FACTOR-PRODUCT MODEL FOR BEEF--A
QUADRATIC PROGRAMMING FORMULATION

A DISSERTATION SUBMITTED TO THE GRADUATE DIVISION OF THE
UNIVERSITY OF HAWAII IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
IN
AGRICULTURAL ECONOMICS
JUNE 1968

By
Chia-lin Cheng Yeh

Dissertation Committee:
Arnold B. Larson, Chairman
Fred C. Hung
Perry F. Philipp
Heinz Spielmann
Jack R. Davidson
ACKNOWLEDGEMENTS

Special acknowledgements are due to Friederich W. von Fleckenstein and L. Barrington Rankine of the Department of Agricultural Economics, University of Hawaii for their assistance throughout the preparation of this manuscript. The author wishes to express appreciation to Annie A. Nobriga for her typographical services.
ABSTRACT

FACTOR-PRODUCT MODEL FOR BEEF--A QUADRATIC PROGRAMMING FORMULATION

This study proposes an operational model for interregional analysis within a quadratic programming framework. The specific objectives are: (1) to present a framework for the interregional analysis (a) where both demand and supply of final products and factors may be expressed by functional relationships (b) where both products and factors can be traded (c) where the quantity of flows and equilibrium prices of the products and factors are simultaneously determined, and (d) where the quadratic programming technique is used; (2) to determine the optimum product and factor flows and regional equilibrium prices of beef, feed grains and roughages through the application of the model.

The study employed Samuelson's net social pay-off concept and used C. Van de Panne's simplex method to arrive at a solution for the quadratic programming model. Perfect competition is assumed to prevail in both the product and factor markets, and the existence of linear demand and supply relationships is also postulated. Regression analysis was used to derive the pertinent data. Empirical analyses for the years 1964 and 1975 were presented. The 1964 estimates were derived to serve as a basis for comparison. Shifts in data involving the transportation rate structure, and distribution of production and consumption were assumed for the year 1975 as a result of expected changes in market organization. Optimal patterns of prices and flows for beef, feed grains and roughages were determined for the
alternatives. The economic explanation of the resulting solutions were also given.

This analysis should contribute to the basic methodology of interregional competition, especially for those types of production activities which require consideration of shipments of both resources and products. It also enables the researcher to use the model for short-run projection since the functional relationships of demand and supply were derived from time series data. In the present study, the empirical findings can serve only as a crude guide, because the data needed is grossly oversimplified. The reliability of this guide, however, can easily be improved if relevant data is made available and a sufficiently large computer is used.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>ACKNOWLEDGEMENTS</th>
<th>ii</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>viii</td>
</tr>
</tbody>
</table>

## CHAPTER I. INTRODUCTION

The Problem .................................................. 1
The Objectives .............................................. 2

## CHAPTER II. GENERAL FRAMEWORK OF ANALYSIS

Discussion of Relevant Works ........................... 3
The Model .................................................... 7
Assumptions .................................................. 7
Notations ..................................................... 7
The Mathematical Model ................................. 11
A More General Formulation ............................. 16
Algorithms for Solving the Quadratic Programme .... 18

## CHAPTER III. A PRICE AND ALLOCATION MODEL FOR BEEF

Regional Demand for Beef .............................. 31
Regional Supply of Beef ................................ 32
Derived Demand for Feeds .............................. 32
Regional Availability of Feeds ....................... 33
The Transfer Costs for Beef, Feed Grains and Roughages 34

## CHAPTER IV. THE DEVELOPMENT OF PERTINENT DATA

Regional Demarcation .................................. 35
Regional Consumption Estimates for Beef .......... 35
Regional Supply Estimates for Beef ................. 38
Regional Derived Demand for Feeds ................. 40
The Availability of Feeds ............................. 42
Function for Transportation Cost ................... 44
The 1975 Projected Data ................................ 45
Assumption A ................................................ 45
Assumption B ................................................ 47
Assumption C ................................................ 48
Assumption D ................................................ 48
TABLE OF CONTENTS (Continued)

<table>
<thead>
<tr>
<th>CHAPTER V. EMPIRICAL ANALYSES</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model I: The Single Product Model</td>
<td>50</td>
</tr>
<tr>
<td>Model II: Factor Product Model</td>
<td>51</td>
</tr>
<tr>
<td>The Price and Flow Analysis for 1964</td>
<td>52</td>
</tr>
<tr>
<td>Price and Spatial Analyses for 1975</td>
<td>66</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CHAPTER VI. CONCLUSIONS</th>
<th>102</th>
</tr>
</thead>
<tbody>
<tr>
<td>APPENDIX A</td>
<td>103</td>
</tr>
<tr>
<td>APPENDIX B</td>
<td>111</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>116</td>
</tr>
</tbody>
</table>
### LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Estimates of Transportation Rates for Beef, Feed Grains and Roughages</td>
<td>46</td>
</tr>
<tr>
<td>II</td>
<td>Adjusted Transportation Rate for Beef, Feed Grains and Roughages</td>
<td>49</td>
</tr>
<tr>
<td>III</td>
<td>Set-Up Tableau for the Single Product Model 1964</td>
<td>54</td>
</tr>
<tr>
<td>IV</td>
<td>Equilibrium Pattern of Production, Consumption, Prices and Flows of Beef in Each Region, 1964</td>
<td>57</td>
</tr>
<tr>
<td>V</td>
<td>Comparison of Estimated Beef Production, Consumption and Prices with the Actual Data 1964</td>
<td>59</td>
</tr>
<tr>
<td>VI</td>
<td>Equilibrium Pattern of Production, Consumption, Prices and Flows of Beef, Feed Grains and Roughages in Each Region 1964</td>
<td>67</td>
</tr>
<tr>
<td>VII</td>
<td>Comparison of Estimated Beef, Feed Grains and Roughages Production, Consumption and Prices with the Actual Data 1964</td>
<td>72</td>
</tr>
<tr>
<td>VIII</td>
<td>Equilibrium Pattern of Production, Consumption, Prices and Flows of Beef in Each Region, 1975 (Model 1)</td>
<td>75</td>
</tr>
<tr>
<td>IX</td>
<td>Equilibrium Pattern of Production, Consumption, Prices and Flows of Beef in Each Region, 1975 (Model 2.1)</td>
<td>78</td>
</tr>
<tr>
<td>X</td>
<td>Equilibrium Pattern of Production, Consumption, Prices and Flows of Beef in Each Region, 1975 (Model 2.2)</td>
<td>85</td>
</tr>
<tr>
<td>XI</td>
<td>Equilibrium Pattern of Production, Consumption, Prices and Flows of Beef in Each Region, 1975 (Model 2.3)</td>
<td>90</td>
</tr>
<tr>
<td>XII</td>
<td>Equilibrium Pattern of Production, Consumption, Prices and Flows of Beef in Each Region, 1975 (Model 2.4)</td>
<td>95</td>
</tr>
</tbody>
</table>
### LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>FIGURES</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Regions and Central Points Used in the Model</td>
</tr>
<tr>
<td>2</td>
<td>Interregional Flows of Beef, 1964 Single-Product Model</td>
</tr>
<tr>
<td>3</td>
<td>Interregional Flows for Beef, Feed Grains and Roughages 1964 Factor Product Model</td>
</tr>
<tr>
<td>4</td>
<td>Interregional Flows of Beef, 1975 Single-Product Model</td>
</tr>
<tr>
<td>5</td>
<td>Interregional Flows of Beef, Feed Grains and Roughages 1975 Factor-Product Model 2.1</td>
</tr>
<tr>
<td>6</td>
<td>Interregional Flows of Beef, Feed Grains and Roughages 1975 Factor-Product Model 2.2</td>
</tr>
<tr>
<td>7</td>
<td>Interregional Flows of Beef, Feed Grains and Roughages 1975 Factor-Product Model 2.3</td>
</tr>
<tr>
<td>8</td>
<td>Interregional Flows of Beef, Feed Grains and Roughages 1975 Factor-Product Model 2.4</td>
</tr>
</tbody>
</table>
CHAPTER I
INTRODUCTION

A long time goal for economists has been to specify a theoretical framework for interregional competition and to design a corresponding operational model. A model of spatial competition should connect the questions of interregional pricing and commodity movement with general equilibrium theory. The developments of Enke\(^1\), Samuelson\(^2\) and others led to the formulation of a spatial model, where space was treated explicitly without violating the postulates of equilibrium theory. Furthermore, they demonstrated how the purely descriptive spatial problem could be converted into an extremum problem in which linear programming was employed as a technique of analysis. Their conceptual framework had made it possible to treat the interregional problem with a newer tool, namely, quadratic programming.

THE PROBLEM

The problem of equilibrium among spatially separated markets can be stated as follows: There are three (or more) regions trading one or two homogeneous products. Some of the factors can also be interchanged among the regions. Each region constitutes a single market and production center. The regions are separated, but not isolated by the unit transportation cost for the products and factors.


For each region, the demand and supply functions for the products and factors can be derived. What then will be the final competitive equilibrium prices in all markets, amount supplied and demanded in each region and shipments among the regions?

THE OBJECTIVES

Given the basic problem stated above, this study attempts to build an operational model of interregional relationships within a quadratic programming framework. The specific objectives are (1) to present a framework for the interregional analysis: (a) where both demand and supply of final products and factors may be expressed by functional relationships; (b) where both products and factors can be traded; (c) where the quantity of flows and equilibrium prices of the products and factors are simultaneously determined; and (d) where the quadratic programming technique is used. (2) to determine the optimum product and factor flows and regional equilibrium prices of beef, feed grains and roughages through the application of the model for the years 1964 and 1975.

This analysis should contribute to the basic methodology of interregional competition, especially for those types of production activities which require consideration of shipments of both resources and products. It also enables the researcher to determine the optimum flows and prices of the factors and products in the past as well as for the future. Although the empirical study is basically for illustrative purposes, the general pattern of pricing and movement for beef, feed grains and roughages can still be detected from the results.
CHAPTER II
GENERAL FRAMEWORK OF ANALYSIS

DISCUSSION OF RELEVANT WORKS

Since the development of linear programming techniques, many spatial equilibrium models have been constructed and applied. The theoretical foundations were laid by Lefeber, Samuelson and many other spatial theorists. Lefeber (1958) incorporated transportation demand and supply functions into the neoclassical production analysis, and thus suggested a general equilibrium approach for treating spatial problems. ³ The point-trading single product model formulated by Samuelson (1952), assumed a fixed number of discrete location points trading a certain product. In addition, supply and demand functions for the product and the transportation costs for shipping the product between any pair of regions were given. The problem was to find the final competitive equilibrium of prices in all the markets and the corresponding quantity and flow involved. ⁴ This contributed to the basic framework of many partial equilibrium analyses for spatial problems.

The transportation model, which is the most commonly used spatial model, enables economists to qualify the locational advantages of different regions by minimizing the cost of flows of goods among regions. Supplies and demands at the respective production points and

³ Louis Lefeber, Allocation in Space: Production, Transport and Industrial Location, (Amsterdam, 1958)

⁴ Samuelson, op. cit.
consumption centers are predetermined. But if spatial models are to simulate interregional competition, they must analyze competition at all levels of consumption and production, not just at the transportation level. This calls for new procedures to modify the rather restricted assumptions of fixed supplies and fixed demands. The modification may take several forms. (1) The augmented transportation model, which uses an explicit demand-supply matrix in connection with a series of transportation models. Judge, Havlicek and Rizek developed a spatial price equilibrium model of this type for the livestock economy.\(^5\) (2) The reactive programming model, which includes explicit demand and supply relations. This model was developed by Tramel and Seale for their study of fresh vegetables.\(^6\) (3) A general activity analysis model with implicit supply relations, whereby these relations are implied by resource availabilities and competing alternative uses. Examples of these are Schrader and King's factor-product model for beef,\(^7\) and Judge's activity analysis model for short-run interregional allocation problems.\(^8\) (4) The quadratic programming model formulated by Takayama and Judge for spatial

---


separated markets of products. Their model assumed explicit demand and supply relations for products. Factors and factor flows were not considered. So far no empirical studies based on their model have been published.9

These models will enable us to attack a large part of the spatial problems, and if appropriately used, should be able to provide us with a set of regional prices consistent with the optimum production and distribution pattern, not just the optimum distribution alone.

However, most of the models concentrate on the demand and supply aspects of the product, and neglect, if not overlook, the demand and supply of the intermediate products and factors. With the few models that deal with intermediate product and factor movements as well as final products, three approaches are used. The first approach specifies the producing sector in detail. Activity units are valued by their prices, and prices of both products and factors are assumed constant with respect to quantity produced or used. The model is stated as minimizing the combined transportation and production costs in meeting regional demand for the product. Snodgrass and French (1958) used this approach in a study of the dairy industry.10

The second approach was used by Schrader and King in their studies of the beef industry. They specified alternative production processes


for the conversion of intermediate products into final product. The
model was postulated as maximizing the value of the final product
minus the cost of transfer for factors and product, subject to the
regional endowment of resources, and demand restraint for the product.
An iterative process was used to bring regional prices and quantities
produced and consumed into equilibrium.11

The third approach which Fox and Taeuber (1955) used in their
analysis for the livestock-feed economy considers a joint equilibrium
for the intermediate product (feed) and the final product (livestock).
For each region, functions were specified for the demand and supply of
livestock and the demand and supply of feed. Production of feed was
assumed to be predetermined, with the resulting inelastic supply
function. The model was solved using two linear programs, one for feed
prices and shipments and the other for livestock prices and shipments.
The solution procedure was to take an assumed set of values for the
unknowns in the livestock market, in order to determine a temporary
set of feed prices and quantities. Then they took the just derived
feed data to solve the livestock problem. The resulting values were
then used in place of the originally assumed value to arrive at another
set of feed figures, and so on until a simultaneous solution of
equilibrium prices and shipments was obtained.12

11 Schrader and King, op. cit.
12 K. A. Fox and R. C. Taeuber, "Spatial Equilibrium Models of the
The present model is a point-trading quadratic programming model which specifies explicit demand and supply relations for the final product and its competitive (or supplementary) products. Explicit supply and demand relations for intermediate products and factors are also derived for each region. The model differs from all the models mentioned above, in the sense that, (1) demand and supply functions for the resources are allowed to be elastic rather than assumed as inelastic; (2) equilibrium prices, quantities and flows are obtained simultaneously, instead of through iterative procedures.

The formulation adopts Samuelson's social pay-off concept and is constructed along the same line as Takayama and Judge's quadratic programming model with the modification of introducing the supply and demand for intermediate products and factors.

THE MODEL

ASSUMPTIONS AND NOTATIONS

Assumptions

1. Perfect competitive markets for the products and factors.
2. The existence of linear regional demand and supply relations in the product and factor markets.

Notations

Let

i, j denote the regional demand and supply points, where i, j = 1, 2, ..., ni.

h, k denote the final products demanded and supplied, where h, k = 1, 2, ..., nj.
l, m denote the intermediate products demanded and supplied, where \( l, m = 1, 2 \ldots \ldots \) \( nk \).

\( n, q \) denote the mobile resources demanded and supplied, where \( n, q = 1, 2 \ldots \ldots \) \( nl \).

\( p^k_{pd} = (p^k_{\text{pd},1} \ldots \ldots p^k_{\text{pd},ni})' \) denotes the regional prices at the demand points for the \( k \)th product.

\( p^1_{td} = (p^1_{\text{td},1} \ldots \ldots p^1_{\text{td},ni})' \) denotes the regional prices at the demand points for the \( 1 \)th intermediate product.

\( p^n_{fd} = (p^n_{\text{fd},1} \ldots \ldots p^n_{\text{fd},ni})' \) denotes the regional prices at the demand points for the \( n \)th mobile factor.

\( p^k_{ps} = (p^k_{\text{ps},1} \ldots \ldots p^k_{\text{ps},ni})' \) denotes the regional supply prices for the \( k \)th product.

\( p^1_{ts} = (p^1_{\text{ts},1} \ldots \ldots p^1_{\text{ts},ni})' \) denotes the regional supply prices for the \( 1 \)th intermediate product.

\( p^n_{fs} = (p^n_{\text{fs},1} \ldots \ldots p^n_{\text{fs},ni})' \) denotes the regional supply prices for the \( n \)th factor.

\( p^k_p = (d^k_{p,1} \ldots \ldots d^k_{p,ni})' \) denotes the linear regional demand relations for the \( k \)th product. The \( (d^k_{p,i}) \) are linear functions of the regional prices \( (p^k_{p,i}) \) such that

\[
d^k_{p,i} = \alpha^k_{p,i} + \sum_{h=1}^{nj} \beta^h_{p,i} p^h_{p,i}
\]

(1)

Where \( \alpha^k_{p,i} > 0, \beta^h_{p,i} \geq 0 \) for \( h = k \), \( \beta^h_{p,i} < 0 \) for \( h \neq k \), for \( i = 1, 2 \ldots \ldots ni \), and \( h, k = 1, 2 \ldots \ldots nj \).

Alternatively the inverse of (1) is

\[
p^k_{p,i} = \lambda^k_{p,i} + \sum_{h=1}^{nj} \theta^h_{p,i} p^h_{p,i}
\]

(1a)
\[ D^1_t = (d^1_{t,1}, \ldots, d^1_{t,n_i}) \] denotes the linear regional demand relations for the 1th intermediate product. The \((d^1_{t,i})\) are linear functions of the regional prices \((p^k_{p, p}, p^1_{t, i})\) such that
\[
d^1_{t,i} = \alpha^1_{t,i} - \frac{n_k}{m=1} \beta^1 m_i \delta^k_{p, i} p^1_{t,i} - \frac{n_j}{k=1} \delta^k_{p, i} p^1_{p, i} \tag{2}\]

Where \(\alpha^1_{t,i} > 0, \beta^1 m_i \geq 0\) for \(1 = m, \beta^1 m_i \leq 0\) for \(1 \neq m, \delta^k_{p, i} \leq 0\), for \(i = 1,2, \ldots, n_i\), and \(1, m = 1,2, \ldots, n_k, k = 1,2, \ldots, n_j\).

