THE
HAWAII GEOTHERMAL PROJECT

MODELLING OF HAWAIIAN GEOTHERMAL RESOURCES

November 1, 1973

TECHNICAL REPORT NO. 1

UNIVERSITY·OF·HAWAII
1907

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ENGINEERING PROGRAM

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November 1, 1973

TECHNICAL REPORT NO. 1

Prepared Under

NATIONAL SCIENCE FOUNDATION

RESEARCH GRANT NO. GI-38319

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I. Introduction

One of the major objectives of the Hawaii Geothermal Project is to locate and identify a suitable geothermal resource on the island of Hawaii and to utilize the heat to produce electricity by means of a research-oriented power plant. As a primary first step of this project the potential geothermal resources on the island of Hawaii must be studied and sufficient information obtained to permit a reasonable prediction to be made of the various parameters of the resource and the methods most suitable for utilizing the heat energy.

The island of Hawaii, youngest of the major islands in the State of Hawaii, consists of porous volcanic rock surrounded by the ocean. Its volcanic origin means that magma chambers may be present and therefore that a large source of heat may be found at a relatively shallow depth. The porosity of the island rock permits free flow of the water from the ocean into the island and thus provides a source of heat-carrying fluid. In addition, rainwater percolating through the volcanic ash forms a water table (the Ghyben-Herzberg lens) on top of the intruded ocean water. It is expected that the primary source of geothermal energy to be found will probably be hot brine.

Except for the shallow layers of the island, the detailed geology of Hawaii is not well known. Prior to the well drilled by Dr. George Keller this past year, only shallow water wells had been drilled. Because of the lack of geological knowledge, the geothermal resource modelling, both theoretical and experimental, must necessarily be developmental. However, it is expected that sufficient information will be obtained so that useful conclusions can be drawn.

II. Results Expected from Modelling

A theoretical analysis of the geothermally interesting portions of the island of Hawaii is well underway. Physical modelling of a possible liquid-dominated geothermal reservoir has been started. Some of the results expected from the modelling studies include the following:

1. temperature profiles surrounding the geothermal resource,
2. distribution and flow of the fluid,
3. salt and other mineral distributions,
4. effects of withdrawing fluid from the bottom of the well,
5. reservoir capacity,
6. effects of fluid recharge.

When these results are obtained, decisions can be made on the suitability of a located geothermal resource, the type of pilot plant to be used, and the number and location of wells.

III. Analytical Modelling

The primary objectives of the analytical modelling are to predict the performance of geothermal wells under different conditions and to study the environmental impact of the geothermal system, especially the stability of the Ghyben-Herzberg lens when perturbed by the extraction of a fluid from a sink below the lens. The results of these studies will aid in the selection of a viable well-site. Specific topics to be included in the modelling are:

1. temperature profiles, heat transfer, and fluid flow characteristics

of geothermal systems on the island of Hawaii,

2. capacity of a geothermal well,
3. expected life span of a geothermal well under different operating and resource conditions,
4. minimum depth required for a geothermal well so that fresh water will not cone downwards to the well bottom as water is pumped out,
5. effect of fluid recharge on the performance of a geothermal well.

A. Free Convection in a Coastal Aquifer with Geothermal Heating from Below

Over the past two decades a considerable amount of work has been done on convective heat transfer in a porous medium. For example, the criterion for the onset of free convection in a porous medium bounded by two parallel isothermal plates at different temperatures was predicted by Horton and Rogers[1] and by Lapwood.[2] A similar analysis was performed by Katto and Masuoka,[3] who also confirmed the prediction experimentally. For the case of a semi-infinite porous medium, the onset of free convection was studied by Wooding.[4] Finite difference solutions for temperature and velocity distributions in a porous medium bounded by isothermal walls were given by Wooding,[5] Chan et al,[6] Donaldson,[7,8] as well as by Holst and Aziz.[9] The effects of a non-isothermal well on free convection in a porous medium was considered by Elder.[10] The related problems of combined free and forced convection in a porous medium were treated by Pratt[11] as well as by Combarnous and Bia.[12] Recently, the more complicated problem of free convection in a porous medium, where density variations are due to both thermal expansion and solute concentration, has attracted considerable attention. For example, the onset of free convection was treated by Nield[13,14] and by Rubin,[15] while finite difference solutions for temperature and velocity distribution were obtained by Henry and Kohout,[16] who studied the problem of waste disposal.

