FREQUENCY DISTRIBUTION OF TSUNAMI RUNUPS IN HAWAII

Proposed for Use In
The National Flood Insurance Program

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The coastal high hazard zone to be established under the National Flood Insurance Program is to be the 100-year tsunami inundation zone. Toward the delineation of the zone Houston et al. (1977) have estimated frequency distributions of tsunami runups at over 500 sites along Hawaiian coasts. The distributions are based in general on synthesized runups at each site of the tsunamis considered significant in Hawaii since 1837. The synthetic runups were based, for the most part, on runup patterns initially computed using a numerical analysis model from a typical tsunami source in each of several Pacific-rim source regions, but adjusted by comparison with the patterns of runups measured at many coastal sites since 1946, and further adjusted in accordance with the intensity of each tsunami in the historic record as indicated by the records since 1837 at a few sites.

The frequency distribution of tsunamis is assumed to be exponential. Solov'ev (1972) found that, with respect to coasts adjacent to tsunami generating areas, the exponential distribution law applies to tsunami intensities. However, with respect to affected coasts in general, Houston et al. assumed the exponential distribution law applies to tsunami runup heights, in accordance with the findings or assumptions of Cox (1964), Wiegel (1965, 1970), Adams (1970), Rascón and Villareal (1975), and Wybro (1976). The distribution for an affected coastal site may be expressed as:

\[ \frac{R^*}{k} = e^{-\alpha - \frac{H}{\beta}} \]

(1)

where \( H \) = runup height (L)

\( R^* \) = mean recurrence interval (T) for tsunami whose height equals or exceeds \( H \).

\( \alpha \) = a site-specific coefficient (L)
\[ \beta = \text{a site-specific coefficient (L}^{-1}\text{)} \]
\[ k = \text{unit time (T)} \]

Recognizing that, numerically, \( R = R^*/k \) is identical to \( R^* \), equation (1) may be expressed in linear form as:

\[ H = \alpha + \beta \ln R \quad (2) \]

or in the form used for least squares regression by Houston et al. as:

\[ H = -B - A \log F \quad (3) \]

where \( F = 1/R = \text{mean recurrence frequency} \)

\[ B = -\alpha \]
\[ A = B/M \]
\[ M = \log e = 0.434 \]

Houston et al. published curves indicating the values of the A and B coefficients along the shorelines of each of the major Hawaiian Islands.

Because the regression analyses of Houston et al. were applied to synthetic rather than measured runup heights, any check of the reasonableness of the frequency distributions they found is useful. Such checks are provided by the findings of Cox (1964) and Wybro (1976).

For 39 tsunamis in the historic record at Hilo dating from 1837, Cox (1964) plotted, against log frequency, the runup heights reported, estimated from descriptions of effects, estimated from marigraphic records, or selected from multiple reports for the same tsunami. He found for the larger tsunamis a linear relation of the sort indicated by equation (3).

Wybro analyzed the frequency distribution of runups of 28 tsunamis observed at Hilo, 24 at Kahului, and 28 at Honolulu during the same period. Although Hilo and Kahului are on the northeast coasts of the islands of Hawaii and Maui, respectively, and Honolulu is on the south coast of the island of Oahu. Wybro found that the distributions at the three sites could be expressed with errors of 10 percent or less by the common formula:

\[ \frac{H}{H_Y} = K + C \ln (R/Y) \quad (4) \]

where \( Y = \text{Duration of period of record} \)
\[ H_Y = \text{Highest recorded runup during period } Y \]

Equation (4) is equivalent to equation (2) if:

\[ \alpha = H_Y (K - C \ln Y) \quad 4(a) \]
\[ \beta = C H_Y \quad 4(b) \]
or equivalent to equation (3) if:

\[ A = C \frac{H_y}{M} \quad 4(c) \]
\[ B = H_y \left( C \log \frac{Y}{M} - K \right) \quad 4(d) \]

\( K \) and \( C \) = constants

If \( H_y \) corresponded to the highest runup predicted by the formula at a site for \( R = Y \), \( K \) should, of course, be 1. By two different methods Wybro estimated the values of the two constants as:

<table>
<thead>
<tr>
<th>Method</th>
<th>I</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K )</td>
<td>1</td>
<td>0.97</td>
</tr>
<tr>
<td>( C )</td>
<td>0.31</td>
<td>0.33</td>
</tr>
</tbody>
</table>

A first check against the analysis by Houston et al. is afforded by comparison of the values of their \( A \) and \( B \) coefficients, and the 100-year runup heights computed from them, with the equivalent values computed from the Wybro formula (4), and indicate by Cox's plot.

