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Congestion and bus frequency

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CONGESTION AND BUS FREQUENCY

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ABSTRACT

This dissertation investigates the issues of bus frequency, modal split, bus service subsidy and bus size in the context of a generalized cost steady-state model. In the literature on transportation, these types of models are often applied to peak period road use, yet have been less than comprehensive in including costs arising from congestion.

In this dissertation a steady-state model is developed and its results critiqued. Then, five sources of congestion are detailed and incorporated into a reconstructed steady-state model. The results of this model are analyzed with regard to the above issues and compared to the steady-state model.

The major conclusion drawn from this inclusion of congestion costs is an obvious one--optimization is consistent with relatively high speeds, indicating very little congestion. It is further concluded that second-best results can closely approximate first-best results, and that smaller buses can generate similar results at much lower subsidy levels.
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LIST OF ABBREVIATIONS AND SYMBOLS

When two abbreviations are listed, the second one refers to the abbreviation used by the computer programs. These programs appear in the two appendices.

$\alpha, A$ - Weight for bus passenger waiting time value.
AOC - Automobile operating costs.
ATC - Auto traveler time costs.
B - Bus company costs, per bus, per hour.
BC - Bus holding capacity.
BOC - Bus operating costs.
BPD - Bus passenger peak duration time.
BTC - Bus passenger time costs.
C - Traffic cycle length.
$c_a, CA$ - Auto running costs.
$C_a$ - Auto operating costs per auto traveler.
CF - Correction factor for fraction of time the road is used.
d, C - Parameter used in calculating $t_a$.
D - Total bus caused delay.
DD - Dispersal delay.
DR - Dispersal rate.
$\epsilon, E$ - The time it takes passengers to board, or exit, a bus.
f, F - The passenger car equivalent, pce, of a bus.
$F_a$ - Auto passenger toll, per mile.
$F_b$ - Bus passenger fare, per mile.
$\gamma, G$ - The number of stops a bus makes while a member of a specific platoon of vehicles.
h, H - Time average passenger spends walking to the road where the bus stop is located.
h_0, HO - Parameter used in calculating $c_a$.
h_1, HI - Parameter used in calculating $c_a$.
h_w, HW - Fraction of the bus headway that the average bus passenger waits for a bus.
IMC - Infrastructure maintenance costs, per mile, per vehicle.
K - Capacity, in the steady-state model; density of vehicles per mile in traffic flow model.
$K_j, KJ$ - Gridlock density per mile.
\( \lambda, D \) - Time a bus spends maneuvering into, and out of, a bus stop.
\( L \) - The number of lanes on the roadway, all one-way.
\( M \) - The average journey length of each traveler.
\( MC_a \) - The auto costs to the marginal traveler.
\( MC_b \) - The bus costs to the marginal traveler.
\( u \) - Average number of passengers per bus stop.
\( \eta, ET \) - Integer value of VD/DR.
\( \eta', ETP \) - Integer value of \( \frac{1}{2} \)VD/DR.
\( N \) - Travelers entering, and exiting, the mile per hour.
\( n_a, NV \) - Average number of travelers per automobile.
\( N_a, NA \) - The number of auto travelers per mile.
\( N_b, NB \) - The number of bus passengers per mile.
\( P \) - The fixed journey costs of using an automobile.
\( pce \) - passenger car equivalent.
\( PD \) - The duration of the peak period.
\( \Psi, PSI \) - The number of platoons of vehicles per bus.
\( Q \) - Flow of vehicles, per lane, per hour.
\( Q^*, QS \) - Desired flow of vehicles, per lane, per hour, when it exceeds the maximum allowable.
\( Q_m, QM \) - Maximum flow allowable, per lane, per hour.
\( r, R \) - The average time a bus spends dropping off, and picking up passengers at a bus stop.
\( RLD \) - Red light delay.
\( t_{0}, T(1) \) - Parameter used in calculating \( t_a \).
\( t_{0}, TI \) - Parameter used in calculating \( t_a \).
\( t_a, TA \) - Auto travel time, per mile.
\( t_b, TB \) - Bus travel time, per mile.
\( t_g, TG \) - Green phase length.
\( t_r, TR \) - Red phase length.
\( t_w, TW \) - Average bus passenger walking and waiting time.
\( U \) - Speed.
\( U_a, UA \) - Base speed derived from traffic flow.
\( U_f, UF \) - Free-flowing speed.
$U_w, \, UW$ - Walking speed.

$V_N, \, VN$ - Highest hourly value of time.

$V_a, \, VA$ - Average value of time for auto travelers.

$V_b, \, VB$ - Average value of time for bus passenger.

$V_D$ - Number of vehicles delayed by a bus stopping.

$X$ - The frequency of buses per hour.

$Y$ - The number of bus stops, and lights, per mile.

$Z$ - The total resource costs of travel for one mile, per hour.
CHAPTER 1
A REVIEW OF THE LITERATURE

1.1 The Nature and Purpose of this Study

The congestion of urban roadways during peak use is a signal that the resource is not being efficiently utilized. The general economic prescription is to price the road to promote its efficient use. Where there are only two modes of travel available, a car or a bus, efficient pricing will generate an efficient modal split among travelers. One common approach to finding this optimized modal split is through the minimization of the generalized cost of travel. A key characteristic of generalized costs is that the time value of travelers is included. One methodology that has been used to minimize these costs is to construct a steady-state model, where the characteristics of a complex transport network are aggregated and averaged into a single unit length of the road. When costs are minimized along this road segment, they are minimized along all road segments in the system.

However, the charging of efficient prices for road use seems a politically unlikely outcome. As a consequence, these steady-state models have been used to find cost minimizing optimizations under a variety of pricing-related constraints. The results of these types of constrained
optimizations are dependent on the way in which congestion effects are, or are not, included in these models. As our point of focus is on peak period use of urban roadways, it will be argued that congestion effects must be adequately identified and included in a steady-state model.

It is the purpose of this dissertation to construct a steady state model that incorporates congestion effects. Specifically, five sources of congestion will be modeled: excess demand for the road, excess demand for the bus, bottleneck effects when a bus stops in a traffic lane to pick up passengers, a bus passenger's effect on bus speed, and the effect of an additional vehicle on traffic flow.

The results of this model will be discussed with regard to three questions. First, how divergent are the results when this model uses a constraint on road pricing and deficit levels from the first-best results, based on the principle of marginal cost pricing of road use? Second, how do the results from this model when using 12 seat minibuses differ from those when using the 55 seat standard bus? Third, how have the results of this model, where congestion effects are explicitly costed out, differ from the results of a more generic steady-state model?

In this chapter, we will review how congestion effects are modeled by various authors using a steady-state model. We will begin by reviewing the problem of congestion and how generalized cost models, where traditional producer
costs are added to consumer time values, are used to deal with this problem. Then we will review the nature of the steady-state model which is an application of generalized costs.

Many authors have used steady-state models to find optimal bus frequencies. We will review a range of these models with regard to how congestion is included in the costs of travel.

In Chapter 2, we will develop a steady-state model that represents the maximum extent to which congestion effects are included. The optimization results will be given and discussed.

In Chapter 3, we will critique the results of the steady-state model. It will be claimed that the unsatisfactory results of this model are due to an unsatisfactory accounting of congestion-related costs. It will be noted that congestion effects are sometimes undercounted, or uncounted, while at other times they are overcounted. A reconstructed steady-state model will be developed specifically to include the costs of congestion.

In Chapter 4, the results of the reconstructed model will be presented. An analysis of these results will follow, with particular focus on costs, modal splits, traffic speeds and bus service deficit levels. A comparison of these results with those presented in Chapter 2 will indicate how the inclusion of congestion
has affected the optimizations. Finally, the model will be subjected to a sensitivity analysis.

Chapter 5 will summarize the focus and results of this dissertation and present some conclusions.

1.2 The Congestion Problem and Road Use Underpricing

Congestion is an example of what economists call an external cost, or a negative externality. In the provision of a good or service, external costs arise when one person's decision raises the costs to others who are not a party to the decision.

This problem has been described by many authors, including Layard and Walters [1978] and Mohring [1976]. If we treat transportation as a factor used to obtain various objectives, optimization of our objectives would require minimizing the costs of this factor, subject to various constraints. As a simplification, we can treat time as the only cost involved in transportation. Beyond a minimal level, additions to the existing traffic stream will cause average speeds to drop, thus increasing travel times. The factor then experiences rising costs with respect to the number of vehicles using the road.

When another journey is made, the new driver causes an increase in journey time for all current drivers, as the average travel time rises. The marginal social cost of this journey is the sum of these additions to travel
time by current drivers plus the travel time of the entering driver. The driver will decide whether to take this journey based on the private cost they face, which is the average travel time. With the marginal social cost greater than the private cost, the number of journeys taken will exceed the optimal number.

The existence of congestion is a signal that the resource, in this case the road, is not being efficiently utilized. Fundamentally, a negative externality can be reduced, if not completely eliminated, through proper pricing of the resource. It is rare, however, that any tolls are charged for roads, and, if so, they are likely to be solely for the purpose of financing the construction of the road, not a "market" pricing of the resource. As a consequence, there is overconsumption of road services, due to the implicit subsidy given to auto travelers.

One early estimate of the underpricing of road use was done by Walters [1961]. He analyzed data from a variety of sources covering different urban U.S. environments. He advanced a mixture of pricing proposals designed to eliminate the divergence between social and private costs. These proposals included mileage taxes, upwards of 15 cents a mile during the peak period, and additional gasoline taxes, with the minimum value given of 30 cents per gallon.
Estimates made by Vickrey [1963] on data from the Lincoln tunnel in New York City showed that "marginal congestion costs" were about five times greater than private, or average, costs. He points out that by pricing road use, the purpose is not to finance the road service, but is rather to promote its economy in use. He estimates that the sum of all taxes and tolls paid by motorists amount to about one-third of real economic costs of urban streets.

In a later paper, Vickrey [1968] constructs a model where roads are provided in competitive environments. He finds that peak period pricing would occur in the short run as a congestion alleviation charge. However, it would be just sufficient to cover the long run cost of providing the road.

A similar type of model for the long run peak load pricing and investment of highways was applied to the San Francisco area by Keeler and Small [1977]. For central city highways they found that the optimal toll per automobile per mile would lie between 6 and 35 cents. This represents a substantial subsidization of commuter auto traffic given their estimate that the typical automobile incurred 1.15 cents per mile in use-related charges.

Using data from Minneapolis, Mohring [1979] estimated that a marginal cost toll for urban arterials would be approximately 70 cents per vehicle per mile. Yet, he
claims that the cost travelers pay, as implied by gasoline
taxes, amounts to about 1.6 cents per vehicle per mile.

If road use is explicitly priced, we will observe
an optimal modal split between autos and buses. When
the relative price of auto travel is lowered, as when
the service of the road is unpriced, bus patronage will
fall. With a decline in revenues, the firm must reduce
service, or raise fares, or both. This, in turn, raises
the relative cost of bus travel to the bus passenger.
As a consequence, there is overconsumption of road services
by auto travelers, and equilibrium levels of mass transit
provision will be unable to eliminate congestion. In
order to better compete against the automobile, mass
transit service levels will have to be increased and fares
either left unchanged, or lowered. This means that the
service will have to be subsidized.

As a second-best justification, Mohring [1979] argues
that if we take the underpricing of road use as given
and, essentially, immutable, we may still achieve the
optimal modal split by lowering the relative price of
bus transit through increases in bus service.

As a practical matter, policymakers, at all levels
of government, have focused on mass transit as a way to
reduce congestion. Most commonly, this has meant the
use of buses.¹ If travelers can be induced, or forced,
to take a bus, then vehicular flows will decrease, reducing congestion and raising traffic speeds.

However, mass transit seems to have had little impact on lessening the problem of congestion. In 1987, mass transit's share of journeys to work was only 6% of the total, down from 12% in 1960.\(^2\) Mass transit systems are generally legal monopolies, and if not operated outright by local governments, heavily regulated by them. Passenger fares cover only a portion of operating costs, and none of the capital costs of these systems.\(^3\) Consequently, subsidies have been necessary to keep the service operating. In 1987, mass transit systems were subsidized at a rate of approximately $1000 per rider.\(^4\)

1.3 Generalized Costs

When road use is not priced, it is likely that, without subsidization, bus service will be underprovided. We can use a cost minimizing model to determine what the efficient level of bus service is going to be. The common approach is to use some form of a "generalized cost" model. This type of model monetizes user inputs and treats them equivalently with producer inputs. In other words, we include not only the operating costs of autos and buses, but also the time costs of the travelers themselves, referred to as "user costs." As Mohring [1972] points out, the traveler performs not only a consuming role in
The provision of the transportation service, but also a producing role. Without the input of the traveler's time, there is no transportation produced.

The importance of these user costs is noted by Jansson [1984] in that transportation is not storable. Its production and consumption occur simultaneously. This makes the choice of production technique crucial, as it directly affects user costs.

The result here is that user costs and producer costs are substitutable. When determining the optimal modal split, this approach can be paraphrased as follows: Increase bus service, raising producer costs, as long as there is an equal, or greater, decline in user costs.

1.4 Steady-State Modeling

The minimization of a generalized cost function can be employed in a range of specific methodologies. At one extreme is a detailed simulation model of a transport network. This has the advantage of characterizing a specific real-world environment. A disadvantage to this type of modeling is that the conclusions drawn can be applied elsewhere only with a great deal of uncertainty.

At the other extreme is a steady-state model, which creates an infinite and uniform transportation network and minimizes costs along one small segment. This has the advantage of being more generally applicable, though
having no real-world environment against which it can be tested.

Our focus will be on the use of a steady-state model to minimize costs and optimize bus frequency. This model is set in a major urban roadway context. There are four important elements to this model. First, demand for trips is perfectly inelastic with regard to price. Here, price refers to explicit, out-of-pocket costs of travel plus the individual's value of their time input to the journey.

Second, travelers all face the same explicit costs. They differ only in the values they place on their time.

Third, there are only two modes of travel, auto or bus. The latter is a consistently slower mode. So, as the total cost of a trip changes, the only effect is to change the modal split among travelers.

Fourth, the artery is an urban one-way street. All trips are the same length, and the origins and destinations of travelers, as well as the traffic signals and bus stops, are uniformly distributed along the street. Signals are presumed synchronized and traffic interruptions are excluded, so as to allow for the smooth flowing of traffic.

A city that would incorporate these characteristics would have residences and businesses uniformly distributed and integrated along the street. There would be no central business district, nor any purely residential community. This is naturally unrealistic of urban areas. However,
insofar as basic traffic conditions in real urban environments can be reasonably represented in this model, it can be used to generate meaningful results. Also, as the purpose is to minimize travel costs and derive an optimal modal split along the street, this need only be done with respect to a single mile. If one mile is optimized, all miles are.

The use of a steady-state model to optimize a traffic network, especially insofar as observing the effects on pricing and bus frequency, has been done by Boyd, Asher and Wetzler [1978], Walters [1982], Mohring [1972, 1979, 1983], Jansson [1980], Oldfield and Bly [1988] and Talley [1989]. In the next section, we will examine how congestion effects have been identified and included in the models of these authors.

1.5 Congestion Effects

By assumption, some congestion effects will not manifest themselves in the steady-state model. For example, when entering travelers join the traffic stream, or exiting travelers leave it, their turning maneuvers, deceleration and acceleration will likely have a downward effect on speeds. Yet, these effects are omitted, largely because of the difficulty in modeling them, and the relatively minor benefits such a sophisticated model would generate. Other congestion effects will be much more
important, and so have been incorporated, to a varying extent, in the models we will consider.

We will focus on five types of congestion effects: the excess demand for the use of the road, the excess demand for the use of the bus, the bottleneck effects of a bus stopping for passengers, the effects on bus speed of a change in the number of passengers and the effects on vehicle speeds by a change in the number of vehicles. While there will be additional effects that are ignored, it will be argued that it is these five types of congestion that account for the bulk of real congestion observed.\(^6,7\)

The order in which we will discuss these models is loosely based on the sophistication of their underlying speed-flow relationship. It is this relationship that would show how the addition of vehicles to the traffic flow will affect travel speeds. This is the fifth type of congestion that we are interested in. We will proceed from models with simple speed-flow relationships to models with much more sophisticated formulations.

The steady-state model used by Boyd, Asher and Wetzler [1972] calculates the peak-period travel generalized costs without any explicit speed-flow relationship. Instead, various modes are considered, from rail to jitneys, where each has a corresponding average speed, exogenously determined. For different flow rates and under different forms of transit, costs are derived
and compared with one another. This model presumes that "street capacity is rationed to achieve fluid traffic conditions." As a consequence, there is no consideration of congestion due to excess demand for the road nor to bus bottleneck effects. This model also constrains the number of passengers to insure that this does not exceed bus capacity and ignores the effects that an additional bus passenger has on travel time for other passengers.

Essentially, this model has no explicit accounting of congestion effects. However, through the use of average speeds an implicit costing of congestion is included. These average speeds come from industry sources and are based on real world observations, which reflect congested urban environments. The result is that for any one mode there is no allowance for the trading off of higher service levels for lower congestion.

In Mohring [1972], a steady-state model is used only to optimize bus frequency. In fact, the model is made up of only bus company costs and bus passenger costs, while all auto-related costs are excluded. This model also utilizes a constant speed, exogenously determined. Consequently, there are no congestion effects due to excess demand for the road, nor due to an increase in the flow of vehicles, nor due to the bottleneck effects of the bus.

This model does not account for any excess demand for a bus, though there is no indication that such a result
actually occurred. There is an explicit accounting of an additional bus traveler's effect on delaying passengers already on the bus. This delay is counted as the time it takes to maneuver to a stop, load passengers, and move away, and is added into the travel time for the bus, so affecting the time costs of other passengers.

The steady-state model used by Jansson [1980] mirrors the variation used by Mohring [1972]. It also aggregates only bus-related costs and ignores auto costs. By calculating the generalized costs of travel, he, too, optimizes bus frequency, where flow speeds are constant, excess demand for the road is ignored, bus stopping bottleneck delays are not calculated, but that the effect of additional bus passengers to the delay of current passengers is included.

The model used by Jansson also does not account for any excess demand for the bus. Indeed, with regard to his simplified optimal frequency formulation, almost identical to the one used by Mohring [1972], he states that it is limited in that it implicitly assumes "that the total capacity of the buses on the route is always sufficient to meet the demand." As it turns out, the optimizations, run over what he considered relevant ranges of passenger flows, yielded small to large amounts of excess bus capacity. He then introduces a size variable into his formulations to optimize bus size as well as
bus frequency, finding that the system yields cost savings with smaller buses.\textsuperscript{10}

In a very simplified version of the steady-state model, Walters [1982] looked at minimizing the generalized costs of travel by focusing on the relationship between costs and bus size. This model also used a constant speed for traffic flow, ignored all auto-related costs, and did not allow for delay effects caused by the addition of another bus passenger. There was no excess demand for buses allowed as the model required fixed bus loads.

In applying this model to data from Washington, D.C. and Minneapolis, Walters obtains optimal bus sizes of about one-quarter the standard 55-seat bus. The optimal frequency of these smaller buses, correspondingly, would be four times greater than was observed in these cities. This led Walters to address the issue of congestion. It might seem that the presence of these more numerous smaller buses would more adversely affect the flow of traffic and lead to high, and uncounted in this model, congestion costs. While he offers a variety of logical arguments to support an opposite view, he concludes that "[c]ongestion and minibuses remain unfinished business."\textsuperscript{11}

In the steady-state model used by Talley [1989], only bus-related costs are modeled, and all auto-related costs are ignored. The effect of an additional bus passenger on bus speeds, and so on other passenger's travel
time, is modeled in the same way as was done by Mohring [1972] and Jansson [1980]. Furthermore, there is still the implicit assumption that bus capacity is sufficient.

In a rather unusual treatment of the speed-flow relationship, Talley incorporates bus speed as a choice variable, on the part of the service provider, subject to optimization. Where the firm has a fixed operating subsidy and passengers evaluate their total costs, the fare plus their time value, when deciding to take the bus, the bus company trades off frequency and speed in providing their service. While it seems clear that a bus could travel slower than the prevailing traffic, creating congestion that is not included in this model, it is left unexplained how a bus could travel faster.

In the steady-state model utilized by Oldfield and Bly [1988], only bus-related costs are calculated. However, this model includes a variable that is designed to account for the congestion effects due to the presence of buses in the traffic stream. The value of this variable is proportional to the total bus-kilometers operated by the bus company. There is an offsetting benefit to operating more buses in that traffic volume is reduced and speeds rise if these buses attract travelers away from automobiles. This benefit is linearly related to the number of bus passengers. The net effect of these
two calculations is to approximate the bottleneck effect of buses in the traffic flow.

