OSCILLATION MODES OF HAWAI'I WATERS FROM THE 2006 KURIL ISLANDS TSUNAMI

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ABSTRACT

The 2006 Kuril tsunami, while not destructive to coastal properties, resulted in prolonged oscillations in Hawai‘i waters. This study reconstructs the tsunami from the seismic source parameters using a nonlinear shallow-water model and uses spectral analysis to examine the oscillation patterns and amplifications around the Hawaiian Islands. The Fast Fourier Transform of the computed sea surface elevation reveals the frequency contents of the tsunami at each grid point. Contour plots of the amplitudes of the Fourier coefficients identify the energy levels and areas where strong amplification occurs. A wavelet filter extracts the free surface time histories of around the high energy periods for examination of the oscillation patterns. On a regional scale, the oscillations consist of resonance in open bays and standing waves across the island chain. Large-scale standing waves appear to be generated by the oscillations of over Penguin Bank. Around O‘ahu, the time-distance diagram separates the wave traveling along the coast from the standing and progressive wave. For periods smaller than 10 minutes, the oscillation patterns are dominated by edge waves capable of traveling long distances. Energetic edge waves of longer periods are also observed. Coastal trapped waves or edge waves appear to be an efficient mechanism in transferring energy and feeding adjacent oscillation patterns.
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CHAPTER 1
INTRODUCTION

On November 15, 2006 at 01:14:17 AM Hawaiian Standard Time (HST) an earthquake with moment magnitude (Mw) 8.3 occurred at 46.607°N, 153.230°E along the Kuril trench, a region prone to large tsunamigenic earthquakes. A tsunami watch was issued for Hawai‘i by the Pacific Tsunami Warning Center (PTWC), but was cancelled at 5:05 AM HST after the Shemya and Amchitka tide stations located along the Aleutians Islands recorded tsunami wave amplitudes of 0.2 and 0.08 m respectively. Around 7:20 AM HST, the tsunami reached the Hawaiian Islands, where rapid changes in sea level and unusually strong currents were observed. The authorities patrolled the beaches and urged Hawai‘i residents and visitors to stay out of the water for the day. Oscillations of water level were especially visible inside harbors, where the measured amplitudes reached 76 cm in Kahului Harbor, Maui and 49 cm in Hilo Bay, Big Island. Hanauma Bay on O‘ahu, heavily frequented by inexperience swimmers, stayed closed for 2 days due to the presence of strong flood and ebb currents. Those observations suggest that a portion of the tsunami energy was amplified in local harbors and trapped in coastal regions, or more generally that the range of frequency of the November 2006 tsunami was sufficiently large to excite natural oscillation modes associated with bathymetric features of different scales.

Trapping and amplification of tsunami waves in coastal areas have been studied by numerous authors. A number of studies have used spectral analysis of tide station records to examine the role of the bathymetry in tsunami response spectra. Loomis (1966) compared the power spectra of different tsunamis at tide gauges in the Hawaiian Island. Van Dorn (1984, 1987) studied the spectral energy decay of 5 tsunamis and the role of local bathymetry features using tide gauge data from the Pacific Ocean and other smaller seas. Abe (1990, 2001) and Rabinovich (1997) studied the contribution of the local
bathymetry and the tsunami source in the frequency spectra recorded at tide gauges in the Pacific Ocean. More recent studies provide estimates of the fault dimensions by spectral analysis of tide gauge data (Abe, 2006) or establish a transfer function for the response of a tsunami at a tide station (Rabinovich, 2006). The results of these studies show that the spectral structure of a tsunami recorded near the coast is mainly determined by the resonant periods of the bathymetry. All previous studies examined tsunami spectral characteristics at discrete locations, typically at tide stations, where water-level data is available.

It is a common practice to use numerical modeling to extend the coverage of sea-surface elevation data when studying harbor oscillation and resonance. Loomis (1970) developed a method to determine the natural oscillation modes of a bay or harbor of arbitrary shape and depth by solving numerically the linearized equation of motion as an eigenvalue problem. Horillo et al. (2007) applied this method to investigate local amplification of the 2006 Kuril tsunami near Crescent City. An alternate approach uses a spectral analysis to extract harbor resonance and oscillation modes from computed time-series of surface elevations using nonlinear models (e.g. Thompson and Demirbilek, 2002; Douyere, 2003). The present study implements this method to examine resonance periods and associated oscillation patterns in Hawai‘i waters from by the 2006 Kuril tsunami. A nonlinear shallow-water model is used to reproduce the water surface elevations of the 2006 Kuril tsunami and a spectral analysis is performed at all grid points in the Hawaiian Islands region. This allows examination of amplitude contour plots for specific wave periods and identification of high energy components. The advantage of this method over the more theoretical approach of Loomis (1970) is the ability to include non-linear processes and the bottom friction.
CHAPTER 2
METHODOLOGY

2.1 Numerical modeling

The increasing confidence in the capability of tsunami modeling was partly made possible by the availability of high resolution bathymetry data and better quality seismic data as well as tsunami observations for validation. This study used a shallow-water wave model for the propagation of the tsunami across the Pacific Ocean. The model employs a staggered finite difference method and an explicit leapfrog time-integration scheme to solve the depth-average governing equations (Liu et al., 1994). Although the shallow-water equations do not take into account wave dispersion, their capability in reproducing water-level measurements and observed inundation have been validated (e.g., Cheung et al., 2006; and Tang et al. 2006).

