The empirical orthogonal function was applied to analyze 40-yr annual rainfall data for 80 stations in the state of Hawaii, which included 21 stations on Hawaii, 18 on Maui, 5 on Molokai, 5 on Lanai, 16 on Oahu, and 15 on Kauai. The chi-square statistic was introduced to test the stabilities of eigenvectors, E-1, E-2, and E-3, for the state and its six major islands. The eigenvectors, especially E-1 and E-2, for the state and individual islands are quite stable within the 30-yr period. Eigenvectors within the 30-yr period with 5-yr data change are more stable than those within a 30-yr period with 10-yr data change. The 40-yr eigenvectors are even more stable than those in the 30-yr period. Both the maximum and conventional power spectral analyses were used to analyze the time-dependent coefficients of E-1, E-2, and E-3. The quasi-biennial oscillation (2-yr cycle) is the predominant cycle for the state and for Maui and Oahu. The El Niño cycle (4- to 4.4-yr cycle) exists in the state and the islands of Hawaii and Kauai, and the 3- to 3.3-yr cycle on Kauai. No significant sunspot cycle (11- and 22-yr cycle) was found on Hawaii.
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SPECTRAL ANALYSIS OF HAWAI'I RAINFALL

Shan-hsin Chiang
Jen-hu Chang
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Associate Investigator: Yu-Si Fok

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The empirical orthogonal function was applied to analyze 40-yr annual rainfall data for 80 stations in the state of Hawai'i, which included 21 stations on Hawai'i, 18 on Maui, 5 on Moloka'i, 5 on Lāna'i, 16 on O'ahu, and 15 on Kaua'i. The chi-square statistic was introduced to test the stabilities of eigenvectors, $B_1$, $B_2$, and $B_3$, for the state and its six major islands. The eigenvectors, especially $B_1$ and $B_2$, for the state and individual islands are quite stable within the 30-yr period. Eigenvectors within the 30-yr period with 5-yr data change are more stable than those within a 30-yr period with 10-yr data change. The 40-yr eigenvectors are even more stable than those in the 30-yr period. Both the maximum and conventional power spectral analyses were used to analyze the time-dependent coefficients of $B_1$, $B_2$, and $B_3$. The quasi-biennial oscillation (2-yr cycle) is the predominant cycle for the state and for Maui and O'ahu. The El Niño cycle (4- to 4.4-yr cycle) exists in the state and the islands of Hawai'i and Kaua'i, and the 3- to 3.3-yr cycle on Kaua'i. No significant sunspot cycle (11- and 22-yr cycle) was found on Hawai'i.
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INTRODUCTION

An extensive rainfall network and long-term rainfall records for many rainfall stations in the state of Hawai'i provide an abundance of long-term rainfall information. Studies of Hawai'i's rainfall climatology by Solot (1948), Taliaferro (1959), Cheng and Lau (1973), Meisner (1976), and Stidd and Leopold (1951), are based on observations of rainfall data. The current trend of climatological statistics applies the orthogonal function method to analyze the characteristics of meteorological elements in one region, such as fields of temperature, pressure, and precipitation. Lyons (1982) applied the empirical orthogonal function analysis in a first attempt to identify quantitatively the dominant eigenvectors (E-1, E-2, E-3), also called spatial patterns, and time coefficients, also known as temporal patterns, of Hawai'i rainfall. Based on a selection of monthly mean precipitation data for 63 stations from the January 1939 through December 1975 period, Lyons concluded that the major rainfall patterns in order of importance (E-1 to E-3) would be trade winds, southwest winds, and convective rainfall on an annual basis; trade winds, southwest winds, and frontal rainfall during the winter, spring, and fall seasons; and trade winds, tropical disturbance, and convective rainfall during the summer.

The empirical orthogonal function method applied to a meteorological element would generate two mutually perpendicular and uncorrelated coordinates that are orthogonal in space and coefficients of different functions that are also orthogonal in time. The application of time coefficients of the empirical orthogonal function is a new method of predicting the behavior of climatological elements. A satisfactory prediction is based on one basic assumption: the eigenvectors or the spatial patterns should be constant within the period of prediction. If spatial functions change with temporal functions, the time coefficients will be unable to introduce future predictions. Based on this assumption, the spatial function can be regarded as a constant within a certain "stable" period. This study determines whether or not the spatial function is a constant in a certain period.

Few studies have focused on investigating the Hawai'i rainfall cycles. Cox (1949) was the first to indicate that 11- and 22-yr rainfall cycles in Hawai'i might be relative to sunspot cycles. Meisner (1976) found a 17-yr
cycle in summer rainfall in Hawai'i. Lyons (1982) mentioned the existence of annual, semi-annual, three-quarter year, and 2- to 2.5-yr cycles in Hawai'i rainfall, and stated that 22 yr sunspot cycles were not found.

The power spectral method is the most common method used to analyze the time series in climatology. This study applies a relatively new method, maximum entropy power spectral analysis, to analyze Hawai'i rainfall data. The advantage of maximum entropy power spectral analysis is especially suitable for short-time data (Chen and Stegen 1974). The cycles derived from the maximum entropy power spectral analysis will be compared to results from the conventional power spectral method.