This model also calculates a congestion cost that arises when there is excess demand for a bus. As bus loads rise, the probability of being passed by a full bus also rises. So, the average wait of bus passengers is calculated as a function of the average bus load.

This model also calculates the effect that an additional bus passenger has on the travel time of other passengers. This is done by including the time it takes to load, and unload, a passenger.

There is no explicit speed-flow relationship, as there is no explicit accounting of auto traffic. The model does have a congestion cost component, as noted, that infers an effect on autos, but this is determined solely by the level of bus service. Consequently, there is no accounting for any excess demand for the road.

In Mohring [1979], a much more sophisticated variant of the steady-state model is used. In this model, auto-related costs and bus-related costs are aggregated into a generalized cost function. An inverse speed-flow relationship is specified that allows for an accounting of congestion-related costs when another vehicle, bus or auto, is added to the traffic flow. While this will then factor in the increasing costs when traffic flows rise, due to falling speeds, the model presumes that the
capacity of the road is not exceeded, and so does not account for the congestion costs that would arise when the demand for the road exceeds its capacity.

This model also had no constraint with regard to bus capacity. However, the optimized results generated bus loads below the capacity of a bus and so were not necessary. This result is very much like the results of Jansson [1980] previously discussed.

Like the model developed in Mohring [1972], this model calculates the delay effect an additional bus passenger has on other bus passengers. This is computed by adding in the time it takes a bus to maneuver toward a bus stop, pick up, or drop off, a passenger, and maneuver away from the stop.

There is no explicit congestion cost due to the bottleneck in traffic flow created when a bus stops. However, in the calculation of traffic flows, the bus is counted as if it had the effect of 3.5 autos, referred to as its passenger car equivalent (pce). This exceeds the value necessary to account for the physical size difference of the bus, and is used as a rough approximation for the congestion effects caused by buses.

This model is used to find optimized bus frequencies under a variety of conditions. Among those conditions is the existence of road use pricing and fixed deficits for the bus service.
In Mohring [1983], the model just described is further extended to the case of minibuses. Again, road use pricing is considered as well as fixed bus deficit amounts. Additionally, a comparison is made between the use of the large bus and the minibus with regard to which generates lower costs and lower subsidies. The issue of the congestion difference between the two bus types is dealt with solely by adjusting the pce for a minibus to 2.75. The results were quite sensitive to this parameter and Mohring concludes that the issue of congestion and minibuses remain "unfinished business."12

In developing the minibus scenario, Mohring used a capacity constraint for bus loads. This prevented any excess demand for these buses, and so this type of congestion would not, by assumption, arise.

1.6 Summary

The existence of congestion along urban arterial roadways during peak periods is a signal that the resource has not been efficiently priced. Empirical investigations show that this underpricing is quite substantial. However, it is generally conceded that efficient pricing is an unlikely option. A second-best solution is to encourage, and subsidize, mass transit alternatives.

To explore the issue of the efficient provision of mass transit--and we will limit our concern to buses--one
A common approach is to construct a generalized cost model. This type of model counts up the costs of time as well as the cost of producer inputs. We can derive an optimal level of bus frequency by finding that level which minimizes these generalized costs.

One particular way in which this is done is through the use of a steady-state model. This type of model creates a single road segment and then identifies its attributes, based on average traffic conditions. When costs in this mile are minimized then the whole system is optimized.

An issue that arises in the context of second-best solutions is how well congestion effects are modeled. This is important because these second-best situations are likely to retain more congestion than a first-best solution.

We identified five types of congestion that are likely to have important consequences on cost minimization. They are the excess demand for the use of the road, the excess demand for the use of the bus, the bottleneck effects of a bus stopping for passengers, the effects on bus speed of a change in the number of passengers, and the effects on vehicle speeds by a change in the number of vehicles.

In this chapter, we have looked at how a variety of authors have dealt with congestion in their steady-state models. Some are quite sophisticated and others are not.
None has been thorough in capturing the effects of our five types of congestion.

In the next chapter, we will develop a detailed steady-state model. This model is a reproduction of that used by Mohring [1983]. It is certainly not representative of other models, but is rather an example of the elite in terms of reckoning with congestion effects.
Notes

1 Pucher, et al. [1983] p. 160, states that rail transit accounts for 27% of mass transit users, inferring that buses account for approximately 70% of mass transit users. Meyer, et al. [1981] reported that, in 1978, buses carried 71% of all urban mass transit users.


5 For some examples of cost-minimizing computer simulation models, see Andersson, et al. [1979], Evans [1988], Glaister [1985] and Glaister [1986].

6 Vickrey [1968] outlines 6 types of congestion with regard to general traffic along a road. His network and control delay and general density delay do not not affect the steady-state, by assumption. The other four types are approximated in this dissertation by the congestion due to excess demand for the road, the bottleneck caused by a stopped bus and the delay caused by an additional vehicle in the traffic flow.

7 McShane and Roess [1990] p. 604 distinguishes between "real" and "apparent" congestion, the latter due to mechanical failure, poor signalization, etc., and easily remedied. By assumption, there is no "apparent" congestion in the steady-state model.


10 Jansson [1980] p. 67. Table 6 shows that most buses used in Stockholm, Sweden, where he applied this model, held from 71 to 80 passengers.


CHAPTER 2
THE STEADY-STATE MODEL AND RESULTS

In this chapter, a general steady-state model will be developed with its results given and interpreted. We will review some of the basic assumptions of this model, and then turn to a detailed elaboration of a steady-state model. We will describe the extent to which congestion is included in the model and discuss how the model generates an equilibrium modal split.

There are seven scenarios under which the model will be run. They span the spectrum from explicit road use pricing to a scenario where there are no explicit charges, zero bus fares and complete deficit financing of the bus service. These scenarios will be run under two different bus-size regimes, one being the 55-seat "standard" bus and the other being a 12-seat minibus.

The results of this model will be presented and interpreted with regard to the issues raised by two questions. First, how do the constrained optimizations compare to the first-best, marginal cost pricing result? Second, to what degree do these results change when using the minibus versus the standard bus?

2.1 The Steady-State Model

The steady-state model is a simplified version of a
traffic network. It uses a set of characteristics to describe a segment of the network. This segment is derived to be representative of all segments. Some key assumptions that have been made with regard to this model are that only one-way traffic is allowed, no turning impediments exist to the flow of vehicles, and the traffic signals are synchronized to allow for the free flow of vehicles.

The way in which this model works is to aggregate the total cost of travel, Z, along this segment of the road over an hour, which can be summarized as totaling explicit and implicit costs. The latter represents the time costs of the travelers. Where our route segment is one mile long, Z is the sum of four component costs. They are auto operating cost, AOC; auto traveler time cost, ATC; bus operating cost, BOC; and bus passenger time cost, BTC.

2.1.1 Auto Operating Cost, AOC.

The operating cost of an automobile is the sum of actual running cost, explicit or implicit tolls, and some fixed cost. These cost components are expressed on a per mile basis. Auto operating cost over a mile of the steady-state route, per hour, is the sum of these three costs multiplied by the number of vehicles flowing along this mile, per hour.
Where $N_a$ is the number of auto travelers entering, as well as the number exiting, each mile per hour, $n_a$ is the number of auto travelers per auto and $M$ is the average journey length, in miles. Therefore, the total flow of autos over the mile, per hour, is $MN_a/n_a$.

The auto running cost per mile is taken to be:

$$c_a = h_0 + h_1[\ln(t_a)],$$

where $h_0$ and $h_1$ are given parameters and $t_a$ is the time, in hours, it takes an auto to travel over the route-mile. The calculation of $t_a$ is as follows:

$$t_a = [t_0 + t_1(Volume/Capacity)^d]/60,$$

where $t_0$, $t_1$ and $d$ are exogenous parameters, chosen to generate the number of minutes it takes to travel one mile. The expression is divided by 60 to convert this into hours. Volume is defined as the sum of autos and buses; the latter expressed in passenger car equivalents, pce's, which flow over the mile, per hour. Where $f$ is the pce of a bus, and $X$ is the number of buses traveling the mile, per hour, the volume of traffic is $MN_a/n_a + fX$.

The capacity of the road, $K$, is fixed, here to a two lane street. Where $N$ is the total number of travelers entering, and exiting, this mile per hour, $MN/K=2$ is taken to be representative of peak period conditions. The total traveler flow per mile, per hour, is $MN$, and so capacity is taken to be half this number.
The implied toll per auto, per mile, is denoted $F_a$. The value of $F_a$ is determined by the portion of taxes paid by auto travelers that is dependent upon their miles driven. For example, the more one drives, the more gasoline one purchases, and the more gasoline taxes one pays. These taxes can then be thought of as an implicit toll for road use.

The final component is the fixed cost, $P$, which will be stated as cost per journey. This, theoretically, represents the opportunity cost of using the car. As a practical matter, it is derived as half the daily fixed costs of a vehicle and is exogenously given. As it is to be expressed in per mile terms, this cost component is written out as $P/M$.

Putting these together, auto operating cost, AOC, can be written out as:

$$AOC = (MN_a/n_a)(c_a + F_a + P/M) = MN_a C_a, \quad (3)$$

where we define $C_a$ as the operating cost per auto traveler.

2.1.2 Auto Traveler Time Cost, ATC.

Auto traveler time cost is calculated as the value of the time input by travelers while traveling over this mile of roadway. Of necessity, an average value of time is used, and these traveler costs depend upon the speed at which the traffic flows. These costs are calculated
as follows:

$$ATC = MN_a t_a \bar{V}_a$$  \hspace{1cm} (4)

where $MN_a t_a$ is the total auto traveler time input over this mile, per hour, and $\bar{V}_a$ is the average value of time for auto travelers. Time values are taken to be uniformly distributed from zero to some maximum value, $V_N$. Where all travelers face the same explicit costs, those travelers with higher time values will travel by the faster mode, i.e., by auto. Conversely, those travelers who use the bus, the slower mode, will have lower time values. Where $N_b$ is the number of bus passengers entering, and exiting, this mile, per hour, then $V_N(N_b/N)$ will be the time value that separates the marginal bus passenger from the marginal auto traveler. The average auto traveler time value will be halfway from this marginal value to the maximum time value. The calculation of the average auto traveler's value of time, in dollars per hour, is:

$$\bar{V}_a = V_N(N_b/N) + \frac{1}{2}[V_N - V_N(N_b/N)] = \frac{1}{2}V_N(1 + N_b/N)$$  \hspace{1cm} (5)

2.1.3 Bus Operating Cost, BOC.

Bus operating cost is derived as the product of the number of buses traveling the route mile per hour, $X$, the time it takes the bus to travel a mile, $t_b$, and the average company cost of providing an hour of bus services, $B$. So, we can write:

$$BOC = BXt_b.$$

\hspace{1cm} (6)
Bus travel time per mile, $t_b$, is defined as the sum of auto travel time per mile, $t_a$, plus the time a bus spends stopped while loading and unloading passengers and plus the time spent maneuvering toward, and away from, bus stops:

$$t_b = t_a + \frac{[2N_b \epsilon/X + \lambda Y(1 - e^{-\mu})]}{3600}. \quad (7)$$

As $N_b$ passengers are loading, and $N_b$ are unloading, per mile, then we must account for the time spent in these activities. Where we assume that loading time is the same as unloading time, and where $\epsilon$ is the length of time for this, measured in seconds, then $2N_b \epsilon$ is the aggregate time, per mile, per hour, spent in this activity. This is divided by $X$ to put it into per bus terms.

Where $\lambda$ is the increased time that a bus faces as it travels, due to the deceleration to, and acceleration from, a bus stop, measured in seconds, and $Y$ is the number of bus stops per mile, then $\lambda Y$ is the aggregate time spent maneuvering, if the bus stopped at every stop. The average number of passengers per stop that either want to get on or get off is $2N_b/(XY)$ and is denoted as $\mu$. Where the number of passengers at a particular stop can be modeled by the Poisson distribution, then $e^{-\mu}$ is the probability that no passengers will want to get on, or get off, at any one stop. We can write, therefore, that the probability that a bus will make a particular stop is $1-e^{-\mu}$, and so weight the maneuvering time by this.
As $t_b$ is measured in hours per mile, then the loading and unloading term and the maneuvering term, measured in seconds, are divided by 3600.

2.1.4 Bus Passenger Time Cost, BTC.

The bus passenger time cost over the mile per hour is the sum, respectively, of the value of a passenger's transiting time, and, for those boarding, the value of their time spent walking to the bus stop and waiting there. The latter two time values are weighted more heavily than the former, based on the presumption that these activities involve more disutility.

We can write this cost as:

$$BTC = MN_b t_b \bar{V}_b + N_b t_w a \bar{V}_b,$$

(8)

where $\bar{V}_b$ is the average value of time for bus passengers, taken to be $\frac{1}{2}V_N (N_b/N)$, and $MN_b t_b$ is time spent transiting by all of the bus passengers over the route-mile per hour.

For those passengers boarding over this mile, we weight their average time value by $a$, where its being greater than one signifies that walking and waiting time is more onerous than transiting time. The time spent walking and waiting is given by $t_w$, as follows:

$$t_w = h_w/X + 1/(2YU_w) + h,$$

(9)

which is the sum of waiting time, time spent walking along the arterial roadway to a bus stop, and time spent walking to the arterial roadway from one's origin, respectively.
Where 1/X is the bus headway time, i.e., the time between each bus, and h_w is the average fraction of bus headway that a passenger spends waiting for a bus, then h_w/X is the average time a passenger spends waiting at a bus stop for a bus. We define h to be the average amount of time it takes a passenger to walk from their origin to the road along which the bus travels. The average walking speed is given as U_w so that 1/U_w is the time it takes to walk a mile. Where the distance between bus stops is 1/Y, the closest bus stop to a person arriving at the arterial roadway would be no further than one half this distance, or 1/(2Y). Where passengers are arriving uniformly along the road, the average distance to walk would be half of this, or 1/(4Y).

We should also include walking time at the end of the journey and so should add in the value of this time for the N_b passengers exiting over this mile. We can reason, similarly, that the average distance walked for these passengers would be 1/(4Y). Ignoring any further walking requirement would make the total walking time for the N_b passengers boarding and the N_b passengers exiting as 1/(4YU_w) + 1/(4YU_w) = 1/(2YU_w).

Summing up all these component costs, and aggregating a bit, yields the total hourly traveler cost, per mile, along the steady-state route as:

\[ Z = MN_a (C_a + t_a \bar{V}_a) + B X t_b + N_b \bar{V}_b (M t_b + \alpha t_w) \] (10)
2.2 Cost Minimization and Model Parameter Values

All travelers face the same explicit costs. The only characteristic that distinguishes one person from another is their value of time. At the optimized modal split, the marginal traveler faces identical costs of taking an auto, $MC_a$, or a bus, $MC_b$.

We can write out the $MC_a$ term as follows:

$$MC_a = c_a/n_a + P/(n_aM) + F_a + V_m t_a, \quad (11)$$

where the first term is the auto running cost per mile, per auto traveler and the second is the vehicle opportunity cost, expressed in per traveler, per mile terms. Both of these terms are determined exogenously. The third term, $F_a$, is an explicit auto toll in per auto traveler, per mile terms. The last term is the time value of traveling this mile for this traveler, where $V_m$ indicates that this is the marginal traveler. The value of $V_m$ is derived as $V_N(N_b/N)$.

The bus costs to this traveler are:

$$MC_b = F_b + V_m t_b + \alpha V_m t_w / M, \quad (12)$$

where the first term is the explicit bus fare, per mile, and the second term is the time value of traveling this mile for the marginal traveler. The third term is the portion of the walking and waiting time value we can apportion to each mile traveled.

The value of $F_a$ is derived by calculating the change in costs when an additional auto traveler is added, without
changing the value of $N_b$. Part of this cost is borne by this additional traveler and part by the existing travelers. The cost imposed on other travelers is $F_a$. With reference to equation (10) we can derive that this is:

$$F_a = N_a M [\bar{V}_a (\frac{\delta t_a}{\delta N_a}) + (\frac{\delta c_a}{\delta N_a})/n_a]$$

$$+ (BX + N_b \bar{V}_b M)(\frac{\delta t_b}{\delta N_a}),$$  \hspace{1cm} (13)

where the partial differentiation of $c_a$, $t_a$ and $t_b$ with respect to $N_a$ can be derived from earlier equations.

Likewise, we derive the value of $F_b$ from the marginal cost of adding an additional bus passenger, leaving $N_a$ unchanged. Again, part of this cost will be borne by the additional bus passenger and part will be imposed on others. Referring to equation (10) we can derive:

$$F_b = 2(B + N_b \bar{V}_b M/X)(\epsilon + \lambda e^{-\mu}).$$  \hspace{1cm} (14)

The parameters and their values appear in Table 1. These values are from those used by Mohring [1979, 1983].

2.3 Congestion in the Steady-State Model

This steady-state model accounts for congestion in four ways—three explicitly and one implicitly. The type of congestion dealt with implicitly is that which arises when the desired, or optimal, bus load exceeds the physical capacity of the bus. As a consequence, a constraint is added into the model that prevents such an outcome. In other words, bus loads cannot exceed capacity limits.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>3</td>
<td>Waiting time value weight as a percentage of travel time value.</td>
</tr>
<tr>
<td>B</td>
<td>$25$</td>
<td>Bus company cost per bus per hour when using a 55-seat bus.</td>
</tr>
<tr>
<td></td>
<td>$6$</td>
<td>Bus company cost per bus per hour when using a 12-seat bus.</td>
</tr>
<tr>
<td>d</td>
<td>2.7</td>
<td>Parameter for auto travel time calculation.</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>1.8</td>
<td>The time, in seconds, it takes to load, or unload, a passenger.</td>
</tr>
<tr>
<td>f</td>
<td>3.5</td>
<td>The pce of a 55-seat bus.</td>
</tr>
<tr>
<td></td>
<td>2.75</td>
<td>The pce of a 12-seat bus.</td>
</tr>
<tr>
<td>$P_a$</td>
<td>.015</td>
<td>The implicit auto toll, per mile, per auto, in dollars.</td>
</tr>
<tr>
<td>h</td>
<td>5</td>
<td>The time, in minutes, it takes an average person to walk from their origin to the arterial roadway.</td>
</tr>
<tr>
<td>$h_0$</td>
<td>.3576</td>
<td>Parameter for auto running cost calculation.</td>
</tr>
<tr>
<td>$h_1$</td>
<td>.0895</td>
<td>Parameter for auto running cost calculation.</td>
</tr>
<tr>
<td>$h_w$</td>
<td>.5</td>
<td>The average fraction of bus headway spent waiting for a bus.</td>
</tr>
<tr>
<td>K</td>
<td>1000</td>
<td>Capacity of the artery, in vehicles per hour.</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>18</td>
<td>The time it takes a bus to maneuver into, and away from, a bus stop, in seconds.</td>
</tr>
<tr>
<td>M</td>
<td>5</td>
<td>Average journey length, in miles.</td>
</tr>
<tr>
<td>N</td>
<td>400</td>
<td>Number of travelers originating per mile, per hour.</td>
</tr>
<tr>
<td>$n_a$</td>
<td>1.25</td>
<td>The average number of travelers per auto.</td>
</tr>
<tr>
<td>P</td>
<td>.30</td>
<td>Opportunity cost per vehicle trip, in dollars.</td>
</tr>
<tr>
<td>$t_0$</td>
<td>2.2</td>
<td>Parameter for auto travel time calculation.</td>
</tr>
<tr>
<td>$t_1$</td>
<td>5.8</td>
<td>Parameter for auto travel time calculation.</td>
</tr>
<tr>
<td>$U_w$</td>
<td>3</td>
<td>Average walking speed, in miles per hour.</td>
</tr>
<tr>
<td>$V_N$</td>
<td>7</td>
<td>Maximum value of time for travelers in dollars per hour.</td>
</tr>
</tbody>
</table>
TABLE 1. (Continued)
PARAMETER DESCRIPTIONS AND VALUES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>8</td>
<td>Number of bus stops per mile for the 55-seat bus.</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>Number of bus stops per mile for the 12-seat bus.</td>
</tr>
</tbody>
</table>

\(^{a}\)Parameter values are taken from Mohring [1979, 1983].

One of the three types of congestion that is dealt with explicitly involves the parameter \( f \), which represents the pce of a bus. This value is not based strictly on the size of the bus, but rather incorporates some attempt to measure the contribution to congestion that a bus adds to flowing traffic.

