The Contract Disclosure Mandate and Earnings Management under External Scrutiny

November 30, 2019

Abstract

This paper studies the effects of mandating compensation disclosure on executive incentive contracts, and the ensuing effects on earnings management, and shareholders' and social welfare. We develop a moral hazard model with multiple principal-agent pairs facing an external inspector who allocates resources across firms to uncover and penalize earnings management. Contract disclosure confers principals with a first-mover advantage, allowing them to design the contract anticipating the inspector's reaction. However, it may also exacerbate a coordination problem among principals, as they do not consider externalities on other principals caused by the effects of their contract choices on the inspector's scrutiny allocation. We find that, if the penalty the inspector imposes on executives is relatively harsher than that imposed on shareholders, contracts become more strongly contingent on reported earnings, earnings are more severely manipulated, and social welfare increases with contract disclosure. However, disclosure improves shareholders' welfare only if the scrutiny resources available to the inspector are not strongly constrained.

1 Introduction

The separation of ownership and control breeds a demand for mechanisms to align the interests of those controlling the firm with those of the owners. Compensation contracts are one of the instruments firms make use of to satisfy such demand. However, the frequent dispersion of ownership, the prevalent asymmetry of information between management and shareholders, and the abundance of opportunities for management opportunistic behavior, among other hurdles, make the alignment difficult and costly for shareholders. Forming a compensation committee, as part of the board, is a usual remedy to overcome the high monitoring costs of dispersed shareholders. However, often, some of the incentive alignment hurdles that apply to managers apply also to compensation committees. To overcome such hurdles, the Securities and Exchange Commission (SEC) mandates the disclosure of management compensation information with the intention of facilitating the monitoring of external interested parties.[1][2] However, little is known of how the disclosure mandate affects the resulting contracts and the incentives of the signing parties (Jensen and Murphy 1990;

[1] See Yeaton (2007) and SEC Press release "SEC Adopts Rule for Pay Ratio Disclosure" (2015).

[2] In assessing materiality of misstatements, the incentive compensation is one of important factors according to SEC Staff Accounting Bulletin: No. 99 – Materiality.

Mas 2016, 2017; Gipper 2017). Are the resulting disclosed contracts more strongly contingent on performance? Do they reduce or increase opportunistic managerial misreporting? Is mandated contract disclosure beneficial for shareholders? Does it increase welfare? In this paper, we intend to answer these questions from an analytical perspective.

We examine a setting with multiple principal-agent pairs, each of which constitutes a firm. Every principal in the economy offers a contract to the corresponding agent, who can accept or reject it. If the agent rejects the contract he obtains his reservation utility, which we normalize to zero. All firms are potentially subject to scrutiny by an external inspector, whose objective is to uncover and punish earnings management. Based on all public information, the inspector scrutinizes reported earnings allocating its resources across all firms in the economy. The inspector represents any third-party who is interested in analyzing corporate financial reporting and detecting earnings management, such as a regulator (e.g., SEC), an auditor, financial analysts, the media, or even investors. The scrutiny of a firm is costly for the inspector. More specifically, the inspector incurs a direct cost, which increases with the level of inspection, and an indirect cost, which reflects the inter-dependencies among the costs of scrutinizing different firms. Specifically, the scrutiny of one firm can make the scrutiny of another firm more costly (i.e., the scrutiny decisions are substitutes) or less costly (i.e., the scrutiny decisions are complements). The substitutability among the scrutiny of different firms may indicate, for instance, that the inspector has limited resources, or that the inspector is forced to use resources of decreasing efficiency as the aggregate volume of inspection increases. The complementarity among scrutiny decisions may reflect the inspector's learning. For instance, inspecting one firm can make the scrutiny of other similar firms more efficient. Alternatively, if the inspector learns about a new way to manipulate earnings while inspecting one firm, this may facilitate the detection of the same kind of manipulation in other firms. Finally, although we relax this assumption later on, we initially assume that the inspector obtains an exogenous benefit from detecting earnings manipulation. In this setting, we compare two regulatory scenarios: one in which contracts are required to be disclosed, which we denote "public contract scenario", and another one in which contracts are kept private, which we denote "private contract scenario".

In the public contract scenario, the inspector can use the disclosed information about contracts to form conjectures about the level of earnings management in each firm in the economy. The observability of the contracts allows each principal to enjoy a first-mover advantage against the inspector. That is, each principal chooses the contract taking into consideration the inspector's reaction. Moreover, the contract choice also affects all other principals and agents in the economy through the inspector's allocation of scrutiny resources. Such externalities lead to a coordination problem among principals. The very incentive each principal has to distort the contract to influence the inspector's behavior ends up affecting all other principals and agents in the economy. In contrast, in the private contract scenario, the inspector cannot rely on any contract information to conjecture earnings management decisions. As a result, the principal cannot directly affect the inspector's choices and, thus, does not enjoy a first-mover advantage. However, the lack of influence on the inspector's scrutiny can potentially reduce inefficient contract distortions by alleviating the associated externalities across firms.

The tradeoff between benefiting from a first-mover advantage and suffering a coordination problem determines which contracting scenario makes principals better off. In extreme cases, such as a one-firm economy or an economy with a very large number of firms, externalities vanish or become immaterial. Therefore, contract disclosure still awards a first-mover advantage to principals, but coordination problems vanish or become inconsequential and, thus, principals are unambiguously better off in the public contract scenario. However, when there are multiple firms in the economy but the number of firms is not too large, the private contract scenario can be better for principals. If the scrutiny of one firm makes the scrutiny of other firms more costly (i.e., substitutes) and such interdependencies are strong enough, then externalities generated by contract disclosure may induce a coordination problem among principals to be severe enough to overcome the benefit of the first-mover advantage. In such situation, principals are better off in the private contract scenario.

We also examine the social welfare implications of contract disclosure. For this analysis, we assume that the penalties imposed by the inspector to both, principal and agent, are simply wealth transfers that do not affect welfare. That is, the social planner's objective is to maximize the aggregate surplus of all firms net of inspection costs. Nevertheless, we still assume that the inspector cannot commit to a level of scrutiny ex ante. Instead, we assume that the aggregate punishment the inspector imposes on firms offsets the inspector's benefit from manipulation detection. In such a setting, the social planner chooses all contracts taking into account externalities among principals and then, after firms report earnings, the inspector scrutinizes them to detect earnings manipulation. The optimal pay-performance sensitivity is larger than those in both, the public and the private, contract scenarios. As a result, mandating contract disclosure increases welfare if only if the marginal cost of scrutiny for the principal is negative. Equivalently, mandating contract disclosure increases welfare if only if the main effect of increasing scrutiny is to directly discipline the agent.

Our paper yields numerous empirical predictions. First, our model predicts that, ceteris paribus, mandating contract disclosure, or providing more information on compensation contracts, increases pay-performance sensitivity, the amount of earnings management, and the total amount of compensation in scenarios in which internal controls are relatively weak, or in regulatory scenarios in which the punishment for earnings manipulation is relatively stronger for agents than for principals. Second, our analysis also yields predictions for the cross-sectional variation of pay-performance sensitivity in both scenarios (i.e., before/after the adoption of a regulation that enforces an increase in contract disclosure). In both scenarios, our analysis predicts that pay-performance sensitivity increases (decreases) with the marginal punishment for the agent (the principal), and increases with the strength of internal controls. The effect of an increase in the cost of inspection is non-monotonic in the public contract scenario. However, an increase in the cost of scrutiny monotonically increases pay-performance sensitivity in the private contract scenario. Finally, in the public contract scenario, pay-performance sensitivity is expected to increase (decrease) with the number of peer firms if the punishment for detected earnings manipulation to principals is large (small) relative to that for agents. In the private contract scenario, on the other hand, the number of peer firms has no impact on pay-performance sensitivity.

Our paper provides some related policy implications. We show that mandating contract disclosure can produce unintended consequences. It can trigger a severe coordination problem among firms in setting their compensation contracts. We show that such coordination problems can lead to a loss of social welfare.

Consequently, the standard setter should take into account the externalities associated with the scrutiny imposed on firms and the interactions of such scrutiny with contract choices at the firm level.

The rest of paper is organized as follows. Section 2 reviews related literature and highlights our contribution. Section 3 presents our model. We first examine the public contract scenario and the private contract scenario separately and then compare across the two regimes. Section 4 provides welfare analysis and Section 5 offers empirical implications. Section 6 concludes the paper. All proofs are in the Appendix.

2 Literature

Our paper contributes to multiple streams of literature. First, it contributes to the literature on disclosure of executive compensation contracts. Executive compensation has been extensively examined in prior literature (Baker, Jensen, and Murphy, 1988; Murphy, 1999, 2013; Bebchuk and Fried, 2003; Frydman and Jenter, 2010; Shue and Townsend, 2017). Because weaker governance structures have greater agency problems leading to higher compensation to CEOs (Core, Holthausen, and Larker, 1999; Betrand and Mullainathan, 2001; Bebchuk, Fried, and Walker, 2002; Armstrong, Ittner, Larcker, 2012), pay transparency, as a disciplining and monitoring tool, has received attention recently and its effects have been closely examined. On the one hand, some studies argue that disclosure may improve stewardship and accountability, and render CEO pay more contigent on firm performance. For instance, Jensen and Murphy (1990) find that the average salary plus bonus for top-quartile CEOs fell in 1974-86 compared to 1934-38 due to political forces, fueled by the public disclosure of executive pay. Also, Mas (2017) shows that pay disclosure can lead to public pressure to decrease the salaries of top managers in the public sector. On the other hand, other studies argue that transparency may have unintended consequences and can even increase compensation (Hayes and Schaefer, 2009; Hermalin and Weisbach, 2012). For instance, Mas (2016) shows that, as a result of the mandated pay disclosure of 1934, even though CEOs with the highest pay saw a decline in their compensation due to its visibility and salience, average CEO compensation increased and pay-performance sensitivity declined (see also Card et al., 2013; Faulkender and Yang, 2013; Shue 2013; Gartenberg and Wulf, 2014). Also, using the introduction of the Compensation Discussion and Analysis (CD&A) in 2006 and its partial rollback in 2012 with the Jumpstart Our Business Startups (JOBS) Act, Gipper (2017) finds that mandating disclosure of management contracts is associated with higher compensation.[3] To a large extent, the above-mentioned studies analyze the impact of pay disclosure on a firm's optimal incentive design. However, the disclosure of executive compensation can also inform other agents in the economy that can ultimately affect the firm's performance. For instance, a firm's incentive system may be relevant to its competitors, its labor force, investors, and regulatory institutions. However, the externalities of pay disclosure have received little attention in the extant literature. Our paper intends to shed some light on one of such externalities. In particular, we examine how contracts are affected by disclosure in the presence of an external inspector that can detect earnings manipulation and punish firms and CEOs accordingly.

Our paper is also related to the literature on strategic delegation (Fershtman and Judd, 1987; Katz, 1991; Aggarwal and Samwick, 1999; Kedia, 2006; Bloomfield, 2018). Fershtman and Judd (1987) examine how

[3]See also Park, Nelson, and Huson (2001), Lo (2003), Craighead, Magnan, and Thorne (2004), and Schmidt (2012).

competing owners provide incentives strategically in an oligopoly economy. They show that principals may distort their agents' incentives to induce beneficial changes in the decisions of competing agents.[4] Also, Katz (1991) studies how unobservable and observable contracts, can serve as precommitments to strategically influence the equilibrium outcome. Bloomfield (2018) empirically shows that, after the introduction of CD&A, Cournot competitors are more likely to use revenue-based pay to commit to more aggressive product market competitive decisions. All these papers examine the use delegation in product-market competition. Instead, our paper examines the strategic interactions among firms that arise through the scrutiny of a common third party, the inspector. In such a setting, mandating the disclosure of contracts, provides a first-mover advantage to principals against the inspector but causes externalities that bring about a coordination problem among firms.

Our study is also related to the broad literature on regulatory enforcement (Kydland and Prescott, 1977; Dye, 2002; Liang, 2004; Chen, Mittendorf, and Zhang, 2010.; Ewert and Wagenhofer, 2016; Laux and Stocken, 2018). Dye (2010) analyzes how accounting standards influence reporting behavior and the evolution of accounting standards. Ewert and Wagenhofer (2016) look into the relationship between public enforcement and auditing, and find that enforcement and auditing can be complements or substitutes depending on the enforcement intensity. Laux and Stocken (2018) study how accounting standards and regulatory enforcement affect entrepreneurial innovation and social welfare and examine implications on the structure of regulatory penalties. Our paper differs from the extant literature in that it examines the coordination problem among firms induced by the allocation of scrutiny resources of a common enforcer. In addition, we specifically study the interaction of contract disclosure and enforcement.[5]

Lastly, our study is related to the literature on endogenous scrutiny as exerted, for instance, by auditors, boards of directors, and financial analysts (Fellingham and Newman, 1985; Dye, Balachandran, and Magee, 1990; Dye, 1993, 1995; Schwartz, 1997; Hillegeist, 1999, Laux and Laux, 2009; Corona and Randhawa, 2010, 2018). Hillegeist (1999) investigates how three alternative damage apportionment rules affect the strategic interaction among owners, auditors, and investors. Laux and Laux (2009) examine the design of incentive contracts and monitoring policies by the board of directors, and their effects on financial reporting, including the level of earnings management. Pae and Yoo (2001) analyze how a firm's internal control and its auditor's effort jointly influence the informativeness of the auditor's report. In contrast, we consider a setting in which an inspector endogenously allocates resources to the simultaneous scrutiny of multiple principal-agent pairs, and compare two regimes that differ only on the observability of the contracts that such pairs sign.

[4] See also Brander and Spencer (1983, 1985), Brander and Lewis (1986) and Maksimovic (1986).

[5] Bozanic et al. (2017) examine how public and private disclosure reguirement affect tax regulator enforcement and firms' public disclosure.

3 Model Setup

Players and Timeline

We consider a single-period contracting setting with three kinds of players: N identical principals (or shareholder, referred to as "she"), N identical agents (or managers, referred to as "he"), and one inspector. All agents are risk-averse and all other parties are risk-neutral. In any firm $i \in \{1,...,N\}$, the principal needs to hire an agent to operate it. All principals simultaneously, each one to a different agent, make a take-it-or-leave-it contract offer. Each agent can either accept or reject the offer. If the agent rejects the offer, the game ends.[6] If the agent accepts the contract offer, he chooses a level of effort, a_i, which is only observed by the agent himself. Productive effort affects true earnings $e_i = va_i + \varepsilon_i$, where $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$ is a random component of earnings that the agent cannot control, and $v > 0$ is the true earnings sensitivity to effort. The agent privately observes a true earnings and issues an earnings report $r_i = e_i + m_i$, which he can manipulate by adding a bias m_i to the true earnings number. The contract, which can only be contingent on reported earnings, r_i, determines the agent's compensation, $w_i(r_i)$. After the principal remunerates the agent according to the terms of the contract, based on all the public information $\mathscr{F}$, the inspector simultaneously scrutinizes all reports allocating resources optimally. The report of each firm i is scrutinized with intensity s_i to detect earnings management. The inspector can represent any third-party interested in and capable of detecting earnings management and directly or indirectly punish the principal and the agent accordingly. A clear example is a regulator such as the SEC. However, auditors, financial analyst, the business press, and even investors can also detect earnings management. Such external parties can reveal the detected manipulation and damage the reputation of the firm and the agent, and also alert other parties with enforcement powers. Both, the agent and the principal, are potentially subject to punishment for misreporting. Punishments are proportional to the amount of earnings management, m_i, times the level of scrutiny allocated to that firm by the inspector, s_i. We often refer to the product of those two magnitudes (i.e., $m_i s_i$) as *detected manipulation*.[7]

Since we want to examine the implications of contract disclosure, we examine and compare two scenarios: a regulatory scenario in which contracts are required to be disclosed and their fulfillment is verified (henceforth, *public contract scenario*), and another regulatory scenario in which contracts are kept private (henceforth, *private contract scenario*).[8] The following timeline summarizes the sequence of events:

1. All principal-agent pairs sign contracts simultaneously
2. Each agent i exerts effort a_i
3. Each agent i privately observes true earnings e_i and issues a public earnings report r_i
4. Principals and agents are remunerated according to the signed contract.