The inverse of (2) is
\[
p^1_{t,i} = \lambda^1_{t,i} - \frac{n_k}{m=1} \theta^1 m_{t, i} \delta^k_{p, i} p^1_{t,i} - \frac{n_j}{k=1} \delta^k_{p, i} p^1_{p, i} \tag{2a}\]

\[ p^n f = (d^n f, i, \ldots, d^n f, n_i) \] denotes the linear regional demand relations for the nth mobile factors. The \((d^n f, i)\) are linear functions of the regional prices \((p^n f, p, p^k_{f, j}, p^1_{t, i})\) such that
\[
d^n f, i = \alpha^n f, i - \frac{n_l}{q=1} \beta^n q f, i p^1_{f, i} - \frac{n_j}{k=1} \delta^k_{f, i} p^1_{p, i} - \frac{n_k}{l=1} \gamma^1_{t, j} p^1_{f, i} \tag{3}\]

Where \(\alpha^n f, i > 0, \beta^n q f, i > 0\) for \(n = q, \beta^n q f, i \leq 0\) for \(n \neq q, \delta^1_{f, i} \leq 0, \gamma^1_{t, j} = 0\) for \(i = 1,2, \ldots, n_i\), \(n, q = 1,2, \ldots, n_l, k = 1,2, \ldots, n_j, k = 1,2, \ldots, n_k\).

The inverse of (3) is
\[
p^n f = \lambda^n f, i - \frac{n_l}{q=1} \theta^n q f, i p^1_{f, i} - \frac{n_j}{k=1} \delta^k_{f, i} p^1_{p, i} - \frac{n_k}{l=1} \pi^1_{t, j} p^1_{f, i} \tag{3a}\]

\[ s^k = (s^k p, p, s^k p) \] denotes the linear regional supply relations for the kth product. The \((s^k p)\) are linear functions of the \(p\) regional prices \((p^k p, p^1 f, i, p^n f)\) such that
\[
s^k p = \epsilon^k p + \phi^k h f, j h, j, f, i p^1_{f, i} + \omega^1 p^1_{t, i} p^1_{p, i} + \sum_{m=1}^{n_1} \xi^m p^1_{f, i} \tag{4}\]
Where $k, j = 0, \phi_{k,j}^h \geq 0$ for $h = k$, $\phi_{k,j}^h \leq 0$ for $h \neq k$,

$\omega^1_{t,j} \geq 0, \xi^k_{f,j} \leq 0$, for $j = 1, 2 \ldots \ldots \ldots n_1, h, k = 1, 2 \ldots \ldots \ldots n_j, 1 = 1, 2 \ldots \ldots \ldots n_k$, and $n = 1, 2 \ldots \ldots \ldots n_l$.

Alternatively the inverse of (4) is

$\Sigma_i^1_{t} = (\Sigma_i^{1,l})$ denotes the linear regional supply relations for the $l$th intermediate product. The $(s_1^{l,t})$ are linear functions of the regional prices $(p_1^{l,t}, p_f^{n,i})$ such that

$s_1^{l,t} = \epsilon_1^{l,j} + \ldots + \omega^{n-1}_{t} + \xi^{n,j}_{f,i}$

Where $\epsilon_1^{l,j} \geq 0, \phi_1^{l,m,j} > 0$ for $l = m, \phi_1^{l,m,j} \leq 0$ for $l \neq m$,

$\omega_1^{n,j} \leq 0$, for $j = 1, 2 \ldots \ldots \ldots n_1, 1, m \ldots \ldots \ldots n_k$, and $n = 1, 2 \ldots \ldots \ldots n_q$.

The inverse of (5) is

$p_1^{l,t} = \gamma^{l,j}_1 + \xi^{l,m,j}_f + \omega^{n,j}_f f + \sum_{q=1}^{n-1} \phi^{n,q,j}_f f_j f_i$

$s_1^{n,f} = (s_1^{n,f})$ denotes the linear regional supply relations for the $n$th factor. The $(s_1^{n,f})$ are linear functions of the regional prices $(p_f^{n,f})$ such that

$s_1^{n,f} = \epsilon_1^{n,j} + \ldots + \omega^{q-1}_{f} + \xi^{q,j}_{f,f}$

Where $\epsilon_1^{n,j} \geq 0, \phi_1^{n,q,j} \geq 0$ for $n = q, \phi_1^{n,q,j} \leq 0$ for $n \neq q$,

for $j = 1, 2 \ldots \ldots \ldots n_1$, and $n, q = 1, 2 \ldots \ldots \ldots n_l$.

The inverse of (6) is

$p_1^{n,f} = \gamma^{n,j}_f + \xi^{n,q,j}_f + \omega^{q,j}_f f_j f_i$
\[ \mathbf{P}^k_{ps} / k = 1,2 \ldots n_j, \mathbf{P}^1_{ts} / 1 = 1,2 \ldots nk, \mathbf{P}^n_{fs} / 1,2 \ldots nl \] denotes the vector of non-negative prices for the final products, intermediate products and factors at the \( i, j \) demand and supply points.

\[ \mathbf{x} = (x^k_{p,ij} / k = 1,2 \ldots nj, i, j = 1,2 \ldots ni, x^1_{t,ij} / 1 = 1,2 \ldots nk, i, j = 1,2 \ldots ni, x^n_{f,ij} / n = 1,2 \ldots nl, i, j = 1,2 \ldots ni \] denotes the unit transportation cost for shipping the commodities from region \( i \) to region \( j \).

The above shows the formulation of demand and supply equations that will be used in the development of the mathematical model below.

**The Mathematical Model**

Samuelson, in a journal article, demonstrated how the purely descriptive problem concerning competitive equilibrium among spatially separated markets can be artificially converted into a maximum problem.\(^{13}\) The problem then can be solved by trial and error or by a systematic procedure of varying shipments in the direction of increasing social pay-off. The social pay-off of any region is defined as the algebraic area under its excess demand curve. It is a kind of consumer's surplus concept. Net social pay-off (NSP) for all regions is defined as the sum of all the separate pay-offs minus the total transportation costs of all shipments.

In mathematical form, Samuelson's net social pay-off for the \( n \) region one product case can be written as

\(^{13}\)Paul A. Samuelson, *op. cit.*
NSP(X) = \sum_{i} \int_{0}^{x_{ij}} \left( (\lambda_i - \tau_j) - (\theta_i + \psi_j) \sum_{j} x_{ij} \right) \, d \left( x_{ij} \right) \\
\quad - \sum_{i} a_i - \sum_{j} x_{ij} x_{ij} \\
= \sum_{i} \left( \lambda_i - \tau_j \right) x_{ij} - \frac{1}{2} \sum_{i} \left( \theta_i + \psi_j \right) x_{ij}^2 - \sum_{i} a_i - \sum_{j} x_{ij} x_{ij} \\

Where a_i is the sum of producer's and consumer's surplus for the product in the i^{th} region before trade.

The problem then is to maximize the quadratic function NSP(X), subject to the non-negative flows X \geq 0. This turns out to be a quadratic programming problem.

The necessary and sufficient conditions for a maximum for the general quadratic programming problem have been given by Kuhn and Tucker.\(^{14}\)

Consider the quadratic programming problem of

Maximize \( f(X) \)

Subject to

\( g_i(X) \leq b_i \quad i = i, \ldots, u \)
\( g_i(X) \geq b_i \quad i = u+1, \ldots, v \)
\( g_i(X) = b_i \quad i = v+1, \ldots, m \)
\( X \geq 0 \)

The Lagrangean function can be formed as follows:

\[ L(X, \Delta) = f(X) + \sum_{i=1}^{m} \Delta_i \left( b_i - g_i(X) \right) \]  \hspace{1cm} (7)

The Kuhn-Tucker necessary condition where \( f(X) \) takes on a global

---

maximum at \( x^* \), for \( x \geq 0 \) is such that there exists a \( \Delta^* \) so

\[
\nabla x L(x^*, \Delta^*) = \frac{\partial f(x^*)}{\partial x_j} - \sum_{i=1}^{m} \Delta_i^* \frac{\partial g_i(x^*)}{\partial x_j} \leq 0 \quad j = 1, \ldots, n \quad (8)
\]

The strict equality holds when \( x^* > 0 \)

Also,

\[
\nabla x L(x^*, \Delta^*) x^* = \sum_{j=1}^{n} x_j^* \left( \frac{\partial f(x^*)}{\partial x_j} - \sum_{i=1}^{m} \Delta_i^* \frac{\partial g_i(x^*)}{\partial x_j} \right) = 0 \quad (9)
\]

Similarly,

\[
\nabla L(x^*, \Delta^*) = (b_i - g_i(x^*), \ldots, b_m - g_m(x^*)) \quad (10)
\]

The first \( u \) components of (10) are non-negative, while components \( u+1, \ldots, v \) are non-positive, and components \( v+1, \ldots, m \) vanish. Furthermore,

\[
\nabla L(x^*, \Delta^*) \Delta^* = \sum_{i=1}^{m} \Delta_i^* (b_i - g_i(x^*)) = 0 \quad (11)
\]

\( \Delta_i^* \geq 0, i = 1, \ldots, u \), \( \Delta_i^* \leq 0, i = u+1, \ldots, v \).

For \( i = v+1, \ldots, m \),

\( \Delta_i^* \) may have either negative or positive sign.

If \( f(x) \) is concave over the non-negative orthant, while \( g_i(x) \) is convex if \( \Delta_i^* > 0 \) and \( g_i(x) \) is concave if \( \Delta_i^* < 0 \), then the above condition is also a sufficient condition for \( x^* \geq 0 \) to be the absolute maximum.

In our problem the objective function is only constrained by the non-negative flows. The constraints \( g_i(x) \) do not exist. The necessary condition becomes

\[
\nabla x L(x^*, \Delta^*) = \frac{\partial LNSP(x^*)}{\partial x_{ij}} \leq 0 \quad \text{From (8)}
\]
That is
\[ \lambda_i - \tau_j - (\theta_i + \psi_j) \sum x_{ij} \leq 0 \]

Since
\[ \lambda_i - \theta_i \sum x_{ij} = p_i \quad \text{Using (1a)} \]
\[ \tau_j - \psi_j \sum x_{ij} = p_j \quad \text{Using (4a)} \]

We can rewrite the condition as
\[ p_i - p_j \leq t_{ij} \quad (12) \]

The strict equality holds when \( x_{ij} = 0 \)

\[ \nabla_{\lambda}(x^*, \Delta^*) x^* = (\lambda_i - \tau_j - (\theta_i + \psi_j) \sum x_{ij} - t_{ij}) \]
\[ x_{ij} = 0 \quad \text{From (9)} \]

Same as
\[ (p_i - p_j - t_{ij}) x_{ij} = 0 \quad (13) \]

Also
\[ X \geq 0 \quad \text{From (10)} \]

The economic interpretation of the above conditions is that the difference in prices between any two regions can differ at most by the unit cost of transportation. Otherwise, the simultaneous buying and selling of the same commodity in different markets will be profitable. In equilibrium, for the region where flow takes place, the price differential will exactly equal to the unit transport cost \( (p_i - p_j = t_{ij}, x_{ij} > 0) \). For the regions where no flow takes place, the price differential will be less than the unit transport cost \( (p_i - p_j < t_{ij}, x_{ij} = 0) \). This price behavior is consistent with that resulting from competitive behavior or the uncoordinated
efforts of the suppliers to sell their output at the maximum possible prices.

If the domain of the integral is converted from \( X \) to \( P \), we can convert the problem into

Maximize

\[
\begin{align*}
    f(P) &= \sum_i \int_0^P d_i \, dp_i - \sum_j \int_0^P (s_j) \, dp_j + \sum_j \int_0^P (s_j) \, dp_j \\
    &= \sum_i \alpha_i p_i - \frac{1}{2} \sum_i \beta_i (p_i)^2 - \sum_j \epsilon_j p_j - \frac{1}{2} \sum_j \phi_j (p_j)^2 + \text{constant}
\end{align*}
\]

Subject to

\[
\begin{align*}
    p_i - p_j &\leq t_{ij} \quad (15) \\
    p_i, p_j &\geq 0 \quad (16)
\end{align*}
\]

Where \( \hat{p}_i \) and \( \hat{p}_j \) are the equilibrium regional prices for the commodity before trade.

Takayama and Judge stated that the demand and supply prices in the same region should always be equal. But according to the formulation, there is no guarantee that the prices will be equal.

For in the case where one region produces the goods in question at a very high cost, if no trade is allowed, the consumers in this region will suffer from very high prices. Once the region is opened up for trade, it may import all the goods that are demanded rather than trying to produce them. If this is the case, the optimum demand price turns out to be so low that at this price the quantity which the region is willing to supply is a negative value. (Actually, there will be no production of the goods in this region). In this particular situation, the dummy supply price will be higher than the
demand price in this region, so as to prevent flows from demand point
to supply point. For example, if there is no flow within region 1,
then $p_1 - p^1 < 0$. Since both $p_1$ and $p^1$ have to be positive, $p^1$ must
be larger than $p_1$.

A MORE GENERAL FORMULATION

Given the single commodity formulation, the model can be extended
to handle the case where there are $nj$ final products, $nk$ intermediate
products and $nl$ factors in $ni$ regions.

The problem is as follows:

Maximize

$$f(P) = \sum_{i,k} \alpha_k \sum_{p \in P} p_{i,k} - \frac{1}{2} \sum_{i,h} \sum_{k \in P} \beta_{h,k} \sum_{i,p \in P} p_{i,p} - \sum_{j,k} \sum_{p \in P} \varepsilon_{j,k} p_{j,k}$$

$$+ \sum_{i,l} \alpha_{l,i} p^1_{i,l} - \frac{1}{2} \sum_{i,m} \sum_{k \in P} \beta_{m,k} \sum_{i,p \in P} p^m_{i,p} + \frac{1}{2} \sum_{i,k} \sum_{p \in P} \delta_k p_{k,p}$$

$$- \sum_{j,l} \sum_{t \in T} \phi_{j,t} p_{l,j} - \frac{1}{2} \sum_{j,m} \sum_{t \in T} \phi_{m,t} p_{m,j} + \frac{1}{2} \sum_{j,n} \sum_{f \in F} \phi_{n,f} p_{n,j}$$

$$+ \sum_{i,n} \alpha_{n,i} p^m_{i,n} - \frac{1}{2} \sum_{i,q} \sum_{f \in F} \phi_{q,f} p^q_{i,f} + \frac{1}{2} \sum_{i,k} \sum_{p \in P} \phi_{k,p} p_{k,p}$$

$$+ \frac{1}{2} \sum_{i,n} \sum_{f \in F} \sum_{p \in P} \phi_{n,f} p_{n,f} - \sum_{i,j} \sum_{n \in N} \phi_{n,f} p_{n,j}$$

Subject to

$$p^k_{i,j} - p^1_{i,j} \leq t^k_{i,j} \quad i,j = 1, \ldots, ni, \quad k = 1, \ldots, nj$$

$$p^1_{i,j} - p^1_{i,j} \leq t^1_{i,j} \quad i,j = 1, \ldots, ni, \quad l = 1, \ldots, nk$$
\[
\begin{align*}
\mathbf{p}_{f,i}^{n} - \mathbf{p}_{f,j}^{n} & \leq t_{f,i,j}^{n}, \quad i,j = 1, \ldots, n_i, \quad n = 1, \ldots, n_l \quad (20) \\
\mathbf{p}_{p,i}^{k}, \mathbf{p}_{t,i}^{1}, \mathbf{p}_{f,i}^{n} & \geq 0, \quad \mathbf{p}_{p}^{k}, \mathbf{p}_{t}^{1}, \mathbf{p}_{f}^{n} \geq 0 \quad (21)
\end{align*}
\]

Since the magnitude of the objective function is artificial in the sense that no competitor in the market will be aware of or concerned with it, there is no need to attach any economic meaning to its value. Therefore, the integral constant can be eliminated.

The problem can be put in a more compact way.

\[ F(\mathbf{P}) = D^T \mathbf{P} + \mathbf{P}^T \mathbf{C} \mathbf{P} \] is to be maximized and which also satisfies the following constraints

\begin{align*}
\mathbf{A} \mathbf{P} & \leq \mathbf{T} \quad (24) \\
\mathbf{P} & \geq 0 \quad (25)
\end{align*}

where: \( \mathbf{P} \) is a column vector of \( 2(n_j+n_k+n_l)n_i \) variables. \( \mathbf{D} \) is a given column vector of \( 2(n_j+n_k+n_l)n_i \) elements. \( \mathbf{A} \) is a \( 2(n_j+n_k+n_l)n_i \times (n_j+n_k+n_l)n_i^2 \) matrix. \( \mathbf{C} \) is a \( 2(n_j+n_k+n_l)n_i \times 2(n_j+n_k+n_l)n_i \) symmetric matrix. \( \mathbf{T} \) is a given vector of \( (n_j+n_k+n_l)n_i^2 \) elements.

