SOME EVIDENCE OF ECONOMICS OF SCALE

IN HAWAIIAN SUGAR PLANTATIONS

by

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ABSTRACT

Of the major sectors in Oahu's economy, sugar growing and milling together use by far the largest quantities of water. The three plantations still operating on Oahu in 1971 accounted for 57 percent of total withdrawals, even though this percentage has steadily declined over the past decade or longer. A persistent trend among plantations, moreover, is for mergers to occur, presumably to take advantage of economies of scale. This study was undertaken to inquire into the effect of scale on sugar production. Data for plantations on four of the Hawaiian Islands is applied several procedures for estimating scale economics and economic efficiency. First, a "survivorship" test is used. Then the efficiency measures developed by M. J. Farrell are calculated. Finally, some regression estimates are determined.

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INTRODUCTION

The proposal which led up to this report envisioned something frankly quite different than what appears in the following pages. As indicated by the project title--"Economic Systems Simulation for Water Supply-Demand Relationships"--the research was undertaken to try to provide a simulation model of Oahu's economy, with an explicit water sector built in. This would not have been the first such model, of course, but would have provided an economically well-grounded device for projecting variables underlying the aggregate demand for water on Oahu.

Some sort of answer to a preliminary question seemed necessary as input for the general model, however. This question concerned use of water by the sugar plantations. The sugar industry is connected to the island's water picture in several direct and indirect ways, but perhaps most fundamental is that sugar production uses great quantities of water--for irrigation of the crops as well as for washing cane and other milling operations. On Oahu, for example, although water withdrawal by plantations has remained almost constant and that of other users has grown substantially in the past decade or so, sugar still accounts, in 1971, for fifty-seven percent of total withdrawals on the island (Oh and Yamauchi 1973). Now, as has been widely discussed in the past few years, sugar production is undergoing considerable adjustment. New technical processes are being developed in response to changing factor costs as well as pressure from environmental protection agencies; and sugar companies have long complained at great length of low rates of profit. One of the more evident forms of response to these pressures is the increase of mergers of old plantations, generally resulting in the closing of one mill and expansion and updating of the remaining mill, presumably taking advantage of economies of scale.

We felt that some inquiry was needed as a prerequisite for a general water planning model into this matter of the effect of scale on sugar production and thence on water use by the sugar industry. Considering the importance of the sugar industry in Hawaii, surprisingly little work has been done on the economics of sugar production. The Experiment Station, Hawaii Sugar Planters' Association, HSPA, does much research on the technology of the process, and collects some information on economic aspects as well. Aside from a handful of regularly published but often incomplete data series on such variables as acreage, labor input, and output, relevant economic data, especially at the level of individual firms, is either not collected or simply not released. As part of the compliance-payment program, the Sugar Division of the U.S. Department of Agriculture periodically surveys each plantation, collecting rather extensive information pertinent to economic analysis of growing and milling operations. Because of nondisclosure agreements, however, none of this data is made available. Our data sources for this study are the HSPA's annual *Sugar Manuals*, for output, acreage, and labor inputs; and annual company financial reports for a rather ill-fitting capital variable (depreciated value of capital stock). (It is interesting, though not very surprising, to note that the capital variable is more often than not insignificant in the regression equations of section 5 below.)

A few studies of particular aspects of cane sugar economics have been done. Labor relations have been carefully watched (Mollett 1965). Hawaiian and Western beet sugar production and marketing, with emphasis on the marketing, was the subject of an Economic Research Center Study by Mund (1966). The best general study of the industry is another ERC report done by Hung (1962), which paid special emphasis to the relation of the general excise tax to the sugar industry's viability. In the past few years, several publications have come through the U. H. Water Resources Research Center dealing with the productivity of irrigation water in sugarcane production (Rankine, Davidson, and Hogg 1972). Finally, the State Department of Planning and Economic Development made some rather rough estimates of profitability criteria (DPED 1973, Res. Rep. 72-2).

At any rate, the inquiry into scale economies turned out to be much more than just a minor preliminary question and in fact occupied our time for longer than the entire formal project period. Even at that, we have been less than entirely successful at answering the question, as will be seen presently. What follows, then, is not at all a simulation model of Oahu's economy with special reference to its water demand and supply characteristics, but rather a somewhat unsatisfactory study of economies of scale in Hawaiian sugar plantations.

SURVIVORSHIP OF PLANTATIONS

In 1950, the Hawaiian sugar industry encompassed 28 plantations with some 220,000 acres planted in cane, and produced 961,000 tons of raw sugar. A decade later, there were still 27 plantations remaining, although acreage had risen very slightly. By 1968, however, when the industry's output reached a peak, the number of plantations had dropped to 25 through mergers, while a 28 percent increase in output, compared to 1950, had been produced from about 10 percent more land, along with changes--by no means proportional--in other inputs. This consolidation process has continued along with several outright closings. Recent announcements indicate that by 1975, only 16 plantations will remain as shown in Figure 1, and will be producing on 222,000 acres (HSPA Sugar Manuals 1951-1969; Gilmore Sugar Manual 1969).

Throughout the past two decades, then, the average size of firm has grown and may be expected to continue to do so. If we can assume that the increased average firm *size* correlates closely with *scale*, and if we can assume that firms survive over time because they are more efficiently organized, at least with respect to scale of operation, than those which disappear, then the trend of firm size may be taken as a rough indication of the existence of economies of scale. This test dates at least to J. S. Mill and was termed, in perhaps a Darwinian spirit, the "survivorship" test (Johnston 1956, pp. 55-56). Though really no more than a formalization of casual observation, the survivorship test hypothesizes that an industry for which economies of scale exist will be characterized over time by a rise in the central tendency of the distribution of firm sizes.

Figure 2 shows a distribution of firms according to size. Firm size is measured by percent of total industry output, and output, in turn, is measured by tons 96° raw sugar. While the appearance of larger modal firm size between 1950 and 1968 is perhaps not so obvious as to provide a textbook-pure example of the survivorship test, the trend does seem to be evident. In 1950, 75 percent of the state's raw sugar output came from firms producing 2.5 percent or more (the median size class) of the industry's total; in 1960, 81 percent of output came from firms in the same relative size categories; and by 1970 the figure was 92.

As mentioned earlier, several plantations are scheduled to close and

HAWAII	1.	PUNA		AMFAC, INC.
	2.	MAUNA KEA		C. BREWER & CO., LTD.
	3.	HILO	JULY 1965	"
	4.	ONOMEA	JULY 1972	"
	5.	PEPEEKEO	JAN. 1963	"
	6.	HAKALAU		"
	7.	LAUPAHOEHOE	JAN. 1957	THEO H. DAVIES & CO., LTD.
	8.	KAIWIKI	JAN. 1973	"
	9.	HAMAKUA		"
	10.	PAAUHAU	JAN. 1973	C. BREWER (DAVIES, 1973)
	11.	HONOKAA		THEO H. DAVIES & CO., LTD.
	12.	KOHALA	(SCHEDULED CLOSING, 1975)	CASTLE & COOKE, INC.
	13.	HUTCHINSON	JUNE 1972	C. BREWER & CO., LTD.
	14.	HAWAIIAN AG.		"
MAUI	15.	PIONEER		AMFAC, INC.
	16.	WAILUKU		C. BREWER & CO., LTD.
	17.	HAWAIIAN COMM.		ALEXANDER & BALDWIN, INC.
OAHU	18.	OAHU	FEB. 1970	AMFAC, INC.
	19.	EWA		CASTLE & COOKE (AMFAC, 1970)
	20.	WAIALUA		CASTLE & COOKE
	21.	KAHUKU	(CLOSED, 1971)	ALEXANDER & BALDWIN, INC.
KAUAI	22.	LIHUE		AMFAC, INC.
	23.	GROVE		GROVE FARM CO., INC.
	24.	MCBRYDE		ALEXANDER & BALDWIN, INC.
	25.	OLOKELE		C. BREWER & CO., LTD.
	26.	GAY & ROBINSON		BISHOP TRUST CO., LTD.
	27.	WAIMEA	FEB. 1969	AMFAC, INC.
	28.	KEKAHA		"
	29.	KILAUEA	(CLOSED, 1971)	C. BREWER & CO., LTD.

FIGURE 1. HAWAIIAN SUGAR PLANTATIONS: STATUS OF COMPANIES SINCE 1950.

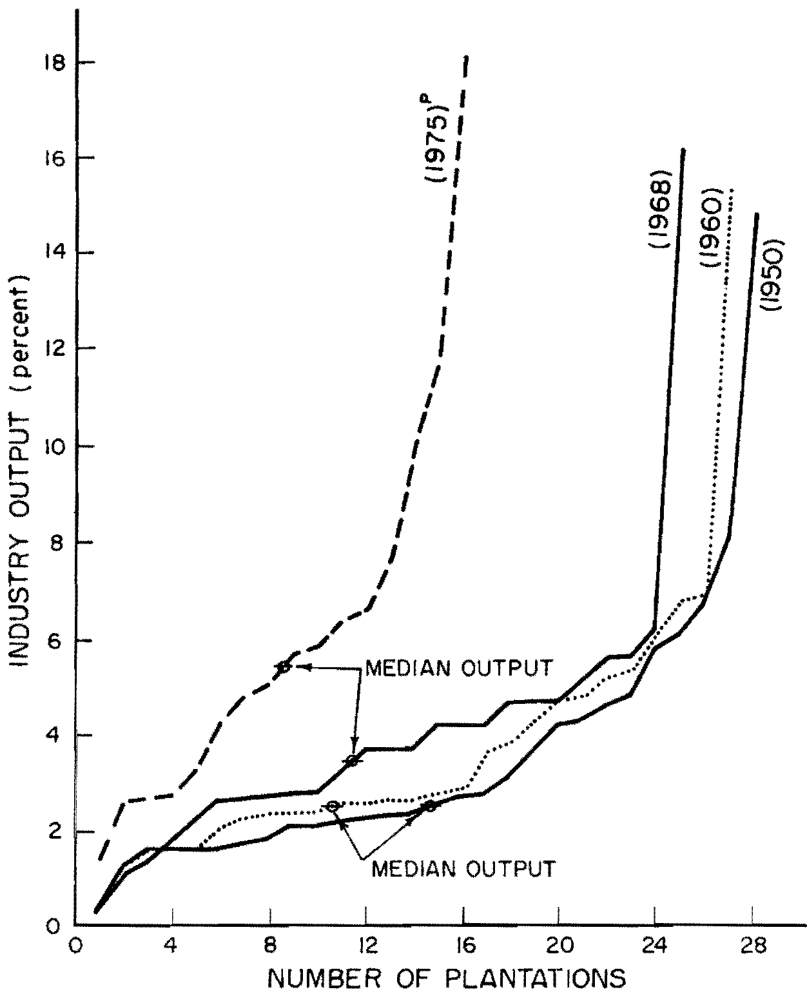

FIGURE 2. CUMULATIVE DISTRIBUTION OF PLANTATIONS BY PERCENT OF INDUSTRY OUTPUT.

P ASSUMES MERGERS ANNOUNCED AS OF SPRING 1972, AND THAT MERGED PLANTATIONS WILL USE SAME ACREAGE AS CONSTITUTENTS.

others are to be absorbed in mergers. If these actions are carried out as announced, and if we assume that the merged unit will produce the same rates of output as the constituent plantations together in 1968, we can show distribution of industry output in 1975 (in Fig. 2). The tendency toward larger size is even more marked than for comparisons of earlier years. The projected median percentage output share is considerably higher in 1975, as can be seen. Indeed, only one surviving firm would have a market share less than the 1950 median.

Taking land input, rather than output, as the measure of size, as we have done in Figure 3, the same trend is shown: 50 percent of the plantations were using 7,000 acres or more in 1950, which the 1970 chart shows 70 percent to be at least this large.

FARRELL EFFICIENCY

Another approach to estimating scale economies will lead to considerably more information than the nonparametric survivorship test provided. The price of such increased information, is, of course, a quantum leap in complexity of the estimation procedure, as well as in data requirements. This method was suggested by M. J. Farrell (1957), and in spite of some shortcomings, has been used in a few subsequent studies, mostly concerned with agricultural applications (Bresler 1966; Seitz 1966; Sitorus 1966).

It seems helpful to review briefly the nature of the production relationship before going into Farrell's procedure. The conventional view of a production function is typified by the classic work of Sune Carlson (1956, pp. 14-15):

> A given amount of output may frequently be produced from a number of different service combinations. It may also be true that the same combination of productive services gives varied amounts of output, depending upon how efficiently the productive services are organized . . . If we want the production function to give only one value for the output from a given service combination, the function must be so defined that it expresses the *maximum product* obtainable from the combination at the existing state of technical knowledge.

