THREE ESSAYS ON INVESTMENT, SAVING, AND THE CURRENT ACCOUNT

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ABSTRACT

The purpose of this dissertation is to answer two major questions. First, can we provide additional evidence pertaining the validity and usefulness of the intertemporal model of the current account? And second, if we say yes to the first question, how will this model perform in the empirical studies related to macroeconomic factors, such as investment and saving, that are closely related to the current account?

The first essay tests the intertemporal model of the current account in a large country framework. Compared to the standard small country model commonly used in the literature, our model shows that only country-specific component of net income will affect the current account, and it generates smoother current account series, implying a stronger connection between current consumption and current net income. Subsequent comparative empirical studies of small- and large-country models, done in the framework of a present value model, show that the large country framework out-performs the traditional small country framework when the object of interest is indeed a large country.

The second essay targets primarily the relationship between degree of persistence of terms of trade (TOT) shock and the current account. Our major contribution is to study this topic in a decomposition framework. To get a measure of the degree of persistence, we use two decomposition techniques, Beveridge-Nelson decomposition and HP filter, to decompose TOT into permanent and transitory components. Empirical results show that the persistence of TOT shocks does play an important role with respect to how the current account respond to such shocks.

The third essay investigates the relationship between investment and saving, or the Feldstein-Horioka (FH) puzzle, by applying regular panel estimation models to time series of investment and saving that are decomposed using a similar approach.
as in the second essay. Our empirical results confirm the findings of the earlier work done using different econometric frameworks that short-run correlation is generally much smaller than long-run correlation. Dependent on particular choice of decomposition technique and panel model, we find some evidence that capital is very mobile for OECD countries 1960-2004.
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Essay 1

Present Value Model of the Current Account in A General Equilibrium Framework

1.1 Introduction

1.1.1 Background

Since the early paper of Sachs (1981), the intertemporal approach has become a popular and standard tool in studying the current account. It provides both microfoundation and dynamics to the analysis. Intertemporal approach to the current account is an extension of the permanent income hypothesis (PIH) in the consumption theory to an open economy framework. First introduced by Milton Friedman, PIH states that a person will consume a constant proportion of permanent, instead of current, income. The theory suggests that consumers will smooth their consumption spending based on the estimation of their permanent income, and they only
change their consumption profile when there exists a permanent shock to the real income. In an open economy, the current account reflects the consequences of the consumers' intertemporal consumption-smoothing behavior. The current account deterioration implies that the consumers' current income cannot meet their current consumption need and therefore they borrow from foreign countries, whereas the current account improvement suggests that the consumers are lending to the rest of the world.

With simplifying assumptions, intertemporal approach yields an easily tractable and analytical expression for the current account that can be conveniently tested in a framework of present value model.\(^1\) Mathematically, the current account \(CA\) can be expressed as the sum of all future expected net income (total output minus government expenditure and investment) changes \(\Delta NO\), discounted at their present values:

\[
CA_t = -E_t \sum_{s=1}^{\infty} \frac{1}{(1+r)^s} \Delta NO_{t+s}.
\]

The model predicts that the current account will deteriorate (improve) when consumers expect their future net incomes to increase (decrease).

### 1.1.2 Literature review

The present value model has been tested extensively in the literature. Earlier works include Sheffrin and Woo (1990), Otto (1992), and Ghosh (1995). Because the model is based on the assumption of a small open economy (SOE), the world interest rate is exogenously given and assumed to be constant. Sheffrin and Woo (1990) test the present value model of the current account using annual data for four small industrial economies.

\(^{1}\)For a good survey of the intertemporal approach to the current account, please refer to Obstfeld and Rogoff (1995).
countries (Belgium, Canada, Denmark, and UK) covering the period 1955-1985. Different statistical testing methodologies provide mixed results for each country, but Canada and UK are the most problematic, which gives momentum to various later modifications of the basic intertemporal model so as to provide better fit. Using data for both Canada and the US, Otto (1992) is the first one to test the present value model with large country data. Surprisingly, the model fails for both countries, and does not perform better for Canada, which should have better statistical results because the testable equations are designed based upon SOE model and thus should fit a small country, like Canada, better than a large one, like the US. In searching for the possible reason for the statistical rejection, the author modifies the model so that households get utility from government expenditure, but this alternative assumption does not improve model performance. Ghosh (1995) finds that the SOE model is rejected when using data from Japan, UK, Canada, and Germany, but not for the case of the US. And for all rejections, the model predicted current accounts are less variable than actual data, implying that the international capital is mobile.

Empirical studies in all three papers provide no statistical support for the validity of the present value model of the current account (PVMCA) using data from major industrial countries. The cross-equation restrictions that the standard PVMCA imposes on the parameters of the unrestricted vector autoregression (VAR) are statistically rejected. And the model predicted current account is much less volatile than that observed in actual data. Since current account is the difference between net income and consumption, deficient current account volatility implies that the model predicted consumption is too volatile.

To reduced the variability in consumption, a common modelling technique is to include nonseparable preference (Sundaresan 1989). Later works on testing PVMCA
accommodate the assumption of nonseparability in different ways. Iscan (1999) introduces durable and non-traded goods into the basic one-good model and tests it against Canada data. The response of consumption to external shocks is dampened because households need to keep a balanced combination of different goods. Ghosh and Ostry (1997) find that the current account volatility can be increased if households engage in precautionary saving. Gruber (2004) offers a solution to the excess volatility problem by incorporating consumption habits into the standard model. The optimal consumption depends not only on permanent income, but also on past consumption. In response to net income shocks, the household tends to maintain his past consumption level, leading to a smoother consumption and a more volatile current account.

An alternative way to improve the fit of the model is by allowing for variable interest rate. Bergin and Sheffrin (2000) extend the basic present value model by introducing variable interest rate and exchange rate. Using three small country (Australia, Canada and UK) data, they find the modified model improves the fit of the intertemporal model. Kano (2003) shows that a standard SOE real business cycle model augmented with a stochastic world real interest rate can produce observationally equivalent result to the habit formation model in Gruber (2004), and therefore he concludes that the future research on the current account should concentrate on the determinants of the world interest rate rather than on alternative specification of the utility function and preference.

Nason and Rogers (2003) conduct a comprehensive investigation into the possible causes of statistical failure of PVMCA in a SOE real business cycle model. Besides nonseparability and variable interest rate as observed earlier, they also present fiscal shock and international capital immobility as potential explanations. Their simu-
lation results indicate that a model with exogenous interest rate shocks performs best.

Since the interest rate plays an important role in validating the PVMCA, it would be a natural question to ask what happens to the current account dynamics if the interest rate is not exogenously given, but endogenously determined in a two-country large open economy (LOE) framework. In this paper, we show that in a general equilibrium framework the present value equation of the current account is

$$ca_t = - \sum_{i=1}^{\infty} \beta^i E_t (\Delta y_{t+i} - \Delta y^w_{t+i})$$

where $\Delta$ is the first difference operator, $y$ is the log form of domestic net output, $y^w$ is the log form of world net output, and $ca$ is $y$ minus $c$, the log form of domestic consumption.

Obstfeld and Rogoff (1995) obtain a similar equation but in absolute, instead of log, form. Without setting up a general equilibrium model, they make pre-assumption that the domestic net output can be decomposed into two parts, the country-specific and the global parts, and that the global part affects the world interest rate, but not the current account. Whereas in my approach, the world net output is naturally derived as the weighted average of both home and foreign countries, instead of being assumed to exist as a predetermined component of domestic net output in an ad hoc way. That global shocks do not matter for the current account is a conclusion naturally derived from my model.

With Obstfeld and Rogoff (1995) being a predecessor, this paper is not the first one to show that the standard model can be modified to examine effects of global and country-specific shocks on the current account, but it is the first one to present an
empirical study on the performance of this LOE intertemporal model in a traditional present value framework,\(^2\) and make further comparison with the basic SOE model.

One major implication of the intertemporal approach is that global shocks will not have any impact on the current account because all countries with symmetric features respond to such disturbances analogously. With a positive shock, every country will have the same intention to borrow in the international capital market, but no country wants to lend. The validity of such proposition, and therefore of the intertemporal approach, is tested formally in Glick and Rogoff (1995). Basically, they get estimates of country-specific and global technology shocks from Solow residuals of individual countries and regress them against the current account variable. They find that the coefficient on the global shock is insignificant. Several later works, including Iscan (2000), Bussiere, Fratzscher and Muller (2005), and Marquez (2002), follow their approach and find similar empirical results.

The SOE present value model cannot accommodate such prediction because the rest of the world, thus the global shock, is literally missing from the model structure. The empirical estimations are made without excluding global factors from domestic net output variables, and therefore may be subject to errors. The LOE general equilibrium framework, on the other hand, integrates Glick and Rogoff's (1995) insight into a present value model that can be conveniently tested by following the rationale and methodologies in a standard SOE model.

As observed earlier, a statistical finding common to all basic SOE present value model papers is that with small industrial country data, the model predicted current account is much less volatile than the actual data. This brings about the interest

\(^2\)The only exception is a very recent paper by Engle and Rogers (2006), which models the current account, in a two-country framework, as the present value of the country's current and expected future shares of the world net output within a general equilibrium framework. But they did not formally test the validity of the present value model.
in later works to improve the fit of the model by introducing additional features. What the literature fails to notice is that for all empirical tests done with the US, the model predicted current account unanimously shows a larger volatility. In Otto (1992), the model current account volatility is statistically greater than the actual data, implying the rejection of the model. In Ghosh (1995), the model is not rejected with US data, however the predicted current account is mathematically more variable. In Gruber (2004), for both the US and Japan, a second largest economy, the model underpredicts the volatility of actual current account even after habit formation is taken into consideration. Are these findings random or do they indicate a systematic difference between large and small countries?

In this paper, we show that the LOE model would predict a smaller current account volatility as compared to the SOE model, for which there exist two explanations. First, the world interest rate is determined within the system. Therefore it moves when home future net income changes. Household will unsmooth consumption by increasing(decreasing) current consumption relative to future consumption when interest rate decreases(increases), making consumption more volatile than in the case of SOE. Therefore, we would expect to see a drop in the volatility of the current account.

The other reason is that in a two-country model, idiosyncratic shock to home net income will more likely work in a way similar to a global shock which will not have any effect on home current account, as pointed out by Glick and Rogoff (1995). And this is especially true in current global environment when advances in information technology spread quickly throughout the whole world and promote economic growth.\footnote{Based on the same reason, Marquez (2002) tests the model of Glick and Rogoff (1995) using more recent data to study if current information technology advances have, more or less, changed} So we should also expect to have smaller predicted current
account volatility in the home country.

In the next section, we present a simple LOE intertemporal model. In section 3, we talk about empirical testing methodologies. In section 4 and 5, we report detailed model testing results. Section 6 concludes.

1.2 The Model

1.2.1 Two-country model with the same time preference

Before we go into model details, let's first make clear variable definitions used in this paper. For each home country variable $X$, we will have a foreign country counterpart defined as $X^*$. Low case $x$ indicates log format of $X$, i.e. $x = \ln X$. $\bar{X}$ indicates steady state value of $X$, therefore $\bar{x} = \ln \bar{X}$. Unless otherwise specified, all variables will follow the above rules.

This is a two-country one-good world. The home representative agent tries to maximize her life-time utility,

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \ln C_s,$$

subject to an intertemporal budget constraint,

$$B_{t+1} = (1 + r_t)B_t + Y_t - C_t.$$

Here $C_t$ is consumption. $\beta$ is time preference. $Y_t$ stands for net income, that is, GDP minus investment and government expenditure, and is taken exogenous. $B_t$ is home country's net foreign assets. $r_t$ is world real interest rate and determined the way how productivity shocks affect current account dynamics.

8
inside the system. The first order condition of maximizing problem is

\[ C_{t+1} = \beta(1 + r_{t+1})C_t. \]  

(1.1)

By log-linearizing the above equation, we get

\[ \Delta c_{t+1} = r_{t+1} + \ln \beta \]  

(1.2)

where \( \Delta c_{t+1} = \ln C_{t+1} - \ln C_t, r_{t+1} \approx \ln(1 + r_{t+1}) \). In steady state, consumption will be constant, so \( \beta = \frac{1}{1 + \bar{r}} \), and \( \ln \beta \approx -\bar{r} \). Assuming the foreign country has the same time preference as in home country, we can write a similar Euler equation for the foreign country

\[ C_{t+1}^* = \beta(1 + r_{t+1})C_t^*. \]  

(1.3)

The world goods market clears when at any time \( t \) we have\(^4\)

\[ Y_t + Y_t^* = C_t + C_t^*. \]  

(1.4)

Using Eq. (1.1) and (1.3) to re-write the above equation as

\[ Y_{t+1} + Y_{t+1}^* = \beta(1 + r_{t+1})(Y_t + Y_t^*) \]  

(1.5)

---

\(^4\)From Walra's law, both bond market and goods market clear when one of them is cleared. So we only need to consider goods market clearing condition here.

\(^5\)For simplicity, we assume home country and foreign country are of equal size in the sense of population. Adding a constant relative size parameter in the model only changes the results marginally. For example, if the foreign country has a population \( n \) time that of the home country, the Eq. (1.6) will be \( r_{t+1} = (1 - \frac{n}{\beta})A_y + \frac{n}{\beta}A_y^* - \ln \beta \).
and do log-linearization (see appendix A.2.1 for details.)

\[ r_{t+1} = (1 - \frac{1}{\rho}) \Delta y_{t+1} + \frac{1}{\rho} \Delta y^*_t + \ln \beta, \] (1.6)

where \( \Delta y_{t+1} = \ln Y_{t+1} - \ln Y_t \), \( \Delta y^*_t = \ln Y^*_{t+1} - \ln Y^*_t \), and \( \rho = 1 + e^{\theta - \bar{\gamma}} \). This equation tells us that the world real interest rate actually is determined by the weighted average of individual country’s net income growth rate, or the world growth rate. This implication is consistent with basic growth theory in which the real interest rate is determined by population, which is omitted from the current model, and technology growth rate, or general economic growth rate if we combine them together.

To see the dynamics of consumption, we plug Eq.(1.6) into Eq.(1.2)

\[ \Delta c_{t+1} = (1 - \frac{1}{\rho}) \Delta y_{t+1} + \frac{1}{\rho} \Delta y^*_t. \] (1.7)

Standard SOE intertemporal model predicts that households smooth their consumption by solely referring to their expectation of all future incomes, or permanent net income. In a circumstance without influences from income shocks, consumption profile will be rather flat, or following a random walk. In comparison, when we extend to a two-country general equilibrium framework, the consumption growth tracks the world growth rate closely and is proportional to home net income growth. The reason is that consumption decision is made according to the current world interest rate which is endogenously determined.

Empirically, low correlation between current consumption and current income, as predicted by permanent income hypothesis, has been refuted by Campbell and Shiller (1987). Theoretically, standard SOE intertemporal model is modified by
adding both saver- and spender-households to allow for larger connection between current income and consumption (see Bussiere et al. (2005) for model details\(^6\)). We can see from the above equation that modelling the current account in a general equilibrium framework is another way to introduce the larger connection between consumption and net income.

To derive dynamics of the current account, we follow the methodology in Bergin and Sheffrin (2000) by doing a log-linearization of the current account equation and the intertemporal budget constraint. The final result, the detailed derivation of which can be found in appendix A.2.2, is

\[
ct = yt + \sum_{i=1}^{\infty} \beta^i E_t (\Delta yt_{t+i} - \Delta yt_{t+i}^w),
\]

or

\[
ca_t = -\sum_{i=1}^{\infty} \beta^i E_t (\Delta yt_{t+i} - \Delta yt_{t+i}^w) = -\sum_{i=1}^{\infty} \beta^i E_t \Delta yt_{t+i}^c,
\]

where \(ca_t \equiv yt - ct\), \(\Delta yt_{t+i}^w = (1 - \frac{1}{\rho}) \Delta yt + \frac{1}{\rho} \Delta yt^*\), and \(\Delta yt^c = \frac{1}{\rho} (\Delta yt - \Delta yt^*).\)

The equations we derived in appendix A.2.2 are under perfect foresight assumption without uncertainty, which is very unlikely in the real world. Therefore, we simply take a short-cut by adding expectation operator into the right hand side of the equations. These equations may not hold exactly if we had solved the problem under uncertainty from the very beginning. But to make it mathematically simple, we just take this as the true relationship between the expected future net incomes

\(^6\)This framework was originally introduced by Campbell and Mankiw (1989) to test the permanent income hypothesis in consumption theory.
and the current account (consumption), and leave the concerns about errors in our future research. Such a modelling technique has also been taken by Iscan (1999) and Engle and Rogers (2006).

Eq. (1.9) is analogous to the current account equation in a SOE PVMCA, except that we have replaced the change of home net income with the relative, or country-specific component of, change in home net income \((\Delta y_t - \Delta y_t^*)\). Besides, the world real interest rate drops off because it has been solved out. This condition says that what matters for the movement of the current account is the idiosyncratic part of net income change whereas the global part common to both countries does not have effect.

We write down some basic equations for a canonical SOE model and compare them with what we have got from a LOE model. All notations, with a superscript ‘s’ to represent SOE results, are of the same definition as in the previous part of the paper unless otherwise defined. These equations, all in log format for easy comparison, are Euler equation

\[
\Delta c_{t+1}^s = r + \ln \beta,
\]

consumption dynamics

\[
c_t^s = y_t^s + \sum_{i=1}^{\infty} \beta^i E_t(\Delta y_{t+i}^s),
\]

and the current account dynamics

\[
\Delta \alpha_t^s = - \sum_{i=1}^{\infty} \beta^i E_t(\Delta y_{t+i}^s).
\]
Comparative statistics for the SOE model are:

\[
\frac{\partial c_t^*}{\partial y_t^*} = 1 - \beta = \frac{r}{1 + r}, \quad \frac{\partial c_t}{\partial y_t} = \beta = \frac{1}{1 + r}.
\]

The majority of the adjustment of a temporary increase in domestic net income occurs through the channel of an improvement in the current account while current consumption only rises marginally (one for \(\frac{1}{1 + r}\)). It is because the consumption is smoothed intertemporally, so the change in consumption over time does not depend on net income. Therefore the SOE model predicts that consumption should be much less volatile than net income.

The comparative statistics in my model are:

\[
\frac{\partial c_t}{\partial y_t} = 1 - \beta = \frac{1}{\rho} + \frac{1}{\rho} \frac{\partial y_t^*}{\partial y_t}, \quad \frac{\partial c_t^*}{\partial y_t} = \beta = \frac{1}{\rho} - \frac{1}{\rho} \frac{\partial y_t^*}{\partial y_t}.
\]

\(\rho\) is defined as \(1 + e^{\theta - \bar{\theta}}\), so \(\frac{1}{\rho}\) is the share of foreign net income in global net income in steady state, and \(0 < \frac{1}{\rho} < 1\). We can see that the temporary increase in net income will have a larger effect on current consumption \((1 - \beta < 1 - \beta \frac{1}{\rho})\) because consumption dynamics is directly determined by the world interest rate which in turn is endogenously determined by change in net income. Further, if the home net income shock can be taken, to some extent, as a global shock, then we would expect \(\frac{\partial y_t^*}{\partial y_t} > 0\). So the effect of net income on consumption will be even greater by the amount of \(\frac{1}{\rho} \frac{\partial y_t^*}{\partial y_t}\). Putting everything together, in a general equilibrium framework, consumption will be much less smoother than in a standard intertemporal model, and we expect to see a larger consumption volatility together with a smaller current account volatility.

To have a better and clearer picture about how different predictions these two
models will have on consumption volatility, we write down some quantitative measures and graph them. We assume that home and foreign net incomes follow the time path:

\[
\begin{bmatrix}
y_t \\
y_t^*
\end{bmatrix} = 
\begin{bmatrix}
\alpha \\
\alpha^*
\end{bmatrix} + 
\begin{bmatrix}
\gamma & 0 \\
0 & \gamma^*
\end{bmatrix} 
\begin{bmatrix}
y_{t-1} \\
y_{t-1}^*
\end{bmatrix} + 
\begin{bmatrix}
e_t \\
e_t^*
\end{bmatrix}
\]

\((e_t, e_t^*) \sim N(0, \Sigma), \Sigma = 
\begin{bmatrix}
\sigma_e^2 & \tau \sigma_e \sigma_e^* \\
\tau \sigma_e \sigma_e^* & \sigma_e^* 
\end{bmatrix}.
\]

We can easily calculate the variance and covariance of net incomes:

\[
\text{Var}(\Delta y_t) = \frac{2}{1+\gamma} \sigma_e^2, \quad \text{Var}(\Delta y_t^*) = \frac{2}{1+\gamma^*} \sigma_e^*.
\]

\[
\text{Cov}(\Delta y_t, \Delta y_t^*) = \begin{cases} 
\tau \sigma_e \sigma_e^* & \gamma = \gamma^* = 0, \\
2 \tau \sigma_e \sigma_e^* & \gamma = \gamma^* = 1, \\
\frac{2-\gamma}{1-\gamma} \tau \sigma_e \sigma_e^* & 0 < \gamma, \gamma^* < 1.
\end{cases}
\]

By taking a first difference of Eq.(1.8), we can get an expression for home consumption volatility:

\[
\text{Var}(\Delta c_t) = [1 + \frac{\beta(\gamma - 1)}{(1 - \beta \gamma)\rho}]^2 \text{Var}(\Delta y_t) + \left[\frac{\beta(\gamma^* - 1)}{(1 - \beta \gamma^*)\rho}\right]^2 \text{Var}(\Delta y_t^*) \\
-2 \left[1 + \frac{\beta(\gamma - 1)}{(1 - \beta \gamma)\rho}\right] \left[\frac{\beta(\gamma^* - 1)}{(1 - \beta \gamma^*)\rho}\right] \text{Cov}(\Delta y_t, \Delta y_t^*),
\]

which can be re-written as a function of parameters \((\beta, \gamma, \gamma^*, \rho, \sigma_e, \sigma_e^*, \tau)\) if we substitute Eq.(1.11) and Eq.(1.12) into it. Similarly, we can obtain volatility measures
\[ \text{Var}(\Delta c_t^*) \text{ and } \text{Var}(\Delta y_t^*) \] for the SOE model, assuming net income follows a time path with the same parameters \( \alpha, \gamma \) and error term \( \varepsilon_t \).

In Figure 1.1, we show and compare four different cases. The \( x \)-axis is value of \( \gamma \), therefore with \( \gamma = 1 \), net income shock is permanent; \( \gamma = 0 \), transitory; and \( 0 < \gamma < 1 \), the impact of net income shock is somewhere in between.

- **SOE with constant interest rate (\( \Delta c, \text{SOE} \)).** As we can see, except for the case with a permanent shock, consumption is much smoother than net income (\( \Delta y \)), with only marginal oscillation. This is because households smooth their consumption intertemporally, and make their consumption decisions based on permanent, instead of current, net income.

- **Large country with zero foreign net income shock and zero correlation between home and foreign net incomes (\( \Delta c, \text{LOE} \)).** Households make their consumption decision based on the world real interest rate, which is determined by world net income growth rate. Home net income shock will work on consumption decision-making through the channel of interest rate. Compared with SOE model, consumption is much more volatile.

- **Large country with non-zero foreign net income shock, but zero correlation between home and foreign net incomes (\( \Delta c, \text{LOE} \tau = 0 \)).** Compared with the second case, positive foreign net income shock, working on home consumption decision through interest rate, adds volatility to consumption.

- **Large country with non-zero foreign net income shock and non-zero correlation between home and foreign net incomes (\( \Delta c, \text{LOE} \tau = 0.8 \)).** With non-zero correlation between net incomes, one country's net income shock can be taken, to some degree, as a global shock, which shifts each country's consumption.
profile without changing its current account. So consumption becomes even more volatile if home country has a strong economic connection with the rest of the world. And it is easy to see that consumption volatility profile shifts more towards net income line when such correlation becomes larger. Consumption and net income volatility lines coincide when correlation coefficient between home and foreign net incomes is 1.

We can also graph the volatility of the current account under different cases, shown in Figure 1.2. Clearly, the current account is expected to be much smoother for large country than for small country, the reason of which, similar to that stated above, will not be repeated here.

What also noticeable is the relationship between $\rho$ and the volatility of consumption. As we said earlier, $\frac{1}{\rho}$ is the steady state share of foreign net income in total world net income. Home country becomes economically more important as $\rho$ gets larger. Same as correlation coefficient between home and foreign net incomes, a larger $\rho$ also tends to increase the volatility of home consumption. This is because when home country takes a larger share of world economy, its idiosyncratic shock are more likely to be taken as a global one. We show in Figure 1.3 that we expect to see a larger consumption volatility with a larger value of either $\rho$ or $\tau$, the correlation coefficient between home and foreign net incomes.

1.2.2 Two-country model with different time preferences

When we have different time preferences for home and foreign countries, Euler equation and market clearing condition will be different from Eq. (1.3) and Eq. (1.5), as shown below:

$$C^*_t = \beta^*(1 + r_{t+1})C^*_t$$  \hspace{1cm} (1.13)
Figure 1.1: Volatility of consumption and net income

Notes: This figure graphs and compares the volatilities of net income (\(\Delta y\)) and consumption in four different cases: the SOE with constant interest (\(\Delta c, \text{SOE}\)), the LOE with zero foreign net income shock and zero correlation between home and foreign net incomes (\(\Delta c, \text{LOE}\)), the LOE with non-zero foreign net income shock and zero correlation between home and foreign net incomes (\(\Delta c, \text{LOE}\) \(\tau = 0\)), and the LOE with non-zero foreign net income shock and non-zero correlation between home and foreign net incomes (\(\Delta c, \text{LOE}\) \(\tau = 0.8\)). All values are calculated assuming constant time preference \(\beta = 0.96\), or steady state interest rate \(\bar{r} = 4\%\). \(\rho\) is assumed to be 2, or home and foreign each takes half of world total net income. Foreign net income is assumed to take the same time path as home net income, \(\gamma^* = \gamma\), and \(\sigma_e^2 = \sigma_c^2 = 1\).
Figure 1.2: Volatility of the current account and net income

Notes: This figure graphs and compares the volatilities of net income (Δy) and the current account in four different cases: the SOE with constant interest (Δca, SOE), the LOE with zero foreign net income shock and zero correlation between home and foreign net incomes (Δca, LOE), the LOE with non-zero foreign net income shock and zero correlation between home and foreign net incomes (Δca, LOE τ = 0), and the LOE with non-zero foreign net income shock and non-zero correlation between home and foreign net incomes (Δca, LOE τ = 0.8). All values are calculated assuming constant time preference β = 0.96, or steady state interest rate  \( \bar{r} = 4\% \). \( \rho \) is assumed to be 2, or home and foreign each takes half of world total net income. Foreign net income is assumed to take the same time path as home net income, \( \gamma^* = \gamma \), and \( \sigma^2_{\gamma} = \sigma^2_{\epsilon} = 1 \).
Figure 1.3: Volatility of consumption, $\rho$, and correlation between net incomes.