The geothermal system in Hawaii can be represented in a very simple fashion as an unconfined coastal aquifer with heating from below as shown in Fig. 1. It has been speculated that as a result of the heating, a warm column of brine rises and penetrates the fresh water lens. After mixing with fresh water the brine flows seaward again, thus forming a large, geothermally-heated, convective flow cycle. An extensive literature search has not located any published work on free convection in a coastal aquifer from the geothermal point of view, although the work by Henry and Kohout[16] is related to the problem.

As a first step in the study, the rectangular aquifer shown in Fig. 2 will be studied. This model represents a simplified one-half portion of the symmetrical island of Fig. 1. To simplify the mathematical formulation of the problem, the following assumptions are made:

1. The flow field is steady and two-dimensional.
2. The variations of salt concentration are ignored.
3. The temperature of the fluid is too low corresponding to its saturated temperature for boiling to take place.
4. Density is assumed to be constant except in the bouyancy force term.
5. Fluid properties such as thermal conductivity, specific heat, kinematic viscosity, and permeability are assumed to be constant.
6. Ocean is at rest; i.e., the effects of tides are neglected.
7. Phreatic surface is a horizontal plane located at $y=h$.

Using these approximations gives the following governing equations:

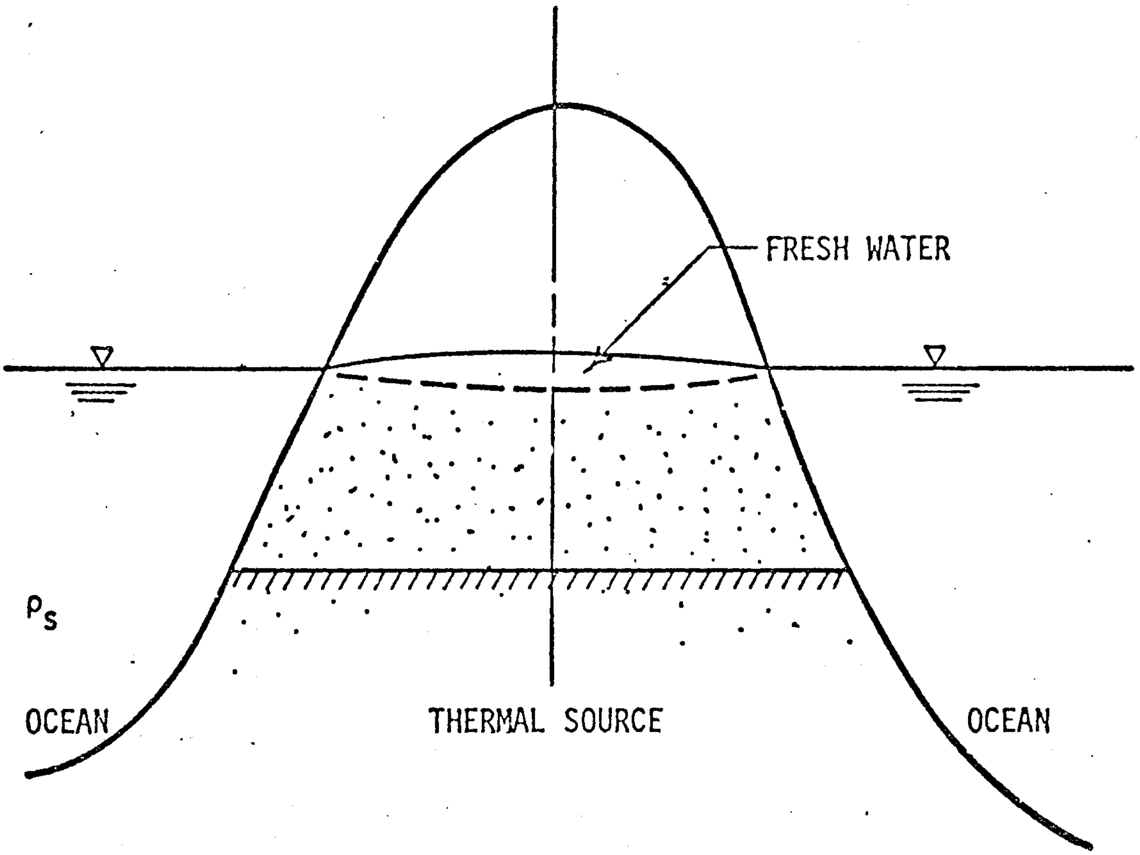

FIG. 1. COASTAL AQUIFER WITH THERMAL SOURCE

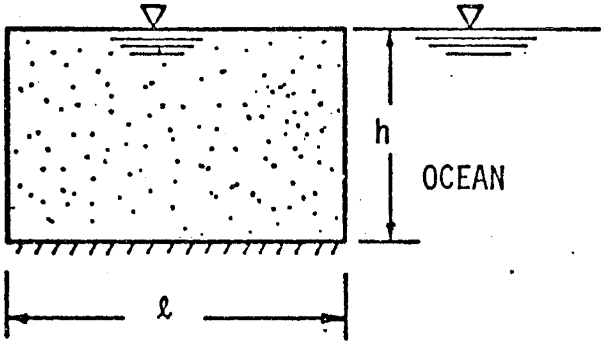

FIG. 2. RECTANGULAR MODEL OF AQUIFER

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u = -\frac{K}{\mu}\frac{\partial p}{\partial x}, \tag{2}$$

$$v = -\frac{K}{\mu}\left(\frac{\partial p}{\partial y} + \rho g\right), \tag{3}$$

