economies.

In the theoretical framework set forth by Drèze and Stern (1987, 1990), the shadow price of a commodity represents the increase in social welfare resulting from the availability of an extra unit of that commodity. A "distorted economy" is then defined as one in which market and shadow prices do not coincide. This definition focuses primary interest on distortion in price signals, rather than on departures from economic organization or market structures presumed to be optimal. As enumerated by Drèze and Stern (1990), sources of distortion in the economy include: (1) indirect or income taxes; (2) uncorrected externalities; (3) quantity controls; (4) controlled prices; (5) tariffs and trade controls; (6) oligopoly; and (7) imperfect markets.

In the context of our basic model of renewable resources, a point of departure for considering second best shadow prices is to refer to efficiency conditions given by equations (4) and (6). We introduce some added notation to simplify the exposition:

\[ r_t = \left[F \kappa^\delta \right], \text{ the return to capital;} \]
\[ \sigma_t = \{\rho - U'(C)C'/U(C)\}, \text{ the consumption rate of interest;} \]
\[ \pi_t = \left[F \kappa - \theta \right], \text{ resource royalty, or rent;} \]
\[ D_t = \frac{1}{r_t} \left\{ F \kappa + \pi_t G'(X) - G(X)\theta'(X) \right\}. \]

Key to the subsequent development is to recognize that each variable defined
above is some function of the basic state and control variables of the model. Specifically, \( r_t, \pi_t, \) and \( D_t \) are each functions of \( K, R, \) and \( X, \) while \( \sigma_t \) is a function of \( C. \) Along the optimal trajectory, the arbitrage conditions hold so that

\[
\sigma_t = r_t \tag{4'}
\]

and

\[
\pi_t = D_t \tag{5'}
\]

But suppose price signals are distorted in the economy, inducing overharvesting of the resource. This distortion will create a "wedge" between royalty and depreciation in the resource market so that \( \pi_t \neq D_t. \) Moreover, as will be demonstrated below, the distortion may be transmitted to the capital market as well, so that \( \sigma_t \neq r_t. \)

Figure 4.1 illustrates the situation in the resource market. The variables \( \pi \) and \( D \) are plotted as schedules of the resource extraction rate, \( R, \) assuming a fixed stock of capital, \( K, \) but allowing the resource stock to adjust according to the dynamic equation \( \dot{X} = G(X) - R. \) In the typical case, resource royalty, \( \pi, \) will be a decreasing function of the extraction rate, \( R; \) in contrast, resource depreciation, or user cost \( D, \) typically increases with \( R. \) A similar graphical representation is given in Clark (1990). The optimum solution is represented by point \( 0^*, \) where the royalty and depreciation schedules cross. The shadow price of the resource at the optimum, i.e., the first best shadow price, is given by the costate variable, \( \psi^*, \) which equals both \( \pi \) and \( D. \)

Distortion in the resource market introduces a wedge between \( \pi \) and \( D, \) represented by the gap, \( B, \) between points \( a \) and \( b, \) in Figure 4.1. The situation
Figure 4.1
Resource Royalty ($\pi$) and Depreciation (D)
illustrated in the figure is one of overharvesting the resource; the extraction rate increases from $R^*$ to $\bar{R}$. Figure 4.1 suggests two alternative ways of modeling the distortion.

We may first impose an exogenous wedge, $B$, as a constraint on the original optimization problem $P1$. In this case, the underlying feasibility set of the economy, determined by the dynamic equations for $K$ and $X$, and the initial stock levels $K_0$ and $X_0$, remains unchanged. The shadow price, $\psi^o$, of the resource arising from this situation is no longer equal to $\psi^*$, but as demonstrated in model $P2$ of Section 4.3, takes a form that includes a weighted average of the values of $\pi$ and $D$ that obtain at resource extraction rate $\bar{R}$. Shadow price, $\psi^o$, is not depicted in Figure 4.1 because its expression involves terms from the capital market as well.

Alternatively, we may consider a government policy, such as imposition of a tax to support public sector production, that contracts the basic feasibility set of the economy. Such a policy might affect the resource market by producing a parametric shift in the royalty and depreciation schedules. In Figure 4.1, this is depicted by a shift in the royalty schedule from $\pi$ to $\tilde{\pi}$ and by a shift in the depreciation schedule from $D$ to $\tilde{D}$. A distorted equilibrium is attained at point $\tilde{0}$, where $\tilde{\pi} = \tilde{D}$. The shadow price of the resource in this tax distorted economy is $\psi^-$. Model $P3$ of Section 4.3 develops this case in more detail and shows that the expression for $\psi^-$ differs substantially from that for $\psi^o$ arising from model $P2$.

The basic model $P1$ and the two variations, $P2$ and $P3$, illustrate important distinctions related to constrained optimization and second best shadow pricing. Model $P1$ represents a first best economy and yields shadow prices that are first best. The
second model, P2, incorporates a distortion that moves the economy along the original feasibility frontier but away from the first best optimum. The appropriate shadow prices are now second best associated with a constrained first best world. In contrast, the third model, P3, involves a tax distortion that pushes the economy inside the original feasibility frontier. Shadow prices are now second best relative to a second best economy.

4.3. Distorted Economies and Second Best Shadow Prices

We now incorporate specific distortions into model P1 and compute the second best shadow prices arising from constrained optimization.

In model P2, we consider a distortion that produces a wedge, $B_t$, between royalty and depreciation at time $t$; that is, $\pi_t = D_t - B_t$. Such a wedge might arise, for example, from a direct tax or subsidy to the resource sector. Following the approach used by Warr (1982) and Dinwiddy and Teal (1985) for the case of static optimization, we incorporate this distortion directly into the model as a problem constraint. We wish to determine the effect on maximum attainable social welfare of depleting the resource stock by one unit in the presence of the distortion. By maximum attainable social welfare, we mean social welfare with respect to the original feasibility set underlying the economy. This equates to solving problem P2:
Max $\int U(C)e^{-p t} \, dt$

s.t. $\dot{K} = F(K, R) - \delta K - \Theta R - C$, $K(0) = K_0$

$\dot{X} = G(X) - R$, $X(0) = X_0$

$D - \pi = B.$

The Lagrangian for the problem is

$$\mathcal{L} = U(c)e^{-p t} + \lambda[F(K, R) - \delta K - \Theta R - C] + \psi[G(X) - R] + \phi[B + \pi - D], \quad (14)$$

which leads to the following first order conditions:

$$\frac{\partial \mathcal{L}}{\partial \lambda} = U'(C)e^{-p t} - \lambda = 0, \quad (15a)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \lambda[F_R - \Theta] - \psi + \phi[\frac{\partial \pi}{\partial R} - \frac{\partial D}{\partial R}] = 0, \quad (15b)$$

$$\dot{\lambda} = - \frac{\partial \mathcal{L}}{\partial K} = -\lambda[F_R - \delta] - \phi[\frac{\partial \pi}{\partial K} - \frac{\partial D}{\partial K}], \quad (15c)$$

$$\dot{\psi} = - \frac{\partial \mathcal{L}}{\partial X} = \lambda \theta'(X)R - \psi G'(X) - \phi[\frac{\partial \pi}{\partial X} - \frac{\partial D}{\partial X}]. \quad (15d)$$

From (15a), it is immediate that

$$\lambda = U'(C)e^{-p t}.$$

Differentiating $\lambda$ with respect to time, equating the result to the right hand side of (15c) and rearranging the terms, yields the following expression for the multiplier $\phi$:  

