A HIGHER-ORDER DEPTH-INTEGRATED MODEL FOR WATER WAVES AND CURRENTS GENERATED BY UNDERWATER LANDSLIDES

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ABSTRACT

A joint theoretical, numerical and experimental study is carried out to develop an improved wave model for predicting water waves and fluid current generated by underwater landslides. In the theoretical study, a fully nonlinear and higher-order dispersive depth-integrated hydrodynamic model by Gobbi and Kirby (1999) and Gobbi et al (2000) is extended to include the time variation in bathymetry. Upon this extension, the new model can be applied to simulate both wave propagation and the dynamic process of wave generation by a submerged moving object such as an underwater landslide. Compared with the lower-order (first- or second-order) traditional long wave models, the higher-order model improves the modeling of the dispersive effect to the fourth-order, thus extending the validity of the wave model from long waves (wavelength-to-water depth ratio larger than 10) to shorter waves (wavelength comparable to the water depth). In addition, it also improves the approximation of the vertical fluid velocity profile from the second-order parabolic assumption to a fourth-order polynomial function for more accurate prediction of the fluid current induced by waves. A finite difference scheme is applied to solve the model equations in one spatial dimension. The new model developed in this thesis is derived independently from the fourth order model by Ataie-Ashtiani and Najafi-Jilani (2007) which is similar but differs from the new model in this thesis.

Experiments also are carried out in a wave flume in the Hydraulics Laboratory of the Department of Civil and Environmental Engineering at the University of Hawaii. Waves are generated by rigid landslide models sliding down an incline with adjustable slopes. The wave elevation is measured by resistance-type wave gauges and the fluid
velocity with particle image velocimetry (PIV). The present higher-order model then is applied to simulate the experimental cases and the numerical results are compared with the experimental data as well as with the results based on two existing lower-order wave models. The results show that the present higher-order model agrees with the experimental measurement better for both the wave elevation and especially the fluid velocity induced by the waves and the landslide motion. Most existing studies focus on wave measurement and prediction. This study is among the first to conduct experiments to measure the landslide induced velocity field and compare the measured velocity with the predicted results.

Tsunami sensitivity to landslide features also is investigated through numerical experiments. Empirical equations are derived for predicting the tsunami wave amplitude and water velocities under the waves, based on the numerical experiments.

With its improved wave dispersion relation and more accurate prediction for the fluid velocity field, the new model developed in this study can be useful to study a wider range of coastal and hydraulic engineering problems including landslide-generated tsunamis and the associated fluid current which is important in the study of sediment transport and seabed erosion during a tsunami attack. Other problems that can also apply the present higher-order model may include prediction of water surface evolution for open channel flows over different bottom disturbances, and surface waves generated by submerged moving vehicles in the shallow ocean in naval applications.
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CHAPTER 1
INTRODUCTION

1.1 Problem Statement

Underwater landslides are the second most common cause for tsunamis. Tsunamis generated by underwater landslides may be as destructive as those generated by earthquakes in certain cases, when the landslides involve large volumes of debris and have lengths of several kilometers. The Grand Banks Tsunami on November 18th, 1929 is one of a few well documented tsunamis which were generated by an underwater landslide. The landslide was triggered by an earthquake of 7.2 magnitude approximately 280 km south of Newfoundland. The mud and sand in a volume approximately 200 km$^3$ flowed eastward up to 1000 km at a speed estimated to be over 100 km/hour (Fine et al., 2005). The tsunami caused runup of 9 to 15 m along the coast of Burin Peninsula, Newfoundland, and claimed 28 human lives, making it one of the most catastrophic natural disasters in Canadian history. Some researchers also have suggested that underwater landslides are the major mechanism for the 1979 Nice tsunami (Assier-Rzadkiewicz, et al., 2000), the 1998 Papua New Guinea tsunami (Bardet et al., 2003), and five Hawaiian local tsunamis from 1813 to 1977 (Cox and Morgan, 1977), among others. Besides the huge landslides in the oceans, small-scale landslides also may occur in reservoirs, lakes and fjords, and cause small-scale local tsunamis, e.g., the Skagway Harbor tsunami on November 3, 1994 (Rabinovich et al., 1999).

Unlike most seismic seafloor deformation, the transitional sliding of underwater landslides is usually much slower. The seafloor deformation and the generation of surface
waves are complicated coupled processes, for which efficient modeling approaches still are being developed. Some researchers have applied the three-dimensional Navier-Stokes (N-S) equations or the Reynolds Averaged N-S equations (RANS) in the simulation of landslide generated tsunamis (e.g. Heinrich, 1992; Rzadkiewicz, et al., 1997; Liu et al, 2005; Yuk et al., 2006). The N-S equations are fundamental fluid mechanics theories, which can predict all the wave properties involved including the nonlinearity, dispersion, viscosity and energy dissipation due to wave breaking. However, the N-S equations are highly complex equations to solve theoretically or numerically, especially considering that the water surface needs to be treated as an unknown moving boundary. For inviscid flows and non-breaking waves, the N-S equations may be simplified into the Euler equations, or further to the potential flow theory if the flow is irrotational (e.g., Grilli and Watts 1999; Grilli et al. 2002; Grilli and Watts, 2005). Although these simplified equations have fewer terms to compute, the numerical solution still needs large computational resources and special schemes to track the moving water surface and sea bottom. In recent years, some researchers have applied the smoothed particle hydrodynamics method (SPH) to solve the complex free surface problems, including the landslide-generated waves (Qiu, 2008). As this method describes the motion of water particles in a Lagrangian frame, no free-surface condition or moving boundary condition is required. On the other hand, the SPH method also requires large computational resources, which may limit its application in real-time simulations due to the current level of available computer resources in the world.

Most of the historical tsunamigenic landslides happened on very mild slopes, with slope angles usually less than 10°. The thickness-to-length ratio and the depth of
submergence-to-length ratio of underwater landslides were normally less than 10% (e.g., Hampton et al., 1996; Watts, et al., 2005). Typical geological features of underwater landslides imply that both landslide motion and water flows are dominantly in the horizontal or mildly sloped plane and that the length of landslides is relatively long, to which the depth-integrated modeling approaches may apply. In the depth-integrated models, the equations become two-dimensional, thus the computational effort is largely reduced. Another advantage of the depth-integrated models is that the varying water surface elevations and the varying water depth are assimilated into the two-dimensional equations as variables, and there is no need to track these moving boundaries. On the other hand, limitations exist on the application of the depth-integrated models. With regard to the problems of wave propagation, the standard Boussinesq model described with depth-averaged water velocity is only valid for relatively long waves in shallow water with negligible velocity variation in the vertical direction. In the recent decades, researchers have made efforts to improve the validity of depth-integrated models by modifying their dispersion relation or by retaining higher-order nonlinear and dispersive terms so that they may be applied to both long and shorter waves (e.g., Nwogu, 1993, Schäffer and Madsen, 1995, Gobbi and Kirby, 1999, Gobbi et al 2000, Liu and Sun, 2005, Ataie-Ashtiani and Najafi -Jilani, 2007).

1.2 Literature Review

The basic problem of wave propagation is that of a small amplitude periodic wave traveling in a constant water depth. For this problem, the water flow may be assumed as ideal, i.e., the water is inviscid, incompressible and the flow is irrotational. The
fundamental solution may be found based on the potential flow theory with linearized water surface boundary conditions as (see Mei, 1989):

\[ \zeta(x,t) = ae^{(kx-\omega t)}, \]  

\[ \phi(x,z,t) = \frac{iga \cosh k(h+z)}{\omega \cosh kh} e^{(kx-\omega t)}, \]  

\[ u(x,z,t) = \frac{gka \cosh k(h+z)}{\omega \cosh kh} e^{(kx-\omega t)}, \]  

\[ w(x,z,t) = \frac{-igk \sinh k(h+z)}{\omega \cosh kh} e^{(kx-\omega t)}, \]

\[ \omega^2 = gk \tanh kh, \]  

\[ c = \frac{\omega}{k} = \frac{\lambda}{T} = \sqrt{\frac{g}{k} \tanh kh}, \]

where \( \zeta \) is the wave elevation from the undisturbed free surface, \( \phi \) is velocity potential, \( a \) is wave amplitude, \( u, w \) are fluid velocities in the horizontal \( x \)- and vertical \( z \)-directions, respectively, \( h \) is water depth, \( g \) is gravitational acceleration, and \( t \) is time.

Wave number \( k \) and frequency \( \omega \) are related to wavelength \( \lambda \) and wave period \( T \) by \( k = \frac{2\pi}{\lambda} \) and \( \omega = \frac{2\pi}{T} \), while wave speed \( c \) is given by equation (1.6). Based on the dispersion relation (1.5) or (1.6), if \( kh < \pi/10 \) or equivalently \( \lambda > 20h \), we have

\[ \omega^2 = gk \tanh kh \approx gk(kh) = gk^2h, \text{ or } \omega = \sqrt{gh}. \]  

This indicates that for sufficiently long waves, wave speed \( c \) may be approximated as independent of wavelength \( \lambda \), or in other words, the waves are non-dispersive. The waves of \( \lambda \geq 20h \) are normally called shallow water long waves. Waves of \( 0.5h < \lambda < 20h \) and \( \lambda < 0.5h \) are defined as intermediate depth waves and deep water short waves, respectively. As the parameter \( kh \) increases, effects of frequency dispersion
become more significant.

In the depth-integrated modeling approaches, long waves may be appropriately described with the nonlinear and non-dispersive shallow water equations, which have been widely employed in the prediction of tsunami propagation and run-up (e.g., Liu et al., 1995, Vasily and Synolakis, 1995), as well as tsunami generation due to underwater landslides (e.g., Jiang and LeBlond, 1993, Imran et al., 2001 a, b). For shorter wavelengths, frequency dispersion becomes more important. Researchers normally define two parameters, \( \varepsilon = a/h \) and \( \mu^2 = (h/\lambda)^2 \), to represent the importance of nonlinearity and frequency dispersion. Peregrine (1967) developed the equations for long waves propagating over uneven bottom. These equations retain the first-order nonlinear and second-order dispersive terms and adopt the Boussinesq assumption of balanced nonlinear and dispersive effects, i.e. \( O(\varepsilon) = O(\mu^2) \). Wu (1981, 1987) extended Peregrine’s model to include both wave generation and propagation over uneven, time-varying water depth. Wu’s (1981, 1987) generalized Boussinesq model in non-dimensional form is

\[
\zeta_t + \nabla \cdot [(h + \zeta) \nabla \Phi] = -h_t + \nabla \cdot \left\{ \left[ \frac{h}{2} (h_t + \nabla \cdot (h \nabla \Phi)) \right] - \frac{1}{3} h^2 \nabla^2 \Phi \right\} \nabla h, \tag{1.8}
\]

\[
\tilde{\Phi}_t + \frac{1}{2} (\nabla \tilde{\Phi})^2 + \zeta + p_e = \frac{h}{2} [h_t + \nabla \cdot (h \nabla \tilde{\Phi})] - \frac{1}{6} h^2 \nabla^2 \tilde{\Phi}_t, \tag{1.9}
\]

where \( \Phi \) is the depth averaged velocity potential

\[
\tilde{\Phi} = \frac{1}{h + \zeta} \int_{-h}^{h} \phi dz.
\]

Wu and Wu (1982) applied Wu’s (1981) model to simulate long waves generated by a surface pressure forcing and discovered that when the external forcing moves at near-
critical speed, a series of run-away solitons may be generated upstream of the moving
disturbances. Wu's (1981) model also is applicable to predicting wave generation by
submerged moving disturbances such as a moving object along the seafloor. Many
studies were carried out in the 1980s and 90s to study water waves generated by
submerged moving disturbances by using the Boussinesq model (e.g., Lee et al., 1989,
Teng and Wu, 1992, 1997) and the Green-Naghdi model (e.g., Ertekin et al., 1986).
Nwogu (1993) developed a new form of the Boussinesq model, i.e., the extended
Boussinesq model, in which the velocity at a reference water level \( z_a \) between the still
water surface and the sea bottom is used instead of the conventional depth-averaged
velocity. It was shown that by defining the reference level at a particular depth, i.e.
\( z_a = -0.531h \), Nwogu’s (1993) model retains the appropriate linear dispersion relation
for the wavelengths from very long to as short as \( \lambda = 2h \). We should point out that even
though the new model improves the dispersion relation by introducing a different velocity
variable, the fundamental assumption is still the same as that for the Boussinesq model by
Peregrine (1967). The model still retains the lowest order of nonlinearity and dispersivity,
and assumes \( O(\varepsilon) = O(\mu^2) \). Following Nwogu’s (1993) definition of a velocity
parameter, Wei et al. (1995) further extended the Boussinesq model by retaining the fully
nonlinear and weakly dispersive effects. This model was later extended again by Lynett
and Liu (2002) to include the bottom disturbance terms, such that the new model has a
wider application to both wave generation and wave propagation. The model equations
by Lynette and Liu (2002) may be described as follows

\[
\frac{H}{\varepsilon} + \nabla \cdot (Hu) + \mu^2 \nabla \cdot \left[ H \left( \frac{z_a^2}{2} - \frac{1}{6} (\varepsilon^2 \zeta^2 - \varepsilon \zeta h + h^2) \right) \right] \nabla S_a
\]
\[ + H \left[ z_a - \frac{1}{2} (\varepsilon \zeta - h) \right] \nabla T_a \right] = O(\mu^4), \quad (1.10) \]

\[ \mathbf{u}_a + \varepsilon (\mathbf{u}_a \cdot \nabla) \mathbf{u}_a + \nabla \zeta + \mu^2 \left\{ \frac{1}{2} z_a^2 \nabla S_a + z_a \nabla T_a \right\}, \]

\[ + \varepsilon \mu^2 \nabla \left\{ \frac{1}{2} S_a^2 \mathbf{u}_a \cdot \nabla S_a + z_a \mathbf{u}_a \cdot \nabla T_a + \frac{1}{2} T_a^2 - \zeta T_a \right\} \]

\[ + \varepsilon^2 \mu^2 \nabla \left\{ - \frac{\zeta^2}{2} S_a - \zeta \mathbf{u}_a \cdot \nabla T_a + \zeta S_a T_a \right\} \]

\[ + \varepsilon^3 \mu^2 \nabla \left\{ \frac{\zeta^2}{2} (S_a^2 - \mathbf{u}_a \cdot \nabla S_a) \right\} = O(\mu^4), \quad (1.11) \]

in which \( H = h + \varepsilon \zeta \) is the total water depth including the wave elevation, \( \mathbf{u}_a = \mathbf{u}(x, y, z_a, t) \), \( S_a = \nabla \cdot \mathbf{u}_a \) and \( T_a = \nabla \cdot (h \mathbf{u}_a) + h_t / \varepsilon \). Other approaches to improve the linear dispersion relation include rearranging the terms in the model equations (e.g., Schäffer and Madsen, 1995; Liu and Sun, 2005) or applying the long wave equations in multiple layers over the water depth (e.g., Lynett and Liu, 2004, Hsiao et al., 2005, Lynett, 2006). To represent shorter waves of \( \lambda < 2h \), models with higher-order dispersive effects \( O(\mu^4) \) also have been developed (e.g., Madsen and Schäffer, 1998, Gobbi and Kirby, 1999, Gobbi et al., 2000, Liu and Sun, 2005, Ataie-Ashtiani and Najafi-Jilani, 2007).