[6] Since all principal-agent pairs are identical, we abstract from any matching process. One can simply assume that principals and agents are randomly paired. Alternatively, one can just assume that there are more agents than principals and that searching for another agent is costless for the principal.

[7] In our main setting, we assume that the contract cannot be contingent on the manipulation detected by the inspector because manipulation detection and punishment take place after the contract is executed. However, we examine an extension in which the intensity of the internal scrutiny is part of the contract, and show that all our results do not change qualitatively.

[8] Under the private contract scenario, the principal can disclose the contract voluntarily. However, she can renegotiate privately afterwards and thus our results remain unchanged.

5. The inspector allocates resources $\{s_i\}$ to scrutinize all firms, and manipulation is punished.

Principal's Preferences

Each principal cares about her terminal terminal payoff V_i at date 5. The principal offers $w_i(r_i)$ to the agent to maximize the expected payoff:

$$\underset{w_i(r_i)}{Max}\, E_P[V_i] \tag{1}$$

The terminal payoff V_i reflects the true earnings of the firm, e_i, minus the penalty the principal pays for any detected earnings manipulation, $d_P m_i s_i$, and minus the compensation $w_i(r_i)$ paid to the agent, i.e., $V_i = e_i - d_P m_i s_i - w_i(r_i)$, where $d_P > 0$ is the intensity with which the principal is punished for earnings management. For tractability, we restrict the contract to be linear, i.e., $w_i(r_i) = \alpha_i + \beta_i r_i$, where α_i is the fixed part of the compensation and β_i measures the sensitivity of the compensation to reported earnings, r_i.

Agent's Preferences

The agent is risk-averse with a CARA utility function, $u(W_i) = -exp[-\rho W_i]$, where ρ is the coefficient of absolute risk aversion, and W_i is the terminal wealth of agent i. Given the restriction to linear contracts, the agent's terminal wealth can be written as,

$$W_i = \alpha_i + \beta_i r_i - \frac{c}{2}a_i^2 - \frac{k}{2}m_i^2 - d_A m_i s_i. \tag{2}$$

In the above expression, c reflects the convexity of the agent's personal cost of effort. The last two terms reflect two different costs that the agent incurs when manipulating earnings. The first term (i.e., $\frac{k}{2}m_i^2$) reflects the punishment imposed by the firm itself on the agent, which is increasing and convex in the amount of earnings management the agent commits. The convexity of such a punishment, k, represents, in a reduced form, the quality of the internal control of the firm, including considerations about the quality of the firm's information system and the intensity of the principal's monitoring. The last term in the expression, reflects the punishment imposed on the agent by the inspector (i.e., $d_A m_i s_i$). Here, the punishment is proportional to the intensity of the punishment $d_A > 0$ and to the detected manipulation $m_i s_i$.

Inspector's Preferences

The inspector allocates resources across firms to uncover earnings management. Given the publicly available information $\mathscr{F}$, the inspector forms conjectures about each agent i's manipulation $m_i^I(\mathscr{F})$, and chooses the level with which to scrutinize each firm, $s = (s_1, s_2, \cdots, s_N)$, to maximize its payoff:

$$\underset{s}{Max}\, b\sum_{i=1}^{N} m_i^I(\mathscr{F})s_i - \frac{w}{2}\sum_{i=1}^{N} s_i^2 - \frac{\gamma}{2(N-1)}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} s_i s_j, \tag{3}$$

where, to ensure an interior solution, we assume that $w^2 - \gamma^2 > 0$. Expression (3) 2reflects the assumption that the inspector benefits from uncovering earnings manipulation, but also needs to spend costly resources to obtain such benefit. The first term in the inspector's objective function indicates that she benefits from the aggregate amount of manipulation detection at a rate b, the marginal benefit of manipulation detection.[9] [10]The cost of scrutiny has two components. The first cost component captures the direct cost of scrutiny, and its convexity is given by w. The second component reflects the interdependencies among the costs of scrutinizing different firms. If $\gamma > 0$, allocating resources to scrutinize a firm is more costly if the inspector is already spending resources scrutinizing other firms. Such substitutability among the inspector's choices may, for instance, reflect the fact that inspection resources are limited. If, on the other hand, $\gamma < 0$, the scrutiny of one firm facilitates the scrutiny of other firms. This may capture the inspector's learning. For instance, the inspection of a firm in a certain industry may make the inspection of another firm in the same industry more efficient. Alternatively, uncovering a certain kind manipulation in one firm may make it easier for the inspector to detect the same kind of manipulation in other firms. Finally, if $\gamma = 0$, the cost of inspecting one firm is independent from the inspection of other firms. Notice that, the actions and choices of the principal and the agent in a firm depend on those of other firms only through the inspector's scrutiny allocation. Indeed, γ is an important parameter in this analysis that shapes the interaction between firms and affects the equilibrium contracts. The coefficient γ is divided by $N-1$ to reflect the fact that, as the number of firms increases, the size of each interaction among firms should be less important. As the number of firms increases, the regulator needs to spend more resources in scrutiny, making the significance of each interaction between a pair of firms less critical. The number of direct cost terms in the inspectors objective (i.e., $\frac{w}{2}\sum_{i=1}^{N} s_i^2$) increases in proportion to the number of firms, N, but the number of interactions ($\frac{\gamma}{2(N-1)}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} s_i s_j$) increases with $N(N-1)$. Thus, without dividing by $N-1$, increasing the number of firms would eventually make the direct cost of inspecting a firm irrelevant, which does not seem reasonable.

3.1 Public Contract Scenario

In this subsection, we analyze the public contract scenario. The timeline outlined above is still descriptive, but with the qualification that all contracts are publicly disclosed once signed. Therefore, all players can observe all contracts after date 1 and, as a result, the publicly available information $\mathscr{F}$ includes all earnings reports and all contracts, i.e., $\mathscr{F} = \{r_i, \alpha_i, \beta_i\}_{i=1}^{N}$. Proceeding by backward induction, we first examine the inspector's scrutiny decisions. At date 5, the inspector forms conjectures about the manipulation decisions of all agents, i.e., $\{m_i^I\}_{i=1}^{N}$ and decides the level of scrutiny for each firm with the goal of maximizing her payoff:

$$\underset{s}{Max}\; b\sum_{i=1}^{N} m_i^I s_i - \frac{w}{2}\sum_{i=1}^{N} s_i^2 - \frac{\gamma}{2(N-1)}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} s_i s_j, \tag{4}$$

[9]Since the inspector represents any external interested parties such as a regulator, an auditor, financial analysts, the media or investors, their benefit of detecting earnings manipulation might be different and, therefore, we assume a general form of benefit. For instance, by detecting earnings manipulation the inspector can gain reputation or promotion.

[10]The inspector's objective function can be seen as the result of delegation. A benevolent social planner without commitment would not try to detect earnings management because it is costly to do so, and since at that point agents have already made their deicions, inspection would only decrease welfare. Therefore, creating a supervising institution and delegating the task of detecting earnings management becomes the only way the social planner can commit to a scrutiny. If instread we allow for commitment, then there is no difference between the private and public scenarios because the inspector's behavior is not affected by the contracts ex post.

where, $s = (s_1, s_2, \cdots, s_N)$. From (4), we obtain the following set of N first order conditions:

$$bm_i^I - ws_i - \frac{\gamma}{(N-1)} \sum_{j=1, j\neq i}^{N} s_j = 0 \; \forall i \in \{1, ..., N\} \tag{5}$$

Solving the system of Nequations we obtain a unique solution for the inspector's scrutiny decisions:

$$s_i = b\frac{(N-1)m_i^I - \frac{\gamma}{w+\gamma}\sum_{j=1}^{N} m_j^I}{(N-1)w - \gamma} \; \forall i \in \{1, ..., N\}. \tag{6}$$

Note that in expression (6) the scrutiny of given firm i, s_i, increases with the inspector's conjecture of the amount of manipulation in that same firm, m_i^I. Indeed, the marginal benefit of scrutinizing firm i increases with the amount of earnings management in that same firm. However, depending on the sign of γ, s_i may increase or decrease with the inspector's conjectured manipulation in other firms, m_j^I with $j \neq i$. The cost the inspector incurs in scrutinizing firm i depends on how much the inspector scrutinizes other firms. If $\gamma > 0$, a larger conjectured manipulation in other firms induces the inspector to scrutinize them more intensely. That in turn makes the scrutiny in firm i more costly and, thus, induces the inspector to reduce s_i. Thus, s_i decreases in the conjectured manipulation in other firms. However, if $\gamma < 0$, increasing the scrutiny of other firms reduces the cost of scrutinizing firm i. As a result, s_i increases in the conjectured manipulation in other firms.

At date 3, the agent observes the realization of true earnings in his firm and issues a report with the goal of maximizing his utility. At this point in time, earnings are realized and, thus, the agent faces no uncertainty. Given that the agent's utility is monotonically increasing in his terminal wealth, the maximization of his utility is equivalent to the maximization of his terminal wealth W_i, as defined in (2). That is, we can consider that every agent i solves the program:

$$\underset{m_i}{Max}\, W_i \tag{7}$$

The first order condition of the program above yields the optimal manipulation choice for the agent as a function of the anticipated inspector's scrutiny decision:

$$m_i = \frac{\beta_i - d_A s_i}{k} \tag{8}$$

Intuitively, the amount of manipulation increases in the slope of the contract and decreases with the anticipated level of scrutiny.

At date 2, agent i makes the productive effort decision anticipating his earnings manipulation decision at date 3 and the inspector's scrutiny decision at date 4. The agent faces now the uncertainty of the realization of earnings, but the CARA utility assumption jointly with the normality of ε_i allows us to express the agent's expected utility maximization program as an equivalent program in which the agent maximizes the certainty equivalent, CE_i. Indeed, we can write, $E[u(W_i)] = u[CE_i]$, where the certainty equivalent is given by $CE_i = E[W_i] - \frac{\rho}{2}Var[W_i]$, more specifically, $CE_i = \alpha_i + \beta_i(va_i + m_i) - \frac{c}{2}a_i^2 - \frac{k}{2}m_i^2 - d_A m_i s_i - \frac{\rho}{2}\beta_i^2\sigma_\varepsilon^2$. Therefore, we can consider that the agent solves:

$$\underset{a_i}{Max}\, CE_i \tag{9}$$

The optimal level of effort is obtained from the first-order conditions of the above program:

$$a_i = \frac{v\beta_i}{c} \tag{10}$$

Since the contract is public, at the time when the agent makes his productive effort decision, neither the agent nor the inspector have any private information. Therefore, the continuation game at this point constitutes a proper subgame that is played by all agents and the inspector. Thus, from equations (8) and (6) we can solve for the subgame-equilibrium earnings manipulation and scrutiny decisions,

$$m_i = \frac{(k(w+\gamma)+bd_A)((N-1)w-\gamma)\beta_i + bd_A\gamma\sum_{j=1}^{N}\beta_j}{(k(w+\gamma)+bd_A)(k((N-1)w-\gamma)+(N-1)bd_A)} \tag{11}$$

$$s_i = b\frac{(k(w+\gamma)+bd_A)(N-1)\beta_i - k\gamma\sum_{j=1}^{N}\beta_j}{(k(w+\gamma)+bd_A)(k((N-1)w-\gamma)+(N-1)bd_A)} \tag{12}$$

Earnings manipulation and scrutiny decisions are interrelated. Expression (6) reveals that the intensity with which firm i is inspected is contingent on the inspector's conjectured amounts of manipulation in all firms in the economy, and expression (8) indicates that any agent i manipulates earnings anticipating the intensity with which firm i will be scrutinized. In the subgame equilibrium, the expressions for the agent's productive effort (10) and manipulation (11), and the inspector's scrutiny (12) are all contingent on the publicly observable contract slopes, β. In fact, all agents and the inspector play a simultaneous-choice game which principals can affect with the contracts they offer. Indeed, although principal i can fully determine agent i's productive effort with the contract slope, β_i, the amount of earnings management and scrutiny in firm i are also contingent on the contract slopes chosen by all other principals in the economy. The following Lemma states such contingency formally:

Lemma 1. *In the public contract scenario, the productive effort, a_i, the amount of earnings management, m_i, and the scrutiny of firm i, s_i, increase with the contract slope in the firm,β_i. Also, m_i increases (decreases) and s_i decreases (increases) with the contract slopes in other firms, β_j with $j \neq i$, if $\gamma > (<)0$. Formally,*

$$\frac{da_i}{d\beta_i} > 0, \frac{dm_i}{d\beta_i} > 0, \frac{ds_i}{d\beta_i} > 0, \tag{13}$$

$$Sign\left[\frac{dm_i}{d\beta_j}\right] = Sign\left[\gamma\right], Sign\left[\frac{ds_i}{d\beta_j}\right] = -Sign\left[\gamma\right] \; with j \neq i. \tag{14}$$

The scrutiny of firm i, s_i, is increasing in the slope of its contract, β_i. Certainly, observing a steeper contract slope, the inspector conjectures that a larger amount of earnings management is committed, and thus increases the scrutiny. Also, if scrutiny decisions are substitutes (i.e., $\gamma > 0$), then s_i decreases with increases in the contract slopes of other firms. Noticing a steeper contract slope in other firms, the inspector conjectures they committed more manipulation. Thus, the inspector increases the scrutiny of other firms, in turn increasing the cost of scrutinizing firm i, and thereby inducing the inspector to lower the intensity of inspection in firm i. If instead the scrutiny decisions are complements (i.e., $\gamma < 0$), then s_i intensifies with increases in the contract slopes in other firms. Those steeper slopes suggest larger manipulation in those firms, to which the inspector reacts with a more intense scrutiny. However, now that actually cheapens the scrutiny of firm i, inducing the inspector to intensify it.