$$\frac{\rho}{\rho_s} = 1 - \beta\,(T - T_s), \tag{4}$$

$$u\,\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right), \tag{5}$$

where u, v are velocity components, ρ the density, K the permeability, μ the viscosity, p the pressure, T the temperature, α the diffusivity, β the coefficient of thermal expansion, and g the gravitational acceleration. The subscript s denotes the condition in the ocean.

For the rectangular aquifer shown in Fig. 2, the boundary conditions are given by

$$\frac{\partial p}{\partial x}(0,y) = 0, \tag{6}$$

$$\frac{\partial T}{\partial x}(0,y) = 0, \tag{7}$$

$$p(\ell,y) = p_a + \rho_s g(h - y), \tag{8}$$

$$T(\ell,y) = T_s \ , \tag{9}$$

$$p(x,h) = p_a \ , \tag{10}$$

$$T(x,h) = T_1 \ , \tag{11}$$

$$\frac{\partial p}{\partial y}(x,0) = -\rho_s[1-\beta(T_o-T_s)]g, \tag{12}$$

$$T(x,0) = T_o(x). \tag{13}$$

Boundary condition (12) follows directly from Equations (3) and (4) for v=0. Since boundary conditions are in terms of p and T, we shall eliminate u,v,ρ from Equations (1) through (5) and express the resultant equations in terms of p and T. Thus we have

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \rho_s g\beta\frac{\partial T}{\partial y} \ , \tag{14}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{K}{\alpha\mu}\frac{\partial p}{\partial x}\frac{\partial T}{\partial x} + \frac{K}{\alpha\mu}\left\{\frac{\partial p}{\partial y} + \rho_s g[1-\beta(T-T_s)]\right\}\frac{\partial T}{\partial y} = 0. \tag{15}$$

It is convenient to express Equations (6) to (15) in dimensionless form. For this purpose, we introduce

$$P = \frac{p - p_a}{\frac{\alpha\mu}{K}} \ , \qquad \Theta = \frac{T - T_s}{T_c - T_s} \ , \tag{16}$$

$$X = \frac{x}{h} \ , \qquad Y = \frac{y}{h} \ , \qquad L = \frac{\ell}{h} \ ,$$

where T_c is the reference temperature at (0,0).

