EXAMINING THE MATHEMATICAL DISCOURSE OF A FIRST GRADE LEARNING COMMUNITY: A CASE STUDY

A DISSERTATION SUBMITTED TO THE FACULTY OF THE COLLEGE OF EDUCATION, UNIVERSITY OF HAWAI‘I AT MĀNOA IN PARTIAL FULLFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

IN

EDUCATION

DECEMBER 2015

By

Seanyelle L. Yagi

Dissertation Committee:

Joe Zilliox, Chairperson
Hannah Slovin
Linda Venenciano
Melfried Olson
Thomas Jackson
Michelle Manes

Keywords: Elementary Mathematics, Discourse, Reform

DEDICATION

To Ed, Aiden and the rest of my family for all their love and support throughout this entire journey

To the Curriculum Research & Development Group and the Hawaii Department of Education, without whose support completion of this project would not have been possible

To my mentors and colleagues who have patiently guided me throughout my career and from whom I have learned so much

Most of all, to a special group of elementary students and their teacher, who opened their hearts and minds to me

ABSTRACT

Mathematical discourse, difficult to characterize, continues to be a challenge for practicing teachers across the country. Although the ideals for changing the way mathematics is taught and learned were published in the NCTM Principles and Standards in 2000, and much research regarding mathematical discourse has been conducted since then, this approach to teaching continues to be daunting for most teachers. As a way to provide practicing teachers with insight into the development of discourse, for young students, this year long research project utilized an ethnographic case study design to study the mathematical discourse in a first grade classroom. Observations were conducted at three points during the course of the year to study the discourse development. The findings indicate the challenges associated with orchestrating whole group interactions with young children, and provide a portrait of a teacher and group of students in development. A challenge continues to be the competing goals of engaging students in student-centered discussions, yet ensuring that they learn the required mathematics described in standards documents, an issue particularly relevant in the current educational climate. Descriptions of first grade students' whole group interactions provide insight into what it means to be student-centered and implications for defining the teacher's role in orchestrating student-centered discussions with the Common Core State Standards for Mathematical Practice in mind.

TABLE OF CONTENTS

CHAPTER 1
INTRODUCTION TO THE STUDY

Within the last century, mathematics education in the United States has undergone several waves of reform. The New Mathematics Reform (Herrera & Owens, 2001) in the 1950s and 1960s, was characterized by the general consensus that education in the U.S. needed to prepare students for a new technological era. This resulted in changes in curriculum development influenced by epistemologists such as Jean Piaget and Jerome Bruner. Immediately following the New Math reform was a back-to-basics movement grounded in behavioral psychology. The next wave of mathematics reform occurred with the National Council of Teachers of Mathematics' (NCTM) 1989, 1991, and 1995 documents, Curriculum and Evaluation Standards for School Mathematics, Professional Standards for Teaching Mathematics and Assessment Standards for School Mathematics. They eventually evolved into the document, Principles and Standards for School Mathematics (NCTM, 2000), which calls for reform in the way people teach and learn mathematics. One of the document's recommendations is the development of a discourse community within the mathematics classroom in which student conjectures and reasoning play a role in determining the direction and validity of the mathematical content in the classroom. In this type of environment, mathematical meaning is negotiated collaboratively, with the teacher taking a facilitative role in guiding the students. Consequently, supporters of reform argued that "The successful implementation of mathematics education reform requires that teachers change traditional teaching practices significantly, and develop a discourse community in their classroom" (Hufferd-Ackles, Fuson, & Sherin, 2004 p. 81), and "students must develop the ability to engage in mathematical thinking, learn to develop conjectures, and frame and solve problems, as well as explain, justify and defend their solutions" (Spillane & Zeuli, 1999 p. 4).

Although the call for change in the way mathematics is taught and learned occurred over ten years ago, shifts toward this vision have been fraught with difficulties. Since the early 1990s, while teachers have attempted to make changes in the way students learn mathematics, the evolution has been uneven. Teachers may have altered certain aspects of their practice to incorporate some reformed ideas, such as the use of manipulatives or different curricular materials, but fail to implement other dimensions which truly capture the essence of reform efforts, such as implementation of reformed discourse patterns (Ball, 1990, Spillane & Zeuli, 1999, Baxter & Williams, 2009).

Importance of the Topic

With the publication of three documents, The Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989), The Professional Standards for Teaching Mathematics (NCTM, 1991) and The Assessment Standards for School Mathematics, (NCTM, 1995) the NCTM presented a vision for mathematics teaching and learning and outlined standards which defined changing roles for teachers and students in a mathematics classroom. They promoted the idea that all students have access to high-quality mathematics instruction through a problem-based, student-centered approach in which students actively construct their understanding of mathematical concepts and skills. This is in contrast to the traditional approach of memorizing arbitrary rules and procedures they learn passively from a teacher. The documents also highlight the notion that *what* students learn is fundamentally related to *how* they learn it (NCTM, 1990).

The latest wave of reform in mathematics education came in the form of a nation-wide collaboration toward a set of agreed-upon literacy and mathematics standards. The National Governors Association for Best Practices (NGA) and the Council of Chief State School Officers (CCSSO) collaborated to publish the Common Core State Standards (CCSS) for English Language Arts and Mathematics, which were released to the public in 2010. This set of mathematics standards, currently adopted by 42 states, contain the content students are expected to learn at each grade level, K-8, and topics, grades 9-12. However, the Common Core State Standards for Mathematics (CCSSM) differ from most states' previous content standards in one respect - they explicitly take into consideration and outline the way in which students are to learn the content. Incorporating aspects of the NCTM (2000) document, Principles & Standards for Mathematics, specifically the Process Standards (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation) and the Strands of Mathematical Proficiency (National Research Council, 2001), the CCSSM includes the Standards for Mathematical Practice (SMPs). These standards "rest on important processes and proficiencies with longstanding importance in mathematics education" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, Standards for Mathematical

Practice) and describe the habits of mind and dispositions important for students to develop over time. They include:

> 1) Make sense of problems and persevere in solving them, 2) Reason abstractly and quantitatively, 3) Construct viable arguments and critique the reasoning of others, 4) Model with mathematics, 5) Use appropriate tools strategically, 6) Attend to precision, 7) Look for and make use of structure, and 8) Look for and express regularity in repeated reasoning. (NGA & CCSSO, 2010, Standards for Mathematical Practice).

Although only SMP 3 explicitly describes incorporating communication in the classroom, I include all of them as related to developing a community of discourse in learning mathematics. Not only have they been accepted as the processes of thinking and doing mathematics by the wider society, they also highlight mathematical processes that may emerge in students' discourse. Although there have been numerous studies on mathematical discourse and teachers' attempts to incorporate mathematical discourse into their practice, because these standards are new, research has yet to take place regarding students' development of these standards within a community of learners in which mathematical discourse takes place. Research is needed on the enacted curriculum, particularly studies which look at the extent to which the standards for mathematical practice are evident in the enacted curriculum (Heck, Weiss, & Pasley, 2011).

Although the NCTM Principles & Standards document and the ideas behind reform-oriented mathematics teaching and learning were developed well over a decade ago, few teachers actually utilize the ideas to drive their instruction. Students are still experiencing a largely traditional mathematics education in which rote procedures and math as memorization is emphasized. Teachers have not changed their practice, or have made significant movement toward incorporating reformed teaching practices into their everyday classrooms, even in light of current research showing improvement in student achievement in classrooms which incorporated some aspects of reformed instruction (Ross, McDougall & Hogaboam-Gray, 2002, Staarman & Mercer, 2010). In a review of empirical research studies on the effects of reform or the difficulty of implementing reformed ideas that were published between 1993 and 2000, researchers found higher student achievement when there "was substantial implementation of reform, a rare event" (Ross et al., 2002, p. 18). In an analysis of classroom interactions videotaped from the 1999 Third International Mathematics and Science Study (TIMSS), (Hiebert et al., 2005) when compared with higher performing countries, instruction within the U.S. was found to focus on low level mathematics–following procedures, providing correct answers, with less time spent on developing students' conceptual understanding and making connections between topics. The findings from this research showed that the typical mathematics classroom in the U.S. is characterized by teaching which is still largely focused on learning rote procedures and recall of facts in a passive manner and has not incorporated ideas of reform as set forth in the NCTM documents.

While student results on international assessments cannot be attributed solely to a single aspect of the classroom, they support the notion that there is still much work to be done for improving students' mathematics education. In the 2011 TIMSS, although the performance of 4th and 8th graders in the US was above the TIMSS scale average; students in the top seven countries far and away outperform other countries, including the United States. These assessments looked at the degree to which students can apply their understanding in a variety of situations and explain their reasoning. In the top six countries, more than half of their 4th graders performed at the Advanced and High benchmarks, compared to less than half of U.S. 4th graders. The 8th grade U.S. students did not fare much better, with just a third scoring at the Advanced and High benchmarks, and a majority of the students scoring at the Intermediate and Low benchmarks. As with the 4th grade scores, the top three countries had the most impressive showing, with more than half of their 8th graders scoring at the Advanced benchmark. Furthermore, in the 2012 Programme for International Student Assessment (PISA), students in the U.S. were found to have weaknesses in working through mathematics tasks with higher cognitive demands, as well as utilizing mathematics in real-world situations (OECD, 2012).

Lastly, motivation for conducting this research originates in my own personal observations and experiences in mathematics. As my interest in mathematics education developed over years of teaching, I was constantly confronted with students' and teachers' fears when asked to engage in mathematical thinking. I recognized that a level of math anxiety exists within many of us who experienced a traditional approach to learning mathematics, where many aspects of our learning experiences fostered a sense of a "fixed mindset" (Dweck, 2006, as cited in Boaler, 2013 p. 143) toward mathematics. In the context of learning mathematics, a fixed mindset means believing there are individuals who are born with the ability to do mathematics as well as those who are not. It means believing one is less capable of solving mathematical problems than others and not viewing mathematical thinking as a sense-making endeavor. In contrast, having a "growth mindset" (Dweck, 2006, as cited in Boaler, 2013, p. 143) means believing that one is capable of growing and developing through facing challenges. All aspects of students' learning experience, including instruction, determine the extent to which they develop a growth or fixed mindset for learning mathematics (Boaler, 2013). As one who had traditional experiences in school, I had a fixed mindset toward mathematics. Through several professional development experiences, I came to view mathematics not as procedures to be memorized whose validity is determined by others, but as genuine processes of personal thinking, communicating and sense-making. Providing students with experiences through which they view themselves as mathematical thinkers as a result of engaging in mathematics as a sense-making activity can lead to students' taking responsibility for their own learning (Ball, 1993).

Believing one's own self to be capable of mathematical thinking impacts perceptions of self-efficacy, which influences thought patterns and emotional reactions (Pajares, 1996). Individuals who have high self-efficacy willingly confront challenging tasks, leading to high expectancy beliefs in one's perceived ability to attain accomplishments. Conversely, individuals who have low self-efficacy negatively view situations, fostering a narrow view of how problems might be approached (Pajares, 1996). Negative attitudes and a sense of inefficacy in being able to do mathematics exist everywhere in the general public - in television shows, commercials, as well as in the prior experiences of many adults (Geist, 2010, Boaler, 2013). Current research presents strong support for a positive relationship between students' attitudes toward mathematics and mathematics achievement (Hattie, 2009). A growing body of research studying the relationship between students' attitudes and mathematics achievement, including the consequences of anxiety toward mathematics on students' ability to do mathematics and continue their education in math-related fields is well documented (Pajares, 1996, Ma, 1999). Students who develop negative attitudes and anxiety toward mathematics will tend to avoid careers that involve mathematical thinking and tend to drop out of mathematics courses prematurely (Ma & Xu, 2004). Furthermore, boys' prior mathematics achievement has been related to the development of anxiety toward mathematics during their junior year of high school (Ma & Xu, 2004). Students' beliefs about themselves as mathematical thinkers influence the choices they make about learning mathematics. At the same time, the choices they make and the way they come to view mathematics is determined by their learning experiences.

Challenges to Reform

Challenges to implementing reformed ideas in the mathematics classroom are well documented in research (Ball, 1990, Ball, 1993, Gabriele & Joram, 2012, Nathan & Knuth, 2010, Ross, et al., 2002, Sherin, 2002). The ambiguity and lack of constructivist models make the aspects of reformed teaching very difficult to implement. Teachers are constantly confronted with conflicting goals, such as honoring students' emerging mathematical conjectures and ideas, while at the same time, having a required curriculum to teach (Ball, 1993). Teachers are also challenged with seeing implications for teaching mathematics as a discipline (Ross, et al., 2002), as well as teachers' inability to feel self-efficacious when adopting reformed approaches to teaching (Gabriele & Joram, 2012). Research has shown that teachers who have strong self-efficacy are generally more willing to adopt new ways of teaching (Charalambous & Philippou, 2010), but the feeling of effectiveness can be impeded when student learning looks different from what a teacher would normally expect to observe. In reformed approaches to teaching, student learning might be less visible and may occur over longer periods of time. According to Smith (1996), one reason the transition from traditional to reform-based mathematics teaching may be particularly difficult for teachers is that the criteria teachers use to judge their own effectiveness are not as evident or familiar as in the traditional classroom. Smith speculates that a teacher's ability to interpret her or his performance as being successful and develop the confidence for teaching reform-based mathematics (i.e. teacher self-efficacy) may be impaired because success is defined differently when teaching reform-based mathematics.

The Challenges of Mathematical Discourse

There are several reasons for which developing mathematical discourse within a community of learners is challenging. Mathematical discourse is difficult to characterize and is academic in nature. There is really no one set of procedures to follow, and can vary for different people, communities, time, settings and purposes (Moschkovich, 2003). Developing mathematical discourse with elementary children is very challenging, as they may not have enough language development to fully explain their thinking in a coherent way (Ball, 1993). For younger children, aged 5-7 years old, engaging in academic dialogue is both new and difficult (Hicks, 1996). Another significant challenge is honoring students' mathematical thinking while, at the same time, having a required curriculum to teach (Ball, 1993). This involves tensions in valuing students' mathematical thinking and still orchestrating the discussion so it is still mathematically productive for students. The actual orchestration is extremely challenging as well, recognizing which student utterance has “grit” (Henningsen & Stein, 1997 p. 526) and will lead to a productive discussion, rather than a show and tell conglomeration of student strategies (Smith & Stein, 2011). There is also the challenge that the discussion can drift back toward focusing on procedures, or the completeness and accuracy of answers rather than remaining on students’ developing conceptual understanding (Henningsen & Stein, 1997).

With all the complexities of implementing reform-oriented instructional practices, including discourse, to life within the classroom, a fundamental challenge for teachers is “reconstructing what it means to know and do mathematics in school, and thus what it means to teach mathematics” (Wood, Cobb & Yackel, 1995, p. 408). The view of what constitutes mathematics can define the nature of the discourse in the classroom (Battista, 1994, Thompson, 1984). The traditional view of mathematics as a static, external body of knowledge students acquire through the manipulation of symbolic representations and following preset procedures persists, often overlooking the idea that mathematics is conceptual and lies in the thinking behind the manipulations and procedures (Rachlin, 1998, von Glasersfeld, 1990). Traditionally teachers have focused their instruction on symbolic manipulation, thinking this is the essence of doing mathematics, when actually, mathematical knowledge exists in the thinking behind the manipulations. Taking a reformed approach to teaching mathematics requires teachers to shift their view of mathematics toward an ambiguous, sense-making activity, involving logical reasoning, argumentation and the negotiation of meanings (Schoenfeld, 1992, Voigt, 1994). When teachers shift their view of what constitutes mathematics to how students think as they engage in mathematical problems and negotiate meanings within a group of learners, the type of discourse which occurs in the classroom will change and be more focused on students’ conceptions of mathematical thinking rather than on symbol manipulation.

As a school-based mathematics coach and complex area resource teacher, I had the opportunity to work with many elementary, middle and high school mathematics teachers. In my experience, teachers who have embraced the essence of reformed mathematics teaching and learning by having their students engage in meaningful mathematical discourse are few and far between. When working with practicing teachers, coaches and curriculum coordinators, I was confronted with questions about how to create a classroom environment in which students communicate to learn mathematics, where they are given latitude to provide explanations about their thinking, make conjectures and provide reasoning for their ideas and yet, learn the required curriculum. What do the SMPs from the CCSSM look like in action in a mathematics classroom? What do they really mean for teaching practice? In an essay comparing constructivist and sociocultural learning theories, Cobb (1994) analyzes each perspective and describes the constructivist theory as a "generally accepted" (p. 13) view, in which students "actively construct their mathematical ways of knowing" (p. 13). While the notion of students being active in their construction of mathematical understanding supports the present study, there is a question of whom Cobb refers to when he uses the words, "generally accepted". Other theorists? Researchers? In my observations, this notion that children actively construct their knowledge is not "generally accepted" by the group for whom it arguably counts the most - the mathematics teachers who work with students on a day-to-day basis.

Purpose of the Study

As mentioned previously, implementing instructional practices consistent with reform ideals can be very challenging for teachers and students, particularly developing mathematical discourse as a way to teach and learn mathematics. Attempts to reform teaching practice have remained un-even; therefore students largely continue to experience traditional approaches to learning mathematics. Furthermore, research is still needed on the enactment of the SMPs. The current climate of reform in mathematics education provides an opportunity to highlight some of the ideas about teaching and learning that have remained within the research community and translate them into actual classroom practice.

The purpose of this study is to describe the development of whole group mathematical discourse in a 1st grade learning community. Specifically, I am interested in investigating the processes by which the members in a 1st grade learning community engage in whole group mathematical discourse in which taken-as-shared mathematical meanings are interactively constituted. I am also interested in seeing how the SMPs manifest themselves in students' discursive experiences in the classroom. Although there has been prior research regarding mathematical discourse, the focus on the manifestation of the standards of mathematical practice within students' discourse makes this study unique. Furthermore, research studying the development of whole group mathematical discourse with young children is sparse at best. My aim is to extend prior research through the study of young children's engagement in whole group mathematical discourse, and provide practicing teachers with insight into the processes for orchestrating discussions in the classroom, including an exploration into possible relationships between discourse and the SMPs.

The Qualitative Paradigm

To further define the nature of this research study, it is helpful to identify its philosophical foundations (Merriam, 2009, Cresswell, 2013). The philosophical foundation of research defines the ontological and epistemological beliefs, which provide the underpinnings of a study. It also provides the foundation for the choice of design and theoretical framework for a study (Cresswell, 2013). Because the purpose of this research is to describe the negotiated meanings interactively constituted by individuals' subjective meanings, an interpretive lens is appropriate for this study. In an interpretive view of doing research, there is no single reality; rather, there are "multiple realities of interpretations of a single event" (Merriam, 2009, p. 8). Furthermore, because interpretation is inherent within all interactions and observations, it is necessary to understand a group's actions and interactions (Miles & Huberman, 1994).

The Role of the Researcher

In descriptive and interpretive qualitative studies, it is important to provide the stance from which the main research instrument, namely the researcher, comes from, as a way show how the researcher's values and perspectives influence the conclusions of the study (Maxwell, 2013). The role of the researcher in conducting observations in qualitative research is varied and can be inclusive within the context of the study. Several possible stances the researcher can take during observations include as a complete participant, participant as observer, or observer as participant (Merriam, 2009). The predominant stance I took during the lessons was that of an observer rather than a participant. Although my active participation during the lessons was minimal, because I served as the lens through which the data were collected, analyzed and interpreted, it is important for me to present my values, assumptions and biases, or the "researcher's position" (Merriam, 2009, p. 219), which I brought to the research.

My view is deeply rooted in my personal teaching practice. This can be both an advantage and a hindrance in analyzing and discussing the findings of research. Although knowledge based on practice can lead to insights in the analysis of data, the questions I ask and conclusions I draw sometimes originate from a didactic perspective, in which dichotomies are present. This can severely limit the analysis and conclusions I draw, but at the same time bring the findings of the research to a relevant level in the classroom. Because of the complexity of a teacher's decision-making processes in the classroom, any "truths" – systems or decisions that have shown success in prior practice - tend to be drawn upon in subsequent actions. Although I believe the best teachers draw on their prior experiences in making subsequent decisions during active instruction, I also believe the best teachers recognize the uniqueness of individual and groups of students, and this is reflected in their flexibility and ability to adapt successfully to situations in which adjustments to suit the unique needs of learners and situations are made. I have always struggled with this latter piece involving flexibility, and continue to grapple with this aspect of instructional decision-making in the classroom. Being aware of my biases allow me to be cognizant of the lens through which I analyze the data and draw conclusions for this study.

My view about teaching practice and the way students learn is informed by theory. In my view, learning involves personal experience and active engagement through which an individual has opportunities to explore, make observations and draw conclusions based on knowledge from prior experience as well as what was observed. That this occurs in coordination with the use of communication, language and interaction with others has always been foundational in my beliefs about how children learn. Seemingly consonant with the amalgam of cognitive learning theories that constitute social constructivism, I try to draw upon those theories in my work as a teacher and professional developer.

In addition, my view is also deeply rooted in personal experience. My beliefs about the way learning occurs are largely based on my own experiences and ways of learning, and I tend to draw on my own experiences as a learner when I think about teaching and learning. Because my view of teaching and learning is deeply rooted in my own personal experience, I am sometimes challenged with understanding ways of thinking and learning that are different from mine. However, because I am cognizant of this bias, I negate the propensity to converge on one way of thinking by being open to ideas and different ways of thinking. This includes further investigation into what is initially observed and probing deeper to gain a fuller and clearer understanding of whatever it is.

I draw from 22 years of working as an educator in the Hawai'i Department of Education, seventeen of which were teaching elementary-aged students in grades two through six. For one and one-half years, I taught mathematics at a middle school. As a practicing teacher, I strove to (and continue to) implement practice from a dialogic perspective, and experienced similar challenges and issues the subject of this research study grappled with. During my time in the department, I served as a math coach and a complex area resource teacher, providing professional development for teachers in mathematics, at both the school and complex level. For the last two years, I was the state resource teacher and lead for formative instruction and data teams, working with administrators and other educators across the state on issues regarding formative assessment (synonymous with formative instruction within the department). Through working in these various positions and levels in the education system, my perspective has evolved during my career, garnering, among other things, a systemic view of public education in this state. These are the lenses through which I conduct this study.

Theoretical Framework

Merriam (2009) states that a theoretical framework provides an underlying frame or structure for a study. This frame includes the theories, beliefs, and assumptions, which support and inform the research (Maxwell, 2013). Because of the interpretive nature of this study, and taking into consideration the theoretical foundations that provide the basis for the development of mathematical discourse in the classroom, this research project will draw from social constructivist and interactionist perspectives. Derived from a blending of constructivism, sociocultural learning theory and symbolic interactionism, social constructivism, along with its epistemological and ontological foundations, will provide the lens through which this study takes place.

Research Questions

Building upon the large body of knowledge regarding mathematical discourse, the purpose of this study is to describe the process by which mathematical discourse develops within a first grade learning community. There are three main research questions I address:

1) What does whole group mathematical discourse look like in a first grade learning community?

2) What aspects of whole group mathematical discourse emerge in the engagement of a first grade learning community in reform-oriented approaches to teaching and learning?

3) How do the Standards for Mathematical Practice in the Common Core State Standards manifest themselves in the whole group mathematical discourse of first grade students?

Definition of Terms

The following terms and definitions are delineated for the purposes of this study.

Development.

The word development is used throughout this document in various ways, as there are differing levels and forms of development one might refer to. Development refers to a process of change or growth, sometimes accompanied by enlightenment or advancement that occurs over a period of time.

Dialogic.

Describes a pattern of interaction characterized by extended back-and-forth dialogue in which the discourse serves as a tool for the development of thought (Wertsch & Toma, 1995).

Facilitation.

Facilitation is the way in which a teacher encourages students to engage in activity, usually without directing them from an authoritarian mode. This might be through providing assistance, adjusting a question, asking a student to use a representation, or to consider new information.

Mathematical Discourse.

Although mathematical discourse is difficult to define, there are general characteristics and dispositions of those who regularly engage in mathematical discourse. Moschkovich (2003), describes these as, "Being precise and explicit, searching for certainty, abstracting and generalizing are highly valued practices in mathematically oriented Discourse communities." Furthermore, mathematical discourse is considered "reasoned discourse" (NCTM, 1991; Moschkovich, 2003), where students are encouraged to not only explain their thinking, but provide logical reasoning and justification, highlighting mathematical content in an effort to make sense of the concepts. The NCTM (1991) describes mathematical discourse as the way people communicate and interact within a classroom community of learners. This involves:

> Ways of representing, thinking, talking, agreeing and disagreeing ... The way ideas are exchanged and what the ideas entail: Who talks? About what? In what ways? What do people write, what do they record and why? What questions are important? How do ideas change? Whose ideas and ways of thinking are valued? Who determines when to end a discussion? (NCTM, 1991).

Discourse involves the discussions that take place in the classroom, including communicating through writing and the use of other visual representations (NCTM, 2014). In this kind of environment, students take a more active role in their learning, rather than the traditional, passive role of receivers of information.

Although discourse involves the type of talk that occurs within the classroom, that is not its defining characteristic. It also involves the type of norms, ways of behaving, and values established collaboratively by the community of learners, (Lampert, 1990, Yackel, E., Cobb, P., Wood, T., Wheatley, G., Merkel, G. 1990). Within a reformed framework, mathematical discourse is socially situated and constructed, where students develop common meanings of whatever it is they are talking about (Hicks, 1996).

Orchestration.

A broad term meant to describe the decisions, processes and actions by which a teacher enacts mathematical discourse in the classroom. This involves the way in which students are engaged in whole group interactions in which the negotiation and co-construction of mathematical meaning occurs (NCTM, 2000).

Triadic.

A form of dialogue in which a question is posed, followed by a response, and an ensuing evaluation occurs. This is one of the most common forms of discourse patterns observed in the classroom (Nathan, Eliam & Kim, 2007).

CHAPTER 2
REVIEW OF THE LITERATURE

The intent in chapter one was to provide a context to define challenges associated with implementing mathematical discourse as well as to provide justification for the purpose of this study. The purpose of a literature review is to not only identify what is known about a topic, but also to provide a "foundation – a theoretical framework – for the problem to be investigated", and demonstrates "how the present study advances, refines or revises what is already known" (Merriam, 2009, p. 72). This chapter explores relevant literature for this study, focusing particularly on the theoretical constructs for engaging in mathematical discourse in the elementary classroom. Consequently, this review will include historical background as well as an explication of the theories and prior research upon which this study is based. A historical view will allow for a well-developed, comprehensive and situated explanation of the constructs, including how they interrelate and provide the underpinnings for the framework. The focus will then move to instructional aspects of mathematical discourse in the classroom, including a description of current and past research relevant to this study.

Several perspectives provide the underpinnings for this study. At first glance, it would seem that discourse would rely heavily on the sociocultural perspective of learning, based primarily on the work of Vygotsky and other activity theorists, such as, Davydov, Leont'ev and Galperin (Cobb, 1994). However, because of the complexities of interactions within the classroom, including disciplinary discourse as an approach through which mathematics might be learned, a single perspective of learning cannot provide the complete theoretical construct for discourse (Cobb, Yackel & Wood, 1992). The next section begins with a catalyst for change in mathematical instruction and the development of mathematical discourse–the constructivist learning theory.

Constructivism

Constructivists view learning as an adaptive reorganization of cognition that occurs within the self, through active engagement as one "strives to be effective by restoring coherence to the worlds of personal experience" (Cobb, 1994, p. 13). Largely based on the work of Piaget, constructivists believe that learning is an individual construction of knowing and self-organization (von Glasersfeld, 1989, Fosnot & Perry, 1996), whereby the learner constructs meaning through active participation. Learning is viewed as a process of adaptation, or making viable, collections of conceptual structures that exist within an individual's "range of experience" (von Glasersfeld, 1989, p. 125). This adaptation occurs through a process of "Equilibration" (von Glasersfeld, 1995, p. 381, Fosnot & Perry, 1996) within the concrete operational stage of development. Described as a dynamic process of "self-regulated behavior" (Fosnot & Perry, 1996, Cognitive Equilibration section, para. 2), equilibration is undergirded by two behaviors - "assimilation and accommodation" (von Glasersfeld, 1989, p. 127). Non-linear in its development, it does not include a sequential series of processes. Rather, it consists of a process of constantly bringing oneself back to equilibrium through fluid "adaptation and organization, growth and change" (Fosnot & Perry, 1996, Cognitive Equilibration section, para. 4). As individuals interact with their environment and encounter inconsistencies with their present schemas, they experience a process by which they engage in activity with it, followed by reflection, integration and accommodation, resulting in self-organization (Fosnot & Perry, 1996).

Although the radical constructivist theory of learning has been criticized by other theorists as focusing too much on the individual and ignoring the role social interaction and the context in which it occurs play on cognitive development, constructivist theorists such as von Glasersfeld, Piaget, Kamii and others have addressed the social aspects of learning in several areas of their work. First, constructivists highlight the important role reflective thinking plays in the development of cognition (Bauersfeld, 1992, von Glasersfeld, 1991). Knowing develops from an individual's reflective ability, whereby it becomes a "major source of knowledge" in mathematics (von Glasersfeld, 1991, p. xviii), and is done so through talking about their mathematical thinking with others. Verbalizing one's thoughts to another ensures that one is reflecting on and examining those thoughts (von Glasersfeld, 1991). For this reason, it is important for students to talk about their mathematical thinking with others, for verbalizing and justifying what one has done requires one to recount and rebuild actions taken and reasoning for others. Through this process of verbal reconstruction, and clarification, students have more opportunities to reflect on their own understanding (Wood, 1998). Furthermore, self-reflection develops only through explicit reflection on others' actions as well, which occurs in the sharing and examination of different ways of thinking about the mathematics (Piaget, 1967, as cited in von Glasersfeld, 1991; Bauersfeld, 1992). Another area in which social interaction plays a role within the constructivist view of learning is its role in accommodation within the process of equilibration. Through interacting with others, perturbations and contradictions to one's way of thinking occur naturally, leading the student to accommodate his thinking (von Glasersfeld, 1989). Engaging in conflict through interacting with others also leads the child from egocentric thinking to viewing the world from others' perspectives, thus facilitating the construction of logic within the child (Kamii & Housman, 2000).

Constructivism contributes toward the basic ontological belief that mathematical knowledge is based upon what we experience, and our perceptions and interpretations of those experiences (Bauersfeld, 1992). Though many people believe that mathematical knowledge lies in the ability to manipulate symbols and abstract representations of how we show mathematical work, considering the constructivist view of what constitutes conceptual understanding, and the relative understanding students develop through equilibration, this view of what constitutes mathematical knowledge is not sufficient. Mathematics is conceptual, based on operational thinking, and therefore invisible to an outside observer (von Glasersfeld, 1990; Rachlin, 1998), except through students' social activity, including manipulating representations. Rather than having the common belief that there exists an external body of mathematical knowledge individuals must attain, the constructivist view is that knowledge is relative to the individual and is based on his own perception of reality (Von Glasersfeld, 1995). Therefore, mathematical truth lies in what the individual perceives to be valid, constructs in his cognitive structure, and does not exist as an external body of knowledge which is transmitted to the student (Fosnot & Perry, 1996). That is not to say that there are no mathematical truths, but these have been agreed upon by the greater society over long periods of time (Ball, 1993). It is the job of the students to re-construct this consensual knowledge base for themselves (Voigt, 1994), and it is the teacher's job to provide the learning environment and the opportunities for students to do so.

Although constructivism presents a well defined and well-articulated epistemological perspective, it does not directly translate itself into mathematical pedagogy (Simon, 1995). Constructivist views do not directly indicate how one should teach a group of children within the context of the classroom. Furthermore, the focus of the constructivist view of cognitive development always turns toward the inner cognition within the individual, even in light of acknowledging the importance of social interaction in development. Because teaching within the context of the classroom has implications for teaching and learning mathematics, constructivism alone is insufficient to account for influences the classroom culture may have on student learning. The next section explicates a learning theory, which attempts to account for this–the Sociocultural Learning Theory.

Sociocultural Learning Theory

In the mid-1980s and early 1990s, dissatisfied with the explanations and focus on the individual within the constructivists' explanation of cognitive development, other theorists' work based on social and collective perspectives came to the fore. The importance of the culture and context in which learners interact and are situated was emphasized, and explanations of cognitive development in which situated activity grounded in cultural, historical and institutional contexts (Wersch & Toma, 1995, Lave & Wenger, 1991) were highlighted. Largely stemming from the work of Vygotsky, these theorists put forth the idea that social activity precedes cognitive development, as they see cognitive development and learning as fundamentally social processes (Simon, 1995). Vygotsky (1986) is very clear in his view of this: "Thought development is determined by language, i.e., by the linguistic tools of thought and by the socio-cultural experience of the child" (p. 94).   Vygotsky was one of the first cognitive theorists to develop a coherent theoretical framework regarding the roles language, culture, and social interactions play in individuals' cognitive development (Edwards & Mercer, 1987). He attempted to provide a theory of intellectual development in which language development within a cultural context played a significant, if not dominant role. In the next few paragraphs, I attempt to explicate some of Vygotsky's theories on cognitive development relevant to this study.

Drawing from Piaget's theories on egocentric speech, Vygotsky and his team conducted their own research in attempts to further investigate what causes egocentric speech and the role it plays in children's development. Egocentric speech, a kind of talking to oneself, was found to play an important role in students' development, and carries out the function of realistic thinking. They found that egocentric speech, presented itself more frequently when children, engaging in an activity, were faced with problems or challenges to complete it. It was determined that for egocentric speech to present itself, "disruption in the smooth flow of activity" (Vygotsky, 1986, p. 30), was an important stimulus. Egocentric speech emerged when the child tried to "comprehend the situation, find a solution or plan a nascent activity" (Vygotsky, 1986, p. 30) and would talk to himself as he was trying to find a paper to write with, or build a tower with ill-fitting blocks. Furthermore, it was found that egocentric speech became a tool for thinking in the "proper sense" (p. 31), where it facilitated the "seeking and planning the solution of a problem"

(p. 31). Egocentric speech was thought to have disappeared completely in school-aged children, but Vygotsky's team hypothesized that it eventually evolved into silent "inner speech" (Vygotsky, 1986, p. 32), the foundation of a child's thinking, and which facilitates the development of logic in the child, as Piaget initially suggests (Vygotsky, 1986). Vygotsky believed that all speech was social; its primary function is to communicate with others. However, somewhere along the path of development, speech breaks off into two branches, egocentric speech and communicative speech, each of which has its own, separate function and purpose. Egocentric speech emerges when the child transfers social collaborative behaviors to "the sphere of inner-personal psychic functions" (Vygotsky, 1986, p. 35). Within this process of transfer, egocentric speech develops into inner speech, the constructs of human thought.

In other writings, Vygotsky (1978) makes a concrete connection in the role speech plays in a child's solving of "practical tasks" (p. 26), where, along with their eyes and hands, "speech and action are part of one and the same complex psychological function" (Vygotsky, 1978, p. 25). I have also witnessed how students rely on forms of egocentric speech to facilitate their thinking as they attempt to solve a problem. I have witnessed, many times, students thinking aloud as they work through a mathematical task, not necessarily in order to communicate a message to someone else, but to think about the activity they are engaging in. There are moments where I have observed adults (myself included) speaking to themselves as a way to clarify ones' thoughts, play out a scenario, or explore possibilities. I have also found in my own teaching practice, in many instances, having students explain their thinking to others verbally assisted them in determining misconceptions or faulty logic. In this latter instance, the act of verbalization of their thoughts helped them reconsider their logic, and they usually ended up saying something to the effect of, "oh, wait, that's not right. I need to rethink that." At some point in a student's development, social speech turns inward, moving from having an interpersonal function to include an intrapersonal function, to become the basis of inner speech. This process facilitates the student's use of guidance provided through social interaction with another person to guide himself (Vygotsky, 1978), employing his inner speech.

Vygotsky's work also included formulating a theory to account for the relationship between children's development and learning. Working from the premise that learning influences and shapes a student's development through interactions with adults and peers in his environment, Vygotsky differentiated between a child's actual developmental level and potential level of development, or "zone of proximal development" (Vygotsky, 1978, p. 86). The process of learning creates a student's zone of proximal development, which is the distance between a student's actual developmental level, as defined by his independent level of problem solving, and his potential developmental level, meaning the level of problem solving activity he can accomplish with assistance from an adult or peer (Vygotsky, 1978). The zone of proximal development defines internal processes that are in the initial stages of development and have not yet matured in the student, but through interactions with others, have the potential to develop through to independence. Furthermore, Vygotsky states that only through learning where students interact with others in his environment, can the internalization of processes within a student's zone of proximal development occur (Vygotsky, 1978).

I include a discussion of Vygotsky's sociocultural learning theory in this literature review not only for its inclusion of social interactions with adults and peers as a central construct for learning, but also because it accounts for the relationship between students' learning and development, considerations central to any instructional approach. Furthermore, it explicates the role language and communication plays in cognitive development, ideas central to the use of discourse within a mathematics classroom. Whereas the constructivist theory of learning accounts for internal processes of cognitive development occurring within the child, the socio-cultural learning theory posits the use of language and social interaction as central to development and learning.