The effect of an increase in the contract slope on manipulation can be best characterized by decomposing it as follows:

$$\frac{dm_i}{d\beta_i} = \frac{\partial m_i}{\partial \beta_i} + \frac{\partial m_i}{\partial s_i}\frac{ds_i}{d\beta_i}, \tag{15}$$

Using expression (8) we can see that the first term is always positive, $\frac{\partial m_i}{\partial \beta_i} = \frac{1}{k} > 0$, and reflects the straightforward intuition that a steeper contract slope increases the marginal benefit of manipulation for the agent, which induces the agent to manipulate more. The second term, however, indicates that the observability of the contracts attenuates this effect. An increase in β_i increases the inspector's scrutiny (i.e., $\frac{ds_i}{d\beta_i} > 0$), thereby decreasing the agent's incentives to manipulate ($\frac{\partial m_i}{\partial s_i} = -\frac{d_A}{k} < 0$). In addition, earnings manipulation is also affected by the contract slopes of other firms. If scrutiny decisions are substitutes, the inspector pays more attention to other firms as they increase their contract slopes, and that reduces the attention the inspector pays to firm i, facilitating earnings manipulation. If instead scrutiny decisions are complements, an increase in the slopes of other firms facilitates the inspection of firm i, attenuating earnings manipulation.

At date 1, all principals simultaneously choose the contracts for their firms. Specifically, each principal i chooses the optimal compensation contract $\{\alpha_i, \beta_i\}$ to maximize her expected payoff, subject to three incentive compatibility constraints, (10), (11), and (12), and a participation constraint for the agent, $CE_i \geq 0$. Since the principal enjoys all the bargaining power, the agent's participation constraint binds. Therefore, the fixed salary, α_i, is determined by the participation constraint, and can be replaced in the principal's objective function, obtaining $E_P[V_i] = va_i - d_P m_i s_i - \frac{c}{2}a_i^2 - \frac{k}{2}m_i^2 - d_A m_i s_i - \frac{\rho}{2}\beta_i^2\sigma_\varepsilon^2$. Thus, the principal of firm i solves the program:

$$\underset{\beta_i}{Max}\, E_P[V_i] \tag{16}$$

$$s.t.(10), (11), and\, (12) \tag{17}$$

Using the constraints we can express the program in the following way:

$$\underset{\beta_i}{Max}\, va_i(\beta_i) - \mathscr{C}(\beta_i, s_i(\beta_i)). \tag{18}$$

In the above program, $\mathscr{C}(\beta_i, s_i(\beta_i)) = \frac{c}{2}a_i(\beta_i)^2 + \frac{k}{2}m_i(\beta_i, s_i)^2 + (d_P + d_A)m_i(\beta_i, s_i)s_i(\beta_i) + \frac{\rho}{2}\beta_i^2\sigma_\varepsilon^2$, denotes the sum of all the costs incurred by the principal. This notation allows us to express our results in a much more succinct and intuitive manner. Obtaining the first order condition for each firm i results in a system of N equations of the form:

$$v\frac{da_i}{d\beta_i} = \frac{\partial \mathscr{C}_i}{\partial \beta_i} + \frac{\partial \mathscr{C}_i}{\partial s_i}\frac{ds_i}{d\beta_i}\, for\, all\, i \in \{1, ..., N\}. \tag{19}$$

Each equation above requires the equilibrium contract slope in each firm i, β_i, to balance the marginal benefit with the marginal costs. The marginal cost has two components. The first component (i.e., $\frac{\partial \mathscr{C}_i}{\partial \beta_i}$)

is attributable to the direct effects that a change in the contract slope has on the agent's productive effort and manipulation decisions keeping the inspector's scrutiny constant. The second component (i.e., $\frac{\partial \mathscr{C}_i}{\partial s_i}\frac{ds_i}{d\beta_i}$) reflects the marginal cost of changing the inspector's scrutiny by changing the contract slope. More specifically, this term results from the multiplication of $\frac{\partial \mathscr{C}_i}{\partial s_i}$, the marginal cost of scrutiny, and $\frac{ds_i}{d\beta_i}$, the change in scrutiny induced by an increase in the contract slope. Solving the system of equations in (19) yields closed form solutions for the the equilibrium contracts $\{\alpha_i, \beta_i\}$ $\forall i \in \{1,...,N\}$, and for all the equilibrium agent actions. The symmetry of the equilibrium allows us to write the marginal costs as $\frac{\partial \mathscr{C}_i}{\partial \beta_i} = \left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o \beta_{Pub}$ and $\frac{\partial \mathscr{C}_i}{\partial s_i} = \left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o \beta_{Pub}$, where $\left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o$ and $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o$ are constants and β_{Pub} is the equilibrium contract slope, symmetric across contracts in the public contract scenario. Proposition 1 describes the equilibrium.

Proposition 1. *(Equilibrium with Public Contracts) There exist a unique equilibrium in which contracts and actions are symmetric across firms. Specifically, $\alpha_i = \alpha_{Pub}$ and $\beta_i = \beta_{Pub}$ for all $i \in \{1,...,N\}$, where*

$$\alpha_{Pub} = \frac{c}{2}a_{Pub}^2 + \frac{k}{2}m_{Pub}^2 + d_A m_{Pub} s_{Pub} - (v a_{Pub} + m_{Pub})\beta_{Pub} + \frac{\rho}{2}\beta_{Pub}^2\sigma_\varepsilon^2, \tag{20}$$

$$\beta_{Pub} = \frac{v\frac{da_i}{d\beta_i}}{\left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o + \left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o \frac{ds_i}{d\beta_i}}. \tag{21}$$

Also, equilibrium effort, earnings management, and scrutiny levels are respectively given by $a_i = a_{Pub}$, $m_i = m_{Pub}$, and $s_i = s_{Pub}$ for all $i \in \{1,...,N\}$, where:

$$a_{Pub} = a_0\beta_{Pub},\ \text{where } a_0 = \frac{v}{c} \tag{22}$$

$$m_{Pub} = m_0\beta_{Pub},\ \text{where } m_0 = \frac{(w+\gamma)}{k(w+\gamma)+bd_A} \tag{23}$$

$$s_{Pub} = s_0\beta_{Pub},\ \text{where } s_0 = \frac{b}{k(w+\gamma)+bd_A} \tag{24}$$

and $\left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o = ca_0\frac{\partial a_i}{\partial \beta_i} + km_0\frac{\partial m_i}{\partial \beta_i} + (d_A+d_P)s_0\frac{\partial m_i}{\partial \beta_i} + \rho\sigma_\varepsilon^2$, $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o = km_0\frac{\partial m_i}{\partial s_i} + (d_A+d_P)(s_0\frac{\partial m_i}{\partial s_i} + m_0)$, $\frac{ds_i}{d\beta_i} = \frac{b(N-1)-s_0 k\gamma}{k((N-1)w-\gamma)+(N-1)bd_A}$, $\frac{da_i}{d\beta_i} = \frac{v}{c}$, $\frac{\partial m_i}{\partial \beta_i} = \frac{1}{k}$, $\frac{\partial m_i}{\partial s_i} = -\frac{d_A}{k}$.

The expression for the equilibrium pay-performance sensitivity in the public contract scenario, β_{Pub}, is quite intuitive. The numerator reflects the sensitivity of the principal's benefit of productive effort to the pay-performance sensitivity, whereas the denominator aggregates the different marginal costs that the principal incurs per unit of the pay-performance sensitivity. The first term in the denominator,

$$\left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o = ca_0\frac{\partial a_i}{\partial \beta_i} + km_0\frac{\partial m_i}{\partial \beta_i} + (d_A+d_P)\frac{\partial m_i}{\partial \beta_i}s_0 + \rho\sigma_\varepsilon^2, \tag{25}$$

aggregates marginal costs directly attributable to an increase in the contract slope keeping the inspector's scrutiny constant. These include the marginal cost of productive effort, $ca_0\frac{\partial a_i}{\partial \beta_i}$, the agent's marginal costs of manipulation enforced by internal controls, $km_0\frac{\partial m_i}{\partial \beta_i} > 0$, the agent's punishment by the inspector, $d_A s_0\frac{\partial m_i}{\partial \beta_i} > 0$, the principal's punishment by the inspector, $d_P s_0\frac{\partial m_i}{\partial \beta_i} > 0$, and the risk premium $\rho\sigma_\varepsilon^2 > 0$. The second term in the denominator, $[\partial\mathscr{C}_i/\partial s_i]_o \frac{ds_i}{d\beta_i}$, aggregates the marginal costs generated indirectly by the inspector's scrutiny response to an increase in the contract slope. From Lemma 1, we know that an increase in the contract slope induces the inspector to increase the scrutiny of the firm, $\frac{ds_i}{d\beta_i} > 0$, but a scrutiny increase can affect the principal's costs positively or negatively depending on the parameter values. Indeed, the term,

$$\left[\frac{\partial\mathscr{C}_i}{\partial s_i}\right]_o = km_0\frac{\partial m_i}{\partial s_i} + (d_A + d_P)(\frac{\partial m_i}{\partial s_i}s_0 + m_0), \tag{26}$$

is a key element in the comparison between scenarios, and its sign is the result of the balance between two countervailing effects. On the one hand, scrutiny decreases the manipulation committed by the agent, $\frac{\partial m_i}{\partial s_i} = -\frac{d_A}{k} < 0$, and that unambiguously decreases the costs of manipulation attributable to both internal controls, $km_0\frac{\partial m_i}{\partial s_i}$, and external controls, $(d_A + d_P)s_0\frac{\partial m_i}{\partial s_i}$. On the other hand, for a fixed manipulation level, scrutiny increases the external punishment directly by increasing the proportion of manipulation detected by the inspector. Such a marginal effect is represented by the term, $(d_A + d_P)m_0$. Therefore, increasing scrutiny can potentially affect the principal's objective positively or negatively. The following lemma provides some insight about the sign of this constant:

Lemma 2. $\left[\frac{\partial\mathscr{C}_i}{\partial s_i}\right]_o > 0 \Longleftrightarrow d_A < \psi(d_P) \Longleftrightarrow k > \bar{k} \equiv \frac{bd_A(d_A+d_P)}{d_P(w+\gamma)}$,
where $\psi(d_P)$ is monotonically increasing in d_P.

An increase in scrutiny increases a principal's cost if the punishment level for the agent, d_A, is relatively small compared to that for the principal, d_P. Increasing d_A reinforces the inhibiting effect of scrutiny on manipulation, and that in turn reduces the cost of internal enforcement and the costs of external enforcement for both principal and agent. However, a higher d_P does not directly change the effect of scrutiny on manipulation. Given a fixed contract, punishing the principal more only increases the direct cost of manipulation for the principal. Therefore, if d_A is relatively large compared to d_P, the inspector's scrutiny reduces the cost of manipulation for the principal. However, if d_P is relatively large compared to d_A, the opposite is true. The third inequality in Lemma 1 reveals that these countervailing effects do not matter if the internal enforcement cost level, k, is large enough. In such a case, increasing scrutiny is always costly for the principal.

Proposition 1 also states that the equilibrium levels of effort a_{Pub}, earnings management m_{Pub}, and inspection s_{Pub} increase with the equilibrium pay-performance sensitivity β_{Pub}. This functional form will prove relevant in the comparison of the levels in the equilibrium decisions across scenarios.

Since in the public contract scenario both outcomes and contracts are public knowledge, at this point one may wonder whether considering some sort of relative performance measurement could improve contractual efficiency. The answer is that, given our assumptions, principals have no incentive to make the contract contingent on the outcome or the contract slope of other firms. First, since the uncertain component of earnings is independent across firms, there is no informational benefit in including the outcomes of other firms in the contract. Second, since all principals choose contracts simultaneously, they cannot observe each other's contracts at the time they make their choices. Even though a firm's contract can still be contingent on the equilibrium slopes of other firms, at the time the contract is determined, principals can only conjecture such contracts. Thus, principals cannot directly affect each other's contract choices. Finally, although each principal's choice of contract affects the payoffs of other principals through the inspector's scrutiny allocation, he does not have an incentive to take into account such externalities.

Given the prominence that β_{Pub} plays in the equilibrium decisions, it is informative to examine how it is affected by changes in model parameters. Corollary 1 states the results of our comparative statics analysis.

Corollary 1. *The equilibrium contract slope in the public contract scenario, β_{Pub}, changes with the parameters in the model as follows:*

1. $\frac{\partial \beta_{Pub}}{\partial d_A} > 0$; $\frac{\partial \beta_{Pub}}{\partial d_P} < 0$; $\frac{\partial \beta_{Pub}}{\partial k} > 0$;
2. *Changes with w:*
   (a) $\frac{\partial \beta_{Pub}}{\partial w} > 0$ *if* $d_A \to 0$;
   (b) $\frac{\partial \beta_{Pub}}{\partial w} < 0$ *if* $d_P \to 0$;
3. *Changes with γ:*
   (a) *If* $d_A \to 0$, *then* $\frac{\partial \beta_{Pub}}{\partial \gamma} > 0$ *for* $\gamma < \frac{w}{2+\sqrt{3}}$, *and* $\frac{\partial \beta_{Pub}}{\partial \gamma} < 0$ *for* $\gamma > \frac{w(N-1)}{2+\sqrt{3}}$;
   (b) *If* $d_P \to 0$, *then* $\frac{\partial \beta_{Pub}}{\partial \gamma} < 0$ *for* $\gamma < \frac{kw+bd_A}{3k}$, *and* $\frac{\partial \beta_{Pub}}{\partial \gamma} > 0$ *for* $\gamma > (N-1)\frac{kw+bd_A}{2k}$;
   (c) *If* $\gamma \to 0$ *then* $\frac{\partial \beta_{Pub}}{\partial \gamma} > (<) 0$ *iff* $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o > (<) 0$;
4. $\frac{\partial \beta_{Pub}}{\partial N} > (<) 0$ *if and only if* $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o > (<) 0$ *and* $\gamma \neq 0$.

The first point in Corollary 1 states that the pay-performance sensitivity, β_{Pub}, is affected by the level of the three different manipulation costs faced by the principal. An increase in k increases the marginal cost or manipulation for the agent, reducing his incentives to manipulate for any given contract slope. Anticipating this, the principal is able to increase β_{Pub} inducing less manipulation. An increase in d_A, however, increases β_{Pub} through three channels. The first effect is analogous to the one generated by an increase in k. In short, an increase in d_A directly increases the marginal cost of manipulation for the agent, curbing manipulation, and thereby allowing the principal to increase incentives. Such a lower manipulation level generates two additional effects. On the one hand, given a level of scrutiny, the cost of such scrutiny is lower because there is less of a reason to punish principal and agent for. On the other hand, conjecturing a lower manipulation level, the inspector reduces the scrutiny of the firm, thereby reducing the punishment for both, principal and agent. These three effects allow the principal to increase β_{Pub}. In contrast, an increase in d_P does not directly effect the agent's incentives to manipulate nor does it affect the inspector's incentives to scrutinize the firm. It simply increases the marginal punishment for the principal at given level of manipulation. As a result, the principal decreases β_{Pub} to reduce the agent's incentives to manipulate.

The second point in Corollary 1 examines how β_{Pub} changes with the extent to which scrutiny is costly for the inspector, w. An increase in w affects β_{Pub} in two main ways. It reduces scrutiny, reducing the detection of manipulation and its associated cost for the principal, allowing the principal to increase β_{Pub}. However, the lower scrutiny also increases the agent's incentives to manipulate, inducing the principal to decrease the contract slope to avoid higher manipulation costs. If d_A is small relative to d_P the direct punishment to the principal takes center stage making the former effect dominant, but if d_P is small relative to d_A then the agent's manipulation cost is central and the latter effects dominates.