Notes: This figure graphs and compares the volatilities of consumption in a LOE model in three different cases: $\tau = 0$ and $\rho = 1.8$, $\tau = 0$ and $\rho = 4$, $\tau = 0.8$ and $\rho = 1.8$, where $\rho$ is the correlation between domestic and foreign net incomes, and $\frac{1}{\gamma}$ is the share of foreign net income in global net income in steady state. All values are calculated assuming constant time preference $\beta = 0.96$, or steady state interest rate $\bar{r} = 4\%$. 

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\[ Y_{t+1}^w = (1 + r_{t+1})Y_t^w[\beta \chi_t + \beta^*(1 - \chi_t)] \]  

(1.14)

where \( Y_t^w = Y_t + Y_t^* \), and \( \chi_t = \frac{C_t}{Y_t^*} \). We combine Eq.(1.13) and Eq.(1.14) to get dynamics of \( \chi_t \)

\[
\frac{1}{\chi_{t+1}} = 1 \frac{\beta^*}{\beta} + \frac{\beta^*}{\beta} \chi_t
\]

(1.15)

or in the form of \( c_t \) and \( y_t^w \)

\[
\Delta y_{t+1}^w - \Delta c_{t+1} = \ln \left[ 1 + \left(1 - \frac{\beta^*}{\beta}\right)(\frac{C_t}{Y_t^*} - 1) \right].
\]

(1.16)

The sign of the right-hand side of Eq.(1.16) will depend on \( \frac{\beta^*}{\beta} \) because \( (\frac{C_t}{Y_t^*} - 1) \) is always negative. When \( \beta^* = \beta \), we will have \( \Delta c_{t+1} = \Delta y_{t+1}^w \), the same result as in the previous model, that is, the home consumption has the same growth rate as the world net income. When \( \beta^* > \beta \), or foreign country is more impatient, foreign country will tend to consume more than home country, therefore, home country consumption growth is smaller than world economic growth. And the reverse is true when \( \beta^* < \beta \), or home country is relatively more impatient.

In a two-country model with different time preferences, there does not exist a valid steady state because one world interest rate, implied by perfect capital mobility, cannot equate two time preferences. To see this, we can solve for steady state \( \chi_t \) from Eq.(1.15) for \( \beta \neq \beta^* \), \( \chi = 1 \), which implies that no matter what values \( \beta \) and \( \beta^* \) take, home country will consume all the world output, leaving foreign country nothing to consume. From literature we know that to include the effect of heterogenous time preference on the current account in a two-country framework, we can assume that the economy is composed of over-lapping-generation households (Buiter 1981), or that the time preference is endogenously determined (Choi, Mark and Sul 2005),
or that the consumer has precautionary saving motivation (Beauchemin and Daniel 2000). Since our model focuses on testing the present value model and is too simple to include any of these features, we will not consider the effect of heterogenous time preference in the following empirical tests.

1.3 Present value model: testing methodologies

The empirical methodology, introduced by Campbell and Shiller (1987) and used for bond and stock prices testing, has been widely utilized in testing PVMCA. Intuitively, if the current account can be expressed as a present value of all future income changes, then it should have embedded all relevant information to forecast future income changes. Therefore after we run an unrestricted VAR on the current account and net income change and test for equality between forecasted and actual current account time series, we are indeed testing for nonlinear restrictions on the VAR parameters. We will talk about these nonlinear restrictions in details later.

This paper first estimates an unrestricted VAR with the current account $ca_t$ and country-specific net income change variable $\Delta y_t^c$ in the following form:

$$
\begin{bmatrix}
ca_t \\
\Delta y_t^c
\end{bmatrix}
= 
\begin{bmatrix}
a_0 & a_1 \\
b_0 & b_1
\end{bmatrix}
\begin{bmatrix}
ca_{t-1} \\
\Delta y_{t-1}^c
\end{bmatrix}
+ 
\begin{bmatrix}
v_t \\
w_t
\end{bmatrix}
$$

or, in a compact notation, $Z_t = AZ_{t-1} + w_t$. $E_t(Z_{t+1}) = A^tZ_t$. The current account variable $ca_t$ is written in a present value format of variable $\Delta y_t^c$:

$$
ca_t = -\sum_{i=1}^{\infty} \beta^i E_t \Delta y_{t+i}^c.
$$

Define two vectors $g' = (1 \ 0)$ and $h' = (0 \ 1)$, and we will have $ca_t = g'Z_t$, and
\[ \Delta y_{t+1} = h' E_t(Z_{t+1}) = h' A^t Z_t \] So we can re-write the above equation as:

\[
 g' Z_t = - \sum_{i=1}^{\infty} (\beta^i h' A^i Z_t) = -h' \beta A (I - \beta A)^{-1} Z_t .
\] (1.18)

From Eq.(1.18) it follows that \( g' (I - \beta A) = -h' \beta A \), which is a compact form of cross-equation restrictions. If we compare elements in the matrices on both sides of the equation, we can get linear restrictions in terms of the individual parameters:

\[
 (a_0 - b_0) \beta = 1 ,
 a_1 = b_1 .
\] (1.19)

The economic interpretation of these restrictions is that change in consumption in the next period should not be predictable given current period information. This can be seen by imposing the restrictions on the VAR (1.17), which results in the variable \( D_t = ca_t - \Delta y_t - \frac{1}{\beta} ca_{t-1} = -\Delta c_t \). Econometrically, \( D_t \) is just the difference between two residuals \( v_t - u_t \), and thus should be orthogonal to any information available at time \( t - 1 \). Economically, intertemporal model tells that all past information \( \Omega_{t-1} = \{ \Delta y_{t-1-i}, ca_{t-1-i}, i \geq 0 \} \) should not be able to predict ex-ante change in consumption \( \Delta c_t \) since consumption dynamics is the result of household’s intertemporal smoothing decision and is decided by the world net income growth in our model. An easy way to test the restrictions (1.19) is therefore to regress \( D_t \) on all past variables contained in \( \Omega_{t-1} \), and these variables should be jointly insignificant when an F-test is performed.

Another form of restriction, which is mathematically equivalent to \( g' (I - \beta A) = -h' \beta A \), can also be obtained from Eq.(1.18) as \( g' = -h' \beta A (I - \beta A)^{-1} \). Because of the inverse operator, we will only get individual parameters restrictions in a non-
linear expression. Given the coefficient matrix estimated from the VAR, $\hat{A}$, we can easily obtain the estimate $c\hat{a}_t$. Comparing variances and covariance of $c\hat{a}_t$ to those of actual current account series can serve as a preliminary test of the non-linear restriction, and thus of the validity of the theory. To have more accurate and convincing test, we need to obtain a Wald-statistics (using the so-called Delta method\(^7\)) to formally test whether $-h'\beta A(I - \beta A)^{-1} = g' = (1 \ 0)$. This Wald test is a typical and popular tool for present value model testing. Though it may be flawed and problematic as pointed out by Mercereau and Miniane (2004),\(^8\) the convenient advantage of being able to generate forecasted $c\hat{a}_t$ still puts it as the top one approach used in the literature.

Most empirical works done so far use two-variable VAR (i.e. the current account and changes in net income) on small country data to test the present value model. Extension to a three-variable VAR has been used by Bergin and Sheffrin (2000) to accommodate the effect of a varying interest rate and exchange rate, and by Iscan (1999) to include a change in durable consumption term. An even larger VAR with five variables (three additional terms being interest rate, exchange rate, and terms of trade) is adopted by Bouakez and Kano (2005). And to my best knowledge, only four papers have tested the SOE present value model using the US data, Otto (1992) for 1950-1988, Ghosh (1995) for 1960-1988, Ghosh and Ostry (1997) for 1919-1990,

\(^{7}\)In our simple case of $A = \begin{bmatrix} a_0 & a_1 \\ b_0 & b_1 \end{bmatrix}$, we can obtain $K = -h'\beta A(I - \beta A)^{-1} = (-\beta_0(1 - \beta_0)(1 - \beta_1), -\beta_1(1 - \beta_0)(1 - \beta_1)\beta_0b_1)$ and test if $K = g$ using Wald-statistics $= (K - g)'(J V J')^{-1}(K - g)$, where $g = (1 \ 0)$, $V$ is the variance-covariance matrix of the VAR parameters, and $J$ is the Jacobian of $K$.

\(^{8}\)Generally, they point out two problems with Wald-test. One is that because of the inverse operation $(I - \beta A)^{-1}$, any small estimation error will be translated into big difference in formation of vector $K$. The other is that with short sample estimation, $J V J'$ may not be an accurate approximation of the variance-covariance matrix of $K$, thus Wald-statistics may be biased. They propose to use F-test, instead of Wald-test.

In this empirical study, we are interested in comparing the statistical results of SOE and LOE models. Our LOE two-variable VAR is structurally based on and mathematically derived from a general equilibrium model with the simplest modelling features. Eliminating other variables, such as exchange rate or interest rate, from our VAR structure will make it easier when making a direct comparison with a basic SOE model. For LOE VAR, we replace net income change variable, as commonly used in SOE model, with country-specific component of net income change variable. We expect to find a better fit of LOE model than SOE model in the sense of a smaller predicted current account volatility.

1.4 Data and Estimation Results

We use both quarterly and annual data for the US to test model performance. The quarterly data comes from various sources spanning the period from the first quarter of 1960 to the second quarter of 2005 (1960q1-2005q2). Please see appendix A.1 for a detailed description. To construct the series of $y_t$, we use US seasonally adjusted current price net output (GDP minus government expenditure and investment) and further adjust it by GDP deflator (2000=100) before taking its log format. To construct the series of $v_t$, we use the sum of net output of all non-US G7 countries, also adjusted for inflation. Since all non-US net output values are denominated in domestic currencies, we convert them into values in US Dollars by current exchange rates. The series of $c_t$ is constructed as $y_t - c_t$ where $c_t$ is the log format of US real private consumption expenditure. The annual data covers a longer horizon from 1889 to 2003. This is the first time that PVMCA is tested against data spanning a
period of over one hundred years. The longest time span ever used in the literature is Canada’s annual data from 1926 to 1995 by Iscan (1999). Due to unavailability of world GDP estimates data in a long horizon, I choose to use 12 western European countries total GDP, which can be found on Maddison’s website (see appendix A.1 for the address), as a proxy for the world net output in my model, and accordingly replace US real net income with US real GDP. A detailed explanation of the data is in appendix A.1. The steady state interest rate is chosen to be 4% following the majority in the literature. Since the US share of G7 total net output is 45% for the period 1960 to 2005, and its share in the sum of 12 western European countries plus the US averages 46% for annual data, we use \( \rho = 0.55 \) for both SOE and LOE estimations.

As a first step, we check for the stationarity of the current account and net income series. Both augmented Dicky-Fuller (ADF) and Philips-Perron (PP) tests are used to test the null hypothesis that the net output and the current account series have unit roots. To ensure a valid ADF test, we use both the Akaike Information Criterion (AIC) and the Schwarz Bayes Information Criterion (SIC) to determine the optimal lag length included in the dynamic process. Then ADF test is applied with the appropriate lag. PP test allows for serial correlation and heteroscedasticity in the regression residuals. It does not require a strict specification of the lag length, but instead, it makes non-parametric correction for heteroscedasticity and autocorrelation in the calculated statistics.

Consistent with the literature, the net output series are found to be first-order integrated, or \( I(1) \). Therefore, the first difference terms \( \Delta y \) and \( \Delta y^c \) are stationary.\(^9\) The current account is stationary with annual data, but not with quarterly data. We

\(^9\)Since the stationarity of \( \Delta y \) is never a problem in the literature, so we choose not to report the statistical results.
see in panel (a) of Table 1.1 that AIC and SIC choose 1 lag for both quarterly and year data. ADF and PP can easily reject unit root in annual current account series if a time trend is included in the regression, but the rejection cannot be attained for quarterly data no matter with or without time trend.

There are two major problems with ADF test that have been researched much in the related literature. First, in face of near unit root, it has low power. When autocorrelation parameter is persistently large (e.g. 0.95), ADF test tends to reject alternative in favor of the null hypothesis of unit root. Therefore the quarterly US current account may be stationary, but the ADF test has given misleading diagnosis.

We apply an adjusted ADF test of Elliot, Rothenberg and Stock (1996), known as GLS-ADF, which has been shown to be more powerful than the original ADF test (Cheung and Chinn (1997)). The t-statistic for quarterly US current account data is -2.052 with 6 lags, and the critical values for 5% and 10% levels are -2.897 and -2.612 respectively. So with this additional test, we still cannot reject unit root.

The second problem with ADF test, and actually with PP test as well, is that because it has unit root as null hypothesis, unit root is prone to be accepted unless there is very strong evidence against it. Therefore it is desirable to apply an alternative test with stationarity as the null hypothesis. To this purpose, we take advantage of the commonly used KPSS (due to Kwiatkowski, Phillips, Schmidt and Shin (1992)) test. The original KPSS test uses Bartlett kernel to calculate autocovariance, and it also suffers from low power in face of highly autoregressive alternatives. In this paper, we use Quadratic Spectral kernel (QS), rather than the traditional Bartlett kernel because it has been shown in the literature (Hobijn, Franses and Ooms (1998), Gil-Alana (2003)) that the use of an automatic bandwidth selection and QS kernel leads to more accurate results for KPSS statistics.
Table 1.1: Unit Root Tests

<table>
<thead>
<tr>
<th></th>
<th>ADF</th>
<th>PP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Z_t$</td>
<td>p-value</td>
</tr>
<tr>
<td>(a) USA with quarterly and annual data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>USA 1960q1-2005q2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No trend</td>
<td>-0.229</td>
<td>0.936</td>
</tr>
<tr>
<td>With trend</td>
<td>-1.881</td>
<td>0.665</td>
</tr>
<tr>
<td>USA 1889-20053</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No trend</td>
<td>-2.498</td>
<td>0.116</td>
</tr>
<tr>
<td>With trend</td>
<td>-3.747</td>
<td>0.020</td>
</tr>
<tr>
<td>USA 1947q1-1999q4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No trend</td>
<td>-2.566</td>
<td>0.100</td>
</tr>
<tr>
<td>With trend</td>
<td>-3.708</td>
<td>0.022</td>
</tr>
<tr>
<td>(b) G6 countries with quarterly data 1960q1-2005q2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>-4.053</td>
<td>0.001</td>
</tr>
<tr>
<td>With trend</td>
<td>-4.351</td>
<td>0.003</td>
</tr>
<tr>
<td>Canada</td>
<td>-3.515</td>
<td>0.008</td>
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<tr>
<td>With trend</td>
<td>-3.974</td>
<td>0.010</td>
</tr>
<tr>
<td>UK</td>
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<tr>
<td>With trend</td>
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<tr>
<td>With trend</td>
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<tr>
<td>With trend</td>
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<td>0.067</td>
</tr>
<tr>
<td>France</td>
<td>-3.106</td>
<td>0.026</td>
</tr>
<tr>
<td>With trend</td>
<td>-3.096</td>
<td>0.107</td>
</tr>
</tbody>
</table>

Notes: The null hypothesis for Augmented Dicky-Fuller (ADF) and Philips-Perron (PP) tests is unit root. Four lags are chosen for PP test based on Newey-West statistics.
The test statistic is 0.207 with bandwidth selection being 3 lags. The critical values for 5% and 10% levels are 0.146 and 0.119. So for a third time, we reject the current account to be stationary.

Referring to the literature, we find that except for Gruber (2004), which does not present statistical result for the US, the other three aforementioned papers all find supportive evidence for stationarity with US data. To find more empirical results on US data, we turn to the literature on current account sustainability which has utilized unit-root and cointegration tests to examine whether or not the long-run intertemporal budget constraint is met. Testing results from this large literature are mixed depending on the countries, the time period, and the testing procedures considered. In the case of the US, Matsubayashi (2005) uses ADF test and finds that the US current account-GDP ratio is stationary. Shibata and Shintani (1998) cannot find statistically significant evidence supporting the stationarity of the US current account. Both Fattouh (2005) and Clarida, Goretti and Taylor (2005) find that there exists a threshold effect in the US current account-GDP series. Christopoulos and Leon-Ledesma (2004) find evidence that US current account follows a non-linear but stationary behavior.

Stationarity, in my understanding, is a concept that should be taken into consideration in a long horizon. Therefore it is not surprising to find that the current account follows a non-stationary path in a short time span, as shown by many papers testing with international data beyond the US. The current account deficits of the US reached record high twice in the last twenty years, once in the mid-1980s with huge government budget deficit and once in the 2000s as shown in Figure 1.4. So the current account is less likely to be stationary with more recent data, though earlier paper on the US data all find the current account to be stationary. In bottom
Figure 1.4: Time series of the US current account as a percentage of GDP

Data source: The US Department of Commerce, BEA online database.

part of panel (a) in Table 1.1, ADF and PP tests show that the US quarterly current account series for 1947q1-1999q4 is stationary. Because pre-1960 data on the other G6 countries is not available, we will proceed the following empirical testings assuming that the US current account is stationary for the period 1960q1-2005q2.

As the first formal test, we run a linear regression to test if $D_t = ca_t - \Delta y_t - \frac{1}{\beta} ca_{t-1}$ is orthogonal to all past information $\Omega_{t-1}$, hereafter called Orthogonality Test. Specifically, we use F-statistics to test the null hypothesis that $a_i = b_i = 0$ for
all \( i = 1, \ldots, k \) in
\[
D_t = a_0 + \sum_{i=1}^{k} a_i c a_{t-i} + \sum_{i=1}^{k} b_i \Delta y_{t-i}^{\epsilon} + \epsilon_t.
\]

We notice that since \( D_t \) is constructed using \( ca_{t-1} \), the above regression may be biased due to such connection. So we also run the same regression without \( ca_{t-1} \) as a regressor, and call it information set \( \Omega_{t-2} \). The results are in panel (a) of Table 1.2. With quarterly data, F-statistics are significant for both information sets and both models, implying that the \( D_t \) series can be predicted from past information and that the present value model should be rejected. The LOE model generates mathematically smaller F-statistics, but these values are not small enough to guarantee the validity of the model. And it does not seem beneficial to have excluded \( ca_{t-1} \) from the past information set. Similarly, with annual data, the F-statistics for LOE model is constantly smaller than those for SOE model. And in all four cases, the LOE model is accepted while the SOE model is rejected at 5% level.

Second, we run two-variable VAR for both SOE and LOE models. In the SOE model, we use the current account (\( c a_t \)) and home net income (\( \Delta y_t \)). In LOE, we use the current account and country-specific home net income (\( \Delta y_{t}^e \)). By constructing the Wald-statistics as explained above, we test if non-linear cross equation restrictions hold, hereafter called Wald Test. We apply AIC and SIC again to determine the optimal number of lags in VAR. Looking at the \( \chi^2 \) statistics of panel (a) in Table 1.3, we find that two data samples generate similar results. The LOE model constantly produces mathematically smaller \( \chi^2 \) statistics as compared to the SOE model. And the former is accepted while the latter is rejected at 5% significance level. That is to say the \( K \) vector calculated from LOE VAR parameters is not significantly different from vector \( g' = (1 \ 0 \ 0 \ 0 \ 0 \ 0) \).\(^{10}\)

\(^{10}\)The reason why we have six elements, instead of two as shown in earlier section, in \( g' \) vector...
Table 1.2: Orthogonality Tests

<table>
<thead>
<tr>
<th>Country</th>
<th>SOE</th>
<th>LOE</th>
<th>p-value</th>
<th>SOE</th>
<th>LOE</th>
<th>p-value</th>
<th>SOE</th>
<th>LOE</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA 1960q1-2005q2</td>
<td>13.597</td>
<td>0.000</td>
<td>7.123</td>
<td>0.000</td>
<td>15.392</td>
<td>0.000</td>
<td>7.910</td>
<td>0.000</td>
<td>6.801</td>
</tr>
<tr>
<td>USA 1988-2005</td>
<td>2.258</td>
<td>0.043</td>
<td>2.189</td>
<td>0.018</td>
<td>3.333</td>
<td>0.047</td>
<td>2.083</td>
<td>0.021</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>1.615</td>
<td>0.15</td>
<td>1.715</td>
<td>0.075</td>
<td>1.895</td>
<td>0.080</td>
<td>1.59</td>
<td>0.051</td>
<td></td>
</tr>
<tr>
<td>Japan</td>
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<td>0.000</td>
<td>21.420</td>
<td>0.000</td>
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<td>0.000</td>
<td>23.600</td>
<td>0.000</td>
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</tr>
<tr>
<td>Canada</td>
<td>11.657</td>
<td>0.000</td>
<td>9.742</td>
<td>0.000</td>
<td>13.181</td>
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<td>10.570</td>
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<td>1.55</td>
<td>0.132</td>
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<td>0.076</td>
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Notes: Numbers reported are F-statistics to jointly test $a_i = b_i = 0$ for $i = 1, ..., k$ in regression

$D_t = a_0 + \sum_{i=1}^{k} a_i y_{t-i} + \sum_{i=1}^{k} b_i y_{t-i}^2 + \epsilon_t$. We additionally select $k = 3, 6$ in the regression.
Table 1.3: Wald test and ratio test

<table>
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<tr>
<th>Country</th>
<th>Period</th>
<th>VAR lag</th>
<th>$\chi^2$</th>
<th>p-value</th>
<th>$\chi^2_{00}$</th>
<th>p-value</th>
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<tr>
<td>USA 1960q1-2003q2</td>
<td>(a) USA with quarterly and annual data</td>
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<tr>
<td>SOE 3</td>
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<tr>
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<tr>
<td>Japan</td>
<td>(b) G6 countries with quarterly data</td>
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<tr>
<td>USA</td>
<td>(c) Higher interest rate with quarterly data</td>
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<td>Japan 1960-1980</td>
<td>(d) Sub-sample estimation with quarterly data</td>
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</table>

Notes: Wald test reports $\chi^2$-statistics used to test the non-linear restrictions in equation $A'(I - \beta A)^{-1} = (1 \beta)$. Ratio test carries out an F-test to test if volatility of predicted current account equals to that of the actual series.
In the third test, known as the Ratio Test, we use estimated $K$ vector to form a series of forecasted current account and then compare its standard deviation to that of the actual series. To statistically test whether these two standard deviations are equal, we use a two-side F-test. The $\sigma_{\text{es}}/\sigma_{\text{y}}$ column of Table 1.3 shows this ratio while the next p-value column reports upper-tail cumulative distribution associated with this ratio. For both datasets, the LOE model generates much smoother forecasted values than does the SOE model. This is consistent with the theoretical analysis in the last section. The SOE model is rejected with high (0.000) significance level, but the LOE model is accepted. These statistical results give support to our earlier argument that in a general equilibrium model, home household consumption will be less smoothing, implying a smoother current account. To give reader a visual impression, we also graph the actual and predicted current account in Figure 1.5, which shows clearly that the current account volatility for LOE is smaller than that for SOE, and that the SOE model consistently overpredicts the current account balance over entire period.

1.5 Sensitivity Test

1.5.1 Testing non-US G7 countries

So far, we have found very encouraging and promising results for the US. The LOE model did a surprisingly good job by surviving almost every statistical test, except for the Orthogonality Test with quarterly data, whereas in sharp comparison the SOE model failed all tests. A natural question to ask now is how the model will perform if we expand our data selection to other large countries. Japan, and probably is because we have 3 lags for each variable in VAR.
Figure 1.5: Actual and in-sample predicted current account: 1889-2003

Note: This figure graphs actual and in-sample predicted current account using US annual data of 1889-2003.
Germany, can also be considered large countries given their current economic importance. Will the LOE model fit them better, as it does with the US, than the SOE model? In the literature, Ghosh (1995) tests the standard SOE model with Japan and Germany data, and find that the predicted current accounts are significantly less volatile than the actual series, and that the Wald Test fails for both countries. Gruber (2004) finds that the predicted current account is significantly more volatile for Japan, and all other statistical tests reject the model as well. Therefore we expect that the LOE model will fit these two countries better because it produces smoother current account.

It will be equally interesting to run the LOE model against small countries data, just like the way that the SOE model has been tested using large country data. As we observed earlier in the paper, researchers tend to find that SOE model generates smaller than one standard deviation ratios for small countries, such as Canada and UK, we expect the LOE model will not out-perform the SOE model because it predicts less volatile time series.

In the following section, we run both SOE and LOE testings for all the non-US G7 countries using quarterly data from 1960q1-2005q2. The construction of the testable time series, $\alpha$, $y$ and $y^*$, for each country follows exactly the same steps as we did for the US. Since all net output series are found to be integrated of one order, their first difference are stationary. We will only present the results for the current account series. Panel (b) of Table 1.1 shows the unit root testing results using ADF and PP tests. For Japan and Canada, both ADF and PP tests reject unit roots at 1% significance level no matter whether or not we include a time trend in the regression. For UK, Germany and France, both ADF and PP tests reject unit root when no time trend is considered. As for Italy, though ADF cannot reject unit root
root hypothesis with a time trend included, PP test gives very significant statistics. Therefore we conclude that non-US G7 countries have stationary current account series.

Having established that the time series are stable, we proceed to examine whether or not the constructed variable $D_t$ can be linearly predicted given the past information sets $\Omega_{t-1}$ and $\Omega_{t-2}$. To make the testing results comparable, we also choose lag length of three and six as we did in the last section. Results are in panel (b) of Table 1.2. Consistent with all other papers, we do not find supportive evidence for the SOE model with small country data. For Canada, UK, Italy and France, all statistics are so large that the null hypothesis of orthogonality is unanimously rejected at zero percentage level. As for Japan and Germany, they are defined to be large countries and therefore do not even fit the small country assumption, so their rejections are of no surprise. It is very interesting that when we switch to the LOE model, the F-values for each country have dropped to such a great magnitude that now we cannot reject the null for Germany. This may be taken as another sign as to the superiority of the LOE model if we take Germany as a large country.

In the next step, we run VAR for each country and calculate $\chi^2$ statistics to test if the non-linear cross equation restrictions hold, and the results are in panel (b) of Table 1.3. The LOE model generally generates much smaller $\chi^2$ statistics than does the SOE model, but statistically both models are rejected at conventional 5% level for all countries, with one exception that UK has one insignificant LOE statistic. $K$ vector is then calculated from VAR parameters and used to predict the implied current account time series. We calculate the ratio of the standard deviation of the implied current account series to that of the actual values, and test if this standard deviation ratio is one. In the literature, this ratio is always found
to be mathematically larger than one for large country, and less than one for small country. For example, in Gruber (2004), this ratio is 2.016 for the US, 2.331 for Japan, but only 0.584 for Canada, 0.621 for Italy, and 0.197 for France. In Ghosh (1995), it is 1.86 for the US, 0.15 for Germany, 0.48 for Canada, 0.19 for UK, but different from Gruber (2004), it is only 0.48 for Japan. In Otto (1992), the ratio is 1.145 for the US and 0.15 for Canada. The only exception is Sheffrin and Woo (1990) who shows graphically that with Canada data the forecasted current account fluctuates dramatically while the actual series is very stable.