The sociocultural learning theory compensates for what is missing in radical constructivism by defining the influences social interaction and the culture and contexts in which they occur have on students' cognitive development. However, it is not without criticism and problems as well. Vygotsky characterizes the process of development within a student's zone of proximal development as internalization, as the student interacts with peers and adults in his environment. However, he does not fully describe the nature of that interaction. Working with a knowledgeable other has been criticized for encouraging the teacher's taking an authoritative stance, where knowledge is considered a static, external body of which a learner would internalize. Instruction within Vygotsky's model can be viewed as an apprenticeship, which might work in other contexts, but not necessarily in mathematics. Upon completion of working with a more knowledgeable other, the learner and adult might look at their product and discuss their shared sense of goals and "effectiveness of their labor" (Confrey, 1995, p. 204). However, this is not always the case in mathematics, where the product might be a concept for which the child has yet to fully grasp its goal, such as development of the associative property of addition, what it means to be equal, or the Pythagorean theorem. Furthermore, an over-reliance on the development of linguistic forms of communication without the development of conceptual understanding may occur (Confrey, 1995).

Although sociocultural learning theories take into account the role language and social contexts play in cognitive development, they do not address the issue of interactions between individuals within a whole group context in which disciplinary-based interactions and collective understandings might develop. In research conducted by Yackel and Cobb (1996) the constructs of negotiated, taken-as-shared meanings and interactive constitution were introduced. Grounded in Symbolic Interactionism, these constructs have implications for the nature of social interactions occurring within the context of the mathematics classroom. Having students and their teacher interactively constitute taken-as-shared meanings define the nature of the social context and the interactions which occur. In this way, the constructs of symbolic interactionism help define the nature of social interactions occurring within a classroom environment. As this has implications for the present study, a brief review of the literature on symbolic interactionism follows.

Symbolic Interactionism

Symbolic interactionism is a relatively new theory in cognitive science and psychology, taking shape and influencing the theories focusing on the individual, predominant in the literature at the time, emerging in mainstream cognitive research around the late 80s and early 90s (Bauersfeld, 1994). Hugh Blumer's 1969 seminal book, Symbolic Interactionism: Perspective and Method, was his (self-proclaimed) attempt at providing a coherent statement on the position of the symbolic interaction theory as well as its methodological position in research. Heavily influenced by George H. Mead's analysis of social interaction (as well as other diverse American theorists), symbolic interactionism is based on the idea of "general similarity" (Blumer, 1969, p. 1). While both sociocultural learning theory and symbolic interactionism highlight the social aspects of cognitive development, focusing on participation and collectivism, Vygotsky's theories view learning as a process of acculturation into a context (Cobb & Bauersfeld, 1995). Complementing that view, symbolic interactionism focuses on the meaning formed through individuals' interpretations and interactions with each other (Blumer, 1969).

Symbolic interactionism rests on three premises:

> 1) Human beings act toward things on the basis of the meanings that the things have for them.
> 2) The meaning of such things is derived from, or arises out of, the social interaction that one has with one's fellows.
> 3) These meanings are handled in, and modified through, an interpretive process used by the person in dealing with the things he encounters (Blumer, 1969, p. 2).

While sociocultural learning theorists view the process by which individuals develop meaning as internalization and appropriation (Fosnot & Perry, 1996) symbolic interactionists view the process as one in which meaning for "objects" (Blumer, 1969, p. 10) raises out of the way individuals regard and act toward objects, influencing the way others regard those objects. Objects are defined as anything that can be referred to, including anything physical, social or abstract (Blumer, 1969). Through this process, a negotiated meaning is formed of the object. Symbolic interactionism provides the theoretical basis for negotiated and taken-as-shared meanings inherent to the discursive activity within a mathematics classroom.

German theorists H. Bauersfeld, G. Krummheuer, & J. Voigt (1988), developed a way to take the symbolic interactionist theory and apply it to the field of mathematics education, in an attempt to mediate psychological and social theories within the discipline. In doing this, they developed their own version of interactionism, which complements the constructivist theory (Cobb & Bauersfeld, 1995) and has provided the theoretical framework for research conducted around mathematical discourse in U.S. classrooms (Cobb, Yackel & Wood, 1992, Simon, 1995, Hufferd-Ackles, Fuson & Sherin, 2004). Focusing on the micro-culture in the classroom (Bauersfeld, Krummheuer & Voigt, 1988, as cited by Yackel & Cobb, 1996), their work includes a genuine consideration and incorporation of the internal, learning-as-adaptation view in radical constructivism, as well as the notion of reflexivity in ethnomethodology (Mehan & Wood, 1975). The principles of their theory and overview can be read in an essay published by Bauersfeld in 1994, and its ideas will be briefly summarized here. In their view, individuals experience a process of mutual adaptation, through communicating with others. This adaptation occurs through a process of modifying interpretations through which individuals negotiate meaning (Cobb & Bauersfeld, 1995). Situating this within the context of mathematics learning, individuals participate in a "culture of mathematizing" (Cobb & Bauersfeld, 1995, p. 8). Participating in this culture involves more than the observable skills and actions that students engage in. Rather, mathematizing lies in the implicit processes of learning mathematics, in the construction of the way in which one learns mathematics including when to do the what and how to do it (Cobb & Bauersfeld, 1995). Within an interaction, what the listener receives undergoes interpretations within the brain, following prior interpretations that occurred in previous interactions. Throughout these interactions, the listener has had to "adapt to the culture by developing viable reactions and trying to act successfully" (Cobb & Bauersfeld, 1995, p. 274). Within this culture, the role of language is "orienting" for the listener, (Bauersfeld, 1995, p. 275). An object is then formed through the process of orienting and interpreting. Furthermore, the object is informed by adaptations made within the culture in which the individual resides, including social interactions and the negotiation of meanings within "consensual domains" (Bauersfeld, 1995, p. 275).

Social Constructivism

Challenging to characterize, social constructivism has an interesting genesis in that it grew out of a blending of multiple perspectives: constructivism, sociocultural learning theory, enthnomethodology, and symbolic interactionism (Cobb, 1994). While there are those theorists who argue that learning occurs within the individual first, followed by social activity, and vice versa, the social constructivist view is that knowledge is constructed within the individual, but is done through social interactions and within a sociocultural context (Au, 1998, Atwater, 1996, Bauersfeld, 1988, 1992, Palinscar, 1998). The term, Social Constructivism, has become common in the literature, and within this theory, there is also wide variability in views and emphases (Fosnot & Perry, 1996). I have chosen to focus my review of the literature on work that seemed to have made genuine attempts at taking into account internal cognitive development as well as the social means by which it develops, not giving preference to either perspective. Merging constructs from the constructivist and sociocultural learning theories is not a trivial matter, and can lead to an overemphasis or mischaracterization of either one (Confrey, 1995). In their analysis, Fosnot & Perry (1996) highlight the idea that humans are social beings, and our ways of knowing are both constructed within the self and within a culture of interaction. "We cannot understand an individual's cognitive structure without observing it interacting within a culture" (Fosnot & Perry, 1996, p. 18). Social constructivism finds value in both the radical constructivist theory and sociocultural learning theory, and views them as complementary, rather than in opposition (Wood, Cobb & Yackel, 1995). At its most basic level, learning is viewed as a coordination of individual and social construction (Simon, 1995). Situated within the context of mathematics, "it is useful to see mathematics as both cognitive activity constrained by social and cultural processes, and as a social and cultural phenomenon that is constituted by a community of actively cognizing individuals" (Wood, Cobb, & Yackel, 1995, p. 402).

In Cobb, Wood, & Yackel's research within a 2nd grade classroom, from which they gathered and analyzed extremely rich data to publish numerous research articles on norms developed within mathematical discourse, they include one other theory of note which has insightful application to learning in the mathematics classroom. They draw upon the notion of reflexivity from Ethnomethodology (Mehan & Wood, 1975), which had several applications in their research. Reflexivity is the notion that as one influences something, you are also influenced by it. For example, each individual living within a culture brings skills, insights and ways of thinking which constitute the culture for that group. Therefore, one could say each individual's beliefs, values and skills make up the culture in the group. However, at the same time, the culture also exists as a negotiated, agreed-upon "taken-as-shared" (Cobb, Yackel & Wood, 1992, p. 118) reality, and in return, defines the current beliefs, values, practices and skills of each individual within the group. The notion of reflexivity is very salient in the social constructivist theory in that it further clarifies the social interactivity between individuals and the group's beliefs within their community.

In summary, social constructivism can be viewed as an integration of a variety of theories, and encompasses what we have currently come to understand about the way children learn. I believe the characterization above captures the essence of social constructivism, and through the study of Radical Constructivism, Sociocultural Learning Theory, Symbolic Interactionism, and Ethnomethodology, one will start to understand the coordination of these theories into what is called social constructivism.

What this Means for Instruction – The Theoretical Framework

With the introduction of the NCTM Principles and Standards (2000), and the recent Common Core State Standards for Mathematics (NGA & CCSSO, 2010), a call for a shift away from the previous behaviorist approach to teaching mathematics, where direct instruction was the recommended application of behaviorist accounts of learning occurred. In the behaviorist approach, teaching was viewed as shaping of learner responses through using procedures such as modeling, demonstration, and reinforcement of correct or close to correct learner responses

(Palinscar, 1998). Characteristics of this view of learning is evident in such interactions as the I-R-E, or Initiate - Respond – Evaluate (Mehan, 1979), where the typical discourse pattern is for the teacher to ask a question, or initiate a response from a student, the student would provide one, and the teacher would provide evaluative feedback, and move on (Cazden, 2001, Edwards & Mercer, 1987). Direct instruction, the teaching model most commonly associated with this view of learning, is characterized by a highly sequenced curriculum, where the teacher is in control of the "pace, sequence, and content of the lesson" (Baumann, 1988, as cited in Palinscar, 1998, p. 347, Fosnot & Perry 1996). Research regarding direct instruction suggests that it is an effective approach for teaching factual content, but less so for reasoning and problem solving (Palinscar, 1998). In this approach, teachers basically "funnel" (Wood, 1998, p. 175) students' thinking into seeing a mathematical situation in only one way and limiting responses to "prepared, unequivocal reactions" (Bauersfeld, 1992, p. 468). In addition to the limitations of this teaching approach, there are also limits to the behaviorist perspective, which is lacking in its account of how learning actually develops (Fosnot & Perry, 1996, Palinscar, 1998).

The blending of cognitive development theories described in previous sections (Constructivism, Sociocultural Learning Theory, Symbolic Interactionism) into social constructivism provides the theoretical framework for the development of mathematical discourse as an instructional approach in the classroom. Defined as "the system of concepts, assumptions, expectations, beliefs and theories that supports and informs your research" (Maxwell, 2013, p. 33), the theoretical framework is "derived from the orientation or stance that you bring to your study" (Merriam, 2009, p. 66), and provides the lens through which a study is viewed (Merriam, 1998). The following literature review will describe aspects of teaching and learning from current and past research related to the development of mathematical discourse in an elementary classroom.

Learning Communities

Drawing from sociocultural learning theory, in which learning cannot be separated from the context in which it occurs, establishing an environment in which mathematical discourse can occur is a key factor in students' mathematical learning. Namely, the establishment of a community of learners*,* in which a goal is to advance the collective knowledge of the group, and of the individual as well, through engaging in discourse (Bielaczyc & Collins, 1998, Hufferd-Ackles, Fuson, & Sherin, 2004). Within this community, there is a "culture of learning, in which everyone is involved in a collective effort of understanding" (Bielaczyc & Collins, 1998, p. 2), and what counts as mathematically valid is based on reasoning rather than on who is perceived to know the most math in the community. Diversity of perspectives is embraced, and sharing of students' mathematical activity and tentative ideas are viewed as learning opportunities for everyone, including the teacher. In this community, the teacher takes a facilitative stance, in which he/she might shift back and forth between "participant in the discussion and that of commentator of the discussion" (Rittenhouse, 1998, p. 173). Students would need to feel comfortable with taking risks and engaging in exploratory communication of their mathematical activity as well as mathematical arguments in which students agree to disagree respectfully (Lampert, Rittenhouse, & Crumbaugh, 1996). This necessitates the need for the establishment of norms, which minimize the social and personal risks that might interfere with mathematical argument (Lampert et al., 1996). Not a trivial endeavor, researchers have found that there may be initial discrepancies between the students' and teachers' expectations for their social roles in the mathematics classroom. Students, from prior classroom experiences, may rely on social-based cues for determining what counts mathematically (Yackel & Cobb, 1996). For example, a student may have the notion that being questioned by a teacher about a solution equates to being told they are incorrect (Cobb, Wood, & Yackel, 1996). Students and teachers also need to navigate the tensions that exist between argumentation promoted by reform-oriented mathematics instruction and their out-of-school conceptions of argumentation (Lampert et al., 1996), necessitating the consideration of how to create a safe environment for students to express thinking that might be different from their peers. Adding to the complexity, the development of such learning communities involves co-establishing ways of communicating and discourse patterns, including norms and deciding what counts as evidence by the teacher and students through their interactions (Lampert, et al. 1996).

Developing a community of learners as an instructional approach (Rogoff, 1994), drawing from Situated Learning (Lave & Wenger, 1991), is rooted in the idea that the environment in which students learn defines and shapes the knowledge that they develop. The mathematics students learn would depend on the context and the manner in which they learn (Ticknor, 2012). Moreover, learning within a community of learners occurs as individuals participate in shared experiences, incorporating a variety of roles and viewpoints within a sociocultural activity. Rogoff (1994) explains:

> Instead of one individual trying to control and address 30 students at once, it is a community working together with all serving as resources to others, with varying roles according to their understanding of the activity at hand and differing responsibilities in the system (p. 214).

The goal in this type of community is to advance the collective knowledge within a culture of learning that involves everyone within that group as they participate in a learning activity (Scardamalia & Bereiter, 1994). Through the process of contributing toward the collective constitution of understanding, each individual's development occurs as well (Goos, 2004, Scardamalia & Bereiter, 1994).

Within the analytical framework for this study, the Math-talk Learning Community framework (Hufferd-Ackles et al., 2004), a classroom community is defined as one in which the teacher and students use discourse to support the mathematical learning of all participants. A primary goal of such a community is to understand and extend one's own thinking as well as the thinking of others in the classroom (Hufferd-Ackles et al., 2004). The literature reviewed in this section provides the theoretical basis for the focus on developing a community of learners as a foundation for reformed-oriented discourse to occur. Furthermore, viewing mathematics as an activity in which we participate, rather than a body of knowledge and skills to be learned (Greeno, 2003), can be considered as appropriate particularly for the learning of young children, whose developing cognitions are closely tied to their experiences (Voigt, 1994). Through participating in experiences in which they do mathematics and engage in discourse in the spirit of the discipline within a community of learners, young children will have the foundation of direct experience from which to draw as they continue their mathematical learning.

All classrooms have cultures established through a variety of ways. Generally, they are established through the patterns of interaction which occur, the questions the teacher asks, the type of and manner in which responses are provided to students, the materials and tasks students work with, the physical classroom environment, school climate, all of which contribute toward the culture of the classroom. The manner in which the teacher initiates and establishes social norms in the classroom also help to define the culture within that classroom community (Wood, 2002). An example of a norm is the expectation that if one is going to present a solution to a problem, one must explain how it was attained and why it is a valid solution. Sharing mathematical thinking is the norm, and following that, the norm that the rest of the group is expected to listen, make sense of what was shared, and ask questions if necessary are other examples as well (Staarman & Mercer, 2010). This leads to developing a shared sense of responsibility for developing the mathematical understanding within the learning community. Norms such as these are co-constructed within the Community of Learners, where all members contribute toward their establishment.

The Role of Argumentation in Discourse

Although having students present their ways of thinking about a mathematical problem provides opportunities for reflection, learning through communicating with others, and investigating "more viable conceptual strategies" (Von Glasersfeld, 1991, p. xviii), the role conflict and mathematical argumentation plays within the discourse is also important to consider. Central to mathematical argumentation is the notion that what is taken to be understood is based on mathematical reasoning. Voigt (1994) makes the argument that by having students engage in the negotiation of what counts as reasoning, the teacher provides a scaffold (assistance, stimulation) for having students base their reasoning on a theoretical stance rather than an empirical one. Moreover, presenting examples of classroom interactions where a taken-as-shared meaning was developed without engaging in conflict, Voigt demonstrated that students developed mathematical meaning different from what the teacher intended. He argued that in order for meaning to develop, the group of individuals needed to go through a process of conflict, and only then can others truly know what an individual is thinking. His classroom examples illustrated this point, where students developed a communal sense of the commutative law of multiplication, which was different from what the teacher intended because they had not engaged in a process of conflict. This process was characterized by students' questioning of each other in order to understand and make sense of each other's ideas. Implicit within this process of questioning to understand, students pushed each other toward presenting reasoning for ideas, as well as making sense of the validity of those ideas and reasoning (Wood, 2002).

Reasoning and its relationship to students relying on themselves and an evidence-based approach lead to deciding what is mathematically valid (Ball, 1993, Lampert, 1990). In Ball's (1993) analysis for students' reflections about participating in a discourse community, children found that they understood and believed in the mathematics once they were convinced to do so. In this way, children saw the validity of mathematics as based on reasoning and evidence, processes rooted in the disciplinary nature of mathematics (Lampert, 1990), and not on the teacher as knowledge-keeper or on whoever is perceived to know the most mathematics. Being able to determine the validity of mathematical ideas on reasoning which makes sense to oneself and has gone through the process of substantiation within a learning community allows students to rely on themselves for what they can learn, and take more responsibility for their own learning (Ball, 1993). Furthermore, through engaging in mathematical discourse, which emphasizes the analysis of ideas, conjectures and reasoning communicated by students, the enhancement of conceptual understanding and procedural skills occurs (Hamm & Perry, 2002).

Questioning

The types of questions teachers ask students and students ask of each other, mediated by the type of tasks presented to students, largely determines the type of discourse students engage in, and can influence the culture of the classroom community (Wood, 2002). Hiebert & Wearne (1993), in their study of the relationship between teaching and learning, utilized the types of questions teachers asked as an indicator of the type of discourse that occurred in the classroom. When teachers asked questions requiring students to describe or explain their strategies or thinking, there were quantitative and qualitative differences in the discourse that occurred. Namely, more time was spent during lessons with students doing the talking and taking responsibility for the direction of the lesson than in those classrooms where recall questions were the predominant type being asked. The nature of the discussion differed as well, as can be expected. Recall questions asked for quick student responses in which the correct answers were sought, while questions that elicited student explanations resulted in student exploration, probing, clarification, and reasoning.

In a reformed approach to developing discourse in the mathematics classroom, the questions a teacher asks are the mainstay of his/her instruction. It is the way in which he/she guides the discussion and thinking of individual students (Davis, 1997, Herbel-Eisenmann & Breyfogle, 2004), and are traditionally linked to "the demand placed on students, with higher order questions leading to greater cognitive challenge for students" (Wood, 2002). The types of questions teachers ask largely determine the type of mathematical thinking and reasoning students engage in (Wood, 2002). Teachers also use questions to guide students through the "mathematical terrain of lessons" (Boaler & Brodie, 2004, p. 780) helping to define the classroom environment. The way the teacher asks questions and the questions that are asked of students also provide a model for students and encourage them to question each other in similar ways (Boaler & Brodie, 2004). However, in order to consider the questions teachers ask which influence the mathematical discourse, one must consider the teacher's stance as he/she encounters students' thinking and the purpose he/she has for asking certain types of questions.

As mentioned before, traditionally, teachers have taken an evaluative stance when listening to student responses, and this typically leads to an IRE pattern of discussion in the classroom, where the teacher asks a question of a student, the student provides a response, and the teacher evaluates what was said and moves on. With this stance, the teacher "sees the students' work in light of how she would approach the problem" and typically results in guiding the students "along the teacher's hypothesized learning trajectory for her students" (Doerr, 2006, p. 6). In this type of interaction, the teacher asks a question for which he/she has a pre-determined answer. Associated with the IRE pattern of questioning, the goal in this orientation is to determine student errors and correct them relative to the teacher's pre-determined learning trajectory.

In contrast to an evaluative stance are interpretive and hermeneutic orientations (Davis, 1997). With these orientations, teachers listen to students' responses with the goal of "accessing their understandings, seeking information through more elaborated responses, and asking for demonstrations or explanations" (Doerr, 2006, p. 6). With these orientations, the teacher's goal is to listen to students' ideas in order to understand their thinking. What is important to consider about the teacher's orientation is not only recognizing the importance of attending to student thinking, but also the mindset from which the teacher listens - for evaluative purposes, or more toward understanding student thinking, because the teacher's orientation influences the type of questions he/she asks (Davis, 1997, Doerr, 2006).

While there have been numerous studies in which the questions teachers asked were studied and categorized into various groups (Boaler & Brodie, 2004, Franke et. al, 2009, Harrop & Swinson, 2003, Hiebert & Wearne, 1993, Ilaria, 2002, Sahin & Kulm, 2008), Wood (1998) focused on looking at patterns of communicative interactions within the classroom. One common communication pattern in the mathematics classroom is "Funneling" (Wood, 1998, p. 170), in which student thinking is directed toward a single, pre-determined path for attaining an expected solution by the questions the teacher asks. An alternative pattern of interaction, "Focusing" (Wood, 1998, p. 172), occurs when the teacher asks clarifying questions about the "discriminating aspects of the solution" (p. 175) in a student's explanation, which the rest of the class may not have understood. In this interaction pattern, the teacher never takes control away from the students' explanation, but rather, spends time helping students make sense of it through asking probing and clarifying questions. By doing this, the teacher communicates to the students that their ideas are as valuable as her own (Wood, 1998). Questions which a teacher asks which lead toward a focusing interaction pattern would be characterized by probing sequences of questions, where teachers are mainly trying to clarify and understand students' thinking. This allows for students to see relationships between their own thinking and movement through the mathematical work, including examining others' ideas (Franke et. al, 2009). Focusing questions provide opportunities for students to respond or develop their explanations, so wherever the discussion leads to, it is in relation to the student's initial thinking. In contrast, Funneling questions direct students toward a correct answer and solution method, but not in relation to what students originally thought (Franke et. al, 2009).

Within a Focusing pattern of interaction, it is logical to associate an interpretive orientation with asking probing questions. If a teacher is interested in understanding student thinking in a non-evaluative way, he/she will naturally ask students probing questions to better understand their initial ideas. Researchers have found probing questions play a significant part in encouraging students' development of mathematical thinking (Ilaria, 2002). Their findings indicated probing questions allowed students to clarify initial thinking that may have been ambiguous or incorrect and sometimes move the students toward providing a correct response (Franke et al. 2009, Smith & Stein, 2011). Moreover, probing questions focusing on clarifying student thinking pushed students toward developing reasoning for their initial ideas, thereby leading students toward thinking based on mathematical reasoning (Boaler & Brodie, 2004, Smith & Stein, 2011). Probing questions, defined as those questions which ask students to further explain, clarify, or elaborate on their ideas (Boaler & Brodie, 2004), would be associated with a more interpretive stance in questioning, as the teacher attempts to make sense of students' thinking for herself and help other students to do so as well. Furthermore, taking an interpretive or hermeneutic stance and utilizing probing questions within a Focusing pattern of interaction supports the use of inquiry and multiple solution methods and diverse ways of thinking about a task or problem. Utilizing probing questions and taking an interpretive stance allows students to revise their individual ideas and develop their solutions, and shifts the evaluation of ideas to the rest of the students (Davis, 1997, Doerr, 2006).

Recent research on teacher questioning has shown that although teachers might know how to initially elicit students' mathematical thinking, they have a more challenging time with following up student thinking in ways which allow students to see relationships and connections between others' ideas and their own, and help them further develop their mathematical ideas (Franke et al., 2009, Smith & Stein, 2011). Franke et al., (2009) found that teachers' asking follow-up questions was not a guarantee that students would elaborate on their initial explanations. What did make a difference, however, was the type of questions teachers asked. These researchers found that leading questions, most associated with a Funneling interaction pattern, from an evaluative orientation, often did not lead to student elaborations, while probing sequences of questions, most associated with a Focusing interaction pattern, from an interpretive or hermeneutic orientation, almost always elicited further elaboration from the student. This provides more opportunities for a student to communicate the reasoning behind his thinking, inviting further questions from the teacher and other students, leading toward a shared understanding of the mathematics under discussion.

Mathematical Tasks

Although it has been the position of this study that a student-centered approach to teaching and learning mathematics allows for students to construct mathematical meaning for themselves, that does not preclude the involvement of the teacher in the education of her students. On the contrary, the teacher has a heavy influence over what the students learn and the manner in which they learn it, particularly in understanding the relationship between the mathematical tasks with which students engage and the learning of mathematics (Arbaugh & Brown, 2005, Hiebert & Wearne, 1993). In studying the relationship between teaching and learning, Hiebert & Wearne (1993) found that two features of the classroom, which had an influence on student learning were the mathematical tasks presented to students and the discourse students participated in. The relationship between the mathematical tasks and discourse and their influence on student learning is a complex one. In their study, the researchers found evidence to support the notion that these two "instructional features influence learning by affecting the kinds of cognitive processes students engage" (p. 422), and are dependent on the learning goals of the classroom. In their study, they found mathematical tasks and discourse to be very "robust features of the classroom" (p. 420), and engaging in certain types of tasks and discourse seemed to encourage particular ways of mathematical thinking. Students in a classroom in which contextualized tasks were posed, models and representations were utilized to solve them, and reflective and analytic talk was the norm, performed better in solving novel tasks in an end-of-unit assessment than did their peers who had had experiences focused on procedures and worked with problems presented symbolically. Because of this relationship between tasks and discourse, and taken together, their influence on student learning, it is necessary to review the literature about mathematical tasks which relates to this study.

The type of mathematical tasks upon which students work each day and the way in which they work on them have a heavy influence on the type of mathematics students learn and how they come to view the discipline over time (Doyle, 1983, Sullivan, Clarke & Clarke, 2013). Mathematical tasks are classroom activities or problems whose purpose is to focus students' attention on particular aspects of the content and the ways of thinking about them (Doyle, 1983, Henningsen & Stein, 1997, Hiebert & Wearne, 1993). In the NCTM's (2000 & 2004) documents, Principles and Standards for School Mathematics and Principles to Actions, educators are encouraged to move away from providing students with exercises in which procedural practice is the focus and utilize non-routine tasks allowing for problem solving and mathematical reasoning to occur. Characteristics of these non-routine tasks include a need for interpretation and formulation of the problem mathematically, where a solution and the ways one might arrive at a solution is intentionally left open for students to determine, or there is a possibility of multiple solutions to the problem (Nohda, 2000, Pehkonen, 1997). The goal, with this approach, is not to produce correct answers, but to "promote students' mathematical ways of thinking and creativity" (Nohda, 2000, p. 8). Engaging with these types of problems allows for each student to use their unique background experiences, different perspectives, and natural dispositions to find their own answers within their range of abilities (Kwon, Park, & Park, 2006, Nohda, 2000). In other words, utilizing open tasks allows diverse learners access into the mathematics embedded within the task, and to participate in the mathematical work (including the discourse development) of the classroom community by drawing upon mental images, prior experiences, and making possible connections to the problem (Martino & Maher, 1999). It also allows students to make connections between how they see mathematics in their own lives and the mathematics they learn in school (Wiliam, 1998, Boaler, 1998, Sullivan, Clarke & Clarke, 2014), as well as engage in high-order thinking and reasoning about mathematical ideas (Kwon et al., 2006, Spangler et al., 2014). Open tasks allow for students to ask questions, make and define their assumptions, and develop initial conjectures. This is in contrast to completing isolated exercises whose main purpose is to have students practice rote procedures without any study of relationships or connections to concepts. In summary, providing open tasks in the mathematics classroom is the starting point of opportunities for students to engage in significant mathematical thinking rather than acquisition and practice of rote procedures.

Adding to the complexity of the relationship between tasks and discourse, there is also the level of rigor of the tasks to consider, including maintaining that rigor during the various phases of implementation (Stein, Grover & Henningsen, 1996, Henningsen & Stein, 1997, Stylianides & Stylianides, 2007). In a study conducted by Stein, Grover & Henningsen, (1996) which looked at different levels of tasks and their enactment in the classroom, the authors identified various types of factors which influence the implementation of the tasks, all of which are determined by the discourse defined by the teacher and students. They include: 1) Classroom norms, 2) Task conditions, and 3) Teachers' and students habits and dispositions (p. 467). In the study, the discourse provided reasons for decline in the cognitive demand of the task as it was implemented, and the cognitive level depended on how the teacher handled the way students engaged with the task. The level and type of mathematics experienced by the students depended on the way in which the tasks were implemented in the classroom. The way the tasks were implemented involved aspects of the discourse within the class, which in turn influenced the level of cognitive demand as the task was being implemented with students. The researchers in this study defined maintaining a "high level" (P. 467) as having students involved in explaining and justifying their thinking, making and testing conjectures, framing problems, and looking for patterns. A "high level" implementation also included whether or not mathematical formulas, procedures and algorithms were connected to concepts and/or meaning. It was found that the maintenance of the high rigor during the implementation of the tasks depended on 4 factors: 1) The degree to which the task built on students' prior knowledge, 2) Competent performance modeled in the form of having students present their solutions to the class, where "multiple representations, meaningful exploration, and appropriate mathematical justifications" (p. 481) were utilized with the group, 3) A sustained focus on justification, explanation and the meaning evident in the teacher's comments, questions, and feedback, and 4) The type and amount of scaffolding provided to students, which did not take away from the complexity of the task, or provide too much assistance. As previously stated, all of these factors fall under the realm of mathematical discourse, and as evident in this study, the relationship between tasks given to students and the discourse involved as they are enacted are heavily interwoven.

What is relevant for this study is the idea that the tasks in which students engage and the mathematical discourse in which they engage as they work on those tasks are heavily intertwined to determine the type of educational experience students have. Selecting an appropriate task depends on one's purpose (Doyle, 1983, Hiebert & Wearne, 1993), and though researchers equate higher cognitive demand and developing a mathematical disposition with the development of conceptual understanding in students (Doyle, 1983, Hiebert & Wearne, 1993, Stein et al., 1996, Martino & Maher, 1999), the Common Core State Standards for Mathematics explicitly state that students should have mathematical experiences in which both procedural, conceptual understanding, and application are the foci of the standards, and equal time and intensity should be spent on each (NGA & CCSSO, 2010). Depending on the teacher's purpose and goal for her students, she will need to develop skill in selecting the appropriate task to help her students work toward that goal (Ball, 2000), as well as in the use of discourse to help students develop their mathematical thinking as they work though the task. Particular tasks lend themselves to more analytic and exploratory discourse, therefore it is extremely important that teachers select tasks that will provide students with opportunities to grapple with significant mathematics and engage in discussions in which their mathematical thinking is pushed forward (Martino & Maher, 1999, Ball, 2000). A task should allow for students to reinvent mathematical ideas for themselves.

Though the selection of tasks is extremely important, students will not learn mathematics from tasks alone, nor will they learn mathematics by simply being provided with challenging tasks. Rather, the way they are enacted in the classroom also heavily influences the type of mathematics student learn. Mathematical tasks and the discourse in which students develop as they engage with those tasks are classroom features, which are almost impossible to separate because of the complexity of the classroom, (Hiebert & Wearne, 1993), and together, provide students with mathematical learning experiences.

The Mathematical Learning of Young Children

Because of this study's focus on studying a 1st grade classroom, a review of the literature regarding the way young children (ages 3-8) come to know mathematics and communicate their mathematical thinking was necessary. With the current focus on preschool and early childhood education (US DOE, 2010), there has been a strong emphasis on research specifically looking into the unique nature of young children's ways of thinking, and how that has informed current classroom practice by educators (US DOE, 2010). This section explicates some of the research pertinent to the present study. The intention here is not to imply that the ideas contained in current research do not apply to other age groups. Rather, the discussion here will focus on that which has been learned about young children's ways of thinking and behaving, but that does not mean the ideas may not apply to other age groups. Beginning with a discussion of what we know about young children's cognitive development delineated in the literature, this section explicates the role children's experiences play in shaping their mathematical understandings, including a look at the way children use language to make sense of the world.

The idea that mathematical knowledge is developed through young children's experiences in and out of school is an important one to consider. Beginning with research conducted by Piaget, the study of the ways young children (birth - age 8) learn and think about mathematics continues (Sophian 2007, Voigt, 1994). Prior research suggests that very young children, from birth, are capable of mathematical thinking (Sophian, 2008), and their mathematical knowledge is heavily dependent on their prior experiences (Sophian, 1999, Voigt, 1994). In a study conducted by Ginsburg, Lee, & Boyd, (2008) it was found that young children, from birth to age 5, develop an understanding of "everyday mathematics" (p. 3), including "informal ideas of more or less, take-away, shape, size and location" (p. 3). This type of experiential knowledge that originates from a child's informal experiences can shape and influence the way he learns and understands mathematics within a structured school setting (Guberman, 1999).

Vygotsky's (1986) theory of concept formation, in which a child's spontaneous concepts influence development of scientific concepts, also supports the notion that a child's experiences outside the classroom influences her understanding of meanings and concepts developed within the classroom. Empirical evidence of this relationship can be found in students' written and spoken language, or "texts" (Lemke, 2004, p. 3). In her analysis of classroom discourse within a science classroom, Hicks (1996) highlights how academic knowledge is constructed through the interweaving of discourses from various places in a child's life. Based on Lemke's (2004) principle of "general intertextuality" (p. 3), which states that because discourses are related, and because one text can be set against another, taken together, they create meanings, which a single text would not otherwise create. For young children in the primary school grades, whose academic discourses are as yet unfamiliar, text appearing in discourse may be heterogeneous, where differing discourses, including those from everyday experiences, may be juxtaposed in ways that might appear to be un-academic (Hicks, 1996). For example, a student may make a scientific observation of a phenomenon, communicate it verbally or in writing, and include other ideas from her social world. In Hicks' (1996) review, she provides an example of a first grade science classroom, where a student made scientific observations of silkworms which might be considered academic in nature, but in the same entry, included ideas from the Teenage Mutant Ninja Turtles movie and cartoon series. This juxtaposition of discourse may be typical of young speakers and writers, and because children's academic knowledge is judged heavily by what they say or write (Hicks, 1996), research into the development of students' mathematical discourse is extremely important. These differing discourses can be seen as diverse paths to the construction of academic knowledge, rather than as simply as evidence that individual children either did or did not possess academic knowledge (Hicks, 1996).

However, construction of knowledge through the intermingling of discourses does not occur without support from the teacher. Research on the practice of teachers in support of bringing together students' everyday experiences in and out of school have shown that allowing students to connect prior knowledge to novel situations and to "generate, apply and verify solutions" (Hamm & Perry, 2002, p. 126), lead to greater conceptual understanding and transfer (Bransford, Brown & Cocking, 1999, Hiebert & Wearne, 1992, Perry 1991, as cited in Hamm & Perry, 2002). Furthermore, the National Research Council (2009) recognizes communicating one's mathematical thinking as a way young children connect their informal experiences and mathematical ways of thinking with the more formal mathematics they need to learn. Strongly heuristic, their knowledge depends on their interactions with others (Mack, 1990, Sophian, 1999, Voigt, 1994, Vygotsky, 1978) including the teacher. Taken together, this research highlights the importance of the teacher's role in facilitating students' communication of their mathematical thinking as a way to connect their informal experiences and ways of thinking with the formal mathematics they need to learn.

Building upon the relationship between children's knowledge and prior experiences, observations of young children have shown that their conceptions are variable, and are bound to the context with which the child associates that knowledge (Lampert, 1986, Sophian, 1999).

This variability that characterizes the reasoning and problem solving abilities of young children includes the notion that their ways of thinking are "qualified" (Sophian, 1999, p. 16), meaning they understand concepts and can solve problems in some situations but not in others. Based on this notion of variability in children's cognitive development, Sophian (1999) describes implications for early mathematics education, one of which calls for "making young children's mathematical knowledge less elusive" (Sophian, 1999, p. 17) through assisting the child with making their thinking explicit. By communicating their solution methods and relating physical, pictorial, verbal and symbolic representations (Hiebert & Wearne, 1992, Lampert, 1986) in developing conceptual understanding, young children's thinking moves from being intuitive in nature to a more explicit state (Sophian, 1999). The degree to which a child is able to make her thinking explicit characterizes the difference between intuitive, emergent knowledge and stable, established knowledge by developmental psychologists (Sophian, 1999).