The third point in Corollary 1 examines the effects of changes in γ on β_{Pub}. An increase in γ produces three main effects on the equilibrium contract slope. The first two effects are analogous to the effects of w. The third effect pertains to the inspector's allocation of scrutiny resources. Essentially, one can see an increase in γ as tightening the inspector's resources. As such, increasing γ makes the inspector's scrutiny more sensitive to an increase in the contract slope. This last effect is muted if γ is small, and that reduces the effects of a change in γ to those of a change in w. However, if γ is large enough, the last effect dominates. An increase in γ then intensifies the inspector's scrutiny reaction to an increase in the contract slope. The effect of such increased scrutiny, however, depends on which punishment is more salient. If d_P is large relative to d_A, then scrutiny directly increases the principal's punishment, who reacts by decreasing the slope of the contract. However, if d_A is large relative to d_P, then an increase in scrutiny reduces the agent's incentive to manipulate, and that allows the principal to increase the contract slope.

Lastly, as N increases, principals see their individual influence on the inspector decrease. This can be clearly seen mathematically because, in the expression for β_{Pub}, the only term affected by a change in the number of firm is $\frac{ds_i}{d\beta_i}$. Intuitively, as the number of firms increases, the inspector's attention is spread among more firms, and each firm's actions have a smaller impact on the inspector's resource allocation decisions. If scrutiny increases the cost for the principal (i.e., $[\partial\mathscr{C}_i/\partial s_i]_o > 0$), then an increase in the number of firms decreases the cost of increasing the contract slope for the principal, who can then increase it. However, if scrutiny decreases the cost for the principal (i.e., $[\partial\mathscr{C}_i/\partial s_i]_o < 0$), then an increase in the number of firms produces the opposite effect, inducing the principal to decrease the contract slope.

3.2 Private Contract

In the private contract scenario, the inspector, as well as other principal-agent pairs, cannot observe the contracts signed by any of the principal-agent pairs. That is, the publicly available information, $\mathscr{F}$, includes only earnings reports, i.e., $\mathscr{F} = \{r_i\}_{i=1}^N$. The derivation of the equilibrium in this scenario is analogous to the one in the public contract scenario but with one important difference: the inspector cannot rely on the contract to conjecture the agent's actions. When deciding the allocation of resources to the scrutiny of different firms, the inspector knows neither the actions of the principals nor the actions of the agents. The inspector can only conjecture them. In essence, with private contracts, there is no proper subgame. Thus, every principal $i \in \{1,...,N\}$ solves the following program,

$$\underset{\beta_i}{Max}\; va_i - d_P m_i s_i - \frac{c}{2}a_i^2 - \frac{k}{2}m_i^2 - d_A m_i s_i - \frac{\rho}{2}\beta_i^2\sigma_\varepsilon^2 \tag{27}$$

$$s.t.\ a_i = \frac{v\beta_i}{c}, \tag{28}$$

$$m_i = \frac{\beta_i - d_A s_i}{k}, \tag{29}$$

$$s_i = b\frac{(N-1)m_i^I - \frac{\gamma}{w+\gamma}\sum_{j=1}^N m_j^I}{(N-1)w-\gamma}. \tag{30}$$

Using the constraints we can again express the program in the following way:

$$\underset{\beta_i}{Max}\ va_i - \mathscr{C}(\beta_i, s_i). \tag{31}$$

Notice that now the inspector's scrutiny decision s_i is not contingent on β_i because the inspector cannot observe the contract. Indeed, as shown in expression (30), the inspector's scrutiny decisions are only contingent on the inspector's conjectures regarding the earnings manipulation in all firms, m_j^I for all $j \in \{1,...,N\}$. All principals solve a program like (31) simultaneously, each one anticipating the decisions of her agent and those of the inspector, and conjecturing the decisions of all other principals and agents. Obtaining the first order condition for each principal i results in a system of N equations of the form:

$$v\frac{da_i}{d\beta_i} = \frac{\partial \mathscr{C}_i}{\partial \beta_i}\ for\, all\, i \in \{1,...,N\}. \tag{32}$$

Comparing the above first order conditions with the analogous ones in the public contract scenario (19), one can notice that now the denominator contains only the partial derivative as opposed to the total derivative. Since in this scenario the inspector cannot observe the contract, the principal cannot affect the inspector's scrutiny with the contract choice. Formally, this can be expressed as $\frac{ds_i}{d\beta_i} = 0$. In other words, while the inspector conjectures the contract correctly in equilibrium, at the time when the principal chooses the contract, an increase in the contract slope does not induce a more stringent inspection. Taking into account that in equilibrium all conjectures are true and using the system of equations specified in (32), we can solve for the equilibrium contracts and decisions in this scenario. The symmetry of the results allows us to write the marginal cost as $\frac{\partial \mathscr{C}_i}{\partial \beta_i} = \left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o \beta_{Pri}$, where $\left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o$ is the same constant we obtained in (25) and β_{Pri} is the equilibrium contract slope common to all contracts in the private contract scenario. Proposition 2 characterizes the equilibrium in the private contract scenario.

Proposition 2. *(Equilibrium with Private Contracts) There exist a unique equilibrium in which contracts and actions are symmetric across firms. Specifically, $\alpha_i = \alpha_{Pri}$ and $\beta_i = \beta_{Pri}$ for all $i \in \{1,...,N\}$, where*

$$\alpha_{Pri} = \frac{c}{2}a_{Pri}^2 + \frac{k}{2}m_{Pri}^2 + d_A m_{Pri} s_{Pri} - (va_{Pri} + m_{Pri})\beta_{Pri} + \frac{\rho}{2}\beta_{Pri}^2\sigma_\varepsilon^2, \tag{33}$$

$$\beta_{Pri} = \frac{v\frac{da_i}{d\beta_i}}{\left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o}. \tag{34}$$

Also, the equilibrium effort, earnings management, and scrutiny are given by $a_{Pri} = a_0\beta_{Pri}$, $m_{Pri} = m_0\beta_{Pri}$, and $s_{Pri} = s_0\beta_{Pri}$.

Notice that the expressions for the equilibrium agent actions are analogous the the ones in Proposition 1. The only difference is that here they are contingent on the equilibrium contract slope in this scenario, β_{Pri}.

Corollary 2. *(i)* $\frac{\partial \beta_{Pri}}{\partial d_A} > 0$*; (ii)* $\frac{\partial \beta_{Pri}}{\partial d_P} < 0$*; (iii)* $\frac{\partial \beta_{Pri}}{\partial k} > 0$*; (iv)* $\frac{\partial \beta_{Pri}}{\partial w} > 0$*; (v)* $\frac{\partial \beta_{Pri}}{\partial \gamma} > 0$*; (vi)* $\frac{\partial \beta_{Pri}}{\partial N} = 0$.

Since the inspector cannot observe the contract, the principal does not choose it anticipating a reaction from the inspector. Therefore, the comparative statics of the pay-performance sensitivity β_{Pri} can now be easily understood as limited to the direct effect that the contract slope has on the principal's cost. For instance, an increase in k or d_A simply increases the marginal cost of manipulation for the agent, reducing his incentives to manipulate. Anticipating this, the principal is able to increase β_{Pri} inducing less manipulation. An increase in d_P, on the other hand, directly increases the punishment for the principal, who, as a result, decreases β_{Pri} to reduce earnings management. Changes in w and γ now have the same effect. An increase in w or γ decreases the inspection level leading to lower expected punishments, and that allows the principal to increase β_{Pri}. Lastly, β_{Pri} does not change with the number of firms. An increase in the number of firms reduces the scrutiny that the inspector can allocate to each firm. However, because the principal anticipates no influence on the inspector's behavior to start with, the number of firms does not affect the principal's equilibrium choice of β_{Pri}.

3.3 Comparison across Regimes

In this section, we examine the economic effects of mandating the disclosure of contracts by comparing the results we obtained under the two scenarios previously analyzed. As stated in Propositions 1 and 2, the difference in the equilibrium contract slopes (i.e., β_{Pub} vs β_{Pri}) drives the contrast between other equilibrium results. For instance, the equilibrium actions, a_i, m_i, and s_i, have symmetrical functional forms across scenarios and they are all linear functions of the equilibrium contract slope in each scenario. Thus, we first focus on the difference of the contract slopes between scenarios. Such a difference is driven by how the observability of the contract determines the influence of the contract slope on the level of inspection s_i. At the time when the principal determines β_i, s_i is contingent on β_i under the public contract, whereas s_i is independent of β_i under the private contract. This is reflected in the denominator of the expressions for β_{Pub} and β_{Pri} in Propositions 1 and 2, respectively. Both denominators contain the term $[\partial \mathscr{C}_i/\partial \beta_i]_o$. This term simply reflect that, in both scenarios, an increase in the contract slope directly increases the incentives for the agent to manipulate, and that increases the cost for the principal. However, the equilibrium slope in public contract scenario also contains an additional term, $[\partial \mathscr{C}_i/\partial s_i]_o ds_i/d\beta_i$. This additional term indicates that, if the contract is observable, the marginal cost of providing incentives to the agent is also affected by the inspector's reaction to the contract change. We can express the difference between the contract slopes of the two scenarios as follows:

$$\frac{\beta_{Pri} - \beta_{Pub}}{\beta_{Pri}\beta_{Pub}} = \frac{1}{v\frac{da_i}{d\beta_i}} \left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o \frac{ds_i}{d\beta_i}$$

Since the inspector always increases the scrutiny of a firm that publicly increases its contract slope, (i.e., $ds_i/d\beta_i > 0$), the difference between contract slopes is mainly driven by the marginal cost of scrutiny (i.e., $[\partial \mathscr{C}_i/\partial s_i]_o$). If the cost for the principal increases with scrutiny (i.e., $[\partial \mathscr{C}_i/\partial s_i]_o > 0$) then, under the public contract scenario, the principal incurs a larger marginal cost of providing incentives to the agent because the inspector increases the scrutiny as a reaction to the increase in the contract slope. Therefore, the principal chooses a smaller slope under the public contract scenario. However, if the cost for the principal decreases with scrutiny (i.e., $[\partial \mathscr{C}_i / \partial s_i]_o < 0$) then the inspector's reaction allows the principal to increase incentives for the agent at a lower marginal cost. Thus, the equilibrium contract slope is larger in the public contract scenario. The following corollary formally states this result:

Corollary 3. *$\beta_{Pub} \geq \beta_{Pri}$ if and only if $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o < 0$.*

Corollary 3 can be reinterpreted using Lemma 2. If the level of punishment for the principal, d_P, is relatively large compared to that for the agent, d_A, then the main effect of an increase in scrutiny is a higher punishment for the principal (i.e., $[\partial \mathscr{C}_i / \partial s_i]_o > 0$). Therefore, the principal reduces the slope to induce a lower scrutiny from the inspector in the public scenario. If instead the level of punishment for the agent, d_A, is relatively large compared to that for the principal, d_P, then the main effect of an increase in scrutiny is an increase of the marginal cost of manipulation for the agent. That is, scrutiny reduces the cost for the principal (i.e., $[\partial \mathscr{C}_i / \partial s_i]_o < 0$), and that allows the principal to increase the contract slope.

We now compare other equilibrium outcomes across the two regimes. As we noted before, the equilibrium actions, a_i, m_i, and s_i, are all linear functions of β_i and are identical across scenarios given the same contract slope. The intuition for this symmetry is that the only difference between scenarios is the observability of the contract. The contract observability determines which economic forces the principal perceives as being are under his influence at the time she chooses the contract. However, in equilibrium all economic forces are present in both scenarios. Therefore, even though the principal chooses different contract slopes across scenarios, they play the analogous roles in equilibrium. This same intuition applies to other equilibrium outcomes, such as the bonus $B_i = \beta_i(va_i + m_i)$ and the total compensation $C_i = \alpha_i + \beta_i(va_i + m_i)$ for the agent. Corollary 4 summarizes the comparison of a_i, m_i, s_i, B_i, and C_i between scenarios.

Corollary 4. *The equilibrium choices compare across scenarios as follows:*
(i) $a_{Pub} \geq a_{Pri}$, $m_{Pub} \geq m_{Pri}$, and $s_{Pub} \geq s_{Pri}$ if and only if $\left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o < 0$;
(ii) $B_{Pub} \geq B_{Pri}$ and $C_{Pub} \geq C_{Pri}$ if and only if $\left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o < 0$.

Again, Lemma 2 facilitates the interpretation of Corollary 4. In general, all equilibrium outcomes are larger under the public contract scenario if the punishment level for the agent is larger than that for the principal and vice versa.

3.4 Pareto optimal contract

So far, we have compared the two scenarios by examining equilibrium contract slopes, agent decisions, and inspector scrutiny intensities. However, we still do not know in which scenario a principal is better off. In this section, we examine a benchmark that is useful to answer such a question. The benchmark examines the maximization of the aggregate welfare of all participants in the economy, excluding the inspector. Since agents always break even in expectation, maximizing the aggregate principals' welfare results in a Pareto optimal set of contracts. That is, given such a set of contracts, no other set of contracts can increase the utility of a principal without reducing the utility of another principal.

One may wonder whether the equilibrium contract sets in the two scenarios previously analyzed are already Pareto optimal. Take, for instance, the private contract scenario. In such a scenario, a principal cannot affect the inspector's scrutiny directly. However, in equilibrium, the inspector's scrutiny is increasing in the contract slope, as Proposition 2 reveals. Therefore, in choosing the contract slope, the principal does not fully consider all the implications that her choice has on her payoff in equilibrium. This consideration alone already reveals that it should be possible to improve upon the contract in the private scenario. In the public contract scenario, the optimal contract does not suffer from this shortcoming. Since the contract is observable by the inspector, the principal already chooses it anticipating the inspector's reaction. However, a change in the contract of given firm i not only affects the inspector's scrutiny of that same firm, s_i, but also the scrutiny of all firms in the economy, s_j for all $j \in \{1,...,N\}$. Thus, each principal's contract choice affects all firms in the economy through its effects on the inspector's scrutiny allocation. Since principals do not take into account these externalities because they only care about maximizing their own profit, this lack of coordination among principals reveals that the contracts in the public contract scenario can also be improved upon.

We obtain the Pareto optimal set of contracts solving a program that maximizes the aggregate payoffs of all principals in the economy, assuming that contracts are publicly observable. We already derived the participation and incentive compatibility constraints for the agents applicable to this case in the public contracts scenario: (10), (11), and (12). Formally, the Pareto optimal set of contract is obtained by solving the program:

$$\underset{\{\beta_i\}_{i\in N}}{Max} \sum_{i=1}^{N} va_i - \mathscr{C}(\beta_i, s_i(\beta_i)). \tag{35}$$

$$s.t.(10),(11), and\,(12)\; for\,all\, i \in \{1,...,N\}, \tag{36}$$

Solving the program above involves obtaining a set of N first order conditions, one for each principal, and then solving them simultaneously. Since in equilibrium all contracts are identical across principal-agent pairs, we simply denote the Pareto optimal contract as $\{\alpha_{PO}, \beta_{PO}\}$. We obtain the following result:

Proposition 3. *(Pareto Optimal Contract) The Pareto optimal contract is symmetric and consists of a fixed salary $\alpha_i = \alpha_{PO}$ and pay-performance sensitivity $\beta_i = \beta_{PO}$* for all $i \in \{1,...,N\}$*, given by*

$$\alpha_{PO} = \frac{c}{2}a_{PO}^2 + \frac{k}{2}m_{PO}^2 + d_A m_{PO} s_{PO} - (va_{PO} + m_{PO})\beta_{PO} + \frac{\rho}{2}\beta_{PO}^2\sigma_\varepsilon^2 \tag{37}$$

$$\beta_{PO} = \frac{v\frac{da_i}{d\beta_i}}{\left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o + \sum_{j=1}^{N}\left[\frac{\partial \mathscr{C}_j}{\partial s_j}\right]_o \frac{ds_j}{d\beta_i}}, \tag{38}$$

where for $j \neq i$ we have $\frac{ds_j}{d\beta_i} = -\frac{s_0 k\gamma}{k((N-1)w-\gamma)+(N-1)bd_A}$.