The second type of congestion explicitly accounted for is the delay bus passengers experience when another passenger boards the bus. This is captured by the bus maneuvering term and the bus loading and unloading term.

The third type of congestion that is included in this model, and certainly the most important, is the delay caused by increased road usage. This is the result of the inverse speed-flow relationship. This has been given in equation (2) and is illustrated in Figure 1.

As Figure 1 shows, additional vehicles added to the traffic flow will depress speeds. This lengthening travel time will then be reflected in equation (10), the calculation of total costs, \( z \).
$V = \text{volume of vehicles over two lanes, per hour.}$

$U = \frac{1}{[(2.2 + 5.8(V/1000)^2.7)/60]}.$

**FIGURE 1**

**STEADY-STATE SPEED-FLOW RELATIONSHIP**
What we observe in this relationship can be described by three traffic conditions: uncongested, congested and heavily congested. For a two lane street with a basic capacity of 1000 vehicles per hour, when the volume is less than about 500 vehicles, the traffic speed will be at, or above, 20 miles per hour. This represents a relatively uncongested situation.

Congestion has a serious dampening effect on speeds as volume grows from 500 to 1000, the latter generating speeds of only about 7.5 mph.

We may consider these volumes as showing congested flows at volumes of 1000, or more, the traffic speed is below 7.5 mph, indicating heavy congestion.

2.4 Model Scenarios and Optimization Results

This model is used to minimize the cost of travel, \( Z \), with respect to bus passengers per mile, per hour, \( N_b \), and the number of buses per hour, \( X \). This can be done by setting the partial derivatives equal to zero and solving for \( N_b \) and \( X \). The resulting derivatives are quite convoluted, and to obtain the optimizing values of \( N_b \) and \( X \) an iteration procedure must be used. As a practical alternative, the optimal values are solved for directly, by varying \( N_b \) and \( X \) and observing the resulting cost, \( Z \). An annotated version of the computer program used to generate these results is given in Appendix 1.
There are seven variations of the basic model that have been run under two different bus regimes. The first regime is where the bus is taken to have 55 seats, the so-called "standard" bus, and able to hold 80, or more, passengers. In the second regime we consider the use of a 12-seat minibus, whose capacity is set to 12. When running through these seven models under the minibus regime, the capacity of the bus is used to explicitly constrain the maximum modal split allowed. This was not done for the standard bus, as optimizations yielded bus loads below capacity limits.

The two bus regimes differ with regard to the values of the hourly bus cost, B, the number of bus stops per mile, Y, and the pce of the bus, f, as outlined in Table 1.

In model I, a first-best optimization is derived, where the principle of marginal cost pricing of the road yields explicit congestion tolls levied on auto travelers and bus passengers. The auto tolls are quite high—in the range of 60 cents per mile, per person. The difference between this auto traveler congestion toll and the fare levied on bus passengers is about 55 cents per mile. This difference is substantial, and in the opposite direction of what one would expect to observe in a typical urban environment. That is, it is common to find that auto travelers pay little, or nothing, in explicit tolls,
while bus passengers pay relatively high fares. In fact, the extent to which auto traveler congestion charges can be imputed from gasoline taxes yields a miniscule 1.2 cents per mile.²

We may still generate the first-best marginal cost pricing solution even without charging auto travelers the tolls mentioned. The driving force of the modal split optimization is not the absolute value of these charges, but the relative difference between the amounts faced by auto travelers versus bus passengers. Where auto tolls are constrained to 1.2 cents per mile, charging a bus fare about 55 cents lower, per mile, will generate the same results. This implies not only complete subsidization of the bus service, but also that explicit payments be made to bus passengers to encourage their patronage.

This alternative is unlikely, and so in model II we constrain auto tolls to that value implied by gasoline taxes, and set the bus fare to zero. The resulting optimization yields higher travel costs than in the first-best case, and also a significant subsidy of bus service.

In models IIIA through IIIE, we continue to constrain auto tolls as above, but remove the constraint on bus fares. In these models, we constrain the bus deficit to 100%, 75%, 50%, 25%, and 0%, respectively, of the figure arrived at in model II.
The results of these optimizations are summarized in Table 2. To analyze these results we will refer to the two questions posed at the beginning of this chapter.

Our first question is, how divergent are the constrained optimizations, those of models II through IIIE, from those of the first-best optimization, model I? For the standard bus, the lowest resource cost occurs in model I while the constrained models have costs from 60% to 190% higher. The higher the subsidy provided to the bus company, the lower the optimized resource cost. For the minibus regime, this same pattern was repeated, with the constrained models generating costs that were 75% to 165% above those of model I.

As we move from model I through model IIIE, for both bus regimes, the bus frequency and bus share of travelers declines. The change in modal splits is especially pronounced, falling by more than half from model I to model II. The bus share of travelers in model IIIE, where there is no bus subsidy, is only 10% and 16% of that for model I when using standard buses and minibuses, respectively.

For the standard bus, the bus load shows quite a bit of variation across models. In model II, where bus fares are set to zero, the bus frequency stays relatively high, at 23 per hour, while the bus load falls by more than half, to 26.1 passengers per bus. This result
TABLE 2

STEADY-STATE MODEL OPTIMIZATION RESULTS

2A. 55-Seat Standard Bus Regime

<table>
<thead>
<tr>
<th>Model</th>
<th>I</th>
<th>II</th>
<th>IIIA</th>
<th>IIIB</th>
<th>IIIC</th>
<th>IID</th>
<th>IIIE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$/Trip:</td>
<td>2.41</td>
<td>4.66</td>
<td>3.92</td>
<td>4.25</td>
<td>4.69</td>
<td>5.32</td>
<td>6.92</td>
</tr>
<tr>
<td>Buses/Hr:</td>
<td>25</td>
<td>23</td>
<td>10</td>
<td>8</td>
<td>6</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Nb/N:</td>
<td>.69</td>
<td>.30</td>
<td>.33</td>
<td>.29</td>
<td>.25</td>
<td>.19</td>
<td>.07</td>
</tr>
<tr>
<td>Pax/Bus:</td>
<td>55.4</td>
<td>26.1</td>
<td>66.7</td>
<td>73.2</td>
<td>82.0</td>
<td>75.8</td>
<td>68.7</td>
</tr>
<tr>
<td>Ua(mph):</td>
<td>17.0</td>
<td>5.1</td>
<td>6.2</td>
<td>5.5</td>
<td>4.9</td>
<td>4.2</td>
<td>3.1</td>
</tr>
<tr>
<td>Ub(mph):</td>
<td>9.3</td>
<td>4.4</td>
<td>4.7</td>
<td>4.3</td>
<td>3.8</td>
<td>3.4</td>
<td>2.7</td>
</tr>
<tr>
<td>Fa/Trip:</td>
<td>3.15</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
</tr>
<tr>
<td>Fb/Trip:</td>
<td>.25</td>
<td>0</td>
<td>-.60</td>
<td>-.45</td>
<td>-.25</td>
<td>.05</td>
<td>.70</td>
</tr>
<tr>
<td>Bus Deficit per hr.($):</td>
<td>0</td>
<td>132.2</td>
<td>132.2</td>
<td>99.2</td>
<td>66.12</td>
<td>33.06</td>
<td>0</td>
</tr>
<tr>
<td>Def./Nb:</td>
<td>0</td>
<td>1.10</td>
<td>1.00</td>
<td>.86</td>
<td>.66</td>
<td>.44</td>
<td>0</td>
</tr>
</tbody>
</table>

2B. 12-Seat Minibus Regime

<table>
<thead>
<tr>
<th>Model</th>
<th>I</th>
<th>II</th>
<th>IIIA</th>
<th>IIIB</th>
<th>IIIC</th>
<th>IID</th>
<th>IIIE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$/Trip:</td>
<td>2.39</td>
<td>4.23</td>
<td>4.23</td>
<td>4.50</td>
<td>4.83</td>
<td>5.29</td>
<td>6.32</td>
</tr>
<tr>
<td>Buses/Hr:</td>
<td>154</td>
<td>65</td>
<td>65</td>
<td>58</td>
<td>51</td>
<td>42</td>
<td>26</td>
</tr>
<tr>
<td>Nb/N:</td>
<td>.92</td>
<td>.39</td>
<td>.39</td>
<td>.35</td>
<td>.31</td>
<td>.25</td>
<td>.15</td>
</tr>
<tr>
<td>Pax/Bus:</td>
<td>12.0</td>
<td>11.9</td>
<td>11.9</td>
<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
<td>11.4</td>
</tr>
<tr>
<td>Ua(mph):</td>
<td>18.0</td>
<td>5.5</td>
<td>5.5</td>
<td>5.1</td>
<td>4.7</td>
<td>4.2</td>
<td>3.4</td>
</tr>
<tr>
<td>Ub(mph):</td>
<td>12.7</td>
<td>4.9</td>
<td>4.9</td>
<td>4.6</td>
<td>4.2</td>
<td>3.8</td>
<td>3.2</td>
</tr>
<tr>
<td>Fa/Trip:</td>
<td>2.95</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
</tr>
<tr>
<td>Fb/Trip:</td>
<td>.40</td>
<td>0</td>
<td>0</td>
<td>.10</td>
<td>.25</td>
<td>.45</td>
<td>.80</td>
</tr>
<tr>
<td>Bus Deficit per hr.($):</td>
<td>0</td>
<td>79.45</td>
<td>79.45</td>
<td>59.59</td>
<td>39.72</td>
<td>19.86</td>
<td>0</td>
</tr>
<tr>
<td>Def./Nb:</td>
<td>0</td>
<td>.51</td>
<td>.51</td>
<td>.43</td>
<td>.32</td>
<td>.20</td>
<td>0</td>
</tr>
</tbody>
</table>

$/Trip - Resource cost (Z) per traveler per journey.
Buses/Hr. - Bus frequency of service, X.
Nb/N - Bus share of travelers.
Pax/Bus - Passenger load per bus.
Ua(mph) - Auto speed.
Ub(mph) - Bus speed.
Fa/Trip - Auto toll per journey per auto traveler.
Fb/Trip - Bus fare per journey per bus passenger.
Bus Deficit per hr.($) - In model II this is BOC, while in models IIIA to IIIE this is 100%, 75%, 50%, 25%, and 0% of the value derived in II.
Def./Nb - Bus deficit, per hour, per bus passenger journey.
illustrates how difficult it is to deter auto travelers without imposing congestion tolls, $F_a$, on them. In this model, even with bus fares at zero, a substantial "service bribe", in the form of low bus loads, is necessary even to generate a bus share of 30%. For the minibus there was no such corresponding large drop in loads.

Where models IIIA through IIIE allow for negative fares to be offered, we observe that this occurs only under the standard bus regime. For both types of buses, these fares rise as the deficit is reduced.

Our second question is, how do the minibus results compare with the standard bus results? With regard to resource costs, the results are quite mixed. Costs are lower for the minibus only for models I, II, IIID and IIIE, and this difference is small to insignificant.

The modal splits generated by the minibus regime are higher in all models with the absolute difference being most pronounced in model I. Here, the minibuses carry 92% of the travelers.

With regard to the subsidy generated in model II, the minibus regime value of $79.45 is only 60% of the value for the standard bus. To generate equivalent modal splits, the minibus deficit is an even smaller proportion of the standard bus deficit. For example, with a deficit of $66.12, the standard bus, in model IIIC, generates a 25% bus share of travelers. The same modal split is
observed using minibuses in model IIID where the deficit is only $19.86, just 30% of the standard bus deficit.

2.5 Summary

In this chapter, we have outlined the characteristics of a steady-state model and identified various issues that we wish to address with regard to the optimization of this model. Then, we turned to the component parts of this model, identified as the operating costs of buses and autos, and the traveler time value spent on this road segment. The model generates an equilibrium modal split by finding where the marginal cost of an auto trip is equal to that of a bus trip. All explicit costs are the same for everyone, while it is just the value of time that distinguishes individuals apart from one another. The distribution of time values is held to be uniform from zero to some maximum.

The particular model developed here is representative of others insofar as the characteristics of the steady-state are concerned. However, it is more sophisticated than other models in that it uses an inverse speed-flow relationship rather than a constant speed-flow relation. This allows for an explicit accounting of the congestion costs associated with higher traffic volumes. In addition to this, there are a few other ways in which this model takes congestion effects into consideration.
There were seven scenarios of this model run under two different bus size regimes. These varied from a first-best optimization where road use was priced according to marginal cost principles to optimization when the road is not priced and the bus service must be completely self-financing.

The first-best optimization results show a large portion of travelers use the bus and that explicit road use tolls would be much higher than is the imputed 1.2 cents per mile. Where road use is not priced and bus fares are set to zero, the bus service will require a substantial subsidy, though the use of minibuses lessens this subsidy significantly. The partial subsidy scenarios generated smaller bus shares of travelers and higher resource costs per mile.

One set of results that have not yet been discussed are the travel speeds of buses and autos. With the exception of the first-best scenario, travel speeds are uniformly low, ranging from 2.7 to 6.2 mph. These results are not satisfactory if these optimizations are taken to be truly cost minimizing. Where congestion is the primary manifestation of the peak-period urban transportation problem, these scenarios have generated this congestion as part of the cost-minimizing optimal solution. As a consequence, an issue is raised of whether or not congestion is being adequately modeled. In the
next chapter this will be considered and a more sophisticated accounting of congestion will be made.
Notes

1 In other words, we are ignoring the equality of $N_b = N - N_a$, so that there is no calculation of a partial derivative of $N_b$ with respect to $N_a$.

2 Mohring [1983] p. 300 uses a value of 1.5 cents per mile, per auto. At 1.25 people per auto this is 1.2 cents per auto traveler, per mile.

3 The results given in Table 2 mirror those shown in Mohring [1979] and Mohring [1983].
CHAPTER 3
CONGESTION AND A RECONSTRUCTED STEADY-STATE MODEL

In the previous chapter, we have seen the results of the steady-state model optimizations. In this chapter, we will begin with a critique of these results, focusing on how that model accounts for our five types of congestion. We will see that sometimes this model undercounts, and sometimes overcounts, congestion.

We will then develop an alternative speed-flow relationship to be used in the steady-state model. This formulation is based on free-flowing traffic and is shown as a parabolic curve. This will serve as the foundation for a reconstructed steady-state model.

We will retain all of the basic elements of the model developed in Chapter 2. The reconstructed model will develop a more sophisticated accounting of bus and auto travel times. These times will now include specific congestion delay, though averaged out across the traveler flow.

To calculate travel times, we will outline six traffic scenarios. These range from a situation where flows are small, and congestion is minute, to traffic gridlock. Each traffic scenario requires a modification in the calculation of congestion.
3.1 A Critique of the Steady-State Model

The optimized results of the steady-state model, presented in Table 2, exhibit a marked dichotomy between the first-best case, where road use is efficiently priced, and the constrained cases. In all of the latter, the speeds for buses and autos vary from 2.7 mph to 6.2 mph. The generation of such low speeds in the context of an optimized traffic network casts doubt on the usefulness of these constrained scenarios. Essentially, where heavy congestion is the primary manifestation of the peak-period urban transportation problem, the steady-state model has indicated that heavy congestion is a constant part of the constrained optimization.

The results presented in Table 2, considered as a whole, lead to a very dramatic conclusion. If the model has adequately accounted for the costs of travel, then any scenario that does not explicitly, and fully, price road usage according to marginal cost principles results in significant congestion. On the face of it, attempts to improve the workings of the traffic system on a piecemeal basis, say by slowly raising the amount of a bus system's subsidy, will be ineffective in reducing congestion.

If the model has not adequately identified and measured congestion costs, then the constrained optimizations may not be cost minimizing in a very
meaningful way. The first-best case, on the other hand, deters congestion effects because road use is explicitly priced. This generates relatively high speeds, high bus patronage and high auto tolls. These results are consistent with what we would expect such an optimized traffic system to look like. Because these tolls are so high, it has motivated us to consider the constrained cases. It is the low speeds, consistent with congestion, that motivates us now to consider how well the steady-state model incorporates congestion costs into the calculation of total resource cost.

In Chapter 1, five types of congestion were identified. The first type of congestion relates to an excess demand for the road, i.e., the road has a maximum volume of traffic that it can accommodate. As the desired demand for the road rises beyond this maximum, the actual volume will decline. In fact, there is a point to which the desired demand could rise that would result in a complete breakdown in the traffic network's ability to work at all, a condition commonly referred to as gridlock.

When demand exceeds capacity, speeds will fall, which is reflected in the steady-state model, but so will the volume. This decline in volume is not recognized in the steady-state model and so is a source of congestion that goes uncounted. That is, in the steady-state model, the desired flow of vehicles is presumed to actually pass
through the mile in one hour, no matter how low the speed. As speeds decline we would expect that the desired flow of vehicles will take longer than one hour to clear, causing a shockwave of congestion on the traffic behind it.

For example, in the standard bus version of model IIID, the traffic volume, ignoring the pce of the bus, is the number of autos plus the number of buses, or 1301 vehicles per hour.¹ If we take 4 mph as the average speed per vehicle, require a minimum headway time of 2 seconds per vehicle², and allow for only 50% use of the road (as lights are green 50% of the time), it would take almost two hours to clear this vehicle flow.³

The time it takes to clear the vehicle flow has been calculated for each scenario presented previously in Table 2 and the results shown in Table 3. Using the auto speed as the average, which biases the time downwards, only the marginal cost pricing scenario, model I, does not exhibit excess demand for the road.⁴

The figures in Table 3 indicate that the constrained optimizations result in significant congestion, the cost of which is not included in the steady-state model. For example, in model IIIC both regimes require about an hour and a half to clear the desired flow. That means that the next hour's traffic must start a half hour late. This is the cost which is not included.
TABLE 3

ALLOCATED TIME TO CLEAR DESIRED FLOW\textsuperscript{a}

<table>
<thead>
<tr>
<th>Model</th>
<th>I</th>
<th>II</th>
<th>IIIA</th>
<th>IIIB</th>
<th>IIIIC</th>
<th>IIIID</th>
<th>IIIE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Bus</td>
<td>.399</td>
<td>1.44</td>
<td>1.23</td>
<td>1.38</td>
<td>1.55</td>
<td>1.84</td>
<td>2.56</td>
</tr>
<tr>
<td>Minibus</td>
<td>.213</td>
<td>1.26</td>
<td>1.26</td>
<td>1.38</td>
<td>1.53</td>
<td>1.75</td>
<td>2.24</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Measured in hours. Values are based on the results given in Table 2 where we set \( f=1 \) and use \( U_a \) as the average speed.

The second type of congestion arises when there is excess demand for the bus. As previously mentioned, this possibility is not allowed to occur. So while this type of congestion may be realistic, it is not exhibited by any of the model optimizations.

The third type of congestion is the bottleneck delay that vehicles experience when a bus stops and constricts the traffic flow. This is exhibited as congestion in that it disrupts the flow of traffic which may lead to vehicles being stopped by a red light. This bottleneck effect also affects bus passengers as the stop may not allow the bus to keep up with the traffic it was with. There is no explicit accounting for this delay but there is a measure for it in the steady-state model. The parameter value for \( f \), the pce of a bus, is designed to approximate a delay effect that the bus causes on other traffic. As used in this model, that effect is constant and does not vary as the traffic flow changes.
The fourth type of congestion is the effect on bus passengers by the addition of one more passenger. The steady-state model accounts for this delay by including the time it takes to pick up, as well as drop off, a passenger in the bus time calculation. However, to the extent that this causes a bus to get caught by a red light is ignored, as was just discussed.

The fifth type of congestion is the effect on the speed of traffic by the presence of another vehicle. The inverse speed-flow relationship, given in equation (2), allows for a depressing effect on speed as the volume of traffic rises. As has already been noted, when volume is quite high, this formulation undercounts the full congestion effects as the model presumes the traffic passes through in an hour.

Under high volume conditions, congestion is undercounted. However, the inverse speed-flow relation has an opposite effect under conditions of moderate traffic volumes. That is, this speed-flow formulation counts congestion when the traffic conditions of the steady-state indicate that little, or none, exist.

3.2 A Revised Speed-Flow Relationship

The steady-state model assumes generally free-flowing conditions. Data on free-flowing traffic suggest a parabolic speed-flow relationship, which is described
by the Greenshields model. The Greenshields model postulates an inverse linear relationship between the density of traffic, measured in vehicles per lane per mile, and the speed at which the traffic flows, measured in miles per hour. This is illustrated in Figure 2a. There is a maximum density, \( K_j \), that results in gridlock and so the speed here is zero. The fastest speed, \( U_f' \), would occur when the density approaches zero.

