RESPONSE TO TIDES OF COASTAL AQUIFERS: ANALOG SIMULATION VS. FIELD OBSERVATION

by

John A. Williams
Ta-Chiang Liu

Technical Report No. 86

June 1975

Project Completion Report
for
ANALOG SIMULATION OF TIDAL EFFECTS

OWRT Project No. A-020-HI, Grant Agreement No. 14-31-0001-3511
Principal Investigator: John A. Williams
Project Period: July 1, 1970 to August 31, 1972

The programs and activities described herein were supported in part by funds provided by the United States Department of the Interior as authorized under the Water Resources Act of 1964, Public Law 88-379.
ABSTRACT

This report presents a summary of the work to date on the response to tides of coastal aquifers. In particular, it presents the results of experiments performed to study the influence of an oscillating water table on storativity (i.e., effective porosity), the application of both harmonic and spectral analyses to water surface time histories measured in the field near Ewa Beach and in Honolulu Harbor, and an electric analog model designed to simulate the shallow, coral-limestone aquifer at Ewa Beach, Oahu, Hawaii. Results indicate that the effective porosity for an oscillating water table will, in general, depend on the frequency of the oscillation with dependence being strong or weak as either or both the specific yield and the transmissivity are larger or smaller, respectively. Also, water surface time histories can be considered to be composed of a fundamental (diurnal) component and a second harmonic (semdiurnal) component and a 24-hr record analyzed and that the resulting average values of the amplitude and phase angles be used. The spectral analysis confirms that the semidiurnal power suffers a greater attenuation than the diurnal power.

It is concluded that the technique of determining aquifer properties from tidal response data is a valid one, but that the results will be less reliable than those ascertained from pump test data. The optimum results are considered to be those based on both tidal data and pump test data, as one method serves as a check on the other.
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>SHORT TERM FLUCTUATIONS IN THE PIEZOMETRIC SURFACE</td>
<td>2</td>
</tr>
<tr>
<td>Fluctuations Caused by Barometric Pressure Changes.</td>
<td>2</td>
</tr>
<tr>
<td>Fluctuations Caused by Ocean Tides.</td>
<td>3</td>
</tr>
<tr>
<td>AQUIFERS COMMUNICATING DIRECTLY WITH THE SEA</td>
<td>4</td>
</tr>
<tr>
<td>AQUIFERS COMMUNICATING INDIRECTLY WITH THE SEA</td>
<td>5</td>
</tr>
<tr>
<td>Fluctuations Caused by Earth Tides.</td>
<td>5</td>
</tr>
<tr>
<td>REVIEW OF PERTINENT RESEARCH WORK.</td>
<td>6</td>
</tr>
<tr>
<td>Aquifers in Direct Communication with the Sea</td>
<td>6</td>
</tr>
<tr>
<td>Aquifers in Indirect Communication with the Sea</td>
<td>10</td>
</tr>
<tr>
<td>AN EXPERIMENTAL STUDY OF THE EFFECTIVE POROSITY FOR AN OSCILLATING</td>
<td>14</td>
</tr>
<tr>
<td>WATER TABLE.</td>
<td></td>
</tr>
<tr>
<td>Introduction</td>
<td>14</td>
</tr>
<tr>
<td>Experimental Considerations</td>
<td>14</td>
</tr>
<tr>
<td>Experimental Results</td>
<td>20</td>
</tr>
<tr>
<td>Summary</td>
<td>24</td>
</tr>
<tr>
<td>EWA BEACH LIMESTONE AQUIFER SIMULATION</td>
<td>25</td>
</tr>
<tr>
<td>Description of the Aquifer, Wells, and Field Data</td>
<td>25</td>
</tr>
<tr>
<td>THE AQUIFER.</td>
<td>25</td>
</tr>
<tr>
<td>THE WELLS.</td>
<td>26</td>
</tr>
<tr>
<td>FIELD DATA: AQUISITION AND ANALYSIS</td>
<td>26</td>
</tr>
<tr>
<td>Description of the Electric Analog Model and Model Data</td>
<td>31</td>
</tr>
<tr>
<td>ELECTRIC ANALOG MODEL.</td>
<td>31</td>
</tr>
<tr>
<td>MODEL DATA: AQUISITION AND ANALYSIS.</td>
<td>39</td>
</tr>
<tr>
<td>The Influence of Changing Aquifer Thickness</td>
<td>43</td>
</tr>
<tr>
<td>ONE-DIMENSIONAL APPROXIMATION OF THE ELECTRIC ANALOG MODEL</td>
<td>43</td>
</tr>
<tr>
<td>ONE-DIMENSIONAL WEDGING EFFECT</td>
<td>44</td>
</tr>
<tr>
<td>DISCUSSION</td>
<td>47</td>
</tr>
<tr>
<td>Results of Field Data Analysis</td>
<td>47</td>
</tr>
<tr>
<td>HARMONIC ANALYSIS</td>
<td>47</td>
</tr>
<tr>
<td>SPECTRAL ANALYSIS</td>
<td>49</td>
</tr>
<tr>
<td>Results of Electric Analog Simulation</td>
<td>49</td>
</tr>
<tr>
<td>ELECTRIC ANALOG MODEL RESULTS</td>
<td>49</td>
</tr>
<tr>
<td>EFFECT OF AQUIFER WEDGING</td>
<td>49</td>
</tr>
</tbody>
</table>
Applicability of the Method ........................................... .50
THE METHOD .......................................................... .50
VARIABILITY OF THE STORATIVITY–TRANSMISSIVITY RATIO. ........ .52
COMPARISON WITH PUMP TEST METHODS. .............................. .54
CONCLUSIONS. ................................................................ .55
ACKNOWLEDGMENTS. ...................................................... .56
REFERENCES ............................................................... .57
APPENDICES .................................................................. .61
   A. Estimate of Response Factors for Dug Wells No. 40 and No. 41. .61
   B. Resonance Frequency for Electric Analog Model .................... .63
   C. Fortran Program for the Evaluation of Variable Permeability
      Aquifer Model for $\alpha_1 > 8$ ....................................... .64

FIGURES

1 Amplitude vs. Distance for a Fluctuating Water Table Adjacent
to the Elbe River .......................................................... 7
2a Schematic Diagram of an Experimental Setup to Determine the
Dewatering Coefficient. ................................................... 16
2b Schematic Diagram of an Experimental Setup to Determine the
Effective Porosity under a Periodic Fluctuation of the
Piezometric Head. .......................................................... 16
3 Plot of Water Volume per Unit Area of Column vs. Drawdown for
Polyurethane Foam and Ottawa Sand .................................. 17
4 Dimensionless Plot of Water Volume per Unit Area of Column
vs. Drawdown for Ottawa Sand. .......................................... 18
5 Grain Size Distribution for Ottawa Sand and Sacramento River
Sand. ......................................................................... 19
6 Plot of Effective Porosity vs. Period for Polyurethane Foam ....... 21
7 Plot of Effective Porosity vs. Period for Ottawa Sand ............... 22
8 Plot of Effective Porosity vs. Period for Ottawa Sand ............... 23
9 Location of Coral Limestone at Ewa Beach, Oahu. .................... 27
10 Aerial Photo of Region around Dug Wells No. 40 and No. 41 ....... 28
11 Water Surface-Time History for Dug Wells No. 40 and No. 41 and
the Tide for Honolulu Harbor. ........................................... 29
12 Autocovariance vs. Time Lag and Spectral Density vs. Frequency
for the Water Surface-Time History at Honolulu Harbor ........... 33
Autocovariance vs. Time Lag and Spectral Density vs. Frequency for the Water Surface-Time History at Dug Well No. 40

Design of the Subsidiary Surface Model

Dimensionless Amplitudes and Phase Angles vs. Time Scale Factor for the Diurnal and Semidiurnal Tidal Components at Dug Well No. 40

Dimensionless Amplitudes and Phase Angles vs. Time Scale Factor for the Diurnal and Semidiurnal Tidal Components at Dug Well No. 41

Dimensionless Amplitudes and Phase Angles vs. Dimensionless Distance for Finite Aquifer of Variable Thickness and Tidal Periods of One and One-Half Day

Dimensionless Amplitudes vs. Storativity/Transmissivity at \( x/L = 0.6 \) for \( 1.0 \leq z_L/z_0 \leq 5 \); and Tidal Periods of One and One-Half Day

### TABLES

1. Significant Components of a Real Ocean Tide
2. Results of Harmonic Analysis
3. A Comparison of Incident and Transmitted Power
4. Summary of Data for Electric Analog Model
5. Comparison of Tidal Efficiencies and Phase Angles
6. Comparison of the Ratio of Storativity to Transmissivity
INTRODUCTION

The Water Resources Research Center at the University of Hawaii in 1968 undertook a research program to study the response of coastal aquifers to tidal fluctuation. In particular, it was desired to determine the feasibility of using tidal response data to help determine aquifer properties. The first phase of the program (OWRT Proj. No. A-015-HI) dealt essentially with modeling techniques, mathematical, electric analog, and hydraulic, and verified the use of the diffusion theory for representation of the physical phenomena involved, provided the appropriate diffusion coefficients were used (Williams, Wada, and Wang 1970).

The results of this initial research provided the impetus for the second study (OWRT Proj. No. A-020-HI) which was conducted in two phases. The first phase concentrated on the development of mathematical models and the corresponding electric analog models for several specific cases of inhomogeneity (Williams and Liu 1971) and for an aquifer system in which leakage was involved (Williams and Liu 1973). The second phase was an attempt to apply the results of the previous work in the construction of an electric analog model of a real aquifer and to verify the model using tidal response data as well as additional information, such as pumping test data, if available.

For all of the models mentioned above, an isotropic shallow aquifer in direct communication with the sea was assumed. However, in Hawaii the primary basal aquifer is quite deep, perhaps as much as 3000 to 5000 ft (Williams and Soroos 1973), which renders the application of the shallow aquifer theory questionable. To investigate this aspect, an additional study was conducted simultaneously with Phases I and II which employed a vertical Hele-Shaw flow model to simulate an unconfined aquifer (Liu 1973). The observations of the tidal response in the model, for a range of increasing depths, were compared with the shallow aquifer theory based on the Dupuit assumptions as well as with the deep aquifer response to tidal fluctuations as derived by Carrier and Munk (1952).

It is the purpose of this report to present the results of the second phase of the work described above and to relate these results to those of previous and current pertinent work.
SHORT TERM FLUCTUATIONS IN THE PIEZOMETRIC SURFACE

The piezometric surface in an aquifer is subject to periodic fluctuations from a variety of sources. Three such sources are fluctuations in barometric pressure, earth tides, and ocean tides. Usually several of these influences are acting simultaneously and are superposed on the longer term seasonal fluctuations.

Fluctuations Caused by Barometric Pressure Changes

Confined aquifer response to barometric pressure fluctuations is characterized by a 180° phase difference with respect to the pressure change. Thus, increases in barometric pressure depress the piezometric head and vice-versa. The barometric efficiency, b.e., of an aquifer is defined as the ratio of a change in barometric pressure, \( \Delta p_o \), to the corresponding change in the piezometric head. Jacob (1940) related this ratio to the bulk modulus of the water, \( \beta_w \), the elastic modulus of the granular skeleton, \( E_s \), and the porosity of the media, \( \varepsilon \). The derivation assumes that the change in barometric pressure is supported partially by the water in the pore space and by the solid skeleton and that the water and the skeleton obey Hooke's law in such a way that the volume changes in both are the same (i.e., the individual solid particles in the skeleton do not change in volume).\(^1\) The resulting relation is

\[
\text{b.e.} = \frac{\gamma \Delta h}{\Delta p_o} = \frac{\Delta p - \Delta p_o}{\Delta p} = \frac{\Delta \sigma_s}{\Delta \sigma_s + \Delta p} = \frac{1}{1 + \frac{\Delta p}{\Delta \sigma_s}}
\]

where \( \sigma_s \) and \( p \) are the compressive stresses in the skeleton and the water, respectively. If the condition of equal changes in volume together with Hooke's law for the water and the skeleton are applied to equation (1), the result is

\[
\text{b.e.} = \frac{\varepsilon E_s}{\varepsilon E_s + \beta_w}
\]

The utility of the barometric efficiency is found in the fact that it may be related to the storativity of the aquifer. Following Todd (1964), the compressibility of the aquifer is taken as the compressibility of the

\(^1\) For the case where this latter assumption is not made, see Tuinzaad (1954).
water plus the compressibility of the skeleton. Thus, if volume changes are with respect to a unit volume of aquifer,¹

$$\frac{1}{E_{aq}} = \frac{\epsilon}{\beta_w} + \frac{1}{E_s} = \frac{\epsilon}{\beta_w(b.e.)}$$

(3)

Finally, since the storativity is defined as the volume of fluid per unit column of aquifer released under a unit decline in head, i.e., \( S = \gamma b / E_{aq} \), then

$$S = \frac{\epsilon \gamma b}{\beta_w(b.e.)}$$

(4)

where \( b \) is the aquifer thickness.

Aquifer response to barometric pressure fluctuations has been well documented by Robinson (1939) and others (Nilsson 1966).

**Fluctuations Caused by Ocean Tides**

The ocean tides consist of a complex wave system composed of a number of harmonic constituents. Eight of the more important constituents have been identified by Defant (1958) and are presented in Table 1 below.

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>PERIOD IN SOLAR HOURS</th>
<th>RELATIVE AMPLITUDE</th>
<th>DESCRIPTION OF COMPONENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_2 )</td>
<td>12.42</td>
<td>100</td>
<td>MAIN LUNAR (SEMIDIURNAL CONSTITUENT)</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>12.00</td>
<td>46.6</td>
<td>MAIN SOLAR (SEMIDIURNAL CONSTITUENT)</td>
</tr>
<tr>
<td>( N_2 )</td>
<td>12.66</td>
<td>19.1</td>
<td>LUNAR CONSTITUENT DUE TO MONTHLY VARIATION IN MOON'S DISTANCE</td>
</tr>
<tr>
<td>( K_2 )</td>
<td>11.97</td>
<td>12.7</td>
<td>SOLI-LUNAR CONSTITUENT DUE TO CHANGES IN DECLINATION OF SUN AND MOON</td>
</tr>
<tr>
<td>( K_1 )</td>
<td>23.93</td>
<td>58.4</td>
<td>SOLI-LUNAR</td>
</tr>
<tr>
<td>( O_1 )</td>
<td>25.82</td>
<td>41.5</td>
<td>MAIN LUNAR</td>
</tr>
<tr>
<td>( P_1 )</td>
<td>24.07</td>
<td>19.3</td>
<td>MAIN SOLAR</td>
</tr>
<tr>
<td>( M_f )</td>
<td>327.86</td>
<td>17.2</td>
<td>FORTNIGHTLY-LUNAR</td>
</tr>
</tbody>
</table>

**SOURCE:** DEFANT 1958.

¹ A more accurate expression is \( 1/E_{aq} = \epsilon/\beta_w + (1-\epsilon)/E_s \). However, eq. (3) is considered to be consistent with the accuracy of the shallow-aquifer approximations. See Bear 1972.
If a fundamental period of 670 hr (i.e., 28 days less 2 hr) is divided into its harmonics, then periods of 25.77, 23.97, 12.64, 12.41, and 11.96 hr correspond to the harmonic numbers 26, 28, 53, 54, and 56, respectively. These periods agree very closely with those of the diurnal and semidiurnal tidal constituents. Thus, the harmonic analysis of a record of 670 hr duration will yield the amplitudes of 5 components which will correspond to the O₁, P₁ plus K₁, H₂, M₂, and S₂ plus K₂ tidal constituents. This technique was first described by Doodson (1922).