We cannot expect to find observations (firms) using given combinations of factor services in producing varying rates of output, of course; a given firm produces at only one output rate at a time. Nor, for the same obvious

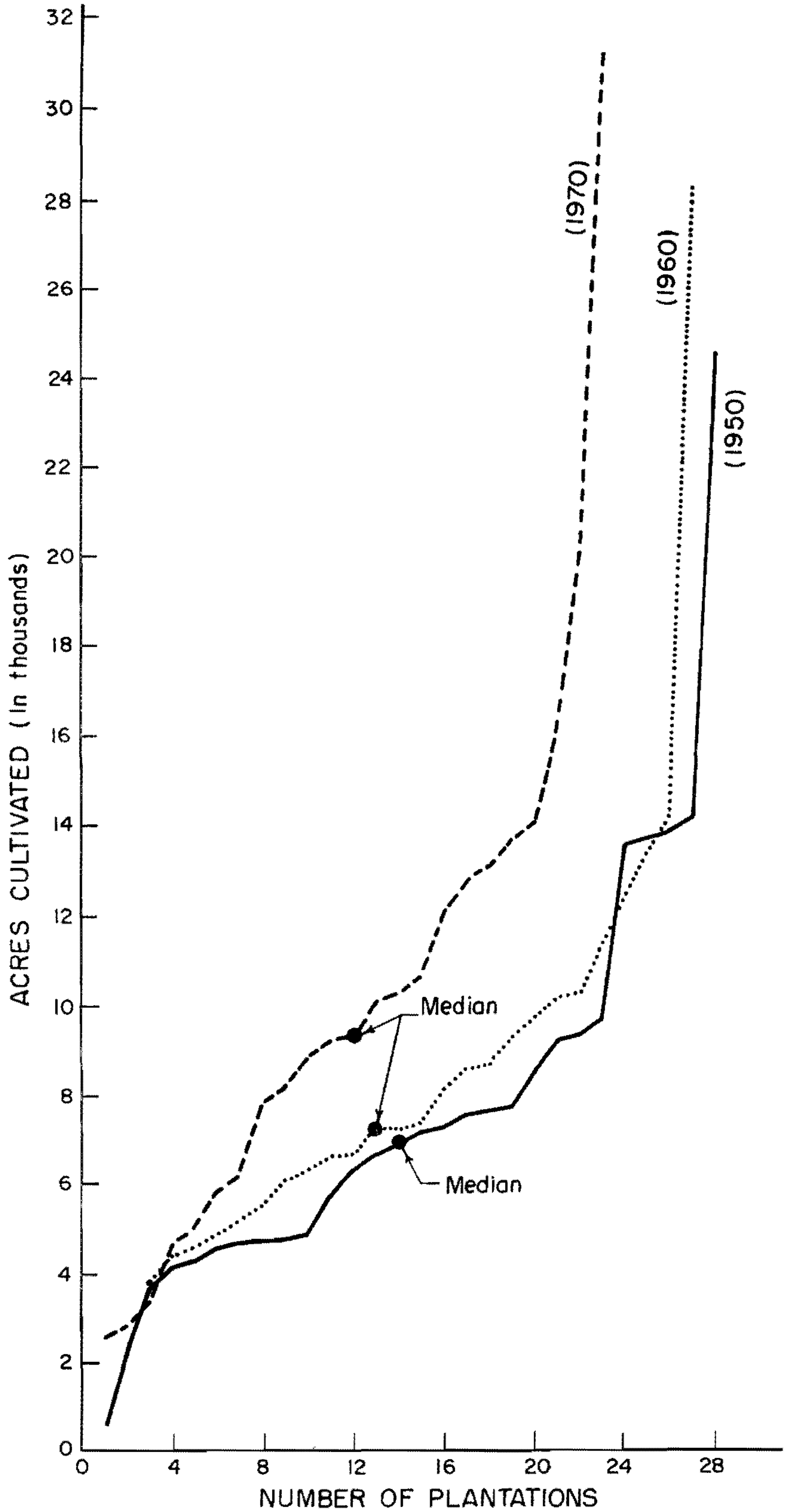

FIGURE 3. CUMULATIVE DISTRIBUTION OF PLANTATIONS BY ACRES CULTIVATED.

reason, can one expect to find varying factor service combinations being used to produce a given output by a firm. Instead we must rely on observations of different firms using varying service combinations and producing various levels of output. But under the assumption of homogeneity, we can at least standardize the output variable, by considering not absolute quantities of factors used, but quantities of each factor per unit of output. Real world data of this nature will not include examples of all technically possible input combinations, of course, but will demonstrate a range of such combinations.

Consider, as Farrell does, some product q produced by n firms, each of which uses two inputs in amounts x_{1_j} and x_{2_j}, where j indexes the firms. Each firm's production function can be written generally as $q_j = f_j(x_{1_j}, x_{2_j})$, $j = 1, \ldots, n$. We wish to consider the production function in the form of an isoquant, in particular a unit output isoquant. Thus, the arguments x_{ij} are replaced by inverses of average products:

$$w_{1_j} = x_{1_j}/q_j \quad \text{and} \quad w_{2_j} = x_{2_j}/q_j$$

In Figure 4, values of w_{ij} are plotted. Each point represents the input combination used by firm j to produce on unit of output.

An estimate of the minimum input quantities necessary to produce one unit of output can be made, as Farrell noticed, by (a) connecting the points nearest the axes; (b) adding a vertical line from the point using the least amount of x_2 per unit of output and a horizontal line from the smallest x_1. The resulting envelope of observed points, labeled qq' in Figure 4, is in the nature of Carlson's production function, although it shows the minimum amounts of factors necessary to produce a given amount (a unit) of the product, rather than the maximum amounts of output producible from given input combinations. The envelope qq' was termed by Farrell the *efficient unit isoquant*.

Firms represented by points inside the envelope can be said to be using whatever factor combinations they do use less efficiently than they might, given current technology. With any given factor proportions--say that represented by the ray Oa in Figure 4--one unit of output can be produced from input levels at point *b*; thus the amounts of factors used by the firm from which we observed *a* are larger than necessary, given the best

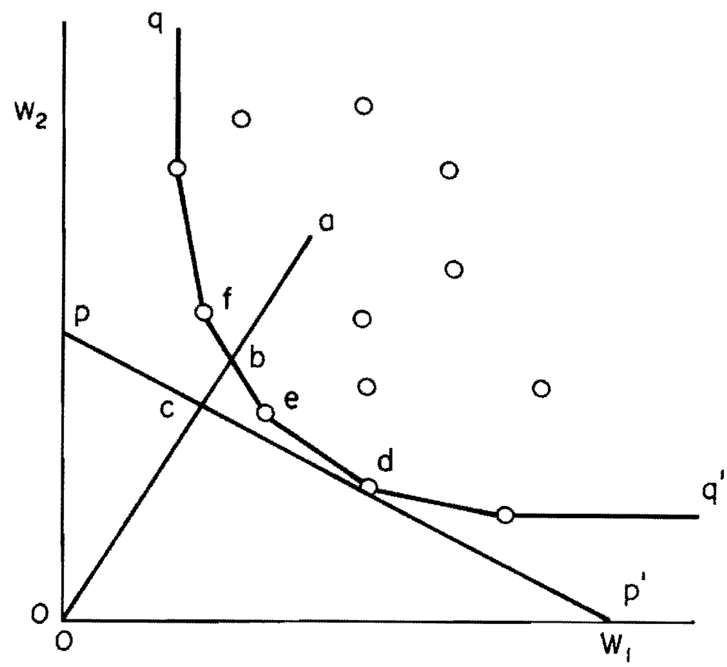

FIGURE 4. FARRELL'S LOWER ENVELOPE ISOQUANT.

current technology. A measure of the degree of efficiency is the ratio Ob/Oa--the ratio between the input combination necessary to efficiently produce one unit with the given factor proportions, and that combination actually observed. This ratio is termed by Farrell the *technical efficiency* rating of point *a*.

Even a firm using a technically efficient input combination may not be producing optimally, of course, depending on prevailing factor prices. Accordingly, a second measure of efficiency can be taken as the ratio between optimal and technically efficient production costs, given factor proportions. This measure is termed *price efficiency* (PE). In Figure 4, pp' is the lowest isocost line, with slope determining as usual factor prices, which just touch production function qq'. The optimal factor combination, given by point *d*, has the same total costs as, for example, point *c*, which represents the same factor proportions as the firm at point *a*. Thus the price efficiency of the input combination represented by the ray Oa is given by PE = Oc/Ob--the ratio between total costs of producing one unit using *actual* factor proportions in a technically efficient manner, and total costs of producing one unit using *optimal* factor proportions in a technically efficient manner.

Finally, the product of technical efficiency and price efficiency indexes yields the ultimate measure, called *overall* or *economic efficiency* (EE). This is intended to relate the cost per unit of output of the optimal input combination, to that of the actual combination. Algebraically,

$$\begin{aligned} EE &= (TE)\ (PE) \\ &= \frac{Ob}{Oa} \cdot \frac{Oc}{Ob} \\ &= Oc/Oa \end{aligned}$$

The essential point of the method is not so much the construction of an isoquant, but rather the comparison of efficiencies of existing firms with real or hypothetical technically efficient firms. If a technically efficient firm exists with the same factor proportions as those of the firm for which an efficiency index is desired, the comparison can be made directly. If not, then a hypothetical firm is constructed as a weighted combination of firms using factor proportions in the neighborhood of the subject firm, with weights determined so as to yield the desired factor proportions. The efficiency comparison is then made between the existing (and generally not technically efficient) firm on the one hand, and the hypothesized, but technically efficient, firm on the other.

Point *b* in Figure 4, for example, is a weighted average of efficiency ratings for firms f and e. Since *b* has the same factor proportions as the firm at point *a*, it (*b*) serves as the efficiency standard for firm *a*.

As Dorfman, Samuelson, and Solow (1958) point out, the production problem can be formulated as a mathematical program equally as well as in the standard Carlson definition. While both approaches answer the full range of problems, there is a difference of emphasis. For a firm which has available more activities (i.e., decision variables in the production plan) than variable inputs, as Dorfman, Samuelson, and Solow cite:

> If we solve the programming problem for that firm in given market circumstances, we shall know the quantities of the inputs that that firm will consume under those circumstances . . . But is the converse true? On the contrary. Carlson's production function (i.e., the conventional one) cannot even be written down until a programming problem has been solved . . . In order to derive the production function we must consider various definit combinations of inputs, and for each of them (i.e., holding the variable inputs temporarily constant) we must determine the program that maximizes output.

Farrell's approach is to observe various combinations of inputs needed to produce a unit of output and, roughly speaking, to find the minimum of such combinations. The similarity to the problem described in the last sentence quoted from Dorfman, Samuelson, and Solow is striking. Strictly speaking the programming formulation of a production problem would search over all technically feasible combinations of inputs to find a production function, while Farrell's method searches only existing and observed combinations. Thus, Farrell's production function is subject to several sources of error: failure to observe any number of relevant production processes, improper measurement of inputs, exclusion of certain differences in quality of factors used, as well as failure to perceive all possible activities. These can be problems of any attempt to empiricize production theory of course, but in some ways they may be more important to Farrell's procedure than to, say, regression procedures. For example, the influence of quirks and unique oddities on particular observations is not "averaged out" as would be the case in a least squares regression estimate.

Using only the extreme observations, the efficient isoquant also seems to "waste" some information, an extravagence for which most economists, given the usual dearth of hard empirical data, would consider a tragedy of near-Shakespearean profundity. But in light of the conventional definition, the extreme points are the only ones relevant.

In principle, then, Farrell's efficient unit isoquant estimate is precisely the function spoken of by Carlson and reworded in programming language by Dorfman, Samuelson, and Solow.

Linear Programming Formulation

Once a problem involves more than two inputs, a graphical presentation of Farrell's method is of course no longer possible. Farrell (1957), however, noted the similarity between the description of the unit isoquants and a linear programming minimization procedure, and designed a computational routine for the n-dimensional case using linear programming. Boles (1971) later wrote efficient computer routines for more up-to-date IBM equipment, solving the linear programming problems and calculating the resulting technical efficiency indices.

The simplest situation restricts the problem to one of no economies of scale. Following Boles (1971), we define Q_j as the rate of output of the j^{th} plantation, and $F_j \equiv [F_{1j}, F_{2j}, \ldots, F_{mj}]$, F_{ij} being the rate of input of each of the m factors by plantation j. Then, for plantation t, the problem is to select a vector X to maximize θ in the expression

$$\sum_{j=1}^{N} X_j Q_j = (1 + \theta) Q_t \tag{1}$$

subject to

$$X_j \geq 0 \qquad j = 1, \ldots, N$$

$$\sum_{J=1}^{N} X_j F_j \leq F_t$$

The technical efficiency index for the t^{th} plantation will be

$$TE_t = \frac{1}{(1 + \theta^*)} \tag{2}$$

where θ^* is the maximum value of θ .

An equivalent formulation of the problem (given the assumption of homogeneity) uses the average per-unit-output values instead of absolute levels of each input. Defining $f_{ij} \equiv F_{ij}/Q_j$, the vector of observations on plantation j for producing one unit of output will be

$$P_j = \begin{bmatrix} 1 \\ f_{1j} \\ f_{2j} \\ \cdot \\ \cdot \\ \cdot \\ f_{mj} \end{bmatrix} \tag{3}$$

and the problem becomes: find X_j to maximize θ in the expression

$$\sum_{j=1}^{N} X_j = (1 + \theta)$$

subject to

$$X_j \geq 0, \qquad j = 1, \ldots, N$$

$$\sum_{J=1}^{N} X_j f_j \leq f_t \tag{4}$$

The technical efficiency index for plantation t remains the same as in equation 2.