At one half the value of \( K_j \), we have the optimal density, \( K' \), which corresponds to the optimal speed, \( U' \). These are optimal in the sense that they generate the maximum flow measured as vehicles per hour. This is shown as \( Q_m \) in Figure 2b, which is the parabolic speed-flow relationship. The form of this relationship is:

\[
Q = K_j (U - U'^2 / U_f').
\] (15)

At speeds above \( U' \), additional vehicles, increasing \( K \), will decrease traffic speed but will increase the flow of traffic, as we move from left to right along the curve. Along the upper portion, slower speeds allow for vehicles to be closer together, and the latter effect dominates, causing the flow of vehicles to rise.

At speeds less than \( U' \) on Figure 2b, additional vehicles continue to reduce speeds, but now the flow of vehicles falls as well, as we move from right to left along the curve. It is along this portion that congestion becomes so costly. The curve shows only the amount of
2a. Speed-Density (U-K) Relationship

2b. Speed-Flow (U-Q) Relationship

2c. Actual Flow, Qa, and Desired Flow, Q*.  

FIGURE 2
THE GREENSHIELDS FREE-FLOWING TRAFFIC MODEL
vehicles that can flow by a point on the road, or over a road segment. Figure 2c shows how we can distinguish the desired flow, \( Q^* \), from the actual flow, \( Q^a \). The desired and actual flows are equal along the uncongested portion of the curve, that is at speeds above \( U' \). Below \( U' \) the continued rise in desired flow causes the backward bend in actual flow, beginning at \( Q_m \). In fact, \( Q^* \) is a mirror image of \( Q^a \) around \( Q_m \). For any desired flow we can find the actual flow and the traffic speed. For example, for a desired flow of \( Q_1 \), the traffic speed will be \( U_1 \) and the actual flow will be \( Q_2 \).

What remains to be done in order to utilize these relationships is to specify the values of \( K_j \) and \( U_f \). As the model is applied to urban roads, we will designate \( U_f \) as the prevailing speed limit. As the inverse speed-flow relationship of equation (2) generates a maximum speed of 27.3 mph, \( U_f \) will be set to 30 mph. Theoretically, the value of \( U_f \) is determined by the particular characteristics of the road and not upon the posted speed limit. Where \( U_f = 30 \) is less than the theoretical maximum, the effect is to reduce the flow of vehicles the model would allow.

At approximately 20 feet per vehicle, the value of \( K_j \) would be \( 5280/20 = 264 \) vehicles per mile. This value is extreme in that it allows for no space between vehicles even though they are stopped. May [1990] reports that
actual observed maximum densities are between 185 and 250. The lower the density, the lower the flow of vehicles, so to continue providing a conservative flow estimate, $k_j = 185$ will be used.

The value of $U'$ is half of $U_f$, so is 15 mph. Substituting the appropriate values into equation (15), the maximum flow is determined to be 1387.5 per lane.

Where the traffic along the urban arterial is using the road only half of the time (because of traffic signals) the maximum hourly flow is half of 1387.5, or 693.75 per lane. Rounding this off at 693, we can now overlay this speed-flow relationship with the one presented in Figure 1.

This is shown in Figure 3, where the Greenshields formulation is given for two lanes of traffic and so the maximum flow is $693(2) = 1386$. The formulation used in the steady-state was derived for two lanes of traffic, making these figures directly comparable. The divergence of these two formulations results in dramatic absolute differences in speeds, and so in costs, especially along the interval from 500 vehicles per hour to 1386 vehicles per hour.

At volumes below 500, the difference in speeds is small enough to ignore. In fact, the calculated volumes for model I, letting the value of $f$ be set to one, are 521 and 282 for the standard bus and minibus, respectively. These two speed-flow relationships are likely to generate similar traffic conditions for using this parabolic model.
At flows above 1386, the steady-state model still understates speeds, so overcounting congestion, but it also overstates flows, thus providing a compensating undercount of congestion. Which effect will dominate is not clear until flows, or their desired level, attain around 2000 vehicles per hour, where the speeds of the two formulations begin to converge quite a bit. For example, the flow for model IIIE, using the standard bus, is 1490. The resource cost given by that model may be acceptable in that underestimation and overestimation of congestion costs are occurring.

The key feature here is that from flows of 500 to about 1400, the Greenshields model, whose flows are conservatively arrived at, shows that traffic is relatively
uncongested and speeds are fairly high, while the steady-state model is generating rising congestion, accompanied by sharply falling speeds. This divergence can be attributed to the fact that the steady-state speed-flow formulation is not a free-flowing relationship, but rather one that has typical urban congestion effects exponentially related to the volume-capacity ratio. With reference to equation (2) the partial derivative of $t_a$ with respect to volume is $15.66(V/K)^{(1.7)}$. That is, the time to travel a mile is exponentially increased as volume is increased. This relationship may adequately cover congestion when flows are very large, but the consequence is the accounting of congestion when traffic flows are moderate and serious congestion is unexpected.

The result of this inspection of congestion costing by the steady-state model is two-fold. First, there are some important sources of congestion that are ignored outright, or underestimated quite a bit. Secondly, there is a systematic overcounting of congestion as it is implicitly incorporated into the speed-flow relationship. The consequence of both these points is to generate second-best optimizations that exhibit too much congestion. This occurs with the first point when, if the congestion had been counted, the optimal solution would have been different. With regard to the second point, this occurs
when moderate volumes generate low speeds and congestion when that is unwarranted.

3.3 A Reconstructed Steady-State Model

The purpose of reconstructing the steady-state model is to account for the costs of congestion in obtaining a cost minimizing modal split and to investigate the issues raised in Chapter 2. While a comprehensive incorporation of congestion characteristics would be well beyond the bounds of such a simplified model, there are changes that can be made that will retain the simplicity of the steady-state, and include much of the essential nature of congestion.

The characteristics that were ascribed to the steady-state will not be altered. The total resource cost, $Z$, will be calculated as it was in equation (10), the sum of auto operating cost, $AOC$, auto traveler time cost, $ATC$, bus operating cost, $BOC$ and bus passenger time cost, $BTC$. The cost minimizing procedure again sets the cost of the marginal auto traveler, $MC_a$, equal to the cost of the marginal bus passenger, $MC_b$, and solves for the optimal modal split and level of service, as previously outlined in Chapter 2.

The major task undertaken by the reconstructed steady-state model, henceforth referred to as RSS, is to more completely include congestion in the calculation
of total costs. This primarily affects the calculations of $t_a$ and $t_b$, the travel times of the two modes. Before turning to the specific formulations of these terms, we will first consider how congestion will manifest itself and be accounted for in the RSS model.

The RSS model will explicitly account for the five types of congestion previously discussed. The first type of congestion occurs when there is excess demand for the road. Referring back to Figure 2c, this occurs at flows above $Q_m$, the maximum capacity, here given as 693 vehicles per lane per hour. This model will evaluate the decline in flows, when speeds fall below $U'$, as the desired flow grows beyond $Q_m$.

Our second type of congestion arises when there is excess demand for the bus. Unlike the steady-state model, the RSS model will allow for this excess demand possibility. This calculation will be incorporated into the cost of the marginal traveler as that individual then determines which mode to take.

The third type of congestion occurs when a bus stops and creates a bottleneck in the traffic flow. The consequence of this is that some vehicles, including the bus, are disrupted out of the flow for which traffic lights have been synchronized. The RSS model evaluates what these costs are and includes them in the total resource cost calculation.
The fourth type of congestion is the effect another bus passenger has on existing passengers' transit time. The RSS model accounts for this delay by measuring how many red lights a bus encounters along the route mile, which is the net result of stopping to pick up, and drop off, passengers.

The fifth type of congestion is due to the addition of another vehicle to the traffic flow, thereby slowing down other auto travelers as well as bus passengers. The speed-flow relationship outlined in Figure 2 is used as the basis for this calculation in the RSS model.

3.4 The Calculation of Bus Travel Time Per Mile

The calculation of bus travel time per mile, $t_b$, is the sum of the auto travel time per mile, $t_a$, and the aggregate delay due to stopping at red lights per mile. The calculation of $t_a$ will be considered in the next section.

With regard to red light delay, we can easily see how this is occurring. As light signals are synchronized to traffic volume, vehicles move through the mile in groups, or platoons. As long as there are no obstructions (left hand turns, right hand turns, etc.), a platoon will pass through the mile without stopping for any red lights. When a bus is a member of such a platoon of vehicles, every time it stops to pick up passengers it falls further
back in the platoon, also disrupting traffic flows in the process. Eventually, it will make a stop that will push it out behind the platoon, and so it will be stopped by the next light. It will then begin flowing with the next platoon of vehicles when the light turns green.

The amount of red light delay a bus encounters per mile will depend on the length of the bus stop, the length of the green phase and the number of stops per mile.

Where \( N_b \) is the number of bus passengers boarding per mile and the same number are getting off the bus per mile, then \( 2N_b \) is the total number of people engaged in this activity per mile per hour. Where \( \varepsilon \) is the time it takes to load, or unload, a passenger, measured in seconds, \( 2N_b \varepsilon \) is the total time that buses spend at bus stops per mile, per hour. We divide that by the number of buses per hour, \( X \), and the number of stops per mile, \( Y \), to get \( r \), the average amount of time a bus spends at a single stop:

\[
 r = \frac{(2N_b \varepsilon)}{(XY)}. \quad (16)
\]

Where \( t_g \) is the green phase length, in seconds, the number of stops a bus makes while a member of a platoon of vehicles is essentially \( t_g/r \). Denoting this number of stops as \( \gamma \), and defining the subscript \( I \) to mean the integer value, we can write:

\[
 \gamma = \lfloor t_g/r \rfloor_I + 1. \quad (17)
\]
Clearly the value of $\gamma$ must be an integer. For example, if the bus stops for 12 seconds at each stop, and the green phase is 20 seconds, then the bus will make only 2 stops while a member of any given platoon. The first stop will occur when the bus leads a platoon, then falling 12 seconds behind the lead vehicle. The second stop occurs when the bus is 12 seconds from the front, and will, at the end of this stop, be 24 seconds behind the lead vehicle. As the green phase is only 20 seconds long, it cannot clear the intersection before the light turns red and the bus will become the lead vehicle in the next platoon. The number of stops are then calculated as $(20/12)_1 + 1 = 2$, and the total delay to the bus is now the green phase plus the red phase.

The number of bus stops per mile is taken to also equal the number of lights per mile, and so is denoted as $Y$. Therefore, the average number of red lights that a bus will stop for, per mile, is $Y/\gamma$.

The total wait is the sum of the red and green phases of the traffic signal which total to $C$, the traffic cycle length. For purposes of simplicity the yellow phase is ignored and the green and red phases, $t_g$ and $t_r$, respectively, are taken to be of equal length. Measuring $C$ in seconds requires that it be divided by 3600 to convert this into hours. Each time a bus waits for a red light it is delayed by a net of $C$ seconds. This includes, then,
the delay experienced as the bus moves from the front
to the back of the platoon, in addition to the actual
red phase time. Therefore, the aggregate red light delay
that a bus experiences per mile is \((C/3600)(Y/y)\).

Putting this altogether, we can write the bus travel
time per mile, \(t_b\), measured in hours, as:

\[
t_b = t_a + \frac{C}{3600}(Y/y).
\]  

(18)

3.5 The Calculation of Auto Travel Time per Mile

The average time it takes an auto to travel one mile,
\(t_a\), is the sum of what will be called the base travel
time, determined by flow conditions, and average delay,
generated by the bottleneck created on the road when a
bus stops to load and unload passengers. The specification
is as follows:

\[
t_a = \frac{1}{U_a} + \frac{D}{3600},
\]  

(19)

where \(U_a\) is the base travel speed, in miles per hour,
and \(D\) is an average delay term, expressed in seconds.
Dividing \(D\) by 3600 will convert it into hours of delay,
per vehicle, per mile traveled.

The calculation of \(U_a\) is based on the Greenshields
model presented in Figure 2 and the speed-flow relationship
shown in equation (15). We will refine that calculation
by writing:

\[
Q = K_f(CF)(U - U^2/U_f),
\]  

(20)

where \(CF\) is a scaling factor. Assuming the green phase
allows for free flowing traffic only over half the hour, then the value of CF is set to one-half.

We can expand this relationship and group terms to obtain the following:

\[ U^2 \left( \frac{K_j CF}{U_f} \right) - U \left( \frac{K_j CF}{U_f} \right) + Q = 0, \]  

which can then be solved for speed, \( U \), using the quadratic equation. With some manipulation of that result we can derive:

\[ U_a = U = \left( \frac{U_f}{2} \right) \left( 1 \pm \sqrt{1 - \frac{4Q}{(U_f K_j CF)^2}} \right). \]

The positive root of this equation generates speeds above 15 mph and represents uncongested flow. This occurs when the desired flow does not exceed the maximum allowable. Where this is not the case, the negative root is used, and speeds will fall to less than 15 mph. It will be this formulation that we will use for \( U_a \) in equation (19).

The only variable left undefined is \( Q \), which is the flow of vehicles per hour per lane. It is essentially the same as the volume calculation discussed in Chapter 2. Where \( MN_a/n_a \) is the number of autos, and \( fX \) is the pce of the number of buses and \( L \) is the number of lanes, we will calculate flow as:

\[ Q = \frac{(MN_a/n_a + fX)}{L}. \]

The average delay can be broken down into appropriate components. We can write:

\[ D = \frac{D_1}{(D_2 D_3)} \]
where $D_1$ is the total delay, in seconds, that a bus causes other vehicles as it passes back through a platoon, $D_2$ is the number of vehicles in a platoon, and $D_3$ is the miles a platoon will travel per bus encountered.

Where $Q$ measures the hourly volume of traffic that flows along the street in discrete groups (i.e. platoons), it is divided by 1800 seconds, which is the usable time the road may be traveled per hour, to obtain the per lane flow of vehicles per second. This is true as long as the red phase is equal to the green phase of the traffic signal. Where the number of lanes is $L$, and the time length of the platoon is $t_g$, the calculation of $D_2$ is quite straightforward. It is written as:

$$D_2 = t_g L Q / 1800.$$  \hspace{1cm} (25)

The miles that a platoon travels for each bus it encounters, $D_3$, is the product of the platoon's speed, $1/t_a$ (the inverse of the travel time of an auto), and the length of time, in hours, it takes the platoon to catch up to the next bus. The time it takes to catch up to the next bus is the distance between buses, in miles, divided by the rate of speed at which the platoon gains on the next bus.

The distance between buses is the average bus speed in miles per hour, $1/t_b$, which is the inverse of the travel time of a bus, multiplied by the bus headway in hours, $1/X$, which is the inverse of bus frequency.
The rate at which the platoon catches up to the next bus is the difference between the platoon's speed, $1/t_a$, and the average bus speed, $1/t_b$. Putting all this together yields:

$$D_3 = (1/t_a)(1/t_b)(1/X)/(1/t_a - 1/t_b), \quad (26)$$

which can be rearranged to get:

$$D_3 = (1/X)/(t_b - t_a). \quad (27)$$

We use the formulation for $t_b$ given in equation (18) to further reduce this to:

$$D_3 = (1/X)/(CY/(3600\gamma)) = 3600\gamma/(XCY). \quad (28)$$

Putting $D_2$ and $D_3$ together we can write:

$$1/(D_2D_3) = CX/(2tgLQ\gamma). \quad (29)$$

The calculation of $D_1$, the total delay a bus causes to a given platoon of vehicles, is much more complicated and will change under different conditions. Before turning to the actual calculation, it will be necessary to describe the congestion process that is being modeled. The key element here is an explicit accounting of the delay caused by the existence of traffic signals.

As a bus travels down the arterial roadway it will make stops to drop off and pick up passengers. When the bus is stopped, it creates a bottleneck in the traffic flow. Vehicles behind the bus will merge into the remaining free flowing lanes, thus reducing the average speed at which these autos are traveling. The platoon perfectly fills up the available time ($t_g$ seconds) in...
order to maximize speed. Therefore, the reduction in speeds means that some vehicles will not pass through the intersection before the light changes.

These delayed vehicles will now interfere with the smooth running of the platoon behind them. Their existence will possibly cause some vehicles at the end of this platoon to be unable to pass through this intersection before the light changes again. Whether this will happen, and if so to what extent, depends on the difference between the maximum flow possible per lane, referred to as \( Q_m \), and the flow corresponding to average traffic conditions, already referred to as \( Q \).

For example, if \( Q_m = 693 \) vehicles/hour/ lane and \( Q = 575.5 \), and there are two lanes, then we have defined the excess capacity of the road as \( 2(693/693 - 575.5/693) = 1/3 \) vehicles per second over both lanes. So, if 8 vehicles were delayed, they could fully disperse over 24 seconds of normal flowing traffic by filling into this excess capacity. However, if the green phase, \( t_g \), is only 21 seconds long, then one vehicle will remain undispersed when the light changes to red. This vehicle will have to disperse with the next platoon.

Generally, then, the delay calculations will be based on the sum of red light delay and dispersal delay. In the above example, red light delay was 8 vehicles multiplied by 21 seconds (\( t_r \) is taken to be the same as \( t_g \)) plus
one vehicle multiplied by 21 seconds, which accounts for the vehicle that got caught by the second red light, for a total of 189 seconds. The dispersal delay was \((0.5)(24 \text{ seconds})(8 \text{ vehicles})\), for a total time spent in continuous dispersal of 96 seconds. The total delay caused by this one stopped bus was, then, 285 seconds.

The computation of \(D_1\) involves identifying and measuring the congestion costs arising from the bottleneck created by a bus stopping to pick up, and drop off, passengers. The RSS model calculates \(D_1\) for six different scenarios, where traffic conditions exhibit varying degrees of congestion. These scenarios range from zero congestion, at very low flows, to a situation where congestion is virtually infinite, and the traffic flow is gridlocked.

### 3.5.1 Scenario 1: No Bottleneck Congestion

There will be no delay if all the vehicles in the bus lane can successfully merge into the remaining lane(s) without causing speeds to fall below \(U'\), that speed where flows are maximized. This will be the case if the desired flow through the bottleneck (due to merging vehicles) is less than, or equal to, the maximum. For example, if the flow of vehicles is 600 per lane, when a bus stops, the desired flow in the remaining lane would be twice that, or 1200, well above the maximum of 693. However, if the flow is only 300 per lane, then, when a bus stops,
the desired flow rises to 600, which is less than the maximum. The case of no delay occurs when the excess capacity of the remaining lane(s), can accommodate the vehicles in the bus lane. When this is true, we set $D_1 = 0$.

This outcome may not seem likely during peak period travel. However, when bus frequencies rise high enough, and a substantial portion of travelers are using the bus, this condition will emerge.

3.5.2 Scenario 2: Bottleneck Congestion Fully Dissipates Before Next Bus

The delay term $D_1$ is the delay caused as the bus passes back through a platoon of vehicles. We can define this as the sum of red light delay, $RLD$, and dispersal delay, $DD$, and so write it out as:

$$D_1 = DD + RLD.$$  \hspace{1cm} (30)

The dispersal delay, as just shown, occurs when delayed vehicles are dispersing into the available excess capacity of the street. The total amount of time spent in this dispersing mode is the product of the average time it takes to disperse the affected vehicles and the number of vehicles so affected.

How many vehicles will be delayed? There are two parts to the answer. Recall that $\gamma$ is the number of stops a bus makes while a member of a platoon. From the first stop, when the bus leads the platoon, until the $(\gamma - 1)^{th}$
stop the bus makes, we have what will be termed full congestion effects of the bus on autos. The number of delayed vehicles, \( V_D \), is the difference between the normal flow and the congested flow for the \( r \) seconds that the bus is stopped. Where \( Q/1800 \) is the normal flow per second, per lane, the normal flow over the time the bus is stopped is \( rL(Q/1800) \). Where there is insufficient excess capacity to accommodate the vehicles in the \( L-1 \) non-bus lanes, the congested flow will be taken to be the maximum, \( Q_m \). All vehicles which could not clear over this interval are then delayed. We can then write the number of delayed vehicles as:

\[
V_D = rL(Q/1800) - r(L-1)(Q_m/1800). \tag{31}
\]

At the last stop a bus makes while a member of a platoon of vehicles, the \( \gamma \)th stop, the amount of vehicles delayed will vary depending on how far the bus is from the end of the platoon. That is, the bus may be anywhere from 0 to \( r \) seconds in front of the last vehicle. As a simple approximation, we will take the number of vehicles delayed at this stop to be \( \frac{1}{2}V_D \).

