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LIFE EXPECTANCY, LABOR FORCE, AND SAVING

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ABSTRACT

Life expectancy increased remarkably in many countries during the 20th century. This dissertation analyzes the effect of the transition in adult longevity on the national saving rate.

The first essay presents a two-period overlapping generations model. During the first period individuals work and during the second period they are retired. Changes in adult survival influence the expected duration of retirement. An increase in adult survival induces working-age adults to save more for their retirement. In steady state, an increase in life expectancy increases the aggregate saving rate if the economy is growing. The simplicity of the two-period model makes it possible to analyze out-of-steady-state aggregate saving. The model shows that the saving rate depends on both the level and rate of change in life expectancy. As a result, rapid increases in life expectancy lead to high national saving rates.

In the second essay, endogenous retirement is investigated. An increase in life expectancy delays retirement in a perfect annuity market and hastens retirement in the absence of an annuity market. Increases in longevity cause a transitory increase in the national saving rate. However, whether a rise in longevity leads to a higher steady state saving rate is ambiguous.

In the final essay, the effect of longevity on the national saving rate is analyzed empirically using both world panel data and longer-term historical data. Analysis based on the world panel data supports the hypothesis that countries with higher adult survival have higher saving rates if GDP is growing. The change in adult survival has an additional effect on the national saving rates, as hypothesized, in advanced economies. We find no evidence that the *change* in adult survival has an effect on saving in developing economies. The trends in saving during the mortality transition of over a century or more provides additional support for the key hypotheses. The onset of the modern transition in mortality was accompanied by a substantial increase in national saving rates. In

Asian countries a period of rapid change in adult survival was accompanied by an elevated rate of saving.

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CHAPTER 1. INTRODUCTION

1.1 Backgrounds and Motivations

The purpose of this research is to investigate the effect of longer life expectancy on saving. In the 20th century, many countries experienced demographic transition. During the first stage, mortality, especially infant mortality, declines rapidly. Fertility does not decline immediately, and population increases dramatically when mortality begins to decline. After a while, fertility begins to decline slowly and population increase becomes moderate gradually. Life expectancy increases with gains in survival at older ages. As a result of demographic transition, many developed countries experience population aging. In this dissertation, we focus on the mortality transition of adults. In our discussion, we use the words life expectancy, longevity, and adult mortality interactively.

Increases in the national saving rate in the 20th century are remarkable in many countries. In the19th century, the saving rate was low and sluggish in most countries. In the early 20th century, the national saving rate began to increase steadily in Western countries. In Asian countries, the saving rate began to rise after World War II, and the increase was much more rapid than in Western countries. In East-Asian countries such as Japan and Taiwan, the national saving rate is now higher than in Western countries.

With such trends of life expectancy and the national saving rate in mind, the following questions are important to solve. First, how does higher life expectancy influence the national saving rate? Second, how does a rapid increase in life expectancy affect the national saving rate? Throughout this dissertation, we seek the answers to these questions both theoretically and empirically.

We use overlapping generations models to represent generations with different economic activities. Our model analyzes the effect of longevity on the national saving rate, investment rate, and current account in both a small open economy and a closed economy. We also describe the dynamic effect of longevity by simulation analysis. Furthermore, the determinant of the national saving rate is analyzed empirically.

1.2 A Summary of the Findings

The work is presented in three essays in Chapters 2, 3, and 4. To begin with, in Chapter 2, an overlapping generation model with lifetime uncertainty is established. In the model, there are two generations in the economy: prime-age adults and the elderly. Prime-age adults work and the elderly are retired. Labor force participation is exogenous. Life expectancy is expressed as the probability of prime-age adult to survive to old age. We analyze the effect of an increase in life expectancy on the national saving rate and investment rate in a small open economy and in a closed economy.

In this chapter, we find that the effect of an increase in life expectancy is interacted with GDP growth in steady state. An increase in survival rate increases both saving of prime-age adults and dissaving of the elderly in steady state. An increase in the survival rate increases (decreases) the national saving rate if GDP is growing (declining) in steady state because saving of prime-age adults is more (less) than the dissaving of the elderly. The implications of the model are summarized as follows, assuming that GDP is growing. As out-of-steady-state effects of longevity on the saving rate, a one-time increase in the survival rate produces a large transitory increase in the national saving rate because only saving of prime-age adult increases at the instant when the survival rate increases. This is followed by a return to the saving rate, which is higher than the original level. A high-sustained increase in life expectancy induces a jump to a high sustained

national saving rate. A current increase in the survival rate increases the wealth ratio to GDP in the next period. In a small open economy, the investment rate is not influenced by life expectancy because it does not affect the growth of labor force. Therefore, an increase in the survival rate brings an increase in the current account balance. In a closed economy, an increase the survival rate increases the investment rate, which is equal to the saving rate. This causes higher GDP growth.

In Chapter 3, the overlapping generations model of Chapter 2 is developed. As in Chapter 2, there are two generations, prime-age adults and the elderly, in the economy. In Chapter 3, retirement of the elderly is endogenous, that is to say, consumers decide when to retire maximizing lifetime utility. We focus on the case of a small open economy. We analyze the effect of an increase in the survival rate on the national saving rate and the national investment rate in a small open economy. Also, examined is the effect of an increase in social security tax financed by pay-as-you-go system.

The following three cases are discussed: (1) perfect annuity market without social security tax; (2) perfect annuity market with social security tax; (3) no annuity market without social security tax. The effect of longevity on retirement decisions, saving decisions, and the national saving rate varies under different assumptions. In perfect annuity market, an increase in life expectancy increases saving and delays retirement. An increase in life expectancy brings a transitory increase in the national saving rate. In steady state, the effect of an increase in life expectancy is interacted with GDP growth as in Chapter 2. However, longevity does not necessarily increase the national saving rate even if GDP growing because of increases in labor force participation of the elderly and the number of surviving elderly. An increase in life expectancy brings a transitory increase in the investment rate, and it does not affect the investment rate in steady state. The effect of longevity on the current account balance is ambiguous.

If a social security tax financed by a pay-as-you-go system is introduced, the sign of the effects of an increase in life expectancy on saving of prime-age adults, retirement of the elderly and the national saving rate are the same as in the case perfect annuity market without social security tax. A current increase in the social security tax rate decreases labor force participation of the elderly during the same period and increases labor force participation in the future.

In the absence of an annuity market, an increase in life expectancy increases saving of prime-age adults and induces earlier retirement. An increase in life expectancy also brings a transitory increase in the national saving rate. The steady-state effect of an increase in life expectancy is ambiguous. It has a transitory effect on the national investment rate, but whether it is positive or negative is ambiguous.

In Chapter 4, based on the models of Chapters 2 and 3, the determinants of the saving rate are analyzed empirically. Some previous studies analyze the effect of life expectancy at birth on the national saving rate. However, we point out that using the data of life expectancy at birth is not appropriate to explain our model because it is greatly influenced by child mortality. We create an undiscounted adult survival index as the ratio of total years lived after age 60 to total years lived from age 30 to 59. As a discounted adult survival index, we assume the years lived in the later stage of life are valued less. The relationship between life expectancy at birth and undiscounted and discounted adult survival indices are calculated from the model life table of Coale and Demeny (1983).

The saving equation is estimated using the world panel data. The effect of an increase in adult survival is interacted with GDP growth. If GDP is growing, an increase in adult survival has a positive effect on the national saving rate. The coefficient of a change of an increase in adult survival is not significant using the whole world data and developing countries. Within the

sub-sample of advanced economies, a rapid increase in adult survival has a positive effect on the national saving rate.

Historical trends of adult survival, and the national saving rate are discussed in relation to our theory. In 20th century, many countries experienced mortality transition. Transition of adult survival in the West is distinctive from Asia. In the West, there are the pre-transition period and the transition period. In the pre-transition period, adult survival was low and stagnant. After the transition period began, adult survival increased at a relatively constant rate. In Asia, mortality transition began later than the West, but there was catch-up period, when adult survival increased quite rapidly. We formalize that there were significant structural change in the time trend of adult survival.

Historical analysis of the national saving rate demonstrates the following implications of our theory. First, the saving rates were very low in countries before the onset of mortality transition. Second, saving rates rose as the proportion of adult years spent in old-age increased. Third, a rapid transition in adult mortality led to higher saving rates. We find that the national saving rate tends to be high if life expectancy grows faster and the level of life expectancy is fixed.

In summary, if life expectancy increases, prime-age adults increase saving. Whether the elderly work more or less depends on assumptions. An increase in life expectancy does not necessarily delay retirement. The effect of an increase in life expectancy on the saving rate in steady state is interacted with GDP growth. Out of steady state, a one-time increase in life expectancy produces a large transitory increase in the national saving rate. Empirical analysis finds that an increase in adult survival has a positive effect on the national saving rate if GDP is growing. Historical data reveal that the national saving rate is high while adult survival increases rapidly.

Therefore, we conclude that a rapid increase in life expectancy contributes to an increase in the national saving rate substantially as well as a high level of life expectancy.

1.3 Literature Reviews

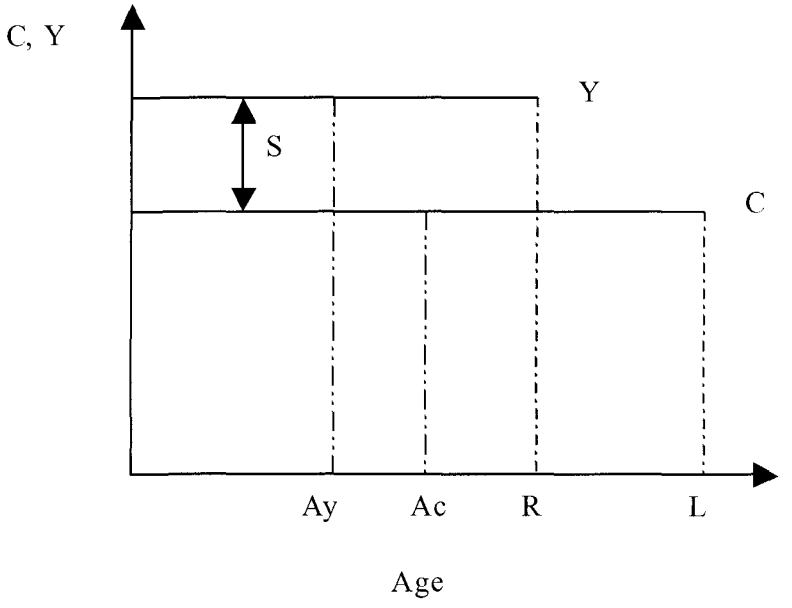
Much effort has been devoted to explain theoretically how saving is determined. The life-cycle model is a predominant theory of the determinant of saving. It implies that consumers determine the amount of saving based on lifetime income in order to maximize their lifetime utility. Most research regarding demographic change and saving is based on the life cycle model. This dissertation extends the life-cycle model by incorporating the effects of life expectancy. This section introduces theoretical literature relating to the life-cycle model. Detailed studies about retirement are introduced in Chapter 3 and empirical studies are discussed in Chapter 4.

(1) A Basic Life-Cycle Model

The concept of the life-cycle model is that consumers determine the amount of consumption that maximizes lifetime utility, not only current utility. Modigliani and Brunberg (1954) illustrate the life-cycle model, calling particular attention to retirement years. Tobin (1967) develops the model and simulates the effects of demographic characteristics on saving. Research by Modigliani and Brumberg and Tobin do not consider lifetime uncertainty. They demonstrate the existence of the rate of growth effect. In their research, length of life is present but not explored. The authors develop detailed relationships of parameters in steady state.

Here, we explain a basic concept of demographic effects on saving using the simplified life-cycle model of Tobin and Modigliani and Brunberg. It is assumed that consumers save for pension motive, that is, they save in order to consume after retirement. Individuals save when they are young, and dissave during their retired years. Tobin does not discuss the effect of life

Figure 1.1 A Simplified Life-Cycle Model



expectancy. However, the life-cycle model could be a core concept of our discussion regarding life expectancy and saving. Moreover, we could find some implications about life expectancy or other demographic variables. Suppose there is no bequest and no uncertainty. In this case growth rate of GDP is equal to population growth. Figure 1.1 introduces the fundamental life-cycle model. Consumers are assumed to retire at age R and die at age L. In the process of aggregation, the effects of population growth can be incorporated. Aggregate saving is obtained by summing the savings of all generations. In the same way, aggregate income is a sum of income earned by all generations. The aggregate saving rate (S/Y) is aggregate saving divided by aggregate income, such as:

$$\frac{S}{Y} = \frac{\sum_{a=0}^{L} N(a)Y(a)s(a)}{\sum_{a=0}^{L} N(a)Y(a)}, \tag{1.1}$$

where a denotes age, $N(a)$ denotes number of consumers at age a, $Y(a)$ denotes earnings of a consumer at age a, and $s(a)$ denotes the share of saving in earnings at age a.

This simple life cycle model assumes that the age distribution of the population depends only on the population growth rate. Thus, the aggregate saving rate can be expressed as:

$$\frac{S}{Y} = b_0 + b_1(n+g), \tag{1.2}$$

where $b_1>0$, and n is growth rate of population and g is technological growth rate. g+n is the growth rate of GDP at the steady state. This is called the rate of growth effect. A decline in the population growth rate leads to an older population and a decline in aggregate saving. An older population leads to lower saving because the young save more than the old.

(2) The Variable Rate of Growth Effect Model

One of the complexities in studying the effects of demographic variables, including life expectancy, is that they influence both the age composition of the population and the consumption and earning profiles of households. Using a steady state model, Mason (1981, 1987) and Fry and Mason (1982) show that equation (1.2) can be generalized to incorporate the effect of variables on consumption and earnings and consumption profiles. The authors derive the variable rate of growth (VRG) model. The VRG effect model implies that the aggregate saving rate is obtained as follows.

$$\frac{S}{Y} \approx -\ln(C) = \beta_0 + (A_C - A_Y)(g+n), \tag{1.3}$$

where C is the consumption share of GDP, A_C is the mean age of consumption and A_Y is the mean age of earning. A_C and A_Y are expressed as follows:

$$A_C = \sum_{a=0}^{L} ac(a) \Big/ \sum_{a=0}^{L} c(a), \tag{1.4}$$

$$A_Y = \sum_{a=0}^{L} aY(a) \Big/ \sum_{a=0}^{L} Y(a) . \quad (1.5)$$

In equation (1.3), the sign and magnitude of the rate of growth effect is determined by A_C - A_Y. If consumers earn income at an earlier age than they consume it, the rate of growth effect is positive. If the pension motive is important, as in the simple life-cycle model, then the condition would generally be met. On the other hand, the dependency rate influences the consumption profile, and A_C - A_Y increases if the dependency rate increases. That is, timing of consumption becomes earlier with an increase in the dependency rate.

The characteristics of the VRG model are as follows. First, it formalizes the relationship between saving and mean ages of consumption (A_C) and earning (A_Y). In the model, changes in the age-consumption and age-earning profiles are expressed in the forms of the changes in A_C and A_Y, respectively. Second, it explains how changes in A_C and A_Y affect saving. Third, the model does not look at how the length of life influences A_C and A_Y. Fourth, the model is based on static and deterministic assumptions.

Population growth causes growth in GDP and changes age structure. The variable rate of growth effect model incorporates such effects effectively. We might well assume that an increase in life expectancy can change saving behaviors of each age and age composition. This insight would be fundamental in the discussion of the effect of longevity.

(3) Saving under Lifetime Uncertainty

The concept discussed above is true only if the survival rate is fixed. It seems important to discuss effects of the change in survival rate. One effect of changes in life expectancy is to a change age composition, hence, aggregate saving. We should also note that changes in life expectancy occur because of changes in child survival early in the demographic transition. In individual

life-cycle models that treat children as separate earners and consumers, the effect of increased life expectancy would be compositional.[1] A decrease in child mortality can change life expectancy at birth and increase the share of survived children, which can change the timing of saving. Also, a change in the infant mortality rate changes population growth and can also change the rate of growth effect on saving.

Then how are the effects of an increase in life expectancy on saving discussed in previous literature? In Figure1.1, longevity can change L and A_C. Several studies have tried to incorporate lifetime uncertainty. In a discussion of life expectancy, it is important to incorporate uncertainty associated with mortality itself. Yaari (1965) proposes the classical model on lifetime uncertainty and consumption. Yaari analyzes how the consumption growth rate differs if bequest motives exist. Moreover, the author investigates how the results are affected depending on the availability of insurance. We might well say that Yaari greatly contributes to establish a model regarding lifetime uncertainty.

Yaari uses the utility functions of Marshall (1920) and Fisher (1930). First, consumer's optimization with certain lifetime is described. Suppose that an individual lives T years. c is a consumption plan during the interval [0, T] of the consumer, and c(t) is consumption at time t. A Fisher utility function is the following form:

$$v(c) = \int_0^T \alpha(t) g[c(t)] dt \,, \tag{1.6}$$

where α is subjective discount function. g is a concave function, that is, $g' \geq 0$ and $g'' \leq 0$. If the optimal plan c^* exists, consumption of the consumer is expressed in the following equation:

---

[1] In this dissertation, the effects of children's mortality are not discussed. However, it will be important to consider.

$$\dot{c}^* = -\left\{ j(t) + \frac{\dot{\alpha}(t)}{\alpha(t)} \right\} \frac{g'[c^*(t)]}{g''[c^*(t)]}, \tag{1.7}$$

where a dot means differentiation with respect to time t, and $j(t)$ is the rate of interest rate at time t.

Marshall utility function incorporates utility obtained from bequest $S(T)$.

$$U = \int_0^T \alpha(t) g[c(t)] dt + \beta(T) \phi[S(T)], \tag{1.8}$$

where $\beta(T)$ is defined for all values of the random variable T.

Yaari introduces the notions of life insurance and annuities. The author defines an actuarial note as a note that can be either bought or sold and which stays on the books until the consumer dies. An actuarial note is automatically cancelled when he dies. In the case where insurance is available, the interest rate of the actuarial note $r(t)$ is assumed to be fair, that is $r(t)$ is equal to $j(t)+\pi_t(t)$. $j(t)$ is the interest rate of regular note, and $\pi_t(t)$ is the probability of dying at time t under the condition that the consumer survive until time t. The necessary premium is $\pi_t(t)$ and total rate on loans is $j(t)+\pi_t(t)$, if the consumer is alive. Suppose consumers do not have bequest motives. If insurance is available and life insurance and annuity markets are fully perfect, consumers will choose to have their entire assets in the form of actuarial notes because the return is higher than regular notes.

Yaari investigates the following four cases:

	Fisher utility function with wealth constraint	Marshall utility function with no constraint
Insurance unavailable	Case A	Case B
Insurance available	Case C	Case D

In case A, there is no bequest motive and insurance is not available. Consumers solve the following problem:

$$\text{Maximize} \int_0^{\bar{T}} \Omega(t)\alpha(t)g[c(t)]dt\,, \tag{1.9}$$

Subject to (i) $c(t) \geq 0$ for all t,

(ii) $c(t) \leq m(t)$ wherever $S(t)=0$,

(iii) $S(\bar{T})=0$.

where Ω is the probability that the consumer will be alive at time t, and $m(t)$ is the consumer's earning. Assuming an interior solution, optimal solution c^* must satisfy the differential equation:

$$\dot{c}^*(t) = -\left\{ j(t) + \frac{\dot{\alpha}(t)}{\alpha(t)} - \pi_t(t) \right\} \frac{g'[c^*(t)]}{g''[c^*(t)]}, \tag{1.10}$$

where $\pi_t(t)$ is the probability of death at time t under the condition that the consumer survives until time t. Comparing equation (1.10) with (1.7), consumption growth of an individual is less in the case of uncertain life time than in the case of deterministic lifetime. In equation (1.10), the subjective rate of discount rate is $\pi_t(t) - \frac{\dot{\alpha}(t)}{\alpha(t)}$, which is greater than the case of (1.7) ($-\frac{\dot{\alpha}(t)}{\alpha(t)}$). Therefore, future consumption is discounted more heavily because survival is uncertain. Equation (1.10) implies that consumers become impatient if the probability of death increases.