Four levels of nested grids were used in the computation to provide wide coverage of the North Pacific Basin as well as sufficient resolution at tide stations, where measured data is available for model validation. The grid arrangement is chosen to maximize the resolution within practical computational time constraints. Figure 1a shows the coverage of the first-level grid from Japan to California with a resolution of 2 minutes (approx. 3700 m). The linear shallow-wave equations were solved in spherical coordinates at the first level, while Cartesian coordinates were used in the next three levels of grids to solve the nonlinear shallow-wave equations. The second grid level, which includes the Hawaiians Islands from Ni‘ihau to Hawai‘i\(^1\) as shown in figure 1b, has a resolution of 15 seconds (463 m). The islands of Kaua‘i, O‘ahu, Maui and Big Island were modeled individually with a finer grid of 3 seconds (92.5 m) resolution. The fourth level covers the coastal regions surrounding the harbor basins at Nawiliwili (Kaua‘i), Honolulu (O‘ahu), Kahului (Maui), Hilo (Big Island), where the tide stations are located. This level

\(^1\) The island of Hawai‘i (correct Hawaiian name) is hereafter called Big Island to avoid confusion.
is resolved with a 0.3 second (9.3 m) resolution grid to capture oscillations of harbor basins.

The bathymetry of the North Pacific was obtained from the 1’ GEBCO (2003) dataset. Although the GEBCO bathymetry shows a smoother sea floor than the Smith and Sandwell (1997) dataset, it was preferred because of its higher accuracy in shallow water (Marks, 2006). For the area around the Hawaiian Islands, a 5” dataset compiled by Japan Marine Science and Technology Center (JAMSTEC) and US Geological Survey (USGS) provides the bathymetry. The individual islands have very good bathymetry and topography coverage that include the SHOALS dataset at 1 to 5 meter resolution in coastal water up to 40 meters depth. In addition, LiDAR data from the Joint Airborne LiDAR Bathymetry Technical Center of Excellence with 3 meter resolution was used for the coastal region of Oʻahu. The different datasets were merged with ArcGIS, converted to the WGS 84 datum and reference to mean-sea level. The final bathymetries were then gridded using the Generic Mapping Tool (Wessel and Smith, 1991). The computation was performed for 10 hours from the initial deformation. The tsunami took approximately 6 hours and 15 minutes to reach the Hawaiian Islands. Table 1 shows the various simulation parameter as well as output intervals for a typical run.
Table 1: Numerical simulation parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Grid 1</th>
<th>Nested grid 2</th>
<th>Nested grid 3</th>
<th>Nested grid 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>North-Pacific</td>
<td>Hawai'i</td>
<td>O'ahu</td>
<td>Honolulu</td>
<td></td>
</tr>
<tr>
<td>Coord. system</td>
<td>Spherical</td>
<td>Cartesian</td>
<td>Cartesian</td>
<td>Cartesian</td>
</tr>
<tr>
<td>S-W equations</td>
<td>Linear</td>
<td>Non-linear</td>
<td>Non-linear</td>
<td>Non-linear</td>
</tr>
<tr>
<td>Grid resolution</td>
<td>2' (~3.7km)</td>
<td>15&quot; (~493m)</td>
<td>3&quot; (~93m)</td>
<td>0.3&quot; (~9m)</td>
</tr>
<tr>
<td>Grid dimension</td>
<td>3901×1561</td>
<td>1081×1561</td>
<td>721×961</td>
<td>841×901</td>
</tr>
<tr>
<td>Time step</td>
<td>0.64 sec.</td>
<td>0.32 sec.</td>
<td>0.16 sec.</td>
<td>0.08 sec.</td>
</tr>
<tr>
<td>Output intervals at grid points</td>
<td>20 min.</td>
<td>2 min.</td>
<td>1 min.</td>
<td>1 min.</td>
</tr>
<tr>
<td>Output intervals at tide and DART stations</td>
<td>30 sec.</td>
<td>30 sec.</td>
<td>30 sec.</td>
<td>30 sec.</td>
</tr>
</tbody>
</table>

2.2 Data processing and spectral analysis

The surface elevation time-series at each grid point is available at the end of the numerical simulation. Contour plot of the discrete Fourier coefficients for specific wave periods are used to identify areas where amplification occurs. In the coastal region, the natural period of bathymetric features can be, in principle, identified as peaks in the frequency spectrum.

The tsunami sea surface history near the coast is highly non-stationary making it particularly suitable for wavelet analysis (Torrence and Compo, 1998). Continuous Wavelet transform provide a time-scale representation of the signal. When time averaged, the global wavelet transforms is similar to the Fourier transform. However, as observed by Hudgins et al. (1993), the time averaged (global) wavelet power spectrum would look like a smoothed Fourier power spectrum losing details especially in the higher frequencies. The Fast Fourier Transform (FFT) algorithm is therefore more appropriate.
than the time-averaged Wavelet Transform for the calculation of the frequency spectra at
the grid points.

Before performing the Fourier transform, a few alterations were made to the raw data. To obtain better frequency resolution, the time-series was extended with zeros until the total number of points was the next integer power of 2. Since few harmonics would fit exactly into the length of the time-series, a Hanning window was also applied to the data to reduce the effect of a finite length time-series (Emery and Thompson, 2001). After FFT, the absolute values of the Fourier coefficients were taken. They were then re-scaled to account for the loss of energy when windowing and multiplied by 2 to account for a one-sided spectrum. The results give the discrete spectral amplitude in the same units as the input data (cm). The square of the amplitude is the energy spectrum (cm²) in terms of angular frequency and the energy density (cm²/s) is obtained when dividing by the frequency resolution (1/Δt).

