OPTIMAL SIGNALING FOR MIMO INTERFERENCE NETWORKS

A THESIS SUBMITTED TO THE GRADUATE DIVISION OF THE UNIVERSITY OF HAWAI'I IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

IN

ELECTRICAL ENGINEERING

AUGUST 2006

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Acknowledgements

I am greatly grateful to my advisors, Professor Gürdal Arslan and Professor M. Fatih Demirkol, for their invaluable help, enormous patience and stimulating encouragement throughout my M.S. program at University of Hawai‘i. Their informative suggestions and useful criticism have enabled me to finish this thesis successfully and enthusiastically. It was my great pleasure to work with them. I also want to thank Professor Anders Høst-Madsen for his kind help and valuable advice on my thesis work.

Also, this work is dedicated to my parents who provided me with selfless love and support during my study as well as enduring my long absence from home.
ABSTRACT

In this thesis, we discuss the issue of overall mutual information maximization in MIMO interference networks. The interaction of links is modeled as a multi-link game in which each link maximizes its own objective function. The properties and performance of Nash equilibrium of the game are analyzed. We show that by imposing a limit on the number of independent data streams for each link, the performance of equilibrium can be substantially improved. A decentralized negotiation mechanism is proposed as an alternative to brute-force type of stream control. The problem of optimal power allocation is also considered. An extension of the negotiation mechanism is presented to incorporate the choice of power levels into the algorithm. Both the basic and extended negotiation processes converge to the optimal profile with arbitrarily high probability.
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Chapter 1

Introduction

1.1 Motivation and Overview

What distinguishes wireless communication is the natural propagation medium in which emitted waves journey through various paths due to reflection and scattering effect. The received signal consists of different replicas varying in amplitude, frequency and direction of arrival, which makes signal recovery a challenging task.

Using antenna arrays is one way of alleviating this effect. Due to the large number of multiple paths, it is unlikely that all the signal replicas along different paths would suffer severe fading in the same time interval. In other words, there would be at least one replica that is in good shape and can be picked by the receiver as the candidate for signal recovery. Besides, an antenna array is also capable of combining different path signals to maximize the signal-to-noise ratio (SNR), which greatly enhances the system's performance.

Another reason for utilizing antenna arrays is the dramatically increased demand for capacity in wireless communication systems. With antenna arrays at both transmitter and receiver ends, Multiple-Input-Multiple-Output (MIMO) systems have emerged as one of the most significant techniques, which tremendously improve system capacity [19]. As the natural extension of so-called smart antenna, MIMO systems explore the spatial signature of signals with space-time signal processing in rich scattering environment. Therefore, multi-path propagation, which is usually a pitfall of wireless communication, is effectively taken advantage of by a MIMO system [14]. Currently, MIMO systems have been broadly used in areas such as broadcasting digital TV, IEEE 802.16 (WiMAX), IEEE 802.11 (WLAN) etc. In addition, MIMO-OFDM technique which exploits the orthogonal subcarriers to overcome frequency selective fading, has been developed as the standard for next generation WLAN and cellular networks.
The trend of increasing users and reducing cell sizes in mobile communication has brought the interference, both from co-channel and adjacent channels, into the center of the stage \([4,7,9,16]\). Recently, spatial multiplexing over multiple MIMO links, named *MIMO Interference Network*, has attracted great attention \([2,5,6,10–12,27]\). In such a network, multiple MIMO links, equipped with antenna arrays at both transmitter and receiver ends, transmit data independently while interfering with each other. This topology is particularly applicable to MIMO ad hoc networks where many users coexist and share the same frequency resource.

The capacity region for single-input-single-output interference channel is an open problem for years \([8]\). So is that of MIMO interference network. With the knowledge of channel state information (CSI), beamforming performed at the transmitter of a link not only affects the capacity performance of the particular link, but also impacts the other links by interference. Each individual link's capacity is dependant on all the others. This intrinsic dependency between links is analyzed in \([2,10–12,27]\). And it is shown that maximizing the total mutual information of all links is a non-convex problem in general. However, several algorithms are introduced to maximize the total mutual information with the knowledge of channel information \([2,10,12,27]\).

In \([10]\), an iterative water-filling method for total mutual information maximization is proposed. Each link maximizes its own mutual information based on knowledge of CSI by adjusting the power distribution among transmitting streams. And this is implemented in a distributed and iterative way until it converges to a steady state. It is also shown that at the steady state, the total mutual information is not necessarily maximized \([2,10–12,27]\). A centralized algorithm based on gradient-ascent approach is introduced in \([27]\). This centralized algorithm yields better performance in terms of total mutual information than the algorithm in \([10]\) especially in large interference scenarios \([2,27]\). However, the gradient computation requires all the channel matrices and covariance matrices of all users. The central control unit is needed to compute the gradient and coordinate the gradient-ascent iterations. Therefore, it is computationally prohibitive with poor scalability which is especially intolerable for ad hoc networks where a large amount of users interact autonomously and no central organizer is available.

In \([12]\), a stream control approach is presented to deal with the inefficiency of equilibrium. By imposing limits on the number of independent data streams of each link, the total mutual information of equilibrium is substantially improved \([10]\). The intuition behind this is that, decreasing the number of streams of a specific link may decrease the interference caused to other links, which is likely to improve the total mutual information of the network. Here the degree of free-
dom is considered as a resource which needs to be allocated prudently among all links to achieve a social-welfare performance.

However, in [12], although the equilibrium is obtained by a decentralized algorithm, i.e. iterative water-filling, the limits on the number of data streams are chosen in a brute-force enumeration fashion, which is not suitable for large scale networks. To tackle this problem, we introduce a negotiation mechanism by which the selfish users can communicate with each other and agree on the best set of limits which yields the equilibrium with the highest overall mutual information in Chapter 3.

Other than the degree of freedom, the total power available in the MIMO interference network is another crucial resource whose allocation correspondingly affects the overall mutual information. Intuitively, we would prefer to allocate more power to a link yielding high mutual information performance while causing low interference to the other links. For instance, we would prefer to allocate more power to a link that has high channel gain but low interference gain. We introduce a decentralized algorithm that finds the optimal power allocation profile in Chapter 4.

In summary, this thesis presents a distributed and hierarchical solution to the following problem: in a large scale $L$ user MIMO interference network with a total power $P_{\text{max}}$ available, how should we utilize the power that the overall mutual information is maximized?

The hierarchical structure of the solution is introduced in Chapter 4. At the higher level, $L$ links in the system negotiate with each other to settle at a power allocation profile as well as a set of limits on the number of streams autonomously. This high level process is expected to be more cooperative in terms of improving the system-wide equilibrium with respect to overall mutual information. At the lower level of hierarchy, however, each link selfishly maximizes its own mutual information subject to the allocation of power and the negotiated limit on the number of streams.

The remaining part of this thesis is organized as follows: a background overview and a system model are provided in Chapter 2. Chapter 3 discusses the equilibrium efficiency improvement in MIMO interference networks by a decentralized stream control approach. Joint negotiation approach that finds the optimal power allocation profile and the limits on the number of data streams is presented in Chapter 4. Concluding remarks as well as future research directions are included in Chapter 5.

