INFLUENCE OF SPATIAL VARIABILITY OF SOIL POTASSIUM AND NONUNIFORMITY OF FERTILIZER APPLICATION ON CROP RESPONSE

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This work is dedicated to my family.

ABSTRACT

Spatial variability of soil properties important in plant growth is common in soils of the tropics and subtropics. The description of this variation is the first step towards understanding and correcting the associated detrimental effects. Geostatistical methods have been used to characterize inherent soil variation, yet the variation resulting from fertilizer or amendment addition can be equally important. Geostatistical techniques have been developed to estimate proportions of a region above or below selected concentrations of elements. These methods have seldom been applied to the description of or the modification of nutrient concentrations in soils but hold promise in dealing with spatial variability of soil properties with inherent variation or that added by fertilizer application.

Conventional statistical analysis and geostatistical analysis were used to investigate the field spatial variability of selected soil properties. Comparison of coefficients of variation (CV) indicated that exchangeable K was the least variable (CV = 12.6) and exchangeable Na the most variable (CV = 34.4). Semi-variograms of exchangeable Ca, Mg, K, and Na revealed strong spatial dependence. Large nugget variances accounted for 41-52% of the total variances. Ranges of spatial dependence were about 17 m for K and Na and more than 20 m for Ca and Mg.

Kriged estimates were used to draw contour maps of selected soil properties.

The structure of spatial dependence of soil K was used to estimate the proportion of a given experimental plot below a specified threshold of exchangeable K. Different patterns of fertilizer distribution were simulated in a field study, and a measure of uniformity (UCF = 1 - CV) of fertilizer distribution was calculated from the CV of K application. The results indicated that fertilizer rates necessary for 90-95% relative yield could be predicted from the response of uniform application and the CV of the actual application. Variable rates of fertilization could then be applied if the deficient zones could be identified in the field.

The response of chinese cabbage to potassium application in the field experiment was strongly affected by the pattern of fertilizer distribution. There was a 9.52% decrease in maximum yield with increased unevenness of fertilizer distribution. The amount of K associated with 95% maximum yield was 97, 113, and 444 kg K/ha for UCF values of 1.0, 0.42, and -0.02, respectively. The corresponding tissue K concentrations associated with maximum yield were 4.00, 4.15, and 4.30%.

The decrease in yield due to nonuniform fertilizer application was quantitatively expressed through the fluctuation response index (FRI) i.e., the product of the variance of fertilizer application and the second derivative of the yield-fertilizer function obtained under uniform fertilizer application. The fluctuation response index holds promise as a way to estimate crop loss due to excessive variability, and therefore financial loss to the farmer. The farmer may then have the opportunity to control the application so that losses are kept within acceptable limits. Nonuniform fertilizer application significantly affected the critical level of exchangeable K for chinese cabbage. Increased variation in exchangeable K from CV = 44.1% to CV = 96.6% resulted in critical levels of soil K of 0.28 and 0.73 cmol(+)/kg, respectively.

The response of maize to residual K was closely related to the degree of spatial variability of residual K. Increased variation of residual K from CV = 37.0% to CV = 48.5% resulted in yield losses and in increased level of exchangeable K associated with 90% maximum yield from 0.16 to 0.24 cmol(+)/kg, respectively. There was a gradual decrease in the correlation coefficient between corn yield and ear-leaf K with increasing spatial variability of residual K.

The potassium retention capacity of the Puaulu soil was investigated. The results indicated that the Puaulu soil has a low affinity for K as evidenced by the very low

Gapon selectivity coefficient (K_G = 0.15 $(mol^{-1}L)^{1/2}$). The topsoil had significantly lower retention capacity than the subsoil.

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I. INTRODUCTION

Application of solid K fertilizers by hand or mechanical equipment (fertilizers applicators) is usually nonuniform. Achieving complete uniformity of fertilizer application is a difficult if not impossible task. Evaluation of the detrimental effects of nonuniformity is useful in order to determine crop loss, and therefore financial loss to the farmer. The farmer may then have the opportunity to control the application so that losses are kept within acceptable limits. There is, however, another major source of variation. Plant growth or yield will depend not only on the distribution of solid K fertilizers over the soil surface but on the spatial variability of indigeneous soil K as well.

Variability of K potassium within a single soil type or field can be large (Ball and Williams, 1968; Schuffelen et al., 1945) and can have a crucial bearing on the determination of field grown crops. Farmers, agronomists, and plant breeders have long recognized the importance of spatial variability in fields of different size and have strived toward uniformity of crop response by mixing and manipulating the soil surface to remove microtopographic irregularities which would interfere with plant development.

Soil variability (lateral and vertical) can be expressed in terms of spatial difference in particular properties of soil or can be revealed and expressed in terms of response of plants growing on the soil. The concept of soil forming factors (Jenny, 1941) provides some explanation why soils vary in both space and time. Practical interest in soil variability has been expressed by studies concerned with soil testing for agronomic purposes (Cameron et al., 1971), soil mapping units (Campbell, 1978), soil physical properties (Cassel and Bauer, 1975; Warrick and Nielsen, 1980), and soil chemical properties (Ball and Williams, 1968).

Variability of soil properties is generally assessed by classical statistical methods e.g., probability density functions with associated moments and coefficients of variation (CV). This type of analysis, however, makes an implicit assumption about the observations of a given soil property, namely that the observations are independent regardless of their location in the field. (Vauclin et al., 1982). Current spatial variability techniques help determine the extent to which this assumption is true. Recently agronomists and soil scientists have been placing more and more emphasis on the fact that variation of a soil property is not completely disordered (random) over the field and that this spatial structure must be taken into consideration in the treatment of soil information. The traditional approach neglects the spatial structure inherent in soils. Classical statistical analysis, thus, provides an incomplete description of the variability of soil properties inasmuch as there is no link between the calculated variance and the distance between the observations.

The theory of regionalized variables, geostatistics, provides new concepts in quantifying variability in soil nutrients and spatial variability. Geostatistics is tailored for the analysis of the spatial variation of continuous geographic distribution.

The objects of this study are to:

1. Evaluate the magnitude of spatial variability of the potassium status of a volcanic ash soil (Typic Hydrandept, medial over thixotropic, isomesic) from the island of Hawaii;

2. Test the usefulness of some geostatistical concepts as a tool for determining the appropriate rates of potassium fertilizer application;

3. Investigate crop response to nonuniform distribution of potassium fertilizers and residual soil potassium.

II. LITERATURE REVIEW

2.1 Spatial variation in fields soils

Soils are known to have both lateral and vertical variability. Two broad categories of field spatial heterogeneity can be distinguished: deterministic heterogeneity (systematic variability) and stochastic (random) heterogeneity (Philip, 1980). Systematic or deterministic variability is a gradual change in soil properties as a function of landform, geomorphic elements and soil forming factors and/or soil management. Random or stochastic variability cannot be related to a known cause (Wilding et al., 1983) and may involve many scales. Variation in soil properties from point to point in landscape results from many causes. Some of these causes affect small volumes of soil and introduce differences over short distances; others introduce long-range soil gradients (Beckett and Webster, 1971). Among the sources of variation in the natural landscape one can distinguish parent materials, climate and topography, physical and chemical processes and biological activities. These sources of variation are also applicable to cultivated landscapes. However, cultivated landscapes might be expected to have additional sources of variability through addition of animal wastes, fertilizers, water, cultivation etc. (Dahiya et al., 1984a). The placement or imperfect broadcasting of fertilizers and farm yard manure tends to superimpose additional heterogeneity in soil properties. There are some indications that variation tends to be greater for soil properties much affected by management than for those less affected. Beckett and Webster (1971) reported that total variances (expressed as coefficients of variation, CV) in different soils were approximately 20% for properties such as sand, clay, or total phosphorus which are relatively unaffected by management; 35% for organic matter, cation exchange capacity and total nitrogen; and 60% for properties such as available K, P, Mg, and Ca which are most affected by management.

Fertilizer application tends to increase the within-field soil variability. Biggar (1978) reported that in general the CV of soil nitrogen was 100-150% greater for fertilized plots than for unfertilized plots. Variability is frequently higher in recently fertilized fields and may rise during and shortly after crop growth as concluded by Beckett and Webster (1971) from literature survey. The high variability due to recent fertilizing or grazing tends to be reduced by cultivation and fallow (Dahiya et al., 1984b).

Most studies of soil variability have been confined to topsoil properties. It is well established that nutrient and water uptake by plants are not confined to the topsoil so that variation of soil properties with depth can also be expected to significantly affect plant growth and development. However, as pointed out by Dahiya et al. (1984a), it is difficult to generalize about the variability of soil properties with depth based on the results of available studies.

Because the variability of soil properties may be great even over small areas many studies carried out in the past have focused mainly on the effect of variability upon the number of samples required to obtain a reasonable estimate of chemical composition (Cline, 1944; Vermeulen et al., 1955). The importance of the distance between samples has been specifically noted only in a few studies (Cipra et al., 1972; Ball and Williams, 1968). Within a given field individuals measurements of soil properties may differ considerably due to a combination of experimental error, temporal variation, and spatial variation (Campbell, 1978). According to Ball and Williams (1968) spatial variation is usually the largest of the three. It should be pointed out, however, that most of the studies where variation in soil properties over distance has been examined have used analysis of variance methods to measure relationships between distance and variance (Cameron et al., 1971; Cipra et al., 1972). However, such classical statistical methods do not permit a complete description of changes over distance (Campbell, 1978). Moreover, even if these methods permit one to determine the number of samples required they do not deal with the optimum sampling distance. On the other hand, the multiplicity of samples supposedly designed to increase the accuracy of evaluation of a given soil property may be a fallacy because, if not properly located, they may indeed repeat indefinitely the same information without yielding anything else (Matheron, 1963).

The theory of regionalized variables, geostatistics, offers another approach to the study of the variability of soil properties by considering the coordinates of the area and the interdependence between neighboring samples. A brief account of this theory will, therefore, be made.

2.2 Theory of regionalized variables

If one were to determine a given soil property e.g., the level of exchangeable potassium (Z), in a field at any place i in class j then according to the classical model of variation this could be written as follows (McBratney et al., 1981).

$$Z_{ij} = \mu + \alpha_j + \varepsilon_{ij} \tag{2.1}$$

where μ = general mean of exchangeable potassium for the whole field

α_j = difference between the general mean and the mean of class j, μ_j

ε_{ij} = spatially-uncorrelated (random) component distributed normally with zero mean and variance σ^2.

The expected value of exchangeable potassium at any place in the jth class is μ_j and can be estimated from a sample of size n_j as (McBratney et al., 1981):

$$\hat{z}_{ij} = \bar{z}_j = 1/n_j \sum_{k=1}^{n_j} z_{kj} \quad (2.2)$$

On the other hand, the estimation variance ^2_E can be defined by

$$\sigma^2_E = E[(Z_{ij} - \hat{Z}_{ij})^2] \quad (2.3)$$

According to McBratney et al. (1981) this is given by

$$\sigma^2_E = \sigma^2 + \sigma^2/n_j \quad (2.4)$$

It can be seen from Eq. (2.4) that the estimation variance is the sum of the within-class variance (σ^2) and the estimation variance of the class mean (σ^2/n_j). The square root of the estimation variance of the class mean i.e. $\sigma\sqrt{n_j}$ is the usual standard error. The estimation variance measures the precision with which any prediction can be made. However, there can be spatial dependence between data points and in these circumstances the classical model is inadequate because it takes no account of spatial correlation and the position of sampling sites (Burgess and Webster, 1980a).

The alternative, therefore, is to use regionalized variable theory. Regionalized variable theory is a set of statistical procedures developed by Matheron (1963) for the analysis of the spatial variation of continuous geographic distributions. A regionalized variable as defined by Matheron (1963) is an actual function, taking a definite value in each point of space.

If the level of exchangeable potassium at location x is represented by a random function Z(x) and at location x+h by Z(x+h) then, according to regionalized variable theory, the relation between these two points a distance h apart can be expressed as the variance of their difference.

$$\gamma(h) = 1/2 \text{ var}[Z(x) - Z(x+h)] \qquad (2.5)$$

where $\gamma(h)$ is the semi-variance and is a measure of the similarity between points a given distance, h, apart. The closer the levels of exchangeable potassium at these points the smaller is $\gamma(h)$, and vice versa. The graph of $\gamma(h)$ against h is the semi-variogram. According to Burgess and Webster (1980a) if the mean of the observations remain constant over distance d, then, provided h is less than d the semi-variance is half the expected squared difference between values at lag:

$$\gamma(h) = 1/2 \text{ E } [Z(x+h) - Z(x)]^2 \qquad (2.6)$$

It should be pointed out that in applying regionalized variable theory it is assumed that Eq. (2.6) applies everywhere, i.e. that the value of the semi-variance depends only on h, the separation between sites, and not on the actual position of the sites. If this assumption holds then the semi-variogram for a given area can be estimated from a single set of data (McBratney et al., 1982).

Journel and Huijbregts (1978, p. 12) have estimated the semi-variance as follows:

$$\gamma^*(h) = 1/2N(h) \sum_{i=1}^{N(h)} [Z(i+h) - Z(i)]^2 \qquad (2.7)$$

where N(h) is the number of experimental pairs [Z(i), Z(i+h)] of data separated by a vector h. Eq. (2.7) can be rewritten as follows:

$$\gamma^*(h) = 1/2N(h) \sum_{i=1}^{N(h)} Z^2(i+h) + 1/2N(h) \sum_{i=1}^{N(h)} Z^2(i) - 1/2N(h) \sum_{i=1}^{N(h)} Z(i+h)Z(i) \qquad (2.8)$$

The negative term on the far right of Eq. (2.8) is the covariance K(h) and each one of the two first terms is equivalent with their sum being the variance K(0). Therefore Eq. (2.8) can be rewritten as: $\gamma^*(h) = K(0) - K(h)$ or simply as: Semi-variance = Variance - Covariance. A plot of an idealized semi-variogram is shown in Fig. 2.1. The semi-variogram can reveal some characteristics of the geographic distribution of exchangeable potassium which are needed to provided kriged estimates at previously unrecorded points. It will be seen from Fig. 2.1 that as the vector distance h becomes infinitesimally small, the covariance K(h) approaches the value of the variance K(0), and as a result, the value of the semi-variance $\gamma(h)$ becomes zero. However, it should be pointed out that in some cases the curve does not pass through the origin. In such cases as h approaches zero $\gamma(h)$ approaches a positive value commonly known as the nugget variance. The nugget variance is due to fluctuations in the soil that occur over distances much shorter than the sampling interval and to experimental uncertainties. Fig. 2.1 also shows that as the vector h becomes larger the correlation between the data points vanishes, and consequently, the value of the semi-variance approaches the value of the variance. However, a small value of ε is accepted as the error when it is assumed that the semi-variance approaches the variance within this arbitrarily selected small value is called the range. The range of the semi-variogram indicates the size of the zone of influence of a sample (Campbell, 1978) and provides an estimate of the minimum distance required for spacing of independent samples since beyond the range the observed levels of exchangeable potassium are considered to be independent of each other and therefore not influenced by spatial dependence. Vieira et al. (1981) reported a range of 50 m for steady state

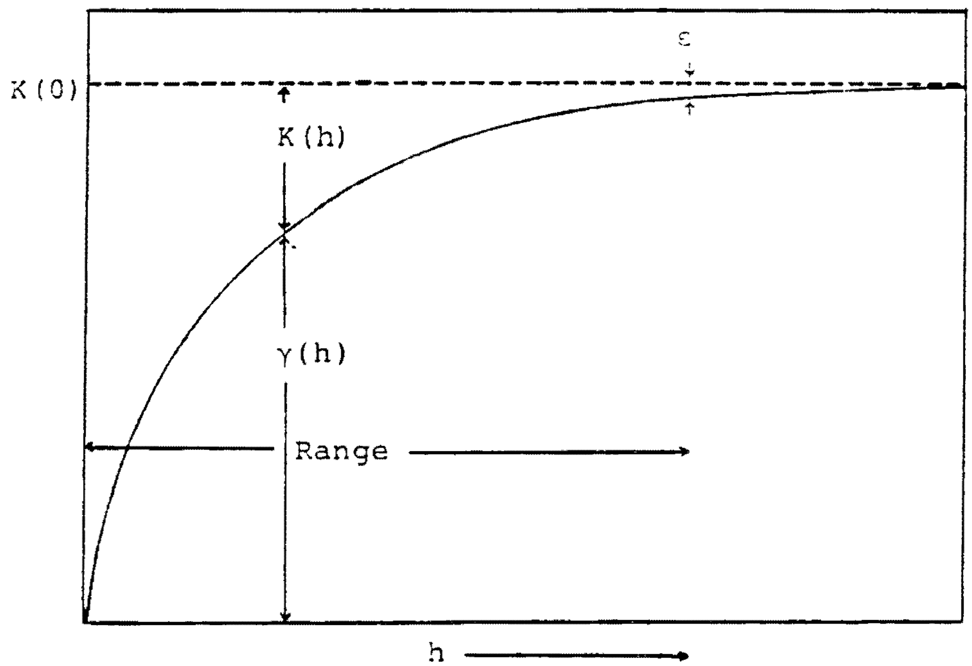


Fig. 2.1 Theoretical spherical model semi-variogram illustrating the relationship between (classical) variance ($K(0) = \sigma^2$), spatial covariance $K(h)$, and the semi-variance $\gamma(h)$.

infiltration rate on a 160 m x 55 m field. Campbell (1978) found ranges of 30 m and 40 m for sand content in two different soils and random variation for pH measurements. Gajem et al. (1981) calculated zones of influence i.e., ranges of spatial dependence for 11 physical parameters going from few centimeters to several tens of meters depending on the spacing. Yost et al. (1982) reported ranges of semi-variograms for pH, Ca, Mg, K, Si, and P sorbed at 0.02 mg P/1 ranging from 32 to 42 km. Hajrasuliha et al. (1980) found that measurements of electrical conductivity on an 80 m grid were spatially related to each other for separation distances of less than 800 m.

Estimation of the value of a spatially distributed variable, e.g., exchangeable potassium, and assessment of the probable error associated with the estimate is called kriging (Davis, 1973). Kriging relies on the variance structure that may exist in the data. The presence or absence of such structure is revealed in the semi-variogram. As a stochastic interpolator, kriging differs from conventional interpolation techniques in that it requires a prior structural analysis. A kriged or estimated value is found by attributing weights λ_i to the neighboring values which are the measured values. However, in order for kriging to be the best linear unbiased estimator, two constraints are placed on the estimates

(Hajrasuliha et al., 1980). First the estimation must be unbiased, which can be written as:

$$E[\hat{Z}(x_0) - Z(x_0)] = 0 \qquad (2.9)$$

Second the squared average error should be a minimum

$$\mathrm{Var}[\hat{Z}(x_0) - Z(x_0)] = \text{minimum} \qquad (2.10)$$

where $\hat{Z}(x_0)$ is the estimation of exchangeable potassium and $Z(x_0)$ is the true value.

Eq. 2.11 can be used to estimate the level of exchangeable potassium, Z, at a fixed location x_0, using a weighted average of the n measured values.

$$\hat{Z}(x_0) = \sum_{i=1}^{N(h)} \lambda_i Z(x_i) \qquad (2.11)$$

where $\hat{Z}(x_0)$ is the estimated value at the location x_0, and λ_i are the weights associated with the data points. Substituting Eq. 2.11 into Eq. 2.9 and developing yields (vieira et al., 1982).

$$\sum_{i=1}^{N(h)} \lambda_i = 1 \qquad (2.12)$$

The type of estimator mentioned above is a linear type of estimator which is unbiased because the weights add up to 1. But, according to Clark (1982) there are in fact an infinite number of such linear unbiased estimators. Among these, at least, one combination of weights must produce a minimum estimation variance, and it is this one that kriging seeks to find. In order to minimize the estimation variance with respect to the weights one must differentiate and set all the resulting partial derivatives to zero i.e.

$$\frac{\partial \sigma_\varepsilon^2}{\partial \lambda_i} = 0 \qquad i = 1, 2, 3, \ldots, n$$

If there are n observations under consideration there will be n unknown weights (λ_i, λ_2, ... λ_n) and n equations. Since the sum of the weights must equal one for an unbiased estimate, an additional equation $\lambda_i = 1$ needs to be included. As a result there will be n+1 equations and in order to obtain a well balanced system one more unknown must be added -- a Lagrangian multiplier.

The kriged estimate can be developed by determining the semi-variances or the covariances among all experimental points within the estimation neighborhood of a point. The covariances form the matrix A which is a n+1 by n+1 matrix where n is the number of points within the estimation neighborhood. The covariances between the point whose value is to be estimated and each experimental point form the n+1 vector B. The set of simultaneous equations for the weights are usually solved by matrix methods:

$$X = A^{-1}B \tag{2.13}$$

where

$$X = \begin{bmatrix} \lambda \\ \mu \end{bmatrix} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \cdot \\ \cdot \\ \cdot \\ \lambda_n \\ \mu \end{bmatrix} \qquad (2.14)$$

where is a Lagrange multiplier

$$A = \begin{bmatrix} \gamma(x_1, x_1)\gamma(x_2, x_1) \ldots \gamma(x_n, x_1) & 1 \\ \gamma(x_1, x_2)\gamma(x_2, x_2) \ldots \gamma(x_n, x_2) & 1 \\ \cdot & \\ \cdot & \\ \cdot & \\ \gamma(x_1, x_n)\gamma(x_2, x_n) \ldots \gamma(x_n, x_n) & 1 \\ 1 \qquad 1 \qquad\qquad 1 & 0 \end{bmatrix} \qquad (2.15)$$

$$\text{and } B = \begin{bmatrix} \gamma(x_1, x_n) \\ \gamma(x_2, x_0) \\ \cdot \\ \cdot \\ \cdot \\ \gamma(x_n, x_0) \\ 1 \end{bmatrix} \qquad (2.16)$$

The associated estimation or kriging variance is computed from

$$\sigma_k{}^2 = \sum_{i=1}^{n} \lambda_i(x_i, x_0) + \mu \qquad (2.17)$$

The kriging procedure described above is known as punctual kriging because measurements are considered as points due to the size of the soil samples. Kriging, however, can also be carried out over areas, a procedure known as block kriging (Burgess and Webster, 1980b). In block kriging, instead of a point χ_0, a portion of a field H_v with its center at χ_0 is considered. The semi-variances between the data points and the interpolated point are replaced by the average semi-variances between the data points and all points in the portion of the field (Burgess and Webster, 1980b). Therefore, each (x_i, x_0) of Eq. 2.16 is replaced by the integral

$\int (x_i, x)\ p(x)\ dx$ where $p(x)$ is given as follows:

$$\begin{cases} p(x) = 1/H_v & \text{if } x \text{ belongs to } v \\ p(x) = 0 & \text{otherwise} \end{cases} \tag{2.18}$$

$$\int p(x)\ dx = 1 \tag{2.19}$$

As a result the coefficients for block kriging can be written as

$$\begin{bmatrix} \lambda \\ \mu \end{bmatrix} = A^{-1} S \tag{2.20}$$

where

$$S = \begin{bmatrix} \int (x_1, x)\ p(x)\ dx \\ \int (x_2, x)\ p(x)\ dx \\ \vdots \\ \int (x_n, x)\ p(x)\ dx \\ 1 \end{bmatrix} \tag{2.21}$$

The estimation variance for the portion of the field H_V is

$$\sigma^2_H = S^T \begin{bmatrix} \lambda \\ \mu \end{bmatrix} - \int\int (x, y)\ p(x)\ dx\ dy \qquad (2.22)$$

It can be seen from Equations (2.17) and (2.22) that the estimation variance depends on the semi-variogram and through it on the configuration of the observation points in relation to the points or block to be estimated, and on the distances between the points and between the points and that which is to be estimated. It does, however, not depend on the observed values themselves. As a result, if the semi-variogram is known before hand then the kriging variance for any sampling scheme can be determined before performing the sampling (McBratney, et al., 1981). Geostatistical techniques described above can also be used to quantify nutrient content and fertilizer rates. This will be illustrated in chapter 6.

2.3 Potassium availability and methods of its evaluation

According to Barber (1968) three mechanisms are involved in K supply to plant roots: root interception, in which the growing root comes into contact with available soil K; mass flow by which K is transported with the flowing water to the root surface; and diffusion of K in the soil solution to the root surface along a concentration gradient.

Interception depends on the soil volume prospected by the plant root during growth. According to Schroeder (1973) interception may be assessed by the proportion of root volume to soil volume. However, it is generally accepted that the root volume does not exceed 2-3% of the soil volume. As a result, interception usually supplies less than 10% of the K requirements of the plant (Schroeder, 1973).

The amount of K supplied by the flow of water depends on the K concentration of the soil solution and quantity of water transpired by the plant. According to Barber et al. (1963), only about 10% of the total K requirements of crop is transported by mass flow, although the contribution can be somewhat greater when the amount of water transpired by the crop is increased. It is generally accepted that diffusion is the main process by which K is transported to plant roots.

There are two main theories with respect to K uptake by plants. According to the first uptake of K by plants is mainly a function of K concentration in soil solution and is either enhanced by the presence of Ca and Mg (Viets, 1942; Marscher, 1964; Epstein, 1972) or independent of Ca and Mg (Mengel, 1968; Barber, 1968; Wild et al., 1969). This theory is in agreement with the experimental finding of plant psychologists that the rate of uptake of an ion varies with its concentration in solution. According to

Nye (1968) expressing this concentration as a concentration ratio with other ions, or using thermodynamic terminology does not give the simple relation between concentration in solution and plant uptake any better justification.

The second theory suggests that because of K/Mg and K/Ca antagonism uptake is mainly a function of K/Ca+Mg ratios in the soil solution (Woodruff, 1955; Beckett, 1964a; Fassbender and Pineres, 1971). The activity ratio (AR^k), or concentration of K and Ca and Mg ions in the soil solution can be expressed as:

$$AR^k = a_K/\sqrt{a_{Ca+Mg}} \qquad (2.23)$$

where a_K = activity of K

a_{Ca+Mg} = activity of Ca+Mg

In acid soils the Al ions have been included (Tinker, 1964). This activity ratio is taken as a measure of the chemical potential of calcium and magnesium. In addition, the amount of potassium gained or lost by the soil (ΔK) in the equilibrium with a potassium salt solution is measured. THus a quantity ($\Delta K=Q$) and an intensity ($AR^k = I$) parameter are obtained. A typical Q/I curve is shown in Fig. 2.2.

ΔK = labile or exchangeable K

AR^k_e = equilibrium activity ratio for K

K_x = specific K sites

PBC^k = potassium buffering capacity

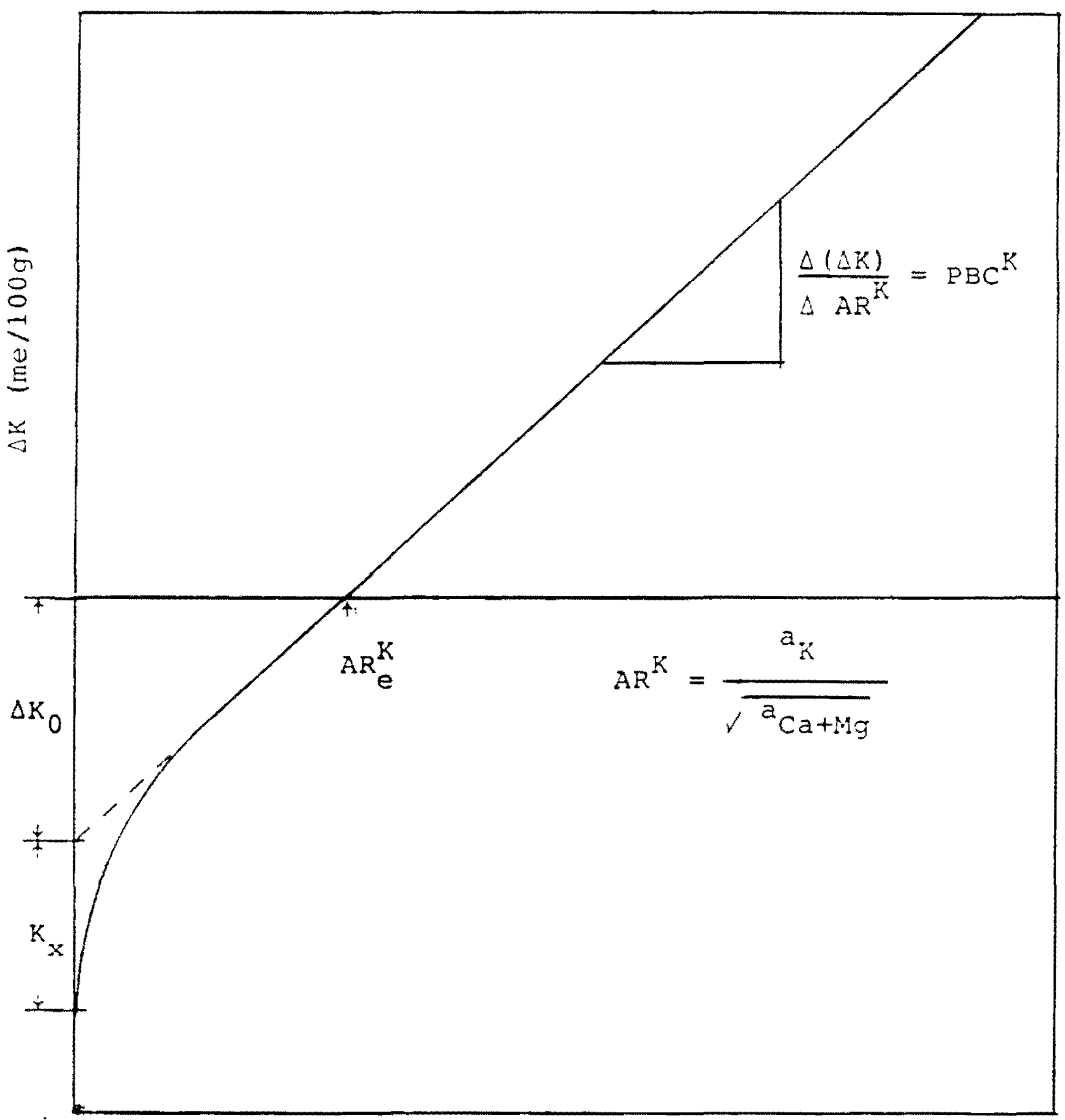


Fig. 2.2 Typical Quantity/Intensity (Q/I) plot.

The $AR^k{}_e$ value is a measure of availability or intensity of labile K in soil. Beckett (1964b) and Le Roux and Sumner (1968) found that K fertilization increase $AR^k{}_e$ values. Le Roux (1966) noted that ΔK^o was a better estimate of soil labile K than exchangeable K.

The PBC^k value is a measure of the ability of the soil to maintain the intensity of K in the soil solution and is proportional to CEC (Lee, 1973). Le Roux (1966) noted that a high soil PBC^k value is indicative of adequate K availability while a low PBC^k would suggest a need for frequent fertilization. The potassium intensity factor (AR^k) is computed from the measured concentrations of K, Ca, and Mg corrected to the appropriate activities by application of the extended Debye-Huckel theory as follows from Eq. 2.25 (Moore, 1972).

$$\log \underline{+} \;=\; -aZ^+Z^- \sqrt{I} / \; 1 \; + \; \alpha\beta\sqrt{I} \qquad (2.24)$$

where

$\gamma\underline{+}$ = the mean activity coefficient of the electrolyte

a = constant (0.5042)

Z^+ = valency of cation

Z^- = valency of anion

$\alpha\beta$ = assumed to be 1

I = ionic strength

where

$$I = 1/2\Sigma C_i Z^2{}_i \qquad (2.25)$$

and where

C_i = concentration of ion i

Z_i = valency of ion i

From the activity ratio the free energy of K - (Ca+Mg) exchange (Woodruff, 1955; Arnold, 1962), can be obtained. This corresponds to the thermodynamic potential of K in soil which is taken as its partial molar free energy relative to that of Ca+Mg. The exchange energy is calculated according to Eq. 2.26.

$$\Delta G_{K,\ Ca+Mg} = RT \ln a_k / \sqrt{a_{Ca+Mg}} \qquad (2.26)$$

where

ΔG = free energy charge

R = gas constant

T = absolute temperature

a_k = activity of K

a_{Ca+Mg} = activity of Ca+Mg

By inserting a numerical value for the gas constant, assuming a temperature of 25 C and changing ln for log the value of 1364 is obtained and the calculated value of ΔG is expressed in calories/equivalent. The range of ΔG is usually between -2000 to -4000 cal/equiv., where the upper value (-2000) indicates a potassium sufficiency, and the lower (-4000) a deficiency (Hagin and Tucker, 1982).

Schofield (1955) was the first to suggest nutrient potentials as measures of the work required to remove nutrients from the soil. According to Page and Talibudeen (1982) extending this hypothesis, a curve relating crop yield to the chemical potential of a nutrient in soil should be a characteristic of the plant, indicating on its ability to remove the nutrient from the soil, under the prevailing environmental conditions. Talibudeen (1974) has presented a schematic curve relating plant response to nutrient potential from which critical values can be identified that are characteristic of a plant during a particular growth phase.

An ideal method for the assessment of K supply to the plants should include quantity, intensity, and mobility parameters. According to Schroeder (1973) only two methods are known which take into account the kinetic aspect of plant nutrient supply by allowing the determination of the above parameters. These are the exchanger method by Tepe and Leidenfrost (1958) and the electroultrafiltration (EUF) method by Nemeth (1972). The interaction between root absorption of K and K fixation/slow release reactions by many soils make it difficult to develop a uniform technique to evaluate soil-K availability.

III. POTASSIUM RETENTION CHARACTERISTICS OF THE PUAULU SOIL

3.1 Introduction

It has long been recognized that large quantities of added soluble K salts in some soils become so transformed that it is not possible to recover them in boiling dilute acids (Agarwal, 1960). This phenomenon has been called K retention. The importance of K retention in soils lies among other things in the fact that it regulates the supply of soil potassium for plants and prevents its leaching. Two mechanisms are considered to be responsible for K retention. In the first mechanism, retention takes place by entrapment of K between basal clay surfaces (Agarwal, 1960; Grim, 1968) where it fits into the hexagonal vacancies formed by tetrahedral oxygen of 2:1 minerals. The second mechanism is believed to take place via the formation of insoluble K compounds, especially aluminosilicates (Agarwal, 1960).

The degree of K retention in clay and soils depends on the type of clay mineral and its charge density, the degree of interlayering, the moisture content, the concentration of K ions as well as the concentration of competing cations, and the pH of the ambient solution bathing the clay or soil (Rich, 1968; Thomas and Hipp, 1968). Cation exchange reactions in soils derived from volcanic ash, with their high contents of amorphous minerals are relatively poorly understood as compared to soils derived from other types of parent materials (Schalscha et al., 1975).

The objective of this study was to investigate K retention characteristics of a volcanic ash soil from the island of Hawaii and to determine a potassium selectivity coefficient.

3.2 Materials and Methods

Soil samples were taken from 0-15 cm and 15-30 cm depth of a volcanic ash soil from the island of Hawaii. The soil was a Typic Hydrandept, medial over thixotropic, isomesic, Puaulu series. Potassium retention was determined by reacting 3g samples of soil with 25 ml of KCl solution. The potassium concentration was 0, 2.24, 5.6, 11.2, 22.4, and 56 mmol/L. The soil solution suspensions were shaken for one hour and then left overnight. The solution was separated from the soil by centrifugation. The separated soil was weighed to correct for cations in the remaining solution and then leached 4 times with 25 ml portions of 1N NH_4OAc pH 7. Cation concentrations were determined by atomic absorption spectrophotometry. Retention was calculated considering K mass balance in the system, whereby K retention (Kf) was defined by the difference between total initial K (consisting of exchangeable, soluble, and added K) and residual K in solution and on the solid phase (exchanged with NH_4 after equilibration). Retention was plotted against K concentration in the equilibrium solution, against Ep and PAR where

$$Ep = K_s/(Ca_s+Mg_s) \quad (3.1)$$

in which s is the exchangeable cation given in mmol/kg and

$$PAR = [K]/([Ca]+[Mg])^{1/2} \quad (3.2)$$

where brackets denote solution concentration in mmol/L.

3.3 Results and Discussion

3.3.1 Potassium retention as related to potassium concentration in equilibrium solution

Potassium retention may be described in different ways. Two possible ways are to relate K retention to the concentration in the equilibrium solution and to the amount of K added to a given weight of soil. In Fig. 3.1 the relationship between K retention (Kf) and K in solution is shown for two different depths. The trend of the two isotherms was very similar and showed that the percentage of added K that was held was higher at low levels of additions than at high levels. Similar behavior was reported by Shaviv et al. (1985a). For a given K solution concentration, the amount of K held by the subsoil was twice that held by the topsoil perhaps because of the increase in CEC with depth.