Our SOE results are not consistent with empirical findings of most other papers.\textsuperscript{11} There is no clear pattern that large countries, Japan, Germany or even the US in panel (a), can produce larger values than small countries. In the literature, the SOE model is always rejected for small countries because the standard deviation ratios are too small. But in this paper, the predicted series are too volatile to accept the theoretic model. All standard deviation ratios drop dramatically when we switch to the LOE model, which is exactly what we have expected. But the magnitude is not big enough for us to accept the model, except for Germany where the standard deviation of forecasted current account is found to be insignificantly different from the actual observed value by a two-side $F$-test.

\subsection*{1.5.2 Higher interest rate and sub-sample estimation}

As another robust test, we want to examine if our test results are sensitive to a different interest rate or sub-sample periods. Otto (1992) finds that using interest

\textsuperscript{11} The reason for large standard ratio for small countries may be due to different definition of VAR variables. We use log format while the other researchers have used absolute values. Or it may be caused by data coming from different source or covering different period. We replicated the standard SOE model test against Canada data as in Gruber (2004) because he also used IFS quarterly database. We still got larger than one standard deviation ratio.
rate ranging 2-8% does not make any qualitative difference for model performance. Sheffrin and Woo (1990) find that the estimation with smaller interest rate (0.04) does slightly better than the model with higher interest rate (0.14) in the sense that it produces larger p-value in orthogonality test. In contrast, higher interest rate significantly reduces in-sample forecasted volatility in Iscan (1999). We re-estimate both models using a higher interest rate (0.14) for the US, Japan, and Canada. The results can be found in panel (c) of Table 1.2 and 1.3. For all countries with both models, a higher interest rate or a smaller discount rate does significantly reduce the values of standard deviations ratios so that based on the ratio test, the SOE model is not rejected with the US data, and the LOE model is accepted with Canada data. But other than that, both orthogonality test and Wald test return statistics similar in magnitude to a standard 4% interest rate model, which are large enough to reject both models for all three countries.

After having compared the model performance across countries of different sizes, we can also make comparison for one country across different sub-sample periods. In Figure 1.6, we graph each country's economic share in G7's total real net output volume. The US has been holding around 40-50% of the total value, which surely qualifies it as a large country. Japan's share has increased from only 3% in the 60s to around 20% in the most recent years. Therefore it may be more appropriate to treat Japan as a small country for pre-80s period and a large country for post-80s. The same applies to Germany whose share increases to around 10% starting from 80s. A comparable but adverse pattern is found in Italy and UK. Accordingly the SOE model may fit them better in the later part of the data period. We report testing results for these sub-samples in panel (d) of Table 1.2 and 1.3. In orthogonality test, for Japan and Italy, the LOE model is correctly accepted for large country
sub-sample and rejected for small country sub-sample, whereas the SOE model is rejected for both sub-periods for Japan, and it is incorrectly accepted for large country sub-sample and rejected for small country sub-sample for Italy. For UK, the LOE model is accepted, but the SOE model is rejected, for both large and small country sub-samples. For Germany, the SOE model is correctly accepted for small country sub-sample and rejected for large country sub-sample, while the LOE model is accepted for both periods.

Standard deviation ratio tests reject the SOE model for any sub-sample country combination. But the LOE model is correctly accepted for two large country sub-samples (Japan and Germany). For Wald tests, both SOE and LOE models are
accepted for three out of four corresponding country size sub-samples. While the SOE model has only one mistakenly predicted point with Italy's large country sub-sample, the LOE model is incorrectly accepted for three small country sub-samples (Japan, UK, and Italy).

1.5.3 Quantitative comparison

In order to have a quantitative comparison of two model's performance, we count the number of cases where the model is correctly accepted (true positive), correctly rejected (true negative), incorrectly accepted (false positive or type I error), or incorrectly rejected (false negative or type II error), and then divide these values by the total number of cases to get a percentage representation. If the SOE model is accepted for a small country or sub-sample, we add one point to true positive; if it is rejected for small country or sub-sample, we add one point to false negative; if it is accepted for a large country or sub-sample, we add one point to false positive; and if it is rejected for a large country or sub-sample, we add one point to true negative. The same rules apply to the LOE model. From the results in Table 1.4, we can see that the SOE model is much more likely to be rejected than the LOE model in every testing method. Though the LOE model has a larger true positive ratio, it also has a larger type I error ratio than the SOE model.

In Figure 1.7, we graph the average performance of two models as compared to a pseudo TRUE model. A TRUE model will have 50% for true positive, total positive, true negative, and total negative, but zero for type I and II errors. In five out of six items, the LOE model ratios are closer to a TRUE model; only for true negative, the SOE model has a larger, thus closer, value to a TRUE model.
Figure 1.7: Quantitative comparison of model performances

![Graph showing quantitative comparison of model performances.](image-url)
Table 1.4: Statistical test performance of SOE and LOE models

<table>
<thead>
<tr>
<th></th>
<th>True Positive</th>
<th>True Negative</th>
<th>Type I Error</th>
<th>Type II Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wald test</td>
<td>15.76%</td>
<td>47.37%</td>
<td>5.26%</td>
<td>31.58%</td>
</tr>
<tr>
<td>Ratio test</td>
<td>0.00%</td>
<td>47.37%</td>
<td>5.26%</td>
<td>47.37%</td>
</tr>
<tr>
<td>Orthogonality test</td>
<td>5.26%</td>
<td>47.37%</td>
<td>5.26%</td>
<td>42.11%</td>
</tr>
<tr>
<td>LOE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wald test</td>
<td>26.32%</td>
<td>26.32%</td>
<td>21.05%</td>
<td>26.32%</td>
</tr>
<tr>
<td>Ratio test</td>
<td>21.05%</td>
<td>42.11%</td>
<td>5.26%</td>
<td>31.58%</td>
</tr>
<tr>
<td>Orthogonality test</td>
<td>31.58%</td>
<td>36.84%</td>
<td>10.53%</td>
<td>21.05%</td>
</tr>
</tbody>
</table>

Notes: The calculation is based on all 19 cases we have tested in Table 1.2 and 1.3. True positive is when SOE (LOE) model is correctly accepted by a small (large) country case. True negative is when SOE (LOE) model is correctly rejected by a small (large) country case. Type I error or false positive is when SOE (LOE) model is incorrectly accepted by a large (small) country case. Type II error or false negative is when SOE (LOE) model is incorrectly rejected by a small (large) country case.

1.6 Conclusion

In this paper, we propose the PVMCA in a two-country general equilibrium framework and test it against the G7 countries data. Compared to the standard SOE model, our model shows that only country-specific component of net income will affect the current account, and it generates smoother current account series, implying a stronger connection between current consumption and current net income.

With annual US data spanning over one hundred years, the LOE model is accepted based on all three statistical tests; and with shorter horizon quarterly data, it is not rejected by two tests. For Japan and Germany which are second test candidates being large countries, more promising results are found with sub-sample rather than with entire 45-year sample. The SOE model is rejected for all countries by all three tests, though there are a couple of acceptance using sub-sample data. Replacing a standard 4% world interest rate with a higher one does not have noticeable impact on our statistical results.
This paper, to my best knowledge, is the first one to formally test a closed-form present value equation of the current account in a two-country framework. And it gives us the opportunity to study how current account reacts not only to domestic macroeconomic variables, but also to international factors. Future work may be done to include effects of exchange rate, heterogenous time preference, and other potentially important variables.
Appendix A.1  Data Description

The quarterly data ranges from 1960Q1-2005Q2. For the US, Japan, Canada, UK, and Germany, macro data on GDP, consumption, government expenditure, and investment are from IMF International Financial Statistics's online CD-ROM data. For Italy and France, since some of the data are missing, we use data from OECD. For Italy, data after 1982Q1 is from OECD online database. Data for periods 1960Q1-1969Q4, 1970Q1-1979Q4, and 1980Q1-1981Q4 are from OECD's publication Quarterly National Accounts vol 1960-1971, vol 1980IV, and vol 1985IV respectively. For France, data after 1980Q1 is from OECD online database. Data for periods 1960Q1-1969Q4 and 1970Q1-1979Q4 are from OECD's publication Quarterly National Accounts vol 1960-1971 and vol 1980IV. When online data is available, GDP deflators (2000=100) for all seven countries are obtained directly. When online data is not available, GDP deflators are my own calculation.

The US quarterly data earlier than 1960 is obtained from the US Department of Commerce, BEA online database.

The yearly data for the US for the period 1889-1921 is from One Hundred Years of Economic Statistics by Thelma Liesner (1989, Facts on File, ISBN 0-8160-2344-1), and for the period 1929-2003 is from the US Department of Commerce, BEA website. The yearly GDP data for 12 western European countries (Austria, Belgium, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Sweden, Switzerland, and United Kingdom) is available from Maddison’s personal website (http://www.ggdc.net/maddison/). For a detailed explanation on methodologies used in estimating GDP for a long historical period, please refer to Angus Maddison's book The World Economy: Historical Statistics, OECD, 2003. Original data is in 1990 International Dollar and is converted into year 2000 Dollar on my own
calculation.

Appendix A.2 Mathematical Derivation of equations

A.2.1 Derivation of Eq.(1.6)

We do a log-linearization of $Y_{t+1} + Y_{t+1}^*$

\[
\ln(Y_{t+1} + Y_{t+1}^*) = \ln\left(\frac{Y_{t+1}}{Y_{t+1}^*} + 1\right) + \ln Y_{t+1}^*
\]

\[
= \ln\left(e^{\ln Y_{t+1} - Y_{t+1}^*} + 1\right) + Y_{t+1}^*
\]

\[
\approx \ln(e^{\bar{y} - \bar{y}^*} + 1) + \frac{e^{\bar{y} - \bar{y}^*}}{e^{\bar{y} - \bar{y}^*} + 1}(Y_{t+1} - Y_{t+1}^*) - \frac{e^{\bar{y} - \bar{y}^*}}{e^{\bar{y} - \bar{y}^*} + 1}((\bar{y} - \bar{y}^*) + Y_{t+1}^*)
\]

where $y_t = \ln Y_t$, $y_t^* = \ln Y_t^*$, and $\bar{y}$ and $\bar{y}^*$ are steady state values. Take log format of Eq.(1.5) and use the above result, we get Eq.(1.6)

\[
\ln(Y_{t+1} + Y_{t+1}^*) - \ln(Y_t + Y_t^*) = \ln Y_{t+1} - \ln Y_t
\]

\[
\approx (1 - \frac{1}{\rho})\Delta y_{t+1} + \frac{1}{\rho}\Delta y_{t+1}^* - \ln \beta
\]

where $\rho = 1 + e^{\bar{y} - \bar{y}^*}$.
A.2.2 Derivation of the current account Eq.(1.9)

Following steps in Huang and Lin (1993) and Bergin and Sheffrin (2000), we write the intertemporal budget constraint as

$$\Phi_0 = \Psi_0 + B_0$$

where $\Phi_0 = C_0 + \sum_{t=1}^{\infty} R_t C_t$, $\Psi_0 = Y_0 + \sum_{t=1}^{\infty} R_t Y_t$, and $R_t = \prod_{s=1}^{t-1} \left(1+r_s\right)$.

Define $\phi_0 = ln\Phi_0$, $\psi_0 = ln\Psi_0$, and $b_0 = lnB_0$. We re-write the above equation in its log format and take a first order Taylor approximation of the r.h.s.

$$\phi_0 - \psi_0 = ln(1 + e^{b_0-\psi_0})$$
$$\approx ln(1 + e^{\bar{b}-\bar{\psi}}) + \frac{e^{\bar{b}-\bar{\psi}}}{1 + e^{\bar{b}-\bar{\psi}}}(b_0 - \psi_0 - \bar{b} + \bar{\psi})$$
$$= (1 - \frac{1}{\omega})(b_0 - \psi_0) + \kappa \tag{A-1.1}$$

where $\omega = 1 + e^{\bar{b}-\bar{\psi}}$ and $\kappa = ln\omega - (1 - \frac{1}{\omega})ln(\omega - 1)$.

From the definition of $\Phi_0$, we can obtain the dynamics of $\Phi$ as

$$\dot{\Phi}_0 = R_1(\Phi_0 - C_0)$$

or

$$\frac{\dot{\Phi}_0}{\Phi_0} = (1 + r_1)(1 - \frac{C_0}{\Phi_0}).$$

Following the same steps as we obtain Eq.(A-1.1), we can get

$$\phi_1 - \phi_0 = r_1 + (1 - \frac{1}{\eta})(c_0 - \phi_0) + \delta$$

46
where \( \eta = 1 - e^{\delta - \psi} \) and \( \delta = \ln \eta + (\frac{1}{\eta} - 1)(1 - \eta) \). Next, we re-arrange the above equation to get

\[
\frac{1}{\eta} (c_0 - \phi_0) = r_1 + \Delta c_1 + (c_1 - \phi_1) + \delta,
\]

and we do a forward iteration to get the intertemporal equation

\[
c_0 - \phi_0 = \frac{\delta \eta}{1 - \eta} + \sum_{t=1}^{\infty} \eta^t (r_t - \Delta c_t) \tag{A-1.2}
\]

For \( \Psi_0 = Y_0 + \sum_{t=1}^{\infty} R_t Y_t \), we can get a similar expression as in equation (A-1.2)

\[
y_0 - \psi_0 = \frac{\gamma \lambda}{1 - \lambda} + \sum_{t=1}^{\infty} \lambda^t (r_t - \Delta c_t) \tag{A-1.3}
\]

where \( \lambda = 1 - e^{\delta - \psi} \) and \( \gamma = \ln \lambda + (\frac{1}{\lambda} - 1)(1 - \lambda) \). If we assume that in steady state, consumption, net income, and interest rate are all constant, then \( \lambda = \eta = \frac{1}{1+\tau} \), or the time preference parameter \( \beta \) in consumption utility function. Plug equations (A-1.2) and (A-1.3) into Eq.(A-1.1) to eliminate \( \phi \) and \( \psi \)

\[
\frac{1}{\omega} y_0 - c_0 + (1 - \frac{1}{\omega}) b_0 + (1 - \frac{1}{\omega}) \frac{\delta \beta}{1 - \beta} + \kappa = -\sum_{t=1}^{\infty} \beta^t \left[ \frac{1}{\omega} \Delta y_t - \Delta c_t + (1 - \frac{1}{\omega}) r_t \right].
\]

To further simplify the equation, we assume that in steady state the net foreign assets is zero, or, \( \omega = 1 \). Then the above equation is written as

\[
y_0 - c_0 = -\sum_{t=1}^{\infty} \beta^t (\Delta y_t - \Delta c_t).
\]

which can be re-written as Eq.(1.9) when Eq.(1.7) is used.
Essay 2

Permanent and Transitory Terms of Trade, Investment and the Current Account

2.1 Introduction

The purpose of this paper is to investigate the relationship between terms of trade (TOT) and the current account in an intertemporal model with investment. The research on the relationship between these two macro variables can be traced back to early works by Harberger (1950) and Laursen and Metzler (1950), who show that in a traditional open-economy Keynesian model, a small country's current account will worsen in response to an exogenous adverse TOT shock. The logic is simple: a TOT deterioration implies a drop in the real income held by the household, thus given a less than unity marginal propensity to consume, savings will decrease. Assuming constant investment, the current account must also decrease. This pos-
itive correlation between TOT and the current account is generally called HLM (Harberger-Laursen-Metzler) effect and known as one of the most important issues in the field of international economics.

The reason why we want to investigate the HLM effect in an intertemporal model is because the intertemporal approach has become one of the most important and standard modelling techniques in international economics since the 1980s and it also challenges the assumed connection between the current account and TOT derived in traditional models. Using a micro-based and forward-looking framework, Obstfeld (1982) and Svensson and Razin (1983) point out that the strength of such a connection between two variables depends primarily on the dynamic behavior of external shocks. Given a constant rate of time preference\(^1\), a permanent TOT shock will lower the real income in both current and future periods, thus shifting consumption instantly to a lower profile, but leaving the current account unchanged. In contrast, when the TOT shock is transitory, there will be both a real income effect and a substitution effect. When real income is lower, the consumption-smoothing household will not decrease his consumption dramatically to cover this windfall loss, but instead he dis-saves, thus worsening the current account balance. In the meantime, there are two different substitution effects. First, the negative TOT shock makes people consume more of exports and less of imports because the former becomes relatively cheaper to consume. This intra-temporal substitution effect leads to an improvement in the current account. And second, with a temporary TOT deteriora-

\(^1\)Svensson and Razin (1983) show that permanent TOT shock has an ambiguous effect on the current account, depending on how rate of time preference responds to welfare change. If rate of preference increases (decreases) with the level of welfare, then the current account improves (deteriorates). Therefore Obstfeld (1982)'s conclusion that a negative TOT shock will improve, instead of worsening as in HLM effect, the current account is just a special case of their model. That permanent shocks do not have effect on the current account is thus based on the assumption that the rate of time preference is constant, which is a very popular modelling method in the literature of the current account.
tion, current consumption becomes more expensive relative to future consumption, therefore household will engage in intertemporal substitution by saving more. This is also called consumption-tilting effect. If the pro-borrowing income effect dominates the pro-saving substitution effect, then the current account is always positively correlated with the TOT movement.

These papers have introduced an alternative way, other than the traditional static models, towards a better and more fundamental understanding of the relationship between the TOT shock and the current account. Later works try to extend the basic model by introducing new elements or by relaxing some of the assumptions and have come up with different conclusions regarding this relationship.

Original works of HLM assume constant investment. The intertemporal approach is an extension of consumption theory into an open-economy framework, therefore it also concentrates on consumption/saving decision as the major transmission mechanism of TOT shocks. However since the current account is the difference between saving and investment in a national accounting equation, it is natural to take investment into consideration when the relationship between TOT shock and the current account is analyzed. In this paper, we consider the role of investment when TOT shock is transmitted into the current account. Several papers have emphasized the importance of taking into consideration such elements as production, investment, and capital accumulation, and thereby the afore-mentioned connection between TOT shock and the current account can be altered.

Serven (1999) considers the case where capital goods have some import content. With a permanent TOT shock, investment can drop down due to permanently lower marginal product of capital, generating an improvement in the current account even when consumption behavior is unchanged. But if there is no import content in cap-
ital goods, his model predicts that the TOT shock does not cause a current account movement, which is consistent with earlier models concentrating on consumption side of story. In case of a transitory shock, besides the income and substitution effect generated by consumption/saving channel, two more ambiguities are added through investment channel. A temporary decrease in marginal productivity of capital can discourage investment and improve the current account; but intertemporally capital goods become cheaper in current periods than in the future, this will encourage investment and worsen the current account. Therefore the ambiguous effect of a transitory TOT shock on the current account becomes even more complex.

Having an import component in capital goods is not the only channel through which permanent TOT shock can have an effect on the current account. Sen and Turnovsky (1989) study the HLM effect in an intertemporal model with consumption-leisure choice and capital accumulation. Besides the afore-mentioned real income effect of the TOT shock, they emphasize the substitution effect among exportables, importables and especially leisure. A negative TOT shock can cause substitution away from imported goods, in favor of domestic goods (exportables and leisure), leading to a reduction in employment and investment; whereas in the case of income effect TOT deterioration lowers real income and goods consumption, but increases leisure consumption, thereby leading to an increase in employment and investment. Consequently, if the substitution effect dominates over income effect, a permanent shock will lead to a reduction in investment and a rise in the current account, just the opposite of HLM effect.

While Sen and Turnovsky (1989) focus on the relative importance of substitution and income effects resulting from TOT changes, Matsuyama (1988) emphasizes the role of relative factor intensity in shaping saving and investment decisions, an
application of Stopler-Samuelson theorem in international approach. If the export sector is relatively more labor intensive, a negative TOT shock will induce a decrease in wage and savings, but an increase in the real rate of return to capital and investment, thus leading to a current account deterioration. However the reverse is true if the export sector is capital intensive. Therefore whether this additional Stopler-Samuelson effect strengthens or weakens the HLM effect really depends on the relative factor intensity in different sectors.

Given the plethora of ambiguities in the ways that TOT shocks may affect the current account as predicted from economic theories, it is surprising that there have been few empirical studies analyzing the relationship between the two. Currently many papers use the standard real business cycle methodology of comparing simulated second moments from calibrated versions of their models with those from the actual data. But in this paper, we will apply econometric-based empirical methodologies, which provide useful additional evidence concerning the empirical plausibility of theoretical models. We briefly review some papers that have adopted econometric approaches in empirical study.

Kent (1997) (KENT) implements a panel-data study of 128 countries in analyzing how the current account responds to TOT shocks of different persistence. He runs regression of the current account against first-differenced TOT and GDP (used as a proxy for productivity shock) with up to four lags variables. For the group of countries with more permanent TOT shocks, he finds the sum of all coefficients on TOT variables is negative, implying that in the intermediate-horizon the current account and TOT is negatively correlated. He reports opposite findings for the group of countries with less persistent TOT shocks. His empirical findings are consistent with predictions from intertemporal models with investment. Despite its
correct intuition and easy implementation, his regression model lacks a structural foundation. It may be understandable to include TOT and GDP as explanatory variables to the current account, but why not also take into consideration other factors, like investment as in Iscan (2000) (ISCAN)? In fact, the author estimates similar reduced-form models, but with investment and productivity, instead of GDP, as the explanatory variables and does not find a significant coefficient on the TOT shock variable for G7 countries.

As an alternative empirical methodology, the structural vector autoregressive model (SVAR) is computationally more complex than the simple regression model, but it allows for more sophisticated inferences regarding the impulse response function and the forecast error decomposition. Both Cashin and McDermott (2002) and Otto (2003) estimate a three-variable SVAR of TOT, output, and the current account, and identify structural shocks by assuming that the TOT is exogenous and that innovation to the current account (taken to be demand shock in the Cashin and McDermott) does not have long-run effect on output. A big drawback of the SVAR model is that it typically involves application of underlying economic theory or intuition to identify shocks and may therefore vary from one model to another. For example, with one SVAR model of investment and the current account, Nason and Rogers (2002) have six different identifying schemes which produce varying and even opposing estimation results.

Our paper follows a procedure similar to ISCAN: we will set up a simple intertemporal model with investment and derive the implied relationship between investment, current account and the TOT in reduced-form equations, which are then examined and tested using the regular regression technique. But our paper differs from the latter in two aspects. First, the main purpose of ISCAN is to ex-
tend GR’s one-good model to include both tradable and non-tradable goods so that the effect of productivity shocks on both investment and the current account can be decomposed into that from tradable and non-tradable goods sectors and investigated separately. As a natural by-product of the model setup, the effect of the real exchange rate (defined as the price of tradable goods relative to non-tradable goods) shock on both investment and the current account is also analyzed. In our paper, we are more interested in the effect of TOT with different degree of persistences than that of productivity shock in different sectors. Therefore, the two-good economy consumes exportables and importables so that relative price is explicitly defined as TOT, as compared to real exchange rate in ISCAN. Though TOT and real exchange rate are both procyclical and closely related\(^2\), they are different. In addition to TOT, the real exchange rate may also be affected by interest differentials between countries, as well as inter-country differences in productivity growth and net foreign asset accumulation. For example, Gruen and Wilkinson (1994) find weak evidence that there exists a stable relationship between TOT and the real exchange rate for Australia 1969-1990. Additionally, we allow investment to be composed of both domestic goods and imports. This feature plays an important role in determining how investment responds to TOT because when investment is using only domestic resources, it will not be affected by TOT shocks. Inclusion of this feature in our model complicates how the current account responds to TOT, but it enables a better understanding of the role that the investment channel plays when the current account adjusts to relative price disturbance.

The second difference lies in the way that the empirical study is carried out.

\(^2\)The importance of TOT in explaining real business cycles can be found in Mendoza (1995) who shows that even though productivity shocks still play an important role, TOT shocks can account for 45-60% of the observed variability of GDP and real exchange rate.
ISCAN takes TOT as single explanatory variable in the regression whereas we distinguish between permanent and transitory TOT. Our empirical setup is consistent with the theoretical prediction of a typical intertemporal model where the degree of persistence will affect how TOT shock is passed onto the current account and other macro factors. Currently, empirical studies (e.g. KENT) always try to investigate the importance of TOT persistence in a horizontal way, i.e. by dividing countries into groups with more or less persistent TOT shocks and then applying cross-sectional analysis. In this paper, we provide an alternative vertical approach, where for each country we decompose its TOT into permanent and transitory components and then do time-series analysis. To our best knowledge, this is the first paper in the literature studying the HLM effect in a decomposed framework. Our work will add to the empirical evidence regarding the validity of the intertemporal approach and the existence of the HLM effect.

The rest of the paper is organized in the following way. Section 2 sets up a structural intertemporal model and derives the reduced-form empirical model. Section 3 presents some stylized facts of TOT and reports empirical results. Section 4 provides concluding remarks.

2.2 TOT effect in a simple intertemporal model with investment

In this section, we derive a simple structural model where households consume both exportable and importable goods and firms produce one exportable good with stochastic productivity. Domestic firms produce one good using capital $K$ and an external stochastic productivity $A$ in a production function of Cobb-Douglas format:
Following GR and ISCAN, the productivity is assumed to evolve in a simple AR(1) process:

\[ A_t = \rho A_{t-1} + \epsilon_t, \tag{2.1} \]

which is a typical assumption in the IRBC literature (see, for example, Stockman and Tesar (1995) and Backus, Kehoe and Kydland (1992)). The produced goods can be used for domestic consumption and investment, or exported. Following Boileau (1999) and Serven (1999), the capital stock is accumulated by adding an investment good composite \( I \), which is composed of both domestic and foreign investment goods contents, therefore \( I_t = K_{t+1} - K_t \).\(^3\) For simplicity, we assume that the investment composite takes the form of \( I = I_1 \omega I_2^{1-\omega} \) where \( I_1 \) (\( I_2 \)) is the home (foreign) goods used in investment, and \( 0 \leq \omega \leq 1 \) represents home share. Installation of new investment involves additional adjustment cost \( \frac{\partial I^2}{2K} \), which is a typical assumption to avoid abnormally large volatility in investment in the literature.

We define \( \pi \) the exogenously given relative price of the exportable in terms of the importable, i.e. the TOT, which also follows a stochastic AR(1) process

\[ \pi_t = \eta \pi_{t-1} + \omega_t. \tag{2.2} \]

Therefore, the domestically produced goods will be evaluated at \( \pi \) while domestic investment at an exact investment price index \( p_I = \pi^\omega \). (For the sake of notational simplicity, we have left out the constant term, see Appendix B.1 for derivation of this index price.)