As can be seen from problem 1, if there is some linear combination of factor sets used by all plantations which is not greater than F_t, and which nevertheless produces more than Q_t units of output, then $\theta_t > 0$. The degree to which plantation t's output level, Q_t, can be exceeded by this linear combination of other firms' resource sets will be measured by the factor θ^*; and so the index $TE_t = 1/(1 + \theta^*)$ represents the degree of "inefficiency" in the use of resources by firm t. In this situation, $TE_t < 1$, of course.

If, on the other hand, plantation t is 100 percent technically efficient, the vector of resources it uses produces at least as much as any linear combination of the factor sets used by other plantations.

To put it another way, no linear combination $\sum X_j F_j$ of other plantations' resource input sets can be found which both uses no greater quantities of the inputs and produces any more than Q_t units of output. Note that since the vector P_t is an activity in the program as well as serving as constraint set, problem 1 or 4 must have a solution; at least Q_t, in problem 1, or one unit, as stated in 4 will be produceable from the constraint set in P_t.

If economies of scale may be present in the data, the problem must be modified somewhat to specify that the technical efficiency calculation is made for each level of scale. Formally, we replace the activity vector P_j in equation 3 with

$$P'_j = \begin{bmatrix} Q_j \\ F_j \\ S_j \end{bmatrix}, \quad j = 1, \ldots, N \tag{5}$$

where P, Q, F are defined as in equation 3, and S_j is the level of scale.[1] Again setting $f_{ij} = F_{ij}/Q_j$, the unit-output activity vector will be

$$P_j = \begin{bmatrix} 1 \\ f_j \\ S_j \end{bmatrix}, \quad j = 1, \ldots, N \tag{6}$$

Then the linear programming problem which will determine the efficiency index for plantation t may be defined as

$$\max \sum_{j=1}^{N} X_j = X_o$$

subject to

$$X_j \geq 0, \quad j = 1, \ldots, N \tag{7}$$

1. Several measures of scale could be imagined. The rate of output itself has been suggested, as well as the rate of any one input. Since our interest at present is in the response of plantations to changes in their land input, we might define scale as total acres under cultivation. However, this also biases the partial unit isoquants, as discussed below, and we adopt instead the rate of output as the scale factor.

$$\sum_{j=1}^{N} f_{ij}X_j \leq f_{it}, \quad i = 1, 2, \ldots, M \tag{7}$$

$$\sum_{j=1}^{N} (S_j - S_t)X_j = (S_t - S_t) = 0$$

The optimal solution to this program yields a value $X_o{}^*$, and the technical efficiency index is now

$$TE_t = 1.0/X_o{}^* \tag{8}$$

Interpretation of these measures is analogous to the constant returns case.

Classifying all plantations as efficient or inefficient thus requires solving one linear programming problem for each plantation. In the process, parameters defining a frontier or envelope of the observed points is specified--a unit isoquant hypersurface.

Technical Efficiency Indices

Table 1 shows the calculated technical efficiency indices for each plantation on which data is available, for each of the years between 1960 and 1967.

The indices TE_t represent what Sietz (1970) called "technical scale efficiency" measures--that is, the technical efficiency of a firm given the level of scale at which it operates. The price and overall economic efficiency indices mentioned earlier would require factor price data on a plantation-by-plantation basis. Such figures are not available, as it turns out, so we will have to be content at drawing whatever implications we can from the technical efficiency measures.

It should be noted here that whatever conclusions we draw must be interpreted very cautiously since the data are neither always exactly the right kind nor of the very best quality. Some plantations release no data on most of their inputs; there are gaps in the time series for some plantations; and for some variables, the figures we have are somewhat misspecified as representations of the intended theoretical variable. Nevertheless, what data we have seems to be all we or any researcher will get.

The usefulness of these ratings could be measured in several ways, one

TABLE 1. TECHNICAL EFFICIENCY INDICES FOR HAWAIIAN SUGAR PLANTATIONS, 1960-1967.

PLANTATION NUMBER	YEAR							
	1960	1961	1962	1963	1964	1965	1966	1967
1	1.0	1.0	1.0	1.0	.984	.998	1.0	1.0
2	--	--	--	--	--	1.0	1.0	1.0
3	1.0	1.0	1.0	1.0	1.0	+	+	+
4	.960	1.0	.979	1.0	1.0	+	+	+
5	.931	.948	.941	1.0	1.0	.943	.919	.869
6	.817	.817	.891	*	*	*	*	*
7	.748	1.0	1.0	1.0	.875	.786	.742	.857
9	.859	.840	.798	1.0	.844	1.0	.849	.951
10	1.0	1.0	1.0	1.0	1.00	1.0	1.0	1.0
11	.718	.706	.645	.637	.704	.904	.859	.894
13	1.0	1.0	1.0	1.0	1.0	1.0	.875	.981
14	1.0	.985	.940	1.0	1.0	1.0	1.0	1.0
16	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
19	.990	--	1.0	.957	.898	.977	.955	1.0
20	--	--	--	1.0	1.0	1.0	1.0	1.0
21	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
24	.843	.892	.896	.891	.882	.930	.979	.945
25	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
28	.975	1.0	1.0	.940	1.0	1.0	--	--
29	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

DASHES INDICATE YEARS FOR WHICH DATA IS MISSING OR BEFORE AMALGAMATION.
* MERGED WITH NO. 5, JANUARY 1963.
+ NO. 3 AND 4 MERGED JULY 1965; NEW PLANTATION CREATED IS NO. 2.

of which is the ability to foreshadow plantation mergers or outright closings. In this respect, at least two of the plantations provide what might seem embarrassing examples of failure: for the eight years represented here, plantations 21 (Kahuku, on Oahu) and 29 (Kilauea, on Kauai) show technical efficiency indices at 1.0, yet both had ceased operating by 1971.

In the case of Kahuku, one plausible hypothesis is that the land being used to grow sugar had other uses which were more profitable than sugar production. Thus the opportunity costs, if not outright monetary costs, of continued sugar production are seen as too high. Technical efficiency indidices might thus indicate a highly efficient operation at the same time as relative input prices, or rather opportunity costs, would reflect in the price and economic efficiency indices not calculated here, a much less sanguine view of the plantations' viability. Opportunity costs, if not actual monetary outlays, for sugar production were not justified by the returns, especially in view of resort development possibilities for the area.

Against this hypothesis must be weighed the fact that relatively little of Kahuku's land was transferred to resort use, and not all the remaining area is yet planned for other purposes. Further, this explanation offers no enlightment about Kilauea Plantation.

An alternative interpretation of what constitutes technological efficiency puts these unexpected results in a somewhat different light. One could argue that Kahuku and Kilauea got high TE ratings *because*, not in spite of their impending departures from the industry.[1] Thus the high TE ratings indicate an exceptionally careful job of managing whatever resources these plantations had. Very loosely speaking, the manager, facing an uphill battle with the innate soil productivity, moisture conditions and so on, ran a very tight ship trying to remain in production for as long as possible, and perhaps began to deplete the immobile capital once the eventual demise was evident.

Interpretation of the Results

Boles' computer programs provide a numerical description of the isoquant hypersurface by enumerating the coordinates of each facet. While this

1. I am indebted for this suggestion to Prof. Peter V. Garrod of the Agricultural and Resource Economics Department, University of Hawaii.

description is unique and exact, it is also difficult to visualize and interpret meaningfully. Consequently, the programs also compute coordinates of two-dimensional "cuts" of the hypersurfaces. For these partial isoquants, the level of output is specified (at 1.0 ton, as usual) and the levels of all inputs except the two to be represented explicitly are fixed at some arbitrarily determined levels within the range of data. The resulting partial unit isoquant shows the transformation between any two inputs, with levels of the other inputs and of the scale factor entering parametrically. Figures 5-12 were produced in this manner.

In these figures, we have chosen to display the trade-off between land and labor, although the two explicitly varying factors might equally well have included capital. We also have some results in which the scale factor was fixed, while a set of land-labor isoquants is constructed while capital input varies parametrically. The poor quality of the capital input data, however, suggests ignoring these results.

The plotting routines use as input only the subset of activities determined efficient in a previous solution. As indicated in the 1960 column of Table 1, for example, eight of the eighteen data points for that year yielded efficiency indices of 1.0. These eight points provide data for calculating the coordinates of Figure 5.

First, the level of capital input is fixed (in this instance, at $65 per ton output) and the scale factor, output of 96° raw sugar, is set (at 20,000 tons, for the first isoquant). The efficiency routines are then used to calculate the minimum level of land, X(3), per unit output, and the amount of labor, X(4), necessary to complement the land input in producing one ton of raw sugar. Amounts of input X(3) are then systematically increased, in increments determined by the simplex procedure, while the amount of X(4) is correspondingly diminished. Eventually, continued decrements in X(3) render additional solutions infeasible. However many points calculated up to that time define a partial unit isoquant. A second level of the scale factor is then specified (25,000 tons, as shown in Figure 5) and the entire process is repeated, resulting in a second isoquant. The magnitude of changes in the scale factor is wholly arbitrary, and in fact differs between Figures 5-12 depending on the data input for each plot (i.e., on the degree of variation among the efficient subset of observations).

With parameters fixed at $65 per ton for capital, and 20,000 tons output for the scale factor, five combinations of land and labor inputs are specified by the programs. These five points and the line connecting them are plotted as the highest isoquant shown in Figure 5. With capital remaining at $65 per ton, the scale variable is parametrically varied--to 25,000 and 45,000 tons, in the present case--and the remaining two curves are determined. These three curves are, of course, only a sample of the many which could be plotted for other values within the range of the scale variable. In general, we attempted to specify four or five different isoquants for each year's data. Capital input is set at some level approximating its mean value among the efficient points, and the scale variable takes on levels approximating the minimum and maximum scale, as well as two or three intermediate values, among the efficient points for the year in question. Not all such values permitted feasible solutions, however, so that for some years as few as two isoquants are plotted. Note also that the capital variable is not at the same level for every year. Some variation was introduced to obtain feasible solutions for more levels of scale than turned out to be possible with capital set at $65 per ton for every problem.

Figure 5 shows relatively well-behaved, almost textbook-pure curves, shifting neatly downward as scale increases,[1] and thereby demonstrating the appearance of economies of scale. The other years' figures, however, show less clear-cut implications. The 1961 results, for example, (Fig. 6) seem to indicate a kind of nonneutral scale economies, with the production process using smaller and smaller units of labor but larger amounts of land as scale rises. The results for 1963, shown in Figure 8, also show another odd result: not only is the factor bias still present--especially noticeable in the radical change in character of the curves between the 30,000 and 35,000 ton levels--but the highest scale value, 54,000 tons, seems to be associated with a considerable decrease in overall technical efficiency as compared to the 35,000 and 45,000-ton curves. A similar result shows up to a lesser degree in several other plots as well.

---

1. It may seem odd, at first glance, to think of increasing efficiency evidenced by isoquants shifting downward. But these curves represent *unit* isoquants, so that lower curves involve smaller amounts of one or both of the factors required to produce one unit of output.

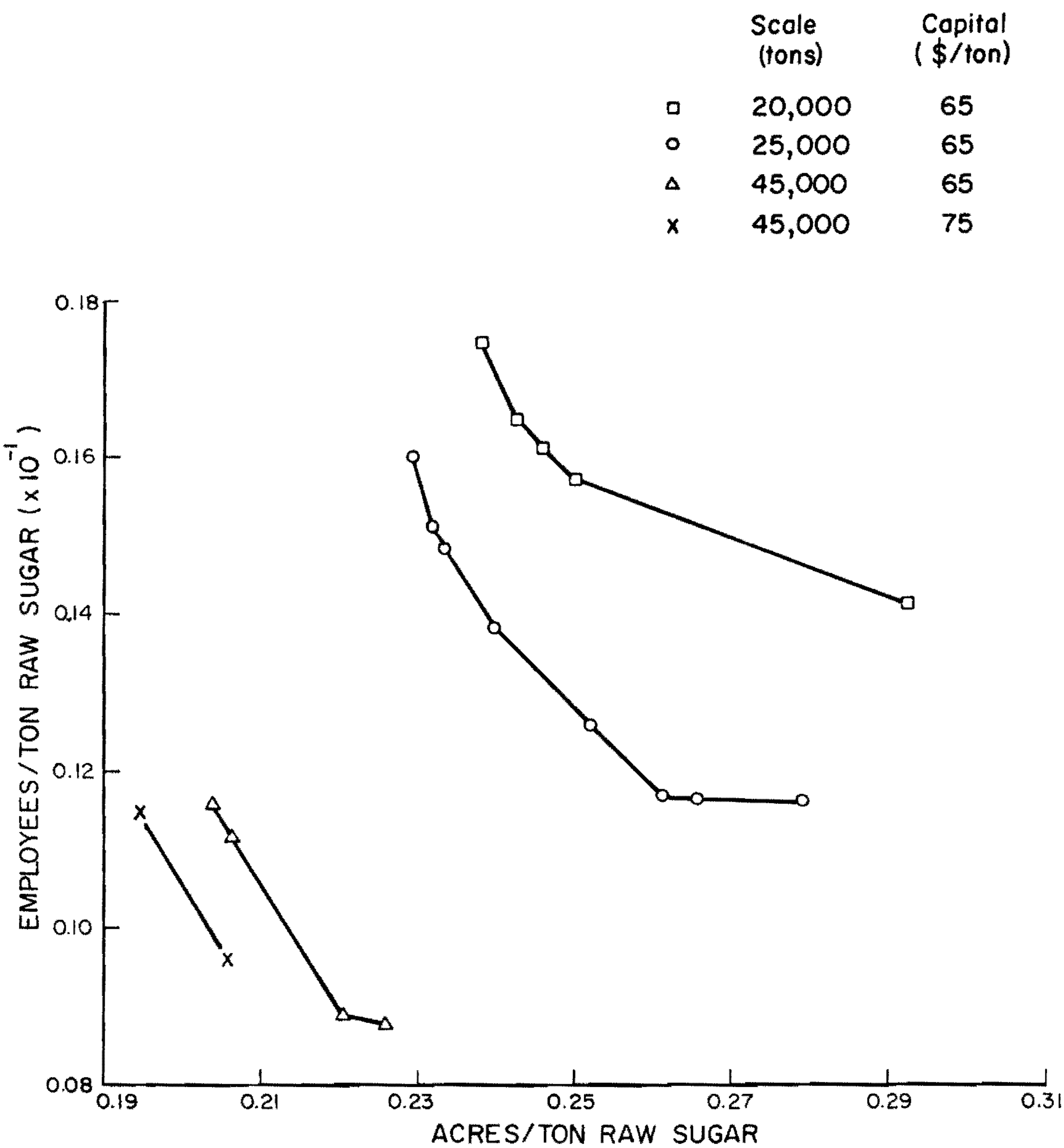

FIGURE 5. PARTIAL UNIT ISOQUANTS. SCALE: OUTPUT. 1960.