Consistent with the notion of variability in young children's thinking, Barnes (2008) presents the idea that students develop their understanding of the world through a special kind of talk in which they try out new ideas in relation to their pre-existing cognitive structures. This type of talk is flexible and allows for modifying ideas as words are spoken. Because this kind of talk is tentative, and is used as a way for students to make sense of ideas, their speech is hesitant, uncertain and may be nonsensical. Calling this "learning talk", or "exploratory talk" (p. 6), Barnes distinguishes this type of talk from "presentational talk" (p. 6), where one would focus on delivering a polished version of one's ideas for others' benefit. With presentational talk, one is most concerned with the delivery of ideas for others, whereas in exploratory talk, one is focused on "sorting out" (p. 7) ideas. Presentational talk also implies that it is meant for others' evaluation, and does not invite exploration of ideas. The emphasis in presentational talk is on "getting it right" (p. 7). Barnes also suggests that when teachers ask students questions that expect students to have the right answers, students engage in a form of presentational talk. Although there may be times when this is appropriate, he suggests that teachers be cognizant of the "learning sequence" (p. 7) in which this occurs, where students are provided with sufficient opportunity to engage in exploratory talk prior to being evaluated. Recognizing both types of talk are important for learning, Barnes suggests that teachers be aware of the different types of talk and when they are appropriate in instruction. Although Barnes' research was not focused specifically on young children, his ideas about the expression of tentative thinking through talk is particularly applicable to first graders, who are still developing their communicative abilities. His research also focused on studying interactions of students in small group situations, but recommends that students have the opportunity to explore their tentative ideas through talk, in a variety of contexts, including whole group interactions.

In summary, taken together, the research presented here regarding the development of young children is meant to describe some of what we know about the way young children learn through a blending of cognitive, social and communicative factors. What we know about the learning of young children; that they bring with them a wealth of experiential knowledge, their development involves multiple processes of connecting their prior knowledge with new and formal knowledge, and the variability in their understandings, along with the grounding in social constructivist ideas, informs the present study.

Discourse as an Approach to Co-Constructing Mathematical Meaning

One salient feature of reform-oriented mathematics teaching involves the use of alternative patterns of discourse in which to learn mathematics, which influences mathematical learning in fundamental ways (Hiebert & Wearne, 1993). There exists a long history of research into classroom discourse, spanning at least two decades. Because mathematical discourse is a heavily researched topic, there exists different ways of defining what constitutes mathematical discourse in the spirit of reform. For the purpose of my study, various perspectives, each of which contributed to the rich description of what constitutes reform-oriented mathematical discourse, will be included. The description which follows attempts to capture the nature of discourse consistent with the spirit of reform, as well as the manner in which it is established and enacted in the classroom. For my research, discourse will be defined as:

1) values, ways of talking and forms of reasoning that characterize mathematics as a discipline (Lampert, 1990, Lemke, 1990, NCTM, 1989, 1991, Yackel, Cobb, Wood, Wheatley & Merkel, 1990).

2) activity which engages students in the negotiation of taken-as-shared meanings of whatever it is they are talking about and occurs within a socially constructed context (Edwards & Mercer, 1987, Hicks, 1996, Voigt, 1994, Yackel & Cobb, 1996).

3) the communication of mathematical explanation, arguments and reasoning through writing and use of other visual representations (Moschkovich, 2003, NCTM, 1991, 2014, Voigt, 1994, Wood, 2002).

In my experience working with mathematics teachers, observing lessons and in my own teaching, I have seen the role communication plays in students' thought development. The act of explaining their thinking to others often facilitated students' clarification of their ideas for themselves. These occurrences are examples of the dialogic nature of discourse, which involves communicating in ways that "generate new meanings" (Wertsch & Toma, 1995, p. 165). This is particularly applicable for young children, whose thought development occurs through the evolvement of egocentric speech into inner speech, the basis of a person's thoughts (Vygotsky, 1986). Beginning as a type of social speech, egocentric speech emerges when the child transfers social, collaborative behavior to inner processes, and can be observed by the child talking to himself when he is forced to stop and think. Citing Piaget, Vygotsky (1986) provided the example of how arguments between children lead to logical reflection in their inner selves. Through the dialogic use of language and communication, young children develop and clarify their thoughts. One can conclude from these theorists, that communication and language play an important role in thought development, particularly for younger children (Vygotsky, 1986).

Discourse involves and determines the way in which lessons are enacted in the classroom. All classrooms have a pattern of discourse established, but in reformed approaches to teaching mathematics, the theme of negotiated understandings is central, where knowledge is socially constructed and agreed upon by the group (Edwards & Mercer, 1987). This is similar to the notion of taken-as-shared meanings described by Cobb,Yackel & Wood (1992) and Fosnot & Perry (1996) and involves the development of intersubjective meanings within a group. Whether or not the source of cognitive development is a social activity or an individual one, meaning is developed through the common agreements of the individual and the community in which that individual interacts and resides. Mathematical meaning making is a sociological process, where meaning is constructed within interactions of individuals (Voigt, 1994) and is inseparable from that sociological context. Another term for this view is "consensual domain" (von Glasersfeld, 1991, p. xvi), which refers to that which multiple individuals agree upon as their perceptions of their subjective worlds, developing what is referred to as intersubjectivity. An example of a consensual domain would be numbers, where mathematical facts are derived from individuals' consensual agreements rather than on the idea that there is an external "objective reality" (von Glasersfeld, 1991, p. xv). The development of intersubjective knowledge was described as a process through which taken-as-shared, mathematical meanings are negotiated within a group

(Voigt, 1994). Negotiated meanings begin as explicit statements or arguments that might contain reasoning and justification, but then eventually become tacit, and are "stable expectations from the individual's point of view" (Voigt, 1994, p. 180). Over time the stable expectations form "commitments between participants" (Voigt, 1994, p. 180) and intersubjectivity emerges.

Through their research, Cobb, Yackel, Wood and McClain further elaborated on the development of taken-as-shared meanings. Beginning with instruction that is "experientially real" (McClain & Cobb, 1998, p. 61) for students, so they can "evoke the imagery of the situations described in problem statements when solving tasks" (McClain & Cobb, 1998, p. 61), students develop personally meaningful mathematical activity. When students communicate their mathematical activity to others, they engage in "reflective discourse" (McClain & Cobb, 1998, p. 59), which is characterized by shifts in the discourse where what is said becomes an "explicit object of discussion" (McClain & Cobb, 1998, p. 59). Through social interactions, in which students engage in the process of explaining and justifying their activities and ideas with reasoning, students "adapt their mathematical activity" (Cobb, Yackel & Wood, 1992, p. 114) and accommodate each other, all the while influencing and being influenced by others. Through this process of negotiation and adaptation, institutionalization of taken-as-shared meanings occurs, and intersubjectivity is developed within the group. Intersubjectivity also occurs within ethnomethodological accounts of everyday life, through descriptive stories in which an event is re-constructed. Through this process, the event is made observable and understandable to oneself or another person, and to make something observable and understandable means to "endow it with the status of an intersubjective object" (Leiter, 1980 as cited in Voigt, 1994, p. 180).

The idea of a co-constructed, negotiated and agreed-upon reality also influences another aspect of the dialogic perspective (Staarman & Mercer, 2010), which looks at the reflexive nature of language. Reflexivity, another ethnomethodological concept (Voigt, 1994), refers to the idea that the way people perceive the social context largely determines the way in which language is used in that context. Reflexively, through interactions, people create the social context, and re-negotiation of that context in an on-going process. All kinds of dialogue within the specified context depend on the participants having a shared understanding of how to make those interactions occur.

In a reformed mathematics classroom, the teacher and students develop the discourse in the classroom, co-establishing the norms, or ways of speaking, sharing, questioning, disagreeing respectfully, as expectations of the mathematics classroom. Normative behavior is characterized by "the interlocking networks of obligations and expectations that exist for the teacher and students that influences the regularities by which they interact and create opportunities for communication" (Goffman, 1974, as cited in Wood, 1998, p. 170). These patterns of interaction influence and can determine the type of mathematical meaning students make (Wood, 1998). Within a community of learners in a mathematics classroom, where reform-oriented discourse takes place, a particular culture and set of normative behaviors need to be established.

Previously mentioned researchers Cobb, Yackel, Wood and their colleagues have conducted extensive research in the area of social and sociomathematical norms (Yackel & Cobb, 1996), and concluded that establishing such norms in the classroom allow for students to take responsibility for their own learning and become empowered to develop social autonomy to learn and do mathematics (Yackel & Cobb, 1996). Furthermore, they found that through the process of negotiating the sociomathematical norms of the classroom during interactions with their peers, students were also developing personal understanding of mathematical meanings (Yackel & Cobb, 1996). Sociomathematical norms are different from general social norms in that they involve negotiating "what counts as mathematically different, sophisticated, efficient, and elegant" (Yackel & Cobb, 1996, p. 461) in student explanations and arguments. General social norms are considered agreements, which might be utilized in any subject area, such as explaining your thinking, providing reasoning, respectfully disagreeing, questioning each other, and listening to and making sense of what others explain. Not to be considered trivial, establishment of general social norms are critical to creating the environment necessary for students to engage in mathematical discussions. Examples of this type of norm are exhibiting genuine respect and consideration for all ideas communicated in the classroom (NCTM, 1989), acknowledging that all comments made by a student were meaningful to that student, and how to regard incorrect answers and misconceptions (Yackel, Cobb, Wood, Wheatley & Merkel, 1990). It involves knowing what to do as a listener while students communicate their explanations, and using that to influence the type of follow up questions asked (Davis, 1997). Both social and sociomathematical norms play different but necessary roles in establishing a community of learners in that through their negotiation by interacting individuals, they establish the culture of the classroom, as well as define the process by which mathematical meanings are interactively constituted (Yackel & Cobb, 1996).

Mathematical Discourse in the Classroom

Regarding this research project, my claim is that having the students and the teacher co-develop mathematical discourse within their classroom is an instructional approach to teaching and learning mathematics. This view is supported in the literature (Yackel et al., 1990, Ball, 1993, Voigt, 1994, Simon, 1995, Hicks, 1996, Hamm & Perry, 2002, Imm & Stylianou, 2011).

In this approach, students would be presented with genuine mathematical problems and tasks for which there are no known solution paths (NCTM, 2000) and for which they would develop their own solution strategies. In addition, the problems would be open and rich enough such that a variety of students could have access into mathematics and develop mathematical meaning through working with them. Students would also have ample opportunity to grapple with the mathematics inherent in the tasks, and an attitude that struggle is a positive aspect of working on mathematical problems would be fostered within the culture of the classroom. Facing and grappling with genuine problems is an important factor for students to experience cognitive dissonance and to construct their learning (Yackel, et al., 1990).

Students would work in small groups to work through tasks and problems, and participate in "teacher orchestrated whole-class" activities and discussions (Yackel et al., 1990, p. 15). During these different groupings, students would be encouraged to explain their solution methods to others and provide reasoning for those methods. The teacher takes two roles in supporting students in shifting from "peripheral participation" to "full participation" (Rittenhouse, 1998, p. 172), one as participant in the discussions, and another as a commentator, highlighting aspects about the discourse. Being a participant in the discussion would involve guiding, probing, and helping students clarify their thinking for themselves and the classroom community. This would be done mainly through asking probing questions and treating each person's thinking as if it was meaningful to that student at the time (Yackel et al., 1990). Students would also be active members of the learning community, encouraged to question each other to understand one another's thinking, with the view that it is the responsibility of everyone in the group to make sense of students' ideas, make conjectures and establish taken-as-shared meanings. Through this clarification and reasoning process of examining each other's mathematical thinking, as well as looking for relationships amongst the way different students thought about their solutions, co-constructed mathematical meaning can occur.

Students would have opportunities to use manipulatives and other mathematical tools in ways which are meaningful to them and which facilitate their mathematical communication and thinking. Current research has shown that student actions on manipulatives and interpretations of the mathematics inherent in the problems co-evolve over time (Martin, 2009). Furthermore, students need opportunities to grapple with the problems and through their actions, make sense of the mathematics (Martin, 2009). Utilizing manipulatives, with flexible structures and interpretations, has also been found to be positively associated with subsequent learning in students (Martin & Schwartz, 2005). Vygotsky (1978)'s theories highlight the relationship speech has on students' actions and manipulations of objects, and through uniting one's perceptions, speech and actions, internalization of the "visual field" (Vygotsky, 1978, p. 26) occurs. These theories on the interplay of acting on representations and mental interpretations support the notion that when students act on physical objects as they are grappling with problems, particularly in their incipient stages of learning a concept, they are more likely to be able to further their subsequent understanding of that topic (Martin & Schwartz, 2005). Within this instructional approach, students would have ample opportunities to act on physical manipulatives as a way to develop their conceptual understanding of topics (Smith & Stein, 2011).

The beginning stages of communicating mathematical thinking are important in the process of developing mathematical discourse in the classroom, and create an environment in which students share in the responsibility for their own learning. Students' initial explanation are usually exploratory in nature, where students verbally present their initial and tentative mathematical ideas to other students and the teacher, at times with the use of manipulatives or other mathematical tools. Putting forth a conjecture into the discourse community naturally assumes that the person making the conjecture is opening oneself up for revision of his/her thinking and assertions (Lampert, 1990). When students use the words, "I think" rather than "it is", they are implicitly making clear that their ideas are open to revision and feedback from others (Lampert, 1990). During this initial sharing, what students share is more exploratory in nature, and allows them to take responsibility for their thinking. This kind of talk is distinguished from the more formal, precise and rehearsed talk a speaker has prepared for judgment by external criteria. Exploratory talk allows the learner to share one's own initial thinking for which one holds responsibility because those ideas are ones' own (Lampert, 1990).

In summary, engaging in mathematical discourse provides the opportunities for students to develop mathematical meaning as set forth in reform ideals. More importantly, engaging in the co-construction of mathematical discourse in the classroom is an instructional approach through which students develop mathematical meaning consistent with current cognitive and sociological theoretical perspectives on learning. Engaging in discursive practices provides opportunities for students to develop mathematical meaning in a way that is individually constructive, and allows students to come to know mathematics as a discipline. Most importantly, it honors students' natural inclinations toward thinking mathematically, and provides them with ways to extend what they empirically understand to be consonant with what the wider society has institutionalized as mathematics. Utilizing mathematical discourse as an approach to learning mathematics allows formal instructive experiences in school to begin with students' thinking and experiences (Hamm & Perry, 2002), what is personally meaningful to them, and develop mathematical meaning as accepted by the wider society. Through these experiences, they come to view themselves as capable of engaging in mathematical thinking.

The Common Core State Standards for Mathematical Practice

Based on the NCTM's (2000) Process Standards and the Strands of Mathematical Proficiency from the National Research Council (2001), the eight Common Core State Standards for Mathematical Practice describe dispositions and processes students ought to engage in and develop as they interact with mathematical contact (NGA & CCSSO, 2010). They are:

> 1) Make sense of problems and persevere in solving them, 2) Reason abstractly and quantitatively, 3) Construct viable arguments and critique the reasoning of others, 4) Model with mathematics, 5) Use appropriate tools strategically, 6) Attend to precision, 7) Look for and make use of structure, and 8) Look for and express regularity in repeated reasoning (NGA & CCSSO, 2010, Standards for Mathematical Practice section).

These standards describe the processes of mathematically proficient students, and do not prescribe a particular teaching approach for students' development of them. However, the nature of the standards implicitly affords an instructional approach in which students communicate their mathematical thinking and provide justification for their reasoning. Students also need opportunities to make sense of problems and solution strategies for themselves, as well as being able to see relationships between abstract mathematical ideas and the contexts in which they occur.

As mentioned previously, teachers find the standards for mathematical practice one of the most perplexing aspects of the Common Core State Standards, and are unsure about how to afford their students opportunities to develop them. Furthermore, research on the enactment of these standards has yet to take place (Heck, Weiss & Pasley, 2011). It is the position of this study that developing mathematical discourse within a community of learners as a teaching approach which will provide opportunities for students to develop the standards for mathematical practice. The question is, to what degree and what is the nature of the development? How are they initiated in students' thinking and developed as related to their discourse? I believe aspects of the standards for mathematical practice will manifest themselves within the mathematical discourse of the classroom community, and therefore a purpose of this study is to look at the nature of those manifestations.

CHAPTER 3
METHODOLOGY

As mentioned in previous chapters, my interactions with classroom teachers, afforded me the opportunity to observe that making a shift toward developing reform-oriented mathematical discourse in the classroom is not prevalent in Hawai‘i's public schools. Furthermore, in professional development sessions I was confronted by skepticism regarding the capabilities of young students to co-construct taken-as-shared mathematical meanings and engage in productive mathematical discourse. I also saw the current climate of reform in mathematics education as an opportunity to have ideas about teaching and learning that have remained within the research community filter into actual classroom practice. These personal observations motivated my desire to study the mathematical discourse, in the process of incorporating reform ideals, within a first grade classroom. The research questions guiding this inquiry are:

1) What does whole group mathematical discourse look like in a first grade learning community?

2) What aspects of whole group mathematical discourse emerge in the engagement of a first grade learning community in reformed-oriented approaches to teaching and learning?

3) How do the Standards of Mathematical Practice in the Common Core State Standards manifest themselves as related to the mathematical discourse of first grade students?

The purpose of this research was to study the processes by which the members in a first grade learning community co-constructed mathematical discourse in which taken-as-shared mathematical meanings were interactively constituted. This study also examined the way in which the Standards of Mathematical Practice from the Common Core State Standards manifested themselves in students' discursive experiences in the classroom.

This chapter will explicate the research design and methods utilized in this study, beginning with a presentation and justification for the selected research paradigm that was most appropriate for this inquiry. This will be followed by a description of the design and methods that were used.

The Qualitative Paradigm

The notion that young children's mathematical learning is rooted in their empirical experiences is supported in the literature (Voigt, 1994, McClain & Cobb, 1998, Sophian, 1999, Copple & Bredekamp, 2009). Concurrently, from the perspectives of the cognitive development theories that provide the theoretical frame for this study, active construction and social processes are intrinsic to children's learning as well (Blumer, 1969, Vygotsky, 1986, von Glasersfeld, 1991, Kamii & Housman, 2000). Taken together, the direct, real-world experiences and the social processes in which children learn, shape their understanding of the world. As such, it was essential to use a research paradigm that captured the social and cognitive processes of students as they individually and interactively developed the discursive environment, taking into account their natural propensities for learning. Consequently, an interpretive epistemological orientation (Erickson, 1985, Merriam, 2009) was utilized for this study. In an interpretive view of doing research, reality is socially constructed, and therefore, there is no single reality, but "multiple realities or interpretations of a single event" (Merriam, 2009, p. 8). Furthermore, the notion that individuals' actions on objects are based on their interpretations of those objects, including social influences by others in their society (Erickson, 1985), and are open to reinterpretation and change, is central to interpretive research. Because the nature of developing mathematical discourse involves the construction of a social context through the interactions of the members of a classroom community, this paradigm supported the purpose of this inquiry; to study the way in which students developed mathematical discourse.

The Research Design & Methods

Interpretive research includes a family of approaches, such as ethnography, case study, phenomenology, narrative analysis, etc., all of whose interests lie in the study of meaning in social life, including how that meaning is socially constructed (Erickson, 1985, Merriam, 2009). In this study, I employed an ethnographic case study design, in which an in-depth and systematic look at the mathematical discourse of an elementary mathematics classroom community served as the "bounded system" (Merriam, 2009, p. 40). A case study is an "empirical inquiry" (Yin, 2014, p. 16), into phenomenon with which the context is inextricably linked. This required an in-depth and systematic study of the mathematical discourse that developed within the natural setting of the classroom, over a period of time. Three special features of case study designs supported the choice of design and methods. First, it is "particularistic" (Merriam, 2009, p. 43), meaning that the focus was on a particular situation, event, or phenomenon. The case is "important for what it reveals about the phenomena and for what it represents" (p. 43). Second, a case study is "descriptive", (Merriam, p. 43) meaning the end product includes a "thick description" (p 43) of the phenomenon under study. From the field of anthropology, thick description includes a "complete, literal description of the incident or entity being investigated" (Merriam, 2009, p. 43). A case study involves the analysis of as many variables as possible, and their interaction over time is also included in the description. Third, a case study is "heuristic"(Merriam, 2009, p. 43), meaning that the study "illuminates the reader's understanding of the phenomenon under study" (p. 43). These three features of case study design aligned with the orientation, purpose, and research questions for this study. They supported the choice of using a case study design and guided the methods for data collection.

Within the classroom community, the main method for data collection was participation-observation fieldwork (Erickson, 1985), where the focus was to describe, analyze, and interpret a "culture-sharing group's shared patterns of behavior, beliefs and language that developed over time" (Plano-Clark & Cresswell, 2010, p. 244). Drawing from the field of cultural anthropology, educational ethnography was conducted with the researcher, as participant-observer, immersed at the site over a period of time conducting fieldwork (Erickson, 1985, Merriam, 2009, Plano-Clark & Cresswell, 2010).

In Erickson's (1985) view, the focus for the participant observer is "immediate and local meanings of actions, as defined from the actors' point of view" (Erickson, 1985, p. 119) rather than solely on the development of "rich description" (Merriam, 2009, p. 16). Local means taking the view that negotiated meanings of the group, having developed through interactions amongst its individuals over time, are specific to that group, resulting in the evolution of a micro-culture (Erickson, 1985). Locality is also based on the notion that "life is continually being lived anew", meaning today's enactment of the warm-up during the math lesson might differ from that of the day before. Because young children's understanding variable, is closely tied to their direct experiences and their real-world interactions (Sophian, 1999), the immediate and local is appropriate for studying these particular subjects. Because the nature of mathematical discourse is specific to a social context constructed by the members of the community, the focus on the mathematics classroom as a micro-culture whose collective meanings are interactively constituted and specific to that "local" group of students makes this view appropriate for this study. However, the view of locality does not preclude the search for patterns across units of study, or for universals. Interpretive research involves the search for "concrete universals" (Erickson, 1985, p. 130), in which specific cases are studied in great detail and subsequently compared with other cases also studied in detail.

The major focus of the fieldwork emphasized study of the group through observation, but also focused on individual meanings of the students and the teacher captured in the researcher's written field notes and reflections. In addition, interviews with the teacher were conducted. Together, these data collection techniques resulted in "thick description" (Merriam, 2009, p. 28), which not only includes observational data, but the researcher's interpretations as well. Each observation was video-recorded and each interview audio-recorded, therefore a reduction in the reliance on looking for repeated occurrences of actions in real-time observations was possible and resulted in a more detailed and complete analysis (Erickson, 1985). Being able to replay video-recordings allowed for the analysis of particular instances clearly without being bound to real-time events, and resulted in a closer look at subtle instances in which the students and teacher interacted.

Selection of the Subjects

The first grade mathematics classroom for this case study was purposefully selected (Merriam, 2009) to be one in which developing mathematical discourse with students was a teacher goal. The purpose of this project was to study the mathematical discourse, and not the individual teacher's capabilities; however, the teacher heavily determines the nature of the discourse that develops within a classroom (Rittenhouse, 1998). This necessitated the careful selection of a teacher who would be open to having an outsider observe her lessons throughout the year, and who would provide her students regular opportunities to develop whole group mathematical discourse. The selected teacher, Mrs. Dennis,[1] with approximately 16 years of experience teaching first graders, had just completed participation in a two-year school-based professional development project, in which developing mathematical discourse through a problem-based approach to learning mathematics had been a focus. Thoughtful planning and consideration about the mathematics she wanted her students to engage in was her norm, and she continued to seek avenues for developing herself professionally. The school's focus at the time was to gain professional development to improve the teaching and learning of mathematics. Part

[1] All identifiable names have been replaced with pseudonyms to protect research subjects' identities.

of the professional development involved participating in a lesson study structure (Lewis, Perry, Hurd, & O'Connell, 2006, Tolle, 2010) whereby teachers within a grade level observed each other's implementation of a collaboratively developed lesson. This particular teacher was selected because her orientation seemed to be consistent with a reform-based stance. She valued a problem-based approach to teaching and learning mathematics and regularly engaged her students in solving open problem and tasks. Furthermore, she exhibited strong levels of openness, flexibility, and a willingness to take risks. She also demonstrated a strong interest in mathematics and the learning of mathematics for herself and her students. All these characteristics contributed toward the selection of this classroom as subjects of this study.

There were 20 students for most of the year in this classroom, all ranging in age from six to seven years old. During the course of the year, two students left and three were added, resulting in a relatively stable group of students. Two students were categorized as English language learners, and were regularly pulled out during portions of math class to attend language-focused lessons. There were no students requiring special education, although one student was referred during the course of the year for needing support with reading and writing.

Description of the Site

A first grade classroom in an urban elementary school on Oʻahu served as the "bounded system" (Merriam, 2009, p. 40) for this study. Located within the city of Honolulu, the school had an approximate enrollment of 600 students, from grades K through 5. Designated a Title I school, meaning they receive supplemental funding from the federal government, approximately 61% of their student population was categorized as having free- or reduced-lunch status. Approximately 14% of the student population was categorized as English Language Learners, and 6% were in Special Education programs. About 60% of their kindergarteners attended preschool, and the school was categorized as a Continuous Improvement School in the school year 2013-2014 under the Hawaii Department of Education's Strive HI index ratings. This rating is one step below the highest rating a school in the HIDOE can receive, indicating a low need for state interventions within this school.

This was the first year the school selected to implement the newly state-adopted mathematics program, Stepping Stones (SS) (Burnett, Irons, DePaul, Stowasser, & Turton, 2013), published by Origo Education. A K-5 program developed specifically for the Common Core State Standards, it incorporates the use of technology as well as activities and manipulatives in its lesson format. The lesson structure typically began with a teacher-led activity or discussion, followed by a guided discussion that transitioned, into the use of the SS student journal. There were 12 modules in the SS first grade program, with 12 lessons in each module. Situated problems were available in supplemental resources of the program from which teachers could draw to use in their lessons. However, as with any other program, the teacher's purpose and intentional use of the materials determines the type of learning experiences the students have (Stein, Grover & Henningsen, 1996). Teachers were provided with opportunities to attend professional development sessions provided by the publisher. Mrs. Dennis attended the introductory sessions, but did not attend any of the follow up workshops.

Although the program was adopted by the state and all teachers were expected to use the materials in their instruction, the manner and degree to which the program materials were used was at the discretion of the teacher. Teachers were encouraged by state leaders to be flexible with the program materials and to utilize them in ways they deemed most effective for their students. The structure of the program included an introductory teacher-led activity, sometimes including the use of manipulatives and interactive digital tools, which transitioned into a teacher-guided "Step-in" (Burnett et al., 2013) discussion within the Student Journal. This Step-In discussion involved a series of teacher-led, digitally displayed questions, pictures and prompts, meant to introduce students to the mathematics being investigated in the lesson. Each student was assigned his/her own journal, which consisted of a book containing colorful two-page layouts of 14-20 exercises for each lesson, which all students were expected to complete. This series of exercises were followed by one or two "Step-Up" (Burnett et al., 2013) exercises meant to be more challenging problems which not all students were expected to complete. In previous years, Mrs. Dennis had utilized a teacher-developed program, which drew materials from a variety of sources. Even with the adoption of new materials, the mathematics-driven activities she previously developed that were integrated throughout various parts of the day, such as during calendar time in the mornings and in transitional activities between other subject areas, were continued. The main portion of the mathematics instruction occurred for approximately 60-90 minutes each day, and served as the focus for this study.

Procedures for Data Collection

Before data could be collected, several preparatory steps needed to occur. First, the necessary approval from the Human Studies Program was attained as well as approval from the

HIDOE to conduct research in a public school classroom. Parents' consent to allow their child to participate in a research project was acquired, as well as student assent to participate. Mrs. Dennis met with each child individually and read from a prepared script seeking student assent. Once the necessary forms were complete, and proper approvals were obtained, the actual data collection commenced. Three rounds of data collection occurred, with the first commencing at the beginning of November, the second in February, and the third in April. Conducting rounds throughout the year afforded the opportunity to collect data of discourse development that occurred during the course of the year. In each of the rounds, observations were conducted approximately twice a week, for seven weeks in round one, four weeks in round two, and three weeks in round three. The start date for data collection was pushed back from September to November because the length of time to gain clearance to conduct research in the HIDOE had been underestimated. This served as a learning experience for the planning, designing and submitting of applications for approvals far in advance from the planned start date. Prior to the start of the data collection period, I visited the class once a week for six weeks as a way to gain entry (Merriam, 2009) into the classroom. The focus during these initial visits was on desensitizing the students and Mrs. Dennis to the video and audio equipment, establishing my role in the classroom and building rapport with the students. Although I did not formally collect data, I used this time to gain a holistic sense of the context (Erickson, 1985), and become familiar with general characteristics of the classroom, such as the lesson pace and flow and class routines.

The following procedures for collecting observational data were identical for each round. Video equipment recorded entire lessons, capturing the introduction, small group work, and whole group discussions. The camera was manipulated so as to capture central points of discussion and interactions throughout the lessons. Field notes were typed on a computer, using a researcher created template (see Appendix A) capturing contextual aspects of the lesson that might be missed by the video, such as the lesson title, goals of the lesson, the types of activities involved, the mood, or occurrences that were outside the scope of the camera. In addition, points of transitions were noted so as to assist with the coding of the data. Directly following each lesson, descriptive observations and initial impressions of the observed lesson were also written on the template. The videos were then manually transcribed and prepared for analysis. Although video recording captured the entire lesson, because the focus of this project was to study the whole group mathematical discourse, only whole group interactions were transcribed. The preparation for analysis involved an initial form of coding (Merriam, 2009), where distinguishing characteristics and contextual factors about the lesson were noted on the template. This included information such as the type of lesson (Problem solving or SS), the overall topic of focus, and who served as presenters. This significantly aided the analysis of the data, and helped define the context of the lesson.

Mrs. Dennis' insights and observations of the nature of the discourse and its development served as a second source of data, against which my data was compared. Semi-structured, post-observation interviews with the teacher occurred approximately once a week or as the teacher's schedule allowed, to collect her observational data of the lessons and discuss the lessons. Each interview was audio-recorded, and began with collecting the teacher's general observations of the lesson, followed by a mutual sharing and discussion of our individual observations. To ensure researcher bias was not introduced into the interviews, they always commenced with the teacher's reflections and observations, after which I shared my interpretations. The interviews were recorded and conducted using a set of general questions (See Appendix C) meant to capture Mrs. Dennis' observations and impressions of the lesson. Following each interview, I reflected on her ideas, observations and insights through writing.

Analysis of the Data

Analysis of the data occurred in three distinct phases, two of which occurred during all three rounds. Overall, the analysis process followed Erickson's (1982) guidelines for starting with a holistic view and moving toward a more refined level of analysis. Three main sources of data were used for analysis - transcriptions from lesson observations, transcriptions and notes from the teacher interviews, and my field notes and reflections. This section will first describe the general process of analysis, followed by an explication of how the analysis occurred for each round.

Inherently an inductive process (Merriam, 2009), the analysis of the qualitative data in this study involved the development of themes and categories through the use of constant comparison (Corbin & Strauss, 2008, Merriam, 2009) methods of analysis. Because the focus of this research involved studying whole group mathematical discourse, analysis focused on only discussions in which the whole group participated. Analysis of the data occurred during the data collection process, allowing for the identification of aspects of instruction on which to focus in subsequent observations and to listen and observe with more sensitivity (Corbin & Strauss, 2008). Initial analysis also allowed me to refine the way I conducted the observations and captured field notes, as I came to recognize what was essential and extraneous. The data went through both a macro- and microanalysis (Corbin & Strauss, 2008), through which a broad analysis of the data was utilized to develop a general portrait of the mathematical discourse as well as a detailed examination of interactions within lessons to develop a specific view of the discourse. The use of microanalysis complemented the general analysis and afforded a comprehensive study of the mathematical discourse in this classroom. Round one data was used in an intense analysis process to develop foundational themes that make up the findings. This was followed by the analysis of data from subsequent rounds.

Phase I.

The analysis process involved several passes through the data, the first of which occurred as the videos were transcribed into written text, for each round. The transcription process served as an initial look at the data, during which initial segmenting and coding occurred. Each segment began with entry into a central topic of discussion, and ended when the focus of discussion switched, signaling a transition to another topic of focus. Transitions may have been signaled through a change in speaker, or through the asking of a question, that shifted the focus of the group. Initial comments, questions and notes were connected to segments, leading to an initial process of coding (Merriam, 2009).

A second pass of the data involved a review of the transcripts at a micro-analytical level, during which codes were assigned to segments, capturing aspects of the associated interaction and raising them to a conceptual level (Corbin & Strauss, 2008). Memo writing was also used in conjunction with the coding, through which exploration and comparisons of the data focused on defining each code by “its analytical properties” (Charmaz, 2006, p. 82). Continuous comparisons of the data occurred, through which broader categories, or themes, (Merriam, 2009) were developed. Memo writing continued, but focused on explicating each theme’s characteristics through the comparison of segments of similar or related codes. Each theme also referenced the coded segments of raw data, which allowed for organization and the easy access to the raw data for memo writing. Throughout the categorization and memo-writing process, observations and codes and conjectures were constantly compared with field notes and information from the teacher interviews.

Phase II.

Once themes were developed using round one data, Phase II commenced with a similar process of coding and memo writing for the subsequent rounds. Codes and segments of raw data from rounds two and three were then compared with the themes and subsumed codes from round one, to which they were added or led to the creation of new themes. This process of coding each round separately followed by comparison and integration into existing themes afforded the opportunity to characterize the uniqueness of each round, as well as the relationships between rounds, including the progression of the discourse through the course of the year. The themes that were descriptive in nature were then pulled out and used to form a description of the discourse, addressing research question one. Interrelationships between the remaining themes and existing theory and literature were then explored through a series of memo writing, through which the conclusions for this study emerged.

During the last stages of analysis, once themes had been developed using all three rounds of data, the application of an analytical framework was utilized as a way to refine the themes and complete the analysis. The Levels of a Math-Talk Learning Community Framework (Hufferd-Ackles et al., 2004) was used for this purpose. The use of the framework afforded the movement of the analysis from a descriptive level to a more theoretical one (Merriam, 2009), through the validation, elaboration and introduction of aspects that defined each theme. The subsequent use of the framework afforded the opportunity for themes based on the observational data to emerge, rather than the imposition of pre-determined categories upon the data. This approach was consonant with the paradigm of the study. To orient the reader to this stage of the analysis, a brief discussion delineating the Math-Talk framework follows.

In an effort to support teachers as they attempt to change their practice toward reform-oriented approaches, researchers Hufferd-Ackles et al. (2004) developed the Levels of a Math-talk Learning Community Framework. Based on a year-long case study of one 3rd grade classroom in an urban Latino school, which exhibited tremendous growth in developing whole-class discourse. The framework emerged in the progression of the classroom community's interactions as they developed mathematical discourse consistent with reform. The framework describes, "key components of a math-talk learning community as well as the "intermediary levels along which the community develops" (Hufferd-Ackles et al., 2004, p. 82). Through their analysis, four components were identified which captured the community's growth over time: 1)

Questioning, 2) Explaining math thinking, 3) Source of mathematical ideas, and 4) Responsibility for learning. (Hufferd-Ackles et al., 2004, p. 87). Within each of these attributes, developmental trajectories defining teacher and student actions are explicated (see Appendix B). Four levels - 0 through 3 - were identified for each trajectory, and emerged in the development of discourse, as the classroom moved from a traditional learning environment toward one that embraced ideas from reform.

In their study, not only were the trajectories at each level captured, but the researchers also included a brief discussion about the nature of the students' transitions and movements through the levels, including how the teacher supported this. They found that movements upward might be disrupted when new topics were introduced, and the teacher needed to take a more central role in the discourse to assist students with being comfortable with the language specific to the topic. To facilitate students' movement from level to level, the teacher shifted her focus toward aspects such as understanding students' mathematical thinking and coaching them as they became more central figures in the math talk.

Comparing the themes, codes and their defining aspects to the trajectories and levels of the framework allowed for their refinement. Comparison to the framework also allowed for my assertions to be refined as well, and provided the bridge from the analysis to the conclusions.

Phase III.

The third phase of analysis involved a third review of the transcripts, in which the focus was on manifestations of the CCSS Standards for Mathematical Practice (SMPs) in the discourse. Unlike the first two phases, this phase involved a mainly deductive mode of analysis (Merriam, 2009), and was conducted at a macro level. This phase was also unique in that I had already completed Phases I and II, when I began the analysis focusing on the SMPs. Having completed a comprehensive analysis of the data, at the macro and micro levels, afforded me the opportunity to draw upon the analyses and themes from the first two phases to aid in this third and final analysis.

Utilizing a deductive mode of thought, interactions were coded according to defining aspects of the standards. This process began with a review of each standard, in which one to three characteristics foundational to that standard was identified. A third review of the transcripts commenced, beginning with a review of interactions attached to themes developed in phases one and two. This review was used to identify segments of interactions that exemplified any one of the standard's characteristics, and were coded accordingly. Utilizing a series of reflective and exploratory memo writing, the way in which aspects of the standards manifested within the coded segments were explicated, resulting in a description of the relationships between the SMPs and the discourse of this class. In addition to using coded segments, overall observations and information from field notes and teacher interviews were incorporated into the description to provide a comprehensive look at how the SMPs manifest within the discourse.

Ensuring Quality and Credibility

Many researchers argue that in considerations of validity and reliability, one must take into account the philosophical assumptions underlying the paradigm of a study (Merriam, 2009). Because this study is based on a view in which there are multiple realities, traditional views of ensuring validity and reliability are not applicable. Rather, taking a view of ensuring credibility and "confirmability" (Merriam, 2009, p. 211) is more congruent with the paradigm of this study. Based on this view, a central consideration involves the notion of congruence, where the findings of the study accurately portray reality.