A representative firm maximizes present value of its current and expected future

\(^3\)Adding capital depreciation factor into the model only complicates mathematical derivation, but does not change main results.
net outputs

$$E_t \sum_{i=0}^{\infty} \frac{Y_{t+i}}{(1 + r)^i} = E_t \left\{ \sum_{i=0}^{\infty} \frac{1}{(1 + r)^i} \left[ \pi_{t+i} A_{t+i} K_{t+i}^s - \pi_{t+i}^s \left( I_{t+i} + \frac{g I_{t+i}^2}{2 K_{t+i}} \right) \right] \right\}, \quad (2.3)$$

subject to $I_t = K_{t+1} - K_t$. The first order Euler equation of this maximization problem is

$$E_t \left[ \pi_t^s \left( -\frac{K_{t+1}}{K_t} + g - 1 \right) + \pi_t^s \left( \frac{A_t+1}{1 + r} K_{t+1}^{s-1} + 1 - \frac{g}{2} + \frac{g K_{t+2}^2}{2 K_{t+1}^2} \right) \right] = 0, \quad (2.4)$$

which can be linearized around the steady state values and solved for both capital stock and investment (see Appendix B.1 for derivation),

$$K_{t+1} = \lambda_0 K_t + \lambda_2 \sum_{i=1}^{\infty} \lambda_i^{-1} E_t A_{t+i} + \lambda_3 \sum_{i=1}^{\infty} \lambda_i^{i-1} E_t \pi_{t+i} + \lambda_4 \sum_{i=0}^{\infty} \lambda_i^i E_t \pi_{t+i-1}, \quad (2.5)$$

$$I_t = \lambda_0 I_{t-1} + \lambda_2 \sum_{i=1}^{\infty} \lambda_i^{-1} (E_t A_{t+i} - E_{t-1} A_{t+i-1})$$

$$+ \lambda_3 \sum_{i=0}^{\infty} \lambda_i^i (E_t \pi_{t+i} - E_{t-1} A_{t+i-1}) + \lambda_4 \sum_{i=0}^{\infty} \lambda_i^i (E_t \pi_{t+i-1} - E_{t-1} A_{t+i-2}), \quad (2.6)$$

where $0 < \lambda_0, \lambda_1 < 1, \lambda_2, \lambda_3 > 0$, and $\lambda_4 < 0$. Substituting equation (2.1) into (2.5) and (2.6) yields

$$K_{t+1} = \lambda_0 K_t + \kappa_1 A_t + \kappa_2 \pi_t, \quad (2.7)$$

$$I_t = \lambda_0 I_{t-1} + \kappa_1 \Delta A_t + \kappa_2 \Delta \pi_t, \quad (2.8)$$

$$\Delta I_t = (\lambda_0 - 1) I_{t-1} + \kappa_1 \Delta A_t + \kappa_2 \Delta \pi_t, \quad (2.9)$$
where

\[ \kappa_1 = \frac{\lambda_2 \rho}{1 - \lambda_1 \rho} > 0, \]
\[ \kappa_2 = \frac{\lambda_3 \eta + \lambda_4}{1 - \lambda_1 \eta}. \]

Equation (2.9) is very similar to equation (A.12) in ISCAN, except that we do not distinguish between world and country-specific productivity shocks as did he. But in his reduced-form model, the coefficient on the relative price variable, i.e., real exchange rate, is positive whereas in our model, the sign of \( \kappa_2 \) is undetermined. We will come to a detailed discussion of this in the next paragraph. Having \( \text{TOT} \) in the model does not change how investment responds to productivity shock. Permanent productivity improvement (when \( \rho = 1 \)) will encourage investment, and since \( \frac{\partial \kappa_1}{\partial \rho} > 0 \), such an effect will diminish as productivity shock becomes less persistent. When \( \rho \) drops to zero, or the productivity shock is pure temporary, there exists no investment response.

The response of investment to \( \text{TOT} \) is a little more complicated because it depends on two factors: the duration of shock and the domestic share in investment. When domestic goods take the full share in investment composite, or \( \omega = 1 \), we have \( \lambda_3 = -\lambda_4 \). Therefore permanent \( \text{TOT} \) shock (with \( \eta = 1 \)) will not have effect on investment. Intuitively when \( \text{TOT} \) shock is permanent and investment uses only domestic goods, the shock will leave investment-based real interest rate unchanged, leaving no incentive for new installation of investment. To the extent that investment has some foreign content, a permanent \( \text{TOT} \) improvement will lead to a rise in investment because the increase in marginal profit \( (\pi) \) is higher than that in user cost \( (\pi^u) \) of an additional unit of capital. When \( \text{TOT} \) shock is transitory, it involves
an extra intertemporal substitution effect between temporarily more expensive current investment and relatively cheaper future investment. Whether a transitory TOT has a positive or negative effect on investment will depend on which one of these two channels dominates. An extreme case is that when investment uses all domestic goods \((\omega = 1)\), intertemporal substitution effect is the only channel and therefore transitory TOT has negative effect on investment. The opposite is true when investment uses only foreign goods \((\omega = 0)\). That is why the coefficient of relative price on investment is positive in ISCAN, but undetermined in our model. In Figure 2.1, we graph the relationship between \(\kappa_2\) and either \(\omega\) or \(\eta\) when the other factor is given. We can see clearly that \(\kappa_2\) is increasing in \(\eta\) but decreasing in \(\omega\).

Households will consume both exportables and importables which make up of the consumption composite \(C\). For simplicity, we assume the consumption function takes a similar form of investment composite so that the consumption price index is \(p = \pi^\theta\) where \(0 < \theta < 1\) is the proportion of exportable goods in consumption basket and assumed to be fixed over time. A representative agent chooses consumption path \(\{C_t\}\) to maximize his life time utility

\[
E_t \left[ \sum_{s=t}^{\infty} \beta^s \ln(C_s) \right]
\]

subject to the intertemporal budget constraint

\[
B_{t+1} = (1 + r) B_t + Y_t - p_t C_t,
\]

where \(B\) is foreign asset holding, \(r\) is world interest rate assumed to be constant, and \(Y\) is net output.
Figure 2.1: $\kappa_2$ as a function of $\omega$ and $\eta$

(a) $\kappa_2$ as a function of $\omega$, given $\eta$

(b) $\kappa_2$ as a function of $\eta$, given $\omega$
Consumption Euler equation yields the random walk dynamics of consumption expenditure

\[ E_t(p_{t+1}C_{t+1}) = p_tC_t, \]  

(2.12)

which can be substituted into the intertemporal budget constraint equation (2.11) to obtain the \textit{ex post} rate of change in consumption expenditure

\[ \Delta(p_tC_t) = \frac{r}{1+r} \sum_{i=0}^{\infty} \frac{1}{(1+r)^i}[(E_t - E_{t-1})Y_{t+i}]. \]  

(2.13)

This is equivalent to the standard result as shown in equation (5) in GR, but in a framework with TOT. Here \((E_t - E_{t-1})Y_{t+i}\) represents revisions of expectations from time \(t - 1\) to \(t\) in response to the new information available at time \(t\). This value is thus unpredictable at time \(t - 1\) under rational expectation. Or we can say that the \textit{ex post} rate of change of consumption depends only on the unanticipated movements in permanent income. For notational convenience, we will define any variable \(\bar{X}_t\) as \((E_t - E_{t-1})X_t\) in the following part.

To solve for consumption, we need to linearize net output around the steady state values (which are represented by suppressing subscript 't' from time variables)

\[ Y_t = \alpha_A A_t + \alpha_K K_t + \alpha_I I_t + \alpha_{\pi}\pi_t \]  

(2.14)
where

\[ \alpha_A = \pi K^\alpha, \]
\[ \alpha_K = \pi \alpha A K^{\alpha - 1} + \pi^2 \frac{gI^2}{2K^2}, \tag{2.15} \]
\[ \alpha_I = -\pi^2 (1 + \frac{gI}{K}), \]
\[ \alpha_\pi = AK^\alpha - \omega \pi^{\omega - 1} (I + \frac{gI^2}{2K}), \]

and substitute it into equation (2.13) to get

\[ \Delta(p_tC_t) = \frac{r}{1 + r} \sum_{t=0}^{\infty} \frac{1}{(1 + r)^t} (\alpha_A A_{t+1} + \alpha_K \ddot{K}_{t+1} + \alpha_I \ddot{I}_{t+1} + \alpha_\pi \ddot{\pi}_{t+1}). \tag{2.16} \]

With equations (2.1), (2.7) and (2.8), we can easily solve for consumption expenditure and (by making a linear approximation) consumption (see Appendix B.1 for derivation)

\[ \Delta(p_tC_t) = \phi_1 (A_t - \rho A_{t-1}) + \phi_2 (\pi_t - \eta \pi_{t-1}), \tag{2.17} \]
\[ \Delta C_t = \phi_1 (A_t - \rho A_{t-1}) + \phi_2 (\pi_t - \eta \pi_{t-1}) + \phi_3 \Delta \pi_t, \tag{2.18} \]

where

\[ \phi_1 = \frac{r}{1 + r - \rho} \alpha_A > 0, \]
\[ \phi_2 = \frac{r}{1 + r - \eta} \alpha_\pi > 0, \]
\[ \phi_3 = -\theta \frac{C}{\pi} < 0. \]

Please note that to simplify the expression for \( \phi_1 \) and \( \phi_2 \), we have used the condition
that $\alpha_K + r\alpha_f = 0$,\(^4\) which is obtained by solving the first order condition of profit maximizing. $-\alpha_f$ is actually the Tobin’s $q$, or the value of an additional unit of capital, and $\alpha_K$ stands for the user cost of capital. Therefore at the steady state, the user cost of capital should be equal to the interest proceed the firm forgoes.

Consumption responds to both permanent and transitory productivity shocks. With a positive permanent shock, it rises by the amount of $\alpha_A$, which equals the change in net output $Y$; whereas with a transitory one, consumption only increases marginally as compared to the net output increase.

TOT works on consumption expenditure in the same way as a productivity shock, and it affects consumption through two channels. There is a real income or consumption-smoothing effect ($\phi_2$) where the TOT shock changes households’ real net income in exactly the same way, with different parameters, as a productivity shock. A permanent TOT shock makes consumption jump instantaneously to its higher new steady state value, while a transitory shock only moves consumption marginally. And second, there is also an intertemporal substitution or consumption-timing effect ($\phi_3$) where the TOT changes consumption price index $p$, thereby making future (current) consumption more desirable when the real consumption interest rate $\frac{p_t}{p_{t+1}}(1+r)$ is greater (smaller) than time preference parameter $\beta$.

Defining the first difference in foreign asset holding as the current account $CA_t = B_{t+1} - B_t$, We can re-write the intertemporal budget constraint as

$$\Delta CA_t = rCA_{t-1} + \Delta Y_t - \Delta(p_tC_t)$$

$$= rCA_{t-1} + C\tilde{A}_t + E_{t-1}Y_t - Y_{t-1} - [E_{t-1}(p_tC_t) - p_{t-1}C_{t-1}], \quad (2.19)$$

\(\text{Having capital depreciation in the model does not change the validity of this condition.}\)
which can be solved in a reduced-form equation as (see Appendix B.1 for derivation)

\[ \Delta CA_t = rCA_{t-1} + \gamma_{11}\Delta A_t + \gamma_{12}A_{t-1} + \gamma_{21}\Delta\pi_t + \gamma_{22}\pi_{t-1} + \gamma_3I_{t-1} \quad (2.20) \]

where

\[
\begin{align*}
\gamma_{11} &= \alpha_A - \phi_1 + \alpha_I\kappa_1, \\
\gamma_{12} &= (\rho - 1)\phi_1, \\
\gamma_{21} &= \alpha_e - \phi_2 + \alpha_I\kappa_2, \\
\gamma_{22} &= (\eta - 1)\phi_2, \\
\gamma_3 &= \alpha_K + \alpha_I(\lambda_0 - 1).
\end{align*}
\]

Productivity shock has an effect on the current account through both consumption and investment channels. The sum of first two items in \( \gamma_{11} \), or \( \alpha_A - \phi_1 \), represents the former channel, whereas \( \alpha_I\kappa_1 \) stands for the latter. If the model is a stochastic endowment model, then the effect of productivity shock on the current account will be through the consumption channel \( \alpha_A - \phi_1 \), which is always positive for transitory or persistent shock and zero for permanent shock. Investment affects the current account negatively, therefore the sign of \( \gamma_{11} \) is undetermined, depending on the relative importance of these two effects. But given \( \frac{\partial\gamma_{11}}{\partial\rho} < 0 \), we know for sure that in the neighborhood of \( \rho = 1 \), the current account responds negatively to the productivity shock because only effect through investment channel will be reflected into current account movement. A positive coefficient will be found in the neighborhood of \( \rho = 0 \).

TOT shock affects the current account in a similar manner as the productivity
shock. Through consumption channel, the effect is always positive, but the magnitude of influence via investment channel is further complicated by $\omega$, the home share in investment. When the shock is permanent, we always have zero consumption effect and negative investment effect, therefore the TOT shock and the current account moving in different directions, just the opposite of the HLM effect. The HLM effect can only be guaranteed when the shock is not permanent and $\kappa_2$ is negative, or $\frac{\nu_{-i_1}}{1+i-\eta} < \omega$. The HLM effect may also be possible with a small positive $\kappa_2$.

Note that in our model the TOT affects the current account only through real income channel. The reason why intertemporal channel is missing is directly related to our model assumption. The log format utility function implies a random walk consumption expenditure $pC$ which precludes the intertemporal effect on the current account because the total consumption expenditure is not affected by the consumption real interest rate. It will enter equation (2.19) if the utility function of a more general format is used.

Since the current account is measured in terms of the importable price and investment in real terms, so we define a new investment variable $I^E$, where $I^E = I^\pi^\omega$, so that coefficients in investment equation (2.9) and the current account equation (2.19) can be comparable. Using the definition of the new variable $I^E$, equation (2.9) can be modified as

$$
\Delta I^E_t = \pi^\omega[(\lambda_0 - 1)I^E_{t-1} + \kappa_1 \Delta A_t + \kappa_2 \Delta \pi_t].
$$

(2.21)

Comparing the effect of productivity shock on investment and the current account, we should have $|\gamma_{11}| > \kappa_1 \pi^\omega$ if $\rho = 1$. Similarly, it is more likely to find $|\gamma_{21}| > \kappa_2 \pi^\omega$. 

65
if TOT shock is permanent.

2.3 Empirical Results

In this part we estimate a simple and an extended model of the reduced-form equations (2.21) and (2.20). In the simple model, we do not distinguish between permanent and transitory components, therefore TOT enters the current account and investment equations as a single item. In the extended model, we use decomposed permanent and transitory TOTs in the regression. Note that when we estimate the current account equation, we will use $\Delta CA_t^* = \Delta CA_t - rCA_{t-1}$ as the dependent variable because we are not interested in evaluating the coefficient on $CA_{t-1}$. Throughout the empirical estimation of this part, we use a constant world interest $r = 0.04$.

2.3.1 Data and dynamic features of productivity and TOT

In order to render empirical estimates comparable with ISCAN, we use annual data from 1950 to 2006 for six Group-7 countries, the United States, Japan, Canada, United Kingdom, Germany and Italy.\footnote{IMF IFS online database has only 20 years annual TOT data for France. So we opt to drop France in this empirical study to avoid small sample bias problem.} Data comes primarily from IMF International Financial Statistics (IFS) online database. Following Backus et al. (1992), the productivity $A$ is measured as the residual from the Cobb-Douglas production function

$$\ln A = \ln Y - \zeta \ln L,$$

$$\Delta CA_t^* = \Delta CA_t - rCA_{t-1}$$
where $Y$ is real GDP, $L$ is total population, and $\xi$ is the labor share of capital as used in GR.\footnote{Following GR, the values we use here are: the US, 0.66; Japan, 0.54; Canada, 0.63; UK, 0.68; Germany, 0.64; Italy, 0.48.} The TOT is constructed as the ratio between export and import prices, or unit value of exports and that of imports if the former is not available. The current account is calculated by subtracting consumption, investment, and government expenditure from GNP, or GDP if the former is not available. Please refer to Appendix B.2 for a detailed data description.

In Table 2.1, we report the AR(1) coefficients for both productivity and the TOT, followed by their corresponding standard deviations in parentheses and an ADF statistic testing the null of unit root. Both productivity and TOT have large, close to one, first order autoregressive coefficients for all six countries. Except for Canada's TOT which is significant at 10\% level, all other time series have large $p$-values so that we cannot reject non-stationarity.

Statistically, unit root testing methodologies, such as ADF test, have a low power in distinguishing between a unit root and a near unit root. Therefore the observed large first order autoregressive coefficient does not allow us to make definite inference about the dynamic properties of the true data generating process. And there is no consensus in the literature regarding whether productivity and TOT truly have unit roots or are just persistently stationary. GR derives reduced-form equations and does empirical estimation separately for each of two possibilities: productivity is a random walk and productivity is stationary with a large AR coefficient. In our paper, we take both productivity and TOT as unit-root processes for the following reason. Since our major focus is on the analysis of TOT's effect on investment and the current account, we will only apply Beveridge-Nelson (BN) decomposition, which is applicable with an AR(1) process, on TOT. Assuming non-stationary pro-
Table 2.1: Dynamic features of productivity and TOT

<table>
<thead>
<tr>
<th></th>
<th>Productivity</th>
<th>Terms of trade</th>
<th>Transitory terms of trade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>b</td>
<td>ADF</td>
<td>b</td>
</tr>
<tr>
<td>US</td>
<td>1.020 (0.006)</td>
<td>2.210</td>
<td>0.939 (0.041)</td>
</tr>
<tr>
<td>JP</td>
<td>0.993 (0.007)</td>
<td>-0.960</td>
<td>0.898 (0.060)</td>
</tr>
<tr>
<td>CA</td>
<td>1.014 (0.007)</td>
<td>1.243</td>
<td>0.790 (0.090)</td>
</tr>
<tr>
<td>UK</td>
<td>1.021 (0.005)</td>
<td>0.294</td>
<td>0.825 (0.083)</td>
</tr>
<tr>
<td>GE</td>
<td>1.001 (0.011)</td>
<td>-0.035</td>
<td>0.872 (0.065)</td>
</tr>
<tr>
<td>IT</td>
<td>0.988 (0.011)</td>
<td>-1.412</td>
<td>0.905 (0.058)</td>
</tr>
</tbody>
</table>

Notes: Regression uses AR(1) model \(x_t = \alpha + bx_{t-1} + u_t\) where \(x_t\) is either productivity or terms of trade. Standard errors are in parentheses. ADF statistics is using ADF test with one lag and no time trend. ** stands for significance at 5% level. * stands for significance at 10% level.
ductivity will make the reduced-form equation simpler and reduce extra noise that decomposing productivity may bring into the empirical results.⁷

The real determinants of TOT shock may be difficult to find. Besides the factors coming from demand side of economy, the supply side, such as the high technology advances of more recent years, can also be a major contributor to TOT variation. Therefore we want to check the correlation between productivity and TOT variables to ensure that our regression results are not contaminated by potential collinearity problem. The signs of correlation coefficient for six countries are mixed, some being positive and others being negative. But the absolute values are generally small, with the average being 0.15. So we say that productivity and TOT are not highly correlated and there exists no collinearity problem.

2.3.2 The simple model

In table 2.2, we report regression results for both investment and the current account equations. We first look at the results for the investment equation.⁸ Lagged investment has the expected negative sign for all six countries, and is significant for most cases at 5% level. The coefficient on productivity is significant for all six countries, and correctly positive for all countries except for the UK. Given the large close-to-one autoregressive coefficient of productivity for all countries, this is consistent with the theoretical prediction outlined in the earlier section. And since the productivity variable used in our estimation equations is what GR and ISCAN used for country-specific productivity, our empirical result also provides additional support to their proposition that country-specific components in productivity shock

⁷The same assumption is made by ISCAN when he runs panel regression with TOT variable.
⁸We use investment to GDP ratio and the current account to GDP ratio in our empirical estimation.
matter for investment movement.

The coefficient on TOT is negative for four countries, but positive for two others. And only the negative ones are significant at either 5% (for three countries) or 10% level (for Japan). ISCAN does not provide empirical results for each individual country, but with panel data, he reports an insignificant and positive TOT coefficient. From the theoretic analysis in the previous part, we know that how TOT affects investment depends on two parameters, the persistency of TOT shock $\eta$ and the importance of domestic content in investment $\omega$. Since all six countries have a large $\eta$, it is more likely to observe a negative TOT coefficient for a large $\omega$ and a positive coefficient for a small $\omega$. We run a simple regression of investment on capital imports (see Appendix B.2 for data and methodology description) and take one minus the coefficient on capital imports as a proxy for $\omega$. We plot the approximated $\omega$ against TOT’s first order autoregressive coefficient in figure 2.2, symbolized by black dots labelled with abbreviated country names. The curve in this figure represents the relationship between $\eta$ and $\omega$ when $\kappa_2$ in equation (2.9) is zero. The coefficient $\kappa_2$ is negative (positive) for any combination of $\eta$ and $\omega$ above (below) the line. Among these six countries, Japan seems to be the country where investment relies least on foreign inputs whereas Germany’s investment depends on resources aboard to the most extent. But when it comes to the effect of $\omega$ and $\eta$ as a whole, all black dots fall into the upper-left space of the $\kappa_2 = 0$ curve, though at a very short distance away, implying a negative, but possibly insignificant, coefficient on TOT. Our empirical results are not consistent with ISCAN who finds that the coefficient on TOT is positive for G7 economies from 1981 to 1987 with panel regression. We also run time-series regression for our six-country 1971-1987 sample and find that except for the US, the other countries all have a positive coefficient
Table 2.2: Regression results: the simple model

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<table>
<thead>
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<tbody>
<tr>
<td></td>
<td>b</td>
<td>c</td>
<td>d</td>
<td></td>
<td>R²</td>
</tr>
<tr>
<td>US</td>
<td>-0.222 (0.088)</td>
<td>0.375 (0.057)</td>
<td>**</td>
<td>**</td>
<td>0.590</td>
</tr>
<tr>
<td>CA</td>
<td>0.152 (0.077)</td>
<td>-0.028 (0.050)</td>
<td>0.003 (0.025)</td>
<td>0.096</td>
<td></td>
</tr>
<tr>
<td>JP</td>
<td>-0.151 (0.053)</td>
<td>0.389 (0.090)</td>
<td>**</td>
<td>-0.056 (0.028)</td>
<td>*</td>
</tr>
<tr>
<td>CA</td>
<td>0.060 (0.034)</td>
<td>-0.097 (0.057)</td>
<td>*</td>
<td>0.056 (0.018)</td>
<td>**</td>
</tr>
<tr>
<td>CA</td>
<td>-0.084 (0.076)</td>
<td>0.426 (0.090)</td>
<td>**</td>
<td>0.025 (0.046)</td>
<td>0.399</td>
</tr>
<tr>
<td>CA</td>
<td>0.036 (0.071)</td>
<td>-0.013 (0.085)</td>
<td>0.042 (0.043)</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>-0.205 (0.088)</td>
<td>-0.142 (0.051)</td>
<td>**</td>
<td>-0.117 (0.046)</td>
<td>**</td>
</tr>
<tr>
<td>CA</td>
<td>0.060 (0.143)</td>
<td>0.251 (0.083)</td>
<td>**</td>
<td>0.273 (0.075)</td>
<td>**</td>
</tr>
<tr>
<td>GE</td>
<td>-0.102 (0.063)</td>
<td>0.246 (0.078)</td>
<td>**</td>
<td>0.006 (0.046)</td>
<td>0.225</td>
</tr>
<tr>
<td>CA</td>
<td>-0.050 (0.051)</td>
<td>-0.264 (0.064)</td>
<td>**</td>
<td>0.023 (0.038)</td>
<td>0.310</td>
</tr>
<tr>
<td>IT</td>
<td>-0.186 (0.053)</td>
<td>0.561 (0.112)</td>
<td>**</td>
<td>-0.164 (0.033)</td>
<td>**</td>
</tr>
<tr>
<td>CA</td>
<td>0.107 (0.086)</td>
<td>-0.438 (0.136)</td>
<td>**</td>
<td>0.148 (0.040)</td>
<td>**</td>
</tr>
</tbody>
</table>

Notes: Investment equation uses $\Delta I_t = a + bI_t - 1 + c\Delta A_t + d\Delta \pi_t + \nu_t$.
The current account equation uses $\Delta CA_t = a + bI_t - 1 + c\Delta A_t + d\Delta \pi_t + \nu_t$.
Standard errors are in parentheses. ** stands for significance at 5% level. * stands for significant at 10% level.
on TOT.\textsuperscript{9} So we conclude that the difference in the sign of TOT between our paper and ISAN is due to different choice of data sample.

We proceed to report results for the current account equation. Lagged investment has a correctly positive effect on the current account, though such an effect is not significant for most countries. Due to extra effect coming from consumption channel, the coefficient of productivity on the current account is undetermined theoretically. But when productivity has a unit root or is very persistent, it is more likely to have a negative effect, which is what we observe for five out of six countries.

Consistent with ISAN's estimation results using panel regression, the coefficients on TOT are mostly positive, but insignificant, indicating the existence of the HLM effect when both investment and consumption channels are considered. Given the range of combination of $\omega$ and $\eta$ for six countries, as symbolized by black dots in figure 2.2, with large but less than 1 AR(1) coefficient observed, the effect of a TOT shock on the current account would be mainly from the investment channel as consumption channel only contributes to the current account improvement marginally. Therefore the negative effect that TOT has on investment, as we find in our earlier empirical results, will be reflected into a positive correlation between TOT and the current account.

To provide an additional justification for the observed positive connection between TOT and the current account, we try to measure the relative importance of transitory and permanent shocks in TOT by a variance ratio (VS), as introduced in

\textsuperscript{9}The estimated coefficients on TOT, with standard errors in parentheses, are -0.024 (0.04) for US, 0.074 (0.028) for Japan, 0.094 (0.07) for Canada, 0.254 (0.111) for UK, 0.069 (0.055) for Germany, and 0.211 (0.045) for Italy.
Figure 2.2: The coefficient of terms of trade on investment equation

This figure shows how $\kappa_2$ in equation (2.9) is affected by two parameters: the persistence of term of trade shock $\eta$ and the importance of import content in investment $\omega$. Any combination of $\eta$ and $\omega$ on the curve gives a zero $\kappa_2$. Combinations on the left side of the curve gives a negative coefficient, and on the right side positive.