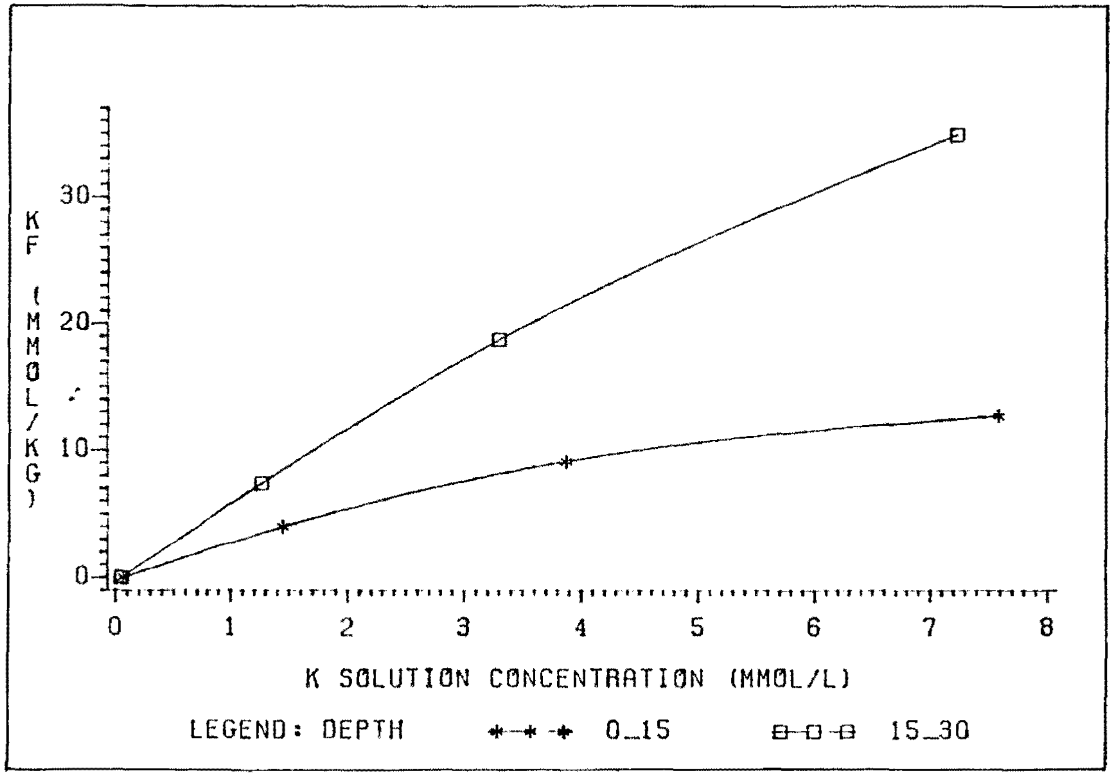


Fig. 3.1 Relationship between K retention (Kf) and K concentration in equilibrium solution.

3.3.2 Relationship between Ep and PAR

The relationship between potassium saturation (Ep) and the potassium adsorption ratio is depicted in Fig. 3.2. The relationship was not linear. Further it is seen that for values of PAR greater than 3.5 EP increased drastically. The value of Ep was higher in the topsoil than in the subsoil for all levels of PAR perhaps because of the increase in surface charge density with depth favoring the adsorption of divalent over monovalent cations according to the double layer cation-exchange theory (Bolt, 1955). The CEC determined by 1N NH_4OAC pH 7 was 36.67 and 54.70 cmol(+)/kg for the topsoil and subsoil, respectively (Ikawa et al., 1985).

3.3.3 Relationship between retention and PAR

The relationship between Kf and PAR is shown in Fig. 3.3. Potassium retention increased with increasing potassium adsorption ratio. Potassium retention, however, can be considered as a reversible process since as shown in Fig. 3.1 K was found in solution even when retention was far from saturation. The subsoil appeared to bind much more potassium than the topsoil for any given value of PAR.

The fractional retention which was defined as (Shaviv et al., 1985)

$$Fkf = Kf/CEC_{in} \tag{3.3}$$

was calculated for each depth. CEC_{in} is the initial

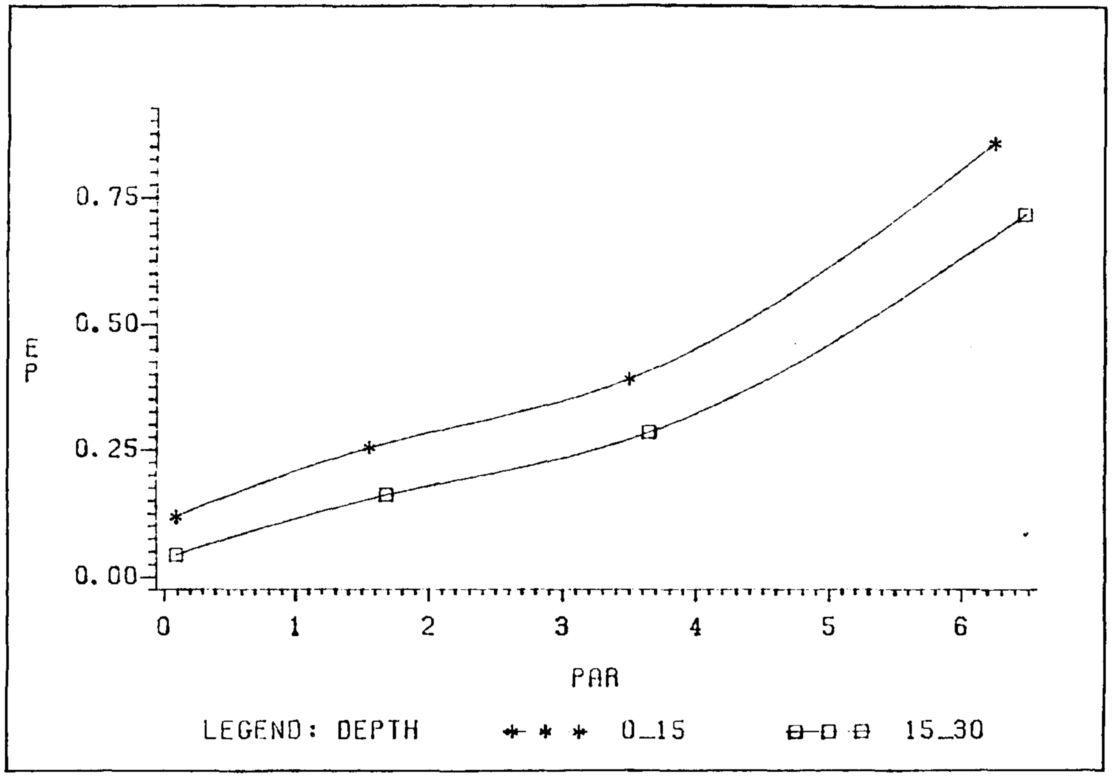


Fig. 3.2 Potassium saturation (Ep) vs K adsorption ratio (PAR).

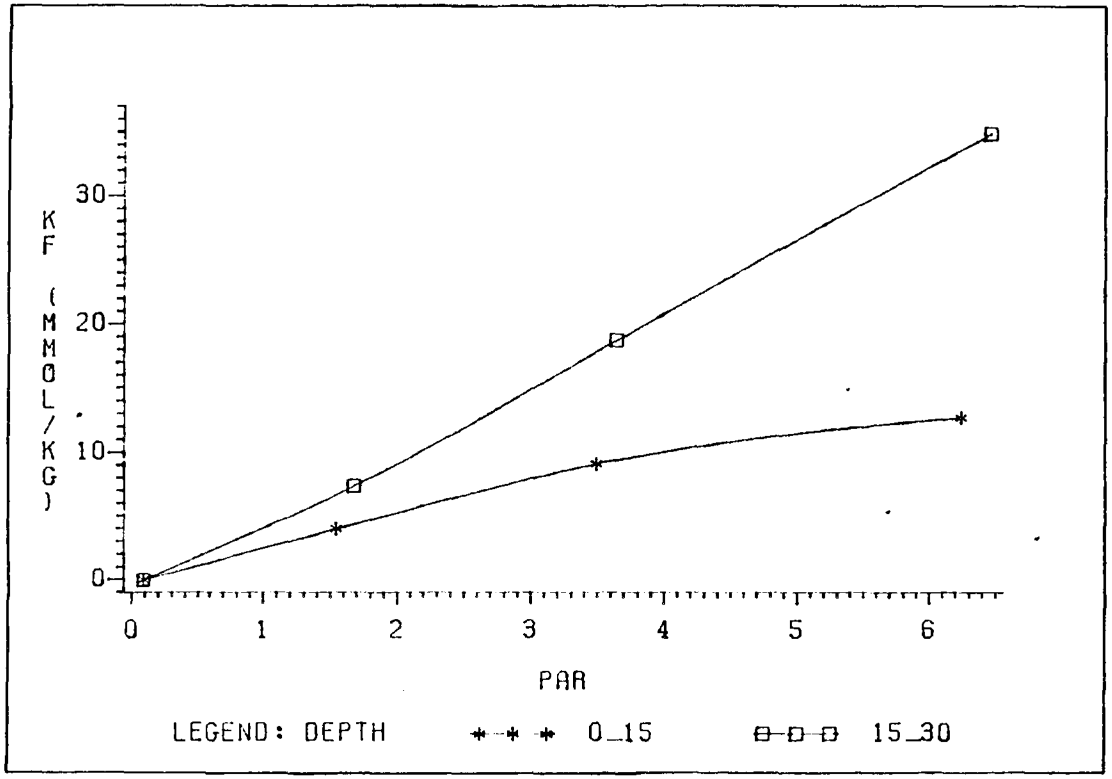


Fig. 3.3 Relationship between K retention (Kf) and K adsorption ratio (PAR).

effective CEC of the soil. A plot of FKf vs EP and FKf vs PAR is shown in Figures 3.4 and 3.5. It is seen that FKf increased with increasing Ep and PAR but more so in the subsoil than in the topsoil. The value of FKf can be interpreted in terms of percentage decrease in initial CEC due to retention. Decreases in exchange capacity as a result of K retention have been reported by some investigators (Kolodny, 1938; Peterson and Jennings, 1938). Levine and Joffe (1947) found from their experiments that at first, the reduction in exchange capacity exceeded the amount of K held. Then a point was reached where the retention of K and the reduction in the exchange capacity were equal and thereafter the reduction in exchange capacity was always less than the amount of K held. In cases where the relationship between FKf and Ep or FKf and PAR can be established it should be possible to estimate the CEC decrease caused by K retention. Such information could be used in an ion transport model (Shaviv et al., 1985a).

3.3.4 Gapon selectivity coefficient

In order to look at the selective adsorption of K, selectivity coefficients were calculated from Eq. 3.4. (Shaviv et al., 1985b)

$$K_G = Ep/PAR \qquad (3.4)$$

Fig. 3.6 shows the relationship between K_G and Ep. The

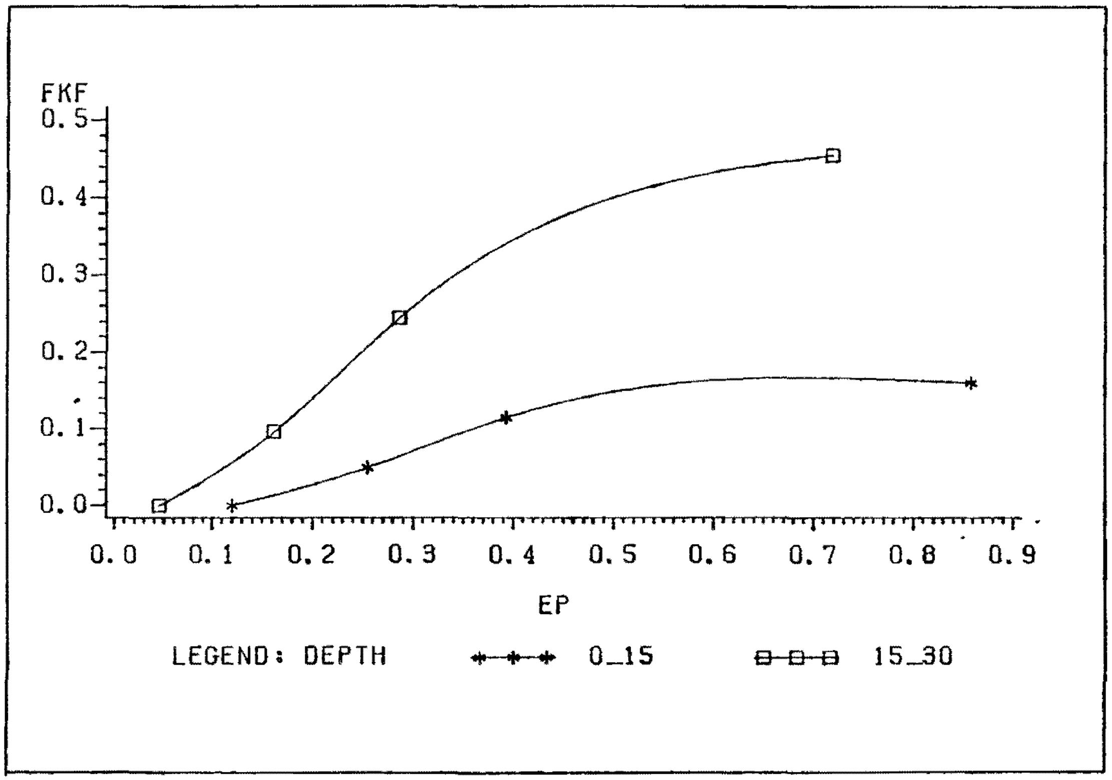


Fig. 3.4 Relationship between K fractional retention (FKf) and K saturation (Ep).

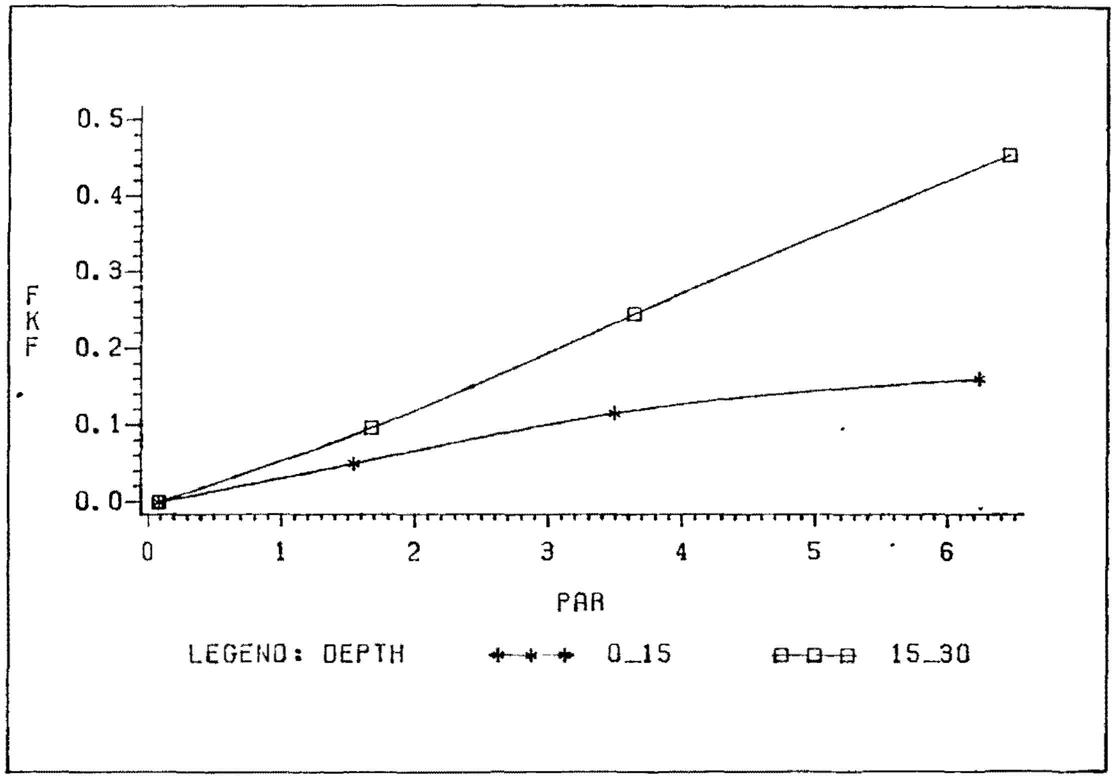


Fig. 3.5 Relationship between K fractional retention (FKf) and K adsorption ratio (PAR).

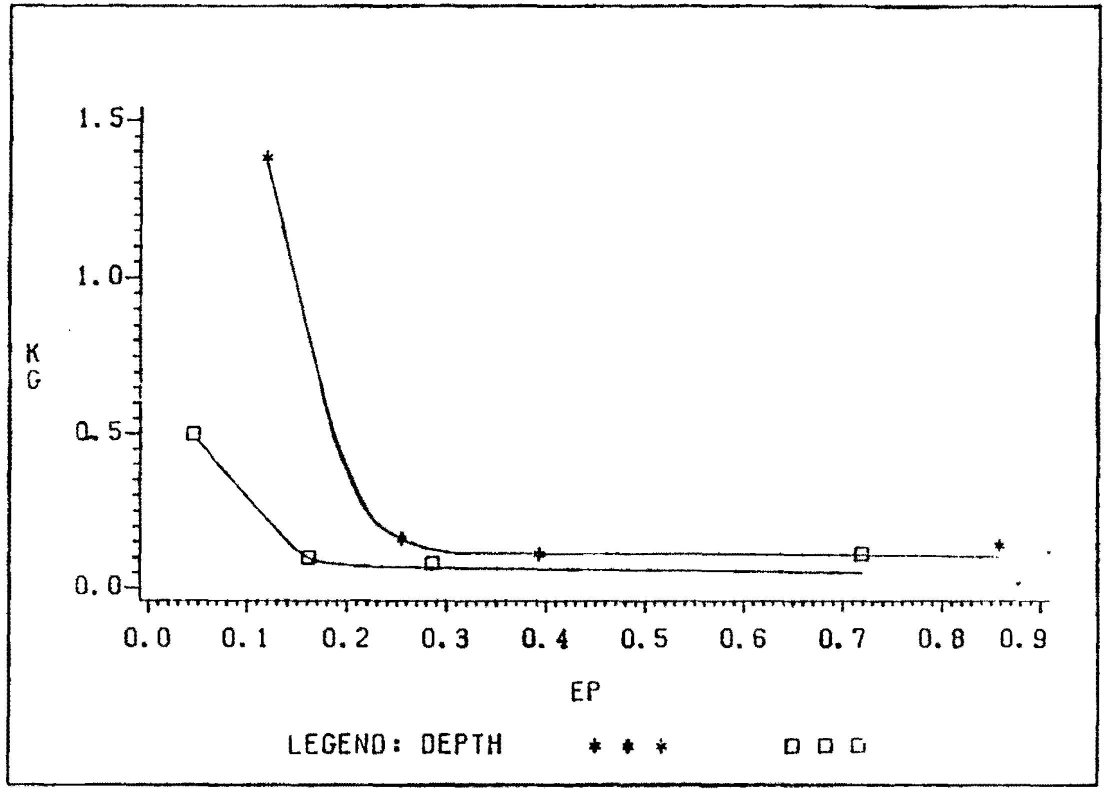


Fig. 3.6 Relationship between K selectivity coefficient and K saturation.

values of K_G decreased with increasing saturation of exchangeable K then leveled off to an almost constant value at Ep values of 1.5 and 2.5 for 0-15 and 15-30 cm depths, respectively. A similar relationship was found by Schalscha et al. (1975). At values of Ep greater than 2.5 the value of K_G may be averaged in order to present K_G as a constant for each depth. This gave a K_G value of about 0.15 for both depths. The selectivity coefficients reported by Carson and Dixon (1972) at high K saturation (>10%) for 20 soils varied from 2.2 to 6.8. Grimme (1980) reported values of K_G ranging from 0.3 to 3 for planar sites in clay and soils. These planar sites are known to have the lowest selectivity. Thus, the Puaulu soil seems to have a very low affinity for K.

Fig. 3.7 shows the relationship between the Gapon selectivity coefficient and the fractional K retention. The fractional retention was defined as the ratio of the amount of K held to the initial CEC. There was a sharp decrease in K_G values with increasing FKf. A constant value of K_G was reached after more than 5 and 10% of the initial exchange sites were occupied by K held in the topsoil and subsoil, respectively.

3.4 Conclusions

1. The topsoil of the Puaulu soil had significantly lower retention capacity than the subsoil. For any given K

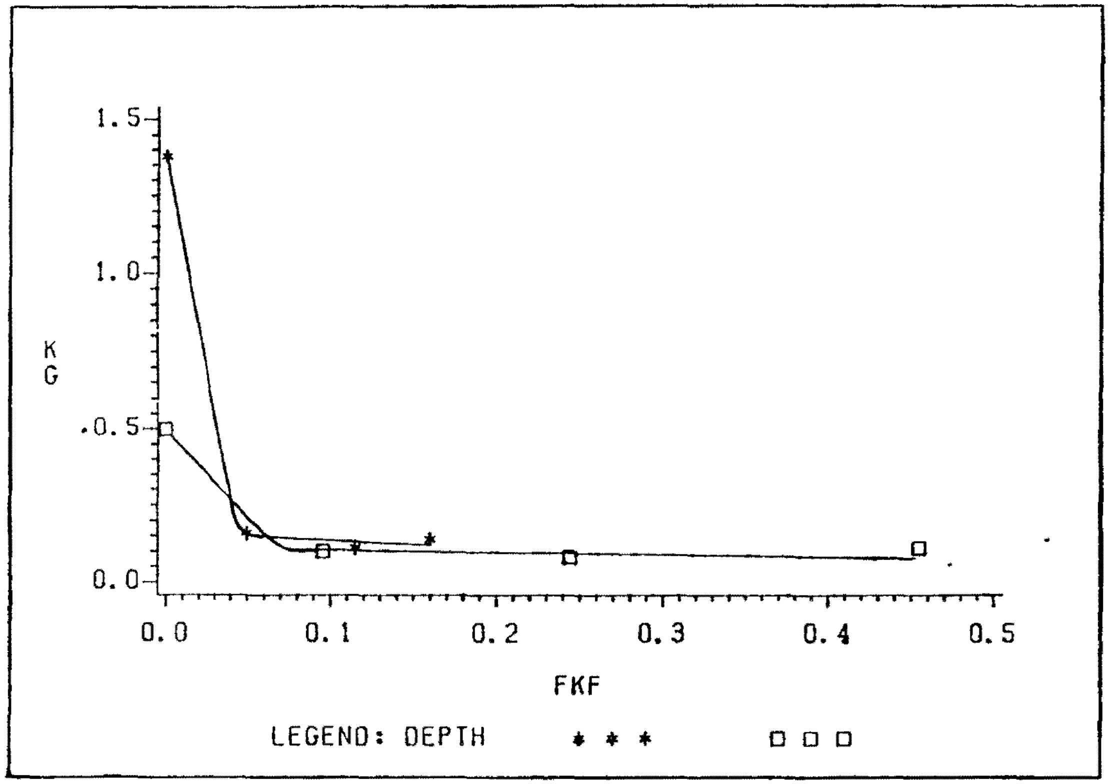


Fig. 3.7 Relationship between K selectivity coefficient and K fractional retention.

solution concentration the amount of K held by the subsoil was twice that held by the topsoil.

2. The relationships between K retention and the ratio of adsorbed Ca+Mg on the one hand, and potassium adsorption ratio on the other hand were nonlinear.

3. The Gapon selectivity coefficient sharply decreased with Ep increasing from 0 to 0.25 and ended with a leveling value of 0.15 $(mol^{-1}\ L)^{1/2}$.

4. There was a strong dependence between K_G and the fractional K retention. A constant value of K_G was reached after 5 and 10% of the initial exchange sites were occupied by K held in the topsoil and subsoil, respectively.

IV. VARIATION AND SPATIAL STRUCTURAL ANALYSIS OF SELECTED PROPERTIES OF THE PUAULU SOIL

4.1 Introduction

Lateral soil variability at scales up to tens of meters occurs in many tropical and subtropical regions. This variability is an important feature in the identification of soil properties relative to crop production, pesticide management etc. (Dahiya et al., 1984). Classical statistical analysis (i.e., probability density function, mean and variance) has been widely used to evaluate spatial variability of soil properties (Cameron et al., 1971; Ball and Williams, 1968; Dahiya et al., 1984). Recently, however, various studies dealing with soil variability have emphasized the spatial dependence or distribution of soil properties over a specific area (Burgess and Webster, 1980; Hajrasuliha et al., 1980; Yost et al., 1982). This latter development was prompted by the need to characterize not only the mean of a property and its deviation, but how it changes over distance, its spatial dependence or structure, and relationship to neighboring values (Nielsen et al., 1982).

The objective of this study was to evaluate the inherent spatial variability and structure of selected properties of a volcanic ash soil (Typic Hydrandept, medial over thixotropic, isomesic) from the island of Hawaii using conventional statistical analysis and the theory of regionalized variables, geostatistics.

4.2 Materials and Methods

4.2.1 Soil sampling and analysis

In order to assess the magnitude of the spatial variability of exchangeable Ca, Mg, K, and Na 161 samples were taken from a 92 m x 42 m field situated at the University of Hawaii Volcano Research Station on the island of Hawaii. Samples were taken with an auger of 7.5 cm diameter at 2 m intervals from the 0-15 cm depth from four transects (N-S, E-W, NE-SW, SE-NW). Field moist soil samples were sieved and kept in sealed plastic bags until laboratory analyses were performed. Exchangeable bases were determined with 1N NH_4OAC pH 7 as follows. The equivalent of 5 g of oven-dried moist soil was shaken for 30 minutes with 100 ml of 1N NH_4OAC and filtered through Whatman N 42 filter paper. Ca, Mg, K, and Na were determined in the filtrate by atomic absorption spectrophotometry.

4.2.2 Conventional Statistical Analysis

Means, variances, and standard deviations were estimated from the 161 samples using standard statistical methods. Distribution functions of exchangeable cations were evaluated using probability plots and the Kolmogorov-Smirnoff D statistic (Barr et al., 1979). Unlike the normal distribution, the mean and variance, expressed in terms of the original data, of a lognormal distribution are:

$$m = \exp(m_{ln} + 0.5\ s^2{}_{ln}) \qquad (4.1)$$

and

$$s^2 = \exp[s^2{}_{ln} + 2m_{ln}(\exp(s^2{}_{ln}) - 1))] \qquad (4.2)$$

where m = mean re-expressed in terms of the original data

m_{ln} = mean of the log transformed data

s^2 = variance re-expressed in terms of original data values

$s^2{}_{ln}$ = variance of the log transformed data values

The coefficient of variation, CV:

$$CV = s/m \times 100\% \qquad (4.3)$$

was used for the purpose of expressing variability on a relative basis. The number of observations needed to obtain a mean value of a given variable with a given precision and confidence level was calculated by Eq. 4.4.

$$n = [(s/d)t_\alpha]^2 \qquad (4.4)$$

where d is the allowable error (precision required within the given limits of the true mean) and t_α is the value of two-tailed Student's t with infinite degree of freedom at the confidence level α.

4.2.3 Geostatistical analysis

The semi-variogram was used to investigate the degree of spatial dependence of exchangeable bases using log-transformed data since these variables were lognormally distributed. The semi-variance is defined by

$$\gamma(h) = 1/2\ E\{[Z(x) - Z(x+h)]^2\} \qquad (4.5)$$

Assuming a constant trend, an unbiased estimate of the semi-variance is obtained by

$$\gamma(h) = 1/2N(h) \sum_{i=1}^{N(h)} [Z(xi) - Z(xi+h)]^2 \qquad (4.6)$$

where E denotes expectation, N(h) is the number of pairs of values [Z(xi), Z(xi+h)] separated by a distance h. The function $\gamma(h)$ or graph of $\gamma(h)$ against h is the semi-variogram. In order to determine if any of the parameters showed anisotropic variation semi-variograms were calculated in four directions from North to South in increments of 45^o (45^o, 90^o, 135^o, 180^o) each subtending an arc of $45\ ^o$ (±22.5). These sectors correspond to the NE-SW, E-W, SE-NW, and N-S axes of direction. Isotropic semi-variograms were also calculated for each variable by considering all samples taken in all directions ($90^o\pm90$).

4.3 Results and Discussion

4.3.1 Conventional Statistical Analysis

The statistics of the soil properties studied are given in Table 4.1. Exchangeable K with a CV of 12% was the least variable of all parameters studied. Exchangeable Na showed the highest lateral variability followed by Mg and Ca. It is further seen from Table 4.1 that the levels of exchangeable Ca, Mg, and Na were relatively high. This may be due to the fact that this soil developed in geologically recent volcanic ash (Ikawa et al., 1985).

It should be pointed out that the variability of the soil properties under study cannot be dissociated from the support size (soil core). This is the classical volume-variance relationship (Froidevaux, 1982), which states that the average values of large samples will be less dispersed (smaller variability) than the average values of small ones. Hence the over-all variability of the properties under study will depend to a great extent on the size of the soil core.

An important problem for the agronomist or soil scientist is determining the number of measurements (samples) required for the estimation of the mean within a given uncertainty band around its real value. Eq. 4.4 was used to estimate the number of samples necessary to produce a given accuracy of estimation of the mean by assuming that the values of the mean and variance of the 161 samples

Table 4.1 Summary of statistical analysis of the data

Statistics	Extractable bases (cmol(+) kg^{-1})			
	Ca	Mg	K	Na
Mean	7.00	2.56	0.20	0.63
Median	6.76	2.48	0.20	0.59
CV	22.30	24.89	12.65	34.45
Range	4.18-11.78	1.01-4.85	0.16-0.29	0.32-1.67

presented in Table 4.1 represented the true mean and variance of all potential sampling sites in the area under study. The results of such calculations are shown in Table 4.2. It is seen from Table 4.2 that only 6 samples would be required to estimate the true mean of exchangeable K to within 10% of the sample mean at 95% probability whereas for exchangeable Na 46 samples would be required.

This information is useful, but it is only a portion of the knowledge required to efficiently sample an area; it is also necessary to estimate the minimum distance for spacing the samples. The geostatistical analysis should provide an estimate of the minimum sampling interval.

4.3.2 Semi-variograms

The results of the spatial structural analysis suggested that exchangeable Ca, Mg, K, and Na could be assumed to vary isotropically. Isotropic experimental semi-variograms of Ca, Mg, K, and Na are shown in Figures 4.1, 4.2, 4.3, and 4.4, respectively. The following general comments can be made.

a) Semi-variograms of Ca, Mg, K, and Na established the presence of strong spatial dependence. They increased with distance until they stabilized around a constant value, the sill, which was different for each variable.

b) All these semi-variograms, however, showed the presence of a large nugget effect which corresponds to the variability that occurs within distances shorter than the

Table 4.2 Number of observations required to attain a sample mean to within 10% of the real mean value

Parameters	Probability level (95%)	Number of samples
Ca	95	19
Mg	95	24
K	95	6
Na	95	46

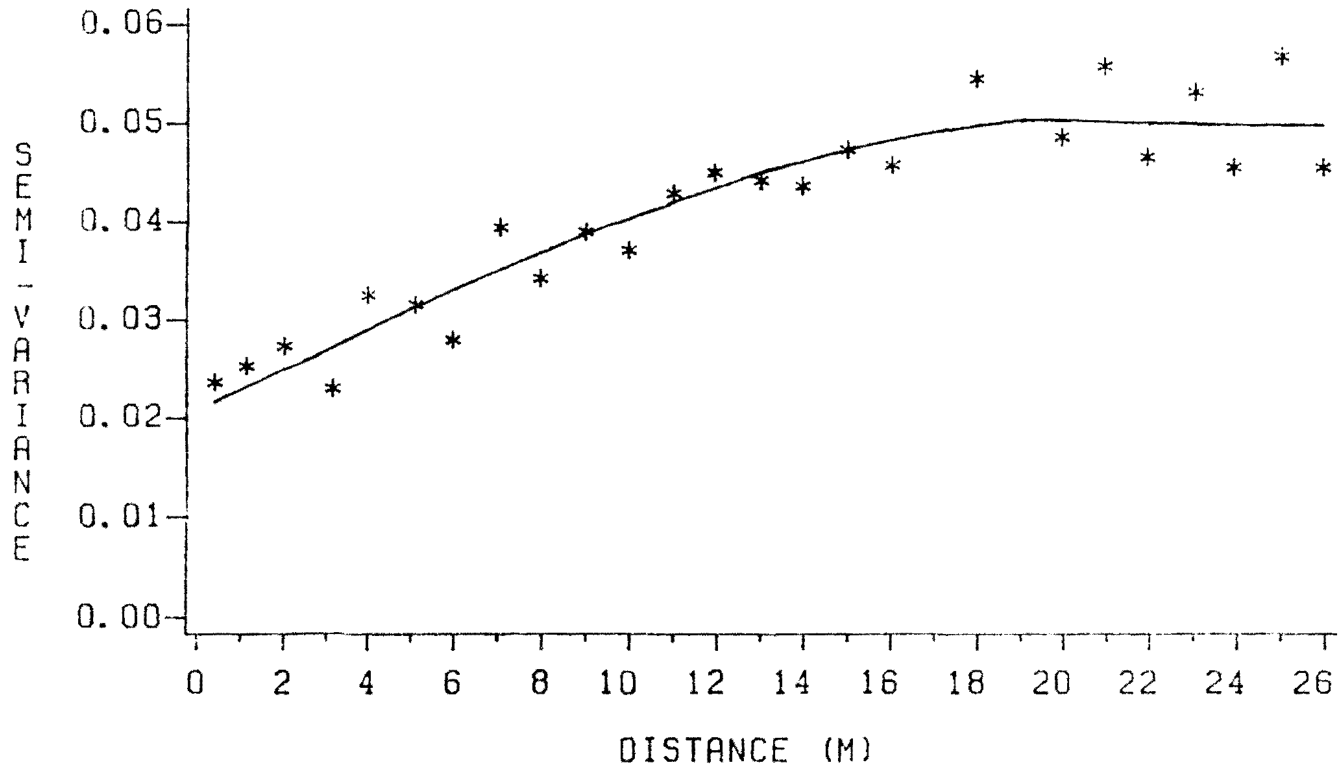


Fig. 4.1 Experimental (stars) and theoretical (solid line) semi-variograms of exchangeable Ca.

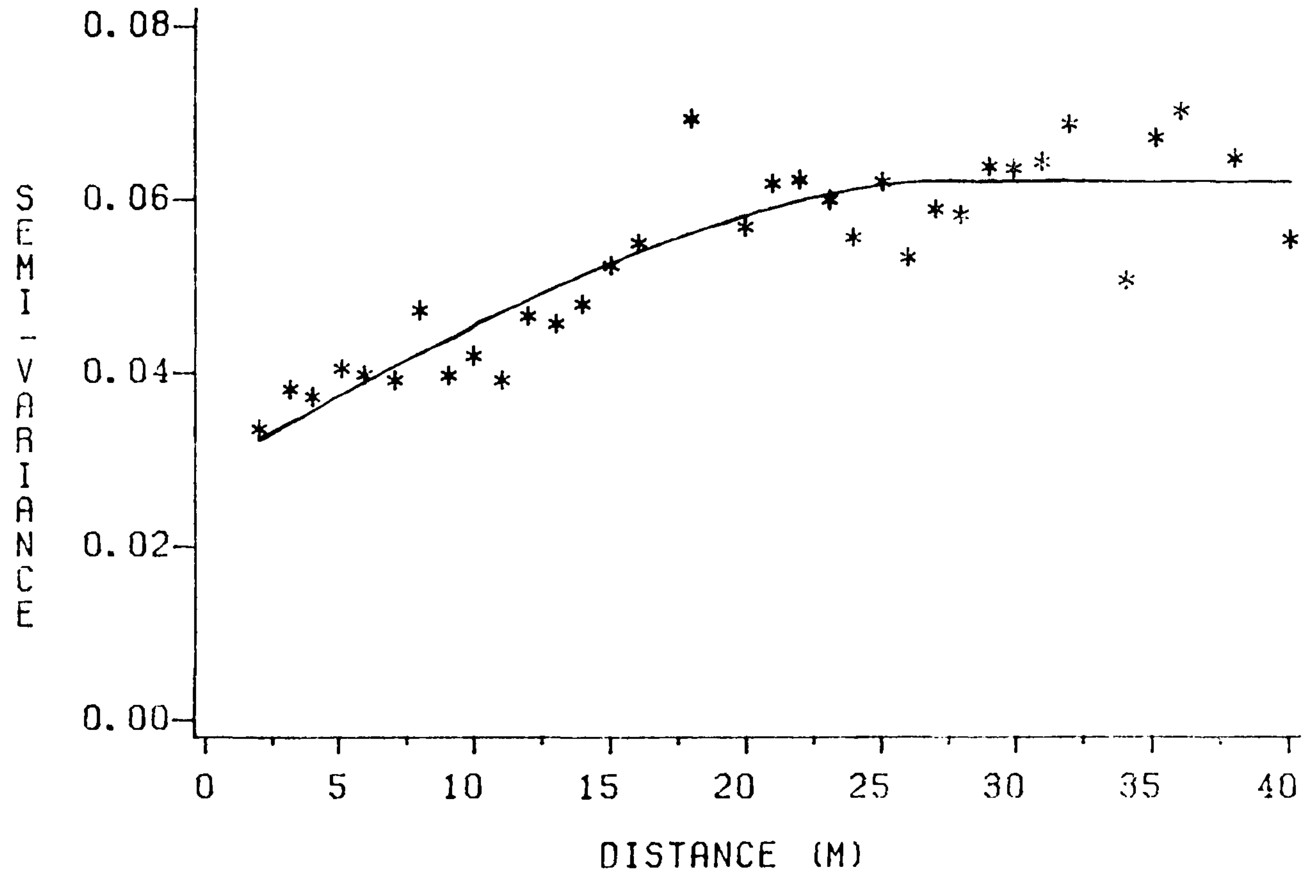


Fig. 4.2 Experimental (stars) and theoretical (solid line) semi-variograms of exchangeable Mg.