\footnote{We can also consider the power distribution by \textit{waterfilling} as an optimal resource allocation solution.}
1.2 Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>(x)</td>
<td>Scalar</td>
</tr>
<tr>
<td>(x)</td>
<td>Vector</td>
</tr>
<tr>
<td>(X)</td>
<td>Matrix</td>
</tr>
<tr>
<td>P.S.D.</td>
<td>Positive semidefinite matrices</td>
</tr>
<tr>
<td>(X^\dagger)</td>
<td>Hermitian transpose of matrix (X)</td>
</tr>
<tr>
<td>(\text{tr}(X))</td>
<td>Trace of matrix (X)</td>
</tr>
<tr>
<td>(\text{det}(X))</td>
<td>Determinant of matrix (X)</td>
</tr>
<tr>
<td>(X^{-1})</td>
<td>Inverse of matrix (X)</td>
</tr>
<tr>
<td>(N_t)</td>
<td>Number of antennas at the transmitter</td>
</tr>
<tr>
<td>(N_r)</td>
<td>Number of antennas at the receiver</td>
</tr>
<tr>
<td>(E[\cdot])</td>
<td>Expectation operator</td>
</tr>
<tr>
<td>((x)^+)</td>
<td>Equals to (\max(x,0))</td>
</tr>
<tr>
<td>(I)</td>
<td>Identity matrix</td>
</tr>
<tr>
<td>(I_k)</td>
<td>The mutual information of (k)th link</td>
</tr>
</tbody>
</table>
Chapter 2

Background

2.1 MIMO Communication Systems

2.1.1 Single Link MIMO Systems

Figure 2.1: Single link MIMO system.

Figure 2.1 is an illustration of a single link\(^1\) MIMO system. The transmitter and the receiver are equipped with \(N_t\) and \(N_r\) antennas, respectively. The channel matrix \(H\) between the transmitter and the receiver, whose dimension is \(N_r\) by \(N_t\), is assumed to have independent identically distributed (i.i.d.) complex Gaussian entries with zero mean and independent real and imaginary parts with equal variance. With the assumption of rich scattering environment and law of large numbers, the channel elements have Rayleigh distributed magnitudes and uniformly distributed phases. The transmitted signal \(x\) is a complex vector of dimension \(N_t\) by 1. At each symbol period,

\(^1\)We assume that each MIMO link is associated with a particular user, and therefore we will use the terms “user” and “link” interchangeably.
x is transmitted through the channel represented by $H$. The received signal $y$ is a $N_r$ by 1 complex vector, and related to $x$ by

$$y = Hx + n$$

where $n$ is a complex Gaussian noise vector that is both spatially and temporally white. In other words

$$E[nn^\dagger] = \sigma^2 I$$

where $I$ is the identity matrix and $\sigma^2$ is the power of noise.

We assume a quasi-static and flat fading environment. Let $Q$ be the covariance matrix of $x$, i.e.,

$$Q = E[xx^\dagger],$$

and $P$ be the power allocated to the link, i.e.,

$$P = tr(Q).$$

The capacity of a single link MIMO system, [8], is given as

$$C = \max_Q \log_2 \det(I + \frac{1}{\sigma^2} HQH^\dagger). \quad (2.1.1)$$

For comparison purpose, we abuse the notation slightly and rewrite (2.1.1) as

$$C = \max_Q \log_2 \det(I + \rho HQH^\dagger), \quad (2.1.2)$$

where $\rho = \frac{P g^2}{\sigma^2}$, $P$ is the transmitted power, $g$ is the scalar channel gain, $Q$ is the normalized covariance matrix with $tr(Q) = 1$, and $H$ is the normalized channel matrix which consists of i.i.d. complex Gaussian entries with zero mean and unit variance.

When channel state information is known at the transmitter side, possibly by a feedback channel, the power $P$ can be allocated optimally across different antennas according to waterfilling solution [8] [25] so as to achieve the capacity $C$. A detailed overview of single link MIMO systems can be found in [18], [19] and [15].

### 2.1.2 Multiple Link MIMO Systems

We consider a MIMO interference network with $L$ links and each link is equipped with $N_t$ transmitting antennas and $N_r$ receiving antennas. Any individual user, say user $k$, is transmitting independent data and producing the interference for other $L - 1$ users. Under the assumption that
single user detection technique is applied, the received signal of user $k$, denoted by $y_k$, is a complex vector with dimension of $N_r \times 1$, and is related to all transmitted vectors $x_1, \ldots, x_L$, by

$$y_k = \sqrt{\rho_k} H_{kk} x_k + \sum_{l=1, l \neq k}^{L} \sqrt{\eta_{k,l}} H_{kl} x_l + n_k,$$  \hspace{1cm} (2.1.3)

where

$$\rho_k = \frac{P_{kg_k^2}}{\sigma^2}, \text{ and}$$  \hspace{1cm} (2.1.4)

$$\eta_{k,l} = \frac{P_{g_{k,l}^2}}{\sigma^2}. \hspace{1cm} (2.1.5)$$

which represents a gauge of interference encountered at $k$th user's receiver and originated from $l$th user's transmitter. Since $\rho_k$ and $\eta_{k,l}$ are ratios of signal power to noise power, we will refer to $\rho_k$ and $\eta_{k,l}$ as the SNR and INR levels.

In (2.1.3), the interference-plus-noise of user $k$ is given as

$$z_k = \sum_{l=1, l \neq k}^{L} \sqrt{\eta_{k,l}} H_{kl} x_l + n_k$$  \hspace{1cm} (2.1.6)

Define the covariance matrix of $z_k$ as

$$R_k = \mathbb{E}[z_k z_k^\dagger] = I + \sum_{l=1, l \neq k}^{L} \eta_{k,l} H_{kl} Q_l H_{kl}^\dagger$$  \hspace{1cm} (2.1.7)

where $Q_l = \mathbb{E}[x_l x_l^\dagger]$ and $\text{tr}(Q_l) = 1$.

The mutual information of link $k$ is now given as

$$I_k = \log_2 \det(I + \rho_k H_{kk} Q_k H_{kk}^\dagger R_k^{-1}).$$  \hspace{1cm} (2.1.8)

From (2.1.8), the mutual information of link $k$ depends on the covariance matrix $Q_k$ as well as $R_k$ which corresponds to the interference coming from other $L - 1$ users. This dependency makes the MIMO interference network an interdependent and complex system. As a consequence, maximizing the overall mutual information $\sum_{k=1}^{L} I_k(Q)$ of all $L$ users is a nonconvex problem, where $Q$ represents the profile of covariance matrices of all transmitters, i.e. $Q = [Q_1, \ldots, Q_L]$.

The interaction among users motivates the utilization of noncooperative game theory and the interpretation of the problem from a game theoretical perspective. Maximization of overall mutual information is the ultimate objective and the interaction among users is modeled as an $N$-person concave game, which will be introduced next.
2.2 A Multi-link Game and Nash Equilibrium

Game theory provides a powerful tool to solve multi-agent decision problems, with conflict and/or cooperation.

To model the problem as a noncooperative game, we interpret the MIMO interference network in such a way that each user wants to maximize his own utility function selfishly based on the choices of other users. A reasonable utility function of selfish user $k$, is naturally the mutual information of link $k$. In other words, user $k$ tries to maximize

$$U_k(Q) = I_k(Q)$$  \hspace{1cm} (2.2.1)

by adjusting his covariance matrix $Q_k$ with the measurement of $R_k$, which is uniquely determined by the rest of $L - 1$ users' covariance matrices under the quasi-static environment assumption. Furthermore, all feasible covariance matrices belong to a common convex set $\Phi$, named individual strategy space, which consists of positive semi-definite (PSD) matrices with unit trace, i.e.

$$\Phi = \{S_{N_tN_r} | S \text{ is Hermitian PSD matrix and } \text{tr}(S) = 1\}$$  \hspace{1cm} (2.2.2)

Thus the profile $Q = \{Q_1, \ldots, Q_L\}$ is chosen from the space of actions $\Phi^L$.