For the contour plots at specific frequencies, the amplitude frequency spectra were choosen rather than the energy frequency spectra because the coefficients can be correlated to the surface elevation. The contour plots showing the total energy for all combined frequencies were made by integrating the energy frequency spectra so high energy components can be readily identified.

To help visualize the oscillation pattern (standing edge wave, bay resonance, etc) responsible for the amplification at a given a period, the sea surface elevation was bandpassed and animated in time. A wavelet filter was used to extract the modulation in time of a certain frequency or a narrow range of frequencies. The advantage of this method over a regular filter such as a Butterworth filter is that noise is removed at all frequencies and a much smoother contour plot is obtained (Torrence and Compo, 1998). The wavelet software is available at http://paos.colorado.edu/research/wavelets/.
3.1 Seismic data

In tsunami modeling, the rupture area of the earthquake is usually approximated as a single or several planar faults. The parameters, which describe the fault geometry, orientation, and direction of movement, are referred to as the source parameters. It is also common practice in tsunami modeling to assume that the initial tsunami wave is identical to the vertical component of seabed deformation. The shape of the modeled initial tsunami wave is therefore a function of the earthquake source parameters and its amplitude is proportional to the slip (displacement) of the rupture.

The magnitude, location, and orientation of the rupture plane of the 2006 Kuril earthquake were obtained from the Harvard Centroid Moment Tensor (HCMT) catalog (http://www.globalcmt.org/CMTsearch.html). The length, width, and slip of the rupture area were adapted from the results of a finite inverse algorithm of teleseismic body waveforms available from the USGS Earthquake Center (http://earthquake.usgs.gov/eqcenter/eqinthenews/2006/usvcam/finite_fault.php). Figure 2 shows the total rupture area (400 km x 137.5 km) in 220 subfaults of 20 by 12.5 km. The inverse algorithm of the seismic data provides the displacement vector (magnitude and direction) at each subfault over the entire rupture area. The magnitude of the earthquake calculated with the inversion is $M_w = 7.66$ which is smaller than the published value of $M_w = 8.3$ by the HCMT.

Sánchez and Cheung (2007) concluded that for far field tsunami modeling, the slip distribution inside the rupture area have little effect on the shape of the leading waves. However, the role of the fault asperities on the frequency spectrum of the tsunami wave train remains unknown. Considering that the frequency spectra of the tsunami in the Hawai‘i waters is expected to be dominated by the local bathymetry, the slip distribution
inside the rupture area was set to a constant value consistent with the total energy release by the earthquake measured by HCMT. The average source parameters are summarized in Table 2 and are illustrated in figure 3. These parameters are applied to the planar fault model of Okada (1985) to generate the seafloor deformation and the initial tsunami wave.

Table 2: Source parameters for tsunami modeling.

<table>
<thead>
<tr>
<th>Strike</th>
<th>Location of ref. point</th>
</tr>
</thead>
<tbody>
<tr>
<td>214° from North</td>
<td>47.575°N</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dip</th>
<th>Location of ref. point</th>
</tr>
</thead>
<tbody>
<tr>
<td>15°</td>
<td>155.69°E</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Length:</th>
<th>Depth of ref. point:</th>
</tr>
</thead>
<tbody>
<tr>
<td>320 km</td>
<td>6.6 km</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Width:</th>
<th>Moment Magnitude Mw</th>
</tr>
</thead>
<tbody>
<tr>
<td>137.5 km</td>
<td>8.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Slip:</th>
<th>Rigidty:</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 m</td>
<td>40 GPa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rake:</th>
<th>Seismic Moment (Mo)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>3.52x10^{21} N·m</td>
</tr>
</tbody>
</table>

3.2 Sea level measurement and observation

Historically tsunami data has been collected from tide stations located in most of the harbors worldwide. Earlier devices consist of a float in a "stilling" well, which damped tsunami signals by filtering out the higher frequency components (Cross, 1968). Newer devices, which consist of acoustic or bubbler systems, provide highly accurate readings. In the United States, all the tide stations are part of the National Water Level Observation Network (NWLO) managed by the National Oceanic and Atmospheric Administration (NOAA)/National Ocean Services (NOS). Water level data at any tide station (6 minutes interval) can be downloaded via the web site (http://tidesonline.nos.noaa.gov/).

In Hawai‘i, the 2006 Kuril tsunami was recorded at 1 minute intervals at Hilo (Big Island), Kahului (Maui), Nawiliwili (Kaua‘i), Honolulu (O‘ahu) and Moku o Lo‘e (O‘ahu) tide stations. The data was made available through the NOAA Pacific Marine Environmental Laboratory (PMEL) (http://nctr.pmel.noaa.gov/kuril20061115_data.html). Tide stations provide abundant data, but the tsunami signal is affected by the near-shore bathymetry in the harbor vicinity. Before the installation of deep ocean buoys in August 2000, tsunami signal had never been recorded in the deep ocean. The DART (Deep-ocean
Assessment and Reporting of Tsunami) buoys were developed for early detection and
detection of tsunamis by NOAA PMEL. Four DART buoys provide good deep ocean
measurements of the Kuril 2006 tsunami (figure 1). The data was recorded at 15 second
intervals and was downloaded from the NOAA website:

In addition to the five tide stations in Hawai‘i waters and the four deep ocean DARTs,
the Kuril 2006 tsunami was also recorded by an upward-looking ADCP deployed at the
Kilo Nalu Near-Shore Reef Observatory, which is a permanent cabled coastal monitoring
station located off Kaka‘ako Waterfront Park on the south-shore of O‘ahu (Hebert et al.,
2007). The ADCP installed at 12 meters depth on a coral reef provides sea level
measurement every second. The recorded tsunami data is used to validate the computed
spectra in Hawai‘i waters.