In case B, let $\bar{U}$ be the expected value of the Marshall utility function of equation (1.8). $\bar{U}$ is expressed as:

$$\bar{U}(c) = EU(c) = \int_0^{\bar{T}} \{\Omega(t)\ \alpha(t)g[c(t)] + \beta(T)\phi[S(T)]\}dt\ . \tag{1.11}$$

Maximizing the expected utility in equation (1.10), c^*, the optimal consumption plan and S^*, the corresponding asset functions are calculated as:

$$\dot{c}^*(t) = -\left\{ j(t) + \frac{\dot{\alpha}(t)}{\alpha(t)} \right\} \frac{g'[c^*(t)]}{g''[c^*(t)]} + \frac{\pi_t(t)}{\alpha(t)} \frac{\alpha(t)g'[c^*(t)] - \beta(t)\phi'[S^*(t)]}{g''[c^*(t)]}, \tag{1.12}$$

$$\dot{S}^*(t) = m(t) - c^*(t) + j(t)S^*(t)\ . \tag{1.13}$$

The first term in the right hand side of equation (1.12) is consistent with the entire right hand side of equation (1.7). The second term in the right hand side of (1.12) can be either positive or negative. If the second term in the right hand side of (1.12) negative, lifetime uncertainty makes the consumer more impatient, and if it is positive, lifetime uncertainty makes him less impatient. That is to say, lifetime uncertainty increases impatience if $\alpha(t)g'[c^*(t)] > \beta(t)\phi'[S^*(t)]$ and decreases it if $\alpha(t)g'[c^*(t)] < \beta(t)\phi'[S^*(t)]$. Yaari concludes that "impatience is greater than in the case of no uncertainty if the marginal utility of consumption exceeds the marginal utility of bequests, and it is less than it would be under no uncertainty if the marginal utility of bequests exceeds that of consumption. (Yaari (1965), p.144.)"

In Case C, it is assumed that there is no bequest motive and insurance is available. The maximization problem can be describes as follows:

$$\text{Maximize} \int_0^{\bar{T}} \Omega(t)\alpha(t)g[c(t)]dt\ , \tag{1.14}$$

Subject to (i) $c(t) \geq 0$ for all t,

(ii) $\int_0^{\bar{T}} \{\exp[-\int_0^t r(x)dx]\}\{m(t) - c(t)\}dt = 0\ .$

Solving the problem, an optimal plan c^* is expressed in the following equation:

$$\dot{c}^* = -\left\{ j(t) + \frac{\dot{\alpha}(t)}{\alpha(t)} \right\} \frac{g'[c^*(t)]}{g''[c^*(t)]}. \tag{1.15}$$

Equation (1.15) is exactly the same as equation (1.7). It seems that insurance can remove uncertainty from the allocation problem. However, this is not quite true. Even though the growth of consumption is the same, the constants of integration that would determine the solutions of these equations are different, therefore, it is possible that the level of consumption is different.

In case D, consumers have a bequest motive and insurance is available. The consumer may allocate his assets or liabilities into regular notes and actuarial notes. Yaari suggests that the consumer solves the following problem:

$$\text{Maximize } \bar{U}(c,S) = \int_0^{\bar{T}} \{\Omega(t)\ \alpha(t) g[c(t)] + \beta(T)\phi[S(T)]\} dt, \tag{1.16}$$

$$\text{subject to (i) } c(t) \geq 0 \text{ for all } t,$$

$$\text{(ii) } \int_0^{\bar{T}} \{\exp[-\int_0^t r(x)dx]\}\{m(t) - c(t) - \pi_t(t)S(t)\} dt = 0.$$

Solving the maximization problem, we obtain the optimal consumption plan c^* and the optimal saving plan S^* as:

$$\dot{c}^* = -\left\{ j(t) + \frac{\dot{\alpha}(t)}{\alpha(t)} \right\} \frac{g'[c^*(t)]}{g''[c^*(t)]}, \tag{1.17}$$

$$\dot{S}^*(t) = -\left\{ j(t) + \frac{\dot{\beta}(t)}{\beta(t)} \right\} \frac{\phi'[S^*(t)]}{\phi''[S^*(t)]}. \tag{1.18}$$

Equation (1.17) is identical to equation (1.7). The important feature of equations (1.17) and (1.18) is that the former does not involve S^* and the latter does not involve c^*. Yaari concludes that when

insurance is available, the consumer can separate the consumption decision from the bequest decision. The author finds that saving plan S affects the level of consumption plan c, but it does not change the growth of consumption.

As we have seen above, Yaari introduces the effect of lifetime uncertainty on the consumption decision. The concept of impatience related to lifetime uncertainty and the methods that address insurance are widely developed by various authors. These issues are fundamental in this dissertation. Here, as is discussed before, an increase in life expectancy can change saving and consumption plan, and, moreover, it will alter age distribution. It would be important to analyze these issues and to investigate the effect of longevity on the aggregate saving.

As an effect of longer life expectancy, many authors agree that longevity brings about higher saving of individuals if there is no bequest motive. For example, Zilcha and Friedman (1985) establish a theory of saving behavior after retirement when the life horizon is uncertain. The authors explore an explanation of why retired people continue to save in retirement for several years. The authors show that the individual increases his saving if he becomes more optimistic about his life probability. They denote this p_k the probability to die at time k. They assume that there is a certain upper limit, T, to lifetime. Thus, it is assumed that $\underline{p} = (p_1, p_2, \ldots, p_T) \geq 0$. It is defined that $\overline{p}$ is more optimistic than $\underline{p}$ if $\overline{q}_j / \overline{q}_k \geq q_j / q_k$ for all k, j (q_i is the probability to survival at age i), satisfying $1 \leq k < j < T$ and if at least one strict inequality holds.

Strawczynski (1993) develops an overlapping generations model with life uncertainty and income uncertainty, analyzing the influence of perfecting income insurance on the size of bequests and demand for annuities. The author analyzes the effect of government intervention. From the

results, we can see that a higher survival rate can decrease consumption in youth, that is, the consumer will save more when the probability of survival increases.

Yakita (2001) establishes an overlapping generations model with lifetime uncertainty and endogenous fertility. The author also considers the effect of life expectancy if social security is financed on a pay-as-you-go basis. He shows that an increase in life expectancy lowers the fertility rate and increases saving for consumption after retirement. An increase in life expectancy increases the probability that one enters the retirement period. It causes individuals to save more for consumption after retirement. Because children are considered to be consumption goods, longer life expectancy decreases the number of children. The author concludes that social security system let individuals save less due to the transfers. It causes an increase in the number of children, which are consumption goods. However, social security does not offset the negative effect of longevity on fertility.

We have introduced just a few examples, but a number of studies address the effect of life expectancy on individual saving incorporating other factors, such as income uncertainty, social security, endogenous fertility, etc. These studies do not address important issues that are examined in this dissertation. (1) The effects of longevity on the aggregate saving rate; (2) The out-of-steady-state effects of an increase in life expectancy; (3) Endogenous labor force participation decision.

(4) Dynamic Analysis of Demographic Change

Most previous studies have focused on steady state relationships and there has been relatively little work on out-of-steady-state properties. This issue is one of the research questions of this

dissertation. It will be important to analyze the process of the changes because demographic variables have transitory effects.

Simulation analysis of Higgins (1994) and Williamson and Higgins (2001) is one example. The analysis is based on an overlapping generation model with three generations: children, prime-age adults, and elderly. The authors assume a plausible demographic transition, that is, a country experiences rinsing fertility or infant mortality. In the analysis, an increase in fertility and a decrease in infant mortality are treated indifferently. Next, fertility declines gradually and reaches a new and lower steady-state value. As fertility changes, the age distribution changes. First, the share of children in the total population increases, next, the share of the prime-age adults increases and the share of children decreases, and finally the share of the old increases.

The authors analyze the cases of a small open economy and a closed economy. In a small open economy, saving rate decreases at the beginning of the demographic change because child dependency increases. As the share of prime age adults increases, the saving rate increases. Eventually, the economy experiences population aging, and at this stage, the saving rate decreases.

Investment jumps in the beginning of the transition due to higher labor force growth. Afterwards, along with the decrease in prime-age adults, the investment rate decreases. The current account balance decreases at the early stage of the demographic transition. As time passes, the current account balance increases as the saving increases. In the aging economy, the current account balance is positive and capital outflow is promoted. They analyze the results empirically, and they found that these effects of the demographic transition discussed above are realistic.

In a closed economy, saving is equal to investment. In the beginning, because child dependency increases, the saving rate decreases. As labor force increases, the saving rate increases. In the later stage of demographic transition, as the share of the old generation increases, the saving

decreases. Comparing with the case of a small open economy, the quantitative swing is smaller because the effect of demographic transition on saving supply and investment demand are at least partly offsetting.

Simulation analysis of Lee, Mason and Miller (2001) contributes to analyze the dynamic effects of demographic transition. The authors investigate the effects of rapid demographic transition in East Asia. They use Taiwan's data in their analysis.

Their theoretical background is a life-cycle model and they assume that consumption and saving behavior are governed by the desire to smooth consumption throughout life. First, the authors assume an essentially steady state and compare pre- and post-transitional age distributions. They found that lower mortality increases saving rate and capital output ratio. They also find that lower fertility has a small effect on saving rates but results in a substantially higher capital-output ratio.

Next, dynamic analysis is attempted by simulating the saving rate. The simulation results show that significant increases in saving rates occur from 1970 to 2005 due to a combination as follows: changes in age structure, lower fertility, and longer life expectancy. The simulated saving rate decreases, over the first half of the twenty-first century as old-age dependency rises. The authors conclude that demographic aging will lead to declines in saving rates in the twenty-first century.

(5) Further Discussion on Life Expectancy and Saving

Discussion on saving and lifetime uncertainty has been made from various points of view. Skinner (1985) incorporates bequest motives in the model. According to the model, the bequest motive may negate and even reverse the positive correlation between longevity and saving implied

by the pure life cycle model. In the model, consumers save for a dual purpose. One is to save for retirement, as discussed in life-cycle model (life-cycle saving motive). The other is to provide bequests. When life expectancy rises, the possibility to leave a bequest at a young age decreases. This can reduce the bequest motive of saving. On the other hand, an increase in life expectancy increases saving because they have a greater incentive to accumulate assets for future consumption. These effects are offsetting and which effect is greater is an empirical question.

It is also possible that life expectancy can change the earning profile, and, as a result, change the saving rate. In Figure 1.1, it is demonstrated by a change in R and A_Y. MaCurdy (1981) suggests a basic approach to discuss intertemporal labor force participation decision. The author set up a model that assumes that individuals decide consumption and hours to work maximizing lifetime utility.

Some studies have addressed life expectancy, labor force participation and saving. Chang (1990) set up a model that addresses uncertain lifetime, saving, and retirement. The author suggests the effect of increased life expectancy on retirement age is ambiguous. In the model, individuals decide retirement based on the old-age saving behavior. Consumers retire when the wage is below the reservation wage. The reservation wage is the shadow price of leisure when one does not work. According to the model, if the annuity market is perfect, the marginal rates of substitution (MRS) between consumption and leisure at the same time, of consumption across time, and leisure across time are independent of the survival rate. Marginal utility of initial wealth is the only factor that determines the effect of longevity on the labor force participation decision. The author shows that if consumption lags behind earnings, longer life expectancy brings about later retirement. If annuity is not available, MRSs are not independent from the survival rate. An increase in the survival rate changes the substitution effect of wage on labor force participation decision. It causes a tilt to the

reservation wage profile and earlier retirement can be induced. This research contributes to the explanation the stylized fact in developed countries that the retirement age is becoming younger while life expectancy is increasing.

Kalemli-Ozcan and Weil (2002) examine savings, life expectancy, and retirement. In their model, individuals make labor and leisure choices, maximizing lifetime utility subject to uncertainty about their date of death. The authors argue that a fall in mortality has an "uncertain effect" and "horizon effect" on retirement. The uncertain effect follows from the decline in uncertainty about the age at death that has resulted form an increase in life expectancy. This leads individuals to attach greater value to retirement (leisure). Thus, they retire at a younger age and save more while young. The horizon effect follows from the effect of longevity on later retirement because longer life means more years of consumption that need to be paid for. Their simulation analysis based on US mortality during the last century, shows that the uncertain effect of declining mortality has been greater than the horizon effect. That is, longevity leads to earlier retirement.

It seems that the effect of longevity on saving incorporating labor force participation decision is very controversial. Longer life expectancy induces either earlier or later retirement. The effect on aggregate saving and dynamic effects of a change in life expectancy has not been discussed. Therefore, there might be much room for further discussion. We would discuss this issue in Chapter 3.

1.4 Organization of the Dissertation

The following chapters are organized as follows. Chapter 2 presents an overlapping generations model that includes life expectancy. We describe the dynamic effect of longevity on the national saving rate using simulation analysis. In Chapter 3, we develop the model of Chapter 2.

Considering the labor force participation decision, it brings us a further understanding of longevity and elements of national income. Chapter 4 addresses empirical analyses. We analyze whether the implications of our theory are supported empirically using both world panel data and household data. Chapter 5 summarizes the accomplishments of this research and draws a conclusion.

CHAPTER 2. AN OVERLAPPING GENERATIONS MODEL OF LONGEVITY AND THE NATIONAL SAVING RATE

2.1 Introduction

The previous chapter reviews studies of demographic effects on saving. Previous studies have not fully analyzed the effects of an increase in life expectancy on the aggregate saving rate and out-of-steady-state effects of longevity. This chapter presents an overlapping generations (OLG) model that incorporates lifetime uncertainty. The OLG model is an extension of the life-cycle model. The basic concept was introduced by Samuelson (1958) and Diamond (1965). In the OLG model, it is assumed that multiple generations exist simultaneously in the economy. Assuming that individuals live for only a few periods simplifies the aggregation of consumption or saving.

Higgins (1994)[2] uses an overlapping generations model with three generations: children, prime-age adults, and the elderly. His model supports the conclusion of Coale and Hoover (1958), who suggest that lower fertility induces high saving. One contribution of Higgins' model is that it describes both the steady state effect of population growth and the out-of-steady-state transition of the saving rate. Another contribution is that the author considers the effects of an increase in fertility on national saving rate in a small open economy and a closed economy. Higgins analyzes the separate effects of population growth on saving supply and investment demand in a small open economy, which has been ignored in previous studies.

Here, we extend Higgins's overlapping generations model to include the effect of lifetime uncertainty. For simplicity, we focus on the effects of changes in the survival rate of adults. We analyze the effect of an increase in the survival rate on the national saving rate in both a small open economy and a closed economy separately. We analytically explore dynamic effects of longer life

[2] See also Higgins and Williamson (1997) and Williamson and Higgins (2001).

using simulation analysis. In the dynamic analysis, we investigate the following two cases: a one-shot permanent increase in survival versus a steady increase in survival. These approaches are important because life expectancy is historically a variable with trend. At first there is little improvement in life expectancy at birth and even when it is increasing most of the gains were at young ages. Improvements in adult mortality are relatively recent, and have been quite steady. The gains may be slower in the future, however there is disagreement among demographers about whether there are binding biological constraints on improvements in life expectancy.

The analysis supports the following conclusions for a growing economy. First, an increase in life expectancy raises the steady state national saving rate in both open and closed economies. Second, an increase in life expectancy increases the steady state current account in a small open economy. Third, in both a small open economy and a closed economy, a permanent and one-time increase in life expectancy produces a large transitory increase in the national saving rate. This is followed by a return to saving rate that is higher than the initial saving rate. Fourth, a rapid, sustained increase in life expectancy induces a jump to a high sustained saving rate. Fifth, a current increase in life expectancy causes a higher ratio of wealth to GDP in the next period. Sixth, in a closed economy, an increase in the survival rate brings about growth in GDP per effective worker.

This chapter is organized as follows. Section 2.2 presents the model. Section 2.3 describes the effect of longevity on the national saving rate, the investment rate, and capital flows in a small open economy. Section 2.4 describes the demand and supply of capital in a closed economy and discusses how longevity affects capital in equilibrium. Section 2.5 elaborates on the effects of longevity on the national saving rate in a closed economy. Section 2.6 discusses the results of detailed simulation analysis regarding longer life expectancy. Section 2.7 concludes.

2.2 Preferences and Production Structure

2.2.1 Consumer's optimization

This model considers a population consisting of two generations of adults. Each person lives two periods, working one period as a prime-age adult and retiring in the next as an elderly adult. It is assumed that all individuals survive through prime-age adulthood, yet some are assumed to die at the end of this first period of life. q_t is the probability that persons born in period t survive to age two. It is assumed that individuals know q_t, but they do not know whether or not they will live into the next period. Those who live until age 2 survive to the end of the period. We should note that all consumers are assumed to retire at age 2, therefore, a change in the survival rate, q, does not influence the size of labor force.

Consumption is governed by a constant intertemporal elasticity of substitution utility function. There is no bequest motive. Preferences are described by the following additively separable utility function.

$$V_t = \frac{c_{1,t}^{1-\theta}}{1-\theta} + \delta q_t \frac{c_{2,t+1}^{1-\theta}}{1-\theta}, \tag{2.1}$$

where $c_{1,t}$ and $c_{2,t+1}$ denote consumption per capita during prime years and during old-age, respectively. V_t is an expected utility function. δ is the discount factor, and is defined as $\delta=1/(1+\rho)$, where ρ is the discount rate. $1/\theta$ is the intertemporal elasticity of substitution.

Prime-age adults of age 1 earn $A_t w_t$. A_t is labor-augmenting technology and w_t is wage per unit of effective labor. They divide their earnings into current consumption and saving for old age, so that:

$$c_{1,t} + s_{1,t} = A_t w_t, \tag{2.2}$$

where $s_{1,t}$ is the saving of prime-age adults. Elderly adults are retired and consume what they saved while young. Insurance against longevity risk is available.[3] An annuity is purchased at the beginning of age 1. If insurance companies are risk neutral, and annuity markets are perfect. The rate of return to survivors is $[(1+r_{t+1})/q_t]$, where r_{t+1} is the riskless interest rate on saving. The return to annuities is $[(1+r_{t+1})/q_t]$. In this situation, returns to insurance are greater than a regular note. Thus, individuals save only in the form of insurance. Therefore, at age 2, consumers consume proceeds from saving. Consumption of the elderly is:

$$c_{2,t+1} = \frac{1+r_{t+1}}{q_t} s_{1,t}. \tag{2.3}$$

From (2.2) and (2.3), the lifetime budget constraint of the individuals at age 1 is:

$$w_t A_t = c_{1,t} + \frac{q_t}{1+r_{t+1}} c_{2,t+1}, \tag{2.4}$$

where $A_t w_t$ is labor income when an individual is young. Consumers maximize utility described in equation (2.1) facing the budget constraint in equation (2.4), controlling $c_{1,t}$ and $c_{2,t+1}$. From the first order conditions, consumption during prime and old age are:[4]

$$c_{1,t} = \Omega_t^{-1} A_t w_t, \tag{2.5}$$

$$c_{2,t+1} = \frac{(1+r_{t+1})}{q_t} \Psi_t \Omega_t^{-1} A_t w_t, \tag{2.6}$$

where Ω^{-1} is the share of labor income consumed in prime age and $\Psi\Omega^{-1}$ is the share saved by prime-age adults. Ω and Ψ are defined by:

$$\Omega_t = 1 + q_t \delta^{\frac{1}{\theta}} (1+r_{t+1})^{\frac{1-\theta}{\theta}}, \tag{2.7}$$

$$\Psi_t = q_t \delta^{\frac{1}{\theta}} (1+r_{t+1})^{\frac{1-\theta}{\theta}}. \tag{2.8}$$

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[3] Yaari (1965) and Blanchard (1985) discuss this issue in detail.