While user $k$ is attempting to maximize his own mutual information, all other users are doing the same. If a steady state exists, any link must not be capable of improving his individual mutual information while others are unchanged. This steady state, as a possible outcome of a noncooperative game, is called a Nash equilibrium named after John F. Nash. Let $Q_{-k}$ denote the covariance matrices of links other than $k$. A specific profile of covariance matrices $Q^* = \{Q_1^*, \ldots, Q_L^*\}$ is called a Nash equilibrium\(^2\) if and only if

$$U_k(Q_k^*, Q_{-k}^*) = \max_{Q_k \in \Phi} U_k(Q_k, Q_{-k})$$  \hspace{1cm} (2.2.3)

The structure and properties of equilibrium in multi-link game introduced above is investigated by [2]. Based on the results of [2], we have the following proposition

**Proposition 1.** The multi-link game characterized by the utility function (2.2.1) and the strategy space $\Phi^L$ has an equilibrium solution. Furthermore, such a game has a unique equilibrium whenever (1) $\max_{k,l \in \{1, \ldots, L\}} \eta_{kl} \leq \eta_0$ for a sufficiently small positive $\eta_0$ and (2) $\text{rank}(\rho_k H_{k,k}) = N_t$ for all $k \in \{1, \ldots, L\}$.

\(^2\)We will simply refer to a Nash equilibrium as equilibrium.
Another important question about the multi-link game model is how good the performance of equilibria is in terms of overall mutual information. Not surprisingly, a certain equilibrium may not necessarily yield the social optimal solution. The performance deficiency is due to the fact that all links are trying to maximize their own individual mutual information rather than the total mutual information. The performance gap due to autonomy and selfishness is recognized as the *Price of Anarchy* in [22] in a network routing scenario.

In the next chapter, we will first introduce two decentralized signaling algorithms converging to Nash equilibria of the multi-link game under mild conditions. The performance deficiency of equilibria is discussed in detail and validated by simulations. Substantial overall throughput improvement of equilibria by stream control is introduced afterwards. A further step in equilibrium efficiency improvement by optimal power allocation will be proposed in Chapter 4.
Chapter 3

Equilibrium Performance Improvement
by Stream Control

3.1 Signaling Approaches to MIMO Interference Network

In the multi-link game model, we assume each user has accurate knowledge of $R_k$ matrix and his own channel information $H_{k,k}$ by feedback. Each user is trying to maximize his own link mutual information by adjusting the covariance matrix $Q_k \in \Phi$. We will introduce two iterative approaches of adjusting covariance matrices under the quasi-static environment assumption.

3.1.1 Iterative Waterfilling with Inertia (IWI)

Define the best response of $k$th link to $Q_{-k}$ as

$$BR_k(Q_{-k}) = \arg\max_{Q_k \in \Phi} U_k(Q_k, Q_{-k})$$

(3.1.1)

Given $R_k$, the unique best response covariance matrix is obtained by traditional waterfilling except that the channel matrix $H_{k,k}$ is replaced by the whitened channel $\hat{H}_{k,k} = R^{-\frac{1}{2}}H_{k,k}$ [25] [7] [12]. The IWI algorithm is then given as

1. Initialize the covariance profile $Q(0)$ such that $Q_k(0) \in \Phi$, $\forall k \in 1, \cdots, L$.

2. For all $L$ users, iteratively update covariance matrices as $Q_k(n + 1) = (1 - \alpha(n))Q_k(n) + \alpha(n)BR_k(Q_{-k(n)})$, where $0 \leq \alpha(n) \leq 1$ is the parameter at iteration step $n$ representing $k$th user's willingness to update. In other words, $1 - \alpha(n)$ denotes the inertia of user $k$ at step $n$. 

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The inertia parameter $\alpha(n)$ here is to prevent users from overreacting. Therefore, the cases where the users are trapped in an everlasting oscillations can be avoided by increasing inertia parameter at a proper rate. The special case where $\alpha(n) = 1$ for all $n$ has been discussed in [10]. Therefore, IWI is a generalization of “iterative waterflling” proposed in [10] and [29] with additional inertia parameter in order to avoid nonconvergent behavior.

**Proposition 2.** If the uniqueness conditions of proposition 1 hold, the IWI process converges to the unique equilibrium of the game as long as inertia parameter $\alpha(n)$ satisfies (1) $\lim_{n \to \infty} \alpha(n) = 0$ and (2) $\sum_{n=0}^{\infty} \alpha(n) = +\infty$.

The proof of this proposition is given in [2].

### 3.1.2 Gradient Play

Gradient play algorithm adopts the well-known gradient-ascent approach. Each user is trying to update his covariance matrix in order to maximize his individual link mutual information with gradient-ascent approach, as follows

1. Initialize the covariance profile $Q(0)$ such that $Q_k(0) \in \Phi \forall k \in 1, \ldots, L$

2. All links $k = 1, \ldots, L$, iteratively update covariance matrices as

   $$ Q_k(n + 1) = (1 - \alpha(n))Q_k(n) + \alpha(n)\mathcal{P}[Q_k(n) + \gamma_k(n)\nabla Q_k U_k(Q(n))] $$

   (3.1.2)

   where

   - $\gamma_k(n)$ is the $k$th user's step size along his gradient ascending direction.
   - $\nabla Q_k U_k(Q(n))$ is the gradient of $k$th user's utility function, i.e. $k$th link's individual mutual information, with respect to $Q_k$.
   - $\mathcal{P}$ is the projection of an arbitrary matrix on the convex set $\Phi$.

   The reason for using projection is that $Q_k(n) + \gamma_k(n)\nabla Q_k U_k(Q(n))$ may not necessarily remain in $\Phi$ (PSD matrices with unity trace) and we need to find a covariance matrix in $\Phi$ with the minimum distance from $Q_k(n) + \gamma_k(n)\nabla Q_k U_k(Q(n))$. The detailed implementation of the projection is provided in [2].

   The gradient play approach given above is quite different from the centralized gradient projection algorithm given in [27]. In gradient play, each user's objective function is the self mutual information rather than the overall network throughput. Gradient play needs only local information and greatly reduces the computational burden.
Proposition 3. If the uniqueness conditions in proposition 1 hold, the gradient play converges to the unique equilibrium of the game as long as inertia parameter $\alpha(n)$ satisfies (1) $\lim_{n \to \infty} \alpha(n) = 0$ and (2) $\sum_{n=0}^{\infty} \alpha(n) = +\infty$, and (3) $\lim_{n \to \infty} \gamma_k(n) = \gamma$, for all $k = 1, \cdots, L$, with a sufficiently small positive $\gamma$.

The proof of this proposition is given in [2] with details, and the performance of IWI and gradient play approaches are demonstrated in the following section.

Although both IWI and gradient play processes exhibit consistently convergent behaviors, the equilibrium generally results in inefficient performance in terms of total mutual information due to the autonomy. This is recognized in [12] [27] and [2]. In [20], the authors use a simple example of a two user MIMO interference network to illustrate the inefficiency of equilibrium. To reduce the performance deficiency of decentralized algorithms, a stream control approach has been proposed in [12], which moderates the effect of selfish nature of links by imposing limits on the number of independent data streams that each user can use.