The problem of periodic fluctuations produced by ocean tides can be separated into two categories: the first is that where the aquifer is in direct communication with the sea by virtue of its outcrop at the coastline, and the second is that where the aquifer is confined and the confining layer is overlain completely or partially by a body of water subject to tidal fluctuations. Within this latter category is a class of more complicated problems in which there exist breaks in the aquitard permitting direct interaction between the aquifer and the overlying ocean.

AQUIFERS COMMUNICATING DIRECTLY WITH THE SEA. The simplest case consists of a one-dimensional flow in a shallow, homogeneous and isotropic aquifer bounded by a vertical coastline at one end and extending to infinity in the direction of x. The solution of the basic (diffusion) differential equation gives

\[ h(x,t) = \xi_0 e^{-\alpha x} \sin(\alpha x - \sigma t); \quad \alpha = \frac{\sqrt{4\pi \tau}}{2T} \]  

where \( \xi_0 \) is the tidal amplitude, \( \sigma = 2\pi T_0 \) is the frequency of a particular tidal component, \( \alpha \) is the damping factor and \( h \) is the piezometric head. From equation (5a) the wave length and speed of the disturbance in the aquifer are, respectively,

\[ \lambda = \frac{2\pi}{\alpha} = \frac{\sqrt{4\pi T_0}}{\tau} \]  \hspace{1cm} (5b)

and

\[ c = \frac{\sigma}{\alpha} = \frac{\sqrt{4\pi T}}{S_0} \]  \hspace{1cm} (5c)

The wave length, \( \lambda \), is also referred to as the "penetration length" and is that distance from the coastline where the fluctuation in the piezometric
is in phase with the tide at the coastline.

It is clear that the response to a given tidal component observed in a well at a known distance from the coastline, together with the amplitude of the tidal component, will permit a calculation of $a$ and a subsequent estimate of the ratio, $S/T$, provided the period is also known.

**Aquifers Communicating Indirectly with the Sea.** The case of a confined aquifer overlain by a body of water subject to tidal fluctuations was first investigated by Jacob (1941). Jacob defined the tidal efficiency as the ratio of the change in piezometric head in the aquifer to the corresponding change in the elevation of the overlying sea water. Thus, using the same assumptions concerning the relative roles of the water and the granular skeleton as in the calculations for the barometric efficiency, the result is

$$t.e. = \frac{\Delta p/\sigma}{\Delta H} = \frac{\Delta p}{\Delta p + \Delta r_s} = \frac{1}{1 + \frac{\Delta \sigma_s}{\Delta p}} = \frac{1}{1 + \frac{\varepsilon E_s}{\beta_w}}$$

(6)

It is clear from equations (1) and (6) that

$$b.e. + t.e. = 1$$

(7)

and that

$$S = \frac{\varepsilon ab}{\beta_w (1 - t.e.)}$$

(8)

**Fluctuations Caused by Earth Tides**

The gravitational attraction between the earth and the moon and the earth and the sun generates a land tide as well as an ocean tide. The effect of a land tide on a confined groundwater body is such that for "high tide" the piezometric surface declines and for "low tide" it rises. This is the result of the aquifer overburden weighing less at "high tide", i.e., when the moon is at upper or lower culmination, than at low tide, when the moon is on the horizon.¹ The fluctuation of the piezometric surface resulting from earth tides has been well documented by Robinson (1909), and by B. Gustafsson and M. Nordstrom in Nilsson (1966). According to Dale

¹ The influence of the land tide was noted as early as the first century by Pliny the Elder in his *Historia Naturalis* in which he notes that "the water in a well rises and falls as in the sea but at other times." See Nilsson (1966).
(1974), earth tides do not noticeably influence the groundwater tides in Hawaii.

REVIEW OF PERTINENT RESEARCH WORK
Aquifers in Direct Communication with the Sea

Work on the tidal response problem dates back at least to Boussinesq (1904) who solved the basic differential equation which bears his name and arrived at the solution given in equation (5). Werner and Noren (1951) extended this solution to include the case of an aquifer of finite length and compared their results with field observations by Prinz (1923). The field data shows a variation in head, h, consistent with equation (5) provided the fluctuations in head at the river (i.e., at x=0) are ignored. The data are presented in Figure 1.

Williams, Wada, and Wang (1970) recast the more general solution of Werner and Noren (1951) into a form more accessible to digital computations and extended it to include a constant head as well as a no-flow condition at the interior boundary of the aquifer. Their results include a comparison of this mathematical model against both an electric analog model and an hydraulic model which employed polyurethene foam as the porous elastic skeleton. Specifically, amplitude and phase angles of the fluctuations were measured at several points in the hydraulic model. The measured amplitudes, the distance to the point of observation, and the tidal period were then substituted into the mathematical model and values of $\alpha$ at each observation position calculated. With $\alpha$ known, the ratio of storativity to transmissivity was estimated.

Since the value of the conductivity depends on the structure of the porous media for a given fluid at a constant temperature, it was concluded that variations in $\alpha$ were essentially variations in the storativity. With the conductivity determined by permeameter tests, the storativity could then be found. It showed no significant dependence on position but some dependence on period, with the larger storativities generally resulting for the longer period waves and the smaller storativities for shorter period waves. This particular aspect of the tidal response problem is discussed in more detail on pp. 14 to pp. 25 of this report.

One important result was found in the fact that for the unconfined
Fluctuations of the Elbe River and water table levels in wells at various distances from the river (after Prinz 1923).

NOTE: After data from Prinz (1923).

FIGURE 1. AMPLITUDE vs. DISTANCE FOR A FLUCTUATING WATER TABLE ADJACENT TO THE ELBE RIVER
aquifer model the storativities were only a small percent of the true porosity. When an average value of $\alpha$ based on the individual values computed for each of the several observation points was used as input, the mathematical and electric analog models produced results which were in generally good agreement with the hydraulic model.

Miller (1941) conducted a set of experiments similar to those described above which employed a Sacramento (Calif.) River sand placed in a tank 1 ft wide x 1.5 ft deep x 9.6 ft long. One test was conducted using a period of 10 mins. and a water depth of 1.105 ft. A 5-min. period was used for the remaining 5 tests while the water depth varied from 0.27 to 1.047 ft. The test results were presented in the form of graphs of amplitude and phase angle vs. position. Analysis of this data by Williams, Wada, and Wang (1970) produced results similar to those for the polyurethane foam. In particular, it indicated that the storativity varied from 1 to 5% of the true porosity, 0.345.

Williams and Liu (1971; 1973) developed mathematical and electric analog models of one-dimensional aquifers influenced by nonhomogeneity and by leakage. Two nonhomogeneous models were considered: the first was an aquifer of infinite length whose permeability (or thickness) changed discontinuously at a distance $x=L$ from the coastline; and the second was an aquifer of finite length, $L$, whose permeability varied linearly with distance from the coast. The results for the discontinuous permeability model showed clearly the influence of the sudden change in conductivity as a discontinuity in the slope of the type-curves of amplitude and phase angle vs. position. The results of the linearly varying permeability model showed type curves which differ in detail but are essentially similar to those curves for constant conductivity. In general, the amplitude curves (for the no-flow boundary condition) seem to flatten out more rapidly and indicate greater attenuation when the permeability increases rather than decreases with distance from the coast.

The leaky aquifer system consisted of two infinite aquifers separated by an aquitard. Results here indicated that a moderate amount of leakage would cause amplitudes and phase angles to vary by 100% or more from those where no leakage was present. Consequently, the presence of leakage in a real aquifer system would, at best, render tidal response data questionable.

One of the most thorough analyses of the aquifer in direct communication with the sea is given by van der Kamp (1973). Using a three-layered aquifer system as a model, he developed the following general solution to the basic
conservation of mass equation, i.e., the (linearized) Boussinesq equation:

\[ h_j(x, z, t) = g_j(z)(A_1e^{px} + A_2e^{-px})e^{j\sigma t} \]  \hspace{1cm} (9)

Here, \( A_1 \) and \( A_2 \) are complex constants, \( \sigma \) is the tidal frequency, \( p \) is a complex number related to the hydrogeologic properties of the layers as well as to the tidal frequency and whose real and imaginary parts are the propagation parameters, and the function, \( g_j(z) \), determines the potential distribution over the depth for the \( j \)-th layer. For a confined aquifer, i.e., the two outer layers are aquicludes and \( g_2(z) \) is a constant; for an unconfined aquifer \( g_2(z) \) is a quadratic equation. The above solution, which is generally not valid in the neighborhood of the coastal boundary, includes supplementary equations relating the hydrogeological properties to the propagation parameters for confined, semiconfined, and unconfined aquifers.\(^1\) In the course of establishing these later relations, a set of criteria is derived for determining which of three aquifer types prevails.

The above relations are then tested with field data taken from a number of locations in the Netherlands and in Canada. Some of the data were analyzed to yield the pertinent hydrogeologic properties of the layers which in turn were applied to the criteria for determining the aquifer type and to the equations for the propagation parameters. The tidal data provided an independent calculation of the propagation parameters. A comparison of the results of these calculations led the author to the general conclusions that the propagation parameters are more reliable when calculated from data other than tidal response data, that the criteria for determining the aquifer types are reasonably reliable,\(^2\) and that vertical gradients were not as significant as initially anticipated.\(^3\) The author also points out that for an

---

\(^1\) In the case of a thin aquifer with no vertical components to the potential gradient, the real and imaginary parts of the complex number \( p = n + im \) reduce to \( m = n = \sqrt{S_2\sigma/T_2} = \alpha_2 \) as defined in eq. (5).

\(^2\) The criteria for a confined aquifer, i.e., \( \sigma S_1(D_1/K_1) > 1 \), appears to be particularly well corroborated by the field data. In this relation, the quantity, \( D_1/K_1 = C_1 \), is the resistivity to vertical flow of the upper layer and it has been assumed that the bottom layer is at least an aquitard if not an aquiclude.

\(^3\) The author points out that for the unconfined case, the influence of vertical gradients may be eliminated if all the observation points are placed at the same elevation within the aquifer.
unconfined aquifer the storativity at the water table appears to increase with the period of the flow, that there seems to be an "extra resistance to vertical flow near the water table," and that it is quite likely that these two phenomena are related.

Liu (1973) found that the shallow aquifer theory as expressed by equations (5a), (5b), and (5c) provided excellent predictions of the amplitudes for aquifers with depths as great as 22% of the penetration length and with the wave amplitude as much as 10% of the depth. Phase angle predictions were not as good, the observed values being 20% to 45% less than those predicted. The linear relation between phase angle and position was retained, however. Liu also observed that the storativity was dependent on the tidal frequency and the aquifer depth, except for the longest tidal period (9 sec) where it remained constant for the range at depths tested.

Recently, Philip (1973) has presented an analysis of the nonlinear diffusion equation by first applying Kirchhoff's transformation and then averaging the resulting equation in time. The second step applies a periodic time variation of the concentration at a boundary. The net result is a transformed problem which involves a solution to Laplace's equation and a transformed boundary condition which requires that the concentration be in excess of its equilibrium value at sufficiently large distances from the boundary. In terms of tidal diffusion this would require that the equilibrium level of the inland groundwater be above that of the mean sea level. This difference amounts of 23% of the tidal amplitude when the amplitude is equal to the thickness of the aquifer.

Aquifers in Indirect Communication with the Sea

The problem of indirect communication with the sea is complicated by the fact that while the aquifer extends continuously across the coastline, the region subjected to variations in loading terminates there.

Carr and van der Kamp (1969) argued that equation (5a) could be used provided that the amplitude \( \zeta_0 e^{-\alpha x} \) was replaced with \( \zeta_0 (t.e.) e^{-\alpha x} \) to account for the efficiency of the granular skeleton. The term, \( (t.e.) e^{-\alpha x} \), is defined as the apparent tidal efficiency, \( (t.e.)_{\text{app}} \). Carr and van der Kamp also observed that from equation (5a) the time lag required for any phase of the tide to penetrate a distance, \( x \), into the aquifer is given by \( t_{\text{lag}} = \frac{x \alpha}{\sigma} \). Thus, the expression for the apparent tidal efficiency can be written
Consequently, a measurement of \((t.e.)_{\text{app}}\) and \(t_{\text{lag}}\) together with a knowledge of the tidal frequency, \(\sigma\), permits the calculation of \(t.e.\) and subsequently, \(S/b\), the specific storativity, from equation (8), provided the porosity is known. As \(\alpha = \sqrt{(S\sigma/2T)} = \sigma t_{\text{lag}}/X\) and \(S\) (or \(S/b\)) has been determined, then \(T\) (or \(K\)) can be found. This is the result of combining equation (8) with equation (5a) and is possible only for the case of indirect communication. Only \(\alpha\) can be estimated for the case where the aquifer outcrops at the coastline. Carr and van der Kamp also point out that the observed lag times must be corrected for the response time of the well and that the observed tidal efficiency must also be corrected for a damping effect produced by the observation hole.

Application of the above theory to tidal data from a confined aquifer near Nine Mile Creek, P.E.I., Canada, produced results which were in reasonable agreement with those from a limited amount of pump test data. Also, their estimated tidal efficiencies and barometric efficiencies summed to .94 which agrees with equation (7) to within the expected error on the field measurements.

van der Kamp (1972) modified the basic differential equation of Boussinesq by accounting for the uniform, but time varying, tidal load on the subsea portion of the aquifer. This amounted to the inclusion of the term, \((t.e.)\partial h_{g}/\partial t\), in the differential equation to be satisfied in the subsea portion of the aquifer. The Boussinesq equation remained valid for that portion of the aquifer extending inland from the coastline. The solutions to these two equations were subjected to two boundary conditions which required the continuity of the piezometric head and the flux of water at the coastline. This resulted in two equations for the piezometric head, one applicable to the subsea portion of the aquifer and the other to the landward portion of the aquifer. Two significant features not present in the Boussinesq solution became evident. First, a seaward-propagating disturbance 180° out of phase with the landward propagating disturbance resulted from the solution to the modified Boussinesq equation and second, the expression for the amplitude of the disturbance in either portion of the aqui-
For the landward portion of the aquifer the expression for the piezometric head is

\[ h(x,t) = \frac{1}{2}(t.e.)_0 e^{-\alpha x} \sin(\alpha x - \sigma t) \]  (11)

van der Kamp also superposed these solutions in such a way as to represent the case of a confined aquifer overlain by a narrow rectilinear channel subjected to tidal fluctuations and a confined aquifer under an ocean traversed by a narrow strip of land. In the case of the narrow channel, the resulting equations predict a phase lead at the channel edge with respect to the tidal change which gradually decreases and becomes a lag as the channel width increases beyond $3/\alpha$.