[4] Detailed calculations are described in Appendix A.

Saving of prime-age adults, $s_{1,t}$, is:

$$s_{1,t} = \Psi_t \Omega_t^{-1} A_t w_t = \frac{q_t \delta^{\frac{1}{\theta}} A_t w_t}{q_t \delta^{\frac{1}{\theta}} + (1+r_{t+1})^{\frac{\theta-1}{\theta}}}. \tag{2.9}$$

The elderly decumulate assets, so that the saving of the elderly, $s_{2,t}$, is negative and: $s_{2,t} = -s_{1,t-1}$.[5] Equations (2.5) and (2.6) imply that an increase in wage increases consumption at both ages 1 and 2, since consumption goods are considered to be normal goods. An increase in the interest rate has both a substitution effect and an income effect on consumption. The substitution effect means that the tradeoff between consumption in two periods becomes more favorable for second period consumption due to an increase in the interest rate. Therefore, saving increases when an individual is young if the substitution effect dominates. The income effect implies that consumers consume more at all ages if the interest rate rises because non-earned income increases. If $(1/\theta)>1$, the substitution effect dominates. If $(1/\theta)<1$, the income effect dominates. If $(1/\theta)>1$, an increase in the interest rate will raise saving. On the other hand, if $(1/\theta)<1$, it will lower saving.

Holding the wage and the interest rate constant in equation (2.9), the effect of an increase in the survival rate on the saving of prime-age adults is:

$$\frac{\partial s_{1,t}}{\partial q_t} = \frac{\delta^{\frac{1}{\theta}} (1+r_{t+1})^{\frac{\theta-1}{\theta}} A_t w_t}{[q_t \delta^{\frac{1}{\theta}} + (1+r_{t+1})^{\frac{\theta-1}{\theta}}]^2} > 0. \tag{2.10}$$

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[5] The elderly consume all of their disposable income, which includes earning from the return to the annuity, but that return is part of the income, so: $s_{2,t} = \left(\frac{1+r_t}{q_t} - 1 \right) s_{1,t-1} - c_{2,t} = -s_{1,t-1}$.

Prime-age adults save more if life expectancy increases. Equation (2.2) implies that $\frac{\partial s_{1,t}}{\partial q_t} = -\frac{\partial c_{1,t}}{\partial q_t}$, so that an increase in q_t has a negative effect on consumption of prime-age adults. From equation (2.6), $\frac{\partial c_{2,t+1}}{\partial q_t} < 0$ holds.

From equations (2.5) and (2.6), the age profile of consumption of those who survive is:

$$\frac{c_{2,t+1}}{c_{1,t}} = \delta^{\frac{1}{\theta}}(1+r_{t+1})^{\frac{1}{\theta}} = \left(\frac{1+r}{1+\rho}\right)^{\frac{1}{\theta}}. \tag{2.11}$$

Equation (2.11) implies that whether the interest rate is greater or less than the discount rate determines whether lifetime consumption is increasing or decreasing.[6] θ determines how much consumption varies in response to differences between the interest rate and the discount rate. Equation (2.11) is independent of the survival rate; The ratio of consumption at age 2 to consumption at age 1 is not influenced by longevity. An increase in q_t increases patience of the young, bur it reduces the return to the annuity. These effects offset each other in a perfect insurance market. This is consistent with the discussion of Yaari (1965). As is introduced in equation (1.15) in Chapter 1, the consumption path is not affected by an increase in the survival rate when insurance is available and the annuity market is perfect. We should also note that the age profile of consumption is not influenced by labor income.

2.2.2 Macroeconomic Setting

Output is determined by a Cobb-Douglas production function, $Y_t = K_t^{\phi} L_t^{1-\phi}$, where Y_t is output and $0 < \phi < 1$. $L_t = A_t N_{1,t}$ is the aggregate labor supply measured in efficiency units. $N_{1,t}$ is the population of prime-age adults and A_t is an index of technology. Technological growth is

[6] Remember $\delta = 1/(1+\rho)$, where ρ is the discount rate. Equation (2.11) implies $c_{1,t} > c_{2,t+1}$ if $r > \rho$ and $c_{1,t} \leq c_{2,t+1}$ if $r \leq \rho$.

exogenous, i.e., $A_t=gA_{t-1}$. The current prime-age population is n times the prime-age population in the previous period ($N_{1,t}=nN_{1,t-1}$), where n is assumed to be constant. The capital stock depreciates at rate ξ per period. Using lower case letters to represent quantities per unit of effective worker, output can be expressed as $y_t = k_t^{\phi}$.

2.3 Saving and Investment in Small Open Economies

Capital is perfectly mobile. The domestic economy can borrow and lend in the international capital market at the world interest rate. The assumption of a small open economy implies that the country is sufficiently small not to influence the world interest rate. According to the arbitrage condition, the marginal product of domestic capital is equal to the world interest rate, such as $\phi k_t^{\phi-1} - \xi = r_{w,t}$. The capital stock per effective worker is:

$$k_t = \left(\frac{\phi}{r_{w,t} + \xi} \right)^{\frac{1}{1-\phi}}. \tag{2.12}$$

The aggregate capital stock is given by $K_t=A_tN_{1,t}k_t$. Equation (2.12) implies that capital per effective worker is determined solely by the world interest rate and the parameters of the production process. Unlike the standard Solow model for a closed economy, the rate of growth of the labor force does not affect k_t, nor does the survival rate affect k_t.[7]

Gross investment at time t, I_t, is expressed as the capital stock of the next period (K_{t+1}) minus the undepreciated portion of capital ($(1-\xi)K_t$).

$$I_t = K_{t+1} - (1-\xi)K_t. \tag{2.13}$$

Noting that $GDP = Y_t = A_tN_{1,t}k_t^{\phi}$, the investment rate with respect to GDP is:

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[7] It is known as Fisher's separation theorem. This does not hold in a closed economy.

$$\frac{I_t}{Y_t} = \left(gn \frac{k_{t+1}}{k_t} - 1 + \xi \right) k_t^{1-\phi}. \tag{2.14}$$

Equation (2.14) indicates that the investment share of GDP depends on the rate of growth of GDP i.e., the combined effect of g and n, depreciation, and capital per effective worker. The investment rate is independent of the survival rate. As discussed before, it is assumed that an increase in life expectancy does not influence the prime-age population; it only increases the number of individuals in the old generation. This result arises from the assumption that the elderly do not earn wages. If we consider a labor force consisting of the old generation, or the survival rate of the children, investment will be affected by the survival rate. Further consideration of this matter is deferred to Chapter 3.

Given the world interest rate, the steady state investment rate is:

$$\left(\frac{I}{Y} \right)^* = \left(gn - 1 + \xi \right) k^{*1-\phi}, \tag{2.15}$$

where $k^{*1-\phi}$ is the equilibrium capital-output ratio. An increase in population growth and technological progress raise investment demand. From equation (2.15), given the equilibrium capital-output ratio, the gross investment rate necessary to maintain equilibrium depends on the rate of growth of GDP less the depreciation factor.

Gross national saving at time t (S_t) is the change in aggregate asset plus depreciation, $S_t = (K_{t+1}+F_{t+1}) - (K_t+F_t)+\xi K_t$, where K is domestic assets and F is foreign assets. Net national saving ($S_t - \xi K_t$) is equal to aggregate national income minus total consumption, thus,

$$(K_{t+1} + F_{t+1}) - (K_t + F_t) = w_t A_t N_{1,t} + r_t (K_t + F_t) - c_{1,t} N_{1,t} - c_{2,t} N_{2,t}. \tag{2.16}$$

Substituting $c_{1,t}$ and $c_{2,t}$ in equations (2.2) and (2.3) into equation (2.16), we get:

$$K_{t+1} + F_{t+1} = s_{1,t} N_{1,t} = S_{1,t}.^{8} \tag{2.17}$$

$S_{l,t}$ is the aggregate saving of prime-age adults. The saving of the young at time t is equal to the aggregate asset at time t+1. The elderly do not want to leave assets when they die, thus they dissave all assets by selling them to prime-age adults. When the domestic private asset ($K_{t+1}+F_{t+1}$) is more than the required capital (K_{t+1}), capital is lent in the international capital market, and in the opposite case, capital is borrowed. The national saving rate is also expressed as the sum of the saving of adults ($S_{1,t}$), the dissaving of the elderly ($S_{2,t}$) and depreciation, yielding $S_t=S_{l,t}+S_{2,t}+\xi K_t= S_{l,t}-S_{l,t-1}+\xi K_t$.

Noting that $\Psi\Omega^{-1}$is the saving share of labor income, the saving rate with respect to GDP is:

$$\frac{S_t}{Y_t} = \frac{(1-\phi)\Psi_t}{\Omega_t} - \frac{(1-\phi)\Psi_{t-1}}{\Omega_{t-1}} \frac{1}{gn}\left(\frac{k_{t-1}}{k_t}\right)^{\phi} + \xi k_t^{1-\phi}. \tag{2.18}$$

The first term of equation (2.18) is the product of the share of labor income in GDP and the ratio of saving to labor income of prime age adults, as derived in equation (2.9). The second term of equation (2.18) is the ratio of dissaving by the generation of the elderly to GDP. This is determined by saving of prime-age adults at time t-1.

The steady state gross national saving rate is:

$$\left(\frac{S}{Y}\right)^{*} = \frac{(1-\phi)\Psi^{*}}{\Omega^{*}} \frac{gn-1}{gn} + \xi k^{*1-\phi}. \tag{2.19}$$

Here, let us see the effect of an increase in q^* on steady state national saving rate. When gn=1, that is, there is no growth in GDP in steady state, the net national saving rate (saving without the depreciated portion) is zero. This implication is consistent with the basic life-cycle models of

[8] For the proof, see Appendix B.

Modigliani and Brumberg (1954), Tobin (1967) and the variable rate of growth (VRG) effect model of Fry and Mason (1982) and Mason (1981, 1987). If GDP is growing, the saving rate increases because the total resources of the young, saving generation rises relative to the total resources of the old, dissaving generation. From equation (2.19),

$$\frac{\partial(S/Y)^*}{\partial q^*} = \frac{(1-\phi)\delta^{\frac{1}{\theta}}(1+r_w^*)^{\frac{\theta-1}{\theta}}(gn-1)}{[q^*\delta^{\frac{1}{\theta}}+(1+r_w^*)^{\frac{\theta-1}{\theta}}]^2 gn}. \tag{2.20}$$

Equation (2.20) shows that an increase in life expectancy raises the national saving rate in the long run if GDP is growing. If individuals live longer, young generations save more, but the elderly dissave even more. Under the assumption that the growth of GDP in steady state is positive, that is, $gn>1$, the saving of prime-age adults exceeds the dissaving by the elderly. If $gn<1$, an increase in the survival rate decreases the national saving rate. This implication corresponds to that of the VRG effect model. As is described in equation (1.3) of the previous chapter, an increase in the mean age of consumption (A_C) increases the saving rate if earning is growing. Here, an increase in q gives rise to an increase in A_C, so that the implication of our model is consistent with the VRG model (Mason (1981,1987) and Fry and Mason (1982)).

The effect of an anticipated increase in the survival rate on current saving rate is:

$$\frac{\partial(S_t/Y_t)}{\partial q_t} = \frac{1-\phi}{\Omega_t^{\,2}}(1+r_w^*)^{\frac{\theta-1}{\theta}} > 0. \tag{2.21}$$

The effect on the saving rate in period t+1 is:

$$\frac{\partial(S_{t+1}/Y_{t+1})}{\partial q_t} = -\frac{(1-\phi)\delta^{\frac{1}{\theta}}(1+r_w^*)^{\frac{\theta-1}{\theta}}}{gn[q_t\delta^{\frac{1}{\theta}}+(1+r_w^*)^{\frac{\theta-1}{\theta}}]^2} < 0. \tag{2.22}$$

Equation (2.21) shows that a current increase in the survival rate will raise the current saving rate and equation (2.22) implies that it will lower the saving rate with a one period lag. Equation (2.21) shows that the effect of an increase in life expectancy on the current saving rate is not

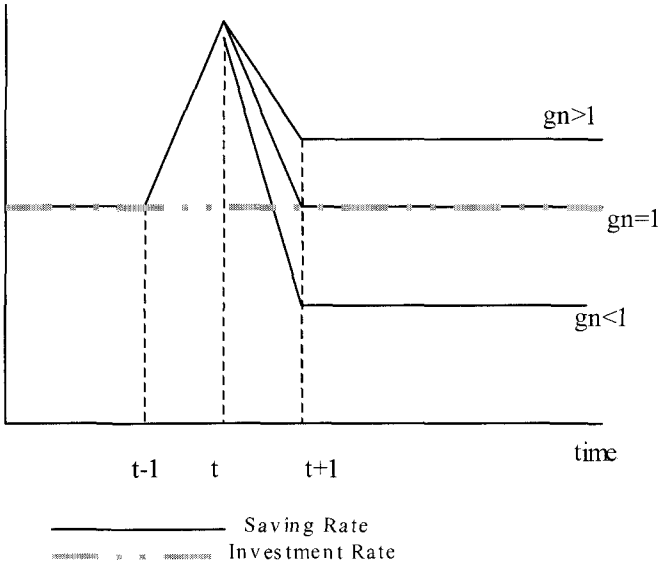
influenced by the growth of GDP. Equation (2.18) shows that the one-time increase in q_t will not influence the saving rate after period $t+1$. The saving rate will remain constant at the value attained in period t+1.

The dynamic effects of a one-time, permanent increase in the survival rate at time t are charted in Figure 2.1 (a). Because young individuals know that they are likely to live longer, they are motivated to save more at time t. The dissaving of the elderly is not affected. Therefore, in equation (2.18), the first term increases but the second term does not. At time t+1 and all future periods, the saving of the young is constant at a level higher than period t. The elderly dissave more because they have saved more while they were young. Thus, the absolute value of the second term of equation (2.18) is greater at time t. As a result, the saving rate is less than that in period t. After time t+1, the amounts of saving of the young and dissaving of the old are constant. The national saving rate reaches a maximum at time t. Whether S_{t+1} is greater or less than the initial saving rate depends on the rate of growth. If $gn>1$, that is, if earning is growing, then the saving rate stabilizes at a higher level than the initial one. If $gn<1$, that is if earning is declining, then the saving after time t+1 is less than the initial level. If $gn=1$, that is earning is not changing, the saving after time t+1 is equal to the initial saving rate. The national saving rate increases regardless of the growth in GDP at time t, the transitory period.

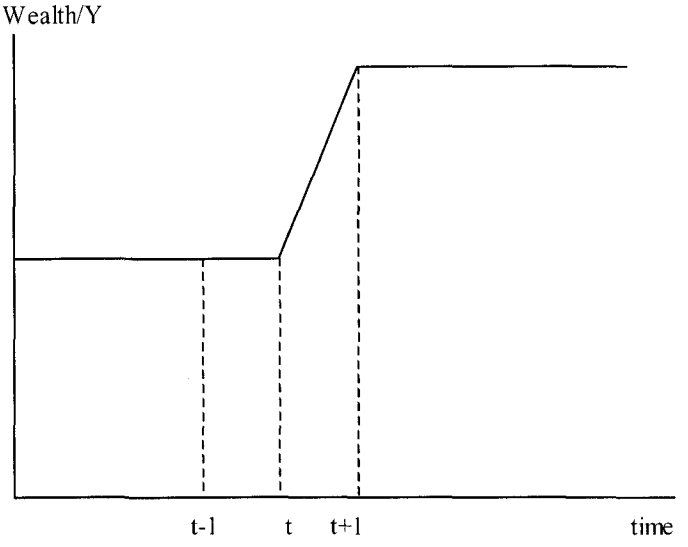
The dynamics of the model can also be explained in terms of wealth. Figure 2.1 (b) presents the dynamics of wealth relative to GDP given an increase in the survival rate at time t. Saving of the young at time t establishes the wealth at time $t+1$. An increase in the survival rate at time t increases the wealth at time $t+1$. After time $t+1$, wealth remains constant and is higher than the initial level. A change in national saving yields a change in national wealth. In Figure 2.1 (b),

Figure 2.1 The Effect of a Permanent One-Time Increase in the Survival Rate in a Small Open Economy

(a) The Effect on the National Saving Rate



(b) The Effect on the Share of Wealth in GDP



wealth increases at time $t+1$, therefore it is clear that the national saving rate produces a large increase at time t. After time $t+1$, the saving rate is constant because wealth is constant. In addition, the growth in wealth is not as high as time t in any case, so the saving rate at time $t+1$ is lower than the initial saving rate. If GDP is growing, aggregate wealth is growing and the saving rate at time t is higher than that of time $t-1$.

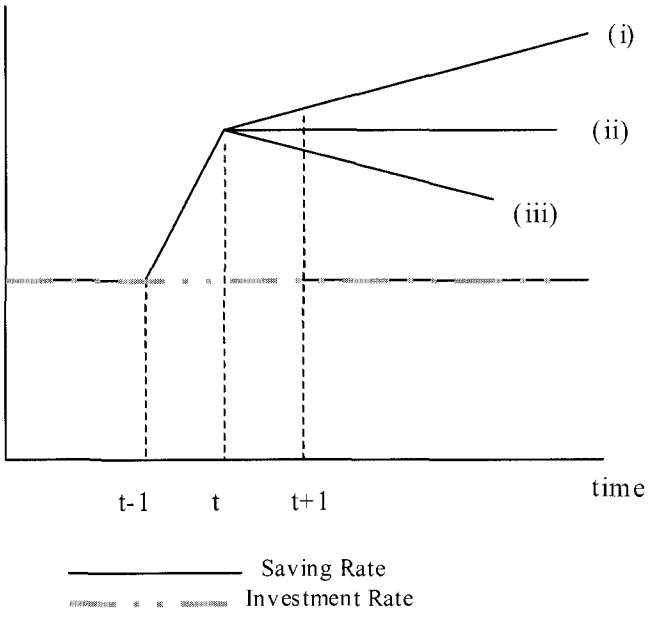
Figure 2.2 (a) displays the dynamics of the national saving rate with a continuing increase in the survival rate. Suppose the survival rate is constant until time $t-1$ and begins to increase continuingly at time t. At time t, the saving rate jumps, as saving of the young increases and the dissaving of the elderly does not change. After time $t+1$, the saving rate continues to increase as shown by pattern (i) if GDP is growing. If GDP is declining, the saving rate is either increasing with flatter slope than when GDP is growing, or constant as illustrated pattern (ii), or declining as shown by pattern (iii) depending on the speed of an increase in the survival rate or the values of parameters. If the survival rate trends upward, the economy never reaches steady state, and the saving rate continues to change. Figure 2.2 (b) presents the dynamic effects of a continuing increase in the survival rate on the share of wealth in GDP. Until time t, wealth is constant. At time $t+1$, it begins to increase continuingly.