DATA

Rainfall stations in Hawai'i number more than 2500. They are maintained by the Division of Water and Land Development (DOWALD), National Weather Service (NWS), Hawaiian Sugar Planters' Association (HSPA), U.S. Geological Survey (USGS), and Pineapple Research Institute (PRI). A list of climatological stations, which includes the state key number, station names, geographic coordinates, and duration of records, was compiled by DOWALD of the Hawaii State Department of Land and Natural Resources in "Climatological Stations in Hawaii" (DOWALD 1973). Rainfall data have been stored on computer magnetic tape by DOWALD.

For this study, 80 rainfall stations were selected based on an even distribution and their representation for the major islands of the state (Kaua'i, O'ahu, Maui, Moloka'i, Lāna'i, and Hawai'i). Forty years of annual rainfall data from 1942 through 1981 were selected because of the absence of monthly data for several stations. A number of stations also lack full years of annual record. To interpolate periods of missing data, a 10-yr average which includes 5 years previous and subsequent was used.

Figure 1 provides a general location of the 80 stations, which include 21 on Hawai'i, 18 on Maui, 5 on Moloka'i, 5 on Lāna'i, 16 on O'ahu, and 15 on Kaua'i.
Figure 1. Locations of 80 rainfall stations selected for analysis
COMPUTATION OF ORTHOGONAL FUNCTION

The empirical orthogonal function was first developed for meteorological purposes by Lorenz (1956) in numerical prediction. The earliest application was made by Gilman (1957). Later Grimmer (1963) applied this function to identify temperature patterns over Europe. Recently, this method is widely used because of the ease of computation using computers.

Detailed computational procedures can be found in the literature (Cattel 1965; Gould 1967). For brevity, the data matrix, $F_{(mxn)}$, can be represented by a set of observations in which $m$ is the number of stations and $n$ the time series, as

$$F_{(mxn)} = \begin{bmatrix}
F_{11} & F_{12} & \cdots & F_{1n} \\
F_{11} & F_{12} & \cdots & F_{1n} \\
\vdots & \vdots & \ddots & \vdots \\
F_{11} & F_{12} & \cdots & F_{1n} \\
F_{m1} & F_{m2} & \cdots & F_{mn}
\end{bmatrix}$$

where $F_{(mxn)}$ is the data matrix.

$F_{(mxn)}$ can be decomposed to two mutually independent, uncorrelated, and perpendicular parts, as

$$F_{(mxn)} = T \times X$$

Let

$$T = \begin{bmatrix}
t_{11} & t_{12} & \cdots & t_{1n} \\
t_{21} & t_{22} & \cdots & t_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
t_{m1} & t_{m2} & \cdots & t_{mn}
\end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix}
x_{11} & x_{12} & \cdots & x_{1n} \\
x_{21} & x_{22} & \cdots & x_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
x_{n1} & x_{n2} & \cdots & x_{nn}
\end{bmatrix}$$

where
\[ T_{(m \times n)} = \text{time coefficient matrix} \]
\[ X_{(n \times n)} = \text{spatial eigenvector matrix}. \]

Then
\[ F_{ij} = \sum_{h=i}^{H} T_{hi} X_{hj} \]
where \( i = 1, 2, \ldots, m \); \( j = 1, 2, \ldots, n \); and \( H = \text{number of eigenvectors}. \)

We also have the following relationships:
\[ \sum_{j=1}^{n} X_{kj} X_{ej} = 0, \quad k \neq e \]
\[ \sum_{j=1}^{n} X_{kj} X_{ej} \neq 0, \quad k = e \]
\[ \sum_{i=1}^{m} t_{ki} t_{ei} = 0, \quad k \neq e \]
\[ \sum_{i=1}^{m} t_{ki} t_{ei} \neq 0, \quad k = e \]

where \( k \) and \( e \) are dummy variables.

**EMPIRICAL ORTHOGONAL FUNCTION FOR HAWAI'I DATA**

For this study, records of annual precipitation were used for a 40-yr period from 1942 through 1981. A total of 80 stations for the six major islands of the state of Hawai'i were selected: 21 on Hawai'i, 18 on Maui, 5 on Moloka'i, 5 on Lāna'i, 16 on O'ahu, and 15 on Kaua'i, for which their respective matrices are \( 40 \times 80 \), \( 40 \times 21 \), \( 40 \times 18 \), \( 40 \times 5 \), \( 40 \times 5 \), \( 40 \times 16 \), and \( 40 \times 15 \) for the analysis of their empirical orthogonal functions. In the seven matrices, \( m = 40 \) (no. of years) and \( n = 80 \) (no. of stas.—21, 18, 5, 16, 15). Because Moloka'i and Lāna'i have only five stations, only five classes are possible.

The first ten eigenvalues for a 40-yr period of rainfall data for the state and individual islands are listed in Table 1. The percentage of variance and cumulative variance for each eigenvalue is listed in Tables 2 and 3.