The higher-order model equations of Gobbi and Kirby (1999) were derived based on the three-dimensional potential flow theory. The seafloor was assumed to be steady. The three-dimensional water velocity potential first was decomposed into two-dimensional components in the horizontal plane, through perturbation expansion. The model equations retain dispersion terms up to \( O(\mu^4) \) and keep all the nonlinear terms. To
retain the (4,4) Pade approximant of the linear dispersion relation, a new variable was defined as the weighted average of water velocity potentials over two reference levels between the initial still water surface and the bottom. Developed based on the three-dimensional Euler equations, Ataie-Ashtiani and Najafi-Jilani’s (2007) model also retains fully nonlinear and fourth-order dispersive terms. Instead of the weighted-averaged water velocity and velocity potential, a weighted-averaged reference level was introduced in the model to reach the improved linear dispersion relation.

In the depth-integrated models, vertical profiles of water velocities need to be approximated by a specific function. The traditional shallow water equations assume that the water velocities distribute uniformly along the water depth. In the weakly dispersive models, the velocity profiles are approximated with a quadratic function of the vertical coordinate. For shorter waves over intermediate water depth, the flow variation in the vertical direction becomes more significant; therefore neither the uniform assumption nor the quadratic assumption is sufficient. In the higher-order models developed by Gobbi and Kirby (1999) and Ataie-Ashtiani and Najafi-Jilani (2007), the vertical profiles of water velocities were approximated by a fourth-order polynomial, which was intended to improve the accuracy of the predicted velocity distribution.

Thus far, existing research studies have been focused more on wave propagation, than on wave generation, and more on wave elevations, than on the water currents due to waves and the landslide motion. Water currents are equally important as the wave elevations to understand the complicated process of tsunami generation due to underwater landslides, as well as to assess the tsunami impact on coastal structures, sediment transport and beach erosion. It is necessary to develop a theoretical and
numerical model which considers the time-varying bathymetry and has improved prediction of both the water surface elevations and the wave induced currents.

1.3 Objectives

The present study is focused on the development of a depth-integrated model for predicting both the waves and currents induced by underwater landslides. The present model is an extension of Gobbi and Kirby’s (1999) theoretical model for wave propagation, which retains the fully nonlinear, higher-order, $O(\mu^4)$, dispersive effects but did not consider the time variation in the water depth caused by underwater landslides. The prediction of water velocity profiles also is improved by introducing a fourth-order polynomial of the vertical coordinate. In the present model, the seafloor is considered as time-varying.

Numerical solutions of the new higher-order model equations are obtained for one-dimensional cases by using a finite-difference scheme. The numerical model is validated against laboratory experiments, which are conducted in this study. The landslide-generated water waves, as well as the water velocities are measured using resistance-type wave gauges and particle image velocimetry (PIV), respectively.

The present numerical model, after being validated, then is applied to investigate the tsunami sensitivity to numerous landslide properties. Based on the simulation results, empirical equations are derived for the near-field wave amplitude and water velocities as a function of landslide properties, such as the maximum thickness, the initial submergence, and the initial acceleration, among other parameters.

We would like to mention that after the new model in this thesis was derived, we
became aware of the recent publication by Ataie-Ashtiani and Najafi-Jiilani (2007) who also derived a fourth-order depth-integrated model for wave generation and propagation. The similarities and differences between the two independently derived fourth-order models will be described in the next chapter. We also note that there were no experiments conducted in Ataie-Ashtiani and Najafi-Jiilani’s study (2007) and the velocity field was not simulated and validated. In the present study, a new model is derived, and also a large set of wave tank experiments has been carried out. The measurement of the landslide induced velocity field by PIV and the application of the PIV data for validation of the 4th order model in this study is among the first to be reported in the subject area.
CHAPTER 2
THEORETICAL DERIVATION

In this chapter, the higher-order depth-integrated model equations are derived. Figure 2.1 is a schematic diagram of the physical problem. The origin of the Cartesian coordinate system is set to the still water level, with the vertical coordinate $z$ pointing upward. The derivation of model equations follows the same procedure as Gobbi and Kirby (1999), except that we consider time-variable seafloor bathymetry, which is assumed to have the same length and time scales as the surface waves. The resulting equations extend those of Gobbi and Kirby (1999) to account for seafloor disturbances such as underwater landslides.

2.1 Governing Equations

The boundary value problem of the waves generated by a bottom disturbance can be described in terms of velocity potential $\phi(x, y, z, t)$ as follows:

$$\phi_{zz} + \nabla^2 \phi = 0, \quad -h \leq z \leq \zeta,$$  \hspace{1cm} (2.1)

$$h_t + \nabla \phi \cdot \nabla h + \phi_z = 0, \quad z = -h,$$  \hspace{1cm} (2.2)

$$\zeta_t + \nabla \phi \cdot \nabla \zeta - \phi_z = 0, \quad z = \zeta,$$  \hspace{1cm} (2.3)

$$g \zeta + \frac{p}{\rho} + \phi_t + \frac{1}{2} \left[ \nabla \phi \cdot \nabla \phi + (\phi_z^2) \right] = 0, \quad z = \zeta,$$  \hspace{1cm} (2.4)

where $\nabla = (\partial/\partial x, \partial/\partial y)$ is the horizontal gradient operator, $p$ is pressure.

Introducing characteristic wavelength $\lambda_0$ as the horizontal length scale, the characteristic water depth $h_0$ as the vertical length scale, and the characteristic wave
amplitude $a_0$ as the scale of wave motion and bottom disturbance, we define the following non-dimensional variables,

$$(x', y') = \left( \frac{x}{\lambda_0}, \frac{y}{\lambda_0} \right), \quad z' = \frac{z}{h_0}, \quad r' = \frac{\sqrt{gh_0} t}{\lambda_0},$$

$$\zeta' = \frac{\zeta}{a_0}, \quad \phi' = \frac{\phi}{a_0 \sqrt{gh_0}}, \quad p' = \frac{p}{\rho ga_0}. \quad (2.5)$$

The nonlinear and dispersive effects are represented by the two parameters $\varepsilon = a_0 / h_0$ and $\mu^2 = (h_0 / \lambda_0)^2$, respectively. In the present study, the fully nonlinear assumption is adopted, i.e., $\varepsilon = O(1)$, while the dispersive effect is accurate to $O(\mu^4)$.

In non-dimensional form, the Laplace equation and the boundary conditions become, after dropping the primes for convenience:

$$\phi_{zz} + \mu^2 \nabla^2 \phi = 0, \quad -h \leq z \leq \varepsilon \zeta, \quad (2.6)$$

$$\frac{\mu^2}{\varepsilon} h_t + \mu^2 \nabla \phi \cdot \nabla h + \phi_z = 0, \quad z = -h, \quad (2.7)$$

$$\zeta_t + \varepsilon \nabla \phi \cdot \nabla \zeta - \frac{1}{\mu^2} \phi_z = 0, \quad z = \varepsilon \zeta. \quad (2.8)$$

$$\zeta + p + \phi_g + \frac{1}{2} \varepsilon \left[ \nabla \phi \cdot \nabla \phi + \frac{1}{\mu^2} (\phi_z)^2 \right] = 0, \quad z = \varepsilon \zeta. \quad (2.9)$$

2.2 Expansion of Velocity Potential

We reduce the dimensionality of the boundary value problem by introducing a series expansion of $\phi$

$$\phi(x, y, z, t) = \sum_{n=0}^{\infty} \zeta^n \phi_n(x, y, t), \quad (2.10)$$
where \( \xi = h + z \). Substituting equation (2.10) into (2.7), we obtain the expression for \( \phi_1 \) in terms of \( \phi_0 \) as

\[
\phi_1 = -\mu^2 G \left( \nabla h \cdot \nabla \phi_0 + \frac{h_1}{\varepsilon} \right),
\]

where,

\[
G = \frac{1}{1 + \mu^2 \nabla h \cdot \nabla h}.
\]

After substituting equation (2.10) into (2.6), a recursive equation is obtained as

\[
(n + 2)(n + 1)\phi_{n+2} + \mu^2 \left( (n + 2)(n + 1)\nabla h \cdot \nabla h \phi_{n+2} \right.
\]

\[
+ (n + 1)\nabla^2 h \phi_{n+1} + 2(n + 1)\nabla h \cdot \nabla \phi_{n+1} + \nabla^2 \phi_n \big] = 0.
\]

This equation is used to obtain the expressions of the remaining components of \( \phi_n \) in terms of \( \phi_0 \)

\[
\phi_1 = -\mu^2 GT_0,
\]

\[
\phi_2 = -\frac{\mu^2}{2} GS_0 + \frac{\mu^4}{2} G^2 T_0 \nabla^2 h + \mu^4 G \nabla h \cdot \nabla (GT_0),
\]

\[
\phi_3 = \frac{\mu^4}{6} \left[ S_0 \nabla^2 h + 2 \nabla h \cdot \nabla S_0 + \nabla^2 T_0 \right] + O(\mu^6),
\]

\[
\phi_4 = \frac{\mu^4}{24} \nabla^2 S_0 + O(\mu^6),
\]

where,

\[ S_0 = \nabla^2 \phi_0 \text{ and } T_0 = \nabla h \cdot \nabla \phi_0 + \frac{h_1}{\varepsilon}. \]

Note that function \( G \) may be rewritten as \( G = 1 + O(\mu^2) \), and that the higher-order part of \( G \) has been assimilated into the truncated terms of \( O(\mu^6) \) in equations (2.16) and
(2.17). Substituting equations (2.14)-(2.17) into (2.10) and truncating at $O(\mu^6)$, the velocity potential $\phi$ may be described as follows

$$
\phi = \phi_0 - \mu^2 \left\{ \xi GT_0 + \frac{1}{2} \xi^2 GS_0 \right\} + \mu^4 \left\{ \xi^3 \left( \frac{1}{2} T_0 \nabla^2 h + \nabla h \cdot \nabla T_0 \right) \right\}
$$

$$
+ \xi^2 \left( \frac{1}{6} S_0 \nabla^2 h + \frac{1}{3} \nabla h \cdot \nabla S_0 + \frac{1}{6} \nabla^2 T_0 \right) + \frac{1}{24} \xi^4 \nabla^2 S_0 \right\} + O(\mu^6). \tag{2.18}
$$

In order to obtain an improved dispersion relation and more accurate prediction for the velocity profile, we follow Gobbi and Kirby's (1999) study in defining a new velocity potential variable as

$$
\bar{\phi} = \gamma \phi_\alpha + (1 - \gamma) \phi_\beta, \tag{2.19}
$$

in which, $\phi_\alpha$ and $\phi_\beta$ are velocity potentials at reference levels $z_\alpha = \alpha h$ and $z_\beta = \beta h$, and $\gamma$ is a weighting factor. Substituting equation (2.19) into (2.18) and inverting the expression, the velocity potential $\bar{\phi}$ can be written in terms of $\phi_0$ as

$$
\bar{\phi} = \phi_0 - \mu^2 \left\{ AhGT_0 + \frac{1}{2} Bh^2 GS_0 \right\} + \mu^4 \left\{ Bh^2 \left( \frac{1}{2} T_0 \nabla^2 h + \nabla h \cdot \nabla T_0 \right) \right\}
$$

$$
+ Ch^3 \left( \frac{1}{6} S_0 \nabla^2 h + \frac{1}{3} \nabla h \cdot \nabla S_0 + \frac{1}{6} \nabla^2 T_0 \right) + \frac{1}{24} Dh^4 \nabla^2 S_0 \right\} + O(\mu^6), \tag{2.20}
$$

in which the parameters $A, B, C, D$ are defined as:

$$
A = \gamma (1 + \alpha) + (1 - \gamma) (1 + \beta), \quad B = \gamma (1 + \alpha)^2 + (1 - \gamma) (1 + \beta)^2,
$$

$$
C = \gamma (1 + \alpha)^3 + (1 - \gamma) (1 + \beta)^3, \quad D = \gamma (1 + \alpha)^4 + (1 - \gamma) (1 + \beta)^4. \tag{2.21}
$$

Inverting equation (2.20) yields the expression of $\phi_0$ in terms of $\bar{\phi}$ as

$$
\phi_0 = \bar{\phi} + \mu^2 \left\{ AhG\bar{T} + \frac{1}{2} Bh^2 G\bar{S} \right\} + \mu^4 \left\{ Ah\nabla h \cdot \nabla \left( Ah\bar{T} + \frac{1}{2} Bh^2 \bar{S} \right) \right\}
$$
where

\[ \phi = \phi^* + \mu \left\{ (Ah - \xi)F_1 + (Bh^2 - \xi^2)F_2 \right\} + \mu^4 \left\{ (Ah - \xi)F_3 + (Bh^2 - \xi^2)F_4 \right\} + O(\mu^5), \]