The current account balance at time t (CA_t) is the economy's net accumulation of foreign assets, that is: $CA_t = S_t - I_t = (S_{1,t} - S_{1,t-1}) - (K_{t+1} - K_t) = F_{t+1} - F_t$. The current account share of GDP is:

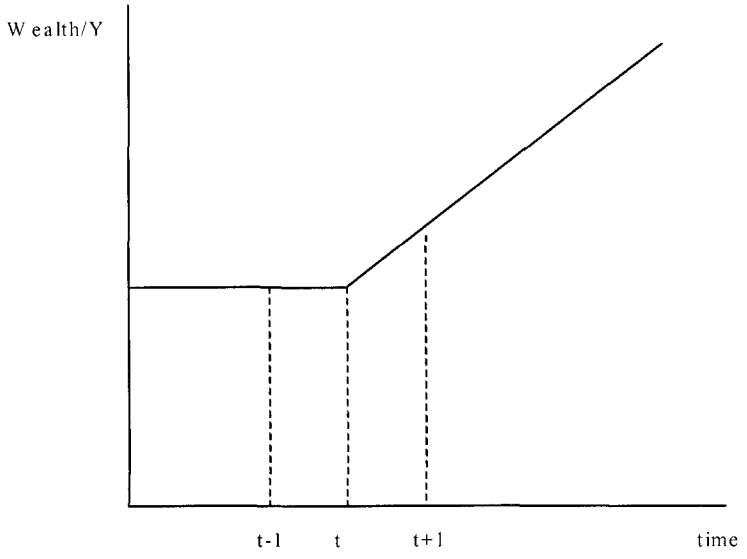
$$\frac{CA_t}{Y_t} = \frac{(1-\phi)\Psi_t}{\Omega_t} - \frac{(1-\phi)\Psi_{t-1}}{\Omega_{t-1} gn}\left(\frac{k_{t-1}}{k_t}\right)^{\phi} - gn\frac{k_{t+1}}{k_t^{\phi}} + k_t^{1-\phi}. \qquad (2.23)$$

Figure 2.2 The Effects of a Continuing Increase in the Survival Rate in a Small Open Economy

(a) The Effect on the National Saving Rate



(b) The Effect on the Share of Wealth in GDP



In steady state,

$$\left(\frac{CA}{Y}\right)^{*} = \frac{(1-\phi)\Psi^{*}}{\Omega^{*}}\frac{gn-1}{gn} + (1-gn)k^{*1-\phi}. \tag{2.24}$$

This implies that the current account share of GDP is constant in steady state. The current account ratio moves in the same direction as the national saving rate because investment is unaffected by longevity. In steady state, if GDP is growing (gn>1), an increase in the survival rate increases saving and does not influence investment, producing capital outflows. If GDP is declining (gn<1), longevity causes capital inflows in steady state.

In Figure 2.1, the gap between the saving rate and the investment rate is the ratio of the current account to GDP. A one-time, permanent increase in the survival rate at time t produces a large transitory increase in the current account. At time t+1, the current account balance decreases and capital inflows are less than that at time t. After time t+1, the ratio of the capital flow to GDP is constant at the new steady state level. In Figure 2.2, the ratio of current account balance to GDP is the difference between the saving rate and the investment rate. If the survival rate is trending upward, the current account balance increases if GDP is growing. If GDP is declining rapidly, it is possible that a rapid increase in the survival rate could expand the current account deficit as time passes.

2.4 Demand and Supply of Capital in a Closed Economy

National saving is the net increase of national wealth. In a closed economy, wealth is equal to domestic capital. In this section, we discuss the determinants of national wealth, and we discuss the national saving in next section.

In a closed economy, there are no international capital flows. Thus, domestic saving equals investment. At equilibrium the demand and supply of capital are equivalent. The interest rate is

determined endogenously. Diamond (1965) examines the long- run competitive equilibrium in a growth model and explores the effects of government debt. The author describes the determinants of capital per effective worker at the equilibrium. Here, we extend the Diamond model by including the effect of an increase in the survival rate.

Consumers decide how much to consume and save as described in section 2. Producers demand capital so that the marginal product of capital net of depreciation equals the interest rate. The demand curve for capital is:

$$r_{t+1} = f'(k_{t+1}) - \xi = \phi k_{t+1}^{\phi-1} - \xi \,. \tag{2.25}$$

The demand for capital $\mathscr{D}$is expressed as $\mathscr{D}_{t+1}=\mathscr{D}_{t+1}(r_{t+1})$.

As is the case in small open economies, gross national saving is the sum of changes in asset holdings of the prime age adults and the elderly and depreciation, so that $S_t=S_{1,t}+S_{2,t}+\xi K_t = S_{1,t} - S_{1,t-1}+\xi K_t$. Gross investment is given by $I_t = K_{t+1} - (1-\xi)K_t$. Because saving is equal to investment, $S_{1,t} = K_{t+1}$. The saving of the young is equal to the capital stock in the next period. Thus, the supply of capital is:

$$K_{t+1} = S_{1,t} = s_{1,t} N_{1,t} = \frac{q_t \delta^{\frac{1}{\theta}} A_t w_t N_{1,t}}{q_t \delta^{\frac{1}{\theta}} + (1+r_{t+1})^{\frac{\theta-1}{\theta}}} \,. \tag{2.26}$$

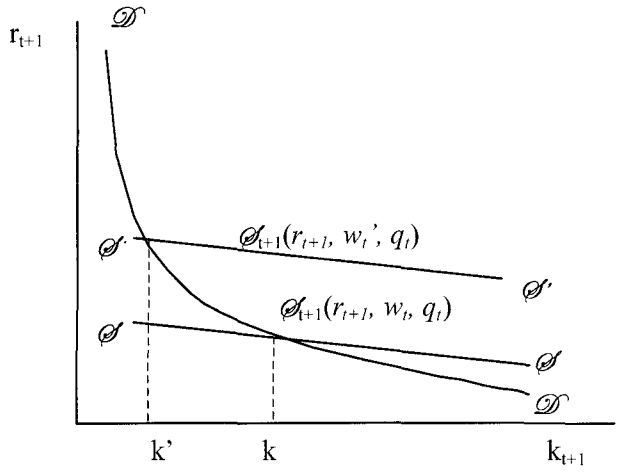
Noting $K_{t+1} = A_{t+1} N_{1,t+1} k_{t+1}$, equation (2.26) can be rewritten as:

$$k_{t+1} = \frac{q_t \delta^{\frac{1}{\theta}} w_t}{gn[q_t \delta^{\frac{1}{\theta}} + (1+r_{t+1})^{\frac{\theta-1}{\theta}}]} = \mathscr{S}_t(r_{t+1}, w_t, q_t) \,, \tag{2.27}$$

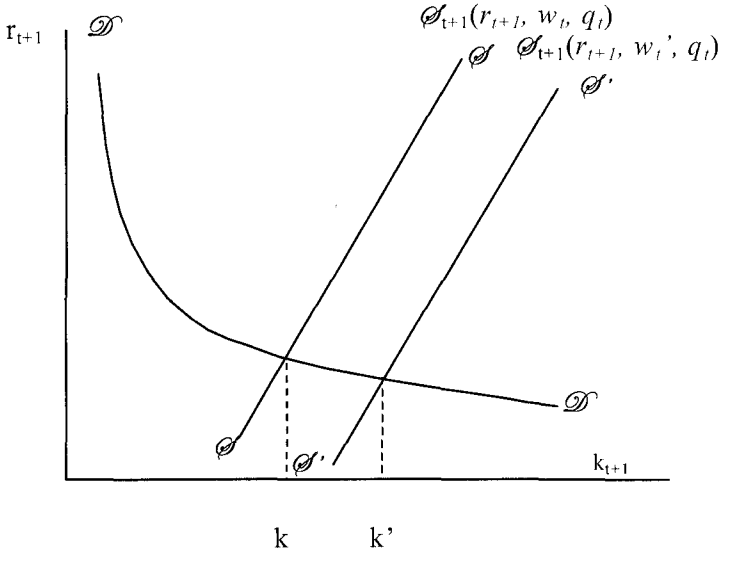
where $\mathscr{S}_t$denotes the supply of capital per unit of effective labor. Equation (2.27) is the supply curve of capital. Equation (2.27) implies that the supply of capital at time $t+1$ is a function of the interest rate at time $t+1$ and wage at time t.

Figure 2.3 Demand and Supply Curves of Capital

(a)



(b)



The wage is equal to marginal product of effective worker and is described as:

$$w_t = f(k_t) - k_t f'(k_t) = (1-\phi)k_t^{\phi}. \tag{2.28}$$

The demand and supply curves in period t+1 are presented in Figure 2.3. The vertical axis is the interest rate at time t+1. $\mathscr{DD}$ is the demand curve, and $\mathscr{SS}$ is the supply curve. In equation (2.25), $f''<0$, so that the demand curve is downward sloping. Equation (2.27) implies that the slope of the supply curve is either positive or negative. Figure 2.3 (a) presents the case where the demand curve is more negatively sloped than the supply curve. In this diagram, the elasticity of saving with respect to the interest rate is large and negative. A sufficient condition for this situation is that the intertemporal elasticity of substitution is much less than unity. An increase in wage at time t shifts the supply curve of capital upwards, which decreases the equilibrium level of capital at time t. In Figure 2.3 (b), the supply curve is more negatively sloped than the demand curve. An increase in the wage at time t shifts the supply curve to the right. As a result, the capital stock at time $t+1$ increases. An increase in the supply of capital increases the equilibrium capital per effective labor. This conclusion seems more usual than Figure 2 (a), therefore we will focus on this case hereafter.

Combining the demand and supply curves in equations (2.25) and (2.27), by equating $\mathscr{S}_t$ and k_{t+1}, the following equilibrium condition is obtained.

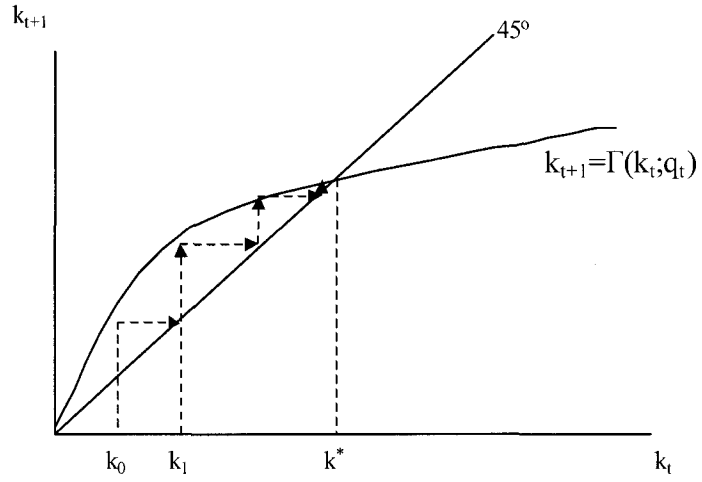
$$(1-\phi)\Psi_t(q_t, r(k_{t+1}))k_t^{\phi} = k_{t+1}gn\Omega_t(q_t, r(k_{t+1})). \tag{2.29}$$

If the survival rate is constant, capital per effective worker will approach the steady state:

$$k^* = \left[\frac{(1-\phi)\Psi(q^*, r(k^*))}{gn\Omega(q^*, r(k^*))}\right]^{\frac{1}{1-\phi}}. \tag{2.30}$$

k^* and q^* denote capital per unit of effective labor and the survival rate in steady state, respectively. Assuming variables other than capital per effective worker at times t and $t+1$ are constant in equation (2.29), the relationship between k_t and k_{t+1} is described as:

Figure 2.4 Dynamics of Capital per Effective Labor



$$\frac{dk_{t+1}}{dk_t}=\frac{(1-\phi)f'(k_t)\Psi_t\Omega_t^{-1}}{gn+[gnk_{t+1}-(1-\phi)f(k_t)](\frac{1-\theta}{\theta})\Psi_t\Omega_t^{-1}[1+f'(k_{t+1})-\xi]^{-1}f''(k_{t+1})}. \qquad (2.31)$$

In equation (2.31), gnk_{t+1} is saving and $(1-\phi)f(k_t)$ is earning per prime age adult at time t. Because saving is less than earning, $gnk_{t+1}<(1-\phi)f(k_t)$. Note that $f''<0$ and that $0<\Psi\Omega^{-1}<1$ hold.

Substituting equation (2.30) into equation (2.31), the numerator of the right hand side of equation (2.31) reduces to ϕgn in steady state. Thus, $0<\frac{dk_{t+1}}{dk_t}<1$ is satisfied at least around the steady state unless the intertemporal elasticity of substitution (1/θ) is much less than 1. If $0<\frac{dk_{t+1}}{dk_t}<1$ holds in equation (2.31), the steady state is stable. Also, if the intertemporal elasticity of substitution is much less than 1, it is possible that the denominator of equation (2.31) is negative. For simplicity, we assume that $0<\frac{dk_{t+1}}{dk_t}<1$ holds; in other words, the economy converges in a non-oscillatory pattern.

The dynamics of k are shown in Figure 2.4.[9] Let us rewrite the relationship between k_t and k_{t+1} in equation (2.29) as:

$$k_{t+1} = \Gamma(k_t;\ q_t). \tag{2.32}$$

We name the function in equation (2.32) the saving locus, as Blanchard and Fischer (1989) do. Suppose the economy is initially at k_0. Because k_{t+1} is greater than k_t, capital per effective worker increases over time until the economy reaches the steady state E. If initial capital per unit of effective worker is greater than in steady state, k falls over time until k attains the steady state value.

Suppose the survival rate is constant in steady state. Then, in steady state, the equilibrium of the capital market in equation (2.29) is given by:

$$(1-\phi)\Psi(q^*, r(k^*))k^{*\phi} = k^* gn\Omega(q^*, r(k^*)). \tag{2.33}$$

Calculating total differentials of both sides of equation (2.33) and simplifying the results yields the following expression of the relationship between capital per of effective worker and the survival rate:

$$\frac{dk^*}{dq^*} = \frac{\Omega^{*-1}\delta^{\frac{1}{\theta}}[1+f'(k^*)-\xi]^{\frac{1-\theta}{\theta}}[(1-\phi)k^{*\phi}-k^* gn]}{gn+[gnk^*-(1-\phi)f(k^*)](\frac{1-\theta}{\theta})\Psi^*\Omega^{*-1}[1+f'(k^*)-\xi]^{-1}f''(k^*)-\Psi^*\Omega^{*-1}(1-\phi)f'(k^*)}. \tag{2.34}$$

Equation (2.30) and $0 < \frac{dk_{t+1}}{dk_t} < 1$ imply that the denominator of equation (2.34) is positive. Because saving of prime age adults is less than wage income, $(1-\phi)k^{*\phi} - k^* gn > 0$ holds, so that the numerator is also positive. Therefore, $\frac{\partial k^*}{\partial q^*} > 0$ holds.

[9] For detailed discussion, see Romer (2001), Blanchard and Fischer (1989) and Barro and Sala-i-Martin (1995).

Consider the effect of an anticipated one-time increase in life expectancy at time t and assume that q stays at q^* in subsequent periods. In equation (2.29), assuming that all variables other than k_{t+1} and q_t are constant, we can characterize the relationship between the changes of q_t and k_{t+1} as:

$$\frac{dk_{t+1}}{dq_t} = \frac{\Omega_t^{-1}\delta^{\frac{1}{\theta}}[1+f'(k_{t+1})-\xi]^{\frac{1-\theta}{\theta}}[(1-\phi)k^{*\phi}-k_{t+1}gn]}{gn+[gnk_{t+1}-(1-\phi)f(k_t)](\frac{1-\theta}{\theta})\Psi_t\Omega_t^{-1}[1+f'(k_{t+1})-\xi]^{-1}f''(k_{t+1})}. \qquad (2.35)$$

The denominator of the right hand side of equation (2.35) is equal to that of equation (2.31). We assume that $0 < \frac{dk_{t+1}}{dk_t} < 1$ holds, so that the denominator of equation (2.35) is positive. As we discussed above, $(1-\phi)k^{*\phi}$ is wage earning at time t, and $k_{t+1}gn$ is the saving per prime-age adult. Because saving of the young does not exceed earning, $(1-\phi)k^{*\phi} - k_{t+1}gn > 0$. Therefore, the numerator is positive and $\frac{dk_{t+1}}{dq_t} > 0$ holds.

It is assumed that the economy is initially in steady state. Suppose the survival rate increases from q^* to q' at time 1 and stays at the same level thereafter. The effects are demonstrated in Figure 2.5. Figure 2.5(a) presents demand and supply of capital. The demand curve of capital is not influenced by the change in the survival rate. Equation (2.27) suggests that capital per effective worker does not change at time 1. At time 1, prime-age adults save more because they face a higher survival rate. The supply of capital increases at time 1 – shown as a shift of the supply curve of capital from $\mathscr{S}^*\mathscr{S}^*$ to $\mathscr{S}_1\mathscr{S}_1$ in Figure 2.5. At the new equilibrium, the capital stock k_2 is higher, the wage is higher, and the interest rate r_2 is lower, causing prime-age adults to save more compared to time 1. The supply curve shifts to the right to $\mathscr{S}_2\mathscr{S}_2$. This induces a further increase in capital per effective worker. A lower interest rate at time 4 and a higher wage at time 3 cause the

supply curve at time 3 to shift to the right. This process continues until a new equilibrium is established.

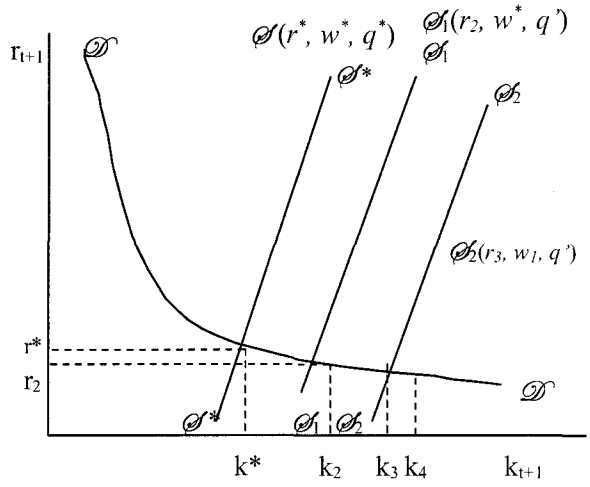
Figure 2.5 (b) demonstrates the relationship of k_{t+1} and k_t at the equilibrium. The economy is initially in steady state E. At time 1, the survival rate increases from q^* to q', which shifts the saving locus Γ upward to Γ'. Capital per effective worker at time t does not change. At time 2, capital per effective worker increases. After time 3, k continues to increase along the saving locus Γ' until it reaches the new steady state value k^{**}.

Suppose that the economy is in steady state at time 0 and, after time 1, the survival rate increases by a constant amount. Then, $q^* < q_1 < q_2 < q_3 \ldots$. Figure 2.6 (a) describes the changes of demand and supply and the equilibrium. The capital per effective worker at time 1 does not change. At time 1, the saving of prime-age adults increases and the supply curve shifts from $\mathscr{S}^*\mathscr{S}^*$ to $\mathscr{S}_1\mathscr{S}_1$. As a result, capital per effective worker at time 2 increases. At time 2, the saving of prime-age adults increases because they have higher wages and are more likely to live longer. Thus, the supply curve shifts to the right. This pattern continues unless the increase in the survival rate stops. Thus, if the survival rate continues to rise, capital per effective worker or the capital-output ratio continues to rise. GDP per effective worker also increases because it is an increasing function of capital per effective worker. Therefore, an increase in the survival rate causes higher economic growth.