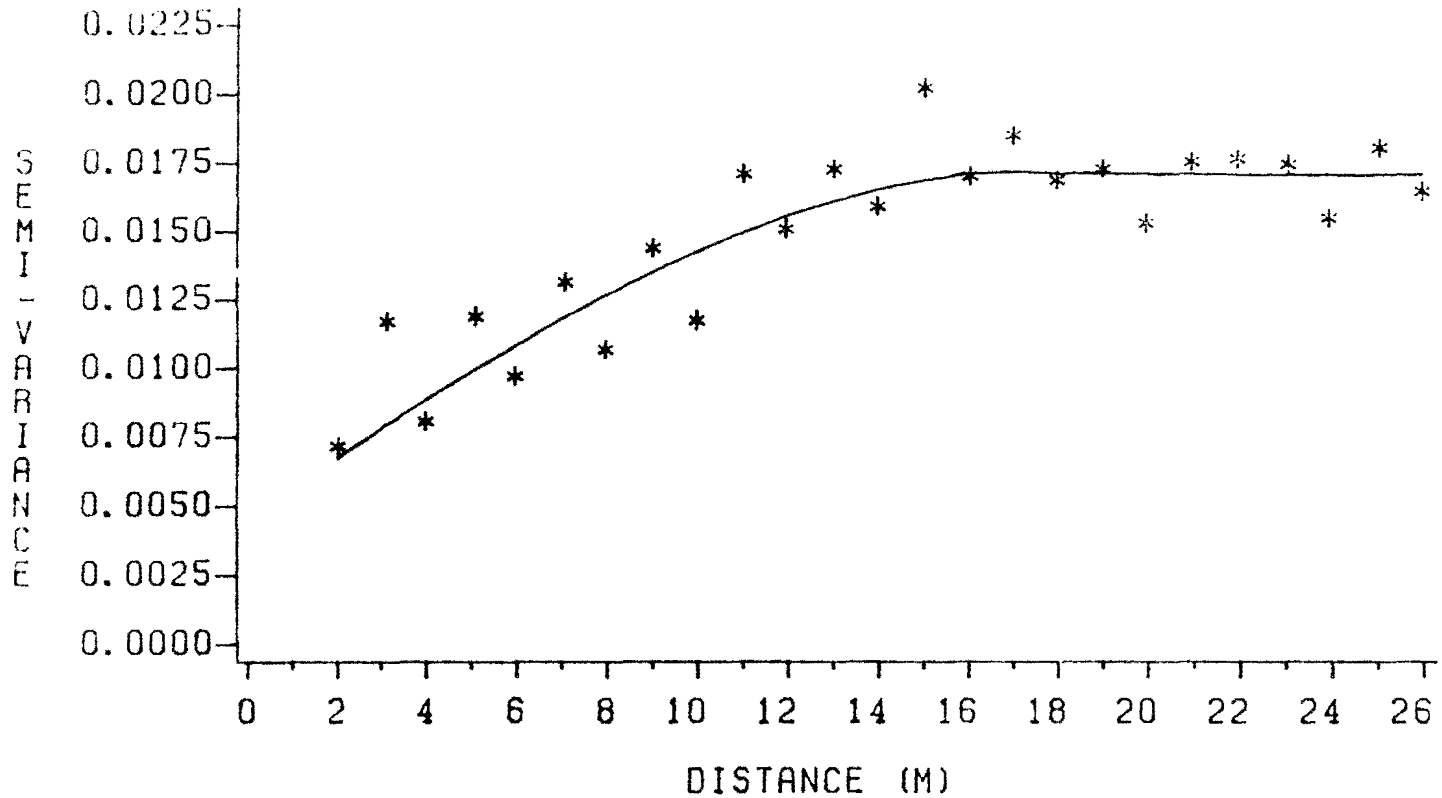


Fig. 4.3 Experimental (stars) and theoretical (solid line) semi-variograms of exchangeable K.

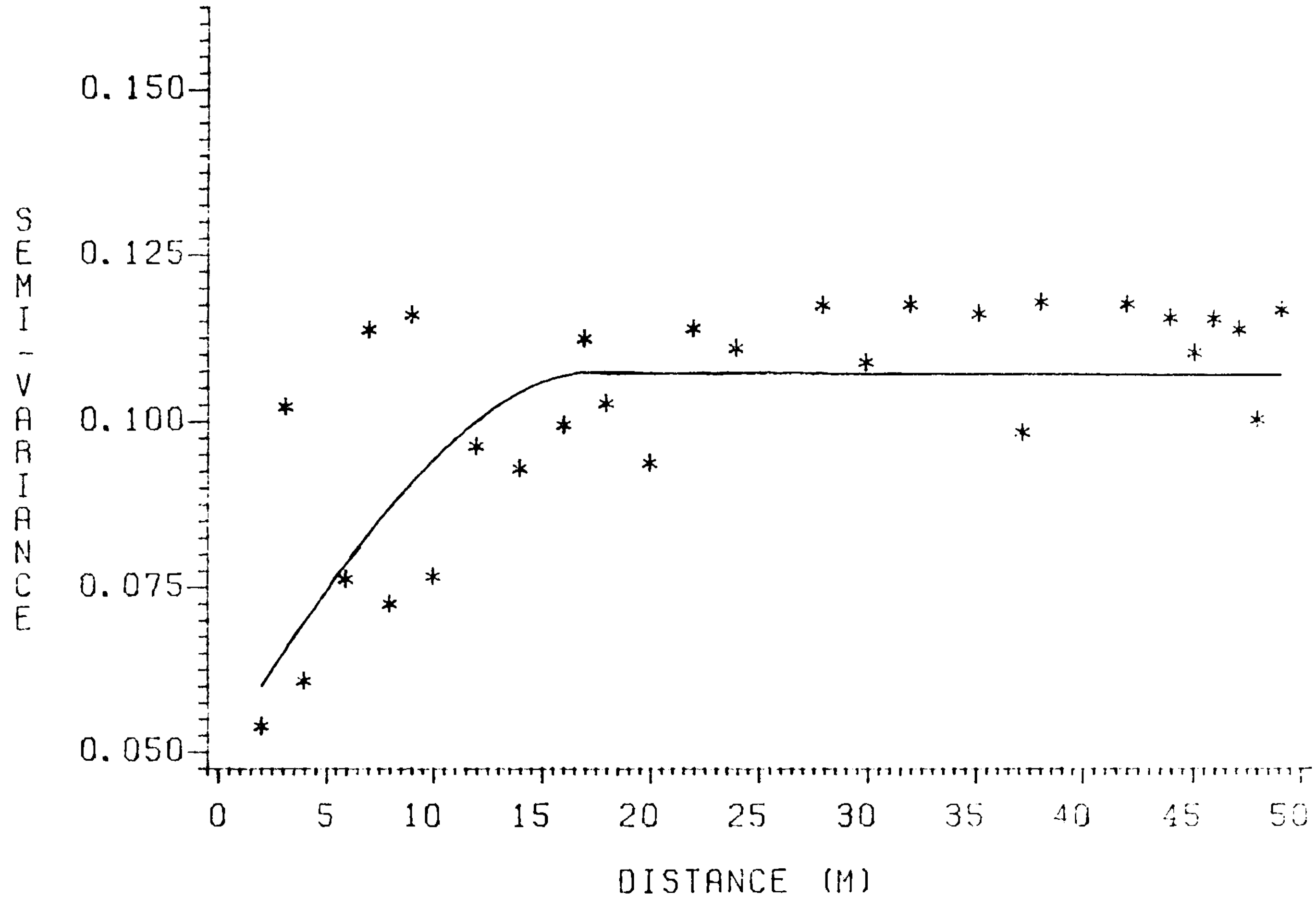


Fig. 4.4 Experimental (stars) and theoretical (solid line) semi-variograms of exchangeable Na.

sampling interval (2 m), and to the presence of experimental uncertainties.

4.3.3 Isotropic modeling of semi-variograms

Isotropic experimental semi-variograms were fitted to the theoretical semi-variograms of a positive definite type by weighted least squares using the SAS nonlinear procedure (barr et al., 1979). The positive definite condition ensures that the variance of any linear combination of values of Z(x) is also positive, a basic requirement of the kriging algorithm. Several theoretical models of conditionally positive definite type could be fitted to the experimental semi-variograms (Spherical, exponential, gaussian, etc.). The selected theoretical semi-variograms were validated using the so-called "jacknifing" procedure with ordinary (simple) kriging (Vauclin et al., 1983). The validation was made by suppressing each observation point one at a time, by providing an estimate in the point ($Z^{\star}(xi)$), by using the remaining N-1 data and analyzing the errors $Z(X_i) - \hat{Z}(X_i)$ where i = 1, 2, ..., N. Two basic conditions must be satisfied for the model to be theoretically consistent:

a) There is no systematic error, i.e., the mean of the error vector $Z(X_i) - \hat{Z}(X_i)$:

$$ME = 1/N \sum_{i=1}^{N} (Z(X_i) - Z(X_i)) \simeq 0 \qquad (4.7)$$

b) The kriging standard deviation KSDi is consistent with the corresponding error $Z(\chi_i) - \hat{Z}(\chi_i)$ and the mean reduced error MRE is

$$\text{MRE} = \{1/N \sum_{i=1}^{N} [Z(\chi_i) - \hat{Z}(\chi_i)/\text{KSDi}]^2\}^{1/2} = 1 \tag{4.8}$$

In addition, one should also check that the frequency distribution of $[Z(\chi_i) - \hat{Z}(\chi_i)]/\text{KSDi}$ is approximately normal. However, according to Gambolati and Volpi (1979) this is not strictly required from a theoretical standpoint. Knowledge of the distribution of the reduced errors is required if confidence intervals must be calculated in which case the classical confidence interval + 2 KSD usually contains approximately 95% of the errors (Froidevaux, 1982). According to Vieira et al. (1983) the requirement of closeness to normal distribution of the reduced errors makes sense, since any deviation from it would mean either systematic underestimation or overestimation.

The spherical model appeared to give a better fit and was therefore selected. The spherical model is

$$\begin{aligned} \gamma(h) &= CO + C(1.5\ h/a - 0.5\ h^3/a^3) && \text{for } h < a \\ \gamma(h) &= CO + C && \text{for } h > a \end{aligned} \tag{4.9}$$

where

C = spatial covariance

a = range of spatial dependence

h = lag distance

The fitted theoretical semi-variograms are shown in Figures 4.1, 4.2, 4.3, and 4.4 for Ca, Mg, K, and Na, respectively. The solid lines in these figures represent the spherical model which was fitted to the experimental semi-variograms. The values of the mean error and mean reduced error presented in Table 4.3 suggest that the selected model was theoretically consistent. It should be pointed out, however, that one of the problems in using equations 4.7 and 4.8 is the lack of a means of determining how close to zero and unity ME and MRE should be, respectively. Note that for each of the semi-variograms in Figures 4.1, 4.2, 4.3, and 4.4 a sill does exist, implying that these variables are not only intrinsic but also second-order stationary (Russo, 1984).

The estimated parameters of the spherical model for Ca, Mg, K, and Na are presented in Table 4.4. The nugget variance accounted for more than 40% of the variance for Ca, Mg, and Na and for as much as 25% for K. These percentages include measurement or procedural error. The range of spatial dependence was shorter for K than for other parameters studied. Exchangeable Mg had the highest range of spatial dependence. The range provides a logical

Table 4.3 Results of the validation test for the spherical model

Variables	Mean of error $cmol(+)\ kg^{-1}$	Mean reduced error
Ca	0.00306	1.300
Mg	0.00474	1.200
K	0.00039	1.100
Na	0.01260	1.100

Table 4.4 Parameters of isotropic semi-variograms for selected soil properties with a spherical model

Variable	Range	Nugget	% Sill	Sill	Sample Variance
Ca	20.5	0.020	41.2	0.049	0.048
Mg	28.4	0.028	46.2	0.062	0.060
K	17.1	0.004	25.1	0.017	0.016
Na	17.2	0.050	46.9	0.107	0.111

estimate of the minimum distance required for spacing soil samples to obtain independent values of the soil property of interest. An optimum sample spacing would be about 17 m for K and Na, 20 m for Ca and 28 m for Mg. In practice, however, samples should be taken at closer spacing, perhaps at 10 m intervals for exchangeable K, to allow for kriging interpolation at low estimation variances and local changes not taken into account by the average semi-variogram. It is interesting to note that the sills of the fitted isotropic semi-variograms were very close to the respective sample variances. This is consistent with the assumption of stationarity for the soil properties under study.

4.4 Conclusions

Conventional statistical analysis and geostatistical analysis were used to investigate the field spatial variability of selected soil properties.

1. Semi-variograms of exchangeable Ca, Mg, K, and Na showed these parameters to be spatially dependent over distances between two observations from 17 to 28 m.

2. Large nugget variances revealed the presence of short range variations that occurred at a distance less than the sampling interval (2 m). The nugget variance accounted for more than 40% of the sill for Ca, Mg, and Na and for 25% of the sill for K.

3. Comparison of coefficients of variation (CV) indicated that exchangeable K was the least variable

(CV=12.6) and exchangeable Na was the most variable (CV=34.4).

V. INTERPOLATION OF SELECTED SOIL PROPERTIES BY KRIGING

5.1 Introduction

Spatial interpolation has long been a major concern in many disciplines. The problem can be formulated as follows. Given a set of spatial data, find the function that will best represent the whole surface and that will predict values at other points or other subareas (Lam, 1983). In soil science it has been common practice to consider the predicted value of a soil property at any unvisited place as the mean value for the class in which it lies (Burgess et al., 1981). However, according to Webster and Beckett (1968) the confidence with which such prediction can be made depends on the homogeneity of the classes mapped. Moreover this approach takes no account of the possible substantial dependence among the data within one class. The theory of regionalized variables offers another approach to the interpolation process through the use of kriging, an interpolation technique which takes into account the correlation between adjacent samples while estimating the interpolated value. The objective of this study was to interpolate the levels of selected soil properties in a 92 m x 42 m field located at the University of Hawaii Volcano Research Station on the island of Hawaii.

5.2 Materials and Methods

Exchangeable Ca, Mg, K, and Na data which were analyzed for spatial dependence were used to krige those parameters at unsampled locations.

5.2.1 Punctual kriging

Using the fitted spherical semi-variogram obtained for each soil property, Ca, Mg, K, and Na were kriged over a 4 m x 4 m grid at 220 unsampled locations. An interpolated value (kriged estimate) is a weighted moving average of measurements made in its neighborhood;

$$\hat{Z}(X_0) = \sum_{i=1}^{N} \lambda_i \, Z(X_i) \tag{5.1}$$

where $\hat{Z}(X_0)$ is the interpolated value at point X_0, $Z(X_i)$ are the measured values at locations X_i, and λ_i are the weights calculated from the semi-variogram under the condition

$$\sum_{i=1}^{N} \lambda_i = 1 \tag{5.2}$$

The weights are calculated such that the kriged estimate is unbiased, i.e. the expected value of $\hat{Z}(X_0) - Z(X_0) = 0$ and the estimation variance, i.e. the variance of $\hat{Z}(X_0) - Z(X_0)$ is minimum, where $Z(X_0)$ is the true value of the property at point X_0. An important feature of kriging is that it provides a measure of the estimation error (kriging variance) associated with kriging estimates. Furthermore, if the kriging errors, i.e. the difference between the true values and the kriging estimates, are normally distributed, this kriging variance can be used to put confidence bands about the contour lines and confidence limits on average values for individual points. The kriging variance can also be used to help select optimal locations in the field to take future measurements (Delfiner and Delhomme, 1975). Kriging is an exact interpolator, which means that if the point to be estimated coincides with the location where a sample was collected, the kriging estimate will equal the observed datum. The estimation variance can be estimated by

$$S^2_k = \psi + \Sigma \lambda_i \gamma(x_i, x_0) \tag{5.3}$$

where ψ is a Lagrangian multiplier and $\gamma(x_i,x_0)$ is the semi-variance between the observed x_i and the interpolated location x_0. Use of Equations 5.1 and 5.3 yields the kriging system, the solution of which yields n weights λ_i and one Lagrangian multiplier which makes it possible to estimate the value of $\hat{Z}(x_0)$ and its corresponding kriging variance.

5.2.2 <u>Block kriging</u>

Apart from its optimality, kriging has a further advantage over other means of interpolation in that the area over which the estimate is made can be varied

(McBratney et al., 1982). Eq. 5.3 refers to estimate for volumes of soil the same size and shape as those on which the original measurements were made (punctual kriging). Optimal estimates can be obtained, however, for larger areas (block kriging). This involves replacing the right hand side of the kriging equations (Equations 5.1 and 5.3) by the average semi-variances between the observation points and the block whose average value is to be estimated. As a result the semi-variogram must be integrated for the area under study. The estimation variance differs from that of punctual kriging in that the within-block variance is subtracted, i.e.,

$$S^2_V = \sum_{i=1}^{N} \lambda_i \gamma(x_i, v) + \psi_V - \gamma(v,v) \qquad (5.4)$$

where $\gamma(x_i,v)$ is the average semi-variance (or covariance) between the sample points and the centroid of the block V; ψ_V is the Lagrangian multiplier and $\gamma(v,v)$ is the average semi-variance (or covariance) between all points within the block, i.e. the within-block variance of classical statistics. Using the spherical model obtained for each soil property interpolation was carried out for 144 blocks each 5 m x 5 m in area. Since soil properties under study were lognormally distributed lognormal kriging was used. This involved estimating the semi-variogram and performing all kriging calculations on the logarithms of the data.

Then log-kriged values and estimation variances were retransformed to the form of the original data as (Trangmar, 1984)

$$Z(x) \quad - \quad \exp(Z' + s'^2_k/2) \qquad (5.5)$$

and

$$s^2_k \quad = \quad m^2 \exp s'^2[1-\exp(-s'^2_k)] \qquad (5.6)$$

where the original data (X) are transformed (Y = log X) and Z(X), s^2 are the kriged value and estimation variance expressed in terms of the original data, Z' is the kriged value performed on Y, s'^2_k is the estimation variance of kriging performed on Y, s'^2 is the variance of Y, and m is the mean of X. Contour maps of punctual and block kriged values were made using the SPLOT algorithm (Bridges and Becker, 1976).

5.3 Results and Discussion

5.3.1 Punctual kriging

Figures 5.1, 5.2, 5.3, and 5.4 show the contour map of exchangeable Ca, Mg, K, and Na respectively. It is seen that in general the levels of Ca, Mg, K, and Na decreased as one goes from the center of the field to the edges whereas K varied in the opposite direction. It is further seen that the contour maps are spotty at some places (near circular contours), especially for Ca and Mg. These discontinuities may be attributed to the large nugget

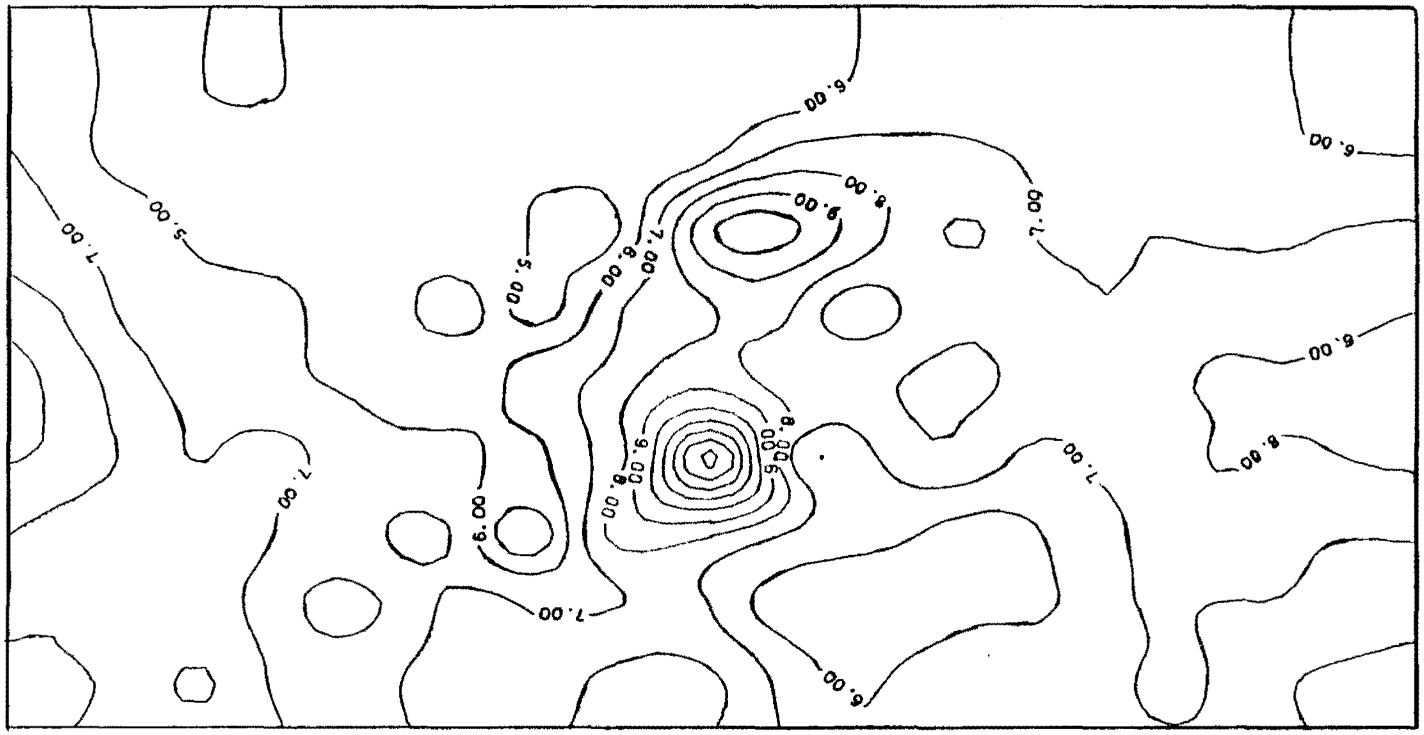


Fig. 5.1 Contour map of punctual kriging estimates of exchangeable Ca.

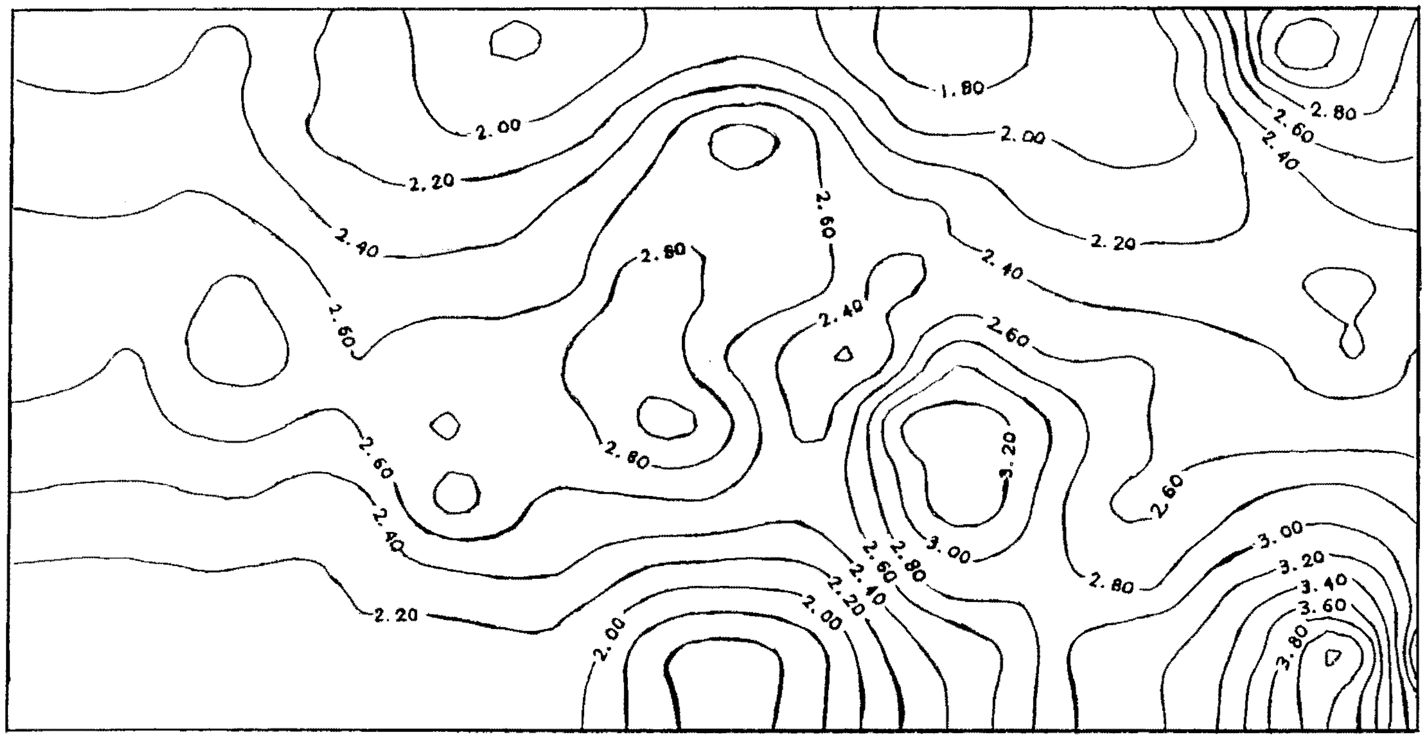


Fig. 5.2 Contour map of punctual kriging estimates of exchangeable Mg.

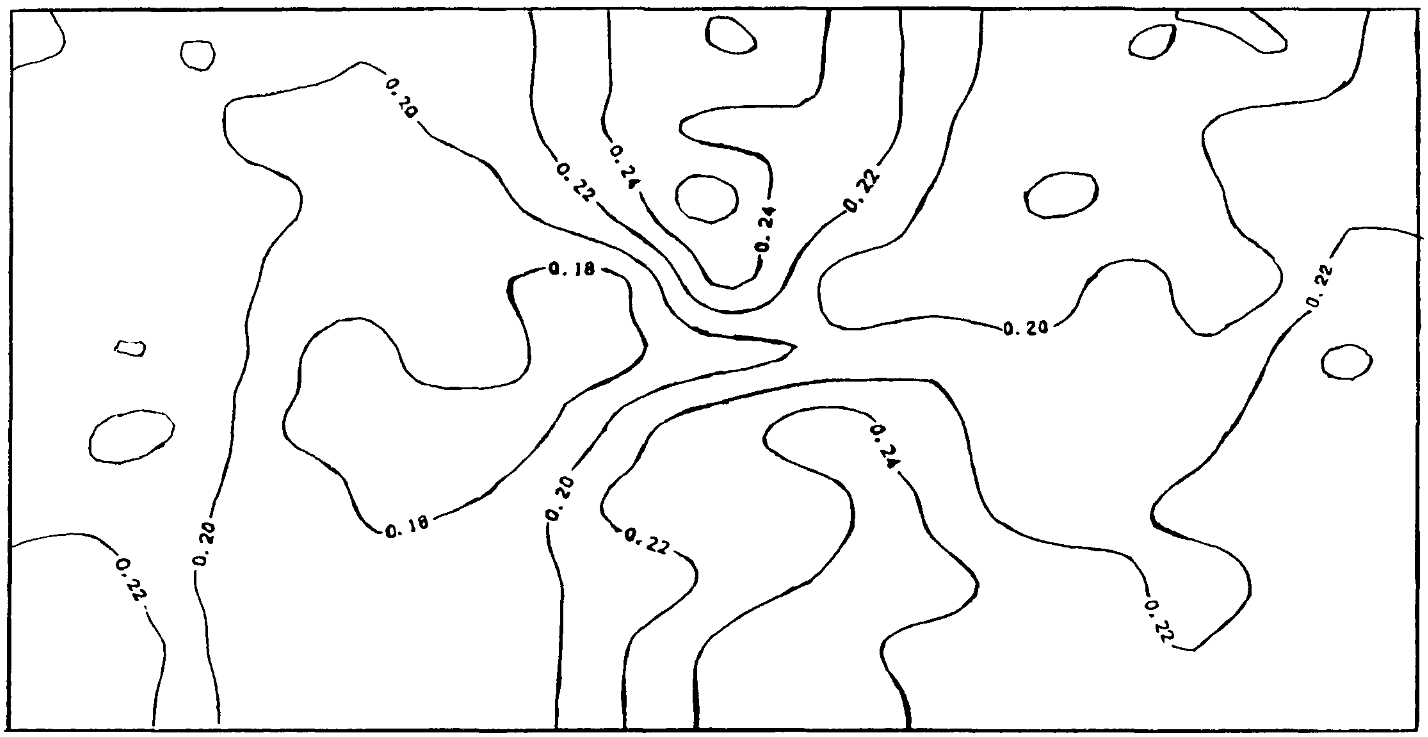


Fig. 5.3 Contour map of punctual kriging estimates of exchangeable K.

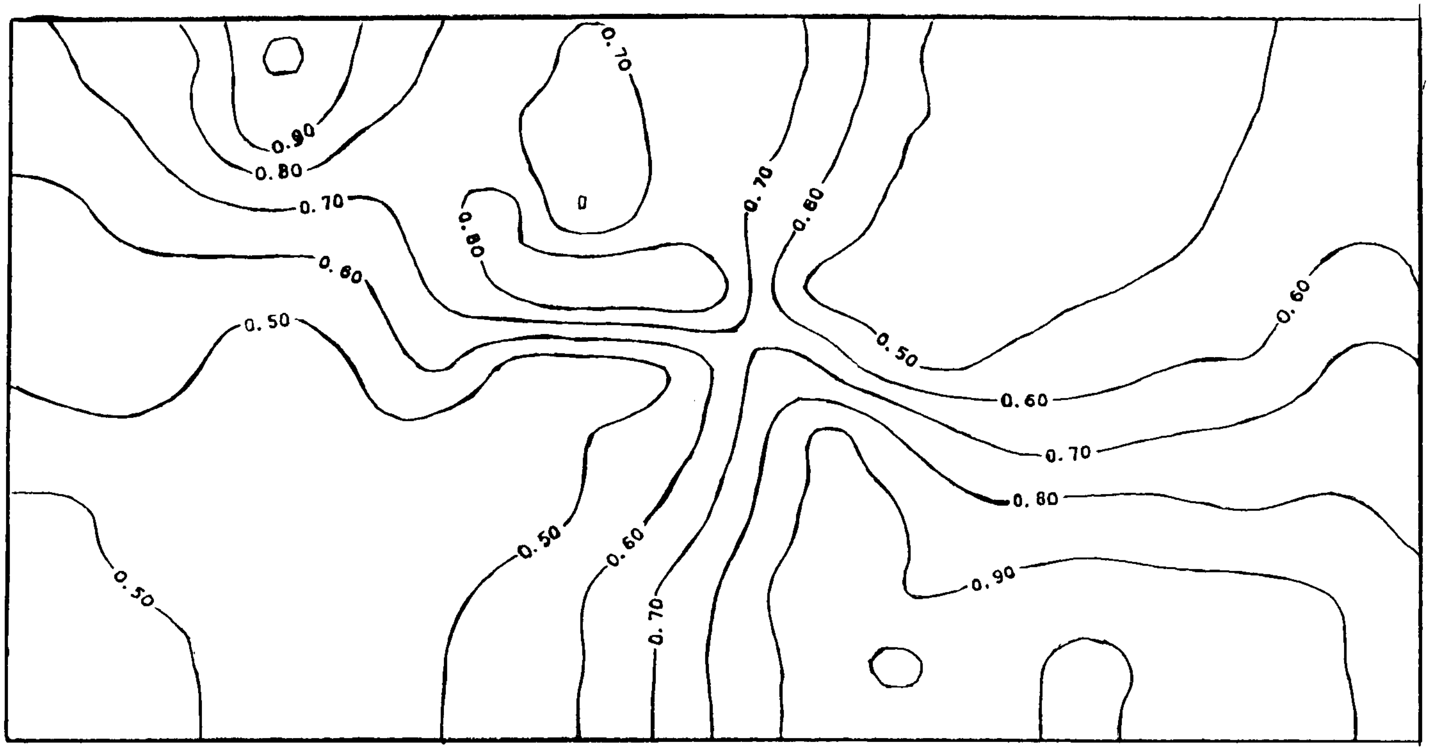


Fig. 5.4 Contour map of punctual kriging estimates of exchangeable Na.

variances of the semi-variograms of these variables. In the case of Ca the relatively large number of concentric isopleths at the center of the field suggests a steep gradient or rapid change in a short distance. The contour maps shown in Figures 5.1, 5.2, 5.3, and 5.4 are useful for a better understanding of soil properties and for practical purposes such as estimating fertilizer needs. Block kriging, however, seems to be a more appropriate tool for estimating fertilizer needs. This will be illustrated in chapter 6.

5.3.2 Block kriging

Figures 5.5, 5.6, 5.7, and 5.8 show contour maps of exchangeable Ca, Mg, K, and Na, respectively. Block kriging resulted in smoother maps but still revealed a decrease in the levels of exchangeable Ca, Mg, and Na as one goes from the center of the field to the edges; the trend being in the opposite direction with K. The smoother maps obtained with block kriging resulted form the fact average values were interpolated for each block and this had the effect of smoothing out local discontinuities among sample values (Trangmar, 1984). Estimation variances obtained with punctual and Block kriging (Appendices 5.1 to 5.8) showed that the variances of 5 meter square blocks were smaller than the variance of point samples. Krige's relation (Journel and Huijbregts, 1978)

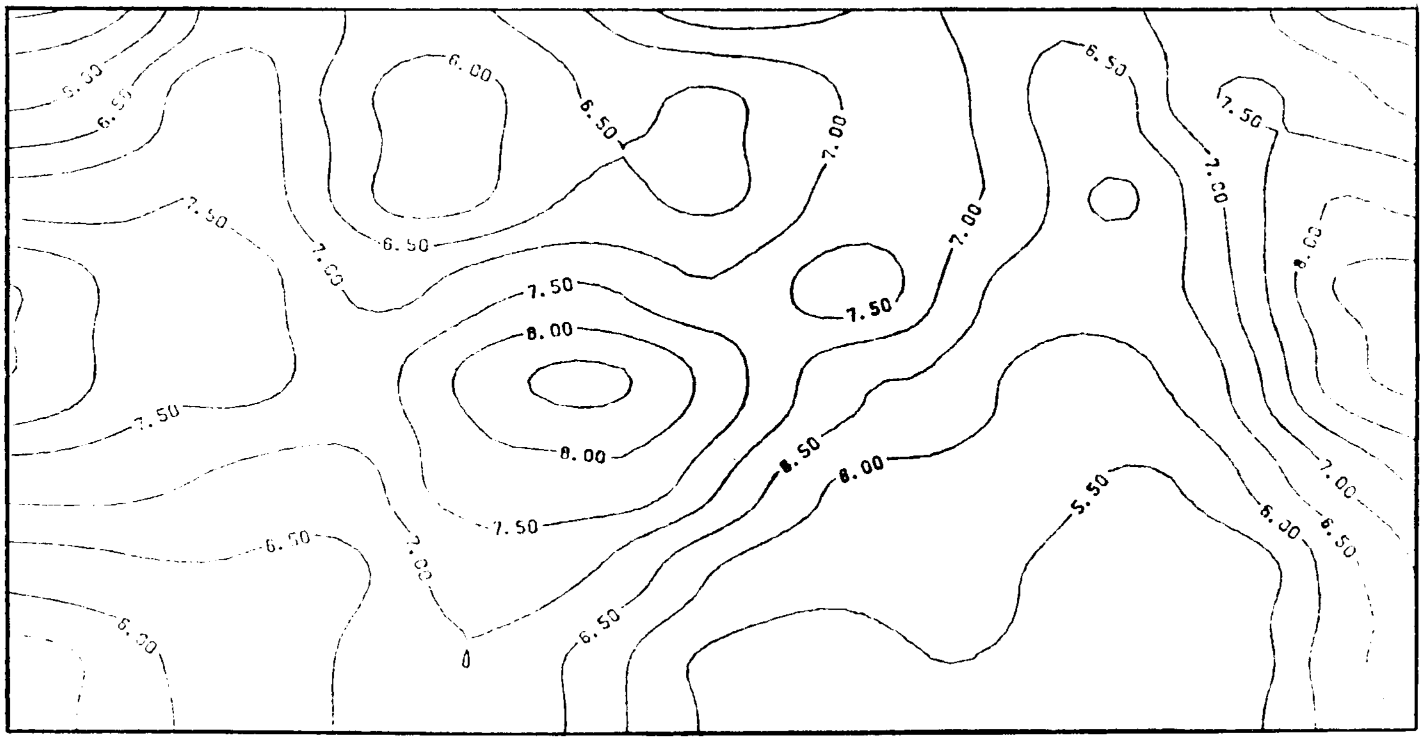


Fig. 5.5 Contour map of block kriging estimates of exchangeable Ca.