Legend labelled "TOT" ("Transitory TOT") stands for combination of $\omega$ and (transitory) terms of trade for each country.
Cochrane (1988), using the formula

\[ VS = \frac{1}{k} \cdot \frac{\text{Var}(TOT_t - TOT_{t-k})}{\text{Var}(TOT_t - TOT_{t-1})}, \quad k \geq 1. \]

This VS index will converge to the variance of a permanent component of TOT, relative to \( \text{Var}(TOT_t - TOT_{t-1}) \), as \( k \) increases. Here we examine VS for six countries for up to 20 years, \( k = 1, 2, ..., 20 \), and graph these VS indices against \( k \) in figure 2.3. For all six countries, the VS settles down to a value ranging from 20% to 40% as \( k \) rises in a time window of 20 years, implying that only 20-40% of the year-to-year changes in the TOT is attributed to the variance of its permanent component whereas the transitory component of TOT can explain most (up to 80%) of the movements in TOT for six countries. Since transitory TOT affects the current account in a positive way and permanent TOT in a reverse way (given the range of \( \omega \) estimated and presented in figure 2.2), the dominance of transitory component in TOT of these six countries can provide further justification of the observed positive coefficients on TOT in the current account equation.

2.3.3 The extended model

There exists a large literature on studying how to decompose integrated macro variables (such as real GDP) in an appropriate way so that the importance of either its trend or its cycle component is not underestimated.\(^{10}\) The unobserved-component (UC) approach, first introduced by Clark (1987) and Harvey (1985), assumes that the observed series \( y_t \) is the sum of an unobserved trend component \( p_t \) which follows a random walk with drift and a stationary component \( c_t \). To identify the parameters

\(^{10}\)See for example, Harvey (1985), Murray and Papanyan (2004).
Figure 2.3: Relative importance of permanent and transitory TOT

For each of six Group-7 countries, we graph a VS index formulated (Cochrane 1988) as

\[ VS = \frac{1}{k} \cdot \frac{\text{var}(TOT_1 - TOT_{1-k})}{\text{var}(TOT_1 - TOT_{1-1})}, \quad k \geq 1 \]

It is an indicator of relative importance of permanent and transitory components of TOT.
involved in the maximum likelihood estimation, it is generally assumed that the innovations to two components are not correlated. The Beveridge-Nelson (BN) approach makes similar assumption about the dynamic features of trend and cycle components, but it uses an ARIMA model to estimate parameters. These two decomposition methods have been found, by many researchers, to yield surprisingly different results for real GDP: UC model suggests a smooth trend together with a highly persistent and very volatile cycle whereas BN implies much of the variation comes from trend component.

In a recent paper, Morley, Nelson and Zivot (2003) show that both BN decomposition approach and UC model can be cast into a state space form and solved empirically with Kalman filter where the observation equation is

\[ y_t = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} p_t \\ c_t \end{bmatrix}, \]

the state equation is

\[
\begin{bmatrix} p_t \\ c_t \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & \phi \end{bmatrix} \begin{bmatrix} p_{t-1} \\ c_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t \\ \xi_t \end{bmatrix},
\]

and the covariance matrix is

\[
Q = \begin{bmatrix} \sigma^2 \epsilon & \sigma \epsilon \xi \\ \sigma \epsilon \xi & \sigma^2 \xi \end{bmatrix}.
\]

They propose, using both simulated and real US GDP data, that solutions to BN decomposition approach are equivalent to those of this Kalman filter problem and that
UC model is just a special case with the restriction $\sigma_{\epsilon} = 0$. Therefore the starkly different results implied by the two approaches lies in the unnecessary restriction that UC model imposes on the correlation between permanent and transitory innovations, which should be left determined by the real data itself.

In this paper, we decompose productivity and TOT into their permanent and transitory components using the BN approach (see appendix B.3 for detailed estimation procedure). The optimum number of lags in the ARMA($p$, $q$) model is chosen to minimize AIC for each country.\textsuperscript{11} The third column of Table 2.1 reports the dynamic properties of the transitory components. As we can see that all series have small AR(1) coefficient and highly significant ADF statistics so that the null of unit root can be easily rejected. If the ARMA($p$, $q$) model uses (1, 0), the first-differenced permanent component will be equivalent to the lagged transitory component. Therefore when both of these two terms enter the extended model, one of them will be dropped. To obtain estimates of all coefficients, we use innovation to the AR(1) model of permanent component as the permanent TOT shock variable $\Delta \pi^P$ in the regression.

Table 2.3 reports estimation results for the extended model. In investment equation, in addition to productivity shock, we have both permanent and transitory TOT shocks. In the current account equation, besides these three variables, we also have lagged transitory, but not permanent, TOT because the theoretical model tells us that lagged permanent components do not affect the current account. The effects of lagged investment on investment and the current account remain practically the same in this extended model as in the simple model in last section, being negative for investment but mostly positive for the current account equation. So does the

\textsuperscript{11}The ($p$, $q$) combinations for TOT are (1, 0) for US and Japan, and (1, 1) for Canada, UK, Germany, and Italy.
coefficient on productivity, which has a significantly positive effect on investment, but negative and mixed effect on the current account.

In the investment equation, permanent TOT tends to have a positive effect for four countries and negative for two other countries. The only significant value is for the UK and positive. On the other hand, transitory TOT is unanimously negative, and but only significant at 10% for Japan, the UK and Italy. Referring to Figure 2.2, all combinations of $\omega$ and $\eta$ for permanent component should be located on the right-hand side of the curve along the vertical line where $\eta = 1$, implying a positive effect, which is consistent with our empirical findings. The empirically found negative coefficient on transitory TOT can be justified by examining $\omega-\eta$ combinations for the transitory component in Figure 2.2, symbolized by black stars labelled with abbreviated country names.

It is also interesting to compare coefficient estimates on different components of TOT. The permanent component should have a mathematically larger effect on investment than does the transitory component because there is a lack of intertemporal substitution effect. In column titled $t_{d_1>d_2}$ of Table 2.3, we report t-statistics used to formally test this inequality hypothesis. All t-statistics are significant at the 10% level.

Because investment channel always contributes negatively to the current account adjustment process in the times of shocks, the sign of coefficients on decomposed TOT variables should be reversed in the current account equation. The current account regression results in Table 2.3 show that transitory and permanent components of TOT affect the current account also in a reversed way, the coefficient on the former being mostly positive and the latter being negative for four out of six countries. The signs on both variables are generally consistent with theoretical
### Table 2.3: Regression results: the extended model with BN filter

<table>
<thead>
<tr>
<th>Country</th>
<th>b</th>
<th>c</th>
<th>d₁</th>
<th>d₂</th>
<th>t_{d₁&gt;d₂}</th>
<th>t_{d₁&lt;d₂}</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>US I</td>
<td>-0.195  (0.086)**</td>
<td>0.379  (0.053)**</td>
<td>0.092  (0.078)</td>
<td>-0.012 (0.202)</td>
<td>3.670 **</td>
<td>0.705</td>
<td></td>
</tr>
<tr>
<td>CA</td>
<td>0.131   (0.131)</td>
<td>-0.184 (0.08)**</td>
<td>-0.075 (0.07)</td>
<td>0.125 (0.305)</td>
<td>4.867 **</td>
<td>0.313</td>
<td></td>
</tr>
<tr>
<td>JP I</td>
<td>-0.074  (0.035)**</td>
<td>0.329  (0.057)**</td>
<td>0.021  (0.023)</td>
<td>-0.142 (0.032)**</td>
<td>10.476 **</td>
<td>0.645</td>
<td></td>
</tr>
<tr>
<td>CA</td>
<td>-0.117  (0.028)**</td>
<td>-0.031 (0.015)**</td>
<td>0.055  (0.058)</td>
<td>0.068 (0.089)</td>
<td>1.367 *</td>
<td>0.591</td>
<td></td>
</tr>
<tr>
<td>CA I</td>
<td>-0.097  (0.088)</td>
<td>0.369  (0.077)**</td>
<td>-0.062 (0.161)</td>
<td>-0.102 (0.089)</td>
<td>1.656 *</td>
<td>0.521</td>
<td></td>
</tr>
<tr>
<td>CA</td>
<td>-0.078  (0.123)</td>
<td>-0.16  (0.097)</td>
<td>-0.549 (0.725)</td>
<td>-0.038 (0.124)</td>
<td>5.291 **</td>
<td>0.122</td>
<td></td>
</tr>
<tr>
<td>UK I</td>
<td>-0.328  (0.109)**</td>
<td>-0.103 (0.045)**</td>
<td>0.511  (0.152)</td>
<td>-0.179 (0.093) *</td>
<td>25.392 **</td>
<td>0.492</td>
<td></td>
</tr>
<tr>
<td>CA</td>
<td>-0.586  (0.167)**</td>
<td>0.125  (0.093) **</td>
<td>0.628  (0.219)</td>
<td>0.26 (0.113) **</td>
<td>-9.792</td>
<td>0.68</td>
<td></td>
</tr>
<tr>
<td>GE I</td>
<td>-0.126  (0.076) *</td>
<td>0.168  (0.077)</td>
<td>0.043  (0.046)</td>
<td>-0.19 (0.141)</td>
<td>10.884 **</td>
<td>0.255</td>
<td></td>
</tr>
<tr>
<td>CA</td>
<td>-0.225  (0.091)**</td>
<td>-0.078 (0.023)**</td>
<td>-0.147 (0.155)</td>
<td>0.023 (0.169)</td>
<td>5.136 **</td>
<td>0.213</td>
<td></td>
</tr>
<tr>
<td>IT I</td>
<td>-0.167  (0.082)**</td>
<td>0.566  (0.108) **</td>
<td>-0.141 (0.134)</td>
<td>-0.181 (0.072)**</td>
<td>3.726 **</td>
<td>0.779</td>
<td></td>
</tr>
<tr>
<td>CA</td>
<td>-0.123  (0.068) *</td>
<td>-0.228 (0.111) **</td>
<td>-0.132 (0.075)</td>
<td>0.32 (0.146) **</td>
<td>20.423 **</td>
<td>0.469</td>
<td></td>
</tr>
</tbody>
</table>

Investment equation uses \( \Delta I_t = a + bI_{t-1} + c\Delta A_t + d_1\Delta \pi_t^p + d_2\Delta \pi_t^r + \nu_t \); the current account equation uses \( \Delta CA_t = a + bI_{t-1} + c\Delta A_t + d_1\Delta \pi_t^p + d_2\Delta \pi_t^r + \nu_t + \epsilon_{1,t-1} \); where \( \pi^p \) and \( \pi^r \) are permanent and transitory TOT shocks.

Standard errors are in parentheses.

** stands for significance at 5% level. * stands for significant at 10% level.

\( t_{d₁>d₂} (t_{d₁<d₂}) \) is the t-statistic used to test if permanent TOT component has a greater (smaller) effect on investment (the current account) than does transitory component.
prediction of our model in the last section and also with the empirical results in Kent (1997). Therefore we have fairly good confidence in positing that persistence does play an important role in determining the dynamics of the current account with respect to TOT shocks. We also test for the inequality hypothesis that permanent TOT has a mathematically larger effect on the current account than does transitory TOT. In column titled $t_{d_1<d_2}$ of the table, we find significant t-statistics for five countries, implying the validity of this inequality condition.

2.3.4 Sensitivity analysis

Because the BN approach does not place any restriction on the correlation between permanent and transitory innovations, it is possible for permanent and transitory components to be negatively correlated with large coefficient, therefore bringing out the potential problem of collinearity in our regression model. To formally test this, we use two statistics, the variance inflation factor (VIF) and the correlation between permanent and transitory components. The average VIF statistic is 1.86 for TOT, which is much smaller than 10, the usual standard indicating serious collinearity problem. But the average correlation coefficient (calculated using absolute values because we have both positive and negative signs) is 0.63, and should be considered a large number.

Generally when there exists a collinearity problem, we would observe insignificant regression coefficients with large t-statistics. To preclude this potential problem that collinearity may exert when we make statistical inferences from regression results, we try to decompose the TOT series using Hodrick-Prescott (HP) filter, a commonly used approach in macroeconomics to de-trend business cycle variables.\textsuperscript{12} The cor-

\textsuperscript{12}The smoothing parameter $\lambda$ is set to 100 for our annual data.
relation coefficient between the HP-filtered permanent and transitory components averages 0.15 for six countries, and when we run the extended investment model with these components, the average VIF values is 1.04. So collinearity is supposedly not a serious problem in the HP filtered series. Another interesting property of the HP filter is that relative to the BN decomposition approach, it tends to over-state the importance of transitory shocks. In Figure 2.4, we graph the permanent and transitory components of TOT for the US. We can see that time series generated by the HP filter is more persistent than that from the BN approach. This is further supported when we run a simple AR(1) regression. The average AR(1) coefficient for the HP filtered transitory TOT is 0.49 for six countries, as compared to 0.17 using the BN approach. Because two approaches generate series of different dynamic feature, it would be interesting to compare their empirical results.

We would like to point out that not only do these two decomposed series have different degrees of persistence, but they also may reflect different aspects of the adjustment process in response to economic shocks. When we inspect figure 2.4, we notice that the up and down trends of transitory TOT actually move in opposite directions for BN and HP. The correlation between transitory TOT of BN and that of HP is, however, country-specific. It is -0.4, -0.5, -0.3, and -0.5 for the US, Japan, Germany and Italy, but 0.8 and 0.9 for Canada and the UK. Therefore when we try to compare the statistical performance of these two decomposition methods, we have to bear in mind that different dynamic features of two decomposed series may reflect different forces driving the variation of TOT and we want to be careful when making comparisons.

Table 2.4 reports regression results using HP filtered TOT components. The

---

13Though the aforementioned equivalence problem between first differenced permanent component and lagged transitory component does not exist with HP filter, we still use innovation to
Figure 2.4: Permanent and transitory terms of trade in the US

(a) Permanent TOT by BN decomposition and HP filter

(b) Transitory TOT by BN decomposition and HP filter
signs of all coefficients for both equations stay pretty much the same as in the BN approach. And the inequality relationship between permanent and transitory components also holds true in most cases. Except for Italy in the investment equation and the US in the current account equation, the permanent TOT component does not have significant coefficients for all countries in both equations while the transitory component seems to have more significant cases.

Even though there is no appropriate statistical test to compare coefficients on transitory components across two decomposition approaches, we still want to make some simple and visual comparisons. When TOT shock becomes more persistent, it will have a mathematically larger effect on investment, but a smaller effect on the current account. Kent (1997) reports similar results using panel data and so does Mendoza (1995) with simulation. In our empirical results, there are five cases (except for the US) where the HP filter has a larger $d_2$ coefficient in the investment equation relative to the BN approach and four cases (US, Japan, UK, and Italy) where the BN decomposition has a larger $d_2$ coefficient on the current account.

2.4 Conclusion

In this paper, we examine the relationship between TOT shocks and investment and the current account based on reduced-form equations derived from a simple intertemporal model with capital accumulation. For six Group-7 countries, our empirical results show that in general there exists a positive correlation between TOT and the current account, though such a relationship is generally low and not statistically significant. Mendoza (1995) reports similar results with a simulation AR(1) regression model as permanent shock variable in the extended model to render comparable estimates.
Table 2.4: Regression results: the extended model with HP filler

<table>
<thead>
<tr>
<th>Country</th>
<th>b</th>
<th>c</th>
<th>d₁</th>
<th>d₂</th>
<th>tₐ₁ &gt; tₐ₂</th>
<th>tₐ₁ &lt; tₐ₂</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>US I</td>
<td>-0.21 (0.077) **</td>
<td>0.403 (0.047) **</td>
<td>-0.068 (0.087)</td>
<td>-0.089 (0.061)</td>
<td>1.505 *</td>
<td>18.499 **</td>
<td>0.76</td>
</tr>
<tr>
<td>CA I</td>
<td>0.153 (0.122)</td>
<td>-0.185 (0.075) **</td>
<td>-0.31 (0.139) **</td>
<td>0.048 (0.049)</td>
<td></td>
<td></td>
<td>0.378</td>
</tr>
<tr>
<td>JP I</td>
<td>-0.085 (0.041) **</td>
<td>0.339 (0.065) **</td>
<td>0.04 (0.073)</td>
<td>-0.039 (0.022) *</td>
<td>7.614 **</td>
<td></td>
<td>0.535</td>
</tr>
<tr>
<td>CA I</td>
<td>-0.105 (0.028)</td>
<td>-0.02 (0.011) *</td>
<td>-0.113 (0.15)</td>
<td>0.058 (0.035) *</td>
<td>1.505 *</td>
<td></td>
<td>0.378</td>
</tr>
<tr>
<td>CA I</td>
<td>-0.121 (0.083)</td>
<td>0.358 (0.078) **</td>
<td>0.034 (0.236)</td>
<td>0.047 (0.043)</td>
<td>-0.413</td>
<td></td>
<td>0.514</td>
</tr>
<tr>
<td>UK I</td>
<td>-0.239 (0.148)</td>
<td>-0.118 (0.063) *</td>
<td>0.447 (0.5)</td>
<td>-0.097 (0.053) *</td>
<td>7.095 **</td>
<td></td>
<td>0.378</td>
</tr>
<tr>
<td>CA I</td>
<td>-0.714 (0.214)</td>
<td>0.148 (0.116)</td>
<td>0.721 (0.738)</td>
<td>0.236 (0.076) **</td>
<td>-4.287</td>
<td></td>
<td>0.607</td>
</tr>
<tr>
<td>GE I</td>
<td>-0.144 (0.081) *</td>
<td>0.181 (0.082) **</td>
<td>0.085 (0.194)</td>
<td>-0.01 (0.051)</td>
<td>3.281 **</td>
<td></td>
<td>0.213</td>
</tr>
<tr>
<td>CA I</td>
<td>-0.233 (0.094)</td>
<td>-0.093 (0.046) **</td>
<td>-0.127 (0.226)</td>
<td>0.04 (0.06)</td>
<td>4.948 **</td>
<td></td>
<td>0.215</td>
</tr>
<tr>
<td>IT I</td>
<td>-0.222 (0.064)</td>
<td>0.58 (0.113) **</td>
<td>-0.277 (0.117) **</td>
<td>-0.145 (0.038) **</td>
<td>-7.958</td>
<td></td>
<td>0.764</td>
</tr>
<tr>
<td>CA I</td>
<td>-0.22 (0.073)</td>
<td>-0.169 (0.089) *</td>
<td>-0.197 (0.134)</td>
<td>0.201 (0.043) **</td>
<td></td>
<td>20.974 **</td>
<td>0.559</td>
</tr>
</tbody>
</table>

Investment equation uses \( \Delta I_t = a + bI_{t-1} + c\Delta A_t + d_1\Delta\pi^p + d_2\Delta\pi^q + \epsilon_t \).

The current account equation uses \( \Delta CA_t = a + bI_{t-1} + c\Delta A_t + d_1\Delta\pi^p + d_2\Delta\pi^q + d_3\Delta TOT_t + \epsilon_t \), where \( \pi^p \) and \( \pi^q \) are permanent and transitory TOT shocks.

Standard errors are in parentheses.

** stands for significance at 5% level. * stands for significant at 10% level.

tₐ₁ > tₐ₂ \((tₐ₁ < tₐ₂)\) is the t-statistic used to test if permanent TOT component has a greater (smaller) effect on investment (the current account) than does transitory component.

Our empirical results also show that the persistence of TOT shocks plays an important role in determining how investment and current account respond to such economic disturbances. Permanent (transitory) TOT shocks tend to have negative (positive) effect on investment and the opposite is true for the current account, which are generally consistent with the prediction of our model and economic intuition. By applying a variance ratio index, we are able to identify transitory shocks as the major contributor of total variation in TOT shocks, which make a further justification of the observed HML effect for six Group-7 countries. These results provide additional support regarding the validity of intertemporal model.

Kent (1997) posits that the connection between persistence of TOT shock and the current account can be used as an indicator of which channel, the consumption or the investment, denominates the adjustment process of the current account. His argument is that if TOT shock is mainly temporary, then a positive correlation between TOT and the current account can be taken as evidence that consumption channel dominates because the investment channel is mainly active only when shock is permanent. In a sharp contrast, our theoretical model seems to suggest that even though investment does not respond to transitory productivity shock, it does react to transitory TOT shock. Our empirical results even show that transitory TOT component tends to have more cases of significant coefficients on the current account than does permanent TOT. Therefore it may be inaccurate and inappropriate to make the inference of domination between consumption and investment channels based on the connection between persistence of TOT shock and the current account.
Appendix B.1 Derivation of Equations

B.1.1 Derivation of investment price index \( p_I \)

Firms will maximize their investment composite \( I = I^1 I^{1-\omega} \) subject to an expenditure constraint \( p_I I = \pi I_1 + I_2 \). The first order Euler equation is

\[
\frac{I_2}{I_1} = \pi \frac{1 - \omega}{\omega},
\]

which can be substituted into the investment composite function. After some simple re-arrangement of terms, we can easily get a solution for investment price index

\[
p_I = \frac{\pi^\omega}{\omega^\omega (1 - \omega)^{1-\omega}}.
\]

B.1.2 Derivations of Eq. (2.5) and (2.6)

We linearize equation (2.4) and re-arrange terms by ignoring constants to obtain

\[
E_t(K_{t+1} + s_1 K_{t+1} + s_2 K_t + s_3 A_{t+1} + s_4 \pi_{t+1} + s_5 \pi_t) = 0 \quad (A-2.1)
\]

where

\[
\begin{align*}
s_1 &= - \left[ 1 + (1 + r)\pi^{1-\omega} + \frac{\alpha(1 - \alpha)AK^{\alpha-1}\pi^{1-\omega}}{g} \right], \\
s_2 &= (1 + r)\pi^{1-\omega}, \\
s_3 &= \frac{\alpha K^{\alpha}g^{1-\omega}}{g}, \\
s_4 &= \frac{K}{g^{\pi^\omega}}(\alpha AK^{\alpha-1} + \omega \pi^\omega - 1), \\
s_5 &= \frac{K}{g^{\pi^\omega}}[-(1 + r)\omega \pi^\omega - 1].
\end{align*}
\]
The linearized Euler equation (A-2.1) can be re-written as

\[ E_t[(K_{t+2} + x_0 K_{t+1}) + x_1 (K_{t+1} + x_0 K_t) + s_3 A_{t+1} + s_4 \pi_{t+1} + s_5 \pi_t] = 0, \quad (A-2.3) \]

where \( x_0 = \frac{s_1 + \sqrt{s_1^2 - 4s_2}}{2}, x_1 = \frac{s_1 - \sqrt{s_1^2 - 4s_2}}{2} \) if \( s_1^2 - 4s_2 \geq 0 \) which can be easily verified by using equations in (A-2.2). By taking a forward iteration, we can then solve for the dynamics of the capital stock as in equation (2.5) where \( \lambda_s \) are defined as \( \lambda_0 = -x_0, \lambda_1 = -\frac{1}{s_1}, \lambda_2 = -\frac{s_2}{s_1}, \lambda_3 = -\frac{s_3}{s_1}, \lambda_4 = -\frac{s_4}{s_1} \). The fact that \( 0 < \lambda_0, \lambda_1 < 1, \lambda_2, \lambda_3 > 0, \) and \( \lambda_4 < 0 \) can be easily proved with equations in (A-2.2). Since \( I_t = K_{t+1} - K_t \), we can easily get equation (2.6) by taking a first difference.

### B.1.3 Derivation of Eq.(2.13)

In a stochastic world, the budget constraint Eq.(2.11) can be re-written as

\[ B_t = -\sum_{i=0}^{\infty} \frac{1}{(1+r)^{i+1}} E_t(Y_{t+i} - p_{t+i} C_{t+i}). \]

With random walk dynamics of consumption, it can be re-written as:

\[ p_tC_t = r \left[ B_t + \frac{1}{1+r} \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} E_t(Y_{t+i}) \right], \quad (A-2.4) \]

or in a first difference format:

\[ \Delta(p_tC_t) = r \Delta B_t + \frac{r}{1+r} \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} [E_t(Y_{t+i}) - E_{t-1}(Y_{t-1+i})]. \quad (A-2.5) \]

We write the intertemporal budget constraint in its first difference form
\[ \Delta B_t = \tau B_{t-1} + Y_{t-1} - p_{t-1} C_{t-1}, \]

and use equation (A-2.4) to re-write it as

\[ \Delta B_t = Y_{t-1} - \frac{1}{1 + \tau} \sum_{i=0}^{\infty} \frac{1}{(1 + \tau)^i} E_{i-1}(Y_{t-1+i}). \]

So we plug the above equation into equation (A-2.5) and do some simplifying mathematics, we can get

\[
\Delta(p_tC_t) = \frac{r}{1 + \tau} \sum_{i=0}^{\infty} \frac{1}{(1 + \tau)^i} [E_{t+i}(Y_{t+i}) - E_{t-1}(Y_{t+i})]
= \frac{r}{1 + \tau} \sum_{i=0}^{\infty} \frac{1}{(1 + \tau)^i} \tilde{Y}_{t+i}.
\]

**B.1.4 Derivation of equation (2.18)**

From (2.1), (2.7), (2.8) and (2.2), we get

\[
\tilde{A}_{t+s} = \rho^s (A_t - \rho A_{t-1})
\]

\[
\tilde{K}_{t+s} = \begin{cases} 
\frac{\kappa_1 (A_t - \rho A_{t-1})}{\rho - \lambda_0} (\rho^s - \lambda_0^s) + \frac{\kappa_2 (\pi_t - \eta \pi_{t-1})}{\eta - \lambda_0} (\eta^s - \lambda_0^s), & s \geq 1 \\
0, & s = 0
\end{cases} 
\]

\[
\tilde{I}_{t+s} = \begin{cases} 
\frac{\kappa_1 (A_t - \rho A_{t-1})}{\rho - \lambda_0} (\rho^s - \lambda_0^s) + \kappa_1 \lambda_0^s (A_t - \rho A_{t-1}) \\
+ \frac{\kappa_2 (\pi_t - \eta \pi_{t-1})}{\eta - \lambda_0} (\eta^s - \lambda_0^s) + \kappa_2 \lambda_0^s (\pi_t - \eta \pi_{t-1}), & s \geq 1 \\
\kappa_1 (A_t - \rho A_{t-1}) + \kappa_2 (\pi_t - \eta \pi_{t-1}), & s = 0
\end{cases}
\]

\[
\tilde{\pi}_{t+s} = \eta^s (\pi_t - \eta \pi_{t-1})
\]
which can be used to obtain the present values of these four variables

\[ \sum_{s=0}^{\infty} \frac{\tilde{A}_{t+s}}{(1+r)^s} = \frac{1+r}{1+r-\rho} \left( A_t - \rho A_{t-1} \right), \]

\[ \sum_{s=0}^{\infty} \frac{\tilde{K}_{t+s}}{(1+r)^s} = \kappa_1 \left( \frac{1+r}{1+r-\rho} \frac{1}{1+r-\lambda_0} \right) \left( A_t - \rho A_{t-1} \right), \]

\[ \sum_{s=0}^{\infty} \frac{\tilde{I}_{t+s}}{(1+r)^s} = \kappa_1 \left( \frac{1+r}{1+r-\rho} \frac{\tau}{1+r-\lambda_0} \right) \left( A_t - \rho A_{t-1} \right), \]

\[ \sum_{s=0}^{\infty} \frac{\tilde{\pi}_{t+s}}{(1+r)^s} = \frac{1+r}{1+r-\eta} \left( \pi_t - \eta \pi_{t-1} \right). \]

Equation (2.18) is the result of a simple substitution of (A-2.7) into (2.16), where the direct solution to \( \phi_1 \) and \( \phi_2 \) are

\[ \phi_1 = \frac{r}{1+r-\rho} \left[ \alpha_A + \frac{\alpha_K + r\alpha_I}{1+r-\lambda_0} \kappa_1 \right], \]

\[ \phi_2 = \frac{r}{1+r-\eta} \left[ \alpha_A + \frac{\alpha_K + r\alpha_I}{1+r-\lambda_0} \kappa_2 \right]. \]

Evaluating production Euler equation (2.4) at steady state yields

\[ \alpha A K^{\alpha-1} = r \pi^{\omega-1} \]

which, combining equations in (2.15), gives us \( \alpha_K + r\alpha_I = 0 \). Therefore, \( \phi_1 \) and \( \phi_2 \) can be further simplified as \( \frac{r}{1+r-\rho} \alpha_A \) and \( \frac{r}{1+r-\eta} \alpha_\pi \).
B.1.5 Derivation of equation (2.19)

With equations (2.1), (2.7), (2.8) and (2.2), we can get

\[ E_{t-1}A_t - A_{t-1} = (\rho - 1)A_{t-1}, \]
\[ E_{t-1}K_t - K_{t-1} = I_{t-1}, \]
\[ E_{t-1}I_t - I_{t-1} = (\lambda_0 - 1)I_{t-1} + \kappa_1(\rho - 1)A_{t-1}, \]
\[ E_{t-1}\pi_t - \pi_{t-1} = (\eta - 1)\pi_{t-1}, \]

which can be substituted into equation (2.14) to get

\[ E_{t-1}Y_t - Y_{t-1} = [\alpha_A(\rho - 1) + \alpha_I\kappa_1(\rho - 1)]A_{t-1} + \]
\[ [\alpha_F(\eta - 1) + \alpha_I\kappa_2(\eta - 1)]\pi_{t-1} + [\alpha_K + \alpha_I(\lambda_0 - 1)]I_{t-1}. \]  

(A-2.8)

From equation (2.11), we know that

\[ \tilde{C}A_t = \tilde{Y}_t - (p_t\tilde{C}_t). \]

Since \((p_t\tilde{C}_t) = \Delta(p_tC_t)\) and \(\tilde{Y}_t = \alpha_A\tilde{A}_t + \alpha_K\tilde{K}_t + \alpha_I\tilde{I}_t + \alpha_{\pi}\tilde{\pi}_t\), so we can solve for

\[ \tilde{C}A_t = (\alpha_A + \alpha_I\kappa_1 - \phi_1)(A_t - \rho A_{t-1}) + (\alpha_F + \alpha_I\kappa_2 - \phi_2)(\pi_t - \eta\pi_{t-1}). \]  

(A-2.9)

\[ E_{t-1}C_t - C_{t-1} = 0 \] because of random walk behavior of consumption. Equation (2.19) therefore boils down to the reduced-form (2.19) when we apply (A-2.8) and (A-2.9).
Appendix B.2 Data Description

Macroeconomic variables, GDP, GNP, consumption, investment, and government expenditure, come from IMF's IFS online database, and are converted into real terms by GDP deflator (year 2000 = 100), and then into USD values using average market exchange rate for year 2000. Population data, which is mid-year values in annual format, comes from the US Census Bureau, International Data Base. Data on export and import prices are also from IFS. We convert them into real terms by setting year 2000 TOT to 1. For Italy, we use unit price of exports and imports since export and import prices are not available.