Figure 2.6 (b) presents the dynamics of capital per effective worker. Initially, the economy is in steady state E. At time 1, the survival rate increases and the saving locus shifts upward. k_2 is determined according to the new saving locus. At time 2, because the survival rate continues to rise, the saving locus shifts upward again. Thus, a higher k is achieved. After time 2, the saving locus

Figure 2.5 The Effect of a One-Time Permanent Increase in the Survival Rate in a Closed Economy

(a)



(b)

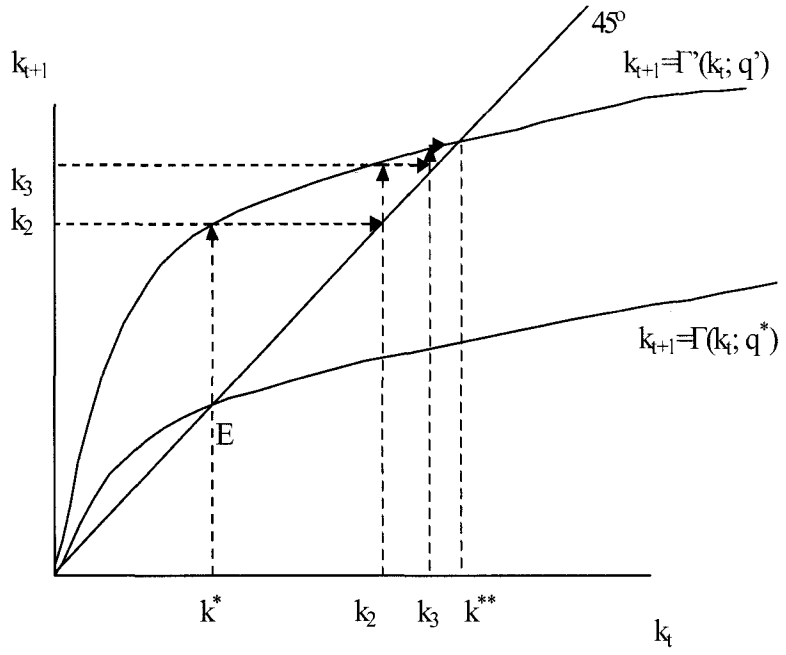
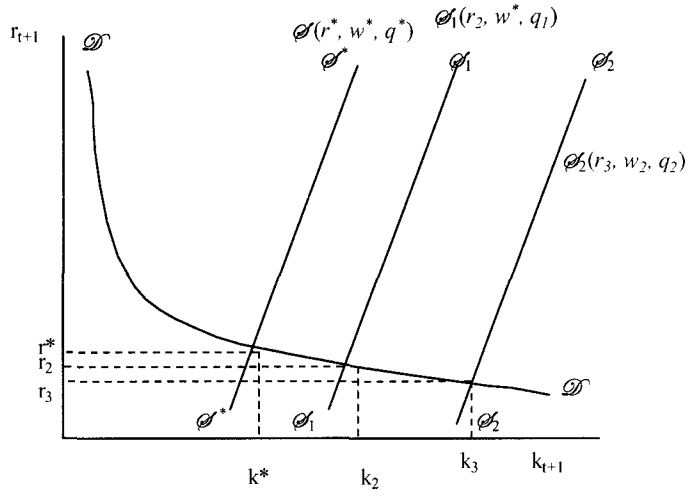
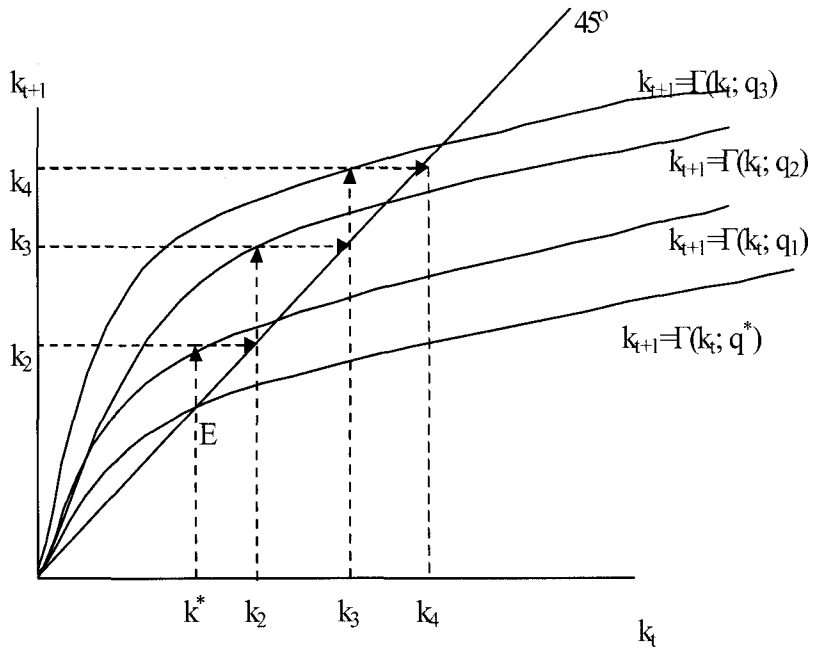


Figure 2.6 The Effect of a Continuing Increase in the Survival Rate in a Closed Economy

(a)



(b)



shifts upward every period, and k continues to increase. If the survival rate increases continuingly, the economy does not reach a steady state, and capital per effective worker keeps growing.

2.5 Saving in a Closed Economy

The saving rate, equal to investment as a share of GDP in a closed economy, is:

$$\begin{aligned}\frac{S_t}{Y_t} &= (1-\phi)\Psi_t\Omega_t^{-1} - (1-\xi)k_t^{1-\phi} \\ &= \left(gn\frac{k_{t+1}}{k_t} - 1 + \xi \right) k_t^{1-\phi}.\end{aligned} \tag{2.36}$$

In steady state $k_t = k_{t+1} = k^*$, and the gross national saving rate in steady state is:

$$\left(\frac{S}{Y}\right)^* = (gn - 1 + \xi)k^{*1-\phi}, \tag{2.37}$$

where $k_t^{1-\phi}$ is the capital-output ratio. The steady state gross saving rate must be sufficient to provide for capital widening and to replace any depreciated capital. Growth in per capita output (g) and population (n) determines the extent of capital widening (gn–1) while ξ determines depreciation. If the saving rate exceeds the steady state rate, capital deepening occurs, i.e. $k_{t+1}/k_t > 1$. The rise in the capital-labor ratio produces a rise in the capital-output ratio. The relationship between the saving rate and capital deepening is clear from equation (2.36). On the other hand, if saving is below the steady state rate, the capital-labor ratio declines, as does the capital-output ratio.

Computing partial derivatives of both sides of equation (2.37) with respect to q^*, the effect of longevity on saving rate is:

$$\frac{\partial (S/Y)^*}{\partial q^*} = (gn - 1 + \xi)(1-\phi)k^{*-\phi}\frac{\partial k^*}{\partial q^*}. \tag{2.38}$$

Because $\frac{\partial k^*}{\partial q^*} > 0$ is satisfied, an increase in the survival rate increases the gross national saving rate if GDP is growing. If $gn - 1 > -\xi$, that is, if GDP is declining at a rate faster than the rate of depreciation, an increase in q^* has a negative effect on the saving rate.

Taking partial derivatives of both sides of equation (2.36) with respect to q_t, we can calculate the effect of the national saving rate of a one-time, anticipated increase of the survival rate:

$$\frac{\partial (S_t/Y_t)}{\partial q_t} = \frac{gn}{k^{*\phi}}\frac{dk_{t+1}}{dq_t} > 0. \tag{2.39}$$

Equation (2.39) implies that an anticipated increase in the survival rate at time t leads to a higher national saving rate. As discussed in the previous section, wealth at time t is not affected by q_t. At time t, saving of prime-age adults increases, which causes an increase in k_{t+1}. The supply of capital increases and the saving rate increases. As in a small open economy, the survival rate at time t only affects the saving of prime-age adults.

The effect of an anticipated increase in the survival rate at time t on the national saving rate at time $t+1$ can be calculated as:

$$\frac{\partial (S_{t+1}/Y_{t+1})}{\partial q_t} = k_{t+1}^{-\phi}\left[gn\left(\frac{\partial k_{t+2}}{\partial k_{t+1}} - \frac{k_{t+2}}{k_{t+1}}\phi\right) - (1-\phi)(1-\xi)\right]\frac{\partial k_{t+1}}{\partial q_t}. \tag{2.40}$$

As is seen above, $0 < \frac{\partial k_{t+2}}{\partial k_{t+1}} < 1$. Also, $\frac{dk_{t+1}}{dq_t} > 0$ holds. For plausible parameter values, this expression is negative. The saving at time $t+1$ is the difference between the wealth at time $t+2$ and $t+1$. An increase in q_t induces capital deepening at time $t+1$, but the wealth of the previous period and depreciation are also higher than at time t. The net increase in wealth at time $t+1$ is less than at time t.

In a closed economy, the survival rate at time t will also influence the saving rate at time $t+2$. This stands in contrast to the case of small open economy.

$$\frac{\partial(S_{t+2}/Y_{t+2})}{\partial q_t} = k_{t+2}^{-\phi}\left[gn\left(\frac{\partial k_{t+3}}{\partial k_{t+2}} - \frac{k_{t+3}}{k_{t+2}}\phi \right) - (1-\phi)(1-\xi) \right] \frac{\partial k_{t+2}}{\partial k_{t+1}} \frac{\partial k_{t+1}}{\partial q_t}. \quad (2.41)$$

$0 < \frac{\partial k_{t+3}}{\partial k_{t+2}} < 1$, $0 < \frac{\partial k_{t+2}}{\partial k_{t+1}} < 1$, and $\frac{dk_{t+1}}{dq_t} > 0$ hold. In the right hand side of equation (2.41), the value inside the bracket is negative for plausible parameters,[10] so that the saving rate at time t has a negative effect on the saving rate at time $t+2$. In the same way, the effects of the survival rate at time t on the saving rate after time $t+2$ can be derived. q_t has a negative effect on the saving rate after time $t+2$.

The intuition of the results above is quite similar to that for small open economy. As long as GDP growth is not very small, specifically, as long as $gn - 1 + \xi > 0$, the gross national saving rate increases in steady state if life expectancy increases. The reason is that an increase in the survival rate increases the saving of prime-age adults and the dissaving of the elderly. An increase in the survival rate at time t increases the saving of the young and does not influence the dissaving of the old at time t. At time $t+1$, the dissaving of the elderly increases because they saved more while they were young, which causes the national saving rate to decrease. Therefore, a permanent increase in the survival rate has a large transitory effect on the saving rate. A different point from the small open economy is that capital stock is determined by equating demand and supply of capital. Capital per effective worker is endogenous in a closed economy, although it is effectively exogenous for the consumption and saving decision in a small open economy. An increase in q at time t induces capital accumulation, and it increases the wage in the next period. The economy does not reach the

[10] For example, the parameters listed in Table 2.1.

steady state immediately. If life expectancy increases permanently, the national saving rate decreases gradually after the transitory increase. Finally, the economy will reach steady state at a higher national saving rate than the initial rate.

2.6 Simulation Analysis of Longevity

In sections 2.3, 2.4, and 2.5, we calculate partial derivatives of the national saving rate with respect to the survival rate, and find that longevity will have various effects from time to time. Simulation analysis is effective to investigate the dynamics of saving. It is important to analyze the effect of an increase in the survival rate on the national saving rate in a closed economy by simulation analysis because we cannot clarify how the national saving rate varies during the transition analytically. In this section, we discuss the transition of the national saving rate in a closed economy and compare the results to those in a small open economy.

We simulate the effect of changes in the survival rate on the national saving rate using the historical survival rates of United States and Japan from 1900 to the projected values for 2020. Our model assumes that individuals live for two 30-year periods and that there are no children. That is to say, prime-age adults are those from 30 to 59 years old, and the elderly are those from 60 to 90 years old. We calculate the simulated saving rates of United States and Japan at 30 year intervals from 1900 to 2020. "Survival rates" are calculated from life tables using $\sum_{x=60}^{89} L_x / \sum_{x=30}^{59} L_x$, where L_x is number of years lived between exact age x and exact age $x+1$.[11] This is the ratio of total years lived from ages 60 to 89 to total years lived from ages 30 to 59. For technological growth (g), we use data on labor productivity growth. For labor force growth (n), we use the rate of growth of the population aged from 30 to 59. g (n) in 1900 represents the growth rate of

[11] When data for the total population are not available, we used the mean of the survival rates for males and females.

Table 2.1 List of Parameters

Parameters	Description	Value
ρ	Discount Rate	0.811
δ	Discount Factor (1/1+ρ)	0.522
ξ	Depreciation	0.785
$1/\theta$	Intertemporal elasticity of substitution	1.3
ϕ	Share of Capital in Total Output	1/3
r_w	World interest rate (Small Open Economy)	4.644

Note: These parameters are for 30 years.

productivity (labor force) from 1900 to 1930. g or n after 1930 are defined in the same way. n in 2020 and 2050 is calculated from projected population by age.

We assume the values for each parameter listed in Table 2.1. The share of capital in total output (ϕ) is set at one-third. The rate of time preference is 0.02 per annum, and 0.811 for 30 years. Following Higgins (1994), an intertemporal elasticity of substitution ($1/\theta$) is 1.3, so that a higher interest rate increases the saving rate modestly. We assume that the economy is in steady state in 1900 in both the United States and Japan. The capital stock depreciates by 5 per cent per annum, so that $\xi=1-0.95^{30}=0.785$. This set of parameters yields an interest rate of 4.653 (an annualized interest rate of 0.059) for the closed economy in the United States in steady state in 1900. This value is chosen as the world interest rate for the open economy cases for both the United States and Japan.

Table 2.2 summarizes survival rates (q), g, n, gn, and the results of simulation analysis for the United States and Japan. Figure 2.7 (a) plots q and gn for the United States. The survival rate increased remarkably from 1930 to 1990 in the US. Figure 2.8 (a) plots q and gn for Japan. In Japan, the survival rate was low and decreased slightly in 1930 compared to 1900. However, the survival rate increased after 1930 and the survival rate in 1990 was almost double that

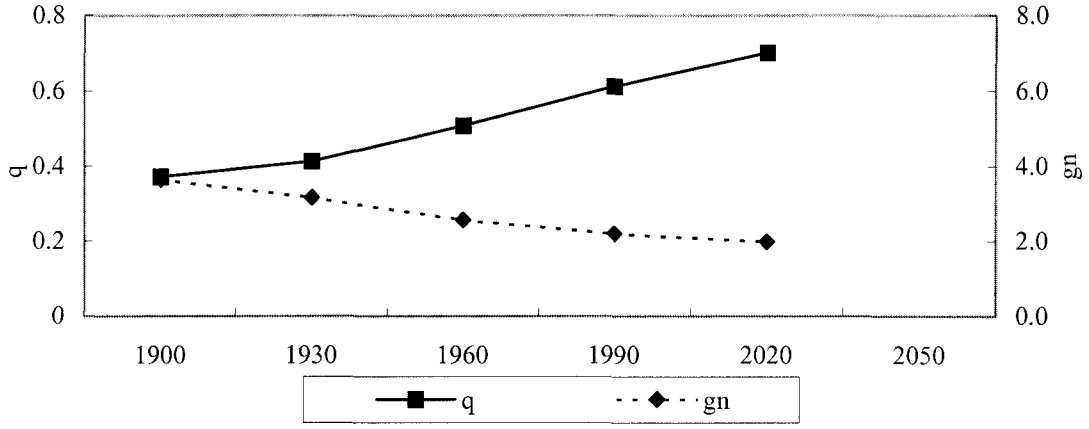
Table 2.2 Simulated Saving Rates of the United States and Japan

The United States									
					Closed Economy			Small Open Economy	
Time	*q*	*g*	*n*	*gn*	*k*	*K/Y*	*S/Y*	*S/Y*	*I/Y*
1900	0.373	1.944	1.878	3.650	0.009	0.044	0.152	0.152	0.152
1930	0.413	2.139	1.479	3.164	0.009	0.044	0.160	0.164	0.130
1960	0.508	1.760	1.461	2.571	0.011	0.050	0.176	0.181	0.104
1990	0.613	1.709	1.290	2.206	0.016	0.064	0.189	0.185	0.088
2020	0.702	1.709	1.167	1.995	0.023	0.082	0.198	0.181	0.079
2050					0.031	0.098			
Japan									
					Closed Economy			Small Open Economies	
Time	*q*	*g*	*n*	*gn*	*k*	*K/Y*	*S/Y*	*S/Y*	*I/Y*
1900	0.374	1.759	1.382	2.431	0.015	0.062	0.137	0.130	0.098
1930	0.371	3.059	1.629	4.983	0.015	0.062	0.150	0.129	0.210
1960	0.477	4.349	1.674	7.279	0.008	0.040	0.198	0.196	0.312
1990	0.680	1.345	0.939	1.263	0.006	0.032	0.204	0.253	0.046
2020	0.781	1.345	0.702	0.944	0.030	0.096	0.188	0.107	0.032
2050					0.069	0.168			

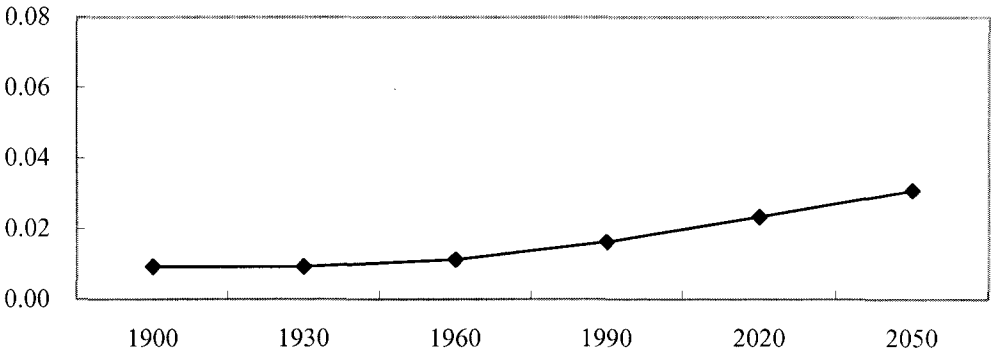
Note: q: the survival rate for those aged 30-59 to become 60-89, g: 1+GDP growth rate, n: 1+growth rate of labor force k: capital per effective labor, K/Y capital-output ratio, S/Y, the national saving rate, I/Y: the national investment rate.
Sources: For the United States; q: Bell and Miller (2002) for 1900 and 1930, Wilmoth and Shkolnikov, *The Human Mortality Database* for 1960 and 1990, United Nations (2000) for 2020. n: *Statistical Abstract of the United States*, various years. g: Maddison (1995) for 1900 and 1930. *Penn World Table* for 1960, 1990. For Japan, q: *Japan Statistical Yearbook* for 1900 and 1930. Wilmoth and Shkolnikov, *The Human Mortality Database* for 1960 and 1990, United Nations (2000) for 2020. n: *Japan Statistical Yearbook* Various years, for 1900-1990. National Institute of Population and Social Security Research for 2020. g: Maddison (1995) for 1900 and 1930. *Penn World Table* for 1960, 1990. For detailed calculation see the text.