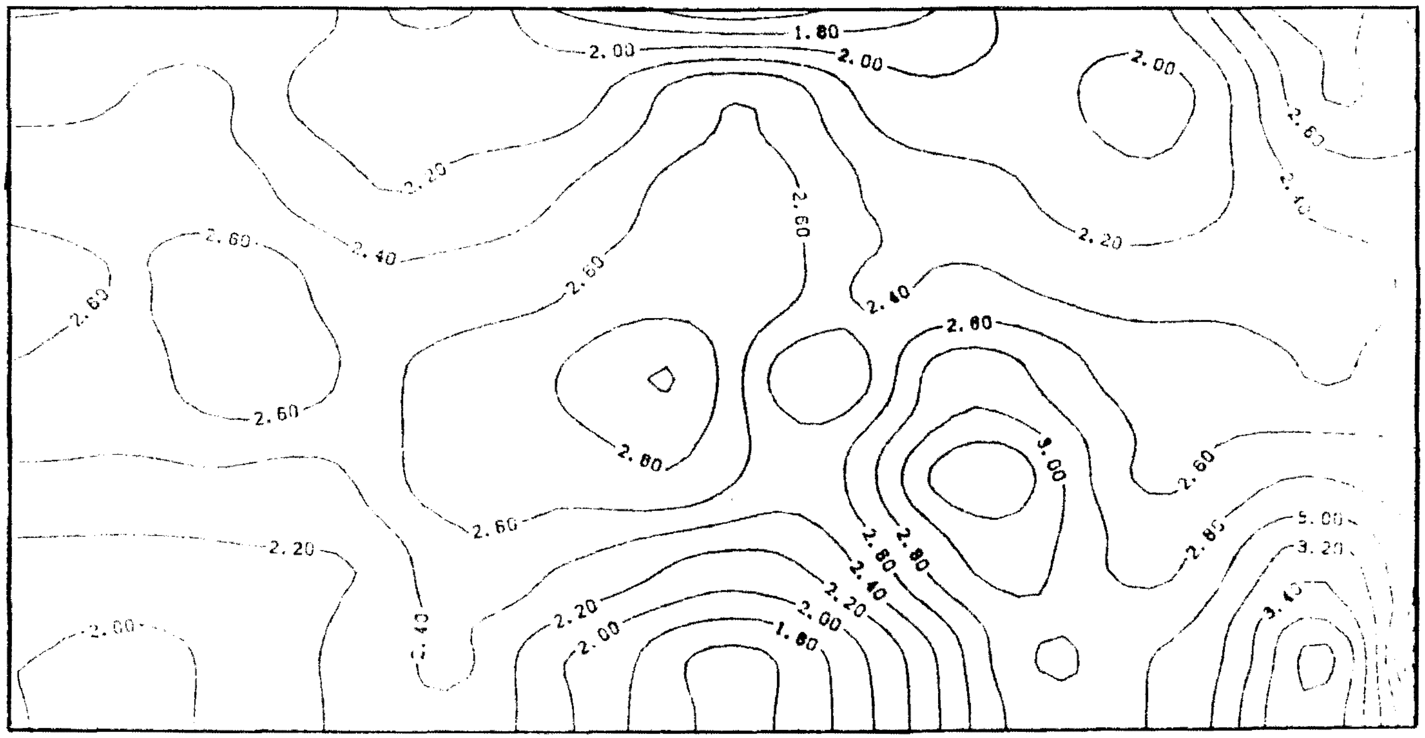


Fig. 5.6 Contour map of block kriging estimates of exchangeable Mg.

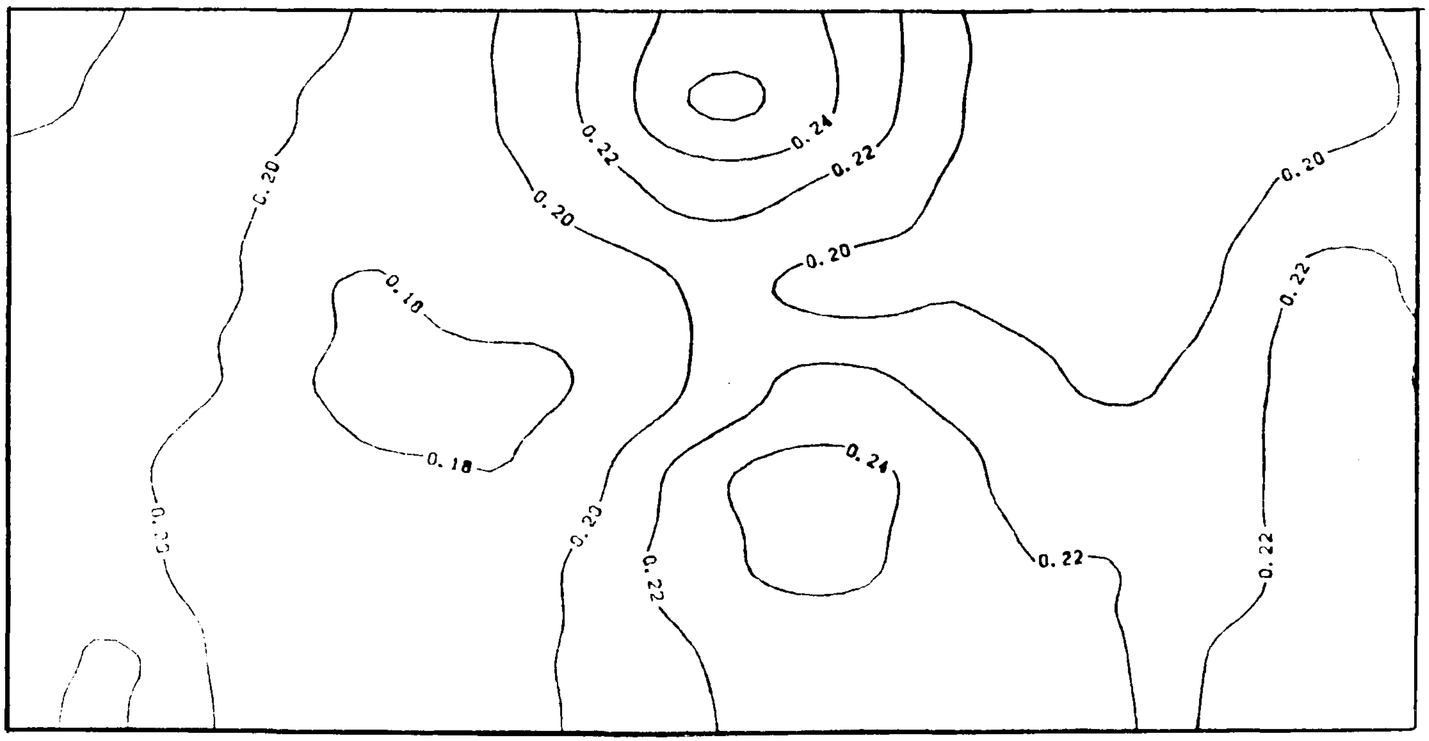


Fig. 5.7 Contour map of block kriging estimates of exchangeable K.

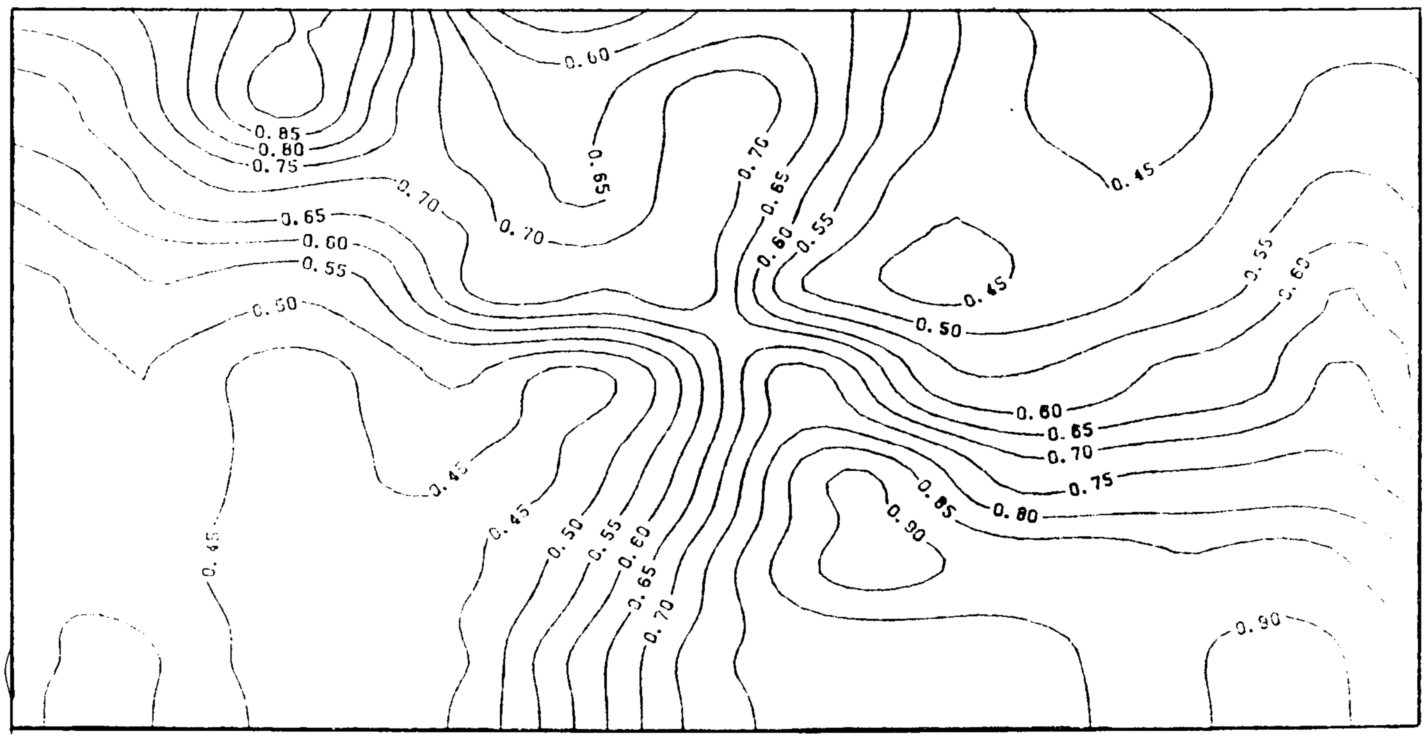


Fig. 5.8 Contour map of block kriging estimates of exchangeable Na.

$$S^2(a/A) = S^2(0/A) - S^2(0/a) \text{ if } 0 < a < A \quad (5.7)$$

describes this. The symbol 0 represents points, i.e. zero area, a represents 5 m square blocks, and A the whole area studied. The variance of 5 m square blocks in the whole area is denoted by $S^2(a/A)$, and the variance of point samples within the area is $S^2(0/A)$.

Contour maps of estimation variances are useful for determining whether or not measurements at additional places in the field would be required to improve the accuracy and precision. Since the estimation variance depends on the configuration of the sampling points and not on the observed values themselves, it is possible to calculate estimation variances for a particular sampling scheme before putting it into practice if the semi-variogram is known a priori. A sampling scheme that ensures that the error is kept within prescribed bounds can therefore be designed. According to Burgess et al. (1981) a regular equilateral triangular grid is the most efficient design for a scheme where variation is isotropic, but a square grid is nearly efficient and will usually be preferred for convenience.

Because of the high nugget variances found in this study relatively smooth maps obtained for each soil property can be a misleading representation of reality. As Webster (1985) pointed out if there is a nugget variance then it follows that the soil does not vary smoothly at the working scale.

5.4 Conclusions

1. Isarithm maps made both by punctual and block kriging showed the spatial variation of exchangeable bases. They revealed the presence of higher levels of exchangeable Ca, Mg, and Na at the center of the field which decreased as one moves towards the edges. An opposite trend was found for K.

2. Isarithm maps of Ca, and Mg had several near circular contours at some places. These near circular contours represented discontinuities in the surface as a result of large nugget variances.

VI. CABBAGE RESPONSE TO NONUNIFORM POTASSIUM APPLICATION

6.1 Introduction

Uniform application of K fertilizer throughout the field of interest is essential to produce maximum yield. The excess of K fertilizer which may appear in some spots within a given field as a result of uneven application can be expected to cause some crop loss, and therefore, some financial loss to the farmer. Investigations concerning the relationship between nonuniform fertilizer application and crop yield are rather scarce. According to Green et al. (1968) machines should be capable of applying fertilizers to meet agronomic standards. They pointed out that the specification for a machine applying fertilizer is "Fertilizer shall be applied so that when the application rate is measured on areas not less than 1 foot by 1 foot and not greater than 1.5 feet by 1.5 feet, the measured variations from the mean rate shall be such that it can be guaranteed that on 95% of the total area the variation from the mean rate will not exceed $\pm$ 20% on high value crops and $\pm$ 30% on low value crops: over the whole area the variations from the mean application rate shall not exceed $\pm$ 30% and $\pm$ 45% for high and low crops, respectively." Hand application of solid fertilizer in experimental plots as well as small farmers' fields may not always meet the above standards. On the other hand even with the best equipment available uneven application can occur due to faulty operation of machine or due to the properties of the fertilizer which can adversely affect the performance of the machine. Nonuniformity of fertilizer application can therefore be expected to affect the optimal rates of fertilizer application.

The rates of fertilizer application are usually determined by comparing a soil test value with a critical value appropriate for the nutrient and situation under study. Although nutrient variability across the field may both reduce yield and result in wasteful application, in many instances too little account is taken of the spatial variability of the soil nutrient. Geostatistical methods of studying two- and three-dimensional phenomena provide new concepts in quantifying nutrient contents and variability and could be useful in determining rates of fertilizer application.

The objects of this study were to:

1. Test the usefulness of some geostatistical concepts as a tool for estimating rates of fertilizer application given the soil sample data and the structure of spatial dependence;

2. Investigate cabbage response to nonuniform potassium application.

6.2 Materials and Methods

6.2.1 Site description

The study was conducted on a 92 m x 42 m fallow field at the University of Hawaii Volcano Research Station on the island of Hawaii. The experimental site is located at 1200 m altitude with a mean annual rainfall of 3000 mm and mean annual temperature of 14°C. The soil is a Typic Hydrandept, medial over thixotropic, isomesic (Puaulu series). The Puaulu series consists of well-drained soils developed in geologically recent volcanic ash (Ikawa et al., 1985). The soils are smeary and dehydrate irreversibly.

6.2.2 A geostatistical approach to the determination of rates of K application

The estimated semi-variogram of exchangeable K determined in chapter 4 was used to estimate the average level of exchangeable K of 8 m x 4 m blocks using block kriging. These blocks were then considered as experimental plots. Using the block variance and taking into consideration the distribution of exchangeable K the proportion of the area of each experimental plot below a specified critical level (0.8 cmol(+)/kg) was estimated with Equation 6.1

$$T_{(X_c)} = G(1/\sigma(\ln X_c/m_0) + \sigma/2) \qquad (6.1)$$

where G(Z) = standard normal probability density function

$$G(Z) = \int_{-\infty}^{\infty} (e^{-t^2/2} / 2\pi) dt \tag{6.2}$$

X_c = critical level of exchangeable K and is considered random variable with a spatial component, i.e., a regionalized variable.

m_0 = arithmetic mean of exchangeable K (kriged value).

σ = logarithmic standard deviation

The variance term, commonly known as dispersion variance or block variance, is uniquely provided by the geostatistical approach. It describes the way in which exchangeable K varies within the field of interest.

The level of exchangeable K in the deficient zone was estimated by Equation 6.3

$$M(X_c) = Q(X_c)/T(X_c) m_0 \tag{6.3}$$

where

$$Q(X_c) = G(1/\sigma(\ln X_c/m_0) - \sigma/2) \tag{6.4}$$

The amount of potassium necessary to bring 0, 25, 50, and 100% of the deficient area up to the critical level was determined. This provided 4 rates of K application (0, 70, 140, and 280 kg K/ha). Another rate of 560 kg K/ha was added.

6.2.3 Experimental design and fertilizer application

The experiment was a 3 x 5 factorial with 3 replications. Treatments were arranged in a split-plot design with indices of fertilizer distribution as main plots and rates of K as subplots. The index of fertilizer distribution was defined as the ratio of the area fertilized in each experimental plot to the total area and was determined as follows. A moving 1 m x 1 m grid was superimposed over each 8 m x 4 m plot and the grid cells to be fertilized were randomly selected so that 100, 75, and 50% of the grid cells received K fertilizer application. This resulted in 3 indices of fertilizer distribution (1.0, 0.75, 0.50) and consequently in 3 uniformity coefficients of fertilizer application.

Each experimental plot received a uniform hand broadcast blanket fertilizer which included: 120 kg N/ha as ammonium sulfate, 235 kg P/ha as treble superphosphate, 15 kg Zn/ha as zinc sulfate, 15 kg B/ha as borax, and 15 kg Cu/ha as copper sulfate. Dolomite was also applied at a rate of 2 metric tons/ha. The blanket fertilizer was incorporated into the soil with a rototiller. Then K fertilizer was applied by hand in randomly selected grid cells and incorporated into the soil with a rototiller. Seedlings of chinese cabbage (Brassica oleracea L. 'Chinensis'), (Nagaoka variety), were transplanted into the experimental plots with a spacing of 90 cm x 40 cm three weeks after fertilizer application. Four weeks after transplanting, cabbage received a topdressing of 120 kg N/ha as urea. At harvest individual cabbage heads were weighed in the field. Plant samples were taken, dried at 60^{o} and nutrient contents were determined.

6.2.4 Soil sampling

Before cabbage was transplanted a 1 m x 1 m moving grid was superimposed over each plot. Soil samples were taken from the 0-15 cm depth at the centroid of each cell with an auger of 7.5 cm diameter.

6.2.5 Laboratory determinations

Exchangeable K was determined with 1N NH_4OAc pH 7 as described in chapter 4. The procedure used for establishing the Quantity-Intensity relationships followed essentially that described by Beckett (1964). 5 grams of soil (< 2 mm) were shaken with 20 ml 0.01 M $CaCl_2$ containing 0, 0.2, 0.5, 1.0, 2.0, 3.0, and 6.0 mM KCl for 8 hours. After equilibration the soil suspension was centrifuged and Ca, Mg, K, and Na were determined in the solution by atomic absorption spectrophotometry. The activity ratio was calculated as follows

$$AR^{k} = a_k / \sqrt{a_{Ca+Mg}} \qquad (6.5)$$

where

a_k = activity of K

a_{Ca+Mg} = activity of Ca + Mg

The energy of exchange of K by Ca and Mg was calculated by Equation 6.6

$$\Delta G_{k,Ca+Mg} = RT \ln (AR^k) \qquad (6.6)$$

where ΔG = free energy, in calories per equivalent

R = gas constant

T = absolute temperature

6.3 Results and Discussion

6.3.1 Yield response to nonuniform K application

One of the goals of fertilizer application is to efficiently apply an appropriate amount of fertilizer, distributed as uniformly as possible. A key requirement then is to be able to characterize the fertilizer distribution that results from the operation of fertilizer application. Various statistical distributions can be used to approximate the observed fertilizer distribution. In the present study the distribution of K fertilizer application was approximated by a uniform distribution whose probability density function is (Haan, 1977)

$$P_x(x) = 1/(\beta - \alpha) \qquad \text{for } \alpha \leq x \leq \beta \quad (6.7)$$

and the cumulative distribution function is

$$P_x(x) = 1/(x - \alpha)/(\beta - \alpha) \quad \text{for } \alpha \leq x \leq \beta \quad (6.8)$$

The mean and variance of the distribution are

$$E(X) = (\beta + \alpha)/2 \tag{6.9}$$

$$VAR(X) = (\beta - \alpha)^2/12 \tag{6.10}$$

According to Haan (1977) α and β can be estimated by applying the method of maximum likelihood. In this case α is the smallest and β the largest sample values.

Using the mean and the variance a uniformity coefficient of fertilizer application (UCF) was defined as:

$$UCF = 1 - \sigma/\bar{Q} \tag{6.11}$$

where σ and $\bar{Q}$ are the standard deviation and mean fertilizer application, respectively. A uniformity coefficient provides an overall indication of the evenness of a fertilizer application pattern. Three uniformity coefficients corresponding to the three indices of fertilizer distribution determined above were calculated.

The relationship between cabbage yield and potassium application is shown in Fig. 6.1 for various uniformity coefficients. It will be seen from Fig. 6.1 that decreases in uniformity of fertilizer application resulted in decreased cabbage yields especially at low rates of K application. Maximum yield also decreased with increased unevenness of fertilizer distribution. The spatial variability in soil properties across the field had a

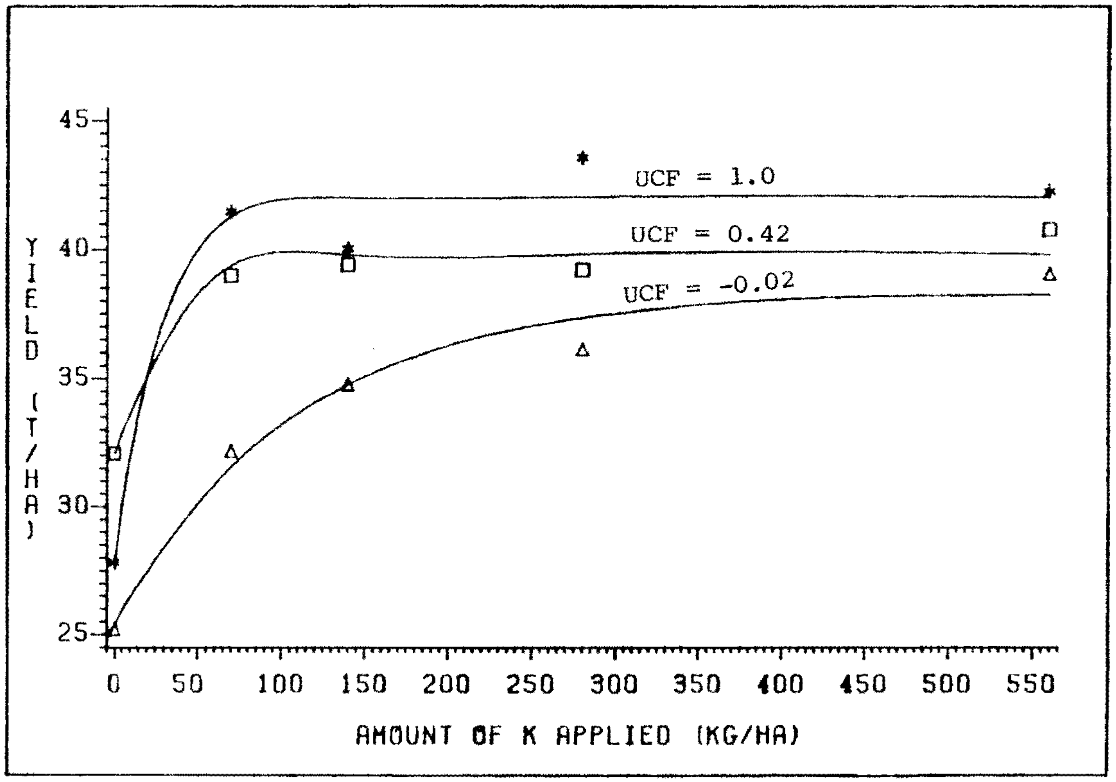


Fig. 6.1 Relationship between cabbage yield and rates of K application for various levels of uniformity of K fertilizer application.

significant effect on yield. This can readily be seen from the differences in yield with no K application.

The response of a crop to fertilizer application can be summarized in a yield-fertilizer function, relating yield to the amount of fertilizer applied. To quantify the relationship between yield and rates of K application the data were fitted to a Mitscherlich model.

$$Y_{(Q)} = A - B \exp(-kQ) \tag{6.12}$$

where $Y_{(Q)}$ = yield (T/ha), A = maximum yield, B and k are coefficients, and Q = amount of K applied.

The following response functions were found for the various uniformity coefficients

$$Y_{(Q)} = 42.03 - 14.18 \exp(-0.0420\ Q) \text{ for UCF=1.0} \tag{6.13}$$

$$Y_{(Q)} = 39.10 - 7.74 \exp(-0.0410\ Q) \text{ for UCF=0.42} \tag{6.14}$$

$$Y_{(Q)} = 38.32 - 12.89 \exp(-0.0092\ Q) \text{ for UCF=-0.02} \tag{6.15}$$

The amount of K required to attain 95% of maximum yield was calculated from Eq. 6.12 as follows

$$KR_{95} = (\ln B - \ln A - \ln 0.05)/k \tag{6.16}$$

The following values of KR_{95} were found for the three levels of variability:

KR_{95} = 97 kg K/ha for UCF = 1.0

KR_{95} = 113 kg K/ha for UCF = 0.42

KR_{95} = 446 kg K/ha for UCF = -0.02

Use of the geostatistical technique employed earlier to determine rates of fertilizer application showed that the application of 97 kg K/ha is equivalent to bringing 35% of the deficient area up to the critical level of 0.8 cmol(+)/kg. If instead of 0.8 cmol(+)/kg a critical level of 0.5 cmol(+)/kg were selected then a rate of 97 kg K/ha would mean bringing 70% of the deficient area up to that critical level. From the above it can readily be seen that the selection of a critical level for the determination of the proportion of each plot below that threshold can have a crucial bearing on the determination of the rates of K application. The geostatistical approach used to determine the rates of K application seems promising. It shows that relatively precise estimates of K content and rates of K application can be obtained provided the semi-variogram of soil K is known and stationary and a moderate number of samples are available. The results suggest that fertilizer rates could be determined on the basis of the proportion of the deficient area. The problem, however, is how to identify the nutrient deficient zones. If a map of the field showing the locations of the variations in the field is available then variable rate fertilization should be possible especially with the new technology being developed for fine-tuned fertility (Luellen, 1985).

Since nonuniformity of K application can be expected to significantly affect yield and a relationship between yield per unit area and the application rate can be established it would be useful to develop an expression for the yield in terms of the application rate and other relevant characteristics of the distribution of K application. In the physical situations involving treatments and their responses, the actual measurements reflect average yield (Y) and average applications of treatment (Q). The determination of average yield involves the evaluation of the following integral (Seginer, 1978).

$$\overline{Y} = \int_0^\infty y_{(Q)}\ f(Q)\ dQ\ ,\quad \int_0^\infty f(Q)\ dQ = 1 \qquad (6.17)$$

where $\overline{Y}$ is the mean yield, Q is the application rate, y is the yield for a given fertilizer application (the yield function), and F(Q) is the fertilizer distribution function. The use of Eq. 6.17 is based on the assumption that the yield-fertilizer function is known for the set of conditions under study. In addition it is assumed that the uniformity of K fertilizer application is indicative of the uniformity of potassium in the root zone, since presumably this is what will influence crop yield. The function f(Q) can take different forms, the most convenient being the normal distribution. The function $y_{(Q)}$ is generally in the form of a polynomial, e.g., quadratic yield function or exponential form, i.e., Mitscherlich's model. Zaslavsky and Mokady (1967) have developed an approximate expression for Eq. 6.17. A brief account of their analytical approach follows.

If one considers a response curve where the yield Y(Qi) is related to the application level (Qi) of a fertilizer and takes into consideration the fact that average ($\overline{Q}i$) and $\overline{Y}$ are measured, then the yield with nonuniform Qi can be assumed to be a function of Qi and the deviation qi from these averages.

$$qi = Qi - \overline{Q}i \tag{6.18}$$

$$Y = Y(Q1, Q2, \ldots, Qn, q1, q2, \ldots, qn) \tag{6.19}$$

The yield (Y_0) is a uniform application ($Qi = \overline{Q}i$) is

$$Y_0 = Y(Q1, Q2, \ldots, Qn, 0, 0, \ldots, 0) \tag{6.20}$$

As a result for a nonuniform application ($Qi \neq Qi$) one can write

$$Y_0 = Y(Q1, Q2, \ldots, Qn) + (\text{correction term}) \tag{6.21}$$

Expanding Y from Eq. 6.19 in Taylor's series gives

$$Y = Y_0(Q1, Q2,\ldots, Qn) + (\partial y/\partial Qi)qi +$$

$$1/2! \sum_{ij} (\partial^2 y/\partial Qi \partial Qj)qiqj +$$

$$1/3! \sum_{ijk} (\partial^3 y/\partial Qi \partial Qj \partial Qk)qiqjqk + \ldots \qquad (6.22)$$

The first term on the right hand side of Eq. 6.22 is a constant; the second term vanishes on averaging. The average yield is found by integrating Eq. 6.22 over the area A and dividing by the area.

$$\overline{Y} = \int (Y\, dA)/A = Y_0(Q1, Q2,\ldots, \overline{Q}n) +$$

$$1/2! \sum_{ij} (\partial^2 y/\partial Qi \partial Qj)\int (qiqj\, dA)/A +$$

$$1/3! \sum_{ijk} (\partial^3 y/\partial Qi \partial Qj \partial Qk)\int (qiqjqk\, dA)/A + \ldots$$

$$(6.23)$$

If one neglects any term beyond the second, then Y can be expressed as the yield Y_0, plus a correction term due to the fluctuations. If a single parameter is considered with an average Q and fluctuations q, then Eq. 6.23 becomes

$$\overline{Y} = Y_0 + 1/2!(\partial^2 y/\partial Q^2)\int (q^2\, dA/A)$$

$$+ 1/3!(\partial^3 y/\partial Q^3)\int (q^3\, dA/A) + \ldots \qquad (6.24)$$

The integral in the third term of Eq. 6.24 expresses the skewness in the q population and it vanishes for a symmetrical population. The second term is divided into two parts:

$$FI = \text{Fluctuation Index} = \sigma_q^2 = \int (q^2\, dA/A) \quad (6.25)$$

$$RI = \text{Response Index} = 1/2(\partial^2 y/\partial^2 Q) \quad (6.26)$$

The whole term is the Fluctuation Response Index (FRI) and expresses changes in average yield due to fluctuations in the application level (qi). Therefore for any twice differentiable yield function the relevant expression relating the degree of uniformity of the fertilizer application treatment to the crop yield could be written as

$$\bar{Y}_{(Q)} = Y_{(Q)} + 1/2(\partial^2 y/\partial Q^2)\sigma_q^2 \quad (6.27)$$

where $Y_{(Q)}$ is the average yield, Q and σ_q are the average and the standard deviation of fertilizer application, and $Y_{(Q)}$ is the yield that would have been obtained with perfectly uniform (no fluctuation in Q) application. In order to calculate the effects of nonuniformity of K fertilizer application on cabbage yield Eq. 6.27 was substituted into Eq. 6.12 to obtain

$$Y_{(Q)} = A - B \exp(-kQ) + 1/2\partial^2 Y_{(Q)}/\partial Q^2 \quad (6.28)$$

Rearranging Eq. 6.11 and substituting for σ in Eq. 6.28 one obtains

$$Y_{(Q)} = A - B \exp(-kQ) + 1/2\partial^2 Y_{(Q)}/\partial Q^2 Q^2 (1-UCF)^2 \quad (6.29)$$

Because of differences in cabbage yield with no K application due to the inherent variability in soil properties the yield-fertilizer function obtained with UCF=1.0 could not be considered as $Y_{(Q)}$ in Equation 6.27. An estimate of $Y_{(Q)}$ was obtained by pooling the data from the entire experiment and fitting them to Eq. 6.12 so that the parameters A, B, and k in Eq. 6.29 are those estimated from the pooled data with A = 39.98, B = 10.17, and k = 0.0172. Using Eq. 6.29 cabbage yield was predicted for UCF values of 1.0, 0.42, and -0.02 and for different rates of K application. A plot of predicted yield against observed yield is shown in Fig. 6.2. A highly significant correlation coefficient of 0.892^{**} was found between observed and predicted cabbage yield.

Uniformity of fertilizer distribution over the soil surface alone may not explain the total variation in yield since tillage operations performed to incorporate fertilizer into the soil introduces another source of variation. Expressing yield in relative terms i.e., relative yield $Y_r = Y_{(Q)}/A$ curves of relative yield as a function of the uniformity of fertilizer application were drawn in Fig. 6.3 using Eq. 6.29. It will be seen from Fig. 6.3 that the decrease in relative yield with decrease in uniformity coefficient was relatively important at low rates of application. At higher rates the areal uniformity of the treatment had less effect on overall yield. Fig.

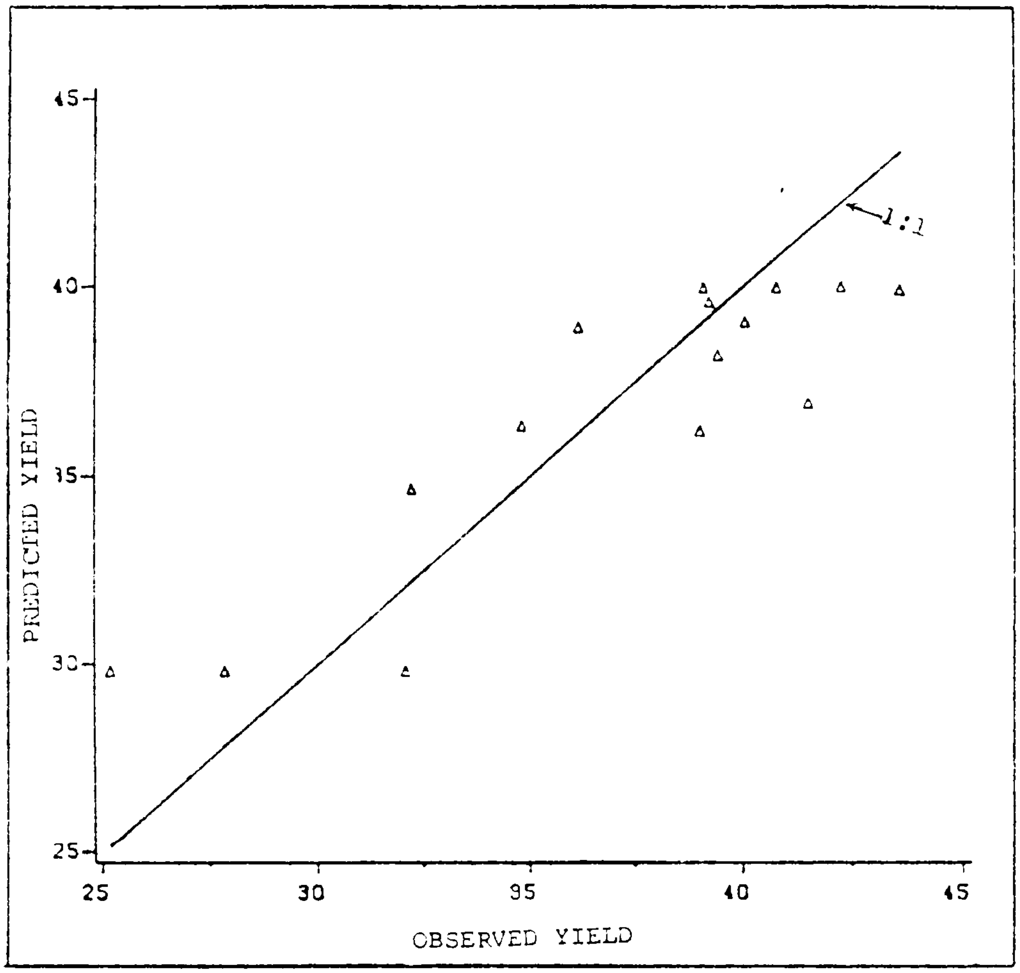


Fig. 6.2 Comparison of predicted and observed cabbage yield.