Equations (2.22) can be substituted into (2.19) to yield the description of \( \phi \) in terms of \( \phi^* \) as

\[ \phi = \phi^* + \mu \left\{ (Ah - \xi)F_1 + (Bh^2 - \xi^2)F_2 \right\} + \mu^4 \left\{ (Ah - \xi)F_3 + (Bh^2 - \xi^2)F_4 \right\} + O(\mu^5), \] (2.23)

where

\[ F_1 = G\tilde{T}, \]
\[ F_2 = \frac{1}{2} G\tilde{S}, \]
\[ F_3 = \nabla h \cdot \nabla (Ah\tilde{T}) + \nabla h \cdot \nabla (Bh^2 \tilde{S}), \]
\[ F_4 = \frac{1}{4} A\nabla^2 (h\tilde{T}) + \frac{1}{4} B\nabla^2 (h^2 \tilde{S}) + \nabla^2 h\tilde{T} - \nabla h \cdot \nabla \tilde{T}, \]
\[ F_5 = -\frac{1}{6} \nabla^2 h\tilde{S} - \frac{1}{3} \nabla h \cdot \nabla \tilde{S} - \frac{1}{6} \nabla^2 \tilde{T}, \]
\[ F_6 = -\frac{1}{24} \nabla^2 \tilde{S}. \]

2.3 Derivation of the Depth-Integrated Equations

Integrating equation (2.6) from \(-h\) to \(\eta\) and applying the boundary conditions
(2.7) and (2.8), we have the depth-integrated equation of mass conservation as follows

\[
\frac{H}{\varepsilon} + \nabla \cdot \mathbf{M} = O(\mu^6), \tag{2.24}
\]

where

\[
\mathbf{M} = H \nabla \tilde{\phi} + \mu^2 H \left\{ \left[ (A-1)F_1 + 2\left( Bh - \frac{H}{2}\right)F_2 \right] \nabla h + \left( Ah - \frac{H}{2}\right) \nabla F_1 + \left( Bh^2 - \frac{H^2}{3}\right) \nabla F_2 \right\} \\
+ \mu^4 H \left\{ \left[ (A-1)F_3 + 2\left( Bh - \frac{H}{2}\right)F_4 + 3\left( Ch^2 - \frac{H^2}{3}\right)F_5 + 4\left( Dh^3 - \frac{H^3}{4}\right)F_6 \right] \nabla h \\
+ \left( Ah - \frac{H}{2}\right) \nabla F_3 + \left( Bh^2 - \frac{H^2}{3}\right) \nabla F_4 + \left( Ch^3 - \frac{H^3}{4}\right) \nabla F_5 + \left( Dh^4 - \frac{H^4}{5}\right) \nabla F_6 \right\}.
\]

Substituting equation (2.23) into equation (2.9) and truncating at \(O(\mu^6)\), we have the approximate unsteady Bernoulli equation defined at the free surface as

\[
\zeta + \tilde{\phi} + \frac{1}{2} \varepsilon \nabla \tilde{\phi} \cdot \nabla \tilde{\phi} + \mu^2 I_2 + \mu^4 I_4 + \frac{1}{2} \varepsilon \mu^2 (2\nabla \tilde{\phi} \cdot \Gamma_2 + J_2^2) \\
+ \frac{1}{2} \varepsilon \mu^4 (\Gamma_2 \cdot \Gamma_2 + 2\nabla \tilde{\phi} \cdot \Gamma_4 + 2J_2J_4) = O(\mu^6), \tag{2.25}
\]

where

\[
I_2 = \left[ (A-1)F_1 + 2(Bh - H)F_2 \right] h + (Ah - H)F_1 + (Bh^2 - H^2)F_2, \\
I_4 = \left[ (A-1)F_3 + 2(Bh - H)F_4 + 3(Ch^2 - H^2)F_5 + 4(Dh^3 - H^3)F_6 \right] h, \\
+ (Ah - H)F_3 + (Bh^2 - H^2)F_4 + (Ch^3 - H^3)F_5 + (Dh^4 - H^4)F_6, \\
J_2 = -(F_1 + 2HF_2), \\
J_4 = -(F_3 + 2HF_4 + 3H^2F_5 + 4H^3F_6), \\
\Gamma_2 = \left[ (A-1)F_1 + 2(Bh - H)F_2 \right] \nabla h + (Ah - H) \nabla F_1 + (Bh^2 - H^2) \nabla F_2.
\]
\[ \Gamma_4 = [(A-1)F_3 + 2(Bh - H)F_4 + 3(Ch^2 - H^2)F_5 + 4(Dh^3 - H^3)F_6] \sqrt{h} \]
\[ + (A(h - H) \nabla F_3 + (Bh^2 - H^2) \nabla F_4 + (Ch^3 - H^3) \nabla F_5 + (Dh^4 - H^4) \nabla F_6]. \]

The depth-integrated continuity equation (2.24) and the unsteady Bernoulli equation (2.25) form the two-equation system for the fully nonlinear and fourth-order dispersive mathematical model.

Parameters \( \alpha, \beta \) and \( \gamma \) were determined by Gobbi et al. (2000) such that the higher-order model retains a linear dispersion relation in forms of \((4,4)\) Padé approximant of the exact solution. The equations relating \( \alpha \) and \( \beta \) to \( \gamma \) read as follows

\[ \alpha = \sqrt{\frac{1}{9} \left[ \frac{8\gamma}{567(1 - \gamma)} \right]^{1/2} + \left[ \frac{8}{567\gamma(1 - \gamma)} \right]^{1/2}} - 1, \] (2.26)

\[ \beta = \sqrt{\frac{1}{9} \left[ \frac{8\gamma}{567(1 - \gamma)} \right]^{1/2}} - 1. \] (2.27)

As there are three parameters to be determined by two equations, there can be an infinite number of possible values. In order for \( \alpha \) and \( \beta \) to have values that are physically meaningful, i.e. real numbers between -1 and 0, the weighting parameter \( \gamma \) should be in the range of \( 0.018 \leq \gamma \leq 0.467 \).

Gobbi et al. (2000) presented the comparison of the linear dispersion relation and the profiles of water velocities among the \( O(\mu^4) \)-model, Nwogu’s (1993) extended Boussinesq model and the exact analytical solution to the linear potential flow theory. The comparison is based on the small amplitude periodic waves over constant water depth. It was shown that good agreement between the \( O(\mu^4) \)-model and potential flow theory can be extended to waves of \( h/\lambda = 1 \).
The non-dimensional equations (2.24) and (2.25) involve parameters $\varepsilon$ and $\mu$, which cannot be determined beforehand in the simulations. For convenience, an alternative group of scaling factors can be introduced to make these parameters disappear from the final equations. The new scaling parameters use the undisturbed characteristic water depth as the length scale in both the horizontal and vertical directions, i.e.

$$(x', y', z', \zeta') = \frac{(x, y, z, \zeta)}{h_0}, \quad t' = \frac{g t}{h_0}, \quad \phi' = \frac{\phi}{\varepsilon h_0 \sqrt{g h_0}}.$$

(2.28)

After the equations are derived by perturbation matching at different orders, a change in the scaling variable will only change the appearance of the final equations but not the orders of the terms in the equations.

If we truncate equations (2.24) and (2.25) at $O(\mu^4)$, and define the velocity potential at the reference depth recommended by Nwogu (1993), i.e., $\phi = \phi(x, y, z_a, t)$ and $z_a = -0.531h$, then the present model becomes equivalent to the fully-nonlinear, weakly dispersive model developed by Lynett and Liu (2002).

2.4 Discussion

In this chapter, we extend Gobbi and Kirby's (1999) fully nonlinear, fourth-order dispersive model by including additional terms describing the seafloor disturbance. The extension made here is to improve the higher-order model such that it can be applied to simulate the water waves and currents generated by the submergence bottom disturbances such as the underwater landslides. The new model equations have the same linear analytical properties as those of Gobbi and Kirby (1999), such as the linear dispersion relation and internal kinematics, for small amplitude periodic waves propagating over
constant water depth. The properties of the new model for water waves generated by seafloor disturbance are not analyzed in this chapter, but will be investigated later through numerical and experimental studies.

The present model is similar to the model by Ataie-Ashtiani and Najafi-Jilani (2007) in the sense that both models are fourth-order models and both include wave propagation and generation. However, the derivation of the two models differs. Instead of the weight-averaged water velocity and velocity potential, Ataie-Ashtiani and Najafi-Jilani (2007) defined a weight-averaged water depth

\[ \bar{z} = \gamma a + (1 - \gamma) \bar{z}_p, \]  

(2.29)

where the parameters \( \gamma, \alpha \) and \( \beta \) are the same as those in Gobbi and Kirby's (1999) study and those in the present study. In order to have the \((4,4)\) Padé approximant of the exact linear dispersion relation, Ataie-Ashtiani and Najafi-Jilani (2007) also required the \( "n" \)-th power of \( \bar{z} \) be given by

\[ \bar{z}^n = \gamma a^n + (1 - \gamma) \bar{z}_p^n, \]  

(2.30)

for the same value of \( \gamma \) in both (2.29) and (2.30). In addition, the final model by Ataie-Ashtiani and Najafi-Jilani (2007) involves three unknowns, namely, the wave elevation and the two velocity components in the horizontal plane, while the present model is derived for two unknowns, i.e., the wave elevation and the velocity potential, which is easier to implement numerically.
Figure 2.1 A sketch of an underwater landslide and the water waves it generates
CHAPTER 3
NUMERICAL ALGORITHM

In this chapter, we present a finite difference algorithm to solve the present \(O(\mu^4)\)-model equations in one spatial dimension. Similar to Wei and Kirby (1996), the fourth-order Adams-Bashforth-Moulton predictor-corrector scheme is employed to approximate the time integration. We discretize the spatial derivatives through a mixed-order central difference scheme. As a result, the nonlinear terms of \(O(1)\) are approximated with fourth-order accuracy, the weakly dispersive terms of \(O(\mu^2)\) and higher-order terms of \(O(\mu^4)\) with third-order and second-order spatial accuracy, respectively. The present discretization method is aimed at reducing the truncation errors of the lower-order terms, such that the simulation converges quickly.

3.1 Model Equations in One Spatial Dimension

The present \(O(\mu^4)\)-model equations may be rewritten in one-dimensional form as

\[ H_t = -M_x, \quad (3.1) \]

\[ U_t = -F_b, \quad (3.2) \]

where

\[ M = H\tilde{\phi}_x + M_2 + M_4, \]

\[ M_2 = H\left\{ (A-1)F_1 + 2\left(Bh-\frac{H}{2}\right)F_2 \right\} h_x + \left(Ah-\frac{H}{2}\right)F_{1x} + \left(Bh^2-\frac{H^2}{3}\right)F_{2x} \],
The Bernoulli equation has been regrouped as in equation (3.2) so that all the time-derivatives of \( \phi \) appear on the left-hand side of the equation only. We have observed in the present study that this grouping strategy significantly improves the stability of numerical simulations. To solve \( \phi \) from equation (3.2), we rewrite \( U \) as follows

\[
M_4 = H \left[ \left( A - 1 \right) F_3 + 2 \left( B h - \frac{H}{2} \right) F_4 + 3 \left( C h^2 - \frac{H^2}{3} \right) F_5 + 4 \left( D h^3 - \frac{H^3}{4} \right) F_6 \right] h_x \\
+ \left( A h - \frac{H}{2} \right) F_{3x} + \left( B h^2 - \frac{H^2}{3} \right) F_{4x} + \left( C h^3 - \frac{H^3}{4} \right) F_{5x} + \left( D h^4 - \frac{H^4}{5} \right) F_{6x} \right),
\]

\[
U = \tilde{\phi} + (A h - H) F_1 + (B h^2 - H^2) F_2 + (A h - H) F_3 + (B h^2 - H^2) F_4 \\
+ (C h^3 - H^3) F_5 + (D h^4 - H^4) F_6,
\]

\[
F_5 = \zeta + p + \frac{1}{2} \phi_x^2 + \frac{1}{2} F_1 + 2 H F_2 + F_3 + 2 H F_4 + 3 H^2 F_5 + 4 H^3 F_6 \phi_x^2,
\]

\[
+ \tilde{\phi}_x \Gamma_2 + \frac{1}{2} \phi_x^2 + \Gamma_2 + \frac{1}{2} \phi_x^2 \Gamma_4 + J_2 J_4.
\]

The Bernoulli equation has been regrouped as in equation (3.2) so that all the time-derivatives of \( \phi \) appear on the left-hand side of the equation only. We have observed in the present study that this grouping strategy significantly improves the stability of numerical simulations. To solve \( \phi \) from equation (3.2), we rewrite \( U \) as follows

\[
U = c_0 + \tilde{\phi} + c_2 \phi_x + c_3 \phi_{xx} + c_4 \phi_{xxx}, \tag{3.3}
\]

where,

\[
c_0 = (A h - H) Gh_x + (A h - H) (A h_x^2 h_x + A h h_{xx}) \\
+ (B h^2 - H^2) \left[ \frac{1}{2} (A - 1) h_x^2 h_x + (A - 1) h_x h_{xx} + \frac{1}{2} A h h_{ext} \right] \\
- \frac{1}{6} (C h^3 - H^3) h_{ext},
\]

\[
c_1 = (A h - H) Gh_x + (A h - H) (A h_x^2 + h h_{xx}) - \frac{1}{6} (C h^3 - H^3) h_{xxx}.
\]
3.2 Fully Reflective Boundary Conditions

In the present study, we assume the simulation domain is bounded by vertical walls and apply the simple fully reflective boundary conditions. To protect the simulation from being affected by the reflected waves, the vertical walls are put far enough from the region of interest. The kinematic condition for the reflective boundary may be stated as

\[ \nabla \phi \cdot \mathbf{n} = 0, \quad -h \leq z \leq \zeta, \]  

(3.4)

where \( \mathbf{n} \) is the unit normal vector pointing outward from the domain. Substituting the definition of \( \phi \) in terms of \( \tilde{\phi} \) as shown in equation (2.23), the following expression may be obtained

\[ \nabla \tilde{\phi} \cdot \mathbf{n} = 0. \]  

(3.5)