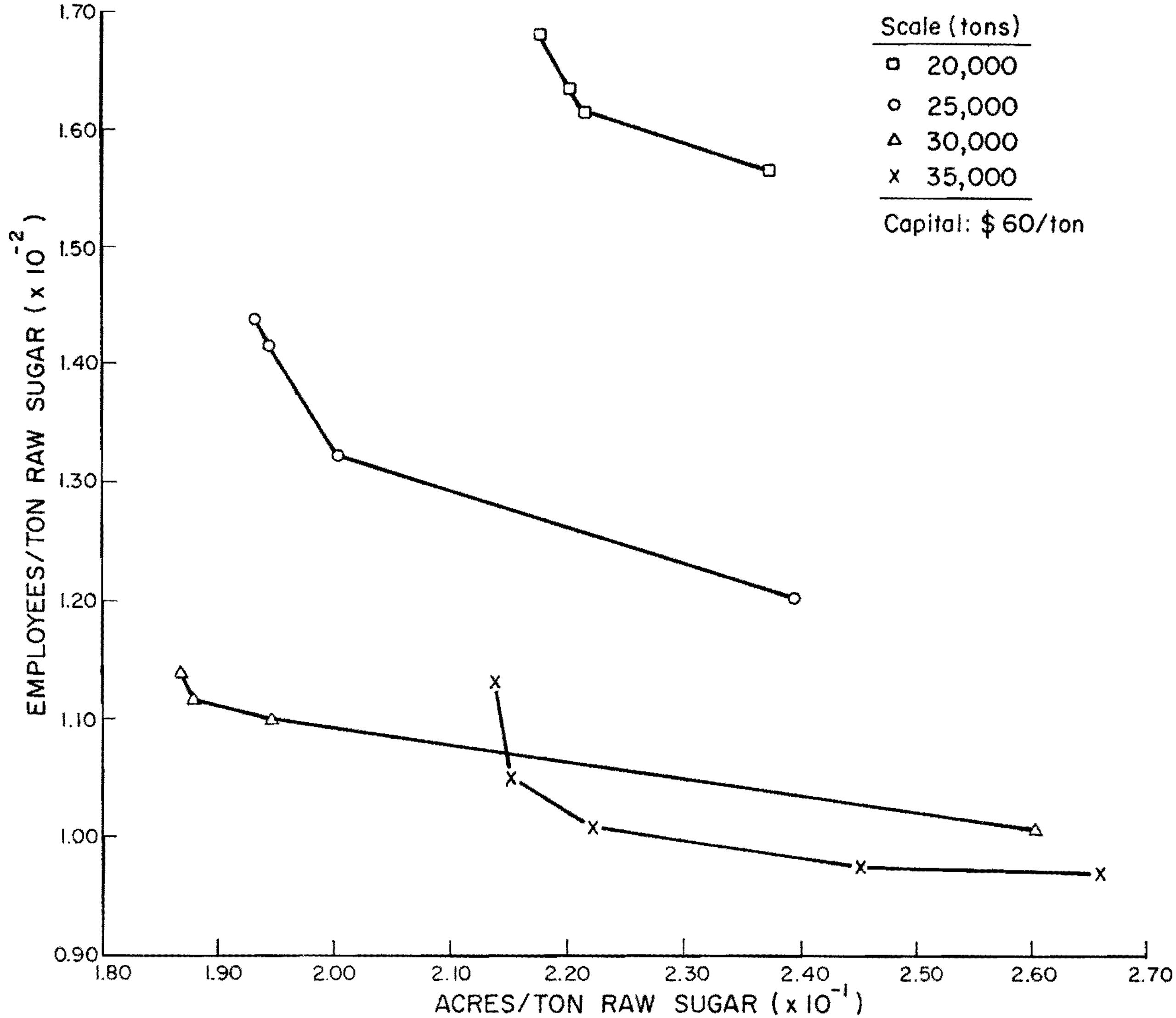

FIGURE 6. PARTIAL UNIT ISOQUANTS. SCALE: OUTPUT. 1961.

FIGURE 7. PARTIAL UNIT ISOQUANTS. SCALE: OUTPUT. 1962.

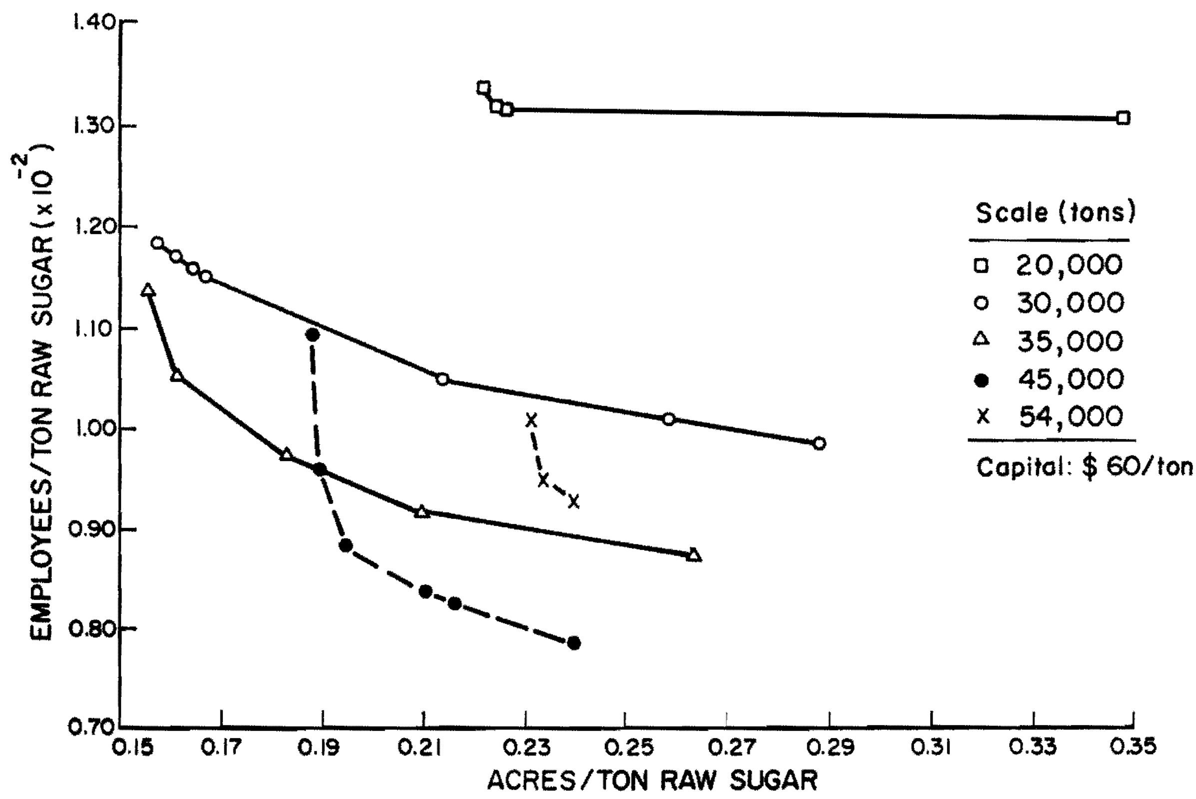

FIGURE 8. PARTIAL UNIT ISOQUANTS. SCALE: OUTPUT. 1963.

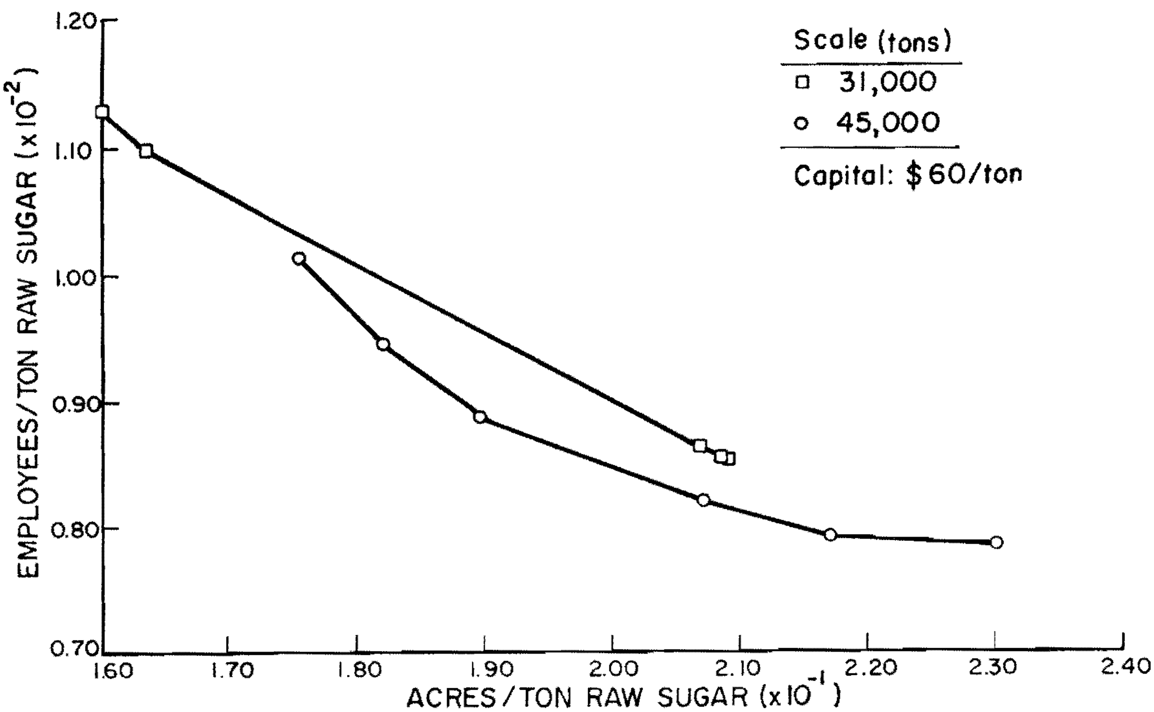

FIGURE 9. PARTIAL UNIT ISOQUANTS. SCALE: OUTPUT. 1964.

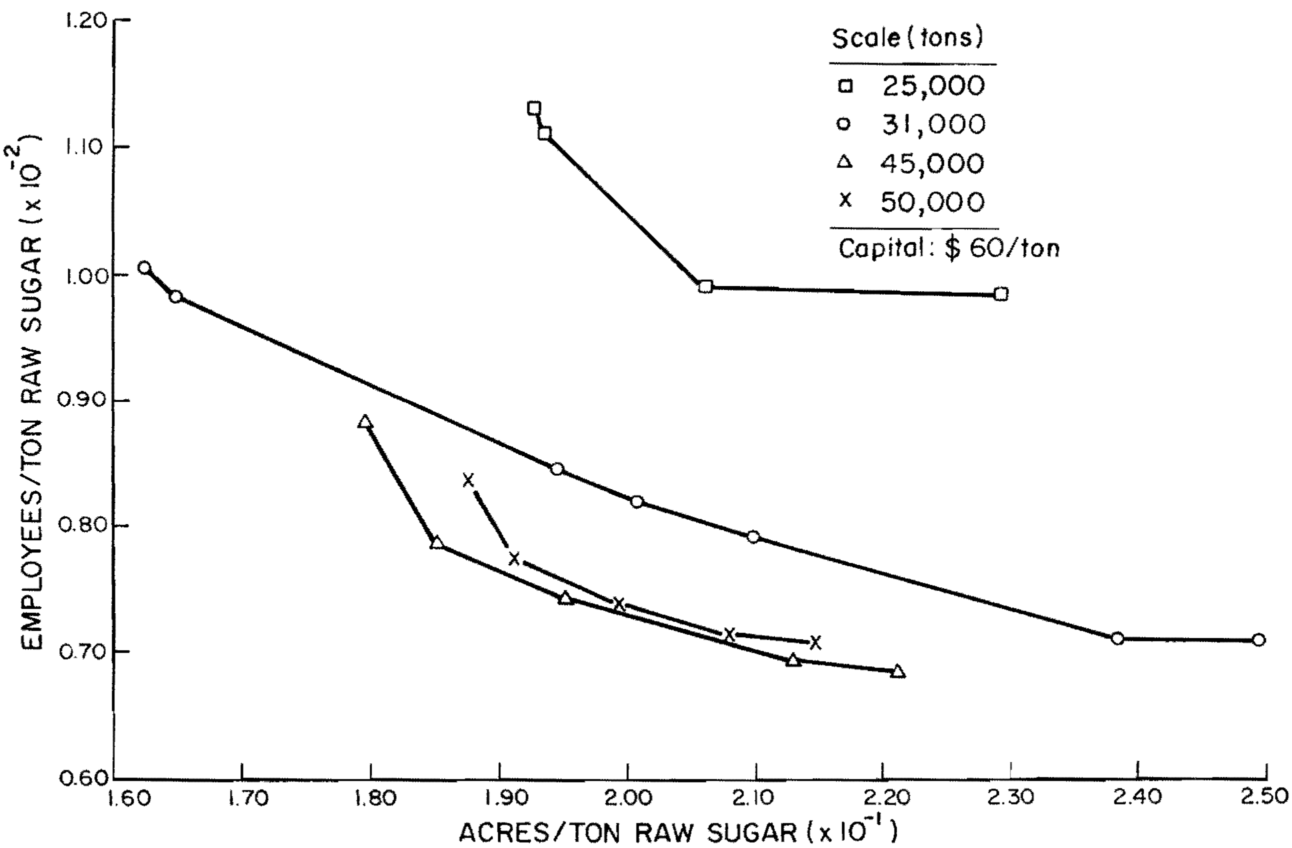

FIGURE 10. PARTIAL UNIT ISOQUANTS. SCALE: OUTPUT. 1965.