Figure 2.7 Simulation Analysis of The United States

(a) Survival Rate (q) and GDP Growth (gn)



(b) Capital per Effective Labor



(c) Saving Rate (S/Y) and Investment Rate (I/Y)

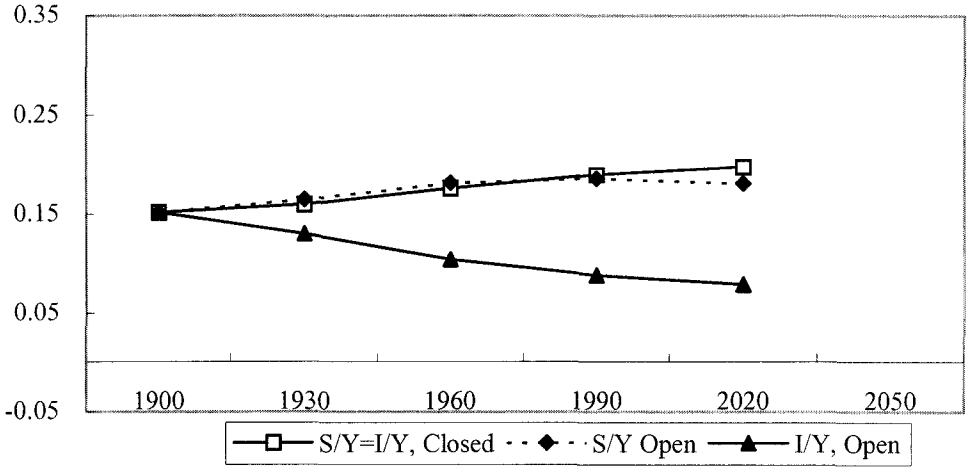
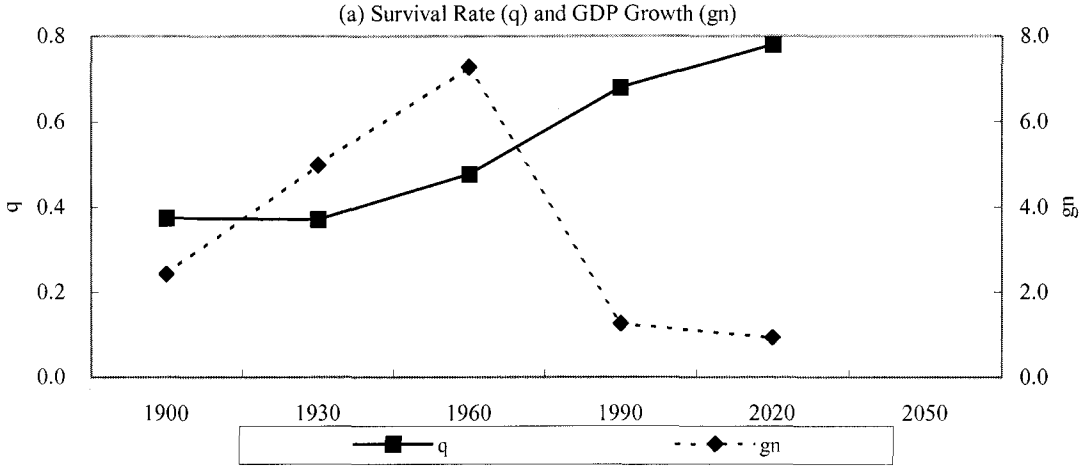
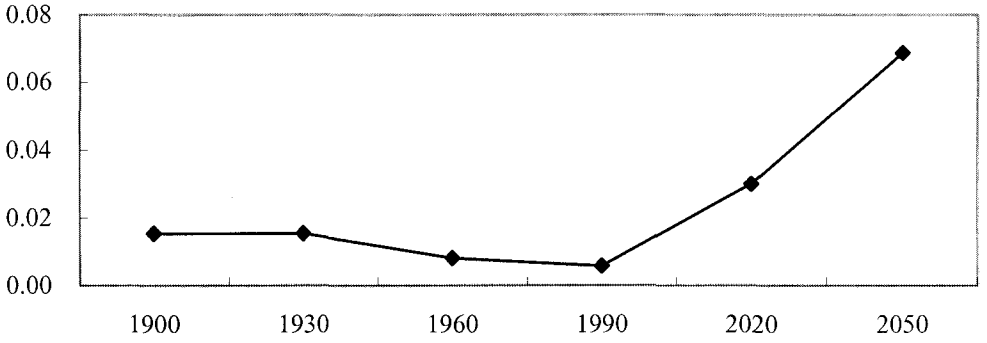


Figure 2.8 Simulation Analysis of Japan

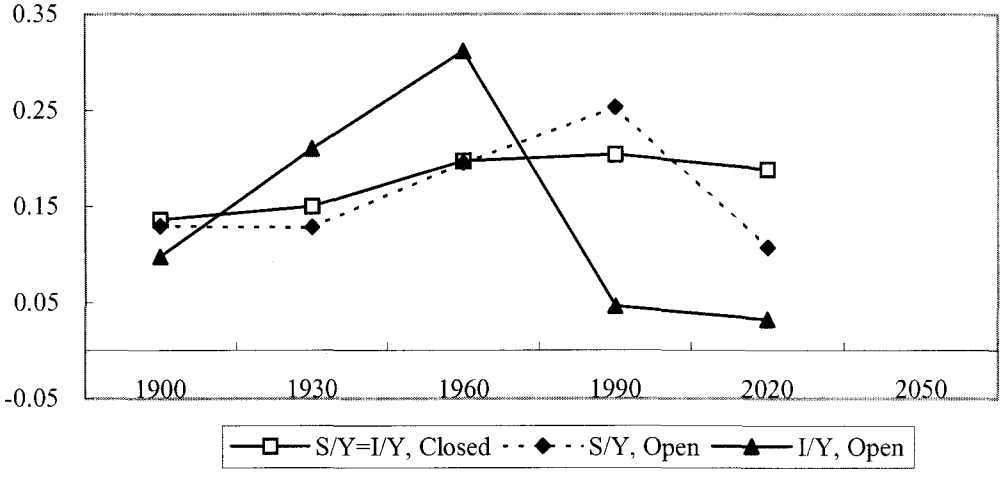
(a) Survival Rate (q) and GDP Growth (gn)



(b) Capital per Effective Labor



(c) Saving Rate (S/Y) and Investment Rate (I/Y)



in 1930. Since 1990, the survival rate has been higher in Japan than the United States, and it is expected that this situation will hold in 2020. The survival rate has changed continuingly since 1930, so the economy does not achieve a steady state.

GDP growth in the US is quite different from that in Japan. In the US, gn has declined moderately over time. From 1960 to 1990, the US experienced large technological growth, but it was not as great as in Japan. Growth in the labor force has declined steadily in the US. High GDP growth before 1990 was characteristic in Japan because Japan experienced a high economic growth period after World War II. After 1990, GDP growth declined. In Japan, the decline in labor force growth after 1990 is remarkable because of low fertility.

Figure 2.7 (b) plots simulated capital per effective worker (k) in a closed US economy. In the United States, simulated k increased at an almost constant rate after 1930. Also, as Table 2.2 shows, the capital-output ratio (K/Y) increased. Figure 2.8 (b) shows simulated k for a closed Japanese economy. Simulated k declined from 1930 to 1990, and the simulated ratio of capital to output declined over the same period. In 2020 and 2050, k and K/Y will increase substantially due to an increase in life expectancy and decrease in labor force.

Figure 2.7 (c) plots the simulated national saving rate (S/Y) of the United States in a closed economy and a small open economy and the investment rate (I/Y) in a small open economy. In the US, S/Y has increased gradually over the period for both a closed economy and a small open economy. In our model, the national investment rate is not affected by the survival rate although it is affected by GDP growth. The simulated national investment rate in a small open economy has trended downward over time. Due to a continuing increase in the survival rate, the simulated saving rate has trended upward. The changes in g, n, and q have brought capital inflows in the United States if we assume a small open economy.

Figure 2.8 (c) plots the changes in the simulated saving rate of Japan in both a closed and a small open economy and the investment rate for a small open economy. The simulated saving rate increased from 1900 to 1990 in a closed economy and declines by a small amount in 2020. In a small open economy, a slight decrease in *S/Y* was calculated for 1930, but simulated *S/Y* increased substantially from 1930 to 1990. In 2020, *S/Y* will fall suddenly because of low economic growth. Simulated *I/Y* in a small open economy increased remarkably from 1900 to 1960. This trend is followed by a great decrease in the investment rate in 1990 and simulated *I/Y* will decline slightly in 2020. Until 1960, investment is higher than saving, which would cause capital outflows.

Our simulation analysis produces results similar to those of Lee, Mason, and Miller (2001). Lee, Mason and Miller analyze the dynamic effects of demographic transition on the saving rate in Taiwan assuming a small open economy. The authors incorporate changes in fertility and mortality.

In terms of the effect of changes in labor force growth, Higgins (1994) and Williamson and Higgins (2001) suggest similar implications. These authors do not incorporate the change in the survival rate. We also estimated the saving rate and investment rate when the survival rate is assumed to be constant over the period. Results are shown in Appendix C, Appendix D, and Appendix E. The results reveal that the saving rates in both the small open economy and the closed economy and capital per effective worker in a closed economy would be much smaller if the survival rate had not changed since 1900. Therefore, the contribution of life expectancy to the saving rate is significant.

2.7 Conclusion

With a few exceptions, countries are experiencing the demographic transition, that is, lower fertility and higher life expectancy. If saving decisions are influenced by characteristics such as age,

and life expectancy, then the demographic transition affects national saving. This research focuses on the effects of a higher adult life expectancy.

We develop an overlapping generation model analyzing the effect of higher life expectancy on the national saving rate. Incorporating lifetime uncertainty into Higgins's (1994) model, we examine the dynamic effects of longevity on the national saving rate, in addition to steady state effects. This study captures transitory characteristics of demographic change and finds remarkable out-of-steady-state effects of longevity. We also use simulations to analyze how the saving rate has changed with an increase in the survival rate. Our findings are summarized as follows.

First, individuals increase saving when they are young if the survival rate increases. If consumers know that they can live longer, they have more time to consume and they save more. If they know they will not live long, they are more likely to consume now. Second, if GDP is growing, the share of the national saving in GDP increases in steady state when the survival rate increases in either a small or a closed economy. Third, in both a small open economy and a closed economy, a permanent and one-time increase in the survival rate brings a large transitory increase in the national saving rate. This is followed by a return to a level that is higher than the initial level if GDP is growing. Fourth, a rapid, sustained increase in life expectancy causes a jump to a higher saving rate. Fifth, an increase in life expectancy increases the ratio of wealth to GDP in the next period. Sixth, an increase in the survival rate causes growth in GDP per effective worker in a closed economy.

We use survival rates and economic growth rates for Japan and the United States to simulate the national saving rate and the national investment rate in a closed economy and a small open economy of every 30 years from 1900 to 2020. We find a substantial influence of the survival rate on the saving rate for both types of economy. The investment rate is not affected by the survival

rate in a small open economy. In a closed economy, the investment rate is equal to the saving rate. In reality, the economy is open, but not perfectly. Therefore, we should think of a situation that integrates the implications of a small open economy and a closed economy.

Many developed countries have experienced demographic transition and are facing population aging. In developed countries, the infant mortality rate is already low, and, due to improved nutrition and medical care, life expectancy of adults may increase more. On the other hand, fertility is declining, causing a decline in labor force or GDP growth. This may induce a further decrease in the saving rate in the future.

This study assumes that all consumers retire at a certain age and labor force participation is not affected by an increase in the life expectancy. We will discuss the effect of longevity on labor force participation in detail in Chapter 3. We have also ignored intergenerational transfers. Longevity could influence both transfers within families and social security. Incorporating intergenerational transfer will provide us further understanding about the effect of longevity on saving. Moreover, our model considers only the adult survival rate. In developing countries, life expectancy is greatly influenced by child mortality. Further discussion of this issue is needed.

CHAPTER 3. LONGEVITY, LABOR FORCE PARTICIPATION, AND THE NATIONAL SAVING RATE

3.1 Introduction

In this chapter, we discuss how the national saving rate changes when life expectancy increases if labor force participation is endogenous. The previous chapter assumes that labor force participation is exogenous, and we find that longer life expectancy induces a higher national saving rate if GDP is growing. On the other hand, it does not influence the investment rate in a small open economy. Here, we extend the model presented in Chapter 2 with an endogenous retirement decision. The retirement age can be determined by institutions, and many companies have mandatory retirement. Therefore, it is important to discuss the case with exogenous retirement, as in Chapter 2. On the other hand, self-employed workers can determine their retirement and some employees work after they retire from their company. So, it is also useful to discuss endogenous retirement. In this chapter, we focus on the case of a small open economy. We analyze the effect of an increase in life expectancy on the national saving rate, the national investment rate, and the current account balance.

In recent years, the retirement age has dropped in many countries. Many studies have described changes in labor forces. One important issue is a change in retirement behavior. Many studies have reported the changes in labor force participation or retirement age of the elderly. For example, Oshio and Yashiro (1997), Gruber and Wise (1998), Costa (1998), Borsch-Supan (2000), Okunishi (2001), and Mason, Lee, and Russo (2000) describe the fact that the retirement age is likely to decrease and the labor force participation of the elderly is declining in developed countries.

Based on the overlapping generations model with two periods and an endogenous retirement decision, we examine the effect of an increase in life expectancy on the national saving rate, the

investment rate, and capital flows in a small open economy. We analyze both steady-state and out-of-steady-state effects of an increase in the survival rate. First, we analyze the effect of an increase in the survival rate when social security is not available. Next, we examine how the results changes if we consider social security financed by a pay-as-you-go system. Also, we analyze the effect of longevity when an annuity is not available.

The findings of this chapter can be summarized as follows. Under a perfect annuity market, an increase in life expectancy increases saving of prime-age adults and raises the retirement age. In steady state, longevity does not influence the national investment rate. Longevity influences the steady state national saving rate, but the sign of the effect is ambiguous. A one-time increase in the survival rate produces a large transitory increase in both the saving rate and the investment rate. The effect of an increase in life expectancy on the current account balance is also ambiguous.

If a social security benefit financed by a pay-as-you-go system is introduced in a perfect annuity market, prime-age adults decrease saving. An increase in the social security tax decreases labor force participation of the elderly in the same period, and increases labor force participation of the elderly in the next period. In steady state, an increase in social security tax rate increases (decreases) labor force participation of the elderly if GDP growth is lower (higher) than the interest rate. An increase in the social security tax decreases the saving rate in steady state if GDP is growing and the growth is less than the interest rate. If the social security tax increases in the next period, the current investment rate decreases. An increase in the current social security tax rate increases the investment rate in the same period. An anticipated increase in the social security tax in the next period decreases the current national saving rate. A current increase in the social security tax rate influences the national saving rate in the same period and the next period, but whether the effects are positive or negative is ambiguous.

If an annuity is not available, an increase in life expectancy increases saving of prime age adults, which causes earlier retirement by the elderly. In steady state, the sign of the effect of an increase in the survival rate on the national saving rate is ambiguous. A one-time increase in the survival rate produces a large transitory increase in the saving rate. An increase in the survival rate does not influence the investment rate in steady state, but it has an out-of-steady-state effect on the investment rate, which varies depending on how much it increases or decreases the growth in total labor force.

Simulation analysis shows that the national investment rate is higher and the national saving rate is lower if we assume endogenous retirement instead of exogenous retirement. Therefore, the effect of longevity on the current account balance is smaller if retirement decision is endogenous.

This chapter is organized as follows. We review previous studies related to our research in section 3.2. In section 3.3, the individual's optimization problem is described. This section presents how the saving of prime age adults and labor force participation of the elderly change if life expectancy increases. It also describes the macroeconomic settings. Section 3.4 analyzes the effects of longevity on the national saving rate, the national investment rate, and capital flows in a small open economy. We discuss both steady-state effects and out-of-steady-state effects. Section 3.5 presents the effects of social security financed by a pay-as-you-go system on individuals' saving and the retirement age. In section 3.6, we analyze the effect of an increase in life expectancy on retirement and the saving rate when an annuity is not available. In section 3.7, we simulate the saving and investment rate in each model using US and Japanese data. In section 3.8, we review the findings of this chapter and summarize the contributions and limitations of this research.

3.2 Fundamentals and Previous Studies

3.2.1 Basics of the Labor Force Participation Decision

If labor force participation is endogenous, individuals decide labor force participation by maximizing their utility from consumption and leisure. In a static model, an increase in the wage has two effects on labor force participation: a wealth effect and a substitution effect. As a result of the wealth effect, labor force participation decreases when wage increases. Because an increase in the wage increases income and because leisure is a normal good, the demand for leisure increases. The substitution effect implies that an increase in the wage decreases labor force participation because it increases the price of leisure. Whether the wealth effect or the substitution effect dominates is ambiguous. An increase in non-wage income always increases the demand for leisure and decreases labor force participation.

The dynamic labor force participation decision was introduced by MaCurdy (1981). The following three conclusions of his study are relevant to this chapter. First, an increase in the wage has a negative effect (wealth effect) and positive effect (substitution effect) on labor force participation in the same period. Which effect dominates is indeterminate. Second, an increase in wage in a different period from t has only a negative effect on labor force participation at time t. Third, an increase in non-wage income decreases labor force participation at all periods. According to the MaCurdy model, consumers determine labor force participation by anticipating their future wage. Consumers maximize the following utility function:

$$\sum_{t=1}^{T} \frac{u_t(c_t, l_t)}{(1+\rho)^t}, \tag{3.1}$$

subject to:

$$E_0 + \sum_{t=1}^{T} \frac{w_t(\bar{l} - l_t)}{(1+r_t)^t} = \sum_{t=1}^{T} \frac{c_t}{(1+r_t)^t}. \tag{3.2}$$

where c_t is consumption, l_t is leisure, w_t is wage, E_0 is non-wage income, r_t is the interest rate at time t, ρ is the discount rate., and $(\bar{l} - l_t)$ is hours worked at time t.

From the first order condition,

$$\partial u(c_t, l_t)/\partial c_t = \left[(1+r_t)/(1+\rho)\right]^t \eta . \tag{3.3}$$

$$\partial u(c_t, l_t)/\partial l_t \geq \left[(1+r_t)/(1+\rho)\right]^t \eta w_t . \tag{3.4}$$

where η is the Lagrange multiplier. From equations (3.3), (3.4) and the budget constraint (3.2), we can solve for the equilibrium c_t^* and l_t^*, for $t = 1 \ldots T$ and η^*, where η^* is the marginal utility of wealth at time 0. η^* is expressed as $\eta^* = \eta(E_0, w_1, w_2, \ldots w_T)$. We assume concavity of preference, which implies that the marginal utility of wealth is decreasing. $\frac{\partial \eta}{\partial E_0} < 0$ and $\frac{\partial \eta}{\partial w_t} \leq 0$ [12] for all t because an increase in non-wage income or an increase in the wage at any time increases lifetime wealth. Labor force participation at time t (z_t) is expressed as:

$$z_t = \bar{l} - l_t = z\left(\tfrac{1+\rho}{1+r_t}, \eta, w_t\right). \tag{3.5}$$

Equation (3.5) implies that only the wage at time t directly affects labor force participation at time t. The wages in other periods affect labor force participation rate indirectly through η.

The effect of w_t on z_t is:

$$\frac{\partial z_t}{\partial w_t} = \left.\frac{\partial z_t}{\partial w_t}\right|_{\eta} + \frac{\partial z_t}{\partial \eta}\frac{\partial \eta}{\partial w_t} . \tag{3.6}$$

The first term of the right hand side of equation (3.6) is the partial derivative of z_t with respect to w_t holding η constant. This term is the substitution effect of an increase in w_t on z_t. An increase in w_t increases the price of leisure at time t and increases labor force participation at time t. The second

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[12] For proof, see Heckman (1974, 1976).

term on the right-hand-side of equation (3.6) is the effect of an increase in w_t on z_t through a change in lifetime wealth (wealth effect). An increase in w_t increases lifetime wealth, which raises the demand for leisure. Therefore, the second term is negative. We cannot determine *a priori* which effect is greater.

The effect of an increase in w_j ($j \neq t$) on z_t is:

$$\frac{\partial z_t}{\partial w_j} = \frac{\partial \eta}{\partial w_j} \leq 0. \tag{3.7}$$

An increase in wage in a different period has a non-positive effect on current labor force participation through an increase in wealth.

The effect of an increase in E_0 on z_t is:

$$\frac{\partial z_t}{\partial E_0} = \frac{\partial \eta}{\partial E_0} < 0. \tag{3.8}$$

An increase in the non-wage income has a negative effect on current labor force participation through an increase in wealth.

3.2.2 Literature Review

In addition to MaCurdy (1981) several authors incorporate the retirement decision in life-cycle models. For example, Chang (1990) and Kalemli-Orzcan and Weil (2002)[13] show that an increase in life expectancy has an ambiguous effect on the retirement age if the length of life is uncertain.

A number of studies relate early retirement to social security. Empirical studies have found a significant negative effect of social security on labor force participation of the elderly. (For example,

[13] For the details of these studies, see Chapter 1.

Feldstein (1974), Clark and Anker (1990, 1993), Rust and Phelan (1997), Gruber and Wise (1998), Clark, York, and Anker (1999), and Anderson, Gustman and Steinmeier (1999).)