Several strategies were utilized throughout this study to ensure the trustworthiness of its assertions and findings. The first was to establish an "evidentiary warrant" (Erickson, 1985, p. 146) to test the validity of the assertions that were made. This was done near the conclusion of the analysis, where the assertions and themes were systematically compared with the entire data corpus (data from observations, field notes, and teacher interview data) to determine if there were any confirming or disconfirming data. This process of comparison and cross-checking occurred throughout the process of analyses, as themes were developed and assertions were generated. For example, the developing themes were compared with pre- and existing data for confirmation or disconfirming evidence in order to clarify and refine them during their development within phase two.

The second strategy employed was the triangulation of data. The process of triangulation involves the cross checking of data from one source to the next (Merriam, 2009). Codes and themes developed from observational data were constantly compared with field notes and teacher interviews to check for consistency throughout the analysis. Reliability in qualitative studies is more concerned with the consistency of the results and the data than with the replicability of the findings (Merriam, 2009).

Lastly, credibility in the interpretations of the data exists in the nature of ethnographic study (Merriam, 2009). As the main research instrument, and in spending extended amounts of time studying the group's interactions in their natural classroom setting, I had the opportunity to be "close" (Merriam, 2009, p. 214) to the reality, more so than if an external instrument had been utilized to capture data. Being immersed in the natural setting allowed me to capture the complexities of developing mathematical discourse, taking into account the many perspectives in the room.

Limitations of the Study

Because this study is an in-depth look at the development of discourse within a bounded case, the ability to generalize is limited. However, the purpose of case-based research is particularly valuable in studying situations in which the context is inherent to the phenomenon, and cannot be separated from it (Yin, 2014). Furthermore, the value in utilizing fieldwork methods lies in highlighting "the invisibility of everyday life" (Erickson, 1985, p. 121), through an in-depth study into the nature of a phenomenon, rather than for the purpose of generalizability. This view was consonant with studying the complexities of mathematical discourse, which was the focus of this study.

In qualitative research, where the observer is the primary research instrument, interaction with the context and participants are assumed (Merriam, 2009). However, there is the danger that investigator bias toward a singular focus consistent with the theoretical frame will be assumed, as is ignoring disconfirming data. Erickson (1985) highlights the common mistake of jumping to premature conclusions too early in the research process. This was remedied by "deliberate searches for disconfirming evidence" (Erickson, 1985, p. 146) in the data during various phases of the analysis.

The initial intent in the planning phases of this study was to have Mrs. Dennis confirm some of the initial findings and assertions throughout the data collection process. However, because of time constraints, it was not possible to do this. Therefore, assertions, codes and findings that emerged during analysis were compared with information contained in teacher interviews as a way to ensure credibility.

CHAPTER 4
FINDINGS

The findings chapter can be broken down into three main sections. In this first section, a general description including aspects influencing the development of the mathematical discourse will be discussed to provide a context for the rest of the results contained in this study. This general description presents a macro perspective of the class' interactions over the course of the school year, and includes a description of instructional aspects that have a strong influence on the discourse. This will be followed by an explication of issues and challenges that arose throughout the year regarding the discourse of the classroom, as viewed through a micro-analytic lens. The chapter will then conclude with a discussion about the students' engagement in the Standards for Mathematical Practice.

Description of the Classroom and General Lesson Structure

This first grade classroom community consisted of 20 students and Mrs. Dennis. A document camera connected to a digital projector, which displayed visual images onto a white board at the front of the class was available for use, and there was a large empty space on the floor near the white-board for students to sit and have their whole group discussions. Six groups of four student desks set up in two rows of three sat behind the projector, and tables were in the back of the room. Three to four computers were set up along one of the walls.

The mathematics lessons in this class lasted anywhere from 60 - 90 minutes of active mathematical work and discussions, and usually took place right after the students' lunch period. The structure of the lessons generally involved an introductory discussion and/or activity, followed by independent work time at the students' desks, where students worked independently, with a partner, or with a small group of four students. Lessons concluded with two types of whole group discussions - the first in which students discussed the work they had completed at their desks, usually followed by a summarizing discussion in which the focus was on what was learned during that day's lesson. Because mathematical discourse involves the norms, values and mathematical sense-making developed collaboratively as a community of learners, and the discussions in the introduction and the summarization had a direct influence on the section in which students presented their work, the analysis focused on the three parts of the lesson structure in which whole group interactions involved mathematical discussions.

The curricular materials used in this classroom largely came from the SS curriculum. There were two general uses of the program materials. First, many of the contextualized problems given to students came directly from the Stepping Stones (SS) student journal or from supplementary problems provided in the program. Teacher-created activities were sometimes used in place of contextualized problems, and were related to the SS core lessons. When these were used, a single task or problem was given to students to work through. The second type of tasks used was the SS core lesson, used in conjunction with the SS student journal.

Introducing tasks & problems to the class.

Mrs. Dennis regularly posed problems to the class through explaining directions and reading the problems with students, sometimes emphasizing what the problems meant, clarifying the information given and what was asked for. Sometimes she also provided direct instruction, in reviewing strategies students could use for the current task, and at times explaining how students might record down their mathematical work. She also used the introduction time to hold a "what do you know" discussion, meant to activate prior knowledge about a concept that would be the focus of the day. Problems and tasks were always posed with verbal explanations, and in a few lessons, the teacher utilized a tool or manipulative to introduce the concept students would be working with that day. If a task involved the use of manipulatives, verbal explanations were provided to explain what the task was, and modeling of the task with the tools to set up the activity for the students was usually absent. Early in the year, the teacher would regularly explain the task or problem to students in a whole group context, and the process of making sense of the problem/task would extend into students' small group work at their desks. Part of the students' sense making of the task occurred as they engaged with the task, while the teacher circulated around to small groups and provided clarification and elaboration about what the task entailed. Later on in the year, as more of the students developed their independence, they usually were able to figure out the task after the whole group discussion and started directly at their desks.

During lessons in which the SS core lesson materials were the focus, the tasks and exercises were introduced using the program's digital materials, which usually consisted of a picture contained in the student journal, and a series of questions from the Step In discussion.

Following an introductory process recommended by the program, the teacher regularly had the students work through the questions through whole group discussions in which she asked the questions and elicited student responses, and sometimes asked students what was meant by vocabulary contained in the title as a way to begin the work. Students would then return to their desks to work on the exercises and tasks in the book independently or with a partner.

Whole group discussions – developing routines and norms.

Aspects of discourse involving cultural values, norms and routines will be implicitly included throughout the findings section of this paper, but a few are highlighted here to serve as precursors for further discussions. They are also included here to explicitly highlight aspects, which may not be apparent in other interactions presented.

During whole group discussions following their work on contextualized problems at their desks, students were regularly expected to share their strategies and work with the class. By share, I mean it was regular practice to be situated in front of the class, have one's work put on display for everyone to see on the document camera, and explain what one had done. This was one of the routines established from the beginning of the year, which set the context for having students engage in mathematical discourse. The focus of these discussions during rounds one and two included making sense of the meaning of numbers, strategies and concepts. In round three, the discussions turned to focus on developing students' facility with solving problems and using a variety of strategies for computing numbers. Rather than focusing on pure procedures for how to arrive at a correct answer, the teacher often asked the students to begin with explaining the pictures they drew and at times, why that was the selected method used. This is illustrated in the following excerpt, where students used interlocking cubes to represent a type of addition fact in which a number is doubled and two is added on to an addend, resulting in a sum that is two greater than the sum of the double (double-plus-two). The focus of the discussion was making sense of a student's thinking that had built a double-plus-two addition fact with cubes:

T - What do you think about what he built? Is this doubles-plus-two?

Sts.- Yes!

T - Can you explain to us how you think this is doubles plus two? How is this doubles-plus-two? Rex.

Rex - because he added two more instead of one.

T - Oh, because he added two more, ok.

S - instead of one.

T - Instead of one, ok. (points to Jerri)

Jerri - it's like he made double eight, and two more.

T - Ohhhh, can you explain that? B & G, can you listen to what Jerri, can you say that again?

Jerri - double eight, and add two more.

T - K, so do you agree that this is doubles, like this? (pointing to a tower of 16) Does this look like doubles plus two?

S - Yes.

T - Is that what you think? I don't think you think that. What do you think it should look like? Can you fix it? (hands the tower to Jerri) Jerri wants to fix it because she didn't think that equation, she said a different equation.

(Jerri takes off the two blue & breaks off the 16 cubes into two towers of 8)

During this interaction, Mrs. Dennis asked students to make sense of what it meant to be a double plus two by looking at the cube tower and written equation. Although there were places in the interactions in which procedures and teacher-determined goals were the main driver of the discussions, there were instances in which discussions on the meaning of concepts and numbers were the focus.

One of Mrs. Dennis' goals for students was to be strategic in their thinking, rather than to rely on following isolated procedures. This was validated in observations, where at times, the teacher asked for divergent ways of thinking and placed an emphasis on strategic thinking, providing opportunities for students to develop their understanding and skill in utilizing a variety of strategies. In this excerpt, the class is discussing different strategies that might be used to determine the sum of five plus four:

T - One, so that you can get nine. Anybody else thought of it in a different way, using a different double?

Ned - two?

T - Jerri used five plus five, she got ten and took one away. Can you use a different double? What's a different double that you could use?

Kerry - ten and six?
T - well, we're trying to figure out this equation. So Jerri said if I use five plus five, but I take one away, then I can get nine. What other double could you use besides five plus five?

Ira - four and four and add one

T – Four, four, because, and you just add one more. Okay? You look at this number, class, you look at this equation, you can use this double, or you can use this double, to help you figure out your answer. So if I had five plus six, which two doubles could I use?

Another apparent routine during mathematical interactions established throughout the year involved the way mathematical outcomes and solutions were validated during discussions. Although during interviews, Mrs. Dennis stated that the attainment of correct answers was not a goal she wished to emphasize for her students, the observational data overwhelmingly indicated that the drive to have correct responses and valid ways of attaining a desired response was reinforced throughout the year. In addition, she predominantly determined whether answers were correct or incorrect, and this was communicated to students in a variety of ways - as explicit utterances of "good job" or "yes" or "no", or sometimes a particular question directed the students toward a correct outcome. Although there were numerous interactions in which students' misconceptions would be the focus of the entire group's discussion, many times this was done as a way to not only highlight the misconception for everyone, but also to correct the student's mistake.

The routines of presenting one's work and initial ideas to the whole group, of focusing on sense-making of the meaning of mathematical concepts and ideas, and the valuing of different strategies were implicitly established throughout the year in many of the lessons, and highlight some of the values developed in the community. Although discussions focused on procedures and attaining single answers at times, these were not the predominant emphasis of the interactions over the course of the year. However, a distinct difference in the patterns of interaction occurred throughout the year, depending on the curricular materials from which the group worked. This will be discussed in the next section.

The relationship between curricular materials, tasks and discourse patterns.

A qualitative difference in the patterns of discursive interactions occurred, in relation to the curricular materials used. When the class worked from the SS student journal in their whole group discussions, interactions usually began with students calling out answers they attained in their work, and there was a tendency toward Initiate-Respond-Evaluate (IRE) patterns of interaction. During these interactions, the goal was to respond to Mrs. Dennis' elicitation with the correct answer to the exercise. Viewed by the teacher as targeted lessons through which students learn specific things, these interactions were mainly teacher-directed or directed by the teacher's interpretation of the program's goals. Once a student provided an answer, Mrs. Dennis would determine the next question and focus of discussion. An IRE pattern of interaction was predominant in the following excerpt, in which the class is reviewing their work in the SS student journal. Students were asked to determine the sum of a double addition fact, given a picture of dominoes containing dot formations. They were required to write the addition fact as an equation, and then add a dot to show a double-plus-one addition fact. They were then asked to write the equation for the double-plus-one addition fact they had just created:

> T - I don't think this page gave you too much trouble, you just had to build it with your counters, if you didn't know it, k. But this page, Ok, what's the first thing that we need to do, class? What's the first thing?
>
> S - make the doubles
>
> T - Make the doubles, ok, so let's see, who could make our double for number one.
>
> T - Shhh. K, Kylie, what double did you use for A?
>
> Kylie - uh, four plus four
>
> T - Ok, everybody looking up at the board.
>
> S - I know what it is. Eight.
>
> T - Greg, do you see the four plus four that they used?
>
> (silence)
>
> T - Here's four here, here's four here. K, did you write that? This is a double - four plus four, and what does that equal?
>
> Sts. – eight

T - Eight. (T writes answers in as sts. say them)

Ira - now draw a circle

T - K, now the second instructions said to…..

Ira - draw a circle

T - Color in. Color in your…..

S - circle.

T - Now what is the equation, Jackie?

Jackie - four and five.

T - Now what is the equation?

S - five plus four

T - Ok, I'm gonna write that one on the bottom one, here, five plus four equals nine, because they want us to do the turn around, but in the picture, what does it show it as?

Ira - four and five?

T - Four plus…..

Ira - five.

S - four plus five equals nine is on the top

T - Boys and Girls, they're already showing you the double (points to next one). What's the double here? What double do you see? Shhhhh. What double do you see?

Ned - Six and six

T - Ok, yes, you can see six and six, here's six, but it has one more, so actually it's not six, what should it be?

(several sts. speak at once) S - seven

T - Seven plus seven, and that equals…..

S - twelve

T - What is seven plus seven?

S - fourteen!

S - fourteen!

T - Fourteen. Very good.

I include this excerpt for several reasons. First, it highlights a typical pattern of interaction that occurred when the focus of a summarizing discussion was on the SS student journal. There was an IRE pattern of interaction, where the exercise was the initiating prompt, a student responded, and Mrs. Dennis provided an evaluation. The way the evaluation was communicated to the students varied - at times it was an overt "good job" or "thank you", but at other times it was a subtle repeating of the desired answer in a validating tone of voice. It also may have been validated with writing down of the answer on the book. Also highlighted is the interaction that occurred in response to an undesirable response, which again was communicated to students in various ways, through the initiation of a question, or an overt correction of the student response. Students' thinking behind their responses was not shared, and the focus was on the answers students provided. Also highlighted in this excerpt, was the use of a verbal fill-in-the-blank technique, where the teacher verbally provided a sentence stem for the purpose of receiving a desired, usually one to two word, response. Different from providing a sentence stem as a communication scaffold to assist students with communicating their thinking to others, the goal of this practice was to keep the students engaged in the discussion, but responding in a way that was predictable and toward a particular desired response. The focus in this interaction was also on the procedure for completing the exercises on the page, rather than on making sense of a double and double plus one, the objective of the lesson.

Although this was the typical pattern of interaction that occurred throughout the year when the focus was a SS lesson, there were interactions that departed from an IRE pattern in that the teacher asked students to explain how they got their answers each time an answer was given. In the following excerpt, the class reviewed their work in comparing two-digit numbers represented as base-ten blocks pictured on a balance in the SS journal. Terry is first called upon to share an answer to one exercise, followed by Rex:

T - Ok soo, Terry, what number do you have here?

Terry - mmmm. Thirty-two.

T - Ok, and for this number here.

Terry - Twenty-three

T - Good job, and which one has less, Terry?

Terry - twenty–twenty-three?

T - K, so we should have circled this one….Um, Terry, how do you know that that one has less?

Terry - because….the thirty-two has more, and the twenty-three, has less.

T - Ok, it's a smaller number, right? Does it have less tens?

Terry - (nods)

T - Yes, it has less tens. Ok, Rex, go ahead. The next one is - what's the next number?

Rex - I have fourteen.

T - Fourteen, how did you get fourteen?

Rex - cause there's a, there's ten on that side, and there's four on the other side, and I took zero out, and I put four, and made fourteen.

T - Ahhhh. look, boys and girls, Rex said this has fourteen. Do you all agree? He said this is a….

In this excerpt, there was a definite IRE pattern, but it varied from the previous example in that the teacher asked students to explain how they knew their answers were valid for both correct and incorrect responses, and subsequently asked a student to show how he would count the cubes to determine it was less. Asking students to explain their strategies shifted the focus onto their thinking, and there was some insight into how this student thought about the exercise, and what his misconception was. Although Rex mistakenly saw five and four ones as ten and four ones on the pan of the balance, the proof for why Rex's initial answer of 14 was incorrect was later validated by the use of the representation - the counting, rather than the teacher telling the class what the correct answer was. The responsibility of validating the answer shifted from the teacher to the rest of the class, with the teacher's encouragement, making it explicitly clear to Rex how the cubes could be counted to arrive at the correct answer. At this point, he teacher discontinued probing the student's thinking beyond the explanation articulated. Regarding Terry's explanation, we were not clear about how she really thought about the comparison. This interaction brought to light the realization of a misconception and how to prove what the correct outcome was. Mrs. Dennis called attention to Rex's mistake, in part to see how he thought about it and how he arrived at an answer of 14, but also to highlight his misconception as an opportunity to learn.

These two excerpts highlight the relationship between the tasks and exercises included in curricular materials and the ensuing discourse. Distinguished from open tasks, the exercises contained in the SS student journal may have been open to the use of differing strategies, but also included exercises for which students needed to determine a single correct answer. Seen by the teacher as "targeted" or meant to have students learn particular ideas and skills for that topic, these types of tasks tended to influence the discourse toward more triadic patterns of interaction. However, the second excerpt certainly highlights the possibility of deviating toward a more dialogic pattern through the use of exploratory questioning about students' thinking behind their answers.

In contrast, when the class worked from a teacher-created activity or a contextualized problem-solving task, the discussion began with a student's explanation about their work regarding that task, including the student's solution outcome. After working on the problem with a partner at their desks, the students returned to the whole group to have a few students called upon to present their work and explain their strategies and what they did to the class. Students' presentations often included explanations of what they did, their solution paths, and their outcomes. Apart from discussing the thinking behind what one did, many interactions began with a student explanation of what was done, but did not necessarily include the student's reasoning and thinking behind what he/she had done. During these episodes, discussions began with the students' presentation of their work and initial ideas, both of which were visible on the document camera and served as the focus of the discussion. Through regularly engaging in this activity, the norm of explaining oneself and one's work was established. This is illustrated in the following excerpt, in which the class has reconvened for a whole group discussion following a two-dimensional shape-sorting activity. Following their work in small groups, students categorized two-dimensional shapes according to their attributes on chart papers, which were displayed for the whole group to analyze and discuss:

> T - Ok, ummm, looks like this group also, and this group also, and this last group, they said short sides and pointy parts.
>
> S - that was mine

T - Pointy parts. What do you mean by the pointy parts?

T - Shhhhh. Greg, or I think it was Kylie, what were you talking about the pointy parts?

Kylie - you know that part, it's all the way like this, over here, it's the pointy parts

T - Ahh, so you're talking about these corners? They look kind of the same. Is that (highlights them with a pen) S - Yeah!

Kylie - mmmhmmm!

T - Ahhh, so they're talking about the corners, and the other groups were talking about the sides, so I'm gonna write that down. We're looking at corners, and we're looking at sides. (writes on board)

In this next interaction, the students worked from the representations, and used hand motions to help with communicating their thinking. This was also observed in the subsequent lesson focused on 2-d shapes. The students were making observations about theirs and others' groupings:

T - Ok, the one up there. K, everyone looking at their group.

Kylie - they don't have any corners and sides.

T - Oh, what did they say, Terry? What did she say? You have to listen. Peter, can you repeat what she said?

Peter - it doesn't have any corners or sides.

T - No corners, no sides. Ahh

Ira - no sides? I know! They don't have any straight lines.

(Peter goes up to share)

Peter - can I share this one? We put them together because other half can make the, the other half can, can make the other half can make a circle.

T - Ah, so these two shapes, this one can fit inside of this one to make another shape just like it. Ahhhhhh. There was another group that said that. You also said that with your, what? Was that this group? You said that this square could make

S - Uhhh, um,
Ira - a rectangle?

T - (nods) The rectangle, right?

Nora - we just put um, like squares together to make a rectangle (uses hands to show)

T - Ahhhhh. So you could make this shape together, like these two. Ok very good. Do you know, that you were looking at the pieces' attributes, okay?

In part because of the task the students engaged in, and because students had the opportunity to explain their groupings, including Mrs. Dennis' stepping in to introduce the mathematical language as they communicated their thoughts, the pattern of interaction was different from what was previously shared. There were only a few evaluative comments made regarding the mathematical thinking, and for the most part, the students' ideas were accepted and used to highlight mathematical concepts. The focus was on the mathematical meaning of the attributes, defining them in relation to how the students saw them as they engaged in the task. The teacher did much of the explaining and relating, and students made comments about how they saw the shapes. This segment also highlights the teacher's facilitation in moving the students toward noticing relationships between different student responses.

During the interactions in which students engaged in teacher created activities and solving open problems, and engaged in whole group discussions about their work, there were numerous opportunities to examine students' initial ideas. Furthermore, there were multiple opportunities for students to share their ideas with others and engage with the mathematics. During discussions, sharing time, and as they worked in small groups, students were continually asked to explain how they got their answers, and in one particular lesson, this practice led to the students' persevering and thinking further about the task. Once a student's idea or strategy was presented to the class, the subsequent interactions varied in pattern, depending on the teacher's decisions and actions. These will be discussed in more depth in subsequent sections.

Because students worked to solve contextual problems or teacher-created open tasks and regularly engaged in whole group presentations of their work throughout the year, provided for multiple opportunities in which examination of student thinking could occur. However, as subsequent discussions will demonstrate, the presence of opportunities did not necessarily mean they were taken advantage of. In addition, it was also observed that when students worked in and from the SS student journal, there was a tendency for triadic patterns and teacher-directed conversations to occur, where a single answer and the use of valid procedures was emphasized.

The lesson structure.

Mrs. Dennis used the Stepping Stones materials in flexible ways, and experimented as a way to get to know the program. She wanted to keep the structure of her lessons to include opportunities for students to work through contextualized problems, while at the same time, implement the SS core lesson. Determining how to do this, what materials to use from the program, how to sequence and relate the contextualized problems with the focus of the student journals in a way that was comfortable and made sense for her and the students took time to develop. This adjustment period, as the teacher worked from her prior way of structuring the activities and flow of the lessons to incorporate new programmatic materials, with new tasks, new structures, and a new digital component occurred through most of the Round 1 observations. When observations commenced in Round 2, the teacher had determined a lesson structure and sequence with which she felt comfortable. The structure evolved into the students engaging in a teacher-created task involving manipulatives or a contextualized problem to solve, which may have originated from the Stepping Stones curriculum, followed by a whole group discussion. Then the focus shifted to the exercises in the SS student journal, where the class engaged in the Step In discussion, and returned to their desks to complete the exercises in their journals. The teacher created tasks and/or contextualized problems related to the day's topic in the student journal, sometimes reviewing previous strategies and concepts. Sometimes they served as an introduction to the student journal exercises, replacing the Step In discussion, and sometimes they were problems with which students utilized strategies that were related to those needed for the journal exercises. In addition, the problem solving tasks tended to be open to various solution methods and different outcomes.

The relationship between the teacher-created tasks/contextualized problems and the SS student journal exercises can be characterized in the following ways, several of which highlight Mrs. Dennis' purpose in providing students with problems to work through and engage in whole group discussions about them. The problems served as a way for students to apply the concepts and skills they had learned within a problem-solving context, and allowed the students to engage in the problem solving process, in which they needed to make sense of a situation, determine concepts and skills which they knew that would assist them in determining an outcome, make sense of the problem's constraints, and determine a solution. Mrs. Dennis saw the importance of engaging in discussions about students' solutions as a way for other students to see and perhaps find starting points for engaging in subsequent related problems. She also used the problems to introduce a concept and/or topic that would be investigated further in the SS lesson, or to provide context for the concepts and ideas contained in the SS lesson or journal.

The description above was meant to provide a general look at aspects of the classroom, lessons and general context in which the mathematical discourse that was the focus of this study occurred. Aspects described above occurred over the course of the year and generally apply to all three rounds of data collection. In the next section, descriptions meant to characterize each of the rounds of observations will be provided. These descriptions do not depict every aspect of the discourse that occurred; rather they highlight the uniqueness characterizing each round.

Round one.

Formal data collection began on November 7 and included 11 observations conducted about one to two times a week for six weeks. The discourse in round one varied in many ways and was difficult to characterize by a single description, as there were several aspects that added to the complexity of the development. As mentioned previously, this was the period where Mrs. Dennis needed time to make sense of new curricular materials while keeping problem solving a part of the regular lesson structure. In addition to making sense of all the components of the program, their purpose, how to select, structure and sequence the activities, and how to relate the problem solving piece with the rest of the math lesson, the teacher had to re-conceptualize her prior experiences in teaching math in terms of the new program. This initiation into using new curricular materials played the largest role in round one interactions–as an adjustment period where the teacher went through a process of making sense of the new materials in light of her goals of having the students engage in problem solving and whole group discussions. Mrs. Dennis also found working with this particular group of students challenging, as they did not readily listen to one another and were all very enthusiastic about being heard at the same time. The immature behavior she observed in this class made orchestrating the discourse very complex, and this period also served as a time for both the students and teacher to develop their group norms and values. In addition, students were still developing their level of comfort with standing in front of others and explaining what they did in the task.

There were a number of topics touched upon during this period, including capacity, place value, equality, comparing numbers, addition strategies, and 2-dimensional geometry. Many of the interactions during this period were teacher-driven and followed an IRE or a version of an

IRE pattern of interaction. However, in December, a definite shift in the discourse patterns occurred during the lessons. This was initiated by a change in topic–geometry, where representations were used heavily, and discourse was more student-centered, meaning the students' ideas drove the direction of the discussions. During this shift, Mrs. Dennis continued to explain student thinking, but her role became more of finding the mathematics in students' thinking, and bringing that to the focus of the discussion, rather than her driving the discussion toward a particular end she had in mind previously. This may also be attributed to the openness of the tasks students engaged in, which involved sorting shapes into groups according to attributes they saw. This allowed for the mathematics under discussion to be based on and driven by the students' thinking, rather than the teacher's.

In the latter part of the round, when the topic returned to place value of two-digit numbers, although teacher-driven interactions occurred, there were interactions in which the teacher probed student thinking more, and asked students to examine the meaning of the numbers they worked with. The focus was rarely on following a single set of pre-determined procedures, except for when directions were being given for the task, the problem, or the workbook. In terms of the mathematics, the focus was on developing conceptual understanding, and not on carrying out math activities as procedures to follow. The focus also may have been on arriving at the correct answers, but there was also an emphasis on the meaning of the mathematics (what it meant to be equal, what it meant to be more or less).

Round two.

Round two began on February 20, and included seven observations conducted about twice a week, for 3-4 weeks. The lesson structure had stabilized somewhat to maintain a routine of students' engaging in a contextualized problem solving task or teacher created activity at the beginning of class, followed by related work in the Stepping Stones Student Journal. The problem or activity usually originated from the SS curriculum in some form, at times from one of the problems in the SS journal, or a problem-solving task in the supplemental materials of the program. A range of topics involving number and operations were studied in this round, including two-digit addition, equality, and fourths as fair shares.

Overall the interactions in round two departed from those in round one in that, except for the lessons, which focused on the SS student journal, there was an absence of an IRE pattern of interaction. In addition, in two of the lessons, several teaching behaviors surfaced which had not occurred in round one and affected the discourse. Although the interactions were largely teacher-led and the teacher's verbal utterances tended to dominate the explanations, she also made an intentional effort to encourage students' direct interactions with other students. As students became more confident in their abilities to explain their thinking, the teacher stepped back more than in previous interactions to allow students to do their own explaining. In the following excerpt, Jackie has just presented his solution to a contextualized problem in which students were provided with the parts to an addition situation, and were required to determine the total. However the problem was written so that one of the parts was being given away:

> T - Kay, so boys and girls, what do you think about….do you have any questions or comments for Jackie's work. Braden?
>
> Braden - Ummmm, the, the, twenty takes two tens
>
> T - The twenty has two tens.
>
> B - he got to take two tens to make
>
> T - Oh, so he's taking two tens away, but, so what are you trying to say by that? K, so say that again. Say that again to Jackie. Jackie, Braden's asking you or telling you something.
>
> B - (to Jackie) You have to minus two tens, right, two tens. Right, two tens. Right? Two tens.

Later on in the discussion, Mrs. Dennis also began to ask students for further explanations about why they agree or disagree with another student's ideas:

> T - He says, this is for Kai's friend. Twenty-five. Do you all agree with that?
>
> S - Yes.
>
> T - Jerri, can you tell us why you agree? You said yes.
>
> J - no
>
> T - Oh, you change your mind, or are you not sure?
>
> J - Uhhhh, I changed my mind.
>
> T - You changed your mind? So you don't think this is right? Okay, can you tell us why you don't think it's right? Jackie, Jerri says she doesn't agree with you on this.
>
> Jackie - twenty plus twenty equals forty because the two tens equals forty,

Jerri - I disagree with you

Jackie - and the five ones equals forty five, so

S - (interrupts) yeah, forty-five take away twenty equals twenty-five.

Rose - because the missing part - twenty and twenty make forty, and then you add the forty to five and that makes forty five, and then you um, take away the twenty from forty and then there's a twenty left and a five left to make twenty five.

S - It's wrong!

Although Jerri never explained what she disagreed with, Mrs. Dennis' elicitation encouraged her to respond to Jackie directly, and the other students entered into the discussion. Mrs. Dennis also began to encourage students to ask questions of one another's ideas, which also impacted the discourse:

Kylie - the forty, and the five, this is what he gave away, and this is how much he still have (points to the twenty). And I put it altogether.

T - Does anybody have any questions or clarification for Kylie?

Ira - Why did Kylie say addition instead of subtraction? (asks T)

Lana - Yeah, that's what I was going to ask - why did she do a plus?

T - Ok, can you answer them?

Kylie - forty-five plus twenty equals fifty, I mean sixty-five.

T - Does that explain to you why she did addition?

Hunter - Oh, yeah, that's perfectly fine.

T - Can you find another way to be more clear? You're telling them what the numbers mean, but why did you…

Eliciting student questions about another student's thinking encouraged Ira to ask a genuine question as he tried to make sense of why Kylie added instead of subtracted, which was his original thinking. The focus naturally shifted onto a student's wondering, and since the teacher continued to facilitate the discussion based on the student's initial question, student thinking became both the basis and the driver of the discussion. Asking one another questions became a valued behavior in mathematical discussions established in round two, and extended into round three.

These teaching behaviors were predominant in two of the lessons in round two, and occurred when students were given open problems, which allowed for multiple solution paths for determining an outcome. The remainder of the lessons was predominantly teacher-driven, and highlights the variability and unevenness in the type of discourse in which the class engaged. Although they occurred in only two of the lessons, these interactions highlighted unique aspects of the round, which encouraged students to engage in substantially alternate forms of mathematical discourse, such as interacting with one another, having one's ideas drive the direction of the class' discussion, and influencing the mathematics the class investigated.

Round three.

The final round of data collection occurred at the very end of the school year, commencing on April 20, and included five observations, over a period of three weeks. This period was meant to capture the culmination of the discourse development for the students' first grade year.

One aspect of the discourse, which distinguished round three from other rounds, involved students' explanations of their mathematical ideas. Many of the students developed confidence and the ability to communicate their ideas in a comprehensible way. This was particularly evident in students who had extreme difficulties with explaining their ideas at the beginning of the year, several of which were English language learners. One of these students in particular contributed significantly toward the discussion in several of the lessons during this period, asking mathematical questions and making observations about relationships he observed between other students' thinking and his own ideas. As a result, students that were not really heard from at the beginning of the year, participated more during the interactions, and contributed toward the discussion. In contrast, however, there were a group of about three to five students whose voices were seldom heard, if at all during the last round. These were students who had contributed significantly during the other rounds of data collection, and for unknown reasons, ceased to verbally participate in the third round of discussions.

As with the other two rounds, there was variability in whose thinking and ideas provided the basis for the discussion. There were interactions in which the students did this, and other interactions in which Mrs. Dennis' goals drove the direction of the discussion and in which she explained student thinking. Although the teacher behavior of eliciting student questions about other students' ideas ceased, students continued to ask each other questions in the lessons.

Students also sometimes made observations about other students' work, but directed their comments to Mrs. Dennis, rather than to their peers.

An interaction during one of the last lessons of the year involving an open-ended problem in which students were asked to find addition equations for combining six pennies with six dimes given there was only six coins used, captured the development of the students, and the role the teacher continued to play throughout the year. The following excerpt comes from the whole group discussion:

> T - Ok, Rose and Ned, can I have your papers? Ok, so what I want to do is, let's take a look at just your first box. K, this is Rose's first box on this side, explain yours, Rose.
>
> Rose - um, I put the six pennies altogether, and it made six.
>
> T - Good, ok, Ned, now go up and do yours
>
> Ned - I did ten ten ten ten ten ten, and equals, and I just put it equals sixty.
>
> Braden - Mrs. Dennis, I have a question. Why, why did Rose and um, Ned has the nine as ten and Rose has ones.
>
> Rose - because um, Ned wanted to start with greatest, and I wanted to start with least.
>
> T - Ahhhh, ok wait, let's stop though. Is that what you did too? You had six pennies altogether, and six dimes altogether, did you do that too? Yes! You did that too.
>
> Braden - um, Rose and Ned are the same like mines
>
> T - So it's the same as yours, right? Now watch what they did after this.
>
> Ned - and then we, and then we figured out, we had something that, and then we both had an idea, so we could whenever we knew one, we switched, so we switched one with each other. so um, after we did this, we um, we were on our second one and then I thought that we should switch one
>
> T - Ok, so everybody, look, watch (T goes up with coins) I'm going to give Ned, he had the, Ned had six dimes. And Rose had….six pennies. (gives each of them their coins) Then what did they do? Watch, watch. What did they do?
>
> Ned & Rose - so we got one, and then we switched it (gives each other one coin) and that was our new one.
>
> T - So now Ned has how many?
>
> Ned - I have fifty one, and she has

Rose - I have, wait, I have sixteen.

T - Did we give you an extra penny? Ok thank you. (goes up & gets it)

Rose - I have fifteen now….

Braden - what happen when they trade again?

T - Braden said, what if you traded again?

(Rose & Ned trade one more)

T - Now what will Ned have?

Ned - I will have forty-two,

T - Forty-two,

Ned - and she will have twenty four.

Rose - it's just switching the numbers.

T - What are you switching? The what?

Ned - we're switching the dimes and the pennies

T - So you're switching dimes and pennies, so that's tens and….

Sts - ones.

Rose - before the two was on the, my two was on the left, but Ned's one was on the….um…

T - The right, the two, when you flipped it, the tens were more, and your's was,

Rose - If I flipped my number, then it will be the same as Ned's.

Jackie - How would they trade, kind of to do like different numbers?

T - So what do you mean by that?

Jackie - like how would they trade like doing like, they trade like three dimes for three pennies?

In the above interaction, Ned and Rose discovered a systematic way for determining a solution for the task. Braden, an English language learner, observed that their process modeled his own thinking, which he had presented earlier. He then asked the question, "what happens next?" which pushed the students to see a symmetric pattern in the numbers that resulted from Ned and Rose switching coins. This was something unexpected that came out of the students' strategies and ideas about the task. The students were confident in explaining themselves and it was done in a way in which the rest of the group made sense of what they did. At the end of the interaction, Jackie asked another question, curious to see what would happen if they switched again. This is what occurred a little later in the discussion:

T - So by them doing it one by one, it helps them to what?

(sev. sts. speak at once) Ned - get new equations.

Ira - see if it works, or no?

T - To see if it works or not? Well, why do you just trade one by one?

Ned - Because um,
Rose - so that we have the same amount of coins?
Ned - so we have different equations
Jackie - it's not too much because -
(sts. speak at once, no one finishes their thought)

T - Hang on, hang on, there's a reason for just, it's helping them do just one at a time.

Kerry - so they don't forget which number they are on.

T - Say it again.

Kerry - so they don't forget which number they are on.

T - So that helps them to keep it in……

Ira - track

T - keep it in order. Or keep track

At the end of this interaction, the direction of the discussion shifts to being teacher led through the question about why the students were trading their coins one by one. Although Jackie asked about what would happen next, discussion about the pattern they observed was not explored any further, and the discussion funneled toward what the teacher wanted to emphasize, which was the keeping track of the numbers. This illustrates a typical way in which the discussion became teacher-directed in its focus, and occurred throughout this last round.

Development of Mathematical Discourse Over the Course of the Year

The discourse development throughout the year can be characterized in a few words - uneven and non-linear. If one considers discourse development as lying on a linear progression, as described in the Hufferd-Ackles et al. (2004) Math-talk Framework, what was observed in this first grade classroom can be described as non-linear in development. An IRE pattern of interaction occurred throughout the year, but was not the sole pattern that occurred within and across lessons. There were versions of an IRE pattern, in which student explanations of their responses were elicited and provided. There were interactions, which were teacher-driven, where the teacher used statements, questions and direct instruction to direct the discussion toward a particular path or show students a way to organize or do something. There were a few interactions, which followed the students' thinking and questions, where students explained what they did and sometimes why they did it. There were also a few interactions between students, in which students' thinking was most visible and provided the basis for the discussion.