To support the two results of his analysis, van der Kamp points out that equation (7) may be rewritten as

\[ \text{b.e.} + (t.e.)_0 = 1 - \frac{1}{2}(t.e.) \]  (12)

where $(t.e.)_0 = \frac{1}{2}(t.e.)$ is the tidal efficiency at the coastline. Thus, the barometric efficiency and the tidal efficiency observed at the coastline should sum to a number less than unity for aquifers extending under the sea. Data from a semiconfined sandstone aquifer at York Point, P.E.I., Canada, and from a confined at Cap-Pelé, N.B., Canada, give values of .79 and .71, respectively, for the sum.

De Cazenove (1971) analyzed aquifers communicating both directly and indirectly with the sea. His analyses are quite extensive and include a variety of boundary conditions as well as the case where leakage takes place at some point in the subsea portion of the aquifer. In particular, his analyses include the same problem as discussed by van der Kamp and his results for the amplitude include the factor of one-half. He also analyzes the case where the landward portion of the aquifer is of finite extent. The expression for $h$ in the finite portion of the aquifer is

\[ h = \frac{(t.e.)_0}{2} \{ e^{-\alpha x} \sin(\sigma t - \alpha x) + e^{\alpha(x-2D)} \sin(\sigma t + \alpha(x-2D)) \} \]  (13)

1. This is the result of the continuity condition at the coastline which requires one-half the disturbance to propagate seaward and the other half to propagate landward with a $180^\circ$ phase difference between the two.

2. This case was apparently first solved by J. Brillant in 1965. See De Cazenove (1971).
where $D$ is the distance which the aquifer extends inland from the coastline.

Dale (1974) has developed a digital modeling technique based upon the finite difference form of the basic differential equations of flow in a shallow aquifer. He identified twelve different aquifer types on the basis of a boundary and loading conditions and computed type-curves, i.e., the variation of amplitude and phase angle with respect to distance from the coastline, for each type. The modeling technique was verified by the comparison with closed form solutions of Williams, Wada, and Wang (1970) for both the one-dimensional and the circular island aquifers.

In addition, field data from four local aquifers were matched to type-curves with varying degrees of success. The $P_1$ plus the $K_1$ and the $M_2$ were the tidal constituents used in matching the type-curves. The harmonic analysis used was based on 28-day records and followed the technique of Doodson (1922). In particular, the type-curve match for the shallow (unconfined) Ewa Beach limestone aquifer resulted in $K/S=2$ to $4 \times 10^7$ sq ft/d. For the confined basaltic aquifer underlying Pearl Harbor, the type-curve match is made for an aquifer which suddenly becomes unconfined in the vicinity of its interior boundary. The digital model simulates this condition by allowing the storativity to increase to unity over a relatively short distance, which effectively imposes a constant head boundary condition or a "standpipe condition" at the interior boundary. The type-curves show a phase lead with respect to the tide in this region which is confirmed by field observations from wells located near the end of the confining layer.\(^1\)

It should be noted that the above modeling technique is applied to unconfined aquifers by taking sufficiently large values of the storativity. Also, all of the above models are based on a one-dimensional flow in that only one space variable is considered.

Lam (1971) has studied the tidal diffusion problem of an atoll. He developed five models which represent various combinations of boundary conditions for the atoll and the enclosed lagoon. The output of the models was compared for several tidal components (viz., $M_2$, $S_2$, $N_2$, $K_1$, and $O_1$) with those measured in the lagoon and at eight observation wells on Swains Island, an atoll in the Tokelau or Union Islands group.\(^2\)

---

1. This observation is also consistent with van der Kamp's (1972) analytical result for a confined aquifer overlain by a narrow channel.
2. The Tokelau Islands group is approximately 2000 miles south and slightly west of the island of Oahu, Hawaii.
One interesting aspect of this study was the computation of the power spectrum for the tide at each observation point. This analysis indicated maximum power for the semidiurnal tide and a lesser amount for the diurnal component, but did not distinguish between the several semidiurnal or diurnal constituents.

His results indicated that the permeability of the atoll varied from \(4.4 \times 10^{-6}\) sq cm to \(14.1 \times 10^{-6}\) sq cm with the larger values resulting from the phase lag measurements and the smaller ones from the amplitude measurements. Considering all five models, extreme values of permeability differ by three orders of magnitude. The porosity of the reef material, based on core samples, varied from .55 to .77.

AN EXPERIMENTAL STUDY OF THE EFFECTIVE POROSITY FOR AN OSCILLATING WATER TABLE

Introduction

The results of the Williams, Wada, and Wang (1970) study indicate some dependence of the storativity on time for both the confined and unconfined cases. Also, Bear, Zaslavsky, and Irmay (1968) point out that in the case of the unconfined aquifer, the amount of water removed per unit of drawdown per unit of horizontal area is the instantaneous dewatering coefficient, \(m\), which after sufficient time may approach the effective porosity which is approximately equal to the specific yield. The basic equation which relates the dewatering coefficient, \(m\), to the instantaneous dewatering rate, \(r\), and the drawdown, \(ds\), which occurs in time, \(dt\), is

\[ mds = rdt. \]  

Experimental Considerations

In order to obtain more information on this phenomenon, several sets of experiments were performed. The first set of experiments involved the measurement of a volume of water drained from a column of porous media together with the corresponding head drop observed in a piezometer tube attached to the column. This was accomplished by collecting water for a given period and noting the elevation of the piezometric head at the end of the period, at which time the partially filled collection container was replaced by an empty one. With a little care, this operation could be carried out without any
spillage. Hence, the cumulative volume of drained water together with the associated elevation of the piezometric head was obtained.\textsuperscript{1} The fluid volume was divided by the area of the column and plotted as a function of $s$, the piezometric head drop. From equation (14), it is clear that the slope of this curve at any point is the instantaneous dewatering coefficient, $m$. Two different materials were used: the first was Ottawa sand and the second was polyurethene foam used by Williams, Wada, and Wang (1970). The plotted results are shown in Figure 3. Figure 4 shows a dimensionless plot of the same quantities for the Ottawa sand where the values of $\psi/A$ and $s$ at the inflection point of the curve have been used to nondimensionalize the ordinate and abscissa, respectively. A schematic representation of the experimental setup for both the foam and the sand is included in Figure 2a. Figure 5 shows the grain size distribution for the Ottawa sand as well as the Sacramento River (California) sand used by Miller (1941) in his experiments on the tidal response of groundwater aquifers.

The second set of experiments was designed to account for the periodic nature of the time variation characteristic of tidal response phenomenon. In this instance, the column was connected to a cylinder of water by a short piece of flexible tubing. The cylinder of water was then placed in rectilinear harmonic motion along its axis. Figure 2b shows a schematic representation of this arrangement. As seen from Figure 2b, the volume of water moving out of, or into, the aquifer column is $s_{A_{r}}$ and the volume of media which is apparently drained or filled is $s_{A_{a}}$. Hence, the effective porosity $\varepsilon' = s_{A_{a}}/s_{A_{r}}$, where $A_{r}$ should be corrected for the area occupied by the two resistance probes used for recording the water surface elevations in the reservoir cylinder. Since these probes are secured to the reservoir cylinder, they record the motion of the water relative to the cylinder and $s_{r}$ may be read directly from the trace on the chart paper. The head, $s_{a}$, was recorded by means of a stationary pressure transducer connected to the aquifer column. Calibration of the resistance probes was accomplished by clamping off the flexible tubing between the aquifer column and the reservoir cylinder and then changing the water surface elevation in the reservoir by a given amount and noting the corresponding change on the recorder chart. With the flexible

\textsuperscript{1} The ratio of the diameter of the piezometer tube to that of the column was less than .05. This results in a contribution to the collected water volume from the piezometer tube of less than one percent. Hence, the measured volumes were not corrected for the small error.
$L = \text{Total sample length}$

$S = \text{Piezometric head drop}$

$\psi = \text{Volume of water collected under head drop, } s$

$A = \text{Area of Column}$

**FIGURE 2a. SCHEMATIC DIAGRAM OF EXPERIMENTAL SETUP FOR FOAM AND SAND TO DETERMINE THE DEWATERING COEFFICIENT, } m$

$\xi_0 = \text{Amplitude of water column oscillation}$

$s_r = \text{Distance water level deflects in water column relative to equilibrium level - measured directly from recorder chart}$

$s_a = \text{Deflection of phreatic surface in aquifer column - measured directly from recorder chart}$

$A_r = \text{Area of water column corrected for area of resistance probes}$

$A_a = \text{Area of aquifer column}$

$L_a = \text{Total length of aquifer column}$

$\bar{z} = \text{Depth to equilibrium level}$

**FIGURE 2b. SCHEMATIC DIAGRAM OF EXPERIMENTAL SETUP TO DETERMINE THE EFFECTIVE POROSITY, } \varepsilon', \text{ UNDER A PERIODIC FLUCTUATION OF THE PIEZOMETRIC HEAD**
FIGURE 3. PLOT OF WATER VOLUME PER UNIT AREA OF COLUMN VS. DRAWDOWN FOR POLYURETHANE FOAM AND OTTAWA SAND
FIGURE 4. DIMENSIONLESS PLOT OF WATER VOLUME PER UNIT AREA OF COLUMN, \( \frac{\psi}{A} \), vs. DRAWDOWN, \( s \), FOR OTTAWA SAND
FIGURE 5. GRAIN SIZE DISTRIBUTION FOR OTTAWA SAND AND SACRAMENTO RIVER (CALIFORNIA) SAND
tubing still clamped off, the pressure transducer was calibrated by raising or lowering the entire aquifer column a given amount and observing the corresponding change on the recorder chart. Both the polyurethane foam and the Ottawa sand were tested.

For both materials, several different column lengths were used, and several water depths, \( \bar{z} \), were used with each column length. Tests were run at each water depth using three different amplitudes, \( \zeta_0 \), for the harmonic oscillation of the reservoir cylinder. Data were recorded for periods of oscillation, \( t_0 \), varying from 2 to 30 sec at each amplitude. The results of these tests and the values of the pertinent parameters involved are presented in Figures 6, 7, and 8.

**Experimental Results**

From the results of the first series of experiments shown in Figures 3 and 4, it is clear that the instantaneous dewatering coefficient is very small until the phreatic surface has dropped 12 to 18 cm in the sand column and about 5 cm in the polyurethane foam column, at which time it increases sharply, i.e., the curve takes a nearly vertical slope. The slope then decreases gradually to what should be close to the specific yield. The sharp increase in \( m \) is particularly evident in the dimensionless plot of Figure 4 and is the point at which the capillary forces can no longer support the volume of water retained in the pore spaces. The curve slopes for the foam and the sand (Trials 1 and 2, 15 February 1972) approach values of .8 and .34, respectively. As the true porosity of the foam is .971 and that of the sand was calculated at .382, these values for the slopes are reasonable estimates of the specific yield for the two materials.

It is interesting to note that the results shown in Figures 3 and 4 can be expressed as a plot of \( s \) vs. \( S_w = 1 - (\Psi/A)/(es) \) where \( S_w \) is the "degree of saturation" of the drained portion of the column. This produces the typical capillary head-saturation curve shown in the insert of Figure 3. It is seen that the head drop which corresponds to the maximum value of \( m = d(\Psi/A)/ds \) is the "bubbling pressure" or the "air entry value."

From the results of the second set of experiments, it is apparent that

---

1. Other pertinent properties of the foam as determined by Williams, Wada, and Wang (1970) are: Young's modulus, \( E = 13.6 \) psi
   
   Hydraulic conductivity, \( K_{avg} = .17 \) fps
   
   Specific storativity, \( S_s = .032 \) ft\(^{-1}\)
FIGURE 6. PLOT OF EFFECTIVE POROSITY, $\varepsilon'$, VS. PERIOD, $t_0$, FOR POLYURETHENE FOAM
FIGURE 7. PLOT OF EFFECTIVE POROSITY, ε', vs. PERIOD, t₀, FOR OTTAWA SAND, L₀ = 57.5 cm
FIGURE 8. PLOT OF EFFECTIVE POROSITY, $\varepsilon'$, vs. PERIOD, $t_0$, FOR OTTAWA SAND, $\bar{z} = 25$ cm
the effective porosity in both materials depends upon the period of oscillation of the piezometric surface. The effective porosity of the foam shows a strong dependence on the amplitude of the oscillation and that of the sand shows very little, if any. The tests on the sand do show, however, a strong dependence of $\varepsilon'$ on the depth of media above the phreatic surface. In the tests on the foam, this distance was held at 4 cm and no such variations were possible. Two additional facts are clear. The magnitude of $\varepsilon'$ is on the order of .01 to .08 for both the foam and the sand, and the slope of the $\varepsilon'$ vs. $t_o$ curve is close to and/or approaching zero for periods greater than 30 seconds.

**Summary**

The effective porosity depends primarily on the amplitude and period of the oscillation in the piezometric head and on the depth of the material above the phreatic surface. For amplitudes of oscillation smaller than the critical distance at which the weight of suspended water overcomes the capillary forces supporting it, relatively little volume will be vacated or filled during the periodic cycle, regardless of how long the period of fluctuation. This is apparent in the tests on the foam using the smallest amplitude of 1.5 cm. However, as this amplitude increases and approaches the critical value, $\varepsilon'$ will increase and show a stronger dependence on the period, as it does for the foam when the amplitudes are increased to 3 and 4.5 cm, respectively. Note that the critical distance for the foam from Figure 3 is about 5 cm, while that for the Ottawa sand varies from 12 to 18 cm. Consequently, the results from the tests on the sand showed little dependence on amplitude as 6 cm was the maximum amplitude tested.

The dependence of $\varepsilon'$ on the depth of the porous media above the phreatic surface seems to diminish as this distance increases. For example, a significant difference is apparent between the test results employing a total column length of 32.5 cm and 37.5 cm with an equilibrium water depth of 25 cm in both cases. However, an increase in column length to 47.5 cm produces a much smaller change in the $\varepsilon'$ vs. $t_o$ curve and a further increase to a column length of 57.5 cm produces no additional change. This is most likely because the capillary fringe region does not get a chance to develop completely when there is not a sufficient depth of media above the phreatic surface. The critical depth of material should be the same as the critical distance.
through which the head must drop to produce the sharp increase in the instantaneous dewatering coefficient as discussed above. For the sand this amounts to an average of 15 cm. Adding 15 cm to the water depth of 25 cm gives 40 cm of approximately the total column length which, if exceeded, will produce no change in the $\epsilon'$ vs. $t_0$ curves of Figure 8.