Given the complexity of the expressions, comparing a principal's expected payoff across scenarios is very hard to do directly. However, there is an indirect way to do that. The expression for the principal's expected payoff in equilibrium is a quadratic function of the equilibrium contract slope. Moreover, the same function applies to both the private and public contract scenarios. They only differ in the slope chosen by the principal. This quadratic function has an inverted-U shape with a maximum at the Pareto optimal slope β_{PO}. The symmetry of quadratic functions around their stationary value allows us to compare the principal's payoffs by simply comparing the distance of the contract slopes with β_{PO}. The smaller the distance between β_{PO} and the slope of the scenario in question, the higher the payoff for the principal.

Comparing the expressions for β_{Pri} and β_{PO}, we can derive the following expression:

$$\frac{\beta_{Pri}-\beta_{PO}}{\beta_{PO}\beta_{Pri}} = \frac{1}{v\frac{da_i}{d\beta_i}} \sum_{j=1}^{N} \left[\frac{\partial \mathscr{C}_j}{\partial s_j}\right]_o \frac{ds_j}{d\beta_i}, \tag{39}$$

The expression above tells us that the distance between β_{Pri} and β_{PO} is proportional to the product of three terms. This product reflects the economic effects that are differential between the private contract scenario and the Pareto optimal derivation. The first term on the right-hand side of the equality is simply the inverse of the principals' marginal benefit of increasing the contract slope, which is always positive. The next two terms are the sum of the marginal costs imposed on all firms in the economy through scrutiny by an increase in the contract slope of firm i. Each of those marginal costs can be decomposed as the product of two terms. The first term in each product is the marginal cost of scrutiny for the principal of firm j. From Lemma 2 we know that the sign of this term can be positive or negative depending on the parameter values. The second term in each product is the marginal effect that an increase in the contract slope of firm i induces on the scrutiny of firm j. Of all the terms in the sum, the term containing $\frac{ds_i}{d\beta_i}$ is present because the contract of firm i is not observable by the inspector in the private contract scenario, whereas we considered it observable in the derivation of the Pareto optimal contract. That is, it is a differential effect. We know from Lemma 1 that $\frac{ds_i}{d\beta_i} > 0$. Indeed, if the contract is observable, increasing the contract slope in firm i induces the inspector to increase the scrutiny of that same firm. The remaining terms in the sum contain $\frac{ds_j}{d\beta_i}$ for $j \neq i$, and they are all the externalities that the contract choice of principal i inflicts on other firms in the economy. They are present in expression (39) because the principal does not take them into account in the private contract scenario, whereas they are considered in the derivation of the Pareto optimal contract. All terms $\frac{ds_j}{d\beta_i}$ for $j \neq i$ have the opposite sign to that of γ. If γ is positive, the increase in the scrutiny of firm i makes the scrutiny of other firms more costly for the inspector. Thus, an increase in the contract slope of firm i induces the inspector to reduce the scrutiny of other firms (i.e., $\frac{ds_j}{d\beta_i} < 0$ for $j \neq i$). If instead γ is negative, the scrutiny of firm i facilitates the scrutiny of other firms. Consequently, an increase in the contract slope of firm i induces the inspector to increase the scrutiny of other firms (i.e., $\frac{ds_j}{d\beta_i} > 0$ for $j \neq i$). Since the marginal cost of scrutiny is symmetric across firms, we can take it out of the sum. The resulting sum, $\sum_{j=1}^{N} \frac{ds_j}{d\beta_i}$, turns out to be always positive. The positivity of the term $\frac{ds_i}{d\beta_i}$ dominates regardless of the sign of γ. Therefore, we can write,

$$Sign\left[\frac{\beta_{Pri}-\beta_{PO}}{\beta_{PO}\beta_{Pri}}\right] = Sign\left[\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o\right]. \tag{40}$$

To compare β_{Pub} and β_{PO} we can derive an analogous expression,

$$\frac{\beta_{Pub}-\beta_{PO}}{\beta_{PO}\beta_{Pub}} = \frac{1}{v\frac{da_i}{d\beta_i}} \sum_{j\neq i,}^{N} \left[\frac{\partial \mathscr{C}_j}{\partial s_j}\right]_o \frac{ds_j}{d\beta_i}, \tag{41}$$

Notice that the only difference between expression (39) and (41) is that the sum in (41) does not contain the term $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o \frac{ds_i}{d\beta_i}$. This is simply because in both, the public contract scenario and the Pareto optimal derivation, we assumed that the contract is observable by the inspector and, as a result, the principal takes into account the inspector's reaction to her contract choice. Therefore, this effect is not differential. Only the externalities considered in the Pareto optimal contract are differential. Since all the terms $\frac{ds_j}{d\beta_i}$ for $j \neq i$ have the opposite sign to that of γ, we can write:

$$Sign\left[\frac{\beta_{Pub}-\beta_{PO}}{\beta_{PO}\beta_{Pub}}\right] = -Sign[\gamma]\, Sign\left[\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o\right]. \tag{42}$$

Essentially, the Pareto optimal derivation takes into account all the effects that the contract choice has on the economy. Principals, however, never take into account the externalities of their choices on other firms. This lack of coordination between principals yields a loss of surplus in all firms in the economy, reducing economic efficiency regardless of the contract observability. Yet, the non-observability of the contract poses another efficiency hurdle for the principal in the private contract scenario. The principal cannot influence the inspector's reaction, and that prevents her from adjusting the agent's incentives to her anticipation of the inspector's scrutiny. Such inefficiencies can only be absent or inconsequential in extreme cases. For instance, if there is only one firm in the economy ($N = 1$), or there are multiple firms but the inspector's cost of scrutiny of one firm is unrelated to the scrutiny of other firms ($\gamma = 0$), then externalities are muted. Similarly, if the number of firms in the economy is very large ($N \to \infty$), the effect of each principal's contract choice on the inspector's scrutiny allocation becomes atomic and, thus, externalities also become immaterial. In all these scenarios, the absence (the less severity) of externalities leads the equilibrium contract in the public contract scenario (closer) to the Pareto optimal contract. However, the equilibrium contract in the private contract scenario still suffers distortions because of the non-observability of the contract. As a result, in these three extreme cases, principals are better off in the public contract scenario than in the private contract scenario. Proposition 4 compares the principal's welfare across scenarios in the absence and the less severity of externalities.

Proposition 4. *In extreme situations in which there are no or less severe externalities, principals obtain a higher payoff in the public contract scenario than in the private contract scenario. Specifically, let* $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o \neq 0$*, if* $\gamma = 0$*,* $N = 1$*,or* $N \to \infty$*, then principals are better off with public contracts than with private contracts. If instead* $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o = 0$*, principals are indifferent between scenarios.*

If there are at least two firms in the economy, the number of firms is finite, and $\gamma \neq 0$, we need to take into account the contract distortions due to both the coordination problems among principals and the non-observability of private contracts. The following proposition states the comparison of the principal's payoffs across scenarios in this case:

Proposition 5. *If the number of firms in the economy is $N \in \mathbb{N}\backslash\{1\}$ and $\gamma \neq 0$, the comparison of the principal's welfare is as follows. If $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o \neq 0$,*

(i) For $\gamma \leq \frac{w}{2}$, principals are better off with public contracts;

(ii) There exists $\bar{\gamma} > \frac{w}{2}$ such that principals are better off with private contracts for $\gamma > \bar{\gamma}$

To interpret the results in Proposition 5 in a intuitive way, it is convenient to first cover the case of a negative γ. If $\gamma < 0$ then from expressions (40) , (42), and Lemma 2 we can derive that if $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o > 0$ then $\beta_{PO} < \beta_{Pub} < \beta_{Pri}$. Indeed, if the marginal cost of scrutiny is positive, the principal chooses a larger contract slope with private contracts. If the contract is observable, increasing the incentives for the agent induces a more intense scrutiny, which is more costly for the principal. Moreover, since $\gamma < 0$, increasing the contract slope facilitates the scrutiny of other firms, which also translates into higher costs for the principals. Therefore, the Pareto optimal contract must have the smallest slope. If $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o = 0$ then $\beta_{Pub} = \beta_{PO} = \beta_{Pri}$. If scrutiny is not costly, the observability of the contract is irrelevant and externalities have no economic impact. Therefore, the three equilibrium contract slopes become identical. Finally, if $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o < 0$ then $\beta_{Pri} < \beta_{Pub} < \beta_{PO}$. If scrutiny saves costs to the principal, then contract observability allows the principal to increase the contract slope by inducing a more intense scrutiny. Also, since $\gamma < 0$, the increased scrutiny facilitates the scrutiny of other firms which, in this case, saves them costs. Therefore, the Pareto optimal slope is the largest one. Overall, if $\gamma < 0$, then principals are always better off in the public contract scenario because the contract distortion is smaller.

The case of a positive γ is more nuanced. If $\gamma > 0$ and $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o > 0$ then $\beta_{Pub} < \beta_{PO} < \beta_{Pri}$. In this case, β_{PO} is smaller than β_{Pri} because the observability of the contract induces a more intense scrutiny, which is costly for the principal. So, the principal tries to mitigate the punishment by reducing the slope. However, β_{Pub} is even smaller than β_{PO} because increasing the contract slope increases scrutiny in the same firm but decreases it in other firms. Therefore, taking into account externalities, the perceived scrutiny increases at a lower pace, which allows for a larger contract slope in the Pareto optimal contract. If instead $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o < 0$, then $\beta_{Pri} < \beta_{PO} < \beta_{Pub}$. In this case, β_{Pri} is smaller than β_{PO} because the more intense scrutiny generated by the contract observability saves costs to the principal. Finally, β_{Pub} is larger than β_{PO} because an increase in the contract slope induces a lower scrutiny in other firms, which saves them less costs. Therefore, internalizing the externalities leads to a decrease in the optimal contract slope. So, if $\gamma > 0$, the Pareto optimal contract slope lies always in the middle, which complicates assessing which scenario provides the highest payoff for the principal. The difference between β_{Pub} and β_{PO} is only due to the externalities. For a small gamma ($0 \leq \gamma \leq \frac{w}{2}$), externalities are small. Indeed, in the limit as $\gamma \to 0$, externalities disappear, and $\beta_{Pub} = \beta_{PO}$. Thus, it is clear that β_{Pub} is very close to β_{PO} for small γ. However, the distance between β_{Pri} and β_{PO} is due to externalities and also to the observability of the contract. While externalities also become irrelevant as $\gamma \to 0$, the contract observability is still relevant, so β_{Pri} does not converge to β_{PO}. In essence, for $\gamma \to 0$, we converge to a setting equivalent to the one single firm setting, with $\beta_{Pub} = \beta_{PO} \neq \beta_{Pri}$. Therefore, the principal is always better off with public contracts. For a large positive γ ($w > \gamma > \bar{\gamma} > \frac{w}{2}$) externalities are important. However, the distance between β_{Pri} and β_{PO} also reflects the fact that the contract is not observable in the private scenario. With a positive γ, while the observability of the contract induces a more intense scrutiny, that same slope increase reduces the scrutiny in other firms. Because of the opposite sign of the two effects, the private contract slope is closer to the Pareto optimal contract slope, and therefore yields a higher payoff for the principal.

4 Welfare Analysis

4.1 Social Welfare

We now examine the set of contracts that maximizes social welfare, and denote such contract as by $\{\alpha_{SW}, \beta_{SW}\}$. We define welfare as the sum of the principals's and the inspector's payoffs. However, once we include the inspector in the economy, external punishments become wealth transfers from principals and agents to the inspector. To keep the setting tractable, we assume that $b = d_A + d_P$. That is, the benefit that the inspector obtains from scrutinizing a firm is equal to the the external punishments the firm suffers. The the set of contracts that maximizes social welfare is achieved by solving the program:

$$\underset{\{\beta_i\}_{i\in N}}{Max} \sum_{i=1}^{N} va_i - \mathscr{C}(\beta_i, s_i(\beta_i)) + I(\beta_i, s_i(\beta_i)). \tag{43}$$

$$s.t.(10), (11), and\, (12)\; for\, all\, i \in \{1, ..., N\}, \tag{44}$$

where $I(\beta_i, s_i(\beta_i)) = b\sum_{i=1}^{N} m_i(\beta_i)s_i(\beta_i) - \frac{w}{2}\sum_{i=1}^{N} s_i(\beta_i)^2 - \frac{\gamma}{2(N-1)}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} s_i(\beta_i)s_j(\beta_j)$ is the inspector's payoff. Taking derivatives with respect to β_i yields a system of N first order conditions. Comparing the first order conditions with the ones in the Pareto optimal contract, there is an additional term $\frac{dI(\beta_i, s_i(\beta_i))}{d\beta_i} = b\sum_{j=1}^{N}\frac{dm_j}{d\beta_i}s_j$ that is derived from the inspector's payoff. This reflects the fact that the inspector's marginal benefit of scrutiny increases with β_i since earnings management increases with β_i. Thus, β_{SW} is greater than β_{PO}.

Proposition 6 characterizes the welfare maximizing contract.

Proposition 6. *(Welfare Maximizing Contract) The contract that maximizes social welfare is symmetric and consists of a fixed salary $\alpha_i = \alpha_{SW}$ and pay-performance sensitivity $\beta_i = \beta_{SW}$ for all $i \in \{1, ..., N\}$ given by,*

$$\alpha_{SW} = \frac{c}{2}a_{SW}^2 + \frac{k}{2}m_{SW}^2 + d_A m_{SW}s_{SW} - (va_{SW} + m_{SW})\beta_{SW} + \frac{\rho}{2}\beta_{SW}^2\sigma_\varepsilon^2, \tag{45}$$

$$\beta_{SW} = \frac{v\frac{da_i}{d\beta_i}}{ca_0\frac{\partial a_i}{\partial\beta_i} + \rho\sigma_\varepsilon^2 + \sum_{j=1}^{N}\left(km_0\frac{dm_j}{d\beta_i} + (w+\gamma)s_0\frac{ds_j}{d\beta_i}\right)} \tag{46}$$

$$= \frac{v\frac{da_i}{d\beta_i}}{\left[\frac{\partial\mathscr{C}_i}{\partial\beta_i}\right]_o + \sum_{j=1}^{N}\left[\frac{\partial\mathscr{C}_j}{\partial s_j}\right]_o\frac{ds_j}{d\beta_i} - bs_0\sum_{j=1}^{N}\frac{dm_j}{d\beta_i}}. \tag{47}$$

Note that the resulting optimal contract chosen by the social planner has a contract slope, β_{SW}, that is greater than β_{Pri} and β_{Pub} because the inspector's payoff increases with a contract slope, leading to larger social welfare. Accordingly, the public contract results in a smaller (larger) distortion in β_i when $\beta_{Pub} > (<$

$)\beta_{Pri}$. Proposition 7 states the comparison of the social welfare between the private contract and the public contract scenarios.