The \( \mathbf{C} \) matrix is the matrix of the direct and cross influences of a change in price in the original demand and supply functions. The "direct influence" is defined as the rate of change in a product or factor with respect to only its own price, i.e., the partial derivatives \( \frac{\partial q_i}{\partial p_j} \), where \( i=j \). The "cross influence" is defined as the rate of change in a product or factor with respect to another price, i.e., \( \frac{\partial q_i}{\partial p_j} \), \( i \neq j \). The elements which lie on the principal diagonal of the matrix are the direct influences, other elements represent the cross influences. J. R. Hicks showed that certain cross influences
are equal, i.e., \( \frac{\partial q_i}{\partial p_j} = \frac{\partial q_j}{\partial p_i} \), \( i \neq j \).\(^{15}\) Hence, the C matrix is symmetrical. But this theory may not be confirmed in real situations, due to several reasons: (1) the inadequacy of the data, (2) the estimation method used which may not be efficient, (3) the time lag involved in the production process, and (4) the existence of inefficient plants in the producing regions. If the C matrix is symmetrical, two methods can be used in handling this situation.

a. If the differences between the cross influences are small, new values of the cross influences can be derived by taking a simple average or a weighted average of the old ones.

b. If the differences are significant, a two step procedure is used in getting the solution. The C matrix is first converted into a symmetrical one by defining new elements \( b_{ij} = b_{ji} = \frac{c_{ij} + c_{ji}}{2} \), so that \( b_{ij} + b_{ji} = c_{ij} + c_{ji} \). Thus, the new symmetrical B matrix emerged where \( B = \| b_{ij} \| = B' \). Note \( c_{ij} \) and \( c_{ji} \) are both coefficients of \( p_i p_j \) when \( i \neq j \), since \( p_i p_j = p_j p_i \). The B matrix then is used instead of the C matrix in the set-up tableau. The equilibrium prices derived are unaffected by this change, but the flows can not be obtained simultaneously. The flows can be derived either by trial and error, or by using a simple transportation model.

ALGORITHMS FOR SOLVING THE QUADRATIC PROGRAMME

The Assumption of Definite Quadratic Form

In order to prevent the existence of various local extrema, the quadratic objective function must be concave in case the desired

---

extremum is a maximum and convex when it is a minimum. This means that the matrix of the quadratic form consisting of the quadratic terms of the objective function must be negative definite or negative semi-definite in the maximization problem and positive definite or positive semi-definite in the minimization case. The quadratic form \( P'CP \) is negative definite, if it is negative \((< 0)\) for every \( P \), except \( P=0 \); \( P'CP \) is semi-definite, if it is non-positive \((\leq 0)\) for every \( P \), and there exists points \( P\neq 0 \), for which \( P'CP=0 \). In the case of positive definite and positive semi-definite the inverse (change the word negative into positive and the symbol \(<\) to \(>\)) of the above conditions are true.

The necessary and sufficient condition for the form \( P'CP \) or the symmetric matrix \( C \) to be negative definite is that

\[
\begin{vmatrix}
  c_{11} & c_{12} & \cdots & c_{1n} \\
  c_{21} & c_{22} & \cdots & c_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  c_{n1} & c_{n2} & \cdots & c_{nn}
\end{vmatrix}
\]

If the determinant \( C = 
\]

\[
\begin{vmatrix}
  c_{11} & c_{21} \\
  c_{21} & c_{22}
\end{vmatrix} > 0
\]

\[
\begin{vmatrix}
  c_{11} & c_{12} & c_{13} \\
  c_{21} & c_{22} & c_{23} \\
  c_{31} & c_{32} & c_{33}
\end{vmatrix} < 0 \quad \cdots \quad (-1)^n \quad |C| > 0 \quad (27)
\]

If \( P'CP \) is positive definite, the above naturally ordered principal minors of \( C \) are all positive.
THE ALGORITHM

In recent years, a large number of algorithms for the solving of concave quadratic programming problems have been developed, some of them as extensions of the Simplex Method, some based on other principles.

In Takayama and Judge's paper, the dual method was used to arrive at the solution. In this thesis, the C. Van de Panne's simplex method is used.

The Idea of the Simplex Method

Let us recall the above quadratic programming problem (24-26). For computational ease, the equation (24) can be rewritten as

\[ F(P) = D'P + P'CP = \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \]

where

\[ -\frac{1}{2}E = C \]

Since C is negative definite or semi-definite, E becomes positive definite or semi-definite.

On changing the inequality constraints of (25) into equality, a vector of slack variables is introduced,

\[ AP + Y = T \]

The non-negative restraints of prices remain the same,

\[ P \geq 0 \]

---

16 T. Takayama and G. G. Judge, op. cit.

Let us form for this constrained problem the following Lagrangean expression

\[ L(P, \Delta) = D'P - \frac{1}{2}P'EP - \Delta'(AP + Y - T) \]  \hspace{1cm} (31)

If we set the partial derivatives of the Lagrangean function \( L \) with respect to \( \Delta \) equals to zero,

\[ \frac{\partial L}{\partial \Delta} = AP + Y - T = 0 \]  \hspace{1cm} (32)

then (31) is not different from (28)

The influence of a change in the \( P \)-variables to the Lagrangean function can be detected by differentiating the Lagrangean expression with respect to \( P \)

\[ \frac{\partial L}{\partial P} = D - EP - A'\Delta = -U \]  \hspace{1cm} (33)

The vector \( U \) is the first derivatives of \( L \) with respect to the \( P \)-variables with a negative sign added.

If for a certain solution \( P^*, \Delta^* \), some \( U \)-variables, say \( u_k^* \), are negative, then the partial derivative of \( L \) with respect to the variable \( p_k^* \) is positive. Thus an increase in the value of \( p_k^* \), holding the other \( P \)-variables constant, will raise the value of the \( L \) function. Such an increase, however small, will generally cause a violation of the constraints (29). Since the constraints are not satisfied, the value of (31) will not equal to (28). Hence, we have to consider, in addition to an increase of \( p_k^* \), changes in the other variables which will compensate for the change in \( p_k^* \), so that the restraints are still satisfied. If it is possible to make such compensating changes only in \( P \)-variables which have zero corresponding \( U \)-variables (this implies that the partial derivatives of the \( P \)-variables are equal to zero), then the value of \( L \) does not change when these variables change.
Furthermore, if these changes make the partial derivatives (negative of U-variables) remain at zero, then the objective function (in this case (28) equals (31)) is increased when $p_k^*$ is increased. At the same time, other variables having zero partial derivatives are adjusted in such a way that the constraints are still satisfied. The idea of the simplex method for quadratic programming is based on the above reasoning.

In order to secure an optimum solution for the problem, the Kuhn-Tucker conditions (8), (9), (10) and (11) have to be satisfied. Applying these conditions to the problem, yields the following results:

$$\frac{\partial L}{\partial p} = D - EP - A' \Delta \leq 0 \quad \text{from (8)}$$

$$-U = D - EP - A' \Delta \leq 0 \quad \text{from (33)}$$

$$U - A' \Delta - EP = -D \quad (34)$$

$$U'P + A'Y = 0 \quad \text{from (9)} \quad (35)$$

$$AP + Y = T \quad \text{from (10)} \quad (36)$$

$$U, P, \Delta, Y \geq 0 \quad (37)$$

Since (11) is satisfied automatically by any feasible solution, it need not be included.

The Set-Up Tableau and the Initial Tableau

Based on the Kuhn-Tucker conditions (34) and (36), the simplex tableau can be set up as below:

**THE SET-UP TABLEAU**

<table>
<thead>
<tr>
<th>Basic Variables</th>
<th>Value of Basic Variables</th>
<th>U</th>
<th>$\Delta$</th>
<th>P</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>-D</td>
<td>1</td>
<td>-A'</td>
<td>-E</td>
<td>0</td>
</tr>
<tr>
<td>Y</td>
<td>T</td>
<td>0</td>
<td>0</td>
<td>A</td>
<td>I</td>
</tr>
</tbody>
</table>
The solution corresponding to this set-up tableau for the basic variables is \( U = -D, Y = T \), all the non-basic variables \( P \) and \( \Delta \) equal to zero. Since the constraints are linear, finding an initial feasible solution for the quadratic programming problem involves the use of the same procedures as that used in solving linear programming problems. If all the elements in the \( T \) vector are non-negative, the solution \( Y = T, P = 0 \) is a feasible solution to our quadratic programming problem, and the initial solution is already on hand. Fortunately, this is exactly the case in our model (note \( T \) is the transportation cost vector). The tableau which contains the initial solution is called the initial tableau, in our special case the set-up tableau is the initial tableau, and trouble is saved from finding the initial solution.

**Rules for the Method**

A standard tableau is a tableau which does not include the \( P \) (or \( Y \)) variable and its corresponding \( U \) (or \( \Delta \)) variable in the same basis. Any tableau which violates the above condition is called a non-standard tableau.

As in the simplex method for linear programming, the column pivot is first selected. This amounts to the choice of a new basic variable. Then, the row of the pivot is selected, which is equivalent to the choice of the basic variable that will be replaced by the new basic variable.

The rule for selecting the incoming variable follows:

If the tableau is standard, select as the new basic variable the p
(or y) variable having the largest negative corresponding basic u (or $\Delta$ ) variable. For example, if $u_1$ is the largest negative u variable in the basis, then $p_1$ is selected to enter the basis, in light of increasing the value of the objective function. In case the tableau is not in standard form, introduce as the new basic variable, the u (or $\Delta$ ) variable from the pair of u-p (or $\Delta$ -y) variable that is not in the basis. For instance, if $u_1$ and $p_1$ are both basic variables, $u_4$ and $p_4$ are both non-basic variables, then $u_4$ is introduced as the new basic variable. This will avoid having more than one pair of u-p (or $\Delta$ -y) variables in the basis.

To select the outgoing variable, the following rule is used:

If the tableau is standard, select as the variable to leave the basis from the set of basic p (or y) variables and the u (or $\Delta$ ) variable of the corresponding new basic variable, the one which first becomes zero upon introducing the new basic variable in a positive amount. If there are ties and the u (or $\Delta$ ) variable is involved, select the u (or $\Delta$ ) variable. If the tableau is non-standard, select the variable to leave the basis from the set of p (or y) variables and the u (or $\Delta$ ) variable of the basic pair, the variable which first becomes zero upon introducing the new basic variable. If there are ties, and the u (or $\Delta$ ) variable is involved, select the u (or $\Delta$ ) variable. Let $r_i$ represent the value of the basic variable in the $i^{th}$ row of the tableau and $g_i$ the element in the $i^{th}$ row in the column of the new basic variable, then the variable to leave the basis is the one associated with $\min(\frac{r_i}{g_i}, \frac{r_i}{g_i} \geq 0, g \neq 0)$, where I is the set of rows associated with the basic p (or y) variables and the u (or $\Delta$ )
variable corresponds to the newly introduced basic variable.

When all the \( u \) variables in the basis become positive, the solution of the problem is obtained, because now the partial derivative of the Lagrangean function with respect to \( p \) becomes negative, and a change in the \( p \) variable will bring about a decrease of the objective function.

**Introducing the Objective Function into the Tableau**

As in linear programming, a row can be introduced into the simplex tableau as the value of the objective function.

The objective function as shown in (28) is

\[
F(P) = D'P - \frac{1}{2}P'EP
\]

where from (33), \( EP = U - A'\Delta + D \)

Substitute this expression into \( F(P) \), so that

\[
F(P) = D'P - \frac{1}{2}P'(U - A'\Delta + D) = D'P - \frac{1}{2}U'P + \frac{1}{2} A'AP - \frac{1}{2}D'P
\]

\[
= \frac{1}{2}D'P - \frac{1}{2}U'P + A'AP
\]

Since \( AP + Y = T \), then

\[
F(P) = \frac{1}{2}D'P - \frac{1}{2}U'P + \frac{1}{2} A'T - \frac{1}{2} A'Y
\]

where \( U'P + Y = 0 \) in a standard tableau,

Let

\[
D'P + A'T = f \tag{38}
\]

such that

\[
f = 2F(P)
\]

The value of equation (38) is assumed to equal to zero in the set-up tableau. Add this equation to the last row of the tableau, we have
The set-up tableau is based on Kuhn-Tucker conditions (34) and (36); and through the iterations, the optimum solution vectors also satisfied the other two conditions (35) and (37). Furthermore, the objective function is assumed to be concave over the non-negative orthant of the constraint variables, the necessary and sufficient
condition for which the solution vectors to be the absolute maximum are both satisfied. So the solution is of the global variety desired.

In the single product model, if the demand and supply functions are well behaved, the existence of an optimum solution is insured. For the product-factor model, strict concavity may not be assured when substitution and complementary terms appear in the demand and supply equations. However, it is most likely that the concavity requirement can be satisfied by the model. For in every demand and supply function, it is probably safe to assume that the direct influence of its own price will be more significant than the cross influence of other product and factor prices to the quantity demanded and supplied. Therefore, the multiplied diagonal values of the determinant would be rather large compared to the other multiplications of the elements.

Uniqueness

The uniqueness of the solution vectors follows immediately from the concavity of the objective function. If the quadratic term \( P'CP \) is negative definite (\( P'EP \) positive definite), the objective function is strictly concave and the global maximum is unique.

Convergence

If degeneracy is excluded, (if not, cycling can be prevented by applying perturbation methods as used in linear programming) the objective function increases in any iteration proceeding from a tableau in standard form, and does not decrease in any iteration proceeding from a tableau not in standard form. Since the number of
different combinations of basic variables is finite, the number of
different tableaux in standard form must be finite. And it is
impossible for the same standard tableau to appear twice, because the
objective function increases whenever an iteration proceeds from a
tableau in standard form. Furthermore, the number of successive
tableaux not in standard form can not exceed \((ni+nj+nk)^2 - 2(ni+nj+nk)\),
so that after a number of tableaux not in standard form a tableau in
standard form must appear. Hence, the procedure must terminate in a
finite number of iterations. In the non-standard tableau, the \(u\)
variable of the basic pair is increasing in successive non-standard
tableaux until it has reached the value zero when this variable leaves
the basis. From this it follows that there is always at least one
positive ratio \(\frac{r_i}{g_i}\), namely, the one connected with the row of the dual
variable of the basic pair. So it is impossible that an infinite
solution occurs when the tableau is not in standard form. The \(p\)
variable of the basic pair can only increase in successive
non-standard tableaux. But if the matrix of the quadratic terms is
not negative definite or semi-definite, then the system may not show
regular convergence. The value of the objective function fluctuates,
and there may not be stability in the regional price and quantity
adjustment processes.

THE COMPUTER PROGRAM FOR SOLVING THE PROBLEM

To arrive at a solution for the quadratic programming problem, a
Fortran IV computer program was written for the IBM 360. The first
part of the program deals with the testing of the negative definiteness
of the matrix of the quadratic form. If the matrix fits the negative
definite requirement, then a subroutine for finding the solution of
the quadratic program is called. The degeneracy problem is coped
with within the program, so as to prevent cycling. The method of
using the computer program is given in Appendix A.
CHAPTER III

A PRICE AND ALLOCATION MODEL FOR BEEF

The quadratic programming model will now be applied to analyze the spatial equilibrium problems in the beef sector of the economy.

Beef production in the United States is concentrated in areas of relatively sparse population. This necessitates the large shipment of beef to the major areas of consumption. The North Central region produced more than half the quantity that is consumed in the U. S., while the most populated Northeastern region has a large deficit in beef supply. The surplus beef production in the Great Plains states is primarily the result of large surplus in feed production, for feed is always the major item in the cost of finishing beef. This geographical location of surplus beef and feed production has important implications for consumers and producers in the surplus regions as well as in the deficit regions. If the supply and demand functions for each region can be derived, it should be possible to determine the efficient allocation for beef, feed grains and roughages. Because of the complexities posed, a model designed to provide answers should not only include the determination of optimum flows and prices of beef, but also the optimum flows and prices of feed grains and roughages. Such a model will give a more accurate picture of the real situation.

In this study the United States is divided into four regions. These regions trade beef and its intermediate products, feed grains, and roughages with each other. Assume there exist points that may be taken to represent regional production and consumption centers. At
each of these localities, domestic demand and supply functions for the product and intermediate products can be derived. The regions are separated but not isolated by a physical unit transport cost for each product and intermediate product. The problem is to determine the equilibrium prices and quantities as well as the least cost shipment patterns for beef, feed grains and roughages under competitive conditions.

Feeder cattle is not included in the model as a variable, because of the obvious time span required in the production process. The cattle cycle and cattle inventory problems will become unavoidable, thus forcing the model to consider the dynamic aspect of the beef industry, which is beyond the scope of this study. Processing capacity is not treated explicitly in the model, but its influence should be reflected by the constant term of the beef supply function in each region.

The model thus consists of five components, each requiring specifications as to

(1) The regional demand for the final products.
(2) The regional supply of the final products.
(3) The derived demand of the intermediate products.
(4) The regional availability of the intermediate products.
(5) The transfer costs for intermediate products and final product.

Regional Demand for Beef

The quantity of beef consumed in each region is dependent upon the price of beef, population and personal disposable income.
Ordinary least squares method is used to obtain the functional relationships. No distinction is made between beef from feedlot finishing and from other sources.

Regional Supply of Beef

The price received by farmers for beef is a decisive factor which governs the supply of beef. Other factors such as the demand prices for feed grains and roughages also have large influence on the volume of beef that will be put on the market. The beef quantity produced in each region can be represented as a function of its own price, and feed grain and roughages prices. Pork price, feed grain and roughages quantities are included as exogeneous variables, since pork production competes with beef production and feed grain and roughages production influence the feed prices. The two stage least squares estimation method is used.18

Derived Demand for Feeds

Feeds are divided into two broad classes, namely, feed grains and roughages. The demand for feeds is derived from the need of feeding the meat animals. This may be reflected in the beef price.