The rate at which these vehicles disperse is given by the excess capacity of the roadway under normal conditions, which is \( L(Q_m-Q)/1800 \) vehicles per second. Where we define the dispersal rate per green phase as \( DR \), we can write this as:

\[
DR = t_gL(Q_m - Q)/1800. \tag{32}
\]
The dispersal rate per second is $\frac{DR}{t_g}$. The time it takes the last delayed vehicle to disperse is $\frac{VD}{(DR/t_g)}$, which can be written as $\frac{1}{2}t_g \frac{VD}{DR}$. The average time to disperse a vehicle is half of this.

It is the product of the delay vehicles, $VD$, and the average time to disperse them that shows the amount of dispersal delay created by one stopped bus. From the 1st to the $(\gamma-1)$th stop, this delay is $V_D \frac{1}{2} t_g \frac{VD}{DR}$ which can be restated as $\frac{1}{2} t_g \frac{VD^2}{DR}$, and for the $\gamma$th stop this is $\frac{1}{2} V_D t_g \frac{1}{2} VD/DR$ which can be written as $\frac{1}{2} t_g (\frac{VD^2}{4})/DR$.

The sum of these delays over all stops a bus makes while a member of a platoon is the dispersal delay:

$$DD = \Sigma \left[ \frac{1}{2} t_g V_D \frac{VD^2}{DR} \right] + \frac{1}{2} t_g (\frac{VD^2}{4})/DR,$$

which is summed for $i=1$ to $\gamma-1$. This can be rearranged to produce:

$$DD = \left( \frac{1}{2} t_g /DR \right) (\Sigma V_D i^2 + .25 VD^2).$$

(34)

The summation term can be reduced to $(\gamma-1)VD^2$ as the number of delayed vehicles is the same over all of these stops. This allows us to further reduce the dispersal delay calculation to the following:

$$DD = \left( \frac{1}{2} t_g /DR \right) (\gamma-.75) VD^2.$$

(35)

The other component in our $D_1$ delay term is the red light delay, $RLD$. This term measures the time that the congested vehicles spend at red lights as they disperse into the available excess capacity of the road. For example, suppose 13 vehicles are delayed by the bottleneck.
and that only 5 can disperse over the course of the green phase. Where the red phase is 30 seconds, the delay at the first red light would be 13 vehicles multiplied by 30 seconds, or 390 seconds. Five vehicles disperse, leaving eight delayed at the second red light, causing an additional 240 seconds of delay. Three vehicles are left for the third red light, for 90 seconds of delay, and all these remaining vehicles clear before the fourth red light. The total red light delay is 720 seconds.

We can imagine this as looking like a step function where the horizontal widths measure the length of the red phase, denoted as $t_r$, and the vertical drops measure the vehicles that disperse between red lights, which has been defined as DR and shown in equation (32). Where we define:

$$\eta = \frac{VD}{DR}$$

(36)

where the subscript I denotes the integer value, then the number of red lights it takes to fully disperse the VD delay vehicles is $(\frac{VD}{DR})_I + 1$, or $\eta + 1$.

With some geometric manipulation of our step function, we can show that, generally, the red light delay created by a bus at stop i, denoted $RLD_i$, is:

$$RLD_i = (\eta + 1)t_r(VD - \frac{1}{2}\eta DR).$$

(37)

In our example, $VD=13$, $DR=5$, $t_r=30$ and we can calculate $\eta=(13/5)_I=2$. The red light delay here is, using equation (37), $3(30)(13-\frac{1}{2}(2))=720$ seconds.
The exception to the formulation given in equation (37) occurs when the bus is at the $\gamma^{th}$ stop, the last one before it exits this platoon. Here, we are using $\frac{1}{2}VD$ as an approximation for the number of vehicles delayed and this will affect the value of $\eta$ given previously. So, defining $\eta'$ as the integer value of $\frac{1}{2}VD/DR$, we can write the delay at the $\gamma^{th}$ stop, $RLD_{\gamma}$ as:

$$RLD_{\gamma} = (\eta' + 1)t_r(\frac{1}{2}VD - \frac{1}{2}\eta' DR)$$

$$= \frac{1}{2}(\eta' + 1)t_r(VD - \eta' DR). \tag{38}$$

The total red light delay over all $\gamma$ stops can be written as:

$$RLD = \sum[(\eta + 1)t_r(VD_i - \frac{1}{2}\eta DR)] + RLD_{\gamma}, \tag{39}$$

which is summed for $i=1$ to $\gamma-1$. This can be reduced to:

$$RLD = t_r(\gamma - 1)(\eta + 1)(VD - \frac{1}{2}DR\eta)$$

$$+ \frac{1}{2}t_r(\eta' + 1)(VD - DR\eta'). \tag{40}$$

When the traffic conditions match scenario 1, the RSS model will calculate the DD and RLD as given in equations (35) and (40), respectively. These will be used to calculate $D_1$, as given in equation (30) which will be used to calculate total delay, $D$, as given in equation (24).

3.5.3 Scenario 3: Bottleneck Congestion Partly Dissipates Before Next Bus

In this case, there is some excess road capacity, allowing congested vehicles to disperse into the flow.
of traffic. However, it is insufficient to allow all the delayed vehicles to disperse before the arrival of the next bus. At this time, all remaining vehicles disperse into the traffic stream in front of the bottleneck. In other words, the bottleneck, which congests vehicles behind it, creates enough excess capacity in front of it to fully disperse the congested vehicles remaining from the previous bus. So, calculating the value of $D_1$ must take into account the time it takes the next bus to arrive. We will, again, calculate $D_1$ as the sum of red light delay, RLD, and dispersal delay, DD.

This scenario will arise when the calculated dispersal time, $VD/(DR/tg)$, is greater than the aggregate green phase time before the next bus. Where the real time between buses is $3600/X$ seconds, the number of Platoons per bus, denoted as $\Psi$, is written out as:

$$\Psi = 3600/(CX) = 1800/(t_g X), \quad (41)$$

where we substitute $2t_g$ for $C$, as it is taken that $t_g = t_r$. This value in equation (41) is the number of platoons between buses, which is the same as the number of traffic signal cycles, or green phases, between buses. The value of $\Psi$ will be an integer as the RSS computer model will choose $t_g$ to insure that $X$ can be uniformly distributed across platoons.

The calculation of RLD is very similar to that done for scenario 2. Here, we clear all remaining vehicles
when the next bus arrives, instead of clearing them up to the \((n+1)\)th stop. As there are \(\Psi\) platoons between buses, so there are \(\Psi\) red lights that the congested vehicles wait for. Where we substitute \(\Psi\) for \(n'+1\) and \(n+1\) in equation (40), we can derive this delay as:

\[
\text{RLD} = \Psi t_r (\gamma - \frac{1}{2})VD - \Psi \frac{1}{2} t_r \gamma (\Psi - 1) DR. \quad (42)
\]

The dispersal delay calculation has similarities to the calculation done for scenario 2 and also to the calculation of red light delay. Here, dispersal takes place until the next bus arrives, and then all remaining vehicles disperse.

As a bus arrives with every \(\Psi\)th platoon, delayed vehicles will then be dispersing in only \(\Psi-1\) platoons. That is because vehicles dispersing in the platoon in which they were originally delayed are not included in this model.

The number of vehicles that disperse over the \(\Psi-1\) platoons is \((\Psi-1)DR\). The average time to disperse these vehicles is \(\frac{1}{2} t_g (\Psi-1)\), giving us a total for these vehicles of \(\frac{1}{2} (\Psi-1)^2 t_g DR\). The amount of vehicles that remain, waiting to disperse, is \(VD - (\Psi-1)DR\) and these vehicles had to wait the entire \(t_g (\Psi-1)\) seconds. Putting these together we can state the dispersal delay at a single stop, denoted \(DD_i\), as:

\[
DD_i = \frac{1}{2} t_g (\Psi-1)^2 DR + t_g (\Psi-1)[VD - (\Psi-1)DR]. \quad (43)
\]
Calculating the dispersal delay over all the stops a bus will make as a member of a platoon yields the following:

\[ DD = (\gamma - \frac{1}{2}) V D t_\gamma (\psi - 1) - \frac{1}{2} \gamma V D t_\gamma (\psi - 1)^2. \]  \hspace{1cm} (44)

To calculate the delay term, \( D_1 \), we sum RLD and DD from equations (42) and (44), respectively. As \( t_g = t_\gamma \), substituting \( t_\gamma \) for \( t_g \), and reducing down, allows us to write this delay term as:

\[ D_1 = t_\gamma (2 \psi - 1) [(\gamma - \frac{1}{2}) V D - \frac{1}{2} \gamma V D (\psi - 1)]. \]  \hspace{1cm} (45)

3.5.4 Scenario 4: Desired Flow Equals Maximum Flow

This fourth scenario arises when there is no excess capacity in any lane under normal conditions. When the bus stops, the vehicles that are delayed cannot disperse until the next bus arrives. The ensuing bottleneck will create only enough room for these previously delayed vehicles to disperse. Of course, now a new group of vehicles will be delayed. This means that there is some permanent congestion as long as the peak period continues. However, each bus creates a fixed amount of delay. For example, if 5 vehicles are delayed under these conditions and a bus comes by every 10 minutes, then the amount of delay caused by a bus making one stop is 50 minutes.

The delay at one stop is the seconds between buses, \( 3600/X \), multiplied by the vehicles delayed, \( V D \). Summed for \( i = 1 \) to \( \gamma \) we can write \( D_1 \) as:
\[ D_1 = \sum (3600/X) V D_i = (3600/X)(\gamma - \frac{1}{2}) V D, \quad (46) \]

where this is used in the calculation of D in equation (24).

3.5.5 Scenario 5: Desired Flow Exceeds Maximum Flow

This scenario arises when the desired flow exceeds the maximum capacity of the road. Referring to Figure 2, where flows are on a per lane basis, \( Q_m \) is 693. In this case, the bottleneck delay is permanent, as was true in scenario 4. The difference here is that actual flows will be less than the desired level and we have an additional source of congestion that may be termed excess demand related.

Where we define PD as the hourly duration of the peak period, then \( 3600(PD) \) is this duration in seconds. When the demand exceeds capacity, the excess must wait until the peak is passed before it can be accommodated. The average wait would be half of this, or \( \frac{1}{2}(3600)PD \). Where \( Q \) is the actual flow, and \( Q^* \) is the desired flow, we will use the simple relationship of \( (1-\frac{Q}{Q^*}) \) to represent the portion of the average wait that is assessed to each vehicle.

The calculation of \( D_1 \) for this scenario is exactly the same as for scenario 4 and is given by equation (46). The calculation of D will now reflect the excess demand congestion and so we will replace the formulation in
equation (24) with the following:

\[ D = \frac{D_1}{(D_2 D_3)} + \frac{1}{2}(3600)PD(1-Q/Q^*). \]  

(47)

3.5.6 Scenario 6: Gridlock Congestion

When the desired flow exceeds twice the maximum flow, the Greenshields formulation used in the RSS model shows that actual flow falls to zero. That is, the system collapses into traffic gridlock. Technically, this will last until the peak period has passed and demand falls. As any such outcome is a signal of inefficiency on a grand scale, the RSS model only monitors for this condition. When flows exceed twice the maximum, the model is not allowed to continue calculating costs, and this outcome is ignored.

3.6 Summary

In this chapter, a close examination of the steady-state results showed that, with the exception of the marginal cost pricing scenario, serious congestion was an integral part of the optimizations. The consequence of this congestion is that these slow speeds yield high travel costs.

We then investigated the extent to which this model incorporated six types of congestion. The model did include an accounting of the effects an additional vehicle has on the speed of the traffic, as well as the delay
effect an additional bus passenger has on the other passengers. The model did not have to contend with excess demand for the bus, as that was constrained against. The model did not include the congestion effects arising from the excess demand for the road, the variability in bottleneck congestion created when a bus stops nor the effect of disrupted traffic being delayed by red lights. The net effect is that congestion effects are undercounted.

However, the inverse speed-flow relation, presented in equation (2) and illustrated in Figure 1, has a complementary tendency to overcount congestion. This can be seen in its comparison to a variation of the Greenshields model, shown in Figures 2 and 3. The Greenshields model is represented to be a reasonable approximation to the speed-flow tradeoff exhibited by free-flowing traffic. The steady-state presumes that traffic is free-flowing, and so it is appropriate to use the Greenshields model as a basis for showing the congestion effects of adding more vehicles to the traffic flow. Under rather conservative traffic flow estimates, the resulting variation points up two shortcomings of the steady-state formulation. The first is that the Greenshields model includes a maximum flow, beyond which increases in the desired flow will actually decrease flow volumes. Secondly, over moderate flow level, speeds are much higher with the Greenshields model. The conclusion
from this point is that the steady-state model has an embedded congestion calculation included in it. This has been shown to make congestion exponentially related to the volume/capacity ratio.

We then set out to reconstruct the steady-state model so that it adequately identifies and costs out congestion. The reconstructed steady-state model, RSS, starts with the Greenshields speed-flow formulation as its basis and then accounts for the various types of congestion outlined.

The major effect on the appearance of the cost model, which is given in equation (10), is to alter the way in which bus travel time per mile, $t_b$, and auto travel time per mile, $t_a$, are computed. The RSS model calculates a delay term under six different traffic conditions, which is then included in the travel time calculations. The results of the RSS model will be given in the next chapter.
Notes

1 From Table 2, the bus frequency is 5 while bus share is .19. The auto share of travelers is .81. At 400 travelers, each traveling 5 miles, the number of auto travelers per mile is (.81)(400)(5) or 1620. At 1.25 people per auto, the number of autos is 1620/1.25 or 1296. The total number of vehicles is 1296 plus 5, or 1301.

2 See May [1990] p. 42 "(O)n most facilities it is difficult to maintain a mean time headway of much less than 2 seconds."

3 Allowing 19 feet for each vehicle (see McShane and Roess [1990] p. 32) at 4 mph with 2 second headways would allocate an additional 11.73 feet to each vehicle. With 1301 vehicles spread over 2 lanes and taking up 30.73 feet, the length, per lane, of this vehicle flow is (1301/2)(30.73)=19,980 feet. The speed at which this traffic clears is 4 mph, or 21,120 feet per hour of usable green time. As only 50% of the time is green, then only 21,120/2=10,560 feet of traffic can clear per hour. The time it takes to clear this vehicle flow is 19,990/10,560=1.89 hours of real time.

4 There is nothing to be inferred from a value less than 1.00 as the calculations are based on a 2 second minimum headway, and this might be longer. That is, the time values on Table 3 are minimums and, with longer headways, these times may be longer.


CHAPTER 4

THE RSS MODEL: OPTIMIZATION AND RESULTS

In the previous chapter, we have developed a reconstructed steady-state model that incorporates an explicit accounting of five types of congestion. In this chapter, we will present the optimization results for this model.

We will begin with a review of the basic model and identify changes made that distinguish this model from the one used in Chapter 2. Besides the difference in which travel times are calculated, we will also change the specifications of auto running cost and the bus passenger's walking and waiting time. We will also review the parameter values used in this model.

The cost minimization procedure repeats that of the steady-state model. We will be optimizing the modal split for a given level of bus service by finding where the cost of the marginal auto traveler equals the cost of the marginal bus passenger. Where this is repeated over all levels of bus service, we can find the optimal modal split and bus frequency.

The results of these optimizations are presented and analyzed with regard to three questions. First, how do the results for the constrained models compare with those of the first-best model? Second, how do the results
differ when using minibuses? Third, how do the results of this model differ from those given for the steady-state model in Chapter 2?

Finally, we will conduct a sensitivity analysis on this model. We will observe what happens to the optimizations of models I and II, for both bus regimes, when certain parameter values change. Some of these changed parameter values are expected to raise resource costs, while others are expected to lower these costs.

4.1 The RSS Model and Parameter Values

The RSS model maintains the format set out in Chapter 2 for the steady-state model. There, total resource costs, $Z$, is derived as the sum of four component cost terms: auto operating cost, $AOC$; auto traveler time cost, $ATC$; bus operating cost, $BOC$; and bus passenger time cost, $BTC$. The formulas for these terms appear in Chapter 2 and are repeated below:

$$Z = AOC + ATC + AOC + BTC,$$  \hspace{1cm} (48)

$$AOC = (MN_a/n_a)(c_a + IMC + P/M),$$  \hspace{1cm} (49)

$$ATC = MN_a t_a \bar{V}_a,$$  \hspace{1cm} (50)

$$BOC = BXT_b,$$  \hspace{1cm} (51)

$$BTC = MN_b t_b \bar{V}_b + N_b t_w \bar{V}_b,$$  \hspace{1cm} (52)

which are virtually identical to the formulas in equations (10), (3), (4), (6) and (8), respectively. The only
exception is that $F_a$ in equation (3) is replaced with IMC in equation (49).

The exogenous variables in the RSS model are $M$, $n_a$, $P$, $B$ and $\alpha$, as was true in the steady-state model. The model will determine an optimal modal split between $N_b$ and $N_a$, where these values sum to $N$, another exogenous variable. The model will also be determining the optimal frequency of bus service per hour, $X$. The calculations of $t_a$ and $t_b$, the auto and bus travel times per mile, are more involved in the RSS model, and have been fully elaborated on in Chapter 3.

The calculation of the average time value for auto travelers and bus passengers, $\bar{V}_a$ and $\bar{V}_b$, respectively, will be the same as used for the steady-state model, where these values depended on $V_N$, the exogenously given maximum value of time. There are three remaining terms that are treated differently than they were for the steady-state model. These are $c_a$, IMC and $t_w$.

The running cost of an auto, per mile, is represented as $c_a$. Its calculation had been dependent on the speed of the vehicle, as given in equation (1). The practical importance of this formulation is minute. From speeds of 10 mph to 30 mph, this cost would vary from 15 cents to 5 cents per mile, barely accounting for 2% of total resource costs. To simplify the model a bit, this term will be treated as an exogenously given value.
The inclusion of auto "tolls" per mile, $F_a$, in equation (3) is unwarranted if $F_a$ is to be derived as a congestion-based toll. When this is the case, $F_a$ is the price charged to auto travelers for their use of the road and it is incorrect to include this in the cost function. When $F_a$ only represents the cost of maintaining the traffic infrastructure, through the payment of gasoline taxes, then it is correct to include this in the AOC term. To insure a consistent interpretation in this regard, we will replace the $F_a$ term with IMC, defined as the infrastructure maintenance cost. The IMC measures the wear and tear on the road caused by auto travelers. This value will be given exogenously.

The third variable to be altered is the walking and waiting time value, $t_{w'}$, presented in equation (9). That formulation will be retained except that $h_w$, the fraction of bus headway that the average person waits for a bus, will be dependent on the bus frequency, $X$. The number of stops per mile, $Y$, the average walking speed, $U_w$, and the average time spent walking to the arterial, $h$, will remain exogenously given.

The $h_w$ value will be recalculated because there is a limit as to how far we can presume that buses will be uniformly spread in the traffic flow. When bus frequency rises, we can expect that the probability that buses bunch together will also rise. The method by which this happens
is quite straightforward. If a bus becomes slightly delayed, then it will face more than the expected number of passengers at the next stop. This means that the bus will be further delayed and there will be even more passengers at the stop beyond this. This process continues, causing this bus to fall back towards the next bus. At the same time, the next bus will be picking up fewer than the expected number of passengers, which means it will advance on its schedule, catching up to the bus ahead of it. Eventually, these two buses will be paired up, as will all later buses. This process may continue, causing more than two buses to be traveling in tandem.

In the case of bus pairing, scheduling 55-seat buses every minute evolves into, essentially, scheduling 110-seat buses every two minutes. This will raise the average traveler resource costs, either by raising bus passenger waiting time costs, or by causing bus passengers to switch to autos, and raising auto traveler costs, or some combination of the two.

The existence of bus bunching will not affect the calculation of bottleneck delay. This delay, described in the previous chapter, aggregates the delay over the route mile, per hour, and then spreads it out over the total flow to obtain an average delay per vehicle. The pairing of buses will not affect the calculation of this aggregate delay and so there is no direct impact on the
congestion analysis. Clearly, where the variation in congestion is a crucial factor in costing out these effects, this bunching effect could have serious consequences. The RSS model utilizes only averages and so this additional dimension is passed over.