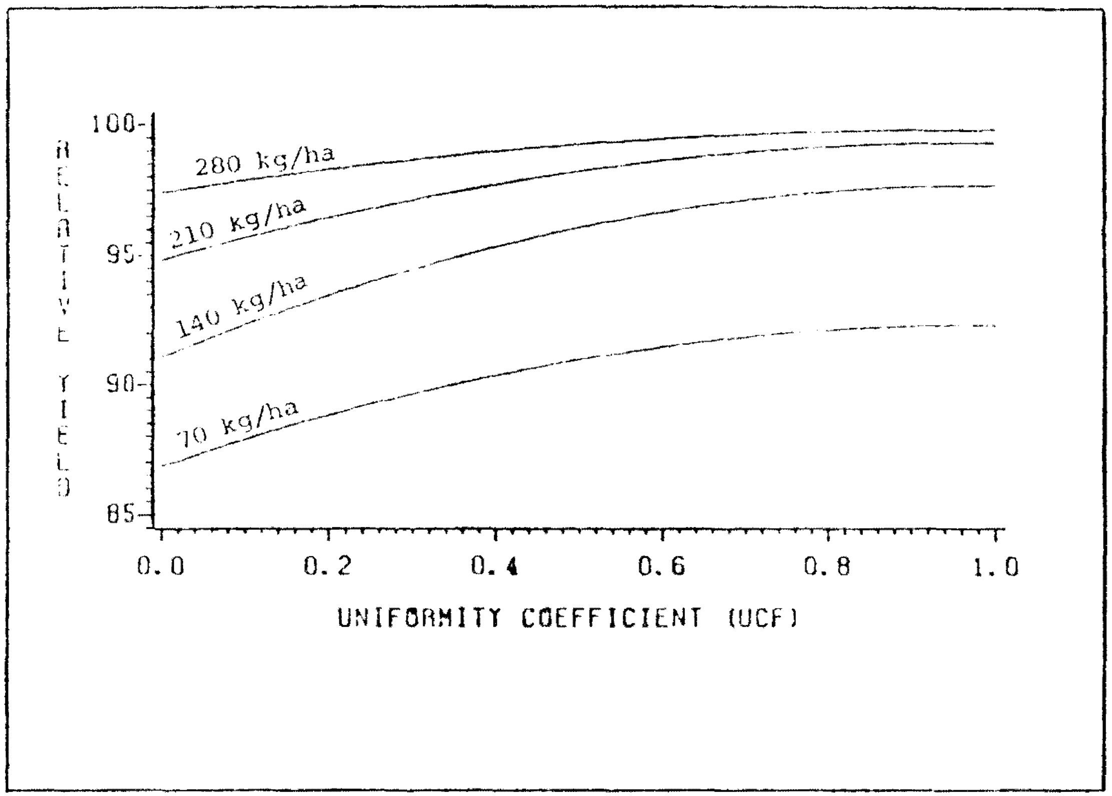


Fig. 6.3 Calculated relative yield of cabbage as a function of uniformity coefficient of K fertilizer application for 4 rates of K application.

6.4 shows the relationship between relative fluctuation index and uniformity coefficient. Such calculations showed that the decrease in relative FRI with increase in UCF was much steeper for lower rates of fertilizer application.

Yield predictions based on Eq. 6.29 will be much influenced by the precision with which the parameters A, B, and k are estimated. It seems, therefore, convenient to have an expression not requiring least squares estimation of certain parameters. We may define relative yield as the ratio of actual yield to the maximum yield in each replication and for each uniformity coefficient. Similarly, application ratio may be defined as the ratio of actual fertilizer rate to the amount of K fertilizer which corresponds to maximum yield. A function relating relative yield to application ratio can be considered fairly general and can be representative of more than one location or year. A plot of relative yield against application ratio showed that this relationship could be described by a parabola of the form

$$Yr = a + bXr + cXr^2 \qquad (6.30)$$

where Yr = the relative yield, Xr = application ratio, a = relative yield without K application, c and b are constants. Using the properties of the parabola maximum value of relative yield can be calculated as follows:

$$Yr_{max} = a - b^2/4c \qquad (6.31)$$

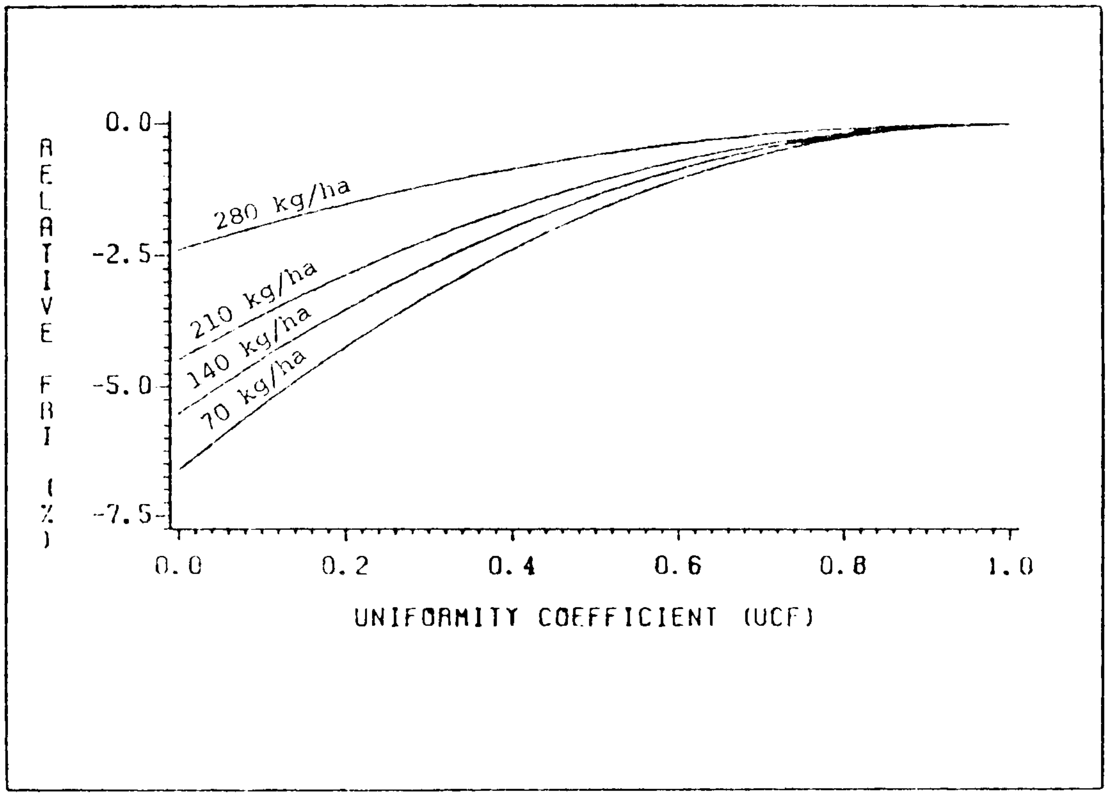


Fig. 6.4 Calculated relative fluctuation response index (FRI) as a function of uniformity coefficient of K fertilizer application for 4 rates of K application.

and will occur at

$$Xr_{max} = -b/2c \tag{6.32}$$

Theoretically maximum value of relative yield is equal to 1 and occurs at a value of application ratio equal to 1. Therefore one can write

$$a - b^2/4c = 1 \tag{6.33}$$

$$-b/4c = 1 \tag{6.34}$$

Solving Equations 6.33 and 6.34 simultaneously yields the following values for b and c

$$b = 2(1 - a) \tag{6.35}$$

$$c = a - 1 \tag{6.36}$$

The relative yield relationship becomes

$$Yr = a + 2(1 - a)Xr + (a - 1)Xr^2 \tag{6.37}$$

The average relative yield can be calculated by taking the second derivative of Eq. 6.37 and expressing Eq. 6.37 in the form of Eq. 6.27. This yields the following expression:

$$Yr = [a+2(1-a)Xr+(a-1)Xr^2]+(a-1)\sigma_{Xr}^2 \tag{6.38}$$

where σ_{Xr}^2 is the variability of fertilizer application and a is the relative yield without fertilizer application.

The term in square brackets gives the relative yield in terms of the application ratio for perfect uniformity. The term $(a-1)\sigma_{Xr}{}^2$ is a correction for nonuniformity in terms of both the application ratio and the variance.

If a uniformity coefficient is defined as UCF = $1 - \sigma/X_r$ then the term $(a-1)\sigma_{Xr}$ becomes $(a-1)Xr^2(1-UCF)^2$. As a result Eq. 6.38 can be written as

$$Yr = [a+2(1-a)Xr+(a-1)Xr^2] + (a-1)Xr^2(1-UCF)^2 \qquad (6.39)$$

With the assumption that relative yield without K application is zero, Eq. 6.39 becomes

$$Yr = (2Xr - Xr^2)-Xr^2(1 - UCF)^2 \qquad (6.40)$$

The theoretical relationship between relative yield and application ratio is depicted in Figures 6.5 and 6.6 using Equations 6.39 and 6.40, respectively. In Fig. 6.5 a relative yield of 40% without K application was assumed. Both figures give an indication of the magnitude of the effects to be expected. In order for the relationship to be as depicted the quadratic yield relationship must continue for rates of application above that associated with maximum relative yield. The advantages of using Eq. 6.39 instead of Eq. 6.29 is that it requires only knowledge of the relative yield without fertilizer application and the variability of the application of a given amount of

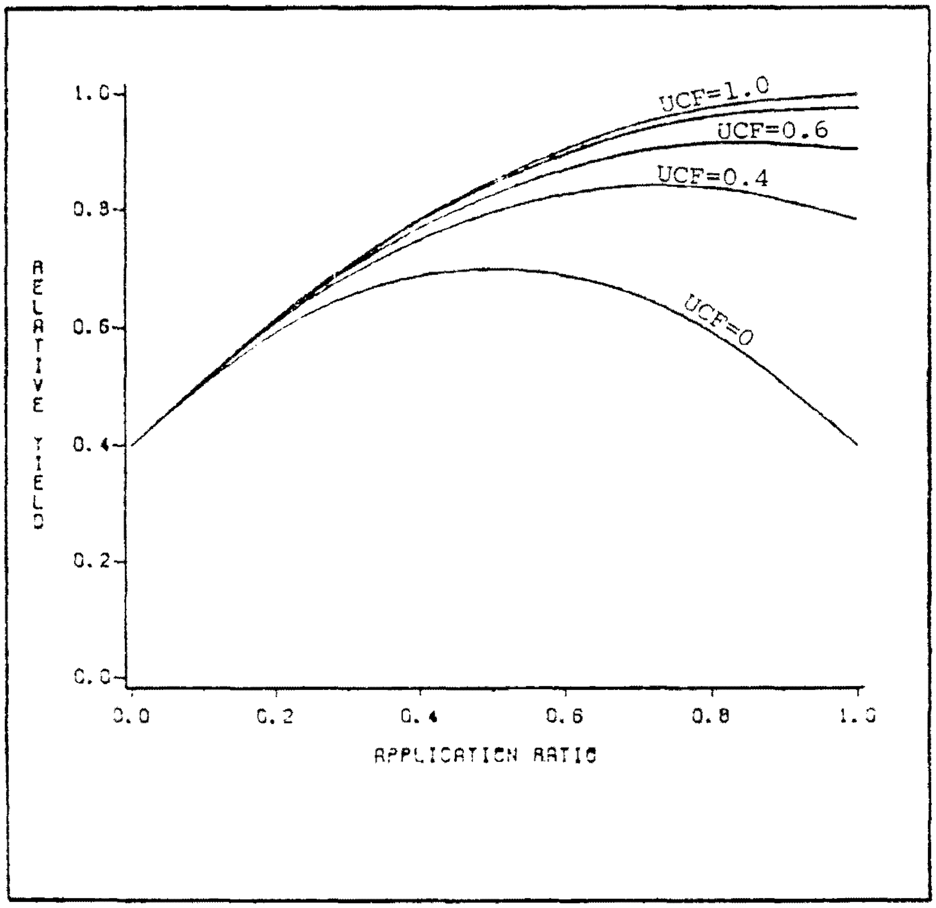


Fig. 6.5 Relationship for the parabolic yield curve between relative yield and application ratio for different uniformity coefficients of K application when relative yield is 40% for zero applied K fertilizer.

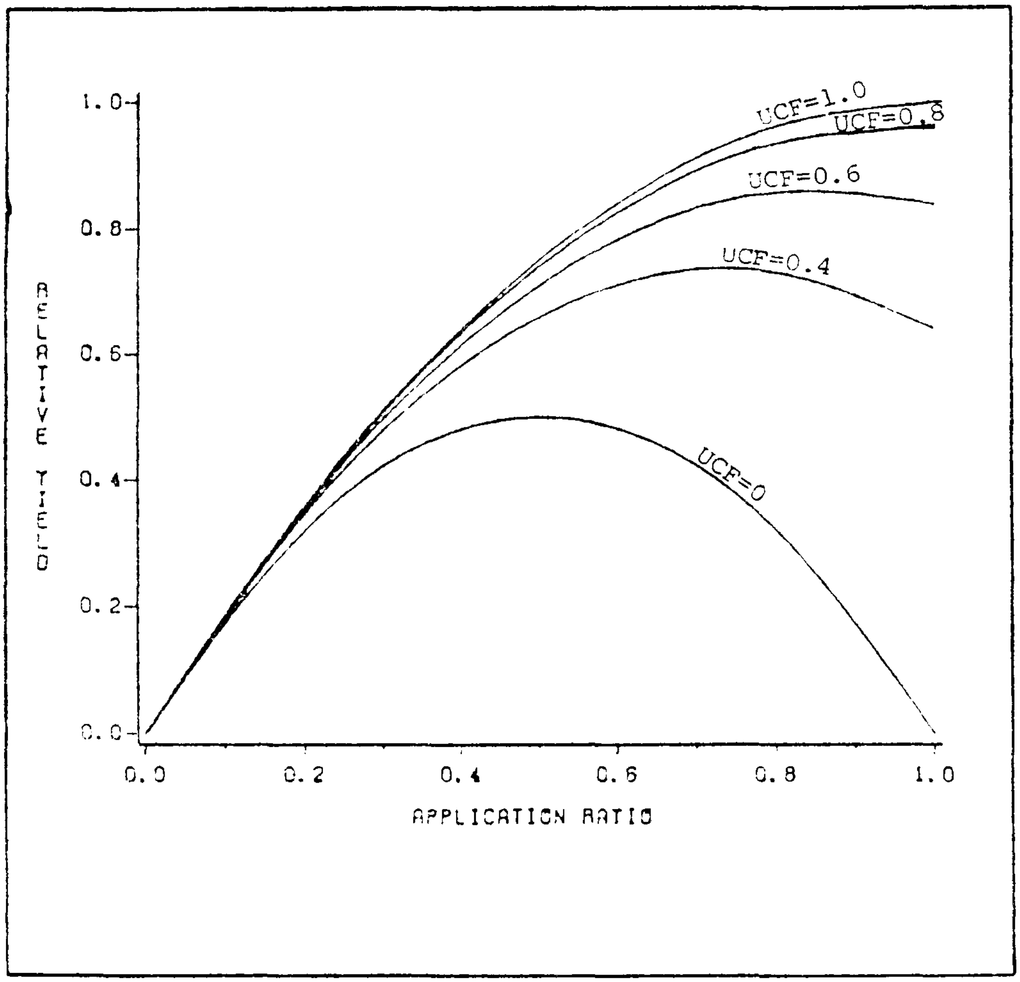


Fig. 6.6 Relationship for parabolic yield curve between relative yield and application ratio for different uniformity coefficients of K application when relative yield is zero for zero applied K fertilizer.

fertilizer expressed as a ratio of the amount associated with maximum yield. Using Eq. 6.39 relative cabbage yield was predicted for the three uniformity coefficients found in this study. The predicted yield was plotted against the observed and the relationship is shown in Fig. 6.7. A highly significant correlation coefficient of 0.781 was found between observed and predicted yield. However, there was a tendency for Eq. 6.39 to underestimate yield when the observed relative yield was above 0.8. It should be pointed out that decreases in yield result essentially from the curvilinear nature of the parabolic yield-fertilizer function. Moreover, the degree of curvature increases as the application ratio approaches that required for maximum yield. As a result the overall yield loss will increase more than proportionately to the degree of nonuniformity (Holmes, 1968).

There are some limitations to the use of data from ordinary experiments measuring crop response to nonuniform application of fertilizers. The first limitation is that if the individual areas are sufficiently small (fluctuation of high frequency), nonuniform application may be unimportant because all individual plants may draw nutrients from areas receiving both high and low rates of fertilizer. In this way all individual plants would effectively receive the mean fertilizer application rate. Little information is available on the maximum area over

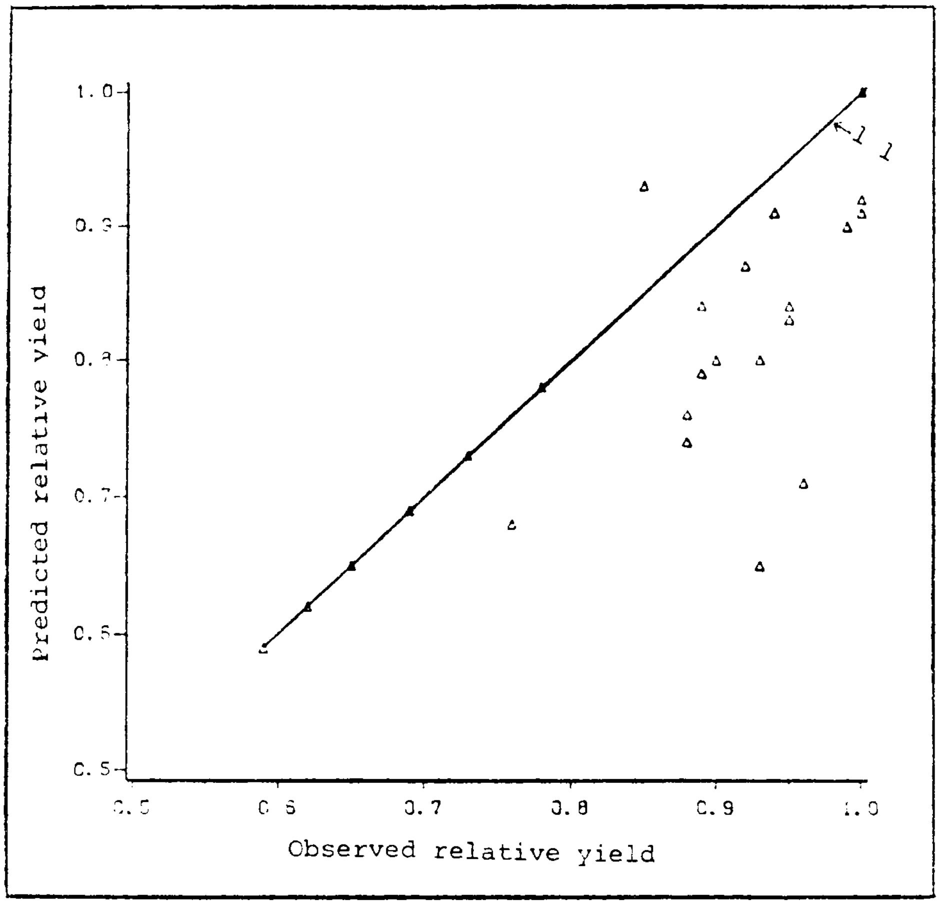


Fig. 6.7 Comparison of predicted and observed relative cabbage yield.

which nonuniformity of application can be rendered unimportant. Prummel and Datema (1962) found that inequalities in rate of application were only important when the individual patches were greater than 0.5 m^2. It seems likely that factors such as plant population and row width will have a considerable influence on the minimum size of area that is important in this respect. According to Holmes (1968) the pattern of fertilizer application should be assessed using areas 30 cm to 45 cm wide as units from which recordings are obtained. Edge effects between areas of low and high application will even then tend to offset the yield losses resulting from nonuniform application, so that calculations of yield losses made from response experiment data which do not provide an allowance for the edge effects will represent the maximum possible effect of nonuniform application. However, if the above limitations are dealt with by reasonable assumptions an approach based on field data can be very useful.

The results obtained in this study suggest that the impact of nonuniformity of fertilizer application will depend to a great extent on the shape and form of the yield-fertilizer production function. If yields are depressed by increments of fertilizer added beyond the point of maximum yield, e.g., quadratic production function then an overestimation of the effect of nonuniformity is likely. If, on the other hand, the production function is such that there is no yield reduction in response to the application of fertilizer in excess of that needed to achieve maximum yield, e.g., Mitscherlich model then the effect of nonuniformity could be estimated more accurately. It should be point out, however, that nonuniformity of fertilizer application on the microscopic level, i.e., within the root zone of a single plant, complicates matters since the response on this level cannot be described by the ordinary yield-treatment curves. The uniformity of fertilizer application as a condition for improved crop yield must be viewed critically. Under certain conditions uniformity of fertilizer application may be achieved at very high cost without a commensurate increase in yield. Under other conditions, however, yield could be greatly increased through uniform application of fertilizers.

6.3.2 Yield-tissue K concentration relationship as affected by nonuniformity of K application

The relationship between yield and K concentration in cabbage is shown in Figures 6.8a, 6.8b, and 6.8c for UCF values of 1.0, 0.42, and -0.02, respectively. It will be seen that the results are in agreement with the generally good relationship reported in the literature between plant nutrient status and crop performance (Dow and Roberts, 1982). However, the scattering of points along each curve points out to the difficulty of determining a specific point experimentally for designation as the critical

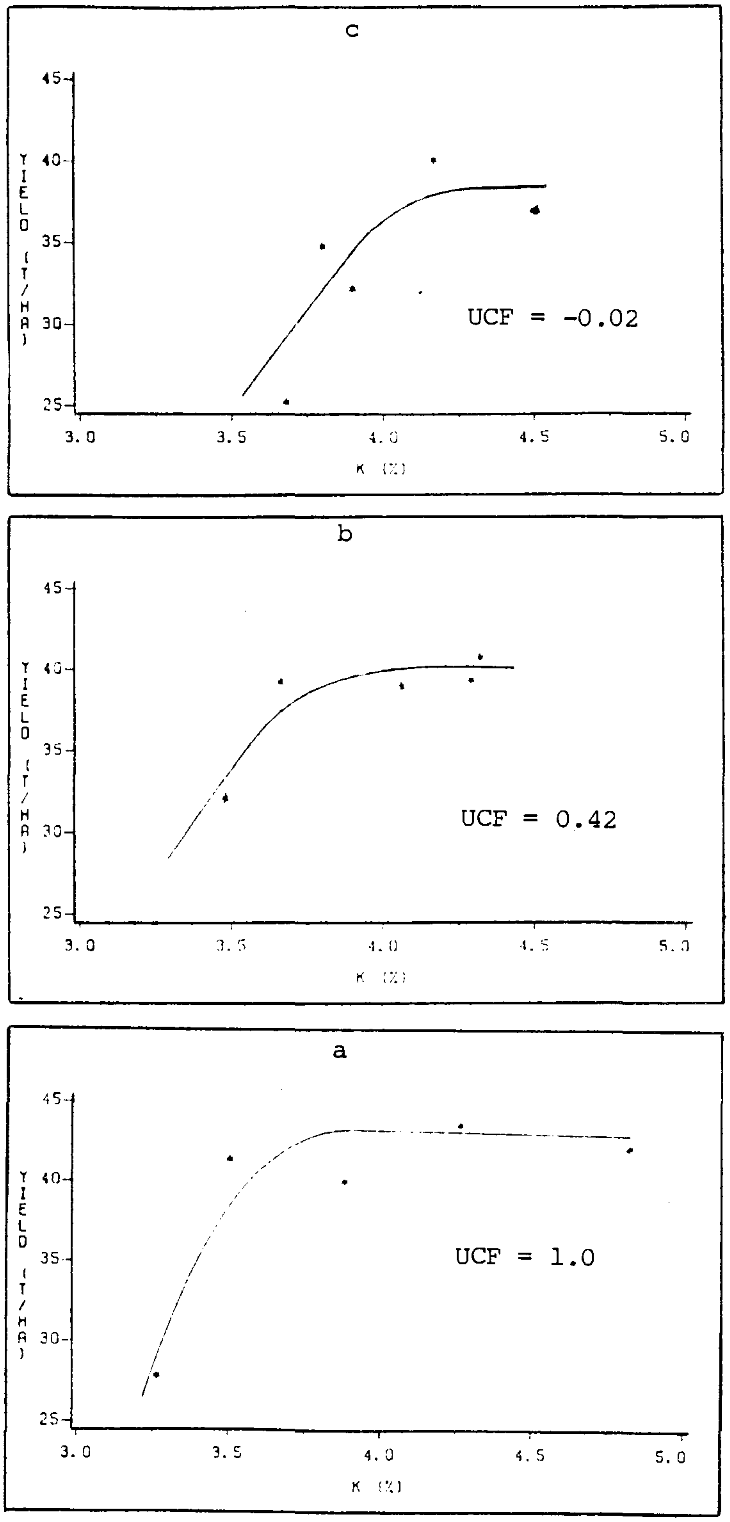


Fig. 6.8 Relationship between cabbage yield and K in dry matter as affected by uniformity of K application.

potassium concentration. The degree of scattering of experimental points along the curve increased with increasing variance of fertilizer application. Moreover, graphical determination of the K associated with maximum yield gave 4.0, 4.15, and 4.30% K for UCF values of 1.0, 0.42, and -0.02, respectively. These results support the contention that a narrow range of concentration rather than a single value seems to be a more appropriate concept for evaluating the nutrient status of crops (Dow and Roberts, 1982). In most agronomic studies tissue nutrient concentrations are expressed as a percentage in the dry matter. The usefulness of this convention was recently questioned by Leigh and Johnston (1983). Leigh and Johnston (1983) argued that percentage K in dry matter is not a reliable measure of the K status of crops because it depends not only on the level of soil K but also on the rates of supply of N, P, and water. Also percentage K in dry matter varies with age so that deficiencies can only be detected if crops are compared at the same age to minimize this time-dependent change. Leigh and Johnston (1983) suggested that tissue water may provide a better basis for calculating nutrient concentrations, since metabolic processes, and hence growth, presumably respond to the concentration of K in tissue water, not to that in the dry matter. Although percentage K in dry matter may be less physiologically relevant than K concentration in tissue water, the relationship between percentage K in dry matter and yield may have predictive value. On any given site, percentage K in dry matter is likely to be related to maximum potential yield for a particular set of treatments.

6.3.3 Cabbage yield and tissue K concentration as affected by spatial variability of soil K

The efficacy of a soil K test is usually measured in terms of its accuracy in predicting the responsiveness of a soil to applied K. The goal of such a test is to measure the quantity of plant-available K and therefore the test is negatively related to responsiveness and the amount of K required to make up the deficit. 1N NH_4OAc pH 7 is a widely used extractant for soil exchangeable K. Figures 6.9a, 6.9b, and 6.9c show the relationship between relative yield and exchangeable K for various coefficients of variation (CV) of soil K. The general form of the relationship was

$$Y = a + bx^{-1} \qquad (6.41)$$

where Y = relative yield, x = exchangeable K, and a and b are constants. It is seen that the relationship between relative yield and exchangeable K became weaker as the CV of soil K increased. It is further seen that the level of exchangeable K associated with 90% maximum relative yield increased with increasing CV: 0.28 cmol(+)/kg for CV = 44.12%, 0.31 cmol(+)/kg for CV = 66.98%, and 0.73

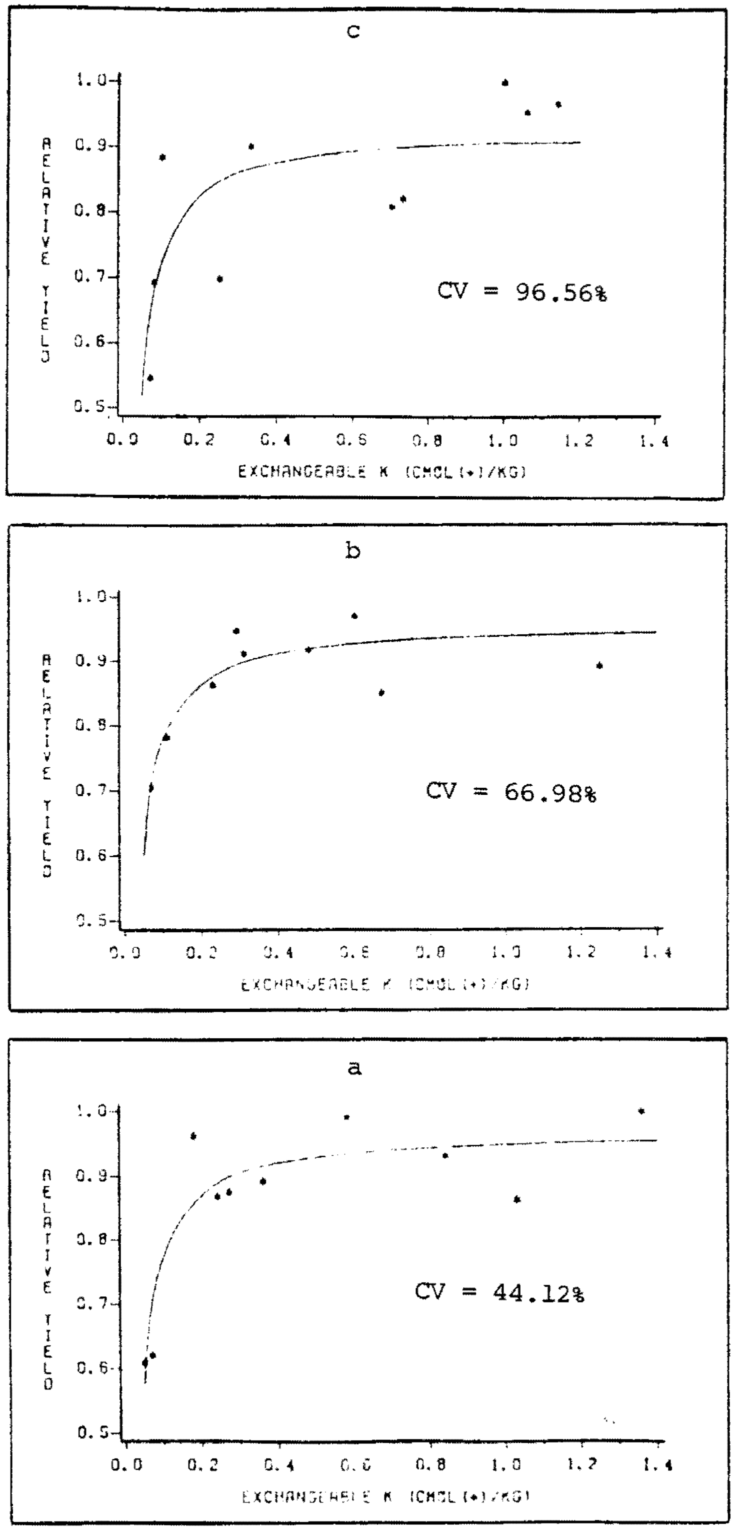


Fig. 6.9 Relationship between relative yield of cabbage and exchangeable K as affected by spatial variability of soil K.

cmol(+)/kg for CV = 96.56%. A critical level of 0.28 cmol(+)/kg is consistent with the range of 0.07-0.20 cmol(+)/kg suggested by Boyer (1972) for tropical agriculture. On the other hand a CV of exchangeable K of 44.12% is consistent with the values reported by Beckett and Webster (1971), and Courtin et al. (1983). However, Trangmar (1984) found a CV of exchangeable K of 105% in a study area of about 0.1 ha in Sitiung, Indonesia. He attributed this high heterogeneity to differences between burn sites, areas of exposed subsoil and intermediate areas of surrounding soil. The results obtained in this study suggest that an average soil K test value may be misleading if the spatial variability of soil K is not taken into consideration. The degree of spatial variability of soil K can have a profound effect on crop growth. Costigan and McBurney (1983) have found large differences in yields of cabbage and lettuce when grown on two similar sandy loam soils. They reported that the most likely explanation for the lower yield in one of the fields was that growth was limited by the heterogeneous spatial distribution of potassium.

Figures 6.10a, 6.10b, and 6.10c show the relationship between tissue K concentration in cabbage leaf and exchangeable K for various degrees of variability of soil K. The correlation coefficient between exchangeable K and percentage K in dry matter decreased with increasing

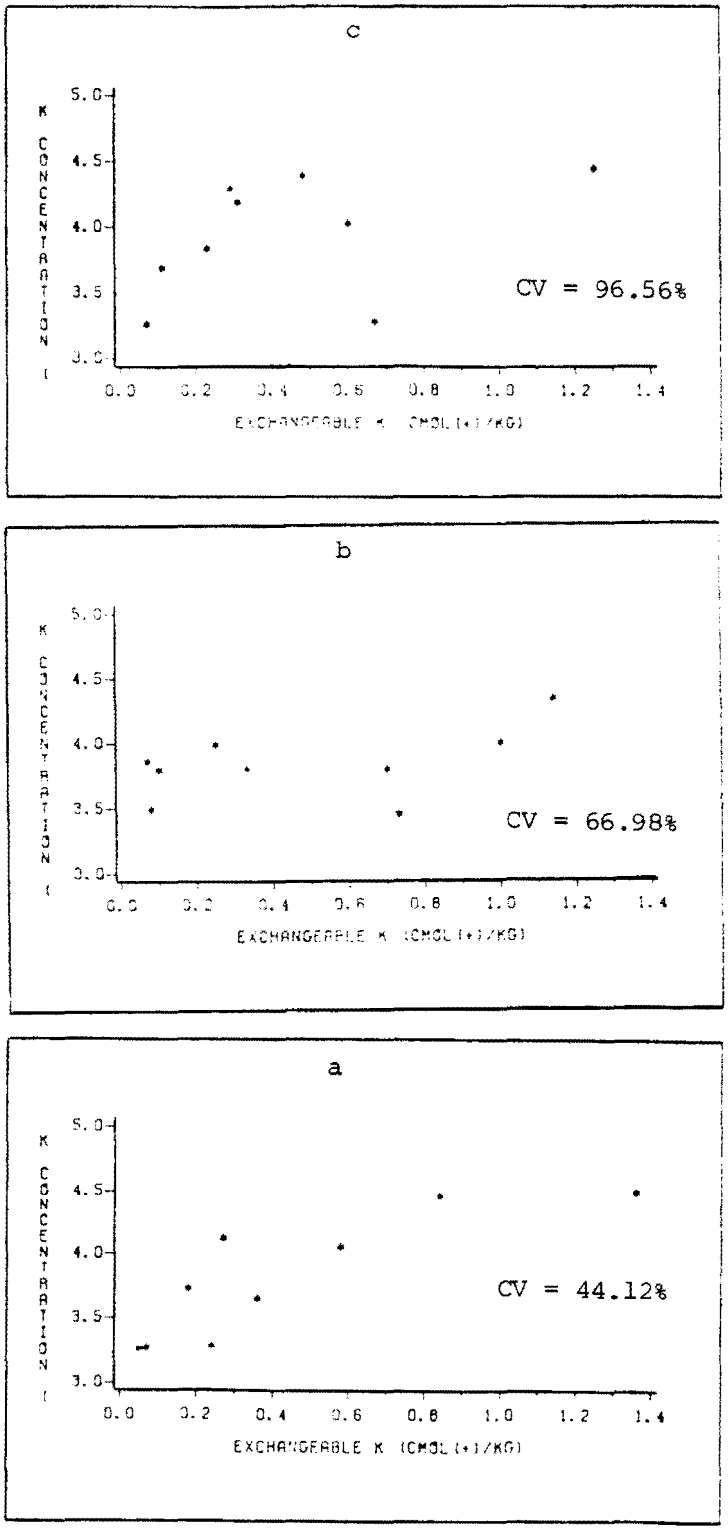


Fig. 6.10 Relationship between K in dry matter and exchangeable K as affected by spatial variability of soil K.

variance of soil K. For example, for CV values of 44.12 and 96.56% the values of the correlation coefficients were 0.854 and 0.585, respectively.