### 3.2 Equilibrium Performance Improvement by Stream Control

The capacity of a single user MIMO link is roughly proportional to the number of independent streams. Accordingly, a selfish user will try to use all the streams available to maximize his mutual information and this is true for all other users. However, utilizing additional stream increases the interference to all users, which in turn may cause a performance degradation from the overall system point of view. Therefore, a mechanism which regulates the number of streams available to users is necessary. We wish to allow more streams to be used by links in “good shape”, where an additional stream yields a net increase in overall mutual information. A brute-force solution is proposed in [12] by imposing a set of limits on the ranks of covariance matrices for all users. However, this makes the strategy space non-convex\(^1\) and the results of [21] is no longer applicable to prove the existence of a Nash equilibrium. To tackle this problem, we will introduce an alternative approach to stream control which preserves the convexity of the feasible solution sets.

\(^1\)For example, matrices $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ are both rank 1, however, their sum $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is rank 2.
3.2.1 Stream Control by Preprocessing

Instead of imposing a rank constraint directly on the covariance matrix $Q_k$, we insert a preprocessing step before the signal vector $x_k$ is generated:

$$x_k = W_k s_k$$  \hspace{1cm} (3.2.1)

where $W_k$ is named the rank selection matrix with dimension $N_t \times r_k$ and $s_k$ is the signal vector with dimension $r_k \times 1$ where $r_k \leq N_t$. Here $r_k$ is the maximum number of streams that user $k$ is allowed to use. Also $W_k W_k^\dagger = I$ is required to maintain that $x_k$ has the same power as $s_k$. We have

$$\text{rank}(Q_k) = \text{rank}(x_k x_k^\dagger) = \text{rank}(W_k s_k s_k^\dagger W_k^\dagger) = \text{rank}(s_k s_k^\dagger) \leq r_k,$$  \hspace{1cm} (3.2.2)

thus the rank selection matrix $W_k$ ensures that the number of independent data streams is at most $r_k$. A conceptual illustration of preprocessing is provided in Figure 3.1.

![Transmitter of user k](image)

Figure 3.1: Conceptual illustration of preprocessing.

By substituting (3.2.1) into (2.1.8), we can obtain

$$I_k = \log_2 \det(I + \rho_k H_{k,k} W_k \Gamma_k W_k^\dagger H_{k,k}^\dagger R_k^{-1})$$  \hspace{1cm} (3.2.3)

where $\Gamma_k$ is the covariance matrix of $s_k$. Therefore, by replacing the channel matrix $H_{k,k}$ with whitened and squashed channel matrix $\widetilde{H}_{k,k} = R_k^{-1/2} H_{k,k} W_k$, the waterfilling solution can still be applied to optimize $I_k$ over $\Gamma_k$ given $R_k$. And since the strategy space that $\Gamma_k$ belongs to is a convex set, proposition 1 can still be used to establish the existence of an equilibrium.
By adding a preprocessing step, the convexity of the strategy space is preserved. The remaining issue is to determine the appropriate \( W_k \) and \( r_k \). Let us focus on the selection of matrix \( W_k \) first.

A heuristic way of choosing rank selection matrix \( W_k \) is proposed in [2]. Assume \( \mathbf{Q}^* \) is an equilibrium without stream control (or the maximum number of streams is set to be \( N_t \)). Let \( \mathbf{Q}_k = \mathbf{U}_k \mathbf{A}_k \mathbf{U}_k^H \) where \( \mathbf{A}_k \) is a diagonal matrix with eigenvalues of \( \mathbf{Q}_k \) and \( \mathbf{U}_k \) is a unitary matrix whose columns are corresponding eigenvectors. We can form the rank selection matrix as

\[
W_k = [u_{k1}^r, \cdots, u_{k r_k}^r]
\]

which consists of the first \( r_k \) columns of \( \mathbf{U}_k \).

Thus, a natural brute-force way of stream control is given as follows

1. Let the \( L \) users reach an equilibrium profile \( \mathbf{Q}^* \) using a decentralized algorithm (such as IWI, gradient-play, etc.) with no stream control applied.

2. For any combination of limits on streams, say \( r = (r_1, r_2, \cdots, r_L) \) and \( 1 \leq r_k \leq N_t \) for all \( k \), choose the rank selection matrices according to (3.2.4) and let the users adjust \( \Gamma_1, \cdots, \Gamma_L \) towards a new equilibrium.

3. Pick a combination profile \( r \) which yields an equilibrium with highest overall mutual information.

Figure 3.2 illustrates the benefit of stream control in terms of average mutual information by simulations. We first generate a simple MIMO interference network with 2 user \((L = 2)\) and each user is equipped with 4 antennas at both transmitter and receiver \((N_t = N_r = 4)\). For simplicity, we assume a symmetric case where \( \rho_k = \rho \) the for all \( k \in \{1, \cdots, L\} \) and \( \eta_{k,l} = \eta \) for all \( k, l \in \{1, \cdots, L\} \). We have investigated all combinations of \((\rho, \eta)\) pairs from \( \rho \in \{0, 5, 10, 15, 20\} \) dB and \( \rho \in \{-5, 0, 5, 10, 15, 20\} \) dB. For each pair, simulations are carried out 1000 times where at each run the channel matrices are randomly generated according to complex Gaussian distribution with zero mean and unit variance. In addition, the per-link mutual information is calculated at each run with four algorithms, which are (1) centralized gradient-ascent algorithm presented in [27], (2) IWI algorithm with inertia parameter \( \alpha(n) = 1/n^{0.2} \) and no stream control is applied, (3) gradient play algorithm with inertia parameter \( \alpha(n) = 1/n^{0.5} \) and step size parameter \( \gamma(n) = 0.01 + 1/n^{0.5} \) for all users with no stream control mechanism and (4) IWI \((\alpha(n) = 1/n^{0.2})\) with stream control by brute-force approach. For fast convergence, we implement the iterative algorithms, both IWI and
gradient play, in an asynchronous fashion. In other words, all users update their covariance matrices in a sequential order. In Figure 3.2, the per-link mutual information averaged over 1000 runs for different \((\rho, \eta)\) pairs is plotted.

As Figure 3.2 shows, centralized algorithm obtains the highest average per-link mutual information in general and it is only provided for performance comparison purposes. As expected, both gradient play and IWI give inferior performance than centralized algorithm because of the inefficiency of equilibria (a.k.a. price of anarchy). However, by stream control, the average per-link mutual information performance of equilibrium attained by IWI is significantly improved, as shown in Figure 3.2. Therefore, the simulation verifies that by imposing appropriate limits on the number of independent streams that users can use, the price of anarchy can be substantially alleviated and the equilibrium of the multi-link game becomes more socially preferable outcome in terms of overall throughput.
In the brute-force type of stream control, exhaustive search is applied over all feasible limit profiles \( r = (r_1, r_2, \cdots, r_L) \). In Figure 3.2, where \( L = 2 \) and \( N_t = N_r = 4 \), 16 combinations are tried. However, the computational burden is extremely high for large scale networks. For example, if the MIMO interference network has 20 users and each of them is equipped with 6 antennas, we have to try \( 6^{20} = 3656158440062976 \) possible combinations of limits to select the best one. The intensive computational demand of brute-force approach motivates a decentralized realization of stream control, which will be introduced next.