Along the boundaries enclosing the computational domain at the free surface, the gradient of the Bernoulli equation may be written as

\[ +(Bh^2 - H^2) \left[ \frac{3}{2}(A-1)h_x h_{xx} + \frac{1}{2} Ahh_{xx} \right], \]

\[ c_2 = \frac{1}{2} (Bh^2 - H^2) G + (A + 2B)hh_x^2 (Ah - H) \]

\[ +(Bh^2 - H^2) \left[ \left( A + \frac{B}{2} - 1 \right) h_x^2 + \left( A + \frac{B}{2} \right) hh_{xx} \right] - \frac{1}{2} (Ch^3 - H^3) h_{xx}, \]

\[ c_3 = (Ah - H) Bh^2 h_x + (Bh^2 - H^2) \left( \frac{A}{2} + B \right) hh_x - \frac{1}{2} (Ch^3 - H^3) h_x, \]

\[ c_4 = \frac{1}{4} (Bh^2 - H^2) Bh^2 - \frac{1}{24} (Dh^4 - H^4). \]
Since the boundaries are fixed at the particular spatial locations, the unit normal vector on the vertical wall satisfies the following conditions:
\[ n_i = 0 \text{ and } n_z = 0. \] (3.7)

It can be shown that equation (3.6) reduces to
\[ \nabla \phi_z \cdot n = (\nabla \phi \cdot n)_z, \] (3.8)
\[ \nabla (\phi_z)^2 \cdot n = 2\phi_z (\nabla \phi \cdot n)_z. \] (3.9)

Making use of the following vector identity
\[ \frac{1}{2} \nabla (\nabla \phi \cdot \nabla \phi) = \nabla \phi \times (\nabla \times \nabla \phi) + (\nabla \phi \cdot \nabla) \nabla \phi, \] (3.10)
we obtain the following equation
\[ \frac{1}{2} \nabla (\nabla \phi \cdot \nabla \phi) \cdot n = (\nabla \phi \cdot n) \nabla^2 \phi. \] (3.11)

According to the kinematic condition along the vertical wall, the terms on the right-hand side of equations (3.8), (3.9) and (3.11) are all eliminated. Therefore, equation (3.6) is reduced to
\[ \nabla \zeta \cdot n = 0. \] (3.12)

Written in one spatial dimension, the boundary conditions (3.5) and (3.12) become
\[ \tilde{\phi}_x = 0 \text{ and } \zeta_x = 0. \] (3.13)

### 3.3 Spatial Differencing

In the present study, we discretize the spatial derivatives through the mixed lower- and higher-order central difference schemes. The first-order derivatives are
approximated to fourth-order accuracy, while the second- and higher-order terms are
discretized to third-order and second-order accuracy, respectively. Let "1" and "N"
denote the grid nodes on the left and right boundaries of the computational domain, \( \Delta x \)
be the uniform grid size and \( u \) the variable to be evaluated. On the interior grid nodes,
i.e., \( i=3, 4, \ldots, N-2 \), the central difference scheme reads

\[
(u_i)_x = \frac{8(u_{i+1} - u_{i-1}) - (u_{i+2} - u_{i-2})}{12\Delta x} + O(\Delta x^4),
\]

\[
(u_i)_{xx} = \frac{16(u_{i+1} + u_{i-1}) - (u_{i+2} + u_{i-2}) - 30u_i}{12\Delta x^2} + O(\Delta x^3),
\]

\[
(u_i)_{xxx} = \frac{u_{i+2} - 2u_{i+1} + 2u_{i-1} - u_{i-2}}{2\Delta x^3} + O(\Delta x^2),
\]

\[
(u_i)_{xxxx} = \frac{u_{i+2} - 4u_{i+1} + 6u_i - 4u_{i-1} + u_{i-2}}{\Delta x^4} + O(\Delta x^2).
\]

The spatial derivatives on the boundary nodes, i.e. \( i=1, 2, N-1 \) and \( N \), are usually
approximated by the off-centered difference scheme (e.g., Gobbi and Kirby, 1999). It is
observed that applying the off-centered difference scheme to the higher-order equations
may cause instability of the numerical model, especially when a highly nonlinear wave
propagates into the vicinity of boundaries. In this study, we introduce image nodes
outside the computational domain and apply central difference approximation to the
boundary nodes. Assume there are four image nodes immediately outside the
computational domain, which are denoted by \( i=-1, -2, N+1 \) and \( N+2 \). To satisfy the fully
reflective conditions on the boundary nodes, we set the quantities on the image nodes in
such a way that for \( u \) being a vector, its magnitude on the image nodes are determined
from the neighboring boundary nodes as

\[
u_{i-1} = -u_2, \quad u_{i-2} = -u_3, \quad u_{N+1} = -u_{N-1}, \text{ and } u_{N+2} = -u_{N-2},
\]

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for \( u \) being a scalar quantity, we have its values on the image nodes as
\[
\begin{align*}
    u_{-1} &= u_2, \quad u_{-2} = u_3, \quad u_{N+1} = u_{N-1}, \text{ and } u_{N+2} = u_{N-2}.
\end{align*}
\]

After discretization, a set of algebraic equations may be constructed from equation (3.3)
\[
A_{N \times N} \{ \phi \}_N = \{ U \}_N - \{ c_0 \}_N, 
\]
where \( A_{N \times N} \) is a diagonal matrix
\[
\begin{bmatrix}
    a_{1,1} & a_{1,2} & a_{1,3} & 0 & 0 & 0 & \cdots & \cdots & 0 \\
    a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & 0 & 0 & \cdots & \cdots & 0 \\
    a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} & a_{3,5} & 0 & \cdots & \cdots & 0 \\
    0 & a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} & a_{4,5} & 0 & \cdots & 0 \\
    0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \cdots & 0 \\
    0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
    0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
    0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
    0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
\end{bmatrix}
\]
\]

Equation (3.14) can be solved through standard LU decomposition method.

3.4 Time Differencing

We employ the fourth-order Adams-Bashforth-Moulton scheme in the estimation of time integration. This scheme involves a predictor-corrector procedure and requires iteration for the corrector stage. To evaluate the equations at a new time level, which is denoted by \( n + 1 \), we make use of the variables at three previous time levels, denoted by \( n, n-1 \) and \( n-2 \), respectively. Let the model equations be represented as \( y_i = f(y, t) \).

If the "cool start" initial condition is employed, i.e., the surface elevation and velocity potential are assumed to be null in the computational domain, the predictor and corrector
stages may be described as
\[
y_{p}^{n+1} = y^n + \frac{\Delta t}{12} (23 f^n - 16 f^{n-1} + 5 f^{n-2}),
\]
\[
y_{n+1} = y^n + \frac{\Delta t}{24} (9 f_{p}^{n+1} + 19 f^n - 5 f^{n-1} + f^{n-2}),
\]
where the subscript \(\text{"p"}\) denotes the predicted value, and \(f_{p}^{n+1} = f(y_{p}^{n+1},(n+1)\Delta t)\).

In case when a "hot start" initial condition is applied, the simulation at the first two time steps needs to be performed with lower-order schemes since the values at the further previous time steps are unknown. At the first time step, the equations are evaluated by the first-order explicit predictor and second-order implicit corrector
\[
y_{p}^{n+1} = y^n + \Delta t f^n,
\]
\[
y_{n+1} = y^n + \frac{\Delta t}{2} (f_{p}^{n+1} + f^n).
\]
At the second time step, we apply the second-order explicit predictor and third-order implicit corrector, which reads as
\[
y_{p}^{n+1} = y^n + \frac{\Delta t}{2} (f^n - f^{n-1}),
\]
\[
y_{n+1} = y^n + \frac{\Delta t}{12} (5 f_{p}^{n+1} + 8 f^n - f^{n-1}).
\]
The corrector stage is iterated until the convergence criterion is satisfied:
\[
\max \left[ \frac{\phi_{i,p}^{n+1} - \phi_{i}^{n+1}}{\phi_{i}^{n+1}}, \frac{\zeta_{i,p}^{n+1} - \zeta_{i}^{n+1}}{\zeta_{i}^{n+1}} \right] < 10^{-6}, \text{ for } i = 1, 2, \ldots, N-1, N.
\]
Evaluation of the right-hand terms in equation (3.2) involves the computation of the time derivatives, such as \(\zeta_{i}\) and \(h_{i}\). Take \(\zeta_{i}\) as an example. If the "cool start" initial condition is employed, the predictor stage utilizes the second-order scheme.
\[ \zeta_t^n = \frac{1}{2\Delta t} (3\zeta^n - 4\zeta^{n-1} + \zeta^{n-2}) + O(\Delta t^2), \]
\[ \zeta_{t}^{n-1} = \frac{1}{2\Delta t} (\zeta^n - \zeta^{n-2}) + O(\Delta t^2), \]
\[ \zeta_{t}^{n-2} = \frac{1}{2\Delta t} (-\zeta^n + 4\zeta^{n-1} - 3\zeta^{n-2}) + O(\Delta t^2). \]

On the corrector stage, \( \zeta_t \) is approximated with third-order accuracy as
\[ \zeta_{t}^{n+1} = \frac{1}{6\Delta t} (11\zeta^{n+1} - 18\zeta^n + 9\zeta^{n-1} - 2\zeta^{n-2}) + O(\Delta t^3), \]
\[ \zeta_{t}^{n} = \frac{1}{6\Delta t} (2\zeta^{n+1} + 3\zeta^n - 6\zeta^{n-1} + \zeta^{n-2}) + O(\Delta t^3), \]
\[ \zeta_{t}^{n-1} = \frac{1}{6\Delta t} (-\zeta^{n+1} + 6\zeta^n - 3\zeta^{n-1} - 2\zeta^{n-2}) + O(\Delta t^3), \]
\[ \zeta_{t}^{n-2} = \frac{1}{6\Delta t} (2\zeta^{n+1} - 9\zeta^n + 18\zeta^{n-1} - 11\zeta^{n-2}) + O(\Delta t^3). \]

If the “hot start” initial condition is applied, lower-order estimates to the time derivatives are obtained in the first two time steps.

3.5 Standard Tests

In this section, we test the numerical model developed in this study, and compare the simulation results between the present \( O(\mu^4) \)-model, Lynett and Liu’s (2002) \( O(\mu^2) \)-model, and Wu’s (1981) Boussinesq model. The \( O(\mu^3) \)-model equations are solved with the same scheme as that of the present \( O(\mu^4) \)-model. Numerical solution to the Boussinesq model is obtained through a two-step predictor-corrector scheme following Teng and Wu (1997). The first case we simulate is a permanent form solitary
wave traveling over a constant water depth. In a nonlinear, dispersive wave model, such as the Boussinesq model, a permanent wave form and constant phase speed may be reached by a solitary wave when a balance between the nonlinear and dispersive effects is achieved. The steady solution to a permanent form solitary wave can serve as a simple and effective indicator for the mass and momentum conservative property of a numerical model. Besides the permanent form of the solitary wave solution, the effects of a seafloor disturbance in generating waves are also tested in this section. The standard case of runaway solitons forced by a submerged moving hump on the seafloor is examined first. We also simulate the waves generated by a bottom upthrust in Hammack’s (1973) experiments.

3.5.1 Permanent Form Solitary Wave

Closed form solutions for the steady solitary wave were obtained for the weakly nonlinear and weakly dispersive models, such as the KdV and the Boussinesq models by previous researchers. We assume the non-dimensional water depth equals unity. In the analytical solution of the Boussinesq equations, the spatial distribution of surface elevation, wave number and wave speed are described in non-dimensional form as follows (Teng and Wu, 1992, Teng, 1997)

\[
\zeta = \frac{a \sec h^2 k(x - ct)}{1 + a \tanh^2 k(x - ct)} \quad (3.15)
\]

\[
k = \frac{3a}{\sqrt{4(1 + 0.68a)}} \quad (3.16)
\]

\[
c = \frac{6(1+a)^2}{\sqrt{a^2(3+2a)[(1+a)\ln(1+a)-a]}} \quad (3.17)
\]
Under this condition, the depth-averaged velocity and the velocity potential may be determined as

\[ \bar{u} = \frac{c\zeta}{1 + \zeta}, \quad (3.18) \]

and

\[ \bar{\phi} = \frac{ca}{k(1 + a)} \tanh k(x - ct). \quad (3.19) \]

For the fully nonlinear models, such as the $O(\mu^2)$- and $O(\mu^4)$-models, analytical solutions are difficult to obtain due to the complicity of the model equations. For these models, researchers usually compute the approximate numerical solutions to the steady solitary wave state through an iterated procedure (Wei et al., 1995, Gobbi, 1998). In the present study, we use equations (3.15) - (3.19) as initial conditions for the $O(\mu^2)$- and $O(\mu^4)$-models, and assume $\bar{\phi} = \tilde{\phi}$ and $u_a = \bar{u}$ initially. As the initial conditions do not satisfy the fully nonlinear model equations, the wave form is not steady at the beginning of the numerical simulations, and small amplitude trailing waves are generated following the main wave. These small amplitude waves travel at a slower speed and will be separated from the leading solitary wave eventually. As a result, the leading solitary wave will reach a steady permanent form after traveling for some distance. The profiles of $\zeta$, $\tilde{\phi}$ and $u_a$ are multiplied by a small factor, usually between 0.95 and 1.05, and input as initial conditions for a new trial. This process is repeated until the steady wave form with the required wave height is reached.

We simulate a permanent form solitary wave with a wave amplitude of $a/h_o = 0.513$. The effective wavelength for this wave is about 12 water depth, which
falls into the long wave category. The grid size and time step are $\Delta x/h_0 = 0.025$ and $\Delta t/\sqrt{g/h_0} = 0.01$, respectively. Figure 3.1 shows the permanent forms of this solitary wave simulated by the $O(\mu^2)$- and $O(\mu^4)$-models, as well as computed by the Boussinesq equation. The profiles of horizontal water velocities are also presented in Figure 3.2. The comparison shows that for long solitary waves, the three numerical models predict very similar water surface elevations and water velocities. The maximum difference is less than 15% between any two models.