Finally, the very small values of $\epsilon'$, i.e., .005 to .025, resulting from the test data confirms the conclusion of Williams, Wada, and Wang (1970) that only 1 or 2% of the available porosity is utilized as storage in polyurethane foam and Sacramento is utilized as storage in polyurethane foam and Sacramento (Calif.) River sand. Furthermore, it is a reasonable speculation that this conclusion may be extended to prototype conditions in view of the results indicating that $\epsilon'$ is weakly dependent or possibly independent of time for periods in excess of several minutes. van der Kamp points out that on the basis of field measurements, the specific yield of unconfined aquifers can be of the order of $10^{-2}$.* It should be observed that the fluctuation in the piezometric head will most likely never exceed the height of capillary rise in a prototype situation, except possibly in a region extremely close to the coastline. Hence, the storativity will be determined essentially by the smallest value of the instantaneous dewatering coefficient, i.e., by the smallest or initial slope of the $\Psi/A$ vs. $s$ curve.

**EWA BEACH SHALLOW LIMESTONE AQUIFER SIMULATION**

**Description of the Aquifer, Wells, and Field Data**

**THE AQUIFER.** The Ewa Beach aquifer is located on the leeward coast of the island of Oahu in the Hawaiian Islands chain. The aquifer is composed of coral and limestone and varies in thickness from approximately 200 ft at the seaward end to approximately 100 ft at the landward boundary which is assumed to be impermeable to water flow. The areal extent of the landward portion of the aquifer is bounded by the coastline from Brown's Camp to the Salt Evaporators adjacent to the West Loch of Pearl Harbor. The interior boundary is taken as the old rail line which extended from Brown's Camp along the Barbers Point Naval Air Station to the Salt Evaporators. These

---

1. The Williams, Wada, and Wang (1970) analysis of Miller's (1941) data shows $0.009 < \epsilon'/\epsilon < 0.048$ and $\epsilon'/\epsilon = 0.026$ for the 6 tests conducted. For the longest period tested, (i.e., $t = 10$ min.) $\epsilon'/\epsilon = 0.020$.

boundaries encompass an area of approximately 20 sq miles. A map of this area is shown in Figure 9. The aquifer is unconfined, and the lower boundary consists of highly impermeable marine mud.

THE WELLS. Two wells for which tidal data were available are located in the area. They were constructed by the Oahu Sugar Company for irrigation purposes and are designated as Dug Wells Nos. 40 and 41. The locations of these wells are indicated on the map shown in Figure 9 and they may be seen in the aerial photograph presented in Figure 10. The significant feature about these wells is that they are located in old coral excavation pits and consist of long channels, hollowed out of the ground with a bulldozer, extending only 6 to 10 ft below the water table. This permits the lower density fresh water to be skimmed off the top of the aquifer by pumping from a point along the channel.

FIELD DATA: ACQUISITION AND ANALYSIS. The field data for the wells were obtained from the files of the district office of the U.S. Geological Survey, Water Resources Division. Data consisted of time-elevation histories for the water surface in Dug Wells Nos. 40 and 41, recorded over a period of 2 weeks from 26 August 1965 to 8 September 1965. During this period, the pumps in both wells were turned on for short periods on several occasions in order to record the drawdown in the wells. Unfortunately, the periods of pumping were so short that essentially only water stored in the channels was extracted during the test. Hence, the drawdown data do not reflect the aquifer properties. Also, the rates of drawdown and recovery in both wells were so rapid with respect to the recorder chart speed that these events appear as essentially straight vertical lines on the chart. Consequently, an analysis of the pumping test data was not feasible. The portions of the time history recorded during pumping and recovery were removed from the record, leaving only the tidal response. This modified time history is presented in Figure 11.

The tide data recorded on the NOAA gage located in Honolulu Harbor was analyzed and used since no tide records were available for the Ewa Beach region. According to Dale (1974), there is no difference in the Pearl Har-

---

1. According to a recently adopted identification system established by the U.S. Geological Survey, Dug Wells Nos. 40 and 41 are now identified as wells 1902-01 and 1900-13, respectively.
FIGURE 9. LOCATION OF CORAL LIMESTONE AT EWA BEACH, OAHU
FIGURE 10. AERIAL PHOTO OF REGION AROUND DUG WELLS NO. 40 AND NO. 41
bor tide and that at Honolulu Harbor except that the former leads the latter by about a quarter of an hour. Hence, the Honolulu Harbor tide was considered a good representation of the actual tide along the Ewa Beach coast.

The data were obtained from the Tides Branch, Oceanographic Division, National Ocean Survey, Rockville, Maryland and consisted of water level elevations, referenced 3.66 ft below MLLW, at hourly intervals for the months of August and September 1965. Figure 11 also shows a plot of this data for the period 26 August to 7 September 1965.

The analysis applied to the tidal records should resolve the basic harmonic constituents present. However, the particular technique employed will be determined essentially by the records themselves. From Figure 11, it is evident that for Dug Well No. 41 a 2-day segment of the record is missing. In addition, there appears to be some drift of the recorder used on Dug Well No. 40 over the period 29 August to 1 September as well as on 27 August, where the records from both wells show an almost discontinuous shift upwards. Tidal constituents of longer periods possibly corresponding to the fortnightly or monthly variations also seem to be present.

From a consideration of these limitations, it was decided to analyze 24-hr segments of the record where each segment was assumed to consist of only the diurnal (24 hr) and semidiurnal (12 hr) components. The particular technique selected was the periodogram analysis, used by Williams (1965), which involves the fitting of an nth order trigonometric polynomial to the 24-hr segments of the record. In general, if the record is composed of a set of harmonics, then it may be represented by

\[ \zeta(t) = \frac{a_0}{2} + \sum_{p=1}^{n} (a_p \cos p\theta + b_p \sin p\theta) \]  

(15a)

where

\[ a_p = \frac{2}{2n+1} \sum_{k=0}^{2n} y_k \cos k\alpha \]  

(15b)

\[ b_p = \frac{2}{2n+1} \sum_{k=0}^{2n} y_k \sin k\alpha \]

and \( \alpha = 2\pi/(2n+1) \). The measurement of the ordinates, \( y_k \), at 2n+1 evenly spaced points along the time axis then provides 2n+1 linear equations from which the 2n+1 unknowns \( a_0, a_1, \ldots, a_n, b_1, \ldots, b_n \) may be found.\(^1\) The ordinates, \( y_k \), at 2n+1 evenly spaced points along the time axis then provides 2n+1 linear equations from which the 2n+1 unknowns \( a_0, a_1, \ldots, a_n, b_1, \ldots, b_n \) may be found.\(^1\) The ordi-

---

1. Equations 15a and 15b for the periodogram analysis are a special case of the more general "least squares" technique where the number of equations may exceed the number of unknowns. In this case, the 2n+1 is replaced by N and \( \alpha = 2\pi/N \), where N>2p. See Whittaker and Robinson (1967).
nates, in this instance, were measured from a baseline which was constructed parallel to what appeared to be the longer period variations in the record. This procedure should minimize or, at best, eliminate the influence of such variations from the record.

Since the wave form was assumed to consist of only the diurnal and semidiurnal components, \( n = 2 \) in equation (15b), and the \( a_p \) and \( b_p \) were calculated using five evenly-spaced ordinates scaled off the records shown in Figure 11. The resulting trigonometric polynomials given by equation (15a) are also plotted in Figure 11 on their corresponding segments of record. The amplitudes and the phase angles calculated from the relations \( (a_p^2 + b_p^2)^{1/2} \) and \( \tan^{-1}(b_p/a_p) \) respectively, for both the diurnal and semidiurnal components are presented in Table 2.

The power spectrums for both the Honolulu Harbor and Dug Well No. 40 records were computed in order to investigate the lumping together of the diurnal constituents and the semidiurnal constituents into a fundamental (24-hr) period and a second harmonic (12-hr) period, respectively. The technique used was that of Blackman and Tukey (1959). The power spectrum for Dug Well No. 40 is limited necessarily by its length of 306 hr or 12.75 days while that of Honolulu Harbor is based on 1464 hr or the entire 61 days of August to September 1965. Even though the time lag should not exceed approximately one-tenth the length of the record, lags up to 60 hr were used for the Dug Well No. 40 spectrum and up to 120 hr for that of Honolulu Harbor. The results of the spectral analysis are presented in the form of graphs of spectral density vs. frequency, and autocovariance vs. time lag. These graphs are shown in Figures 12 and 13 for Honolulu Harbor and Dug Well No. 40, respectively.

The power associated with the 24-hr segments of record were estimated from the relation \( \sum_{p=1}^{n} (a_p^2 + b_p^2) \). These estimates of the power together with those from the spectral analysis are presented in Table 3.\(^1\)

Description of the Electric Analog Model and Model Data

ELECTRIC ANALOG MODEL. The model was laid out on U.S.G.S. maps of the Ewa Beach and Puuloa regions of Oahu. Since the map scale was 1 in. = 2000 ft, a grid spacing of \( a = 2000 \) ft was selected. This left a one-inch spac-

\(^1\) The power in the spectrum is the area under the spectral density curve and is equal to the autocovariance at zero lag.
TABLE 2. RESULTS OF HARMONIC ANALYSIS

<table>
<thead>
<tr>
<th>SEGMENT OF RECORD ANALYZED</th>
<th>8/27-28/65</th>
<th>8/27/65</th>
<th>8/28/65</th>
<th>9/04/65</th>
<th>9/05/65</th>
<th>9/06/65</th>
<th>AVG.*</th>
<th>6/22 to 7/22/65†</th>
</tr>
</thead>
<tbody>
<tr>
<td>HONOLULU HARBOUR</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>aₐ, ft</td>
<td>.4747</td>
<td>.4620</td>
<td>.2688</td>
<td>.6441</td>
<td>.5753</td>
<td>.5606</td>
<td>.5022</td>
<td>.63</td>
</tr>
<tr>
<td>aₛ, ft</td>
<td>.7632</td>
<td>.8456</td>
<td>.7903</td>
<td>.4199</td>
<td>.4367</td>
<td>.5223</td>
<td>.6030</td>
<td>.58</td>
</tr>
<tr>
<td>θₐ, deg</td>
<td>-14.54</td>
<td>225.86</td>
<td>241.08</td>
<td>193.32</td>
<td>203.58</td>
<td>211.47</td>
<td>215.06</td>
<td>235.1</td>
</tr>
<tr>
<td>θₛ, deg</td>
<td>7.60</td>
<td>146.26</td>
<td>168.35</td>
<td>11.96</td>
<td>32.76</td>
<td>57.30</td>
<td>83.33</td>
<td>167.4</td>
</tr>
<tr>
<td>DUG WELL NO. 40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>aₐ, ft</td>
<td>.0404</td>
<td>.1001</td>
<td>.0498</td>
<td>.1104</td>
<td>.0978</td>
<td>.0998</td>
<td>.0916</td>
<td>.15</td>
</tr>
<tr>
<td>aₛ, ft</td>
<td>.0798</td>
<td>.0750</td>
<td>.0744</td>
<td>.0416</td>
<td>.0508</td>
<td>.0594</td>
<td>.0602</td>
<td>.04</td>
</tr>
<tr>
<td>θₐ, deg</td>
<td>52.77</td>
<td>321.23</td>
<td>300.13</td>
<td>242.34</td>
<td>249.74</td>
<td>255.06</td>
<td>273.70</td>
<td>293.5</td>
</tr>
<tr>
<td>θₛ, deg</td>
<td>74.37</td>
<td>214.25</td>
<td>226.66</td>
<td>62.63</td>
<td>71.58</td>
<td>82.05</td>
<td>131.43</td>
<td>224.1</td>
</tr>
<tr>
<td>DUG WELL NO. 41</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>aₐ, ft</td>
<td>.0330</td>
<td>.0670</td>
<td>.0451</td>
<td>.0904</td>
<td>.0784</td>
<td>.0732</td>
<td>.0708</td>
<td>.08</td>
</tr>
<tr>
<td>aₛ, ft</td>
<td>.0660</td>
<td>.0688</td>
<td>.0699</td>
<td>.0320</td>
<td>.0342</td>
<td>.0394</td>
<td>.0489</td>
<td>.04</td>
</tr>
<tr>
<td>θₐ, deg</td>
<td>75.53</td>
<td>314.58</td>
<td>323.41</td>
<td>260.00</td>
<td>277.72</td>
<td>287.30</td>
<td>292.60</td>
<td>324.5</td>
</tr>
<tr>
<td>θₛ, deg</td>
<td>88.10</td>
<td>227.94</td>
<td>248.29</td>
<td>103.15</td>
<td>119.22</td>
<td>146.72</td>
<td>169.06</td>
<td>275.7</td>
</tr>
</tbody>
</table>

† FOR ENTRIES HERE aₐ IS BASED ONLY ON C₂₈ OR THE P₁ PLUS THE K₁ TIDAL COMPONENT AND aₛ IS BASED ONLY ON C₅₄ OR THE M₂ TIDAL COMPONENT FOR THE INDICATED DATES. AFTER R.H. DALE (1974).
FIGURE 12. AUTOCOVARIANCE vs. TIME LAG AND SPECTRAL DENSITY vs. FREQUENCY FOR THE WATER SURFACE-TIME HISTORY AT HONOLULU HARBOR
FIGURE 13. AUTOCOVARIANCE vs. TIME LAG AND SPECTRAL DENSITY vs. FREQUENCY FOR THE WATER SURFACE-TIME HISTORY AT DUG WELL NO. 40
**TABLE 3. A COMPARISON OF INCIDENT AND TRANSMITTED POWER**