Based on the values presented in Tables 1 to 3, we may conclude that the larger the island area and the greater the number of stations, then the more eigenvalues are required. If we take 80% of the total variance into consideration, the first seven eigenvalues are needed for the state of Hawai'i; the first five for Hawai'i Island; the first three for Maui; the
### Table 1. Forty-Year Rainfall Eigenvalues for Hawai'i

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>State</td>
<td>33.32</td>
<td>11.28</td>
<td>5.78</td>
<td>5.52</td>
<td>3.45</td>
<td>2.58</td>
<td>2.12</td>
<td>1.85</td>
<td>1.61</td>
<td>1.43</td>
</tr>
<tr>
<td>Hawai'i</td>
<td>7.92</td>
<td>3.38</td>
<td>2.19</td>
<td>1.87</td>
<td>1.22</td>
<td>1.06</td>
<td>0.68</td>
<td>0.44</td>
<td>0.35</td>
<td>0.25</td>
</tr>
<tr>
<td>Maui</td>
<td>10.21</td>
<td>3.46</td>
<td>0.94</td>
<td>0.80</td>
<td>0.51</td>
<td>0.43</td>
<td>0.36</td>
<td>0.24</td>
<td>0.21</td>
<td>0.19</td>
</tr>
<tr>
<td>Moloka'i</td>
<td>3.03</td>
<td>0.95</td>
<td>0.66</td>
<td>0.21</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lāna'i</td>
<td>4.64</td>
<td>0.19</td>
<td>0.08</td>
<td>0.06</td>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O'ahu</td>
<td>10.52</td>
<td>2.23</td>
<td>0.90</td>
<td>0.68</td>
<td>0.42</td>
<td>0.29</td>
<td>0.22</td>
<td>0.18</td>
<td>0.17</td>
<td>0.13</td>
</tr>
<tr>
<td>Kaua'i</td>
<td>9.29</td>
<td>2.52</td>
<td>1.00</td>
<td>0.60</td>
<td>0.44</td>
<td>0.34</td>
<td>0.20</td>
<td>0.17</td>
<td>0.14</td>
<td>0.11</td>
</tr>
</tbody>
</table>

### Table 2. Forty-Year Percent of Variance for Each Eigenvalue

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
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<tbody>
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<td>State</td>
<td>42</td>
<td>14</td>
<td>7</td>
<td>7</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Hawai'i</td>
<td>38</td>
<td>18</td>
<td>11</td>
<td>9</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Maui</td>
<td>57</td>
<td>9</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Moloka'i</td>
<td>61</td>
<td>19</td>
<td>13</td>
<td>4</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lāna'i</td>
<td>93</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O'ahu</td>
<td>66</td>
<td>14</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Kaua'i</td>
<td>62</td>
<td>17</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table 3. Forty-Year Cumulative Percent of Variance for Each Eigenvalue

<table>
<thead>
<tr>
<th>Eigenvalue</th>
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<th>3</th>
<th>4</th>
<th>5</th>
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<th>7</th>
<th>8</th>
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<th>10</th>
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</thead>
<tbody>
<tr>
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<td>63</td>
<td>70</td>
<td>74</td>
<td>77</td>
<td>80</td>
<td>82</td>
<td>84</td>
<td>86</td>
</tr>
<tr>
<td>Hawai'i</td>
<td>38</td>
<td>56</td>
<td>67</td>
<td>76</td>
<td>81</td>
<td>86</td>
<td>90</td>
<td>92</td>
<td>93</td>
<td>95</td>
</tr>
<tr>
<td>Maui</td>
<td>57</td>
<td>76</td>
<td>81</td>
<td>86</td>
<td>88</td>
<td>91</td>
<td>93</td>
<td>94</td>
<td>95</td>
<td>96</td>
</tr>
<tr>
<td>Moloka'i</td>
<td>61</td>
<td>80</td>
<td>93</td>
<td>97</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lāna'i</td>
<td>93</td>
<td>97</td>
<td>98</td>
<td>99</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O'ahu</td>
<td>66</td>
<td>80</td>
<td>85</td>
<td>90</td>
<td>92</td>
<td>94</td>
<td>95</td>
<td>96</td>
<td>97</td>
<td>98</td>
</tr>
<tr>
<td>Kaua'i</td>
<td>62</td>
<td>79</td>
<td>85</td>
<td>89</td>
<td>92</td>
<td>95</td>
<td>96</td>
<td>97</td>
<td>98</td>
<td>99</td>
</tr>
</tbody>
</table>
first two for Moloka‘i, O‘ahu, and Kaua‘i; and only the first for Lāna‘i. Since the first three eigenvalues account for at least two-thirds of the total variance for the state and the island of Hawai‘i and for more than 80% of the total variance for Maui, Moloka‘i, Lāna‘i, O‘ahu, and Kaua‘i, the first three eigenvalues were selected to test their stabilities.