3.5.2 Water Waves Generated by a Moving Submerged Hump

A submerged hump moving at critical speed $U$, i.e. $U/\sqrt{gh_0} = 1$, generates run-away solitons upstream of the hump. Following these solitons is a region of depression, which is followed by a train of oscillating waves. The phenomenon of run-away solitons generated by resonant forcing was first discovered in the numerical experiments based on the Boussinesq model by Wu and Wu (1982) and Lee et al. (1989), and investigated later in the laboratory and numerical experiments by Teng and Wu (1997), among many others. In this section, we simulate the generated waves applying Wu’s (1981) Boussinesq model, Lynett and Liu’s (2002) $O(\mu^2)$-model and the present $O(\mu^4)$-model.

Considering a cosine-shape bottom hump, the still water depth at an arbitrary time may be described as

$$h(x,t) = h_0 - \frac{b_s}{2} \left[ 1 + \cos \left( \frac{2\pi}{L_s}(x + ct) \right) \right] H_c \left[ \frac{L_s}{2} - |x + ct| \right],$$

(3.20)

where $L_s/h_0 = 10$ and $b_s/h_0 = 0.1$ are the length and maximum height of the hump,
and $H_c$ is the Heaviside step function. The numerical simulations are conducted with the same uniform grid size of $\Delta x/h_0=0.05$ and constant time step of $\Delta t\sqrt{g/h_0}=0.01$ for all the numerical models. Figure 3.3 shows the free surface profiles forced by the moving hump after it travels rightward over a distance of $100h_0$ with the critical speed. Comparison indicates that the weakly dispersive models -- namely, the Boussinesq model and the model by Lynette and Liu (2002) -- simulate the waves with almost the same phase. The difference between these two models exists in the wave heights. The present $O(\mu^4)$-model predicts different wave phase as well as wave heights when compared with the weakly dispersive models.

3.5.3 Water Waves Generated by a Bottom Upthrust

Hammack (1973) conducted experiments on wave generation by a simple bottom upthrust. In the experiments, a bottom wave generator was installed at the upstream end of a wave tank. The wave generator had a flat plate which initially sat on the same level as the bottom of the wave tank. Therefore, the initial water depth was uniform along the entire wave tank. The wave generator was operated in such a way that the water depth over the wave source changes according to

$$h(x,t) = h_0 - \delta H_c(L_x - x), \quad t \geq 0,$$

in which $h_0$ is the initial undisturbed water depth, $\delta = \delta_0(1-e^{-\alpha t})$ is the upthrust displacement, $\delta_0$ is the maximum upthrust, $L_x$ is the length of the wave source, $\alpha$ is a constant controlling the upthrust speed. The motion of the bottom wave generator may be categorized as impulsive, transitional and creeping, with regard to the values of $\alpha$. In
Hammack's (1973) experiments, the parameters were set as \( a \sqrt{h_0 / g} = 1.305, 0.231 \) and 0.010, along with \( \delta_0 / h_0 = 0.2, 0.1 \) and 0.3 for impulsive, transitional and creeping motions, respectively. The length of the wave source was the same for all the three trials, i.e., \( L_s / h_0 = 12.2 \). During the experiment, the time-series of water surface elevation were measured at the upstream end of the wave tank, \( x / h_0 = 0 \), and the edge of wave source, \( x / h_0 = L_s / h_0 \).

We simulate the waves generated in Hammack's (1973) experiments with the present \( O(\mu^4) \)-model. The computational domain is discretized with a uniform grid size of \( \Delta x / h_0 = 0.05 \). The constant time step is \( \Delta t \sqrt{g / h_0} = 0.01 \). The discontinuity of water depth at the edge of wave source may introduce a singular point, over which the higher-order spatial differences of water depth become infinite and the numerical model may become unstable. To remove the point of singularity, we modify the bathymetry as

\[
h(x, t) = h_0 - \delta H_e (L_s - x) - \frac{\delta}{1 + \tanh \left[ 1 - \tanh \left( \frac{x - x_i}{L_i} \right) \right]} H_e (x - L_s),
\]

(3.22)

where \( L_i \) is the distance from the edge of wave source to the inflection point of hyperbolic tangent, \( x_i = L_s + L_i \) is the location of the inflection point. In the present study, we set \( L_i = 1.5 h_0 \).

Good agreement is observed between the numerical simulations and the experimental measurements as shown in Figure 3.3. For the impulsive motion of the bed, both the simulation and experiment indicate that the initial water surface has the maximum displacement equal to \( \delta_0 \) at the upstream end. The maximum surface elevation
at the edge of the wave source is about \( \delta_y/2 \). In Figure 3.4, we plot the ratios of \( \zeta/\delta \) at

\[ x/L_s = 0, 0.5, 0.75 \text{ and } 1.0, \] respectively. In most of the wave source region, the water surface follows the deformed tank floor during the fast impulsive thrust. The time scale of the impulsive motion is representative of the seismic seafloor deformation. This experiment shows that to predict the initial water surface profiles in the seismic tsunami, we can transform the seafloor displacement onto the water surface. For the transitional and creeping motions, the evolution of water surface is controlled by the maximum height of bottom thrust and also the time history of their motions.
Figure 3.1 Surface profiles of a solitary wave \((a/h_0=0.513)\). The present \(O(\mu^4)\)-model: solid; the \(O(\mu^2)\)-model: dash; the Boussinesq model: dot. The vertical dashed line indicates the position of wave peak.

Figure 3.2 Vertical profiles of horizontal velocities under the peak of solitary wave \((a/h_0=0.513)\). The present \(O(\mu^4)\)-model: solid; the \(O(\mu^2)\)-model: dash; the Boussinesq model: dot.
Figure 3.3 Water waves forced by a moving submerged hump traveling over $100 h_0$ with critical speed. The present $O(\mu^4)$-model: solid; the $O(\mu^2)$-model: dash; the Boussinesq model: dot. The right-moving submerged hump is depicted by a dotted curve.
Figure 3.4 Time-series of surface elevations in Hammack’s (1973) experiment of bottom upthrust. From top to bottom: impulsive, transitional and creeping motions. Left column is for the upstream end of flume. Right column is for the edge of wave source. The present $O(\mu^x)$-model: solid; experiment: circle.

Figure 3.5 Relationship between the bottom deformation and water surface elevation for the impulsive bottom upthrust. (a) Ratio of surface elevation to upthrust height. $x/L_s=0$ (solid), 1/2 (dash), 3/4 (dash-dot), 1 (dot). (b) Time history of upthrust height.
CHAPTER 4

EXPERIMENTAL STUDY AND MODEL VALIDATION

In this chapter, the present $O(\mu^4)$-model is validated against laboratory experiments on water waves and water velocities generated by rigid landslide models. Several laboratory studies on the tsunamis generated by underwater landslides have been conducted by other groups, for example, Weigel (1955), Watts (1998, 2000), Fritz et al. (2004), Liu et al. (2005), Sue et al. (2006), and Enet and Grilli, (2007), among others. However, in most of the reported experiments, the landslide models were rectangular or triangular blocks with sharp edges which may induce flow separation and vortices, or were relatively short compared with the local water depth. Therefore, the data from these experiments cannot be used to validate the depth-integrated models. In this study, we conduct quasi two-dimensional experiments in a rectangular flume in the Hydraulics Laboratory of the Department of Civil and Environmental Engineering at the University of Hawaii. The landslide models have smoothly curved surfaces. In this chapter, most of the results will be presented in dimensionless form, and both the local water depth and the landslide length will be used as the scaling parameters.

4.1 Experiments of Landslide-Generated Waves and Currents

In this section, we first describe the configuration of the experiments, and then discuss the features of the generated waves and the water velocities based on the experimental measurements.
4.1.1 Experimental Set-up

A schematic diagram of the experimental set-up is plotted in Figure 4.1. The quasi two-dimensional experiments are conducted in a rectangular water flume, which is 0.15 m (6 in.) wide, 0.41 m (16 in.) deep and 9.75 m (32 ft) long. The bottom and sidewalls are mostly constructed with Plexiglas. One sidewall in the middle section is made with glass to enhance the light transmission for particle image velocimetry (PIV). Installed in the middle section is an artificial incline with adjustable slope angle. The artificial incline is also made of Plexiglas and painted in black with waterproof paint to absorb incoming laser light.

Two rigid landslide models are utilized. Both have a smooth-curved top surface, which follows truncated cosine shape curves. The geometry of the landslide models is defined as

\[ b(x) = \frac{b_m}{2} \left[ 1 + \cos \left( \frac{2\pi}{L_0} x \right) \right], \quad \text{for} \quad -L_m/2 \leq x \leq L_m/2, \]

where \( b_m \) is the maximum height, \( L_m \) is the length of model and \( L_0 \) is the length of the full period cosine shape. The first model is 0.305 m long, 0.033 m thick and made of Plexiglas. The second model is made of aluminum with a length of 0.394 mm and a maximum thickness of 0.019 mm in the midsection. Both models have a width slightly smaller than that of the flume, and are painted black.

A pulley system is utilized to drive the landslide model sliding along the artificial incline at mild slopes (5°-15°). A steel wire is used to connect the landslide model to the pulley system. The model is held through the steel wire by a pin on top of the incline before it is released to move. Connected with the pin, we install an electronic switch to
record the time instant when the pin is raised up (i.e., released) and the model starts sliding.

Two resistance-type wave gauges, denoted as “g1” and “g2” (see Figure 4.1), are installed along the flume. The distances from the wave gauges to the still waterline on the slope are denoted as $x_{g1}$ and $x_{g2}$, respectively. In the present study, wave gauge “g1” is always above the middle of the initial landslide position.

The velocities of water particles are measured by the PIV system from TSI, Inc. This system includes a dual Nd: YAG laser generator, laser sheet optics which converts the laser beam into a laser sheet, a high speed video camera, a laser pulse synchronizer and the PIV processing software Insight 3G™. The laser generator has two laser heads. Each emits a laser pulse of 120 mJ at a frequency of 29 Hz, which also is the net sampling frequency for the PIV. The laser sheet is shot into the water flume from the downstream end of the flume to illuminate the vertical two-dimensional PIV measuring plane above the artificial incline. For each measurement, the video camera captures a pair of images with a time delay, which is usually set from 5000\(\mu\)s to 8000\(\mu\)s in the present experiments. The images are input into the computer and processed to give the instantaneous water velocity field. The displacement of the landslide model also is derived from the images taken by the high-speed video camera.

We choose 5°, 10° and 15° as the slope angles for the artificial incline. Water is filled in or drained out of the flume to vary the initial submergence of the landslide. A total of ten experimental cases of different combination of slope angles, water depths and landslide models were carried out. The configurations are listed in Table 4.1.
4.1.2 Repeatability of Experimental Results

Sometimes, experimental measurement may be contaminated by unexpected random factors, and may not represent the physical process accurately. In the present study, we repeat each experiment for at least five times, and check the repeatability of the measurements. Only those consistent with the majority of the trials in a specific experiment are considered as valid. One of the valid trials is chosen to represent the experiment. All measurements for this experiment are processed from the trial, which is chosen. No averaging or smoothing technique is employed in the data processing. As an example, plotted in Figure 4.2 are the time-series of landslide displacements, and in Figure 4.3 are the water surface elevations, both are measured in five trials of experiment case 1. Very good repeatability is observed for the landslide motion and generated waves among the different trials. This procedure is repeated for all experiments.

4.1.3 Motion of Landslide Models

In the present experiments, we adjust the traveling distance and speed of the landslide models such that they generate non-breaking waves with wave heights measurable by the resistance-type wave gauges. The Plexiglas model has a higher thickness. It usually moved slower than the thinner aluminum model. Driven by the pulley system, the Plexiglas landslide model accelerates first until the ballast weight touches the ground. Then, the model decelerates due to the drag forces and stops eventually. Accordingly, the entire process of the landslide motion may be separated into an accelerating and a decelerating stage. The rates of acceleration are found to be nearly constant during the two stages. Therefore, the landslide displacements can be fitted by a
function of time as

\[
s(t) = \begin{cases} 
\frac{a_1 t^2}{2}, & t \leq t_1, \\
\frac{a_1 t_1^2}{2} + a_1 t_1 (t - t_1) + \frac{a_2}{2} (t - t_1)^2, & t_1 < t \leq t_2,
\end{cases}
\]

where, \( s \) is the landslide displacement, \( a_1 \) and \( a_2 \) are the rates of acceleration in the two stages, \( t_1 \) and \( t_2 \) are the times when the motion in the present stage terminates. For the aluminum model, there is only an acceleration stage. The motion of the aluminum model is stopped immediately when the model reaches the toe of incline. The displacement of the aluminum model can be fitted by the following equation:

\[
s = \frac{a_1 t_1^2}{2}, \text{ for } t \leq t_1.
\]

The parameters of landslide motion in all cases are listed in Table 4.2. Shown in Figures 4.4 are the time-series of landslide displacements, measured in cases 1 and 7, as well as fitted by equations (4.2) and (4.3), respectively. The prediction by equations (4.2) and (4.3) is very accurate in both cases, with determination coefficients being 0.997 and 0.998, respectively.

4.1.4 Generated Waves and Currents

In Figures 4.5 - 4.8, we plot the time-series of water surface elevations in all the experiments. In the near field, the landslide first generates a depression over the middle of its initial position. The maximum amplitude of the depression increases with the initial acceleration \( a_1 \), and decreases with the initial submergence of landslide \( h_{bc} \). In the far field, the generated waves include a leading peak followed by a train of oscillatory waves with rapidly decaying amplitude. The amplitude of waves also is proportional to the
inverse of initial submergence of landslide. The wavelength is mainly determined by the duration of landslide motion. Listed in Table 4.2, we also show the ratio of maximum wave amplitude to local water depth at wave gauges "g1" and "g2", denoted as $h_1$ and $h_2$, respectively. In all the cases, this ratio is less than 0.1 in both near field and far field, indicating weakly nonlinear phenomena.

The PIV measurement of water velocities may be inaccurate near the surfaces where intensive reflection of laser from these surfaces often overshadows that reflected by the seeding particles. In the PIV processing software Insight 3G™, users are provided with an option to define a rectangular "region of interest" (ROI) in an image and restrict the data processing within this region. In the present study, we define an ROI for each experiment, and obtain the time-series of water velocity fields in a fixed area in every case. The ROI's are always located away from any reflecting subjects, such as the water surface, landslide model and artificial incline.