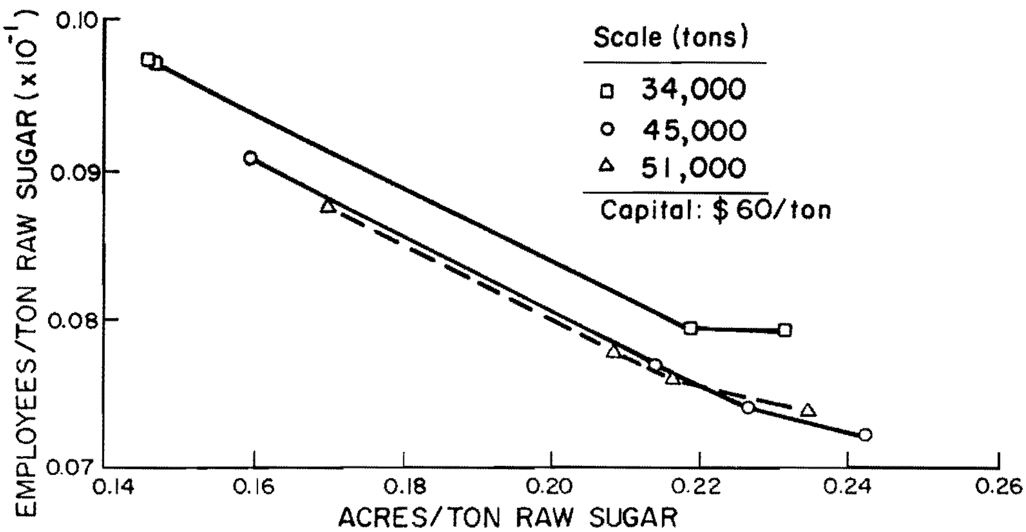

FIGURE 11. PARTIAL UNIT ISOQUANTS. SCALE: OUTPUT. 1966.

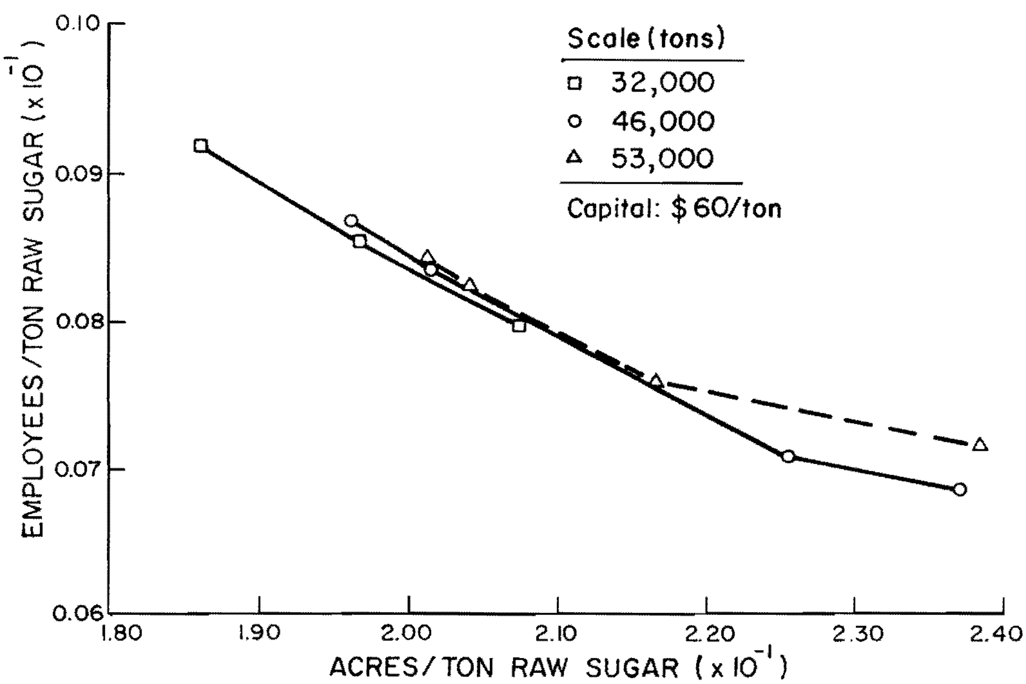

FIGURE 12. PARTIAL UNIT ISOQUANTS. SCALE: OUTPUT. 1967.

Figure 13 is similar to the previous eight plots, except that a different use is made of the data. Rather than using only one year's data, all 138 observations from the eight year period were pooled. The initial efficiency computations indicated that 24 of the points were 100 percent technically efficient; these points make up the input for the plot routines, resulting in Figure 13. As before, the transformation demonstrated is between land and labor, per unit output, for a given level of capital input, with the scale factor, output, varying parametrically.

One striking feature of all these plots is the tendency of the isoquants to move downward very considerably as the scale factor is increased from its initial (lowest) value. At higher levels, movements of the frontier generally seem to follow no particular pattern, sometimes increasing, in other instances decreasing. In many cases, this shifting occurs in such a way as to indicate factor bias between levels of scale.[1] Note especially the 35,000-ton unit isoquant in Figure 8, as compared to the 45,000-ton curve in the same figure.

As mentioned in the footnote (p. 13), plots similar to Figures 5 to 13 were also derived from a series of efficiency problems in which land (total cultivated acres) was used as the scale factor, instead of output. The evidences of land-biased economies of scale were even more pronounced than with output serving in the role of scale. Figure 14 is included to demonstrate this tendency. The underlying data is the set of 138 pooled observations for the period 1960-1967, the same as for Figure 13 except that land rather than output is now the scale factor. The two displays should not be expected to be identical. No given rate of output is uniquely associated with any given level of land, so that, for example, the highest curve in Figure 14 (for which 5000 acres is the scale index) is not simply the highest curve in Figure 13 (where 22,000 tons output represents scale) translated into acreage. Nevertheless, the nonneutral shifts in scale $\equiv$ land plot are far more obvious than in the scale $\equiv$ output figure.

---

1. At some ratio of labor/land, the increase in scale of operation causes a greater increase in the marginal productivity of land than in that of labor, so that the slope of the isoquant increases, in absolute value, at that labor/land ratio.

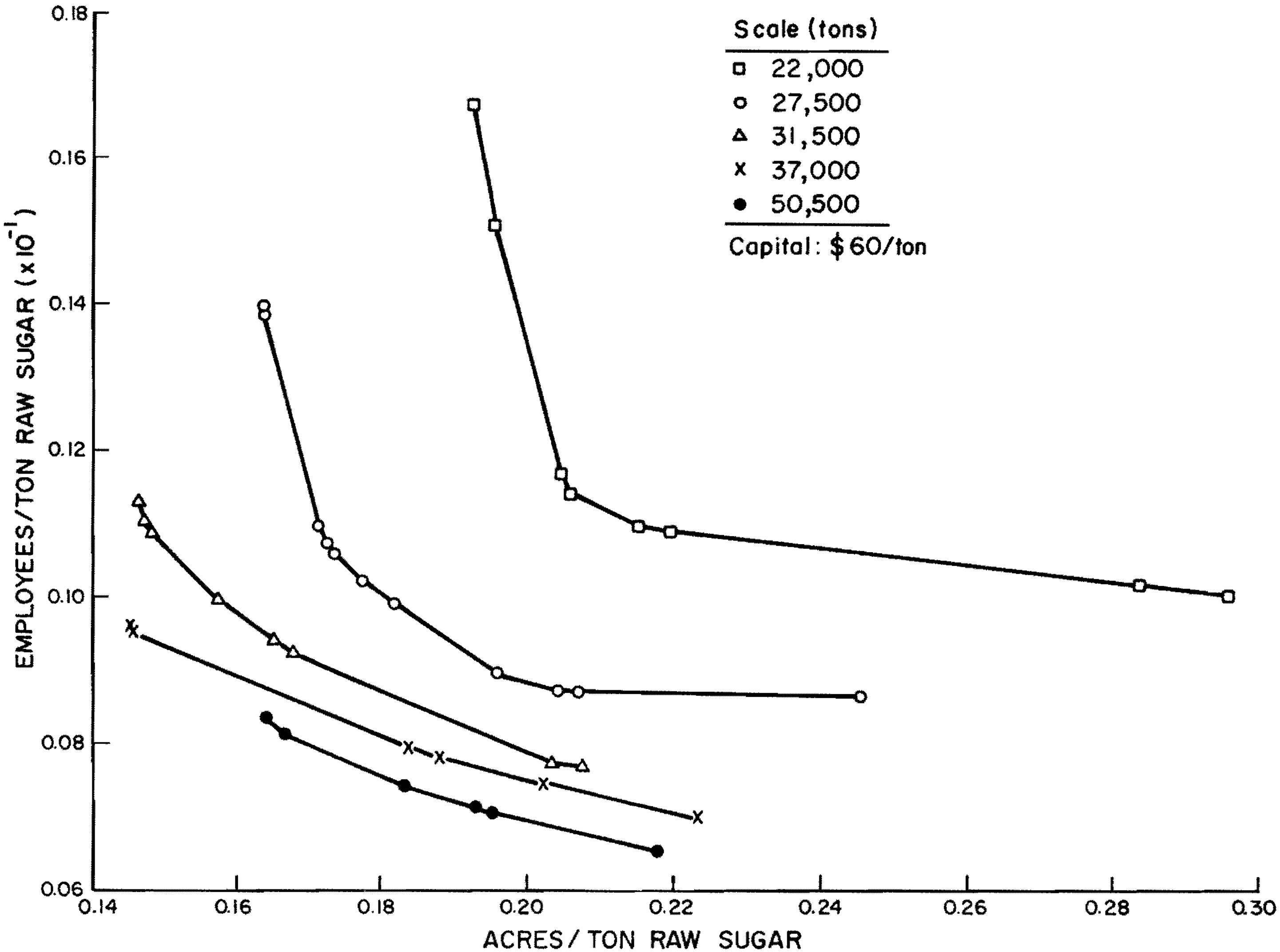

FIGURE 13. PARTIAL UNIT ISOQUANTS. 1960-1967. SCALE: OUTPUT.