To obtain an estimate of $\omega$, the measure of the importance of domestic content in investment, we run a simple regression of logged investment on logged capital imports and use the coefficient on FDI as a proxy for $1 - \omega$. Annual capital imports data comes from NBER's International trade data website http://cid.econ.ucdavis.edu/data/undata/undata.html. Data is available for year 1962-2000 and in current USD. We add up SITC code starting with 7 to obtain total capital imports for each country.

Appendix B.3 Beveridge-Nelson Decomposition

Suppose variable $x_t$ is first difference stationary

$$\Delta x_t = x_t - x_{t-1} = \mu + \sum_{i=0}^{\infty} \lambda_i \nu_{t-i}, \quad a_0 = 1.$$  \hfill (A-2.10)
We can obtain an expression for \( x_{t+k} \) in the form of first-difference items:

\[
E_t(x_{t+k}) = x_t + \sum_{i=1}^{k} E_t(D_{t+i}), \quad \text{where } D_t = \Delta x_t = x_t - x_{t-1}.
\]  

(A-2.11)

Using equation (A-2.10), we know that

\[
E_t(D_{t+k}) = \mu + \sum_{i=k}^{\infty} \lambda_i v_{k+i-1}
\]

which can be substituted into equation (A-2.11) to get

\[
E_t(x_{t+k}) = x_t + k\mu + \sum_{j=1}^{k-1} \sum_{i=j}^{\infty} \lambda_i v_{k+1-j}.
\]

We define the permanent component of \( x_t \) as

\[
x_t^p = E_t(x_{t+k}) - k\mu, \quad \text{where } k \to \infty,
\]

and the transitory component as

\[
x_t^c = x_t - x_t^p.
\]

Since

\[
x_t^p - x_{t-1}^p = \mu + \sum_{i=0}^{\infty} \lambda_i v_t,
\]

and

\[
x_t^c = -\left( \sum_{j=1}^{\infty} \sum_{i=j}^{\infty} \lambda_i v_{t+1-j} \right), \quad \text{(A-2.12)}
\]
we say that any first-difference stationary time series can be decomposed into a permanent part, which follows a random walk with a drift, and a transitory part, which is stationary.

Next we show how to empirically estimate these two components. We know that instead of the MA representation in equation (A-2.10), we can equivalently write \( D_t \) in an ARMA\((p,q)\) form

\[
D_t = \sum_{i=1}^{p} \phi_i D_{t-i} + \mu(1 - \sum_{i=1}^{p} \phi_i) + \nu_t + \sum_{i=1}^{q} \theta_i v_{t-i}.
\]

Empirically, we choose the value of \( p \) and \( q \) to minimize AIC statistic. Now suppose we estimate the above equation in ARMA\((2,0)\) so that

\[
D_t = a_0 + a_1 D_{t-1} + a_2 D_{t-2} + \nu_t,
\]

which can be re-written in a matrix format

\[
\begin{bmatrix}
1 \\
D_t \\
D_{t-1}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 \\
a_0 & a_1 & a_2 \\
0 & 1 & 0
\end{bmatrix}\begin{bmatrix}
1 \\
D_{t-1} \\
D_{t-2}
\end{bmatrix} + \begin{bmatrix}
0 \\
\nu_t \\
0
\end{bmatrix},
\]

or compactly

\[
\Gamma_t = \Lambda \Gamma_{t-1} + \varepsilon_t.
\]

Using the above equation and (A-2.12) and allowing for a large enough \( k \), we can solve for the transitory component as

\[
x^*_k = k \frac{a_0}{1 - a_1 - a_2} - \begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix} \sum_{i=1}^{k} \Lambda^i \Gamma_t.
\]
The estimation procedure will be a little more complicated if the moving average part is not zero. Suppose the AIC suggests empirical estimation in the form of

\[ D_t = a_0 + a_1 D_{t-1} + a_2 D_{t-2} + \epsilon_t + b_1 \epsilon_{t-1}, \]

which can be written in a compact matrix form

\[
\Gamma_t = A\Gamma_{t-1} + \epsilon_t + B\epsilon_{t-1}, \quad \text{where } B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.
\]

The above ARMA matrix equation can be written in an equivalent AR equation

\[ M_t = CM_{t-1} + \Omega_t, \]

where \( M_t = [\Gamma_t \quad \Gamma_{t-1} \ldots \quad \Gamma_{t-n}]' \) for \( n \to \infty \), \( \Omega_t = [\epsilon_t \quad 0 \ldots \quad 0]' \), and

\[
C = \begin{bmatrix} A + B & (-B)(A + B) & (-B)^2(A + B) & \ldots & (-B)^{n-1}(A + B) \\ 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 & 0 \end{bmatrix}.
\]

Then the transitory component of \( x_t \) can be written as an expression of all estimated coefficients

\[
x_t^c = k \frac{a_0}{1 - a_1 - a_2} - [0 \quad 1 \quad 0][I_{3 \times 3} \quad 0 \ldots \quad 0] \sum_{i=1}^{k} C^i M_t.
\]
For both cases, the permanent component of $x_t$ is always obtained by subtracting transitory component from the original time series $x_t^p = x_t - x_t^c$. 
Essay 3

The Feldstein-Horioka Puzzle
Revisited

3.1 Introduction

In their renowned paper, Feldstein and Horioka (1980) (FH) documented the large correlation between national saving and investment. They interpreted this high association between two macro variables as implying low capital mobility. The economic intuition behind their reasoning is simple: in a world of perfect or high capital mobility, resources should be able to flow freely to the most efficient sector world-wide, thereby loosening the constraint that confines domestic investment to domestic saving. Under this circumstance, zero, or at least a considerably small, correlation should be obtained. Subsequent studies on the saving-investment correlation, despite their uses of different econometric tools and data sets spanning more recent years, continue to produce correlation coefficients that lie in parallel with FH's estimates. These results stand in apparent and sharp contrast to the
conventional wisdom on globalization that the international market is becoming increasingly integrated. This FH puzzle is one of the six puzzles in international macroeconomics identified by Obstfeld and Rogoff (2000b), and as put in the words of Obstfeld and Rogoff (2000a), is regarded by international economists as “posing the ‘mother of all puzzles’”.

The large literature on FH puzzle falls into two major categories. The first strand focuses on the interpretation of the estimation results. High correlation between saving and investment can be explained either by the effects on both investment and saving stemming from certain common factors, such as productivity shocks (Obstfeld 1986, Baxter and Crucini 1993), exchange risk (Frankel 1992), demand shock (Wen 2007), and monetary shock (Schmidt 2007), or by current account targeting policies taken by central governments (Bayoumi 1990). Some of these theoretical interpretations have been tested empirically (e.g. Kim 2001), while others have not.

Since high correlation between saving and investment can be justified even with a perfectly mobile capital market, it may not provide a reliable reference to the degree of capital mobility. Examples of alternative measures suggested in the literature are: real interest parity (Frankel 1992, Moosa 1996), correlation between output and consumption (Shibata and Shintani 1998)\(^1\), and the current account volatility predicted based on an intertemporal model as compared to that from actual data (Ghosh 1995)\(^2\).

The second strand tries to find new evidence on FH puzzle by extending technical

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\(^1\)The author derives a reduced-form equation based on an intertemporal model and proposes that the regression coefficient of output on consumption can be taken as a measure of capital mobility. In empirical study, he concludes that perfect capital mobility is found for half of the countries, but not for US and Japan.

\(^2\)For a detailed literature review and more detailed technical explanation, please refer to my first paper.
aspects of the puzzle. Using 16 OECD countries for 1960-1974, the original FH paper estimates a cross-sectional equation

\[
\bar{I}_t = \alpha + \beta^{BE} \bar{S}_t + e_t,
\]

(3.1)

where \(\bar{I}\) and \(\bar{S}\) are investment and saving expressed as ratios to GDP and averaged over time period. The between-estimator (BE) \(\beta^{BE}\), or the saving's retention coefficient as is known in the literature, is roughly 0.65. Later works replicate FH's estimation by applying up-to-date econometric methodologies, e.g. fixed-effect (FE) model (Krol 1996), mean-group (MG) model (Coakley, Fuertes and Spagnolo 2004), error-correction model (ECM) with time-series data (Jansen 1996), pooled mean-group model (PMG) (Pelgrin and Schich 2004), and by using data sets spanning more recent years or covering less developed countries (Mamingi 1997).

This paper is more akin to the second category. We do not intend to argue over the validity of whether or not the saving's retention coefficient is a good indicator of capital mobility. Instead, we try to make inferences about capital mobility from both long-run and short-run correlation coefficients, and especially the latter, between investment and saving. We are particularly interested in the short-run coefficient because economic theory usually provides some guidance regarding the long-run coefficient, but is typically silent on the short-run idiosyncratic dynamics. Based on the intertemporal general equilibrium model of the open economy, the current account should be stationary in the long-run to reflect the intertemporal budget constraint or solvency constraint. Therefore saving and investment will always move together in the long-run since the current account is simply the difference between these two variables. High correlation in the long-run reflects more the condition of
cointegration than the degree of capital mobility. The short-run correlation between saving and investment, on the other hand, is a relatively more reliable indicator of short-run capital mobility because the intertemporal budget constraint is less likely to be binding.

Traditional methodology of estimating long-run and short-run correlations is mostly based on the application of various forms of ECM. For example, Coiteux and Olivier (2000) estimate an unrestricted error correction model, allowing the long-term equilibrium relationship between saving and investment to be estimated, instead of being imposed a priori. To obtain the short-run coefficient, they use Engle and Granger two-step method. First, they estimate a fixed effect model on panel data to get the error correction terms for each country. Second, they use pooled data to estimate the short-run coefficient $\beta$ using a model

$$\Delta I_t = \alpha + \beta \Delta S_t + \gamma(S_{t-1} - \hat{d}_i - \hat{b}I_{t-1}) + \epsilon_t. \quad (3.2)$$

Equation (3.2) can be re-arranged to accommodate the theoretically prescribed $(1, -1)$ cointegration vector between investment and saving

$$\Delta I_t = \alpha + \beta \Delta S_t + \gamma(S_{t-1} - I_{t-1}) + \delta S_{t-1} + \epsilon_t. \quad (3.3)$$

Equation (3.3), used in Jansen (1996) and Taylor (1997), is a general form of ECM that can be estimated with only one step (as compared to Engle-Granger's two step ECM). Additionally, various model specifications can be tested by making restrictions on parameters in equation (3.3). For example, by making restrictions $\gamma = 1$, $\beta - \delta = 1$, equation (3.3) encompasses level regression as in equation (3.1); and if $\gamma = \delta = 0$, it embodies OLS regression in first difference.
The validity of ECM builds upon the assumption that dependent and independent variables are cointegrated. The statistical power of cointegration tests is renownedly low, especially in the case of panel data. Even though economic theory supports the long-run connection between investment and saving, empirical results regarding cointegration between investment and saving for OECD countries are limited and mixed (Coakley et al. 2004). From an econometric point of view, a first-differenced model should be used in place of ECM if no cointegration is found statistically. 3

In the most recent years, some works start to investigate the short-run correlation using a PMG model, which originates from an autoregressive, distributed lag (ARDL) dynamic process, typically of order (1,1), as assumed in Pelgrin and Schich (2004) and Bebczuk and Schmidt-Hebbel (2006), and takes the transformed form of an error correction model in empirical estimation. The PMG model first estimates the long-run correlation coefficient using pooled data

\[ I_{i,t} = a + b S_{i,t} + v_{i,t}, \]

then an error correction model is estimated for each country using

\[ \Delta I_{i,t} = \alpha + \beta \Delta S_{i,t} + \gamma (I_{i,t-1} - b S_{i,t}) + e_{i,t}. \]

3 Though a level regression would render superconsistent estimator and is considered superior to first-differenced model in case of cointegration, empirical study by Jansen (1996) shows that using first-differenced model produces very similar results to an ECM. 4 Assume that investment and saving are connected through a simple ARDL(1,1) process,

\[ I_{i,t} = \lambda_0 + \lambda_1 S_{i,t} + \lambda_2 S_{i,t-1} + \lambda_3 I_{i,t-1} + e_{i,t}, \]

which can be rearranged in the form of equation (3.4) where \( \alpha_i = \lambda_0, \beta_i = \lambda_3, \gamma_i = \lambda_1 - 1, \) and \( b = \frac{\lambda_1 + \lambda_3}{\lambda_2}. \) Please note the mathematical format difference between traditional ECM and PMG model. The error correction term in the former is \( I_{i,t-1} - b S_{i,t-1} \) while in the latter is \( I_{i,t-1} - b S_{i,t}. \) The will make a difference when it comes to size of estimated coefficient. We will have a more
The PMG estimator is simply the average of $\hat{\beta}_i$. Though there is an increasing amount of FH-related work done using dynamic models, we will stick to static panel models that see most instances in the literature because first they would render estimation results readily comparable to earlier empirical results, and second, Herwartz and Xu (2006) show, based on cross-validation criteria, that dynamic models are generally outperformed by static panel models.

In this paper, we take an alternative method which enables us to estimate both long-run and short-run correlation while circumventing the problem of cointegration. In particular, we decompose saving and investment into their respective permanent and transitory components. We define long-run correlation the estimated coefficient on permanent saving, and short-run correlation the coefficient on transitory saving. A similar approach is also used in Sarno and Taylor (1998) who studied the case of the UK using quarterly data for 1955:1-1994:4. Our paper differs from Sarno and Taylor in several perspectives. First, they use a bi-variate vector autoregressive (BVAR) model, attributed to Blanchard and Quah (BQ), to implement decomposition, but we use two univariate decomposition models, one being Beveridge-Nelson (BN) and the other one being HP filler. In Sarno and Taylor's (1998) BVAR model, one variable is investment or saving and the second one is the GDP growth rate. To identify permanent and transitory shocks, the authors assume that temporary shocks have zero effect on investment (saving) in the long-run. Though BQ decomposition has the advantage of integrating economic theory for identifying purposes, as claimed by the authors, what they failed to notice is that BQ decomposition actually implies that permanent and transitory series follow a similar dynamic path.

detailed discussion on the difference in section 3.4.4.

Examples are dynamic OLS (Hussein 1998) autoregressive dynamic linear (Abbott and Vita 2003), and dynamic GLS (Caporale, Panopoulou and Pittis 2006)
(see appendix for technical explanation), which in our view does not fit in the particular study of correlation between saving and investment. There are economic and empirical reasons to believe that long-run and short-run correlations follow different time-series paths. Second, we use panel econometric models for 24 OECD countries with annual data for 1960-2004 while they use time-series model for only one country. Third, the definition of the saving-investment correlation is different. In our paper, it is defined as regression coefficient of investment on saving, which is commonly used in the literature. In Sarno and Taylor, they use statistical correlation between changes in saving and investment. To the best of our knowledge, this is the first paper in the literature that studies short-run and long-run saving-investment correlation with panel data using a decomposition framework.

Section 2 reviews statistical properties of three static panel estimators, BE, FE and MG, in a simple Monte Carlo exercise, where the data generating process is calibrated to our particular data set. Section 3 describes data sources and some stylized facts that are typically found for such data in the literature. Section 4 investigates the long-run and short-run saving-investment correlations and reports our major empirical findings. Section 5 concludes.

### 3.2 Three panel estimation models

We want to briefly review three static panel models which we will use in our empirical study, the BE model, which is originally used in the FH in the 80s, the FE model, which sees frequent empirical applications in FH puzzle related works in the 90s, and the MG model, which becomes a popular tool in late 90s and the new millennium.

The BE model is not a widely used panel estimation method. It is applied
especially when the time-series data for each individual are thought to be somewhat inaccurate or when they are assumed to contain random deviations from their long-run means. This is why the estimator in equation (3.1) is typically termed the long-run correlation. And FH believe that by using a BE estimator, the endogenous problem, i.e. correlation due to some factors, such as productivity shocks, having common effect on both investment and saving, can be taken fairly good care of when time-series data are averaged over a period of a fairly long time-span. But a BE estimator imposes equality in both slope and intercept of the regression model across all group members, thereby ignoring all the individual-specific variation in the relation between saving and investment.

The FE model uses regression equation

\[ I_{i,t} = \alpha_i + \beta^{FE} S_{i,t} + e_{i,t}, \]  

(3.5)

where intercept \( \alpha_i \) is allowed to vary across individuals, therefore captures all idiosyncratic features, if there are any. The slope coefficient, though, is still restricted to be homogenous for all individuals. The FE model seems to be a better estimator than the BE estimator because it preserves the theoretic prediction that saving and investment should move together in the long-run due to the intertemporal budget constraint while reconciling with ample empirical evidence that there exists country effect (e.g. Ho and Chiu 2001), or considerable cross-country heterogeneity (e.g. Jansen 1998, Taylor 1997). Ho (2002) shows that FE panel model reduces the correlation's estimate by about 0.12, as compared to BE in original FH paper.

Introduced by Pesaran and Smith (1995), the MG estimator starts to be used in examining FH puzzle in the new millennium (e.g. Coakley et al. 2004, Bebczuk
and Schmidt-Hebbel 2006). Both intercepts and slope are considered heterogeneous across all individuals, and the MG estimator is calculated as the group average. In formula, the MG estimator and its associated standard error can be expressed as

\[
\hat{\beta}_{MG} = \frac{\sum_{i=1}^{N} \hat{\beta}_i}{N},
\]

(3.6)

\[
\sigma_{\hat{\beta}_{MG}} = \sqrt{\frac{\sum_{i=1}^{N} (\hat{\beta}_i - \hat{\beta}_{MG})^2}{N(N-1)}},
\]

(3.7)

where \(N\) is the total number of units in the panel and \(\hat{\beta}_i\) is the coefficient on saving from a regression using each individual's time-series data.

Next we would like to outline two econometric issues related to these panel estimators that are particularly relevant to our interest in investigating saving and investment correlation. First, there is the problem of non-stationary regressors. As a stylized fact related to FH puzzle, both saving and investment series are generally found to be non-stationary in levels, which we will verify again in the next section. With time-series data, the OLS estimator will not be consistent unless the regressors and regressand are cointegrated. Phillips and Moon (1999) show (in equations (5.12) and (5.13)) that under weakly exogenous conditions, the pooled least square estimator, either across individuals or as a time average, can be consistent for the true coefficient in a large \(N\) or \(T\) panel. The potential danger of spurious regression is reduced by pooling or averaging over groups. Additionally, pooled OLS, FE and MG estimators are all unbiased and consistent (Coakley et al. 2004).

Second, there is the issue of the small sample. General panel models call for large \(N\) and/or large \(T\) asymptotics. The performance of these estimators in small sized data sets are subject to practical application. Evaluating the BE, FE, and MG estimators in rigorously expressed equations and formula is beyond the purpose of
this paper. Instead, we will give reader a visual presentation based on the results from a simple Monte Carlo simulation exercise, where the data generating process (DGP) is calibrated to match the panel dimensions and some other specific properties of our data set. There are three simulated panels. In panel 1, all variables are \( I(1) \) and cointegrated, therefore the regression residuals are stationary or \( I(0) \). The slope coefficient is homogenous across all groups. In panel 2, all variables are \( I(1) \), but not cointegrated. Therefore residuals are \( I(1) \). The slope coefficient is still homogenous. In panel 3, all variables are \( I(0) \) (there is no cointegration problem in this case), but with heterogenous slope. The number of time periods, \( T \), takes values in \( \{10, 30, 50\} \), and the number of units, \( N \), is set to 20. Data are generated on a dependent variable \( \bar{y}_{i,t} \) and an exogenous regressor \( \bar{x}_{i,t} \) for units \( i = 1, 2, ..., N \) and time periods \( t = 1, 2, ..., T \), according to:

\[
\bar{y}_{i,t} = \beta_i \bar{x}_{i,t} + \zeta_{i,t} 
\]

(3.8)

\[
\bar{x}_{i,t} = \gamma \bar{x}_{i,t-1} + \zeta_{i,t}, \quad \zeta_{i,t} \sim IN(0, \sigma_{\epsilon,t}^2) 
\]

(3.9)

\[
\zeta_{i,t} = \rho_i \zeta_{i,t-1} + \epsilon_{i,t}, \quad \epsilon_{i,t} \sim IN(0, \sigma_{\epsilon,t}^2). 
\]

(3.10)

In the case of panel 1, we will set \( \beta_i = 1, \gamma = 1, \) and \( \rho_i = 0.5^6 \). In panel 2, all parameters stay the same except that we set \( \rho_i = 1 \) so that the residual from \( \bar{y}, \bar{x} \) regression is non-stationary. In the case of panel 3, we use \( \beta_i \sim U[0.5 \pm 0.2]^7, \gamma = 0.5^8, \) and \( \rho_i = 0.5 \) for all units. Additional heterogeneity is introduced by allowing

---

6For 10 out of 24 OECD countries in our sample, saving and investment appear to have a cointegration relationship. For these cointegrated countries, the AR(1) coefficient for saving-investment regression residuals is roughly 0.5.

7We impose \( \gamma=1 \) in panel 1 to accommodate the intertemporal budget constraint that economists would usually impose when estimating an ECM, like the one in equation (??). In our empirical estimations, the short-run correlation is about half the size of long-run correlation on average. Therefore, we just assume that on average \( \gamma=0.5 \) in panel 3.

8This roughly matches the AR(1) coefficient on permanent saving series decomposed using HP
for heteroscedasticity across units. This is an important assumption because MG and FE estimator will be equivalent if $\sigma^2_{e,t}$ is uniform in homogenously sloped panels (Coakley, Fuertes and Smith 2001). We use $\sigma_{e,t} \sim U[0.0014 \pm 0.0005]$ for panel 1 and 2, and $U[0.0021 \pm 0.0007]$ for panel 3. For simplicity, $\sigma_{e,t}$ is assumed to be the same across units, $\sigma_e = 0.0015$ for panel 1 and 2, and $\sigma_e = 0.0038$ for panel 3.\(^9\)

We run 2000 scenarios for each $(N, T)$ combination and report the estimated $\beta$s and their associated $t$-statistics in table 3.1. When the slope is homogenous and variables are cointegrated (panel 1), all three estimators are unbiased and in terms of efficiency, BE has the smallest standard deviation in all $T$ cases, followed by FE and then MG. In the case of non-cointegration (panel 2), all three estimators remain unbiased, but their standard deviations are much larger than in panel 1. The dispersion increases most for BE with $T = 50$ (by a factor of 30), and FE and MG are similar in the order of magnitude. The superiority of FE over BE estimator in terms of efficiency implies that spurious problem can be considerably weakened in demeaned series. The most interesting results come from panel 3 where DGP assumes heterogenous slope with stationary variables. Presumably, we would expect MG to perform the best because after all it is the only estimator among all three that integrates the heterogeneity assumption. We are a little surprised that all three estimators are equally unbiased and MG is actually the least efficient one.