At the beginning of the year, Mrs. Dennis described frustration with student behavior and having students listen and attend to one another during whole group discussions. While it was true that not all of the students attended to the presenter at the same time, there is evidence that some students listened to other students' presentations of their work early in the year. Some students responded to each other's thinking, summarizing and paraphrasing what they understood about what was communicated. At times, the teacher encouraged students to consider other students' thinking through eliciting student agreements and disagreements to student ideas, to which students responded. There were instances where students spontaneously summarized someone else's thinking, and where a few students explained what another student thought. This provides evidence that there were students who were listening and making observations about one another's mathematical work, one of the first steps to having student-student interactions during whole group discussions. These behaviors increased in number by the end of the third round of observations, and included more students exhibiting them. Within and across lessons in each of the rounds, there were interactions that were student-centered, and interactions in which the teacher drove the direction and the focus of the discussion. In Round 2, Mrs. Dennis made a concerted effort to elicit student questions about other students' work, and explicitly encouraged students to communicate directly with one another, but then these teaching behaviors ceased by Round 3, although the effects of this are observed in students continuing to ask questions in Round 3.

One aspect of the discourse, which followed a linear development from the beginning of the year through to the end, was the students' comfort with and ability to communicate their ideas with others in a whole group situation. The beginning of the year involved students becoming accustomed to getting up in front of the class to explain their work. It was evident that they were not comfortable with doing this, as they were often challenged with explaining what they did in a way that was comprehensible to others. However, by the end of the year, some students were extremely comfortable with communicating their ideas and current understanding to others, asking questions, and could do so in a comprehensible and clear way. This development was particularly evident in students who had extreme challenges with explaining themselves at the beginning of the year. For other students, hearing different ideas encouraged them to reflect on their initial ideas and consider other ways of thinking before drawing conclusions.

Based on the prior descriptions, developing alternate forms of mathematical discourse in a classroom is a very complex and difficult endeavor for a teacher to take on and implement. The prior sections have so far provided a general description of the types of interactions and the development of mathematical discourse in this classroom over the course of the school year. Several issues and challenges involved in developing an alternative form of mathematical discourse were touched upon in the discussion above. The next section explicates some of the challenges and issues that arose as the discourse developed in this classroom community.

Issues with Orchestrating Students' Discussion

Just as a conductor, in orchestrating a symphony must facilitate the musicians who make the music in a beautiful piece, through communicating when to play, how much, when the focus is where, for how long, the manner in which that piece of music might be played, so it is with orchestrating a discussion in a classroom. Deciding who will be the focus of the discussion, for how long, when to ask a student to elaborate on a comment, how the different students' ideas and ways of thinking relate to each other illustrate the complexities in orchestrating mathematical discourse. Inherent within these decisions are challenges and dilemmas a teacher must work through in order to have meaningful discussions occur. Very difficult to implement because of the complexity involved, it is not surprising that the teacher and students in this class experienced challenges with various aspects of engaging in discourse. The interactions in this classroom highlighted six main issues dealing with orchestrating and engaging in mathematical discussions: 1) Establishing focus and exploring student thinking, 2) Opportunities to explain one's thinking, 3) The challenge of students explaining their thinking, 4) Interpreting and imposing on student thinking, 5) Unexpected student responses as opportunities to learn, 6) Students responding to students' thinking vs. student-to-student interactions. Although the first three themes could have been collapsed into one large category focused on student explanations, they were separated into smaller chunks so as to make it easier on the reader. These themes, describing issues that emerged for both the teacher and students will be discussed in a dynamic way, meaning focus will sometimes be on the pedagogical aspects of the discourse development and other times will focus on the students' behaviors and perspectives, including interactions between them. This highlights the complexities a teacher is required to keep in mind in developing discursive interactions. Each of the six issues will be discussed in the following sections.

Establishing focus and exploring student thinking.

One of the first aspects of the discourse observed was how quickly the focus shifted from one student to another during discussions. Although this did not occur inordinately, during the year, it was noticeable enough to have a significant impact on the discourse. There were times where the focus of the conversation would switch to another student before the initial person's ideas were truly explicated and the rest of the class could make sense of them. One type of situation in which this occurred was when the teacher tried to have more students actively contribute toward the discussion, which will be in the following discussion and serve as an introduction to the rest of the findings.

A challenging aspect of facilitating the discussions was to have more than just a few of the students actively contributing toward the discussion, and to have a variety of student voices contributing toward the exploration of students' ideas. Reported by the teacher as one of the challenges she faced, having more than a few students engage in a mathematical conversation was a point of struggle particularly in the earlier part of the year. This is illustrated in the following excerpt, where the class is investigating the addition strategies of doubles and doubles plus one. The students have created addition equations to fall under three different categories– doubles, doubles-plus-one and doubles-plus-two:

> T - Nora, I'm going to put it back because you erased it. (writes 5 +1 = 6 under Doubles +1) I'm going to put that back cause that's what you had, okay? (adds in 3 + 2 = 5 as well) Ok, Nora, can you go ahead and explain what you put, and why you put those? So wait, let's check her doubles first. Do you agree with all of her doubles?
>
> Sts. - yes
>
> T - Nora, do you think that these are correct?
>
> Nora - yes
>
> T - How do you know that they are correct?
>
> Nora - because um, they're all the same numbers and they're all
>
> T - Cause you're adding…..
>
> N - them all together.
>
> T - Adding them together, very good. Ok, now let's look, I'm not sure what is doubles plus one, can you explain why you picked that first one? Why did you pick this first one, Nora? Can you explain why you picked this first one?
>
> N - because I thought it was doubles plus one, but it's not.
>
> T - Ah, how do you know it's not now. What makes you say it's not? Why are you thinking that?
>
> S - because….(inaudible) Nora - because it doesn't has a double
>
> T - What do you mean by that? Very good.
>
> Nora- (silent)

Here Nora was challenged with explaining why she placed a particular equation in the doubles plus one category, and stated her confusion about what made a doubles plus one fact. The teacher shared an equation that Nora no longer believed was a doubles plus one, nor could she explain why. The conversation continued on, as the teacher attempted to have Nora explain her thinking behind what she did, but was unsuccessful. The discussion progressed to the following:

> T - Boys and girls, are you looking at her work? K, so I know Ira is wanting to share, but I want to hear other children who don't get to share often too. Nora, look at what she

wrote, I mean Terry, look at what she wrote, what do you think about those two equations? Terry, what do you think about those two equations?

Terry - (silent)

T - Do you think they are doubles plus one?

Terry - (nods)

T - So what are you thinking? What do you think about those two equations?

Terry- (silence)

T - Or are you confused and not sure? You're not sure?

Terry - (nods)

T - Jackie, can you help to explain?

J - the first one, you have doubles plus one because it's a doubles plus one because you have five plus four equals….uh, nine.

T - ahhhh. Uh, Kekoa, what did Jackie just say?

Kekoa- uhhhh, (silence)

T - K, can you ask him, say Jackie, what did you say, and we need to focus.

Kekoa - Jackie, what did you say?

Jackie - it's not a - if it was a doubles plus one, it would be five plus four equals nine.

Kekoa - If it was a doubles plus one

T - If it was a doubles plus one,

Kekoa - one, it would be five plus four

T - why do you think he picked those two numbers, Kekoa? Why do you think he picked those two numbers?

Kekoa - Because

T - Why did he say five and four?

Kekoa - because that's a doubles plus one

T - What do you mean by that? How do you know that's a doubles plus one?

Kekoa - (silent)

T - How come five and four is a doubles plus one, and five and one is not a doubles plus one? (3 sts. raise their hands)

T - Oh I love, children who have your hands up, thank you because I know you're thinking. So Kekoa, you don't understand, are you not sure why five and four? Cause I want you to listen to Rose, look at Rose, Kekoa.

Rose - I know it's a five plus four is a double one there's four, and you can add one more.

T - Ahhhhhh. So can you show them what you mean by that on the board? So Kekoa, I want you to look, go and draw that, Rose. So Kekoa look at, it might be hard to understand what she's saying, but

Rose - (goes up and draws dots, a group of five, and another group of four) and one more, add one (results in two groups of five)

Kerry - that's doubles plus two

T - Ohhhk, so he's saying well, he added the one more, try erase the one in the middle you drew in there again, that's a four, right? Ok, so he did add one, Rose, But what is he saying when you're adding one? What does that mean? Which one is the one that you're adding?

These excerpts illustrate several issues. In Mrs. Dennis' struggle to have more than one student engage in the mathematical conversation, a movement of the focus from one student to the next, without the exploration of anyone's ideas in depth, occurs. Other issues worthy of discussion are present in these two excerpts, however for the present discussion, the movement of focus from one student to another is the one that will serve as an initial analysis point. While this aspect of the interaction may seem trivial, it creates an interaction in which the mathematical discussion remains at a procedural level. The mathematical discussion does not go beyond the procedures the students used nor does it probe their thinking of understanding of the doubles-plus-one strategy. Since Nora was unable to explain why she no longer believes the equation is a doubles plus one addition fact, the teacher called on Terry to explain, but she is not sure either, so Jackie is asked to explain. Although it seems Jackie provided a desirable response, rather than exploring his ideas, Mrs. Dennis asked for Kekoa to repeat what Jackie has just shared, as a way to have a different student engage in the mathematical conversation. Kekoa was also unsure about why it is or is not a doubles-plus-one, and therefore the teacher called on Rose to go up to the board and draw how she saw it. Through this interaction, we see that Rose had her own misconception about a doubles-plus-one. Actually thinking of it as adding one to make a double, rather than the double within (or as a part of) the doubles plus one, she drew 5 + 4 and added one more, resulting in 5 + 5. This interaction highlights the switching of focus from one person to the next, and students' misconceptions about doubles plus one emerge, rather than a discussion that elevated to focusing on a relationship between doubles and doubles-plus-one addition facts. One possible explanation that might account for the shifting of focus from student to student is the issue of exploring, probing and clarifying the presenter's thinking for the rest of the students, the teacher and the person presenting. Once a teacher has selected a person's ideas to be the focus, whether or not the focus is truly established can depend on the degree to which that person's ideas are explored and understood. Probing and clarifying student thinking once they have presented their idea helps everyone focus on the idea and/or decide to switch focus to another person or topic. In other words, true establishment of an area of focus cannot occur if someone simply presents an idea, as Jackie did in the excerpt above. In developing discourse, students can speculate about and paraphrase others' thinking, but doing it without really explicating someone's thinking to begin with leads to making assumptions about others' thinking and can make it difficult to build upon someone else's thinking.

These excerpts also illustrate the challenge and complexity involved in having young children explain their thinking within a whole group setting, bringing forth several dilemmas a teacher has to work through in orchestrating discourse. First it involves having students present and explain their ideas to the class. Then several issues may arise, one of which is what to do when a student is unable to explain her reasoning. What does one do to keep the discussion moving forward when a student gets stuck? If ideas are presented, how does one facilitate the exploration of them in a way that will still engage the rest of the class and assist the presenter with clarifying her ideas for herself? Is the presenter's thinking explored enough? How does one decide when to change the focus? Answering these questions also involves the teacher's goal and purpose for the entire discussion, and how to weave and use students' thinking to help each individual develop some sort of understanding of the processes and concepts that may be a part of that goal is also a large consideration. Many of these issues will be discussed throughout this findings chapter.

Opportunities to explain one's thinking.

Exploring students' thinking includes the issue of whether or not students were provided with opportunities to share their thinking behind the first response given. Distinguished from being selected to share one's thinking, this notion involves whether or not, once chosen, a student has the opportunity to explain his/her thinking fully to the rest of the class. As provided in a few of the excerpts, although the student is called upon to be a presenter, he or she might provide an initial explanation, but then did not have the opportunity to fully explain what he or she thought. In addition, very little probing occurs, for everyone in the community to really understand the presenter's thinking. The following discussion occurs subsequent to students playing a game in which they made two-digit numbers from rolling two dice, each numbered zero-five. They then colored pictures of base-ten blocks to represent their number, and compared it to their partner's to decide which was greater:

T - k, Jackie, tell them what you did on this one.

Jackie - I, I wrote uh, one and I colored one

T - You wrote a one and a what?

Jackie - and a...zero.

T - Ahhh, and

Ira - that's only one

T - Oh, he chose a one and a zero, and the number that you chose was…

S - one, one

S - it would be…

S - he put a one first, then he could have a ten

Ira - I was about to say that

T - (goes up to board) Ahhh, class, look, Look at the way Jackie wrote his number. He wrote a…

S - zero and a one.

T - Zero and one. Is this a ten? It looks like a ten, but is it a ten?

Sts. - nooo

T - Ahhh, because the zero is first, the zero is in the what place?

S - first

In this excerpt, Jackie was never really given the opportunity to explain his thinking behind writing 01 and 10. The teacher and the rest of the students took over the explanation and the conversation was directed toward emphasizing the place value aspect of the numbers Jackie wrote, the idea that 01 really means zero tens and a one. However, one cannot help but wonder, was that really how Jackie thought about it? There was a strong possibility he did, but uncertainty exists unless he explained it for himself. Hearing it from Jackie directly would not only have provided other students the opportunity to make sense of his ideas based on his own explanation, but it would have also afforded him the opportunity to engage in self-reflective thought through the examination of his own thinking. Furthermore, because Jackie did not re-enter the discussion, discourse remained at an observational level, rather than shifting into negotiation and development of his mathematical thinking, characteristic of student-to-student interactions.

This highlights the notion that, although the opportunity to examine student thinking might be present, it will not necessarily occur unless the teacher makes, or allows it to happen. Although opportunities were present throughout interactions, the closer examination of student thinking by the speaker or the audience did not occur. This highlights an important aspect regarding the facilitation of the discourse - that student presentation of mathematical work provides a starting point for discourse to develop, but does not automatically result in the development of a collaboratively constructed discourse unless students' are afforded the opportunity to explain their mathematical thinking behind their employed strategies, through which negotiation of ideas can occur.

Having said that, other instances occurred in which students spontaneously explained their thinking, not as the focus of the discussion, but as a part of the discussion. Here is one example in which the class is engaged in a whole group discussion about place value. In this lesson, the students are creating a human place value chart, using their bodies and fingers to represent two-digit numbers. The teacher has established that the tens place is to her immediate right, and the ones place is to her immediate left:

T - So you can put all the numbers, but once you get to ten, you have to move it to the….

S - tens side

T - Tens side. (nods) Ok, now look, how many tens, or how many people, do I have here on this side?

Sts. - one

T - And how many

S - zero

T - ones do I have? Zero. Let's look at that number ten, guys. Here's the number ten. I have one person here, he is the ten, and I have (T makes motions on the right side of her chair - the "ones place")

Ned - It's like this, but it's like in people, so the one is standing where Matt is, and the zero is on this side, and the zero is on this side, and the one is on that side, where he's standing, that's where the one, and he's one.

T - Right. So what does this mean, then?

Ned - It's the same!

T - What does it mean? It explains something, what does it explain? (points to Kylie)

Kylie - like that one, it's a ten, and that one is a zero ones.

It is difficult to see in the transcript, but this point in the conversation signaled a connection between representations that became very salient for students, to which they responded with great excitement. Ned seemed to state the obvious, but his explanation of the way he saw the representation served as his way to make sense of and comprehend his observation. He spontaneously explained his current thinking as a way to make sense of the relationship between representations.

As with any classroom, although Ned was a student who felt comfortable and spontaneously communicated his mathematical thinking with others, there were other students who did not have the level comfort that he exhibited. Students ranged in their confidence and inclination to explain and communicate their thinking with others, with some students having extreme difficulty and others who were fluent, and some in-between. This will be explored further in the next section.

The challenge of students' explanation of their thinking.

In my experience as a resource teacher, one of the most common challenges teachers cited in developing mathematical discourse was having students explain their thinking, particularly if one is working with English language learners. While it is natural to exhibit difficulty with explaining oneself if one is still learning the language, it is not impossible. The issue teachers have with the challenges of student explanations may lie in their purpose for having students do them; in seeing explanations as an end goal for students to strive toward - to explain how and why they did something. This view misses an important role of communicating one's thinking rooted in the social constructivist theory of communicating as thinking, where one develops one's thinking *through* communicating one's ideas. That is not to say the teacher of this class held one view or the other, but this highlights an aspect of communication as a part of discourse development that can impact the nature of the discourse that occurs in a classroom.

Though not predominant during the discussions, there were instances in the lessons where, for whatever reason, a student had difficulty with explaining his/her thinking, even when the teacher attempted to probe a little deeper and make sense of what was initially shared. This is highlighted in the excerpt above of Nora's presentation (see Establishing Focus section, p. 83). Nora hit a point where she stopped trying to explain her thinking, and the teacher became uncertain about how to proceed. This illustrates a dilemma teachers face in orchestrating discourse - what to do in the moment, if a student stops or is unable to elaborate on her thinking, not unheard of for first graders. It is possible to speculate, that in Nora's case, the context in which she tried to explain her thinking may have inhibited her ability to do so. Because she was attempting to explain her thinking in terms of being correct or incorrect rather than as an exploration into her thinking, she may have not felt completely comfortable with elaborating her ideas. This is an instance in which an interview with Nora might have shed light into how she felt about it. In any case, there is insufficient evidence to know for sure why she stopped explaining herself, and might be a place for further study.

Interactions in which students were challenged with explaining their thinking sometimes highlighted their cognitive processes as they attempted to communicate their ideas. The view of thought development through the act of communicating was relevant in the following interaction with Kai. In this interaction, the class is engaged in the Step In discussion of the SS student journal, in which they are presented with a pan balance showing 2 tens and 6 ones on the left pan and 1 ten and 9 ones on the right pan. The pan with the 2 tens and 6 ones is lower than the other, and students are asked to decide which number is greater:

T - Nineteen, alright, now let's go to the next question. It says, which number is greater?

Kai - twenty six

T - How do you know? Kai?

K - it's more than nineteen.

T - It's more than nineteen. It has more what?

K - More, more cubes

T - It has more cubes. Ok, lets count the number of cubes that it has. (counts all the blocks as ones, including the tens blocks. 26 ends up with 8 blocks, and 19 ends up with 19. Sts. count with the T) Is that what you mean? Is that what you mean? It has more cubes when you count like that?

Kerry - I know why, I know why (sev. sts. speak at once)

T - Shhhhh, Wait, stop. I'm asking Kai. Kai, you said this one has more cubes, so I counted it. Do you mean more cubes like that? Is that what you mean? Just to clarify.

Kai - more - (points to board)

T - So how are you counting it? Can you go up to the board and show us how you counted it, and how did you know it was more?

K - (goes up to board and points to 19) Ten and one two three four five….nine

T - Ok, so Kai, what do you mean? How does this side have more than this side? Tell us what you mean by that.

Kai - this one is (inaudible), it's close to the number.

Kai was a student who had a tendency to begin his explanations with where he was currently in his thinking, rather than explaining himself from the beginning of his thought process so others could make sense of what he had done. Therefore, at times, Mrs. Dennis had to facilitate his presentation by walking him through his thought process in order to make sense of what he had done and continued to think about. He was a prime example of a student who, with the teacher's facilitation, sometimes became clearer about his thinking as he communicated. In this interaction, Kai knew one thing; that 26 was more than 19, but could not articulate how he knew. At the point where Mrs. Dennis asked him to go up to the board and count, his thinking became explicit, including what he was challenged with - the actual counting to arrive at 19. Although we were not certain how much of his ability to count related to his conception of 19, it may have had an impact. Rather than asking Kai a question which facilitated his thinking in the moment to make sense of 19, such as “what does that tell you about 19?”, or “What do you notice about 19 based on your counting?” broad questions about how 19 compared to 26 were asked. This did not help him articulate his emerging understanding of the numbers and how they compared. Kai’s interaction represents an important purpose for having students explain their thinking to others, a notion central to the development of discourse - that communication is a reflective process, a thinking process through which students reflect upon what they did, why they did it, and develop what they thought.

Although several excerpts highlighting students’ challenge with explaining their thinking and possible related issues have been discussed above, there is still the possibility that there will be instances where a student will explain and be understood as much as he is able to explain and be understood, and that brings us to Braden. Braden was a student who was particularly challenging to understand. He was very soft-spoken and did not enunciate or project well, so it was very difficult to figure out what he was trying to say. In the following Step In discussion, digital pictures of dot formations on dominoes are being shown to the students. In this particular interaction, a domino with six and four dots is projected, with the question, “What doubles (addition fact) is going to help you figure out this problem?” posed to the students:

T - Braden. Which double are you going to use?

Braden - six and six.

T - Six and six. How will you use six and six, Braden?

Braden - six and six - two dots, six, two dots, and one dot. There’s two reds, there’s two red dots (points to board as he talks from where he is sitting. It makes it very difficult to really understand what he is talking about) and the black, and four dots going across.

T - Ooook, (goes to board) so Braden, you said six plus six (draws two more dots on 4 domino). Is that what you would picture? Ok, so what is six plus six? What did you get?

S - twelve.

Braden - uh, you have to get two and like six and six, T - Ok, what is six plus six?

B - eighteen?

T - Ok, how did you get that number?

B - uh, it go two dots, like that, and then you have

T - Ok, can you count it then? Can you count it, to show us?

B - twelve, there's twelve (B stands up and counts dots from where he was sitting. We still can't see which ones he is really pointing at)

T - Ok, so is it eighteen? It's what?

B - Twelve.

T - Twelve. Ok, you got twelve, and you said you would use six plus six. But our question is six plus four. So if it's like this, what are you going to do with the twelve to now help you get this? (erases the two she had drawn in to illustrate what Braden was talking about. Now shows the original domino)

Ira - take away two?

Sts. - take away

S - break it in half

Braden - there's six and then four, and then four, five, six, seven, eight nine, ten, and now ten, then six and four equals ten (counts from where he is sitting; difficult to see what he is exactly talking about)

T - So how would you use the twelve? Cause you said you would use six plus six. You were visioning that there's a dot here, and there's a dot there, so you were trying to make it be a double, ok, very good, but Greg, can you help him? If he used the six, but you're trying to get the four, what would you do, Greg?

Greg - take away

T - Take away, Ok, so he could use the twelve, Jackie, and then take away…..

S - two

T - Two, so that he could get ten. Ok, Is this a double?

Sts. - yes

T - Yes, could you do, so he made up a new rule, he did not doubles plus two, but what did he do? Doubles

S - minus

T - Take away two. (writes on board) Okay. That's a new one. Okay. Well, that's what he would use, six plus six, that's his double.

Although he was difficult to understand, Braden had different ways of thinking and at times asked questions which moved the discussion forward (See Round 3 sec, p. 77). He was also one of the students who benefitted from having the opportunity to communicate his thinking, even when he didn't always make sense. Mrs. Dennis always gave Braden time to express his ideas, even when they were unexpected and challenging to understand as in the above interaction. In this excerpt, she extrapolated Braden's thinking at the beginning of her interaction with him, as it was difficult to understand what he said. She also asked him questions, directing his observation toward seeing the six and six in terms of an addition equation. It would have been helpful to have him go up to the board and actually point and draw exactly what he had been thinking, but he was not given that opportunity. However, Mrs. Dennis accepted Braden's double, and although it was not the expected answer, made it work, calling it "his own" strategy.

This interaction highlights several questions about Mrs. Dennis' actions. How did directing Braden's observations to the six and six addition, even though he had not specifically explained it in that way, affect the interaction? Why might a teacher draw in and erase the circles to show his thinking? These questions and the issues they bring forth illustrate the complex decisions a teacher must make as a student's explanation unfolds. That she did these things in order to move the discussion forward and make Braden's tentative ideas useful for the discussion highlights the notion that in some cases such actions may be necessary to keep a student's presentation intact, comprehensible and seen as contributing toward the discussion. However, the degree to which a teacher plays a role in extrapolating student thinking can sometimes lead to an imposition of ideas on to students' thinking. What this means will be the topic of discussion in the next section.

A major issue of orchestrating discourse involves the management and facilitation of students' explanations in coordination with the thinking in which they engage and bring forth through those explanations. Therefore, at this point, it would be useful to briefly discuss the relationships between the first three sections of this chapter, as a way to summarize the findings thus far and highlight aspects pertinent to orchestrating mathematical discourse. These relationships are highlighted in the overlap between themes. For example, the first excerpt from the theme, Establishing Focus, includes an interaction in which a student was challenged with explaining her thinking, which initiated a movement from student to student within the discussion. As another example, the exploration of student thinking can provide opportunities for students to explain their own thinking. Taking into consideration the purpose for asking students to explain their thinking, in summary, these findings inform aspects of instruction central to the facilitation of students' mathematical thinking through their explanations. Interactions in which students, including those who are challenged with explanations, are afforded the opportunity to explicate their own thinking, in conjunction with the exploration of their ideas by the classroom community, contribute toward providing a context in which not only a focused mathematical discussion might occur, but also a place for students to reflect on, clarify and develop their mathematical ideas.

Interpreting and imposing on student thinking.

In teaching, we are constantly making interpretations and inferences about students' thinking, as much as we can, based on our observations, interactions, and conversations with students. We make our best predictions based on the data we have - both qualitative and quantitative about student thinking, and it is how we decide to act on our current understanding that can have an impact on the discourse developed in the classroom. Within this first grade classroom, there were instances where, the teacher paraphrased or repeated students' comments in a way that was different from what the student initially stated. At times she imposed some of her ideas on to student thinking, based on what she understood about what the student communicated.

Imposing ideas on student thinking can occur when the drive to highlight particular aspects of the mathematical concepts that are the focus of the lesson outweighs students' ways of thinking. This can occur through asking a simple question, or summarizing student thinking without actually knowing what the student thought. The following interaction occurs following a task in which students used interlocking cubes to represent doubles facts. They also recorded their pictures and wrote an equation that represented their facts in their math tablets:

T - But, Kerry said, what if you did fifty plus fifty,

S - plus fifty one
S - equals a hundred and one

T - oh yes, she said fifty plus fifty one

T - But we asked Kerry, how did she know that so quickly, what did you say, Kerry, what did you do?

Sts. - a hundred and one

T - Shhhh, Kerry what did you use?

Kerry - I added on

T - But which double did you use to get this?

Kerry - (silence)

T - Which double did you use to get -

S - fifty plus fifty

T - Fifty plus fifty. Ok (writes on board)

It is evident here that Kerry did not use a double to figure out an equation for attaining 101. During the discussion, she posed the possibility that a double might be used, and Mrs. Dennis misinterpreted her explanation, thinking that she used a double. Kerry explained that she added on, and was asked the question, "What double did you use?" although she did not say she used it. It seems implausible that Kerry actually counted on 50, but because Kerry's thinking is not probed a bit more, her strategy for how she counted was unknown. Here, the message seems to be, "You needed to have used a double. What was it?" and in doing so, the valued behavior, to use a double to attain the answer, is imposed onto Kerry's ideas.

Besides imposing through asking a question, imposing on student thinking might occur through an elaboration of what one thinks one understood about student thinking. In the following excerpt, Hunter is presenting a list of doubles facts he determined from looking at a set of dominoes:

T - Ok, so Hunter, let's go to the next one. What do you have here?

S - one and zero?

Hunter - one plus zero equals one

T - So is that a double? Is that why you put that there?

H - yes

T - You think that's a double? Ok, boys & girls, do you agree with one plus zero?

Sts. - no

T - No? Ok, why not? Jerri, why not?

S - because they're not the same amount

Jerri - because um, double means the same number, and the two numbers is not the same.

T - Ok, so she said doubles means you add the same number.

….

Ira - he even has zero plus five

T - Oh, ok, so, Ira said look at this one. (points to the equation) Is that a double?

S - no

T - What is Hunter think - I see the pattern or mistake that he's making. What do you notice about this one? They both have a what?

S - zero
Greg - zero plus one is not a

T - Well, look at this one, look at what he sees in this.

Hunter - one plus zero equals one

Kerry - he thinks it's a double because one plus zero equals one, but he thinking that's a double

T - Because he sees a one...and a one Kerry- a one and a one. Look, he has some thinking. He sees the one and….

W - the zero

T - He sees a one and a…..

S - he switched the numbers

T - Ahhhhh, but does that make it a double?

Although Hunter was at the board with his work displayed, he did not really have the opportunity to explain how he thought about his doubles. Mrs. Dennis noticed a pattern in his equations and used that to explain Hunter's misconception about what a double was, imposing her observations onto Hunter's thinking. Hunter was never asked to validate or elaborate on his thinking. It is highly probable that he saw a double in the way she described, but by not being able to communicate himself, Hunter loses the opportunity to reflect on his thinking, communicate himself to others, and receive assistance with clarifying his own misconception.

Imposing through the summarization of students' thinking also occurred in this excerpt, in which students represented the number, four, with their fingers. When asked what number was represented, a student responds with "forty!" which results in the following interaction:

> T - Is this forty, class?
>
> Sts. - no!
> S - cause it's zero, four
>
> T - It's just zero, right, and four. It's not four and a….zero.
>
> Nale - if you switched it to the other side, it would be forty.
>
> Rose - it's only zero because it's only there for the higher numbers, like sixteen.
>
> T - Ahhhh, Ok wait, thank you, you can take a seat. So Rose says -
>
> Rose - the zero is only there cause it's only for higher numbers, like sixteen.
>
> T - K, so this zero, she said, is holding a place, just to show you, that actually, there is no number here, right now. It's a zero. But we don't normally write the zero when we write four. Because we know, if there is no number here, that means that there are…...zero tens, there are no tens, ok? She said only if you write like the number sixteen, then you put the one there, because one holds the place for the….

Is this imposing or is it interpreting? The teacher interpreted and made assumptions of what she thought Rose meant in her comment. It is evident that we really don't know what Rose was thinking when she made her comment about the 16 and the teacher chooses not to investigate her thinking further. Rather she kept the discussion going in the present direction, through elaborating what she thought Rose meant or wanted to highlight from Rose's statement. By elaborating on Rose's comment, Mrs. Dennis both imposed her ideas onto Rose's thinking and maintained the present direction of the discussion.

In other instances, Mrs. Dennis may have mistakenly heard what students said, at times replacing what they actually said with what she thought they said or may have wanted them to say. This illustrates the challenge of, with all that is occurring in the classroom, attending to and listening carefully to what students say and mean. This also illustrates the need to assist students with making their thinking clear for others to understand, not through a solely verbal explanation, but through using and manipulating their diagrams and mathematical tools to assist them with communicating and clarifying their thinking for others, and at times, for themselves. In this brief interaction, Peter explained why his group categorized a group of shapes in the way they did:

> Peter - can I share this one
>
> T - Ok.
>
> Peter - we put them together because other half can make the, the other half can, can make the other half to make a circle.
>
> T - Ah, so these two shapes, this one can fit inside of this one to make another shape just like it. Ahhhhhh. I had another group that said that. You also said that with your, what? Was that this group? You said that this square could make
>
> S - Uhhh, um,
>
> Ira - a rectangle?
>
> T - (nods) The rectangle, right?

In this excerpt, Mrs. Dennis paraphrased what she thought Peter shared, but her version is a little different from what he seemed to describe. No fault to the teacher–this was probably what she thought she heard, but this interaction highlights the challenge of being clear about students' thinking, for the teacher and the rest of the students.

Within previous sections of the findings, aspects central to facilitating students' explanations as a part of the orchestration of mathematical discourse were presented. This was followed by a way in which the teacher in this class managed challenges inherent in asking student to explain their thinking as the discussions unfolded. Building on to what has been found thus far regarding student explanations, particularly in the possibilities for developing student thinking, the findings turn next to unexpected student explanations and responses as potential spaces for student development.

Unexpected student responses as opportunities to learn.

Student misconceptions, seemingly “incorrect” or unexpected student responses, if facilitated well, can become powerful opportunities to learn for students. Although Mrs. Dennis handled “incorrect” and unexpected responses in a variety of ways, as evident in some of the previous interactions she was also unafraid to ask a student who had an unexpected response or a different way of thinking to share his ideas with the class, even if it seemed to be an “incorrect” response. Rather than simply telling students they were incorrect, Mrs. Dennis sometimes called upon these students to present their ideas to the class. Her purpose in doing this was threefold - first, to make sense of student thinking - “to see how a student was thinking”. Second, to highlight to the class that people have similar ways of thinking, including misconceptions, and third, to correct a student’s thinking and ensure that the student saw how to attain the expected or correct answer or way of getting the correct answer.

In this next excerpt, this discussion followed a previous discussion in which Ned is the only one in the class who saw that blocks can be added to both sides of a balance scale in order to make a picture true. The SS exercise presented students with a picture of a balance scale with an equal number of cubes shown, but with the balance in an unequal position. The task was to add blocks to the existing picture in order to make the position of the balance true, and then to write the numbers that represent the new quantities. The rest of the students saw that blocks added to the lower side of the balance will make the balance stay down and be in the correct position, but Ned saw that one may add blocks to both pans to manipulate the balance:

T - Ok, so let’s look at Ned’s. K, Ned, can you share what you did?

Ned - I was thinking, the sentence didn’t say anything about having, um, it doesn’t care which side that you add ones, so I added one more to here, and I wrote thirty four, and thirty three, because there’s, there’s ten twenty thirty, and three more, so that equals thirty three. I added one more to this, but it was thirty-three, because it was equal, and I tried to make it go down, so I added one so three plus one more equals four. But it’s in thirty.

T - Ok, so do you folks agree with Ned?

Sts. - Nooo.
S - No, I don’t.

T - Ok, so can somebody explain why? Jackie, can you tell him why?

Jackie - because even the sentence says that, uh, you still have to do it because the picture says on this side, you have to do it, more.

T - Ok, so Ned, did you hear - look at the picture. Which side has to have more, Ned?

Jackie - it stays the same. He was just thinking what about the sentence, but both of them are important.

T - Ned - ok, so look at the picture, Ned. Which side should be heavier, Ned?

Sts. - the right
Ned(points to right side)
(sev. sts. speak at once)

T - How come?

S - because it shows down, it shows down.
Ned - it's not-
S - it's not the same

T - So Ned, if this side had thirty-four and this side had thirty-three, would this side go up?

Ned- (silent)
S - your picture wouldn't go up.
S - Yes

T - Would thirty-four be up and thirty-three be down?
S - no.

T - Because thirty-four has more.

(Ned nods)

T - So how should it be?

(Ned points to right side)

T - On that side. K, so Ned, you have to pay attention to this…..

S - picture.

T - Picture. Ok, can you fix that? Use your red pen and fix that. Thank you for sharing that with us. It was important, because he was right, it does not say, where to draw the

Jackie - but the picture says. Because the picture is a symbol.

T - Oh, wait, oh wait,

Jackie - it said, it has something to do with symbols.

T - Ahhhh. Jackie just said, this is a.,... (points to picture of balance)

S - symbol

T - Pictures are symbols. And the picture is telling you….

Jackie - a story.

T - A story…..and the story is that…..

Ned- you should, you should look at the picture.

T - This side is greater, this side is l - Wow.

Jackie - I know that because Ms. uh, our Hawaiian teacher telled us, I asked her, are all pictures a symbol, and she said yes.

The idea that, even in mathematics, a picture might be considered a symbol, which tells a story and provides information that one needs to pay attention to, was very salient for the students in this excerpt. Through his conversation with the Hawaiian Studies Kūpuna, Jackie developed an understanding of the role pictures play in making sense of a situation. Through the discussion, he made a connection between his prior conversation and the present mathematical situation. Furthermore, his idea about the role of the picture in the problem was shared with the entire group through the discourse. The realization of the importance of looking at the "story" pictures tell, even in mathematics, came about through the ensuing discourse of the class.

Although this interaction resulted in a great learning opportunity about the role of the pictures in solving problems, it also highlighted a missed opportunity for exploring more mathematics that illustrates the complexities of orchestration that manifested as the discussion unfolded. Ned had an interesting insight in the previous discussion, that one might be able to add cubes to both sides of the balance and maintain the illustrated relationship. Had this observation been explored further, more mathematics would have been investigated. Rather, his mistake of not noticing the picture is corrected, and represents a missed opportunity for exploring not only a foundational algebraic concept, but also a non-standard solution to the exercise. In deciding whether to pursue the mathematics further, Mrs. Dennis needed to consider to what extent the first graders were ready to move on and focus on more mathematics related to the task. As part of the many decisions she had to make during active instruction, Mrs. Dennis also needed to consider when to continue the discussion and when to move on. These are issues teachers face and quickly make decisions about during discussions.