Figures 4.9 presents the velocity vector field at different time instants in the ROI in case 9 as a sample. In stages (a) and (b), the water particles near the landslide model move faster than those near the water surface. The motion of water particles becomes dominantly upward when the water surface rises up to develop a leading wave peak. In stages (c) and (d), after the leading wave peak passes, the water surface drops down and a depression is developed following the leading wave. The motion of water particles becomes dominantly backward in the horizontal direction. When a peak in the trailing waves is developing in stages (e) and (f), the motion of water particles in the upper layer is strongly upward. In the lower layer near the bottom, the motion of water particles is weak and mainly in the horizontal direction. The variation of velocities along the water
depth is more rapid under the shorter trailing waves. In the entire process, no flow separation or strong horizontal vortex is observed. This is because the waves are small amplitude non-breaking waves, and the surface of the landslide model is smooth. In Figures 4.10, we also plot the time-series of water velocities measured on five fixed points along a vertical section in the ROI.

4.2 Validation of Numerical Model

The experiments are simulated with the present $O(\mu^4)$-model, as well as Lynett and Liu's (2002) $O(\mu^2)$-model and Wu's (1981) Boussinesq model for comparison. The landslide models have geometric discontinuity at the front and rear tips. This discontinuity should be removed in the numerical simulations, or it may cause singularity in the higher-order derivatives of water depth and make the numerical models highly unstable. The geometry of the landslide is modified to have a full-period cosine shape. Having the same maximum height, its length is rescaled such that the modified geometry has the same volume as the prototype, i.e.

$$
\int_{L_m/2}^{L_s/2} \frac{b_m^2}{2} \left[ 1 + \cos \left( \frac{2\pi}{L_s} x \right) \right] dx = \int_{L_m/2}^{L_s/2} \frac{b_m^2}{2} \left[ 1 + \cos \left( \frac{2\pi}{L_0} x \right) \right] dx,
$$

where $L_s$ is the length of the modified geometry, and $L_m < L_s < L_0$. The numerical models in this study do not include a moving boundary (i.e., a run-up) algorithm. Instead, we modify the geometry near the initial shoreline by assuming the slope starts from an upstream shallow water channel. The channel has a thin constant water depth of $h/L_s = 0.01$. 

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4.2.1 Numerical Convergence

The finite difference schemes have truncation errors, which are proportional to the grid sizes and time steps. This means that decreasing the grid size may reduce the truncation errors. On the other hand, the condition of numerical stability usually requires the value of the non-dimensional time step \( \Delta t \sqrt{g/L} \) to be less than that of the non-dimensional grid size \( \Delta x/L \), which indicates that while a small grid size can improve the scheme accuracy, it may reduce the stability of the scheme. In addition, finer grid size requires more grids to be applied in the computational domain, which in turn requires more computational resources. Therefore, in order to choose an appropriate grid size for a numerical model, convergence test needs to be conducted.

We discretize the one-dimensional computational domain with grid sizes of 0.02L, 0.04L, and 0.08L, respectively, and employ constant time step of 0.0002 \( \sqrt{L/g} \). The very small time step we use here is to satisfy the stability requirement of the numerical model when the grid size is 0.02L. The length of the computation domain is 16L, and the other parameters, such as the landslide geometry and the slope of underwater incline follow those in laboratory experiment case 2. All the computations are conducted on a PC of Intel Pentium IV with a 2.4GHz CPU and 512MB memory. The numerical model is run for 60000 steps. CPU times consumed by the simulations are listed in Table 4.3. Figures 4.11 show the water surface profiles simulated by the present \( O(\mu^4) \)-model with different grid sizes. Comparison indicates a trend of convergence as the grid sizes decrease and very good consistence between the simulations with grid sizes of 0.02L and 0.04L. Convergence tests are also conducted for the \( O(\mu^2) \)-model and the Boussinesq model. Water surface profiles are presented in Figures 4.12 and 4.13, and
numerical parameters are listed in Table 4.3.

4.2.2 Numerical Results

In the present study, we simulate numerically the experimental cases with a uniform grid size of $0.04L_s$, and constant time step of $0.0005\sqrt{L_s/g}$. Shown in Figures 4.14-4.23 is the comparison between the measured water surface elevations and those simulated with the three numerical models. Determination coefficients are computed by calculating the difference between the simulated wave elevation and the measured wave elevation in the far field at wave gauge “g2”, and are listed in Table 4.4. The reason that the determination coefficients are not evaluated at “g1” is that the wave field at “g1” is not fully developed and also that the viscous effects at “g1” are greater due to its shallower depth. All the three wave models implemented numerically are inviscid models.

Comparisons of the waves at g2 indicate that the $O(\mu^4)$-model agrees the best with the experiments. Quantitative results in Table 4.4 show a significant improvement by the present fourth-order model compared with the lower-order models such as the Boussinesq model. For the leading long wave whose wavelength is usually about 5 to 6 times the water depth, all three models provide quite accurate predictions for both the wave height and the phase. The interesting observation is from the trailing wave region. In this region, the waves are relatively short with wavelength about twice the local water depth. The results from this study (e.g., results in Figure 4.15) clearly show that for shorter waves, the present fourth-order model predicts both the wave height and the phase much more accurately than the traditional lower-order Boussinesq model. Theoretically,
the extension of the Boussinesq model to higher order such as the present fourth-order model is intended to improve the dispersion relation so as to expand the validity of the depth-integrated models from long waves to include both long and shorter waves. The experimental results in this study validate this extension and improvement.

Besides the water surface elevations, we also compute the water velocities by applying the three numerical models. As mentioned in the earlier chapters, the present fourth-order model not only improves the dispersion relations, it also extends the second-order approximation of the velocity profile by the Boussinesq model to a fourth-order polynomial prediction. So far, to our knowledge, the improvement in velocity prediction by the fourth-order model has not been examined through experiments by other groups. The present study is among the first to investigate this issue.

We also would like to point it out that the accuracy of the simulated water velocities depends on both the polynomial approximation of water velocity profiles in the mathematical models and the accuracy of the predicted water surface elevations in the numerical simulations. In Figure 4.24, we compare the vertical profiles of the water velocities, simulated by the numerical models and measured in experiment case 1. The profiles are processed at a section fixed between the two wave gauges. In stage (a), the section is under the leading wave. The wave height is low and the length is long. Both the horizontal and vertical velocities are of small magnitude. In this case for relatively long waves, all the three numerical models predict the water velocities with good accuracy. In the later stages, the wavelength becomes shorter and the wave elevation becomes higher. The weakly dispersive models deviate from the measurements, while the $O(\mu^4)$-model still has good agreement with the experiment. This again, validates the present fourth-

47
order model for its improvement for both the wave and the velocity predictions. In Figures 4.25-4.34, we also present the time-series of horizontal and vertical velocities on five fixed points between the bottom and still water surface along a section. Consistent with the surface elevations, the simulated water velocities by the higher-order dispersive model show significant improvement in cases with deeper landslide submergence and steeper slope.

4.3 Discussion

The present $O(\mu^4)$-model is validated against the two-dimensional laboratory experiments conducted in this study. In all the experiments, we use mild slopes of $5\degree$-15\degree, and smooth-curved landslide models with the thickness-to-length ratios less than 10\%. This configuration reflects realistic events. According to the field surveys, most historical events of the landslide-generated tsunamis happened on very mild slopes, usually less than $5\degree$, and the landslides have very small thickness compared with their lengths. Under this condition, most of the depth-integrated wave models should be applicable. The present study shows that the fourth-order model provides better predictions for both the wave height and the fluid velocity for both the leading and trailing wave regions compared with the traditional Boussinesq model. However, we do notice that the higher-order models usually cost more CPU time while the lower models are simpler to implement and use less computer resources. Based on the particular physical wave problem at hand and the accuracy required, a researcher or engineer can make a proper choice of the different wave models as their prediction tools.

In the present experiments, the water flume has a width of 0.15 m (6 in.) between
the two sidewalls. The frictional effect of the sidewalls may be significant near the wall surface, but less significant away from the walls. In the present study, all measurements - wave measurement and velocity measurement - are done near the centerline of the flume so as to minimize the wall effect. Viscous effects may be more important where the water is extremely shallow, for example, at the initial landslide position near the waterline. In the validation of the numerical models - which are inviscid, we see the agreement at wave gauge "g1" in the shallower water is always poorer than that at "g2" in the deeper water. In some cases, such as case 3, the numerical simulation becomes poorer than those cases of deeper water depth. This difference is most likely due to the viscous effect.

Carrier et al. (2003) investigated the runup and drawdown of different types of tsunamis, included in which is that generated by an offshore underwater landslide. In their study, the initial water surface deformation is composed of a leading N-wave shaped depression. Similar waveforms also are observed in the experiments and numerical simulations. In Figure 4.11, an N-wave shaped depression is found above the landslide in early stages as $t\sqrt{g/L_s}=2$ and 4. As the downslope motion continues, dispersive waves are developed upstream the landslide, and the waveforms become more complicated. In the real cases, the landslide may translate over a short distance and the leading N-wave shaped depression reflects the reality.
Table 4.1 Parameters of the experimental setup

<table>
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<tr>
<th>Case #</th>
<th>$\theta$ (°)</th>
<th>$I_m$ (m)</th>
<th>$b_m$ (m)</th>
<th>$h_{oz}$ (m)</th>
<th>$x_{g1}$ (m)</th>
<th>$x_{g2}$ (m)</th>
<th>$h_1$ (m)</th>
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Table 4.2 Landslide kinematics and generated waves

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<th>$a_2$ (m/s)</th>
<th>$t_2$ (s)</th>
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<th>$\max(\frac{\zeta}{h_2})$</th>
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Table 4.3 Computational efficiency of the numerical models

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<th>CPU time (min)</th>
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$O(\mu^4)$-model

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<th>Time steps</th>
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Boussinesq model

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Table 4.4 Determination coefficients ($R^2$) for the numerical simulations

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Figure 4.1 Schematic diagram of the experimental setup.
Figure 4.2 Landslide displacements in case 1. Individual trials are indicated by different symbols.

Figure 4.3 Time-series of water surface elevations in case 1. Top: wave gauge "g1", bottom: wave gauge "g2". Individual trials are indicated by different symbols.
Figure 4.4 Landslide displacements and fitting curves in cases 1 and 7. Experiments: symbols; fitting curves: solid lines.
Figure 4.5 Time-series of water surface elevations measured in cases 1-3. Top: wave gauge “g1”; bottom: wave gauge “g2”. Case 1: dash-circle; case 2: dash-cross; case 3: dash-triangle.

Figure 4.6 Time-series of water surface elevations measured in cases 4-6. Top: wave gauge “g1”; bottom: wave gauge “g2”. Case 4: solid; case 5: dash; case 6: dot.
Figure 4.7 Time-series of water surface elevations measured in cases 7 and 8. Top: wave gauge “g1”; bottom: wave gauge “g2”. Case 7: dash-circle; case 8: dash-cross.

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Figure 4.15 Time-series of water surface elevations in case 2. Top: wave gauge “g1”, local water depth is $h_1 = 0.207 L_s$, bottom: wave gauge “g2”, local water depth is $h_2 = 0.308 L_s$. The present $O(\mu^4)$-model: solid; the $O(\mu^2)$-model: dash; the Boussinesq model: dot; experiment: circle.
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Figure 4.17 Time-series of water surface elevations in case 4. Top: wave gauge “g1”, local water depth is \( h_1 = 0.224 L_s \), bottom: wave gauge “g2”, local water depth is \( h_2 = 0.408 L_s \). The present \( O(\mu^4) \)-model: solid; the \( O(\mu^2) \)-model: dash; the Boussinesq model: dot; experiment: circle.
Figure 4.18 Time-series of water surface elevations in case 5. Top: wave gauge “g1”, local water depth is \( h_1 = 0.207L_s \), bottom: wave gauge “g2”, local water depth is \( h_2 = 0.391L_s \). The present \( O(\mu^4) \)-model: solid; the \( O(\mu^2) \)-model: dash; the Boussinesq model: dot; experiment: circle.

Figure 4.19 Time-series of water surface elevations in case 6. Top: wave gauge “g1”, local water depth is \( h_1 = 0.189L_s \), bottom: wave gauge “g2”, local water depth is \( h_2 = 0.372L_s \). The present \( O(\mu^4) \)-model: solid; the \( O(\mu^2) \)-model: dash; the Boussinesq model: dot; experiment: circle.
Figure 4.20 Time-series of water surface elevations in case 7. Top: wave gauge "g1", local water depth is $h_1 = 0.257L_s$, bottom: wave gauge "g2", local water depth is $h_2 = 0.469L_s$. The present $O(\mu^4)$-model: solid; the $O(\mu^2)$-model: dash; the Boussinesq model: dot; experiment: circle.

Figure 4.21 Time-series of water surface elevations in case 8. Top: wave gauge "g1", local water depth is $h_1 = 0.214L_s$, bottom: wave gauge "g2", local water depth is $h_2 = 0.426L_s$. The present $O(\mu^4)$-model: solid; the $O(\mu^2)$-model: dash; the Boussinesq model: dot; experiment: circle.
Figure 4.22 Time-series of water surface elevations in case 9. Top: wave gauge “g1”, local water depth is \( h_1 = 0.177 L_s \), bottom: wave gauge “g2”, local water depth is \( h_2 = 0.507 L_s \). The present \( O(\mu^4) \)-model: solid; the \( O(\mu^2) \)-model: dash; the Boussinesq model: dot; experiment: circle.

Figure 4.23 Time-series of water surface elevations in case 10. Top: wave gauge “g1”, local water depth is \( h_1 = 0.153 L_s \), bottom: wave gauge “g2”, local water depth is \( h_2 = 0.483 L_s \). The present \( O(\mu^4) \)-model: solid; the \( O(\mu^2) \)-model: dash; the Boussinesq model: dot; experiment: circle.
(a) $t \sqrt{g/L_s} = 1.2$
(b) $t\sqrt{g/L_s} = 2.1$
\[ t \sqrt{g/L_z} = 3.4 \]
Figure 4.24 Predicted velocity profiles as compared with measurements in case 10. The present $O(\mu^4)$-model: solid; the $O(\mu^2)$-model: dash; the Boussinesq model: dot; experiment: circle. The bottom row sketches the locations of landslide and wave profiles simulated by the present $O(\mu^4)$-model. Dashed line indicates the section, on which the velocity profiles are obtained.