Proposition 7. *The social welfare is larger (smaller) under public contract if and only if* $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o < 0$.

Proposition 7 states that the welfare implications of mandating contract disclosure are mainly determined by the marginal cost of scrutiny for the firm. Indeed, if the firm is marginally harmed by the inspector's scrutiny, it is better to mandate contracts to be disclosed.

5 Empirical Implications

Our findings provide new insights on the growing literature on executive compensation which has come under enormous scrutiny and received lots of attention recently. The U. S. Securities and Exchange Commission (SEC) has been exerting effort to improve transparency of executive compensation and provide investors with comprehensive and comprehensible information on a company's financial transactions with management. The disclosure of executive compensation has been required since 1934 (Securities Exchange Act of 1934) and its revision was enacted in 1992 (SEC, 1992). In 2006, SEC Chairman Christopher Cox noted that "Our job is to ensure that investors have available to them all of the compensation information they need, presented in a clear and understandable form that they can use" in his speech (Cox, 2006), and the SEC introduced a new requirement that companies prepare a thorough and comprehensive Compensation Discussion and Analysis (CD&A) (SEC, 2006A, 2006B). Although our model discusses the public and private contract scenarios explicitly, we can extend our results and intuition to the cases in which firms provide more or less information about executive contracts.

First, (more) disclosure of contract increases pay-performance sensitivity, the amount of earnings management, and the total amount of compensation for firms whose internal controls are relatively weak or punishment for detected earnings manipulation to principals is weak relative to that to agent, compared to nondisclosure (or less disclosure) of contract. One can compare pay-performance sensitivity, the amount of earnings management, and the total amount of compensation before and after regulations requiring disclosure or more detailed information of contracts.[11] Second, our model offers a rationale for a cross-sectional variation of pay-performance sensitivity in each contract scenario (i.e., before/after the adoption of contract disclosure, or when providing less or more information on compensation contract). In both contract scenarios, we predict that pay-performance sensitivity is larger when punishment for detected earnings manipulation to the agent (the principal) is larger (smaller) and the internal control is stronger. For tests, one may compare pay-performance sensitivity in firms that have different qualities of audit committee and governance or when there is a change in penalty for managers or companies (e.g., the Sarbanes-Oxley Act of 2002). The effects of the cost of inspection including the direct cost and externality are not monotonic in the public contract scenario. When punishment for detected earnings manipulation to the agent (the principal) is

[11]Ferri, Zheng, and Zou (2017) find no evidence of a change in discretionary accruals for their full-sample regression. They argue that this might result from a change of the cost of earnings manipulation. We provide more explanations in this regard and offer ways to conduct empirical tests for a change of earnings management.

negligible, pay-performance sensitivity increases (decreases) with the direct cost of inspection. If the internal control is relatively strong (weak), pay-performance sensitivity increases (decreases) with the magnitude of externality as the magnitude of externality is very small or the number of peer firms is very large. Also, pay-performance sensitivity decreases with the magnitude of externality as the internal control is very weak. In addition, in the presence of externality of scrutiny, pay-performance sensitivity is expected to increase (decrease) with the number of peer firms when the internal control is relatively strong (weak) or punishment for detected earnings manipulation to principals is large (small) relative to that to agent in the public contract scenario. In contrast, in the private contract scenario, pay-performance sensitivity monotonically increases with the cost of inspection including both the direct cost and externality and are independent of the number of peer firms. For tests, the direct cost of inspection can be proxied by the resources/difficulties with which the inspector can scrutinize. The level of requirements for derivative suit can be an example. The level of externality can be measured by the qualities or ways of earnings management commonly committed in industries or in the market. For instance, if the inspector is investigating a new type of earnings management, the inspector can learn by doing it and the inspection of one firm can be complement to that of other firms. The number of peer firms can be proxied by the number of firms under the same inspector or supervisor, or the number of peer firms in the same industry or in a competing market.

6 Conclusion

This paper examines the effect of contract disclosure on the optimal incentive contract in multiple principal-agent relationships. After signing the contract, each agent exerts productive effort and engages in earnings management. The inspector allocates its resources to uncover earnings management and the scrutiny is interdependent across firms in that the inspection of one firm can complement or substitute that of other firms. In the public contract scenario, the principal can commit to a Stackelberg leader action but observability of contract can induce more severe coordination problems across firms in that their contracts influence the inspector's scrutiny. In a private contract scenario, on the other hand, the principal has no means of committing to a Stackelberg leader action but unobservability can alleviate coordination problems. We discuss the factors that affect the tradeoff between the two contract scenarios.

Standard setters have been pushing forward to enhance transparency of executive contracts, which has been presumed to improve corporate governance and prevent excessive pay. On the contrary to conventional wisdom, our paper highlights that contract disclosure may result in unintended consequences because coordination problems may induce losses to shareholders and social welfare. Contract disclosure needs to be viewed as a two-edged sword. It has benefits of providing more information to the market as documented in prior literature. However, it encompasses costs of exacerbating coordination problems among firms. Our study suggests that regulators need to carefully consider the effect of externality of executive pay disclosure in that it may create coordination problems among firms in designing their optimal contracts.

References

[1] Aggarwal, R. K. and A. A. Samwick, 1999, Executive Compensation, Strategic Competition, and Relative Performance Evaluation: Theory and Evidence, Journal of Finance 54 (6): 1999-2043.

[2] Armstrong, C.S., C. D. Ittner, and D. F. Larcker, 2012, Corporate Governance, Compensation Consultants, and CEO Pay Levels, Review of Accounting Studies 17 (2): 322-351.

[3] Baker, G. P, M. C. Jensen, and K. J. Murphy, 1988, Compensation and Incentives: Practice vs. Theory, Journal of Finance 43 (3): 593-616.

[4] Bebchuk, L. A. and J. M. Fried, 2003, Executive Compensation as an Agency Problem, Journal of Economic Perspectives 17 (3): 71 -92.

[5] Bebchuk, L.A., J. M. Fried, and D. I. Walker, 2002, Managerial Power and Rent Extraction in the Design of Executive Compensation, The University of Chicago Law Review 69 (3): 751- 846.

[6] Bertrand, M., and S. Mullainathan, 2001, Are CEOs Rewarded for Luck? The Ones without Principals Are, Quarterly Journal of Economics 116 (3): 901-932.

[7] Bloomfield, M. J., 2018, Compensation Disclosures and the Weaponization of Executive Pay: Evidence from Revenue-Based Performance Evaluation, Working Paper, University of Pennsylvania.

[8] Bozanic, Z., J. L. Hoopes, J. R. Thornock, and B. M. Williams, 2017, IRS Attention, Journal of Accounting Research 55 (1): 79–114.

[9] Brander, J. A. and T. R. Lewis, 1986, Oligopoly and Financial Structure: The Limited Liability Effect, American Economic Review 76 (5): 956–970.

[10] Brander, J. A. and B. J. Spencer, 1983, Strategic Commitment with R&D: the Symmetric Case, Bell Journal of Economics 14 (1): 225–235.

[11] Brander, J. A. and B. J. Spencer, 1985, Export Subsidies and International Market Share Rivalry, Journal of International Economics 18 (1-2): 83-100.

[12] Card, D., A. Mas, E. Moretti, and E. Saez, 2012, Inequality at Work: The Effect of Peer Salaries on Job Satisfaction, American Economic Review 102 (6): 2981-3003.

[13] Chen, Q., B. Mittendorf, Y. Zhang, 2010, Endogenous Accounting Bias When Decision Making and Control Interact, Contemporary Accounting Research 27 (4): 1–29.

[14] Core, J. E., R. W. Holthausen, and D. F. Larker, 1999, Corporate Governance, Chief Executive Officer Compensation, and Firm Performance, Journal of Financial Economics 51 (3): 371-406.

[15] Corona, C. and R. Randhawa, 2010, The Auditor's Slippery Slope: An Analysis of Reputational Incentives, Management Science 56 (6): 924-937.

[16] Corona, C. and R. Randhawa, 2018, The Value of Confession: Admitting Mistakes to Build Reputation, The Accounting Review 93 (3): 133-161.

[17] Cox, C., 2006, Speech by SEC Chairman: Chairman's Opening Statement; Proposed Revisions to the Executive Compensation and Related Party Disclosure Rules, Available at https://www.sec.gov/news/speech/spch011706cc.htm.

[18] Craighead, J. A., M. L. Magnan, and L. Thorne, 2004, The Impact of Mandated Disclosure on Performance-Based CEO Compensation, Contemporary Accounting Research 21 (2): 369-98.

[19] Dye, R., B. Balachandran, and R. Magee, 1990, Contingent Fees for Audit Firms, Journal of Accounting Research 28 (2): 239-266.

[20] Dye, R., 1993, Auditing Standards, Legal Liability, and Auditor Wealth, Journal of Political Economy 101 (5): 887-914.

[21] Dye, R., 1995. Incorporation of the Audit Market, Journal of Accounting and Economics 19 (1): 75-114.

[22] Dye, R. A., 2002, Classifications Manipulation and Nash Accounting Standards, Journal of Accounting Research 40 (4): 1125–1162.

[23] Ewert, R. and A. Wagenhofer, 2016, Effects of Increasing Enforcement on Financial Reporting Quality and Audit Quality, Journal of Accounting Research 57 (1): 121-168.

[24] Faulkender, M. and J. Yang, 2013, Is Disclosure an Effective Cleansing Mechanism? The Dynamics of Compensation Peer Benchmarking, Review of Financial Studies 26 (3): 806-839.

[25] Fellingham, J., and P. Newman, 1985, Strategic Considerations in Auditing, The Accounting Review 60 (4): 634-650.

[26] Ferri, F., R. Zheng, and Y. Zou, 2018, Uncertainty about Managers' Reporting Objectives and Investors' Response to Earnings Reports: Evidence from the 2006 Executive Compensation Disclosures, Journal of Accounting and Economics 66 (2-3): 339–365.

[27] Fershtman, C. and K. L. Judd, 1987, Equilibrium Incentives in Oligopoly. American Economic Review 77 (5): 927–940.

[28] Frydman, C. and D. Jenter, 2010, CEO Compensation, Annual Review of Financial Economics 2: 75-102.

[29] Gartenberg, C. M. and J. Wulf, 2014, Pay Harmony: Peer Comparison and Executive Compensation, Harvard Business School Working Paper No. 13-041.

[30] Gipper, B., 2017, The Economic Effects of Mandating Expanded Compensation Disclosures, Working Paper, Stanford University.

[31] Hayes, R. M. and S. Schaefer, 2008, CEO pay and Lake Wobegon Effect, Journal of Financial Economics 94 (2): 280-290.

[32] Hermalin, B. and M. Weisbach, 2012, Information Disclosure and Corporate Governance, Journal of Finance 67 (1): 195-233.

[33] Hillegeist, S. A., 1999, Financial Reporting and Auditing under Alternative Damage Apportionment Rules, The Accounting Review 74 (3): 347-369.

[34] Jensen, M. C. and K. J. Murphy, 1990, Performance Pay and Top-Management Incentives, Journal of Political Economy 98 (2): 225-264.

[35] Katz, M. L., 1991, Game-playing Agents: Unobservable Contracts as Precommitments, RAND Journal of Economics 22 (3): 307–328.

[36] Kedia, S., 2006, Estimating Product Market Competition: Methodology and Application, Journal of Banking and Finance 30 (3): 875–894.

[37] Kydland, F. E., and E. C. Prescott, 1977, Rules Rather than Discretion: The Inconsistency of Optimal Plans, Journal of Political Economy 85 (3): 473–491.

[38] Laux, C. and V. Laux, 2009, Board Committees, CEO Compensation, and Earnings Management, The Accounting Review 84 (3):869-891.

[39] Laux, V. and P. C. Stocken, 2018, Accounting Standards, Regulatory Enforcement, and Innovation, Journal of Accounting and Economics 65 (2-3): 221–236.

[40] Lo, K., 2003, Economic Consequences of Regulated Changes in Disclosure: The Case of Executive Compensation, Journal of Accounting and Economics 35 (3): 285–314.

[41] Liang, P. J., 2004, Equilibrium Earnings Management, Incentive Contracts, and Accounting Standards, Contemporary Accounting Research 21(3): 685–718.

[42] Maksimovic, V., 1988, Capital Structure in a Repeated Oligopoly, Rand Journal of Economics 19 (3): 389-407.

[43] Mas, A., 2016, Does Disclosure Affect CEO Pay Setting? Evidence from the Passage of the 1934 Securities and Exchange Act, Working Paper, Princeton University.

[44] Mas, A., 2017, Does Transparency Lead to Pay Compression?, Journal of Political Economy 125 (5): 1683-1721.

[45] Murphy, K. J., 1999, Executive compensation, Handbook of Labor Economics 3: 2485-2563.

[46] Murphy, K.J., 2013, Executive Compensation: Where We Are, and How We Got There, Handbook of the Economics of Finance: 211-356.

[47] Pae, S. and S. Yoo, 2001, Strategic Interaction in Auditing: An Analysis of Auditors' Legal Liability, Internal Control System Quality, and Audit Effort, The Accounting Review 76 (3): 333-356.

[48] Park, Y. W., T. Nelson, and M. R. Huson, 2001, Executive Pay and the Disclosure scenario: Canadian Evidence, Journal of Financial Research 24 (3): 347-365.

[49] Schmidt, C., 2012, Does Transparency Increase Executive Compensation?, Working paper, Norwegian School of Economics.

[50] Schwartz, R., 1997, Legal Regimes, Audit Quality, and Investment, The Accounting Review 72 (3): 385-406.

[51] SEC, 1992, Executive Compensation Disclosure (Final Rules), Release No. 33-6962, Federal Register 57 No. 204, 57 CFR 48126-56.

[52] SEC, 1999, SEC Staff Accounting Bulletin: No. 99 – Materiality: Release No SAB 99, Available at https://www.sec.gov/interps/account/sab99.htm.

[53] SEC, 2005, SEC Adopts Rule for Pay Ratio Disclosure, Press Release, Available at https://www.sec.gov/news/pressrelease/2015-160.html.

[54] SEC, 2006A, Executive Compensation and Related Party Disclosure: Release No 33-8655, Available at http://www.sec.gov/rules/proposed/33- 8655.pdf.

[55] SEC, 2006B, Executive Compensation and Related Person Disclosure: Release No 33-8732A, Available at http://www.sec.gov/rules/final/2006/33-8732a.pdf.

[56] Shue, K., 2013, Executive networks and firm policies: Evidence from the random assignment of MBA peers, Review of Financial Studies 26 (6): 1401-1442.

[57] Shue, K. and R. R. Townsend, 2015, Growth Through Rigidity: An Explanation of the Rise of CEO Pay, Journal of Financial Economics 123 (1): 1-21.

[58] Yeaton, K., 2007, The SEC's New Rules on Executive Compensation, The CPA Journal: 26-33.

Appendix: Proof

Proof of Lemma 1

It follows immediately from (10), (11), and (12).