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\[ \phi = \frac{U'(C)e^{-\nu'(\sigma-r)}}{(\frac{\partial \pi}{\partial K} - \frac{\partial D}{\partial K})} \]  

(16)

The current value second best shadow price of the resource is then

\[ \psi^o = U'(C)\pi + \frac{U'(C)[\sigma-r][\frac{\partial \pi}{\partial R} - \frac{\partial D}{\partial R}]}{[\frac{\partial \pi}{\partial K} - \frac{\partial D}{\partial K}]} \]  

(17)

\[ = U'(C)(-D\frac{\partial \pi}{\partial R} - \frac{\partial D}{\partial R}) + (\pi - D)\frac{\partial \pi}{\partial R} + (\sigma - r)[\frac{\partial \pi}{\partial R} - \frac{\partial D}{\partial R}]^2 \]

Several observations may be offered regarding this shadow price. Aside from the factor \( U'(C) \), the shadow price, \( \psi^o \), is comprised of two terms. The first is a Harberger weighted average of the prevailing royalty and resource depreciation in the resource market (see Harberger, 1969, 1972). This measures the direct effect of the distortion. The second terms is a weighted combination of the distortions in the capital and resource markets and represents indirect effects. The general equilibrium nature of these effects is evident. The wedge in the resource market generates a distortion in the capital market, which is then reflected back into the second best shadow price for the resource.

In partial equilibrium analysis, the indirect effects are typically neglected, and the second best shadow price is taken to be the Harberger weighted average. Boadway and Bruce (1984) present a standard development of this approach for static cost-benefit analysis. The presumption is that the indirect effects, when summed, are negligible and the direct effect, represented by the Harberger type formula, captures the impact of the
market distortions on social welfare. In this example, the weights in the Harberger average are the slopes of the resource royalty and depreciation schedules. A royalty schedule with a steep slope relative to that of the depreciation schedule will give added weight to resource depreciation in the computation of the second best shadow price.

Note that if the distortion in the capital market is small, or its weighting insignificant, the second best shadow price can be approximated by $U'(C)\pi$. While this may appear to be equivalent to the best shadow price, it is not. As emphasized by Dinwiddy and Teal (1987), the formulae may be the same, but their values are not. In this example, first best and second best royalties differ. However, this case does serve as an example of using "producer prices," i.e., resource royalty, for second best shadow prices, as advocated by Diamond and Mirrlees (1976).

In model P3, we change the fundamental nature of the economy by incorporating public sector production and a general economy-wide tax. Specifically, output, net of capital depreciation, and resource extraction cost, is taxed at rate $\tau$. This is similar to the proportional tax rate on profits used in the two-period model of Sandmo and Drèze (1971). The tax revenue is used to support provision of public sector output, $Z$, which is measured in consumption units. Total consumption, $Y$, is then the sum of $Z$ and $C$, the consumption of private sector output. The productivity of the public sector relative to that of the private sector is measured by a productivity index $\gamma$. When $\gamma < 1$, the public sector is less productive than the private sector. As an example, we might think of an economy where the private sector and public sector both provide health, education, and postal services, but where one sector is more productive than the other. In this
model, the planner solves P3:

\[
\max \int_0^\infty U(Y)e^{-pt} \, dt
\]

s.t. \( \dot{K} = F(K, R) - \tau(F(K, R) - \delta K - \theta R) - \delta K - \theta R - C, \quad K(0) = K_0 \)

\[
\dot{X} = G(X) - R, \quad X(0) = X_0
\]

\[
Z = \gamma \tau(F(K,R) - \delta K - \theta R)
\]

\[
Y = C + Z.
\]

Following the recommendation by Chiang (1992), the equality constraint is handled through direct substitution rather than through use of a Lagrange multiplier. The present value Hamiltonian is written:

\[
H = U(Y)e^{-pt} + \lambda[(1 - \tau)F(K,R) - (1 - \tau)\delta K - (1 - \tau)\theta R - C]
\]

\[+ \psi[G(X) - R].\]  \hspace{1cm} (19)

The maximum principle yields the following first order conditions

\[
\frac{\partial H}{\partial C} = U'(Y)e^{-pt} - \lambda = 0, \hspace{1cm} (20a)
\]

\[
\frac{\partial H}{\partial R} = U'(Y)e^{-pt}\gamma \tau[F_R - \theta] + \lambda(1 - \tau)[F_K - \theta] - \psi = 0, \hspace{1cm} (20b)
\]

\[
\dot{\lambda} = -\frac{\partial H}{\partial K} = -U'(Y)e^{-pt}\gamma \tau[F_K - \delta] - \lambda(1 - \tau)[F_K - \delta], \hspace{1cm} (20c)
\]

\[
\dot{\psi} = -\frac{\partial H}{\partial X} = \lambda(1 - \tau)\theta'(X)R - \psi G'(X). \hspace{1cm} (20d)
\]

Using equations (20a) through (20d) to solve for the costate variables in current value
terms yields

\[ \bar{\lambda} = U'(Y) \] \hspace{1cm} (21a)

and

\[ \Psi = [\gamma \tau + (1-\tau)][F_R - \theta]. \] \hspace{1cm} (21b)

The arbitrage conditions which now govern the economy reveal the parametric shifts that have occurred in the schedules for the consumption rate of interest, \( \sigma \), the return to capital, \( r \), resource royalty, \( \pi \), and depreciation, \( D \). The Ramsey condition becomes

\[ \frac{U''(Y)}{U'(Y)} = \rho - [\gamma \tau + (1-\tau)][F_K - \delta], \] \hspace{1cm} (22)

while the generalized Hotelling Rule includes an extra term as well as coefficients depending on \( \tau \) and \( \gamma \):

\[ [F_R - \theta] = \frac{1}{[\gamma \tau + (1-\tau)][F_K - \delta]} \left[ \dot{F}_K + (F_K - \theta)G'(X) - \theta'(X)G(X) \right] + \frac{\gamma \tau \theta(X)R}{[\gamma \tau + (1-\tau)][F_K - \delta]} \] \hspace{1cm} (23)

Note that for \( \tau = 0 \), all first order conditions and resulting arbitrage conditions revert to those associated with problem P1. An interesting special case arises for the situation where an economy-wide tax is imposed, but the private sector and public sector are equally productive, that is, \( \tau \neq 0 \), but \( \gamma = 1 \). Intuition might suggest that for this case we revert back to the arbitrage conditions arising from problem P1. But this is not the case when unit extraction costs for the resource rise as the stock decreases. The arbitrage condition in the resource market now becomes
First observe that \( \theta'(X) \) is negative. The second term on the right-hand side of equation (24) is thus negative and represents an offset to depreciation of the resource. Harvesting the marginal unit of resource at time \( t \) raises extraction cost. But this rising cost is netted out in the computation of the tax. The result is that the tax distortion induces overharvesting, so that the benefits of the tax write-off can be enjoyed in the present.

The two models described here produce second best shadow prices that are substantially different from each other. Model P2 yields a second best shadow price for the resource with a Harberger type weighted average component. Model P3 generates a second best resource shadow price with a much simpler expression. The two problems are not equivalent. For example, if we take resource arbitrage condition (23) from Problem P3 and incorporate it as the constraint in the resource market in Problem P2, second best shadow prices \( \psi^0 \) and \( \psi^- \) will still differ. In the next section, we consider the implications of these different second best shadow prices for national income accounting.