(d) $t\sqrt{g/L_s} = 5.3$
Figure 4.25 Time-series of water velocities in case 1. From top to bottom: $z/h_1=-0.151$, $-0.226$, $-0.301$, $-0.377$, $-0.452$. Local water depth is $h_1/L_s=0.311$. The present $O(\mu^4)$-model: solid; the $O(\mu^2)$-model: dash; the Boussinesq model: dot; experiment: circle.
Figure 4.26 Time-series of water velocities in case 2. From top to bottom: $z/h = -0.153$, -0.239, -0.325, -0.410, -0.496. Local water depth is $h/L_s$. The present $O(\mu^4)$-model: solid; the $O(\mu^2)$-model: dash; the Boussinesq model: dot; experiment: circle.
Figure 4.27 Time-series of water velocities in case 3. From top to bottom: $z/h_l=-0.165$, -0.261, -0.357, -0.453, -0.549. Local water depth is $h_l/L_s=0.244$. The present $O(\mu^4)$-model: solid; the $O(\mu^2)$-model: dash; the Boussinesq model: dot; experiment: circle.
Figure 4.28 Time-series of water velocities in case 4. From top to bottom: $z/h = -0.204$, -0.328, -0.452, -0.576, -0.700. Local water depth is $h/L_s = 0.373$. The present $O(\mu^4)$-model: solid; the $O(\mu^2)$-model: dash; the Boussinesq model: dot; experiment: circle.
Figure 4.29 Time-series of water velocities in case 5. From top to bottom: $z/h = -0.112$, -0.243, -0.373, -0.504, -0.635. Local water depth is $0.354L_z$. The present $O(\mu^4)$-model: solid; the $O(\mu^2)$-model: dash; the Boussinesq model: dot; experiment: circle.
Figure 4.30 Time-series of water velocities in case 6. From top to bottom: $z/h_i=-0.126$, -0.230, -0.334, -0.437, -0.541. Local water depth is $0.334L_y$. The present $O(\mu^4)$-model: solid; the $O(\mu^2)$-model: dash; the Boussinesq model: dot; experiment: circle.
Figure 4.31 Time-series of water velocities in case 7. From top to bottom: $z/h_l=-0.131$, -0.311, -0.492, -0.672, -0.852. Local water depth is $h_l/L_s=0.380$. The present $O(\mu^4)$-model: solid; the $O(\mu^2)$-model: dash; the Boussinesq model: dot; experiment: circle.
Figure 4.32 Time-series of water velocities in case 8. From top to bottom: $z/h_t = -0.229, -0.329, -0.504, -0.679, -0.829$. Local water depth is $h_t/L_s = 0.342$. The present $O(\mu^4)$-model: solid; the $O(\mu^2)$-model: dash; the Boussinesq model: dot; experiment: circle.
Figure 4.33 Time-series of water velocities in case 9. From top to bottom: $z/h_l$=-0.126, -0.300, -0.473, -0.646, -0.820. Local water depth is $h_l/L_z$=0.395. The present $O(\mu^4)$-model: solid; the $O(\mu^2)$-model: dash; the Boussinesq model: dot; experiment: circle.
Figure 4.34 Time-series of water velocities in case 10. From top to bottom: \( z/h_t = -0.125, -0.288, -0.451, -0.614, -0.777 \). Local water depth is \( h_t/L_y = 0.368 \). The present \( O(\mu^4) \)-model: solid; the \( O(\mu^2) \)-model: dash; the Boussinesq model: dot; experiment: circle.
CHAPTER 5
TSUNAMI SENSITIVITY TO LANDSLIDE PARAMETERS

In this chapter, the present validated $O(\mu^4)$-model is applied in a series of numerical experiments to study the tsunami sensitivity to the physical characteristics of underwater landslides. The properties of a rigid underwater landslide affecting the generated waves may include the length, thickness, initial submergence and acceleration of the landslide, as well as the slope of underwater incline. A clear relationship between the landslide parameters and near-field waves has been observed in the previous studies by other groups (e.g., Hammack, 1973, Watts, 1998, Goldfinger, et al., 2000, Watts et al., 2005). In most of those studies, the landslide models were sharp edged rectangular or triangular blocks. Due to the strong turbulent wake generated by these blocks, their effective shape and volume in water was difficult to be estimated accurately. In the present study, the landslide models have smaller thickness and smoother surfaces. We would like to develop a new empirical equation for predicting the wave amplitude based on the results from the present study. Following Watts et al. (2005), we define the maximum depression over the initial landslide position as the characteristic wave amplitude, and denote it as $\zeta_0$. A diagrammatic sketch of the characteristic wave amplitude is shown in Figure 5.1.

5.1 Setup of Numerical Experiments

We consider an underwater landslide initially sitting on a gentle uniform slope. The rigid underwater landslide has the same shape of those in the laboratory experiments of previous chapter. The slope is inclined from a shallow water shelf to a deepwater
ocean. All the depths and lengths are presented in the non-dimensional forms scaled with respect to the landslide length $L_s$. The shallow water shelf has a constant depth of 0.01$L_s$. The length of the computation domain is determined in such a way that the generated waves never reach its boundaries in all the simulations. To record the time-series of wave elevations, a numerical wave gauge is installed above the middle of the initial landslide position. In all the simulations, a uniform grid size of $\Delta x/L_s = 0.04$ and a constant time step of $\Delta t \sqrt{g/L_s} = 0.001$ are employed.

The motion of the underwater landslide is estimated following the wavemaker formalism by Watts (1998). The displacement $s$ is described as a function of $t$ as

$$s = s_0 \ln[\cosh(t/t_0)],$$

where the characteristic length $s_0$ and the characteristic time $t_0$ are defined as

$$s_0 = \frac{u_t^2}{a_0}, \text{ and } t_0 = \frac{u_t}{a_0}. \quad (5.2)$$

Both the initial acceleration $a_0$ and the terminal landslide speed $u_t$ are dependent upon the forces due to inertia, gravity, basal friction and the hydrodynamic drag acting on the landslide. Complete experiments are needed to determine these parameters (e.g., Watts, 1998), and are beyond the scope of the present study. We chose $a_0$ that can cover most of the possible values in nature. The terminal speed of landslide is chosen conveniently to be $u_t = \sqrt{gL_s}$. Note that the maximum depression always happens at a very early stage; the terminal speed of landslide has no significant impact on the characteristic wave amplitude.

Previous studies suggested that the characteristic wave amplitude $\zeta_0$ might be
determined by a function of five non-dimensional parameters as (Hammack, 1973, Watts, 1998)

\[
\frac{\zeta_0}{s_0 \sin \theta} = f \left( \sin \theta, \frac{h_{0z}}{L_z}, \frac{b_z}{L_z}, S_8, Ha_0 \right),
\]

(5.3)

in which, \( S_8 = s_0 \sin \theta / h_{0z} \) is the ratio of the vertical length scale of the landslide acceleration to the initial landslide submergence, and the Hammack number \( Ha_0 = t_0 \sqrt{gh_{0z}} / L_z \) denotes the ratio of the time scale of wave generation to the duration of long wave propagation over the length of landslide. We note that both \( S_8 \) and \( Ha_0 \) are functions of the initial landslide acceleration \( a_0 \). Therefore, we modify equation (5.3) as

\[
\frac{\zeta_0}{s_0 \sin \theta} = f \left( \sin \theta, \frac{h_{0z}}{L_z}, \frac{b_z}{L_z}, \frac{a_0}{g} \right).
\]

(5.4)

In the above equation, all the parameters in the right-hand side are independent of each other.

5.2 Numerical Results

To investigate the effect of an individual landslide parameter, we vary the value of this parameter while keeping the others constant in the numerical experiments. The input parameters and resulting characteristic wave amplitudes are listed in Table 5.1. The data points in the experiments on a specific parameter are fitted with exponential equations. For some parameters, the exponents of the functions are close to one, suggesting nearly linear relationships between these parameters and the normalized characteristic wave amplitude \( \zeta_0 / (s_0 \sin \theta) \).
5.2.1 Slope Angle

In cases 1-5, we vary the slope angle within a range of $\theta \in [4^\circ, 12^\circ]$, while the other parameters are kept constant. These numerical experiments compose the first series to investigate the effect of varying slope angle. Two more series of numerical experiments are conducted in cases 6-10 and 11-15, in which the slope angle varies within the same range. The other parameters are constant for each series, i.e. $b_s/L_s=0.086$, $a_0/g=0.04$ and $h_{0c}/L_s=0.10$ for series 1; $b_s/L_s=0.086$, $a_0/g=0.08$ and $h_{0c}/L_s=0.10$ for series 2; and $b_s/L_s=0.086$, $a_0/g=0.06$ and $h_{0c}/L_s=0.10$ for series 3. Figure 5.2 shows that the values of $\zeta_0/(s_0 \sin \theta)$ decrease for increased $\sin \theta$.

5.2.2 Initial Submergence

Initial submergence of landslide is another principal factor for the generated waves. Increasing the initial submergence of landslide will involve more water volume above the landslide and in the generated waves, thus generating a lower wave amplitude. We conduct three series of numerical experiments, i.e. cases 12, 17 and 29-32 for series 1; cases 14, 19 and 33-36 for series 2; and cases 8, 28 and 37-40 for series 3. In every experimental series, the initial submergence of the landslide increases from 0.08$L_s$ to 0.18$L_s$, with an interval of 0.02$L_s$. The other parameters are set to be constant in each series as: $\theta=6^\circ$, $b_s/L_s=0.086$, $a/g=0.06$ for series 1; $\theta=10^\circ$, $b_s/L_s=0.086$, $a/g=0.06$ for series 2; and $\theta=8^\circ$, $b_s/L_s=0.086$, $a/g=0.08$ for series 3. Shown in Figure 5.3, the normalized characteristic wave amplitude $\zeta_0/(s_0 \sin \theta)$ decreases when $h_{0c}/L_s$ increases.
5.2.3 Landslide Thickness

Watts et al. (2005) indicated that the characteristic wave amplitude is a linear function of the landslide thickness, given that the thickness is the only parameter that varies. The numerical experiments to investigate the dependence of $\zeta_0$ on $h_s$ are cases 29 and 41-44 for series 1; cases 23 and 45-48 for series 2; and cases 37 and 49-52 for series 3. In every series of experiments, the landslide thickness is $h_{oc}/L_s=0.070, 0.080, 0.089, 0.100$ and 0.110, respectively. The other parameters are $\theta=6^\circ$, $\alpha_0/g=0.06$ and $h_{oc}/L_s=0.12$ in series 1; $\theta=10^\circ$, $\alpha_0/g=0.05$ and $h_{oc}/L_s=0.10$ in series 2; and $\theta=8^\circ$, $\alpha_0/g=0.12$ and $h_{oc}/L_s=0.12$ for series 3. It shows that $\zeta_0/(s_0 \sin \theta)$ becomes higher for larger landslide thickness. The simulation results are plotted in log-log scale in Figure 5.4. Similar to Figures 5.2 and 5.3, the plotted results in Figure 5.4 appear as straight lines which enable us to develop simple power laws as presented in equation (5.5). The power for the landslide thickness $h_s$ in (5.5) is 0.976, very close to one, which confirms the linear relationship between the wave amplitude and the landslide thickness.

5.2.4 Initial Acceleration

Under the same conditions, the higher magnitude of $\alpha_0$ indicates faster motion of landslide, which involves more energy transferred into surrounding water and thus generates higher waves. The numerical experiments in series 1-3 have initial accelerations increasing from $\alpha_0/g=0.04$ to $\alpha_0/g=0.08$ with a constant increment of 0.01. The slope angles are constant for each series as $\theta=6^\circ$, $10^\circ$ and $8^\circ$ in series 1, 2 and 3, respectively. The constant initial submergence is $h_{oc}/L_s=0.10$ for series 1 and 2 and
\( h_{0c} / L_x = 0.20 \) for series 3. In all the cases, the thickness of landslide is \( b_x / L_x = 0.086 \). In Figure 5.5, we see that the value of \( \zeta_0 / (s_0 \sin \theta) \) decreases for smaller values of \( \alpha_0 / g \).

### 5.3 Predicting Equations

An empirical equation is derived from the 52 numerical experiments through a least-square method, and is written as

\[
\frac{\zeta_0}{L_x} = 0.3022 \left( \frac{b_x}{L_x} \right)^{0.9763} \left( \frac{\alpha_0}{g} \right)^{0.8916} \left( \sin \theta \right)^{0.0940} \left( \frac{h_{0c}}{L_x} \right)^{-0.9240}.
\] (5.5)

The coefficient of 0.3022 is used to consider the effects of other factors not covered in the numerical experiments, such as the shape of the landslide. In Figure 5.6, the validation of equation (5.5) is investigated by comparing the characteristic wave amplitudes simulated in the numerical experiments and predicted by the empirical equation. Also plotted in this figure are the laboratory experiment cases 1-6, which have parameters within the application range of this equation, i.e., \( \theta \in [4^\circ, 12^\circ] \), \( \alpha_0 / g \in [0.04, 0.08] \), \( h_{0c} / L_x \in [0.08, 0.18] \) and \( b_x / L_x \in [0.07, 0.11] \). A very good agreement is found in Figure 5.5, with an overall determination coefficient of \( R^2 = 0.954 \) for the numerical and laboratory experiments.

Equation (5.5) indicates a quasi-linear relationship between the characteristic wave amplitude and the landslide parameters including the landslide thickness and initial acceleration, as well as the inverse of initial submergence. Varying slope angle may not affect the wave amplitude significantly. Therefore, we further simplify the predicting equation as

\[
\frac{\zeta_0}{L_x} = 0.299 \frac{b_x}{h_{0c}} \frac{\alpha_0}{g}.
\] (5.6)
The determination coefficient for this equation is $R^2 = 0.934$. In equation (5.6), we truncate the coefficient at the fourth digit. In fact, the equation is not quite sensitive to the number of digits. For example, the determination coefficient is almost the same if we round up the digits of coefficient to 0.3.