3.3 Comparison between modeled and measured water-levels

The validity of this study rests on the quality of the tsunami frequency content
reproduced by the numerical model. It is important to identify the frequency band in
which the agreement between computed and observed tsunami spectra is satisfactory. In
preparation for the spectral analysis, all tsunami observations were detrended and high-
passed at 120 minutes using a Butterworth digital filter to remove tide. Frequency
spectra for observed and computed wave forms were calculated using Fast Fourier
Transform after a Hanning window was applied to the data. Proper normalization was
then applied to the Fourier coefficients to account for the lost of energy when the window
is applied as well as two-sided spectrum (Emery and Thomson, 2001).

Comparisons between the computed and recorded data are shown in figure 4. The sea
surface elevation is shown on the left column and amplitude frequency spectra in the
right. The first four sets of plots show the comparison at the DART buoys off the Alaska
coast (DARTs 21414, 46408, 46413) and the one west of Big Island (DART 51407). The
agreement between the observed and computed surface elevation degrades quickly after the first few waves as it is often the case in tsunami modeling. However, the amplitude and the period of the modeled first waves show good agreement with observations. It is interesting to notice that the observed data with sufficiently long durations shows a broad array of frequencies. Some peaks are consistent from one location to another, such as those around 15 and 30 minutes, which are probably associated with the fault geometry. Computed and observed spectra show the same general features but the energy seems to be distributed differently among the peaks. At all DART locations the computed spectra deviate from the observations at periods larger than 60 minutes.

The DART buoys located near the Aleutian Islands (DARTs 21414, 46408, 46413) are in the first level grid with 2 arc-minute resolution. Small bathymetric features such as smaller islands are not well resolved at this grid resolution. Non-linear wave interactions may also not be resolved properly. Small bathymetric features, non-linear effects and tsunami dispersion may play an important role in altering the frequency content of the tsunami. In some cases (DART 21414 and 51407), the length of the available observed data were very short, and may contribute to the difference observed between computed and recorded spectra. DART 51407 is located on the west side of Big Island, Hawai‘i, more than 5,000 km away from the source. Both the observed and calculated tsunami spectra show reduced higher energy components as well as an organization of the energy in fewer frequency bands compared to other DART buoys.

Figure 4 shows the comparison of measured and computed tsunami waveforms at 5 gauge stations. Each of those locations was modeled at high resolution (0.3 arc-second). The modeled and recorded data show good agreement in the time and frequency domains at all locations except for Nawiliwili. The source of the discrepancies at Nawiliwili is unknown but similar results were obtained by NOAA PMEL (http://nctr.pmel.noaa.gov/kuril20061115.html). The principal peaks as well as the distribution of the energy in the observed spectra were reproduced by the numerical
model with the exception of the 15-min peak at Hilo and Kahului. Kahului and Hilo are harbors protected by breakwaters which are difficult to represent in a bathymetry grid of 10-meter resolution. Comparison of the computed and observed tsunami spectra at Kilo Nalu show very good agreement at period longer than 10 minutes. All the observed peaks were successfully rendered by the model as well as their relative energy distribution. However it seems like the model had more difficulties to resolve higher frequency components. Possible sources of error in the spectra include, the source parameters of the earthquake, bathymetry resolution, complex non-linear wave-wave interactions, seabed friction and tsunami dispersion. However, the comparison of the computed and recorded spectra gives enough confidence in the model results to study oscillation of period of 10 to 60 minutes in the Hawai'i region.
CHAPTER 4
RESULTS AND DISCUSSION

The frequency content and the duration of the excitation are important factors that contribute to wave resonance within coastal features. This chapter describes the propagation of the 2006 Kuril tsunami across the Pacific Ocean. Spectral analysis of model data is then used to describe the oscillation patterns of the waves trapped in the coastal region near Hawai‘i.

4.1 Initial deformation and trans-oceanic propagation

The initial sea surface deformation produces a pulse that propagates as the leading wave across the ocean. However, tsunami observations in the deep ocean clearly show that the tsunami signal is not composed of one pulse, but rather consists of a broad frequency spectrum as shown in Figure 4. In the coastal region, a tsunami containing a large range of frequency components is likely to excite different modes of the oscillation at the coastal features such as a shelf or a bay that may lead to large amplifications and coastal hazards.

The dominant frequencies of the initial tsunami waves are related to the geometry of the submarine fault. Longer periods are observed in the direction of the strike angle and shorter periods normal to the fault (Abe, 2006). Spectral analysis of DART buoys measurements during the Kuril 2006 tsunami reveal two dominant periods around 15 and 30 minutes. The shape of the leading wave is asymmetric and is shaped more like a solitary wave than a sine wave. As a result, sine and cosine wave of many frequencies are necessary to reconstruct the leading wave and may explain why the tsunami spectrum is broad-banded. The fault area is located under the steep eastern flank of the Kuril trench at about 200 km from 20 islands. When the tsunami was generated, multiple reflections...
occurred from the Kuril Islands producing significant scattering of the initial tsunami wave.