6.3.4 Cabbage yield and tissue K concentration as related to equilibrium activity ratio and potassium potential

The work of Beckett (1964) has shown that equilibrium soil activity ratios and the relationship between these ratios, (I) and the labile potassium of the soil, (Q), may be useful in studies of the potassium nutrition of plants. On the other hand, critical potassium potentials have been reported in the literature for a range of crops (Page and Talibudeen, 1982). In this study the relationship between cabbage performance, activity ratio, and energy of exchange of K with Ca + Mg was investigated for the case where UCF = 1.0. Fig. 6.11 shows the relationship between cabbage yield and equilibrium activity ratio. The relationship was described by a Mitscherlich type model (Equation 6.12) where Y denotes the yield at a given activity ratio (Q), A is the asymptotic maximum yield as Q approach infinity, and B and K are coefficients. The estimates of the parameters A, B, and K were 42.89, 18.86, and 301.05, respectively. The activity ratio associated with 95% maximum yield was calculated with Eq. 6.16, in which KR_{95} was replaced by AR_{95}. The calculated value of AR_{95} was 0.0072 $M^{1/2}L^{-1}$. The value of activity ratio associated with maximum yield

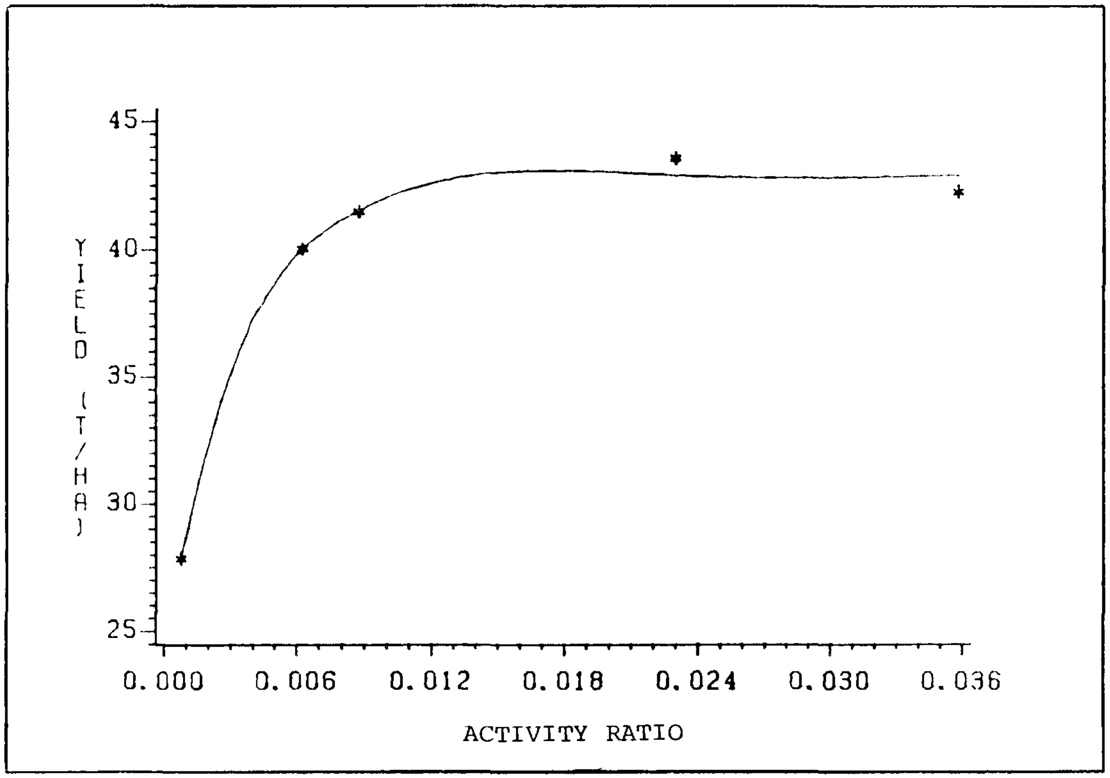


Fig. 6.11 Relationship between cabbage yield and potassium activity ratio.

found in this study is 4 times smaller than the critical value of 0.029 reported by Barrow (1969) for soils of low buffer capacity. A similar relationship was found between cabbage yield and soil K potential (Fig. 6.12). It is seen from Fig. 6.12 that maximum yield occurred at G = -2500 cal/equiv. A similar value was reported for a range of crops (Talibudeen and Page, 1978). Yield reductions at high K potentials may be caused by inhibition of other nutrients rather than toxic levels of K contents in the plant. The relationship between K potential, activity ratio and the concentration of other nutrients rather than toxic levels of K contents in the plant. The relationship between K potential, activity ratio and the concentration of other nutrients shown in Appendices 6.1 to 6.5 seem to support this assumption. The decrease in the Mg concentration in plant tissue brought about by the addition of K is consistent with the results reported by Hossner and Doll (1970) and Ologunde and Sorensen (1982). Potassium concentration in cabbage dry matter was almost linearly related to the activity ratio and potassium potential (Figures 6.13 and 6.14) suggesting the possibility of luxury consumption since yield reductions were observed at high activity ratios and potassium potentials.

6.4 Conclusions

1. The response of chinese cabbage to K application was strongly dependent on the pattern of fertilizer

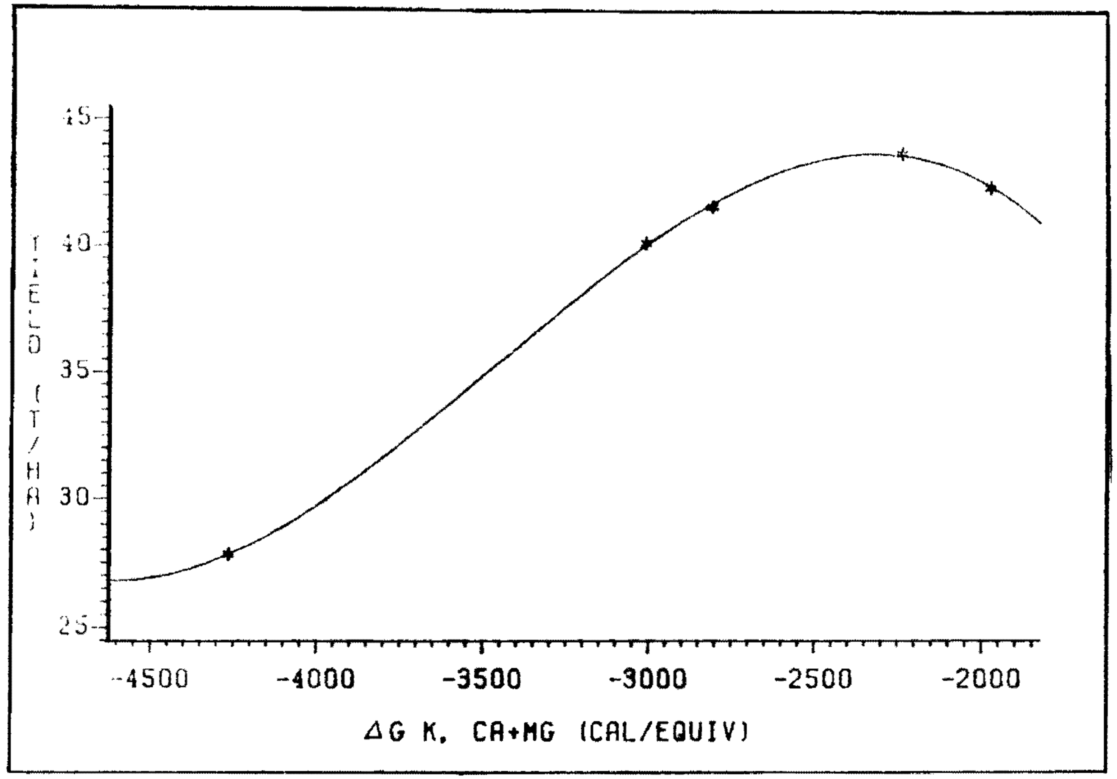


Fig. 6.12 Relationship between cabbage yield and potassium potential.

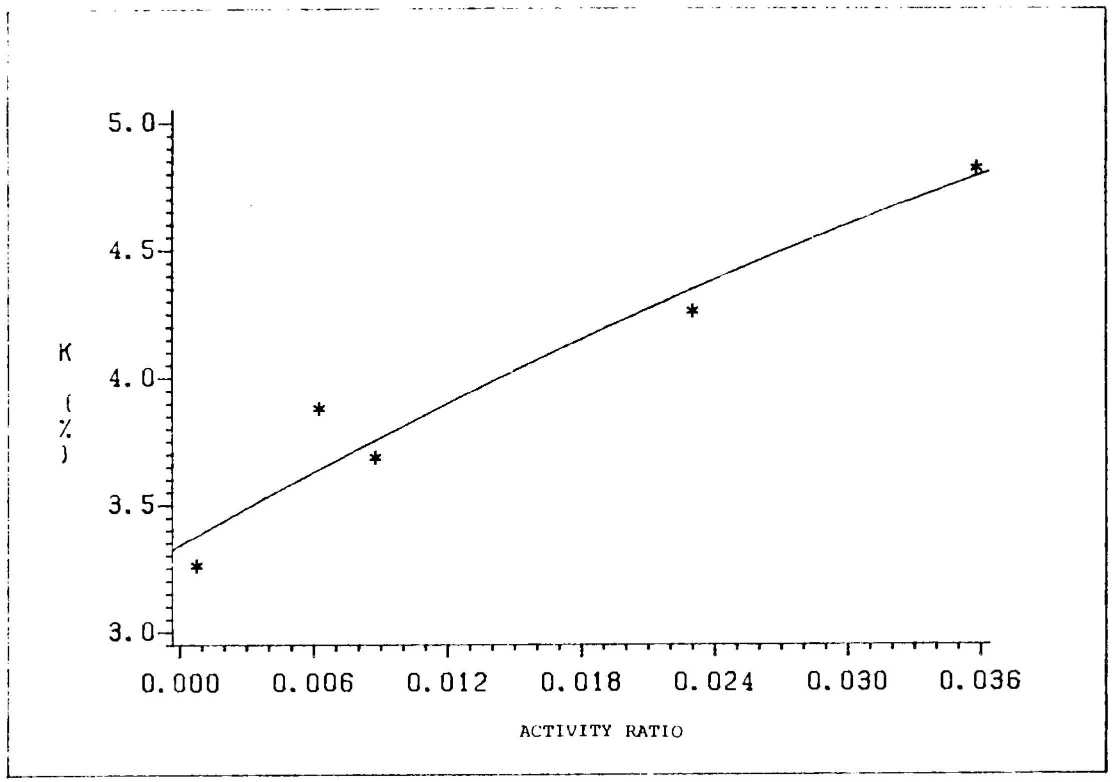


Fig. 6.13 Relationship between K in cabbage dry matter and activity ratio.

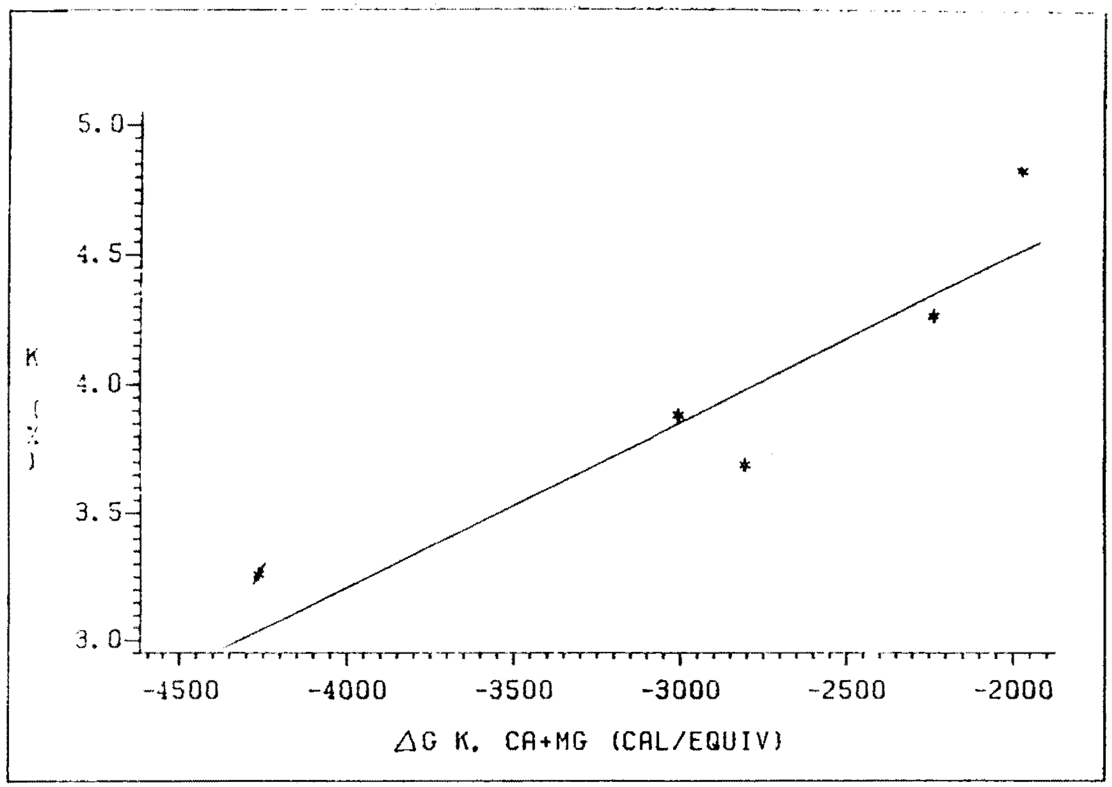


Fig. 6.14 Relationship between K in cabbage dry matter and potassium potential.

distribution. Different maximum yields were obtained under different uniformity conditions. With a uniformity coefficient of -0.02 the K requirement was four times that found under more uniform conditions.

2. The decrease in yield due to nonuniform application of fertilizer was quantitatively expressed through the Fluctuation Response Index. Its value decreased with increasing rates of K application suggesting that at higher rates of application the areal uniformity of fertilizer application reduced yield less than at low rates of application.

3. The impact of nonuniformity of fertilizer distribution will depend to a great extent on the shape and form of the yield-fertilizer production function.

4. The geostatistical approach used to determine rates of K fertilizer seems to be a promising technique if an appropriate critical level can be selected for a given crop and situation to determine the area below that threshold. Variable rates of fertilization could be applied if the deficient zones could be identified in the field.

5. The critical K concentration in plant tissues slightly increased from 4.0% for UCF=1.0 to 4.3% for UCF=-0.02.

6. The level of exchangeable K associated with 90% of maximum relative yield increased from 0.28 cmol(+)/kg for a

CV of 44.12% to 0.73 cmol(+)/kg for a CV of 96.56%. The results suggest that an average soil K test value may be misleading if the spatial variability of soil K is not taken into consideration.

7. Maximum cabbage yield occurred at G = -2500 cal/equiv. and at an activity ratio of 0.0072 $M^{1/2}$ L^{-1}.

VII. CORN RESPONSE TO SPATIAL VARIABILITY OF RESIDUAL POTASSIUM

7.1 Introduction

For years agricultural experimenters have used field experiments to try to understand variation in crop yield from place to place and under different systems of culture. Lack of uniformity in both the physical and chemical characteristics of the soil is one of the foremost factors causing variation in crop productivity. Even within the bounds of a small experiment soil variation may be sufficiently high to cause discordant and anomalous results (Balmunkand, 1928). Surface soils that are apparently uniform may be underlain with heterogeneous subsoil. With a sound experimental design differences in crop yield due to soil variation from place to place can be somewhat reduced for the purpose of a fertilizer trial. However, the task of eliminating or controlling these differences cannot be solved by statistical means alone. Further investigations are necessary in order to determine the cause of yield changes, and how the factors influence plant growth to cause these variations in yield. The previous treatment of the soil frequently results in variations which may not be apparent at the time a field trial is initiated. The initial pattern of K fertilizer distribution over the soil surface and the variability in soil chemical and physical properties which affect its retention by soils can have a profound effect on the spatial distribution of residual potassium. On the other hand, spatial variability of residual potassium can be expected to significantly affect crop performance.

The objective of this study was to investigate the effects of level and spatial variability of residual potassium on maize performance.

7.2 Materials and Methods

This study was conducted at the University of Hawaii Volcano Research Station on the island of Hawaii. A detailed description of the site, the soil, and the experimental design was given in chapter 6. The experiment was initiated after a cabbage crop. After cabbage harvest a 1 m x 1 m moving grid was superimposed over each plot. Soil samples were taken at the 0-15 cm depth from the centroid of each cell with an auger of 7.5 cm diameter. Soil samples were also taken from the 15-30, and 30-45 cm depths from selected plots. Soil samples were kept in sealed plastic bags until laboratory determinations were performed. Each 8 m x 4 m experimental plot received a blanket application of 120 kg N/ha as ammonium sulfate, 175 kg P/ha as treble superphosphate, 10 kg Cu/ha as copper sulfate, 15 kg B/ha as borax, and 15 kg Zn/ha as zinc sulfate. Fertilizers were broadcast by hand and thoroughly mixed into the soil with a rototiller. Maize (*Zea mays* L.), Waimea Dent variety, was planted at a spacing of 80 cm x 25 cm. At 50% silking corn ear leaf samples were collected and analyzed for nutrient contents. 1N NH_4OAc pH 7 extractable potassium, Quantity-Intensity parameters, and energy of exchange of K with Ca + Mg were determined as described in chapter 6.

7.3 Results and Discussion

7.3.1 Corn yield as affected by spatial variability of residual potassium

Fig. 7.1 shows the relationship between corn yield and the amount of K applied prior to the previous crop. There were significant increases in yield with increasing amount of K applied for the three uniformity coefficients determined in chapter 6. Corn yield, however, was higher with a uniformity coefficient of 1.0 than with UCF values of 0.42 and -0.02. A plot of corn yield against rates of K fertilizer applied to previous crop (cabbage), however, may be misleading because it does not reflect the level of soil K actually available prior to planting corn. In addition to the potassium taken up by the previous crop (cabbage) relatively large amounts of K were moved into deeper layers. For example, after the first crop the levels of exchangeable K in the control treatment were 0.05, 0.06, and 0.07 cmol(+)/kg for the 0-15, 15-30, and 30-45 cm depths, respectively. The corresponding values in the highest treatment with UCF=1.0 were 0.15, 0.31, and 0.32 cmol(+)/kg. Fig. 7.2 shows the relationship between corn

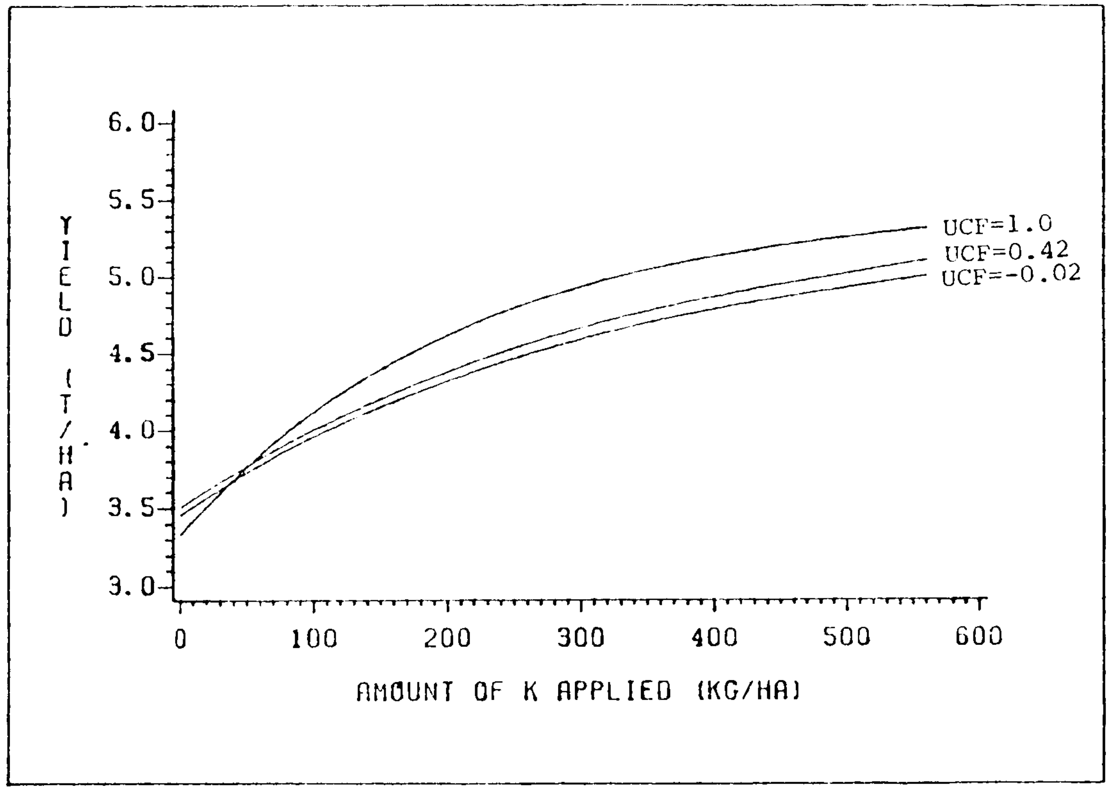


Fig. 7.1 Residual effect of previously applied K fertilizer as measured by corn grain yield.

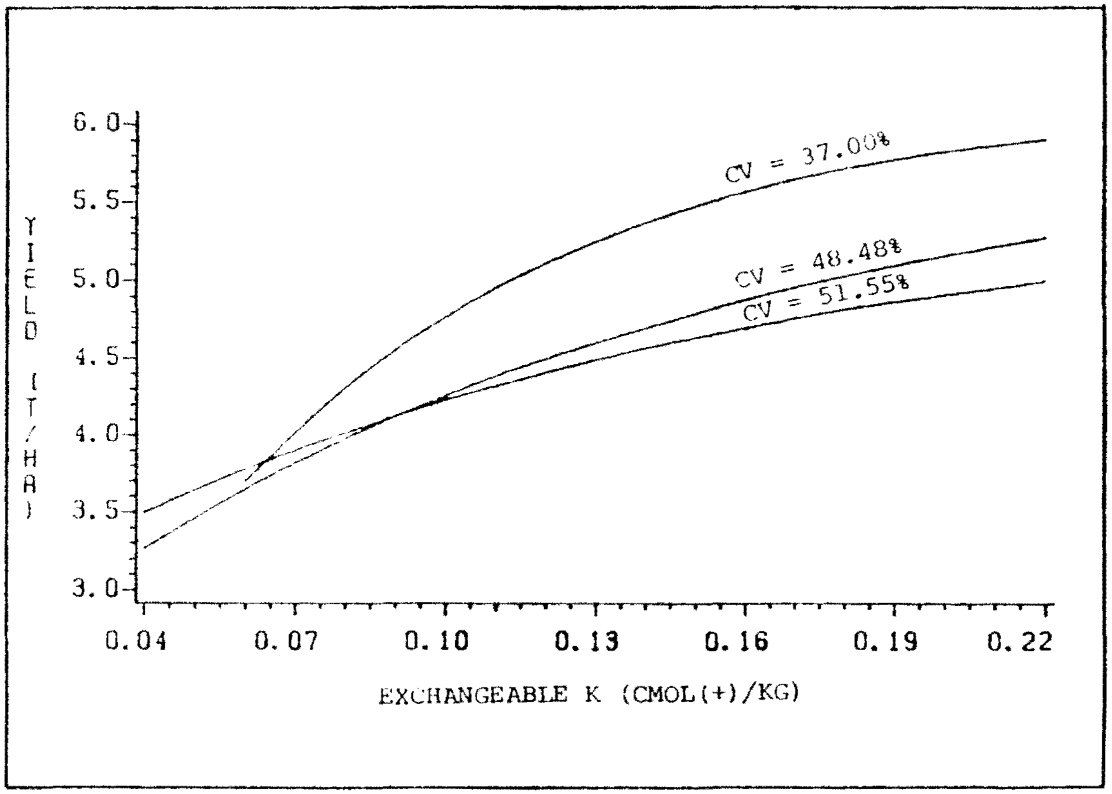


Fig. 7.2 Relationship between corn grain yield and residual K as measured by exchangeable K.

yield and residual exchangeable K. It is seen that corn grain yield increased with increasing levels of exchangeable K. It is further seen that yield decreased with increasing variability of residual K. The coefficients of variation (CV) in Figure 7.2 represent average values of each response function. To quantify the yield-exchangeable K relationship the data were fitted to a Mitscherlich type model.

$$Y_Q = A[1 - \exp(-b(Q - Q_0))] \qquad (7.1)$$

in which A = maximum possible yield, Q_0 is the value of exchangeable K (Q) when Y approaches 0, and b is an empirical constant.

The following yield functions were found for each average CV.

$$Y = 6.17[1 - \exp(-14.10(Q + 0.005))] \text{ for CV=37.00\%} \qquad (7.2)$$

$$Y = 5.97[1 - \exp(- 7.56(Q + 0.065))] \text{ for CV=48.48\%} \qquad (7.3)$$

$$Y = 5.52[1 - \exp(- 7.54(Q + 0.093))] \text{ for CV=51.55\%} \qquad (7.4)$$

The level of exchangeable K associated with 90% maximum yield can be calculated from Eq. 7.5

$$KR_{90} = \ln 0.1/-b + Q_0 \qquad (7.5)$$

The following results were obtained.

$$KR_{90} = 0.16 \text{ cmol(+)/kg for CV = 37.00\%}$$

KR_{90} = 0.24 cmol(+)/kg for CV = 48.48%

KR_{90} = 0.21 cmol(+)/kg for CV = 51.55%

The value of 0.16 cmol(+)/kg is consistent with the results reported in the literature. According to Agboola and Corey (1976) the critical threshold value of exchangeable K in the soil below which maize response to applied K may be expected in the humid tropics is about 0.16 cmol(+)/kg. On the other hand Boyer (1976) concluded that for tropical agriculture the absolute minimum level of exchangeable K was close to 0.10 cmol(+)/kg, but varied from 0.07 to 0.24 cmol(+)/kg depending on the kinds of soils and plants involved. The results obtained in this study suggest that the critical level of exchangeable K for a given crop may be significantly affected by the degree of spatial variability of soil K.

The theoretical model developed by Zaslavsky and Mokady (1967) and which was described in chapter 6 could be used in situations where information on exchangeable K and its spatial variability is available. This, however, will require that the yield-exchangeable K function be known for a particular situation. The data from the entire experiment were pooled and fitted to Eq. 7.1. The resulting yield-exchangeable K function was

$$Y(Q) = 5.28[1 - \exp(-15.27(Q + 0.022))] \quad (7.6)$$

According to Zaslavsky and Mokady (1967) and neglecting generic factors, crop yield depends on both the spatial average of Q ($\overline{Q}$), e.g., exchangeable K, and the spatial deviations from that average. Expanding the Y(Q) function (Equation 7.6) about $Q = \overline{Q}$ by Taylor series and neglecting third-order and higher terms the yield function can be written as follows:

$$\overline{Y} = Yu(Q) + 1/2(\partial^2 Y/\partial Q^2)S^2{}_q \qquad (7.7)$$

where $\overline{Y}$ is the spatial average crop yield, Yu(Q) is the yield that would have been obtained with perfectly uniform level of $Q = \overline{Q}$, (no fluctuations in Q), and $S^2{}_q$ is the spatial variance of Q. As long as the Y(Q) relationship is convex, i.e. $\partial^2 Y/\partial Q^2 < 0$, spatial variability in the soil properties, e.g., exchangeable K (expressed through $S^2{}_q$), will cause a reduction in the spatial average crop yield relative to Yu. On the other hand, when $\partial^2 Y/\partial Q^2 > 0$, spatial variability in exchangeable K will cause a higher average crop yield as compared with Yu. In Equation 7.7 Yu(Q) is the same as Y(Q) in Equation 7.6. Therefore Eq. 7.7 becomes

$$\begin{aligned}\overline{Y} = {} & 5.28[1-\exp(-15.27(Q + 0.022))] + \\ & 1/2[(-1231.14)\exp(-15.27(Q+0.022))][S_q{}^2] \qquad (7.8)\end{aligned}$$

where $S_q{}^2$ is the variance of exchangeable K. The variance of exchangeable K could be used to define a uniformity coefficient as in Eq. 6.11. In calculating the variance of exchangeable K the scale of observation was the soil core taken with an auger of 7.5 cm diameter. The standard deviation characterizes the total variability of the measurements. However, individual corn root-zones can be expected to have an extent much larger than the scale of observation (7.5 cm) and this must be taken into consideration in calculating a uniformity coefficient for exchangeable K. Cogels (1983) has presented a modification of Eq. 6.11 by introducing a nondimensional correction function r(Si) as follows:

$$UCK_{Si} = 1 - S/\overline{Q}\ r(Si) \qquad (7.9)$$

with the condition that r(Si) varies from 1 to 0 and where Si is the scale of influence of individual plant root zones and UCK is the effective uniformity coefficient of exchangeable K. Since the scale of influence (Si), i.e., the diameter of the horizontal projection of individual plant root-zones, is larger than the scale of observation (So) the total variability characterized by the standard deviation may be divided into two components. One is the within root-zones variability and the other the between root-zones variability. The latter is the effective variability. Cogels (1983) has presented a method for estimating the correction factor r(Si) for a systematic sampling. His method was used to calculate the effective uniformity coefficient of exchangeable K for each experimental plot. Fig. 7.3 shows the grid of observation points in each plot. This grid is determined by a number of lines Nl, a number of rows Nr and a constant grid spacing d. A total of 7 units were defined whose sides were a multiple of the spacing interval (kd) with k = 1. The number of units Nu(k) could be calculated as follows:

$$Nu(k) = (Nl - k)(Nr - k) \qquad (7.10)$$

for $1 \leq k \leq$ smallest of Nl and Nr

The number of observation points No(k) belonging to each unit is

$$No(k) = (k + 1)^2 \qquad (7.11)$$

the within unit variability component can be calculated as follows:

$$\hat{w}^2_{(k)} = 1/Nu(k).No(k) \sum_{i}^{Nu(k)} \sum_{j}^{No(k)} (Xij - \overline{X}i.)^2 \qquad (7.12)$$

with

$$\overline{X}i. = 1/No(k) \sum_{j}^{No(k)} Xij \qquad (7.13)$$

Xij is the measurement j of unit i.

The parameter $\hat{w}^2_{(k)}$ is believed to include measurements errors i.e., within unit variability. The between unit

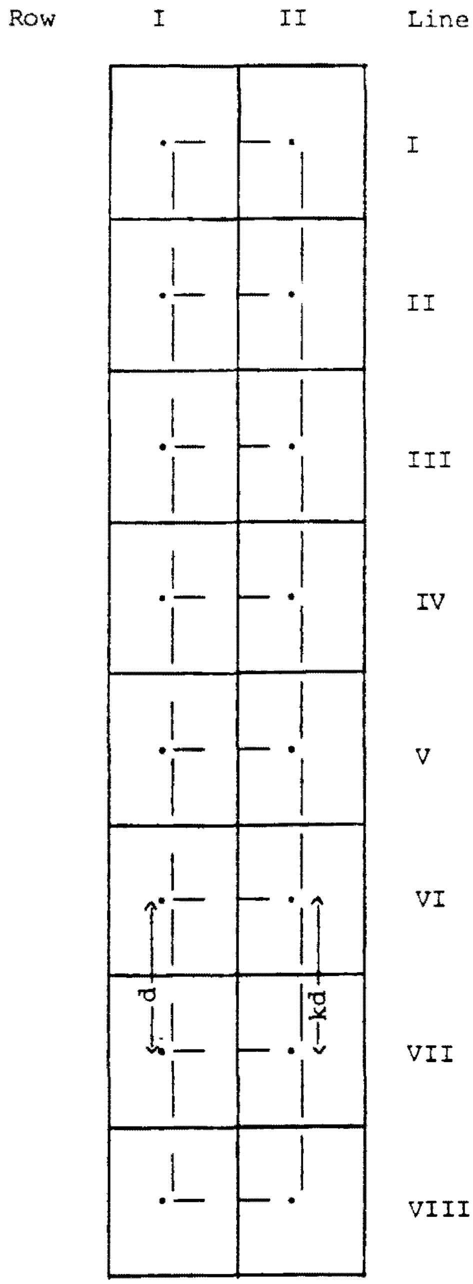


Fig. 7.3 Arrangement of soil samples in a matrix Xij for a grid of Nl = 8 lines and Nr = 2 rows and a k value of 1.

variability component is characterized by a parameter $\hat{b}^2_{(2)}$ estimated by

$$\hat{b}^2_{(k)} = 1/Nu(k) \sum_{i}^{Nu(k)} (\overline{X}i. - \overline{X}..)^2 \qquad (7.14)$$

with

$$\overline{X}.. = 1/Nu(k) \sum_{i}^{Nu(k)} \overline{Xi}. \qquad (7.15)$$

The total variability is estimated by

$$\hat{t}^2_{(k)} = 1/Nu(k).No(k) \sum_{i}^{Nu(k)} \sum_{j}^{No(k)} (Xij - \overline{X}..)^2 \qquad (7.16)$$

On the other hand

$$\hat{t}^2_{(k)} = \hat{w}^2_{(k)} + \hat{b}^2_{(k)} \qquad (7.17)$$

The nondimensional correction function r(Si) is estimated by what Cogels (1983) called the scalogram i.e., the ratio between $b_{(k)}$ and $t_{(k)}$:

$$r(Si) = \hat{b}_{(k)}/\hat{t}_{(k)} \qquad (7.18)$$

with

$$Si = So + kd \qquad (7.19)$$

Rearranging Eq. 7.9 and substituting for S^2 in Eq. 7.8 yields

$$Y = 5.28[1-\exp(-15.27(Q + 0.022))] + 1/2[(-1231.14)\exp(-15.27(Q + 0.022))] [Q^2(1 - UCK)^2/((r(si))^2] \quad (7.20)$$

Eq. 7.20 was used to predict corn grain yield using the observed effective uniformity coefficients and levels of exchangeable K in each experimental plot. Fig. 7.4 shows the relationship between predicted and observed yield. A highly significant correlation coefficient of 0.848 was obtained between observed and predicted yield. Since absolute crop yield will depend on factors other than level of exchangeable K it is convenient to express yield in relative terms. Using Eq. 7.20 theoretical curves of relative yield Yr (Yr = Y/A, with A = 5.28) as a function of effective uniformity for three levels of exchangeable K were drawn in Fig. 7.5. It will be seen from Fig. 7.5 that the decrease in relative yield with decrease in UCK was much steeper for lower levels of exchangeable K. It is further seen that when the level of exchangeable K is high the uniformity coefficient can be decreased without causing much loss in yield. For example, a decrease in UCK from 1.0 to 0.5 is predicted to result in an 18% decrease in yield for a level of exchangeable K of 0.10 cmol(+)/kg, but only a 1% decrease in yield is expected for 0.5 cmol(+)/kg. The decrease in yield due to the variability of soil K is readily seen in Fig. 7.6 where the Fluctuation Response

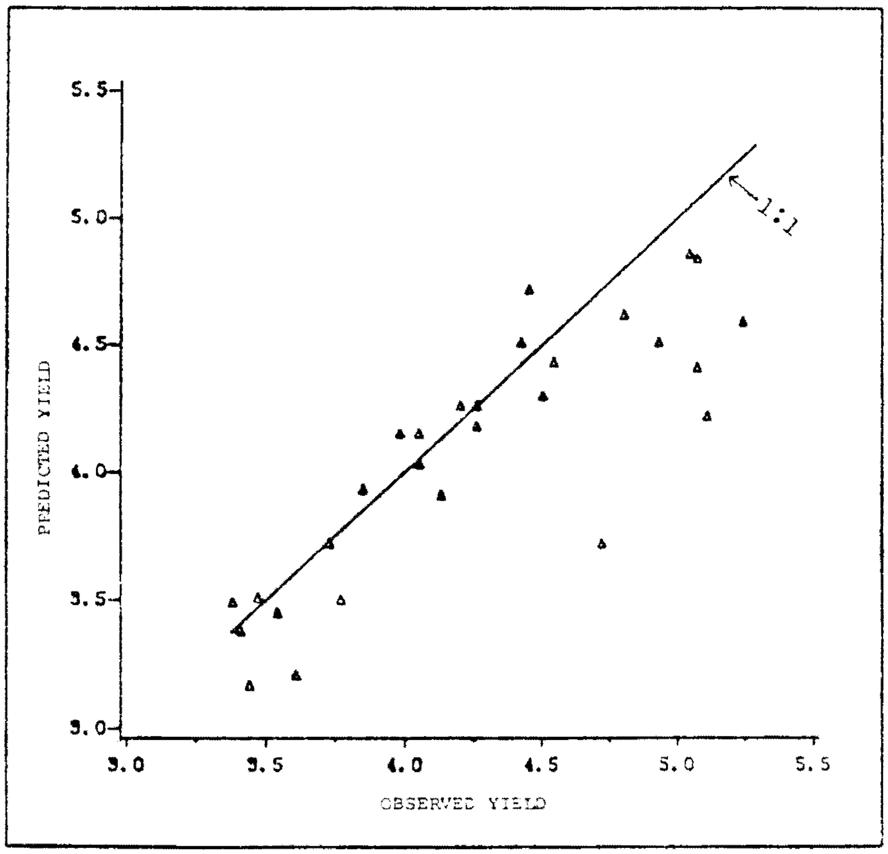


Fig. 7.4 Comparison of predicted and observed corn grain yield.