Ordinary least squares method was first tried, but the signs obtained were not as anticipated and the errors were awfully large, so that the relationships were rejected. Even lagging the feed grains and roughages prices did not help. The reason for this situation may have been the neglect of the fact of the simultaneous nature of the economic variables. Thus pork price, feed grains and roughages quantities were included in the model as exogeneous variables. The pork price is treated as an exogeneous variable in order to limit the quadratic programming model to a one product case and reduce computation burden. But this can be easily relaxed.
Demand for Feed Grains

Feed grains include corn, barley, oats and sorghum grains. They are considered perfect substitutes for each other. The grain price in each region is derived by weighting the price of each kind of grain by its quantity produced in that region. The quantity demanded is represented as a function of its own price, the price of its competing goods, roughages, and the price of beef. Two stage least squares is used in arriving at the relationships.

Demand for Roughages

Roughages include all hay and sorghum forages. Its price in each region is a quantity weighted average. The quantities required in a region depend on its own price and prices of beef and feed grains. The functional relationship is obtained through the use of the two stage least squares method.

Regional Availability of Feeds

Supply of Feed Grains

The supply of feed grains in a certain region depends on its own price and on the availability of crop land.

Supply of Roughages

Roughages supplied depends on its own price and the quantity of hay land available in the region.

The ordinary least squares estimation method is used to obtain the supply functions for both feed grains and roughages.
The Transfer Costs for Beef, Feed Grains and Roughages

In order to obtain freight rates between all areas, one city in each area was chosen as the shipping and receiving point. The reason for assigning the same point to represent consumption center and production center in each region is to reduce the computation burden. However, without difficulty, they can be treated at different points if desired. Intraregional transfer costs are allowed by assuming standard average distances from the center to border of the region.
CHAPTER IV

THE DEVELOPMENT OF PERTINENT DATA

REGIONAL DEMARCATION

The continental United States was divided into four regions: Northeast, North Central, West and South. The reasons for selecting such a small number of regions were many: (1) The thesis was oriented toward finding a new approach for interregional competition, hence the study might be entirely illustrative; (2) The computation burden increases drastically when the number of regions increases, and the storage capacity of the computer limited the number of regions that could be considered; (3) The availability of certain data, especially regional time series data for consumption estimates and prices. Each regional market or source of supply was represented by a point that is identified with a city near the geographical center of each area. The regions and trading points are shown in Figure 1.

REGIONAL CONSUMPTION ESTIMATES FOR BEEF

The demand functions were estimated at the retail level. The regional data for quantities demanded were derived from the 1955 Household Food Consumption Survey. From this survey, the share of the quantity demanded for each region was obtained. Using these percentages, the quantity demanded for each year for each region was calculated, assuming that the same pattern of consumption existed

FIG. 1 REGIONS AND CENTRAL POINTS USED IN THE MODEL
throughout the years. Quantity demanded was then estimated as a function of retail beef price, disposable personal income and population.

The general linear relationship was

\[ D_{p,i} = b_{0,i} + b_{1,i}p_{p,i} + b_{2,i}p_{o,i} + b_{3,i}I_{i} \]

where

- \( D_{p,i} \) = per capita consumption of beef in the \( i^{th} \) region
- \( b_{0,i} \) = constant in \( i^{th} \) region
- \( p_{p,i} \) = retail price of beef in \( i^{th} \) region
- \( p_{o,i} \) = population in the \( i^{th} \) region
- \( I_{i} \) = disposable personal income in the \( i^{th} \) region

Ordinary least squares method was applied to obtain the individual regional demand function. In order to change the per capita retail demand into carcass demand, the per capita difference between retail and carcass consumption was deducted from the right hand side of the retail and carcass consumption was deducted from the right hand side of the retail demand equation. Since the spread was assumed constant throughout the years, it only affected the constant term, and this constant term had to be adjusted for the quantity effect of the spread.

The carcass demand function for each region was as follows:

Northeast 1

\[ D_{p1} = -169.746 - 0.813767p_{p1} + 0.00001031I_{1} + 7.18758p_{o1} \]  

(108) \hspace{1cm} (0.16) \hspace{1cm} (0.0001) \hspace{1cm} (5.23)  

North Central 2

\[ D_{p2} = 43.3705 - 0.715735p_{p2} + 0.00082122I_{2} + 0.596256p_{o2} \]  

(19) \hspace{1cm} (0.40) \hspace{1cm} (0.0005) \hspace{1cm} (0.51)
West 3

\[ D_{p3} = 91.2189 - 0.91051p_3 + 0.000762757I_3 + 1.74628p_03 \]  
(56) (0.18) (0.00028) (1.10)  

South 4

\[ D_{p4} = -37.0736 - 0.513474p_4 + 0.000008714I_4 + 2.4048p_04 \]  
(18) (0.09) (0.000005) (1.14)  

where \( D_p \) stands for the quantity of carcass beef demanded

The per capita consumption was then transformed into total consumption (million pounds). Retail price was converted into farm price (cents). The 1964 disposable income and population figures were substituted into the equations 39-42 for \( I \) and \( p_0 \). The following relationships result:

\[ D_{p1} = 6374.3063 - 38.74833p_1 \]  
(43)  

\[ D_{p2} = 7795.0087 - 38.19878p_2 \]  
(44)  

\[ D_{p3} = 5111.385 - 27.85529p_3 \]  
(45)  

\[ D_{p4} = 4279.0935 - 30.17224p_4 \]  
(46)  

where

\( D_p \) represents the total quantity of carcass beef demanded

\( p_p \) represents the farm price of beef

**REGIONAL SUPPLY ESTIMATES FOR BEEF**

The quantity supplied in carcass weight was derived from the cattle slaughter data for each region. Beef prices were prices received by farmers. The prices for feeds were those that prevailed at the demand point for the respective region.
The general functional relationship was

\[ S_p^j = a_0^j + a_1^j p_j + a_2^j p_{gi} + a_3^j p_{ri}, \]

where

- \( S_p^j \) = quantities supplied for beef in region \( j \)
- \( a_0^j \) = constant in the \( j \)th region
- \( p_j \) = farm price for beef in region \( j \)
- \( p_{gi} \) = demand price for feed grains in region \( j \)
- \( p_{ri} \) = demand price for roughages in region \( j \)

In order to obtain good estimates for the parameters of the supply functions, two stage least square estimation method was used. Regional pork price and quantity were included as exogenous variables.

The statistical supply functions for beef in each region are shown below:

- \( S_p^1 = 1270.913 + 18.651 p_j - 9.15 p_{gi} - 7.201 p_{ri} \) \( (590) \) \( (7.3) \) \( (2.4) \) \( (4.5) \) \( (47) \)
- \( S_p^2 = 8832.173 + 193.413 p_j^2 - 176.238 p_{gi} - 50.26 p_{ri} \) \( (1259) \) \( (84.5) \) \( (96.3) \) \( (49.5) \) \( (48) \)
- \( S_p^3 = 501.837 + 92.684 p_j^3 - 16.127 p_{gi}^2 - 36.52 p_{ri}^2 \) \( (248) \) \( (36.1) \) \( (10.2) \) \( (18.9) \) \( (49) \)
- \( S_p^4 = 1545.682 + 40.374 p_j^4 - 8.23 p_{gi}^4 - 1.315 p_{ri}^4 \) \( (924) \) \( (11.8) \) \( (6.4) \) \( (1.2) \) \( (50) \)

The supply functions for the single product model were obtained by substituting the 1964 actual data of feed grains and roughages prices into the functions 47-50 for the variables \( p_{gi} \) and \( p_{ri} \).

The functions are

- \( S_p^1 = 614.928 + 18.651 p_j \) \( (51) \)
\[ S_p^2 = 805.263 + 193.413p^2 \]  
\[ S_p^3 = 1045.933 + 92.684p^3 \]  
\[ S_p^4 = 1161.774 + 40.374p^4 \]

**REGIONAL DERIVED DEMAND FOR FEEDS**

The functional relationship for the derived demand for feeds is

\[ D_{f,i} = c_{oi,i} + c_{1,i}p_1 + c_{2,i}p_2 + c_{3,i}p_3 \]

where

\[ D_{f,i} = \text{quantities demanded for feed grains or roughages in region } i \]

\[ c_{oi,i} = \text{constant in region } i \]

The two stage least squares was again employed to derive the following statistical functions

**For Feed Grains**

\[ D_{g1} = 4146.00 + 9.49p_1 - 15.939pg_1 + 13.578P_{r1} \]
\[ (1057) \quad (5.6) \quad (3.4) \quad (7.1) \]

\[ D_{g2} = 117839 + 178.645p_2 - 1027.12pg_2 + 221.412P_{r2} \]
\[ (35493) \quad (102.9) \quad (412.0) \quad (137.6) \]

\[ D_{g3} = 16301 + 16.06p_3 - 181.818pg_3 + 74.134P_{r3} \]
\[ (572) \quad (7.4) \quad (65.1) \quad (36.8) \]

\[ D_{g4} = 40546 + 7.902p_4 - 381.43pg_4 + 3.9P_{r4} \]
\[ (9967) \quad (3.1) \quad (100.3) \quad (1.3) \]

**For Roughages**

\[ D_{r1} = 18231 + 10.412p_1 + 12.882pg_1 - 296.042P_{r1} \]
\[ (6739) \quad (8.7) \quad (5.4) \quad (111.7) \]

\[ D_{r2} = 74408 + 62.14p_2 + 224.412pg_2 - 944.12P_{r2} \]
\[ (36812) \quad (31.1) \quad (187.4) \quad (312.5) \]

\[ D_{r3} = 27960 + 40.792p_3 + 74.080pg_3 - 410.24P_{r3} \]
\[ (15369) \quad (19) \quad (51.9) \quad (108.7) \]
It was noted that the coefficient of feed grains (roughages) price in the supply function for beef was not equal to the coefficient of beef price in the derived demand function for feed grains (roughages) in the respective region. Under normal conditions, as in our case, the difference between the values of the coefficients should be small. Therefore, it was safe to take a simple or weighted average of the two in order to adjust the supply and demand functions. But if the difference is rather significant, and if this is due to some basic structural reasons, such as the existence of inefficient firms at the supply point, then the perfect competitive hypothesis is violated. However, this condition can still be handled by using a two step procedure. First the equilibrium prices are obtained and then substituted into the supply and demand equations. This will enable us to locate the surplus and deficit regions. As a result of this, the quantities of flows can be obtained either by trial and error, or by using a simple transportation model.

In our models, the supply and demand functions were adjusted by taking a simple average of the coefficients.

The adjusted functions for beef, feed grains and roughages are as follows:

Supply for Beef

\[
S_p^1 = 1270.913 + 18.651p_p^1 - 9.32p_{g1} - 8.807p_{r1} \quad (63)
\]

\[
S_p^2 = 8832.173 + 193.413p_p^2 - 177.5p_{g2} - 56.2p_{r2} \quad (64)
\]

\[
D_r4 = 36967 + 1.563p_p^4 + 2.781p_{g4} - 671.66p_{r4} \quad (62)
\]

\[
\text{Supply for Beef}
\]

\[
S_p^1 = 1270.913 + 18.651p_p^1 - 9.32p_{g1} - 8.807p_{r1} \quad (63)
\]

\[
S_p^2 = 8832.173 + 193.413p_p^2 - 177.5p_{g2} - 56.2p_{r2} \quad (64)
\]
\[ S_p^3 = 501.837 + 92.684p_p^3 - 16.904p_g^3 - 38.656p_r^3 \]  \hspace{1cm} (65) \\
\[ S_p^4 = 1545.682 + 40.374p_p^4 - 8.066p_g^4 - 1.439p_r^4 \]  \hspace{1cm} (66) \\

Demand for Feed Grains \\
\[ D_{g1} = 4146 + 9.32p_p^1 - 15.939p_g^1 + 13.23p_r^1 \]  \hspace{1cm} (67) \\
\[ D_{g2} = 117839 + 177.5p_p^2 - 1027.12p_g^2 + 222.912p_r^2 \]  \hspace{1cm} (68) \\
\[ D_{g3} = 16301 + 16.094p_p^3 - 181.818p_g^3 + 74.107p_r^3 \]  \hspace{1cm} (69) \\
\[ D_{g4} = 40546 + 8.066p_p^4 - 381.43p_g^4 - 3.34p_r^4 \]  \hspace{1cm} (70) \\

Demand for Roughages \\
\[ D_{r1} = 18231 + 8.807p_p^1 + 13.23p_g^1 - 296.042p_r^1 \]  \hspace{1cm} (71) \\
\[ D_{r2} = 74408 + 56.2p_p^2 + 222.912p_g^2 - 944.12p_r^2 \]  \hspace{1cm} (72) \\
\[ D_{r3} = 27960 + 38.656p_p^3 + 74.107p_g^3 - 410.24p_r^3 \]  \hspace{1cm} (73) \\
\[ D_{r4} = 36967 + 1.439p_p^4 + 3.34p_g^4 - 671.66p_r^4 \]  \hspace{1cm} (74) \\

**THE AVAILABILITY OF FEEDS** \\

The general form of the supply function for feeds is \\
\[ S_g(r)^j = d_0^j + d_1^j p_g(r)^j + d_2^j I_g(r)^j \]

where \\
\[ S_g^j = \text{quantity of feed grains supplied in region } j \] \\
\[ S_r^j = \text{quantity of roughages supplied in region } j \] \\
\[ d_0^j = \text{constant in the } j^{th} \text{ region} \] \\
\[ p_g^j = \text{supply price for feed grains in the } j^{th} \text{ region} \] \\
\[ p_r^j = \text{supply price for roughages in the } j^{th} \text{ region} \]
\( L_{g}^{j} \) = cropland available in region \( j \)

\( L_{r}^{j} \) = hay land available in region \( j \)

Ordinary least squares was used to derive the statistical equations which are as follows:

**Feed Grains**

\[
S_{g}^{1} = -855.2 + 6.432p_{g}^{1} + 0.26L_{g}^{1} \\
(134) \quad (2.5) \quad (0.04)
\]

\[
S_{g}^{2} = 87908.2 + 338.92p_{g}^{2} + 0.034L_{g}^{2} \\
(19147) \quad (261.3) \quad (0.01)
\]

\[
S_{g}^{3} = 1982.3 + 72.3p_{g}^{3} + 0.052L_{g}^{3} \\
(564) \quad (30.1) \quad (0.03)
\]

\[
S_{g}^{4} = 10524.592 + 76.4p_{g}^{4} + 0.1056L_{g}^{4} \\
(4315) \quad (28.5) \quad (0.05)
\]

**Roughages**

\[
S_{r}^{1} = 10143.815 + 8.105p_{r}^{1} + 0.015L_{r}^{1} \\
(2711) \quad (3.0) \quad (0.01)
\]

\[
S_{r}^{2} = 61531.164 + 4.156p_{r}^{2} + 0.012L_{r}^{2} \\
(24713) \quad (2.2) \quad (0.003)
\]

\[
S_{r}^{3} = 22743.928 + 2.154p_{r}^{3} + 0.00149L_{r}^{3} \\
(10014) \quad (0.35) \quad (0.0008)
\]

\[
S_{r}^{4} = 13673.888 + 10.145p_{r}^{4} + 0.3081L_{r}^{4} \\
(75) \quad (76) \quad (77) \quad (78)
\]

The 1965 data for available cropland was then substituted into the feed grain supply functions, thus resulting in the following equations:

\[
S_{g}^{1} = 2504.947 + 6.432p_{g}^{1} \\
(83)
\]

\[
S_{g}^{2} = 94426 + 338.92p_{g}^{2} \\
(84)
\]

\[
S_{g}^{3} = 4743.5 + 72.3p_{g}^{3} \\
(85)
\]

\[
S_{g}^{4} = 18622 + 76.4p_{g}^{4} \\
(86)
\]
The supply equations for roughages with hay land available held constant at the 1964 level are shown below:

\[ S_r^1 = 10247 + 8.105P_r^1 \]  \hspace{1cm} (87)
\[ S_r^2 = 61977 + 4.156P_r^2 \]  \hspace{1cm} (88)
\[ S_r^3 = 22762 + 2.154P_r^3 \]  \hspace{1cm} (89)
\[ S_r^4 = 18060 + 10.145P_r^4 \]  \hspace{1cm} (90)

**FUNCTION FOR TRANSPORTATION COST**

The transportation cost functions were adopted from studies by King and Schrader.\textsuperscript{20, 21} Their functions are indicated below. Transportation rates are expressed in cents per hundred pounds.

**Meat**

\[ r = 5.921 + 0.095836M + 2.48552M^2 \]

**Corn**

\[ r = 4.692 + 0.035548M + 0.604269M^2 \]

**Hay**

\[ r = 0.0 + 0.047341M + 2.238693M^2 \]

where \( M \) represents short-line rail mileage.

\textsuperscript{20} G. A. King and L. F. Schrader, \textit{op. cit.}

\textsuperscript{21} Although it would be desirable to use the published transportation rates, the data problem had made it very difficult to do so. Since there are no published rates for transportation between the trading points, it would be necessary to sum the rates between intermediate cities to obtain such data. Since these rates include loading and unloading charges, deductions would have to be made for them. These charges, however, are not specified in the published rates. Thus, it would be too difficult to handle the problem in this way.
The cost functions were then adjusted to the unit of measurement used in the analysis. The unit of measurement for beef was cents per pound, and for corn and hay it was dollars per ton. Corn and hay rates were used for feed grains and roughages respectively. There was no adjustment for intraregional beef shipments. However, intraregional transfer costs for feeds were deducted from the interregional costs to arrive at a net rate, assuming a radius of two hundred miles from the trading point.

The adjusted transfer cost functions are as follows:

**Meat**
\[ r = 0.05921 + 0.0009583M + 0.0248552M^2 \]  
(91)

**Corn**
\[ r = -3.85937 + 0.0071096M + 0.1208538M^2 \]  
(92)

**Hay**
\[ r = 11.78541 + 0.0094682M + 0.4477386M^2 \]  
(93)

The estimated transfer costs between the regions are shown in Table I.