During the propagation, the tsunami waves remain undisturbed by bathymetric features that are small compared to the wavelength of the tsunami (typically 100 to 500 km). However numerous islands in the Pacific Ocean are large enough to disturb the wave pattern. The scattering initiated by the diffraction of the wave is likely responsible for the late tsunami wave arrivals at distant coast as suggested by Kowalik (2007), Hebert (2001) and Rabinovich and Thomson (2007). Scattering will also contribute to the spreading of the tsunami energy in different directions across the Pacific basin. Mofjeld (2001) concluded that the Hawaiian Islands are an important source of scattering in the Pacific. A good example of redirection of tsunami energy due to scattering is illustrated in figure 5. When the 2006 Kuril tsunami propagates along Hawaiian Islands chains, the wave train bent as it propagates over the shallow guyots and atolls. The sea floor on the south side of the island chain presents more irregularities than on the northern shore, slowing and refracting the waves. The bending of the wave train will cause a second delayed arrival of the tsunami. As shown in figure 5, the first waves hit the Hawaiian shores from the north-west and the second arrival approached the shore from the west.

The frequency content of the initial tsunami wave can also be modified during the propagation of the tsunami in shallow waters. Sub-harmonics (lower frequencies) and super-harmonics (higher frequencies) are observed in tsunami spectrum probably due to nonlinear wave-wave interactions.

4.2 Oscillations around the Hawaiian Islands

Integration of the power spectrum over all the frequencies gives the total energy of the tsunami. When evaluated at all the grid points, the contour plot of energy serves to identify amplification and resonance of the tsunami. Figure 6 shows the energy distribution in Hawai‘i waters of the 2006 Kuril tsunami. Localities with strong
amplification and the corresponding peak periods are also indicated in the figure. Most of those areas consist of large open bays, where the tsunami triggered the fundamental mode of oscillation. A high energy area is the shallow Penguin Bank south-west of Moloka'i, where a combination of the first and second modes of oscillation of an open basin is evident. The amplification at these locations is considerable comparing to the low energy level in the open ocean north and south of the island chain.

To further investigate the resonance, contour plots of the amplitude spectra for various frequencies were inspected. The periods of maximum amplification were compared with results of spectral analysis made by other authors when available. A good study area for bay oscillations is Hilo Bay on the Big Island. This study estimates that the first mode of oscillation has a period of approximately 28 minutes. Loomis in 1970 published a report in which he examines the oscillation mode of several bays and harbors in Hawai'i by solving the linear shallow water-wave equation as an eigenvalue problem. He found that Hilo Bay has a natural mode of oscillation of 26.16 minutes. Houston (1977) investigated the transformation of tsunamis with the Hawaiian Islands and using spectral analysis of previous tsunamis, found that the fundamental period of oscillation of Hilo Bay is 28.9 minutes. The three different approaches produce similar results.

Figure 7 presents contour plots of the amplitude of the Fourier coefficients at periods of 17, 27, 35 and 43 minutes. As the period and therefore the wavelength of the tsunami increase, the amplification occurs over larger bathymetric features. The regional distribution of the tsunami energy is mostly coastal-trapped, concentrated in large bays, channels, and narrow shelf. The light blue and white pattern in the background indicates small amplitude standing waves generated by reflection of the tsunami off the islands shelf. Little energy is visible below 17 minutes mainly because the bathymetric features where the amplification occurs are too small to be represented on a regional scale. Coastal amplification between the islands of Maui County, on the north shore of O'ahu and on the north-east coast of Big Island can be seen at a period of 17 minute. The nodal
line (line of no energy) visible on the south-west flank of Maui seems to suggest that a standing wave developed in the channel separating Maui from Lāna'i and Kaho'olawe. The amplification on the north shore of O'ahu as further discussed in the next section is mainly due to large scale edge waves and standing waves. A standing wave pattern is visible over Penguin Bank with an anti-node developed at the Moloka'i shoreline and nodes along open banks.

Penguin Bank is a shallow, nearly flat plateau lying in approximately 55 meters of water. The bank, which extends south-west approximately 43.5 km from the west shore of Moloka'i, is surrounded by steep flanks. The computed standing waves over the bank exhibit at least 2 different modes of oscillation at 17 and 27 minutes. Penguin Bank may be approximated as an open-ended basin where the Moloka'i coast is the solid boundary to the east and the bank edge is the open boundary to the west. As stated in the Coastal Engineering Manual (2006), the general equation of natural periods of oscillation in open-basin is given by:

\[ T_n = \frac{4L}{(1+2n)\sqrt{gh}} \]  \hspace{1cm} (1)

where \( n \) is the number of nodes without counting the one at the open boundary, \( L \) is the length of the basin, \( g \) is the gravitational acceleration, and \( h \) is the water depth. A closer look at the position of the node indicates that the effective length over which the oscillation occurs is approximately 30 km. Using an average water depth of 55 meters, the natural oscillation periods of the bank are 17 minutes for \( n = 2 \) and 29 minutes for \( n = 1 \) which is close to the 17 and 27 minutes observed in this study. The surface elevation of the simulation was bandpassed around 17 and 27 minutes and animated in time. Figures 8 and 9 present snapshot in time of the bandpassed signal. The \( n=1 \) and \( n=2 \) oscillation over Penguin Bank can be clearly visualized with an antinode at the Moloka'i coast and a node at the west boundary.
The energy pattern between Moloka‘i and O‘ahu, seems to indicate that the oscillation over Penguin Bank may also affect the south shore of O‘ahu (figures 7-9). A nodal line oriented north-south connecting the western tip of Penguin Bank to the south-east shore of O‘ahu is visible at a period of 27 minutes. Nodal lines are associated with strong horizontal currents which may be the reason why unusually strong currents were observed at Hanauma Bay for two days following the tsunami. The contour plot of the amplitude of the 27 minute period also highlights the amplification along the Maui and Big Island coasts. The embayment of Hilo, Kahului, Mā‘alaea and Kohala (figure 6) have approximate widths of 6 to 10 km (measured offshore) and depths from 15 to 50 meters. Using Eq. (1) with an average water depth of 30 meters, the range of periods to trigger the fundamental mode of oscillation is between 23 to 39 minutes. These periods are typically in the frequency range of tsunamis. However, only Hilo bay is known historically for high runup and strong oscillations during tsunamis. For the 2006 Kuril tsunami, the tide stations at Kahului and Hilo respectively recorded large wave amplitude of 72 cm and 49 cm respectively as a result of bay resonance.