Equations (14) and (15) in terms of dimensionless variables are

$$\frac{\partial^2 P}{\partial X^2} + \frac{\partial^2 P}{\partial Y^2} = Ra \frac{\partial \Theta}{\partial Y} , \tag{17}$$

$$\frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} + \frac{\partial P}{\partial X}\frac{\partial \Theta}{\partial X} + \frac{\partial P}{\partial Y}\frac{\partial \Theta}{\partial Y} + D\frac{\partial \Theta}{\partial Y} - Ra\Theta\frac{\partial \Theta}{\partial Y} = 0. \tag{18}$$

where $R_a \equiv \dfrac{\rho_s g\beta K(T_c - T_s)h}{\mu\alpha}$ is the Rayleigh number and $D \equiv \dfrac{\rho_s Kgh}{\alpha\mu}$ which is

called the "discharger number" by Elder. Boundary conditions (6) to (13) are given by

$$\frac{\partial P}{\partial X}(0,Y) = 0, \tag{19}$$

$$\frac{\partial \Theta}{\partial X}(0,Y) = \quad , \tag{20}$$

$$P(L,Y) = D(1 - Y), \tag{21}$$

$$\Theta(L,Y) = 0 , \tag{22}$$

$$P(X, 1) = 0 , \tag{23}$$

$$\Theta(X,1) = \Theta_1 , \tag{24}$$

$$\frac{\partial P}{\partial Y}(X,0) = -D + R_a\Theta_o(X), \tag{25}$$

$$\Theta(X, 0) = \Theta_o(X) , \tag{26}$$

$$\text{where } \Theta_1 \equiv \frac{T_1 - T_s}{T_c - T_s} \; ,$$

$$\Theta_o(x) \equiv \frac{T_o(x) - T_s}{T_c - T_s} \; . \qquad (27)$$

Equations (17) and (18) are a set of coupled non-linear partial differential equations of elliptic form. Numerical solutions to these equations with boundary conditions (19) through (26) can be obtained by standard finite difference methods. The resulting set of algebraic equations can be solved by iteration; that is, solving for P in Equation (17) by assuming values for Θ. With the nodal values of P thus obtained, Equation (18) will be used for the solution of nodal values of Θ.

A more refined analysis using the finite element method is currently being pursued and will permit an irregular geometry to be used. This analysis of the problem will provide a more realistic picture of the actual situation.

B. Coning of Fresh Water and Salt-Water Interface in a Coastal Aquifer

The problem of upconing was studied by Muskat[17] for the case of brine intrusion into oil wells. The same problem was later considered by Meyer and Garder,[18] and by Kidder[19] using different analytic techniques. The similar problem of upconing of sea water in a fresh water well was considered by Dagan and Bear[20] theoretically, and confirmed by Schmorak and Mercado[21] in their field investigations.

The related problem of salt water intrusion in a coastal aquifer without a sink or a source was considered by Glover.[22] The shape and location of steady state interface was obtained by Henry,[23] whereas the corresponding transient problem was considered by Bear and Dagan[25] as well as by Shamir and Dagan.[26] In all of these studies, the assumption of interface is employed. This assumption has greatly simplified the problem since the fluids on either side of the interface can be assumed to be incompressible. Recently, the movement of the salt water front including the effects of dispersion was considered by Pinder and Cooper.[27] However, the related problem of coning including the effects of dispersion has never been attempted in the published literature, even for isothermal conditions. The formulation of this problem for the geothermal resource is now in progress.

IV. Physical Modelling

A literature survey has disclosed that there has been very little physical modelling in the field of geothermics. H. R. Henry and F. A. Kahout[16] have been conducting related investigations for waste disposal purposes but in their studies the heat source has not exceeded 43°C. It is expected that the Hawaii model will operate at the magma temperature, 1100°C, with a well-head hot water temperature of about 275°C. The Stanford University Geothermal Group is also involved with physical modelling,[28,29] although their modelling has been restricted at this time to the active "chimney" portion of the reservoir.