4.4. Distorted Economies and Second Best NNP

To facilitate discussion of net national produce in distorted economies, we employ a graphical representation based on Weitzman (1976). Figure 4.2 illustrates the basic situation for the simple case of an economy with a single type of capital stock, \( K \). This could represent physical capital, as in standard models of economic growth, or ecological...
capital in the context of a cake-eating economic model. Curves are plotted in utility-investment space, with points representing pairs \((U(C), \dot{K})\).

Curve (aa) represents the frontier of the feasibility set for the economy, which is assumed to be convex. Added to the Weitzman framework is a family of welfare indifference curves that depict the trade-off between present utility and future consumption possibilities, i.e., investment. Specifically, welfare, \(W_t\), at time \(t\) is given by

\[
W_t = \int_{t}^{\infty} U(C_\tau)e^{-\rho \tau} d\tau,
\]  

so that \(W_t\) is a functional defined over consumption trajectories, \((C_\tau)\). Harris, Heady, and Mitra (1989) give a similar graphical representation of welfare indifference curves in their analysis of investment in a growing economy with tax restrictions.

Point \(0^\star\) in Figure 4.2 is a point along the optimum trajectory of the first best economy, and may be related to the solution of problem P1. The first best shadow price of capital, \(\lambda^\star\), is determined by the slope of the tangent line (bb) common to the feasibility frontier and welfare indifference curve, \(W_t\). First best NNP, expressed in units of utility of aggregate consumption, is then represented by the intercept of tangent line (bb) to the vertical axis, so that

\[
NNP^\star = U(C^\star) + \lambda^\star \dot{K}^\star.
\]  

Weitzman (1976) emphasizes the idea that \(NNP^\star\) represents a hypothetical maximum level of consumption at time \(t\). The maximum level of consumption that could actually be achieved is represented by the point where the feasibility frontier (aa) meets the vertical axis.

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Figure 4.2
NNP in the Weitzman Framework
It is apparent from the diagram that NNP*, based on first best shadow prices, λ*, is the proper welfare measure for assessing small projects, or minor perturbations about the optimum point 0°. By considering lines parallel to (bb) through points close to, but distant from 0°, one can see that NNP increases or decreases in the same direction as aggregate welfare.

However, the first best shadow price, λ*, does not support a true NNP measure of welfare for a distorted economy represented by 0° = (C°, K°) on the feasibility frontier. This point is analogous to the solution of problem P2 and represents a constrained optimum yielding aggregate welfare W² < W¹. Consider line (dd) through point 0°, parallel to line (bb) with slope λ*. The intercept of line (dd) with the vertical axis would give net national product as

\[ NNP = U(C°) + λ*K°. \] (27)

Point x, representing a small perturbation of the economy away from 0°, lies to the left of and below line (dd), but above welfare indifference curve W². Thus, point x is associated with a project that would increase aggregate welfare, but decrease net national product at shadow price, λ*. This project would be erroneously rejected.

A similar analysis shows that taking "producer prices" based on the marginal rate of transformation between consumption and investment at 0°, would also be inappropriate. For the case of a static economy involving multiple commodities, Diamond and Mirrlees (1976) identify conditions under which producer prices serve as appropriate second best shadow prices for cost-benefit analysis. While Diamond the
Mirrlees do not require that taxes be optimal, they do assume that all markets clear and that government shows no concern for the division between public and private production. The basic model in this paper represents a dynamic production economy that supports consumption of a single aggregate commodity and thus differs substantially from the Diamond and Mirrlees (1976) framework. Nonetheless, the Diamond and Mirrlees assumptions seem compatible with the structure of problem P3, and for that model, second best shadow prices are equivalent to producer prices, as determined by the tangent to the second best feasibility frontier. But for the case of problem P2, whose solution at time \( t \) is represented by point \( 0^\circ \) in Figure 4.2, the wedge in the capital market, arising from the resource market distortion, violates the Diamond and Mirrlees market clearing assumption.

In Figure 4.3, we offer a graphical depiction of the second best shadow price, \( \lambda^\circ \), appropriate for project evaluation and national income accounting in an economy at the constrained optimum, \( 0^\circ \). The construction is certainly not a rigorous proof, but is suggestive of the differential welfare approach to shadow pricing used by, for example, Boadway (1975), and Dievert (1983). Suppose we draw one unit of capital out of the private sector at time \( t \), and in so doing, perturb the feasibility frontier from \( (aa) \) to \( (a'a') \). We then seek 'compensated' shadow prices similar to those advocated by Sieper (1981). That is, we conceptually return the economy to the level of welfare, \( W^2 \), that prevailed at \( 0^\circ \), and ask for what shadow prices would the differential welfare change \( dW = 0 \) be reflected in a change of zero in net national product. The second best shadow price, \( \lambda^\circ \), we are looking for is that determined by the slope of the line \( (gg) \).
Figure 4.3
First Best vs. Second Best NNP
through 0° and point P°, which is located at the intersection of the perturbed frontier (a’a’) and welfare indifference curve, W².

Inspection of Figure 4.3 suggests that λ° is, in fact, a weighted average of producer prices and consumer prices, or more precisely, the marginal rate of transformation and the marginal rate of substitution between consumption and investment. Graphically, line (gg) falls between lines (ee) and (ff). Finally, we mention the analysis of Dinwiddy and Teal (1987), which establishes the principle, implied in the work of Dréze and Stern (1985), that the differential welfare approach and the constrained optimization approach to shadow pricing are equivalent: they yield the same formula for second best shadow prices. This suggests a close correspondence between the second best shadow price formula of problem P2 and the graphical depiction above.

The case of problem P3 is represented by the point ŵ in Figure 4.4. The introduction of public sector production and an economy-wide tax moves the feasibility frontier of the economy from (aa) to (hh). The public sector may be less productive than the private sector and the tax may be nonoptimal; thus, the economy is less efficient than that associated with model P1. Nonetheless, assuming convexity of the new feasibility set, a suboptimal trajectory is determined by the tangency at each time t of the feasibility frontier and a welfare indifference curve. The associated shadow price of capital, ŵ, is the slope of the tangent, depicted in Figure 4.4 by line (jj). And net national product, ŴNP, is represented by the intersection of line (jj) with the vertical axis, so that

\[ \text{ŴNP} = U(\tilde{C}) + \tilde{\lambda}\tilde{K}. \]  

(28)

The three versions of NNP developed in the previous discussion are compared in
Figure 4.4
NNP in First Best and Second Best Economies
Figure 4.4. NNP$^*$ may be designated the first best net national product, because it is associated with a first best economy that follows an optimum trajectory. In contrast, NNP$^o$ is the second best net national product for a constrained optimum economy. Finally, NNP is the second best net national product for a suboptimal economy. Once the second best net national product has been determined using the appropriate shadow prices, the linearization process advocated by Dasgupta (1993) and Måler (1991) may be then applied as a local approximation for use in evaluating small projects. This allows net national product to be expressed in units of aggregate consumption numeraire.

4.5 Concluding Remarks

The objective of this chapter was to derive second best shadow prices for both man-made and ecological capital in economies subject to market distortion. This objective was pursued by investigating two variations of a basic model of a production economy with renewable resources. These models permitted consideration of two distinct situations: first, distortions that are in principle removable by improvement of market incentives; and second, distortions that are essentially permanent, because political or institutional constraints prevent their correction.