Another interesting feature is the high energy bands between the islands of Maui County visible on the 27 and 35 minutes amplitude contour plots (figure 7). When the tsunami surface elevation was bandpassed around 27 minutes and animated in time, large coastal trapped waves traveling on the shelf of West-Maui are visible (figures 9 and 10). The incident tsunami waves are initially coming from the north-west. As the tsunami wave refracts on the coast of Moloka‘i the waves enter the channel between West-Maui and Moloka‘i. Approximately one half hour after the first waves reached Hawai‘i, a delayed arrival approached the island chain from the west (figure 5). The waves from the west propagate between the 3 islands west of Maui and move toward the Maui shoreline. Because the waves refract when entering the shallow water of Moloka‘i, Lāna‘i and Kaho‘olawe, the wave crest becomes perpendicular to the coast of West-Maui and becomes trapped to the shelf. The fact that the trapped wave are rotating along the coast
with a maximum amplitude at the Maui coast line seems to indicate that the trapped waves are edge waves. The presence of nodes (figure 7) indicates that the initially traveling edges waves become mostly standing being reflected and further amplified from the large bays of Kahului and Mā'alaea on the north and south shore of Maui. Edge waves and surrounding oscillations, such as standing waves between Moloka'i and Lāna'i, bay oscillations and incident waves, interact in a complex fashion making the edge waves traveling on the West-Maui shelf rotating sometimes counterclockwise and sometimes clockwise during the simulation depending of the direction and strength of the incident waves.

The 35 minute amplitude contour plot shown in figure 7 illustrates the transition between the coastal trapped waves and the large scale regional resonance. At this wave period and greater, only the largest coastal features such as Kahului (Maui) and Kohala (Big Island) still resonate. The oscillation originating at Kohala extends to the east shore of Maui with an anti-node to the south-west. The waves that are amplified in Kahului Bay are redirected westward due to refraction and seem to supply energy to the oscillation modes on the southern side of the island. No nodes or antinodes remain visible over Penguin Bank. Instead, a uniform distribution of the energy, lowering and rising at unison is visible for a range of period going from 34 to 55 minutes.

At the 42 minute period, the wavelength of the tsunami is too long for the wave to be trapped on the shelf (figure 7). The amplitude contour plot shows standing waves over the Hawaiian Ridge. The surface elevation was band passed between 40 and 44 minutes and animated in time to investigate the oscillation pattern. Figure 11 shows snap shots of the surface elevation at 8:00 hours after the generation of the tsunami. The results show a larger system of standing waves with nodes at Kaua'i, O'ahu, East-Maui, and the Big Island and a smaller system with nodes at O'ahu, Moloka'i, West-Maui, and East-Maui. The two systems are coupled with the same oscillation period and give rise to the highest amplitude over Penguin Bank.
4.3 Oscillations around O'ahu

This section investigates the oscillation patterns at the island scale. The Island of O'ahu was chosen because of its irregular coastlines which are likely to trap and amplify tsunami waves and also because it is the most populated of the Hawaiian Islands. Figure 12 presents the contour plot of the spectral energy around O'ahu. Most of the energy is concentrated on North-Shore, Kahana Bay, Bellows Airfield-Waimanalo, and Pearl Harbor channel. The amplification is significant in comparison to the low energy level just offshore of the island. Figure 13 shows the contour plots of the amplitude of the Fourier coefficients of the significant modes of oscillation at 10, 15, 18 and 24 minutes period. Figure 14a shows the time-distance diagrams of the energy bandpassed around these periods to infer where the edge waves are originated and which directions they propagate. The time-distance diagram is a 2D plot commonly used in geophysics and oceanography to analyze wave celerity in a particular medium. Each horizontal row represents a sea surface elevation time series, which is color-coded according to its amplitude. The vertical axis indicates the location in km along the 10 meters contour counter-clockwise from the Makapu‘u, which is the eastern-most point of O‘ahu. Kilometer markers are reference in figure 14b. Vertical patterns represent fast traveling waves that hit a large section of the coast at the same time such as in the cases of progressive tsunami waves. Inclined patterns represent slower traveling waves along the depth contours such as edge waves.

At the 10 minute period, the contours on the north shore of O‘ahu have a semi-circular shape with the amplitude decreasing rapidly seaward to zero (nodal line) and then slowly increase. This pattern is the signature of a standing mode 0 or 1 for edge waves as illustrated in Figures 15 and 16 (Komar, 1998). Edge waves are coastal trapped waves that propagate in the alongshore direction. Their amplitude, which is maximum at the shoreline, decreases exponentially offshore (Munk, 1956; Komar, 1998, etc). The
The celerity of traveling edge waves over a uniform slope angle $\beta$ is given by Ursell (1959) as:

$$C = \frac{gT}{2\pi} \sin[(2\pi + 1)\beta]$$

Two progressive edge waves with the same period and mode traveling in the opposite direction can become standing edge waves. Standing edge waves have well-defined nodes and antinodes along the shore such as those along the north shore of the island. When standing, the edge waves are amplified by resonance and appear clearly in the contour plots as zone of high energy separated by nodes. Traveling edge waves will generally have smaller amplitude and may not clearly be visible in the amplitude contour plot. Figure 17 shows the modeled sea surface elevation bandpassed between 8 and 12 minutes. When this is animated in time, many traveling edge wave are visible all around the island. On the North-Shore, standing edge wave episodes are interrupted by period of traveling edge wave, sometimes traveling clockwise, sometimes counterclockwise.