As the bunching up of buses will cause longer de facto headway intervals, we will allow the $h_w$ term to exceed one, as follows:

$$h_w = 0.5 + (0.5)(X/30)_I,$$  \hspace{1cm} (53)

where the subscript I stands for the integer value of the final term, $X/30$. The average proportion of bus headway spent waiting will be 0.5 when bus frequency, $X$, is between 1 and 29, 1.0 when $X$ is between 30 and 59, 1.5 when $X$ is between 60 and 89, 2 when $X$ is between 90 and 119, and so forth. While there is little that is sophisticated about this term, it will be claimed that it is likely to understate true resource costs from high bus frequencies. As a practical matter, this formulation mostly affects the minibus scenarios, where bus frequency is often quite high.

The parameter values to be used in the RSS model are presented in Table 4. As is seen on the table, many of these parameter values are the same as was used in the steady-state model. This allows us to more reliably compare the results of that model with the RSS model.

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<table>
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<th>Parameter</th>
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<th>Description</th>
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<td>Waiting time value weight as a percentage of travel time value.</td>
</tr>
<tr>
<td>B</td>
<td>$25$</td>
<td>Standard bus cost per hour.</td>
</tr>
<tr>
<td>BC</td>
<td>$6$</td>
<td>Minibus cost per hour.</td>
</tr>
<tr>
<td>BPD</td>
<td>12</td>
<td>Standard bus holding capacity.</td>
</tr>
<tr>
<td>$c_a$</td>
<td>$0.056$</td>
<td>Minibus holding capacity.</td>
</tr>
<tr>
<td>BPD</td>
<td>3</td>
<td>Bus peak period delay, in hours.</td>
</tr>
<tr>
<td>CF</td>
<td>0.5</td>
<td>Automatic running cost per mile.</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>1.8</td>
<td>Correction factor for unused road capacity due to red phase.</td>
</tr>
<tr>
<td>f</td>
<td>2</td>
<td>Time it takes a passenger to get on, or off, a bus, in seconds.</td>
</tr>
<tr>
<td>1</td>
<td>The pce of a standard bus.</td>
<td></td>
</tr>
<tr>
<td>$h$</td>
<td>5</td>
<td>Time bus passenger spends walking from origin to arterial, in minutes.</td>
</tr>
<tr>
<td><strong>IMC</strong></td>
<td>$0.012$</td>
<td>Infrastructure maintenance cost, per vehicle, per mile.</td>
</tr>
<tr>
<td>$K_j$</td>
<td>184.8</td>
<td>Gridlock density of vehicles, in pce's per mile.</td>
</tr>
<tr>
<td>L</td>
<td>2</td>
<td>Number of lanes of one-way traffic.</td>
</tr>
<tr>
<td>M</td>
<td>5</td>
<td>Average journey length, in miles.</td>
</tr>
<tr>
<td>N</td>
<td>400</td>
<td>Number of travelers entering or exiting per mile per hour.</td>
</tr>
<tr>
<td>$n_a$</td>
<td>1.25</td>
<td>Auto travelers per auto.</td>
</tr>
<tr>
<td>P</td>
<td>$0.30$</td>
<td>Opportunity cost of vehicle/trip.</td>
</tr>
<tr>
<td>PD</td>
<td>3</td>
<td>Bus peak period delay, in hours.</td>
</tr>
<tr>
<td>$Q_m$</td>
<td>693</td>
<td>Maximum flow of vehicles per lane, per hour.</td>
</tr>
<tr>
<td>$U_f$</td>
<td>30</td>
<td>Maximum speed for free-flowing traffic (miles per hour).</td>
</tr>
<tr>
<td>$U_w$</td>
<td>3</td>
<td>Walking speed of arriving bus passengers (mph).</td>
</tr>
<tr>
<td>$V_N$</td>
<td>$7$</td>
<td>Maximum value of time for travelers, per hour.</td>
</tr>
<tr>
<td>Y</td>
<td>8</td>
<td>Standard bus stops per mile.</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>Minibus stops per mile.</td>
</tr>
</tbody>
</table>

* Values used in the steady-state model of Chapter 2.  
** Replaces $F_a$ in equation (49).
There are four parameters that relate to the underlying speed-flow relationship used by the RSS model. These are the correction factor, CF, the gridlock density, $K_j$, the maximum flow, $Q_m$ and the maximum free-flowing traffic speed, $U_f$. The values of these four parameters were given in Chapter 3 and were chosen to conservatively represent free-flowing conditions for the urban arterial road modeled here.

There are four other parameters listed on Table 4 that do not appear directly in the cost calculation of AOC, ATC, BOC and BTC. These parameters influence the calculations of $t_a$ and $t_b$, the auto and bus travel times. Primarily this is done through their effect on the calculation of the delay term, $D$, that appears in equation (24). These parameters are the holding capacity of a bus, $BC$, the pce of a bus, $f$ and the duration of the peak, $PD$ and $BPD$.

The bus holding capacity, $BC$, is the sum of sitting and standing passengers. This value is used to identify when the bus capacity is fully utilized. This sum depends on how much space is allocated to sitting. For the standard bus, the range of values given by Quinby [1982] is 40 to 115. He further relates that the designed maximum load for a typical urban transit bus (nonarticulated), the General Motors RTS II, is 68 passengers, where there are 45 seats. This bus is said
to have a "crushed capacity" of 92 persons. In the RSS model, the standard bus seating capacity is taken to be 55, and the maximum load is set to 65. Under the minibus regime, capacity will be set to 12, as was true in the steady-state model.

The pce value of a bus, \( f \), is now taken to reflect only the size difference between a bus and a car. The data given by Quinby [1982] shows the GM RTS II is 40 feet long\(^2\), twice what is being used for the average car length. He also shows that a minibus that holds from 10 to 16 passengers would be 15 to 18 feet long\(^3\), approximately the length of an auto. The value of \( f \) is then set to 2 for the standard bus and 1 for the minibus. In the steady-state model, this value represented a proxy for the congestion effects of a bus. As that process is being explicitly modeled here, this value will now be used solely as a weight for the additional physical length of a bus.

The peak duration, \( PD \), and bus peak duration, \( BPD \), account for the fact that the peak period does not persist forever. When the desired traffic flow exceeds the maximum, or when the amount of bus passengers exceeds bus capacity, the delay is assumed to last an average of half the peak duration. Both of these values are exogenous, and fixed in this model to three hours.\(^4\)
4.2 Cost-minimization

The cost-minimizing procedure for the RSS model is identical to that used for the steady-state. The model calculates the bus cost and auto cost for the marginal traveler, adjusting the modal split until these costs are equal. The cost of the marginal auto traveler, $MC_a$, was shown in equation (11), and that formulation will be used by the RSS model.

The cost of the marginal bus traveler, $MC_b$, shown in equation (12) will be revised to show the expected, or average, cost due to excess demand for the bus. This new formulation expressed in per mile terms is:

$$MC_b = F_b + V_m t_b + \alpha V_m t_w / M$$
$$+ \alpha V_m [(BPD/2)(1 + MN_B - X(BC))/(MN_B + 1)] / M$$

where the first three terms are exactly the same as shown in equation (12). The final term is the average of the delay due to excess demand for the bus. The value of this time is $\alpha V_m$ as it is an addition to the wait at a bus stop. The average wait is half the bus peak duration, or BPD/2. The number of these delayed individuals is the desired passenger flow, $MN_B + 1$, which now includes the marginal passenger, less the aggregate capacity of the bus flow, $X(BC)$. This aggregate wait time is averaged over the $MN_B + 1$ individuals and then the value of this delay is averaged over the journey length, $M$, to put this into per mile terms.
The RSS model has been written up as a BASIC language program. An annotated copy of this program appears in Appendix 2. The cost-minimization process starts, in this program, with a given value of bus frequency, $X$. The program then sorts through the various modal splits and iterates its way to the cost minimizing modal split. For each modal split it considers, the program identifies the appropriate traffic scenario, as outlined in the previous chapter, and evaluates congestion costs accordingly.

Following the selection, and identification, of the optimal modal split for this value of $X$, the program loops through various values for bus frequency, repeating the process outlined above. What is occurring is that the model is tracing out a cost function with respect to bus frequency. The cost-minimizing level of bus frequency, and the accompanying optimal modal split can then be identified by inspection of the results. This same procedure was used to generate the results of the steady-state model in Chapter 2.

4.3 Results

The results from the RSS model are summarized in Table 5. As was done with the steady-state model, the RSS model has been run through seven variations, identified as Models I through IIIE. These models were optimized.
### TABLE 5
THE RSS MODEL OPTIMIZATION RESULTS

#### 5A. 55 Seat Standard Bus Regime

<table>
<thead>
<tr>
<th>Model:</th>
<th>I</th>
<th>II</th>
<th>IIIA</th>
<th>IIIB</th>
<th>IIIC</th>
<th>IIID</th>
<th>IIIE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$/Trip:</td>
<td>$1.40</td>
<td>1.56</td>
<td>1.49</td>
<td>1.54</td>
<td>7.70</td>
<td>9.45</td>
<td>9.45</td>
</tr>
<tr>
<td>Buses/Hr:</td>
<td>17</td>
<td>19</td>
<td>11</td>
<td>10</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Nb/N:</td>
<td>.34</td>
<td>.18</td>
<td>.19</td>
<td>.17</td>
<td>.04</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Pax/Bus:</td>
<td>40.2</td>
<td>19.1</td>
<td>35.2</td>
<td>34.0</td>
<td>81.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ua(mph):</td>
<td>21.6</td>
<td>17.2</td>
<td>18.2</td>
<td>17.1</td>
<td>2.4</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Ub(mph):</td>
<td>16.4</td>
<td>15.3</td>
<td>14.6</td>
<td>14.0</td>
<td>2.3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Fa/Trip:</td>
<td>.76</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
</tr>
<tr>
<td>Fb/Trip:</td>
<td>.11</td>
<td>0</td>
<td>-16</td>
<td>-08</td>
<td>-29</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Deficit:</td>
<td>$11.34</td>
<td>31.04</td>
<td>31.04</td>
<td>23.28</td>
<td>15.52</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Def./Nb:</td>
<td>.08</td>
<td>.43</td>
<td>.40</td>
<td>.34</td>
<td>.96</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

#### 5B. 12 Seat Minibus Regime

<table>
<thead>
<tr>
<th>Model:</th>
<th>I</th>
<th>II</th>
<th>IIIA</th>
<th>IIIB</th>
<th>IIIC</th>
<th>IIID</th>
<th>IIIE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$/Trip:</td>
<td>$1.34</td>
<td>1.45</td>
<td>1.45</td>
<td>1.48</td>
<td>1.51</td>
<td>1.52</td>
<td>9.45</td>
</tr>
<tr>
<td>Buses/Hr:</td>
<td>59</td>
<td>37</td>
<td>34</td>
<td>33</td>
<td>29</td>
<td>29</td>
<td>0</td>
</tr>
<tr>
<td>Nb/N:</td>
<td>.35</td>
<td>.21</td>
<td>.20</td>
<td>.19</td>
<td>.17</td>
<td>.17</td>
<td>.0</td>
</tr>
<tr>
<td>Pax/Bus:</td>
<td>12.0</td>
<td>11.1</td>
<td>11.8</td>
<td>11.2</td>
<td>12.0</td>
<td>11.7</td>
<td>0</td>
</tr>
<tr>
<td>Ua(mph):</td>
<td>21.5</td>
<td>18.3</td>
<td>18.2</td>
<td>17.7</td>
<td>17.2</td>
<td>17.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Ub(mph):</td>
<td>19.5</td>
<td>17.0</td>
<td>16.8</td>
<td>16.4</td>
<td>15.9</td>
<td>15.7</td>
<td>-</td>
</tr>
<tr>
<td>Fa/Trip:</td>
<td>.77</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
</tr>
<tr>
<td>Fb/Trip:</td>
<td>.05</td>
<td>0</td>
<td>-01</td>
<td>.03</td>
<td>.06</td>
<td>.11</td>
<td>-</td>
</tr>
<tr>
<td>Deficit:</td>
<td>$10.38</td>
<td>13.09</td>
<td>13.09</td>
<td>9.82</td>
<td>6.55</td>
<td>3.27</td>
<td>-</td>
</tr>
<tr>
<td>Def./Nb:</td>
<td>.07</td>
<td>.16</td>
<td>.16</td>
<td>.13</td>
<td>.09</td>
<td>.05</td>
<td>-</td>
</tr>
</tbody>
</table>

$/Trip - Resource cost ($) per traveler per journey.  
Buses/Hr. - Bus frequency of service, X.  
Nb/N - Bus share of travelers.  
Pax/Bus - Passenger load per bus.  
Ua(mph), Ub(mph) - Auto and bus speed, respectively.  
Fa/Trip - Auto toll per journey per auto traveler.  
In models II-IIIE this is given as the IMC value.  
Fb/Trip - Bus fare per journey per bus passenger.  
Bus Deficit per hr.($) - In model II this is BOC,  
while in models IIIA-IIIE this is 100%, 75%,  
50%, 25%, and 0% of the value derived in II.  
Def./Nb - Bus deficit, per hour, per bus passenger journey.
under two bus regimes. The first regime uses a standard bus, having 55 seats, while the second regime uses a 12-seat minibus. These two regimes differ with regard to some parameter values, as detailed in Table 4.

Model I is a first-best optimization where the principle of marginal cost pricing of the road yields explicit congestion tolls levied on auto travelers and bus passengers. In model II, the auto "toll" is fixed to the value implied by gasoline taxes, bus fares are set to zero and a full subsidization of bus service is provided.

In models IIIA through IIIE, only a portion of the subsidy generated by model II is provided. Bus fares are unconstrained, but the bus company must break even. The subsidy provided in the five models is 100%, 75%, 50%, 25% and 0%, respectively, of the value from model II.

There are three broad questions we want to address in interpreting these results. First, how divergent are the constrained optimizations from the MC-pricing results? Second, how do the results under the minibus regime differ from those of the standard bus? And, third, how do these results differ from the steady-state model results?

As to the difference between model I and the constrained models, the higher the subsidy, the closer are the resource costs to the MC-pricing model. For the standard bus, with the subsidy set to 75% of the value
derived in model II, the resource costs were only 10% higher than under the MC-pricing model. With 100% subsidization, whether bus fares were constrained or not, the resource costs were higher than model I by 11% or less. For the minibus regime, with a subsidy of only 25% of the value derived from model II, shown by model IIID, the resource costs were less than 15% above the MC-pricing cost of model I. The higher subsidy models yielded optimized costs even closer to the value for model I.

The low subsidy models, IIID and IIIE for the standard bus, and IIIE for the minibus, generated all-auto corner solutions that were heavily congested. The resource cost of $9.45 shown by these models is the highest that the RSS model will produce for the parameter values given in Table 4. In model IIIE, the combination of no subsidy, positive fares, and heavy flows means that the presence of any buses will only raise costs. Essentially, what is happening is that travel times for the two modes converge and the waiting time value for the bus added to the bus fare makes the bus the more costly mode across all values of time. The standard bus model IIIC provides an intermediate result, where the subsidy is enough to provide for only a very low bus frequency. Costs are very high at $7.70, but below the all-auto corner solution.

The number of buses per hour shows more variation across models than do resource costs. Generally, the
constrained scenarios have fewer buses per hour. This would be expected as, with the auto toll eliminated, the incentive to take the bus is reduced. The one exception to this pattern is the standard bus model II where the bus frequency is higher than for model I. A closer comparison between the results of these two models indicates why this difference has occurred. The effect of the elimination of the auto toll can be countered, to some extent, by reducing fares and improving service. As fares are restricted to being zero in this model, the only option available is to offer higher than usual bus frequencies. That this is the case can be verified by observing the passenger load per bus. In model II, this is 19.1, less than half the 40.2 shown for model I. In models IIIA and IIIB, the bus frequencies are around 50% of the level in model II, yet they generate about the same modal split by offering negative fares.

While the difference between the results of models II and IIIA, using the standard bus, illustrated a wide flexibility between offering higher bus frequencies or lower bus fares, this same flexibility was not present in the minibus models. Under this regime, model II showed a decline in bus loads of less than 10% over those in model I, yet it also had about a 40% decline in bus frequency. The decline in bus frequency mirrored the 40% decline in the bus share of travelers. The limited
ability to trade off higher frequencies, and lower bus loads, for lower resource costs is verified in model IIIA, where bus loads barely rise, while bus fares barely fell into the negative range, illustrating a complementary limitation in trading off lower fares for lower resource costs. Additionally, models IIIB, IIIC and IIID all yield positive, and rising, fares.

The optimal modal splits in Table 5 show a sharp drop from model I to the low resource cost constrained models. The decline, in the sense of a falling bus share of the traveler flow, is between 40% and 50% for both bus regimes. This decline mirrors a decline in the speeds for buses and autos, though not of the same magnitudes. These declines in speed are in the range of 10% to 20%, where the relative decline is larger for auto speeds than for bus speeds. The rather small decline in speeds relative to the decline in the bus share of travelers is not unexpected in light of the tradeoff between speed and flow shown by the Greenshields model in Figure 2. It should be noted, however, that the reported speeds on Table 5 are derived as the inverse of the travel times shown in equations (18) and (19), which include the amount of congestion delay.

Our second major point of focus is a comparison of the minibus results with those of the standard bus. With regard to resource costs, the minibus values are lower
than the corresponding standard bus values, but the
difference for the low cost scenarios range from 2.7%
for model IIIA to 7% for model II. Certainly the major
difference with regard to resource cost is that the minibus
regime produces low cost results for models IIIC and IIID
while the standard bus generated very high costs for both,
with the latter model yielding an all-auto corner solution.

Besides the comparable resource costs for the two
bus regimes, models I, II, IIIA and IIIB also generate
strikingly similar modal splits and auto travel speeds.
The similarity of auto speeds is not unexpected, as given
the similarity of modal splits, the traffic flows will
be approximately the same. The fact that there are more
than twice as many minibuses as there are standard buses,
except for model II, whereas the pce of the former is
only half that of the latter, is not enough to
significantly affect the traffic flow. The corresponding
bus speeds are not nearly so equivalent as the minibuses
are carrying far fewer passengers, so making fewer stops.

The nearly identical values for the modal split in
the first four models raise the question of whether the
modal split is dependent upon the bus size. It may well
be argued that changing bus sizes, and frequencies, do
little to affect the bulk of a bus passenger's costs.
For example, in model IIIB the minibus is carrying only
one-third as many passengers as the standard bus, yet

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its speed is only 2.3 mph above the standard bus' 14 mph. And certainly the bus operating cost is only a small fraction of total resource costs, so that significant changes in the BOC term will have little affect on total costs. For example, where the total resource cost per mile, per hour, would be the resource cost per person, per mile, multiplied by the total traveler flow, per hour, then for model II this would be \((\$1.56/5)(2000) = \$624\) for the standard bus, and \((\$1.45/5)(2000) = \$580\) for the minibus. The BOC for this model is the same as the deficit, as bus fares are zero, and so is \$31.04\) for the standard bus and \$13.09\) for the minibus. This accounts for only 5% and 2.25% of total resource costs for the respective bus regimes.

While resource costs, modal splits and speeds are little changed when using the minibus, the size of the deficit is quite dependent on the bus regime. As just noted, the minibus deficit is \$13.09\) while the standard bus deficit is nearly two and a half times greater, \$31.04\). When taken by itself, the small reduction in resource costs when using minibuses would make the choice between these bus regimes arbitrary. However, the minibus regime correspondingly costs less to finance which, in the context of these models, could be the determining factor. At subsidy levels between \$3.27\) per mile, per hour, to \$13.09, the minibus regime generates low resource cost.
optimizations. The standard bus, on the other hand, generated a very high cost, heavily congested optimization when the subsidy level was as low as $15.52.

There is one more characteristic that is common to both regimes and that is the existence of a deficit for the MC-pricing model. The size of the deficits are almost the same, $11.34 for the standard bus and $10.35 for the minibus. These deficits arise because the congestion charge bus passengers pay, $F_b$, is insufficient to cover the bus operating cost. There is nothing in this model that would preclude such an outcome. The level of auto tolls collected are significant and certainly could be used to finance the bus service.

Our third point of focus is a comparison of these results with those of the steady-state model presented in Table 2. The most obvious difference is that the low resource cost optimizations for the RSS model yield costs per trip of 40% to 70% below those of the corresponding steady-state models. The reason for this can be directly linked to the speed-flow relationships used. The high speeds of the RSS models indicate that the flow levels are on the relatively uncongested portion of the curve shown in Figure 2. As long as the desired flow is less than the maximum, speeds will remain relatively high. In the RSS models, these speeds vary from 14 mph to 21.6
A comparison of the desired flows for these two models is shown in Table 6.