STABILITY OF EMPIRICAL ORTHOGONAL FUNCTION

To test the stability of the eigenvectors derived from the empirical orthogonal function, the 40-yr period of rainfall data is divided into three groups. Each group contains a 30-yr period and only 5-yr data do not overlap each other. The first 30-yr data group contains rainfall data from 1942 to 1971; the second, from 1947 to 1976; and the third, from 1952 to 1981. Overlapping 25-yr data indicates that 83% of the total data are identical. For the first and third groups, the 10-yr data do not overlap and, thus, only two-thirds of the total data are identical. The cumulative percent of total variance of the first three 30-yr eigenvalues for each group of the six major islands of Hawai‘i are computed and listed in Table 4.

In Table 4 we note that the first eigenvalue, with the exception of Hawai‘i Island, accounts for more than 50% of the cumulative percent of total variance of eigenvalues. Up to the third eigenvalue, 85% of the total variance is accounted for and only 70% for Hawai‘i Island.

In a comparison of Tables 4 and 3 it is interesting to note that the eigenvalues for each island are almost identical.

To quantitatively test the differences between each group of eigenvectors, the chi-square method is used. The quantity of chi-square is defined as

$$
\chi^2 = \frac{(O_1 - H_1)^2}{H_1} + \frac{(O_2 - H_2)^2}{H_2} + \ldots
$$

$$
= \sum_{i=1}^{m} \frac{(O_i - H_i)^2}{H_i}
$$

where

- $m = \text{total number of stations in each group}$
- $O_i = \text{eigenvector's value at station } i \text{ in one group}$
- $i = 1, \ldots, m \text{ stations}$
H = eigenvector's value at station i in another group, 
i = 1, ... m stations.

**TABLE 4. CUMULATIVE PERCENT OF TOTAL VARIANCE OF**
**FIRST THREE 30-YR EIGENVALUES PER GROUP**

<table>
<thead>
<tr>
<th>FIRST THREE 30-YR EIGENVALUES</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Group</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hawai'i Is.</td>
<td>34</td>
<td>36</td>
<td>43</td>
<td>56</td>
<td>56</td>
<td>61</td>
<td>70</td>
<td>68</td>
<td>71</td>
</tr>
<tr>
<td>Maui</td>
<td>55</td>
<td>57</td>
<td>55</td>
<td>78</td>
<td>78</td>
<td>75</td>
<td>84</td>
<td>84</td>
<td>81</td>
</tr>
<tr>
<td>Moloka'i</td>
<td>63</td>
<td>63</td>
<td>61</td>
<td>81</td>
<td>83</td>
<td>81</td>
<td>93</td>
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<td>97</td>
<td>97</td>
<td>99</td>
<td>98</td>
<td>98</td>
</tr>
<tr>
<td>O'ahu</td>
<td>68</td>
<td>68</td>
<td>69</td>
<td>83</td>
<td>83</td>
<td>81</td>
<td>89</td>
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</tr>
<tr>
<td>Kaua'i</td>
<td>63</td>
<td>63</td>
<td>66</td>
<td>79</td>
<td>79</td>
<td>82</td>
<td>85</td>
<td>87</td>
<td>88</td>
</tr>
</tbody>
</table>

The computed chi-square values of E-1, E-2, and E-3 for each group by islands are listed in Tables 5 to 7.

The limiting values of chi square for accepting or rejecting eigenvectors at the 5% and 1% significant level are given in Table 8.

It is apparent in Tables 5 to 7 that the first eigenvector E-1 is quite stable, with the exception of Hawai'i Island, and that the second eigenvector E-2 is still stable, except for Moloka'i. However, E-3 is less stable because the Hawai'i, Maui, and Kaua'i values exceed the acceptable level for constant eigenvectors.

The chi-square values between the 40- and 30-yr eigenvectors were also computed and the results listed in Tables 9 to 11.

Based on the values in Tables 9 to 11, the 40-yr eigenvectors for E-1 to E-3 are very stable, with the exception of Moloka'i's E-2 value. Thus, the 40-yr eigenvectors for E-1 to E-3 are the best representatives for each island and the state. Forty-year spatial patterns for E-1, E-2, and E-3 are respectively shown in Figures 2, 3, and 4. Because these patterns are based on annual rainfall data, they differ markedly from the spatial patterns of Lyons (1982) who used monthly data.
### TABLE 5. COMPUTED CHI-SQUARE VALUE FOR E-1, E-2, AND E-3 BETWEEN FIRST AND SECOND GROUPS

<table>
<thead>
<tr>
<th>Chi Square</th>
<th>Hawai'i</th>
<th>Maui</th>
<th>Moloka'i</th>
<th>Lāna'i</th>
<th>O'ahu</th>
<th>Kaua'i</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1</td>
<td>10.44</td>
<td>3.90</td>
<td>3.01</td>
<td>0.06</td>
<td>0.20</td>
<td>0.75</td>
</tr>
<tr>
<td>E-2</td>
<td>9.09</td>
<td>5.08</td>
<td>19.76</td>
<td>1.74</td>
<td>2.12</td>
<td>1.65</td>
</tr>
<tr>
<td>E-3</td>
<td>78.98</td>
<td>39.72</td>
<td>7.39</td>
<td>7.23</td>
<td>8.28</td>
<td>14.13</td>
</tr>
</tbody>
</table>