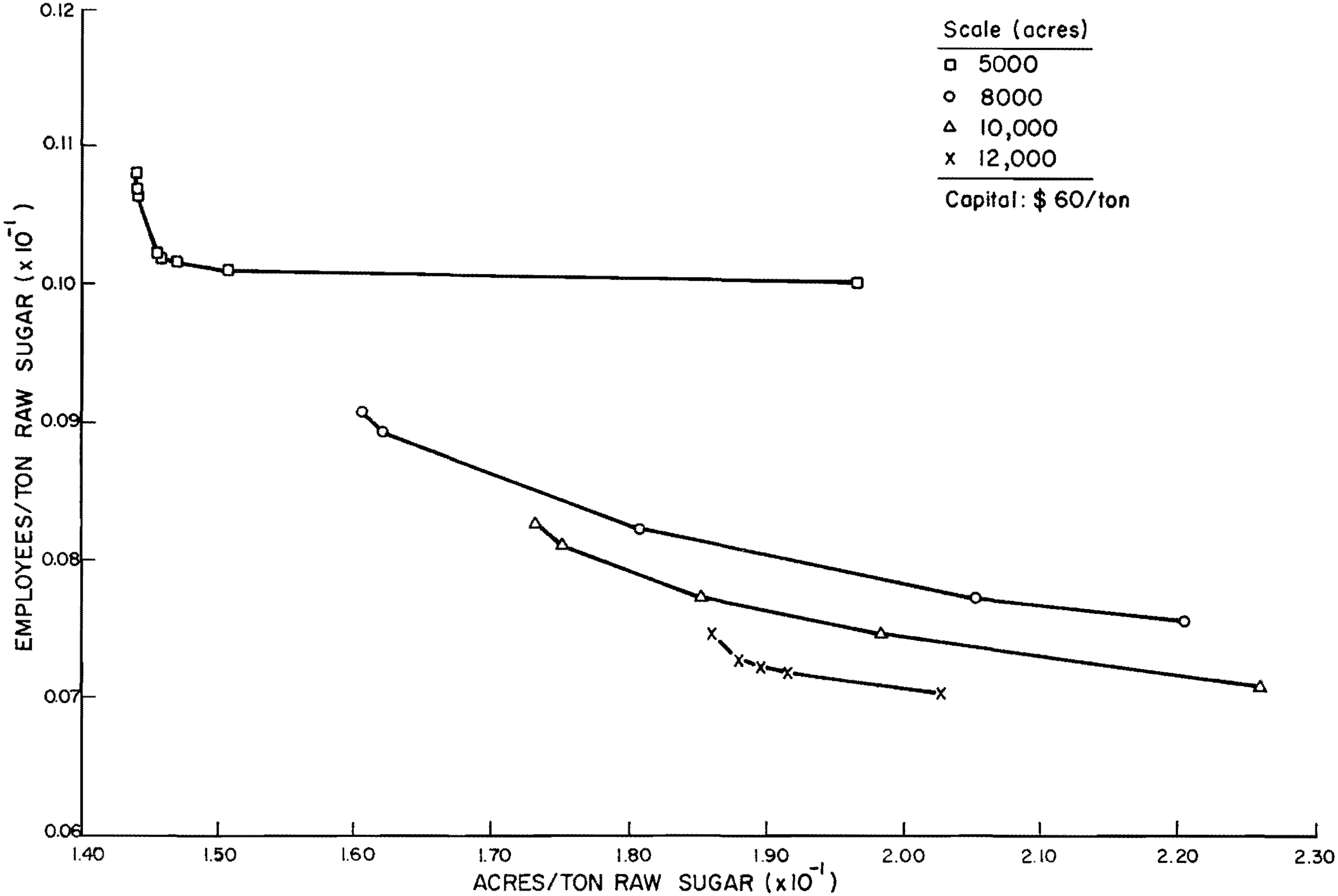

FIGURE 14. PARTIAL UNIT ISOQUANTS. SCALE: ACRES. 1960-1967.

The same tendency shows up in individual years as well as the pooled-data version, to a greater or lesser degree, as indicated by comparing Figure 15 in which land is the scale factor, with Figure 9.

Yet another way to interpret the data involves constructing a "time series" of isoquants by plotting curves for successive years. In Figures 16 and 17, such a series of partial unit isoquants is plotted. Each curve shows the substitution between labor input per ton of raw sugar produced, and total acres cultivated per ton output, for the specified year. All eight isoquants in each figure are based on a fixed level of capital input ($60 and $65 per ton, roughly bracketing median value). The scale factor (output) is set at 30,000 tons in Figure 16 and 35,000 tons in Figure 17. Fixing the scale factor and the capital input level at one value for all eight years results in only a few points (for some years), but has been done here to make the curves comparable in every other way.

As can be seen, the general tendency is for isoquants to shift toward the origin, indicating greater efficiency through the years. Equally obvious is that this general tendency is not uninterrupted throughout the period, and that some factor bias of technical change shows up.

REGRESSION ESTIMATES

The information contained in the foregoing plots is somewhat unsatisfying in the way that nonnumerical results always are. Given the nature of the problem, this is hardly avoidable. But the numerical results such as are described in the preceding section, are too grossly numerical to be comprehendible; and the graphical form of presentation, while helping to interpret the numerical results, leaves something to be desired in precision and comprehendibility. A brief excursion into a conventional statistical approach was undertaken in an attempt to partially overcome this fuzziness.

Since the data base is the same, we would anticipate roughly similar kinds of answers. With no firm-specific price information, a satisfactory, fully identified Cobb-Douglas or similar production function cannot be estimated in the usual fashion (Nerlove 1965, Chap. 6). To whatever extent we can put any faith in a straight forward regression on the theoretical CD function, however, ignoring the problem of identification, we may proceed

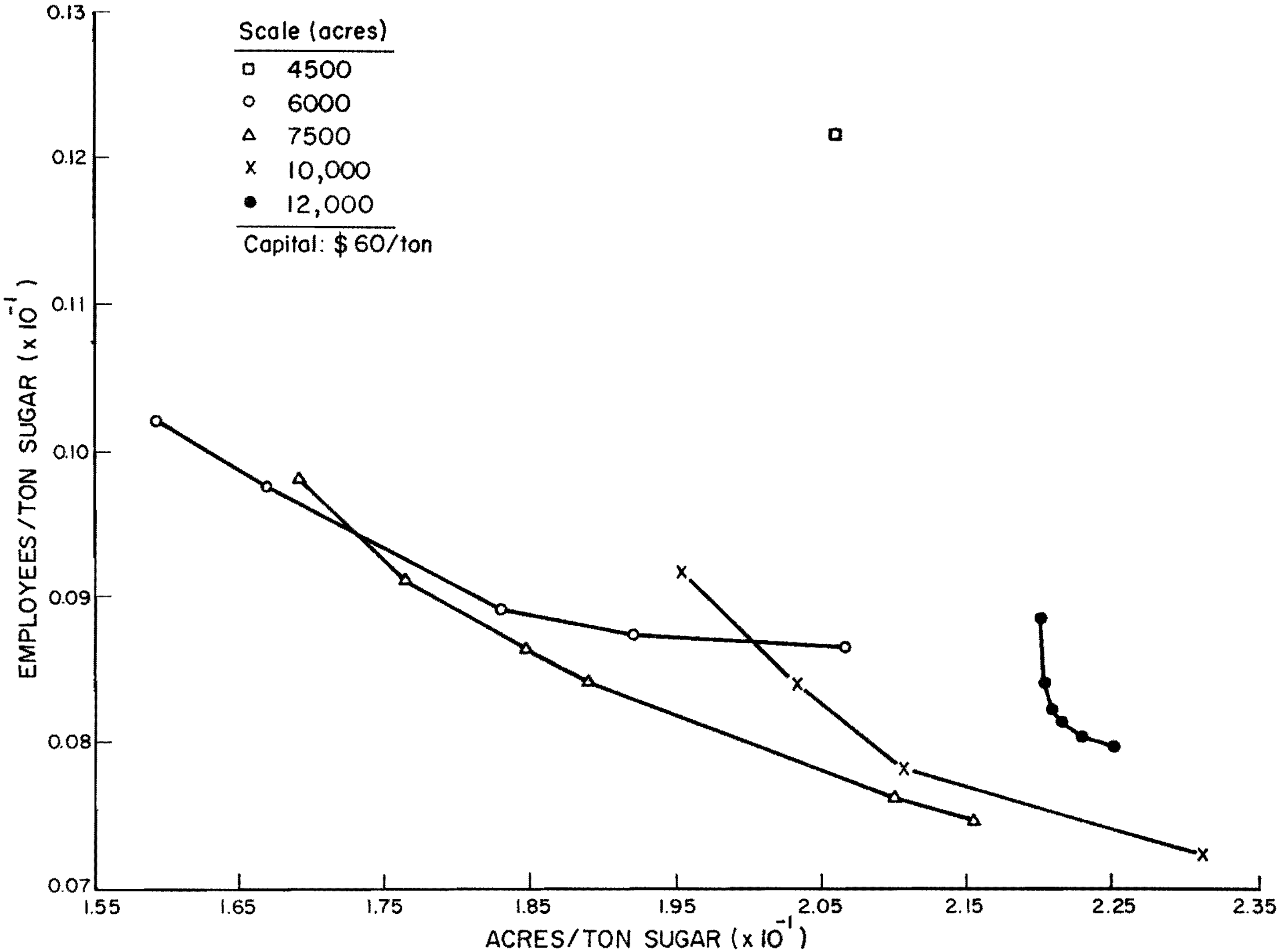

FIGURE 15. PARTIAL UNIT ISOQUANTS. SCALE: LAND ACRES. 1964.

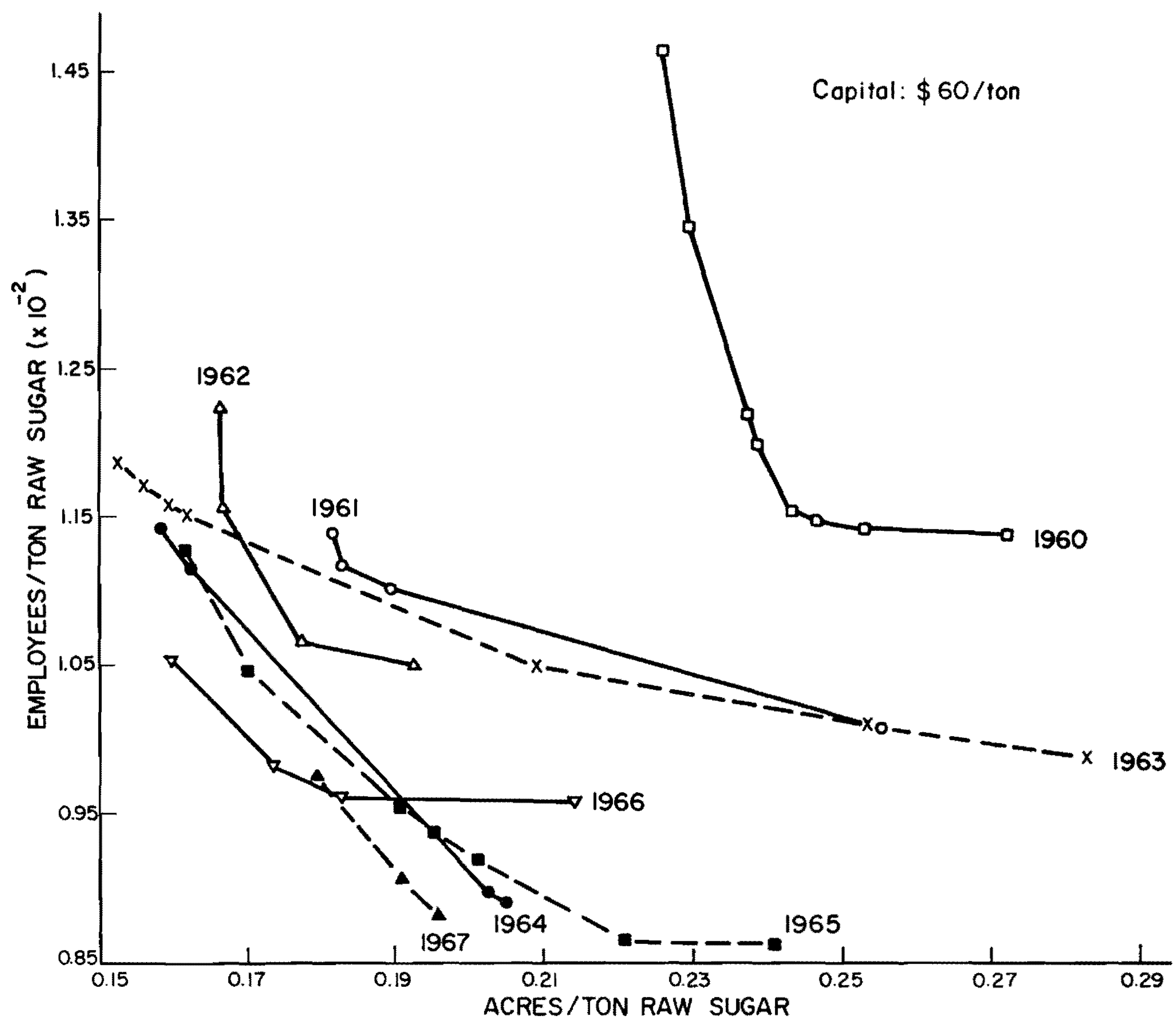

FIGURE 16. TIME SERIES OF ISOQUANTS. SCALE = 30,000 TONS.