### 3.2.2 Stream Control by Decentralized Negotiations

To decentralize the stream control, we hierarchically divide the problem into two subproblems. In the higher level, \( L \) users negotiate with each other and agree on a limit profile \( r^* = (r^*_1, r^*_2, \cdots, r^*_L) \) which yields the highest total mutual information. And, in the lower level, each user maximizes his own mutual information with the limit on the number of streams that he agreed in the higher level, using gradient play, IWI, etc.

We model the higher level subproblem as a finite game among \( L \) users with a common strategy space \( \Omega = \{1, \cdots, N_t\} \). In addition, each user's utility function is set to the overall mutual information of the system instead of his individual mutual information. Therefore, the higher level game belongs to a particular type of games named *identical interest games* where all the players share the same utility function.

The identical interest games belong to a larger class of games named *potential games*. A game with utility functions \( U_1, \cdots, U_L \) is called a potential game if

\[
U_k(x_k', x_{-k}) - U_k(x_k, x_{-k}) = P(x_k', x_{-k}) - P(x_k, x_{-k}) \quad \text{for all } k \text{ and all } x_k', x_k, x_{-k}.
\]

where function \( P \) is called a *potential function* for the game. In identical interest games, a potential function is clearly the common interest, i.e. the overall mutual information in our model.

The potential games have many appealing properties [17] and there are several available learning algorithms which converge to the equilibrium in potential games. One of them is the well known decentralized algorithm named *adaptive play* [28]. Adaptive play enables users to agree on the strategy profile which maximizes their common utility function with arbitrarily high probability. However, the computational complexity of adaptive play is linear with the cardinality of the strategy space. That is to say, at each step of iterations, the updating user \( k \) will evaluate the common utility function \( N_t \) times to select the best value of \( r_k \). In other words, the user has to use IWI or gradient play (or any other low level algorithms) for \( N_t \) times to pick the best value of \( r_k \), which is still a great
computational burden. To alleviate this computational burden, we propose a variant of adaptive play which converges to the best limits combination with arbitrarily high probability:

- Initialize $r(0) \in \Omega^L$ randomly and calculate the total mutual information $I(0)$ with $r(0)$
- At step $n$
  1. Uniformly pick an updating user $m$ from user 1 to user $L$
  2. User $m$ uniformly picks a value $z_m$ from $1$ to $N_t$.
  3. Calculate the total mutual information $\tilde{I}$ with $\tilde{r} = [r_1(n - 1), r_2(n - 1), \ldots, r_{m-1}(n - 1), z_m, r_{m+1}(n - 1), \ldots, r_L(n - 1)]$
  4. Set $r(n) = \tilde{r}$ and $I(n) = \tilde{I}$ with probability $\frac{e^{I(r)}}{e^{I(r)} + e^{I(\tilde{r})}}$ where $\tau > 0$ is a smoothing factor, and set $r(n) = r(n - 1)$ and $I(n) = I(n - 1)$ otherwise.

In this variant of adaptive play, the negotiation process is carried out in a decentralized fashion. Moreover, at each step of negotiations, the updating user will calculate the equilibrium of lower level game only once, instead of $N_t$ times as in the original version of adaptive play. Consequently, the computational burden is substantially reduced. Note that after each updating step, the designated user broadcasts his choice of limit and corresponding total mutual information to other users. However, this information exchange is in scalars thus the extra overhead by broadcasting is negligible compared with the excessive computational burden in the original adaptive play approach.

**Proposition 4.** Let $r(n)$ be the profile at step $n$ of the variant of adaptive play introduced above, then

$$\lim_{\tau \to 0} Pr\{r(\infty) = \underset{r \in \Omega^L}{\text{argmax}} I(r)\} = 1$$

where $I(r)$ is the total mutual information calculated by lower level algorithm with stream constraint $r$ and $Pr$ stands for probability.

**Proof.** We first prove that for a given positive smoothing factor $\tau$, the negotiation process will have a stationary distribution

$$Pr(r^*) = \frac{e^{I(r^*)/\tau}}{\sum_{r \in \Omega^L} e^{I(r)/\tau}}$$

for any state $r^* \in \Omega^L$. 

17
Assume that (3.2.6) is satisfied. Let \( r_1 \) and \( r_2 \) be two arbitrary profiles in \( \Omega^L \). When negotiation process reaches the steady state, we can express the detailed balance condition as

\[
Pr(r_1) \times Pr(r_1 \to r_2) = Pr(r_2) \times Pr(r_2 \to r_1)
\]

(3.2.7)

If profile \( r_1 \) and \( r_2 \) differs in exactly one position, for example

\[
r_1 = [r_1, r_2, \ldots, r_m^a, \ldots, r_L], \quad r_2 = [r_1, r_2, \ldots, r_m^b, \ldots, r_L].
\]

(3.2.8)

Then the transition probability from \( r_1 \) to \( r_2 \) is

\[
Pr(r_1 \to r_2) = \frac{1}{L} \times \frac{1}{N_t} \times \frac{e^{f(r_2)/\tau}}{e^{f(r_1)/\tau} + e^{f(r_2)/\tau}}
\]

(3.2.9)

Thus

\[
\frac{Pr(r_1) \times Pr(r_1 \to r_2)}{Pr(r_2) \times Pr(r_2 \to r_1)} = \frac{e^{f(r_1)/\tau} \times e^{f(r_2)/\tau}}{e^{f(r_2)/\tau} \times e^{f(r_1)/\tau}} = 1
\]

(3.2.10)

\[
\Rightarrow Pr(r_1) \times Pr(r_1 \to r_2) = Pr(r_2) \times Pr(r_2 \to r_1)
\]

(3.2.11)

If profile \( r_1 \) and \( r_2 \) differs in more than one position, \( Pr(r_1 \to r_2) = 0 \) and the balance equations still hold. Therefore, due to the uniqueness of stationary distribution in irreducible and aperiodic Markov chains [3], assumption (3.2.6) is valid for all \( r \in \Omega^L \).

Based on the stationary distribution given in (3.2.6), taking the limit as \( \tau \) approaching 0 will give the equality (3.2.5) in Proposition 4.

\[
\begin{array}{c|c|c}
& \text{Left} & \text{Right} \\
\hline
\text{Up} & 2, 2 & 0, 0 \\
\text{Down} & 0, 0 & 1, 1 \\
\end{array}
\]

Figure 3.3: Illustration of exploration by smoothing factor \( \tau \).

The smoothing factor \( \tau \) takes an important role in the negotiation process. By introducing \( \tau \), we deliberately invite some randomness into the decision step. This can be intuitively explained
on Figure 3.3. If $\tau$ is 0, which means there is no randomness in the decision making process, the users may get trapped at a low efficiency equilibrium (Down, Right). However, with the randomness caused by nonzero $\tau$, they may reach (Down, Left) and get to the high efficiency equilibrium (Up, Left) ultimately. The difference between big $\tau$ and small $\tau$ is exploration versus efficiency. In our simulation, we choose $\tau$ inversely proportional to $n^2$ where $n$ is the number of iteration steps. Hence the negotiation process can explore the majority of $\Omega^L$ when $\tau$ is relatively big and converge when $\tau$ gets sufficiently small as iterations go.

![Figure 3.4](image)

**Figure 3.4:** Average per-link mutual information for $L = 3$ and $N_t = N_r = 4$.

To validate the effectiveness of the stream control negotiations, we compare the average per-link mutual information performance in Figure 3.4. The simulation setting is similar to that in Figure 3.1 except that here we consider a 3 user MIMO interference network with 4 antennas at each transmitter and receiver. The stream control is performed both by negotiation and brute-force approaches where IWI is used for the lower level equilibrium computation.