The first case, that of correctable market distortion, leaves open the possibility that the economy could eventually attain the first best golden rule steady state. In the second case, distortions are presumed to be irremovable so that the feasibility set for the economy, and ultimately, the golden rule steady state, if it exists, are second best. In either case, attaining the constrained first best optimum or the second best optimum,
depend directly on the shadow prices that are used to guide economic activity. This paper argues that second best shadow prices are the appropriate prices for national income accounting, as well as cost-benefit analysis and policy design.

As a final remark, we note that the analysis presented in this paper does not attempt to address the issue of convexity. Harris, Heady and Mitra (1989) point out that there can be no guarantee that feasibility sets will be convex in second best situations. Drèze and Stern (1990) offer the reminder that, "In distorted economies it is possible to provide counter-examples to most propositions concerning shadow prices." Nonetheless, throwing up one's hands and relying on first best shadow prices as a basis for net national product in distorted economies would not be compatible with the goal of "green accounting", i.e., promoting sustainable economic growth.
CHAPTER 5

EXTENSIONS AND CONCLUSIONS

5.1. Introduction

The focus of this study has been an examination of sustainable economic growth, with specific concern directed at the management of ecological capital. This focus has led to the development in earlier chapters of golden rules and second best shadow prices to guide policy toward sustainability. This final chapter presents several extensions of the earlier work in an attempt to generalize results and then offers concluding remarks on sustainable growth and means to its attainment.

In Chapter 2, we developed a modified golden rule and then a golden rule for the case of an economy with an exhaustible resource that is eventually replaced by a higher priced substitute, based on a backstop technology. In Section 5.2, we extend these results to the case of a renewable resource and consider alternative steady state solutions that yield golden rule variations. This extension draws on work done by Roumasset and Wang (1992) in developing a unified approach to natural resource modeling. Section 5.3 presents added justification for setting the utility discount factor at zero as a means of meeting society’s obligation to future generations in managing economic growth. This analysis is based on a recent paper by Burton (1993), which considers the distinction between personal discount rates and generational discount rates. In Section 5.4, we
attempt to synthesize the results of the previous chapters and relate golden rules to the net national product. Section 5.5 follows with a brief discussion of golden rules in the context of distorted economies. The study is concluded in Section 5.6 with a summary of major findings and suggestions for further extensions and future research.

5.2. Golden Rules for a Renewable Resource Economy

Chapter 2 presented a model of economic growth for an economy with an exhaustible resource and a backstop resource technology. Incorporation of the backstop into the model admitted the possibility of deriving golden rules for efficient resource management compatible with intergenerational equity. In this section, we extend these earlier results to the case of a renewable resource. Since this represents an extension of the basic models outlined in Chapters 1 and 2, we refer to those chapters for details on variables and notation.

As in Chapter 1, the basic dynamic equations for the economy are

\[ \dot{K} = F(K, R) - \delta K - \theta R - C \quad (1) \]

\[ and \quad \dot{X} = G(X) - R. \quad (2) \]

To account for population, \( L \), as in Chapter 2, we assume exponential growth at rate \( n \). Expressing key variables in per capita terms, using lower case notation, the basic optimization problem is then:
\[
\text{Max } \int_0^\infty u(c)e^{-\rho t} \, dt
\]  
\(\text{s.t. } \dot{k} = f(k, r) - \mu k - \theta r - c, \ k(0) = k_0\)  
\[\dot{X} = G(X) - rL, \ X(0) = X_0\]
\[\theta \leq \theta_b\]
\[X \geq 0.\]

The parameter \(\mu = n + \delta\), is the sum of the population growth rate and the rate of capital depreciation. As before, \(\theta\) is the per capita unit cost of resource extraction, presumed to increase as the resource stock \(X\) is drawn down. Per capita extraction cost is constrained to go no higher that \(\theta_b\), the unit extraction cost of the backstop resource. The Hamiltonian for this problem becomes

\[
H = u(c)e^{-\rho t} + \lambda [f(k, r) - \mu k - \theta r - c] + \psi [G(X) - rL].
\]

Owing to the inequality constraints imposed on the problem, we form the Lagrangean

\[
\mathcal{L} = H + \gamma [\theta_b - \theta] + \phi [X].
\]

The complementary slackness conditions associated with the inequalities are

\[
\gamma \frac{\partial \mathcal{L}}{\partial \gamma} = \gamma [\theta_b - \theta] = 0 \quad (6a)
\]

and \(\phi \frac{\partial \mathcal{L}}{\partial \phi} = \phi X = 0.\) \(\quad (6b)\)

Application of the maximum principle leads to the two basic efficiency conditions
that guide the economy along the optimal trajectory to the steady state, which is attained at some endogenous time $T$:

\[ \eta(c) \frac{\dot{c}}{c} = f_k - (\mu + \rho), \quad (7a) \]

\[ (f_r - \theta) = \frac{1}{(f_k - \delta)} [f_r - \theta (X) - \theta'(X)G(X)]. \quad (7b) \]

At time $T$, the economy enters the steady state, and $\dot{c} = \dot{f}_r = 0$. The basic steady state conditions then become

\[ f_k = (\mu + \rho), \quad (8a) \]

\[ (f_r - \theta) = \frac{1}{(f_k - \delta)} [(f_r - \theta(X)) - \theta'(X)G(X)]. \quad (8b) \]

Substitution of (8a) into (8b) yields the basic form of the steady state modified golden rule for renewable resources:

\[ f_r = \theta + \frac{\theta'(X)G(X)}{G'(X) - (n + \rho)}. \quad (9) \]

We distinguish three types of steady state according to the taxonomy presented in Roumasset and Wang (1992):

(1). \textit{Economically renewable resources}. This is the case in which the steady state stock is positive and the resource extraction cost $\theta$ is less than the cost of the backstop; that is, $X(T) > 0$ and $\theta(X(T)) < \theta_b$. Equations (8a) and (9) represent the modified golden rule for this situation. By setting $\rho = 0$, we obtain the golden rule for an economically renewable resource:
\[ f_k = \mu, \]  
\[ f_r = \theta + \frac{\theta'(X)G(X)}{G'(X) - n}. \]  

(2). **Exhaustible resources.** The case of nonrenewable resources is covered by setting \( G(X) = 0 \) in equation (9). More generally, however, we consider the possibility that renewable resources might be extinctable along the optimum trajectory. In this type of steady state, \( X(T) = 0 \) and \( \theta(T) = \theta_b \). Therefore for the case of exhaustible resources, the golden rule becomes

\[ f_k = \mu, \]  
\[ f_r = \theta_b. \]  

This is precisely the rule established in Chapter 2 for exhaustible resources.

(3). **Replaceable resources.** In this case the backstop price, \( \theta_b \), becomes binding before the stock level \( X \) is allowed to reach the golden rule level, or before it is exhausted. Now \( X(T) > 0 \), but \( \theta(X(T)) = \theta_b \). For the case of replaceable resources, the golden rule is again represented by conditions (11a) and (11b).