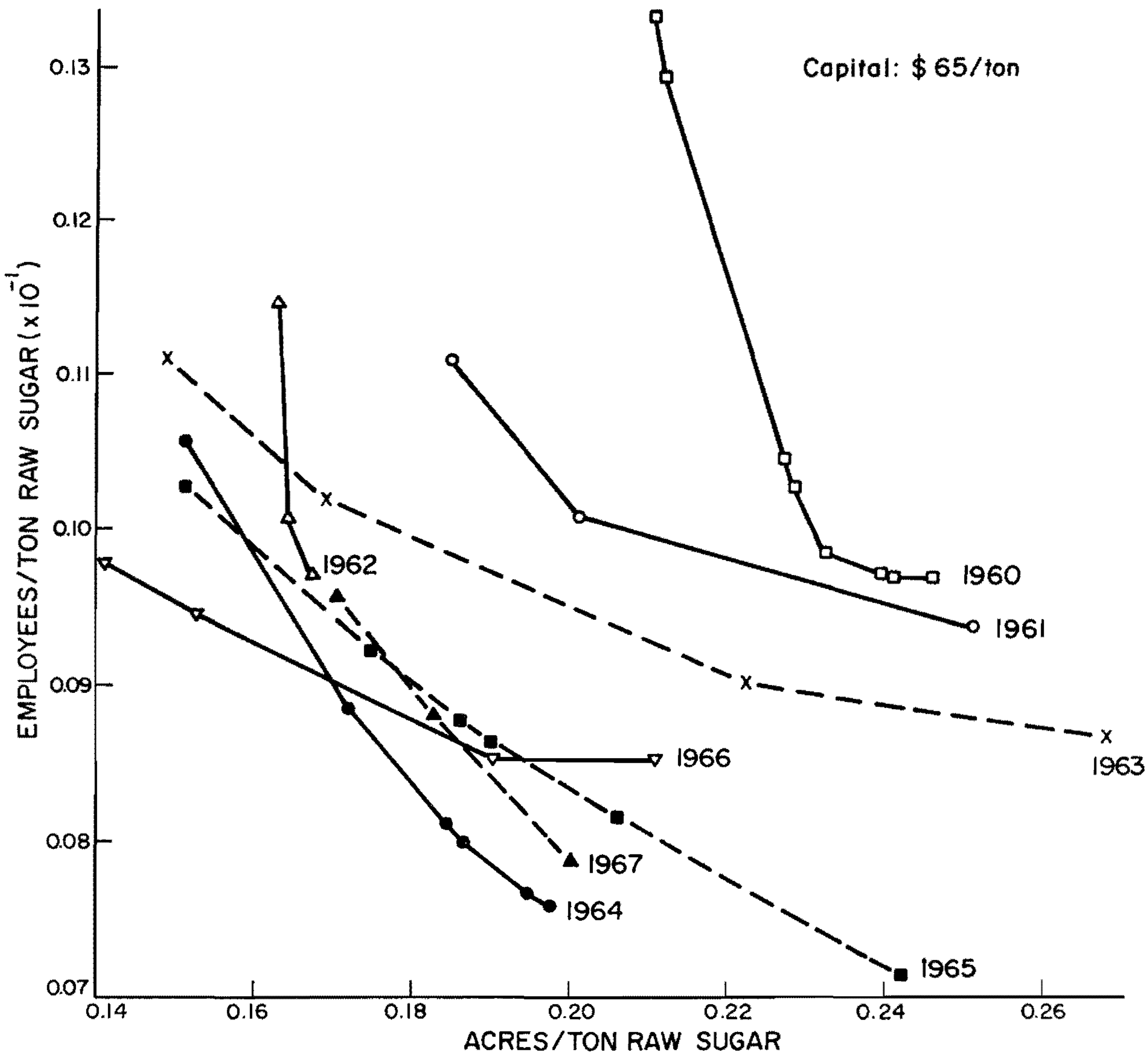

FIGURE 17. TIME SERIES OF ISOQUANTS. SCALE = 35,000 TONS.

as follows:

$$Q = f\ (L,\ N,\ K)$$
$$= \beta_0 L^{\beta_1} N^{\beta_2} K^{\beta_3} \qquad (9)$$

where Q represents output (tons 96° raw sugar); and L, N, and K represents the rates of input of labor, land, and capital, respectively. Adding a stochastic term and taking logs, the function to be estimated becomes

$$\log Q_{it} = \log \alpha_1 + \alpha_2 \log L_{it} + \alpha_3 \log N_{it} + \alpha_4 \log K_{it} + \log \varepsilon_{it}, \qquad (10)$$

where ε is the stochastic term, i indexes the plantation, and t the time period, and where the switch from β's to α's is in recognition of the fact that the regression will not give us parameters of the production function itself, but rather of an unidentified relationship between quantities of input and output.

The theoretical version of the model (9) indicates decreasing, constant or increasing returns to scale as the sum of the factor input coefficients $c'\beta \lesseqgtr 1$, where $c' = (0,1,1,1)$ and $\beta' = (\beta_1,\beta_2,\beta_3,\beta_4)$. Even though the stochastic version is admittedly devoid of econometric integrity as an estimate of the true production function, the corresponding stochastic version, as stated in equation 10, has been estimated, yielding values α_k.

Data for each of the years 1960-1967 were used to compute annual cross-section estimates of the function; then the figures for all eight years were pooled and a combination cross-section-time-series version was estimated. Coefficients of the resulting nine equations are shown in Table 2.

Adjusted R^2 values for each equation are generally of respectable magnitude, and in fact are rather high for the years 1965, 1966, and 1967 in particular. Also, the F-statistics allow us to reject the hypothesis that all coefficients of any one equation are zero. All the same, there are variations in the statistical quality of the estimates. While all but two of the nine labor input coefficients, and all but one of the land input values are statistically significant at the five percent level, only one of the capital input coefficients (that for the pooled-data equation) passes the same test.

Our present interest, however, is not so much whether the individual coefficients are significant, as whether their sum is significantly

TABLE 2. REGRESSION ANALYSIS OF PLANTATION DATA.

PERIOD	(INTERCEPT)	(CAPITAL)	(LAND)	(EMPLOYMENT)	n	$\overline{R}^2$	F-RATIO	$\sum_{k=2}^{4} \alpha_k$	95% CONFIDENCE LIMITS ON $\sum_{k=2}^{4} \alpha_k$
1960-67	1.110* (.452)	.160* (.056)	.508* (.068)	.411* (.094)	138	.763	148.096	1.080	.934 - 1.225
1960	.091 (1.402)	.047 (.160)	.763* (.199)	.452 (.303)	18	.743	17.407	1.261	.707 - 1.815
1961	2.355 (1.141)	.091 (.112)	.411* (.146)	.506* (.233)	17	.744	16.533	1.008	.543 - 1.461
1962	2.133 (1.425)	.122 (.158)	.443* (.199)	.430 (.270)	18	.652	11.617	.995	.356 - 1.634
1963	1.486 (2.287)	-.065 (.310)	.361 (.309)	1.120* (.513)	18	.581	8.855	1.415	.672 - 2.158
1964	1.481 (1.035)	.107 (.145)	.500* (.143)	.507* (.300)	18	.860	35.728	1.113	.789 - 1.437
1965	1.525 (.723)	.128 (.106)	.521* (.103)	.417* (.163)	17	.931	72.533	1.066	.831 - 1.301
1966	2.516* (.823)	-.015 (.111)	.341* (.129)	.871* (.183)	16	.908	50.300	1.198	.935 - 1.477
1967	1.733* (.654)	.144 (.076)	.485* (.099)	.393* (.132)	16	.936	73.787	1.022	.825 - 1.219

* SIGNIFICANT AT 5 PERCENT LEVEL, AS JUDGED BY t-STATISTIC.
STANDARD ERRORS ARE SHOWN IN PARENTHESES WITH EACH COEFFICIENT.

different from unity. Using a t-statistic for linear combinations of multiple regression equations (Johnston 1958, p. 131), confidence limits on the sum $c'\alpha$ of the factor-input coefficients were constructed for each of the nine equations. These are shown in the last column of Table 2. None of the intervals, for which the set confidence level is 95 percent, conclusively shows that the corresponding sum of coefficients exceeds unity; the lower confidence limits all dip below one. But by the same token, we could not reject any number of hypotheses that $c'\alpha$ exceeds one by some small amount.

FURTHER DETAILS OF THE FARRELL EFFICIENCY RESULTS

The results mentioned in "Technical Efficiency Indices" are only a part of the conclusions implied by the Farrell efficiency programs. Since these details are subsidiary to our main interest, they are brought out separately in the following, drawing inspiration from Boles (1971).

Table 3 displays a printout of the solution to the Farrell efficiency problem using the data for the 18 plantations on which we have complete sets of observations for 1960. The first item of substance is the data matrix, printed here as the transpose P'_j the matrix of P_j in expression 6 and implied by problem 7. Note that the observation index is a four digit number, the first two of which reference the year and the last two, the plantation number. Also note the output is always one unit, as in vector P_j of expression 6; each input is stated as input per unit of output; and the last item is the scale factor, output. The row indices and scalars following the data matrix are the smallest levels of each factor among all observations; the data matrix will be scaled by these numbers to produce a working matrix and improve computational accuracy.

Following the scalars is the first RHS solution, as Boles terms it. The label of the activity (6010) presently serving as the right hand side of the problem is given first, along with the technical efficiency index which has been calculated from information printed subsequently. The next information includes the substance of the solution. The first column printed contains the row indices, followed by activity levels of the activities whose labels appear in the third column (where negative numbers refer to slack activities). The fourth column is a sequential index corresponding

TABLE 3. OUTPUT OF BOLES' FARRELL-EFFICIENCY PROGRAMS.

MODE 3--TECHNICAL EFFICIENCY INDEXES. SCALE: OUTPUT. 1960.

MODE = 3

NUMBER OF OUTPUTS = 1
NUMBER OF FACTORS = 3
NUMBER OF OBSVNS = 18

OBJ FCN ROW = 1
MAXIT = 40
CUM ITER: NCT = 0

SWITCHES:
NSW1 = 2
NSW2 = 2
NSW3 = 2

TOLERANCES:
TOL1 = 0.10000D-03
TOL2 = 0.10000D-03
TOL3 = 0.10000D-03
TOL4 = 0.10000D-03

NO.	OUTPUT	CAPITAL	ACRES	EMPL	ALE=Q
6001	1.00000	54.95065	0.19608	0.00706	63480.00000
6003	1.00000	52.05288	0.25777	0.01303	25018.00000
6004	1.00000	65.82431	0.26705	0.01298	24726.00000
6005	1.00000	94.48566	0.26289	0.01425	23576.00000
6006	1.00000	80.75536	0.31601	0.01580	23287.00000
6007	1.00000	86.65343	0.31032	0.01576	33182.00000
6009	1.00000	129.20639	0.35837	0.01477	20786.00000
6010	1.00000	46.73824	0.34821	0.01463	15927.00000
6011	1.00000	157.79773	0.35779	0.01841	22653.00000
6013	1.00000	71.93161	0.26261	0.01093	25151.00000
6014	1.00000	73.00765	0.21653	0.00814	45098.00000
6016	1.00000	74.46089	0.21575	0.01694	22962.00000
6019	1.00000	123.94208	0.19407	0.01466	44283.00000
6021	1.00000	72.91121	0.24053	0.02035	16264.00000
6024	1.00000	145.32812	0.23670	0.02077	22577.00000
6025	1.00000	102.48620	0.18786	0.01632	24385.00000
6028	1.00000	98.77862	0.19531	0.01618	37023.00000
6029	1.00000	31.64173	0.29875	0.01637	14969.00000

ROW	SCALAR
1	1.00000
2	31.64173
3	0.18786
4	0.00706
5	14969.00000

LAMBDA = 1.02598

ACTIVITY 6010 1.000

ITERATION COUNT = 6 CUM ITER = 0 PHASE = 5 RHS = 6010

SCALE = 1.00000 SCALE RHS = 0.06400

1	0.0	-1	19	0.39573	1.00000	2
2	0.0	6029	18	0.04116	0.0	1
3	0.0	6013	10	1.58601	1.02183	1
4	0.0	-2	20	-57.83109	44.03396	1
5	1.00000	6010	8	-1.23144	0.00003	1

LAMBDA = 1.36832

ACTIVITY 6013 1.000

ITERATION COUNT = 6 CUM ITER = 0 PHASE = 5 RHS = 6013

SCALE = 1.00000 SCALE RHS = 0.68021

1	0.0	-1	19	0.39573	1.00000	2
2	0.0	6029	18	0.04116	0.0	1
3	1.00000	6013	10	1.58601	1.36278	1
4	0.0	-2	20	-57.83109	58.72659	1
5	0.0	6010	8	-1.23144	0.00004	1

ACTIVITY 6029 1.000

ITERATION COUNT = 6 CUM ITER = 0 PHASE = 5 RHS = 6029

SCALE = 1.00000 SCALE RHS = 0.0

1	0.0	-1	19	0.39573	1.00000	2
2	1.00000	6029	18	0.04116	0.0	1
3	0.0	6013	10	1.58601	1.36278	1
4	0.0	-2	20	-57.83109	58.72659	1
5	0.0	6010	8	-1.23144	0.00004	1

LAMBDA = 1.18375

ACTIVITY 6009 0.859

ITERATION COUNT = 7 CUM ITER = 0 PHASE = 5 RHS = 6009

SCALE = 1.00000 SCALE RHS = 0.38860

1	0.16455	-1	19	0 39944	1.00000	2
2	0.00537	-3	21	-0.00373	0.0	1
3	0.61346	6013	10	1.58135	0.0	1
4	59.32237	-2	20	-58.50863	78.84787	1
5	0.55109	6010	8	-1.18190	0.00003	1

LAMBDA = 1.22598

ACTIVITY 6005 0.931

ITERATION COUNT = 9 CUM ITER = 0 PHASE = 5 RHS = 6005

SCALE = 1.00000 SCALE RHS = 0.57499

1	0.07363	-1	19	0.32057	1.00000	2
2	0.14174	6029	18	-1.19098	0.0	1
3	0.60844	6013	10	0.96109	2.21526	1
4	13.08531	-2	20	-87.86334	34.46974	1
5	0.32345	6025	16	0.55047	0.00003	1