One major system of social security is the fully funded system. Prime-age adults pay social security tax and the amount they pay plus interest is returned during old age. If social security is financed by a fully funded system, an increase in social security tax does not affect retirement decision because social security tax is just a substitute for private saving.[14]

Another major system of social security is the pay-as-you-go system. In an economy with pay-as-you-go social security, prime-age adults pay social security tax, which is transferred to the elderly at the same period. If the social security is financed by a pay-as-you-go system, an increase in the social security benefit lowers the retirement age because non-wage income increases. Many developed countries have a pay-as-you-go social security system. A number of studies have addressed social security and retirement. For example, Hu (1979), Siddiqqui (1995), Michel and Le Pestieau (1999), and Matsuyama (2000) develop overlapping generations models of Samuelson (1958) and Diamond (1965) and assume that the labor force participation of the elderly is endogenous. The authors find that an increase in social security benefit induces earlier retirement.[15]

3.3 Individual Decision Making and Production

3.3.1 Consumer's Optimization

We develop an overlapping generations model with lifetime uncertainty, including labor force participation decision of the elderly. In the model, consumers determine the amount of

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[14] If there is a borrowing constraint, it is possible that fully funded social security induces retirement. For detailed discussion, see Crawford and Lilien (1981) and Fabel (1994).

[15] Hu (1979) assumes that the social security tax is financed by a payroll tax and that the substitution effect of an increase in wage is greater than the income effect. Siddiqui (1995) assumes that the payroll tax is not imposed in the old age.

consumption during prime-age and old age and the retirement age maximizing lifetime utility. In this subsection, we find that an increase in life expectancy increases saving by prime age adults and delays retirement.

First, we assume that social security is not available. As in Chapter 2, there are two generations, prime-age adults (age 1) and the elderly (age 2). All individuals survive to age 1, but only q can survive to age 2. Here, we analyze only the change in labor force participation of the elderly as Hu (1978), Siddiqqui (1995), Michel and Pestieau (1999), Matsuyama (2000), and Crettez and Maitre (2002). Each individual is capable of providing one unit of labor per period. Prime-age adults work full time, but the elderly choose the amount to work.[16] Adding an endogenous variable increases the complexity of the model. For simplicity, we assume that utility is log-linear in form. Thus, the intertemporal elasticity of substitution (1/θ) is one. The elderly provides z unit of labor and retires. $1 \geq z \geq 0$. An individual's lifetime utility function is:

$$V_t = \ln c_{1,t} + \delta q_t [\ln c_{2,t+1} + \gamma \ln(1 - z_{t+1})], \tag{3.9}$$

where $c_{1,t}$ and $c_{2,t+1}$ denote consumption of prime-age adults and the elderly, respectively. δ is a measure of time preference, and defined as $\delta = 1/(1+\rho)$, where ρ is the discount rate. γ represents the preference for leisure.[17] δq_t is interpreted as a measure of effective time preference. An increase in the survival rate increases patience because a consumer is more likely to consume at age 2. We assume that insurance is available and that the annuity market is perfect. A consumer provides one unit of labor at age 1, earns labor income $A_t w_t$, where A denotes the technological

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[16] In many labor economics textbooks, it is assumed that consumers choose how much to work in one day. Here, we do not discuss labor force participation within a day, and only consider leisure after retirement.

[17] $1 \geq z_{t+1} \geq 0$ implies $\log(1-z_{t+1}) < 0$ and $\partial u_t / \partial \gamma \leq 0$. To avoid this problem, we would need to add a large number to $\log(1-z_{t+1})$ so that $\partial u_t / \partial \gamma \geq 0$. Because the utility function is ordinal, this does not affect the results.

level, and w is wage per unit of effective labor. The consumer allocates earning between consumption and saving, so that the following constraint is satisfied:

$$c_{1,t} + s_{1,t} = A_t w_t . \tag{3.10}$$

At age 2, consumers consume proceeds from saving and labor income from z unit of labor. Consumers choose how much to work, so that:

$$c_{2,t+1} = \frac{1+r_{t+1}}{q_t} s_{1,t} + A_{t+1} w_{t+1} z_{t+1} . \tag{3.11}$$

We assume that the productivity of prime-age adults and the elderly are equal in the same period and that prime-age adults know this future labor income at age 2. From (3.2) and (3.3), an individual's lifetime budget constraint is:

$$c_{1,t} + \frac{q_t}{1+r_{t+1}} c_{2,t+1} = A_t w_t + \frac{q_t}{1+r_{t+1}} A_{t+1} w_{t+1} z_{t+1} . \tag{3.12}$$

Maximizing lifetime utility in equation (3.9) facing the budget constraint (3.4), we can calculate the optimal consumption of prime-age adults and the elderly, and the labor force participation of the elderly. When $z_{t+1}>0$, $c_{1,t}$, $c_{2,t+1}$ and z_{t+1} are:

$$c_{1,t} = \frac{1}{\lambda} = \frac{(1+r_{t+1}) A_t w_t + q_t A_{t+1} w_{t+1}}{(1+r_{t+1})(1+q_t\delta + q_t\gamma\delta)} , \tag{3.13}$$

$$c_{2,t+1} = \frac{(1+r_{t+1})\delta}{\lambda} = \frac{\delta[(1+r_{t+1}) A_t w_t + q_t A_{t+1} w_{t+1}]}{1+q_t\delta + q_t\gamma\delta} , \tag{3.14}$$

$$z_{t+1} = 1 - \frac{(1+r_{t+1})\gamma\delta}{A_{t+1} w_{t+1} \lambda} = \frac{(1+q_t\delta) A_{t+1} w_{t+1} - \gamma\delta(1+r_{t+1}) A_t w_t}{A_{t+1} w_{t+1}(1+q_t\delta + q_t\gamma\delta)} , \tag{3.15}$$

where λ is the Lagrangian multiplier.

From equations (3.5) and (3.6), the age profile of consumption is given by:

$$\frac{c_{2,t+1}}{c_{1,t}} = \delta(1+r_{t+1}) . \tag{3.16}$$

Equation (3.16) is the consumption profile for those who survive, and is the same as the age profile of consumption derived in Chapter 2 (equation (2.9)) if the intertemporal elasticity of substitution is one. As is discussed in Chapter 2, whether a consumer increases or decreases consumption over the lifetime depends on the discount factor and the interest rate. It is independent of the survival rate. From equation (3.13) and (3.14), an increase in the survival rate decreases consumption at all ages, but the ratio of consumption at age 2 to consumption at age does not change. Consumers become patient when the survival rate increases; on the other hand, the return to an annuity decreases. These effects offset. Also, labor income does not affect the age profile of consumption, although labor force participation is endogenous.

The saving of prime-age adult is

$$s_{1,t} = A_t w_t - \frac{1}{\lambda} = \frac{(1+r_{t+1})(q_t\delta + q_t\gamma\delta)A_t w_t - q_t A_{t+1} w_{t+1}}{(1+r_{t+1})(1+q_t\delta+q_t\gamma\delta)}. \tag{3.17}$$

Equation (3.17) implies $\frac{\partial s_{1,t}}{\partial \gamma} > 0$, so that a high preference for leisure induces prime-age adults to save for retirement.

We can see $\frac{\partial(1/\lambda)}{\partial q_t} < 0$ for plausible values of the parameters.[18] Taking the partial derivative of $s_{1,t}$ with respect to q_t in equation (3.9), the effect of an increase in the survival rate on the saving of prime-age adult is:

$$\frac{\partial s_{1,t}}{\partial q_t} = -\frac{\partial(1/\lambda)}{\partial q_t} > 0. \tag{3.18}$$

Equation (3.10) implies that saving of a prime age adult increases when the survival rate rises. Under a perfect annuity market, a higher survival rate reduces the return to saving. On the other hand, an increase in the probability of surviving to age 2 causes an individual to save more to use at

[18] Detailed discussion is provided in Appendix F.

age 2. If consumers can work at age 2, they may reduce saving because they receive a wage in the next period. For plausible parameters, the positive effects of an increase in the survival rate on saving dominate. An increase in the interest rate increases saving. The present wage has a positive effect on saving of prime-age adults, but the wage at age 2 has a negative effect.

Taking the partial derivative with respect to q_t of both sides of equation (3.7), the effect of longevity on labor force participation of the elderly is:

$$\frac{\partial z_{t+1}}{\partial q_t} = -\frac{(1+r_{t+1})\gamma\delta}{A_{t+1}w_{t+1}}\frac{\partial(1/\lambda)}{\partial q_t} > 0. \tag{3.19}$$

The elderly work more if the survival rate increases. In summary, prime-age adults save more if life expectancy increases. On the other hand, the return to the annuity decreases. An increase in q can decrease labor force participation at age 2 because it increases lifetime income. For plausible parameters, the positive effect of an increase in the survival rate on labor force participation dominates. Equation (3.7) shows that an increase in labor income at time t increases lifetime wealth and decreases z_{t+1}, as discussed in subsection 3.2.1. An increase in the wage at time t+1 increases labor z_{t+1}. An increase in the wage at time $t+1$ has both a substitution effect and a wealth effect, but $\frac{\partial z_{t+1}}{\partial w_{t+1}} > 0$ implies that the substitution effect dominates any time in this case. A higher wage in the future shifts the age earning profile upwards. An increase in the preference for leisure γ lowers z_{t+1}.

We assume a small open economy, where the wage per unit of effective labor is determined exogenously. Details of the small open economy are described in the next section. Let us denote the wage in steady state w^*. Labor force participation in steady state z^* is:

$$z^* = \frac{g(1+q^*\delta)-\gamma\delta(1+r_w^*)}{g(1+q^*\delta+q^*\gamma\delta)}. \tag{3.20}$$

In our special case of a log-linear utility function, the wage level in steady state w^* does not influence labor force participation of the elderly, but technological growth g has a positive effect on on labor force participation. The wages at both ages 1 and 2 have a wealth effect on labor force participation of the elderly, increasing lifetime income and the demand for leisure. Wage at age 2 has a substitution effect on labor force participation at age 2. An increase in wage at age 2 raises the price for leisure and decreases labor force participation in the same period. These effects cancel out in our model. If a prime-age adult knows that there will be rapid technological growth, he prefers to work at age 2, reducing saving at age 1. Equations (3.19) and (3.20) imply that an increase q^* increases z^*.

We assume an interior solution for consumption at age 1 and 2. However, it is important to consider the corner solution for labor force participation at time $t+1$. If $z_{t+1}=0$, consumers are retired during age 2. In this case, consumption during age 1 and age 2 is:

$$c_{1,t} = \frac{A_t w_t}{1+q_t\delta}, \tag{3.21}$$

$$c_{2,t+1} = \frac{(1+r_{t+1})\delta A_t w_t}{1+q_t\delta}. \tag{3.22}$$

The saving of prime-age adults is:

$$s_{1,t} = \frac{q_t\delta A_t w_t}{1+q_t\delta}. \tag{3.23}$$

Equations (3.13), (3.14) and (3.15) imply that $c_{1,t}$, $c_{2,t+1}$, and $s_{1,t}$ are the same as those obtained in Chapter 2, with an exogenous retirement assumption, if the intertemporal elasticity of substitution is one. In the case of a corner solution, consumption and saving are influenced by neither labor income at age 2 nor the preference for leisure. An increase in q increases the saving of prime-age adults. Whether the corner solution is optimal or not depends on the parameters. A high preference for leisure would cause a corner solution. If the corner solution is optimal, the effect of longevity

can be described in the same way as in Chapter 2. Hereafter, we will assume that the interior solution is optimal.

3.3.2 Macroeconomic Setting

We assume a Cobb-Douglas production function as in Chapter 2, $Y_t = K_t^{\phi} L_t^{1-\phi}$, where Y_t is gross domestic product (GDP) and K_t is capital stock. $L_t = A_t(N_{1,t} + N_{2,t} z_t)$ is aggregate effective labor supply. A_t denotes the level of technology, $N_{1,t}$ denotes the population of prime-age adults, and $N_{2,t+1}$ denotes the elderly population. Lower case letters denote per effective labor force, that is $y_t = Y_t / L_t$ and $k_t = K_t / L_t$. Output per effective worker is expressed as $y_t = k_t^{\phi}$. The capital-output ratio is $K / Y = k^{1-\phi}$. Let us denote the growth rate of population by n-1 and the technological growth rate by g-1. $N_{1,t} = nN_{1,t-1}$, $A_t = gA_{t-1}$ and $N_{2,t} = (q_t / n)N_{1,t}$ hold. Therefore, aggregate effective labor is:

$$L_t = A_t N_{1,t}(1 + \frac{q_{t-1}}{n} z_t). \tag{3.24}$$

We assume a small open economy, that is, the economy is so small that it does not affect the world interest rate. As in Chapter 2, we assume perfect capital mobility, meaning that the domestic economy can borrow and lend in the international capital market at the world interest rate. According to the arbitrage equation, the marginal product of domestic capital is equal to the world interest rate, so that $\phi k_t^{1-\phi} - \xi = r_{w,t}$ where ξ is the depreciation rate and $r_{w,t}$ is the world interest rate at time t. Capital per effective worker is:

$$k_t = \left(\frac{\phi}{r_{w,t} + \xi} \right)^{\frac{1}{1-\phi}}. \tag{3.25}$$

Aggregate capital is given by $K_t=L_tk_t$. Equation (3.17) implies that capital stock per unit of effective labor is determined exogenously by the world interest rate and the parameters of the production process.

3.4 Saving and Investment in Small Open Economies

3.4.1 The National Investment Rate

Gross investment at time t, I_t, is equal to capital at time $t+1$ minus the undepreciated portion of capital at time t.

$$I_t = K_{t+1} - (1-\xi)K_t . \tag{3.26}$$

Noting that $Y_t = L_t k_t^{1-\phi}$, the investment rate is:

$$\begin{aligned} \frac{I_t}{Y_t} &= \left[\frac{L_{t+1}k_{t+1}}{L_t k_t} - (1-\xi) \right] k_t^{1-\phi} \\ &= \left[gn \frac{1+\frac{q_t}{n} z_{t+1}}{1+\frac{q_{t-1}}{n} z_t} \frac{k_{t+1}}{k_t} - (1-\xi) \right] k_t^{1-\phi}. \end{aligned} \tag{3.27}$$

Equation (3.27) has implications that differ from the results obtained in Chapter 2. In Chapter 2, we show that the survival rate does not influence the investment rate. Here, because the labor force participation decision is influenced by the survival rate, a change in the survival rate affects the investment rate.

We denote the world interest rate, capital stock per unit of effective labor and the survival rate in steady state by r_w^*, k^* and q^*, respectively. The wage is equal to the marginal product of labor, so that,

$$w_t = (1-\phi)k_t^{\phi} . \tag{3.28}$$

Wage per effective labor is constant at w^* in steady state.

The gross investment rate in steady state is:

$$\left(\frac{I}{Y}\right)^{*} = \left(gn - 1 + \xi\right) k^{*1-\phi}. \tag{3.29}$$

Equation (3.29) implies that the survival rate does not affect the steady state investment rate. The investment demand increases in order to equip an increase in labor force. If the survival rate is constant, labor force participation of the elderly is constant and the number of surviving elderly grows at rate n–1.[19] Therefore, aggregate labor force is growing at rate n–1 and the effective labor force grows at rate gn–1 in steady state.

The survival rate has out-of-steady-state effects as well, in contrast to the case of Chapter 2 with exogenous retirement. An anticipated one-time increase in the survival rate at time t is obtained by taking the partial derivative of both sides of equation (3.19) with respect to q_t:

$$\frac{\partial(I_t / Y_t)}{\partial q_t} = \frac{gn}{(1 + \frac{q_{t-1}}{n} z_t)} k^{*1-\phi} \left(z_t + q_t \frac{\partial z_{t+1}}{\partial q_t} \right) > 0. \tag{3.30}$$

An increase in q_t has a positive effect on the investment rate at time t. q_t does not affect labor force participation of the elderly at time t, yet reduces retirement of the elderly at time t+1. The labor force increases from time t to t+1 at a rate greater than n, which causes a higher investment at time t.

From equation (3.19), the effect of an increase in q_t on the investment rate at time t+1 is:

$$\frac{\partial(I_{t+1} / Y_{t+1})}{\partial q_t} = -gk^{*1-\phi} \left(1 + \frac{q_{t+1}}{n} z_{t+2} \right) \frac{z_t + q_t \frac{\partial z_{t+1}}{\partial q_t}}{\left(1 + \frac{q_t}{n} z_{t+1} \right)^2} < 0. \tag{3.31}$$

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[19] An increase in child survival would increase n in the absence of a compensating fertility increases. This would result in a higher $(I/Y)^{*}$.

An increase in q_t lowers the investment rate at time t+1 because it increases z_{t+1} but does not affect z_{t+2}. A one-time increase in q at time t only increases the labor force at time t+1 and does not increase labor after time t+2. The growth of the labor force decreases form time t+1 to time t+2. Therefore, the investment rate decreases at time t+1.

Suppose the increase in q_t is anticipated, one-time and permanent. z_t is not affected by q_t. z_{t+1} increases if q_t increases. Therefore, the investment rate I_t increases. After time t+1, q is constant. Labor force participation of the elderly is higher than before q changes, but does not change after time t+1. Therefore, the investment rate at time t+1 decreases to the same value at time t-1. Therefore, a one-time permanent increase in the survival rate at time t produces a transitory increase in the investment rate at time t, which is followed by a return to the previous level.

If the survival rate increases continuingly, the labor force participation of the elderly increases continuingly, which increases the growth of labor force. This causes a sustained increase in the investment rate.

3.4.2 The National Saving Rate

Recall that gross national saving at time t (S_t) is the sum of the change in total wealth of prime-age adults ($S_{1,t}=s_{1,t}N_{1,t}$) and the elderly ($S_{2,t}= -s_{t-1}N_{1,t-1}$), plus depreciation ($\xi K_t$). Thus, $S_t=S_{1,t}+S_{2,t}+\xi K_t$. Gross national saving can be expressed as $S_t= S_{1,t} -S_{1,t-1}+\xi K_t$. Thus, saving by prime-age adults at time t constitutes wealth at time t+1. Therefore, $S_{1,t}=K_{t+1}+F_{t+1}$. If the supply of domestic capital is more than the domestic demand, capital flows out of the economy and vice versa. Suppose the world interest rate is constant, so that the wage is constant at w^* and capital per effective labor is constant at k^*. Let us define X_t, the share of saving in labor income of prime-age adults, as:

$$X_t = \frac{s_{1,t}}{w^* A_t} = \frac{(1+r_w^*)(q_t\delta + q_t\gamma\delta) - q_t g}{(1+r_w^*)(1+q_t\delta+q_t\gamma\delta)}, \tag{3.32}$$

so that $\partial X_t / \partial q_t > 0$. The gross national saving rate is:

$$\frac{S_t}{Y_t} = \frac{s_{1,t}N_{1,t} - s_{1,t-1}N_{1,t-1}}{L_t k^{*\phi}} + \xi k^{*1-\phi} = \frac{(1-\phi)\left(X_t - \dfrac{X_{t-1}}{gn}\right)}{1+\dfrac{q_{t-1}}{n}z_t} + \xi k^{*1-\phi}. \tag{3.33}$$

The gross national saving rate in the steady state is:

$$\left(\frac{S}{Y}\right)^* = \frac{(1-\phi)X^*}{1+\dfrac{q^*}{n}z^*}\frac{gn-1}{gn} + \xi k^{*1-\phi}. \tag{3.34}$$

When gn=1, that is, there is no growth in GDP in the steady state, the net national saving rate (saving net of depreciation) is zero. This is consistent with the results in Chapter 2, and the basic life-cycle models of Modigliani and Brunberg (1954), Tobin (1967) and Mason (1981, 1987).