Unexpected student responses were utilized as opportunities to learn, at times intentionally through Mrs. Dennis bringing them into the class' focus, and at other times, spontaneously. The next spontaneous interaction highlights two themes - students making observations about other students' thinking and the use of a misconception as an opportunity to learn. This interaction occurred during a task in which students were seated in a circle on the floor, each with a place-value mat and base-ten blocks. They used the blocks to build two-digit numbers the teacher provided to them:

> T - This is ten, and one more (writes 11 on board) Eleven, but I didn't say one ten and one one. I said eleven, but you knew how to build it.
>
> Rose - Greg said there would be two tens, but there's actually not because it would make twenty.
>
> T - Oh, Greg, you had put two over there? Why did you put two here? Why did - did you have it like this? Did you have it like that?
>
> (Greg shakes head, no)
>
> T - Did you do that?
>
> Rose - what?
>
> T - Is that what he had, Rose?
>
> R - I thought so, but he took it away.
>
> T - Oh, ok. No, but tell us why did you put it like that? What were you thinking?
>
> G - (silent)
>
> Rose - because it looks like a one.
>
> T - Yeah, is that why, Greg? Greg, did you think that looked like eleven?
>
> G - (silent)
>
> Rose- I was almost going to do that though.
>
> T - Ahhhh, B & G, stop, Greg had pulled these out like this, and Rose thought I was thinking the same thing. She was going to put it like this. Because what was she

thinking?

Ira - It look like eleven!

T - It looks like the number! You're right, but, Greg, now what do you know? Now what do you know?

G - (silent)

T - In this number, let's just say it was a number, it's not just strips, but if this was a number, now what do you understand? This stands for the….

G - tens

T - And this one stands for the…..

G - ones

T - Ones! Right! So when we do it in a picture form, the number eleven is going to look like that, yeah. (points to one ten and a one) Ok, very good. How many of you your thinking was the same, like ooo, you pictured this, so you grabbed the two tens, and you were going to put it like that.
(sts. raise hands)

T - Yeah, that might have been, right? That might have been.

During this interaction, Rose, sitting next to Greg, noticed what he did when they were asked to build the number 11 with their base ten blocks. She saw that he took out two tens at first, and volunteered that information to the teacher. She noticed it because she had thought in a similar way, initially putting out two tens because they looked like the number eleven. This also illustrated Rose's reflection on her own thinking, in relation to Greg' way of thinking. She noticed Greg doing something and guessed at his thinking behind that because she had a similar way of thinking. Rather than shying away from this interaction, Mrs. Dennis brought it to the attention to the class, as others may have had similar thoughts as well. This was presented as an opportunity to learn from each other, although one might speculate, based on his silence, that Greg was not entirely comfortable with the discussion around his thinking.

At other points during the discussions, unexpected student responses provided for opportunities to facilitate students' mathematical thinking, not only through highlighting novel ways of thinking about a task, but also through making connections between students' informal prior experiences and more formal ways of thinking about mathematics. This was highlighted in the following interaction in which the teacher re-introduces a pan balance to the class:

> T - It's not, it's like a scale, but a scale would tell you how heavy something is. This is a balance. You want to see which is heavier than the other, or, you might want to try and make it balanced. And what does balanced mean?
>
> Jackie - balance means do not fall down, but stay on one feet, or
>
> T - Right, so how do you balance? What makes you balance?
>
> Jackie - Balance, uh, you use, uh, you try, uh
>
> T - Can I balance if I'm like this? (tips body way over to one side)
>
> S - no
>
> Ned - No, you have to be like this (holds arms out from sides)
>
> T - So what is like that?
>
> S - the same
>
> Ned - equal!

Although Mrs. Dennis' initial question eliciting students' thinking about what it meant to be balanced was within the context of a balance as a mathematical tool, Jackie drew from his informal experience of balancing one's body in his response. Through Mrs. Dennis' facilitation, the connection to what it meant to be balanced, being equal on either side, was made by Ned. This is a great example of how Mrs. Dennis brought to light mathematical ideas related to students' life experiences and informal knowledge, and helped them make the connection between their prior experiences and mathematical ideas. Through this facilitation, the students were able to consider the mathematical concept, equality, in relation to their experiences in trying to balance their bodies. Through these types discussions, students were able to connect what they knew with formal mathematical ideas.

Mrs. Dennis responded to unexpected student responses in a variety of ways. In some interactions, if valid or unexpected responses arose during conversations or as the class went through their SS student journals, she responded directly in an evaluative manner, and continued on in the pattern of discussion in which it took place. In other situations, Mrs. Dennis intentionally called the class' attention and focused the discussion around an examination of the idea that had been put forth as a way to create a learning opportunity for all the students. Both "correct" and "incorrect" responses were the focus of discussion, but the discussion lasted longer when there was a controversial idea put forth, and more mathematics was discussed. Those interactions seemed to include profound interactions and thinking, although they were not always her initial goal. In some cases, such as with Ned, student thinking would be explicated to a certain extent, and then driven to move the student toward seeing what was viewed as a better solution or way of attaining a solution. In any case, the highlighting of unexpected student responses for the group's consideration provided opportunities for the development of students' mathematical thinking through the discourse.

The section discussed one way Mrs. Dennis responded to student thinking within the whole group discussion, which was to highlight unexpected ways students thought about the task. In the following section, an alternative response to student thinking that emerged in the data will be discussed–students responding to students. This construct will be discussed as related to another different, yet related construct, student-to-student interactions.

Students responding to students vs. student-to-student interactions.

A goal for Mrs. Dennis was to have students directly interact and communicate with other students about their mathematical thinking during whole group discussions. This proved to be very challenging with this group of first graders. It also became very challenging when a strong desire to have particular ways of thinking about and working with the mathematics as a goal for student learning determined the direction and focus of the discussions.

Overall, the students responded to one another's thinking in different ways throughout the year. They routinely worked with a partner and within small groups at their desks, and in this way interacted and responded to each other as they grappled with a mathematical task. During the first data collection period, both the teacher and I were under the impression that there was very little student-to-student interaction occurring during whole group discussions. The observational data I collected supported this impression, however, there were indications that a few students were responding to other students' work and ideas early in the year. From the beginning of the year, Mrs. Dennis elicited student responses about other students' mathematical ideas through asking for summarizing statements or to share what they thought about someone's ideas. She also asked students on a regular basis if they agreed or disagreed with what had been presented, and sometimes asked students to repeat what they heard to emphasize what had been said for others' consideration. These elicitations provided the space for students to make observations about and respond to other students' thinking. Some of these are contained in excerpts included throughout this document.

An interesting aspect illustrating the challenges of orchestrating discourse is how elicitations between students were sometimes used and the implicit messages that may have been sent to students. Mrs. Dennis sometimes asked students for agreements or disagreements knowing a student response was incorrect, as a way to tell the student he or she had a misconception that needed to be corrected. Once students disagreed with a student's presentation, they went on to make observations about why the response was incorrect, which both explained why it was incorrect, but also emphasized the erroneous thinking to the person who had initially shared it. The interaction between Ira, Kerry and Hunter (Interpreting and imposing section, p. 96), illustrated the role making observations about student thinking without input from the speaker can play in emphasizing misconceptions to a speaker. During that interaction, Ira and Kerry both saw patterns and commented on Hunter's work, but addressed their observations to Mrs. Dennis. Hunter was never asked to respond or provide his thinking for the class. Lastly, Kerry's comment that Hunter thought a double was a one plus zero equals a one communicates to Hunter that he is incorrect in his thinking. Although she is right, she indirectly told him about his misconception, rather than asking him a question that would encourage him to reflect on his thinking. Throughout the interaction, the message to Hunter is clear–he was erroneous in his thinking.

One might question, are first graders capable of asking each other questions about one's mathematical thinking? As mentioned previously, Mrs. Dennis made a concerted effort to have students communicate directly to other students during the second data collection period. She did this through directing students to communicate and ask questions, not to her, but to the student for whom it was meant. In this interaction, Hunter is presenting his solution to a problem involving reasoning, based on a series of pictures of pan balances containing various figures in different positions, about the values of those figures. The task involved determining equations to represent the relationships presented in the pictures:

T - What equations did you write?

Hunter - (pointing to the equations & the picture) Three plus three equals six, and six plus six equals twelve, and four plus four plus four that's twelve too.

T - Is there anybody that has a question for him? Like you're not sure you understand why he did something. Jackie, do you have a question for him? Or something that you want him to explain?

J - (to Hunter) Why did you do three equations?

Hunter - why did I make three equations?

T - So show him, he doesn't know why there are three; he wants to know what matches up with what. ok, so show on the picture, where that matches to. Jackie, take a look.

H - three plus three equals six. (points to balance w/ 3 circles on one pan)

T - Do you see why it matches that?

J - yes

T - Why does it match that?

Mrs. Dennis explicitly guided the students to ask each other a question, listen to each other, and held Jackie accountable for his response that he understood Hunter's explanation. This, as well as her address to the rest of the class, encouraged students to listen to each other. In addition, she explicitly asked the student to make connections between his equation and the representation, based on Jackie's question. Unfortunately, Hunter was unable to explain why he had three equations, instead repeating what he had shown on his paper, but it is through this type of encouragement that students began to ask questions about each other's work. However, students did not address each other directly without Mrs. Dennis' explicit direction that they do so.

The discussion contained in this theme of the findings demonstrates the complexities involved in orchestrating students' mathematical discourse within a whole group. Encouraging students to explain their thinking, engaging a range of learners in active contribution toward the discussion, managing and facilitating challenging student explanations, and dealing with unexpected student responses are just a few of the issues which make facilitating these types of discussions extremely complex. For some of these issues, the way in which Mrs. Dennis grappled and worked through them lies in the goals and purposes she holds for the lesson, discussion and students. This will be explicated in the next section.

The Role of Goals in Determining the Direction of the Discourse

Throughout the year, Mrs. Dennis and/or programmatic goals both implicitly and explicitly drove interactions. To a large extent, this determined the degree to which interactions were teacher-driven or student-driven. During the interactions in which the Stepping Stones materials were the focus, the discussions were driven by the need to complete the exercises shown as was the attainment of expected answers and the method for attaining those answers. The need to have correct answers, to have completed the exercises, and to see the concepts or skills in a particular way drove the interactions when the SS student journals were used. In contrast, when the discussion centered around a situational problem or a teacher-created task, more of the student thinking was explored and visible during the interactions. However, the drive for students to arrive at a particular response and to use a particular solution method was predominant throughout the interactions and implicitly emphasized through questioning and discursive patterns, no matter what type of materials or tasks were the focus of discussion.

Teacher-driven discussions.

This next excerpt is an example of Mrs. Dennis directing the discussion toward an expected outcome through a question she asks. In this excerpt, students have completed a series of exercises in their SS journal, in which they are presented with pictures of base-ten blocks representing two-digit numbers, and they must determine the smaller number:

T - Terry, what number do you have here?

Terry - mmmm. Thirty-two.

T - Ok, and for this number here.

Terry - Twenty-three

T - Good job, and which one has less, Terry?

Terry - twenty – twenty-three?

T - K, so we should have circled this one. K, I'm sorry, yeah, I told you the wrong one, cause you were supposed to circle the one that is…..

Rex - less.

T - less. So can you cross that one out, Rex? It should not be that side, it should be the one that is less. (points to correct answer) Um, Terry, how do you know that that one has less?

Terry - because….the thirty two has more, and the twenty three, has less.

T - Ok, it's a smaller number, right? Does it have less tens?

Terry - (nods)

T - Yes, it has less tens.

In this interaction, when the teacher asked Terry if the number 23 had less tens, she is essentially giving her the correct answer in the form of a question, and all Terry had to do was agree with the teacher. Through asking this direct type of question, Mrs. Dennis ensured that the correct answer was stated for the class, but it was she who explained why twenty-three is less than thirty-two, rather than the student. In addition, a teacher or program's goals are a determinant of the pattern of interaction that occurs. An IRE or funneling pattern of interaction is inherently based on a teacher's pre-determined goals for a lesson or interaction. Therefore, interactions in which Mrs. Dennis' or a program's goals drove the direction of the discussion, sometimes resulted in a funneling or and IRE pattern of questioning by the teacher and a directed or targeted interaction in which student thinking is not always explicated for the group. This is illustrated in the following excerpt in which students are analyzing the attributes of an irregular polygon:

T - Ok, so B & G, this is what they're talking about. Look at this corner here.

S - it's inside!

T - Ah, it's an inside corner. So you're not sure - somebody's calling it an inside corner, is it a corner, or it's not a corner,

Jerri - not a corner!
S - inside corner
(sev. sts argue about it)

T - Ok why is it not a corner?

Jerri - because its….(pause)

T - Well, let's look back at our paper. Let's look back at our paper. (Referring to their chart paper where they had previously written some definitions and drawings)

The amount of time in the pause following Jerri's attempt to explain why she thinks the corner is not truly a corner is very brief–not more than two seconds. Rather than allowing Jerri to complete her thought and explain how what she saw was not a corner, the students are directed to a chart paper, where they had previously written some definitions and corresponding drawings. This both corrects Jerri's incorrect response (without allowing everyone to see her possible misconception) focusing the group on one way of seeing a corner, and prevented her from explaining her own thinking to the class.

The two interactions above illustrate two ways in which Mrs. Dennis' goal of emphasizing an expected outcome drove the direction of the lessons. However, there were also interactions in which a negotiation of students' ideas in relation to the programmatic goals of a lesson also occurred. In the following interaction, Rose presented her ideas about why she thought an equation fell under a doubles-plus-two category:

> Rose - I know it's sixteen because one two three four five six seven, eight nine….sixteen
>
> T - Ok, so when you count all, it's sixteen, right? How did you use doubles, cause it's doubles-plus-two? What double did you use? What doubles did you use?
>
> Rose - Seven plus -
>
> T - How did you know that was doubles-plus-two?
>
> R - um, because I looked up here
>
> T - And what made you know it was, how did you know it was going to go under doubles-plus-two?
>
> R - (silent)
>
> T - You got it right, but I'm wondering how did you know it went here?
>
> R - (silent)
>
> T - K, boys and girls, if it says doubles, you have to be able to find the…..if it says doubles, you have to
>
> S - you have to know where it goes.
>
> T - find the doubles in that number. So what's the doubles in that number? Rose. What is the doubles that you would use, or that you see in that number?
>
> R - seven and seven?

As stated in the program materials, the goal for this lesson was skill-driven; it involved the students being able to use the doubles-plus-two strategy to solve addition facts. Implicit in this goal was having students investigate and see the relationships between doubles, double-plus-one and double-plus-two addition facts and their corresponding sums. Focusing on the goals set by the program materials, Mrs. Dennis centered the discussion around how Rose knew a particular fact was a doubles-plus-two fact. This was brought to the class' attention to assist other students with making sense of what it means to be a double-plus-two. However, when it was determined that Rose did not really use a double strategy to arrive at an answer, Rose did not know how to respond. Mrs. Dennis then addressed the class, stating that they needed to find the double in the equation, and asked Rose again what doubles she *would* use for that equation, to which Rose responded. This interaction highlights several things. First, it highlights the tensions that can occur for teachers in both honoring students' thinking and having them meet program-driven goals. It also highlights how Mrs. Dennis navigated this tension, through the simple adjustment of her question, from "what did you use" to a "what would you use", to which Rose responded, and the discussion was able to move forward. This illustrated a strategic move on Mrs. Dennis' part to have students explore possibilities in light of a student's response and negotiate the interaction between a student's strategy and a lesson's goal.

The preceding excerpt highlights Mrs. Dennis' attempt to include the students' voices and mathematical ways of thinking into the discourse of the classroom. Navigating pre-determined teacher goals or goals determined by programmatic materials in light of students' thinking is no trivial matter. It is therefore not surprising that the interactions in which teacher driven discussions occurred were predominant throughout the year, where the teacher's questions, statements, observations, etc. determined the direction and focus of discussions, no matter the type of curricular materials utilized.

Student-driven discussions.

In a few interactions during the year, students' thinking provided the basis for and drove the direction in which the discussion went. Mrs. Dennis put aside her agenda for a brief period and the discussion followed the students' ideas and thinking. The following example comes from the task described previously, in which students used their fingers and bodies to represent two-digit numbers and emphasize their place values. After establishing that the tens place is to the students' left, and the ones place is to the students' right, Mrs. Dennis asked how the number twenty-four can be represented using the students' hands. The discussion evolved into deciding how many people need to be called upon to represent the number 24 and different combinations, which total the number:

T (counts w/ sts.) - Ten, twenty, twenty one,....twenty four

Peter - I thought supposed to be five people.

T - So, was Kekoa right? He said

Sts. - yes!

T - four, he said three people. Cause he said two for the…..

S - he said two for the tens, and

T - tens and one person for the….ones. Okay, thank you very much.

Peter - it can be five people, Mrs. Dennis

S - It can be five people

Kylie - we can have five people, one person can hold five, and another person can hold five and five and five

T - Oh, so if one person uses one hand, they're doing another way of thinking about this. They said what if Kekoa held up only five, Kekoa hold up five. And he held up five, hold up five, and the two of you come and hold up five

(four students hold up five fingers)
T - Ok, now let's count by fives.

Sts. - five, ten, fifteen, twenty

T - Twenty-one

Sts. - twenty-one, twenty-two...twenty four.

T - Ahhh, I like your math thinking. You split the ten into fives.

In this interaction, although Mrs. Dennis' intent had been to represent two-digit numbers with students' fingers, and emphasize the place value of those numbers, the students became interested in deciding how many students would be needed to represent the numbers, and that evolved into the above discussion, where she followed Peter's suggestion that five people could represent the number 24. This flexibility not only allowed for a different way of determining a combination for 24, but it allowed the students' ideas to drive the direction of the discussion, providing a different way of thinking. The interactions in this excerpt illustrated how student thinking can not only provide the basis for and determine the direction of a discussion, but also incorporate divergent ways of thinking into the discussion.

Although it didn't occur often throughout the year, interactions in which students questioned each other and had direct student-to-student interaction also naturally resulted in student-driven interactions. As students exchanged a few sentences with each other or asked each other a question followed by a response, their ideas and inquiries became the focus of the discussion, and had the potential to drive the discussion. However, in this class, because those types of interactions did not occur for sustained periods of time, student-to-student interactions did not predominantly determine the direction of the discussion. Nonetheless, it is interesting to consider the added complexities student-student interactions might contribute toward an already complex situation. Based mainly on student thinking, these types of conversations can make it difficult for a teacher to have singular control over the way students engage with the mathematics. This does not preclude teachers from having pre-determined learning goals and objectives, however, ideas upon which the discussion are based originate from the students' own mathematical conceptions and the apprehension of new ideas occur in relation to their current ways of thinking. Along with other aspects, including student thinking as the basis for discussions, this characterizes the nature of student-driven discussion.

The analysis of the data suggested that the role in which a teacher's or a program's goals played in determining the direction of the discussion impacted the degree to which the discussion is teacher-driven or student-driven. In addition, the distinction of teacher-driven discussions from student-driven discussions lies in the degree to which a teacher's goals determine the focus and nature of the mathematics emphasized.

The Role of Representation in Developing Discourse

The role in which representations played in the discourse development of this classroom comes at the end of this document, not because it is less important, but because many of the interactions which provided insight into the role and use of representations in mathematical discourse have already been included and discussed in previous sections of this paper.

Throughout the year, students in this classroom had access to manipulatives and tools with which to do mathematical work. They used manipulatives to model mathematical situations and as thinking tools with which to solve problems, see relationships, and explore mathematical ideas. Manipulatives were used to represent their mathematical ideas in a variety of contexts - during teacher-led whole group discussions on the floor as well as when they worked through a problem at their desks, and students had multiple opportunities within and outside of the mathematics lessons to explore and utilize manipulative tools for a variety of purposes, including games and during problem solving.

Following the presentation of a task or problem, students typically worked with manipulatives at their desks and recorded their thinking on an answer sheet using pictures, diagrams and equations to represent their mathematical thinking. The answer sheet would then be placed under the document camera and displayed for everyone in the class to see. The presenter spoke from the front of the room, using his/her displayed work as a reference, while the teacher controlled the student's work at the document camera. This was the typical way in which representations were used during whole group discussions, as a reference for presentations, but still under control of the teacher. Along with students' verbal explanations, the displaying of student work during presentations focused the discussion on student thinking from the outset of the discussions.

There were a few lessons throughout the year, in which students' representations were incorporated into their explanations during whole group presentations. During these interactions, students' mathematical thinking became explicitly clear for the entire class. One occurred when Rose illustrated how she saw the relationship between a double and a double-plus-one addition fact (see Establishing focus section, p. 83) by going up to the board and drawing groups of circles. We had not known how she thought about it until she actually drew in the circle to show how she changed a double plus one into a double. This action brought to light her understanding of the relationship between a double and double plus one addition fact, and what she understood a double plus one fact to be. Another such interaction occurred with Ned and Rose as they explained how they collaborated to systematically find different addition combinations using six coins consisting of dimes and pennies (see Round 3, p. 77). Although the rest of the students were not able to directly see the coins as Ned and Rose made their switches, using the manipulatives to illustrate their thought process made the explanation salient for the rest of the students, and engaged the entire class in making sense of their thinking. As with Rose's interaction, the use of the coins and the actions the students used to recreate their thought processes during their explanations made their thinking clear for everyone in the class, and encouraged everyone to engage in the discussion. In addition, all of the interactions in which there was sustained student explanation, interaction and in which the mathematics seemed especially salient for both the speakers and the rest of the students involved the use of representations and the students' actions on the representations.

The role representations played in validating mathematical ideas was also brought to light in a teacher-led discussion in which Mrs. Dennis used a balance and base-ten blocks to engage students in comparing two-digit numbers:

> T - Ok, well, let me try something different. Ok, I'm going to do this, this time. I'm going to put, okay look three (tens) - how many do I have in there right now?
>
> Sts. - three
>
> T - Three tens, okay, look, I have all these ones. I'm gonna stick this on this side.
>
> (sev. sts. predict what will happen) That's equal
>
> T - Ok, let me tell you how many there are. (counts by twos) twenty two - I'm going to put twenty two in there,
>
> Rose - it's still going to be - it's still going to be, this one is going down because it has more cubes than it. That's only twenty-two, and it's twenty nine in there.
>
> T - Ok, let's take a vote - who thinks this side is going to….what's going to happen when I put this in? Here, let me pick a name. Jackie. I have twenty-two of these, I'm going to throw that in here. What do you think is going to happen when I put this in here?
>
> Jackie - (pauses) uh, uh, the, uh the red is one still gonna be up.
>
> T - The red's going to stay up?
>
> J - because there's three tens, but the three tens is thirty, and it's twenty-two, and the thirty is more.
>
> T - Ah, anybody thinks the opposite of Jackie? Anybody think that this side is going to go down, and this side is going to go up?
>
> (several sts. raise hands)
>
> T - Jerri, you think this one's going to go down?
>
> Jerri - (nods)
>
> T - Ok, how come? (looks at Rex who also has his hand raised)

Rex - because you have twenty-two of them.

The use of the balance allowed Mrs. Dennis to remain neutral as the students made their predictions. This interaction was followed by the action of pouring the 22 ones onto the balance, and the students drew conclusions about which number was greater based on their own observations, rather than relying on Mrs. Dennis' validation. Furthermore, with the use of the balance and Mrs. Dennis' elicitation of student predictions, more student thinking emerged in the discussion. It was important to note that Rex thought the group of ones would be more than the three tens because that group contained more blocks, and in the subsequent discussion, Mrs. Dennis asked the students to consider reasons for what occurred. Several ideas were presented, including the notions that there were eight more in 30 than in 22, and that "it looks like a lot, but it's not really a lot".

The use of representations in a few of the whole group discussions had a dramatic impact on the discourse, influencing aspects such as students making sense of their own mathematical thinking, making their mathematical thinking explicit to others, and providing validation for mathematical ideas. Providing students with the opportunity to use and manipulate their representations as they explained what they had done resulted in richer discussions in which student thinking was a predominant focus.

Students' Engagement in the Standards of Mathematical Practice

It is important to consider that as they are written, the SMPs describe mathematical processes and ways of thinking that proficient students exhibit. In other words, they present aspects of mathematical dispositions as end goals, but do not delineate how students go about developing them, nor do they provide teachers with guidance for providing learning experiences, which facilitate students' development of them.

Although authors of the CCSSM state that the SMPs were derived from the NCTM's Process Standards, they do not explicitly support the approach of developing mathematical discourse as a way to have students develop the SMPs. However, developing mathematical discourse in the classroom provides the context and the opportunity for students to develop each of the SMPs in the spirit that was envisioned by the authors of current reform. Mathematical discourse provides a forum in which students' development of the mathematical processes described in the SMPs can occur. Furthermore, development of the SMPs can be observed through the discourse.

As would be expected, the observational data collected in this classroom did not suggest students engaged in developing the all of the SMPs during all of the lessons, or some of them in some of the lessons. Part of the issue involved the degree to which students were provided with opportunities to engage in the SMPs, and whether or not opportunities were facilitated and capitalized upon by Mrs. Dennis and the students. There were instances in which a few emerged in the discourse, in various ways, but the data do not suggest that development of the SMPs was predominant during the course of the year. At times Mrs. Dennis engaged in a few of them as a way to model the thinking involved for the students. At times the students engaged in them directly, and through reflecting on their experience in that lesson, was able to explain something that was learned from the experience. There were also instances where words that might be construed as aligned with an SMP were used in a discussion, but the thinking behind those words did not really capture the essence of the MP.

That students were regularly presented with contextualized problems to grapple with and solve without being presented with a pre-determined solution path means they regularly engaged with several of the MPs, such as MP1, Make sense of problems and persevere in solving them, MP2, Reason abstractly and quantitatively, and MP4, Model with mathematics. The following interaction was one in which two of the MPs (MP 1 and MP 4) became salient. The class had worked on a problem in which they needed to determine the number of eggs eaten on Saturday and Sunday knowing that the total eaten was 12. The only constraint was that more eggs had been eaten on Saturday than on Sunday. Several students had already presented their ways of thinking, and the discussion was coming to a close when Mrs. Dennis quickly placed Lane's paper on display to reinforce their previous discussion about the constraint in the problem:

> T- Ok, so look, I wanted to share, this was actually Lane and Greg. Look at their equation. Why did they have 12 and with a minus and question, minus a question?
>
> Lane - because we don't see the numbers, we can't see the numbers that we need to make twelve.
>
> T - Ah, so Lane said, it doesn't say about the other numbers. But Lane knew that you had to make….
>
> Lane - twelve.
>
> T - Twelve. They had to equal up to twelve. So that's how she was thinking about this equation, that something and something has to make…..

This interaction highlighted how Lane attempted to make sense of the problem situation, and actually wrote an equation, which modeled what she grappled with in the problem. Although Lane was not able to come to a complete solution to the problem, she wrote an equation, which modeled the situation and illustrated her mathematical thinking. This brief interlude showed her development of MP 1, Making sense of problems and persevering in solving them, and MP 4, Modeling with mathematics. Mrs. Dennis did not explicitly discuss this aspect of Lane's thought process, but called the class' attention to it.

In several other interactions, the notion of making sense of problems as paying attention to information provided and a problem's constraints were highlighted. Two interactions, one involving Ned (see Unexpected student responses, p. 77), who saw the importance of paying attention to the information pictures provide as part of a problem, and another interaction involving Rose, in which she states that it was important to recognize the information provided in the problem both illustrated the students' realization about this aspect in solving problems. During both of these discussions, Mrs. Dennis elevated their observations of their learning in these particular situations to the general process of making sense of problems in order to solve them.

Another way in which making sense of problems played a role in the students' mathematical work and the way in which Mrs. Dennis facilitated that came to light during one lesson in which the students engaged in a particularly challenging problem. The class went through their usual routine of being presented with the problem in a whole group and then returned to their desks to work. As the students worked on the problem, it became evident that it was very challenging for them. Rather than stopping the lesson to provide more direction, Mrs. Dennis moved from group to group, and each time asked students, "how do you know that?" and "why did you get that?". This action encouraged the students to consider the problem further, and though most of them never came to a solution at the end, their thoughts about the problem developed from their initial interpretations.

Although MP 3 explicitly describes incorporating mathematical communication in the classroom, students' development of aspects characteristic of this standard was not predominantly observed. For the most part, students did not present arguments; rather, their utterances included presentations and explanations about what they had done. Although there were instances in which the students were prompted to provide reasons for what they had done, the discussion never took on the form of an argument supported by reasoning. In most situations, if the student's answer and reasoning was correct, Mrs. Dennis validated it, and if it was not, she responded with a question or comment which resulted in a funneling pattern of interaction toward the correct response. In only one interaction, that with Ned (see Unexpected student responses, p. 100) was there an argument put forth by a student. Throughout that interaction, Ned insisted that there was no reason blocks couldn't be added to both sides of a balance scale and presented reasoning for why this was so based on the written directions for the exercise. Although one student, Jackie, challenged Ned a little, the interaction mainly involved Mrs. Dennis illustrating to Ned why he was mistaken.

This brings to mind the possible need for student-to-student interaction in order to "justify their conclusions, communicate them to others, and respond to the arguments of others" (NGA & CCSSO, 2010). Although not explicitly stated, in order for students to engage in mathematical arguments in which their reasoning is presented as justification, it is necessary to have a back and forth communicative process with one's peers. The term, argument itself implies the process of presenting one's "logical progression of statements" (NGA & CCSSO, 2010) for others' response, whether it is verbal or in writing. For young children who are still developing their communication abilities, this process would naturally occur verbally prior to presenting clearly written positions. In this first grade class, because students were at the beginning stages of questioning each other's mathematical thinking, and these interactions were never sustained beyond asking an initial question, the students never reached the level of presenting an argument and justifying oneself to one's peers.

The final practice, which was exemplified in the interactions, was MP 7, Look for and make use of structure. Several interactions during the latter part of the year illustrated students' engagement with aspects of this practice. Ned and Rose's discovery of a pattern in the sums and combinations of pennies and dimes as they systematically interchanged coins is an example of discerning a pattern from their actions. Another interaction involved students seeing that selecting the greatest addends given multiple options would yield the greatest sum:

> T - That's what Peter said. So Peter, can you tell Braden how did you pick your coins? Braden, look at Peter, look at Peter. How did you know how to choose your coins? When it said the greatest?
>
> Peter - um, cause, because twenty-five plus twenty-five equals fifty, then you add ten, then it makes sixty.

T - But why did you choose the twenty-five the twenty-five and the ten?

Kekoa - I know why.
Peter - Because that's the' that's the greatest number, that's the greatest coin in the wallet.

T - Ah, Kekoa, what were you going to say?

Kekoa - (looks up, apparently forgot)

T - Were you thinking the same way as Peter?

Kekoa - yeah, because uh twenty-five and twenty-five was the greatest number, and I added the second greatest number, ten, and then it made sixty.

Seeing structure in mathematics involves being able to "step back for an overview and shift perspective" (NGA & CCSSO, 2010, Look for and Make Use of Structure). In this excerpt, Kekoa and Peter intuitively saw that in order to have the greatest sum, one would add the greatest values of a given set of addends. Rather than adding numbers at random, they were able to take a broader look at the relationship between the size of the addends and their corresponding sums and use that to help them solve the problem. We were never clear about how Kekoa and Peter came to see this relationship, and their reasoning was not explicated for themselves or the other students, but was validated by Mrs. Dennis. If their reasoning about this relationship had been explored, the students could have engaged in MP 8, Look for and express regularity in repeated reasoning, since that would have naturally led to looking for relationships in the numbers and equations that supported their thinking. Discussing their reasoning would also have brought the relationship to light for other students to think about. As it was, it remained a mystery for those students who did not immediately see it.

There were no observations in which the remaining practices, MP 5, Use appropriate tools strategically, MP 6, Attend to precision, and MP 8, Look for and express regularity in repeated reasoning, emerged within the discussion. Students had regular access to mathematical tools with which to work as they solved problems, and had some choice regarding their use, but neither strategic selection nor consideration of the strengths and limitations of different tools emerged in the discussion. Although the students cared very much about being correct in their calculations and answers, their goal was to be correct, not necessarily to be precise and accurate. In addition, they were still developing their ability to communicate precisely. Lastly, as with the interaction above, if the students saw a relationship or pattern in the numbers they worked with, the discussions never reached the point of exploring the reasoning behind the observations that were made.

CHAPTER 5
CONCLUSIONS

This last and final chapter includes an explication of the implications and conclusions based on the findings of this study. Because the ideas behind developing mathematical discourse are not new, and there has been extensive research in this area, the discussion here will focus on aspects that emerged which were salient to this particular study, are pertinent to the current climate of education and highlight aspects of developing whole group discourse with young children. The discussion focuses on implications for broad, general aspects of teaching and learning regarding discourse, then moves to the more specific issues surrounding students and the role of teachers in educating their students. The chapter will end with a discussion on the relationship between discourse and the SMPs, as this is where further research is most needed. Implications for five areas will be discussed, forming the conclusions of the study. They are: 1) Insights into the Analytical Framework, 2) The Role of a Lesson's Learning Goals, 3) Developing Intersubjectivity with First Graders, 4) What it Means to be Student-Centered, 5) The Role of the Teacher, and 6) Relationships between Mathematical Discourse and the Standards of Mathematical Practice. These six areas describe insights gained by the researcher, taking into account issues that currently exist in education, and at the same time, builds upon prior research.

Conclusion I: Insights into the Analytical Framework

This research project largely drew its inspiration from the Levels of Math-Talk Learning Community (Hufferd-Ackles et al. 2004) framework (See Appendix B), which delineates mathematical discourse in the classroom. Used as an analytical framework, it provided guidance and a lens through which the analysis of the present study's data took place. In the following discussion, the findings from this present study will be compared with aspects of mathematical discourse described in the Math-talk framework, as a way to build upon prior research. Because the context in which the Math-talk framework was developed is different from the context for the present study, the findings of this study provides additional insight into that research. The beginning of the discussion will focus on ways the present study departed from the ideas contained in the framework, to help define the context in which the present study took place. This will be followed by insights into aspects of discourse this study contributes toward the framework.

The first point of departure from the framework involves the linear nature of the trajectory from a traditional mathematics classroom toward the reform-oriented discourse of a math-talk learning community. The trajectory defines development across four components, Questioning, Explaining mathematical thinking, source of mathematical ideas and responsibility for learning. While the components and their levels individually provided an analytical framework for the present study, the linear nature of the developmental levels did not. Although the Hufferd-Ackles et al. (2004) study presents the idea that development up the trajectory does not mean classes will remain at the highest level once achieved, their evidence showed that the class had indeed progressed up the levels of the trajectory in a somewhat linear fashion, experiencing some oscillations at the highest levels when new material was under study. In contrast, the teacher and students from the present study did not progress along a linear trajectory of development. Rather, it was uneven and non-linear, which seems to be consistent with other current research on teachers' attempts to implement mathematical discourse (Ball, 1990, Baxter & Williams, 2009, Nathan & Knuth, 2003, Sherin, 2002, Ross, et al., 2002, Spillane & Zeuli, 1999, Stein et al. 1996). Furthermore, there were variations within the components of the framework, where some aspects of a component were present, but others were not. For example, two aspects define the Explaining mathematical thinking component at level two - the teacher probes student thinking more deeply, supporting detailed descriptions from students. The teacher is also open to and elicits multiple strategies from students. Although the teacher in the present study was open to multiple student strategies, she did not always probe student thinking deeply. Therefore although she exhibited some aspects of a component, she did not with others. Reinforcing the shift in epistemological basis for learning mathematics necessary for transforming practice, the data from the present study highlights the challenge and complexity involved in changing the way students learn and think about mathematics. This sends a signal to professional developers, administrators and teachers themselves, that adopting and implementing reformed approaches to mathematical discourse takes time, perseverance, and flexibility on the part of the teacher. The data in this research project highlights the magnitude of the shift teachers need to make in order to have their students engage in reform-oriented mathematical discourse.

In designing this study, I was particularly interested in the development of mathematical discourse of young children in terms of the Math-Talk framework. Were first graders capable of engaging in this kind of mathematical discourse? How consistent would the mathematical discourse that developed in a 1st grade classroom be with the ideas delineated in the framework? The framework provided a lens for describing and analyzing the data collected in this study. The purpose of the study was not to evaluate the teacher's practice or the students, rather, to gain insight into first graders' mathematical discourse and its development.

Predominantly, aspects of the mathematical discourse that emerged from this study can be described by the framework, but at varying levels. For example, highlighted in the findings section was the tendency toward an IRE pattern of interaction when the SS program materials were used in the whole group discussions. This is an example of the interactions being categorized at a level zero across most of the components while looking at those lessons as a whole. However, in some of the interactions involving the SS student journal, there was some departure from a complete IRE pattern, where the teacher began to ask questions such as, "how do you know?" or "Why did you think that?". When this occurred in the classroom, the interactions moved toward Level one in the framework. Variations also occurred within a lesson. For example, two radically different patterns of interactions occurred within the same lesson, related to the materials used. When a contextualized problem was the focus, some of the aspects of some of the components in Level two were observed, and within the same lesson but at a different point in the discussion, aspects from Level one were observed.

An important consideration to make involves the level at which the analysis of the data occurred (Sherin, 2002) and its alignment to the framework. In the present study, the findings include interactions analyzed within lessons (micro) as well as across lessons (macro). In the Hufferd-Ackles et al. (2004) study, it is unclear how variations in the interactions were accounted for. Although not addressed explicitly, it can be inferred that the interactions were categorized at a particular level over a longer period of time and at a macro level. It is possible that, because the analysis in the present study involved analysis at a micro level, variations within and across levels occurred. One way to mediate the variation would have been to focus on a particular sub-set of lessons at a micro level, which would account for the interactions within lessons. The Math-Talk Learning framework does not address differing analytical levels, therefore, a different framework would need to be applied or be developed through future research.