### TABLE 6. COMPUTED CHI-SQUARE VALUE FOR E-1, E-2, AND E-3 BETWEEN FIRST AND THIRD GROUPS

<table>
<thead>
<tr>
<th>Chi Square</th>
<th>Hawai'i</th>
<th>Maui</th>
<th>Moloka'i</th>
<th>Lāna'i</th>
<th>O'ahu</th>
<th>Kaua'i</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1</td>
<td>47.00</td>
<td>6.08</td>
<td>2.58</td>
<td>2.81</td>
<td>0.95</td>
<td>1.38</td>
</tr>
<tr>
<td>E-2</td>
<td>8.42</td>
<td>11.50</td>
<td>6.48</td>
<td>4.95</td>
<td>2.77</td>
<td>3.81</td>
</tr>
<tr>
<td>E-3</td>
<td>57.45</td>
<td>89.62</td>
<td>2.59</td>
<td>0.07</td>
<td>15.24</td>
<td>11.78</td>
</tr>
</tbody>
</table>

### TABLE 7. COMPUTED CHI-SQUARE VALUE FOR E-1, E-2, AND E-3 BETWEEN SECOND AND THIRD GROUPS

<table>
<thead>
<tr>
<th>Chi Square</th>
<th>Hawai'i</th>
<th>Maui</th>
<th>Moloka'i</th>
<th>Lāna'i</th>
<th>O'ahu</th>
<th>Kaua'i</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1</td>
<td>41.72</td>
<td>2.84</td>
<td>1.91</td>
<td>0.00</td>
<td>1.00</td>
<td>1.56</td>
</tr>
<tr>
<td>E-2</td>
<td>0.95</td>
<td>10.13</td>
<td>10.07</td>
<td>1.53</td>
<td>4.10</td>
<td>1.42</td>
</tr>
<tr>
<td>E-3</td>
<td>27.15</td>
<td>37.87</td>
<td>4.48</td>
<td>1.12</td>
<td>15.45</td>
<td>59.46</td>
</tr>
</tbody>
</table>

### TABLE 8. CRITICAL VALUES OF CHI-SQUARE FOR EACH ISLAND

<table>
<thead>
<tr>
<th>Degree of Freedom</th>
<th>Hawai'i</th>
<th>Maui</th>
<th>Moloka'i</th>
<th>Lāna'i</th>
<th>O'ahu</th>
<th>Kaua'i</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20</td>
<td>17</td>
<td>4</td>
<td>4</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>$\chi^2_{0.05}$</td>
<td>31.4</td>
<td>27.6</td>
<td>9.5</td>
<td>9.5</td>
<td>25.0</td>
<td>23.7</td>
</tr>
<tr>
<td>$\chi^2_{0.01}$</td>
<td>37.6</td>
<td>33.4</td>
<td>13.3</td>
<td>13.3</td>
<td>30.6</td>
<td>29.1</td>
</tr>
</tbody>
</table>
TABLE 9. COMPUTED CHI-SQUARE VALUE FOR E-1, E-2, AND E-3 BETWEEN FIRST 30-YR AND 40-YR GROUPS

<table>
<thead>
<tr>
<th>Chi Square</th>
<th>Hawai'i</th>
<th>Maui</th>
<th>Islands of Moloka'i</th>
<th>Lāna'i</th>
<th>O'ahu</th>
<th>Kaua'i</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1</td>
<td>17.64</td>
<td>2.15</td>
<td>0.95</td>
<td>0.02</td>
<td>0.35</td>
<td>0.40</td>
</tr>
<tr>
<td>E-2</td>
<td>6.54</td>
<td>5.23</td>
<td>3.09</td>
<td>0.03</td>
<td>0.72</td>
<td>5.46</td>
</tr>
<tr>
<td>E-3</td>
<td>8.45</td>
<td>0.35</td>
<td>0.77</td>
<td>2.07</td>
<td>28.60</td>
<td>2.85</td>
</tr>
</tbody>
</table>

TABLE 10. COMPUTED CHI-SQUARE VALUE FOR E-1, E-2, AND E-3 BETWEEN SECOND 30-YR AND 40-YR GROUPS

<table>
<thead>
<tr>
<th>Chi Square</th>
<th>Hawai'i</th>
<th>Maui</th>
<th>Islands of Moloka'i</th>
<th>Lāna'i</th>
<th>O'ahu</th>
<th>Kaua'i</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1</td>
<td>7.51</td>
<td>0.84</td>
<td>0.93</td>
<td>0.05</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>E-2</td>
<td>7.39</td>
<td>10.50</td>
<td>0.67</td>
<td>1.34</td>
<td>4.70</td>
<td>1.03</td>
</tr>
<tr>
<td>E-3</td>
<td>16.66</td>
<td>0.02</td>
<td>2.55</td>
<td>1.19</td>
<td>19.60</td>
<td>15.82</td>
</tr>
</tbody>
</table>