Q.E.D.

Proof of Proposition 1

The principal maximizes her expected payoff in (16) subject to the constraints in (6), (8), and (10). The principal's problem can be expressed as follows:

$$\underset{\beta_i}{Max}\; va_i(\beta_i) - \mathscr{C}(\beta_i, s_i(\beta_i)), \tag{48}$$

where $\mathscr{C}(\beta_i, s_i(\beta_i)) = \frac{c}{2}a_i(\beta_i)^2 + \frac{k}{2}m_i(\beta_i, s_i)^2 + (d_P + d_A)m_i(\beta_i, s_i)s_i(\beta_i) + \frac{\rho}{2}\beta_i^2\sigma_\varepsilon^2$. Taking derivatives of (48) with respect to β_i yields a system of N equations of the form: for all $i \in \{1, ..., N\}$,

$$v\frac{da_i}{d\beta_i} = \frac{\partial \mathscr{C}_i}{\partial \beta_i} + \frac{\partial \mathscr{C}_i}{\partial s_i}\frac{ds_i}{d\beta_i}, \tag{49}$$

where $\frac{\partial \mathscr{C}_i}{\partial \beta_i} = ca_i\frac{\partial a_i}{\partial \beta_i} + km_i\frac{\partial m_i}{\partial \beta_i} + (d_A + d_P)s_i\frac{\partial m_i}{\partial \beta_i} + \beta_i\rho\sigma_\varepsilon^2$, $\frac{\partial \mathscr{C}_i}{\partial s_i} = km_i\frac{\partial m_i}{\partial s_i} + (d_A + d_P)(s_i\frac{\partial m_i}{\partial s_i} + m_i)$, and $\frac{ds_i}{d\beta_i} = \frac{b(N-1) - \frac{b}{k(w+\gamma)+bd_A}k\gamma}{k((N-1)w-\gamma)+(N-1)bd_A}$.

Using the fact that $a_i = \frac{v\beta_i}{c}$, $m_i = ((N-1)w - \gamma)\Psi\beta_i + bd_A\Gamma B$ and $s_i = b(N-1)\Psi\beta_i - bk\Gamma B$, where $\Psi = \frac{(k(w+\gamma)+bd_A)}{(k(w+\gamma)+bd_A)(k((N-1)w-\gamma)+(N-1)bd_A)}$, $\Gamma = \frac{\gamma}{(k(w+\gamma)+bd_A)(k((N-1)w-\gamma)+(N-1)bd_A)}$, and $B = \sum_{j=1}^N \beta_j$, the above equations can be rearranged to show that the expressions of β_i are a function of B and are identical for all $i \in \{1, ..., N\}$, which confirms that the equilibrium β_i is symmetric for all $i \in \{1, ..., N\}$.

The symmetry of the equilibrium yields $a_i = a_0\beta_i$, $m_i = m_0\beta_i$, and $s_i = s_0\beta_i$, where $a_0 = \frac{v}{c}$, $m_0 = \frac{(w+\gamma)}{k(w+\gamma)+bd_A}$, and $s_0 = \frac{b}{k(w+\gamma)+bd_A}$ and thus $\frac{\partial \mathscr{C}_i}{\partial \beta_i} = \left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o \beta_i$ and $\frac{\partial \mathscr{C}_i}{\partial s_i} = \left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o \beta_i$. Finally, the unique symmetric equilibrium is

$$\beta_i = \frac{v\frac{da_i}{d\beta_i}}{\left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o + \left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o \frac{ds_i}{d\beta_i}}. \tag{50}$$

The second order condition is satisfied: $\frac{d^2E_P[V_i]}{d\beta_i^2} = -c\frac{\partial a_i}{\partial \beta_i}\frac{\partial a_i}{\partial \beta_i} - k\frac{dm_i}{d\beta_i}\frac{dm_i}{d\beta_i} - (d_A + d_P)(\frac{dm_i}{d\beta_i}\frac{ds_i}{d\beta_i} + \frac{dm_i}{d\beta_i}\frac{ds_i}{d\beta_i}) - \rho\sigma_\varepsilon^2 < 0$.

α_i is obtained from the participant constraint for the agent. Substituting the equilibrium β_i into the expressions for a_i, m_i, and s_i yields a_{Pub}, m_{Pub}, and s_{Pub} as stated in the proposition.

Q.E.D.

Proof of Lemma 2

Note that

$$\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o = km_0\frac{\partial m_i}{\partial s_i} + (d_A + d_P)(\frac{\partial m_i}{\partial s_i}s_0 + m_0) > 0 \tag{51}$$

$$\Longleftrightarrow (d_A + d_P)m_0 > -(km_0 + (d_A + d_P)s_0)\frac{\partial m_i}{\partial s_i} \tag{52}$$

$$\Longleftrightarrow d_P m_0 > \frac{d_A}{k}(d_A + d_P)s_0 \tag{53}$$

$$\Longleftrightarrow d_P(w+\gamma)k > d_A(d_A + d_P)b \tag{54}$$

$$\Longleftrightarrow k > \frac{bd_A(d_A + d_P)}{d_P(w+\gamma)} \tag{55}$$

$$\Longleftrightarrow d_A < \psi(d_P) = -\frac{d_P}{2} + \frac{1}{2}\sqrt{d_P^2 + \frac{4d_P k(w+\gamma)}{b}} \tag{56}$$

where $\frac{\partial \psi(d_P)}{\partial d_P} = -\frac{1}{2} + \frac{\sqrt{b}\sqrt{d_P}}{4\sqrt{bd_P + 4k(r+w)}} + \frac{\sqrt{bd_P + 4k(r+w)}}{4\sqrt{b}\sqrt{d_P}} > 0$.

Q.E.D.

Proof of Corollary 1

(i) From Proposition 1, we obtain

$$\beta_{Pub} = \frac{v\frac{da_i}{d\beta_i}}{ca_0\frac{\partial a_i}{\partial \beta_i} + km_0\frac{dm_i}{d\beta_i} + (d_A + d_P)(\frac{dm_i}{d\beta_i}s_0 + m_0\frac{ds_i}{d\beta_i}) + \rho(\sigma_\eta^2 + \sigma_\varepsilon^2)} \tag{57}$$

where $\frac{dm_i}{d\beta_i} = \frac{(w-\frac{\gamma}{(N-1)})(k(w+\gamma)+bd_A)+b\frac{\gamma}{(N-1)}d_A}{(k(w+\gamma)+bd_A)(k(w-\frac{\gamma}{(N-1)})+bd_A)}$ and $\frac{ds_i}{d\beta_i} = \frac{b((k(w+\gamma)+bd_A)-k\frac{\gamma}{(N-1)})}{(k(w+\gamma)+bd_A)(k(w-\frac{\gamma}{(N-1)})+bd_A)}$. Then it follows that $\frac{\partial \beta_{Pub}}{\partial d_A} > 0$, $\frac{\partial \beta_{Pub}}{\partial d_P} < 0$, and $\frac{\partial \beta_{Pub}}{\partial k} > 0$.

(ii) (a) It follows from the fact that $Sign[\lim\limits_{d_A \to 0} \frac{\partial \beta_{Pub}}{\partial w}] > 0$;

(b) It follows from the fact that $Sign[\lim\limits_{d_P \to 0} \frac{\partial \beta_{Pub}}{\partial w}] > 0$.

(iii) (a) It follows from the fact that $Sign[\lim\limits_{d_A \to 0} \frac{\partial \beta_{Pub}}{\partial \gamma}] = Sign[\left(-\gamma^2(I-3) + (I-1)^2 w^2 - 4\gamma(I-1)w\right)]$;

(b) It follows from the fact that $Sign[\lim\limits_{d_P \to 0} \frac{\partial \beta_{Pub}}{\partial \gamma}] = Sign[-(b^2(I-1)^2 d_A^2 + b(I-1)kd_A(\gamma(I-5) + 2(I-1)w) + k^2\left(-2\gamma^2(I-2) + (I-1)^2 w^2 + \gamma\left(I^2 - 6I + 5\right)w\right))]$;

(c) It follows from the fact that $Sign[\lim\limits_{\gamma \to 0} \frac{\partial \beta_{Pub}}{\partial \gamma}] = Sign[-(bd_A(d_A + d_P) - kwd_P)]$.

(iv) It follows from the fact that $Sign[\frac{\partial \beta_{Pub}}{\partial N}] = Sign[-\gamma^2(bd_A(d_A + d_P) - k(w+\gamma)d_P)]$.

Q.E.D.

Proof of Proposition 2

The principal maximizes her expected payoff in (27) subject to the constraints in (28) - (30). Taking derivatives of (31) with respect to β_i yields a system of N equations of the form: for all $i \in \{1, ..., N\}$,

$$v\frac{da_i}{d\beta_i} = \frac{\partial \mathscr{C}_i}{\partial \beta_i}. \tag{58}$$

Using the fact that $a_i = \frac{v\beta_i}{c}$, $m_i = ((N-1)w-\gamma)\Psi\beta_i + bd_A\Gamma B$ and $s_i = b(N-1)\Psi\beta_i - bk\Gamma B$, where $\Psi = \frac{1}{k((N-1)w-\gamma)+(N-1)bd_A}$, $\Gamma = \frac{\gamma}{(k(w+\gamma)+bd_A)(k((N-1)w-\gamma)+(N-1)bd_A)}$, and $B = \sum_{j=1}^{N}\beta_j$, the above equations can be rearranged to show that the expressions of β_i are a function of B and are identical for all $i \in \{1,...,N\}$, which confirms that the equilibrium β_i is symmetric for all $i \in \{1,...,N\}$.

The symmetry of the equilibrium yields $a_i = a_0\beta_i$, $m_i = m_0\beta_i$, and $s_i = s_0\beta_i$, where $a_0 = \frac{v}{c}$, $m_0 = \frac{(w+\gamma)}{k(w+\gamma)+bd_A}$, and $s_0 = \frac{b}{k(w+\gamma)+bd_A}$ and thus $\frac{\partial \mathscr{C}_i}{\partial \beta_i} = \left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o \beta_i$. Finally, the unique symmetric equilibrium is

$$\beta_i = \frac{v\frac{da_i}{d\beta_i}}{\left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o}. \tag{59}$$

The second order condition is satisfied: $\frac{\partial^2 E_P[V_i]}{\partial \beta_i^2} = -c\frac{\partial a_i}{\partial \beta_i}\frac{\partial a_i}{\partial \beta_i} - k\frac{\partial m_i}{\partial \beta_i}\frac{\partial m_i}{\partial \beta_i} - \rho\sigma_\varepsilon^2 < 0$.

α_i is obtained from the participant constraint for the agent. Substituting the equilibrium β_i into the expressions for a_i, m_i, and s_i yields a_{Pri}, m_{Pri}, and s_{Pri} as stated in the proposition.

Q.E.D.

Proof of Corollary 2

Using $\frac{da_i}{d\beta_i} = \frac{v}{c}, \frac{\partial m_i}{\partial \beta_i} = \frac{1}{k}$, $a_0 = \frac{v}{c}$, $m_0 = \frac{(w+\gamma)}{k(w+\gamma)+bd_A}$, and $s_0 = \frac{b}{k(w+\gamma)+bd_A}$, rearranging the expression of β_{Pri} in Proposition 2 yields $\beta_{Pri} = \frac{\frac{v^2}{c}}{\frac{v^2}{c}+\frac{1}{k}+\frac{bd_P}{k(k(w+\gamma)+bd_A)}+\rho\sigma_\varepsilon^2}$, which immediately leads to (i)-(vi).

Q.E.D.

Proof of Corollary 3

From Proposition 1 and 2, the difference between the contract slopes of the two scenarios can be expressed as follows:

$$\frac{\beta_{Pri}-\beta_{Pub}}{\beta_{Pri}\beta_{Pub}} = \frac{1}{v\frac{da_i}{d\beta_i}}\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o \frac{ds_i}{d\beta_i} \tag{60}$$

Because $\frac{ds_i}{d\beta_i} > 0$, the difference between contract slopes is mainly driven by the marginal cost of scrutiny (i.e., $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o$). Therefore, $\beta_{Pub} \geq \beta_{Pri}$ if and only if $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o < 0$.

Q.E.D.

Proof of Corollary 4

(i) It follows immediately from Proposition 1 and 2 and Corollary 3.

(ii) Note that $B_i = \beta_i(va_i+m_i) = \beta_i^2[\frac{v^2}{c}+\frac{(w+\gamma)}{k(w+\gamma)+bd_A}] > 0$ and $C_i = \alpha_i + \beta_i(va_i+m_i) = \frac{1}{2}\beta_i^2[\frac{v^2}{c} - \frac{k(w+\gamma)^2}{(k(w+\gamma)+bd_A)^2} + \frac{2(w+\gamma)}{k(w+\gamma)+bd_A} + \rho\sigma_\varepsilon^2] > 0$. By Corollary 3, we have $B_{Pub} \geq B_{Pri}$ and $C_{Pub} \geq C_{Pri}$ if and only if $\left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o < 0$.

Q.E.D.

Proof of Proposition 3

The Pareto optimal set of contracts solves a program that maximizes the aggregate payoffs of all principals in the economy, assuming that contracts are publicly observable.

$$\underset{\{\beta_i\}_{i\in N}}{Max} \sum_{i=1}^{N} va_i - \mathscr{C}(\beta_i, s_i(\beta_i)) \tag{61}$$

Taking derivatives with respect to β_i yields a system of N equations of the form: for all $i \in \{1, ..., N\}$,

$$v\frac{da_i}{d\beta_i} = \frac{\partial \mathscr{C}_i}{\partial \beta_i} + \sum_{j=1}^{N} \frac{\partial \mathscr{C}_j}{\partial s_j}\frac{ds_j}{d\beta_i}. \tag{62}$$

Using the fact that $a_i = \frac{v\beta_i}{c}$, $m_i = ((N-1)w - \gamma)\Psi\beta_i + bd_A\Gamma B$ and $s_i = b(N-1)\Psi\beta_i - bk\Gamma B$, where $\Psi = \frac{(k(w+\gamma)+bd_A)}{(k(w+\gamma)+bd_A)(k((N-1)w-\gamma)+(N-1)bd_A)}$, $\Gamma = \frac{\gamma}{(k(w+\gamma)+bd_A)(k((N-1)w-\gamma)+(N-1)bd_A)}$, and $B = \sum_{j=1}^{N}\beta_j$, the above equations can be rearranged to show that the expressions of β_i are a function of B and are identical for all $i \in \{1, ..., N\}$, which confirms that the equilibrium β_i is symmetric for all $i \in \{1, ..., N\}$. Finally, the unique symmetric equilibrium is

$$\beta_i = \frac{v\frac{da_i}{d\beta_i}}{\left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o + \sum_{j=1}^{N}\left[\frac{\partial \mathscr{C}_j}{\partial s_j}\right]_o \frac{ds_j}{d\beta_i}}. \tag{63}$$

The second order condition is satisfied: $\frac{d^2\sum_{i=1}^{N}E_P[V_i]}{d\beta_i^2} = -c\frac{\partial a_i}{\partial \beta_i}\frac{\partial a_i}{\partial \beta_i} - k\frac{dm_i}{d\beta_i}(\frac{dm_i}{d\beta_i} + \sum_{j\neq i}^{N}\frac{dm_j}{d\beta_i}) - (d_A + d_P)(\frac{dm_i}{d\beta_i}\frac{ds_i}{d\beta_i} + \sum_{j\neq i}^{N}\frac{dm_j}{d\beta_i}\frac{ds_j}{d\beta_i} + \frac{dm_i}{d\beta_i}\frac{ds_i}{d\beta_i} + \sum_{j\neq i}^{N}\frac{dm_j}{d\beta_i}\frac{ds_j}{d\beta_i}) - \rho\sigma_\varepsilon^2 < 0$.