In Figure 3.4, we observe that the average per-link mutual information generated by the negotiation process coincide with the brute-force approach for almost all the $(\rho, \eta)$ pairs. However, the computational burden of the stream control negotiations is much less compared to that of the brute-force approach.
We also investigate the convergence of the negotiation process. Figure 3.5 plots the evolution of negotiated limits on the number of independent data streams for a MIMO interference network with $L = 5$, $N_t = N_r = 6$, $\rho = 15\text{dB}$, and $\eta = 7.8\text{dB}$.

Figure 3.6 plots the trajectory of total mutual information in correspondence to the negotiations in Figure 3.5. When all links converge to the optimal set of limits, the total mutual information shown in Figure 3.6 meets the performance of stream control by brute-force approach, outperforming IWI with no stream control. Furthermore, this sample negotiation process converges to an agreement on limits within 100 iterations, as shown in Figure 3.5 and 3.6, while brute-force approach requires an exhausting search over all $6^5$ possibilities.

To sum up, by imposing a limit on the maximum number of streams that each user can use, the performance of equilibrium of multi-link game is remarkably improved. Instead of the original brute-force type of stream control, we propose a variant of the well-known adaptive play to enable the autonomous users negotiate with each other and settle down with the best set of limits after convergence. By numerical simulation, we verify the benefits of stream control and the efficiency of this negotiation process. In the next section, we will consider a scenario where a large number
3.2.3 Localized Negotiations

In a large scale MIMO interference network, an individual user may not care about the users far from him since the impact of link parameter adjustment on remote user is relatively slim. This indifference to remote users motivates a localized implementation of stream control negotiations. In other words, each user negotiates with only the users in his neighborhood and takes the neighborhood total mutual information as the utility function to be maximized. The neighborhood of a user can be defined as a circle centered at the user's location with a radius parameter $d$. Note that the users' utility functions may not be identical, hence the high level games may not be identical interest games anymore. However, they still fall into the category of potential games with the over-

Figure 3.6: Evolution of total mutual information during link negotiations for $L = 5$ and $N_t = N_r = 6$ with $p = 15$dB and $\eta = 7.8$dB.
all network mutual information being a potential function. Therefore, the proof of convergence of stream control negotiations still holds for localized implementation of negotiations with any radius parameter $d$.

Since an individual user includes the mutual information of more of the other users as $d$ increases, we expect the overall performance to increase as $d$ increases, but at the expense of more communication between the users.

To validate this reasoning, we set up a simulation scenario as follows:

- There are 5 users in the network and each user is equipped with 4 antennas at both transmitter and receiver.
- All users share the same radius parameter $d$.
- The distance between two arbitrary users $d_{i,j}$ is an integer from one to ten.
- $\eta_{k,j} = 20 \text{dB} / d_{k,j}^2$, where $d_{k,j}$ is the distance between user $k$ and user $j$.
- $\rho_k = 20 \text{dB}$ for all users.

The radius parameter $d$ increases from one to ten and for each value of $d$, the localized negotiations are run a thousand times. The network topology is set and the corresponding distance matrix is shown in Figure 3.7. Figure 3.8 shows the increase of per-link mutual information averaged over 1000 runs for each $d$ as $d$ increases.

By assigning different users distinct utility functions (i.e. neighborhood mutual information), the broadcast overhead is avoided. At each iteration, the IWI or gradient play algorithm is executed only within the neighborhood of a particular user. And, when this user updates, the covariance matrices of those users outside his neighborhood stay fixed as identity matrices. In short, the computational complexity and overhead of negotiation process are substantially reduced by assigning a negotiation radius parameter $d$ to all users, at the cost of inferior total mutual information, as shown in Figure 3.8.

---

2$\eta_{k,j}$ is the distance between transmitting and receiving antennas are negligible compared with users' distances.

3We assume that this topology is feasible.
<table>
<thead>
<tr>
<th>distances</th>
<th>User 1</th>
<th>User 2</th>
<th>User 3</th>
<th>User 4</th>
<th>User 5</th>
</tr>
</thead>
<tbody>
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<td>3</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>User 2</td>
<td>3</td>
<td>0</td>
<td>9</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>User 3</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>User 4</td>
<td>5</td>
<td>1</td>
<td>9</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>User 5</td>
<td>7</td>
<td>8</td>
<td>10</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 3.7: Distance Matrix.
Figure 3.8: Per-link mutual information as a function of the radius parameter $d$. 
Chapter 4

Equilibrium Performance Improvement by Joint Power Adjustment and Stream Control

4.1 Optimal Power Allocation

In the previous chapter, we have discussed the issue of equilibrium performance improvement by a stream control approach. We have shown that by imposing a limit on the number of independent data streams on each user, the equilibrium performance can be substantially improved. We have also introduced a decentralized negotiation mechanism that enables selfish users to settle down on the set of optimal limits with probability 1.

Let us now consider the following problem

*Main Problem:* In a large scale $L$ user MIMO interference network with a total power $P_{\text{max}}$ available, how should we utilize the power in such a way that the overall mutual information is maximized?

In other words, we are interested in solving the following optimization problem:

$$\text{maximize} \quad \sum_{k=1}^{L} (I_k)$$
$$\text{such that} \quad \sum_{k=1}^{L} (P_k) \leq P_{\text{max}}, \ k = 1, \ldots, L$$

where $I_k$ denotes the mutual information of $k$th link at the equilibrium and $P_k$ is the power allocated for link $k$.

$^1$Or noise-normalized power if we assume the unit variance of noise.
As mentioned in the introduction, the main problem is especially practical in a MIMO interference network with a total power constraint. Power is a particularly valuable resource for ad hoc networks. Therefore, the optimal power allocation among users has to be taken into account.

To single out the power variable, we write the mutual information in form of

$$I_k = \log_2 \det(I + P_k g_{k,k}^2 H_{k,k} Q_k H_{k,k}^\dagger R_k^{-1})$$  \hspace{1cm} (4.1.1)

And $R_k$ is defined as

$$R_k = I + \sum_{l=1, l \neq k}^L P_l g_{k,l}^2 H_{k,l} Q_l H_{k,l}^\dagger$$  \hspace{1cm} (4.1.2)

where we normalize the noise power to 1.

Therefore, the overall mutual information is written as

$$I(P) = \sum_{k=1}^L \log_2 \det(I + P_k g_{k,k}^2 H_{k,k} Q_k H_{k,k}^\dagger R_k^{-1})$$  \hspace{1cm} (4.1.3)

where vector $P = [P_1, P_2, \ldots, P_L]$ is the power allocation profile among all $L$ users such that $\sum_{k=1}^L P_k \leq P_{\text{max}}$. And the overall mutual information $I$ is calculated by the lower level algorithms with the portion of power designated for each user.

Let us revisit the numerical simulation in the last chapter where the performance of IWI and gradient play are compared. In that scenario, we plotted the average per-link mutual information for different $(\rho, \eta)$ pairs. Recall that $\rho_k = P_k g_{k,k}^2$, thus we actually adopted a power allocation strategy that is a "constant $\rho$ policy". This means that the total power is allocated inversely proportional to the square of the associated channel gain. For easy comparison purposes, we assume that all the channels have unit gain. Therefore, at the transmitter end, the total power $P_{\text{max}}$ is equally allocated among $L$ users. Equal power allocation is obviously not the optimal power allocation profile since we can transfer some power from a user who is suffering from severe interference, to a particular user with a "good" channel causing less interference to other users in order to improve the total mutual information. Consequently, a decentralized solution for the optimal power allocation problem is imperative.