TABLE 11. COMPUTED CHI-SQUARE VALUE FOR E-1, E-2, AND E-3 BETWEEN THIRD 30-YR AND 40-YR GROUPS

<table>
<thead>
<tr>
<th>Chi Square</th>
<th>Hawai'i</th>
<th>Maui</th>
<th>Islands of Moloka'i</th>
<th>Lāna'i</th>
<th>O'ahu</th>
<th>Kaua'i</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1</td>
<td>18.63</td>
<td>2.02</td>
<td>0.62</td>
<td>0.05</td>
<td>0.65</td>
<td>0.82</td>
</tr>
<tr>
<td>E-2</td>
<td>1.66</td>
<td>2.86</td>
<td>16.84</td>
<td>7.19</td>
<td>0.14</td>
<td>0.47</td>
</tr>
<tr>
<td>E-3</td>
<td>12.09</td>
<td>3.24</td>
<td>0.62</td>
<td>0.57</td>
<td>7.11</td>
<td>9.42</td>
</tr>
</tbody>
</table>

COMPUTATION OF MAXIMUM ENTROPY POWER SPECTRUM

In meteorology and climatology, the most commonly used spectral analysis is the conventional power spectral method, based on the Fourier transformation of the autocorrelation function of the original observed data multiplied by a certain window function. First proposed by Blackman and Tukey (1958), this method is based on the assumption that the data outside the observations should be either zero or periodic values, and that does not provide an estimation of the true spectrum. Thus, the conventional power spectral estimation has certain drawbacks to the analysis of short-term series (Ulrych and Bishop 1972; Chen and Stegen 1974).
Figure 2. Forty-year E-1 rainfall patterns for six major islands of Hawai‘i
Figure 3. Forty-year E-2 rainfall patterns for six major islands of Hawai‘i
Figure 4. Forty-year E-3 rainfall patterns for six major islands of Hawai‘i
Burg (1967, 1968, 1975) developed a new and different approach, maximum entropy spectral analysis, for estimating the power spectrum of a set of short-term data. A detailed description of the procedure (Smylie, Clarke, and Ulrych 1973) is briefly summarized as the maximum spectrum of a stationary, random and uniformly sampled process that results in a spectrum from maximizing the entropy of that process. The entropy of a Guassian band-limited time series is defined as

\[
\frac{1}{4f_N} \int_{-f_N}^{f_N} \log S(f) \, df
\]

where

- \( S(f) \) = power spectrum
- \( f_N \) = Nyquist frequency
- \( \Delta t \) = uniform sampling interval

The maximum entropy spectrum can be obtained from

\[
P_E(f_N) = \frac{P_M}{f_N \sum_{j=1}^{M-1} \gamma_j \exp(-i2\pi jf\Delta t)}^2
\]

and \( P \) is determined by

\[
\begin{pmatrix}
P_M \\
0 \\
\end{pmatrix}
= 
\begin{pmatrix}
\rho_0 & \rho_1 & \cdots & 1 \\
\rho_1 & \rho_0 & \cdots & \gamma_1 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & \rho_M \\
\rho_1 & \cdots & \cdots & \gamma_M \\
\end{pmatrix}
\]

where \( \rho_0, \rho_1, \rho_M \) are autocorrelation functions and \( \gamma_j \) is the prediction error coefficient determined by Akaike's final prediction error (FPE) (1969a, 1969b, 1970).

**MAXIMUM ENTROPY SPECTRAL ANALYSIS OF HAWAI'I TIME SERIES**

Because the first three eigenvalues can account for 80% of the total variance, this study analyzes only the first three time coefficients. The 40-yr annual rainfall time series for E-1 to E-3 is shown in Figure 5 for the state of Hawai'i. The 40-yr time series for E-1 to E-3 is shown in Figures 6 to 11 for the individual islands.

The maximum spectral densities for entropy power and conventional
Figure 5. Forty-year annual rainfall time series, state of Hawai'i
Figure 6. Forty-year annual rainfall time series, Hawai‘i Island
Figure 8. Forty-year annual rainfall time series, Moloka‘i
Figure 9. Forty-year annual rainfall time series, Lāna'i
Figure 10. Forty-year annual rainfall time series, O'ahu
Figure 11. Forty-year annual rainfall time series, Kaua‘i
power were computed for the state and for each island. The densities were plotted along the coordinates against the frequency and periodic time along the abscissa (Figs. 12-18).