That aspects from the Math-Talk framework were observed, leads me to believe that it is certainly possible for first graders to engage in and co-construct a mathematical discourse community, if they are provided the opportunity and facilitation. What might differ, however, is the *way* in which the discourse is developed, in the nuances of teacher decisions involving the environment and facilitation. Although the way in which a teacher might facilitate discourse development is somewhat implicit in the framework at a general level, it would not provide enough guidance for a teacher to define the conditions under which particular groups of students might develop mathematical discourse. For instance, a well-established recommendation involving making young children's thinking explicit through the use of students' representations and their explanations, as a part of instruction (Sophian, 1999), is not explicit in the framework. The use of representation was also an aspect integral to students' engaging in mathematical discourse that emerged from the present study. Although the Math-Talk Learning Community framework provides components of mathematical discourse delineated by teacher and student behaviors at a general level, further insight into specific aspects of facilitating and orchestrating students' discourse in different contexts, such that is provided in this study, is needed.

Conclusion II: The Relationship Between Mathematical Discourse and Lesson Goals

Two themes in the findings bring to light the relationship between mathematical discourse and lesson goals - their role in the development of mathematical discourse and in influencing the mathematics emphasized to students. The role lesson goals played both at the outset of a lesson and during their enactment impacted the mathematical discourse that developed in the classroom and hence, the nature of the mathematics students interacted with. It is no surprise that the way a lesson's learning goals are enacted have a direct relationship with the mathematics emphasized during a lesson. However, observing the *way* in which the discussions unfold lends insight into the role learning goals and targets play in influencing the mathematical discourse and the mathematics students interact with. This section will begin with looking at how the discourse influenced both the nature of the discourse and the mathematics that became the focus of the discussion. This will be followed by a discussion about implications for the wider educational community.

During the first period of data collection, two interactions highlight how studying discourse can lend insight into the role learning goals play in influencing the mathematics that is emphasized to students. The first one involved Rose, who drew her dots to show doubles and doubles plus two. There was an instance where Kerry comments that what Rose has made is a doubles plus two rather than a doubles plus one. Throughout that interaction, there were multiple goals that were intended and enacted as the lesson unfolded. First was the programmatic goal of using a double to solve a double plus one addition fact. Implicit within this goal was the need to see the relationship amongst doubles, doubles plus one and doubles plus two addition facts. The teacher was forced to navigate these multiple goals, as well as her own for the students as the lesson is implemented. As was observed in the discussion, the teacher drove the direction of the discussion toward making and seeing a double and double plus one in the pictures and exercises the students complete and discuss. In subsequent lessons, the focus is also on how to turn a double into a double plus one or a double plus two, and how to find the doubles within the other addition facts. In her desire to have students meet the programmatic goal of being able to use a double to solve a double plus one and double plus two addition facts, study of the relationships between these addition fact categories and what it meant to be a double plus one or two fact was not emphasized in the lesson. Rather than investigate Kerry's observation of Rose's inadvertent action in making a double plus two, the discussion is directed by the teacher toward emphasizing the procedure for turning a double into a double plus one fact. This highlights how the way the lesson's goals were enacted influenced both the nature of the mathematical discourse and the mathematics that was emphasized to the students.

The second interaction involved Ned, as he attempted to defend his notion that blocks might be added to two sides of a balance scale, which showed one pan lower than the other. Again, the teacher's drive to have students meet what she perceived as the programmatic goal, and to have students complete the worksheets correctly, highlights a missed opportunity to investigate a different way of thinking which could have provided an important foundation for mathematics students will investigate in the future. In this interaction, the notion that in order to complete the exercise correctly, cubes needed to be added to one side of the balance, the side that is already lower, to represent a larger quantity was communicated to students. It is unfortunate that the discussion did not include Ned's idea that one might add cubes to both sides of the balance and still have the right side showing a greater quantity. This interaction again influenced the mathematics that was emphasized - that being greater meant you add to one side which will make a side go lower - valid thinking, but it is not the only way to arrive at a valid response, representing a missed opportunity to see a different way of thinking leading to the investigation of more mathematics.

These two excerpts and what they highlight about teaching and learning show the interaction between learning goals and discourse development within a lesson and how they influence the mathematics that is emphasized. Ned's interaction also introduces the possibility that student thinking can be the driver of the discussion direction and still remain within the intended goals of the lesson. If Ned had been asked to provide an example that illustrated his idea that blocks could be added to both sides of the balance and still be valid, another way of thinking about reducing and increasing quantities might have been introduced, but the discussion would still have remained within the intended goals of the lesson, which was to compare quantities illustrated on a balance scale. A possible area for future study, research into ways divergent student thinking and alternative forms of discourse might occur and stay within the intended goals of the lesson would be helpful to practicing teachers as well as contribute toward existing literature.

These interactions also highlight a reflexive relationship that may exist between the goals of the lesson and the discourse as the lesson is enacted. Intended goals can determine the discourse of the lesson, depending on how they are enacted. Reflexively, the evolution of a lesson's goals can also occur as the lesson unfolds, through the discursive actions of the teacher and students. In Stepping Stones, the intended goal of a series of lessons was to use understanding of a double to determine the sum of doubles plus one and doubles plus two addition facts. However, during their enactment, the ideas evolved toward focusing on how to make a double into a double plus one, or how to see a double within a double plus one. In this way, the goals of the enacted lesson evolved from what was intended, mediated by the way the discussion unfolded. Had the teacher further explored Kerry's observation that Rose had created a doubles plus two fact that was also another double, the mathematical discourse would have turned toward student-driven patterns of interaction and ideas that were the focus of the discussion would have been different.

In the case of this particular study, a complex array of goals, pre-determined and enacted, influenced both the mathematical discourse and the nature of the mathematics that was emphasized. First there were explicit and implicit goals set forth by the Stepping Stones program materials the teacher used in the classroom. Each lesson included at least one-two learning goals explicitly stated within each lesson, including those rooted in the MPs. Within the student journals, goals such as attaining a single correct answer, filling in the blank spaces and completing the exercises on the entire page were implicitly emphasized and determined the pattern of interaction when those materials were used. The goals for the lessons emerged and evolved through the interactions of the teacher and the students and were apparent in their verbal utterances and actions. In addition, the teacher enacted her own goals through the discourse, mainly focusing students on the attainment of valid responses using valid methods. At times, the goals morphed into versions of what had been initially planned, in response to students' interaction with the activities of the lesson, however, the goal to have a valid response in the eyes of the teacher mainly drove the interactions. This complex movement of establishment and evolution of goals before and during enactment illustrate the complexities of working with learning goals and outcomes for a particular lesson. This has implications for the current climate of education, in which pre-determining learning outcomes and targets play a large role.

Across the U.S., a strong movement involving the use of formative assessment as a part of instruction currently exists. This movement is supported by a body of research demonstrating the effectiveness of formative assessment in students' academic achievement in a variety of areas (Black & Wiliam, 2009, Hattie, 2009, Heritage, 2010, Sadler, 1998, Stiggins, 2005). One aspect of formative assessment that has garnered a strong emphasis is the notion of setting pre-determined learning goals and targets in support of teachers and students gaining clarity for what the focus of a lesson ought to be. For a lesson to be successful, it is believed, the teacher needs to be clear about the learning outcomes of a lesson. While I do not dispute this notion, the teacher's use of learning goals in enacting a lesson needs to be mediated to involve and engage students in constructing their understanding of concepts and ideas contained within the goals of the lesson.

For the Hawaii Department of Education, and other educational systems which adhere strictly to a standards-based education in which teaching with the end in mind and determining learning targets and communicating those targets to students are paramount in instruction, developing student-centered discourse will be a challenge, as these views seem to be based on conflicting ideas at their most foundational levels. Standards-based education can promote a teacher-directed approach where students attain knowledge pre-determined by experts (Deboer, 2002). In contrast, a student-directed approach resembles mathematics as a discipline in a way that teacher-directed approaches do not (Deboer, 2002). For teachers who exist in standards-driven environments, this highlights a struggle rooted in epistemological beliefs, among other things they will have to navigate. However, with new standards, such as the SMPs in the CCSS, and the ideas contained in teaching frameworks such as the Teaching and Learning Framework (Danielson, 2009) being used for teacher evaluation, the idea of driving student discussion toward a pre-determined end in mind is called into question and highlights a conflict educational systems will encounter, but may never address. Complexities inherent to the enactment of learning goals rather than their over-simplification need to be embraced at all levels of educational systems. A re-conceptualization of the role goals and standards play in lesson enactment is strongly needed. Furthermore, practicing teachers need to be given the space to deviate from outcomes-based approaches to teaching and learning that allow for inquiry and investigation of student thinking by teachers and students as lesson goals are enacted.

Conclusion III: Developing Intersubjectivity with 1st Graders

The role of language and communication in the development of students' mathematical conceptions does not occur in isolation. Consonant with sociocultural and interactionist views, student explanations and construction of thought occurs within the context of whole group discussions, through interacting with others. This involves the processes of negotiating meaning through communicating with others, in which individuals experience adaptations in cognition (Cobb & Bauersfeld, 1995). These adaptations occur through the development of intersubjectivity (Voigt, 1994, von Glasersfeld, 1991), in which the members of a learning community negotiate taken-to-be-shared mathematical meanings within a group of individuals. In the present study, although the students in this class regularly engaged in open-ended tasks in which they grappled with the mathematics and communicated their strategies to others in whole group situations, the negotiation and co-construction of collective mathematical meanings were absent in the classroom. Rather, the mathematical concepts and meanings that were accepted by the class, while they may have been based on students' initial ideas, were largely the teacher's. Furthermore, the role of communication in this class was to provide a way for the teacher and other students to formatively become acquainted with students' ways of thinking, rather than as a way for students to collaborate and develop collective mathematical meanings. This was apparent during observations and validated during interviews with the teacher, in which she stated that having students present their work allowed her to "see" their thinking.

Negotiated meanings begin as explicit statements or arguments, which contain reasoning and justification (Voigt, 1994). Although the students never presented formal arguments and the teacher heavily guided the development of mathematical meaning, I would argue that their observations and responses to each other's ideas served as emergent versions of explicit statements about mathematical thinking. The data highlighted students' observations about others' mathematical thinking including responses in the form of summarizing statements, agreements, disagreements and emergent forms of reasoning. Students' observations about others' ideas shared during whole group discussions served as explicit statements about mathematical thinking. Although their observations and comments on student thinking were never elevated to the investigation of mathematical objects, nor was there a true negotiated consensus developed through the validation and reasoning about mathematical models, I would argue that the students exhibited emergent forms of negotiated meanings as they responded to others' thinking in various ways. Negotiated meanings begin with making observations or commenting on some form of mathematical thinking.

Furthermore, in order for negotiated meanings to occur, one must have the participation and cooperation of the members of the group. The students in the group were largely challenged with listening to each other's mathematical thinking, and were still developing their abilities to engage in mathematical conversations in which they interacted with their peers. The development of the social context took a lot of the class' time and the teacher's energy. As mentioned previously, lessons in which all of the students actively engaged in the conversations at the same time did not always occur. It took a long time for most of the students to become acculturated into the context of the mathematical discourse. This highlights another challenge teachers of young children will need to be prepared for, and further supports the need for teachers to explicitly teach students how to engage in whole group mathematical conversations, including the norms and agreed-upon disciplinary values of engaging in mathematical discourse.

Conclusion IV: What it Means to be Student-Centered

Many supporters of reformed approaches use the words, student-centered, to characterize discursive interactions in the classroom (Deboer, 2002, Mehan, 1979, Sherin, 2002). In addition, the practice of having students talk and question one another is positively viewed in the Math-Talk Learning Community framework (Hufferd-Ackles et al., 2004), as well as in the Framework for Teaching (Danielson, 2007). In some studies, students' engagement in discourse with each other seems to be a hallmark of reform-oriented discourse. But what does it mean to be student-centered? What does it entail? What is the relationship between students interacting with other students and developing a student-centered classroom? These are questions I encountered in conducting this study. Several themes that emerged within the findings section coalesce and inform what it means to be student-centered–Student-driven discussions and several categories within Issues with Orchestrating Students' Discussion. Ideas within these themes in relation to relevant cognitive development theories and their implications will be discussed below.

Conducting a deep analysis of the interactions in this study pushed me to consider aspects of interactions, which would be characterized as student-centered. There were interactions I observed which were obviously student-centered in that the students' ideas determined the direction and provided the foundation of the discussions. When a student asked a question or made an observation, which became the focus of subsequent conversation, this action moved the discussion's direction to follow student thinking, thus making it the driver of the discussion.

However, through my observations, I also realized that this alone did not result in a student-centered discussion. Having a student-centered view of teaching and learning also involves being grounded in epistemological beliefs consonant with social constructivist learning theories. An example of this was in the degree to which students were given the opportunity to explain their own thinking and communicate their own ideas. Grounded in social constructivism, students' explanation of their thinking for others is a way for students to reflect on and construct their ideas and thoughts. Students develop thinking through communicating their ideas, as tentative as they may initially be, making connections between their prior understandings and new, more formal concepts. In the present study, students were given the opportunity to present their initial ideas and strategies for solving a problem to the group, but were not always given the opportunity to explicate their thinking further. The impact of this on students' developing conceptions of the mathematical ideas with which they were investigating is outside the scope of this research project, and highlights an area for future study.

Although interactions in which students explicated their own thinking was not predominant in the data, there were a few instances in which students' verbal contributions, questions and thought processes provided support and insight into existing research on the development of students' mathematical thinking through engaging in discursive interactions. The first set of examples involved two of Jackie's interactions, in which he made a connection between what it meant to be balanced with the mathematical tool one uses to show equal quantities. In the other, he conjectured that mathematical pictures could be thought of symbols and needed one's attention when solving problems. These unexpected responses to the teacher's elicitations drew from Jackie's prior life experiences outside of the mathematics classroom, and brought to light their mathematical connections for Jackie and the other students through the discussion. The second set of interactions involved Kai and Braden as students who were challenged in communicating their thinking clearly. Through the process of explicating their tentative thinking, with the teacher's facilitation, these students clarified their tentative mathematical observations and in doing so, developed their mathematical thinking. These two sets of examples illustrate the development of mathematical thinking through discursive interactions, first in the relating of new mathematical ideas with students' experiences, and second in facilitating students' clarification of their initial ideas. These occurred through the discourse, involving the students' active participation and construction. In this way, I propose the discursive processes through which students make connections within their schemas and clarify their thinking further informs what it means to be student-centered.

One aspect of both communicating to make sense of one's own thinking and to illustrate one's ideas to the wider learning community involves the use of representation in student explanations. Several interactions in which representations played a major role in making sense of students' thinking were observed, first in Rose's example in which she went up to the board and drew what made a double plus one addition fact. The second interaction occurred at the end of the year, and involved Rose and Ned's explanation of how they switched coins and found a symmetric pattern in their actions. In these interactions, students utilized the actual models and tools they worked with in constructing their ideas in their presentations, and were able to act upon them as a way to illustrate and re-create their thought processes. These actions afforded complete clarity into their thinking by the community, and allowed others to see relationships between different ways of thinking. Although this provided the audience with explicit insight into Rose and Ned's mathematical thinking, there is less certainty about the role the use of models and representations had in helping students develop their mathematical understanding. However, I would propose that, drawing from Barnes' (2008) notion of trying ideas out through the use of language, similar processes might be said of students' use of representational models. Use of tools, including blocks and drawings, are ways students experiment with their ideas. Coupled with explaining their tentative ideas verbally, I posit that modeling and manipulating tools as they explicate their ideas to the whole group allows students to connect new ideas with pre-existing ones. Providing students with opportunities to use representations during their whole group explanations allows for students to develop their mathematical thinking and make their thinking explicit to others.

The previous discussion highlights a possible area of disconnect for teachers in relating cognitive development theories that underpin instructional approaches and their instructional practice. Although documents such as the NCTM Principles and Standards (2000) and the CCSS (NGA & CCSSO, 2010) recommend development of mathematical discourse within the classroom as an approach to instruction, the theoretical foundations for these recommended visions are not necessarily those that teachers draw from in their work with students. The findings from this research suggest that further study into the learning and pedagogical theories teachers draw from in enacting aspects of mathematical discourse is needed, involving other factors which influence a teacher's practice. Furthermore, this study highlights the need for professional development for both teachers in- and pre-service that make explicit connections between theory and practice. Although teachers in service may have a sense that students learn through communicating, they may not have a complete understanding of the role language and communication play in cognitive development. In addition, misunderstanding about instructional practice grounded in constructivist theory may also exist. Teachers may understand students come to them with a wide range of prior experiences and ideas, which have bearing on their current learning. However, teachers may not be so sure about how to use students' ideas as a way for students to actively construct their understandings during mathematically productive discussions. The teacher from the present study valued students' communicating with others, but her practice did not reflect the wider role communication plays in students' cognitive development. Furthermore, the belief that students construct their own understanding through interacting with their environment was also not always central to her daily practice. This highlights the need for teachers to engage in professional development activities which encourage them to develop their understanding of cognitive learning theories as they relate to the recommendations of policy makers and their everyday practice.

Another consideration to investigate further involves the developmental levels of the children and the view that negotiated meanings developed over time within a context is specific to that particular group (Erickson, 1985). The facilitation and development of mathematical discourse at various levels will look different, based not only on the personal characteristics of the teacher, but the developmental levels of the students. That Mrs. Dennis, an experienced 1st grade teacher with several years of professional development in reform-oriented mathematics pedagogy, tended to exhibit behaviors which demonstrated teacher-centered interactions may signal a characterization of facilitating young children within whole group situations. It is definitely not unreasonable to predict that first graders' ability to focus on and participate in extended student-to-student mathematical discussions will differ from other age groups. Furthermore, the students' propensity to fade in and out in their focus during the whole group discussion, resulting in a number of interactions in which some students were actively contributing toward the whole group discussion and others were not, leads one to wonder if that is characteristic of young children's whole group interactions. Further, possibly comparative study into the facilitation and development of young children's whole group mathematical discourse, taking into account domain specific interactions, would lend insight into the tendencies of developing discourse with young students.

A final aspect of what it means to be student centered that emerged from this research involves the theme of student-to-student interactions. The epitome of what it means to be student centered, student-to-student academic dialogue involves multiple processes, including student explanation, students taking responsibility for their own learning, and providing a forum in which genuine mathematical argumentation might take place. These interactions also provide a process through which the negotiation and co-construction of mathematical meanings might occur. When students interact directly with each other, the negotiation of ideas occurs between students as well as with the teacher. Partnered with the teacher's involvement and facilitation, the group can develop intersubjectivity (Voigt, 1994). Although in initial interviews, the teacher in the present study included the idea of "developing mutual understanding" through a "talking together" (T Interview 11.10.15) where the students share and make sense of each other's ideas in her description of mathematical discourse, this aspect garnered little emphasis during the year. One can only speculate the reasons for this, but a major challenge the teacher faced early in the year was engaging more than the same few students to actively contribute toward the whole group discussions. It took time for students to actively contribute toward the whole group discussions, and that may have shifted her focus away from facilitating taken-as-shared meanings. However, for the first graders, it was evident that they needed explicit facilitation and support for engaging in academic dialogue with each other, as this did not come naturally to them. Another layer to instruction, the added challenge of balancing the time it takes to facilitate students' academic dialogue with each other with focusing on the content the students need to learn would be an aspect the teacher would have to navigate.

Facilitating student-to-student interactions is challenging, takes time, and requires that the teacher function in a different role than most teachers have experienced in the past. Student-to-student interactions are student-driven, whereby students explain their thinking, while other students respond to and ask questions of the presenter. It involves discussing their observations and engaging in academic dialogue with one another. In this type of interaction, student thinking is the most visible and drives the direction of the discussion. What then, is the role of the teacher? This will be the focus of discussion in the next section.

The ideas contained in this section do not provide a comprehensive definition of what it means to be student-centered; rather they describe the aspects pertaining to what constitutes student-centered interactions that emerged from this research. For teachers who are at the beginning stages of developing mathematical discourse in their classrooms, an important implication from this study involves the notion that student-centered discussions go beyond having students stand in front of the class and present their work and their initial ideas to the whole group. It is not enough to elicit students' ideas and have students present their ideas to the class. Rather, being student-centered includes further exploration of students' thinking behind their actions, including their reasoning, justification, seeking relationships, and focusing on significant mathematical ideas, through interactions involving everyone in the classroom community. Through this research, questions about what it meant to be student centered came to the fore and were explored, with the following aspects emerging: 1) student-driven discussions, 2) development of mathematical thinking, and 3) student-to-student interactions. Although these aspects do not completely characterize what it means to be student centered within mathematical discourse, they should be included in considerations about developing student-centered discussions. My hope is this will provide practicing teachers with some insight into aspects of student behaviors and ways of thinking which might assist them in adopting more reformed approaches to teaching mathematics. Throughout this section, questions about the teacher's role in facilitating student-centered discussions were highlighted. This issue will be discussed in the next section.

Conclusion V: The Role of the Teacher

The role the teacher plays in establishing the environment, orchestrating student discussions and supporting students in communicating their ideas is critical in determining the type of discourse that is developed. However, as discussed throughout the findings, the role the teacher plays in orchestrating the discourse and the issues she must grapple with are both challenging and complex. Two aspects under the direct influence of the teacher that provide insights into the role teachers might play in orchestrating mathematical discourse emerged from the findings of this project: 1) Gaining Clarity into Student Thinking, and 2) Bringing Mathematical Objects into Focus Through the Discourse.

Gaining clarity into student thinking.

Gaining clarity into student thinking, as distinguished from clarifying student thinking, implies the role the teacher takes in becoming clear about student thinking that is consistent with a social constructivist view. While clarifying student thinking implies the teacher acting upon students in order to better understand their explanations, gaining clarity suggests students' active construction of their thinking, to which others gain access. This section will explore the teacher's role in moving beyond the selection of students and the presentation of student ideas for the class, and into, what happens next in the discussion? Several of the issues that emerged from the findings lend insight into one of the roles the teacher plays in developing mathematical discourse: gaining clarity into student thinking, for the speaker, the teacher, and the classroom community.

Gaining clarity and insight into young children's thinking is not always easy. It takes work to instruct students in how to explain their thinking for others, at times using tools to illustrate one's ideas and make clear what one thought. For students who are still developing in their use of egocentric speech and predominantly communicate as a way to think out loud for themselves (Vygotsky, 1986), communicating for others' comprehension is no trivial task. This adds another layer of complexity into the teacher's already full curricular agenda. Understandably, for students who are still developing their comfort with and ability to clearly communicate their thoughts to others, explanations may be limited and challenging to listen to. However, that does not preclude providing students with as many opportunities as possible to communicate themselves to others, nor does it mean leaving students to learn how to explain themselves on their own. All students, particularly younger students, benefit from the facilitative skills of the teacher in teaching them how to explain themselves and interact within a whole group mathematical discussion. It involves facilitating students' communication about their thinking, at times helping them clarify their thinking for themselves, and being cautious about imposing one's views on student thinking. As mentioned in the findings, when teachers begin to impose their views and ways of thinking upon students, a singular way of thinking is emphasized and valued, and the students' ideas are lost. In these instances, students are no longer constructing or negotiating meaning; rather, they become passive receivers of knowledge.

Teachers of young children are renowned in their ability to make sense of student thinking. I have been amazed at the skill of some teachers in reading and finding meaning in young children's emergent drawings, writings and invented spellings. This originates from the need to make sense of children who are still developing their abilities to communicate one's thoughts clearly and coherently for others to understand. I believe teachers' predilection to make inferences about students' sometimes incoherent thinking also occurs as students communicate verbally as well as in writing. This process of finding meaning in students' emergent verbal explanations was validated throughout the findings, where the teacher made inferences and assumptions about students' thinking, both when students had trouble communicating themselves and not.

There will always be instances when student explanations are unclear and incoherent. There are different ways a teacher might respond to that. Deciding to make inferences about student thinking and acting upon one's assumptions about student thinking is just one possible response, and can lead to imposing one's ideas on to students' ways of thinking. Arguably, in some of the interactions in which students were particularly challenged with communicating their thinking, the need to play a more influential role into explicating student thinking may be warranted. However, other responses might include asking a question, continuing to probe student thinking, referring the student to a representation, or withholding a direct response until a later time. As teachers, we will always make inferences and interpretations about students' thinking and understanding at some level. However, we can gain insight into what and how students think through the opportunities and tools we provide students to communicate their thinking to themselves and others. Minimizing the degree to which we (as teachers) make assumptions and impose our ideas on students during our instruction will mean that we are clearer about how students are thinking about the mathematics and be better prepared to assist with their development. It will mean we have heard about their thinking from their own lips, in their own words, from their own minds. We will do less imposing of our own ideas on them, and will be informed about how to facilitate their growth and provide the conditions under which they continue to grow and develop.

Clarity into student thinking was particularly salient in the excerpt with Rose, in which she was asked to draw how she saw the relationship between a double and a double-plus-one (see Establishing focus and exploring student thinking, p. 84). Mentioned in the Findings section, after hearing Rose's verbal explanation for why 5 + 4 was a double plus one addition fact that seemed consistent with what was desired, the teacher called her to the front of the class to share her thinking. It wasn't until she actually went up and drew what she meant that we really understood her conception of a doubles plus one, and that it differed from what was desired. The teacher's (and the program's) focus had been on investigating how a double turned into a double plus one fact by increasing one addend and the sum of a double fact by one. Rather than conceiving of it that way, Rose saw 5 + 4 as a double plus one fact because it could be turned into a double fact. She conceived a double plus one fact in its ability to transform into a double fact, very different from what was intended. Bringing students' thinking to light, not to determine its validity or consistency with the lesson's goal, but to use in the development of mathematical meaning through the discourse underscores the importance of the teacher's role in assisting students with communicating their thinking. Facilitating students through the re-creation of their thought process is involved, not only to gain clarity for themselves, but for others' comprehension as well. I would be as bold to say that the major work of the teacher in the mathematics classroom, particularly for young children, is to facilitate students' ability in communicating their mathematical thoughts and engage in whole group discussions around those thoughts. This involves explicitly teaching students how to start and continue their thought processes, using the tools with which they worked. It involves explicitly instructing them on how to refer to and manipulate their tools, pictures and diagrams, and to be prepared to provide their reasoning for what they did or thought using those tools. It involves instructing the entire group to focus on one idea at a time rather than proliferate an accumulation of ideas, and to develop mathematical meaning from exploring relationships in divergent ideas and different ways of thinking. It involves explicitly teaching students that all members in the community think in different ways, and that includes providing sufficient time for everyone to consider their own and others' ideas. It involves teaching students how to question and interact with each other, and facilitating them through the actual process. This includes teaching students how to disagree and agree, and what it means to respect all ideas brought forth into the learning community.

These ideas illustrate a major shift in teachers' views about their own role in the classroom, from an authoritative leader toward a participant-leader. This means that the teacher views herself as being the explicit leader and facilitator in orchestrating discussions, but at the same time, taking a participant role as an equal with the rest of the students in listening to and making sense of students' ideas. This provides the space students need in order to communicate, think mathematically, and engage in the discourse for themselves. This conception of the teacher's role in discourse is not a new one in the literature; however, it is particularly salient for working with young children, whose lives are under the strong influence of adults. In addition, it requires that the teacher see her role as a creator-of-situations in which students develop mathematical understanding for themselves, rather than the transmitter of information. It requires teachers to recognize what they genuinely control, such as the selection of tasks students engage with, their manner of engagement, what ideas become the impetus for mathematical investigation and learning, and how students interact with relationships between ideas. Furthermore, it is important to see the relationship between communication and what one thinks one understands about what a student knows, and that this understanding varies. As the teacher, this involves understanding that what one knows about student thinking is mediated by their ability to communicate and illustrate their thinking (and one's skill in facilitating that), and that students, through communicating their ideas, are in continual development.

For policy makers, administrators and professional developers, it is important to recognize the variability in the shifts teachers make, both in their beliefs about mathematics teaching and learning and implementation of reform-oriented pedagogy. When researchers and the general public discuss "the new way of teaching math" as opposed to traditionalist views, the general discussion revolves around an all or nothing, dichotomous characterization of a teacher's practice. However, this research illustrates the unevenness and variability in a teacher's implementation of reform-oriented mathematical discourse. Supporting current research highlighting the complexities of teachers changing their practice, (Ball, 1990, Sherin, 2002, Spillane & Zeuli, 1999) the findings of this study are consistent with the notion that teachers reform some aspects of their instruction, but not others (Spillane & Zeuli, 1999), missing, at times, the true essence of reformed approaches to teaching mathematics. Just as students are viewed as "in development" of their understanding of mathematics, so teachers' practice can also be viewed as continuous practice-in-development.

Mathematical objects as the focus of the discourse.

Thus far in this conclusions chapter, the organizational structure has predominantly been to present ideas and constructs from the literature followed by a discussion of how the findings in this study relate to those ideas. Perhaps a symptom of researcher bias, the explication of related research prior to a discussion about the findings is the usual practice. To mediate this, in this section the opposite will take place, where the discussion will begin with a discussion regarding the findings, then will move into an explication of their relationship to the literature, followed by a summary of the relationship between focusing on mathematical objects and insight into the teacher's role.

There were several interactions observed in which there seemed to be missed opportunities for exploring genuine mathematical ideas, whether they involved pushing toward mathematical reasoning, movement toward a conceptual focus within the discussion or investigating mathematical bases for generalized, abstract ideas. These were instances in the discussion in which the mathematical ideas, procedures or whatever was the current focus could have shifted from the students' empirical observations toward more objective or generalizable considerations. This would have involved a pushing of students' consideration from their immediate experiences toward a generalizable state in which their reasoning would play a larger role, and mathematical ideas would be placed within a larger context. Moving the focus from what students did toward an examination of the mathematical ideas behind what one did would have also been involved. There were other interactions in which symbolic representations were the focus of discussion, with no meaningful referent for what they meant. Some interactions involved the focus on abstract, generalized ideas, without an examination of their underlying mathematical concepts.

This issue came to light in several interactions. The first one involved Kekoa and Peter and the class' discussion around why it would make sense to choose the coins of largest value to attain the largest possible total when purchasing a group of items (See Student Engagement in Standards of Mathematical Practice section, p. 120). Kekoa and Peter intuitively saw that by choosing the coins with the largest value, they could attain the greatest total. This strategy was shared with the rest of the class, but stopped short in explicating the reasons for why that was a valid relationship in the numbers. In another sequence of lessons, the teacher emphasized to the students that they focus on the number of tens groupings when comparing two-digit numbers without really having the students investigate why it would make sense to look at the number of tens first. These examples illustrate how the mathematics emphasized remained at the procedural level, focusing on how to attain the greatest total, or how to determine the larger number, rather than an investigation of the mathematics underlying these procedures.

These examples illustrate several shifts for teachers in the way they view mathematics, and how those mathematical ideas come to light for students. The first involves a shift from viewing students' mathematical work as determining products through carrying out procedures toward an investigation of structures underlying "mathematical objects" (Davis & Hersh, 1995). Shifting one's focus from students' engagement with tasks for the purpose of completing those tasks toward investigating mathematical ideas and concepts and underlying structures through the engagement in problem solving was especially salient in the findings of this study. That the teacher's focus and goals predominantly involved completing tasks in a valid way, or investigating students' thinking, but did not focus on the mathematics behind the students' thinking, as in Kekoa & Peter's interaction, illustrates this point. Based on these observations and my past interactions with teachers, I believe this to be a major culture shift, in the way teachers see the mathematical work of students, and in how they view mathematics in general.

The issue of Mathematical Objects as the Focus of Discourse is included in this section to highlight the role of the teacher in focusing the students on investigating significant mathematics. An underlying notion may be that a teacher's view of what constitutes mathematics drives the degree to which mathematical ideas is investigated by students. It is beyond the scope of this project to demonstrate conclusively that the way this teacher viewed mathematics was the determinant for not pursuing the mathematics further in the examples provided. But that these missed opportunities occurred at least four times during the observations, with the teacher seemingly unaware of it, in addition to the recurrent emphasis on completing tasks in a valid way, led me to consider that her conception of what constitutes mathematics and the mathematical work of the students might be involved. Not uncommon amongst elementary teachers, it highlights another challenge a teacher must negotiate involving the way one views mathematics and how that view influences the mathematical experience of one's students. Engaging in mathematical thinking and making sense of mathematical concepts through the examination of the behavior of mathematical objects can be viewed as a major purpose for developing discourse in the classroom. However, this is a major shift for teachers who have not experienced this in their own schooling, and can be particularly daunting for those who teach very young students. It highlights a need for professional development for teachers that includes strong pedagogical foundations, but at the same time encourages teachers to re-conceptualize what constitutes the mathematics their students are to learn.

Conclusion VI: Relationships Between Mathematical Discourse and the Standards of Mathematical Practice

Interesting considerations regarding the relationship between the Common Core's Standards of Mathematical Practice and Mathematical Discourse emerged through this research. First, and perhaps not surprisingly, involved the role of mathematical discourse in students' development the standards of mathematical practice. The way students engaged (or could have engaged) in development of the SMPs through the discourse became evident in the findings. The interactions involving Ned and Rose highlighted in the last section of the Findings chapter, particularly illustrate how, through the discourse, they came to see what was important to consider in solving problems. Another example involved Peter and Kekoa's conjecture that selecting the largest coin values resulted in the greatest sum. Had they been asked for their reasoning, they and the other students might have engaged in investigating patterns and reasoning about the patterns, MP 8. Whether it be Ned, who saw the importance of considering the pictures shown, or Rose, who realized the importance of the constraints of a problem, it was evident that students developed these understandings through engagement in the discourse. This highlights a relationship in which mathematical discourse might provide a context through and in which students develop the mathematical processes of the MPs. That students did not engage in sustained student-to-student interactions and at the same time, did not engage in argumentation with supportive reasoning to the group also highlights this relationship. As there was no student-to-student interaction occurring, there was no structure or forum in place for students to engage in mathematical argumentation and respond to others' arguments. Furthermore, because students were in the beginning stages of asking questions of each other, justification was not necessary unless the teacher asked for it. Not to be confused as a cause-effect relationship, student-to-student interactions are one aspect of discourse in which mathematical argumentation might take place.

Another way in which engaging in mathematical discourse provides a forum for developing the MPs is in the processes of interactive constitution that might develop within the group. This begins with having students' observations and ideas brought to the attention of the entire class, such as the one in which the teacher presented Lane's equation modeling her thinking about the problem. Although this instance was very brief and the teacher did the sharing and highlighting, students might learn from seeing Lane's thinking. Within whole group mathematical discourse, the potential exists for students to participate in and learn from the explication of another student's ideas and develop aspects of the MPs through the processes of interactive constitution.

These examples illustrate how, through actively participating in aspects of discourse, students might engage in mathematical processes contained in the MPs. In this way, mathematical discourse can provide the context in which development of processes within the MPs might occur. Certainly the MPs provide one vision that defines aspects of mathematical thinking and processes for all students to attain in their education. Although they may not be necessary to the development of mathematical discourse as defined in the Math-Talk Learning framework, they certainly strengthen the discursive processes and provide ways of mathematical thinking that might be the focus of interactions as defined by the framework. For example, through students' explanations, discussions may involve a student making reference to how his symbolic equation relates to contextual aspects within a problem, a key aspect of MP2, Reasoning abstractly and quantitatively. Taken together, the engagement in mathematical discourse and the MPs strengthen the overall mathematical experience of students.

Reflexively, an interesting question to consider is whether or not the development of mathematical discourse is necessary for students to develop the MPs, or whether students can foster them in other ways. In order for students to develop arguments based on reasoning and to judge the arguments of others, is it necessary for students to begin with explaining their strategies and mathematical thinking? An instinctive reaction might be in the positive. After all, the epistemological beliefs that ground mathematical discourse also supports the notion that students are the ones meant to develop the processes contained in the MPs. However, there is hesitancy in declaring that the processes of mathematical discourse in its entirety are needed for students to develop the MPs. In my interactions with teachers, I have heard of attempts at altering one's practice so as to have students meet the MPs. However, these seem to take place as disjointed, isolated occurrences in student behaviors and in the teacher's practice. My bias is that developing mathematical discourse as envisioned by reformed authors provides opportunities for students to develop all of the MPs within a coherent and connected context rather than in a fragmented manner. However, whether or not there are other approaches that provide students with opportunities to develop the MPs is an area for future study.

Overall, the findings regarding mathematical discourse and the MPs further support implications for professional development that challenge teachers' conception of what constitutes mathematical thinking as well as epistemological beliefs about the way students learn mathematics. In addition, professional development that highlights the relationship between mathematical discourse and development of the MPs would also benefit teachers. Research studying the relationships between mathematical discourse and other aspects of reform-oriented mathematics and students' development of the mathematical processes of the MPs are also needed, including the study of different approaches teachers take in engaging students in development of the MPs.