### PERIODOGRAM ANALYSIS

<table>
<thead>
<tr>
<th>SEGMENT OF RECORD</th>
<th>LOCATION</th>
<th>( \frac{2}{p=1} \sum (a^2+b^2), \text{ ft}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 PM: 27 AUGUST 1965 to 5 PM: 28 SEPTEMBER 1965</td>
<td>HONOLULU HARBOR</td>
<td>4039.07 \times 10^{-4}</td>
</tr>
<tr>
<td></td>
<td>DUG WELL NO. 40</td>
<td>40.00 \times 10^{-4}</td>
</tr>
<tr>
<td></td>
<td>DUG WELL NO. 41</td>
<td>27.22 \times 10^{-4}</td>
</tr>
<tr>
<td>27 AUGUST 1965</td>
<td>HONOLULU HARBOR</td>
<td>4642.24 \times 10^{-4}</td>
</tr>
<tr>
<td></td>
<td>DUG WELL NO. 40</td>
<td>78.22 \times 10^{-4}</td>
</tr>
<tr>
<td></td>
<td>DUG WELL NO. 41</td>
<td>46.12 \times 10^{-4}</td>
</tr>
<tr>
<td>28 AUGUST 1965</td>
<td>HONOLULU HARBOR</td>
<td>3484.14 \times 10^{-4}</td>
</tr>
<tr>
<td></td>
<td>DUG WELL NO. 40</td>
<td>40.07 \times 10^{-4}</td>
</tr>
<tr>
<td></td>
<td>DUG WELL NO. 41</td>
<td>34.60 \times 10^{-4}</td>
</tr>
<tr>
<td>04 SEPTEMBER 1965</td>
<td>HONOLULU HARBOR</td>
<td>2955.90 \times 10^{-4}</td>
</tr>
<tr>
<td></td>
<td>DUG WELL NO. 40</td>
<td>69.59 \times 10^{-4}</td>
</tr>
<tr>
<td></td>
<td>DUG WELL NO. 41</td>
<td>45.98 \times 10^{-4}</td>
</tr>
<tr>
<td>05 SEPTEMBER 1965</td>
<td>HONOLULU HARBOR</td>
<td>2608.38 \times 10^{-4}</td>
</tr>
<tr>
<td></td>
<td>DUG WELL NO. 40</td>
<td>60.72 \times 10^{-4}</td>
</tr>
<tr>
<td></td>
<td>DUG WELL NO. 41</td>
<td>36.58 \times 10^{-4}</td>
</tr>
<tr>
<td>06 SEPTEMBER 1965</td>
<td>HONOLULU HARBOR</td>
<td>2935.35 \times 10^{-4}</td>
</tr>
<tr>
<td></td>
<td>DUG WELL NO. 40</td>
<td>67.44 \times 10^{-4}</td>
</tr>
<tr>
<td></td>
<td>DUG WELL NO. 41</td>
<td>34.56 \times 10^{-4}</td>
</tr>
<tr>
<td>AVERAGE POWER†</td>
<td>HONOLULU HARBOR</td>
<td>3325.20 \times 10^{-4}</td>
</tr>
<tr>
<td></td>
<td>DUG WELL NO. 40</td>
<td>63.20 \times 10^{-4}</td>
</tr>
</tbody>
</table>

### SPECTRAL ANALYSIS

| 01 AUGUST 1965 TO 30 SEPTEMBER 1965 | HONOLULU HARBOR | 3600.00 \times 10^{-4} |
| 26 AUGUST 1965 TO 08 SEPTEMBER 1965* | DUG WELL NO. 40 | 62.68 \times 10^{-4} |

† The average values do not include the "power" estimated for 27-28 August 1965.

* The power associated with periods longer than 24 hours (see Figure 13) is not included in this total.
ing in which to fit the 1/2 watt carbon resistors (± 10% tolerance) and the 50 WVDC ceramic capacitors (± 10% tolerance). Values of 1000 ohms and 0.1 x 10^{-6} farads were selected as the typical electrical sizes for the resistors and capacitors, respectively. These values represent the resistivity, (i.e., the reciprocal of the transmissivity) and storativity, respectively, of one complete grid square. Values of resistance and capacitance for partial grid squares located along the boundaries were determined by the vector area technique (Karplus 1958). The coastline from Brown's Camp to the Salt Evaporators formed the boundary along which the head fluctuated with the tide while at the interior a no-flow boundary was constructed.

The time scale factor (Walton and Prickett 1963), K_t, for the model is

\[ K_t = \frac{a^2(S/T)}{RC} = 4 \times 10^{10} (S/T) \text{ days/sec} \]  

where transmissivity, T, is expressed in sq ft/d.

At the outset, modeling the wells appeared to pose a problem since the wells possess a characteristic phase lag and amplitude decay factor dependent upon their geometry and independent of the tide. Subsequently, two separate techniques were employed. The first approach was to model the well electrically by constructing a separate electric analog model for the surface layer (approximately 10 ft thick) of that grid square which contained the well. This subsidiary surface model had a refined grid spacing of \( a = 200 \) ft, and the well was represented by a single wire shaped to the general geometry of the real well and provided a direct electrical short between those resistors representing the region traversed by the well.

The design of the subsidiary surface model was based on an average aquifer depth of 150 ft and an average well (channel) depth of 10 ft or 6.5% of the average depth. The resistance per square of the surface layer is then \( R_2 = (h/h_2)R = 1000/0.065 = 15,500 \) ohms and the resistance per square of the base layer is \( R_1 = (h/h_1) \) 1000 = 100/0.935 = 1070 ohms. If the typical resistors of \( R = 1000 \) ohms bounding the grid square containing the well are replaced with resistors of value \( R_1 \) ohms, that square and one-half of the square adjacent to each of its sides will represent a section of aquifer thickness, \( h_1 \). The numerical value of \( R_1 = 1070 \) ohms is well within the 10% tolerance of resistor, \( R \). Hence, in the basic model, the resistors for the grid square

containing the well were left at R ohms. The subsidiary surface model consisted of 15,500 ohm resistors on the interior and 31,000 ohm resistors along the boundaries of the square. In order to distribute the head around the boundary of the subsidiary surface layer, a second set of 3100-ohm resistors was placed in parallel with each of the boundary resistors of the surface model. This second set of resistors simulates the influence of the surface layer directly adjacent to the grid square which contains the well. To connect the subsidiary surface model, terminals at the corner grid points were connected at the grid points of the square on the basic model which contained the well. The steps outlined above are defined in Figure 14.

The storativity of both the surface and the underlying regions remained concentrated in the capacitors connected at the grid points of the basic model. The storativity of the channel itself was accounted for by a small capacitor whose electrical size was determined from the proportion

\[
\frac{C_w}{(\text{surface area})_{\text{well}}} = \frac{C}{a^2 S}
\]  

(17)

The surface area of Dug Well No. 40 is estimated at 18,750 sq ft and the storativity at a conservative value of .02, which results in approximately 3 \text{ \mu F} for \( C_w \). (This value is small and had no observable influence on the model output.)

The second technique for estimating the lag time and decay factors for the well channels is based on analytical considerations. Assuming that the changes in elevation of the phreatic surface near the well are approximated by a series of steady state flows leads to the following relations for the amplitude decay and the time lag: \(^1\)

\[
\xi_0/\xi = 1/[1 + (bx_o/(2T)\sigma)]^{1/2}
\]  

(18a)

\[
\tan \theta_p = -[bx_o/(2T)\sigma]
\]  

(18b)

In these equations \( b \) is the width of the channel, \( T \) the transmissivity, \( \sigma \) the tidal frequency, and \( x_o \) the distance from the channel bank beyond which no influence of the channel is observed as the level of the phreatic surface rises or falls. The distance \( x_o \) may be estimated from the relation

\[
x_o = F\sqrt{\theta K/\varepsilon}
\]  

(19)

\(^1\) Derivation details of the two factors are given in Appendix A.
The outlined region represents the lower layer of aquifer, i.e. \( h_1 = 140' \), modeled by the four resistors, \( R_1 \).

Note: \( R_1 \leq 0.1R = 100\Omega \)

These resistors, \( \frac{2R_2}{10} = 3100\Omega \), distribute the voltage from the adjacent upper layer, i.e. that portion of the upper layer delineated by crosshatching on 14c.

Subsidiary surface model is connected to basic model by connecting similarly identified pairs of lead wires.

**FIGURE 14. DESIGN OF THE SUBSIDIARY SURFACE MODEL**
where \( t \) is one-fourth the tidal period, \( \epsilon' \) is an effective porosity, and \( F \) is a function of time whose value falls in the range \( 1.23 \leq F \leq 1.73 \).\(^1\) The upper and lower bounds of \( F \) represent the limiting case where a phreatic aquifer drains into an empty channel and the limiting case where a full channel discharges to an aquifer whose depth is zero at distances \( x \geq x_o \) from the channel, respectively. Equation (19) assumes that the bottom of the channel and that of the aquifer coincide. In the case where the aquifer extends below the channel, some flux can be expected to pass through the channel bottom; however, the assumption of no flow through the bottom is entirely consistent with the shallow aquifer theory assumed here. Hence, equation (19) with the depth of the channel \( h = h_2 \) and \( F = 1.73 \) should give a reasonable estimate for an upper bound on \( x_o \). Selecting \( t = 1/2 \) day, \( h = 10 \) ft, \( K = 10^5 \) ft/day, \( \epsilon' = 0.1 \) results in an \( x_o = 340 \) ft, and, subsequently, \( \theta_p < 1^0 \) and \( \zeta/\zeta_o < 0.01 \). Thus, from analytic considerations, the presence of the well apparently does not influence significantly the response of the piezometric surface. It is worth noting that \( x_o \) may either be doubled or halved and the above inequalities for \( \theta_p \) and \( \zeta/\zeta_o \) will still be satisfied.

**MODEL DATA: ACQUISITION AND ANALYSIS.** Data were acquired using the basic model both in conjunction with the subsidiary surface model for Dug Well No. 40 and without it. The electric analog model was driven by a General Radio (No. 1310-A) audio oscillator and the output from the model monitored on a Hewlett-Packard dual trace (No. 122A) oscilloscope: one trace indicating the tidal oscillation at the coastline and the other trace monitoring the response at any selected nodal point. The procedure followed was the same for both model configurations. The time scale factor, \( K_a \), was varied from approximately 600 to 6000 days/sec and the amplitudes and phase angle recorded from the traces on the oscilloscope screen for both the fundamental and second harmonic components (i.e., for the diurnal and semidiurnal components of the tide). The data is recorded in Table 4 where the amplitudes have been normalized with respect to the tidal amplitude at the coast. The dimensionless amplitudes and the phase angles are also plotted as functions of the time scale factor, \( K_a \), and are shown in Figures 15 and 16. It should be noted that when the subsidiary surface model was not used, the

---

\(^1\) Equation (19), derived J. Bear et al. (1968) is based on the assumption that flow can be considered as a series of steady state flows.
### TABLE 4. SUMMARY OF DATA FOR ELECTRIC ANALOG MODEL

<table>
<thead>
<tr>
<th></th>
<th>DUG WELL NO. 40</th>
<th>DUG WELL NO. 41</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SUBSIDIARY SURFACE MODEL DISCONNECTED</td>
<td>SUBSIDIARY SURFACE MODEL CONNECTED</td>
</tr>
<tr>
<td>$K_4$ day/sec</td>
<td>$\rho_D$</td>
<td>$\rho_S$</td>
</tr>
<tr>
<td>5980</td>
<td>0.044</td>
<td>0.023</td>
</tr>
<tr>
<td>598</td>
<td>0.392</td>
<td>0.266</td>
</tr>
<tr>
<td>2800</td>
<td>0.125</td>
<td>0.049</td>
</tr>
<tr>
<td>1780</td>
<td>0.200</td>
<td>0.096</td>
</tr>
<tr>
<td></td>
<td>SUBSIDIARY SURFACE MODEL FOR DUG WELL NO. 40 CONNECTED</td>
<td></td>
</tr>
<tr>
<td>5980</td>
<td>0.063</td>
<td>0.027</td>
</tr>
<tr>
<td>598</td>
<td>0.425</td>
<td>0.313</td>
</tr>
<tr>
<td>3500</td>
<td>0.125</td>
<td>0.050</td>
</tr>
<tr>
<td>2500</td>
<td>0.175</td>
<td>0.089</td>
</tr>
<tr>
<td>5980</td>
<td>0.063</td>
<td>0.022</td>
</tr>
<tr>
<td>5980</td>
<td>0.044</td>
<td>0.018</td>
</tr>
<tr>
<td>598</td>
<td>0.425</td>
<td>0.310</td>
</tr>
<tr>
<td>598</td>
<td>0.380</td>
<td>0.270</td>
</tr>
<tr>
<td>3500</td>
<td>0.125</td>
<td>0.050</td>
</tr>
<tr>
<td>3500</td>
<td>0.098</td>
<td>0.033</td>
</tr>
<tr>
<td>2500</td>
<td>0.178</td>
<td>0.083</td>
</tr>
<tr>
<td>2500</td>
<td>0.145</td>
<td>0.075</td>
</tr>
<tr>
<td>1850</td>
<td>0.198</td>
<td>0.089</td>
</tr>
<tr>
<td>2750</td>
<td>0.135</td>
<td>0.050</td>
</tr>
</tbody>
</table>
FIGURE 15. DIMENSIONLESS AMPLITUDES, $\rho$, AND PHASE ANGLES, $\theta_D$, VS. TIME SCALE FACTOR, $K_4$, FOR THE DIURNAL AND SEMIDIURNAL TIDAL COMPONENTS AT DUG WELL NO. 40
Figure 16. Dimensionless amplitudes, $\rho$, and phase angles, $\theta_p$, vs. time scale factor, $K_4$, for the diurnal and semidiurnal tidal components at dug well No. 41.
model response was measured at the nodal point nearest the well being monitored.

From the amplitude plots in Figures 15 and 16, it is clear that the subsidiary surface model produces results which are not significantly different from those of the basic model alone, as predicted by the analysis in the previous section. The plots of the phase angles observed on the basic model define relatively good curves, while the phase angles from the combined model show considerable scatter. However, the scatter appears to be more the result of experimental error than a difference in trends between the two sets of data. For the phase angle measurements, the least count on the oscilloscope screen was 12°. Also, the subsidiary model seemed to generate some instability in the basic RC circuit which could easily account for the scatter.

It is also clear from Figures 15 and 16 that different time scale factors are required to produce the observed amount of damping of the two tidal components. This discrepancy raises the question of the influence of the wedging effect caused by the aquifer thickness decreasing by a factor of approximately one-half between the coastline and the interior boundary.

The Influence of Changing Aquifer Thickness

ONE-DIMENSIONAL APPROXIMATION OF THE ELECTRIC ANALOG MODEL. The aquifer region observed in a plan view shows a fair amount of variation in width with Dug Well No. 40 being located at one end of the narrower section approximately halfway between the entrance to Pearl Harbor and Barbers Point. This narrow portion persists for a distance of 4 or 5 miles and suggests that the aquifer model may behave in an essentially one-dimensional manner in this region. To establish this behavior, amplitudes and phase angles were measured at each nodal point along the cross section passing closest to Dug Well No. 40. These data are plotted in Figure 15 together with the one-dimensional solution for an aquifer of constant depth and a no-flow interior boundary. The time scale factor was based on $e^{\gamma /T} = 0.87 \times 10^{-7}$ day/sq ft, which yields $\rho_{D} = 0.125$ at Dug Well No. 40. The results show excellent agreement between the predicted and ob-

1. The electric analog model should be operated at frequencies which are considerably different from the natural frequency of the RC network. To ascertain the limits on the frequency, the amplitude at Dug Well No. 40 was observed for a set of values of $K_{a}$. The results indicate that error can be expected in the observed amplitude for values of $K_{a} > 6$ or $7 \times 10^{3}$ day/sec. The plot of $\rho$ vs. $K_{a}$ for these data is shown in Appendix B.
served amplitude but the observed phase angles are as much as 30% smaller than the predicted values. The observed phase angle data do conform to the general geometry of the theoretical curves, however, and the results are considered to be sufficiently good to justify the use of the one-dimensional flow theory in studying the wedging effect on the aquifer response to tides.