α_i is obtained from the participant constraint for the agent. Substituting the equilibrium β_i into the expressions for a_i, m_i, and s_i yields a_{PO}, m_{PO}, and s_{PO} as stated in the proposition.

Q.E.D.

Proof of Proposition 4

First, if $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o = 0$, $\beta_{Pub} = \beta_{Pri} = \beta_{PO}$ by (39) - (42) and thus principals are indifferent between thr two scenarios. Second, suppose $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o \neq 0$. When there is no externality of inspection (i.e., $\gamma = 0$) or the number of firms in the economy N is 1, $\beta_{Pub} = \beta_{PO}$ and $\beta_{Pri} \neq \beta_{PO}$ by (39) - (42). Hence, the principal is better off under public contracts if $\gamma = 0$ or $N = 1$. Next, consider the case in which the number of firms in the economy N is ∞ and $\gamma \neq 0$. Note that

$$\frac{\beta_{Pri}-\beta_{PO}}{\beta_{PO}\beta_{Pri}} = \frac{1}{v\frac{da_i}{d\beta_i}}\sum_{j=1}^{N}\left[\frac{\partial \mathscr{C}_j}{\partial s_j}\right]_o \frac{ds_j}{d\beta_i} = \frac{1}{v\frac{da_i}{d\beta_i}}\left[\frac{\partial \mathscr{C}_j}{\partial s_j}\right]_o \sum_{j=1}^{N}\frac{ds_j}{d\beta_i} \tag{64}$$

$$= \frac{1}{v\frac{da_i}{d\beta_i}}\left[\frac{\partial \mathscr{C}_j}{\partial s_j}\right]_o (b(N-1)\Psi - Nbk\Gamma) \tag{65}$$

$$= \frac{1}{v\frac{da_i}{d\beta_i}}\left[\frac{\partial \mathscr{C}_j}{\partial s_j}\right]_o \frac{b}{k(w+\gamma)+bd_A} \tag{66}$$

$$\frac{\beta_{Pub}-\beta_{PO}}{\beta_{PO}\beta_{Pub}} = \frac{1}{v\frac{da_i}{d\beta_i}}\sum_{j\neq i,}^{N}\left[\frac{\partial \mathscr{C}_j}{\partial s_j}\right]_o \frac{ds_j}{d\beta_i} = \frac{1}{v\frac{da_i}{d\beta_i}}\left[\frac{\partial \mathscr{C}_j}{\partial s_j}\right]_o \sum_{j\neq i,}^{N}\frac{ds_j}{d\beta_i} \tag{67}$$

$$= \frac{1}{v\frac{da_i}{d\beta_i}}\left[\frac{\partial \mathscr{C}_j}{\partial s_j}\right]_o (-1)(N-1)bk\Gamma \tag{68}$$

because $s_i = b(N-1)\Psi\beta_i - bk\Gamma B$, where $\Psi = \frac{(k(w+\gamma)+bd_A)}{(k(w+\gamma)+bd_A)(k((N-1)w-\gamma)+(N-1)bd_A)}$, $\Gamma = \frac{\gamma}{(k(w+\gamma)+bd_A)(k((N-1)w-\gamma)+(N-1)bd_A)}$, and $B = \sum_{j=1}^{N}\beta_j$. This confirms that (40) and (42) hold when $N \to \infty$.

If $\gamma < 0$, $\beta_{PO} > \beta_{Pub} > \beta_{Pri}$ for $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o < 0$ and $\beta_{PO} < \beta_{Pub} < \beta_{Pri}$ for $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o > 0$, by Corollary 3, (39) - (42), and the properties above. Thus, the principal is better off under public contracts if $\gamma < 0$. If $\gamma > 0$, $\beta_{Pri} > \beta_{PO} > \beta_{Pub}$ for $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o > 0$ and $\beta_{Pri} < \beta_{PO} < \beta_{Pub}$ for $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o < 0$. Observe that as $N \to \infty$,

$$Sign[(\beta_{Pri}-\beta_{PO})-(\beta_{PO}-\beta_{Pub})] = Sign\left[\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o\right] > 0 \; for \left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o > 0 \tag{69}$$

$$Sign[(\beta_{Pub}-\beta_{PO})-(\beta_{PO}-\beta_{Pri})] = Sign\left[\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o\right] < 0 \; for \left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o < 0. \tag{70}$$

Thus, the principal is better off under public contracts if $\gamma > 0$. In sum, the principal is better off under public contracts when the number of firms in the economy N is ∞ and $\gamma \neq 0$.

Proof of Proposition 5

If $\gamma < 0$, $\beta_{PO} > \beta_{Pub} > \beta_{Pri}$ for $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o < 0$ and $\beta_{PO} < \beta_{Pub} < \beta_{Pri}$ for $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o > 0$, by Corollary 3, (39) - (42), and the properties above. Thus, the principal is better off under public contracts if $\gamma < 0$. If $\gamma > 0$, $\beta_{Pri} > \beta_{PO} > \beta_{Pub}$ for $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o > 0$ and $\beta_{Pri} < \beta_{PO} < \beta_{Pub}$ for $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o < 0$. Note that

$$Sign[(\beta_{Pri}-\beta_{PO})-(\beta_{PO}-\beta_{Pub})] = Sign\left[\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o\right] Sign[\Gamma(\gamma)] \; for \left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o > 0 \tag{71}$$

$$Sign[(\beta_{Pub}-\beta_{PO})-(\beta_{PO}-\beta_{Pri})] = Sign\left[\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o\right] Sign[\Gamma(\gamma)] \; for \left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o < 0. \tag{72}$$

where $\Gamma(\gamma) = (N-1)[c(k(w+\gamma)+bd_A)^2((k(w-\gamma)+bd_A) - k\frac{\gamma}{(N-1)})(\frac{v^2}{c}+\frac{1}{k}+\rho(\sigma_\varepsilon^2)) - \frac{cbd_A}{k}\{2k(k(w+\gamma)(w-\frac{\gamma}{(N-1)})+bwd_A)+bd_A(k(w+\gamma)+bd_A-k\frac{\gamma}{(N-1)})\} + 2cb(d_A+d_P)(k(w+\gamma)(w-\frac{\gamma}{(N-1)})+bwd_A)]$. First, observe that $\Gamma(\gamma) > 0$ if $0 < \gamma \leq \frac{w}{2}$. Second, if $w > \gamma > \frac{w}{2}$, there exists $\bar{\gamma}(> \frac{w}{2})$ such that $\Gamma(\gamma) < 0$ for

$\gamma > \bar{\gamma}$, because $\Gamma(\gamma)$ is a cubic function of γ and its coefficient on γ^3 is negative. Therefore, if $0 < \gamma \leq \frac{w}{2}$,

$$Sign[(\beta_{Pri} - \beta_{PO}) - (\beta_{PO} - \beta_{Pub})] = Sign\left[\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o\right] > 0 \; for \; \left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o > 0 \tag{73}$$

$$Sign[(\beta_{Pub} - \beta_{PO}) - (\beta_{PO} - \beta_{Pri})] = Sign\left[\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o\right] < 0 \; for \; \left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o < 0. \tag{74}$$

and if $\gamma > \bar{\gamma}(> \frac{w}{2})$,

$$Sign[(\beta_{Pri} - \beta_{PO}) - (\beta_{PO} - \beta_{Pub})] = Sign\left[\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o\right] < 0 \; for \; \left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o > 0 \tag{75}$$

$$Sign[(\beta_{Pub} - \beta_{PO}) - (\beta_{PO} - \beta_{Pri})] = Sign\left[\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o\right] > 0 \; for \; \left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o < 0. \tag{76}$$

Proof of Proposition 6

The social planner maximizes the sum of principals' expected payoffs

$$\underset{\{\beta_i\}_{i \in N}}{Max} \sum_{i=1}^{N} va_i - \mathscr{C}(\beta_i, s_i(\beta_i)) + I(\beta_i, s_i(\beta_i)) \tag{77}$$

where $I(\beta_i, s_i(\beta_i)) = b\sum_{i=1}^{N} m_i(\beta_i)s_i(\beta_i) - \frac{w}{2}\sum_{i=1}^{N} s_i(\beta_i)^2 - \frac{\gamma}{2(N-1)}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} s_i(\beta_i)s_j(\beta_j)$ is the inspector's payoff.

Taking derivatives with respect to β_i yields a system of N equations of the form: for all $i \in \{1,...,N\}$,

$$v\frac{da_i}{d\beta_i} = \frac{\partial \mathscr{C}_i}{\partial \beta_i} + \sum_{j=1}^{N} \frac{\partial \mathscr{C}_j}{\partial s_j}\frac{ds_j}{d\beta_i} - b\sum_{j=1}^{N} \frac{dm_j}{d\beta_i}s_j. \tag{78}$$

where the last term is derived by applying the envelop theorem to the inspector's expected payoff, i.e., $\frac{dI(\beta_i,s_i(\beta_i))}{d\beta_i} = \frac{\partial I(\beta_i,s_i(\beta_i))}{\partial \beta_i} = b\sum_{j=1}^{N} \frac{dm_j}{d\beta_i}s_j$. Similarly, the above equations can be rearranged to show that the expressions of β_i are a function of B and are identical for all $i \in \{1,...,N\}$, which confirms that the equilibrium β_i is symmetric for all $i \in \{1,...,N\}$. Finally, the unique symmetric equilibrium is

$$\beta_i = \frac{v\frac{da_i}{d\beta_i}}{\left[\frac{\partial \mathscr{C}_i}{\partial \beta_i}\right]_o + \sum_{j=1}^{N}\left[\frac{\partial \mathscr{C}_j}{\partial s_j}\right]_o \frac{ds_j}{d\beta_i} - bs_0\sum_{j=1}^{N}\frac{dm_j}{d\beta_i}}. \tag{79}$$

The second order condition is satisfied provided that $b = d_A + d_P$, $\frac{d^2\sum_{i=1}^{N} E_P[V_i]}{d\beta_i^2} = -c\frac{\partial a_i}{\partial \beta_i}\frac{\partial a_i}{\partial \beta_i} - k\frac{dm_i}{d\beta_i}(\frac{dm_i}{d\beta_i} + \sum_{j\neq i}^{N}\frac{dm_j}{d\beta_i}) - (d_A + d_P)(\frac{dm_i}{d\beta_i}\frac{ds_i}{d\beta_i} + \sum_{j\neq i}^{N}\frac{dm_j}{d\beta_i}\frac{ds_j}{d\beta_i} + \frac{dm_i}{d\beta_i}\frac{ds_i}{d\beta_i} + \sum_{j\neq i}^{N}\frac{dm_j}{d\beta_i}\frac{ds_j}{d\beta_i}) - \rho\sigma_\varepsilon^2 + b\sum_{j=1}^{N}\frac{dm_j}{d\beta_i}\frac{ds_j}{d\beta_i} + \sum_{j=1}^{N}\frac{\partial^2 I}{\partial s_j^2}(\frac{ds_j}{d\beta_i})^2 < 0$.

α_i is obtained from the participant constraint for the agent. Substituting the equilibrium β_i into the expressions for a_i, m_i, and s_i yields a_{SW}, m_{SW}, and s_{SW} as stated in the proposition.

Q.E.D.

Proof of Proposition 7

First, observe that

$$\frac{\beta_{Pri}-\beta_{PO}}{\beta_{PO}\beta_{Pri}} = \frac{1}{v\frac{da_i}{d\beta_i}}(\sum_{j=1}^{N}\left[\frac{\partial \mathscr{C}_j}{\partial s_j}\right]_o \frac{ds_j}{d\beta_i} - bs_0\sum_{j=1}^{N}\frac{dm_j}{d\beta_i}) \tag{80}$$

$$= \frac{1}{v\frac{da_i}{d\beta_i}}[(km_0\frac{\partial m_i}{\partial s_i}+(d_A+d_P)(\frac{\partial m_i}{\partial s_i}s_0+m_0))\sum_{j=1}^{N}\frac{ds_j}{d\beta_i} - bs_0\sum_{j=1}^{N}\frac{dm_j}{d\beta_i}] \tag{81}$$

$$= \frac{1}{v\frac{da_i}{d\beta_i}}[(-d_Am_0+(d_A+d_P)(-\frac{d_A}{k}s_0+m_0))s_0 - bs_0m_0] \tag{82}$$

$$= \frac{1}{v\frac{da_i}{d\beta_i}}(-d_Am_0+(d_A+d_P)(-\frac{d_A}{k}s_0))s_0 < 0 \tag{83}$$

$$\frac{\beta_{Pub}-\beta_{PO}}{\beta_{PO}\beta_{Pub}} = \frac{1}{v\frac{da_i}{d\beta_i}}(\sum_{j\neq i,}^{N}\left[\frac{\partial \mathscr{C}_j}{\partial s_j}\right]_o \frac{ds_j}{d\beta_i} - bs_0\sum_{j=1}^{N}\frac{dm_j}{d\beta_i}) \tag{84}$$

$$= \frac{1}{v\frac{da_i}{d\beta_i}}[(km_0\frac{\partial m_i}{\partial s_i}+(d_A+d_P)(\frac{\partial m_i}{\partial s_i}s_0+m_0))\sum_{j\neq i,}^{N}\frac{ds_j}{d\beta_i} - bs_0\sum_{j=1}^{N}\frac{dm_j}{d\beta_i}] \tag{85}$$

$$= \frac{1}{v\frac{da_i}{d\beta_i}}[(-d_Am_0+(d_A+d_P)(-\frac{d_A}{k}s_0+m_0))(-\frac{\gamma bk}{(k(w+\gamma)+bd_A)(k(w-\frac{\gamma}{N-1})+bd_A)}) - bs_0\frac{(w+\gamma)}{k(w+\gamma)+bd_A}] \tag{86}$$

$$= \frac{1}{v\frac{da_i}{d\beta_i}}[(d_Pm_0))(-\frac{\gamma bk}{(k(w+\gamma)+bd_A)(k(w-\frac{\gamma}{N-1})+bd_A)}) \tag{87}$$

$$= +(d_A+d_P)s_0(\frac{\gamma bd_A}{(k(w+\gamma)+bd_A)(k(w-\frac{\gamma}{N-1})+bd_A)} - \frac{(w+\gamma)}{k(w+\gamma)+bd_A}] < 0 \tag{88}$$

Therefore,

$$\beta_{SW} > \beta_{Pub} \tag{89}$$

$$\beta_{SW} > \beta_{Pri}. \tag{90}$$

Next, from Corollary 3, $\beta_{Pub} > (<) \beta_{Pri}$ if and only if $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o < 0$. Thus, the social welfare is larger (smaller) under public contract if and only if $\left[\frac{\partial \mathscr{C}_i}{\partial s_i}\right]_o < 0$.