LAMBDA = 1.21675

LAMBDA = 1.19697

ACTIVITY 6011 0.718

ITERATION COUNT = 9 CUM ITER = 0 PHASE = 5 RHS = 6011

SCALE = 1.00000 SCALE RHS = 0.51333

1	0.39335	-1	19	0.32057	1.00000	2
2	0.32195	6029	18	-1.19098	0.0	1
3	0.80708	6013	10	0.96109	2.16284	1
4	62.46673	-2	20	-87.86334	33.65417	1
5	0.26432	6025	16	0.55047	0.00003	1

LAMBDA = 1.25259

ACTIVITY 6025 1.000

ITERATION COUNT = 9 CUM ITER = 0 PHASE = 5 RHS = 6025

SCALE = 1.00000 SCALE RHS = 0.62903

1	0.0	-1	19	0.32057	1.00000	2
2	0.0	6029	18	-1.19098	0.0	1
3	0.0	6013	10	0.96109	2.26333	1
4	0.0	-2	20	-87.86334	35.21779	1
5	1.00000	6025	16	0.55047	0.00003	1

LAMBDA = 1.03492

ACTIVITY 6021 1.000

ITERATION COUNT = 11 CUM ITER = 0 PHASE = 5 RHS = 6021

SCALE = 1.00000 SCALE RHS = 0.08651

1	0.0	-1	19	0.39000	1.00000	2
2	0.0	-4	22	-0.00076	0.0	1
3	1.00000	6021	14	-1.39105	4.15746	1
4	0.0	-2	20	-81.11026	0.0	1
5	0.0	6025	16	1.78105	0.00003	1

LAMBDA = 1.24722

ACTIVITY 6024 0.843

ITERATION COUNT = 11 CUM ITER = 0 PHASE = 5 RHS = 6024

SCALE = 1.00000 SCALE RHS = 0.50825

1	0.18595	-1	19	0.39000	1.00000	2
2	0.00035	-4	22	-0.00076	0.0	1
3	0.26403	6021	14	-1.39105	5.01034	1
4	31.59316	-2	20	-81.11026	0.0	1
5	0.92192	6025	16	1.78105	0.00003	1

LAMBDA = 1.10148

ACTIVITY 6016 1.000

ITERATION COUNT = 12 CUM ITER = 0 PHASE = 5 RHS = 6016

SCALE = 1.00000 SCALE RHS = 0.53397

1	0.0	-1	19	0.17253	1.00000	2

2	0.0	-4	22	0.00406	0.00295	1
3	0.0	6021	14	-2.15162	3.61579	1
4	1.00000	6016	12	2.90153	0.0	1
5	0.0	6025	16	-0.57738	0.00001	1

LAMBDA = 0.94801

ACTIVITY 6001 1.000

ITERATION COUNT = 14 CUM ITER = 0 PHASE = 5 RHS = 6001

SCALE = 1.00000 SCALE RHS = 3.24076

1	0.0	-1	19	-0.01692	1.00000	2
2	0.0	-4	22	0.00386	0.0	1
3	1.00000	6001	1	0.38696	5.10002	1
4	0.0	-2	20	20.12901	0.0	1
5	0.0	6025	16	-0.40389	-0.00000	1

LAMBDA = 0.96792

ACTIVITY 6019 0.990

ITERATION COUNT = 14 CUM ITER = 0 PHASE = 5 RHS = 6019

SCALE = 1.00000 SCALE RHS = 1.95831

1	0.01055	-1	19	-0.01692	1.00000	2
2	0.00293	-4	22	0.00386	0.0	1
3	0.51434	6001	1	0.38696	5.20716	1
4	44.82352	-2	20	20.12901	0.0	1
5	0.49622	6025	16	-0.40389	-0.00000	1

LAMBDA = 0.97567

ACTIVITY 6028 0.975

ITERATION COUNT = 14 CUM ITER = 0 PHASE = 5 RHS = 6028

SCALE = 1.00000 SCALE RHS = 1.47331

1	0.02516	-1	19	-0.01692	1.00000	2
2	0.00252	-4	22	0.00386	0.0	1
3	0.33140	6001	1	0.38696	5.24887	1
4	9.46691	-2	20	20.12901	0.0	1
5	0.69376	6025	16	-0.40389	-0.00000	1

LAMBDA = 1.21675

LAMBDA = 1.13615

ACTIVITY 6003 1.000

ITERATION COUNT = 22 CUM ITER = 0 PHASE = 5 RHS = 6003

SCALE = 1.00000 SCALE RHS = 0.67132

1	0.0	-1	19	0.17850	1.00000	2
2	0.0	6029	18	-1.31555	0.00184	1
3	0.0	6025	16	-0.20310	2.31440	1
4	0.0	6013	10	-1.30293	23.61961	1
5	1.00000	6003	2	3.00008	0.00001	1

LAMBDA = 1.13167

ACTIVITY 6004 0.960

ITERATION COUNT = 22 CUM ITER = 0 PHASE = 5 RHS = 6004

SCALE = 1.00000 SCALE RHS = 0.65181

1	0.04150	-1	19	0.17850	1.00000	2
2	0.03172	6029	18	-1.31555	0.00183	1
3	0.06946	6025	16	-0.20310	2.30528	1
4	0.44046	6013	10	-1.30293	23.52654	1
5	0.49987	6003	2	3.00008	0.00001	1

LAMBDA = 1.21675

LAMBDA = 1.16753

LAMBDA = 1.31125

ACTIVITY 6014 1.000

ITERATION COUNT = 25 CUM ITER = 0 PHASE = 5 RHS = 6014

SCALE = 1.00000 SCALE RHS = 2.01276

1	0.0	-1	19	0.11793	1.00000	2
2	1.00000	6014	11	0.69147	0.00221	1
3	0.0	6025	16	-0.03824	2.81934	1
4	0.0	6013	10	-0.94071	28.07702	1
5	0.0	6003	2	0.40540	0.00001	1

ACTIVITY 6006 0.817

ITERATION COUNT = 5 CUM ITER = 25 PHASE = 5 RHS = 6006

SCALE = 1.55568 SCALE RHS = 0.0

1	0.22428	-1	19	0.19816	1.00000	2
2	0.16233	6025	16	-0.22546	0.00180	1
3	0.65974	6013	10	-1.44639	2.26137	1
4	0.20940	6029	18	-1.46041	23.07838	1
5	0.19281	6003	2	3.33042	0.00001	1

LAMBDA = 1.09102

ACTIVITY 6007 0.748

ITERATION COUNT = 7 CUM ITER = 31 PHASE = 5 RHS = 6007

SCALE = 2.21671 SCALE RHS = 0.0

1	0.33722	-1	19	0.10015	1.00000	2
2	0.52378	6014	11	-0.44483	0.00277	1
3	0.52096	6003	2	-0.20257	2.69310	1
4	0.26777	6016	12	0.09954	16.60527	1
5	0.02470	6001	1	0.64800	0.00001	1

FSOL3, ALL POINTS CLASSIFIED

to each activity label in column 3. Then comes a "parametric vector" which need not concern us, and then the dual solution, or shadow prices for each row. The final column contains an indicator of whether the row is a "free" one (as the objective function) or "restricted" (as with constraint equations).

The technical efficiency index itself is calculated from the activity levels in the second column of the RHS solution. In the case of activity 6010, the solution indicates that six simplex iterations were needed to obtain the efficiency index of 1.000 with activity 6010 serving as the right hand side vector. The LP has determined that no linear combination of the other 17 observations using no greater inputs could produce more than the 1.0 unit turned out by plantation number 10 in 1960. Thus, the objective value of problem 7 is simply $\sum_{j=1}^{18} X_j = 1.0 = X_o^*$, and the technical efficiency index for activity 6010 is $TE_{6010} = 1.0/X_o^* = 1.0$. For all other activities, $X_j = 0.0$. In similar fashion, consider the fourth solution printed, which is the first with TE < 1.0. In this case, the technical efficiency index is $TE_{6009} = 1.0/(.61346 + .55109) = .85870$ (slack activities do not enter the denominator of TE_j).

Note also that the resource requirements for productive activities in the basis should price out at 1.0 when evaluated at the shadow prices in column 6 of the RHS solution. In the notation of expression 6, with Π_j representing the vector of shadow prices, we should have $\Pi_j' F_j = 1.0$. Thus for activity 6010, for example,

$$\Pi_{6010}' F_{6010} = (0.0 \quad 1.02183 \quad 44.03396) \begin{bmatrix} 46.43824 \\ 0.34821 \\ 0.01463 \end{bmatrix} = 1.00003 \simeq 1.0$$

On the other hand, if an activity is inefficient, as compared with a linear combination of nearby activities, the resources it uses will price out to something greater than 1.0--as with activity 6009 in the present problems:

$$\Pi_{6009}' F_{6009} = (0.0 \quad 0.0 \quad 78.84787) \begin{bmatrix} 129.20639 \\ 0.35837 \\ 0.01477 \end{bmatrix} = 1.16458 > 1.0$$

The second column of the printed RHS solution contains the nonzero elements of the optimal solution vector for problem 7. Plugging this vector into problem 7 helps to clarify the meaning of these elements. The optimal solution for a given RHS vector f_t may be thought of as a set of factors by which input levels of those activities in the (optimal) basis can be weighted so that the resulting weighted average of basic activities uses the same quantities of input as f_t while producing greater output than f_t at a weighted average scale equal to that of f_t.

For example, consider the solution for activity t = 6011, an inefficient activity. A combination of .32195 units of activity 6029, 0.80708 units of activity 6013, and .26432 units of activity 6025 use the same amounts of inputs as 6011, while producing 1.39335 times as much output at a weighted average scale of operation the same as that of 6011. With X_t^* denoting the solution vector as in problem 7, with the matrix of coefficient vectors of the optimal basis for this problem denoted f_t^*, and where "1" is a unit vector, we have for t = 6011,

$$\left[\frac{1}{f^*_{6011}}\right] X^*_{6011} = \left[\frac{1}{f_{6029}}\ \frac{1}{f_{6013}}\ \frac{1}{f_{-2}}\ \frac{1}{f_{6025}}\right] X^*_{6011} = f_{6011}$$

$$= \begin{bmatrix} 1.0 & 1.0 & .0 & 1.0 \\ 31.64173 & 71.93161 & 1.0 & 102.48620 \\ 0.29875 & 0.26261 & .0 & 0.18786 \\ 0.01637 & 0.01093 & .0 & 0.01632 \end{bmatrix} (.32195 \quad .80708 \quad 62.46673 \quad .26432)$$

$$\begin{bmatrix} 1.39335 \\ 157.79749 \\ 0.35449 \\ 0.01840 \end{bmatrix} \simeq \begin{bmatrix} 1.0 \\ 157.79773 \\ 0.35779 \\ 0.01841 \end{bmatrix} = f_{6011}$$

Each efficient observation, of course, lies on the frontier for a given level of scale, while inefficient activities are inside the frontier. Points on the frontier corresponding to each inefficient observation may be of some interest. For such observations, one or more factors are used in "excess", as evidenced by a zero shadow price from the LP dual. If we subtract such excess inputs from the original observation vector, and multiply all input levels by the index of technical efficiency, we have a point on the efficient frontier. Using activity 6011 again as an example, we see from column 6 of the printed RHS solution that the second row (capital input) has a shadow price of zero, so that capital was used to "excess" by plantation 11 in 1960. The optimal level of the slack variable, indexed as -2, corresponding to the capital input row is 62.46673.

Thus

$$\begin{bmatrix} 1.0 \\ 157.79773 \\ 0.35779 \\ 0.01841 \end{bmatrix} - \begin{bmatrix} 0.0 \\ 62.46673 \\ 0.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} 1.0 \\ 95.33100 \\ 0.35779 \\ 0.01841 \end{bmatrix}$$

Multiplying this vector by the technical efficiency index gives

$$.718 \begin{bmatrix} 1.0 \\ 95.33100 \\ 0.35779 \\ 0.01841 \end{bmatrix} = \begin{bmatrix} 1.0 \\ 68.44765 \\ 0.25689 \\ 0.01322 \end{bmatrix}$$

SUMMARY AND CONCLUSION

We have examined data on Hawaiian sugar plantations with an eye toward inferring whether or not economies of scale prevail and if so to what degree. Three approaches have been pursued: First, the survivorship test indicates that the median percentage share of total state output produced by surviving firms has been growing over the past two decades and if announced plans for mergers occur, will continue to do so for the foreseeable future.

Secondly, we derived efficient unit isoquants using theoretical procedures of M. J. Farrell and computational algorithms of J. N. Boles. Interpreted in any of several ways, the resulting efficiency indices and isoquants are somewhat ambiguous, with high technical efficiency ratings assigned to at least two plantations, Kilauea and Kahuku, which ceased operations shortly after the period of our study. Whatever our estimates of their technical efficiency, the plantations' owners apparently felt them unprofitable ventures.

Finally, we obtained regression estimates of the relationship between output and the three inputs for which we had reasonably complete data. Being unidentified, the meaning of these regression coefficients is unclear and their statistical significance is not such as to allow us to unequivocally accept the hypothesis that their sum exceeds unity. The most we can say is that as best linear unbiased estimates, the regression equations nearly all predict that a proportionate increase in all inputs would lead to a slightly greater than proportionate increase in raw sugar output.

In the end, then, we have rather little more certainty about the problem than what we began with: the appearance of scale economies seems evident but not unequivocal. We have gathered a body of data and organized a set of computer programs, but the conclusions falling from it all remain more than usually uncertain.

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