**THE 1975 PROJECTED DATA**

Data used in the analyses for 1975 were based on the alternative projections for the demand relations, production patterns and transportation rates. Five alternative assumptions were made:

**Assumption A.**

New demand functions for beef were derived, because changes in consumption pattern, income and population were anticipated. The 1975
### TABLE I. ESTIMATES OF TRANSPORTATION RATES FOR BEEF, FEED GRAINS AND ROUGHAGES BETWEEN REGIONS

<table>
<thead>
<tr>
<th></th>
<th>BEEF  $/lb</th>
<th>FEED GRAINS $/ton</th>
<th>ROUGHAGES $/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3 4</td>
<td>1 2 3 4</td>
<td>1 2 3 4</td>
</tr>
<tr>
<td>NE</td>
<td>0 2.21 3.46 2.30</td>
<td>0 9.71 18.32 10.42</td>
<td>0 16.79 31.52 17.86</td>
</tr>
<tr>
<td>NC</td>
<td>2 0 1.89 1.65</td>
<td>0 7.63 5.07</td>
<td>0 12.86 10.01</td>
</tr>
<tr>
<td>W</td>
<td>3 0 2.96</td>
<td>0 14.92</td>
<td>0 25.72</td>
</tr>
<tr>
<td>S</td>
<td>4 0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: Computed from equations (91), (92) and (93)
population and income projections were substituted into demand functions (39-42) for the po and I variables. Further, the share of the quantity demanded for each region was calculated using data from the 1965 consumption survey. Thus the demand function follows the 1965 consumption pattern. The demand relationships are shown below:

\[
D_{p1} = 10134.754 - 43.12699p_{p1} \tag{94}
\]

\[
D_{p2} = 10874.905 - 41.068422p_{p2} \tag{95}
\]

\[
D_{p3} = 6084.654 - 28.09921p_{p3} \tag{96}
\]

\[
D_{p4} = 8454.291 - 44.09787p_{p4} \tag{97}
\]

**ASSUMPTION B.**

An increment of five percent was assumed for both beef production and feed consumption. The shift in beef production may result from increased efficiency in the beef industry. However, the shifts in the demand curves for feeds are brought about by the increase demand in the other livestock industries. The functions obtained are:

**Supply for Beef**

\[
S_{p1}^1 = 1334.459 + 19.584p_{p1}^1 - 9.786p_{g1} - 9.247p_{r1} \tag{98}
\]

\[
S_{p2}^2 = 9273.782 + 203.084p_{p2}^2 - 186.3p_{g2} - 59.01p_{r2} \tag{99}
\]

\[
S_{p3}^3 = 526.929 + 97.318p_{p3}^3 - 16.899p_{g3} - 40.589p_{r3} \tag{100}
\]

\[
S_{p4}^4 = 1622.966 + 42.939p_{p4}^4 - 8.469p_{g4} - 1.511p_{r4} \tag{101}
\]

**Demand for Feed Grains**

\[
D_{g1} = 4353.3 + 9.786p_{p1}^1 - 16.736p_{g1} + 13.892p_{r1} \tag{102}
\]

\[
D_{g2} = 123730.95 + 186.3p_{p2}^2 - 1078.476p_{g2} + 234.058p_{r2} \tag{103}
\]

\[
D_{g3} = 17116.05 + 16.899p_{p3}^3 - 190.909p_{g3} + 77.812p_{r3} \tag{104}
\]
\[ D_{g4} = 42573.3 + 8.469p_p^4 - 400.502p_g4 + 3.507p_r4 \]  
\[ (105) \]

Demand for Roughages

\[ D_{r1} = 19142.55 + 9.247p_p^1 + 13.892p_g1 - 310.844p_r1 \]  
\[ (106) \]

\[ D_{r2} = 78128.4 + 59.01p_p^2 + 234.058p_g2 - 991.326p_r2 \]  
\[ (107) \]

\[ D_{r3} = 29358 + 40.589p_p^3 + 77.812p_g3 - 430.752p_r3 \]  
\[ (108) \]

\[ D_{r4} = 38815.35 + 1.511p_p^4 + 3.507p_g4 - 705.243p_r4 \]  
\[ (109) \]

**ASSUMPTION C.**

The production of feed grains in the South was assumed to have a six percent increase, in view of the increase production of sorghum grain in the South. The following supply function was obtained

\[ S_{g3} = 19739.32 + 80.98p_g^4 \]  
\[ (110) \]

**ASSUMPTION D.**

A reduction in long-distance transportation rate was assumed due to economies of scale in the distance dimension of rail service. Long-distance rates beyond 1,000 miles were reduced in the following manner: 0.1¢ per pound for beef, $0.5 per ton for feed grains and $1.0 per ton for roughages for each additional 100 miles. The new transportation rate obtained are shown in Table II.
<table>
<thead>
<tr>
<th></th>
<th>BEEF</th>
<th>FEED GRAINS</th>
<th>ROUGHAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>NE</td>
<td>1</td>
<td>0</td>
<td>1.902</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.923</td>
<td>5.07</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>10.42</td>
<td>16.72</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>13.71</td>
<td>18.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14.09</td>
<td></td>
</tr>
<tr>
<td>NC</td>
<td>2</td>
<td>0</td>
<td>1.828</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>10.42</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16.72</td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>3</td>
<td>0</td>
<td>2.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>14.09</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: calculated by author
CHAPTER V
EMPIRICAL ANALYSES

In the previous chapters the theoretical model and the data relating to regional demands, supplies and transport costs were presented. Attention will now be given to an empirical study of the spatial equilibrium problems for beef, feed grains and roughages for the year 1964 and 1975. For each of the two years, two types of models were discussed. One was the single product model which gave the optimum flows and prices of the product only. The other was the factor-product model, where optimum flows and prices of the product as well as the factors were determined. Since the functional relationships of demand and supply were derived from time series data, the influence of prior trade is already included in the data. Thus, the flows obtained were subject to this influence.

The two types of the models can be described briefly as follows:

Model I: THE SINGLE PRODUCT MODEL

Only the demand and supply functions for beef were included. Prices for feed grains and roughages in the beef supply function were assumed constant for the respective years. No account was taken of demand and supply for feed grains and roughages. The solution yielded optimal interregional beef flows and prices to satisfy a given set of needs from a given set of sources, while the total transport cost was automatically minimized. The need in each location was represented by its demand curve, and the capacity of each source was demonstrated by its supply schedule.
Model II: FACTOR-PRODUCT MODEL

Model II differs from Model I in that the demand and supply functions for feed grains and roughages were also included. Feeds and beef were allowed to flow between regions. Prices and flows as well as quantities of beef, feed grains and roughages were determined within the system.

The quadratic programming solutions provided the following information for each of the regions studied:

1. Quantities supplied and demanded for beef.
2. Quantities supplied and demanded for feed grains and roughages. (Model II only).
3. Shipment pattern for beef.
4. Shipment pattern for feed grains and roughages. (Model II only).
5. Equilibrium prices for beef.
6. Equilibrium prices for feed grains and roughages. (Model II only).

For 1964, two sets of flows and prices were derived, one from each model. The purpose for obtaining this data is to provide a basis for comparison when the demand situation changes. Five submodels were formed based on alternative assumptions regarding consumption pattern, production pattern and transport cost for the year 1975. Only one submodel was of the single product type, the others were of the factor-product variety. Analyses of the results obtained under the models for 1964 and 1975 will be presented below. A comparison between the estimated results and the actual data were made for the 1964 models.
For the 1975 analyses, the influences of alternative assumptions on the solution were isolated.

THE PRICE AND FLOW ANALYSES FOR 1964

The two sets of prices and flows for 1964 will now be discussed.

1. Single Product Model

The main objective of this analysis was to derive the optimum prices and flows of beef. Price and quantity of beef were the two variables used in the demand and supply functions, (43-46, 51-54). The problem became one in which the net social pay-off is maximized subject to transportation cost. The quadratic programming problem in matrix notation is shown below:

Maximize

\[ D'P - \frac{1}{2}P'EP \]

Subject to

\[ AP \leq T \]
\[ P \geq 0 \]

where

\[ D' = \begin{bmatrix} 6374.306 \\ 7795.009 \\ 5111.385 \\ 4279.094 \\ -614.928 \\ -805.623 \\ 1045.933 \\ -1161.774 \end{bmatrix} \]

\[ P = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} \]
The set-up tableau of this problem is shown in Table III.

The solutions to the problem are presented in Table IV. Interregional flows derived are shown in Figure 2. The North Central and South were surplus regions, while the Northeast and West were deficit regions. The nature of the solutions may be illustrated by the Northeast region. The total demand for beef was 4539.2645 million pounds. Besides the 1498.2085 million pounds produced at home, 2979.2085 million pounds were received from the North Central region and 61.351 million pounds from the South.

A comparison of the programming solutions with the actual data (1964) was made in Table V. Since no account was taken of the beef imported in the production data used, there was a discrepancy between quantities demanded and supplied. The estimates are quite close to
TABLE III. SET-UP TABLEAU FOR THE SINGLE PRODUCT MODEL

<table>
<thead>
<tr>
<th>B.V. Value</th>
<th>B.V.</th>
<th>u₁</th>
<th>u₂</th>
<th>u₃</th>
<th>u₄</th>
<th>u₅</th>
<th>u₆</th>
<th>u₇</th>
<th>u₈</th>
<th>A₁₁</th>
<th>A₁₂</th>
<th>A₁₃</th>
<th>A₁₄</th>
<th>A₂₁</th>
<th>A₂₂</th>
<th>A₂₃</th>
<th>A₂₄</th>
<th>A₃₁</th>
<th>A₃₂</th>
<th>A₃₃</th>
<th>A₃₄</th>
<th>A₄₁</th>
<th>A₄₂</th>
<th>A₄₃</th>
<th>A₄₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>u₁</td>
<td>-6374.306</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>u₂</td>
<td>-7795.009</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>u₃</td>
<td>-5111.385</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>u₄</td>
<td>-4279.094</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>u₅</td>
<td>614.928</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>u₆</td>
<td>805.623</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>u₇</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>u₈</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y₁₁</td>
<td></td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y₁₂</td>
<td></td>
<td>2.21</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y₁₃</td>
<td></td>
<td>3.46</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y₁₄</td>
<td></td>
<td>2.30</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y₁₅</td>
<td></td>
<td>2.21</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y₁₆</td>
<td></td>
<td>1.89</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y₁₇</td>
<td></td>
<td>1.65</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y₁₈</td>
<td></td>
<td>3.46</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y₁₉</td>
<td></td>
<td>1.89</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y₂₀</td>
<td></td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y₂₁</td>
<td></td>
<td>2.96</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y₂₂</td>
<td></td>
<td>2.30</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y₂₃</td>
<td></td>
<td>1.65</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y₂₄</td>
<td></td>
<td>2.96</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y₂₅</td>
<td></td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f</td>
<td></td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B.V. Value</th>
<th>B.V.</th>
<th>u₁</th>
<th>u₂</th>
<th>u₃</th>
<th>u₄</th>
<th>u₅</th>
<th>u₆</th>
<th>u₇</th>
<th>u₈</th>
<th>A₁₁</th>
<th>A₁₂</th>
<th>A₁₃</th>
<th>A₁₄</th>
<th>A₂₁</th>
<th>A₂₂</th>
<th>A₂₃</th>
<th>A₂₄</th>
<th>A₃₁</th>
<th>A₃₂</th>
<th>A₃₃</th>
<th>A₃₄</th>
<th>A₄₁</th>
<th>A₄₂</th>
<th>A₄₃</th>
<th>A₄₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
### TABLE III. (Continued) SET-UP TABLEAU FOR THE SINGLE PRODUCT MODEL

<table>
<thead>
<tr>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$p_3$</th>
<th>$p_4$</th>
<th>$p^1$</th>
<th>$p^2$</th>
<th>$p^3$</th>
<th>$p^4$</th>
<th>$y_{11}$</th>
<th>$y_{21}$</th>
<th>$y_{31}$</th>
<th>$y_{41}$</th>
<th>$y_{12}$</th>
<th>$y_{22}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-38.748</td>
<td>-38.199</td>
<td>18.651</td>
<td>-193.413</td>
<td>-92.684</td>
<td>-40.374</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>-27.855</td>
<td>-30.172</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>-30.775</td>
<td>-51.113</td>
<td>-3.854-42.79</td>
<td>0.94+614.928+805.623</td>
<td>1045.933+1161.774</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
TABLE III. (Continued) SET-UP TABLEAU FOR THE SINGLE PRODUCT MODEL

<table>
<thead>
<tr>
<th>$y_{32}$</th>
<th>$y_{42}$</th>
<th>$y_{13}$</th>
<th>$y_{23}$</th>
<th>$y_{33}$</th>
<th>$y_{43}$</th>
<th>$y_{14}$</th>
<th>$y_{24}$</th>
<th>$y_{34}$</th>
<th>$y_{44}$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>REGION</td>
<td>NORTHEAST</td>
<td>NORTH CENTRAL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------</td>
<td>-------------------------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>PRODUCTION</td>
<td>CONSUMPTION</td>
<td>PRICE</td>
<td>PRODUCTION</td>
<td>CONSUMPTION</td>
<td>PRICE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flow</td>
<td>FROM</td>
<td>TO</td>
<td></td>
<td>FROM</td>
<td>TO</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>mil. lb.</td>
<td>mil. lb.</td>
<td>¢</td>
<td>mil. lb.</td>
<td>mil. lb.</td>
<td>¢</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. E.</td>
<td>1498.2085</td>
<td>1498.2085</td>
<td>47.36</td>
<td>0</td>
<td>9537.5061</td>
<td>45.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(within)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. C.</td>
<td>2979.7048</td>
<td></td>
<td></td>
<td>6070.3672</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(within)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(within)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>0</td>
<td>W</td>
<td></td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>61.3512</td>
<td>S</td>
<td></td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>WEST</td>
<td>SOUTH</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>3313.6960</td>
<td>3801.1301</td>
<td>47.04</td>
<td>0</td>
<td>2980.9493</td>
<td>2919.5981</td>
<td>45.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. C.</td>
<td>487.4341</td>
<td></td>
<td></td>
<td>N. C.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>3313.6960</td>
<td>W</td>
<td></td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>0</td>
<td>S</td>
<td></td>
<td>2919.5981</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: calculated by author
FIG. 2 INTERREGIONAL FLOWS OF BEEF
1964 SINGLE-PRODUCT MODEL
<table>
<thead>
<tr>
<th>REGION</th>
<th>NORTHEAST</th>
<th></th>
<th></th>
<th>NORTH CENTRAL</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ESTIMATED</td>
<td>ACTUAL</td>
<td></td>
<td>ESTIMATED</td>
<td>ACTUAL</td>
<td></td>
</tr>
<tr>
<td>PROD.</td>
<td>1498.209</td>
<td>1306.017</td>
<td>9537.506</td>
<td>10231.43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CONS.</td>
<td>4539.265</td>
<td>4909.35</td>
<td>6070.367</td>
<td>6464.228</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PRICE</td>
<td>47.36</td>
<td>50.04</td>
<td>45.15</td>
<td>46.47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>REGION</td>
<td>WEST</td>
<td></td>
<td></td>
<td>SOUTH</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PROD.</td>
<td>3313.696</td>
<td>3687.419</td>
<td>2980.949</td>
<td>2898.979</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CONS.</td>
<td>3801.13</td>
<td>3986.793</td>
<td>2919.598</td>
<td>3999.744</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PRICE</td>
<td>47.04</td>
<td>47.58</td>
<td>45.06</td>
<td>48.06</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: calculated by author
actual figures, thus, they are used as a basis for comparison with the 1975 projections.

2. **Factor-Product Model**

In this analysis the principal interest was to determine the optimum patterns of interregional flows and prices of beef, feed grains and roughages. Using demand and supply functions (43-46, 63-74). The programming model formulated is shown as follows:

To maximize

\[ F(P) = D'P - \frac{1}{2}P'EP \]

Subject to

\[ AP \leq T \]

\[ P \geq 0 \]

where

\[
D' = \begin{bmatrix}
6374.306 \\
7795.009 \\
5111.385 \\
4279.094 \\
4146.000 \\
117839.000 \\
16301.000 \\
40546.000 \\
18231.000 \\
74408.000 \\
27960.000 \\
36967.000 \\
1270.913 \\
8832.173 \\
501.837 \\
1545.682 \\
2504.947 \\
94426.000 \\
4743.500 \\
18622.000 \\
10247.000 \\
61977.00 \\
22762.000 \\
18060.000
\end{bmatrix}
\]

\[ P = \begin{bmatrix}
p_{P1} \\
p_{P2} \\
p_{P3} \\
p_{P4} \\
p_{pg1} \\
p_{pg2} \\
p_{pg3} \\
p_{pg4} \\
p_{Pr1} \\
p_{Pr2} \\
p_{Pr3} \\
p_{Pr4}
\end{bmatrix} \]
\[
E = \begin{bmatrix}
38.748 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 38.199 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 27.855 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 30.172 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 15.939 & 0 & 0 & 0 & -13.23 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1027.12 & 0 & 0 & 0 & -222.912 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 181.82 & 0 & 0 & 0 & -74.107 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 381.43 & 0 & 0 & 0 & -3.34 \\
0 & 0 & 0 & 0 & 0 & -13.23 & 0 & 0 & 0 & 296.04 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -222.912 & 0 & 0 & 0 & 944.12 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -74.107 & 0 & 0 & 0 & 410.24 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -3.34 & 0 & 0 & 0 & 671.66 \\
0 & 0 & 0 & 0 & 0 & -9.32 & 0 & 0 & 0 & -8.807 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -177.5 & 0 & 0 & 0 & -56.2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -16.094 & 0 & 0 & 0 & -38.656 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -8.066 & 0 & 0 & 0 & -1.439 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
\[
\begin{bmatrix}
0 & 0 & 0 & -1.439 & 0 & 0 & 0 & 0 & 0 & 0 \\
18.651 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 193.413 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 92.684 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 40.374 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 6.432 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 338.92 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 72.3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 76.4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8.105 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.156 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.154 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10.145
\end{bmatrix}
\]
\[
T = \begin{pmatrix}
0 \\
2.21 \\
3.46 \\
2.30 \\
0 \\
9.71 \\
18.32 \\
10.42 \\
0 \\
16.79 \\
31.52 \\
17.86 \\
2.21 \\
0 \\
1.89 \\
1.65 \\
9.71 \\
0 \\
7.63 \\
5.07 \\
16.79 \\
0 \\
12.86 \\
10.01 \\
3.46 \\
1.89 \\
0 \\
2.96 \\
18.32 \\
7.63 \\
0 \\
14.92 \\
31.52 \\
12.86 \\
0 \\
25.72 \\
2.30 \\
1.65 \\
2.96 \\
0 \\
10.42 \\
5.07 \\
14.92 \\
0 \\
17.86 \\
10.01 \\
25.72 \\
0
\end{pmatrix}
\]
The problem was then solved by using the simplex method for quadratic programming.