The type of steady state ultimately attained is largely dependent on the relative growth rates of the resource and of population. This can be seen by rearrangement of equation (10b):
\[ \theta = f_r - \frac{\theta'(X)G(X)}{G'(X) - n}. \]  

(12)

Noting that \( \theta'(X) < 0 \), we observe that an increase in the population growth rate \( n \) tends to raise the steady state resource extraction cost \( \theta \). A high enough population growth rate, \( n \), relative to \( G'(X) \) will push \( \theta \) up to the backstop price \( \theta_b \). An interesting special case arises when \( \theta'(X) = 0 \), that is, unit extraction cost is constant. Assuming \( \theta < \theta_b \), for otherwise the economy would move straight to the backstop, the golden rule becomes \( f_r = \theta \), or equivalently, \( G'(x) = n \). If, in addition, population growth is zero, the golden rule says that the resource stock should be maintained at the level consistent with maximum sustainable yield.

The extension of golden rules to the case of a renewable resource gives added power and generality to the use of these rules as a foundation for sustainability. In the next section, we consider the theoretical and ethical justification for setting \( \rho = 0 \) in deriving golden rules of capital accumulation and resource management.

5.3. The Rate of Time Preference and Intergenerational Equity

A recent paper by Burton (1993) in the resource economics literature makes the argument that the standard approach to utility discounting confuses two key, but distinct concepts: the discount rates that reflect the intertemporal preferences of members of society and issues of intergenerational equity. Burton incorporates both considerations into models of optimal resource harvesting by postulating two separate discount rates: a personal discount rate, designated in the subsequent development by \( \beta \), which reflects the
rate of pure time preference of individuals, and a generational discount rate, $\rho$, which addresses society's degree of concern for intergenerational equity.

While Burton (1993) confines his analysis to simple cake-eating models of a resource based economy, we apply his methodology to our basic model of a production economy with renewable resources. Following Burton, we assume that the economy is made up of overlapping generations of otherwise identical individuals. In the simplest representation, each generation contains one individual who lives to age $N$. At each time $t$, society is made of individuals who range in age from 0 to $N$, and no two individuals have the same age. An individual of age $\tau$ is allotted consumption good in amount $c(t, \tau)$ and enjoys utility $u_t(c(t, \tau))$. An individual born at time $T$ measures lifetime utility $U_T$ according to the following formula

$$U_T = \int_{\tau=0}^{N} u_t(c(t, \tau)) e^{-\rho t} d\tau. \quad (13)$$

Social welfare is then a weighted sum of the lifetime utilities of individual members of society, where the weights are based on the generational discount rate, $\rho$:

$$W = \int_{T-N}^{T} U_T e^{-\rho t} dT. \quad (14)$$

Within this framework, an argument similar to that given by Burton shows that society's welfare at time $t$, $V(t)$, is the discounted sum of utilities of the individual members of society. In continuous time, we integrate individual utility over the age range $0 \leq \tau \leq N$. Thus,
where $\beta$ is the personal discount rate and $\rho$ is the generational discount rate. Distribution of consumption goods to different age groups at any time $t$ is constrained by the aggregate level of output available for consumption at time $t$, $C(t)$. Therefore,

$$\int_0^N c(t, \tau) \, d\tau \leq C(t).$$

As usual, the problem is to maximize aggregate welfare over an infinite time horizon subject to the dynamic constraints of the economy. For our basic model of renewable resources, this becomes

$$\text{Max } \int_0^\infty U(C)e^{-pt} \, dt$$

s.t. $\dot{K} = F(K, R) - \delta K - \theta R - C, \ K(0) = K_0$

$$\dot{X} = G(X) - R, \ X(0) = X_0,$$

where

$$U(C(t)) = \text{Max } V(t)$$

s.t. Condition (16).

We first look at the constrained maximization of $V(t)$. The Lagrangean for this problem can be written as

$$\mathcal{L} = \int_0^N u(c(t, \tau))e^{-(\beta-\rho)t} \, d\tau + \phi[C(t) - \int_0^N C(t, \tau) \, d\tau].$$

Here we assume that utility functions do not vary with time or age of individual.
Inspection of the Lagrangean indicates that a necessary condition for maximization is that weighted marginal utilities be equated, so that,

\[ u'(c(t, 0)) = u'(c(t, \tau))e^{-(\beta - \rho)\tau}, \quad 0 \leq \tau \leq N. \tag{20} \]

Suppose \( \beta > \rho \), i.e., the personal discount rate is greater than the generational discount rate. Then from equation (20), it is apparent that marginal utility is lower for younger individuals. These younger individuals are therefore responsible for a larger share of society’s aggregate consumption at time \( t \). The implications of this result may be developed further by specifying the form of the individual utility function \( u \). As in our basic model presented in Chapter 1, we assume a functional form with constant elasticity of marginal utility:

\[ u(c(t, \tau)) = -[c(t, \tau)]^{(\eta-1)}, \quad \eta > 1. \tag{21} \]

Equation (20) then implies that at the optimum,

\[ c^*(t, \tau) = c^*(t, 0)e^{-(\beta - \rho)\tau/\eta}. \tag{22} \]

Condition (16) permits derivation of \( c^*(t, 0) \) from equation (22):

\[ c^*(t, 0) = C(t)[\frac{\eta}{(\beta - \rho)}(1 - e^{-(\beta - \rho)N/\eta})]^{-1}. \tag{23} \]

Next write

\[ U(C(t)) = \int_0^N u(c^*(t, \tau))e^{-(\beta - \rho)\tau} \, d\tau. \tag{24} \]

Substituting from equations (22) and (23) then gives,
\[ U(C) = -\frac{\eta}{(\beta - \rho)}(1 - e^{-\frac{(\beta - \rho)N}{\eta}})^{\eta} C^{-(\eta - 1)}. \]  

For notational convenience, we omit explicit reference to the functional dependence of aggregate consumption on time \( t \). Let \( A \) represent the "aggregation" coefficient in (24) and include the discount rates \( \beta \) and \( \rho \) as arguments in the resulting expression:

\[ U(C, \beta, \rho) = -A(\beta, \rho)C^{-(\eta - 1)}. \]  

(26)

A few observations are in order with respect to this derivation. First, consider the effect on aggregate marginal utility of an increase in the personal discount rate \( \beta \):

\[ \frac{\partial U(C, \beta, \rho)}{\partial C} = A(\eta - 1)C^{-\eta}, \]  

(27)

\[ \frac{\partial^2 U(C, \beta, \rho)}{\partial \beta \partial C} = \frac{\partial A}{\partial \beta}(\eta - 1)C^{-\eta}. \]  

(28)

A tedious but straightforward computation shows that

\[ \frac{\partial A}{\partial \beta} < 0, \text{ so that } \frac{\partial^2 U}{\partial \beta \partial C} < 0. \]

This comparative static provides analytical validation of Burton's (1993) remark that a higher personal discount rate results in lower marginal utility of aggregate consumption for society. Similar comparative static analysis shows that

\[ \frac{\partial A}{\partial \rho} > 0, \text{ so that } \frac{\partial^2 U}{\partial \rho \partial C} > 0. \]

An increase in the generational discount rate increases the marginal utility of society.