Optimal power allocation problem has been studied extensively in the CDMA context [26] [23] [24] [13] [1]. The continuous power range from 0 to $P_{\text{max}}$ is quantized into discrete "power levels" for ease of tractability. Inspired by the finite cardinality of power levels and the variant of adaptive play introduced in the previous chapter, we present a new two-level hierarchical negotiation process. The total power $P_{\text{max}}$ is quantized into $M$ discrete power levels, i.e. each user has a power level strategy space $\varphi = \{0, 1, 2, \ldots, M\}$ and any total power allocation profile is
from the strategy space $\Phi^L_M$ which consists of power level profiles $p = \{l_1, l_2, \cdots, l_L\}$ satisfying the constraint $\sum_{k=1}^L l_k \leq M$. At the higher level, users not only negotiate the limits on the number of data streams, but also the power levels that they wish to use under the sum power constraint. With the negotiated power levels and limits on the number of independent data streams, IWI or gradient play algorithm is used at the lower level of the hierarchy to calculate the total mutual information. The procedure of this joint negotiation algorithm, which is similar to the variant of adaptive play presented before, is as follows

- Initialize $r(0) \in \Omega^L$ and $p(0) \in \Phi^L_M$ randomly and calculate the total mutual information $I(0)$ with $r(0)$ and $p(0)$.

- At step $n$
  1. Uniformly pick an updating user $m$ from user 1 to user $L$
  2. User $m$ uniformly picks a value $z_m$ from 1 to $N_t$.
  3. User $m$ uniformly picks a power level $l_m$ such that the new power allocation profile $\hat{p} = [l_1(n-1), l_2(n-1), \cdots, l_{m-1}(n-1), l_m, l_{m+1}(n-1), \cdots, l_L(n-1)] \in \Phi^L_M$.
  4. Calculate the total mutual information $\hat{I}$ with $\hat{r} = [r_1(n-1), r_2(n-1), \cdots, r_{m-1}(n-1), z_m, r_{m+1}(n-1), \cdots, r_L(n-1)]$ and the new power allocation profile $\hat{p}$.
  5. With some positive smoothing factor $\tau$, set $r(n) = \hat{r}$, $p(n) = \hat{p}$ and $I(n) = \hat{I}$ with probability $\frac{e^{r(lr)}}{e^{r(lr)} + e^{r((n-1)l)}}$. Otherwise, set $r(n) = r(n-1)$, $p(n) = p(n-1)$ and $I(n) = I(n-1)$.

Clearly, this negotiation algorithm is merely the extension of the variance of adaptive play introduced in Chapter 3. At the higher level, $L$ users negotiate the limits on the number of independent data streams as well as the power levels. At each negotiation step, instead of a single value of limit on the number of independent streams, the selected updating user chooses a (power level, limit) pair from the joint strategy space $\Phi \times \Omega$.

Let $S$ denote the set of all (power level, limit) pairs, i.e.

$$S = \Phi^L_M \times \Omega^L$$

(4.1.4)

Let

$$s = [(l_1, r_1), (l_2, r_2), \cdots, (l_L, r_L)]$$

(4.1.5)

be a profile in $S$ and $I(s)$ is the total mutual information associated with $s$. We have
Proposition 5. Let \( s(n) \) be the profile in step \( n \) produced by joint negotiation presented above, then

\[
\lim_{\tau \to 0} \Pr\{s(\infty) = \arg\max_{s \in S} I(s)\} = 1 \tag{4.1.6}
\]

Proof. We first prove that for a given positive smoothing factor \( \tau \), the negotiation process will have a stationary distribution

\[
Pr(s^*) = \frac{e^{I(s^*)/\tau}}{\sum_{s \in S} e^{I(s)/\tau}} \tag{4.1.7}
\]

for any state \( s^* \in S \).

Assume (4.1.7) is satisfied. Let \( s_1 \) and \( s_2 \) be two arbitrary profiles in \( S \). When negotiation process reaches the steady state, we can express the detailed balance condition as

\[
Pr(s_1) \times Pr(s_1 \rightarrow s_2) = Pr(s_2) \times Pr(s_2 \rightarrow s_1) \tag{4.1.8}
\]

If profile \( s_1 \) and \( s_2 \) differ in exactly one position, for example

\[
s_1 = [(l_1, r_1), (l_2, r_2), \cdots, (l_m, r_m), \cdots, (l_L, r_L)], s_2 = [(l_1, r_1), (l_2, r_2), \cdots, (l_m^*, r_m), \cdots, (l_L, r_L)].
\]

the maximum power level that user \( m \) can pick is

\[
\max_{\text{level}} = M - \sum_{k \neq m} (l_k) \tag{4.1.9}
\]

Then, the transition probability from \( s_1 \) to \( s_2 \) is

\[
Pr(s_1 \rightarrow s_2) = \frac{1}{L} \times \frac{1}{\max_{\text{level}} + 1} \times \frac{1}{N_L} \times \frac{e^{I(s_2)/\tau}}{e^{I(s_1)/\tau} + e^{I(s_2)/\tau}} \tag{4.1.10}
\]

Thus

\[
\frac{Pr(s_1) \times Pr(s_1 \rightarrow s_2)}{Pr(s_2) \times Pr(s_2 \rightarrow s_1)} = \frac{e^{I(s_1)/\tau} \times e^{I(s_2)/\tau}}{e^{I(s_2)/\tau} \times e^{I(s_1)/\tau}} = 1 \tag{4.1.11}
\]

\[
\Rightarrow Pr(r_1) \times Pr(s_1 \rightarrow s_2) = Pr(s_2) \times Pr(s_2 \rightarrow s_1) \tag{4.1.12}
\]

If profile \( s_1 \) and \( s_2 \) differ in more than one position, \( Pr(s_1 \rightarrow s_2) = 0 \) and the balance equations still hold. Therefore, due to the uniqueness of stationary distribution in irreducible and aperiodic Markov chains [3], (4.1.7) is valid and unique for all feasible \( s \in S \).

Therefore, as in the proof of Proposition 4, taking the limit as \( \tau \) approaching 0 will give the equality (4.1.6). \( \square \)
In Chapter 3, we show that with stream control applied, the equilibrium efficiency in terms of mutual information is improved. It is evident that by optimally allocating the power among $L$ users, the equilibrium efficiency will be further enhanced, which will be verified by numerical simulations in the next section.

**4.2 Simulation Results**

To validate the performance improvement by optimal power allocation, we provide three simulation scenarios with different network parameter settings.

**4.2.1 Scenario 1**

- A MIMO interference network with $L = 2$ and $N_t = N_r = 4$.
- We assume for any user $k \in \{1, \cdots, L\}$, the channel matrix $H_{k,k}$ has unit power gain.
- The quantization level $M$ is 50.
- The smoothing factor is $\tau = 10/n^2$ where $n$ is the iteration number.

Figure 4.1 shows the strategy space of power levels in a 2 user MIMO interference network. Any user has the option to choose from power level 0 to $M$ as long as the sum power constraint $\sum_{k=1}^{L} l_k \leq M$ satisfies. The boundary of achievable regions is marked with circles. The triangular region with "X" is the restricted region that is excluded from the strategy space of updating users in the joint negotiation algorithm. We conjecture that the joint negotiation algorithm will always converge to a power allocation profile that uses all the power available.