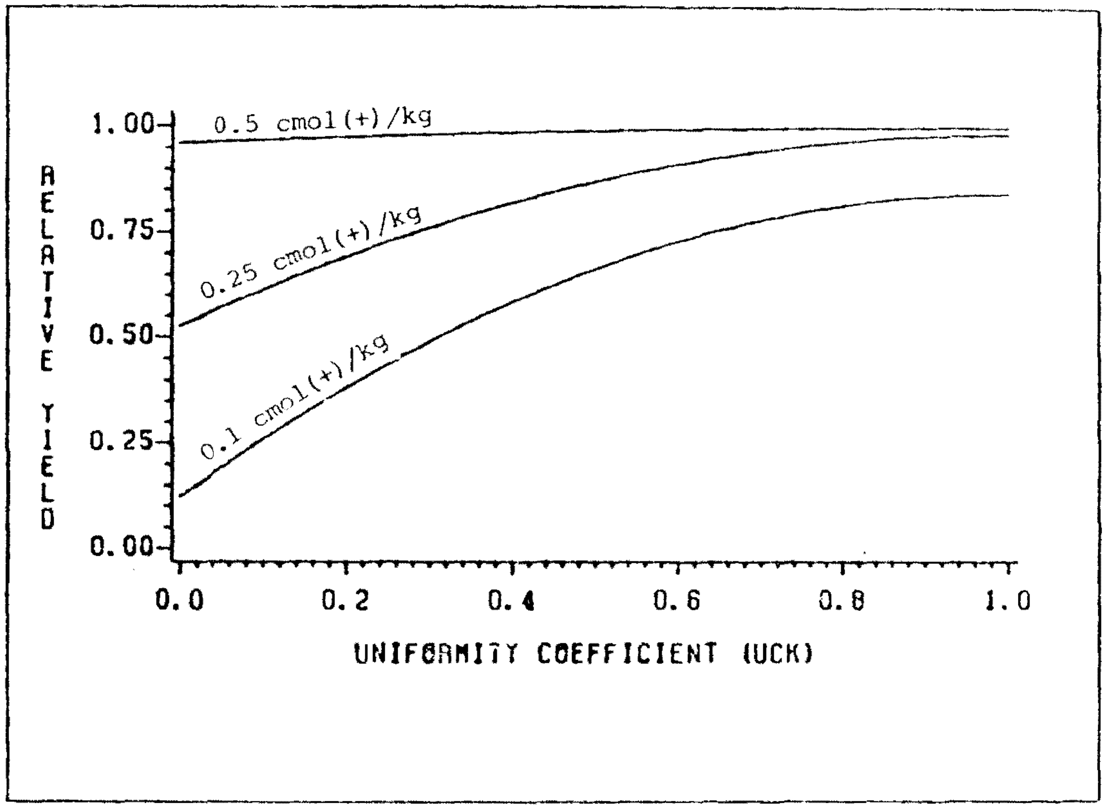


Fig. 7.5 Calculated relative yield as a function of uniformity coefficient of soil K for 3 levels of exchangeable K.

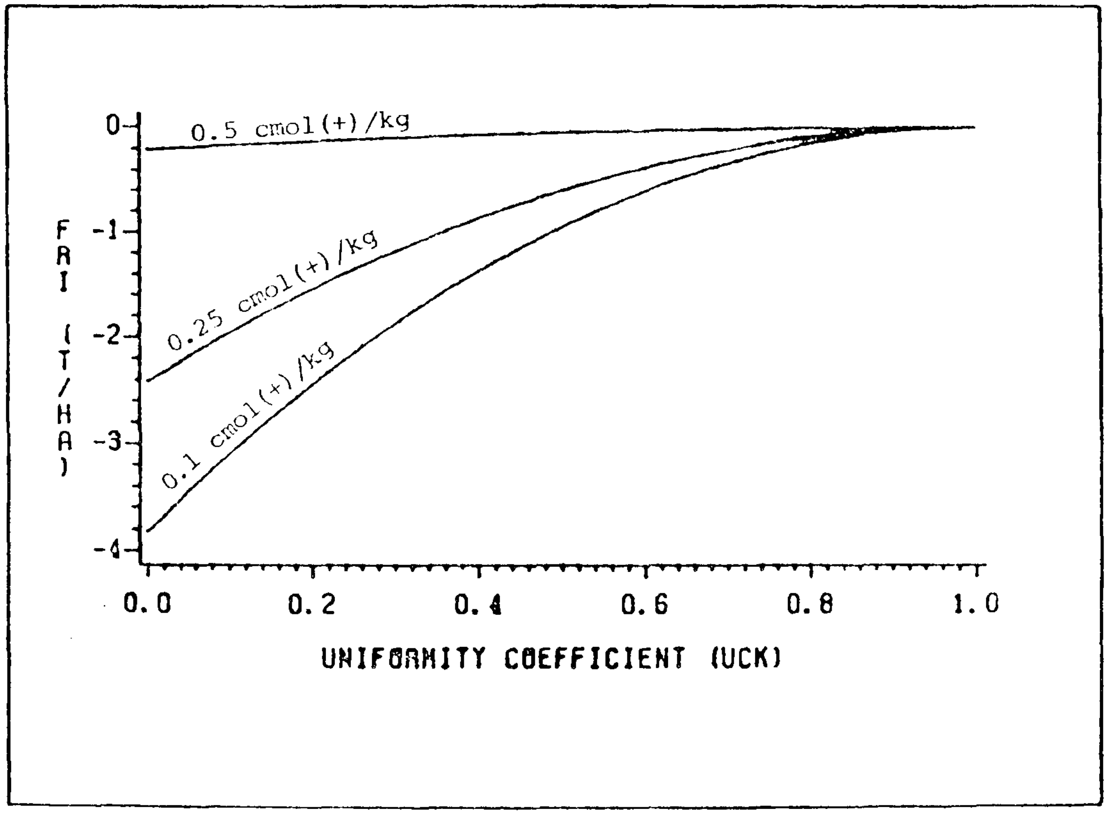


Fig. 7.6 Calculated fluctuation response index (FRI) as a function of uniformity coefficient of soil K for 3 levels of exchangeable K.

Index (FRI) quantitatively expresses the change in yield due to nonuniform distribution of soil K. These results suggest that different yields are likely to be obtained with the same level of exchangeable K but with different magnitude of spatial variability of soil K.

7.3.2 Yield and ear-leaf K in relation to the spatial variability of soil K

Figures 7.7a, 7.7b, and 7.7c show the relationship between exchangeable K and K concentration in corn ear leaves at silking time for different coefficients of variation (CV) of soil K. The percent K in corn leaves was highly correlated with the exchangeable K content of the field-moist soil samples. Ear-leaf K increased with increasing level of exchangeable K. It is further seen that the shape of curve relating K concentration to exchangeable K was significantly affected by the magnitude of spatial variability of soil K as characterized by the coefficient of variation. For CV values of 37.0% and 48.44% the relationship between percent K and exchangeable K was curvilinear. Similar results were reported by Hanway et al. (1962). At a CV value larger than 50% the curvilinear nature of the relationship was less obvious. The results of this study show that serious consideration should be given to the magnitude of spatial variability of soil K in studies where the amount of K in the leaf is

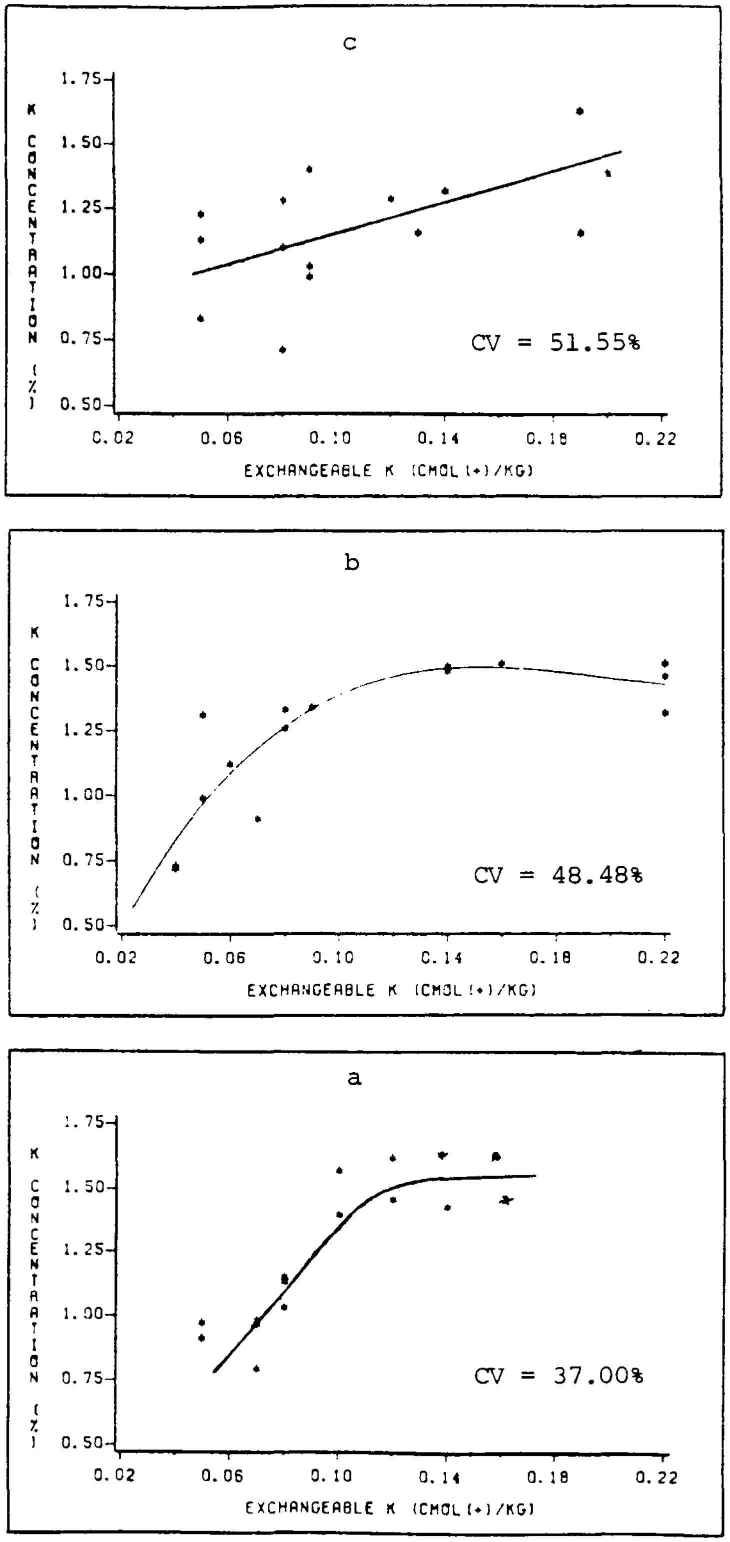


Fig. 7.7 Relationship between exchangeable K in field-moist soil samples and percent K in corn leaves at silking time as affected by spatial variability of soil K.

considered as a criterion for correlation purposes with potassium soil test values.

The relationship between corn grain yield and ear-leaf K is depicted in Figures 7.8a, 7.8b, and 7.8c for three CV values of exchangeable K. Corn yield increased with increasing concentration of K in ear leaves. The relationship was linear within the range of experimental data. This seems to be a departure from the curvilinear relationship between yield and leaf K reported in the literature (Loue, 1963). However, the range of K concentration over which the curvilinear relationship was observed was 0.4-2.4% which is much wider than the one found in this study. It also will be seen from Figures 7.8a, 7.8b, and 7.8c that the scattering of the points along the curve relating corn yield to ear-leaf K increased with increasing CV of exchangeable K. Loue (1963) has suggested that rather than referring to a critical level it would be better to refer to a critical zone which he determined to be between 1.7 to 2.0% K. Although many factors can affect the yield-K concentration relationship, the results of this study suggest that spatial variability of soil K can have a significant effect on the predictable functional relationship between crop yield and nutrient concentration in plant tissues on which the concept of critical nutrient concentrations in plant tissues is dependent (Blamey and Mould, 1979). The relationships

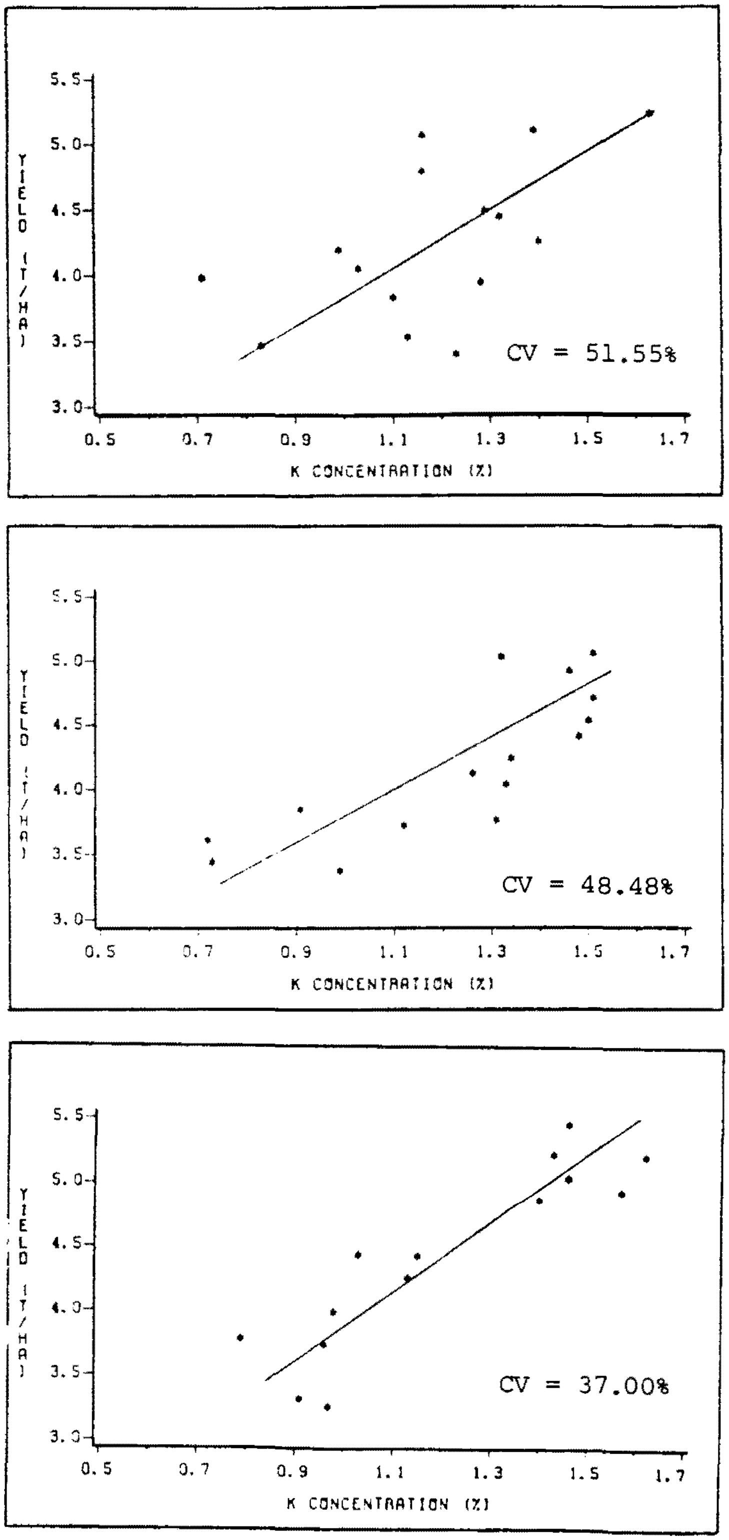


Fig. 7.8 Relationship between corn yield and ear-leaf K as affected by spatial variability of soil K.

between nutrient concentrations in corn ear leaves, yield and exchangeable K are summarized in the form of correlation coefficients in Table 7.1. For K, Ca, Mg, and S there was a gradual decrease in the correlation coefficient between these parameters, yield, and exchangeable K with increasing variability of soil K.

7.3.3 Leaf K and yield as related to equilibrium activity ratio and potassium potential

Study of the relationship between yield, ear-leaf K, and activity ratio and potassium potential is of importance from the standpoint of evaluating the Q/I technique and the concept of potassium potential for defining potassium availability to corn on a field basis for the Puaulu soil. The relationship between corn yield and activity ratio is depicted in Fig. 7.9. The relationship was described by a Mitscherlich type model

$$Y = A(1 - B\, e^{-kx}) \qquad (7.21)$$

where Y denotes the yield at a given activity ratio (x), A is the asymptotic maximum yield as x approaches infinity, B and k are coefficients [B = $(A - y_0)/A$ where y_0 is the yield at activity ratio of zero. The activity ratio associated with maximum yield can be calculated from Eq. 7.22

$$AR_{90} = (\ln B - \ln 0.1)/k \qquad (7.22)$$

Table 7.1 Linear correlation coefficients for the relationship between yield, exchangeable K and nutrient concentrations in corn ear leaves at 50% silking as affected by variability of residual K.

Nutrient	Yield			exchangeable K		
	CV of exchangeable K			CV of exchangeable K		
	37.00	48.48	51.55	37.00	48.48	51.55
N	0.28	0.42	-0.54	0.12	0.38	-0.44
P	-0.46	-0.11	-0.16	-0.49	-0.17	-0.23
K	0.88	0.80	0.56	0.84	0.72	0.57
Ca	-0.79	-0.70	-0.67	-0.70	-0.61	-0.62
Mg	-0.92	-0.76	-0.61	-0.86	-0.71	-0.63
S	-0.74	-0.73	-0.62	-0.75	-0.68	-0.63

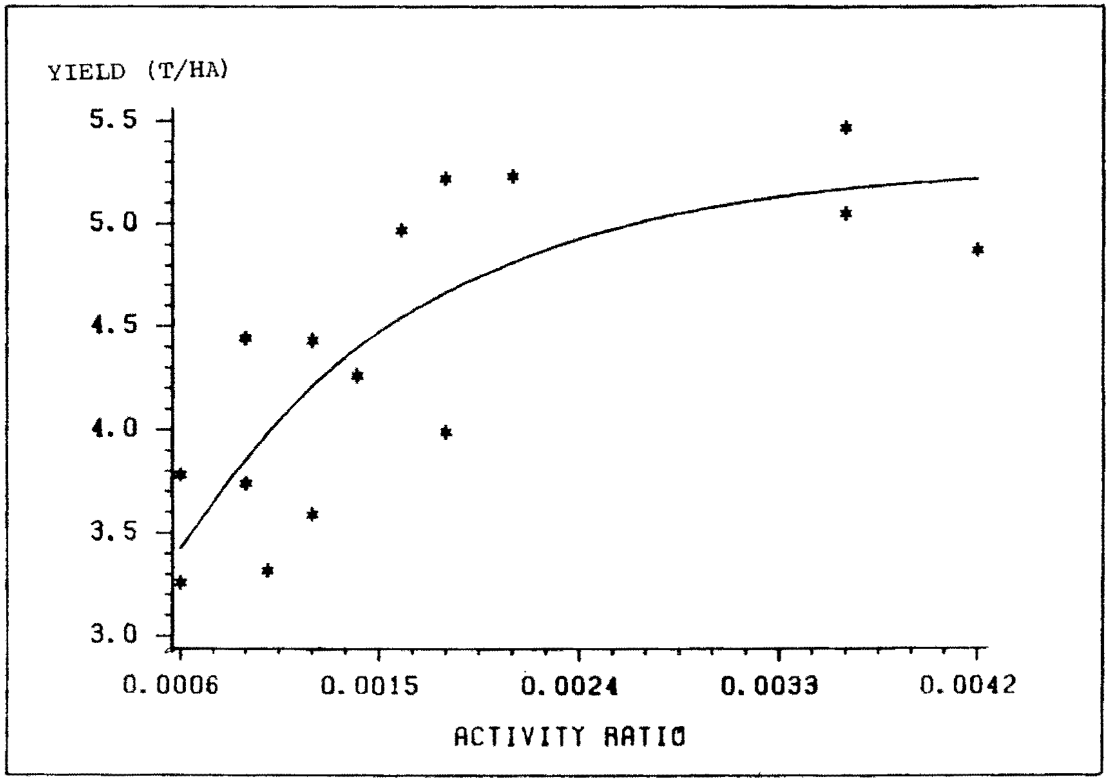


Fig. 7.9 Relationship between corn grain yield and activity ratio.

The estimates of the parameters k, A, and B were 910.18, 5.29, and 0.608, respectively. The calculated value of AR_{90} was 0.0020 $(M)^{1/2}L^{-1}$. A similar relationship was found between ear-leaf K and activity ratio (Appendix 7.1).

Corn yield was also related to the energy of exchange of K with Ca + Mg (Fig. 7.10). Maximum yield occurred at DG = -3400 cal/equiv. Ear-leaf K was similarly related to potassium potential (Appendix 7.2).

7.4 Conclusions

1. The response of maize to residual K was strongly related to the degree of its spatial variability. Corn yield decreased with increasing variation in soil K. The level of exchangeable K associated with 90% maximum yield increased from 0.16 cmol(+)/kg to 0.24 cmol(+)/kg for CV of 37.00 and 48.48%, respectively.

2. The decrease in yield due to increased variability of residual K was quantitatively expressed through the fluctuation response index (FRI). Its values indicated that at high levels of exchangeable K the decrease in yield due to spatial variation in soil K became less.

3. The functional relationship between corn yield and ear-leaf K was significantly affected by the variability of residual K. There was a gradual decrease in the correlation coefficient between yield and ear-leaf K with increasing variance of residual soil K.

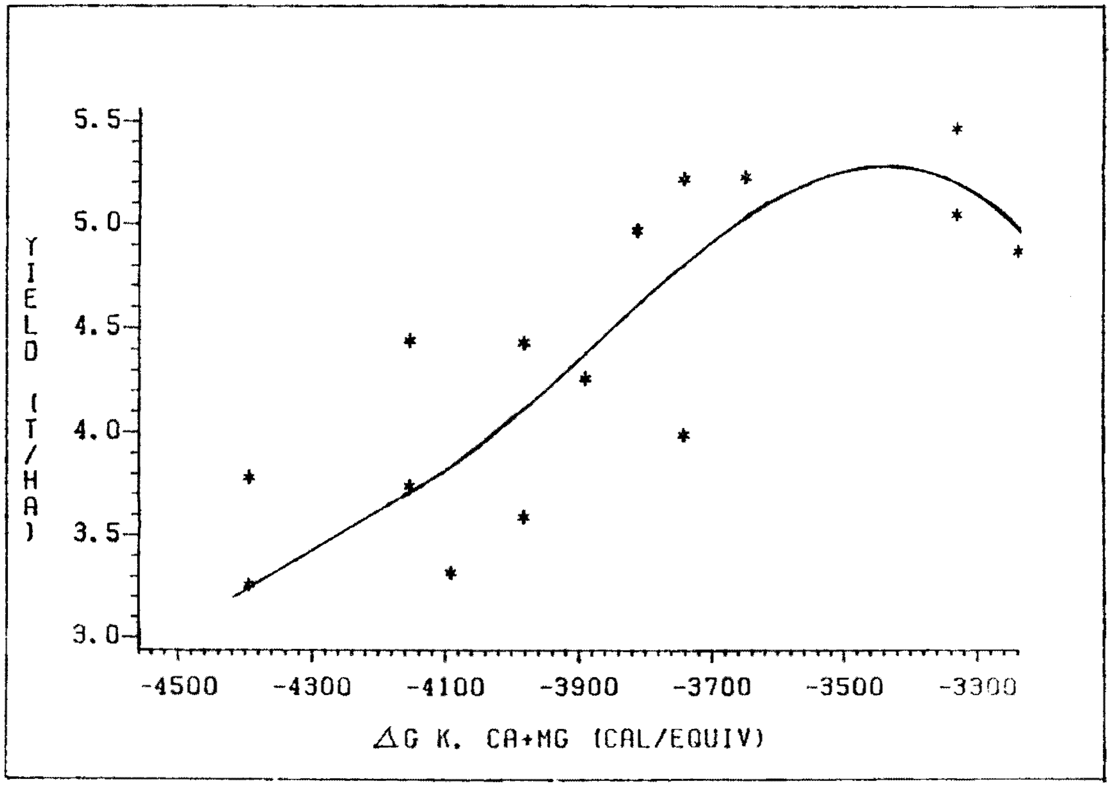


Fig. 7.10 Relationship between corn grain yield and potassium potential.

4. The correlation coefficient between exchangeable K and ear-leaf K also decreased with increasing variability of soil K.

5. The activity ratio associated with 90% maximum yield was 0.0020 $(M)^{1/2}L^{-1}$ for a CV of soil K of about 37.00%.

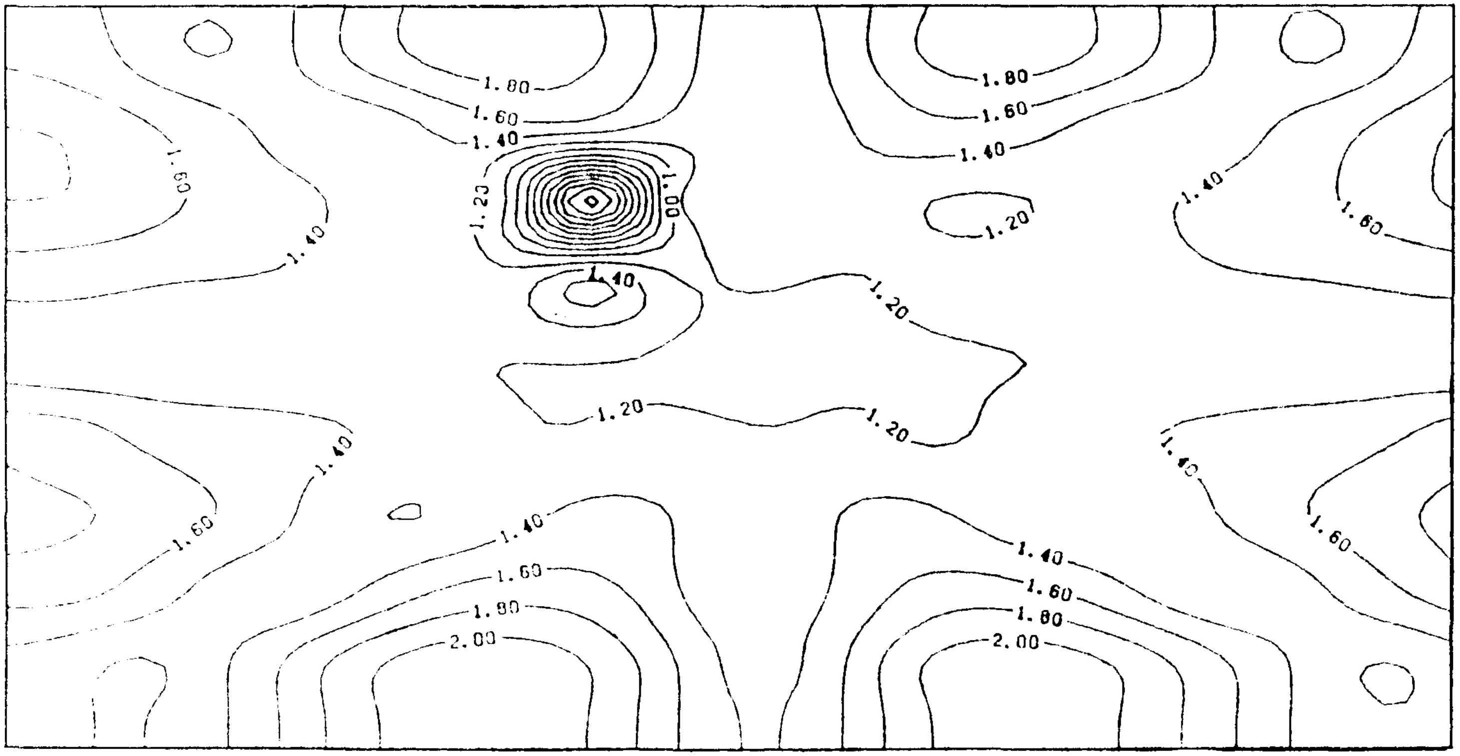
VIII. SUMMARY

The potassium retention capacity of the Puaulu soil was investigated. The results indicated that the Puaulu soil has a low affinity for K as evidenced by the very low Gapon selectivity coefficient. More potassium was retained in the subsoil than in the topsoil. The amount of K adsorbed (Kf) increased with Ep (the ratio of adsorbed K to adsorbed Ca and Mg) and with PAR ($PAR = [K]/([Ca] + [Mg])^{1/2}$, where brackets denote solution concentration in mmol/L).

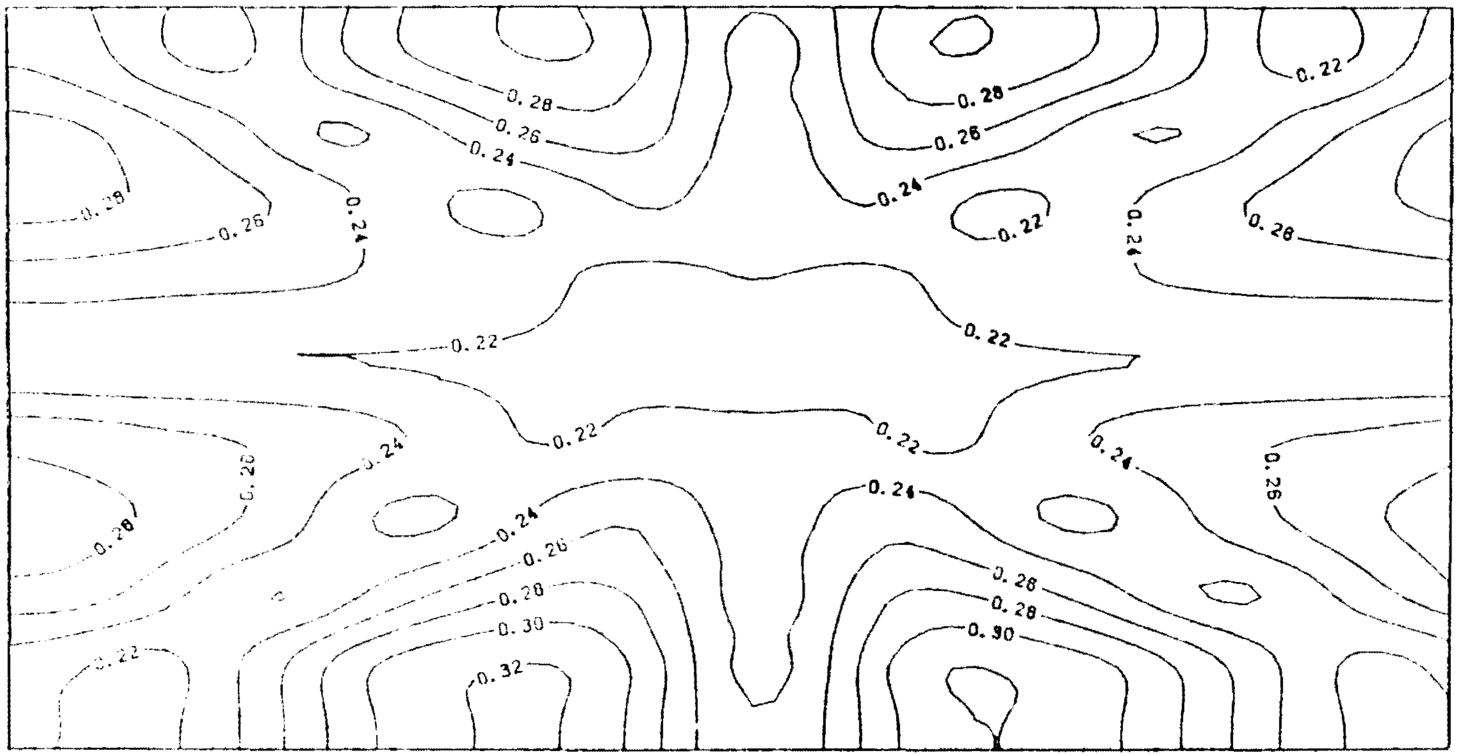
Conventional statistical analysis and geostatistical analysis were used to investigate the field spatial variability of selected soil properties. Comparison of coefficients of variation (CV) indicated that exchangeable K was less variable and exchangeable Na the most variable. Isotropic semi-variograms were calculated to identify spatial dependence of exchangeable Ca, Mg, K, and Na. They revealed the presence of strong spatial dependence but also of short-range variations that occurred at a distance less than the sampling interval (2 m). The semi-variograms were used to estimate exchangeable Ca, Mg, K, and Na at unsampled locations using both punctual and block kriging. The structure of spatial dependence of exchangeable K was also used to estimate the proportion of each experimental plot below a specified threshold of soil K and fertilizer rates.

The effect of uneven distribution of K fertilizer on cabbage response was investigated. The results indicated that different maximum yields of Chinese cabbage were likely to be obtained depending on the pattern of fertilizer distribution. The amount of potassium required to attain 95% of maximum yield increased with increasing variance of fertilizer application. An analytical approach showed that the fluctuation response index (FRI) i.e., the product of the variance of fertilizer application and the second derivative of the yield-fertilizer function obtained under uniform application could be used to predict yield losses due to nonuniform fertilizer application. With low rates of fertilizer application it is important to distribute the fertilizer uniformly otherwise there may be appreciable yield loss. The critical level of exchangeable K increased with increased spatial variability of soil K.

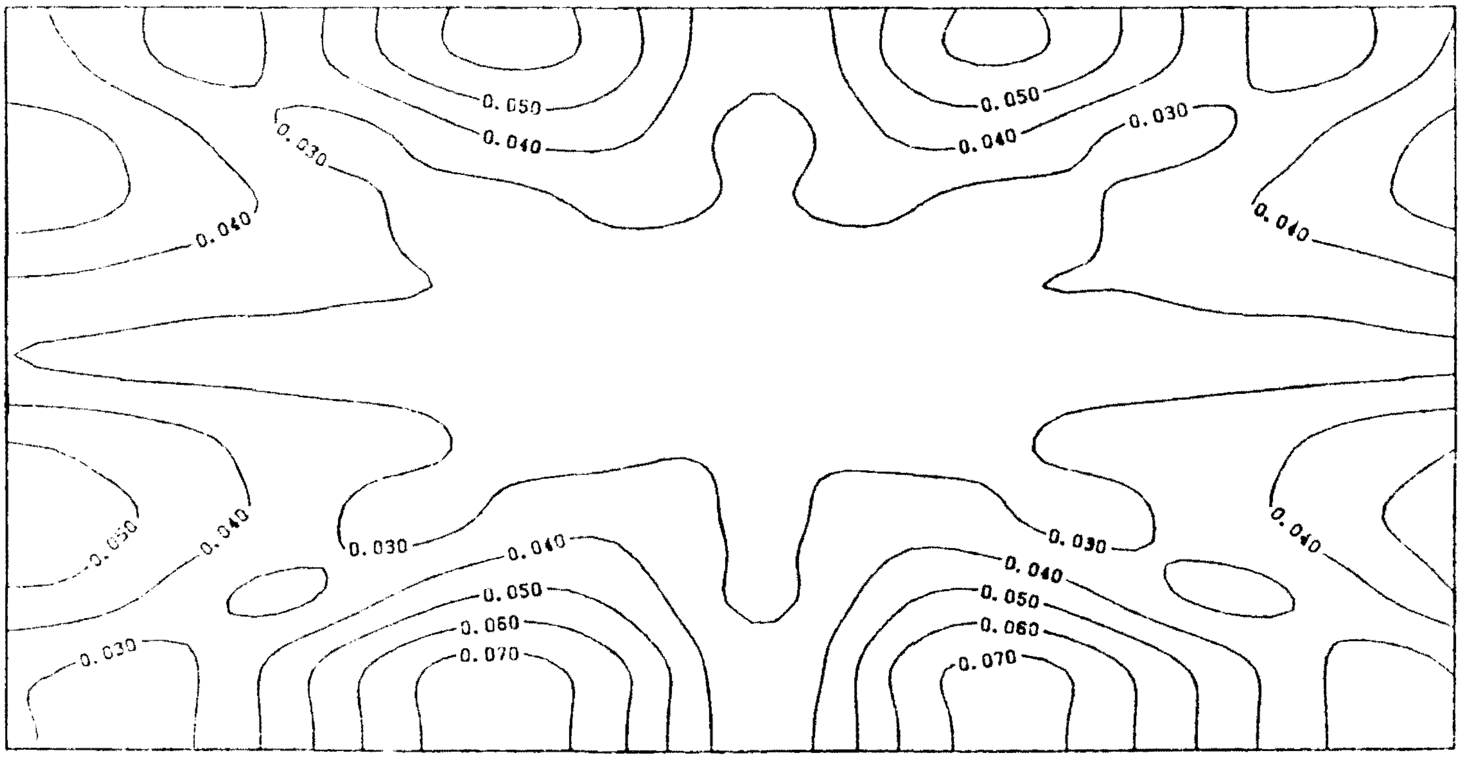
The effect of spatial variability of residual K on corn response was investigated. The results showed significant decreases in corn grain yield with increasing variability of residual K. The level of exchangeable K associated with 90% maximum yield increased with increased variation in soil K. The functional relationship between corn grain yield and ear-leaf K was significantly affected by the variability of residual K.