Solutions are shown in Table VI and Figure 3. To make it more understandable, take North Central for example; the total beef demanded in this region was 6124.4 mil. lb., total beef production was 9971.5 mil. lb.. The surplus amount was exported to Northeast (3130.9 mil. lb.) and West (716.17 mil. lb.). Feed grains produced approached 105,664.08 thousand tons of this amount, 96,202.3 thousand tons was demanded at home, and 1,486.8 thousand tons was shipped to Northeast, 3,578.07 thousand tons to West and 4,596.86 thousand tons to South. The region was just self-sufficient in roughages production.

A comparison of beef price, production and consumption between this model and the single product one shows that the equilibrium beef price in each region was lower than that of the single product model, thus permitted more beef consumption in each region.

The estimated results were also compared with the 1964 actual data. This is shown in Table VII.

PRICE AND SPATIAL ANALYSES FOR 1975

All the demand relationships for beef in the analyses for 1975 were represented by the adjusted functions given in equations (94-97). Other functions regarding production pattern and transportation costs were made to depend on alternative assumptions.

1. Single Product Analysis

New demand functions were derived for 1975. Population and
### TABLE VI. EQUILIBRIUM PATTERN OF PRODUCTION, PRICES AND FLOWS OF BEEF, FEED GRAINS AND ROUGHAGES IN EACH REGION, 1964

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>1463.1853</td>
<td>1463.1853</td>
<td>4604.0999</td>
<td>45.943</td>
</tr>
<tr>
<td></td>
<td>(within)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. C.</td>
<td>3130.9146</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**FEED GRAINS thous. ton $/ton**

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>2784.5020</td>
<td>2784.5020</td>
<td>4271.3340</td>
<td>43.461</td>
</tr>
<tr>
<td></td>
<td>(within)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. C.</td>
<td>1486.8320</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**ROUGHAGES thous. ton $/ton**

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>10485.8242</td>
<td>10485.8242</td>
<td>10485.8242</td>
<td>29.471</td>
</tr>
<tr>
<td>N. C.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE VI. (Continued) EQUILIBRIUM PATTERN OF PRODUCTION, PRICES AND
FLOWS OF BEEF, FEED GRAINS AND ROUGHAGES IN EACH REGION, 1964

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BEEF mil. lb.</td>
<td></td>
<td>$/lb.</td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>9971.5072</td>
<td>6124.4219</td>
<td>43.733</td>
</tr>
<tr>
<td>N. C.</td>
<td>6124.4219</td>
<td>(within)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>FEED GRAINS thous. ton</td>
<td>$/ton</td>
<td></td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>105664.0749</td>
<td>96202.3125</td>
<td>33.751</td>
</tr>
<tr>
<td>N. C.</td>
<td>96202.3125</td>
<td>(within)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>ROUGHAGES thous. ton</td>
<td>$/ton</td>
<td></td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>62074.9766</td>
<td>62074.9766</td>
<td>29.471</td>
</tr>
<tr>
<td>N. C.</td>
<td>62074.9766</td>
<td>(within)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE VI. (Continued) EQUILIBRIUM PATTERN OF PRODUCTION, PRICES AND FLOWS OF BEEF, FEED GRAINS AND ROUGHAGES IN EACH REGION, 1964

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE c/lb.</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>0</td>
<td>3124.3738</td>
<td>3840.5445</td>
<td>45.623</td>
</tr>
<tr>
<td>N. C.</td>
<td>716.1707</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>3124.3738 (within)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>FEED GRAINS thous. ton</th>
<th>$/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>0</td>
<td>7735.3320</td>
<td>41.381</td>
</tr>
<tr>
<td>N. C.</td>
<td>3578.0710</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>7735.3320 (within)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>ROUGHAGES thous. ton</th>
<th>$/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>0</td>
<td>22814.3594</td>
<td>24.317</td>
</tr>
<tr>
<td>N. C.</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>22814.3594 (within)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE VI. (Continued) EQUILIBRIUM PATTERN OF PRODUCTION, PRICES AND FLOWS OF BEEF, FEED GRAINS AND ROUGHAGES IN EACH REGION, 1964

<table>
<thead>
<tr>
<th>REGION</th>
<th>SOUTH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BEEF mil. lb.</td>
</tr>
<tr>
<td></td>
<td>N. E. 0</td>
</tr>
<tr>
<td></td>
<td>N. C. 0</td>
</tr>
<tr>
<td></td>
<td>S. 2958.8638 (within)</td>
</tr>
</tbody>
</table>

|        | FEED GRAINS thous. ton | $/ton |
|        | N. E. 0 | 21587.9180 | 29184.7774 | 38.821 |
|        | N. C. 7596.8594 | 0 |
|        | W. 0 | 0 |
|        | S. 21587.9180 (within) |

|        | ROUGHAGES thous. ton | $/ton |
|        | N. E. 0 | 18344.1406 | 18344.1406 | 28.013 |
|        | N. C. 0 | 0 |
|        | W. 0 | 0 |
|        | S. 18344.1406 (within) |

Source: calculated by author
FIG. 3 INTERREGIONAL FLOWS FOR BEEF, FEED GRAINS AND ROUGHAGES
1964 FACTOR-PRODUCT MODEL
<table>
<thead>
<tr>
<th>REGION</th>
<th>ESTIMATED</th>
<th>ACTUAL</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PRODUCTION</td>
<td>CONSUMPTION</td>
<td>PRICE</td>
<td>PRODUCTION</td>
<td>CONSUMPTION</td>
<td>PRICE</td>
</tr>
<tr>
<td>NORTH</td>
<td>BEEF mil. lb.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CENTRAL</td>
<td>price ¢/lb.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>REGION</td>
<td>FEED GRAINS thous.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ton</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>price $/ton</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ROUGHAGES thous.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ton</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>price $/ton</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| REGION       |          |          |          |          |          |          |
| NORTHEAST    |          |          |          |          |          |          |
|              | 1463.1853| 4604.099 | 45.943   | 1306.017 | 4909.35   | 50.04     |
|              | 2784.5020| 4271.3340| 43.461   | 3140.520 | 3140.520  | 46.972    |
|              | 10485.8242| 10485.8242| 29.471 | 10828.000| 10828.000| 31.411 |
| NORTH        | CENTRAL  |          |          |          |          |          |
|              |          |          |          |          |          |          |
|              | 9971.5072| 6124.4219| 43.733   | 10231.43 | 6464.228  | 46.47     |
|              | 105664.075| 96202.3125| 33.751 | 104888.004| 104888.004| 39.567 |
|              | 62074.9766| 62074.9766| 29.471 | 62293.000| 62293.000| 20.965 |
TABLE VII. (Continued) COMPARISON OF ESTIMATED BEEF, FEED GRAINS AND ROUGHAGES PRODUCTION, CONSUMPTION AND PRICES WITH THE ACTUAL DATA, 1964

<table>
<thead>
<tr>
<th>ESTIMATED</th>
<th>ACTUAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRODUCTION</td>
<td>CONSUMPTION</td>
</tr>
<tr>
<td>REGION</td>
<td>WEST</td>
</tr>
<tr>
<td>BEEF mil. lb.</td>
<td></td>
</tr>
<tr>
<td>price ¢/lb.</td>
<td></td>
</tr>
<tr>
<td>3124.3738</td>
<td>3840.5445</td>
</tr>
<tr>
<td>FEED GRAINS thous. ton</td>
<td></td>
</tr>
<tr>
<td>price $/ton</td>
<td></td>
</tr>
<tr>
<td>7735.3320</td>
<td>11313.4030</td>
</tr>
<tr>
<td>ROUGHAGES thous. ton</td>
<td></td>
</tr>
<tr>
<td>price $/ton</td>
<td></td>
</tr>
<tr>
<td>22814.3594</td>
<td>22814.3594</td>
</tr>
<tr>
<td>REGION</td>
<td>SOUTH</td>
</tr>
<tr>
<td>BEEF mil. lb.</td>
<td></td>
</tr>
<tr>
<td>price ¢/lb.</td>
<td></td>
</tr>
<tr>
<td>2958.8638</td>
<td>2958.8638</td>
</tr>
<tr>
<td>FEED GRAINS thous. ton</td>
<td></td>
</tr>
<tr>
<td>price $/ton</td>
<td></td>
</tr>
<tr>
<td>21587.918</td>
<td>29184.7774</td>
</tr>
<tr>
<td>ROUGHAGES thous. ton</td>
<td></td>
</tr>
<tr>
<td>price $/ton</td>
<td></td>
</tr>
<tr>
<td>18344.1406</td>
<td>18344.1406</td>
</tr>
</tbody>
</table>

Source: calculated by author
income were used as exogeneous variables. The civilian population projected for 1975 was 222 million. Personal disposable income was assumed to increase 2.5 percent per year. The demand functions previously derived were then adjusted by this population and income data. In addition, they were also modified according to the 1965 Household Food Consumption Survey.\(^{22}\) The new demand functions were equations (94-97). The supply relationships for beef and transport costs were assumed to remain unchanged. Thus, the functions (51-54) adopted for the 1964 single product analysis were again used. The equilibrium of beef production, consumption, prices and flows are shown in Table VIII and Figure 4. The nature of the solution may be illustrated with the North Central region, which produced 13873.9 million pounds of beef. The demand within the region was 8099.97 million pounds. Of the excess amount produced, 4328.47 million pounds were shipped to Northeast and 1445.47 million pounds to the South.

The resultant estimates were then compared with the ones for 1964. The new demand relationships showed that by 1975 the beef consumption in the South will increase drastically, relative to the demand in other regions. Since only demand functions were assumed to have changed in this single product model, the influence of the change in regional consumption pattern can be detected from the results. Due to the increase in population and income, the total consumption in each region has increased. This in turn will raise the prices to a higher

TABLE VIII. EQUILIBRIUM PATTERN OF PRODUCTION, CONSUMPTION, PRICES AND FLOWS OF BEEF IN EACH REGION, 1975 (MODEL 1)

<table>
<thead>
<tr>
<th>FLOW</th>
<th>FROM</th>
<th>TO NORTHEAST</th>
<th>FROM</th>
<th>TO NORTH CENTRAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PRODUCTION</td>
<td>CONSUMPTION</td>
<td>PRICE</td>
<td>PRODUCTION</td>
</tr>
<tr>
<td></td>
<td>mil. lb.</td>
<td>¢/lb.</td>
<td>mil. lb.</td>
<td>¢/lb.</td>
</tr>
<tr>
<td>N. E.</td>
<td>1916.369</td>
<td>1916.369</td>
<td>7125.3809</td>
<td>69.779</td>
</tr>
<tr>
<td>N. C.</td>
<td>4328.4767</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>880.5352</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

REGION WEST SOUTH

<table>
<thead>
<tr>
<th></th>
<th>WEST</th>
<th>SOUTH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PRODUCTION</td>
<td>CONSUMPTION</td>
</tr>
<tr>
<td></td>
<td>mil. lb.</td>
<td>¢/lb.</td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>5100.6875</td>
</tr>
<tr>
<td>N. C.</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>5100.6875</td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Source: calculated by author
FIG. 4 INTERREGIONAL FLOWS OF BEEF
1975 SINGLE-PRODUCT MODEL
level in order to stimulate more production. The changes in the share of each region were reflected by its prices and flows. The South region has shifted from being a surplus region to a deficit region. The western region has changed from an importing region to an exporting region. The North Central now exports to the Northeast and South. The Northeast imports beef from North Central and West rather than from North Central and South.

2. Factor-Product Analyses

Five solutions of the factor-product type were obtained under assumed changes that may be expected to occur by the mid-seventies. The alternatives selected for this study were given in the previous chapter and designated as A, B, C, D. In some cases, only one function, and in others, more than one functions were assumed to change. The expected equilibrium patterns resulting from these changes were simulated on the IBM 360.

The spatial solutions of the alternative projections were as follows:

2.1 Changes in consumption pattern, income and population.

(Assumption A)

The new demand relationships resulting from increase in population and income and changes in consumption pattern were given in equations (94-97). Other functions (supply functions for beef, supply and demand functions for feeds) were assumed to persist throughout the years. The solutions are shown in Table IX and Figure 5.
TABLE IX. EQUILIBRIUM PATTERN OF PRODUCTION, PRICES AND FLOWS OF BEEF, FEED GRAINS AND ROUGHAGES IN EACH REGION, 1975 (MODEL 2.1)

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BEEF mil. lb.</td>
<td></td>
<td></td>
<td>¢/lb.</td>
</tr>
<tr>
<td></td>
<td>N. E. 1879.7051</td>
<td>1879.7051</td>
<td>7115.1968</td>
<td>70.015</td>
</tr>
<tr>
<td></td>
<td>within</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>N. C. 4522.8828</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>W. 712.6089</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S. 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>FEED GRAINS thous. ton</td>
<td></td>
<td></td>
<td>$/ton</td>
</tr>
<tr>
<td></td>
<td>N. E. 2801.9473</td>
<td>2801.9473</td>
<td>4463.2508</td>
<td>46.172</td>
</tr>
<tr>
<td></td>
<td>within</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>N. E. 1661.3035</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>W. 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S. 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ROUGHAGES thous. ton</td>
<td></td>
<td></td>
<td>$/ton</td>
</tr>
<tr>
<td></td>
<td>N. E. 10492.4531</td>
<td>10492.4531</td>
<td>10492.4531</td>
<td>30.286</td>
</tr>
<tr>
<td></td>
<td>within</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>N. C. 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>W. 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S. 0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE IX. (Continued) EQUILIBRIUM PATTERN OF PRODUCTION, PRICES AND FLOWS OF BEEF, FEED GRAINS AND ROUGHAGES IN EACH REGION, 1975 (MODEL 2.1)

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BEEF mil. lb.</td>
<td></td>
<td></td>
<td>$/lb.</td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>14030.1521</td>
<td>8090.2578</td>
<td>67.805</td>
</tr>
<tr>
<td>N. C.</td>
<td>8090.2578 (within)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>FEED GRAINS thous. ton</td>
<td>$/ton</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>106783.0483</td>
<td>98150.9375</td>
<td>36.462</td>
</tr>
<tr>
<td>N. C.</td>
<td>98150.9375 (within)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ROUGHAGES thous. ton</td>
<td>$/ton</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>62083.5547</td>
<td>62083.5547</td>
<td>25.699</td>
</tr>
<tr>
<td>N. C.</td>
<td>62083.5547 (within)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>REGION</td>
<td>FLOW</td>
<td>PRODUCTION</td>
<td>CONSUMPTION</td>
<td>PRICE</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td>------------</td>
<td>-------------</td>
<td>-------</td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>4926.1128</td>
<td>4213.5039</td>
<td>66.555</td>
</tr>
<tr>
<td>N. C.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>4213.5039</td>
<td>(within)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>FEED GRAINS thous. ton</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>0</td>
<td>7931.3320</td>
<td>11338.8947</td>
<td>44.092</td>
</tr>
<tr>
<td>N. C.</td>
<td>3407.5627</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>7931.3320</td>
<td>(within)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>ROUGHAGES thous. ton</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>0</td>
<td>22819.6367</td>
<td>22819.6367</td>
<td>26.766</td>
</tr>
<tr>
<td>N. C.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>22819.6367</td>
<td>(within)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>REGION</td>
<td>FLOW</td>
<td>PRODUCTION</td>
<td>CONSUMPTION</td>
<td>PRICE</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td>------------</td>
<td>-------------</td>
<td>-------</td>
</tr>
<tr>
<td></td>
<td>BEEF mil. lb.</td>
<td>¢/lb.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>3974.4299</td>
<td>5391.4414</td>
<td>69.455</td>
</tr>
<tr>
<td>N. C.</td>
<td>1417.0115</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>3974.4299</td>
<td>(within)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>FEED GRAINS thous. ton</td>
<td>$/ton</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>21795.0430</td>
<td>25358.2876</td>
<td>41.532</td>
</tr>
<tr>
<td>N. C.</td>
<td>3563.2446</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>21795.0430</td>
<td>(within)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ROUGHAGES thous. ton</td>
<td>$/ton</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>18344.8320</td>
<td>18344.8320</td>
<td>28.081</td>
</tr>
<tr>
<td>N. C.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>18344.8320</td>
<td>(within)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: calculated by author
FIG. 5 INTERREGIONAL FLOWS FOR BEEF, FEED GRAINS AND ROUGHAGES
1975 FACTOR-PRODUCT MODEL 2.1
Compared with the 1964 factor-product results, the influence of a change in demand functions for beef was quite obvious. The expansion in total beef consumption in each region due to population and income increase has shifted the demand curve to the right, thus resulting in higher prices and more production. Since no import from outside the United States was considered, the equilibrium beef production should equal total beef consumption. The higher level of beef price and production will stimulate more demand in feed grains and roughages, thus increasing the beef price and supply of feeds. But the shift of the demand curve need not be a parallel one, due to changes in the consumption pattern. Although the beef consumption in the West has increased, its relative share in the total U. S. beef consumption decreased. Hence, the West has changed from a deficit region into a surplus region. In contrast, because of the drastic increase in the share of beef consumption in the South, it has shifted from an exporting region to an importing region.