We now apply these results to the original problem, stated as (17). Setting up the
Hamiltonian in the usual way, we have

\[ H = U(C)e^{-pt} + \lambda[F(K, R) - \theta R - C] + \psi[G(X) - R]. \]  \hspace{2cm} (29)

Application of the maximum principle yields the standard first order conditions which can be manipulated to generate the classic arbitrage conditions, the Ramsey condition and a generalized Hotelling Rule for renewable resources:

\[ \frac{U''(C)}{U'(C)} \dot{C} = \rho - [F_K - \delta] \]  \hspace{2cm} (30)

and

\[ [F_R - \theta] = \frac{1}{[F_K - \delta]}[\dot{F}_R + (F_R - \theta)G'(X) - \theta'G(X)G(X)]. \]  \hspace{2cm} (31)

Further development of equation (27) is especially revealing. Computing derivatives and then forming the ratio \( U''/U' \) results in cancellation of the coefficient A(\( \beta, \rho \)). The Ramsey condition can then be written in the standard way,

\[ \eta \frac{\dot{C}}{C} = F_K - (\delta + \rho). \]  \hspace{2cm} (30a)

Arbitrage conditions (30a) and (31) show that even in this overlapping generations model, the optimum trajectory is governed at each time t by the relationships among aggregate quantities and the generational discount rate \( \rho \), but not the personal discount rate, \( \beta \).

Next consider the current value Hamiltonian,

\[ \tilde{H} = U(C, \beta, \rho) + \tilde{\lambda}k + \tilde{\lambda}x. \]  \hspace{2cm} (32)

The current value shadow prices are given by:
\[ \bar{\lambda} = U_1(C, \beta, \rho) \]  

\[
\text{and } \bar{\psi} = U_1(C, \beta, \rho)[F_R - \theta].
\]

The current value Hamiltonian appears to be dependent on both \( \beta \) and \( \rho \), since these parameters are arguments in \( U \) as well as \( U_1 \) by virtue of the coefficient \( A(\beta, \rho) \). This dependence, however, is not significant, because the coefficient \( A(\beta, \rho) \) can be regarded as a simple scaling factor in the Hamiltonian. In other words, \( \beta \) and \( \rho \) do not affect relative current value shadow prices, and relative shadow prices are the prices that matter.

We believe that this finding has important implications for modeling economic growth in a manner compatible with intergenerational equity. Stewardship for the future can be accommodated by setting the generational discount rate, \( \rho \), equal to zero. Such an approach to intergenerational equity would not conflict with the possible existence of a positive personal discount rate, \( \beta \), governing intertemporal preferences over the lifetime of the individual.

5.4 Sustainable Growth and Golden Rule Net National Product

The results of Sections 5.2 and 5.3 demonstrate the general applicability of the golden rule as a basis for attaining sustainable economic growth. In particular, Section 5.2 shows how golden rules may serve in a broad range of cases, involving either renewable or non-renewable resources, with or without population growth. In the overlapping generations model of Section 5.3 personal discount rates only affect relative
shadow prices and therefore do not directly influence the movements of aggregates in the economy. In the interests of intergenerational equity we may set the generational rate of time preference, \( \rho \), equal to zero in aggregate growth models without negating the existence of personal discount rates that are positive. This finding gives added merit and legitimacy to the implementation of the golden rule.

Consideration of the golden rule, in fact, helps clarify the connection between net national product and sustainable growth. In recent discussions of sustainability, Mäler (1991) and Johansson (1993) consider the time profile of net national product. If the maximized current value Hamiltonian is

\[
\tilde{H}^* = U(C) + \lambda^* \dot{K}^* + \psi^* \dot{X}^* ,
\]

it is a simple exercise to show that

\[
\frac{d\tilde{H}^*}{dt} = \rho [\lambda^* \dot{K}^* + \psi^* \dot{X}^*].
\]

This result, incidentally, is contrary to the occasional assertion that the current value Hamiltonian along the optimum trajectory is constant over time (see e.g., Chiang (1992)). This is not the case for optimal control problems with time discounting, even those that would otherwise be classified as autonomous.

Mäler (1991) then offers a definition of sustainability based on the time profile of the current value Hamiltonian; specifically, sustainable growth, according to Mäler, is growth for which \( d\tilde{H}/dt \geq 0 \). For \( \rho > 0 \), this condition implies that
which says that the value of the total capital stock, measured in current year prices, never decreases. A special case is that for which $d\tilde{H}^*/dt = 0$. Then the associated condition on capital is

$$[\lambda^* \dot{K}^* + \psi^* \ddot{X}^*] = 0.$$  \hspace{1cm} (37)

This is Hartwick's rule, which keeps net investment equal to zero; all resource rents are invested in capital accumulation, but no additional saving is pursued.

Pezzey (1994) evaluates this as well as other possible sustainable constraints constant on aggregate growth. An alternative, and perhaps more direct approach to sustainability is based on a comparative evaluations of consumption trajectories. In Chapter 2, we presented the concept of relative sustainability as characterizing economic growth that supports a consumption trajectory eventually meeting or exceeding maximin consumption or some other floor level of consumption chosen by the planner. This approach to sustainable growth is most readily addressed by returning to the golden rule for resource management and capital accumulation.

As a point of departure, consider utility maximization without time discounting. Solving this problem requires that we employ the technique discussed in Chapter 2, in connection with a Ramsey type "bliss" point. In the context of our basic model, the golden rule steady state level of consumption $\hat{C}$ is given by

$$\hat{C} = F(\hat{K}, \hat{R}) - \delta \hat{K} - \theta \hat{R}$$  \hspace{1cm} (38)

where $\hat{K}$ and $\hat{R}$ are determined by the golden rule conditions for $F_K$ and $F_R$. As
discussed earlier, these conditions depend on whether the resource is economically renewable, exhaustible or replaceable. The bliss point for the economy in terms of aggregate utility is then \( U(\hat{C}) \). The Ramsey problem with respect to our basic model becomes

\[
\text{Max } \int_0^\infty [U(C) - U(\hat{C})] \, dt
\]

\( s.t. \quad \dot{K} = F(K, R) - \delta K - \theta R - C, \quad K(0) = K_0 \)

\( \dot{X} = G(X) - R, \quad X(0) = X_0. \)

The Hamiltonian for this problem is

\[
H = [U(C) - U(\hat{C})] - \lambda[F(K, R) - \delta K - \theta R - C] + \psi[G(X) - R].
\]  

Application of the maximum principle yields the familiar Ramsey condition and generalized Hotelling Rule for the transition to the steady state, but now with \( \rho = 0 \):

\[
\eta(C) \frac{\dot{C}}{C} = (F_K - \delta), \quad (41a)
\]

\[
F_R - \theta = \frac{1}{(F_K - \delta)} [\dot{F}_R - (F_R - \theta)G'(X) - \theta'(X)G(X)]. \quad (41b)
\]

Of special interest, however, is the Hamiltonian. Since the Ramsey problem can be classified as autonomous with no time discounting, the Hamiltonian along the optimum trajectory remains constant, that is, \( dH^*/dt = 0 \), where

\[
H^* = [U(C^*) - U(\hat{C})] + \lambda^* \dot{K}^* + \psi^* \dot{X}^*.
\]  

Now in the steady state, \( \dot{K}^* = \dot{X}^* = 0 \) and \( C^* = \hat{C} \). This implies that \( \dot{H} = 0 \) and,
consequently, \( H^* = 0 \) for all time \( t \). Rearranging the expression for \( H^* \) gives

\[
U(C*) = U(C^*) + \lambda^* \dot{K}^* + \psi^* \dot{X}^*.
\] (43)

Bliss point utility, \( U(\hat{C}) \), is therefore serves as the golden rule net national product. It remains constant over time; as \( C^* \) increases toward \( \hat{C} \), \( \dot{K}^* \) and \( \dot{X}^* \) approach zero, and \( \lambda^* \) and \( \psi^* \) decrease monotonically\(^1\). We illustrate the situation in Figure 5.1, which is based on the Weitzman (1976) framework. As in Chapter 4, we depict only one type of capital, which we designate \( K \). Curves are plotted in utility-investment space, rather than consumption-investment space as in Weitzman (1976). This facilitates graphical representation of the dynamic change in the economy over time.