With the exception of model I, the steady-state flows and RSS flows are of similar magnitudes. However, the steady-state imposes a congestion penalty on these moderate flow levels causing speeds to be very low. It is directly as a result of these low speeds that these models generate high resource costs. The RSS model presumes that these flows are consistent with free-flowing conditions and so generates higher speeds and lower costs.

The high cost optimized results of the RSS model differ from the steady-state in the opposite direction. Here, the RSS costs are much higher than the steady-state, while speeds are a bit lower. A general implication that may be drawn from this is that, with regard to the RSS model, optimizations along the congested portion of the speed-flow curve are largely achieved as all-auto corner solutions. When the desired flow exceeds the maximum, it appears that buses contribute more to congestion, and costs, than they alleviate. As a consequence the RSS model shows a sharp break between the low cost optimizations and the high cost optimizations, while the steady-state exhibits a more gradual change.

The frequency of bus service is higher for nearly all of the steady-state models. As the RSS model adds in the bottleneck congestion delay caused by the bus this
### TABLE 6

**DESIRED FLOWS PER LANE**

#### 6A. Standard Bus Regime

<table>
<thead>
<tr>
<th>Model:</th>
<th>I</th>
<th>II</th>
<th>IIIA</th>
<th>IIIB</th>
<th>IIIC</th>
<th>IIID</th>
<th>IIIE</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSS model</td>
<td>558.5</td>
<td>692.8</td>
<td>680.5</td>
<td>696.7</td>
<td>800.5</td>
<td>833.3</td>
<td>833.3</td>
</tr>
<tr>
<td>Steady-State</td>
<td>270.8</td>
<td>594.8</td>
<td>563.3</td>
<td>595.7</td>
<td>628.0</td>
<td>677.5</td>
<td>776.0</td>
</tr>
</tbody>
</table>

#### 6B. Minibus Regime

<table>
<thead>
<tr>
<th>Model:</th>
<th>I</th>
<th>II</th>
<th>IIIA</th>
<th>IIIB</th>
<th>IIIC</th>
<th>IIID</th>
<th>IIIE</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSS Model</td>
<td>571.2</td>
<td>676.8</td>
<td>683.7</td>
<td>691.5</td>
<td>706.2</td>
<td>766.2</td>
<td>833.3</td>
</tr>
<tr>
<td>Steady-State</td>
<td>143.7</td>
<td>540.8</td>
<td>540.8</td>
<td>570.7</td>
<td>600.5</td>
<td>646.0</td>
<td>721.3</td>
</tr>
</tbody>
</table>

*Values derived from Tables 2 and 5. The calculation of flow is \((1 - N_b/N_MN_a + X)/L\).*

The expectation of fewer buses and smaller loads for the RSS model leads us to expect that the bus share of the traveler flow will be lower for the RSS model than for the steady-state model. Yet, there is an additional reason why the RSS generates lower bus shares of traveler flow. This is due to the way in which congestion affects bus travel times. The steady-state measures the bus stop length, while the RSS model basically doubles this amount by including the delay experienced at red lights. The
consequence of this calculation is that the disincentive to bus passengers is greater in the RSS model.

The difference in speeds between the two models also generate the widely divergent subsidies of model II. The RSS model subsidies are only one-fourth to one-sixth the level of the steady-state subsidies, for the standard bus and minibus, respectively.

4.4 Sensitivity Analysis

The choice of parameter values used in the RSS model were generally restricted to the values used by the steady-state model so as to facilitate a direct comparison of these two models. However, of additional importance is how sensitive the RSS model results are to a change in these parameter values. By observing the optimization under alternative conditions, we get a clearer understanding of how the RSS model functions.

From the list of parameters in Table 4, eight have been chosen that will be claimed to represent a full spectrum of effects on the RSS model. The parameter changes are divided into two groups. It is expected that one group of changes will reduce flows and lower resource costs, while it is expected that the other group raises flows and raises resource costs. With only one exception, these expectations are borne out by the results presented in Tables 7 and 8.
### TABLE 7

**RSS MODEL SENSITIVITY ANALYSIS:**

**STANDARD BUS REGIME**

**7A. Model I Results**

<table>
<thead>
<tr>
<th>Change</th>
<th>Res. Cost</th>
<th>X</th>
<th>%Nb</th>
<th>Bus Load</th>
<th>Ua</th>
<th>Ub</th>
<th>Fa</th>
<th>Fb</th>
<th>Def.</th>
</tr>
</thead>
<tbody>
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<td>.56</td>
<td>43.8</td>
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<td>15.7</td>
<td>$1.49</td>
<td>$0.15</td>
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</tr>
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<td>.81</td>
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<td>20.0</td>
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<td>.09</td>
<td>13.66</td>
</tr>
<tr>
<td>Y=4</td>
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<td>.31</td>
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<td>15.9</td>
<td>.86</td>
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<td>4.84</td>
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<td>8.01</td>
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</table>

Res. Cost - Resource cost per journey; X - bus frequency per hour; %Nb - Bus share of travelers; Bus Load - passengers per bus; Ua - auto speed (mph); Ub - bus speed (mph); Fa - auto toll per trip; Fb - bus fare per trip; Def. - hourly bus deficit

*Optimization values taken from Table 5.*

**7B. Model II Results**

<table>
<thead>
<tr>
<th>Change</th>
<th>Res. Cost</th>
<th>X</th>
<th>%Nb</th>
<th>Bus Load</th>
<th>Ua</th>
<th>Ub</th>
<th>Fa</th>
<th>Fb</th>
<th>Def.</th>
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</thead>
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</tbody>
</table>

Res. Cost - Resource cost per journey; X - bus frequency per hour; %Nb - Bus share of travelers; Bus Load - passengers per bus; Ua - auto speed (mph); Ub - bus speed (mph); Fa - auto toll per trip; Fb - bus fare per trip; Def. - hourly bus deficit

*Optimization values taken from Table 5.*
TABLE 8

RSS MODEL SENSITIVITY ANALYSIS:
MINIBUS REGIME

8A. Model I Results

<table>
<thead>
<tr>
<th>Change</th>
<th>Res.</th>
<th>Cost</th>
<th>X</th>
<th>%Nb</th>
<th>Load</th>
<th>Ua</th>
<th>Ub</th>
<th>Fa</th>
<th>Fb</th>
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<td>.02</td>
<td>10.57</td>
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</table>

8B. Model II Results

<table>
<thead>
<tr>
<th>Change</th>
<th>Res.</th>
<th>Cost</th>
<th>X</th>
<th>%Nb</th>
<th>Load</th>
<th>Ua</th>
<th>Ub</th>
<th>Fa</th>
<th>Fb</th>
<th>Def.</th>
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</thead>
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<tr>
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<td>24.1</td>
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</table>

Res. Cost - Resource cost per journey; X - bus frequency per hour; %Nb - Bus share of travelers; Bus Load - passengers per bus; Ua - auto speed (mph); Ub - bus speed (mph); Fa - auto toll per trip; Fb - bus fare per trip; Def. - hourly bus deficit

*Optimization values taken from Table 5.
There are five parameters used to illustrate rising flows. Two that raise traveler flows directly are $N$, the number of travelers entering, and exiting, per mile and $M$, the length of the journey. The sensitivity analysis raises the value of $N$ to 600 and $M$ to 10 miles to observe the effects of rising flows.

Three parameters may be expected to change the vehicle flow by adversely affecting the modal split. These are the number of bus stops, and lights, per mile, $Y$, the scaling factor for the disutility felt by bus travelers from walking and waiting, $\alpha$, and the cost of bus service, $B$. This analysis lowers $Y$ to 4 when using the standard bus and 8 when using the minibus, raises $\alpha$ to 6 and raises $B$ to $40$, for the standard bus, and $12$, for the minibus.

Six parameters are used to show declining flows in the standard bus case, and four are used for the minibus case. To show a decrease in the traveler flows directly, we again change the values of $N$ and $M$. In this situation, we will lower $N$ to 200 and lower $M$ to 2.5 miles.

To show what is expected to be a decline in the vehicle flow, we will increase the value for the number of travelers riding in an auto, $n_a$, to 2.0, and raise the number of lanes, $L$, to 3. Additionally, for the standard bus only, we will raise the carrying capacity of the bus, $BC$, to 85 passengers, and raise the number of stops, per mile, $Y$, to 16.
4.4.1 Standard Bus Regime: Model I

These results are shown on Table 7. This table also includes the optimized values for model I which were shown previously on Table 5. The changes that are expected to increase costs generally do so. Reducing bus stops by half, to 4, and raising costs by 60%, to $40, had only a very small effect on costs, rising by only a few cents per journey. Also, speeds and the congestion-based charges, $F_a$ and $F_b$, were changed very little. Both changes resulted in a fall in bus service, $X$, and a rise in bus loads. The reason for this is that both changes raise costs on the bus side. Increasing operating costs raises BOC directly, while reducing the number of stops generally causes bus passenger times to rise. The resulting subsidies change accordingly. As stops fall, and service falls, so the deficit declines. As operating cost rises, the decline in service is not enough to keep the deficit from rising.

Doubling the disutility of walking and waiting time, $\alpha$, to 6 deters bus passengers in a similar way to decreasing the number of stops. Costs rise here and the bus frequency falls, though passenger loads do not change much. This is understandable as the pattern of reducing frequency and obtaining higher bus loads is not easy to do with the higher disutility of waiting time. This also results in a higher auto toll, $F_a'$, as travelers are willing
to pay more for these trips. While the bus fare is mostly unchanged, the fall in service reduces the deficit.

Increasing the number of travelers entering, and exiting, per mile, $N$, by 50%, to 600, has all the consequences that we would expect. This heavy traveler volume has an increasing effect on bus frequency, bus loads, bus share of travelers, bus deficits, congestion tolls and resource costs. Speeds fall slightly as the higher bus share of travelers keeps traffic flow from rising very much and the bus loads are increased by less than 10%.

When journey length, $M$, is doubled to 10 miles, the traveler flow will double. However, the results reported in Table 7 have the resource cost, auto toll and bus fare expressed for a 5 mile journey. This is done to allow for a direct comparison between these results and the others on the table. For this value of $M$, the actual resource cost for the 10 mile journey is double the $1.39 on the table. The values of $F_a$ and $F_b$ are also expressed for 5 miles, and so their true values are double the values given on the table.

The results on Tables 7 and 8 for $M=10$ and $M=2.5$ have been adjusted to show resource costs, auto tolls, and bus fares for a 5 mile journey. That is, to find the true value of these costs and tolls the values in the tables should be doubled for $M=10$ and halved for $M=2.5$.  

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To facilitate a comparison of results across all these changed parameter values, the 5 mile journey will be the unit of measure. The other values on the tables are not dependent upon the journey length, so no other adjustments need to be made.

The exception to rising costs with rising flows occurs when we raise the journey length, $M$, to 10 miles. Here costs actually fall, though slightly, as the bus share more than doubles to 81% of all travelers and bus frequency more than triples to 59 buses per hour. As most cost components in the model are proportional to the miles traveled, this result must be due to the effect of spreading out fixed costs over more miles. The single largest fixed cost is the bus passengers' walking and waiting cost. The effect of doubling the journey length is to reduce this cost on a per mile basis. Accompanying this rise in bus share is a rise in bus loads, though this is not enough to cause a decline in traffic flow and so there is a rise in speeds for both modes. The deficit is greater, but not by a very large amount. The cost of the increased frequency has been offset, to some extent, by the rise in bus share, the increased bus loads and the faster speeds.

For the parameter values that were expected to lower costs, the results of Table 7 exhibit a great deal of symmetry. The one change that raised costs was changing
the journey length, M, to 2.5 miles. Now, the fixed costs are spread over fewer miles and the bus trip has become costlier, on a per mile basis. The consequence is that bus frequency falls dramatically, to only 4 per hour, bus share falls to 8% and bus loads fall by half, to 20 passengers per bus. The decline in the bus share does not overwhelm the decreasing effect on flows that shortening the journey length has, and so speeds have risen for both modes.

When the number of travelers entering, and exiting, N, falls to 200 we also observe opposite effects. The effects of this change can be seen by noting that the auto toll, F_a, has fallen to 18 cents per journey. This follows as the level of congestion is being reduced. The consequences of this is that auto travelers are not greatly discouraged, and we observe that bus share falls to 12%, bus frequency falls to 4 per hour and bus loads fall.

The effect of raising the number of stops, Y, to 16 per mile has almost no effect on the modal split, resource costs, speeds, tolls and the deficit. Bus frequency remains the same. This mirrors the results seen when we reduce the number of stops to 4 per mile.

When we raise bus capacity, B, to 85 passengers, the optimized values remained unaffected. This is not surprising as excess capacity already existed in the optimized results.
We use two other parameters to show the effects of falling costs. These are the number of travelers per auto, \( n_a \), and the number of lanes, \( L \). We raise the value of \( n_a \) to 2, which reduces the per person auto costs and the flow of vehicles. These reductions show up as a fall in the auto toll and so there is less deterrence to using an auto. Consequently, bus share falls, bus frequency falls, and the bus deficit falls.

When we raise \( L \) to 3, a direct reduction in flow per lane occurs. This reduces the auto toll, again due to a reduction in congestion effects. Bus share falls, frequency falls and the bus deficit falls.

4.4.2 Standard Bus Regime: Model II

This model imposed a constraint on the bus fare and auto toll so these optimizations can no longer rely on fluctuating congestion changes to mitigate the effects of congestion. Of the five changes designed to raise costs, only two do so with relatively high speeds. Lowering \( Y \) to 4 and raising \( B \) to $40 caused little change from the optimized values, which is the same result we observed for model I.

Raising \( a \) to 6 for model I led to a 60% increase in \( F_a \). In model II, there are no auto tolls and the effect is to deter bus use, causing the bus share to decline to 10%. This results in a deterioration of the traffic
network as congestion reduces traffic speeds to about 3.5 mph. Resource costs more than triple, rising to $5.76, and bus deficits soar to $122.40 per mile, per hour, a nearly four-fold increase over the optimized results.

When we raise M to 10 miles or raise N to 600 the system is on the verge of collapse as average speeds fall to less than 1 mph. To prevent gridlock conditions from occurring, bus frequency is increased, and the deficits grow in dramatic fashion. The resource costs, too, have grown dramatically, to levels 15 to 20 times greater than the optimized value.

The effects of the parameter changes that reduce flows reflect those for model I. Where the value of $F_a$ and $F_b$ in the model I results were low, as was the case when N=200, M=2.5, $n_a=2$ and $L=3$, so the corresponding results for model II are very similar. Increasing bus capacity to 85 again has no effect on the optimized values.

Raising the number of stops per mile to 16 has a modest effect on lowering cost. More unusual is that it increases bus frequency and the bus share of travelers, while reducing bus loads. So, while raising Y had a minimal impact on the optimization when we price road use, when we have something less then the first-best solution this change has an increased impact.
4.4.3 Minibus Regime: Model I

The minibus results are shown in Table 8. The parameter changes that we expect to raise costs are \( N=600, M=10 \) and \( a=6 \). As the minibus models are run with \( Y=16 \), the sensitivity analysis uses \( Y=8 \) as the parameter change. The minibus costs are $6 and so we consider double this value, $12, for this analysis.

For the parameter changes that we expect to reduce costs, we again consider \( N=200, M=2.5, n_a=2 \) and \( L=3 \). We will omit consideration of a higher value for \( Y \) and an increase in bus capacity.

The results for the minibus parallel what we have seen in the standard bus case. The effect of the flow-increasing changes are to raise resource costs, except in the case \( M=10 \). In that result, costs fall slightly, measured over 5 miles, while bus frequencies and the bus share of travelers rise dramatically. The changes that reduce flows also reduce costs, except, again, the changed journey length.

4.4.4 Minibus Regime: Model II

These results, shown in Table 8, also generate the same type of results as we have seen for model I. When raising \( N \) to 600, \( M \) to 10 and \( a \) to 6 the model shows that the traffic network is heavily congested, speeds are low and costs are high. When we lower \( Y \) to 8 and raise \( B \)
to $12, the changes are very slight, except for the resulting deficit.

The changes that lower flows result in lower costs, lower deficits and higher speeds. Raising the number of lanes, $L$, to 3 has no consequent effect on the frequency of service nor the bus share of travelers. This result differs from what occurred in the standard bus regime. However, resource costs are approximately the same for both bus regimes.

4.5 Summary

In this chapter, the optimization results of the RSS model have been presented. They indicate that relatively low cost modal splits can be generated under pricing and deficit constraints. This differs sharply from the results of the steady-state model presented in Chapter 2.

The low cost results, shown on Table 5, exhibit relatively high speeds, placing flows along the upper portion of the speed-flow diagram. This is an indication that the RSS model, by adding in congestion delay and monetizing it, has a natural bias towards uncongested results.

The RSS model does not uniformly generate low cost results. In the model variations where the bus service subsidy was quite low, the resulting optimizations show
little, or no, bus service and heavy congestion. It is clear, within the scope of this model, that subsidy levels are of crucial importance in overcoming congestion. Unless, that is, road use is priced.

Not surprisingly the minibus regime generated low costs at subsidy levels far below that for the standard bus. An advantage that minibuses have is that their low loads keep travel time down and results in higher bus fares.

The RSS model was then optimized for a variety of parameter changes. The results were consistent with our expectations and with our understanding of how this model works.
Notes


4 The effect of an excess demand for the road adds a congestion delay to the calculation of auto travel time per mile. This effect was shown in equation (47). Similarly, when there is an excess demand for the bus, this will cause a delay only to bus passengers. The effect of this is to add a delay component, incorporating BPD, to the calculation of BTC.

5 It should be noted that the calculation of the congestion-based bus fare is not the same as in equation (14). In the RSS model this fare will differ according to the traffic scenario. Its calculation appears in the annotated computer model presented in Appendix 2.
CHAPTER 5
SUMMARY AND CONCLUSIONS

The general economic prescription for the existence of congestion along roadways is to price its use according to marginal cost principles. A variety of authors who have studied this issue conclude that an efficient price, charged to peak period users, would be substantially above imputed values that travelers currently pay.

It seems unlikely that municipalities are going to charge efficient prices for road use. Consequently, heavy subsidization of mass transit is often put forward as a second-best solution to congested roadways. To investigate this issue a common methodological approach has been to construct a generalized cost model to represent a segment of the roadway. These "steady-state" models utilize average traffic conditions and have been used to generate optimal bus frequencies.

While the motivation for this analysis stems from the desire to find ways to reduce congestion, the steady-state models we have surveyed are not especially thorough in identifying and measuring congestion effects. An exception is the model developed in Mohring [1979], which explicitly included auto traveler costs as well as bus passenger costs.
There are five types of congestion that we have identified as being important in a full accounting of travel cost. They are excess demand for the road, excess demand for the bus, bottleneck effects when a bus stops in a traffic lane to pick up passengers, a bus passenger's effect on bus speed, and the effect of an additional vehicle on traffic flow.

We developed a steady-state model and derived optimized results under two bus regimes. We considered seven variations of the model that allowed for marginal cost pricing, various levels of bus deficits and zero bus fares. The results of these optimizations were quite remarkable. Under all of the constrained models, speeds were very low, symptomatic of heavy congestion, and costs correspondingly high.

A reconstruction of the steady-state model was undertaken to explicitly include congestion costs. We began by specifying a revised speed-flow relationship, based on the free-flowing nature of the steady-state. We then developed new calculations for auto and bus travel time, based on our different types of congestion. This model, referred to as the RSS, was used to obtain optimized results to the same seven variations done for the steady-state model.

Our analysis of these results involved comparing the first-best model with the constrained models, comparing
the minibus results with the standard bus, and comparing the RSS results to those of the steady-state done in Chapter 2.

Our analysis concluded with a look at how sensitive the model was to changes in various parameter values. The results were generally anticipated. Variations that generated extremely high costs and heavy congestion involved parameter changes that were so dramatic as to cause the traffic network to degenerate into total gridlock.

There are four main conclusions that we can draw from results of the RSS model optimization.

First, a thorough accounting of congestion effects generates optimizations with high speeds and low costs. This result is compatible with the notion that congestion is a signal of inefficient resource cost. While the model developed here is far from comprehensive in its treatment of congestion, it does account for each of the five types listed. It is not until the traffic network is overwhelmed by demand, as occurred when we raised traveler flows to 600 per hour and journey lengths to 10 miles, that we are unable to find optimizations at high speeds.

Second, the results of second-best models, where there is no pricing of roads, can yield resource costs nearly as low as the first-best model. This result is quite the opposite of the steady-state model, where the
constraints caused much higher costs. This difference is attributable to the specification of the speed-flow relationship. The RSS model uses a free-flowing formulation, which is compatible with the nature of the steady-state.