The Hawaii geothermal model will be constructed for the island of Hawaii on a total systems basis; that is, a study designed initially for the actual situation--the entire reservoir bounded on the bottom by an impermeable layer, on the sides by the sea, and on the top by the producing geothermal field and the Ghyben-Herzberg lens--beginning at first with the geothermal reservoir and progressing eventually to the simulation of a fully operating geothermal field. The two initial regions to be investigated will be the Ghyben-Herzberg lens and an element of the porous medium.

Some of the variables requiring consideration in a physical geothermal model are:

1. permeability--not of the microscopic rock, but of the macroscopic system; that is, fractures will most probably dominate over vesicular or layer porosity in determining permeability,
2. temperature,
3. pressure,
4. fluid composition,
5. fluid flow rate,
6. porosity--to obtain fluid volume fraction.

The size of the physical model and the exact features to be modelled require careful study. If the island of Hawaii is reshaped into a square, each side will be approximately 64 miles in length. (A circular reshape would result in a diameter of 72 miles.) The impermeable bottom layer is at a depth one mile below sea level.[30] (Note that the ocean floor in the vicinity of the Hawaiian Islands is considerably below this layer.) The permeable medium under consideration is thus 64 miles by 64 miles by 1 mile. A physical model on a 64:64:1 scale will be of dubious value, as it is the depth dimension that is of greatest interest. It is therefore far more suitable to physically model a portion of the Big Island, rather than the whole island itself, although all areas impacting on the reservoir must be included.

The element selected for this study will extend from the lower impermeable layer up to, but not including, the Ghyben-Herzberg lens. The horizontal dimension should be greater than the vertical. As the location of the well will probably be a point several miles inland, a 4:1 ratio appears reasonable with 4:4:1 for a three dimensional model. Figure 3 is a schematic of the planned experimental set-up and is similar to the rectangular aquifer shown in Fig. 2. One face (A) interfaces with the sea. If desired, the heat source can be moved or the size and configuration altered to obtain streamline symmetry conditions at the left boundary (B). As the initial analytical model is two-dimensional, with one boundary condition being set by symmetry relations, the physical model is being designed accordingly. However, flexibility will be designed into the tank so that conversion can easily be made to a three-dimensional model by increasing the size of the tank in the z-direction.

The preliminary design of the physical model is for a 4 mile wide (from sea to inland boundary) by 1 mile deep two-dimensional element. Tentatively, the actual size of the physical model will be 2 feet by 8 feet with the dummy third dimension being 1 foot. The height of the tank will be 6 feet to allow for later consideration of different ratios and elements.

Questions related to the porous and fluid media which should be used, whether porosity should match that of the natural rock in the area, and how fissures and/or fractures can be manipulated to obtain the desired permeability remain unanswered at this time. An important problem is the overwhelmingly large scale-down factor involved (5280:1). Dimensional analysis will be used to help determine some of the more necessary scale factors.