ONE-DIMENSIONAL WEDGING EFFECT. As mentioned previously, Williams and Liu (1971) have derived the relation for both the amplitude and phase angle with the position, \( x/L \), for a one-dimensional flow in an aquifer with linearly changing conductivity. These same relations can be applied to the case of a linearly, but slowly varying thickness, \( z(x) \), by utilizing the transmissivity instead of the conductivity to calculate a modified diffusion coefficient, \( \alpha_1 \).

The computer program VAMP-NF (Williams and Liu 1971), written specifically to evaluate the output of the mathematical model, would not converge for the relatively large values of \( \alpha_1 \), because the terms in the power series representation of the several modified Bessel functions became too large. Subsequently, a second computer program was written. This program employed an asymptotic representation of the modified Bessel functions which is valid for arguments greater than or equal to 8 (McLachlan 1934, p. 152). The steps required to calculate the coefficient \( \alpha_1 \), together with the new program and a sample output, are included in Appendix C.

To determine what influence the wedging effect had on the aquifer response, the mathematical model was used to evaluate the dimensionless amplitude \( \rho \) at \( x/L=0.6 \), the relative position of Dug Well No. 40, over the range \( 0.50 \leq (e'/\bar{T}) \times 10^7 \leq 1.25 \) for \( z_L/z_0 \) equal to 1, 1.2, 1.5, 2, 3, 4, and 5 and for both the diurnal and semidiurnal tides. The results of this evaluation are shown in Figure 18. The mathematical model was also used to compute the amplitudes and phase angles as functions of \( x/L \) for \( e'/\bar{T}=0.87 \times 10^{-7} \) day/sq ft and \( z_L/z_0=2 \) and \( 4 \). These results have been included in Figure 17 for comparison with those for \( z_L/z_0=1 \).

It is seen from Figure 17 that as the ratio of \( z_L/z_0 \) increases, the amplitude does not decay as rapidly while the phase angles tend to decrease at a given \( x/L \). This trend holds for both the diurnal and semidiurnal tides. From Figure 18, it is clear that the ratio of the amplitudes at \( x/L=0.6 \) for the two tidal components remains essentially constant with respect to variations in \( z_L/z_0 \). For example, the ratio is 0.125/0.053 for values of \( z_L/z_0=1 \),
Figure 17. Dimensionless amplitudes, $\rho$, and phase angles, $\theta_p$, vs. \(x/L\) for finite aquifer of variable thickness: $z_L/z_0 = 1, 2, 4$; \(\varepsilon'/T = 0.87 \times 10^{-7}\) day/sq ft and tidal periods of one and one-half day.
FIGURE 18. DIMENSIONLESS AMPLITUDES, $\rho$, vs. $\epsilon' / T \times 10^7$ day/sq ft at $x/L = 0.6$ for $z_L / z_o = 1, 1.2, 1.5, 2, 3, 4, 5$; and tidal periods of one and one-half day.
2, and 4 while $\varepsilon'/\bar{T}$ takes on the (approximate) values $0.87 \times 10^{-7}$, $1.08 \times 10^{-7}$ and $1.26 \times 10^{-7}$ day/sq ft, respectively. Thus if it is required that the aquifer exhibit the same ratio of $\varepsilon'/\bar{T}$ for both the diurnal and semidiurnal components, then for a given diurnal tidal efficiency, $\rho_D$, the semidiurnal tidal efficiency, $\rho_S$, is fixed. At the same time, these values of $\rho_D$ and $\rho_S$ remain constant with respect to the wedging factor, $z_L/z_0$. The electric analog model based on a uniform thickness will give an erroneous value of $\varepsilon'/\bar{T}$ for a wedging aquifer. However, Figure 18 may be used to determine the correct value, i.e., in the above example if $z_L/z_0=2$, the corrected value is $\varepsilon'/\bar{T}=1.08 \times 10^{-7}$ day/sq ft.

**DISCUSSION**

**Results of Field Data Analyses**

**HARMONIC ANALYSIS.** The results of the field data analyses are summarized in Figures 11, 12, and 13 and in Tables 2, 3, and 5. From Table 2, it is seen that the average of the amplitudes $a_D$ and $a_S$ and the relative phase angles $\theta_D$ and $\theta_S$ are in reasonable agreement with those of Dale (1974) based on a 28-day record. A comparison of the values of the dimensionless amplitudes, $\rho_D$ and $\rho_S$, and phase angles, $\theta_{p,D}$ and $\theta_{p,S}$, with the averages for those quantities is presented in Table 5. There is a good degree of consistency between the results for each of the five 24-hr segments of the record and again, the average values are in reasonable agreement with those obtained by Dale (1974), particularly for Dug Well No. 41. The standard deviations indicate that two-thirds of the time both $\rho_D$ and $\rho_S$ are within 10% of their average value, while $\theta_{p,D}$ and $\theta_{p,S}$ are within 16% of their average values, except $\theta_{p,S}$ at Dug Well No. 40 where the standard deviation is about 30%. It should be pointed out that the harmonic analysis for the 24-hr period starting from 5:00 P.M. on 27 August 1965, was not included in the average since the trigonometric polynomial for this period produced a poor fit with the given record. This is the result of a poor choice of

---

1. The daily variations of $\rho$ and $\theta$ are the result of both the diurnal and the semidiurnal tidal constituents having slightly different periods and phases which produce the spring and neap tides and cause a beat in the tidal record. For the diurnal constituents, $K_1$ and $D_1$, the period of the beat is 13.66 days and for the semidiurnal constituents, $M_2$ and $S_2$, the period is 14.785 days.
TABLE 5. COMPARISON OF TIDAL EFFICIENCIES AND PHASE ANGLES

<table>
<thead>
<tr>
<th>DUG WELL NO.</th>
<th>$\rho_D$</th>
<th>$\rho_S$</th>
<th>$\theta_{p,D}$ deg</th>
<th>$\theta_{p,S}$ deg</th>
<th>DATE OF RECORD</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>.085</td>
<td>.104</td>
<td>68.3</td>
<td>66.8</td>
<td>27-28 AUG. 1965</td>
</tr>
<tr>
<td>41</td>
<td>.070</td>
<td>.086</td>
<td>88.1</td>
<td>80.5</td>
<td>&quot;</td>
</tr>
<tr>
<td>40</td>
<td>.217</td>
<td>.089</td>
<td>65.4</td>
<td>68.0</td>
<td>27 AUG. 1965</td>
</tr>
<tr>
<td>41</td>
<td>.145</td>
<td>.081</td>
<td>58.7</td>
<td>81.7</td>
<td>&quot;</td>
</tr>
<tr>
<td>40</td>
<td>.185</td>
<td>.094</td>
<td>59.1</td>
<td>58.3</td>
<td>28 AUG. 1965</td>
</tr>
<tr>
<td>41</td>
<td>.168</td>
<td>.088</td>
<td>82.3</td>
<td>80.0</td>
<td>&quot;</td>
</tr>
<tr>
<td>40</td>
<td>.171</td>
<td>.099</td>
<td>49.0</td>
<td>50.7</td>
<td>04 SEPT. 1965</td>
</tr>
<tr>
<td>41</td>
<td>.140</td>
<td>.076</td>
<td>66.7</td>
<td>91.2</td>
<td>&quot;</td>
</tr>
<tr>
<td>40</td>
<td>.170</td>
<td>.116</td>
<td>46.2</td>
<td>38.8</td>
<td>05 SEPT. 1965</td>
</tr>
<tr>
<td>41</td>
<td>.136</td>
<td>.078</td>
<td>74.1</td>
<td>86.5</td>
<td>&quot;</td>
</tr>
<tr>
<td>40</td>
<td>.178</td>
<td>.114</td>
<td>43.6</td>
<td>24.8</td>
<td>06 SEPT. 1965</td>
</tr>
<tr>
<td>41</td>
<td>.131</td>
<td>.075</td>
<td>75.8</td>
<td>89.4</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DUG WELL NO.</th>
<th>$\rho_D$</th>
<th>$\rho_S$</th>
<th>$\theta_{p,D}$ deg</th>
<th>$\theta_{p,S}$ deg</th>
<th>DATE OF RECORD</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVG. VALUES</td>
<td>.184</td>
<td>.102</td>
<td>52.7</td>
<td>48.1</td>
<td>&quot;</td>
</tr>
<tr>
<td>STD. DEVIATION</td>
<td>.017</td>
<td>.011</td>
<td>8.3</td>
<td>15.1</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DUG WELL NO.</th>
<th>$\rho_D$</th>
<th>$\rho_S$</th>
<th>$\theta_{p,D}$ deg</th>
<th>$\theta_{p,S}$ deg</th>
<th>DATE OF RECORD</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELECTRIC ANALOG MODEL DATA</td>
<td>FROM: FIG. 13</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>.184</td>
<td>.090</td>
<td>70</td>
<td>115</td>
<td>&quot;</td>
</tr>
<tr>
<td>41</td>
<td>.144</td>
<td>.045</td>
<td>135</td>
<td>195</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DUG WELL NO.</th>
<th>$\rho_D$</th>
<th>$\rho_S$</th>
<th>$\theta_{p,D}$ deg</th>
<th>$\theta_{p,S}$ deg</th>
<th>DATE OF RECORD</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIELD DATA AFTER R. H. DALE</td>
<td>&quot;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>.24</td>
<td>.07</td>
<td>58.4</td>
<td>56.7</td>
<td>25 JUNE to &quot;</td>
</tr>
<tr>
<td>41</td>
<td>.13</td>
<td>.07</td>
<td>89.3</td>
<td>108.2</td>
<td>22 JULY 1965</td>
</tr>
</tbody>
</table>

---

a ENTRIES ARE CALCULATED FROM DATA PRESENTED IN TABLE 2, e.g., FOR DUG WELL NO. 40, 05 SEPT. 1965, $\rho_D = 0.0978/.5753 = .170$, and $\theta_{p,D} = 249.74 - 203.58 = 46.16$.

b ENTRIES ARE BASED ON THE DASHED-LINE CURVES FAIRED THROUGH THE DATA OF FIGURES 13 AND 14. THE AVERAGE VALUES OF $\rho_D$ AND $\rho_S$ ARE BOTH USED AS ENTRY POINTS TO THESE FIGURES.

c ENTRIES ARE AFTER R. H. DALE 1974. HERE, $\rho_D$ IS BASED ONLY ON $C_{28}$ OR THE $P_1$ PLUS THE $K_1$ TIDAL COMPONENT, WHILE $\rho_S$ IS BASED ONLY ON $C_{54}$ OR THE $M_2$ TIDAL COMPONENT.
starting points, viz., the peak tide on the 27th as 24 hr later the peak tide is about 15% lower, thus presenting a decidedly nonperiodic fluctuation.

SPECTRAL ANALYSIS. The results of the spectral analysis shown in Figures 12 and 13 confirm the fact that the bulk of the incident "power" as well as that transmitted inland is concentrated at the diurnal and semi-diurnal frequencies. It is also clear that more dissipation is experienced by the semidiurnal power than the diurnal, as expected.

In particular, the periods at peak power are 24.00 hr and 12.00 hr, for Dug Well No. 40, and 24.00 hr and 12.63 hr for Honolulu Harbor. It should be pointed out that a spectral analysis of the Honolulu Harbor record, based only on the tidal record for the period of 27 August through 7 September resulted in peak power at 12.00- and 24.00-hr periods.

Table 3 indicates that the power estimated from a 24-hr segment of record varies somewhat from day to day for both Honolulu Harbor and Dug Well No. 40, but the average values of the power for the five 24-hr segments of records agree with the results of the spectral analysis to within 8% or less.

Both spectral density functions show some power at periods greater than 24 hr. For Dug Well No. 40, this amounts to approximately 25% of the total power while at Honolulu Harbor it is only 1 or 2% of the total. This is most likely the result of the longer period tidal constituents. In order to resolve the fortnightly tidal component, 4 to 5 months of records are required for analysis.

Results of Electric Analog Simulation

ELECTRIC ANALOG MODEL RESULTS. The results of the electric analog model tests are summarized in Table 4 and in Figures 15 and 16. The significant result from these tests is that different ratios of storativity to transmissivity are required to produce the observed amount of damping of the diurnal and semidiurnal tidal components. From Figures 15 and 16, the range of variation is $0.50 \times 10^{-7} \leq \epsilon'(T) \leq 0.55 \times 10^{-7}$ day/sq ft at Dug Well No. 40 and $0.29 \times 10^{-7} \leq \epsilon'(T) \leq 0.36 \times 10^{-7}$ day/sq ft at Dug Well No. 41. These differences are small but consistent in that the semidiurnal tide yields the smaller value of $\epsilon'(T)$.

EFFECT OF AQUIFER WEDGING. The electric analog model represents an aquifer of uniform thickness, whereas the observed aquifer is known to de-
crease in thickness as it extends inland from the coastal boundary. The possible influence of this changing thickness on the observed amplitudes and phase angles was considered in terms of a "wedging-factor," \( (z_L/z_o) \). However, Figure 18 shows that if the same value of \( (\varepsilon'/T) \) is to describe the aquifer for each tidal frequency, then the ratio of \( (\rho_D/\rho_S) \) is independent of aquifer wedging. Consequently, the linear wedging theory offers no explanation to the question of the dependence of \( (\varepsilon'/T) \) on frequency. Figure 18 does, however, provide a means of correcting the results from an electric analog model based upon a uniform aquifer thickness for a linear wedging in the prototype.

**Applicability of the Method**

**THE METHOD.** The use of tidal response data to estimate the aquifer properties involves two steps. First, the input and response must be measured in the form of water surface-time histories and the harmonic components of the records resolved. Second, the particular relationship between the input and the response which produces the best estimate of the ratio of storativity to transmissivity must be selected. The relationship selected depends upon the nature of the boundaries of the aquifer. If the aquifer is one-dimensional and of infinite extent, it takes the form of equation (5a). For more complicated boundaries, e.g., a finite one-dimensional aquifer with a no-flow boundary condition, a more complicated expression must be used. (See Werner and Noreen 1944; Williams, Wada, and Wang 1970.) For general boundaries no closed-form solution is available, and it is necessary to resort to models such as the electric analog or digital model devised by Dale (1974).