Third, as long as minibus costs are relatively low, they can generate similar modal splits and resource costs as the standard bus, but at much lower deficit levels. If costs are proportional to size, this conclusion should hold. In our model, the standard bus cost four times as much as the minibus to operate, yet held more than five times as many passengers. In the sensitivity analysis for zero bus fares, when we raised minibus costs to $12, the deficit rose to $26.17. This is relatively close to the deficit of $31.04 created by the standard bus when it charges zero fares and costs $25 to operate per hour, per mile.

Fourth, as long as road use is unpriced, it appears that a low cost, high speed, optimized traffic network will require bus company subsidies. All of the zero subsidy models yielded heavily congested traffic. This confirms the common notion that without mass transit subsidies, congestion would be worse. However, the RSS model was only used to look at two competing bus regimes. A far more comprehensive analysis of competing modes would
need to be done before such a conclusion could more forcefully be made.

An obvious concern, that is not addressed, is that how the subsidy is to be financed. There certainly would be limits as to how large a subsidy would be acceptable. This model has been applied strictly to the transportation of individuals along a roadway during peak periods. As the model generates deficit levels, clearly a more comprehensive treatment would include the benefits and costs to taxpayers of providing such a subsidy.

The steady-state model developed in this dissertation, where congestion costs are included, is certainly not limited to the analysis presented here. There are three extensions of this model that would pick up where we have left off.

First, this model can be used to more comprehensively evaluate congestion-reducing policies. This may be done through the adjustment of parameter values. For example, a successful ride-sharing program would raise the number of travelers per auto. In the sensitivity analysis, we have observed the results of such a change.

Second, this model can be used to compare a wider spectrum of bus sizes. The results of the analysis presented in this dissertation are indicative of what further study would yield, but are still only a start in this regard.
Third, this model can be extended to cover different traffic characteristics. We considered the effect of increasing the number of lanes, which would fall into this category. Other changes that may bear further study are changing the length of the red and green phases and changing the maximum speed to accommodate different types of roads.
APPENDIX 1

THE STEADY-STATE MODEL:
AN ANNOTATED BASIC-LANGUAGE PROGRAM

10 HO=.3576 : HI=.0895 : T(1)=2.2 : TI=5.8 : C=2.7 : D=18
20 H=4.8 : HW=.5 : E=1.8 : B=25 : N=400 : K=1000

These three lines initialize the parameter values.
Their description appears on the Table of Symbols.

25 DIM J(50)

As part of the iteration procedure an array is kept
that measures the difference in costs between the
marginal bus passenger and marginal auto traveler.
These values are stored under the J label.
Henceforth, these types of statements will be referred
to as "Iteration statements."

50 FOR X=1 TO 100 STEP 1

This begins the loop for each value of X, the number
of buses per hour. The program locates the
cost-minimizing modal split for each value of X.
The program will then print out the information for
these modal splits for each value of X.

60 LOW=1 : HIGH=N : R=HIGH-LOW
65 Q=1
70 FOR NB=LOW TO HIGH STEP R/4
75 GOSUB 500
80 NEXT NB
85 GOSUB 700

Iteration statements. The program picks a range
of values for the number of bus passengers, from
a minimum of one, to a maximum of N and calculates
resource costs. The subprogram at statement 500
does the basic cost calculations, while the subprogram
at statement 700 sorts through the results, narrowing
the range over which the cost-minimizing modal split
is located. The Q value in statement 65 serves as
a counter.
90 IF R<=2 THEN GOTO 110
95 GOTO 70

Iteration statements. The range of values for NB which the program is searching is denoted by R. When the range is at, or below, two, the iteration process is completed.

110 IF MLOW < MMID THEN GOTO 118
112 IF MMID < MHIGH THEN GOTO 116
114 NB=HIGH : GOTO 150
116 NB=MID : GOTO 150
118 IF MLOW > MHIGH THEN GOTO 114
120 NB=LOW
150 GOSUB 500

Iteration statements. These statements identify the cost-minimizing modal split.

155 AOC = NA*(M*CA+P)/NV
160 ATC = M*NA*VA*TA/60
165 BOC = B*X*TB
170 BTC = VB*NB*(M*TB + A*TW)
175 Z = AOC + ATC + BOC + BTC

These are the final resource cost calculations, equivalent to equations (3), (4), (6), (8) and (10), respectively.

179 PRINT ...
.
186 PRINT ...

Various statements to print the results of the model.

190 NEXT X
200 END

The program continues to loop through to the next X value.

500 NA=N-NB

Once the value of NB is identified, the value of NA, the number of auto travelers, is calculated.

505 V=M*NA/NV+F*X

This statement calculates the volume of traffic.
Statement 510 shows the auto travel time, per mile, equivalent to equation (2). Statements 515 and 520 show the average time value for auto travelers and bus passengers, respectively.

Statement 530 evaluates the effect of an additional traveler on speeds, used to calculate the tolls, based on congestion.

Statement 532 is used for imposing the constraints on auto tolls and bus fares given by model II. This statement is skipped during the calculation of model I results, as it is a remark statement. When model II is run, the "REM" is deleted.

These two statements calculate the congestion costs imposed on other travelers by an additional auto traveler and an additional bus passenger, respectively. Statement 535 is equivalent to equation (13), while statement 540 is equivalent to equation (14).

Statement 545 establishes the auto running cost, which is equivalent to equation (1). Statement 550 shows the cost of the marginal auto traveler, and is equivalent to equation (11). Statement 555, showing the walking and waiting time for bus passengers, is equivalent to equation (9).

This statement shows the bus travel time, per mile, which is equivalent to equation (7).
565 \[ MCB = FB + VN \times (NB/N) \times (A \times TW + M \times TB) \]

This statement shows the calculation of the journey costs for the marginal bus traveler. This statement is equivalent to equation (12).

570 \[ J(Q) = |MCA - MCB| \]
575 \[ J(Q+10) = NB \]
580 \[ Q = Q + 1 \]
585 RETURN

These statements save modal split information for use in the sorting subprogram, which begins at statement 700.

700 IF \[ J(2) < J(5) \] THEN GOTO 750
715 \[ LOW = J(13) \]
720 \[ MLOW = J(3) \]
725 GOTO 820
750 IF \[ J(1) < J(4) \] THEN GOTO 800
760 \[ LOW = J(12) \]
765 \[ MLOW = J(2) \]
770 GOTO 820
800 \[ LOW = J(11) \]
805 \[ MLOW = J(1) \]
820 \[ R = HIGH - LOW \]
835 \[ Q = 1 \]
840 RETURN

Iteration subroutine. These statements sort through the modal split information to find the cost-minimizing solution.
APPENDIX 2

THE RSS MODEL:
AN ANNOTATED BASIC-LANGUAGE PROGRAM

These three lines initialize the parameter values. Their description, except for J, appears on the Table of Symbols. This term is used for the auto speed calculation done in line 300. If desired flows exceed the maximum allowable, J changes to -1 to correctly derive speeds. This is used to properly place the speed of traffic along either the upper portion, or the lower portion, of the speed-flow relationship shown in Figure 2.

As part of the iteration procedure an array is kept that measures the difference in costs between the marginal bus passenger and marginal auto traveler. These values are stored under the MCD label. The variable K is a counter used in the iteration procedure. Henceforth these types of statements will be referred to as "Iteration statements."

This begins the loop for each value of X, the number of buses per hour. The program locates the cost-minimizing modal split for each value of X. The program will then print out the information for these modal splits for each value of X.

Statement 103 establishes the waiting time for a bus passenger, and is used to calculate the total walking and waiting time in statement 105. The INT command means "integer value." These two statements are equivalent to equations (53) and (9), respectively.
These statements fix the green and red phases as close to 30 seconds as possible, while insuring that buses are uniformly spread across traffic platoons. The total cycle length is denoted as $C$.

$$\text{PSI}=\frac{3600}{(C \times X)}$$

$\text{PSI}$ is the number of platoons per bus, and is equivalent to equation (41).

Iteration statements. The program picks a range of values for the number of bus passengers, from a minimum of one, to a maximum of $N$ and calculates resource costs. The subprogram at statement 240 does the basic cost calculations, while the subprogram at statement 800 sorts through the results, narrowing the range over which the cost-minimizing modal split is located. Statement 147 reinitializes parameter values.

$$\text{IF RELP}=1 \ \text{THEN GOTO} \ 142$$

$$\text{IF } R \leq 6 \ \text{THEN GOTO} \ 160$$

$$\text{GOTO} \ 180$$

Iteration statements. RELP is a counter to determine when the new iteration is using the same high or low value. If so, the range is extended. The range
of values for NB which the program is searching is denoted by R. When the range is at, or below, six, the iteration process is completed.

160 IF MLOW < MMID THEN GOTO 168
162 IF MMID < MHIGH THEN GOTO 166
164 NB=HIGH : GOTO 172
166 NB=MID : GOTO 172
168 IF MLOW > MHIGH THEN GOTO 164
170 NB=LOW
172 PD=0 : BPD=0 : QS=0 : J=1
173 GOSUB 240
174 GOSUB 520

Iteration statements. These statements identify the cost-minimizing modal split and then reinitialize parameter values. Statement 173 calculates costs. The subprogram at statement 520 prints out the final results and then sends the program to the next value of X, bus frequency.

180 CTR=1
185 FOR NB=LOW TO HIGH STEP R/4
190 IF CTR=1 THEN GOTO 210
195 GOSUB 240
200 CTR=1 : K=K+1
205 GOTO 215
210 CTR=0 : K=K+1
215 PD=0 : BPD=0 : QS=0 : J=1
217 NEXT NB
220 GOSUB 800
222 IF RELP=1 THEN GOTO 142
225 GOTO 152

Iteration statements. These statements are purely for internal time saving for the computations.

240 NA=N-NB

Once the value of NB is identified, the value of NA, the number of auto travelers, is calculated.

242 IF NB <= BC*X/M THEN GOTO 250
244 BPD=3

When available bus capacity, BC*X/M, exceeds the desired number of bus passengers, a congestion delay, BPD, is imposed on bus passengers. This will affect their waiting time.
Traffic flow calculation, equivalent to equation (23).

To calculate the costs of adding an additional auto traveler the program keeps track of how this additional traveler affects different variable values. These type of statements are used to calculate the cost of the marginal auto traveler. This statement calculates the effect of an additional traveler on traffic flow.

When desired flow exceeds the maximum, actual flow must be calculated and the speed term must reflect that we are on the lower portion of the speed-flow curve. Thus, \( J \) becomes \(-1\) for statement 300. The program also sets the peak delay term, \( PD \), to three hours, and this affects resource costs.

Statement 300 is the calculation of the base travel speed of traffic, equivalent to equation (22). Statement 305 is the effect an additional traveler has on traffic speed.

Calculations of the length of the bus stop, equivalent to equation (16), average time value for auto travelers, equivalent to equation (5), average time value for bus passengers, and the number of stops a bus makes while in a platoon, equivalent to equation (17), respectively.
350 IF QS<=QM THEN GOTO 380
360 \[ D = \left( \frac{1800 \cdot C \cdot Y}{(Tg \cdot L)} \right) \cdot \left( \frac{R \cdot Q}{1800} \right) / Q \cdot \left( 1 - \frac{1}{2G} \right) + \frac{3600 \cdot PD}{2 \cdot (1 - Q/Qs)} \]
361 \[ DDA = \frac{5 \cdot 3600 \cdot PD \cdot L}{2 \cdot (-DA)} \]
362 \[ ADB = \left( \frac{1800 \cdot C \cdot Y}{(Tg \cdot L)} \right) \cdot \left( \frac{(R+2E) \cdot Q}{1800} \right) / Q \cdot \left( 1 - \frac{1}{2G} \right) \]
365 TC= 5
370 GOTO 500

Scenario 5 traffic conditions, where the desired flow exceeds the maximum allowable. The delay calculation, shown in statement 360, is equivalent to equation (47). Statement 361 shows the additional delay due to the marginal auto traveler, DDA, while statement 362, ADB, does the same for the marginal bus passenger.

380 IF Q*L/(L-1) > QM THEN GOTO 400
390 D=0 : ADB=0
391 DDA=0
392 TC=1
395 GOTO 500

Scenario 1 traffic conditions, where there is no congestion delay due to the bus stopping to pick up passengers.

400 \[ VD = R \cdot \left( \frac{Q}{1800} \right) \cdot L \cdot (1 - ((L-1)/L) \cdot \frac{Q}{Q}) \]
401 \[ DVDA = DQA \cdot R \cdot L / 1800 \]
402 \[ VDB = (R+2E) \cdot \left( \frac{Q}{1800} \right) \cdot L \cdot (1 - ((L-1)/L) \cdot \frac{Q}{Q}) - VD \]
410 \[ DR = TG \cdot L \cdot (QM - Q) / 1800 \]
411 \[ DDRA = -DQA \cdot TG \cdot L / 1800 \]
412 \[ ET = INT(VD/DR) : ETP = INT(0.5*VD/DR) \]
413 \[ DEA = INT((VD+DVDA)/(DR+DDRA)) - ET \]
415 \[ DEPA = INT(0.5*(VD+DVDA)/(DR+DDRA)) - ETP \]

These statement calculate the number of congested vehicles, when a bus stops, and their dispersal rates. Statement 400 is a rearranged version of equation (31), while statements 410 and 412 are equivalent to equations (32) and (36), respectively. Statements 401, 411, 413 and 415 are related to the calculation of the cost of the marginal traveler.
Scenario 4 traffic conditions, where the flow of vehicles equals the maximum, so that the bottleneck delay is permanent. Statements 441 and 442 relate to the calculation of the cost of the marginal traveler.

\[
\text{IF VD*TG/DR} \leq \frac{1800}{X} \text{ THEN GOTO 470}
\]

\[
\begin{align*}
\text{D1} &= \text{TR} \times (2\times\text{PSI}-1) \times ((G-.5)\text{VD}-.5\times\text{DR} \times (\text{PSI}-1)) \\
\text{D1A} &= \text{TR} \times (2\times\text{PSI}-1) \times (\text{DVDA} \times (G-.5) - .5 \times \text{DDRA} \times G \times (\text{PSI}-1)) \\
\text{ADB} &= \text{TR} \times (2\times\text{PSI}-1) \times ((G-.5)\text{VDB}-.5\times\text{DR} \times (\text{PSI}-1)) \\
\text{TC} &= 3 \\
\end{align*}
\]

GOTO 480

Scenario 4 traffic conditions, where there is insufficient excess capacity to disperse all the delayed vehicles before the next bus arrives. Statement 460 is equivalent to equation (45). Statements 461 and 462 relate to the calculation of the cost of the marginal traveler.

\[
\text{DD} = .5 \times \text{TG} \times (G-.75) \times \text{VD}^{**2} / \text{DR}
\]

\[
\begin{align*}
\text{DDDA} &= .5 \times \text{TG} \times (G-.75) \times (2\times\text{VD}\times\text{DVDA}/\text{DR} - \text{DDRA}\times\text{VD}^{**2}/(\text{DR}^{**2})) \\
\text{DDB} &= 2 \times (\text{DD}/(G-.75) \times ((1+E)/R)^{**2}-1) \\
\text{RLD} &= \text{TR} \times (G-1)^{**} \times (\text{ET}+1)^{**} \times (\text{VD}-.5\times\text{DR} \times \text{ET}) \\
& \quad + .5 \times \text{TR} \times (\text{ETP}+1)^{**} \times (\text{VD}-\text{DR} \times \text{ETP}) \\
\text{DRDA} &= \text{TR} \times (G-1)^{**} \times (\text{VD}-.5\times\text{DR} \times \text{ET}) \times \text{DEA} \\
& \quad + (\text{ET}+1)^{**} \times (\text{DVDA}-.5\times(\text{DR} \times \text{DEA} + \text{ET}\times\text{DDRA})) \\
& \quad + .5 \times \text{TR} \times ((\text{VD}-\text{DR} \times \text{ETP}) \times \text{DEPA} + (\text{ETP}+1)^{**} \\
& \quad \times (\text{DVDA} \times \text{DR} \times \text{DEPA} \times \text{ETP} \times \text{DDRA})) \\
\text{RDB} &= 2 \times (\text{ET}+1)^{**} \times \text{TR} \times \text{VD} \times E/R \\
\end{align*}
\]

\[
\text{D1} = \text{DD} + \text{RLD}
\]

\[
\begin{align*}
\text{D1A} &= \text{DDDA} \times \text{DRDA} \quad \text{ADB} = \text{DDB} \times \text{RDB} \\
\text{TC} &= 2
\end{align*}
\]

Scenario 2 traffic conditions, where all congested vehicles fully disperse before the next bus arrives. Statements 470 and 473 are equivalent to equations (35) and (40), respectively. Statement 477 is equivalent to equation (30). Statements 471, 472, 475, 476 and 478 relate to the calculation of the cost of the marginal traveler.

\[
\text{D2} = \text{TG} \times \text{L} \times \text{Q}/1800
\]

\[
\begin{align*}
\text{D2A} &= \text{DQA} \times \text{TG} \times \text{L}/1800 \\
\text{D3} &= (3600/C) \times (G/Y)/X \\
\text{D} &= \text{D1}/(\text{D2} \times \text{D3}) \\
\text{DDA} &= \text{D1A}/(\text{D2} \times \text{D3}) - \text{D1} \times \text{D2A}/(\text{D3} \times \text{D2}^{**2})
\end{align*}
\]
These are the final delay calculations, culminating with the calculations of auto travel time and bus travel time, shown in statements 500 and 503, which are equivalent to equations (19) and (18), respectively. Statements 480, 490, and 495 are equivalent to equations (25), (28) and (24), respectively. Statements 481, 496 and 501 relate to the calculation of the cost of the marginal traveler.

This statement is used for imposing the constraints on auto tolls and bus fares given by model II. This statement is skipped during the calculation of model I results, as it is a remark statement. When model II is run, the "REM" is deleted.

These two statements calculate the congestion costs imposed on other travelers by an additional auto traveler and an additional bus passenger. Statement 506 is equivalent to equation (13), except that the partial derivative of auto costs has dropped out due to its becoming an exogenous parameter.

These statements finish up the calculation of the journey costs for the marginal auto traveler, shown in 512, and the marginal bus passenger, shown in 513. These two statements are equivalent to equations (11) and (54), respectively, except that they are stated in per journey terms, rather than as per mile. Statements 514 and 515 store these results into the MCD array.
AOC = (M*NA/NV)*(CA+P/M+IMC)
ATC = M*NA*VA*TA
BOC = B*X*TB
BTC = (M*NB*VB*TB)+NB*A*VB*(TW+.5*BPD*(1-X*BC/(M*NB))
Z = AOC + ATC + BOC + BTC

These are the final resource cost calculations, equivalent to equations (49), (50), (51), (52) and (48), respectively. The calculation of bus passenger time costs, shown in statement 535, includes a bus congestion component. If there is no excess demand for the bus, then BPD is zero and the latter portion to this term drops out.

PRINT . . .

Various statements to print out the results of the model.

PD=0 : QS=0 : J=1 : BPD=0 : K=1 : W=0
NEXT X
END

The program reinitializes parameter values and counters. The program then continues to loop through to the next X value. When this looping is complete the program ends.

IF MCD(2) < MCD(5) THEN GOTO 850
LOW=MCD(13) : MID=MCD(14) : HIGH=MCD(15)
MCD(1)=MCD(3) : MCD(3)=MCD(4)
MCD(11)=MCD(13) : MCD(13)=MCD(14)
MLOW=MCD(3) : MMID=MCD(4) : MHIGH=MCD(5)
GOTO 920

IF MCD(1) < MCD(4) THEN GOTO 900
LOW=MCD(12) : MID=MCD(13) : HIGH=MCD(14)
MCD(1)=MCD(2) : MCD(5)=MCD(4)
MCD(11)=MCD(12) : MCD(15)=MCD(14)
MLOW=MCD(2) : MMID=MCD(3) : MHIGH=MCD(4)
GOTO 920

LOW=MCD(11) : MID=MCD(12) : HIGH=MCD(13)
MCD(5)=MCD(3) : MCD(3)=MCD(2)
MCD(15)=MCD(13) : MCD(13)=MCD(12)
MLOW=MCD(1) : MMID=MCD(2) : MHIGH=MCD(3)
GOTO 930
R=HIGH-LOW
K=1
Iteration subroutine. These statements sort through the values saved in the MCD array in order to narrow the range of the next search for the cost-minimizing modal split.
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