2.2 Changes in production of beef and demand for feed grains and roughages. (Assumption B)

The production of beef and the derived demand for feeds were assumed to have increased five percent uniformly. The increase in beef production may result from increase efficiency in the beef industry, while the increase in feed consumption may be due to increase demand for other livestock and products. Alternatives A and B were both brought into this analysis. The new functions derived are given in equations (94-109). All the other data remained at their previous
levels. The equilibrium patterns are shown in Table X and Figure 6.

A comparison between this analysis and the one given above (2.1) demonstrated how the introduction of assumption B influenced the solution. An expansion in production of beef will ordinarily bring about an increase in feed demanded. Since an increase in the demand of feed was already assumed, combining the two forces, the increase in prices of feeds will be quite large. The higher level of feed prices will induce more feed grains and roughages production, but it will also curtail beef production. The influence of the increase in the feed price may be able to offset the expansion in beef production, thus actually reducing the total beef supply. This situation was evident in the North Central region, since the coefficients that denote the effect of a change in feed grains and roughages on the beef supply function in this region were quite significant. As a result, production in the North Central contracted rather than expanded.

2.3 Changes in the production of feed grains in the South.

(Assumption C)

The production of feed grains in the South was assumed to have increased six percent, in view of the increase production trend of sorghum grain in the South. The new function was given in equation (100). Combining this assumption with the previous two assumptions (A and B), the equilibrium prices and flows of beef, feed grains and roughages are as presented in Table XI and Figure 7.

If these results are compared with those derived from 2.2, the influence of the change can be singled out. Because the total feed
TABLE X. EQUILIBRIUM PATTERN OF PRODUCTION, PRICES AND FLOWS OF BEEF, FEED GRAINS AND ROUGHAGES IN EACH REGION, 1975 (MODEL 2.2)

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BEEF mil. lb.</td>
<td></td>
<td></td>
<td>¢/lb.</td>
</tr>
<tr>
<td>N. E.</td>
<td>1919.4182</td>
<td>1919.4182</td>
<td>7117.2464</td>
<td>69.967</td>
</tr>
<tr>
<td>N. C.</td>
<td>4442.4102</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>755.4180</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FEED GRAINS thous. ton</th>
<th>$/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>2826.0149</td>
</tr>
<tr>
<td>N. C.</td>
<td>1822.3303</td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ROUGHAGES thous. ton</th>
<th>$/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>10507.0703</td>
</tr>
<tr>
<td>N. C.</td>
<td>0</td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
</tr>
</tbody>
</table>
TABLE X. (Continued) EQUILIBRIUM PATTERN OF PRODUCTION, PRICES AND FLOWS OF BEEF, FEED GRAINS AND ROUGHAGES IN EACH REGION, 1975 (MODEL 2.2)

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BEEF mil. lb.</td>
<td></td>
<td></td>
<td>ç/lb.</td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>13790.7415</td>
<td>8092.2188</td>
<td>67.757</td>
</tr>
<tr>
<td>N. C.</td>
<td>8092.2188 (within)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REGION</th>
<th>FEED GRAINS thous. ton</th>
<th>$/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>0</td>
<td>108052.8823</td>
</tr>
<tr>
<td>N. C.</td>
<td>99938.0625 (within)</td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REGION</th>
<th>ROUGHAGES thous. ton</th>
<th>$/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>0</td>
<td>62100.2578</td>
</tr>
<tr>
<td>N. C.</td>
<td>62100.2578 (within)</td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
TABLE X. (Continued) EQUILIBRIUM PATTERN OF PRODUCTION, PRICES AND FLOWS OF BEEF, FEED GRAINS AND ROUGHAGES IN EACH REGION, 1975 (MODEL 2.2)

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BEEF mil. lb.</td>
<td></td>
<td>c/lb.</td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>4970.2657</td>
<td>4214.8477</td>
<td>66.507</td>
</tr>
<tr>
<td>N. C.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>4214.8477 (within)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>FEED GRAINS thous. ton</th>
<th>$/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>0</td>
<td>8202.2930</td>
<td>47.840</td>
</tr>
<tr>
<td>N. C.</td>
<td>3244.4536</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>8202.2930 (within)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>ROUGHAGES thous. ton</th>
<th>$/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>0</td>
<td>22826.7266</td>
<td>30.071</td>
</tr>
<tr>
<td>N. C.</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>22826.7266 (within)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE X. (Continued) EQUILIBRIUM PATTERN OF PRODUCTION, PRICES AND FLOWS OF BEEF, FEED GRAINS AND ROUGHAGES IN EACH REGION, 1975 (MODEL 2.2)

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BEEF mil. lb.</td>
<td></td>
<td>ç/lb.</td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>4137.4297</td>
<td>5393.5422</td>
<td>69.407</td>
</tr>
<tr>
<td>N. C.</td>
<td>1256.1125</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>4137.4297 (within)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>FEED GRAINS thous. ton</td>
<td>$/ton</td>
<td></td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>22081.355</td>
<td>25129.3914</td>
<td>45.280</td>
</tr>
<tr>
<td>N. C.</td>
<td>3048.0359</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>22081.355 (within)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>ROUGHAGES thous. ton</td>
<td>$/ton</td>
<td></td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>18358.0352</td>
<td>18358.0352</td>
<td>29.381</td>
</tr>
<tr>
<td>N. C.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>18358.0352 (within)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: calculated by author
FIG. 6 INTERREGIONAL FLOWS FOR BEEF, FEED GRAINS AND ROUGHAGES
1975 FACTOR-PRODUCT MODEL 2.2
TABLE XI. EQUILIBRIUM PATTERN OF PRODUCTION, CONSUMPTION, PRICES AND FLOWS OF BEEF, FEED GRAINS AND ROUGHAGES IN EACH REGION, 1975 (MODEL 2.3)

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BEEF mil. lb.</td>
<td></td>
<td>c/lb.</td>
</tr>
<tr>
<td>N. E.</td>
<td>1920.0837</td>
<td>1920.0837</td>
<td>7130.8229</td>
<td>69.652</td>
</tr>
<tr>
<td>(within)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. C.</td>
<td>4477.5820</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>733.1572</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>FEED GRAINS thous. ton</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>2821.7795</td>
<td>2821.7795 4655.8247</td>
<td>49.257</td>
</tr>
<tr>
<td>(within)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. C.</td>
<td>1834.0452</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>ROUGHAGES thous. ton</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>10506.7891</td>
<td>10506.7891 10506.7891</td>
<td>32.055</td>
</tr>
<tr>
<td>(within)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. C.</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE XI. (Continued) EQUILIBRIUM PATTERN OF PRODUCTION, CONSUMPTION, PRICES AND FLOWS OF BEEF, FEED GRAINS AND ROUGHAGES IN EACH REGION, 1975 (MODEL 2.3)

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BEEF mil. lb.</td>
<td></td>
<td>$/lb.</td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>13860.4758</td>
<td>8105.1406</td>
<td>67.442</td>
</tr>
<tr>
<td>N. C.</td>
<td>8105.1406 (within)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>FEED GRAINS thous. ton</th>
<th>$/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>0</td>
<td>107828.4798</td>
</tr>
<tr>
<td>N. C.</td>
<td>100552.9375 (within)</td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>ROUGHAGES thous. ton</th>
<th>$/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>0</td>
<td>62099.5195</td>
</tr>
<tr>
<td>N. C.</td>
<td>62099.5195 (within)</td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
TABLE XI. (Continued) EQUILIBRIUM PATTERN OF PRODUCTION, CONSUMPTION, PRICES AND FLOWS OF BEEF, FEED GRAINS AND ROUGHAGES IN EACH REGION, 1975 (MODEL 2.3)

<table>
<thead>
<tr>
<th>REGION</th>
<th>WEST</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FLOW</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
</tr>
<tr>
<td>N. C.</td>
<td>0</td>
</tr>
<tr>
<td>W.</td>
<td>4223.6914 (within)</td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
</tr>
</tbody>
</table>

|        | FEED GRAINS thous. ton | $/ton |
| N. E.  | 0 | 8154.3867 | 11556.3113 | 47.177 |
| N. C.  | 3401.9246 |             |             |       |
| W.     | 8154.3867 (within) | | | |
| S.     | 0 |             |             |       |

<p>|        | ROUGHAGES thous. ton | $/ton |
| N. E.  | 0 | 22826.4141 | 22826.4141 | 29.923 |
| N. C.  | 0 |             |             |       |
| W.     | 22826.4141 (within) | | | |
| S.     | 0 |             |             |       |</p>
<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>0</td>
<td>4129.6680</td>
<td>5407.4212</td>
<td>69.092</td>
</tr>
<tr>
<td>N. C.</td>
<td>1277.7532</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>4129.6680</td>
<td>(within)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SOUTH</th>
<th>BEEF mil. lb.</th>
<th>$/lb.</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. C.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FEED GRAINS thous. ton</th>
<th>$/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>23352.4180</td>
</tr>
<tr>
<td>N. C.</td>
<td>2039.5725</td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
</tr>
<tr>
<td>S.</td>
<td>23352.4180</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ROUGHAGES thous. ton</th>
<th>$/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>18357.9883</td>
</tr>
<tr>
<td>N. C.</td>
<td>0</td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
</tr>
<tr>
<td>S.</td>
<td>18357.9883</td>
</tr>
</tbody>
</table>

Source: calculated by author
FIG. 7  INTERREGIONAL FLOWS FOR BEEF, FEED GRAINS AND ROUGHAGES
1975 FACTOR-PRODUCT MODEL 2.3
grains production in the South was not abundant, an increase in six percent did not have a significant effect. The increase in the supply of feed grains in the South would bring about a reduction in price of feed grains, thus strengthening the competitive status of the South on the factor market. This would decrease the quantity of feed grain imported from the North Central region.

2.4 A change in long-distance transportation rate. (Assumption D)

The long-distance (beyond 1,000 miles) transportation rates of beef, feed grains and roughages were assumed to decrease for each additional 100 miles in the following manner: 0.1¢ per pound for beef, $0.5 per ton for feed grains and $1 per ton for roughages. This might be the result of the existence of economies of scale in the distance dimension of rail service. The new transportation rates are shown in Table II. All the alternative assumptions A, B, C and D were used in this analysis. The optimum pattern of flows and prices are presented in Table XII and Figure 8.

Reduction in long distance transportation rates will reduce the price differentials between the regions that are far apart. This enables the regions with geographical disadvantage to better their competitive status. The results in this analysis supported this reasoning. The Northeast region increased its beef import from the West and reduced the amount imported from the North Central region. This made it possible for the South to increase its imports from the North Central region. The same was true for the feed grains shipment;
**TABLE XII. EQUILIBRIUM PATTERN OF PRODUCTION, CONSUMPTION, PRICES AND FLOWS OF BEEF, FEED GRAINS AND ROUGHAGES IN EACH REGION, 1975 (MODEL 2.4)**

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BEEF mil. lb.</td>
<td></td>
<td></td>
<td>$/lb.</td>
</tr>
<tr>
<td>N. E.</td>
<td>1925.0662</td>
<td>1925.0662</td>
<td>7153.8921</td>
<td>69.117</td>
</tr>
<tr>
<td></td>
<td>(within)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. C.</td>
<td>4395.5742</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>833.2517</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>FEED GRAINS thous. ton</th>
<th></th>
<th>$/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>2812.1140</td>
<td>2812.1140</td>
<td>4674.6069</td>
</tr>
<tr>
<td></td>
<td>(within)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. C.</td>
<td>1862.4929</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>ROUGHAGES thous. ton</th>
<th></th>
<th>$/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>10506.1211</td>
<td>10506.1211</td>
<td>10506.1211</td>
</tr>
<tr>
<td></td>
<td>(within)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. C.</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE XII. (Continued) EQUILIBRIUM PATTERN OF PRODUCTION, CONSUMPTION, PRICES AND FLOWS OF BEEF, FEED GRAINS AND ROUGHAGES IN EACH REGION, 1975 (MODEL 2.4)

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>NORTH CENTRAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BEEF mil. lb.</td>
<td>PRODUCTION</td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>13807.7554</td>
</tr>
<tr>
<td>N. C.</td>
<td>8114.4727 (within)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>W. 0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>S. 0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>FEED GRAINS thous. ton</td>
<td>$/ton</td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>107841.0615</td>
</tr>
<tr>
<td>N. C.</td>
<td>100470.000 (within)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>W. 0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>S. 0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>ROUGHAGES thous. ton</td>
<td>$/ton</td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>62099.4961</td>
</tr>
<tr>
<td>N. C.</td>
<td>62099.4961 (within)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>W. 0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>S. 0</td>
<td>0</td>
</tr>
</tbody>
</table>
TABLE XII. (Continued) EQUILIBRIUM PATTERN OF PRODUCTION, CONSUMPTION, PRICES AND FLOWS OF BEEF, FEED GRAINS AND ROUGHAGES IN EACH REGION, 1975 (MODEL 2.4)

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION (BEEF mil. lb.)</th>
<th>CONSUMPTION</th>
<th>PRICE (¢/lb.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>0</td>
<td>5035.3376</td>
<td>4202.0859</td>
<td>66.961</td>
</tr>
<tr>
<td>N. C.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>4202.0859 (within)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION (FEED GRAINS thous. ton)</th>
<th>CONSUMPTION</th>
<th>PRICE ($)/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>0</td>
<td>8134.6445</td>
<td>11623.2522</td>
<td>46.904</td>
</tr>
<tr>
<td>N. C.</td>
<td>3488.6077</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>8134.6445 (within)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION (ROUGHAGES thous. ton)</th>
<th>CONSUMPTION</th>
<th>PRICE ($)/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. E.</td>
<td>0</td>
<td>22826.4570</td>
<td>22826.4570</td>
<td>29.377</td>
</tr>
<tr>
<td>N. C.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>22826.4570 (within)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE XII. (Continued) EQUILIBRIUM PATTERN OF PRODUCTION, CONSUMPTION, PRICES AND FLOWS OF BEEF, FEED GRAINS AND ROUGHAGES IN EACH REGION, 1975 (MODEL 2.4)

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BEEF mil. lb.</td>
<td></td>
<td>c/lb.</td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>4119.7305</td>
<td>5417.4390</td>
<td>68.865</td>
</tr>
<tr>
<td>N. C.</td>
<td>1297.7085</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>4119.7305</td>
<td>(within)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>FEED GRAINS thous. ton</td>
<td>$/ton</td>
<td></td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>23355.4063</td>
<td>25375.3672</td>
<td>44.654</td>
</tr>
<tr>
<td>N. C.</td>
<td>2019.9609</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>23355.4063</td>
<td>(within)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REGION</th>
<th>FLOW</th>
<th>PRODUCTION</th>
<th>CONSUMPTION</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>ROUGHAGES thous. ton</td>
<td>$/ton</td>
<td></td>
</tr>
<tr>
<td>N. E.</td>
<td>0</td>
<td>18357.9883</td>
<td>18357.9883</td>
<td>29.377</td>
</tr>
<tr>
<td>N. C.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.</td>
<td>18357.9883</td>
<td>(within)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: calculated by author
FIG. 8 INTERREGIONAL FLOWS FOR BEEF, FEED GRAINS AND ROUGHAGES
1975 FACTOR-PRODUCT MODEL 2.4
the export from North Central to the nearest region, the South, had decreased, while the export to the farther regions, the Northeast and West, increased. The costs for shipping roughages were still too high to permit flows among the regions.
CHAPTER VI
CONCLUSIONS

This study proposed an operational framework for interregional analysis to determine the equilibrium spatial price and allocation of beef and feeds when both the demand and supply functions in each region are considered. It has been realized that the adjustment in any one region is dependent on the adjustment in other regions via factor and commodity markets, thus a procedure which concentrates on the product only will not likely give realistic results.

The study employed Samuelson's net social pay-off concept and used the quadratic programming as a basic tool for arriving at a solution. Perfect competition is assumed to prevail in both the product and factor markets, and the existence of demand and supply relationships is also postulated. Regression analysis was used to derive the pertinent data. Empirical analyses for the years 1964 and 1975 were presented. The 1964 estimates were derived to serve as a basis for comparison, because they are not far from the 1964 actual data. Shifts in data involving the transportation rate structure and redistribution of production and consumption were assumed for the year 1975 as a result of expected changes in market organization. Optimal patterns of prices and flows for beef, feed grains and roughages were determined for the alternatives. The economic explanation of the resulting solutions were also given.

But the empirical findings can only be used as a crude guide, and should be viewed with considerable caution. Since the number of
regions selected is very small, the data used is grossly oversimplified. Yet this can be easily corrected if relevant data is made available and a sufficiently large computer is used. Due to the nature of the quadratic programming, the number of variables and equations grows rapidly as the number of regions increases. The storage capacity of the computer will likely limit the number of regions selected. One way to alleviate this situation is to try to store the information on tape and read in the data from the tape whenever computation is needed. Another limitation is the use of a particular mathematical function to represent demand and supply relationships. For better or worse, linear functions are assumed, since these are required in this application of quadratic programming.