Generally, critical values, readily available from statistical softwares such as STATA and SAS, are calculated from asymptotics. When sample size decreases, $t$-statistics will become larger, implying potentially increasing inaccuracy in direct comparison of calculated $t$ statistics with those traditional critical values. This size effect can be shown in the last three columns of table 3.1, where $t$-statistics are

\(^9\) $\sigma_e$ and $\sigma_e$ values are also calibrated to match the dynamic features of our data.
Table 3.1: Monte Carlo simulation for panel regression

<table>
<thead>
<tr>
<th></th>
<th>Regression results</th>
<th>Size effect</th>
<th></th>
<th></th>
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<tr>
<td></td>
<td>T=10</td>
<td>T=30</td>
<td>T=50</td>
<td>T=10</td>
<td>T=30</td>
<td>T=50</td>
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<tr>
<td></td>
<td>SM SSD</td>
<td>SM SSD</td>
<td>SM SSD</td>
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<tr>
<td>Panel 1</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>β</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BE</td>
<td>0.9995 (0.041)</td>
<td>0.9999 (0.016)</td>
<td>0.9999 (0.009)</td>
<td>4.741</td>
<td>1.863</td>
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</tr>
<tr>
<td>FE</td>
<td>0.9976 (0.084)</td>
<td>0.9996 (0.034)</td>
<td>0.9990 (0.022)</td>
<td>3.971</td>
<td>1.572</td>
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<tr>
<td>MG</td>
<td>0.9982 (0.108)</td>
<td>0.9996 (0.042)</td>
<td>0.9994 (0.028)</td>
<td>3.892</td>
<td>1.500</td>
<td>1</td>
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</tr>
<tr>
<td>t-stat</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BE</td>
<td>-0.0173 (1.070)</td>
<td>0.0051 (1.033)</td>
<td>-0.0069 (1.040)</td>
<td></td>
<td></td>
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<tr>
<td>FE</td>
<td>-0.0401 (1.418)</td>
<td>-0.0152 (1.567)</td>
<td>-0.0760 (1.627)</td>
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<tr>
<td>MG</td>
<td>-0.0185 (1.084)</td>
<td>-0.0115 (1.028)</td>
<td>-0.0286 (1.076)</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Panel 2</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>β</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BE</td>
<td>1.0029 (0.251)</td>
<td>1.0088 (0.247)</td>
<td>1.0017 (0.251)</td>
<td>1.001</td>
<td>0.984</td>
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</tr>
<tr>
<td>FE</td>
<td>0.9994 (0.155)</td>
<td>0.9994 (0.151)</td>
<td>1.0009 (0.144)</td>
<td>1.078</td>
<td>1.050</td>
<td>1</td>
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</tr>
<tr>
<td>MG</td>
<td>0.9971 (0.163)</td>
<td>0.9992 (0.154)</td>
<td>1.0017 (0.144)</td>
<td>1.136</td>
<td>1.070</td>
<td>1</td>
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</tr>
<tr>
<td>t-stat</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BE</td>
<td>0.0101 (1.039)</td>
<td>0.0391 (1.036)</td>
<td>0.0039 (1.054)</td>
<td></td>
<td></td>
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<tr>
<td>FE</td>
<td>-0.0022 (1.994)</td>
<td>-0.0117 (3.505)</td>
<td>0.0273 (4.329)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>MG</td>
<td>-0.0204 (1.038)</td>
<td>-0.0052 (1.075)</td>
<td>0.0043 (1.017)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Panel 3</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>β</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>BE</td>
<td>0.4998 (0.070)</td>
<td>0.5005 (0.029)</td>
<td>0.4990 (0.016)</td>
<td>4.298</td>
<td>1.782</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>FE</td>
<td>0.4969 (0.148)</td>
<td>0.4996 (0.064)</td>
<td>0.4991 (0.039)</td>
<td>3.784</td>
<td>1.630</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>MG</td>
<td>0.4986 (0.184)</td>
<td>0.5006 (0.075)</td>
<td>0.4984 (0.048)</td>
<td>3.819</td>
<td>1.561</td>
<td>1</td>
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</tr>
<tr>
<td>t-stat</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BE</td>
<td>0.2061 (1.041)</td>
<td>-0.0112 (1.080)</td>
<td>0.0199 (1.123)</td>
<td></td>
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<tr>
<td>FE</td>
<td>-0.0259 (1.398)</td>
<td>-0.0134 (1.640)</td>
<td>-0.0333 (1.560)</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>MG</td>
<td>-0.0062 (1.077)</td>
<td>0.0129 (1.080)</td>
<td>-0.0367 (1.045)</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Notes: Panel 1 is homogenous slope with cointegrated I(1) variables. Panel 2 is homogenous slope with non-cointegrated I(1) variables. Panel 3 is heterogenous slope with I(0) variables. For each panel, we run 2000 scenarios. In each scenario, we estimate β and its associated t-statistics in equation $y_{t,4} = \beta x_{t,4} + u_{t,4}$ using between-estimation (BE), fixed effect (FE), and mean group (MG). SM (SSD) stands for sample mean (sample standard deviation) over 2000 scenarios. Size effect evaluates SSD of β as a ratio to the base, $T = 50$ case.
reported as ratios to their corresponding \( T=50 \) cases. On average, \( t \)-statistics for \( T = 30 \) (\( T=10 \)) are twice (four times) as much as \( T =50 \) for panel 1 and 3. Interestingly, this size effect almost disappears in panel 2, where variables are not cointegrated.

Figure 3.1 shows the estimated cumulative distribution function of \( t \)-statistics for all three panels for \( T = 10 \) and \( T = 50 \) cases. Theoretically, \( t \)-statistics should follow a normal distribution with zero mean and unit standard deviation. For all cases, \( t \)-statistics from FE model seem to always have a fatter tail than BE and MG models. With homogenous slope, both BE and MG have \( t \)-statistics tracking closely to a standard \( N(0,1) \) curve, suggesting reasonably accurate inference. But in the case of a heterogenous slope, especially when \( T \) is small, the BE curve shifts to the right of the normal distribution curve while the MG curve remains unbiased.

To summarize the results from our simple Monte Carlo simulation exercise, MG may be considered the best estimator among all three model candidates when we need to take into account several factors that are particularly relevant to investigating the FH puzzle in OECD countries, such as non-stationary variables of interest, non-cointegration between regressor and regressand, and the small sample effect. Though the superiority of the MG model, over traditional POLS or FE models, has been tested and confirmed by quite a few authors (e.g. Coakley et al. (2004)) in large panels, we provide here additional evidence that even in very small samples, the model can have a relatively sound performance. And it is interesting to see that BE estimator, though having been put under much criticism for imposing unrealistically stringent assumptions on homogeneity, performs surprisingly well even in the presence of heterogeneity across panel groups. However, this result needs to be interpreted with caution. With the existence of heterogeneity of group units, the
Figure 3.1: Empirical cumulative distribution function of t-statistics

(a) Panel 1, Homogenous slope with cointegrated I(1) variables

(b) Panel 2, Homogenous slope with non-cointegrated I(1) variables

(c) Panel 3, Heterogenous slope with I(0) variables
unobserved idiosyncratic attributes will be picked up by the error term \( e_i \) in equation (3.1), making \( e_i \) and \( \bar{S}_i \) correlated with each other.\(^{10}\) Therefore BE is inconsistent, even though it seems to be unbiased in our simulation exercise in most cases.

### 3.3 Data and some stylized facts

Annual observations on saving, investment and GDP for the period 1960-2004 are gathered for 24 OECD countries\(^{11}\). Data come from various issues of OECD's publication on National Account Composite. Saving is defined as gross saving (net saving\(^{12}\) plus consumption of fixed capital\(^{13}\)) over GDP. Investment is defined as gross capital formation over GDP. All series are in current local currencies.

Figure 3.2 plots period-average saving and investment for all countries. Saving and investment, as ratios to GDP, generally fall between 0.1 to 0.4, and their combination follows a nice linear relationship with a positive slope between 0 and 1, except for the case of Luxembourg. The problem of Luxembourg being an outlier is discussed in detail in Coiteux and Olivier (2000) and Jansen (2000) where the authors try to refute Krol's (1996) proposition that the FE model can reduce saving's retention coefficient remarkably. They show that the big drop in correlation is actually due to the inclusion of Luxembourg in the panel regression. Some of the potential explanations to the outlier problem of Luxembourg can be traced

\(^{10}\)Suppose the true model between \( I \) and \( S \) is \( I_{it} = \alpha_i + \beta_i S_{it} + u_{it} \), but a BE model is estimated as in equation (3.1). Then the residual term \( e_i = (\alpha_i - \alpha) + (\beta_i - \beta^BE) \bar{S}_i + u_i \), and it is correlated with \( \bar{S} \).

\(^{11}\)They are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, UK and USA.

\(^{12}\)The main components of the OECD definition of net saving are: personal saving, business saving (undistributed corporate profits), and government saving (or dissaving).

\(^{13}\)The main components of the OECD definition of consumption of fixed capital are the capital consumption allowances (depreciation charges) for both the private and the government sector.
Notes: Investment and saving are expressed as ratios to GDP, and averaged over the period of 1960-2004.

down to its large international banking sector (Als 1988) and its current account surplus that runs consistently on the order of 30 percent of its GDP (Blanchard and Giavazzi 2002). Therefore it has traditionally been excluded in estimation. In the paper, we show that Luxembourg does not have to be excluded if the appropriate estimation technique is applied. Ho (2002) also revisited the Luxembourg issue using dynamic OLS and fully-modified OLS models. He found that inclusion of Luxembourg does not make a significant difference in estimation results, and that what matters is the power of the estimation technique.

Next we verify that some of the particular features that we impose on DGP used in our Monte Carlo simulation exercise are present in the time-series of saving and investment for OECD countries. In the first four columns of table 3.2, we show the
Dickey-Fuller (DF) unit root test results for saving and investment. Both variables are insignificant in level, but significant after taking a first difference. Therefore saving and investment are generally non-stationary \( I(1) \) variables for OECD countries. In the next step, we check the empirical evidence on cointegration relationship between saving and investment, and the results are reported in the last column of the table. We use Johansen's (1995) trace test based on a bi-variate VAR model.\(^{14}\) Half of the countries have significant \( z \) statistics at 10%, suggesting cointegration for these economies, while the others have non-stationary residuals from level regression, an indication of non-cointegration. In order to estimate saving-investment correlation, Coiteux and Olivier (2000) formed two panel data sets, using ECM for those OECD countries where cointegration is found and using first-differenced model for non-cointegrated countries. Here we can avoid this trouble by pooling all countries into one data set with mixed \( I(0) \) and \( I(1) \) residuals as we have shown in the simulation exercise that panel estimators can be reasonably accurate in presence of both.\(^{15}\)

\(^{14}\)The order of lags used in VAR is determined by a combination of three information criteria, Akaike's (AIC), Schwarz's Bayesian (SBIC), and Hannan Quinn's (HQIC). We pick the order of lags that two or more of these three information criteria agree. It is 2 for Iceland, Japan, New Zealand, and Portugal, and 1 for the rest of 20 OECD countries.

\(^{15}\)Coakley et al. (2004) show, in a Monte Carlo simulation that MG performs well in presence of mixed \( I(0) \) and \( I(1) \) residuals in large sample. We also run our small sample Monte Carlo simulation with mixed residuals and the results stand in between those with \( I(0) \) and \( I(1) \) residuals. So for expositional simplicity, we did not report these results in section 3.2.
Table 3.2: Unit root and cointegration tests of investment and saving for 24 OECD countries

<table>
<thead>
<tr>
<th></th>
<th>Unit root test</th>
<th>Cointegration test</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>I</td>
<td>S</td>
</tr>
<tr>
<td>Australia</td>
<td>-2.727*</td>
<td>-1.583</td>
</tr>
<tr>
<td>Austria</td>
<td>-1.600</td>
<td>-1.364</td>
</tr>
<tr>
<td>Belgium</td>
<td>-1.771</td>
<td>-1.419</td>
</tr>
<tr>
<td>Canada</td>
<td>-2.140</td>
<td>-1.747</td>
</tr>
<tr>
<td>Denmark</td>
<td>-1.898</td>
<td>-1.716</td>
</tr>
<tr>
<td>Finland</td>
<td>-1.840</td>
<td>-2.079</td>
</tr>
<tr>
<td>France</td>
<td>-2.415</td>
<td>-0.838</td>
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<tr>
<td>Germany</td>
<td>-1.383</td>
<td>-1.826</td>
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<tr>
<td>Greece</td>
<td>-2.284</td>
<td>-1.785</td>
</tr>
<tr>
<td>Ireland</td>
<td>-2.032</td>
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<tr>
<td>Italy</td>
<td>-1.990</td>
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<td>Japan</td>
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<td>-0.457</td>
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<tr>
<td>Netherlands</td>
<td>-1.671</td>
<td>-2.488</td>
</tr>
<tr>
<td>New Zealand</td>
<td>-3.090**</td>
<td>-2.742*</td>
</tr>
<tr>
<td>Norway</td>
<td>-1.575</td>
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<td>Portugal</td>
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<tr>
<td>Spain</td>
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<td>Sweden</td>
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<td>Turkey</td>
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<tr>
<td>UK</td>
<td>-2.445</td>
<td>-2.003</td>
</tr>
<tr>
<td>US</td>
<td>-1.682</td>
<td>-1.166</td>
</tr>
</tbody>
</table>

Notes: For unit root test, we report $Z_t$ from Dickey-Fuller unit root test. For cointegration test, we use Johansen's (1995) trace test. The null hypothesis is no cointegration. Critical value at 5% is 15.41.
3.4  Empirical estimation

3.4.1 A benchmark estimation

To create a benchmark, we estimate saving’s retention coefficient for the whole sample period using BE, FE, and MG models\(^{16}\). Panel (a) of table 3.3 reports estimation results using three models for OECD data with or without Luxembourg. When Luxembourg is excluded, three models generate correlation estimates of the same magnitude, around the level of 0.6, which is consistent with the two-third rule typically reported in all earlier works. Inclusion of Luxembourg drags down coefficient estimates for BE and FE models dramatically by 40-50\%, however it has only marginal effect on MG estimator. And all three estimators are significantly different from zero. Our estimation confirms the empirical results of Krol (1996) who reports a significant drop of saving’s retention coefficient when FE, instead of BE, is used, though such is refuted by Jansen (2000) and Coiteux and Olivier (2000) who claim that the drop is more attributed to inclusion of Luxembourg in the panel estimation rather than choice of different estimation model. We will examine the problem of Luxembourg in further detail in the following sections.

3.4.2 Estimation with decomposed series

In this paper, we define long-run and short-run correlation in a way different from what’s been traditionally used in the literature. We decompose investment and saving series into their respective permanent and transitory components, and regress permanent and transitory saving on its investment counterpart. The coefficient

\(^{16}\text{Being inputs to MG estimator, the individual country’s correlation estimates can be obtained by either an OLS regression, or a SUR regression. We used the latter because Coakley et al. (2004) show that OLS is inferior.}\)
Table 3.3: Investment-saving correlation for OECD countries: 1960-2004

<table>
<thead>
<tr>
<th></th>
<th>Without Luxembourg</th>
<th>With Luxembourg</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>( \beta )</strong></td>
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<tr>
<td>Panel (a) original saving and investment series</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BE</td>
<td>0.628 (0.094) **</td>
<td>0.306 (0.065) **</td>
</tr>
<tr>
<td>FE</td>
<td>0.303 (0.026) **</td>
<td>0.409 (0.023) **</td>
</tr>
<tr>
<td>MG</td>
<td>0.625 (0.072) **</td>
<td>0.596 (0.076) **</td>
</tr>
<tr>
<td>Panel (b) decomposed saving and investment series</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BN(p)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BE</td>
<td>0.627 (0.094) **</td>
<td>0.303 (0.065) **</td>
</tr>
<tr>
<td>FE</td>
<td>0.591 (0.026) **</td>
<td>0.402 (0.023) **</td>
</tr>
<tr>
<td>MG</td>
<td>0.624 (0.072) **</td>
<td>0.594 (0.074) **</td>
</tr>
<tr>
<td>HP(p)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BE</td>
<td>0.628 (0.094) **</td>
<td>0.306 (0.065) **</td>
</tr>
<tr>
<td>FE</td>
<td>0.594 (0.024) **</td>
<td>0.439 (0.022) **</td>
</tr>
<tr>
<td>MG</td>
<td>0.614 (0.072) **</td>
<td>0.61 (0.105) **</td>
</tr>
<tr>
<td>BN(t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BE</td>
<td>-0.005 (0.833)</td>
<td>-0.087 (0.512)</td>
</tr>
<tr>
<td>FE</td>
<td>0.225 (0.035) **</td>
<td>0.218 (0.033) **</td>
</tr>
<tr>
<td>MG</td>
<td>0.201 (0.347)</td>
<td>0.225 (0.382)</td>
</tr>
<tr>
<td>HP(t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BE</td>
<td>-0.039 (0.188)</td>
<td>0.029 (0.178)</td>
</tr>
<tr>
<td>FE</td>
<td>0.421 (0.030) **</td>
<td>0.364 (0.028) **</td>
</tr>
<tr>
<td>MG</td>
<td>0.491 (0.069) **</td>
<td>0.479 (0.069) **</td>
</tr>
</tbody>
</table>

Notes: This table reports \( \beta \) as in regression \( I = \alpha + \beta S \), where \( I \) and \( S \) are original rates of investment and saving in panel 1, permanent components of investment and saving in BN(p) and HP(p) of panel 2, and transitory components of investment and saving in BN(t) and HP(t) of panel 2. Standard errors are in parentheses. BE is between-estimator, FE is fixed effect estimator, and MG is mean group estimator. BN(p) refers to regression using permanent saving and investment decomposed by BN method. BN(t), HP(p) and BN(t) have similar definitions.
estimated from a permanent regression model is called long-run correlation, and that from a transitory model is called short-run correlation. There may be some conceptual concerns about the appropriateness in defining a regression estimate on the permanent component model as long-run saving-investment correlation, and that on the transitory model as short-run correlation because these definitions are different from traditional ones. Though Sarno and Taylor (1998) used a similar approach to ours, they did not give any explanations regarding this conceptual concern.

The permanent component represents the trend of a time series. It shows the dynamics of the time series variable if short of economic shocks. Therefore it would make perfect sense to examine the long-run relationship between two variables by evaluating the correlation between their trends or permanent components. The transitory component represents the cycle or the deviation of the series from their trend (permanent component). Therefore, defining the short-run relation as the correlation between transitory components of two variables is comparable to defining the short-run relation as the correlation between first-differenced variables as in Jansen (1997). The approach taken in this paper may not be a better measurement of long-run and short-run correlations than those in the literature, but it provides, at least, an alternative approach that can be comparable to the current ones.

Following Sarno and Taylor (1998), we will use the BN decomposition method. The basic intuition behind it is simple: any non-stationary \( I(1) \) variable can be decomposed into a permanent part, which follows a random walk with a drift, and a transitory part, which is stationary. As a sensitivity check, we also use the HP filler\(^{17} \) as an alternative decomposing tool. We will refrain from providing further technical

\(^{17}\text{The parameter in HP filler is set to 100 to accommodate annual data.}\)
details of how these two decomposition methods can be applied in empirical work, which can be found in my second paper. From Dicky-Fuller unit root test results reported in Table 3.2, we know that both saving and investment are \( I(1) \) variables, therefore they meet the condition for valid decomposition. Table 3.4 reports unit root test results for decomposed series. Both decomposition methods did a good job in isolating stationary transitory components from their corresponding non-stationary counterpart. Permanent saving and investment mostly have insignificant statistics while transitory series are unanimously stationary. The non-cointegration problem still exists in permanent series decomposed using the BN method, and gets even worse in HP filtered series. Of course, the co-integration problem does not exist for transitory components as they are stationary time-series themselves.

Now we are ready to launch a series of panel regressions using the decomposed saving and investment. Panel (b) of table 3.3 provides a complete set of regression estimates using decomposed series. The long-run estimates are very close to those shown earlier in panel (a) for benchmark models. When Luxembourg is taken as an outlier, all three models produce estimators of similar level, still around 0.6 and significant. But if Luxembourg is included, BE and FE models under-estimate the coefficient by a significant amount. Both decomposition methods confirm this pattern.

When we turn to short-run correlation estimates, we see quite different pictures depending on the choice of decomposition technique, panel model, and data. First, whether or not including Luxembourg in the panel regression now stops making a difference. Estimated short-run correlations are similar for data set with or without Luxembourg, in both mathematical magnitude and statistical significance. And this is true for both BN and HP filtered components. Second, FE and MG estimators are
Table 3.4: Unit root test of permanent and transitory investment and saving for 24 OECD countries

<table>
<thead>
<tr>
<th>Country</th>
<th>$I_{p}^{BN}$</th>
<th>$S_{p}^{BN}$</th>
<th>$I_{p}^{KN}$</th>
<th>$S_{p}^{KN}$</th>
<th>$I_{p}^{HP}$</th>
<th>$S_{p}^{HP}$</th>
<th>$e_{p}^{BN}$</th>
<th>$e_{p}^{HP}$</th>
<th>$e_{p}^{BN}$</th>
<th>$e_{p}^{HP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>-1.617</td>
<td>-1.990</td>
<td>-9.299**</td>
<td>-5.651**</td>
<td>0.382</td>
<td>-0.288</td>
<td>-5.681**</td>
<td>-3.684**</td>
<td>-1.415</td>
<td>4.879</td>
</tr>
<tr>
<td>Canada</td>
<td>-1.359</td>
<td>-1.515</td>
<td>-4.556**</td>
<td>-5.901**</td>
<td>0.079</td>
<td>-0.598</td>
<td>-2.958**</td>
<td>-3.989**</td>
<td>-3.566**</td>
<td>-0.782</td>
</tr>
<tr>
<td>Greece</td>
<td>-2.176</td>
<td>-2.724*</td>
<td>-5.262**</td>
<td>-4.694**</td>
<td>0.270</td>
<td>-0.600</td>
<td>-3.629**</td>
<td>-2.973**</td>
<td>-2.090</td>
<td>4.448</td>
</tr>
<tr>
<td>Ireland</td>
<td>-1.901</td>
<td>-2.015</td>
<td>-6.935**</td>
<td>-5.408**</td>
<td>-0.142</td>
<td>0.017</td>
<td>-4.021**</td>
<td>-3.644**</td>
<td>-1.859</td>
<td>4.97</td>
</tr>
<tr>
<td>Italy</td>
<td>-2.168</td>
<td>-2.323</td>
<td>-6.293**</td>
<td>-6.139**</td>
<td>-0.510</td>
<td>0.713</td>
<td>-3.204**</td>
<td>-3.304**</td>
<td>-1.709</td>
<td>-1.193</td>
</tr>
<tr>
<td>Japan</td>
<td>-1.465</td>
<td>-1.774</td>
<td>-5.892**</td>
<td>-5.293**</td>
<td>-1.644</td>
<td>-0.931</td>
<td>-3.562**</td>
<td>-3.018**</td>
<td>-1.562</td>
<td>11.281</td>
</tr>
<tr>
<td>Spain</td>
<td>-1.194</td>
<td>-1.044</td>
<td>-12.497**</td>
<td>-6.538**</td>
<td>0.086</td>
<td>-0.201</td>
<td>-6.870**</td>
<td>-3.774**</td>
<td>-3.513**</td>
<td>-1.18</td>
</tr>
<tr>
<td>Switzerland</td>
<td>-1.594</td>
<td>-1.260</td>
<td>-6.972**</td>
<td>-6.787**</td>
<td>-0.528</td>
<td>-0.224</td>
<td>-4.050**</td>
<td>-3.840**</td>
<td>-1.511</td>
<td>0.369</td>
</tr>
<tr>
<td>UK</td>
<td>-2.520</td>
<td>-2.155</td>
<td>-6.080**</td>
<td>-5.839**</td>
<td>0.126</td>
<td>0.035</td>
<td>-3.853**</td>
<td>-3.710**</td>
<td>-2.993**</td>
<td>-1.826</td>
</tr>
</tbody>
</table>

Notes: This table reports $Z_t$ from Dickey-Fuller unit root test. $I_{p}^{BN}$ stands for permanent investment decomposed using BN method. All other saving and investment variables have similar definitions. $e_{p}^{BN}$ is the residual from regression $I_{p}^{BN} = \alpha + \beta S_{p}^{BN}$. All other residuals have similar definitions. ** and * stands for significance at 5% and 10% level.
about the same level, around 0.2 for BN series and 0.4 for HP series, while the BE estimator is much smaller than the former two and statistically insignificant. Third, though FE and MG models generate estimates of similar magnitude for both decomposition methods, the MG estimate is statistically indifferent from zero when BN series are used while FE estimate is not. Out of 12 short-run correlation estimates we have in total, six, or 50%, which are significant. And even for those statistically significant short-run correlation estimates, they are still mathematically much smaller than their corresponding long-run estimate. Our result is different from Sarno and Taylor (1998) who find that using quarterly data 1955-1994, the short-run correlation for the UK is mathematically larger than the long-run correlation. And the authors take their findings as empirical support to Obstfeld’s (1986) proposition that investment only responds to long-run shocks and therefore the correlation between investment and saving is larger to accommodate this potentially smaller amount of capital flow in the short-run. Since both their decomposition model and estimation method are different from this paper, it may be of little practical use to make direct comparison between their results and our empirical findings. If we believe that saving-investment correlation is a good measure of capital mobility, our empirical results seem to support another theory that capital flow enjoys a greater mobility in the short-run, which is well justified using Feldstein’s (1983) argument (pp.147) “...the short run capital flow is part of a once-for-all adjustment of the international portfolio. When adjustment is complete, the rate of capital flow returns to a lower level governed by the rate of growth of the world capital stock and the share of international assets in the equilibrium portfolio”. More interesting results can be found when we look at the dynamics of long-run and short-run correlations.
3.4.3 Dynamics of long-run and short-run correlations

Apparently there exists no perfect capital mobility in the real world. What many economists are more interested in is the relative degree of capital mobility over the time horizon. Investigating the FH issue across different periods of time will give economists a much easier time because they do not need to dig further into the theory to find an answer to the question why empirically observed value of 0.6 is so different from theoretical predicted value of 1 (which of course is an important issue,) or to the question at what level the empirical correlation should be as implied by the intertemporal budget constraint.\(^{18}\) Rather, by comparing estimated correlations across stages of time, especially for periods before or after the world capital market is commonly believed to be more integrated, they can be comfortable enough to say that though the original FH proposition isn’t flawless and complete, it at least provides a partial and valuable reference to the degree of capital mobility.

In this section, we estimate the long-run and short-run correlations over a period of ten years, starting from 1960. For each new run, we move one year forward and keep the time span at a constant of ten years\(^ {19}\). Following the procedure, we will have 35 estimated points for each saving-investment series. We plot these numbers in figure 3.3 for FE and MG models with data sets including or excluding Luxembourg.

First, we will exploit the visual convenience of these graphs to examine the out-

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\(^{18}\)Kim (2001) estimates the regression after adjusting saving and investment variables for productivity and fiscal shocks, so that the effects of common factors on high correlation can be eliminated or at least minimized. The correlation between adjusted saving and investment for 19 OECD countries is still high (ranging from 0.42 to 0.75 for models with different control variable). This correlation may be one candidate measuring the magnitude of effect of pure cointegration on saving-investment correlation.