The ratio of storativity to transmissivity for the Ewa Beach aquifer, determined by the electric analog model together with the values found by Dale (1974), are presented in Table 6. There is excellent agreement for the results based upon the diurnal tidal efficiency, \( \rho_D \), while those based upon the semidiurnal tidal efficiency, \( \rho_S \), differ by a factor of three with the electric analog model giving the smaller values. Correcting for aquifer wedging increases \( \varepsilon'/T \) about 30%. It should be recalled that Dale's (1974) digital model is limited to a one-dimensional aquifer while the electric analog model accounts for horizontal (areal) variations in the aquifer boundaries. This additional capability seems to have little influence on
### Table 6. Comparison of the Ratio of Storativity to Transmissivity.

<table>
<thead>
<tr>
<th>Well No.</th>
<th>Diurnal Component</th>
<th>Semidiurnal Component</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho_D$</td>
<td>$K_n$</td>
</tr>
<tr>
<td></td>
<td>day/sec</td>
<td>$z_L / z_0 = 1$</td>
</tr>
<tr>
<td>DUG WELL NO. 40</td>
<td>0.184</td>
<td>2200</td>
</tr>
<tr>
<td>DUG WELL NO. 41</td>
<td>0.144</td>
<td>1450</td>
</tr>
</tbody>
</table>

AFTER R.H. DALE$^b$

<table>
<thead>
<tr>
<th>Well No.</th>
<th>$\rho_D$</th>
<th>$K / L^2 S$</th>
<th>$(S / K)$</th>
<th>$\rho_S$</th>
<th>$K / L^2 S$</th>
<th>$(S / K)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(day)$^{-1}$</td>
<td>day/ft$^2$</td>
<td>(S/K)</td>
<td>(day)$^{-1}$</td>
<td>day/ft$^2$</td>
<td>(S/K)</td>
</tr>
<tr>
<td>DUG WELL NO. 40</td>
<td>0.24</td>
<td>0.20</td>
<td>0.50x10$^{-7}$</td>
<td>--</td>
<td>0.07</td>
<td>0.067</td>
</tr>
<tr>
<td>DUG WELL NO. 41</td>
<td>0.13</td>
<td>0.24</td>
<td>0.40x10$^{-7}$</td>
<td>--</td>
<td>0.07</td>
<td>0.12</td>
</tr>
</tbody>
</table>

$^a$ Value of $\varepsilon' / T$ corrected for wedging effect from Figure 16.

$^b$ Entries are based on analysis after R.H. Dale 1974. Values of $(K / L^2 S)$ have been estimated from type-curves of $\rho$ vs. $X / L$. L has been taken as 10,000 ft for both wells.
the model results, however, particularly in the region of Dug Well No. 40, as is evident from Figure 17. Furthermore, the fact that each model yields essentially the same value for the diurnal \( (\varepsilon'/\bar{T}) \) at Dug Well No. 41 as well as at Dug Well No. 40, indicates that this limitation on the digital model may not be significant for the unconfined Ewa Beach aquifer.¹

It should be noted, however, that Dale's (1974) results show a considerably greater difference between the diurnal and the semidiurnal storativity-to-transmissivity ratio than does the electric analog model, viz., factors of about 3 and 2 at Dug Wells No. 40 and No. 41, respectively.

VARIABILITY OF THE STORATIVITY-TO-TRANSMISSIVITY RATIO. The experimental results presented in Figures 7 and 8 suggest that the storativity, \( \varepsilon' \), is weakly dependent or possibly even independent of the period of oscillation for periods greater than a few minutes, say, 15 to 30. The results from the electric analog model show that the diurnal \( (\varepsilon'/\bar{T}) \) is only 10 to 20% larger than the semidiurnal \( (\varepsilon'/\bar{T}) \), where the standard deviation on \( \rho_P \) and \( \rho_S \) is 10%. If Prinz's (1923) observations of tidal fluctuation adjacent to the Elbe River are analyzed on the assumption that equation (5a) is the most appropriate relationship between input and response, the resulting semidiurnal \( (\varepsilon'/\bar{T}) \) is about 30% greater than the longer period--approximately 9 days--\( (\varepsilon'/\bar{T}) \).² These results suggest that \( (\varepsilon'/\bar{T}) \) is not basically frequency dependent and that these relatively small differences may be primarily the result of errors in the field measurements or errors in resolving the harmonics present in the time histories of the piezometric surface.

On the other hand, Dale's (1974) results show the semidiurnal \( (\varepsilon'/\bar{T}) \) to be 100 to 200% greater than the diurnal \( (\varepsilon'/\bar{T}) \), while van der Kamp (1973) gives the diurnal storativity as 100% greater than the semidiurnal storativity for an unconfined sandstone aquifer at Cap-Pelé, N.B., Canada. van der Kamp (1973) also suggests that the storativity is dependent on frequency with the lower frequencies producing the larger values of storativity.

A comparison of the Ewa Beach and Cap-Pelé aquifer properties with those of the Ottawa sand provides some insight into the dilemma. For the

---

¹ Since Dug Well No. 41 is approximately the same distance from the Pearl Harbor Channel as it is from the Ewa Beach coast, a one-dimensional representation of the aquifer in this region should be less reliable than in the vicinity of Dug Well No. 40.
² The fluctuations at the river bank were excluded from these calculations.
Cap-Pelé aquifer, van der Kamp (1973) reports the specific yield $^1 = 0.015$ and the transmissivity $= 280$ sq m/day. For the Ewa Beach aquifer, the specific yield is estimated to be 0.2 while the conductivity is estimated to be $1.1 \times 10^4$ fpd.$^2$ For the average aquifer thickness of 150 ft, the transmissivity is $1.7 \times 10^6$ sq fpd or $1.5 \times 10^5$ sq m/day. The Ottawa sand used in the experiments described above has a specific yield of 0.34 and a conductivity of $0.42$ cm/sec giving a transmissivity of $0.55 \times 10^5$ sq m/day for a 150-ft thick layer. From a comparison of these properties, it is clear that the Ottawa sand and the Ewa Beach limestone are quite similar with respect to the pore volume available for storativity and the ease with which the water may move through the media. Thus, the response to tides of the Ewa Beach aquifer can be expected to be similar to that of the Ottawa sand, and therefore $(\varepsilon' / \overline{T})$ exhibits relatively little, if any, dependence on the tidal frequency. In this instance, a good share of the 10 to 20% difference between the semi-diurnal and diurnal values of $(\varepsilon' / \overline{T})$ is attributed to errors in the records and/or errors inherent in the harmonic analysis of the data. On the other hand, the significantly smaller values of specific yield and transmissivity for the Cap-Pelé aquifer apparently result in drainage and imbibition characteristics sufficiently different from the Ottawa sand to cause the storativity to show a decided dependence on time--$\varepsilon'$ increasing for the lower tidal frequencies. The fact that Dale's (1974) analysis results in the semi-diurnal $(\varepsilon' / \overline{T})$ being substantially larger than the diurnal $(\varepsilon' / \overline{T})$ is contrary to both field and laboratory observation and may well be an error.

It is interesting to note that using the values of $(\varepsilon' / \overline{T})$ corrected for wedging and $\overline{T} = 1.7 \times 10^6$ sq fpd gives for the diurnal and semi-diurnal storativities, $\varepsilon'_D = 0.12$ and $\varepsilon'_S = 0.11$. Thus, $\varepsilon'_D / \varepsilon = 0.31$ and $\varepsilon'_S / \varepsilon = 0.28$, indicating that the storativity is, at the most, only slightly dependent on frequency.

The question of the variation of storativity with both the amplitude and the phase of the tidal components may be raised in view of the hyster-

1. The specific yield was ascertained by a pump test of 20-hrs duration and would probably increase somewhat for longer periods of pumping. The porosity of the Cap-Pelé sandstone is given as 22%.
2. This value of the conductivity was determined by Khan (1974) for a coral reef limestone aquifer at Waimanalo, on the island of Oahu, which is considered to be similar to the Ewa Beach aquifer. The porosity of coral reef limestone is 0.30 to 0.35.
etic behavior of the available pore space during dewatering and imbibition. However, such variations do not appear to be significant since the amplitude and phases of the diurnal and semidiurnal components varied considerably for the five segments of record analyzed while the tidal efficiencies remained remarkably consistent (see Tables 2 and 5).

COMPARISON WITH PUMP TEST METHODS. The procedure for the evaluation of aquifer properties by means of pump test data is similar to that using tidal response data: an input (a drawdown applied at the well) and a response (a drawdown at an observation well or wells) must be measured, and the appropriate relationship (pump test formula) between the two selected in order to calculate the aquifer properties. However, no preliminary analysis of the pump test data similar to the resolution of the harmonic components of the tide records is generally required unless the pumped well itself is used as an observation well. In this case, the well losses must be determined and subtracted from the total drawdown at the pumped well. In both methods, a knowledge of the nature of boundaries is required to select the best relationship between input and response. Finally, both methods give values for aquifer properties which are averaged over the distance separating the points of input and response. In general, the two methods will yield different results (van der Kamp 1973). The pump test method probably produces the more reliable results since the observed data usually requires no preanalysis and, in cases of nonequilibrium data, early time drawdown measurements are relatively uninfluenced by boundaries.

However, the two methods can give results which show a similar statistical distribution of the aquifer properties. For example, a pump test of the step drawdown type described by Williams and Soroos (1973)\(^1\) involves five different discharges and three observation wells. The Theim equation was applied and a transmissivity determined for each discharge from the slope of the drawdown vs. distance-to-the-observation well curve. The transmissivities fell in the range $0.28 \times 10^6 \leq T \leq 0.44 \times 10^6$ sq fpd and $T_{avg} = 0.37 \times 10^6$ sq fpd with a standard deviation of $0.059 \times 10^6$ sq fpd. From Table 5 and Figure 15, five values of $(\varepsilon'/T)$ based on the diurnal tidal efficiency

---

1. Step drawdown pump test conducted 10 October 1965 at the Wilder Avenue Pumping Station, Honolulu, Hawaii.
at Dug Well No. 40 may be estimated. Using \( \epsilon' = 0.1 \), five values of \( \overline{T} \) result which \( 1.74 \times 10^6 \leq \overline{T} \leq 2.21 \times 10^6 \) sq fps, \( (\overline{T})_{\text{ave}} = 1.89 \times 10^6 \) sq fps and the standard deviation = \( 0.18 \times 10^6 \) sq fps.

**CONCLUSIONS**

In general, the use of tidal response data is a viable means of estimating aquifer properties in coastal areas or in any region where an aquifer is in communication with a body of water whose free surface is subjected to periodic fluctuations. The technique requires essentially the same amount of field work as a pump test and can be expected to produce results which are generally not as reliable as those from a pump test. A knowledge of the nature of the boundaries, the presence of leakage, etc., is important in interpreting results and, as a rule, it is preferable to work with the amplitudes of the oscillations rather than with the phase angles. The optimum procedure for the evaluation of aquifer properties would include analyses of both tidal response and pump test data since one serves as an independent check on the other.

Specific conclusions drawn from the discussion above are summarized.

1. Tidal efficiencies based on an analysis of a 24-hr water surface-time history which considers only one diurnal and one semidiurnal component are reasonably consistent with those based on a 28-day record and composed of two diurnal and three semidiurnal components. It is recommended, however, that several 24-hr segments be analyzed and that care be taken to select 24-hr segments for which the record is essentially periodic (e.g., see the 24-hr period starting 5:00 P.M., 27 August 1965).

2. In the case of a phreatic aquifer, the storativity appears to have some dependence on frequency. This dependence will be weaker or stronger as the specific yield and/or the transmissivity are larger or smaller, respectively. The storativity (i.e., the effective porosity) will generally be only a fraction of the specific yield (about 30% for the Ewa Beach coral limestone). More research is required in this area.

3. The ratio of the diurnal to the semidiurnal tidal efficiency \( \rho_D / \rho_S \) is independent of a linear decrease in aquifer thickness, i.e., of the wedging effect, at least for wedging factors \( 1 \leq z_L / z_0 \leq 5 \).

4. The location and geometry of boundaries, the presence of leakage
and the overall geology of the aquifer region are all important in making the correct interpretation and analysis of tidal response data.

5. The tidal response theory provides a viable method for determining aquifer characteristics. The optimum procedure, however, would include analyses of both tidal response and pump test data.

ACKNOWLEDGMENTS

The authors would like to express their appreciation to Dr. R.H. Dale of the U.S. Geological Survey, Honolulu Office for his assistance in obtaining the field data for Dug Wells Nos. 40 and 41 and for his review of the manuscript. We should also like to thank Dr. R.E. Green of the Department of Soil Science, University of Hawaii for his review of and comments on the section dealing with the effective porosity for an oscillating water table.
REFERENCES


A long, two-dimensional ditch intersects an aquifer in which the elevation of the phreatic surface is subjected to periodic fluctuations. The water surface in the ditch will also fluctuate, but will lag behind the phreatic surface of the aquifer and have a smaller amplitude. This phase lag and amplitude decay are characteristic of the ditch geometry and the media and may be estimated in the following way.