With the exception of one, all of the curves in Figure 4.2 are extracted from Figure 3.2 for the case where $\rho = 10$dB. The curve marked by diamonds is generated by the joint power level-stream control negotiation algorithm. Note that by the unity channel gains assumption, the curves other than the one marked by diamonds represent a "equal power allocation" strategy and the total power is $P_{\text{max}} = L \times \rho = 20$, which is divided into 50 power levels in the joint negotiation process. As seen in Figure 4.2, the performance improvement by joint negotiation is not dramatic in this case. This is because that there are only two users in the network, and all the channel gains are unity. However, if there are more users with different channel gains, the quality of the users may differ considerably and as a result the performance improvement due to power level negotiations may be significant.
### Figure 4.1: Strategy space of power levels represented in matrix form.

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<tr>
<td>M-1</td>
<td>O</td>
<td>X</td>
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<td>X</td>
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</table>

4.2.2 Scenario 2

Let us consider the simple network topology shown in Figure 4.3 with heterogeneous channel gains. There are three users in the network and each user has four antennas at both transmitter and receiver. The transmitters are denoted in circle while “X” represents the receivers. The mutual interference channels are marked as dashed lines. Figure 4.4 shows the value of channel gains, and the other simulation parameters are given as

- The quantization level $M$ is 50.
- The smoothing factor is $\tau = 10/n^2$ where $n$ is the iteration number.

As observed in Figure 4.3, user 2 is the bottleneck node in the network since he causes severe interference to the other two users due to the location of his transmitter. Intuitively, forcing user 2 to “turn down the volume” by a proper amount can improve the overall performance of the network substantially. To substantiate this, we first allocate each user 100 unit of power equally and
use IWI for the lower level adjustments.\textsuperscript{2} The mutual information for user 1, 2 and 3 is obtained as \{8.9506, 4.0109, 6.6830\} bps/Hz, respectively, with total mutual information being 19.6446 bps/Hz. Then we deliberately suppress the power usage of user 2 and transfer the power to the other two users. For illustration purposes, we arbitrarily allocate the power as \{126, 6, 168\} (the total power is still 300) and IWI is applied thereafter. The corresponding mutual information for user 1, 2 and 3 are \{12.0470, 0.7879, 13.2367\}, respectively, with total mutual information being 26.0715 bps/Hz. Hence a 33\% of performance improvement is attained by transferring the power of user 2 to other users.

4.2.3 Scenario 3

In this part, we consider a more complicated MIMO interference network where

\textsuperscript{2}For simplicity, we do not apply stream control here.
Figure 4.3: Network topology for scenario 2.

- $L = 5$ and $N_t = N_r = 4$.
- The channel gain of each channel is uniformly generated between 0 and 1.
- The quantization level $M$ is 50.
- The total available power in the network are $\{50, 100, 150, 200, 250, 300\}$.
- The smoothing factor is $\tau = 10/n^3$ where $n$ is the iteration number.

The simulations are run 1000 times and the average per-link mutual information is compared. In each run, we first apply the "equal power policy" and allocate each user the same share of power. Then, the total power is allocated by the joint negotiation algorithm and the per-link mutual information is calculated.

Figure 4.5 demonstrates the benefit of joint negotiation algorithm. Equal power policy is used to obtain the two curves marked by 'O's and 'X's. The curve with triangles denotes the average per-link mutual information obtained by joint negotiation process, which yields superior performance in all settings of maximum power.

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3The unit of power is irrelevant thus omitted here.
Figure 4.4: Values of channel gains in Figure 4.3.

<table>
<thead>
<tr>
<th>gains</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.9</td>
<td>0.6</td>
<td>0.9</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>0.2</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Figure 4.6 and 4.7 show the evolution of the negotiated power levels and limits on the number of data streams for all 5 users, in a sample run of the simulation. In each iteration, one of the users is selected as the updating user who picks a power level and a value of limit randomly and decides whether or not to update according to the probability determined by the smoothing factor $\tau$. The summed power level is plotted in black on the top of Figure 4.6. When $\tau$ gets sufficiently small, all users come to an agreement on a profile of power levels, and limits and the overall mutual information remains unchanged.

4.3 Solution to the Main Problem

We have shown that by optimally allocating the available power, the performance of equilibrium in terms of overall mutual information can be substantially increased. Combined with the performance enhancement by stream control mechanism, we propose a systematic solution to the main problem introduced in Chapter 1, which has a two-level hierarchical structure, as follows

- **Higher level**
  - Quantize the total power $P_{\text{max}}$ into discrete $M$ power levels.
  - Let the users jointly negotiate the power levels and limits on independent data streams.
Figure 4.5: Average per-link mutual information with different power allocation policies.

- **Lower level**
  
  Given the power levels and the limits agreed in higher level, let each user maximize his individual mutual information using IWI or gradient play until an equilibrium is reached.

The hierarchical solution is illustrated in Figure 4.8.
Figure 4.6: Evolution of the negotiated power levels for 5 users in a sample run of simulation.
Figure 4.7: Evolution of the limits on the number of data streams for 5 users in a sample run of simulation.
Figure 4.8: Illustration of two-level solution to the main problem.
Chapter 5

Conclusion

While MIMO technique has been broadly applied in many areas of wireless communication, the capacity of MIMO interference network is still an open problem. The existence of interference among users makes the problem challenging. Beamforming done at one link's transmitter affects the mutual information of its own as well as other link's mutual information through interference channels. The total mutual information of the network is dependent on all the channel matrices and signal covariance matrices of all users. If there is no interference, the problem is uncoupled and the socially optimal solution is obtained as the sum of individual optimal solutions, which are obtained by waterfilling approach. The performance gap between the socially optimal solution and equilibrium solutions owing to the mutual interference is called the "Price of Anarchy". Thus, alleviating this performance gap motivates the research of this thesis. And, the large number of coexisting users requires distributed algorithms.

We model the interactions between selfish users as a multi-link game. The steady state of interactions is interpreted as a Nash equilibrium of the game. In Chapter 3, we show that by imposing a limit on the maximum number of independent data streams that each user is allowed to use, the performance of equilibrium with respect to the overall mutual information is significantly improved. Although decreasing the number of streams diminishes the mutual information of a particular link, the corresponding interference is reduced at the same time, hence the system's performance is likely to be improved. While the original stream control is implemented in a brute-force fashion, we propose a decentralized negotiation algorithm which enables autonomous users to negotiate the limits that they are willing to impose on themselves. It is proven that the negotiation process will converge to the optimal set of limits with arbitrarily high probability, which is also validated by numerical simulations in Chapter 3.
Chapter 4 discusses the issue of optimal power allocation. Given a total power for the network, different power allocation policies result in distinct equilibrium performance. Inspired by the quantization of power and the negotiation algorithm for stream control, we introduce a joint negotiation approach to choose the power levels as well as the limits on the number of streams. The joint negotiation algorithm converges to the optimal power allocation profile and set of limits on the number of streams simultaneously with arbitrarily high probability. We also demonstrate the superiority of optimal power allocation over "equal power allocation" policy by extensive simulations.

In this thesis, we used a heuristic approach to choose the rank selection matrices $W_k$. Finding the "optimal" rank selection matrices remains as an open problem.
Bibliography