Curve (aa) of Figure 5.1 represents the feasibility frontier of the economy at time \( t = 0 \). Consumption level \( \hat{C} \) is the maximum attainable level of consumption at time \( t = 0 \), if no investment were to take place, and \( U(\hat{C}) \) is the associated level of utility. The utility-investment pair \( (U(C^*), K^*) \) lies on the optimal trajectory to the steady state. With positive investment, capital is accumulated, and the feasibility frontier moves outward and upward toward the right. Because \( U(C) \), rather than \( C \), is plotted on the vertical axis, equal increments in maximum attainable consumption starting from \( \hat{C} \) are reflected as diminishing increments in maximum attainable utility. Therefore, movement of the feasibility frontier is not symmetric. As the frontier moves outward, consumption and utility of consumption increase over time. The shadow price of capital, \( \lambda^* \), is represented at each time, \( t \), by the slope of the line tangent to the prevailing feasibility frontier and passing through \( U(\hat{C}) \) on the vertical axis. As the economy evolves, the
Figure 5.1
Golden Rule NNP
feasibility frontier advances until maximum attainable consumption reaches the golden rule level, \( \hat{C} \), and \( \dot{K}^* = 0 \), as depicted by curve (bb).

We introduce an important caveat in the interpretation of golden rule net national product. When expressed in terms of utility of consumption, golden rule net national product is constant over time. However, when measured relative to the aggregate consumption numeraire at each time \( t \), golden rule net national product actually increases. Dividing \( \text{NNP} \) by \( U'(C^*) \) yields \( \frac{U(C)}{U'(C)} \); while \( U(C) \) is constant, \( U'(C) \) decreases as \( C^* \) approaches \( C \) along the optimum trajectory to the golden rule steady state.

Throughout this study, we have employed the services of the family of utility functions \( U(C) = -C^{-(\eta - 1)} \), \( \eta > 1 \), exhibiting constant elasticity of marginal utility. Dasgupta and Heal (1979) observe that the parameter \( \eta \) reflects the degree to which society is egalitarian in the distribution of consumption across generations (alternatively, \( \eta \) may be interpreted as a measure of relative risk aversion). As \( \eta \) gets larger, the initial level of consumption increases and the consumption trajectory becomes flatter. This is illustrated in Figure 5.2. Note, however, that there is an upper limit to the initial level of aggregate consumption. Compatible with the feasibility set underlying the economy at \( t = 0 \), this level is \( \tilde{C} \) in Figure 5.2, the level of maximum attainable consumption.

Consider the limiting case where \( \eta \) approaches infinity. By rearranging the Ramsey condition for consumption as

\[
\frac{\dot{C}}{C} = \frac{F_k - \delta}{\eta},
\]

(44)

it is apparent that as \( \eta \to \infty \), \( \dot{C}/C \to 0 \). This is precisely the maximin outcome. With
Figure 5.2

Effect of Elasticity of Marginal Utility on Golden Rule Trajectories

\[
0 = \eta_0 < \eta_1 < \eta_2 < \eta_\infty = \infty
\]
infinite elasticity of marginal utility, society chooses the point \((U(\bar{C}), 0)\) on the feasibility frontier at time \(t = 0\). At this point, net investment is zero, implying Hartwick's rule for the case of an economy with both man-made capital and ecological capital.

How does maximin relate to the golden rule steady state and the bliss point of the economy? The possibility of capital accumulation guarantees that \(\dot{C} < \bar{C}\). However, for the case of infinite elasticity of marginal utility, \(U(\bar{C}) = U(\bar{C})\); the utility of maximum attainable consumption at time \(t = 0\) now becomes the economy's bliss point. This observation, however, does not establish maximin welfare as the correct approach to intergenerational equity. Imposition of maximin welfare on a society whose preferences could be represented by an aggregate utility function with \(\eta < \infty\), would constitute a case of economic distortion. But it would represent a major distortion of justice as well, by robbing future generations of the golden rule standard of living.

The other extreme is represented by the case, \(U(C) = C\), for which the elasticity of marginal utility is zero. The Ramsey problem now becomes

\[
\text{Max } \int_0^\infty [C - \dot{C}] \, dt
\]

s.t. Dynamic equations for the economy.

Geometrically, this is equivalent to choosing the feasible consumption trajectory \(C(t)\) that minimizes the area between the horizontal line \(\bar{C}\) and the trajectory \(C(t)\). To minimize this area, the economy should accumulate capital as quickly as possible up to the point that \(F_k = \delta\) to permit the most rapid rise of consumption toward the golden rule steady state. Such a program of capital accumulation might entail totally impoverishing the
present, so that output at time $t = 0$, net of capital depreciation and extraction costs, is fully allocated to investment.

The contrast between the two extremes is striking. For the case $\rho = 0$ and $\eta = \infty$, the present gains at the expense of future generations, who forgo the opportunity to enjoy the golden rule steady state level of consumption, $\hat{C}$. At the other extreme, we have $\rho = 0$ and $\eta = 0$. In this case future generations reap the benefits of golden rule consumption owing to the profound sacrifice of the present generation.

5.5. Golden Rules in Distorted Economies

In Chapter 4 of this study, second best shadow prices were constructed for use in distorted economies to preserve net national product as a true measure of aggregate welfare. The results of that chapter serve as a basis for implementing golden rules in distorted economies as a means to attain sustainable growth.

Two types of economic distortion may be distinguished in principle: those which are irremovable in practice, because of institutional rigidities in the economy, and those that might be eliminated by correcting inadequate market incentives. The former type of distortion belongs to the realm of political economy, while the latter is the subject of information cost and transaction cost microeconomic analysis. Model P3 of Chapter 4 might be considered an example of an economy with an irremovable distortion: a general profits tax is imposed on the economy as a source of revenue for public sector production. Taxes need not be optimal and the public sector might be less productive than the private sector. In contrast, model P2 of Chapter 4 serves as an example of an
economy where inadequate economic incentives, arising from, say, incomplete information or misguided but alterable policies, lead to distortions in one or more markets. For both types of distortion, second best shadow prices are the appropriate market signals to guide economic policy and to support national income accounting in the interests of sustainable growth.

Consider model P3 with the economy-wide tax. As discussed in Chapter 4, the resulting distortion shrinks the feasibility set of the economy and pushes the feasibility frontier inside the frontier that would otherwise have prevailed without the distortion. Second best shadow prices in this case can be taken as producer prices; they are based on tangency conditions with respect to the new feasibility frontier. For model P2, however, the distortion generates movement of the economy along the prevailing feasibility frontier, but away from the optimal point at each time \( t \). Resulting second best shadow prices are not based on tangency conditions, but rather on weighted averages.