For this study, the conventional power spectra were smoothed using the Hamming-Tukey coefficients, which are defined as

\[
\hat{E}(o) = 0.54 \ E(o) + 0.46 \ E(1)
\]

\[
E(k) = 0.23 \ E(k - 1) + 0.54 \ E(k) + 0.23 \ E(k + 1), \ 1 \leq k \leq m
\]

\[
E(m) = 0.46 \ (m - 1) + 0.54 \ E(m)
\]

where

\[\hat{E}(k) = \text{last smoothed spectrum}\]

\[E(k) = \text{raw spectrum}.\]

The last smoothed spectra of E-1, E-2, and E-3 for each island and for the state were further tested by the following equation,

\[
\frac{\hat{E}(k)}{E(k)} = \frac{\chi^2}{\nu}
\]

where

\[\hat{E}(k) = \text{individual spectrum}\]

\[E(k) = \text{average spectrum}\]

\[\chi^2 = \text{chi square}\]

\[\nu = \text{freedom of degree} = \frac{2N - \frac{1}{2}m}{m}\]

\[N = \text{sample size}\]

\[m = \text{maximum time lag}.\]

For this study, \(N = 40\) and \(m = N/2 = 20\); therefore, \(\nu = 3.0\). The critical value of \(\chi^2\) for \(\nu = 3\) is 11.341 at \(\alpha = 0.01\), 7.815 at \(\alpha = 0.05\), and 6.251 at \(\alpha = 0.10\).

For Hawai'i rainfall the significant E-1, E-2, and E-3 cycles passed the statistical significance test at \(\alpha = 0.10\) or \(\alpha = 0.05\) as shown in Table 12.

Based on the maximum entropy method (MEM) results and the conventional power spectral results (Table 12), we concluded that the MEM has a better resolution for small-size samples.

In the state of Hawai'i, the predominant periodicity of E-1 is a 2-yr cycle and the second cycle is 4.4 years. The important cycle of E-2 for the state is also 2-yr cycle. Two weak cycles of E-2 are 3.0 and 4.0 years. The E-3 time series has only a very weak 2.5-yr cycle.
Figure 12.1. Annual rainfall, maximum spectral density for entropy power, state of Hawai'i

Figure 12.2. Annual rainfall, maximum spectral density for conventional power, state of Hawai'i
Figure 13.1. Annual rainfall, maximum spectral density for entropy power, Hawai'i Island

Figure 13.2. Annual rainfall, maximum spectral density for conventional power, Hawai'i Island
Figure 14.1. Annual rainfall, maximum spectral density for entropy power, Maui

Figure 14.2. Annual rainfall, maximum spectral density for conventional power, Maui
Figure 15.1. Annual rainfall maximum spectral density for entropy power, Moloka'i

Figure 15.2. Annual rainfall, maximum spectral density for conventional power, Moloka'i
Figure 16.1. Annual rainfall, maximum spectral density for entropy power, Lāna'i

Figure 16.2. Annual rainfall, maximum spectral density for conventional power, Lāna'i
Figure 17.1. Annual rainfall, maximum spectral density for entropy power, O'ahu

Figure 17.2. Annual rainfall, maximum spectral density for conventional power, O'ahu
Figure 18.1. Annual rainfall, maximum spectral density for entropy power, Kaua' i

Figure 18.2. Annual rainfall, maximum spectral density for conventional power, Kaua' i
<table>
<thead>
<tr>
<th>State</th>
<th>EIGEN-VECTOR</th>
<th>MEM CYCLE</th>
<th>CONVECTIONAL POWER SPECTRAL ANALYSIS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E-1</td>
<td>2.0</td>
<td>4.4</td>
</tr>
<tr>
<td></td>
<td>E-2</td>
<td>2.0</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>E-3</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>Hawai’i</td>
<td>E-1</td>
<td>2.4</td>
<td>4.4</td>
</tr>
<tr>
<td></td>
<td>E-2</td>
<td>2.0</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>E-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maui</td>
<td>E-1</td>
<td>2.0</td>
<td>4.4</td>
</tr>
<tr>
<td></td>
<td>E-2</td>
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<td>10.0</td>
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<tr>
<td></td>
<td>E-3</td>
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<td>2.1</td>
</tr>
<tr>
<td>Moloka’i</td>
<td>E-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>E-2</td>
<td></td>
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</tr>
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<td></td>
<td>E-3</td>
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<td></td>
</tr>
<tr>
<td>Lāna’i</td>
<td>E-1</td>
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<td></td>
<td>E-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>O'ahu</td>
<td>E-1</td>
<td>2.0</td>
<td>3.3</td>
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<td></td>
<td>E-3</td>
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<tr>
<td>Kaua’i</td>
<td>E-1</td>
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</tr>
<tr>
<td></td>
<td>E-3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** MEM = maximum entropy method.

The highest peak of Hawai’i Island’s power spectral density indicates that the most important cycle of E-1 is 4 to 4.4 years. The 2-yr cycle of E-1, however, is not very important compared with the 4-yr cycle. Figure 13 shows that the 3.1- and 2-yr cycles are important for E-2. No prominent cycles exist in E-3 time series coefficients.

Maui’s E-1 spectrum also has a 2-yr peak. The other peaks for E-2 to E-3 are so weak that no important cycles exist (Fig. 14).

The spectral densities of E-1 to E-3 are too small on Moloka’i and
Lāna'i so that no outstanding cycles exist for these two islands (Figs. 15, 16).