Figure 14a shows the time-distance diagram of the waves bandpassed between 8 and 12 minutes. Standing edge waves will appear like high-intensity small vertical lines and traveling edge waves are identifiable by incline lines. Even though most of the oscillation consists of traveling edge waves the majority of the energy seems to be attributable to the standing edge waves amplified by resonance.

Bricker et al. (2007) analyzed the sea surface elevation and velocity components of the Kuril 2006 tsunami recorded at Kilo Nalu on the south shore of O'ahu and concluded that edge waves traveling both clockwise and counterclockwise were generated by the tsunami. Analysis of the diagram indicates that the first edge waves might be generated at the northern most tip of the island at Kahuku Point (km 180 in figures 14a and 14b). From this region, two sets of edge waves propagate in the opposite directions as indicated by the "<" pattern and are then partly reflected ">' pattern and dissipated by nearby bays. About 30 minutes later, edge waves traveling eastward and southward are generated...
at Ka‘ena Point (km 125). Other less energetic edge waves were generated at the Mōkapu peninsula (km 240) and Ko‘Olina area (km 80). An interesting observation is that edge waves of 10 minutes seem to be able to travel long distances and around headlands as sharp as Ka‘ena Point. From the time-distance diagram, the velocity of a 10 minute edge wave near Kahuku is approximately 32 m/s. Using equation 2 for a edge wave traveling over an average beach slope of 0.01, the velocity calculate is 9 m/s for a mode 0 edge wave and 28 m/s for a mode 1 edge wave indicating that the incline lines in the time-distance diagram are produce by mode 1 traveling edge waves.

At the wave period of 15 minutes, the energy is concentrated in fewer areas. The North-Shore coast has the same width as 3 edge waves of 15 minute periods resulting in resonance and amplification. The modeled sea surface elevation at each grid point was bandpassed between 14 and 16 minutes to investigate the propagation of the edge waves (figure 18). The animation and the time-distance diagram show that the edge waves might be generated at the northern tip of the island at Kahuku Point (km 180) as well as in the Moku‘ula area (km 135). The edge waves generated then travel along the North-Shore coastline until they are reflected or redirected toward the ocean at the two headlands. Fewer edge waves are visible in other areas of the island, mainly because concave coastlines such as the North-Shore are more effective in trapping edge waves (Munk, 1956). The time-distance diagram shows a sharp discontinuity in wave amplitude at 125 km indicating that the edge waves at this period are not capable of crossing Ka‘ena point. The same observation is also made for periods larger than 15 minutes and may also explain why the energy is concentrated in fewer areas around the island. It seems those larger edge waves are less capable of crossing headland and are more likely radiated back to the ocean. This is due to the relation between the shelf width over which the edge waves travel and their wavelength. The shelf on the west side of the island is narrow which may explain why no edge waves are visible on that side of the Island.
The 18 minutes period amplitude contour plot represents in addition to large traveling edge waves, small standing waves in the Hale‘iwa area (km 150). The strong vertical pattern in the time-distance diagram bandpassed between 17 and 19 minutes (figure 14a and figure 19) indicates that the Hale‘iwa area is very energetic. Kahuku Point (180 km) continues to generate edge waves traveling in both directions. The two systems are coupled and edge waves traveling toward Hale‘iwa (km 150) seem to energize the standing waves. The west shore of O‘ahu show very little wave activity and here again the energy is concentrated in area were the coastline is concave and/or where the shelf is wide.

The maximum amplification of the north shore of O‘ahu occurs at 24 minutes period. This is probably the natural period of oscillation of the bay-shaped northern shore. The time-distance diagram is dominated by vertical patterns with the exception of Kahuku Point indicating a change in the oscillation regime from predominantly edge waves to standing waves. The surface elevation was bandpassed between 22 and 25 minutes and animated in time (figure 20). At the beginning of the simulation the waves enter the North-Shore from the north-west and are amplified by resonance in the embayment. The west arrival of incident waves seems to generate large edge wave rotating around Kahuku striking the north-east shore. It is not clear if the standing waves in the North-Shore embayment and the large edge waves are couple or independent. In any cases, the system reach an equilibrium and the north and east shore have opposite phase with what seems to be a node at Kahuku Point like a large standing wave spanning on the two shores. Consequently, the east shore of O‘ahu seems to receive a portion of the energy amplified in the North-Shore embayment through this standing wave. This system illustrates complex interaction between the different wave patterns and how energy amplified by resonance can be transferred by the wave trapped on the shelf to other part of the island.
CHAPTER 5
CONCLUSIONS AND RECOMMENDATIONS

Spectral analysis of computed sea surface elevations of the 2006 Kuril tsunami reveals strong-energy oscillation patterns in Hawai‘i waters at the regional and island scales. The computed wave spectra are validated over the frequency range of interest with measured data around the Hawaiian Islands. The spectral energy of the sea surface elevation at each grid point is extracted by the Fast Fourier Transform and contour plots of the amplitude of the Fourier coefficient identify areas of strong amplification. A wavelet filter extracts the free surface time histories around the high energy periods for examination of the oscillation patterns.