Final Reflections

The purpose of this research project was to investigate the development of mathematical discourse within a first grade learning community, as a way to provide practicing teachers with a portrait of the complexities involved in this challenging approach to teaching and learning. It is important to highlight that implementing mathematical discourse, particularly with early elementary students, is a challenging endeavor, and takes time to develop. As a researcher, this project afforded the opportunity to develop a perspective conducive to systematic investigation of this phenomenon, and this also took time to develop.

One of the most challenging aspects of the data collection process involved the logistical and procedural issues regarding the teacher interviews. The interviews directly followed the observed lesson and took place during the students' Hawaiian studies time, when there was another adult in the room leading the students through a lesson. In theory, this allowed the teacher to be available for the interviews. However, because Mrs. Dennis was required to remain in the room with the students, it was extremely difficult to capture her verbal utterances clearly on the recorder. Furthermore, her role during these lessons was to monitor the students' behavior, and at times, she became distracted and her train of thought was interrupted. Meant to be a time for mutual reflection, collaboration and discussion about the lesson just enacted, the timing and place of the interviews was not conducive for those processes to occur. In future investigations, although challenging, the best venue for conducting teacher interviews is before or after school, after a brief amount of time following the lesson has passed so as to allow the teacher time to engage in self-reflection about the lesson. Other research studies employed the use of a teacher journal (Sherin, 2002) as a venue for teacher reflection about their lessons, which served as another method of data collection to consider in future research projects.

This research project presents a view into developing whole group mathematical discourse with early elementary students, bringing forth research of a teaching approach grounded in social constructivism and disciplinary notions of what it means to teach and learn mathematics. More research is needed in this worthwhile and important endeavor if the visions of reformists are to be realized in every-day classrooms.

APPENDIX A: OBSERVATION TEMPLATE

Date:	Lesson Topic:	Number of Students:	Materials used:
Lesson Objective/Focus:	Stepping Stones Lesson:		

Description of Environment/Context:

Time	Teacher verbal utterances/actions	Students' utterances/actions

Notes:

APPENDIX B: ANALYTICAL FRAMEWORK

Levels of the Math-Talk Learning Community: Action Trajectories for Teacher and Student

---

Overview of Shift over Levels 0-3: The classroom community grows to support students acting in central or leading roles and shifts from a focus on answers to a focus on mathematical thinking

---

A. Questioning	B. Explaining Mathematical Thinking	C. Source of Mathematical Ideas	D. Responsibility for Learning
Shift from teacher as questioning to students and teacher as questioners.	Students increasingly explain and articulate their math ideas.	Shift from teacher as the source of all math ideas to students' ideas, also influencing the direction of the lesson.	Students increasingly take responsibility for their learning and evaluation of others and self. Math sense becomes the criterion for evaluation.

Level 0: Traditional teacher-directed classroom with brief answer responses from students.

A. Questioning	B. Explaining Mathematical Thinking	C. Source of Mathematical Ideas	D. Responsibility for Learning
Teacher is the only questioner. Short, frequent questions function to keep students listening and paying attention to the teacher.	No or minimal teacher elicitation of student thinking, strategies, or explanations. Teacher expects answer-focused responses. Teacher may tell answers.	Teacher is physically at the board, usually chalk in hand, telling and showing students how to do math.	Teacher repeats student responses (originally directed to her) for the class. Teacher responds to students' answers by verifying the correct answer or showing the correct method.
Students give short answers and respond to the teacher only. No student-student math talk.	No student thinking or strategy-focused explanation of work. Only answers are given.	Students respond to math presented by the teacher. They do not offer their own math ideas.	Students are passive listeners; they attempt to imitate the teacher and do not take responsibility for the learning of their peers or themselves.

Level 1: Teacher beginning to pursue student mathematical thinking. Teacher plays central role in the math-talk community.

A. Questioning	B. Explaining Mathematical Thinking	C. Source of Mathematical Ideas	D. Responsibility for Learning
Teacher questions begin to focus on student thinking and focus less on answers. Teacher begins to ask follow-up questions about student methods and answers. Teacher is still the only questioner.	Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in explanations herself.	Teacher is still the main source of ideas, though she elicits some student ideas. Teacher does some probing to access student ideas.	Teacher begins to set up structures to facilitate students listening to and helping other students. The teacher alone gives feedback.
As a student answers a question, other students listen passively or wait for their turn.	Students give information about their math thinking usually as it is probed by the teacher (minimal volunteering of thoughts). They provide *brief descriptions* of their thinking.	Some student ideas are raised in discussions, but are not explored.	Students become more engaged by repeating what other students say or by helping another student at the teacher's request. This helping mostly involves students showing how *they* solved a problem.

Level 2: Teacher modeling and helping students build new roles. Some co-teaching and co-learning begins as student-to-student talk increases. Teacher physically begins to move to side or back of the room.

A. Questioning	B. Explaining Mathematical Thinking	C. Source of Mathematical Ideas	D. Responsibility for Learning
Teacher continues to ask probing questions and also asks more open questions. She also facilitates student-to-student talk, e.g., by asking students to be prepared to ask questions about other students' work.	Teacher probes more deeply to learn about student thinking and supports detailed descriptions from students. Teacher open to and elicits multiple strategies.	Teacher follows up on explanations and builds on them by asking students to compare and contrast them. Teacher is comfortable using student errors as opportunities of learning.	Teacher encourages student responsibility for understanding the mathematical ideas of others. Teacher asks other students questions about student work and whether they agree or disagree and why.

Students ask questions of one another's work on the board often at the prompting of the teacher. Students listen to one another so they do not repeat questions.	Students usually give information as it is probed by the teacher with some volunteering of thoughts. They begin to stake a position and articulate more information in response to probes. They explain steps in their thinking by providing *fuller descriptions* and *begin to defend* their answers and methods. Other students listen supportively.	Students exhibit confidence about their ideas and share their own thinking and strategies even if they are different from others. Student ideas sometimes guide the direction of the math lesson.	Students begin to listen to understand one another. When the teacher requests, they explain other students' ideas in their own words. Helping involves clarifying *other* students' ideas for themselves and others. Students imitate and model teacher's probing in pair work and in whole-class discussions.

Level 3: Teacher as co-teacher and co-learner. Teacher monitors all that occurs, still fully engaged. Teacher is ready to assist, but now in more peripheral and monitoring role (coach & assister).

A. Questioning	B. Explaining Mathematical Thinking	C. Source of Mathematical Ideas	D. Responsibility for Learning
Teacher expects students to ask one another questions about their work. The teacher's questions still may guide the discourse.	Teacher follows along closely to student descriptions of their thinking, encouraging students to make their explanations more complete; may ask probing questions to make explanations more complete. Teacher stimulates students to think more deeply about strategies.	Teacher allows for interruptions from student during her explanations; she lets students explain and "own" new strategies. (Teacher is still engaged and deciding what is important to continue exploring) Teacher uses student ideas and methods as the basis of lessons or mini-extensions.	The teacher expects students to be responsible for co-evaluation of everyone's work and thinking. She supports students as they help one another sort our misconceptions. She helps and/or follows up when needed.
Student-to-student talk is student initiated, not	Students describe more complete strategies; they	Students interject their ideas as the teacher or other	Students listen to understand, then initiate clarifying

dependent on the teacher. Students ask questions and listen to responses. Many questions are "Why?" questions that require justification from the person answering. Students repeat their own or other's questions until satisfied with answers.	defend and justify their answers with little prompting from the teacher. Students realize that they will be asked questions from other students when they finish, so they are motivated and careful to be thorough. Other students support with active listening.	students are teaching, confident that their ideas are valued. Students spontaneously compare and contrast and build on ideas. Student ideas form part of the content of many math lessons.	other students' work and ideas for themselves and for others during whole-class discussions as well as in small group and pair work. Students assist each other in understanding and correcting errors.

Adapted from "Describing Levels and components of a Math-Talk Learning Community," by Hufferd-Ackles, K., Fuson, K., Sherin, M., 2004, Journal for Research in Mathematics Education 35:2, p. 88. Copyright 2004 by the National Council of Teachers of Mathematics.

APPENDIX C: SEMI-STRUCTURED TEACHER INTERVIEW QUESTIONS

1. What did you think about the day's lesson? What went well? What would you change? Why?

2. What observations did you make about the students' discussion? Did the discussions occur as you anticipated?

3. What was the most challenging part of the lesson for this class?

4. Here's some of what I observed. How does this compare with what you observed?

5. What if you ask this?

6. What are you noticing about the students' discussion over the last few weeks?

APPENDIX D: SAMPLE OF STUDENT ASSENT PROTOCOL AND STUDENT ASSENT FORM

Student Assent Script

Please read this to each student in _________________’s class. Each student will need the form below to complete.

Say: Ms. Yagi is a student at a school. She wants to see how first graders talk in math. She wants to watch _____________’s class. She wants to videotape. She might also ask you about math. I am going to read sentences to you. You can say yes or no. It is ok to say no. You will not get a bad grade. No one will think badly of you.

Read the statements below with the student. Please have him/her check an answer.

Student Assent Form

NO

YES

1. It is ok for Ms. Yagi to watch me do math.

2. It is ok for Ms. Yagi to ask me questions about how I do math.

3. It is ok for Ms. Yagi to look at my work.

4. It is ok for Ms. Yagi to take pictures of me when I do math.

5. I know it is ok for me to say “no” or “pass” when Ms. Yagi asks me a question.

6. I know it is ok for me to say “no” or “pass” when Ms. Yagi takes pictures in the class.

APPENDIX E: SAMPLE OF PARENT CONSENT FORM

University of Hawaii at Manoa
Parental or Guardian's Consent for Child to Participate in Research Project:
Examining the Mathematical Discourse of a 1st Grade Learning Community: A Case Study

Aloha, I am Seanyelle Yagi. I work for the Hawaii Department of Education and I attend the University of Hawaii. To earn my doctoral degree, I am studying how 1st grade students learning and communicate in a math classroom.

I am asking your permission for your child to participate in this project. I am conducting my study in ________________________'s class.

Project Description: If your child participates in this project, here's what will happen and how long it will take.

1. I will be observing your child's math lessons 3 times during the year – 2 months at the beginning of the year, 2-4 weeks at the middle, and 2-4 weeks at the end. I will take notes about how students talk and think about math 2-3 times a week. I may talk with your child during lessons.
2. I will also be videotaping the math lessons to help record my observations. These videos will be written into text and used as data in my research.
3. I will also be looking through students' written work to help me see how they are thinking. I will be sharing all of my observations with the classroom teacher, Mr./Mrs. _________________________.

Benefits and Risks: I believe that your child will not directly benefit by participating in this study. However, my research project will add to the general body of research about how 1st graders talk in their math classrooms.

I believe there is little to no risk in your child's participation in this study. All math lessons will be done as they normally would each day. If you choose not to have your child participate in the research, he/she will **not** suffer any consequences, and your child's images and work will not be used as part of the project. Taking part in this project will have no effect on any grades or class standing.

Video-recording equipment will not interfere with your child's learning experience. During the project, video images will be stored on a password-protected computer. If you give your permission, video images of your child may be used to help teachers understand how students learn math. They will not be used for any other purpose. If permission is not granted, video images will be destroyed at the conclusion of this research project.

Confidentiality and Privacy: During this project, my University of Hawaii advisor and I will be the only people outside of your child's school who will have access to the data. The University of Hawaii Human Studies program and the Hawaii State DOE Data Governance

Office have the right the review research records for this study.

When I report the results of my research project, I will not use your child's name. Instead, I will use a pseudonym (fake name) of your child. If you would like a summary of my final report, please contact me at the number listed near the end of this consent form.

Voluntary Participation: Your child's participation in this project is voluntary, and so is your decision about permitting or not permitting him or her to participate.

At any time, your child can stop participating in this project and you can withdraw your permission without any loss of benefits or rights.

If you have any questions about this project, please contact me at (808)752-7273 or seanyelle@gmail.com.

You can also contact my University of Hawaii advisor, Dr. Joseph Zilliox, at (808)956-5358, or zilliox@hawaii.edu.

If you have any questions about your rights or the rights of your child in this research project, you can contact the University of Hawaii, Human Studies Program, by phone at (808)956-5007 or by email at uhirb@hawaii.edu.

Please keep the prior portion of this consent form for your records. If you agree for your child to participate in this study, please sign the following signature portion of this consent form and return it to your child's teacher.

Signature(s) for Consent:

________ I give permission for my child to participate in the research project entitled, *Examining Discourse of a 1st Grade Learning Community: a Case Study.*

________ I understand that, in order to participate in this project, my child must also agree to participate.

________ I understand that my child and /or I can change our minds about participating, at any time, by notifying the researcher to end participation in this project.

OR

________ I do **not** give permission for my child to participate in the research project entitled, *Examining Discourse of a 1st Grade Learning Community: a Case Study.*

Name of Child (Print): ____

I give permission for my child's video images to be utilized for professional development of teachers and educators.

__________ Yes, I give permission

__________ No, I do not give permission

Name of Parent/Guardian (Print): ____

Parent's /Guardian's Signature: ____

Date: _________________________________

REFERENCES

Arbaugh, F. & Brown, C. (2005). Analyzing Mathematical Tasks: A Catalyst for Change? Journal of Mathematics Teacher Education. 8. 499-536. DOI: 10.1007/s10857-006-6585-3.

Atwater, M. (1996). Social Constructivism: Infusion into the Multicultural Science Education Research Agenda. *Journal of Research in Science Teaching.* 33:8. 821-837.

Au, K. (1998). Social Constructivism and the School Literacy Learning of Students of Diverse Backgrounds. Journal of Literacy Research. 30:2. 297-319. DOI: 10.1080/10862969809548000.

Ball, D. (1990). Reflections and Deflections of Policy: The Case of Carol Turner. *Educational Evaluation and Policy Analysis*. 12:3. 247-259.

Ball, D. (1993). With an Eye on the Mathematical Horizon: Dilemmas of Teaching Elementary School Mathematics. *The Elementary School Journal*. 93:4. 373-397.

Ball, D. (2000). Forward in *Implementing Standards-Based Mathematics Instruction, A Casebook for Professional Development.* Stein, M, Smith, M., Henningsen, M. & Silver, E. NY:Teachers College Press, Columbia University.

Barnes, D. (2008). Exploratory Talk for Learning. In Mercer, N. & Hodgkinson, S. (Eds.) *Exploring Talk in School: Inspired by the Work of Douglas Barnes.* California:Sage Publications. 1-16.

Battista, M. (1994). Teacher Beliefs and the Reform Movement in Mathematics Education. *The Phi Delta Kappan*. 75:6. 462-463, 466-468, 470.

Baxter, J. A., & Williams, S. (2009). Social and analytic scaffolding in middle school mathematics: Managing the dilemma of telling. *Journal of Mathematics Teacher Education*, *13:1*. 7-26.

Bauersfeld, H. (1992). Classroom Cultures from a Social Constructivist's Perspective. *Educational Studies in Mathematics.* 23:5. 467-481.

Bauersfeld, H. (1994). Theoretical Perspectives on Interaction in the Mathematics Classroom. In Biehler, R., Scholz, R, Strasser, R., & Winkelmann, B. (eds.). *Didactics of Mathematics as a Scientific Discipline.* Dordrecht:Kluwer Academic Publishers. 133-146.

Bauersfeld, H. (1995). "Language Games" in the Mathematics Classroom: Their Function and Their Effects. In Cobb, P. & Bauersfeld, H. (Eds.). *The Emergence of Mathematical Meaning: Interaction in Classroom Cultures.* NY:Routledge. 271- 291.

Bielaczyc, K. & Collins, A. (1998). Learning Communities in Classrooms: A Reconceptualization of Educational Practice. In Reigeluth, C.M. (Ed.). *Instructional design theories and models.* Mahwah, NJ: Lawrence Erlbaum Associates.

Black, P. & Wiliam, D. (2009). Developing the Theory of Formative Assessment. *Educational Assessment, Evaluation and Accountability*. 21:1. 5-31

Blumer, H. (1969). *Symbolic Interactionism: Perspective and Method*. Cliffs, NJ:Prentice-Hall.

Boaler, J. (1998). Open and Closed Mathematics: Student Experiences and Understandings. *Journal for Research in Mathematics Education.* 29:1. 41-62.

Boaler, J. (2013). Ability and Mathematics: the mindset revolution that is shaping education. *Forum.* 55:1. 143-152.

Boaler, J. & Brodie, K. (2004). The Importance, Nature, and Impact of Teacher Questions. *Proceedings of the twenty-sixth annual meeting, North American Chapter of the International Group for the Psychology of Mathematics Education* (PME-NA).

Burnett, J., Irons, C., DePaul, D., Stowasser, P, & Turton, A. (2013). *Stepping Stones*. MO:Origo Education.

Cazden, C. (2001). *Classroom Discourse: The Language of Teaching and Learning.* Portsmouth, NH:Heinemann.

Charalambous, C. & Philippou, G. (2010). Teachers' concerns and efficacy beliefs about implementing a mathematics curriculum reform: integrating two lines of inquiry. *Educational Studies of Mathematics.* 75. 1-21. DOI 10.1007/s10649-010-9238-5.

Charmaz, K. (2006). *Constructing Grounded Theory: A Practical Guide Through Qualitative Analysis.* CA:Sage Publishing.

Cobb, P. (1994). Where is the Mind? Constructivist and Sociocultural Perspectives on Mathematical Development. *Educational Researcher.* 23:7. 13-20.

Cobb, P, & Bauersfeld, H. (1995). Introduction: The Coordination of Psychological and Sociological Perspectives in Mathematics Education. In Cobb, P. & Bauersfeld, H. (eds). *The Emergence of Mathematical Meaning: Interaction in Classroom Cultures*. NY:Routledge.

Cobb, P., Yackel, E., & Wood, T. (1992). Interaction and Learning in Mathematics Classroom Situations. *Educational Studies in Mathematics.* 23:1. 99-122.

Cobb, P., Wood, T., & Yackel, E. (1996). Discourse, Mathematical Thinking, and Classroom Practice. *Contexts for Learning: Sociocultural Dynamics in Children's Development.* Forman, E., Minick, N., & Stone, C. (eds.). Cary, NC:Oxford University Press. 91-119.

Confrey, J. (1995). How Compatible are Radical Constructivism, Sociocultural Approaches, and Social Constructivism? In Steffe, L. & Gale, J. (Eds.). *Constructivism in Education.* Hillsday, NJ:Lawrence Erlbaum Assoc. 185-225.

Copple C. & Bredekamp, S. (2009). *Developmentally Appropriate Practice in Early Childhood Programs*. Washington, D.C.:NAEYC.

Corbin, J. & Strauss, A. (2008). *Basics of Qualitative Research.* Los Angeles, CA:Sage Publications.

Cresswell, J. (2013). *Qualitative Inquiry and Research Design* (3rd ed.). CA:SAGE Publications.

Danielson, C. (2007). *Enhancing Professional Practice: A Framework for Teaching, 2nd edition*. VA:ASCD.

Davis, B. (1997). Listening for Differences: An Evolving Conception of Mathematics Teaching. *Journal for Research in Mathematics Education.* 28:3. 355-376.

Deboer, G. (2002). Student-Centered Teaching in a Standards-Based World: Finding a Sensible Balance. *Science & Education*. 11: 405-417.

Doerr, H. (2006). Examining the Tasks of Teaching When Using Students' Mathematical Thinking. *Educational Studies in Mathematics*. 62: 3-24. DOI: 10.1007.s10649-006-4437-9.

Doyle, W. (1983). Academic Work. *Review of educational research.* 53:2. 159-199.

Edwards, D. & Mercer, N. (1987). *Common Knowledge: The Development of Understanding in the Classroom.* NY:Methuen & Co.

Erickson, F. (1982). Audiovisual Records as a Primary Data Source. *Sociological Methods & Research*. 11. 213-232. DOI: 10.1177/004912418201100208.

Erickson, F. (1985). Qualitative Methods in Research on Teaching. *Handbook of Research on Teaching.* Wittrock, M.C. (ed.). NY:Collier-Macmillan. 119-161.

Fosnot, C. T., & Perry, R. S. (1996). Constructivism: A psychological theory of learning. In Fosnot, C. (Ed.). *Constructivism: Theory, perspectives, and practice*. NY:Teachers College Press. 8-33.

Franke, M, Webb, N., Chan, A., Ing, M., Freund, D., Battey, D. (2009). Teacher Questioning to Elicit Students' Mathematical Thinking in Elementary School Classrooms. *Journal of Teacher Education.* 60:4. 180-192.

Gabriele, A. & Joram, E. (2012). Teachers' Reflections on Their Reform-Based Teaching in Mathematics: Implications for the Development of Teacher Self-Efficacy. *Action in Teacher Education.* 29:3. 60-74. DOI: 10.1080/01626620.2007.10463461.

Geist, E. (2010). The Anti-Anxiety Curriculum: Combating Math Anxiety in the Classroom. *Journal of Instructional Psychology.* 37:1. 24-31.

Ginsburg, H., Lee, J., Boyd, J. (2008). Mathematics Education for Young Children: What It is and How to Promote It. *Social Policy Report*. 22:1. 3-22.

Goos, M. (2004). Learning Mathematics in a Classroom Community of Inquiry. *Journal for Research in Mathematics Education.* 35:4. 258-291.

Greeno, J. (2003). Situative Research Relevant to Standards for School Mathematics. *A Research Companion to Principles and Standards for School Mathematics.* Kilpatrick, J., Martin, W., Schifter, D. (eds.). Reston, VA:NCTM. 304-332.

Guberman, S. (1999). *Cultural Aspects of Young Children's Mathematics Knowledge.* Retrieved from: http://files.eric.ed.gov/fulltext/ED438892.pdf.

Hamm, J. & Perry, M. (2002). Learning Mathematics in First-Grade Classrooms: On Whose Authority? *Journal of Educational Psychology*. 94:1, 126-137. DOI: 10.1037//0022-0663.94.1.126.

Harrop, A. & Swinson, J. (2003). Teachers' Questions in the Infant, Junior & Secondary School. *Educational Studies.* 29:1. 49-60.

Hattie, J. (2009). *Visible Learning: A Synthesis of Over 800 Meta-Analyses Relating to Acheivement*. NY:Routledge.

Heck, D., Weiss, I., Pasley, J. (2011). *A Priority Research Agenda for Understanding the Influence of the Common Core State Standards for Mathematics.* Chapel Hill, NC:Horizon Research Inc. Retrieved from Horizon Research Inc.: http://www.horizon-research.com/reports/2011/CCSSMresearchagenda/research_agenda.php

Henningsen, M. & Stein, M. (1997). Mathematical Tasks and Student Cognition: Classroom-based Factors That Support and Inhibit High-Level Mathematical Thinking and Reasoning. *Journal for Research in Mathematics Education*. 28:5, 524-549.

Herbel-Eisenmann, B. & Breyfogle, L. (2005). Questioning Our Patterns of Questioning. *Mathematics Teaching in the Middle School.* 10:9. 484-489. Reston, VA: NCTM.

Heritage, M. (2010). *Formative Assessment: Making it Happen in the Classroom*. CA:Corwin.

Hicks, D. (1996). Discourse, Learning and Teaching. *Review of Research in Education.* 21. 49-95.

Hiebert, J., Stigler, J., Jacobs, J., Givvin, K., Garnier, H., Smith, M., Hollingsworth, H., Manaster, A., Wearne, D., & Gallimore, R. (2005). Mathematics Teaching in the United States Today (and Tomorrow): Results From the TIMSS 1999 Video Study. *Educational Evaluation and Policy Analysis.* 27:2. 111-132.

Hiebert, J. & Wearne, D. (1992). Links between Teaching and Learning Place Value with Understanding in First Grade. *Journal for Research in Mathematics Education.* 23:2. 98-122.

Hiebert, J. & Wearne, D. (1993). Instructional Tasks, Classroom Discourse, and Students' Learning in Second-Grade Arithmetic. *American Educational Research Journal*. 393-425

Hufferd-Ackles, K., Fuson, K., & Sherin, M. (2004). Describing Levels and Components of a Math-Talk Learning Community. *Journal for Research in Mathematics Education.* 35:2. 81-116.

Ilaria, D. (2002). Questions That Engage Students in Mathematical Thinking. *Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Athens, Greece.

Imm, K. & Stylianou, D. (2011). Talking Mathematically: An analysis of discourse communities. *The Journal of Mathematical Behavior.* 31. 130-148.

Kamii, C. & Housman, L. (2000). *Young Children Reinvent Arithmetic: Implications of Piaget's Theory.* NY:Teachers College Press.

Kwon, O., Park, J., Park, J. (2006). Cultivating Divergent Thinking in Mathematics through an Open-ended Approach. *Asia Pacific Education Review*. 7:1. 51-61.

Lampert, M. (1986). Knowing, Doing, and Teaching Multiplication. *Cognition and Instruction*. 3:4. 305-342.

Lampert, M. (1990). When the Problem is Not the Question and the Solution is Not the Answer: Mathematical Knowing and Teaching. *American Educational Research Journal.* 27:1. 29-63.

Lampert, M., Rittenhouse, P., & Crumbaugh, C. (1996). Agreeing to disagree: Developing sociable mathematical discourse. *The handbook of education and human development: New models of learning, teaching, and schooling*. Olson, D. & Torrance N. (eds.). Cambridge:Blackwell. 731-764.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. NY:Cambridge University Press.

Lemke, J. L. (2004). Intertextuality and Educational Research. In Shuart-Faris, N. & Bloome, D. (Eds.). *Uses of Intertextuality in Classroom and Educational Research.* USA:Information Age Publishing. 3-16.

Lewis, C., Perry, R., Hurd, J., O'Connell, M. P. (2006). Lesson Study comes of Age in North America. *Phi Delta Kappan*. 88:4. 273-281.

Ma, X. (1999). A Meta-Analysis of the Relationship between Anxiety toward Mathematics and Achievement in Mathematics. *Journal for Research in Mathematics Education.* 30:5. 520-540.

Ma, X. & Xu, J. (2004). The causal ordering of mathematics anxiety and mathematics achievement: a longitudinal panel analysis. *Journal of Adolescence.* 27. 165-179. doi:10.1016/j.adolescence.2003.11.003.

Mack, N. (1990). *Learning Fractions with Understanding: Building Upon Informal Knowledge.* Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA.

Martin, T. (2009). A Theory of Physically Distributed Learning: How External Environments and Internal States Interact in Mathematics Learning. *Society for Research in Child Development.* 3:3. 140-144.

Martin, T. & Schwartz, D. (2005). Physically Distributed Learning: Adapting and Reinterpreting Physical Environments in the Development of Fraction Concepts. *Cognitive Science.* 25. 587-625.

Martino, A., & Maher, C. (1999). Teacher Questioning to Promote Justification and Generalization in Mathematics: What Research Practice Has Taught Us. *Journal of Mathematical Behavior.* 18:53-71.

Maxwell, J. (2013). *Qualitative Research Design: An Interactive Approach.* (3rd ed.). CA:SAGE Publications.

McClain, K. & Cobb, P. (1998). The Role of Imagery and Discourse in Supporting Students' Mathematical Development. In Lampert, M. & Blunk, M. Eds. *Talking Mathematics in School: Studies of Teaching and Learning.* UK: Cambridge University Press. 56-81.

Mehan, H. (1979). *Learning Lessons*. MA:Harvard University Press.

Mehan, H. & Wood, H. (1975). *The Reality of Ethnomethodology*. New York:Wiley.

Merriam, S. (1998). *Qualitative Research and Case Study Applications in Education*. CA:Jossey-Bass Publishers.

Merriam, S. (2009). *Qualitative Research: A Guide to Design and Implementation.* CA:Jossey-Bass Publishers.

Miles, M. & Huberman, M. (1994). *Qualitative Data Analysis: An Expanded Sourcebook.* (2nd ed.). CA:SAGE Publications.

Moschkovich, J. (2003). What Counts as Mathematical Discourse? *International Group for the Psychology of Mathematics Education*. 3. 325-332.

Nathan, M., Eilam, B., & Kim, S. (2007). To Disagree, We Must Also Agree: How Intersubjectivity Structures and Perpetuates Discourse in a Mathematics Classroom. *The Journal of the Learning Sciences.* 16:4. 523-563.

Nathan, M. & Knuth, E. (2010). A Study of Whole Classroom Mathematical Discourse and Teacher Change. *Cognition and Instruction*. 21:2 175-207. DOI: 10.1207/S1532690XCI2102_03.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA:NCTM.

National Council of Teachers of Mathematics (1991). *Professional standards for teaching mathematics*. Reston, VA:NCTM.

National Council of Teachers of Mathematics (1995). *Assessment Standards for School Mathematics.* Reston, VA:NCTM.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

NGA & CCSSO. (2010). *Common Core State Standards* (Mathematics). Washington, D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.

National Research Council. (2001). *Adding it Up: Helping Children Learn Mathematics.* Kilpatrick, J., Swafford, J. & Findell, B. (Eds.). Washington D.C.:National Academy Press.

Nohda, N. (2000). Teaching by Open-Approach Method in Japanese Mathematics Classroom. *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)*, Hiroshima, Japan.

OECD. (2012). *Programme for International Student Assessment*. Retrieved from http://www.oecd.org/pisa/home/.

Pajares, F. (1996). Self-Efficacy Beliefs in Academic Settings. *Review of Educational Research.* 66:4. 543-578.

Palinscar, A. (1998). Social Constructivist Perspectives on Teaching and Learning. *Annual Review of Psychology.* 49. 345-375.

Pehkonen, E. (1997). *Use of Open-Ended Problems in Mathematics Classroom* (Report No. 176). Helsinsky: Dept. of Teacher Education.

Plano-Clark, V. & Cresswell, J. (2010). *Understanding Research: A Consumer's Guide.* NJ:Pearson.

Rachlin, S. (1998). Learning to See the Wind. *Mathematics Teaching in the Middle School.* 3:7. 470-473.

Rittenhouse, P. (1998). The Teacher's Role in Mathematical Conversation: Stepping In and Stepping Out. In Lampert, M. & Blunk, M. (Eds.). *Talking Mathematics in School.* NY:Cambridge University Press. 163-189.

Rogoff, B. (1994). Developing Understanding of the Idea of Communities of Learners. *Mind, Culture and Activity.* 1:4. 209-229. DOI: 107-9039/94/209-229.

Ross, J., McDougall, D., & Hogaboam-Gray, A. (2002). Research on Reform in Mathematics Education, 1993-2000. *Alberta Journal of Educational Research.* 48:2. 122-138.

Sadler, D. (1998). Formative Assessment: Revisiting the territory. *Assessment in Education.* 5:1. 77-85.

Sahin, A. & Kulm, G. (2008). Sixth grade mathematics teachers' intentions and use of probing, guiding, and factual questions. *Journal of Mathematics Teacher Education.* 11:3. 221-241. DOI: 10.1007/s10857-008-9071-2.

Scardamalia, M. & Bereiter, C. (1994). Computer Support for Knowledge-Building Communities. *The Journal of the Learning Sciences.* 3:3. 265-283.

Schoenfeld, A. (1992). Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics. In D. Grouws (ed.). *Handbook for Research on Mathematics Teaching and Learning.* New York:Macmillan.

Sherin, M. (2002). A Balancing Act: Developing a Discourse Community in a Mathematics Classroom. *Journal of Mathematics Teacher Education*. 205-233.

Simon, M. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. *Journal for Research in Mathematics Education.* 26:2. 114-145.

Smith, J. (1996). Efficacy and Teaching Mathematics by Telling: A Challenge for Reform. *Journal for Research in Mathematics Education*. 27:4. 387-402.

Smith, M. & Stein, M. (2011). *Five Practices for Orchestrating Productive Mathematics Discussions.* Reston, VA:NCTM.

Sophian, C. (1999). Children's Ways of Knowing: Lessons from Cognitive Development Research. In Copley, J. (Ed.). *Mathematics in the Early Years.* Reston, VA:NCTM.

Sophian, C. (2007). *The Origins of Mathematical Knowledge in Childhood.* NY:Lawrence Erlbaum Associates.

Spangler, D., Kim, J.S., Cross, D., Kilic, H., Iscimen, F. A., & Swanagan, D. (2014). Using Rich Tasks to Promote Discourse. In Karp K. & McDuffie, A. (Eds). *Annual Perspectives in Mathematics Education - Using Research to Improve Instruction*. Reston:NCTM

Spillane, J. & Zeuli, J. (1999). Reform and Teaching: Exploring Patterns of Practice in the Context of State and National and State Mathematics Reform. *Educational Evaluation and Policy Analysis*. 20:1. 1-27.

Staarman, J. & Mercer, N. (2010). The Guided Construction of Knowledge: Talk between Teachers and Students. Ch. 3. *International Handbook of Psychology in Education.* Littleton, K., Wood, C., & Staarman, J.K. eds. UK:Emerald Group Publishing, Ltd. 75-104.

Stein, M.K., Grover, B., & Henningsen, M. (1996). Building Student Capacity for Mathematical Thinking and Reasoning: An Analysis of Mathematical Tasks Used in Reform Classrooms. *American Educational Research Journal.* 33:2. 455 - 488.

Stiggins, R. (2005). From Formative Assessment to Assessment FOR Learning: A Path to Success in Standards-Based Schools. *Phi Delta Kappan*. 87:4. 324-328.

Stylianides, A. & Stylianides, G. (2007). Studying the classroom implementation of tasks: High-level mathematical tasks embedded in 'real-life' contexts. *Teaching and Teacher Education.* 859-875. doi: 10.1016/j.tate.2007.11.015.

Sullivan, P., Clarke, D., & Clarke, B. (2013). *Teaching With Tasks for Effective Mathematics Learning*. New York: Springer.

Thompson, A. (1984). The Relationship of Teachers' Conceptions of Mathematics and Mathematics Teaching to Instructional Practices. *Educational Studies in Mathematics*. 15:2. 105-127.

Ticknor, C. (2012). Situated learning in an abstract algebra classroom. *Educational Studies in Mathematics.* 81:3. 307-323. DOI: 10.1007/s10649-012-9405-y.

Tolle, P. (2010). Lesson Study: Still a Work in Progress in America. *Mathematics Teacher*. 104:3. 181-185.

U.S. Department of Education. (2010). *Early Learning: America's Middle Class Promise Begins Early*. Retrieved from http://www.ed.gov/early-learning.

Voigt, J. (1994). Negotiation of Mathematical Meaning and Learning Mathematics. *Educational Studies in Mathematics.* 26:⅔. 275-298.

von Glasersfeld, E. (1989). Cognition, Construction of Knowledge and Teaching. *Synthese: History, Philosophy, and Science Teaching.* 80:1. 121-140.

von Glasersfeld, E. (1990). Chapter 2: An Exposition of Constructivism: Why Some Like It Radical. *Journal for Research in Mathematics Education.* 4. 19-29, 195-210.

von Glasersfeld, E. (1991). *Radical Constructivism in Mathematics Education.* Hingham, MA: Kluwer Academic Publishers. xiii-xix.

von Glasersfeld, E. (1995). Sensory Experience, Abstraction, and Teaching. *Constructivism in Education.* Steffe, L. & Gale, J. (Eds.). Hillsdale, NJ:Lawrence Erlbaum Assoc. 369-383.

Vygotsky, L. (1978). *Mind in Society.* In Cole, M., John-Steiner, V., Scribner, S. & Souberman, E. (Eds.). Cambridge, MA:Harvard University Press.

Vygotsky, L. (1986). *Thought and Language.* In Kozulin, A. (Ed.). Cambridge, MA:MIT Press.

Wertsch, J. & Toma, C. (1995). Discourse and Learning in the Classroom: A Sociocultural Approach. In Steffe, L. & Gale, J. (eds). *Constructivism in Education.* NJ:Lawrence Erlbaum Assoc. 159 - 174.

Wiliam, D. (1988). Open Ends and Open Beginnings. *Unpublished paper.*

Wood, T. (1998). Alternative Patterns of Communication in Mathematics Classes: Funneling or Focusing? In Steinbring, H., Bussi, M., & Sierpinska, A. (eds.). *Language and Communication in the Mathematics Classroom*. Reston, VA: NCTM. 167-178.

Wood, T. (2002). What Does it Mean to Teach Mathematics Differently? *Mathematics education in the South Pacific; proceedings of the 25th Annual Conference of the Mathematics Education Research Group of Australasia Inc.* 1. 61-67.

Wood, T., Cobb, P., & Yackel, E. (1995). Reflections on Learning and Teaching Mathematics in Elementary School. In Steffe, L. & Gale, J. (Eds.). *Constructivism in Education.* Hillsdale, NJ:Lawrence Erlbaum Assoc., Inc. 401-422.

Yackel, E. & Cobb, P. (1996). Sociomathematical Norms, Argumentation, and Autonomy in Mathematics. *Journal for Research in Mathematics Education.* 27:4. 458-477.

Yackel, E., Cobb, P., Wood, T., Wheatley, G., & Merkel, G. (1990). The Importance of Social Interaction in Children's Construction of Mathematical Knowledge. In Cooney, T. & Hirsch, C. (Eds.). *Teaching and Learning Mathematics in the 1990s.* Reston, VA:NCTM. 12-21.

Yin, R. (2014). *Case Study Research: Design and Methods.* CA:SAGE Publications.