If it is assumed that the oscillations are of sufficiently small amplitude to justify the use of the Dupuit assumptions and the shallow aquifer theory, then $q = -K\partial h/\partial x = -K\bar{h} \partial h/\partial x$. This last expression may be integrated and solved for $q$ to give

$$q = Kh/x_o (h_o - h)$$

The conservation of mass principle applied to the boundaries of the ditch requires that

$$q = b/2 \frac{dh}{dt}.$$  

If it is assumed that the rise or fall of the phreatic surface produces a series of steady state flows into or from the ditch, then $q$ may be eliminated between these last two equations to give

$$bx_o/2K\bar{h} \left(\frac{dh}{dt}\right) = \alpha \left(\frac{dh}{dt}\right) = h_o - h; \quad \alpha = bx_o/2K\bar{h}.$$  

The particular integral for this equation represents the steady state solution. If $h = \bar{h} + \zeta \sin(\sigma t + \theta p)$, then at $x = x_o$, $h_o = \bar{h} + \zeta_o \sin \omega t$, where $x_o$ is the distance beyond which no influence of the ditch is observed in the
deflection of the phreatic surface. Substituting this solution into the differential equation gives

\[ a \zeta \sigma \cos(\sigma t + \theta_p) = \zeta_0 \sin \sigma t - \zeta \sin(\sigma t + \theta_p) \]
\[ = (\zeta_0 - \zeta \cos \theta_p) \sin \sigma t - \zeta \sin \theta_p \cos \sigma t \]

or

\[ \cos \theta_p - \alpha \sigma \sin \theta_p = \zeta_0 / \zeta \]
\[ \alpha \sigma \cos \theta_p + \sin \theta_p = 0 \]

This last pair of equations may be solved for \( \theta_p \) and \( \zeta / \zeta_0 \), i.e.,

\[ \tan \theta_p = -\alpha \sigma \]

\[ \frac{\zeta}{\zeta_0} = \frac{1}{1/\sqrt{1 + (\alpha \sigma)^2} + (\alpha \sigma)^2 / \sqrt{1 + (\alpha \sigma)^2}} \]
\[ = \frac{1}{\sqrt{1 + (\alpha \sigma)^2}} \]
APPENDIX B. RESONANCE FREQUENCY FOR ELECTRIC ANALOG MODEL

\[ K^4 = 7 \times 10^4 \text{ day/sec} \approx \text{RESONANCE FREQUENCY} \]

- X - SUBSIDIARY SURFACE MODEL IN CIRCUIT
- O - SUBSIDIARY SURFACE MODEL DISCONNECTED
Appendix C. Fortran Program for the Evaluation of Variable Permeability Aquifer Model for $\alpha_1 \geq 8$

The mathematical model, VAMP-NF, may be applied to the case of a variable thickness $z(x)$ by making a modification of the coefficient $\alpha = \epsilon' \sigma / m^2 \tilde{z} K_0$. Here $\epsilon'$ is the effective porosity, $\tilde{z}$ is the uniform thickness of the aquifer, $K_0$ is the conductivity at the interior boundary, and $m$ is determined from the relation $K(x) = K_0(1 + mx)$ evaluated at $x = L$, the interior boundary. In terms of the transmissivity, this latter expression is $T(x) = T_0(1 + mx)$; $T = \tilde{z} K$, and $m = (T_L/T_0 - 1)/L$. If it is the thickness which changes rather than the conductivity, then $m = (z_L/z_0 - 1)/L$. Also, $\bar{T} = (T_0 + T_L)/2 = T_0(1 + z_L/z_0)/2$. Substitution of these last expressions into that for $\alpha$ gives

$$\alpha_1 = \epsilon' \sigma L^2 / 2 \bar{T} \cdot (1 + z_L/z_0)/(z_L/z_0 - 1)^2$$

$$= \alpha \left[ (1 + z_L/z_0)/2 \right] ; \quad \tilde{z} K_0 = \bar{T}$$

The coefficient, $\alpha_1$, may be substituted into the mathematical model, VAMP-NF, for the determination of $\rho$ and $\theta$ as functions of position, $x/L$. Note that $\bar{T} = \tilde{z} K_0$ is now the product of the average thickness and the constant conductivity.

In Figure 18 the smallest value of $\alpha_1$ occurs for $z_L/z_0 = 5$ and $\epsilon' / \bar{T} = 0.50 \times 10^{-7}$ day/sq ft and is approximately 6. Hence, the curve for $z_L/z_0 = 5$ in Figure 18 may be slightly in error. For the remaining curves, $\alpha_1$ exceeds 8. In fact, for $z_L/z_0 < 1.2$, the resulting $\alpha_1$ generates numbers in the asymptotic expansions which cause an "overflow" in the IBM 360.
VAMP - NF

**This program is used to calculate the amplitude and the phase angle for coastal aquifer with linear increase or decrease in hydraulic conductivity, no flow at internal boundary.**

**DOUBLE PRECISION AL, CL, CKL, CKO, M, CD, PERIOD, SIGMA, ALPHA,**

1. TENX, XI, P, PB, PI, BEROP, BEIOP, A12, A1, A2, CKROP,
2. CK1OP, A32, A3, A4, BER1PB, BEI1PB, B12, B1, B2, CK1PB,
3. CK1PB, B32, B3, B4, BER1PI, BEI1PI, CI2, CI, C2, CK1ROI,
4. CK10P1, C32, C3, C4, A, B, C, D, RT, RXI, RL, RCL, R, RHQ,
5. SCA, SIN, AS, SCB, S1NB, BS, SCC, SINC, CS, SCD, SIND, DS,
6. TS, COSA, AC, COSB, BC, COSC, CG, COSD, TC, TC, THM, THETAP

**DEGREE, S02, PI**

**READ(5,20)PERIOD,CD**

**FORMAT(F15.0,O15.7)**

**IFIPERIOD .EQ. 9.0**

**STOP**

**Al=10000.0**

**PI=3.1415927**

**Cl=2.0**

**M=(ICl-1.0)/AL**

**SIGMA=2.0*PI/PERIOD**

**ALPHA=13.0*SIGHA·COSI·12.0·H.H)**

**WRITE(6,30)PERIOD,CD,ALPHA**

**FORMAT (1H15,9H PERIOD,CD)**

**1 1HO,8ZL/I0 = ,F6.3/IHO,8HALPHA**

**2 4X,9HAMPLITUDE,1X,11HPHASE ANGLE**

**TENX=20.0**

**X=TENX/20.0**

**XI=1.0+(M+AL)*XI**

**P=DSORT(4.0·ALPHA·XI)**

**PB=OSORT(4.0·ALPHA)**

**Pl=DSORT(4.0·ALPHA·Cl)**

**SO2=2.0**

**SO2=DSORT(SO2)**

**A1=(DEXP(P/SO2)/DSORT(T2.0*P*P1))**

**1/1.0/15.0*P**

**-15.0/1256.0*P1*P1)**

**-133.0/12048.0*P**

**+3.0/12048.0*P1*P1)**

**A2=A1*P**

**A2=P/SO2-P1/8.0-1.0/(8.0*SO2*P1)-1.0/16.0*P**

**A3=DSORT(T2.0*P))**

**1/1.0/15.0*P**

**-15.0/1256.0*P1*P1)**

**+3.0/12048.0*P**

**-133.0/12048.0*P1*P1)**

**A32=A3*A3**

**A4=1.0-P/SO2-P1/8.0+1.0/(8.0*SO2*P1)-1.0/16.0*P**

**A5=25.0/(154.0*SO2*(P**3.0))**

**B1=(DEXP(PB/SO2)/DSORT(T2.0*P*P1))**

**1/1.0/15.0*P**

**-15.0/1256.0*P1*P1)**

**+3.0/12048.0*P**

**-133.0/12048.0*P1*P1)**

**B1Z=B1*P**

**B2=P/SO2+3.0*PI/8.0+3.0/(8.0*SO2*P1)-3.0/16.0*P**

**B3=DSORT(PB/SO2)**

**1/1.0/15.0*P**

**-15.0/1256.0*P1*P1)**

**+3.0/12048.0*P**

**-133.0/12048.0*P1*P1)**

**B32=B3*B3**

**B4=1.0-P/SO2-P1/8.0+3.0/(8.0*SO2*P1)+3.0/16.0*P**

**B5=2.0/(128.0*SO2*(P**3.0))**

**B6=2.0/(128.0*SO2*(P**3.0))**

**C1=(DEXP(P/PI)/DSORT(T2.0*P*P1))**

**1/1.0/15.0*P**

**-15.0/1256.0*P1*P1)**

**+3.0/12048.0*P**

**-133.0/12048.0*P1*P1)**

**C1Z=C1*C1**

**C2=(P/SO2-P1/8.0-1.0/(8.0*SO2*P1)-1.0/16.0*P**

**C3=DSORT(T2.0*P))**

**1/1.0/15.0*P**

**-15.0/1256.0*P1*P1)**

**+3.0/12048.0*P**

**-133.0/12048.0*P1*P1)**

**C32=C3*C3**

**C4=1.0-P/SO2-P1/8.0+1.0/(8.0*SO2*P1)-1.0/16.0*P**

**C5=25.0/(154.0*SO2*(P**3.0))**

**A=Al*C32*B3**

**B=B32*C32**

**C=Cl*At*B3Z**

**O=C1*81*A32*B3**

**RI=I(Al-2)*-I(A4-4)**

**RXI=I(B32*A32)**

**RL=I(C2-B1-C4)**

**PCL=(B32*C12+B1*C12)**

**R=RXI/RCL**

**RHO=DSORT(DAB5(R))**

**SCA=(A2-N2)-C4**

**SIN=DSIN(SCA)**

**AS=A1*SINA**

**SCB=A4-C4**

**SINB=DSIN(SCB)**
BS=B*SINB  
SCC=A2-C2  
SINC=DSN(SCC)  
CS=C*SINC  
SCD=((A4-RS)-(R2-RS))  
SIND=DSN(SCD)  
OS=D-SIND  
TS=A5+BS+CS-DS  
COSIN=DCOS(SCA)  
AC=A*COSA  
COSB=DCOS(SCB)  
AC=A*COSB  
CC=C*COSC  
COSD=DCOS(SCD)  
DC=D*COSD  
TC=AC+BC+CC-DC  
TA=TS/TC  
THEAT=DATA(TAM)  
IF(TS,GT,0.0,AND.TC,GT,0.0) THEAT=THEAT-2.0*PI  
IF(TS,LT,0.0,AND.TC,LT,0.0) THEAT=THEAT-PI  
WRITE(6,401,X,RHO,DEGREE  
FORMAT('D9.3,015.6,016.6')  
IF(TENX,LT,0.0) GO TO 10  
TENX=TENX-1.0  
GO TO 2  
END

*OPTIONS IN EFFECT* ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP  
*OPTIONS IN EFFECT* NAME = MAIN ; LINECNT = 58  
*STATISTICS* SOURCE STATEMENTS = 82 ; PROGRAM SIZE = 4520  
*STATISTICS* NO DIAGNOSTICS GENERATED

F88-LEVEL LINKAGE EDITOR OPTIONS SPECIFIED NONE  
DEFAULT OPTION(S) USED = SIZE=96256,43008  
****USERPROG Does not exist but has been added to DATA SET
<table>
<thead>
<tr>
<th>LOCATION</th>
<th>AMPLITUDE</th>
<th>PHASE ANGLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1000 01</td>
<td>0.10000000</td>
<td>-0.2035510-12</td>
</tr>
<tr>
<td>0.9500 00</td>
<td>0.60135000</td>
<td>-0.1305520 02</td>
</tr>
<tr>
<td>0.9000 00</td>
<td>0.64037700</td>
<td>-0.2627960 02</td>
</tr>
<tr>
<td>0.8500 00</td>
<td>0.51026500</td>
<td>-0.3968080 02</td>
</tr>
<tr>
<td>0.8000 00</td>
<td>0.40537600</td>
<td>-0.5326670 02</td>
</tr>
<tr>
<td>0.7500 00</td>
<td>0.32106100</td>
<td>-0.6704570 02</td>
</tr>
<tr>
<td>0.7000 00</td>
<td>0.25348500</td>
<td>-0.8102530 02</td>
</tr>
<tr>
<td>0.6500 00</td>
<td>0.19949300</td>
<td>-0.9521040 02</td>
</tr>
<tr>
<td>0.6000 00</td>
<td>0.15648800</td>
<td>-0.1096010 03</td>
</tr>
<tr>
<td>0.5500 00</td>
<td>0.12232800</td>
<td>-0.1241880 03</td>
</tr>
<tr>
<td>0.5000 00</td>
<td>0.95247000</td>
<td>-0.1389580 03</td>
</tr>
<tr>
<td>0.4500 00</td>
<td>0.73789500</td>
<td>-0.1539890 03</td>
</tr>
<tr>
<td>0.4000 00</td>
<td>0.56769800</td>
<td>-0.1690260 03</td>
</tr>
<tr>
<td>0.3500 00</td>
<td>0.43250100</td>
<td>-0.1044500 03</td>
</tr>
<tr>
<td>0.3000 00</td>
<td>0.32537000</td>
<td>-0.2004970 03</td>
</tr>
<tr>
<td>0.2500 00</td>
<td>0.24191200</td>
<td>-0.2178100 03</td>
</tr>
<tr>
<td>0.2000 00</td>
<td>0.18032800</td>
<td>-0.2373690 03</td>
</tr>
<tr>
<td>0.1500 00</td>
<td>0.14068500</td>
<td>-0.2595120 03</td>
</tr>
<tr>
<td>0.1000 00</td>
<td>0.12164700</td>
<td>-0.2814190 03</td>
</tr>
<tr>
<td>0.0500 01</td>
<td>0.11646500</td>
<td>-0.2972350 03</td>
</tr>
<tr>
<td>0.0</td>
<td>0.11608600</td>
<td>-0.3029090 03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LOCATION</th>
<th>AMPLITUDE</th>
<th>PHASE ANGLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1000 01</td>
<td>0.10000000</td>
<td>-0.1846110 02</td>
</tr>
<tr>
<td>0.9500 00</td>
<td>0.52960800</td>
<td>-0.3716040 02</td>
</tr>
<tr>
<td>0.9000 00</td>
<td>0.38305700</td>
<td>-0.5610760 02</td>
</tr>
<tr>
<td>0.8500 00</td>
<td>0.27586700</td>
<td>-0.7531240 02</td>
</tr>
<tr>
<td>0.8000 00</td>
<td>0.19778000</td>
<td>-0.9478580 02</td>
</tr>
<tr>
<td>0.7500 00</td>
<td>0.14113300</td>
<td>-0.1145390 03</td>
</tr>
<tr>
<td>0.7000 00</td>
<td>0.10021800</td>
<td>-0.1345850 03</td>
</tr>
<tr>
<td>0.6500 00</td>
<td>0.70799600</td>
<td>-0.1549390 03</td>
</tr>
<tr>
<td>0.6000 00</td>
<td>0.49749900</td>
<td>-0.1756180 03</td>
</tr>
<tr>
<td>0.5500 00</td>
<td>0.24160500</td>
<td>-0.2180170 03</td>
</tr>
<tr>
<td>0.5000 00</td>
<td>0.16696200</td>
<td>-0.2397450 03</td>
</tr>
<tr>
<td>0.4500 00</td>
<td>0.11467100</td>
<td>-0.2617800 03</td>
</tr>
<tr>
<td>0.4000 00</td>
<td>0.80007000</td>
<td>-0.2730610 03</td>
</tr>
<tr>
<td>0.3500 00</td>
<td>0.52383300</td>
<td>-0.3066690 03</td>
</tr>
<tr>
<td>0.3000 00</td>
<td>0.34327800</td>
<td>-0.3302950 03</td>
</tr>
<tr>
<td>0.2500 00</td>
<td>0.22136400</td>
<td>-0.3569990 03</td>
</tr>
<tr>
<td>0.2000 00</td>
<td>0.15311300</td>
<td>-0.3877170 02</td>
</tr>
<tr>
<td>0.1500 00</td>
<td>0.13136300</td>
<td>-0.5779610 02</td>
</tr>
<tr>
<td>0.1000 00</td>
<td>0.12967700</td>
<td>-0.6908580 02</td>
</tr>
</tbody>
</table>

**PERIOD = 1000 DAYS**

**CD = 0.8700-07DAY/FT**

**ZL/Z0 = 2.00**

**ALPHA = 0.82000 02**

**PERIOD = 0.500 DAYS**

**CD = 0.8700-07DAY/FT**

**ZL/Z0 = 2.00**

**ALPHA = 0.16400 03**