The values of the \( A \) and \( B \) coefficients determined by Houston et al. may be read from their graphs from the sites numbered:

- for Hilo Hawaii No. 104
- for Kahului Maui No. 39
- for Honolulu Oahu No. 66

Recognizing that the period of record, \( Y \), was 140 years at the time of the analysis of Houston et al. and that no higher runups, \( H_y \), had been experienced than those tabulated by Wybro: 34.2 feet for Hilo, 21.2 feet for Kahului, and 8.25 for Honolulu; comparative values produced by the two Wybro methods may be calculated from equations 4e and 4d, and derived from Cox's plot.

From the \( A \) and \( B \) coefficients, 100-year runup estimates may be computed as:

\[ H_{100} = -B - A \log \left( \frac{1}{100} \right) = 2A - B \quad (5) \]

Additional 100-year runup estimates in meters may be read from Wybro's graphs of the results of the application of his second method and converted to feet for comparison.

The values estimated by the several methods are compared in Table 1. (The \( H_{100} \) values in parentheses are those read from Wybro's graphs).
Table 1. Estimates of runup distribution coefficients and 100-year runups for Hilo, Kahului, and Honolulu.

<table>
<thead>
<tr>
<th>Site</th>
<th>Source</th>
<th>A (ft.)</th>
<th>B (ft. per log yr.)</th>
<th>$H_{100}$ (ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hilo</td>
<td>Houston et al.</td>
<td>20</td>
<td>14</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>Wybro I</td>
<td>24.4</td>
<td>18.4</td>
<td>30.4</td>
</tr>
<tr>
<td></td>
<td>Wybro II</td>
<td>26.0</td>
<td>22.3</td>
<td>29.7 (30.2)</td>
</tr>
<tr>
<td></td>
<td>Cox</td>
<td>29</td>
<td>13</td>
<td>29</td>
</tr>
<tr>
<td>Kahului</td>
<td>Houston et al.</td>
<td>12</td>
<td>6</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>Wybro I</td>
<td>15.2</td>
<td>11.4</td>
<td>18.9</td>
</tr>
<tr>
<td></td>
<td>Wybro II</td>
<td>16.1</td>
<td>14.1</td>
<td>18.1 (16)</td>
</tr>
<tr>
<td>Honolulu</td>
<td>Houston et al.</td>
<td>3</td>
<td>1.5</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>Wybro I</td>
<td>5.9</td>
<td>4.4</td>
<td>7.4</td>
</tr>
<tr>
<td></td>
<td>Wybro II</td>
<td>6.3</td>
<td>5.5</td>
<td>7.0 (6.6)</td>
</tr>
</tbody>
</table>

In the case of each coefficient, the value estimated by Houston et al. is distinctly lower than the corresponding values derived from the other analyses. Although the differences are somewhat compensated in the estimation of the 100-year runup values, the 100-year runups estimated by Houston et al. are significantly lower for Hilo and Honolulu than those estimated by the other investigators.

A second check against the analysis of Houston et al. derives from Wybro's finding of a common relationship between the frequency distributions at the three sites of different exposures.

It will be noted from 4(c) and 4(d) that, if the common relationships found by Wybro for Hilo, Kahului, and Honolulu applies at all sites:

$$\frac{B}{A} = \log Y - \frac{M}{C} K$$

Inserting for $Y$ the value 140 years, and for $C$ and $K$ the values obtained by Wybro methods I and II,

$$B = 0.74 A \text{ for method I}$$

$$B = 0.87 A \text{ for method II}$$

The corresponding ratio indicated by Cox's plot for Hilo is smaller, only 0.53. The ratios indicated by coefficients taken from the graphs of Houston et al. are 0.70 for Hilo, 0.50 for Kahului, and 0.50 for Honolulu.

Even casual comparison of the Houston et al. curves for the A and B coefficients indicates that there is a high positive correlation between them. However, from a sample of coefficients for 20 percent of the sites on the islands of Hawaii, Maui, and Oahu (Figure 1), it appears that the Kahului and Hilo values at least, are unusually low. In general, approximately:

$$B = 1.31 A - 5.6$$
Figure 1. Correlation between A and B coefficients of Houston et al.
If Wybro's findings may be taken to indicate that the ratios B/A are not site dependent, designating:

\[ \gamma = \frac{B}{A} = \text{constant} \]

\[ H = -A - \gamma A \log F \]

Equation (3) then becomes:

\[ H = D \log \left( \frac{F}{f} \right) \]

where \( D = \gamma A \)

\( f = 10 \left( \frac{1}{\gamma} \right) \)

The question may be raised whether it would be better to fit, by least squares, to the synthetic heights estimated by Houston et al., the line defined by equation (6) rather than the line defined by equation (3).

If the value of \( \gamma \) were that indicated by Wybro's method I, \( \gamma = 0.74 \), \( f \) would be 13.7 years, and equation (6) would become:

\[ H = D \log \left( \frac{F}{13.7} \right) \]

In equation (7) only the coefficient \( D \) is site-specific. The least-squares criterion would be satisfied by estimating:

\[ D = \frac{\sum (H)}{\sum \left[ \log \left( \frac{F}{13.7} \right) \right]} \]
References