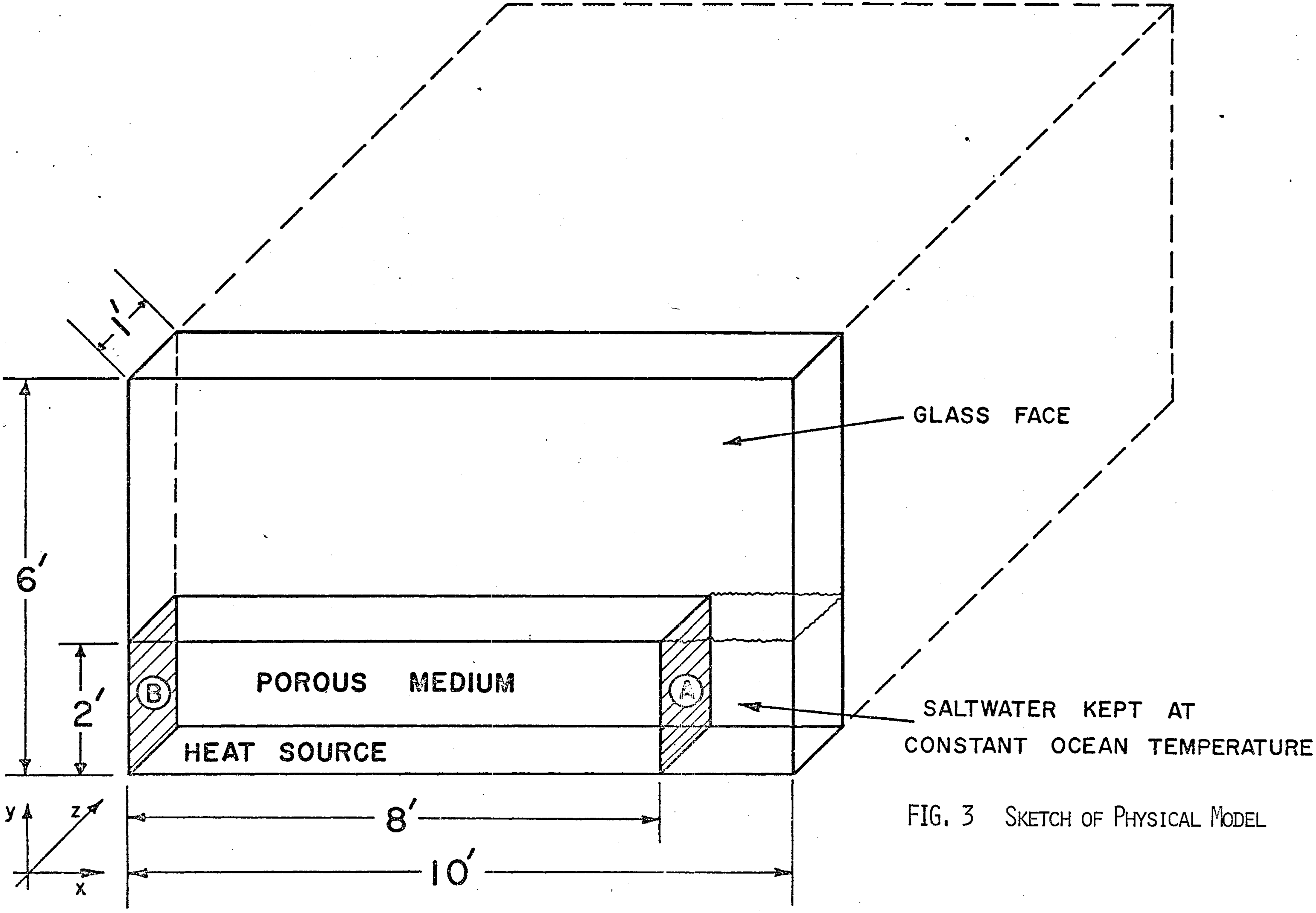

FIG. 3 SKETCH OF PHYSICAL MODEL

The location, configuration and temperature of the heat source must also be specified. There is agreement that magma is generated at the top of the mantle, 20 to 60 miles below the surface of the earth.[31] There will therefore be very large reservoirs of magma at this depth range. There is also agreement that magma reservoirs exist at shallow depths from 1 to 12 miles below the surface of volcanic regions. The shape and size of these shallow reservoirs have been estimated by gravity and magnetic methods to be ellipsoidal or vertically-flattened spheroidal and in the order of 1 to 3 miles along the longer axis.[32] It is expected that one of these magma chambers will provide the large geothermal heat source.

To provide for comparison with the analytical model the heat source will initially be placed at the bottom of the physical model; size, configuration and actual effective temperature of the heat source will be determined later. For design purposes it will be necessary to allow for flexibility; adequate range (100 to 1200°C), size (from "point" source to plane source), and movability in three directions. The scaling of temperature also needs to be considered carefully.

The exact parameters to be measured and the measuring techniques to be used must be specified. Earlier works in related fields have used probes in a two or three dimensional matrix pattern. Temperature, pressure, flow rate, and fluid composition can be measured using appropriate probes interfacing multi-channel recorders or indicators. The more sophisticated infrared, magnetic, nuclear, electronic (resistance or capacitance), and acoustic (sonic or ultrasonic) techniques will be evaluated for possible use in the Hawaii geothermal model.

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