\(^{19}\)The choice of ten years is arbitrary, but not unreasonable. First, it is in the same order of magnitude as FH’s original estimation, where a group average is taken over 12 years. Second, usually business cycle is around 8-10 years. Third, the Monte Carlo simulation results suggest that in case of I(1) errors, sample size has least effect on t-statistics. Using ten years is just as good as using 50 years.
Figure 3.3: Time properties of long-run and short-run correlations

(a) Original saving and investment series

(b) Permanent saving and investment, BN

(c) Transitory saving and investment, BN

(d) Permanent saving and investment, HP

(e) Transitory saving and investment, HP
lier problem of Luxembourg again, this time, within a time framework. Panel (a) of figure 3.3 graphs the estimation results using original saving and investment series before decomposition. Excluding Luxembourg does not produce a major difference in MG estimators over time, but the FE curve with the country lies consistently below that without the country. Panels (b) and (d) report time properties of long-run correlation estimates for BN and HP series. Again the outlier problem is only marginal for the MG estimator, but quite severe for the FE estimator. Two panels on the right (c) and (e) depict dynamics of short-run correlation coefficients. It seems that having Luxembourg in the panel regression does not produce any systematic bias in the estimates. The two correlation curves track each other closely, except for the case of the FE model with HP-filtered data. To give reader a more accurate picture of how big the difference is between two curves, we report the mean and standard deviation of the difference for all ten sets of estimators in table 3.5. None of the difference in the MG estimators are significant in a standard $t$-test which compares two sample means. But most of the difference in FE estimators are significant, the only exception is for short-run correlation series decomposed using the BN method. The value for short-run correlation series decomposed using HP filler (0.06), though much smaller than the other series under FE model (ranging from 0.16 to 0.24), is still significant at a traditional 5% level.

To summarize our findings so far about the outlier problem of Luxembourg as reported in many earlier studies, we agree with Ho (2002) that estimation technique does matter. Traditional panel estimation models, such as BE and FE, tend to underestimate the saving's retention coefficient when Luxembourg is included. But with MG estimation technique, inclusion or exclusion of the country produces similar correlation estimates that are statistically equivalent. But what's more in-
Table 3.5: Differences between correlations with or without Luxembourg

<table>
<thead>
<tr>
<th></th>
<th>MG model</th>
<th>RE model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SEM</td>
</tr>
<tr>
<td>Full</td>
<td>0.021 (0.021)</td>
<td>0.169 (0.018) **</td>
</tr>
<tr>
<td>BN(p)</td>
<td>0.022 (0.020)</td>
<td>0.164 (0.019) **</td>
</tr>
<tr>
<td>BN(t)</td>
<td>0.034 (0.068)</td>
<td>0.010 (0.026)</td>
</tr>
<tr>
<td>HP(p)</td>
<td>0.030 (0.031)</td>
<td>0.242 (0.047) **</td>
</tr>
<tr>
<td>HP(t)</td>
<td>0.022 (0.025)</td>
<td>0.060 (0.026) **</td>
</tr>
</tbody>
</table>

Notes: Mean stands for over-time average of differences between estimator when Luxembourg is included and that when the country is excluded in the panel regression. SEM is its corresponding standard error of mean. Full model uses original saving and investment series. BN(p) refers to estimator based on permanent components decomposed using BN method. BN(t), HP(p) and HP'(t) have similar definitions.

Interesting is that after reviewing the dynamic features of different sets of correlation curves, we notice that the outlier problem actually can be attributed to the big difference in the estimated long-run, rather than short-run, coefficients. There must be some systematic distinction between the ways that Luxembourg and other OECD economies operate (recall the two reasons we mentioned earlier) that have caused this major difference in long-run saving’s retention coefficients, which makes the FE model’s homogenous slope assumption problematic.

Next, let’s focus on the dynamics of long-run and short-run correlations. Short-run correlations are relatively more variable over a given time span than long-run correlations. Take the BN decomposed series. The standard deviations of the former are 0.28 and 0.11 for MG and FE estimators, and 0.09 and 0.08 respectively for the latter. It seems that there exists some underlying power, or the intertemporal budget constraint as most economists would believe, that confines the long-run correlation within the boundary of 0.4 to 1 over time stretch, but the short-run correlation enjoys a relative greater freedom to jump up and down between 0 and
1. Jansen (1998) reports similar results using a cross-sectional regression evaluated on an annual basis. But he follows the traditional definitions of long-run and short-run correlations, i.e., the coefficient on a level saving-investment regression being the former and the coefficient on an ECM or a first-differenced model being the latter. In this sense, we say that our newly defined methodology did a decent job in matching the performance of traditional method by capturing the relative dynamic relationship between long-run and short-run correlations.

In table 3.6, we show the statistical results of estimated long-run and short-run correlations using MG model and excluding Luxembourg. The coefficients themselves are already graphed in figure 3.3. On average, the long-run coefficients are always higher than short-run coefficients. This is consistent with those found with traditionally defined long-run and short-run correlations. In the literature, correlation estimated using a cross-sectional level model is always greater than that using time-series first-difference model (Taylor 1997). Looking at the t-statistics, long-run coefficients are mostly significantly different from zero. Depending on the decomposition method, the short-run coefficient can become insignificant. For the BN series, the short-run coefficient is insignificant even in periods (the 70s) with values as large as 0.7. This must be due to high heterogenous features across countries that lead to a large standard deviation of the group averaged coefficient.

Referring to figure 3.3, we see that long-run and short-run coefficients do not always move together. Take the BN decomposed series. The long-run correlation tends to decrease starting in around 1980 when less stringent capital policies started to take effect in many OECD countries. At roughly the same period, the short-run correlation reaches its bottom and starts to pick up in the 80s. The disagreement between the dynamics of two correlations can be confirmed if we calculate the cor-
Table 3.6: Saving-investment correlations for 23 OECD countries 1960-2004: MG

<table>
<thead>
<tr>
<th>year</th>
<th>BN(p)</th>
<th>BN(t)</th>
<th>HP(p)</th>
<th>HP(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1965</td>
<td>0.687 (0.12)**</td>
<td>0.364 (0.56)</td>
<td>0.884 (0.27)**</td>
<td>0.609 (0.11)**</td>
</tr>
<tr>
<td>1966</td>
<td>0.693 (0.11)**</td>
<td>0.397 (0.57)</td>
<td>0.822 (0.26)**</td>
<td>0.659 (0.12)**</td>
</tr>
<tr>
<td>1967</td>
<td>0.682 (0.12)**</td>
<td>0.397 (0.57)</td>
<td>0.604 (0.32) * 0.579 (0.11)**</td>
<td></td>
</tr>
<tr>
<td>1968</td>
<td>0.594 (0.12)**</td>
<td>0.642 (0.74)</td>
<td>0.472 (0.35)</td>
<td>0.472 (0.13)**</td>
</tr>
<tr>
<td>1969</td>
<td>0.504 (0.16)**</td>
<td>0.499 (0.68)</td>
<td>0.539 (0.31) * 0.509 (0.16)**</td>
<td></td>
</tr>
<tr>
<td>1970</td>
<td>0.662 (0.13)**</td>
<td>0.589 (0.72)</td>
<td>0.630 (0.27)**</td>
<td>0.559 (0.14)**</td>
</tr>
<tr>
<td>1971</td>
<td>0.554 (0.12)**</td>
<td>0.746 (0.73)</td>
<td>0.638 (0.24)**</td>
<td>0.496 (0.13)**</td>
</tr>
<tr>
<td>1972</td>
<td>0.499 (0.13)**</td>
<td>0.734 (0.73)</td>
<td>0.560 (0.19)**</td>
<td>0.422 (0.14)**</td>
</tr>
<tr>
<td>1973</td>
<td>0.527 (0.13)**</td>
<td>0.716 (0.73)</td>
<td>0.540 (0.18) ** 0.443 (0.14)**</td>
<td></td>
</tr>
<tr>
<td>1974</td>
<td>0.486 (0.13)**</td>
<td>0.758 (0.78)</td>
<td>0.607 (0.08)**</td>
<td>0.393 (0.14)**</td>
</tr>
<tr>
<td>1975</td>
<td>0.454 (0.12)**</td>
<td>0.595 (0.80)</td>
<td>0.686 (0.14)**</td>
<td>0.374 (0.12)**</td>
</tr>
<tr>
<td>1976</td>
<td>0.467 (0.10)**</td>
<td>0.502 (0.57)</td>
<td>0.736 (0.15)**</td>
<td>0.364 (0.12)**</td>
</tr>
<tr>
<td>1977</td>
<td>0.473 (0.10)**</td>
<td>0.480 (0.57)</td>
<td>0.729 (0.19)**</td>
<td>0.387 (0.12)**</td>
</tr>
<tr>
<td>1978</td>
<td>0.600 (0.10)**</td>
<td>0.390 (0.57)</td>
<td>0.650 (0.29)**</td>
<td>0.487 (0.12)**</td>
</tr>
<tr>
<td>1979</td>
<td>0.734 (0.13)**</td>
<td>0.193 (0.56)</td>
<td>0.671 (0.28)**</td>
<td>0.618 (0.14)**</td>
</tr>
<tr>
<td>1980</td>
<td>0.605 (0.12)**</td>
<td>0.362 (0.53)</td>
<td>0.900 (0.10)**</td>
<td>0.550 (0.11)**</td>
</tr>
<tr>
<td>1981</td>
<td>0.620 (0.11)**</td>
<td>0.041 (0.13)</td>
<td>0.995 (0.10)**</td>
<td>0.495 (0.11)**</td>
</tr>
<tr>
<td>1982</td>
<td>0.581 (0.10)**</td>
<td>-0.262 (0.20)</td>
<td>0.934 (0.13)**</td>
<td>0.523 (0.11)**</td>
</tr>
<tr>
<td>1983</td>
<td>0.583 (0.09)**</td>
<td>-0.238 (0.25)</td>
<td>0.793 (0.18)**</td>
<td>0.590 (0.12)**</td>
</tr>
<tr>
<td>1984</td>
<td>0.570 (0.09)**</td>
<td>-0.218 (0.25)</td>
<td>0.611 (0.18)**</td>
<td>0.639 (0.11)**</td>
</tr>
<tr>
<td>1985</td>
<td>0.527 (0.13)**</td>
<td>0.078 (0.27)</td>
<td>0.474 (0.23) * 0.677 (0.10)**</td>
<td></td>
</tr>
<tr>
<td>1986</td>
<td>0.562 (0.11)**</td>
<td>0.306 (0.39)</td>
<td>0.528 (0.24)**</td>
<td>0.642 (0.10)**</td>
</tr>
<tr>
<td>1987</td>
<td>0.632 (0.11)**</td>
<td>0.469 (0.50)</td>
<td>0.649 (0.22)**</td>
<td>0.668 (0.10)**</td>
</tr>
<tr>
<td>1988</td>
<td>0.557 (0.11)**</td>
<td>0.509 (0.48)</td>
<td>0.743 (0.20)**</td>
<td>0.755 (0.11)**</td>
</tr>
<tr>
<td>1989</td>
<td>0.615 (0.13)**</td>
<td>0.440 (0.48)</td>
<td>0.663 (0.20)**</td>
<td>0.759 (0.12)**</td>
</tr>
<tr>
<td>1990</td>
<td>0.514 (0.16)**</td>
<td>0.499 (0.48)</td>
<td>0.717 (0.25)**</td>
<td>0.667 (0.13)**</td>
</tr>
<tr>
<td>1991</td>
<td>0.552 (0.13)**</td>
<td>0.444 (0.49)</td>
<td>0.743 (0.22)**</td>
<td>0.607 (0.16)**</td>
</tr>
<tr>
<td>1992</td>
<td>0.586 (0.10)**</td>
<td>0.363 (0.42)</td>
<td>0.493 (0.20)**</td>
<td>0.549 (0.10)**</td>
</tr>
<tr>
<td>1993</td>
<td>0.621 (0.09)**</td>
<td>0.311 (0.43)</td>
<td>0.453 (0.24) * 0.664 (0.10)**</td>
<td></td>
</tr>
<tr>
<td>1994</td>
<td>0.579 (0.08)**</td>
<td>0.299 (0.39)</td>
<td>0.572 (0.33) * 0.643 (0.09)**</td>
<td></td>
</tr>
<tr>
<td>1995</td>
<td>0.433 (0.10)**</td>
<td>0.338 (0.37)</td>
<td>0.640 (0.38)</td>
<td>0.579 (0.09)**</td>
</tr>
<tr>
<td>1996</td>
<td>0.454 (0.12)**</td>
<td>0.257 (0.32)</td>
<td>0.696 (0.41)</td>
<td>0.560 (0.08)**</td>
</tr>
<tr>
<td>1997</td>
<td>0.426 (0.13)**</td>
<td>0.063 (0.34)</td>
<td>0.801 (0.45) * 0.455 (0.08)**</td>
<td></td>
</tr>
<tr>
<td>1998</td>
<td>0.440 (0.17)**</td>
<td>-0.056 (0.28)</td>
<td>0.780 (0.38)**</td>
<td>0.454 (0.09)**</td>
</tr>
<tr>
<td>1999</td>
<td>0.413 (0.17)**</td>
<td>-0.037 (0.18)</td>
<td>0.318 (0.26)</td>
<td>0.346 (0.10)**</td>
</tr>
</tbody>
</table>

Average 0.624 (0.07)** 0.201 (0.38) 0.614 (0.07)** 0.491 (0.07)**

Notes: We report here only estimation results using mean group (MG) model in a recursive ten-year moving window. BN(p) refers to estimator based on permanent components decomposed using BN method. BN(t), HP(p) and HP(t) have similar definitions. Year stamp is the median number in the ten-year time span that we used for each stepwise estimation. Luxembourg is excluded. Standard errors are in parentheses. ** and * stand for significance at 5% and 10% level.
relation between them, which is as low as 0.034 for both BN and HP series (MG model). Both visual and quantitative evidence show us that it may be different forces driving the long-run and short-run saving-investment correlations. For the long-run, it may be the intertemporal budget constraint, and for the short-run, it may be the response to temporary shocks.

The potential effect of common responses to various economic shocks on saving-investment correlation is empirically investigated in Kim (2001). After controlling for effects of productivity shock, terms of trade shock, and fiscal shock, the saving-investment correlation, in the vicinity of 0.5, remains significantly different from zero. Explaining or interpreting the sources of saving-investment correlation is beyond the main purpose of this paper. But it would be interesting to follow Kim’s methodology in our decomposition framework. We expect that the controlling for temporary economic shocks will have a significant effect on estimated short-run saving-investment correlation, whereas it will only marginally affect the long-run correlation. We will leave this for further research in the future.

3.4.4 Comparing with other measures of short-run correlation

We compare the dynamics of short-run correlation with those of ECM and PMG, two typical models in the literature that allow researchers to examine short-run correlations. We estimate an ECM in the form of equation (3.2) where the short-run correlation $\beta$ is an estimate from pooled data. For the PMG model, a regression is estimated for each country in the form of equation (3.4), and the estimated coefficients are averaged to obtain the PMG estimator. Note the difference between equations (3.2) and (3.4). The former imposes a homogenous coefficient and is
mainly based on the cointegration assumption while in the latter, the short-run coefficient is allowed to be different across countries and is derived from a dynamic model of saving and investment.

Table 3.7 reports short-run correlation estimates using PMG and ECM in a recursive ten-year moving window. The short-run ECM estimator for 1960-2004 is 0.335 and very significant (with a t-ratio 10.90). And all coefficients in the recursive time framework are also significant. The short-run PMG estimator for the whole sample for 1960-2004 is 0.28 and significant. This value is very close to those in the literature, 0.25 in Pelgrin and Schich (2004) and 0.19 in Bebczuk and Schmidt-Hebbel (2006). When we estimate PMG in a recursive ten-year moving window, we find that PMG estimator is insignificant in most of the time periods, very similar to the results for the short-run correlation with BN decomposition and MG model. To allow easy comparison of dynamics of short-run correlations across different models, we plot four short-run correlations in Figure 3.4. Two of the curves, already reported in the last section, are estimators from MG models with BN and HP decomposed series. The other two are ECM and PMG. Interestingly, even though four series of short-run correlations are estimated using different methodologies and their averaged values over time and related statistical significance are also different, their relative dynamic features are matched on a fairly good basis. Starting in the 60s, they all tend to be steadily going down over time until they hit the bottom in early 80s (it is a little bit earlier for HP series; it reaches its bottom around mid 70s) and then begin to increase. The up-rising tendency stops around the end of 80s and the beginning of 90s when the down tendency takes its place. And when we calculate the correlation between our measures of the short-run saving's retention coefficient and those traditionally used in the literature, we find fairly large values. The correlation
Table 3.7: Saving-investment correlations for 23 OECD countries 1960-2004: PMG and ECM

<table>
<thead>
<tr>
<th>Year</th>
<th>PMG</th>
<th>ECM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1965</td>
<td>0.236 (0.123) *</td>
<td>0.347 (0.061) **</td>
</tr>
<tr>
<td>1966</td>
<td>0.263 (0.106) **</td>
<td>0.369 (0.061) **</td>
</tr>
<tr>
<td>1967</td>
<td>0.302 (0.112) **</td>
<td>0.410 (0.058) **</td>
</tr>
<tr>
<td>1968</td>
<td>0.342 (0.126) **</td>
<td>0.463 (0.064) **</td>
</tr>
<tr>
<td>1969</td>
<td>0.221 (0.138)</td>
<td>0.309 (0.061) **</td>
</tr>
<tr>
<td>1970</td>
<td>0.236 (0.152)</td>
<td>0.353 (0.060) **</td>
</tr>
<tr>
<td>1971</td>
<td>0.224 (0.146)</td>
<td>0.355 (0.060) **</td>
</tr>
<tr>
<td>1972</td>
<td>0.227 (0.140)</td>
<td>0.418 (0.063) **</td>
</tr>
<tr>
<td>1973</td>
<td>0.125 (0.134)</td>
<td>0.404 (0.065) **</td>
</tr>
<tr>
<td>1974</td>
<td>0.158 (0.136)</td>
<td>0.386 (0.066) **</td>
</tr>
<tr>
<td>1975</td>
<td>0.068 (0.149)</td>
<td>0.406 (0.087) **</td>
</tr>
<tr>
<td>1976</td>
<td>0.004 (0.111)</td>
<td>0.375 (0.068) **</td>
</tr>
<tr>
<td>1977</td>
<td>0.028 (0.102)</td>
<td>0.385 (0.066) **</td>
</tr>
<tr>
<td>1978</td>
<td>0.043 (0.112)</td>
<td>0.369 (0.069) **</td>
</tr>
<tr>
<td>1979</td>
<td>0.019 (0.109)</td>
<td>0.324 (0.067) **</td>
</tr>
<tr>
<td>1980</td>
<td>0.087 (0.102)</td>
<td>0.434 (0.066) **</td>
</tr>
<tr>
<td>1981</td>
<td>-0.008 (0.091)</td>
<td>0.309 (0.068) **</td>
</tr>
<tr>
<td>1982</td>
<td>-0.060 (0.097)</td>
<td>0.324 (0.067) **</td>
</tr>
<tr>
<td>1983</td>
<td>-0.065 (0.117)</td>
<td>0.311 (0.089) **</td>
</tr>
<tr>
<td>1984</td>
<td>0.030 (0.127)</td>
<td>0.329 (0.067) **</td>
</tr>
<tr>
<td>1985</td>
<td>0.103 (0.136)</td>
<td>0.374 (0.064) **</td>
</tr>
<tr>
<td>1986</td>
<td>0.132 (0.115)</td>
<td>0.417 (0.06) **</td>
</tr>
<tr>
<td>1987</td>
<td>0.273 (0.126) **</td>
<td>0.462 (0.058) **</td>
</tr>
<tr>
<td>1988</td>
<td>0.290 (0.149) *</td>
<td>0.458 (0.064) **</td>
</tr>
<tr>
<td>1989</td>
<td>0.250 (0.132) *</td>
<td>0.461 (0.062) **</td>
</tr>
<tr>
<td>1990</td>
<td>0.282 (0.133) **</td>
<td>0.451 (0.060) **</td>
</tr>
<tr>
<td>1991</td>
<td>0.305 (0.115) **</td>
<td>0.393 (0.056) **</td>
</tr>
<tr>
<td>1992</td>
<td>0.367 (0.091) **</td>
<td>0.433 (0.052) **</td>
</tr>
<tr>
<td>1993</td>
<td>0.350 (0.097) **</td>
<td>0.409 (0.053) **</td>
</tr>
<tr>
<td>1994</td>
<td>0.308 (0.093) **</td>
<td>0.340 (0.052) **</td>
</tr>
<tr>
<td>1995</td>
<td>0.205 (0.096) **</td>
<td>0.265 (0.051) **</td>
</tr>
<tr>
<td>1996</td>
<td>0.118 (0.077)</td>
<td>0.255 (0.051) **</td>
</tr>
<tr>
<td>1997</td>
<td>0.204 (0.117) *</td>
<td>0.229 (0.051) **</td>
</tr>
<tr>
<td>1998</td>
<td>0.093 (0.093)</td>
<td>0.170 (0.051) **</td>
</tr>
<tr>
<td>1999</td>
<td>0.052 (0.120)</td>
<td>0.155 (0.052) **</td>
</tr>
</tbody>
</table>

Average: 0.280 (0.073) ** 0.335 (0.031) **

Notes: We report $\beta$ estimates for pool mean group (PMG) and error correction (ECM) models using a ten-year recursive time window. The estimated PMG model is

$$\Delta I_{t,t} = \alpha_t + \beta_t \Delta S_{t,t} + \gamma_t (S_{t-1} - \bar{S}_{t-1}) + \epsilon_{t,t},$$  

The ECM model is

$$\Delta I_{t,t} = \alpha + \beta \Delta S_{t,t} + \gamma (S_{t-1} - \bar{S}_{t-1}) + \epsilon_{t,t}.$$  

Year is the median in the ten-year time window. Luxembourg is excluded. Standard errors are in parentheses. ** and * stand for significance at 5% and 10% level.
coefficient between estimates using BN and PMG (ECM) is 0.53 (0.60), which is higher than that using HP and PMG (0.44) or ECM (0.37). High correlation provides additional support that difference models may render comparable estimation results.

There are at least two potential explanations of the decreasing saving-investment correlation before early 1980s. First, the breakdown of the Bretton Woods system in early 70s pushed many industrial countries to remove capital control constraints, which leads to a large capital mobility in OECD countries. And second, the oil price shocks in 1973 and 1979 may have contributed to a drop in saving-investment correlation. Jansen and Schulze (1996) explain why negative saving-investment correlation is likely to be found during the 1973 oil shock for the case of Norway. Their main argument is that investment will increase when there is an oil price jump to increase future production and to exploit the profitability. But consumption also goes up in anticipation of increasing revenues from this new source of income. Therefore saving will temporarily drop down.

3.5 Conclusion

The purpose of this paper is to re-examine the FH puzzle by providing an alternative measure of long-run and short-run correlations. We decompose saving and investment into their permanent and transitory components, and run panel regressions separately on two sets of components. By doing so, we are able to re-investigate the outlier problem of Luxembourg and found that it is the long-run correlation, not the short-run correlation, that led to remarkable under-estimation of saving-investment
Figure 3.4: Comparing short-run correlations from different models

Notes: PMG is pooled mean group estimator, which is a group-average of short-run correlation estimates from regression $\Delta I_{i,t} = \alpha_i + \gamma(i_{i,t-1} - bS_{i,t}) + \beta_i \Delta S_{i,t} + \epsilon_{i,t}$ where the long-run correlation $\hat{b}$ is estimated from a POLS in the format of $I_{i,t} = a + bS_{i,t} + u_{i,t}$. ECM uses a pooled regression as in equation (3.2). BN (HP) correlation is defined as regression coefficient on transitory saving decomposed using BN (HP) method. Luxembourg is not included in the panel regression.
correlation when BE and FE models are used. Monte Carlo simulation shows that MG should probably be considered the best static panel estimator when multiple factors, such as non-stationary, non-cointegrated variables, heterogeneity, and small sample, need to be taken into account.

The short-run correlations estimated using traditional ECM and more recent PMG model are generally found smaller than long-run correlations, but still significantly different from zero. With our decomposition framework, we provide further empirical evidence that the saving-investment correlation does have a smaller size in the short-run than in the long-run. In addition, depending on the decomposition method and the estimation model, the short-run correlation estimated under our decomposition framework can be insignificantly different from zero. Comparative studies show that BN decomposition method provides a more accurate description of the dynamics of short-run correlation relative to traditional measures, than does the HP filter. So the insignificant short-run correlation, estimated using BN decomposition and MG model, provides some tentative evidence that capital is very mobile for OECD countries 1960-2004, if we believe that capital mobility is better reflected in the short-run correlation.

As we said earlier in section 3.4.3, the main purpose of this paper is to examine the magnitude of long-run and short-run correlation in a new decomposition framework. However it would be interesting to further explore the question by asking what explains long-run and short-run correlations. The marginal correlation between these two correlations apparently implies different mechanism working beneath them. We will leave this interesting topic to future research project.
Appendix C.1 Blancard and Quah (BQ) decomposition model

We show in this appendix that permanent and transitory components from BQ decomposition model follow a similar path. Suppose that a bi-variate structural VAR (SVAR) model in the format of

\[ AZ_t = BZ_{t-1} + \varepsilon_t, \]

where \( Z_t = [xz_t \ y_t]' \), \( \varepsilon_t = [\varepsilon^P_t \ \varepsilon^T_t]' \), and \( \varepsilon^P_t, \varepsilon^T_t \) are permanent and transitory shocks. This SVAR model can be re-arranged and defined as a structural moving average model (SMA)

\[ Z_t = D_0 \varepsilon_t + D_1 \varepsilon_{t-1} + D_2 \varepsilon_{t-2} + D_3 \varepsilon_{t-3} + ..., \]

where \( D_0 = A^{-1} \), and \( D_j = (A^{-1}B)^jA^{-1} \), \( \forall j \geq 1 \). The permanent component of \( Z \), \( Z^P_t \), is defined as that comes from permanent shock, therefore

\[ Z^P_t = D_0 G \varepsilon_t + D_1 G \varepsilon_{t-1} + D_2 G \varepsilon_{t-2} + D_3 G \varepsilon_{t-3} + ..., \]

where \( G = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \). Or using parameters in SVAR, we can re-write it as

\[ Z^P_t = (A^{-1}B)Z^P_{t-1} + A_{t-1}G \varepsilon_t. \]

Similarly transitory components of \( Z \) can be expressed as

\[ Z^T_t = (A^{-1}B)Z^T_{t-1} + A_{t-1}M \varepsilon_t, \]
where $M = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. We can easily see that the permanent and transitory components of variable $Z$ follow a very similar dynamic path, both having $A^{-1}B$ as AR(1) coefficient.
Bibliography