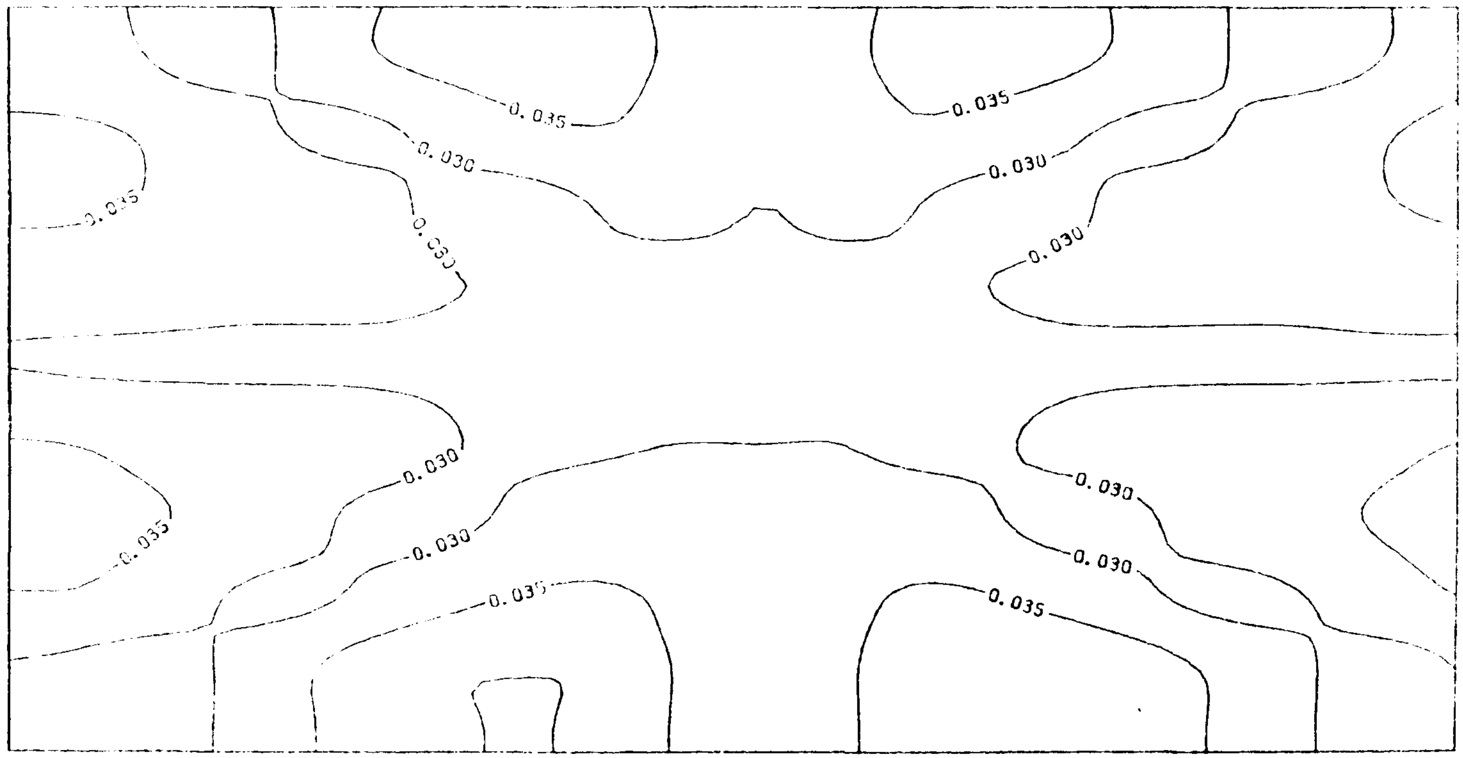
Appendix 5.1 Estimation variances of exchangeable Ca for punctual kriging.



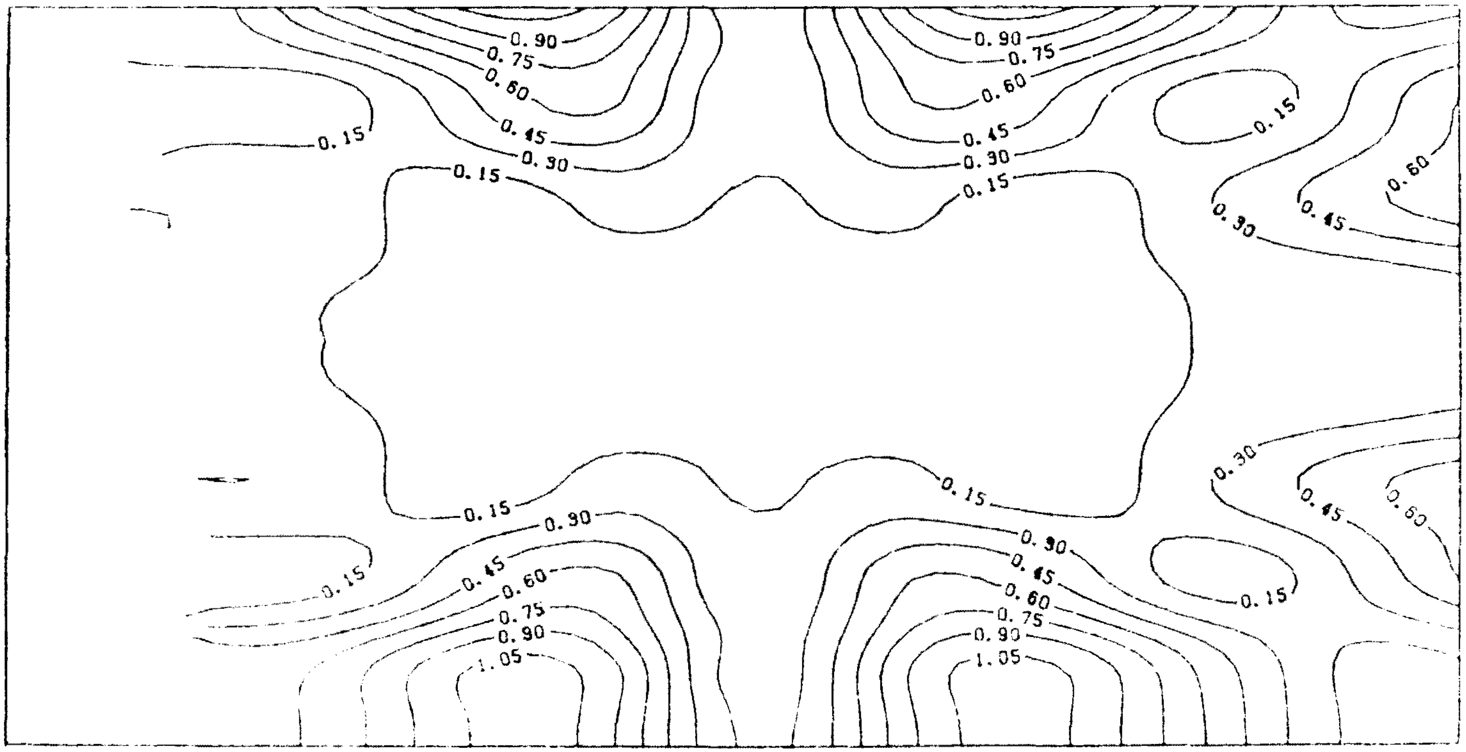
Appendix 5.2 Estimation variances of exchangeable Mg for punctual kriging.



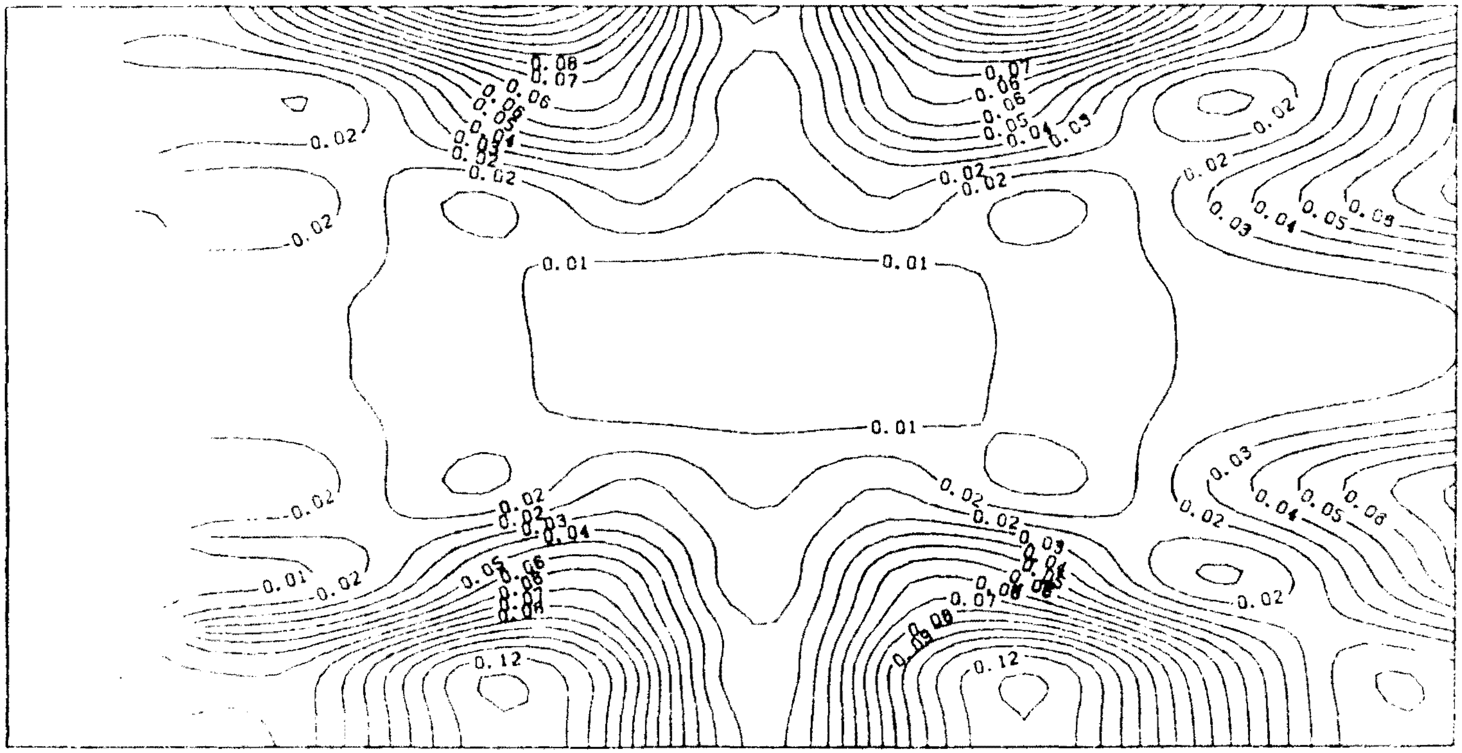
Appendix 5.3 Estimation variances of exchangeable K for punctual kriging.



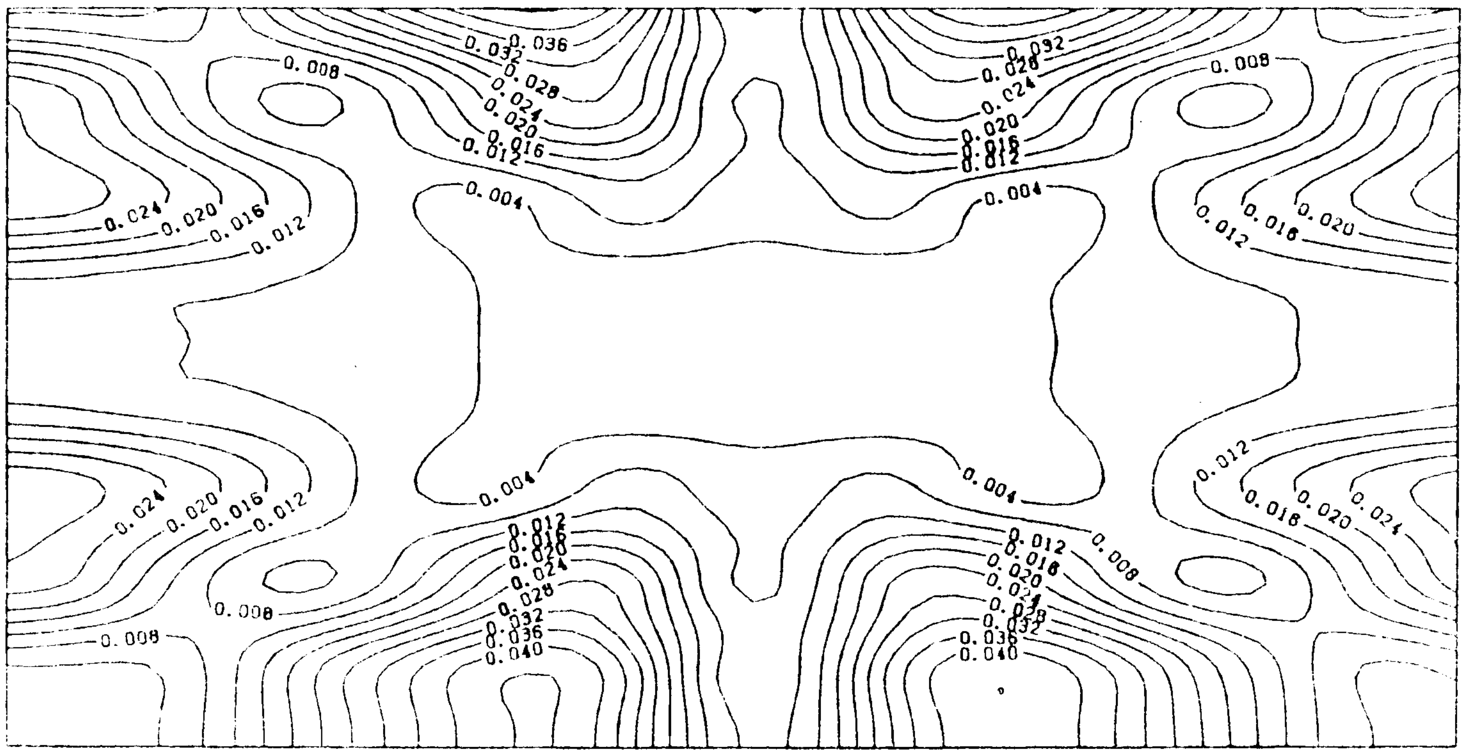
Appendix 5.4 Estimation variances of exchangeable Na for punctual kriging.



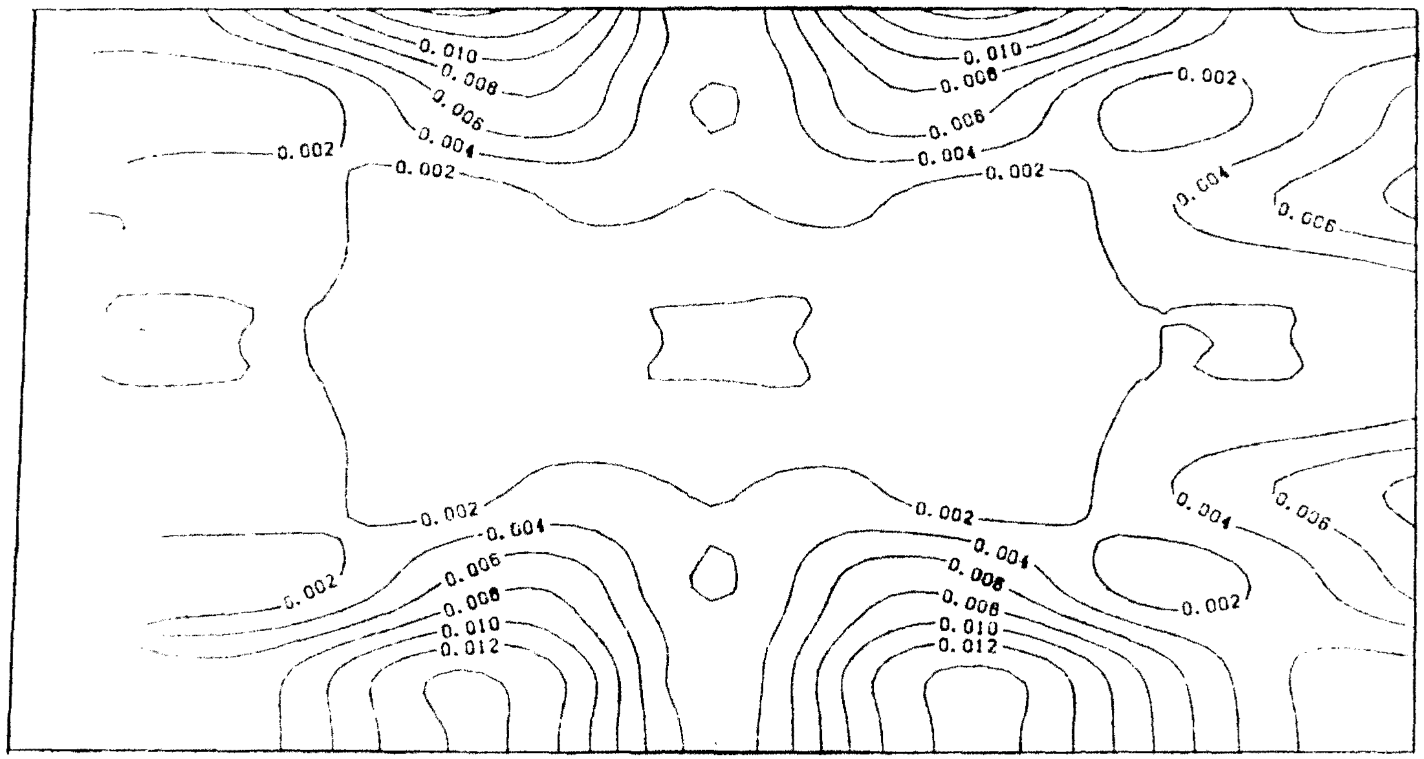
Appendix 5.5 Estimation variances of exchangeable Ca for block kriging.



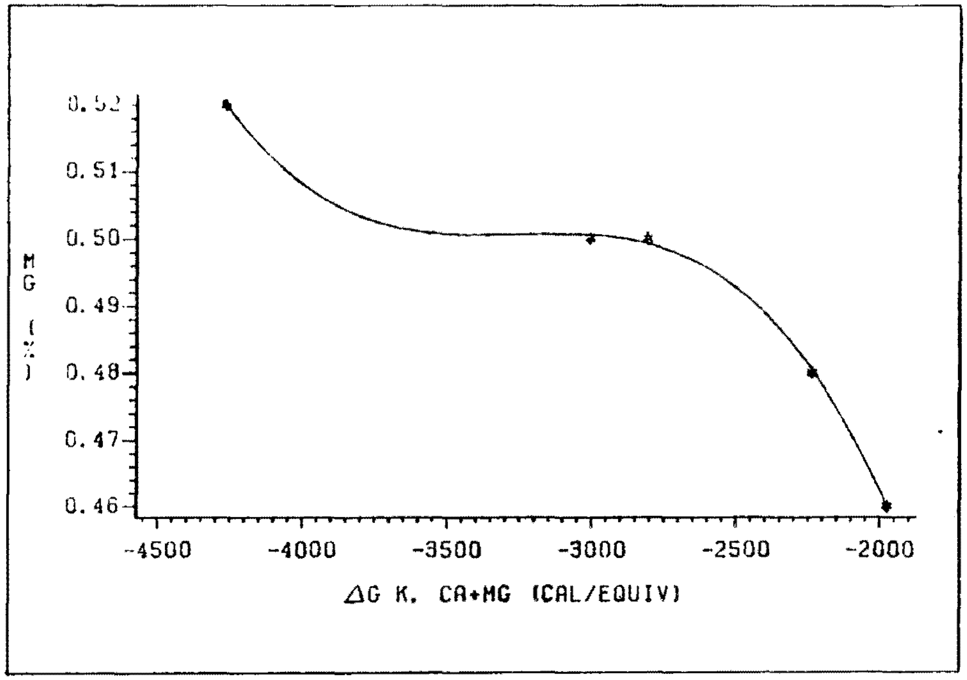
Appendix 5.6 Estimation variances of exchangeable Mg for block kriging.



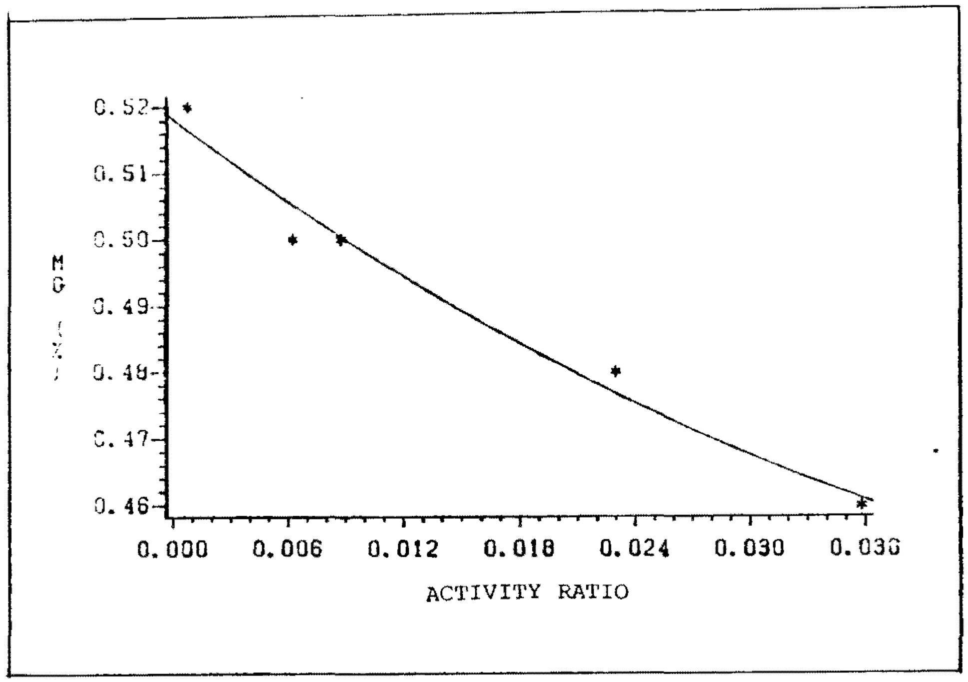
Appendix 5.7 Estimation variances of exchangeable K for block kriging.



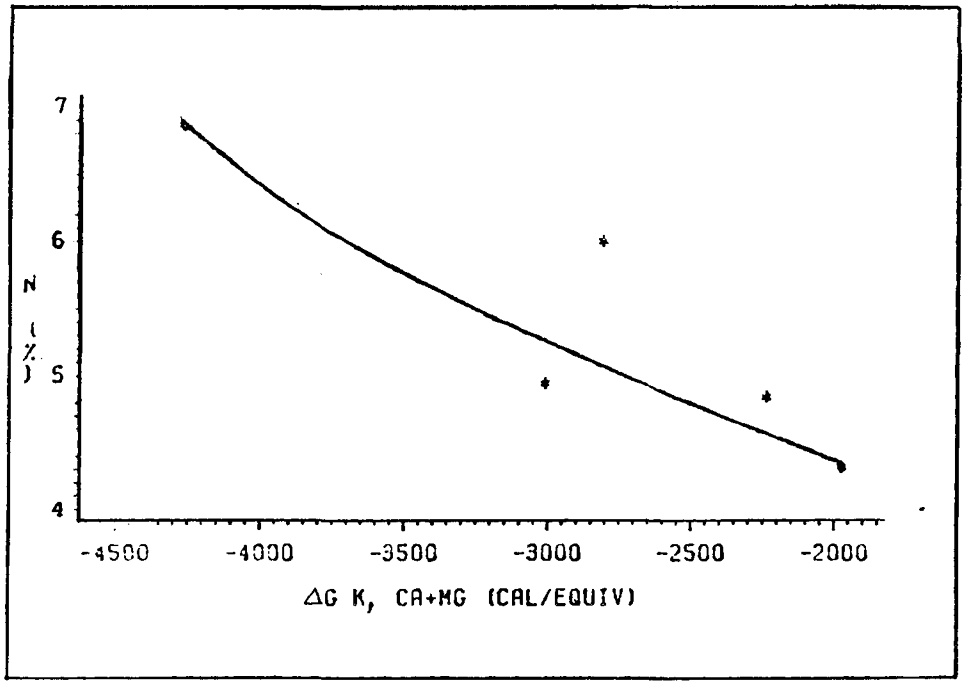
Appendix 5.8 Estimation variances of exchangeable Na for block kriging.



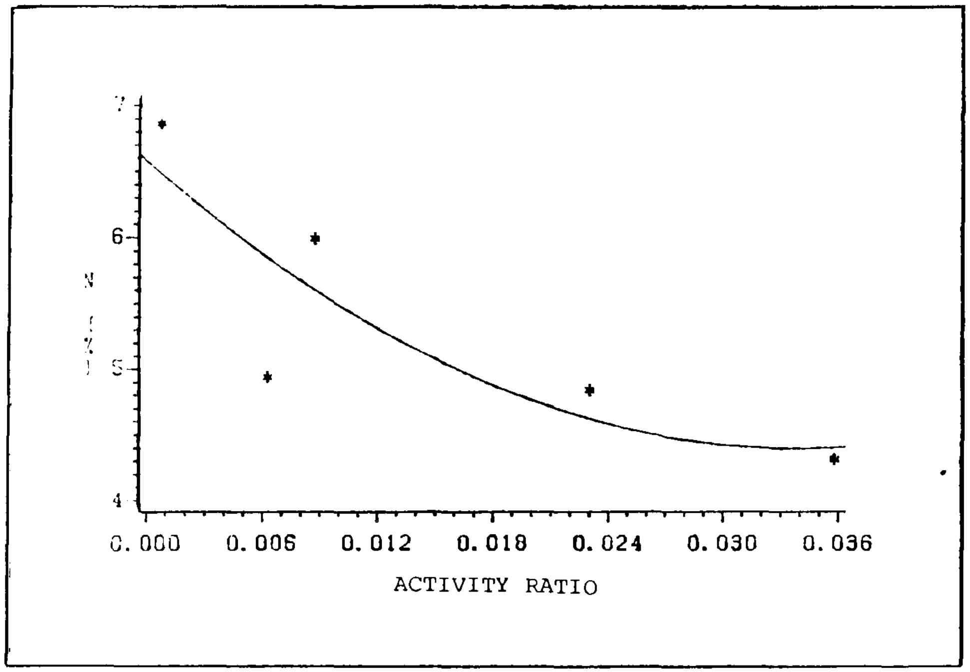
Appendix 6.1 Relationship between Mg in cabbage dry matter and potassium potential.



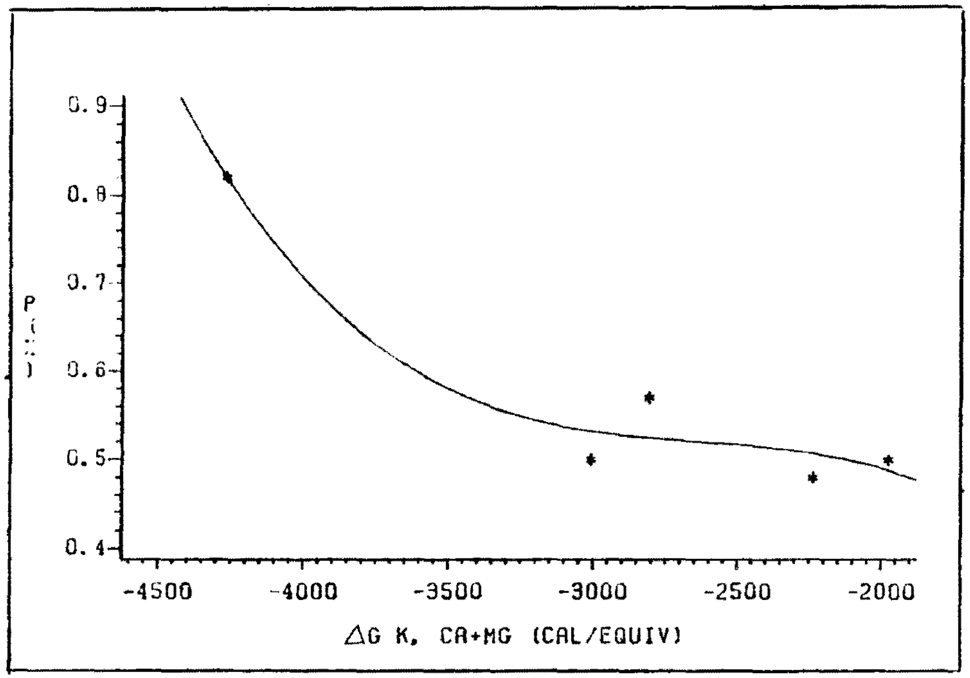
Appendix 6.2 Relationship between Mg in cabbage dry matter and activity ratio.



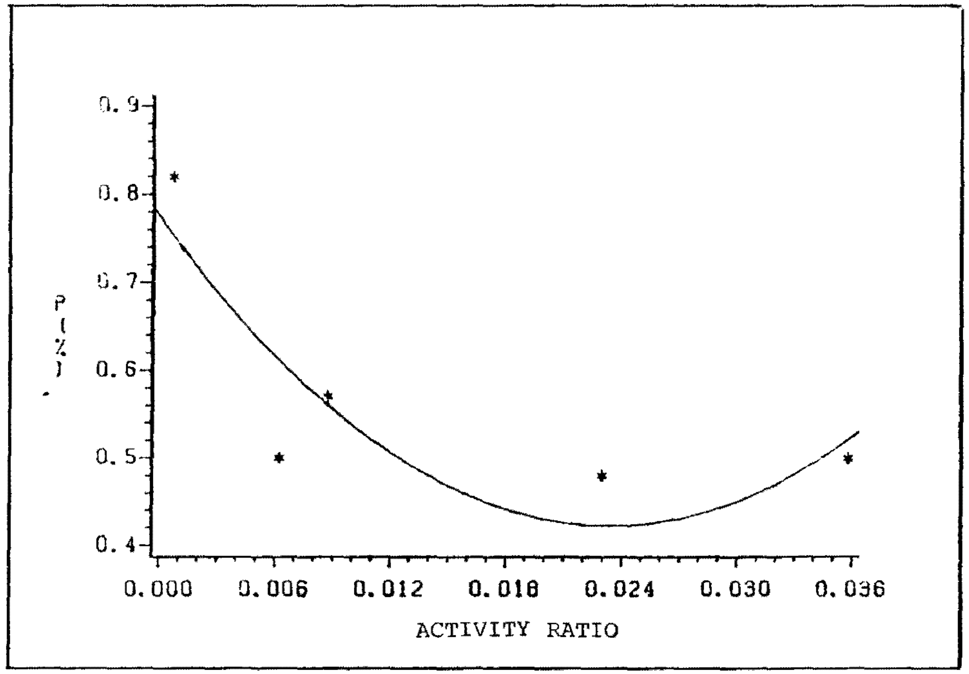
Appendix 6.3 Relationship between N in cabbage dry matter and potassium potential.



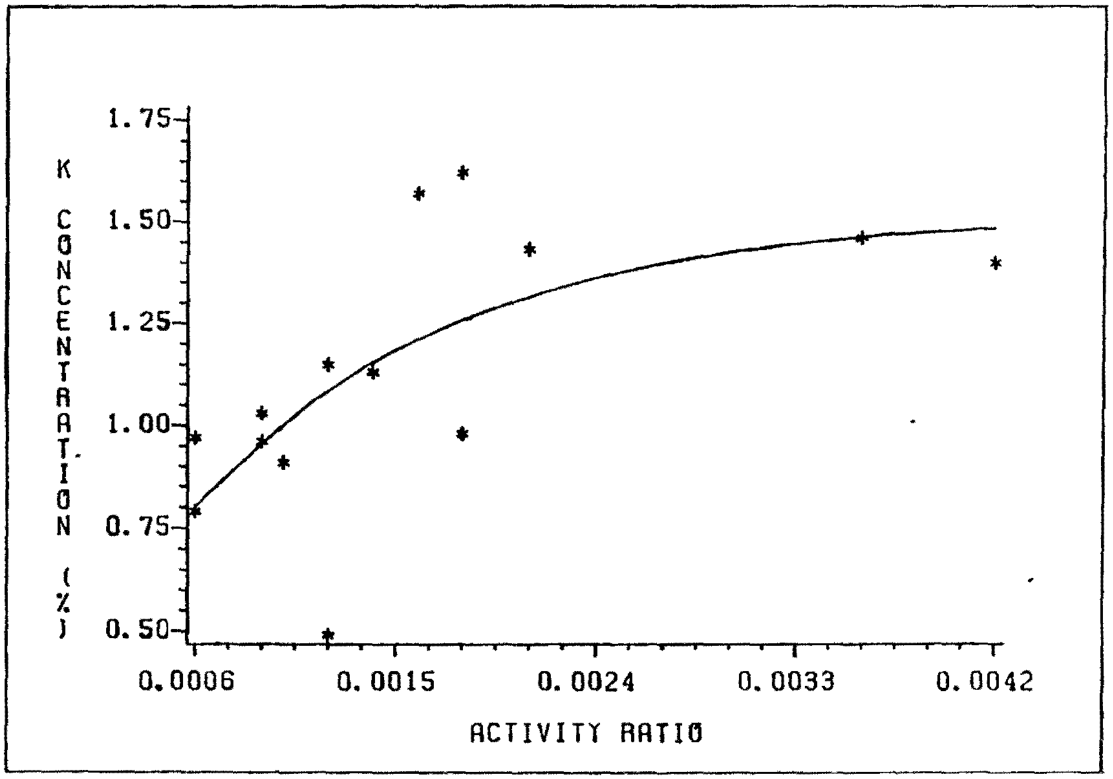
Appendix 6.4 Relationship between N in cabbage dry matter and activity ratio.



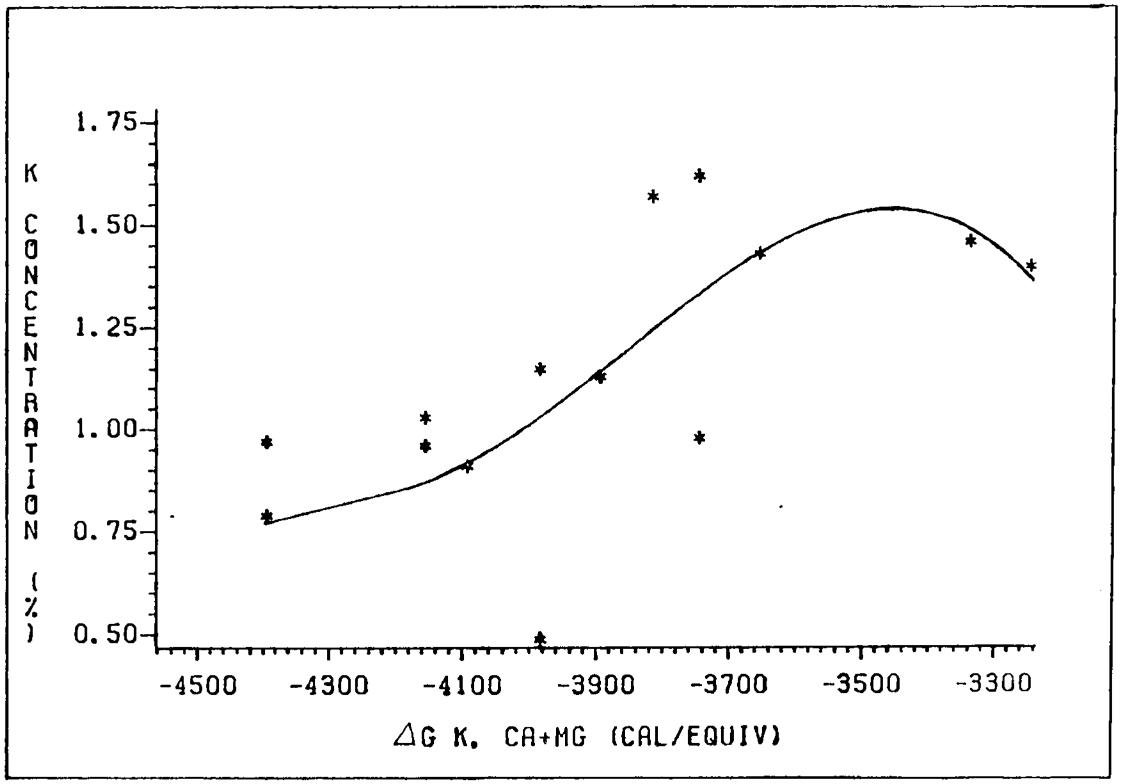
Appendix 6.5 Relationship between P in cabbage dry matter and potassium potential.



Appendix 6.6 Relationship between P in cabbage dry matter and activity ratio.



Appendix 7.1 Relationship between ear-leaf K and activity ratio.



Appendix 7.2 Relationship between ear-leaf K and potassium potential.

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