In the numerical experiments, all the lengths are scaled based on $L_\alpha$. The effect of the length of landslide is not investigated separately. Within its application limit, equation (5.6) suggests the characteristic wave amplitude be proportional to the length of landslide. To investigate this effect, we conduct another numerical experiment with the length of landslide being $2.0L_\alpha$ and the other parameters the same as those in case 52. The characteristic wave amplitude in this experiment is $\zeta_0 / L_\alpha = 0.0249$. Recalling that the characteristic wave amplitude in case 52 is $\zeta_0 / L_\alpha = 0.0134$, we have the ratio in the two experiments as 0.0134: 0.0249, which is very close that of the landslide lengths, i.e., 1.0: 2.0.

5.4 Maximum Water Particle Velocities

In the numerical experiments, we also compute the maximum water velocities above the middle of the initial landslide position. It is observed that there is a unique peak of water velocities at the seafloor. The occurrence of this peak usually follows immediately the maximum near-field depression. Thereafter in this section, we choose the magnitudes of water velocities at the water bottom as the characteristic velocities, and denote them as $|u_0|$ and $|w_0|$, respectively in the horizontal and vertical directions. Listed in Table 5.1 are the maxima of characteristic water velocities. A very good correlation is found between the characteristic wave amplitude $\zeta_0$ and $\max|u_0|$ and $\max|w_0|$ in all the
numerical experiments.

Following the same approach as for the characteristic wave amplitudes, we derive the empirical equations for the maximum characteristic water velocities as

\[
\frac{\max |u_b|}{\sqrt{gL_x}} = 0.2188 \frac{(b_x/L_x)^{0.7299} (a_0/g)^{0.2959} (\sin \theta)^{0.1563}}{(h_{0c}/L_x)^{0.7014}},
\]

and

\[
\frac{\max |w_b|}{\sqrt{gL_x}} = 2.1450 \frac{(b_x/L_x)^{0.0640} (a_0/g)^{0.5012}}{(h_{0c}/L_x)^{0.0999} (\sin \theta)^{0.0467}}.
\]

The validation of above equations is investigated by comparing the simulated and predicted \(\frac{\max |u_b|}{\sqrt{gL_x}}\) and \(\frac{\max |w_b|}{\sqrt{gL_x}}\) in Figures 5.7 and 5.8. The determination coefficients are computed to be \(R^2=0.9031\) and \(0.9751\), respectively. By equations (5.7) and (5.8), we conclude that the water velocities increase for higher thickness and initial acceleration of landslide; but decrease for increased initial submergence. The horizontal water velocity is more sensitive to the landslide thickness and initial submergence; while the vertical velocity is mainly determined by the landslide thickness, as well as the initial acceleration. The change of slope angle does not affect the water velocities significantly.
Table 5.1 Input parameters and results of numerical experiments

| Case # | $\theta$ (°) | $b_s/L_s$ | $a_o/g$ | $h_{oc}/L_s$ | $\zeta_0/L_s$ | $\max|u_p|/\sqrt{gL_s}$ | $\max|w_p|/\sqrt{gL_s}$ |
|-------|--------------|-----------|---------|--------------|--------------|----------------|----------------|
| 1     | 4°           | 0.086     | 0.04    | 0.10         | 0.0104       | 0.0505         | 0.0420         |
| 2     | 6°           | 0.086     | 0.04    | 0.10         | 0.0107       | 0.0524         | 0.0415         |
| 3     | 8°           | 0.086     | 0.04    | 0.10         | 0.0102       | 0.0491         | 0.0394         |
| 4     | 10°          | 0.086     | 0.04    | 0.10         | 0.0110       | 0.0542         | 0.0398         |
| 5     | 12°          | 0.086     | 0.04    | 0.10         | 0.0117       | 0.0581         | 0.0396         |
| 6     | 4°           | 0.086     | 0.08    | 0.10         | 0.0194       | 0.0609         | 0.0590         |
| 7     | 6°           | 0.086     | 0.08    | 0.10         | 0.0199       | 0.0635         | 0.0584         |
| 8     | 8°           | 0.086     | 0.08    | 0.10         | 0.0185       | 0.0592         | 0.0552         |
| 9     | 10°          | 0.086     | 0.08    | 0.10         | 0.0200       | 0.0658         | 0.0559         |
| 10    | 12°          | 0.086     | 0.08    | 0.10         | 0.0212       | 0.0708         | 0.0557         |
| 11    | 4°           | 0.086     | 0.06    | 0.10         | 0.0151       | 0.0572         | 0.0513         |
| 12    | 6°           | 0.086     | 0.06    | 0.10         | 0.0155       | 0.0594         | 0.0508         |
| 13    | 8°           | 0.086     | 0.06    | 0.10         | 0.0146       | 0.0555         | 0.0481         |
| 14    | 10°          | 0.086     | 0.06    | 0.10         | 0.0158       | 0.0614         | 0.0486         |
| 15    | 12°          | 0.086     | 0.06    | 0.10         | 0.0166       | 0.0663         | 0.0484         |
| 16    | 4°           | 0.086     | 0.06    | 0.08         | 0.0190       | 0.0698         | 0.0541         |
| 17    | 6°           | 0.086     | 0.06    | 0.08         | 0.0180       | 0.0665         | 0.0512         |
| 18    | 8°           | 0.086     | 0.06    | 0.08         | 0.0198       | 0.0736         | 0.0517         |
| 19    | 10°          | 0.086     | 0.06    | 0.08         | 0.0188       | 0.0707         | 0.0489         |
| 20    | 12°          | 0.086     | 0.06    | 0.08         | 0.0222       | 0.0829         | 0.0503         |
| 21    | 6°           | 0.086     | 0.05    | 0.10         | 0.0129       | 0.0564         | 0.0464         |
| 22    | 6°           | 0.086     | 0.07    | 0.10         | 0.0175       | 0.0619         | 0.0547         |
| 23    | 10°          | 0.086     | 0.05    | 0.10         | 0.0132       | 0.0584         | 0.0444         |
| 24    | 10°          | 0.086     | 0.07    | 0.10         | 0.0178       | 0.0641         | 0.0524         |
| 25    | 8°           | 0.086     | 0.04    | 0.08         | 0.0136       | 0.0650         | 0.0422         |
| 26    | 8°           | 0.086     | 0.05    | 0.08         | 0.0168       | 0.0700         | 0.0473         |
| 27    | 8°           | 0.086     | 0.07    | 0.08         | 0.0224       | 0.0766         | 0.0558         |
| 28    | 8°           | 0.086     | 0.08    | 0.08         | 0.0250       | 0.0788         | 0.0594         |
| 29    | 8°           | 0.086     | 0.06    | 0.12         | 0.0134       | 0.0551         | 0.0506         |
| 30    | 8°           | 0.086     | 0.06    | 0.14         | 0.0104       | 0.0422         | 0.0468         |
| 31    | 6°           | 0.086     | 0.06    | 0.16         | 0.0097       | 0.0411         | 0.0472         |
| 32    | 6°           | 0.086     | 0.06    | 0.18         | 0.0093       | 0.0405         | 0.0476         |
| 33    | 10°          | 0.086     | 0.06    | 0.12         | 0.0134       | 0.0559         | 0.0485         |
| 34    | 10°          | 0.086     | 0.06    | 0.14         | 0.0119       | 0.0520         | 0.0487         |
| 35    | 10°          | 0.086     | 0.06    | 0.16         | 0.0107       | 0.0494         | 0.0488         |
| 36    | 10°          | 0.086     | 0.06    | 0.18         | 0.0090       | 0.0390         | 0.0457         |
| 37    | 8°           | 0.086     | 0.08    | 0.12         | 0.0168       | 0.0577         | 0.0563         |
| 38    | 8°           | 0.086     | 0.08    | 0.14         | 0.0157       | 0.0571         | 0.0574         |
| 39    | 8°           | 0.086     | 0.08    | 0.16         | 0.0130       | 0.0471         | 0.0545         |
Continuation of Table 5.1

| Case # | θ   | $b_s / L_s$ | $a_0 / g$ | $b_{0c} / L_s$ | $\zeta_0 / L_s$ | $\max |\mu_h| / \sqrt{gL_s}$ | $\max |\omega_h| / \sqrt{gL_s}$ |
|--------|-----|-------------|-----------|----------------|-----------------|-----------------------------|-----------------------------|
| 40     | 8°  | 0.086       | 0.08      | 0.18           | 0.0128          | 0.0483                      | 0.0559                      |
| 41     | 6°  | 0.110       | 0.06      | 0.12           | 0.0150          | 0.0559                      | 0.0617                      |
| 42     | 6°  | 0.100       | 0.06      | 0.12           | 0.0147          | 0.0575                      | 0.0578                      |
| 43     | 6°  | 0.080       | 0.06      | 0.12           | 0.0113          | 0.0461                      | 0.0447                      |
| 44     | 6°  | 0.070       | 0.06      | 0.12           | 0.0106          | 0.0450                      | 0.0399                      |
| 45     | 10° | 0.110       | 0.05      | 0.10           | 0.0187          | 0.0781                      | 0.0615                      |
| 46     | 10° | 0.100       | 0.05      | 0.10           | 0.0153          | 0.0653                      | 0.0528                      |
| 47     | 10° | 0.080       | 0.05      | 0.10           | 0.0121          | 0.0538                      | 0.0404                      |
| 48     | 10° | 0.070       | 0.05      | 0.10           | 0.0113          | 0.0522                      | 0.0357                      |
| 49     | 8°  | 0.110       | 0.08      | 0.12           | 0.0204          | 0.0642                      | 0.0714                      |
| 50     | 8°  | 0.100       | 0.08      | 0.12           | 0.0214          | 0.0715                      | 0.0689                      |
| 51     | 8°  | 0.080       | 0.08      | 0.12           | 0.0158          | 0.0553                      | 0.0522                      |
| 52     | 8°  | 0.070       | 0.08      | 0.12           | 0.0134          | 0.0477                      | 0.0444                      |
Figure 5.1 Definition of characteristic wave amplitude in the time-series of near-field waves.

Figure 5.2 Tsunami sensitivity to slope angle. Numerical experiments: circle (series 1), triangles (series 2), squares (series 3); fitting curves: solid lines.
Figure 5.3 Tsunami sensitivity to initial submergence. Numerical experiments: circle (series 1), triangles (series 2), squares (series 3); fitting curves: solid lines.

Figure 5.4 Tsunami sensitivity to landslide thickness. Numerical experiments: circle (series 1), triangles (series 2), squares (series 3); fitting curves: solid lines.
Figure 5.5 Tsunami sensitivity to initial acceleration. Numerical experiments: circle (series 1), triangles (series 2), squares (series 3); fitting curves: solid lines.

Figure 5.6 Characteristic wave amplitudes in numerical and laboratory experiments fitted by the predicting equation. Numerical experiments: circles; laboratory experiments: dots. Solid line indicates 1:1 correspondence.
Figure 5.7 Maximum horizontal characteristic water velocities, computed in the numerical experiments and fitted by the empirical equation. Numerical experiments are denoted by circles and solid line indicates 1:1 correspondence.

Figure 5.8 Maximum vertical characteristic water velocities, computed in the numerical experiments and fitted by the empirical equation. Numerical experiments are denoted by circles and solid line indicates 1:1 correspondence.
6.1 Conclusions

In the present study, we extend Gobbi et al.'s (1999, 2000) fully nonlinear, fourth-order dispersive wave propagation model to a wave generation model by adding the time variation in the water depth, i.e., in the seafloor bathymetry. The new model is capable of simulating not only wave propagation but also importantly the wave generation by bottom disturbances, such as underwater landslides. The higher-order dispersive terms and the fourth-order approximation to the velocity profiles improve the linear dispersion relation, as well as the velocity distribution.

The model equations are solved in one spatial dimension through a finite difference scheme. Validation of the numerical model is conducted against the laboratory experiments of landslide-generated waves and currents conducted in a two-dimensional water flume with smooth-curved rigid landslide and mild slopes. Both wave elevations and water velocities are measured with resistance-type wave gauges and PIV, respectively. The experiments are simulated with the present $O(p^4)$-model, Lynett and Liu's (2002) $O(p^2)$-model and the Boussinesq model by Wu (1981). Comparisons between the numerical simulations and the experimental measurements clearly show that the higher-order improvement of the dispersive properties improves the accuracy of simulated wave elevations, phase, as well as the water velocities.

By applying the present validated higher-order model, we also conduct a series of numerical experiments to investigate the tsunami sensitivity to the principal
landslide parameters. The experimental results indicate that the maximum near-field depression increases with the landslide thickness and initial acceleration, as well as the slope angle of underwater incline, but decreases with the initial landslide submergence. An empirical equation for the characteristic tsunami wave amplitude based on the landslide parameters is developed. Very good correlation is found between the maximum depression and maximum water velocities in the near field. Empirical equations also are derived to relate the maximum water velocities with the landslide parameters based on the numerical experiments. These easy-to-use empirical formulas can provide coastal engineers as a simple tool to make a quick estimate for the tsunami wave height and the fluid velocity.

6.2 Recommendations

While deriving the present higher-order model equations, we assume the seafloor disturbance has the same time and length scales as the water surface waves. This assumption is valid for landslides of long and thin shapes. In reality, a shorter disturbance may generate waves whose wavelengths are far longer than the disturbance itself. In such cases, this assumption becomes insufficient. Different length scales may be applied to the seafloor disturbance and the water surface waves separately, such that the model may achieve even wider applicability.

The model equations include higher-order spatial derivatives of water depth. These higher-order terms may induce singularity where bathymetry varies, and make the numerical model very unstable in certain cases. The higher-order derivatives of water depth may be treated with a better scheme in the numerical algorithm, or the
model equations can be modified to drop some of these terms through an approach such as adopting the mild slope assumption.

In order for the model to be applicable to the real oceans, the numerical model needs to be extended into two spatial dimensions. Moving boundary condition – namely, the run-up condition – needs to be developed along the shoreline. In addition, to make the computation more efficient, the absorbing condition along the far field open boundaries needs to be developed in order to reduce the size of the computational domain. The hydrodynamic model may be coupled with another numerical model, which predicts the landslide debris flows, making it possible to simulate the more complicated coupled process of bathymetry change and water surface waves.
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