O'ahu's spectral densities indicate that the 2-yr cycle is the most important, including two weaker 3.3- and 8-yr cycles for the E-1 time series coefficient. The 2- and 4.4-yr cycles for E-2 are marginally important. No significant cycle exists for E-3 on O'ahu (Fig. 17).

The most important 2-yr cycle does not occur on Kaua'i. Instead, the predominant cycle for E-1 is 3 years. Both E-2 and E-3 spectral densities do not exhibit important cycles (Fig. 18).

The 2- to 2.1-yr cycle, which is the most important for E-1 and E-2 for the state and the islands of Hawai'i, Maui, and O'ahu, may be related to the quasi-biennial oscillation that has been proved to exist in the tropical stratosphere (Reed 1964; Lindzen and Holton 1968). In the equatorial regions, the zonal winds, which are easterlies and westerlies between 50 and 25 mb pressure surfaces, have a pronounced oscillation of approximately 26-mo period (2.2 yrs). This 2.2-yr oscillation has been known as the quasi-biennial oscillation (QBO). The presence of QBO in the atmosphere is widely recognized and it has been related to different atmospheric parameters and different regions (Angell and Korchover 1962, 1964, 1968, 1975; Miller, Woolf, and Teweles 1967; Rasmusson, Arkin, and Chen 1981; Reed 1965; Trenberth 1975). The relationship between Hawaiian spectra and the QBO requires further study.

The 4- to 4.4-yr cycle, which is second in importance to the state and to the islands of Hawai'i, Maui, and O'ahu is relative to the El Niño cycle. The relationship between rainfall in the state of Hawai'i and El Niño/southern oscillation (ENSO) was studied as early as 1932 (Walker and Bliss 1932). Meisner (1976) found a negative correlation between Hawai'i rainfall and the sea surface temperature (SST) at equatorial eastern Pacific. Lyons (1982) found that Hawai'i has anomalously low rainfall during most—not all—El Niño winters, and above normal rainfall during summers with an anomalously warm SST at equatorial eastern Pacific. Cheo (1984) found Hawai'i annual rainfall after El Niño was below normal, especially after a strong El Niño, and winter season rainfall of an El Niño and summer season rainfall after El Niño was also below normal. He also found that the correlation between windward rainfall in Hawai'i and El Niño is not very good. Haraguchi and Matsunaga (1985) found that almost all major
droughts in Hawai'i were correlated with the negative southern oscillation index (SOI).

The 3-yr cycle, which is prominent on Kaua'i and slightly important for the state and O'ahu, requires advanced investigation. The reason Kaua'i has no QBO may be because of its geographical location. Kaua'i is farther north, less affected by tradewind disturbances and tropical storms during the summer, and is also more often affected by winter fronts even during droughts.

The 11-yr sunspot cycle, which is very common in other places, is absent in Hawai'i.

**SUMMARY AND CONCLUSIONS**

The empirical function was applied to Hawai'i rainfall analysis to delineate it into two noncorrelated components: eigenvectors and time-dependent coefficients, also known respectively as spatial characteristics and time series. The chi-square statistic was introduced to test the stabilities of eigenvectors, E-1 to E-3, for the state and individual islands. The time-dependent coefficients were analyzed by using maximum and conventional power spectral methods.

The results obtained from this study include the following:

1. The convergence of eigenvalues for each island are almost identical
2. The first eigenvector can account for more than 50% of the total variance, except for Hawai'i Island which is 34%
3. The first three eigenvectors can account for almost 85% of the total variance, except for Hawai'i Island which is 70%
4. The eigenvectors, especially E-1 and E-3, for the state and for each island are quite stable within the 30-yr period
5. The 40-yr eigenvectors are much more stable than the 30-yr eigenvectors
6. The eigenvectors within the 30-yr period with 5-yr data change are more stable than those within the 30-yr period with 10-yr data change
7. Only E-1 and E-2 time-dependent coefficients have important power spectral peaks
8. No significant power spectral peak was found in the E-3 time series

9. The quasi-biennial oscillation (2- to 2.1-yr cycle) is the predominant cycle for the state, Maui, and O'ahu

10. The El Niño cycle (4- to 4.4-yr) is the second important cycle for the state and for the islands of Hawai'i and Kaua'i

11. The 3- to 3.3-yr cycle is the most important cycle for Kaua'i

12. No significant sunspot cycle (11-yr) was found for Hawai'i.

Based on the above results, planners making water-resource decisions can expect major variations in Hawai'i's rainfall, for which a 3-yr cycle can be generally expected. The super-position of the El Niño cycle on the quasi-biennial oscillation will produce the long-term rainfall variation estimation for the state of Hawai'i.

ACKNOWLEDGMENTS

The authors are especially grateful to Dr. Thomas A. Schroeder of the University of Hawaii at Manoa Department of Meteorology for reviewing the manuscript and providing many helpful suggestions. We also wish to thank Dr. L. Stephen Lau for his encouragement and interest in this research. Our appreciation is also extended to the Division of Water and Land Development of the Hawaii State Department of Land and Natural Resources for providing the rainfall data.

REFERENCES CITED