On a regional scale, high energy regions are located mostly along the coast with periods less than 35 minutes. Resonance occurs in open bays as well as over Penguin Bank. At 42 minutes, the results show a large standing wave pattern with the islands acting as nodes. The standing waves appear to be forced by the oscillation of the water over Penguin Bank. This underwater bank is very effective in trapping and amplifying waves. The oscillatory flow at its open boundaries may easily excite and force oscillations at adjacent coastlines on O‘ahu and Moloka‘i.

On a smaller scale, the time-distance diagram separates the wave traveling along the coast from the standing and progressive waves. For periods smaller than 10 minutes, oscillation patterns are dominated by edge waves capable of traveling long distance and crossing sharp headlands. Edge waves at longer periods, are not capable of passing Ka‘ena Point and are radiated back to the ocean or reflected eastward. However, edge waves with periods larger than 15 minutes are more energetic and less susceptible to scattering by small coastal features. Coastal trapped waves or edge waves are observed to be efficient in transferring energy and feeding adjacent oscillation patterns.

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The contour plots of Fourier coefficients indicate areas with high tsunami energy. High energy, however, may not necessarily translate into high run-up heights, but rather indicates prolonged oscillation of the coastal waters. Tsunami mitigation efforts have primarily focused on delineation of tsunami inundation and the evacuation of the residents in coastal areas. Few tools exist to evaluate the hazards in coastal waters. This method is a tool to understand strong currents associated with resonance oscillation, which may persist for several days. It could be used to identify dangerous areas, where water related activities and navigation would be restricted in the aftermath of a tsunami.

In addition, the amplitude contour plots identify the natural periods of oscillation of large open bays. Hilo Bay with a natural period of oscillation of 28 minutes and O'ahu North-Shore with a period of 24 minute, have historically responded strongly to tsunamis, perhaps because tsunamis often have energy in those frequency bins. The Kuril 2006 tsunami, however, excited Kahului Bay with the highest energy because a large amount of energy of the initial tsunami was in the 30 minute bin, which is close to the natural period of oscillation of Kahului bay.

This study shows that the tsunami wave response at the shoreline is strongly correlated to the length scale of the underlying bathymetry as well as the period and the direction of the incoming waves. Improvement can be made by using a dispersive numerical model for the propagation of the tsunami. The next step would be to conduct sensitivity test with other (real or hypothetical) tsunamis to understand the effect of wave direction and frequency contents on the resulting oscillation patterns in Hawaii waters. The results indicate that edge waves may be sensitive to the incoming wave direction since their generation is related to refraction. Resonance in open bays is expected to be less sensitive to the azimuth and more sensitive to the frequency content.
Figure 1: Bathymetry grid and computational domains. a) Pacific Ocean bathymetry grid, DART Buoy (△), extend of the second level of grid □; b) Hawaii bathymetry grid, location of tide stations (★), extend of the third level of grid □.
Figure 2: Finite Fault Model from USGS. Distribution of the amplitude and direction of slip in mm. Perimeter of fault area for tsunami modeling.

Figure 3: Definition sketch of a finite rectangular fault model
Figure 4: Comparison of computed (---) and measured (----) wave forms and amplitude spectra at DART buoys, tide stations and Kilo Nalu.
Figure 5: Surface elevation (m) of the computed Kuril 2006 tsunami, 5:00, 6:00 and 7:00 hours after the initial deformation.
Figure 6: Kuril 2006 tsunami Power Spectra (cm²) integrated for all frequencies in the Hawai‘i region. Red arrows indicate areas of high energy with period of maximum amplification.
Figure 7: Amplitude of Fourier Coefficients at the Hawaii Islands at 17-min, 27-min, 35-min, and 42-min.
Figure 8: Amplitude of the surface elevation bandpassed between 16 and 18 minutes in the Hawaii Islands region.
Figure 9: Amplitude of the surface elevation bandpassed between 26 and 28 minutes in the Hawaii Islands region.
Figure 10: Amplitude of the surface elevation bandpassed between 33 and 37 minutes in the Hawaii Islands region.
Figure 11: Amplitude of the surface elevation bandpassed between 40 and 46 minutes in the Hawaii Islands region.
Figure 12: Kuril 2006 tsunami Power Spectra (cm$^2$) integrated for all frequencies around O‘ahu. Red arrows indicate areas of high energy with corresponding period of maximum amplification.
Figure 13: Amplitude of Fourier Coefficients around O’ahu at 10min., 15min., 18 min. and, 24min.
Figure 14a) Time–distance diagram of amplitude of the surface elevation at the 10 meters depth contour bandpassed between 9 and 11 min., 14 and 16 min., 17 and 19 min. and 22 to 25 min. The vertical axis is the distance in km traveling counter clockwise from the eastern-most point of Oahu. The blue line is the theoretical shallow wave velocity in 10 m of water.
Figure 14b) Kilometer markers for the time-distance diagram

Figure 15: Cross-shore variation in the amplitude of the series of edge waves of modes \( n = 0-3 \). (Adapted from Komar 1998, pp.252)

Figure 16: The pattern of standing edge wave of mode \( n = 0 \). (Adapted from Komar 1998, pp.254)
Figure 17: Amplitude of the surface elevation bandpassed between 8 and 12 minutes around O‘ahu.
Figure 18: Amplitude of the surface elevation bandpassed between 14 and 16 minutes around O'ahu.
Figure 19: Amplitude of the surface elevation bandpassed between 17 and 19 minutes around O'ahu
Figure 20: Amplitude of the surface elevation bandpassed between 22 and 25 minutes around O'ahu
REFERENCES


Wessel, P. and Smith W.H.F. (1991), Free software helps map and display data. EOS Transcripts, American Geophysical Union, 72(41), 441.