A second best golden rule can be derived by considering the necessary arbitrage conditions that emerge from the solution of problem P2 of Chapter 4:

\[
\eta \frac{\dot{Y}}{Y} = F_K^{-}(\delta + \rho) \tag{46a}
\]

and

\[
[F_R - \theta] = \frac{1}{[\gamma \tau + (1 - \tau)][F_K - \delta]} \left( \dot{F}_R + [F_R - \theta]G'(X) - \theta'(X)G(X) \right) \tag{46b}
\]

\[
+ \frac{\gamma \tau}{[\gamma \tau + (1 - \tau)]^2} \cdot \frac{\theta'(X)R}{[F_K - \delta]}.
\]

For this particular model, \( Y \) is the sum of consumption from private output and that from
public output. In the steady state, $\dot{Y} = \dot{F}_R = 0$, $F_K \delta = \rho$ and $R = G(X)$. Substituting into equation (46b) and rearranging terms yield

$$[F_R - \theta] \left[ \frac{G'(X)}{\gamma \tau + (1 - \tau)} - \rho \right] = \frac{\theta'(X)G(X)(1 - \tau)}{[\gamma \tau + (1 - \tau)]^2}. \quad (47)$$

Expression (47) is a modified golden rule for a tax distorted economy with renewable resources. Setting $\rho = 0$ yields a second best golden rule:

$$[F_R - \theta] = \frac{\theta'(X)G(X)(1 - \tau)}{[\gamma \tau + (1 - \tau)]G'(X)}. \quad (48)$$

The analysis of this model in Chapter 4 concludes that with rising extraction costs, the tax induces overharvesting of the resource. The primary reason is that extraction costs are netted out of the economy’s output in the computation of the generalized profits tax. The prospect of a tax "write-off" in the current period serves as an incentive to harvest more now. The overharvesting that results tends to drive up the unit extraction cost that prevails in the steady state. Comparative static analysis of Chapter 2 shows that the golden rule steady state level of consumption declines as steady state unit extraction cost is increased. Therefore, the golden rule steady state in this economy is second best.

For the second type of distortion, i.e., that which could potentially be eliminated by correction of market incentives, the first best golden rule steady state remains an attainable goal. Required specifically are market signals that induce economic activity to correct the distortion. Those signals are precisely the second best shadow prices derived in Chapter 4 for a constrained first best economy. Use of these second best shadow prices as a basis for policy, cost-benefit analysis, and national income accounting
will tend, over time, to move the economy from the constrained optimum and along the prevailing feasibility frontier at each time $t$ to the first best optimum. The economy can then "ride" the optimum trajectory toward the first best golden rule steady state.

5.6. Concluding Remarks

This study has attempted to put the concept of sustainable growth on a firmer theoretical foundation. Drawing on the fields of neoclassical growth theory and public economics, this study has made several contributions toward this objective. First, the study derives, under relatively general conditions, golden rules and modified golden rules as a basis of optimal capital accumulation and natural resource management in growing economies. Next, the study develops second best shadow prices for cost-benefit analysis and national income accounting in economies with market distortions. We show, in particular how second best shadow prices enable net national production to serve as a true measure of social welfare.

To resolve some of the ambiguity in the literature concerning second best shadow prices, the study emphasizes important distinctions in the types of distortion that may prevail in an economy. Distortions that are irremovable because of rigidities in the economy are distinguished from those that may, in principle, be eliminated by adjusting market incentives. Also, distortions that move the economy inside the feasibility frontier are distinguished from this that move the economy along the frontier but away from the optimal point. These distinctions help clarify the relationship between golden rules and second best shadow prices in the pursuit of sustainable growth.
An especially controversial issue in the sustainable framework literature is that of utility discounting over time. Ramsey claimed that it would be "ethically indefensible to discount utilities of future generations." At the same time, compelling arguments can be made for the existence of personal discount rates that reflect impatience in the intertemporal preferences of members of society. Through use of an overlapping generations model incorporating distinct personal and generational discount rates, the study makes the case that the generational discount rate may be legitimately set at zero in models of aggregate growth. This result added further support to the use of golden rules as a means to achieve intergenerational equity as well as economic efficiency.

The findings of this study are derived from a general model of natural resources that treats renewable and nonrenewable resources as special cases. Extraction cost is assumed to rise as the resource is depleted until an exogenously fixed backstop price is reached, at which time the backstop technology is activated to provide a substitute resource. No technological change is considered, and the population growth rate, \( n \geq 0 \), is taken to be exogenous. This basic model can be readily extended to address more complex situations. For example, if amenity value is assigned to the natural resource, the resource stock, \( X \), can be included in the utility function as an aggregate good (see Krautkraemer (1985) for a discussion of resource amenities and preservation of natural environments).

Pollution, in addition to the management of natural resource stocks, is emerging as an important issue in the sustainability literature. The linkage between growth and pollution has been explored by several authors, including Keeler, Spence, and
Zeckhauser (1971), d’Arge and Kogiku (1973), Heal (1982), and Tahvonen and Kuuluvainen (1993). In the context of our general model, pollution can be addressed by adding pollution discharge, $D$, as an input to production, where we assume that $D$ has a well-defined marginal product, $F_D$. Some of the economy’s output may then be allocated to social abatement of pollution, $A$, at unit cost $\Phi$. The basic dynamic equation for capital accumulation now becomes

$$\dot{K} = F(K, R, D) - \delta K - \theta R - \Phi A - C.$$ 

If, in addition, we wish to consider production discharge as contributing to a stock pollutant (e.g., greenhouse gases), we can include a dynamic equation governing the concentration of the pollutant, $M$, in the environment along the lines of Nordhaus (1991):

$$\dot{M} = \alpha(D - A) - \beta M,$$

where $\alpha$ and $\beta$ are parameters. Untreated emissions contribute to the buildup of the stock pollutant, while growth in the stock is diminished by a natural decay process.

A comprehensive survey of the future research agenda in the area of sustainable development is given in Toman et. al. (1993). Included in the research agenda is a consideration of how endogenous growth theory could illuminate the sustainability debate. In terms of our general model, endogenous technological change, induced by rising resource prices as stocks are depleted, would lead to reduction in the backstop resource cost, $\phi_b$, thereby increasing the modified golden rule and golden rule steady state levels of per-capita consumption. Aspects of endogenous population growth could also be
incorporated in the model. A dynamical systems representation of population and the environment, such as that discussed in Nerlove (1991), is one possible approach.

Increasing the complexity of models, however, is not likely to change the basic conclusions of this study. Ultimately, the elements of sustainable growth are efficient use of the economy's resources and stewardship for the future. Sustainability is best achieved, not by the imposition of artificial constraints on economic growth, but by prudent policies based on golden rules of capital accumulation and resource management. Policies compatible with the golden rule enable economies to travel the path discovered by the pioneers of sustainability over 50 years ago - Frank Ramsey and Harold Hotelling.
Notes

1. An exception to monotonic decrease in the value of costate variables over time is the case $U(C) = C$. In this example $\lambda^*_t = 1$, for all time $t$.

2. Weitzman (1976) distinguishes this from the maximum *hypothetical* level of consumption at time $t$, which is represented by net national product.

3. This utility function is not formally a member of the family of functions $U(C) = -C^{-(\eta+1)}$, where $\eta > 1$. Nonetheless, the elasticity of marginal utility is zero, and we write $\eta = 0$.

4. This result was established for the case of an economy with overlapping generations of individuals who live to age $N$, each of whom has identical preferences specified by utility function with constant elasticity of marginal utility. However, the result can be shown to go through for other functional forms, for example, $U(c) = -(1/\alpha)e^{-\alpha}$. 
BIBLIOGRAPHY