Considerable further analysis is required to improve the model. The direction of the analysis should focus on the problem of incorporating the cobweb adjustment process into the model, thus changing the static model into a dynamic one.
APPENDIX A

COMPUTER PROGRAM FOR QUADRATIC PROGRAMMING

The program is divided into two parts, the first part is used to test the definiteness of the matrix of the quadratic form, the second part is the program for arriving at a solution for the problem. The program is a general one, and in order to illustrate the method of using it, an example is shown below.

The factor-product problem on page 56 is used as an illustration:

To Maximize

\[ F(P) = D'P - \frac{1}{2} P'EP \]

Subject to

\[ AP \leq T \]
\[ P \geq 0 \]

where

- \( D \) is a column vector of 24 elements
- \( P \) is a column vector of 24 elements
- \( E \) is a symmetrical 24 x 24 matrix
- \( A \) is a 48 x 24 matrix
- \( T \) is a column vector of 48 elements

The matrix \( E \), is first tested for its positive definiteness, if the matrix is positive definite, then the subroutine of the quadratic programming is called. The JS in the main program denotes the number of rows or columns of the matrix \( E \), while the NI in the subroutine QP denotes the number of rows in the set-up tableau (excluding the objective function row) NIU denotes the number of rows in the \( E \) matrix, NJ denotes the number of columns in the set-up tableau and
NIJ denotes the number of rows in the set-up tableau including the objective function. In this example, JS = 24, NI = 24 + 48 = 72, NIU = 24, NJ = (24+48+1) x 2 = 146, NIJ = 24 + 48 + 1 = 73. When using the program, the values of JS, NI, NIU, NJ, NIJ need to be specified in the main program.

The program is as follows:

```
DIMENSION A(36,36), L(36), M(36), AA(36,36)
DO 1 I = 1,JS
  DO 1 J = 1,JS
    READ (5,11) A(I,J)
11 FORMAT (8F10.3)
    IF (A(I,I).LE.0.) GO TO 501
    AA2 = A(I,1)*A(2,2) - A(1,2)*A(2,1)
    IF (AA2.LE.0.) GO TO 501
    DO 2 K = 3,JS
      DO 3 I = 1,K
        DO 3 J = 1,K
          3 AA(I,J) = A(I,J)
          CALL MINV(AA,K,D,L,M)
          IF (D.LE.0) GO TO 501
          2 CONTINUE
          501 CALL QP(NI,NIU,NJ,NIJ)
            STOP
            END
```

The data is then read in according to the set-up tableau, the set-up tableau is divided into two parts, the first part contains
th U variables $U(I,J,1)$, the second part contains the $Y$ variables $U(I,J,2)$. The index controls the variables to come in or going out. Index $(I,1)$ is punched in such a way that the first $NIU$ elements are 1's, and the rest elements from $NIU + 1$ to $NI$ are zeros, the index $(I,2)$ is punched in such a way that the first $NIU$ elements are zeros, and the rest elements from $NIU + 1$ to $NIJ$ elements are 1's. The $U(I,J,1)$ and $U(I,J,2)$ can be read any form you like, but in this program it is read in by columns. The name $(I,K,1)$ refers the variable names of the set-up tableau horizontal from $U_1$ to $\Delta$, and name $(I,K,2)$ refers the variable names of the rest of the tableau.
SUBROUTINE QP(NI,NIU,NJ,NIJ)

C PROGRAM FOR SOLVING QUADRATIC PROGRAMMING PROBLEM USING SIMP

DIMENSION INDEX(73,2),U(73,146,2),NAME(73,4,2)

READ(5,499) (INDEX(I,1),I=1,NI)

NII=1+NI
READ(5,499) (INDEX(I,2),I=1,NII)

NI=NIJ+NIU
DO 401 J=1,N1
401 CONTINUE

NS1=1+N1
DO 714 J=NS1,NJ

DO 715 I=1,NIU
U(I,J,1)=0
715 CONTINUE

NIV=1+NIU
READ(5,402) (U(I,1,2),I=NIV,NI)

DO 703 I=NIV,NI
703 CONTINUE

U(I,J,2)=0
F=0.

405 CONTINUE

IF(INDEX(I,1).NE.1) GO TO 409
IF(INDEX(I,2).NE.1) GO TO 409
G=U(I,1,1)*U(I,1,2)
G=F=G

G=U(I,1,1)*U(I,1,2)
0048 WRITE (6, 410)
0049 410 FORMAT (14X1HG)
0050 WRITE (6, 411) G
0051 411 FORMAT (1XF20.7)
0052 WRITE (6, 412)
0053 412 FORMAT (///)
0054 GO TO 300
0055 409 CONTINUE
0056 WRITE (6, 810)
0057 810 FORMAT (14X1HF)
0058 WRITE (6, 411) F
0059 DO 413 I = 1, NI
0060 IF (INDEX (I, 1).NE. 1) GO TO 413
0061 IF (U(I, 1, 1).GE. 0.) GO TO 413
0062 WRITE (6, 412)
0063 GO TO 200
0064 413 CONTINUE
0065 DO 406 I = 1, NI
0066 IF (INDEX (I, 1).NE. 1) GO TO 406
0067 WRITE (6, 407) (NAME(I,K,1), K=1,4), U(I, 1, 1)
0068 407 FORMAT (1HO, 5X4A4, 2X1F20.7)
0069 WRITE (6, 720) (U(I, J, 1), J=2, NJ)
0070 720 FORMAT (2X6F20.7)
0071 406 CONTINUE
0072 DO 408 I = 1, NI
0073 IF (INDEX (I, 2).NE. 1) GO TO 408
0074 WRITE (6, 407) (NAME(I,K,2), K=1,4), U(I, 1, 2)
0075 WRITE (6, 720) (U(I, J, 2), J=2, NJ)
0076 408 CONTINUE
0077 99 RETURN
0078 200 WMIN = 0.
0079 DO 201 I = 1, NI
0080 IF (INDEX (I, 1).NE. 1) GO TO 201
0081 IF (U(I, 1, 1).GE. WMIN) GO TO 201
0082 WMIN = U(I, 1, 1)
0083 MINW = 1
0084 201 CONTINUE
0085 J = MINW + NJ
0086 IF (U(MINW, J, 1).NE. 0.) GO TO 289
0087 WMIN = 999999999999999999.
0088 GO to 299
0089 289 WMIN = WMIN/U(MINW, J, 1)
0090 299 XMIN = 999999999999999999.
0091 DO 202 I = 1, NI
0092 IF (INDEX (I, 2).NE. 1) GO TO 202
0093 IF (U(I, J, 2).EQ. 0.) GO TO 202
0094 Z = U(I, J, 2)/U(I, J, 2)
0095 IF (Z - 0.) 202, 203, 204
0096 203 IF(U(I,J,2).LE.0.) GO TO 202
0097 204 IF(XMIN-Z) 202,853,854
0098 853 CONTINUE
0099 DO 851 K=2,NJ
0100 ST=U(MINX,K,2)/U(MINX,J,2)
0101 SS=U(I,K,2)/U(I,J,2)
0102 IF(SS.LT.0.) GO TO 854
0103 IF(SS=ST) 855,851,857
0104 855 XMIN=Z
0105 MINX=I
0106 GO TO 202
0107 857 CONTINUE
0108 IF(ST.GE.0.) GO TO 854
0109 XMIN=Z
0110 MINX=I
0111 GO TO 202
0112 851 CONTINUE
0113 854 XMIN=Z
0114 MINX=I
0115 202 CONTINUE
0116 IF(WMIN,LT.0.) GO TO 205
0117 IF(WMIN.GT.WMIN) GO TO 205
0118 INDEX(MINW,1)=0
0119 INDEX(MINW,2)=1
0120 DO 206 K=1,NJ
0121 206 U(MINW,K,2)=U(MINW,K,1)/U(MINW,J,1)
0122 GO TO 207
0123 205 INDEX(MINX,2)=0
0124 INDEX(MINW,2)=1
0125 IF(MINX.NE.MNW) GO TO 208
0126 WRITE(6,209)
0127 209 FORMAT(1X7HCYCLING)
0128 GO TO 99
0129 .208 DO 210 K=1,NJ
0130 210 U(MINW,K,2)=U(MINW,K,2)/U(MINW,J,2)
0131 207 CONTINUE
0132 DO 211 I=1,NI
0133 IF(INDEX(I,1).NE.1) GO TO 211
0134 U1=U(I,J,1)
0135 IF(U1.EQ.0.) GO TO 211
0136 DO 212 K=1,NJ
0137 212 U(I,K,1)=U(I,K,1)-U1*U(MINW,K,2)
0138 211 CONTINUE
0139 DO 213 I=1,NI
0140 IF(INDEX(I,2).NE.1) GO TO 213
0141 IF(I.EQ.MINW) GO TO 213
0142 U2=U(I,J,2)
0143 IF(U2.GE.0.) GO TO 213
0144 DO 214 K=1,NJ
0145 214 U(I,K,2)=U(I,K,2)-U2*U(MINW,K,2)
0146 213 CONTINUE
0147 IF(U(NII,1,2).GE.F) GO TO 215
0148 216 WRITE(6,217)
0149 217 FORMAT(1X29HOBJECTIVE FUNCTION DECREASING)
0150 GO TO 99
0151 215 F=U(NII,1,2)
0152 GO TO 405
0153 300 MINW=I
0154 J=MINX+1
0155 MINK=MINX
0156 IF(U(MINW,J,1).NE.0.) GO TO 389
0157 WMIN=999999999999999999.
0158 GO TO 399
0159 389 WMIN=U(MINW,1,1)/U(MINW,J,1)
0160 399 XMIN=999999999999999999.
0161 DO 301 I=1,NI
0162 IF(INDEX(I,2).NE.1) GO TO 301
0163 IF(U(I,J,2).EQ.0.) GO TO 301
0164 Z=U(I,1,2)/U(I,J,2)
0165 IF(Z<0.) 301,302,303
0166 302 IF(U(I,J,2).LE.0.) GO TO 301
0167 303 IF(XMIN<Z) 301,953,954
0168 953 CONTINUE
0169 DO 951 K=2,NJ
0170 SST=U(MINY,K,2)/U(MINY,J,2)
0171 SSS=U(I,K,2)/U(I,J,2)
0172 IF(SSS.LT.0.) GO TO 954
0173 IF(SSS=SST) 955,951,957
0174 955 XMIN=Z
0175 MINY=I
0176 GO TO 301
0177 957 CONTINUE
0178 IF(SST.GE.0.) GO TO 954
0179 XMIN=Z
0180 MINY=I
0181 GO TO 202
0182 951 CONTINUE
0183 954 XMIN=Z
0184 MINY=I
0185 301 CONTINUE
0186 IF(WMIN<0.) GO TO 304
0187 IF(WMIN.GT.XMIN) GO TO 304
0188 INDEX(MINW,1)=0
0189 INDEX(MINK,1)=1
0190 DO 305 K=1,NJ
0191 305 U(MINK,K,1)=U(MINW,K,1)/U(MINW,J,1)
0192    GO TO 306
0193    304 INDEX(MINY,2)=0
0194    INDEX(MINK,1)=1
0195    IF(MINK.NE.MINY) GO TO 307
0196    WRITE(6,209)
0197    GO TO 99
0198    307 CONTINUE
0199    DO 309 K=1,NJ
0200    309 U(MINK,K,1)=U(MINY,K,2)/U(MINY,J,2)
0201    306 CONTINUE
0202    DO 310 I=1,NI
0203    IF(INDEX(I,1).NE.1) GO TO 310
0204    IF(I.EQ.MINK) GO TO 310
0205    U11=U(I,J,1)
0206    IF(U11.EQ.0.) GO TO 310
0207    DO 311 K=1,NJ
0208    311 U(I,K,1)=U(I,K,1)-U11*U(MINK,K,1)
0209    310 CONTINUE
0210    DO 312 I=1,NII
0211    IF(INDEX(I,2).NE.1) GO TO 312
0212    U22=U(I,J,2)
0213    IF(U22.EQ.0.) GO TO 312
0214    DO 313 K=1,NJ
0215    313 U(I,K,2)=U(I,K,2)-U22*U(MINK,K,1)
0216    312 CONTINUE
0217    IF(U(NII,1,2).LT.F) GO TO 216
0218    F=U(NII,1,2)
0219    MINX=MINY
0220    GO TO 405
0221    END
APPENDIX B

COMPUTER PROGRAM FOR TWO STAGE LEAST SQUARES

The Computer Program is written for the IBM 7040.

First, read in Y1, which stands for the independent variables in the function we want to estimate, it is a column vector, the number of elements in this vector depends on how many years of data we have.

Second, read in Y2, which is the matrix of the dependent variables.

Third, read in X, which is the matrix of the exogeneous variables (except the first column), the value of the first column are all equal to 1. The other columns denote the variables, thus depend on the data. When call TSLS, we only need to specify the value of N1, N2, N3, N4, and N, where N1 denotes the number of years involved, N2 denotes the number of endogeneous variables which actually appeared in the function, N3 denotes the number of pre-determined variables, N4 equals N2+1
DIMENSION Y1(10,1), Y2(10,10), X(10,10), A(36,36), L(36), M(36)
DO 332 I=1,10
READ(5,201) Y1(I,1)
201 FORMAT(1F20.11)
READ(5,202) (Y2(I,J),J=1,3)
202 FORMAT (4F20.3)
READ(5,301) (X(I,J),J=1,4)
301 FORMAT(1F20.8,1F20.8,1F20.7,1F20.8)
332 CONTINUE
CALL TSLS(Y1,Y2,X,10,3,4,4,4)
STOP
END

SUBROUTINE TSLS(Y1,Y2,X,N1,N2,N3,N4,N)
DIMENSION Y1(10,1), Y2(10,10), X(10,10), B(7,7), A(36,36),
C(7,10), E(7,110), F(7,7), G(10,1), Y3(1,10), A1(36,36), G1
(10,1), CO(10), L(36), M(36)
DO 231 J=1,N2
DO 232 L1=1,N3
P=0
DO 233 I=1,N1
P=P+Y2(I,J)*X(I,L1)
233 CONTINUE
B(J,L1)=P
232 CONTINUE
231 CONTINUE
DO 234 I=1,N3
DO 235 J=1,N3
Q=0.
DO 236 M1=1,N1
Q=Q+X(M1,L)*X(M1,J)
236 CONTINUE
A(I,J)=Q
235 CONTINUE
234 CONTINUE
CALL INVERT(A,N,D,L,M)
DO 237 I=1,N3
DO 238 M1=1,N1
R=0.
DO 239 J=1,N3
R=R+A(I,J)*X(M1,J)
239 CONTINUE
C(I,M1)=R
238 CONTINUE
237 CONTINUE
DO 240 J=1,N2
DO 241 M1=1,N1
S=0.
DO 242 L1=1,N3
S=S+B(J,L1)*C(L1,M1)
242 CONTINUE
E(J,M1)=S
241 CONTINUE
240 CONTINUE
DO 243 J=1,N2
DO 244 I=1,N2
T=0.
DO 245 M1=1,N1
T=T+E(J,M1)*E(I,M1)
245 CONTINUE
F(J,I)=T
244 CONTINUE
243 CONTINUE
DO 246 J=1,N2
U=0.
DO 247 M1=1,N1
U=U+E(J,M1)*Y1(M1,1)
247 CONTINUE
G(J,1)=U
246 CONTINUE
DO 248 J=1,N2
P1=0.
DO 249 I=1,N1
P1=P1+Y2(I,J)
249 CONTINUE
Y3(1,J)=P1
248 CONTINUE
P2=0.
DO 250 I=1,N1
P2=P2+Y1(I,1)
250 CONTINUE
DO 251 I=1,N2
DO 252 J=1,N2
A1(I,J)=F(I,J)
252 CONTINUE
251 CONTINUE
DO 253 I=1,N2
A1(I,N4)=Y3(1,1)
253 CONTINUE
DO 254 I=1,N2
A1(N4,I)=Y3(1,1)
254 CONTINUE
A1(N4,N4)=10.
CALL INVERT(A1,N4,D,L,M)
SUBROUTINE INVERT (A, N, D, L, M)

PROGRAM FOR FINDING THE INVERSE OF A NXN MATRIX

DIMENSION A(36,36), L(36), M(36)

SEARCH FOR LARGEST ELEMENT

D = 1.0

DO80 K = 1, N

L(K) = K

M(K) = K

BIGA = A(K, K)

DO20 I = K, N

DO20 J = K, N

IF (ABS(BIGA) - ABS(A(I, J))) 10, 20, 20

BIGA = A(I, J)

L(K) = I

M(K) = J

CONTINUE

INTERCHANGE ROWS

J = L(K)

IF (L(K) - K) 35, 35, 25

25 DO30 I = 1, N

HOLD = A(K, I)

A(K, I) = A(J, I)

30 A(J, I) = HOLD

INTERCHANGE COLUMNS

I = M(K)

IF (M(K) - K) 45, 45, 37

37 DO40 J = 1, N

HOLD = - A(J, K)

A(J, K) = A(J, I)
DIVIDE COLUMN BY MINUS PIVOT

DIVIDE ROW BY PIVOT

CONTINUED PRODUCT OF PIVOTS

REPLACE PIVOT BY RECIPROCAL

FINAL ROW AND COLUMN INTERCHANGE

END
A. Books


B. Government Documents


United States of Labor, Retail Prices of Food Index and Average Prices. Washington, 1955-1966


C. Reports


Goodwin, J. D., Optimum Distribution Patterns of Feeder Cattle from the Southeast. Southern Cooperative Series, October 1965.


Kansas Agricultural Experimental Station, The Competitive Position of Kansas in Marketing Beef. Manhattan: Kansas State University, August 1963.


Purcell, J. C. and Ford, K. E., Long-Run Consumption Trends and Comparative Shares by Regions for Fresh and Processed Food Products. Georgia: University of Georgia, June 1966.


D. Periodicals