Taking the partial derivative of both sides of equation (3.27) with respect to q^* yields:

$$\frac{\partial(S/Y)^*}{\partial q^*} = \frac{(1-\phi)\left[\dfrac{\partial X^*}{\partial q^*}\left(1+\dfrac{q^*}{n}z^*\right) - X^*\left(\dfrac{z^*}{n}+\dfrac{q^*}{n}\dfrac{\partial z^*}{\partial q^*}\right)\right]}{\left(1+\dfrac{q^*}{n}z^*\right)^2}\left(\frac{gn-1}{gn}\right). \tag{3.35}$$

Equation (3.28) implies that longevity increases the saving of prime-age adults and increases the dissaving of the elderly. Also, an increase in q^* implies an increase in the number of the elderly who rely on earnings rather than saving to finance their consumption. Therefore, an increase in the survival rate has both positive and negative effects on the gross national saving rate. Which effect dominates depends on the parameters. It is possible that an increase in the survival rate decreases the saving rate in the steady state.

Let us remind the implications of the variable rate growth effect (VRG) model of Fry and Mason (1982) and Mason (1981, 1987). The authors suggest the following equation, as introduced in Chapter 1:

$$\frac{S}{Y} \approx -\ln C = \beta_0 + (A_C - A_Y)(gn - 1), \tag{3.36}$$

where C is aggregate consumption, β_0 is a constant term, A_C denotes the mean age of consumption, and A_Y denotes the mean age of earning. The results of our model are related to those of the VRG model. An increase in the survival rate increases both A_C and A_Y, so that whether its effect on the saving rate is positive or negative is ambiguous.

An increase in the technological growth rate can decrease the saving of prime-age adults. It also increases the labor force participation of the elderly, that is, it shifts the age-earning profile upward. Therefore, the growth effects interact with $(A_C - A_Y)$, changing the timing of saving. An increase in the survival rate increases the share of the elderly in total population, which affects the national saving rate.

Then what are the out-of-steady-state effects of an increase in the survival rate? Here, we discuss the effect of an anticipated increase in the survival rate at time t. We assume that the survival rate is q^* for all other periods. The effect of an increase in the survival rate on the current saving rate can be obtained by taking the partial derivative with respect to q_t as follows:

$$\frac{\partial(S_t / Y_t)}{\partial q_t} = \frac{(1-\phi)\partial X_t / \partial q_t}{1 + \frac{q_{t-1}}{n} z_t} > 0 \tag{3.37}$$

An increase in q_t increases saving by prime-age adults, but affects neither the saving nor labor force participation of the elderly.

The effect of an increase in q_t on the saving rate at time t+1 is:

$$\frac{\partial(S_{t+1}/Y_{t+1})}{\partial q_t} = \frac{(1-\phi)\left[-\frac{\partial X_t/\partial q_t}{gn} - \left(X_{t+1} - \frac{X_t}{gn}\right)\left(\frac{z_{t+1}}{n} + \frac{q_t}{n}\frac{\partial z_{t+1}}{\partial q_t}\right)\right]}{\left(1+\frac{q_{t-1}}{n}z_{t+1}\right)^2} < 0 \qquad (3.38)$$

Equation (3.31) shows that q_t has a negative effect on the saving rate at time t+1. q_t raises the dissaving of the elderly because they have saved more at age 1. Also, it raises Y_{t+1} by delaying the retirement of the elderly and increasing the number of the elderly.[20] Equation (3.26) implies that q_t does not affect the saving rate after time t+2.

Based on the results above, the dynamic effects of an anticipated one-time increase in the survival rate can be described as follows. Suppose the survival rate increases at time t. The immediate result is an increase in the national saving rate. At time t+1, the saving rate declines. Whether the saving rate is greater than the initial level depends on the parameters. Given a sufficient labor force response, national saving would return to a level lower than the initial one even though GDP is growing.

Suppose the survival rate continues to increase from time t. At time t, the national saving rate increases because only the saving of prime-age adults increases. After time t, it is ambiguous whether the saving rate continues to increase or begins to decline. Continuing increases in the survival rate proceed to increase the saving of prime-age adults, but also continue to increase the dissaving of the elderly and the share of the elderly in the total population. It is possible that the saving rate begins to decline at time t+1 and continues to decrease.

[20] If net saving is negative at time t+1, it is possible that an increase in q_t increases S_{t+1}/Y_{t+1}.

3.4.3 The Current Account Balance

The current account balance at time t (CA_t) is the difference between saving and investment. That is, $CA_t = S_t - I_t = (S_{l,t} - S_{l,t-1}) - (K_{t+1} - K_t) = F_{t+1} - F_t$. If saving exceeds investment, the economy is a net creditor and if saving is less than investment, the economy is a net borrower. Suppose the world interest rate is constant, and capital per effective worker is constant at k^*. The current account share of GDP is given by:

$$\frac{CA_t}{Y_t} = \frac{S_t}{Y_t} - \frac{I_t}{Y_t}. \tag{3.39}$$

In steady state, CA/Y is:

$$\left(\frac{CA}{Y}\right)^* = \left(\frac{S}{Y}\right)^* - \left(\frac{I}{Y}\right)^*. \tag{3.40}$$

Investment is not influenced by an increase in q^* in equation (3.29), so that $\frac{\partial (I/Y)^*}{\partial q^*} = 0$. Therefore,

$$\frac{\partial (CA/Y)^*}{\partial q^*} = \frac{\partial (S/Y)^*}{\partial q^*}. \tag{3.41}$$

We cannot determine whether q^* has positive or negative effect on the current account balance. In Chapter 2, which assumes that everyone retires at age 2, longevity causes an increase in the current account if GDP is growing. However, under the assumption that labor force participation of the elderly is endogenous, steady state capital outflow may change even when GDP is increasing .

Out-of-steady-state effects of longevity also differ from those found in Chapter 2. Differentiating equation (3.32) with respect to q_t yields:

$$\frac{\partial (CA_t / Y_t)}{\partial q_t} = \frac{\partial (S_t / Y_t)}{\partial q_t} - \frac{\partial (I_t / Y_t)}{\partial q_t}. \tag{3.42}$$

From equation (3.30), $\frac{\partial (I_t / Y_t)}{\partial q_t} > 0$ and from equation (3.37), $\frac{\partial (S_t / Y_t)}{\partial q_t} > 0$. That is, an increase in the survival rate at time t increases the saving and investment rates at time t. We cannot

tell which effect is greater. Whether an increase in the survival rate at time t increases or decreases the current account balance is ambiguous.

The effect of an increase in the survival rate at time t on the share of current account balance in GDP is given by:

$$\frac{\partial(CA_{t+1}/Y_{t+1})}{\partial q_t} = \frac{\partial(S_{t+1}/Y_{t+1})}{\partial q_t} - \frac{\partial(I_{t+1}/Y_{t+1})}{\partial q_t}. \tag{3.43}$$

From equation (3.31), $\frac{\partial(I_{t+1}/Y_{t+1})}{\partial q_t} < 0$ and from equation (3.38), $\frac{\partial(S_{t+1}/Y_{t+1})}{\partial q_t} < 0$. That is, an increase in the survival rate at time t increases the saving and investment rate at time t+1. Which effect dominates is ambiguous. Therefore, we cannot determine whether a transitory effect of an increase in the survival rate on the current account balance is positive or negative.

3.5 Social Security and Longevity

3.5.1 Consumer's Optimization under Social Security System

Social security plays an important role in the saving and labor force participation decisions. We investigate the effect of an increase in a pay-as-you-go social security tax on the saving rate incorporating responses in labor force participation. We analyze the effect of an increase in social security on saving and retirement behaviors of individuals, the national saving rate, the national investment rate, and the current account balance.

Social security financed by a payroll tax has two kinds of effects. First, there are wealth effects or intergenerational transfer effects. The wealth of current retirees increases while the wealth of future retirees declines. Second, there are price effects. Because payroll taxes are raised, the price of leisure declines.[21] This has substitution and income effects as described in section 3.2.

[21] In our model, we cannot deal with the price effects, as described later.

We assume that the utility function of individuals is the same as the one without social security introduced in equation (3.1):

$$V_t = \ln c_{1,t} + \delta q_t[\ln c_{2,t+1} + \gamma \ln(1 - z_{t+1})]. \tag{3.1}$$

Social security is financed by a payroll tax. At age 2, an individual receives the benefit after retirement.

We adopt the approach of Michel and Pestieau (1999). Consumers determine how much to consume at age 1 and 2 and how much to work at age 2. The budget constraint of prime-age adults is given by:

$$c_{1,t} + s_{1,t} = A_t w_t (1 - \tau_t), \tag{3.44}$$

where τ_t is the payroll tax rate. At age 2, individuals pay the social security tax before they retire. After they retire, they receive the social security benefit. The budget constraint of the elderly is:

$$c_{2,t+1} = \frac{(1 + r_{t+1})}{q_t} s_{1,t} + A_{t+1} w_{t+1} z_{t+1} (1 - \tau_{t+1}) + p_{t+1}(1 - z_{t+1}), \tag{3.45}$$

where p_{t+1} is the retirement benefit. Under the pay-as-you-go system, the payroll tax at time t is received equally by those who retire at the same period. As Michel and Pestiau (1999) suggest, "there is a double tax on continued work: the payroll tax, $\tau_{t+1} w_{t+1} \Delta z_{t+1}$, and the forgone pension benefits, $p_{t+1} \Delta z_{t+1}$." There is no adjustment to the pension for early or late retirement. In some systems, the US being one, this approach overstates the penalty of continuing to work. Therefore, the revenue constraint is:

$$\begin{aligned} p_{t+1}(1 - z_{t+1}) &= \frac{A_{t+1} w_{t+1} \tau_{t+1} N_{1,t+1} + A_{t+1} w_{t+1} \tau_{t+1} z_{t+1} N_{2,t+1}}{N_{2,t+1}} \\ &= A_{t+1} w_{t+1} \tau_{t+1} \left(\frac{n}{q_t} + z_{t+1} \right) \end{aligned} \tag{3.46}$$

From equations (3.44), (3.45), and (3.46) the lifetime budget constraint is given by:

$$c_{1,t} + \frac{q_t}{1+r_{t+1}} c_{2,t+1} = (1-\tau_t) A_t w_t + \frac{q_t}{1+r_{t+1}} A_{t+1} w_{t+1} z_{2,t+1} + \frac{\tau_{t+1} A_{t+1} w_{t+1} n}{1+r_{t+1}}. \quad (3.47)$$

In equation (3.47), the coefficient of z_{t+1} is $q_t A_{t+1} w_{t+1}/(1+r_{t+1})$ and is not related to τ_{t+1}. The social security tax that the elderly paid is returned to himself or herself after retirement, therefore, it does not affect the price of leisure. Therefore, a change in the social security tax affects labor force participation of the elderly only as a change in non-wage income.[22]

The net expected value of benefits received by prime-age adults at time t is:

$$T_t = -\tau_t A_t w_t + \frac{\tau_{t+1} A_{t+1} w_{t+1} n}{1+r_{t+1}}. \quad (3.48)$$

Equation (3.48) implies that the survival rate does not affect the net expected benefits from social security. A higher survival rate means a higher probability of receiving benefits at age 2, but the benefit received is actually lower. An increase in τ_t decreases and an increase in τ_{t+1} increases net benefit for prime-age adults at time t. Because the social security tax does not change the price of leisure at age 2, the net benefit has a similar effect to that of non-wage income on labor force participation of the elderly. If T_t increases, the elderly decrease labor force participation at time t+1.

Consumers maximize lifetime utility in equation (3.1) under the budget constraint in equation (3.40), controlling $c_{1,t}$, $c_{2,t+1}$, and z_{t+1}. If $z_{t+1}>0$, the optimal consumption at age 1 and 2, and labor force participation at age 2 are given by:

$$c_{1,t} = \frac{1}{\mu} = \frac{(1+r_{t+1})(1-\tau_t) A_t w_t + A_{t+1} w_{t+1} (\tau_{t+1} n + q_t)}{(1+r_{t+1})(1+q_t\delta + q_t\gamma\delta)}, \quad (3.49)$$

$$c_{2,t+1} = \frac{(1+r_{t+1})\delta}{\mu} = \frac{\delta[(1+r_{t+1})(1-\tau_t) A_t w_t + A_{t+1} w_{t+1} (\tau_{t+1} n + q_t)]}{1+q_t\delta + q_t\gamma\delta}, \quad (3.50)$$

[22] This is one problem caused by the assumption that consumers can live for only two periods.

$$z_{t+1} = 1 - \frac{\gamma\delta(1+r_{t+1})}{A_{t+1}w_{t+1}\mu} = \frac{A_{t+1}w_{t+1}(1+q_t\delta) - A_t w_t \gamma\delta(1+r_{t+1})(1-\tau_t) - \gamma\delta A_{t+1}w_{t+1}\tau_{t+1}n}{A_{t+1}w_{t+1}(1+q_t\delta+q_t\gamma\delta)}, \quad (3.51)$$

where μ is the Lagrangian multiplier.

The saving of prime-age adults is

$$s_{1,t} = (1-\tau_t)A_t w_t - \frac{1}{\mu} = \frac{(1+r_{t+1})(q_t\delta+q_t\gamma\delta)(1-\tau_t)A_t w_t - A_{t+1}w_{t+1}(\tau_{t+1}n+q_t)}{(1+r_{t+1})(1+q_t\delta+q_t\gamma\delta)}. \quad (3.52)$$

Equations (3.49), (3.50), and (3.51) are the same as (3.13), (3.14), and (3.15), respectively, except for the Lagrangean multiplier.

Equation (3.51) implies that $\frac{\partial z_{t+1}}{\partial \tau_t} > 0$ and $\frac{\partial z_{t+1}}{\partial \tau_{t+1}} < 0$. An increase in τ_t decreases the net benefit of social security for prime-age adults at time t, which causes an increase in z_{t+1}. An increase in τ_{t+1} increases the net benefit from the social security tax for prime-age adults at time t, so that it has a negative effect on z_{t+1}.

Suppose that $\tau_t = \tau_{t+1} = \tau^*$, $w_t = w_{t+1} = w^*$, $r_{t+1} = r_w^*$, and $T = T^*$. Net transfers in steady state T^* is:

$$T^* = \frac{A_t \tau^* w^* [gn - (1+r_w^*)]}{1+r_w^*}. \quad (3.53)$$

From equation (3.53), if $ng > 1+r_w^*$, i.e., if aggregate economic growth is higher than the interest rate, the net benefit from social security in a pay-as-you-go system is positive. If $ng = 1+r_w^*$, the net benefit from social security is zero, and if $ng < 1+r_w^*$, the net benefit from social security is negative.

In steady state, from equation (3.51), labor force participation of the elderly is given by:

$$z^* = \frac{g(1+q^*\delta) - \gamma\delta(1+r_w^*) - \gamma\delta\tau^*[ng-(1+r_w^*)]}{g(1+q^*\delta+q^*\gamma\delta)}. \quad (3.54)$$

The effect of an increase in τ^* on z^* is:

$$\frac{\partial z^*}{\partial \tau^*} = \frac{-\gamma\delta[ng - (1 + r_w^*)]}{g(1 + q^*\delta + q^*\gamma\delta)}. \tag{3.55}$$

An increase in the social security tax has a negative effect on labor force participation of the elderly if $ng>1+r_w^*$; that is, if GDP growth is higher than the interest rate. An increase in the social security tax increases the burden on prime-age adults and the benefit to the elderly. If $ng>1+r_w^*$, the net benefit is greater. Consumers retire earlier at age 2 because lifetime non-wage income increases. If $ng<1+r_w^*$, the net loss from an increase in the social security tax is greater, and the labor force participation of the elderly increases. Therefore, an increase in the social security tax does not necessarily induce earlier retirement.

Equation (3.52) implies $\frac{\partial s_{1,t}}{\partial \tau_t} < 0$ and $\frac{\partial s_{1,t}}{\partial \tau_{t+1}} < 0$. An increase in the social security tax either at age 1 or age 2 lowers saving of prime-age adults. A higher tax at age 1 reduces disposable income and reduces saving. If prime-age adults expect higher social security benefits at age 2, they would not like to save now; rather, they would like to consume now. It is also clear that an increase in τ^* decreases saving of prime-age adults, s_1^*.

The sign of the effect of an increase in q_t on $c_{1,t}$, $c_{2,t+1}$, z_{t+1}, and $s_{1,t}$ can be determined by calculating $\frac{\partial(1/\mu)}{\partial q_t}$. Equations (3.51) and (3.52) show that an increase in the survival rate affects the labor force participation, saving, and $-1/\mu$ in the same directions. Taking the partial derivative of $1/\mu$ with respect to q_t yields:

$$\frac{\partial(1/\mu)}{\partial q_t} = \frac{A_{t+1}w_{t+1} - (\delta + \gamma\delta)[(1 + r_{t+1})(1 - \tau_t)A_t w_t + \tau_{t+1} n A_{t+1} w_{t+1}]}{(1 + r_{t+1})(1 + q_t\delta + q_t\gamma\delta)^2}. \tag{3.56}$$

An increase in the survival rate has a negative effect on $1/\mu$ under plausible parameter values. This implies that $\frac{\partial(1/\mu^*)}{\partial q^*} < 0$ holds, where $1/\mu^*$ is the value of $1/\mu$ in steady state. The effect of an increase in the survival rate on the labor force participation of the elderly is given by:

$$\frac{\partial z_{t+1}}{\partial q_t} = -\frac{(1+r_{t+1})\gamma\delta}{A_{t+1}w_{t+1}}\frac{\partial(1/\mu)}{\partial q_t} > 0. \tag{3.57}$$

An increase in the survival rate reduces retirement age of the elderly when the social security tax rate is constant. In the same way, labor force participation of the elderly increases when the survival rate increases in steady state.

From equation (3.52), the effect of an increase in q_t on the saving of prime-age adults is:

$$\frac{\partial s_{1,t}}{\partial q_t} = -\frac{\partial(1/\mu)}{\partial q_t} > 0. \tag{3.58}$$

An increase in the survival rate increases the saving rate in steady state, as is the case without social security.

An increase in the social security tax decreases labor force participation of the elderly and the saving of prime-age adults if GDP growth is higher than the interest rate. However, under a certain level of the social security tax financed by pay-as-you-go system, an increase in the survival rate increases the saving of prime-age adults and labor force participation of the elderly. Therefore, the dynamic effects of an increase in the survival rate on the national saving rate and the investment rate are similar to those in the case without social security.

If z=0, that is, if the corner solution is optimal, consumption during age 1 and 2 is:

$$c_{1,t} = \frac{A_t w_t(1-\tau_t)(1+r_{t+1}) + A_{t+1}w_{t+1}\tau_{t+1}n}{(1+r_{t+1})(1+q_t\delta)}, \tag{3.59}$$

$$c_{2,t+1} = \frac{\delta[A_t w_t(1-\tau_t)(1+r_{t+1}) + A_{t+1}w_{t+1}\tau_{t+1}n]}{1+q_t\delta}, \tag{3.60}$$

The saving of prime-age adults is:

$$s_{1,t} = \frac{A_t w_t(1-\tau_t)(1+r_{t+1})q_t\delta - A_{t+1}w_{t+1}\tau_{t+1}n}{(1+r_{t+1})(1+q_t\delta)}. \tag{3.61}$$

Equation (3.61) shows that q_t has a positive effect on $s_{1,t}$ for plausible parameters. Just as in the case of an interior solution, an increase in the payroll tax at time t and t+1 decreases saving. As we have

discussed in the previous section, consumption and saving are not influenced by the preference for leisure in the case of a corner solution.

3.5.2 The Effect of Social Security on the Saving and Investment Rates

Here, we assume that the interior solution is optimal. We discuss the effect of an increase in pay-as-you-go social security on the national saving rate, the national investment rate, and the current account balance in a small open economy. Let us assume that the survival rate and capital per unit of effective labor is constant at q^* and k^*, respectively. In the same way as in section 3.3, labor force participation of the elderly is constant in steady state.

Suppose the increase in the social security tax rate is anticipated. We assume that the world interest rate is constant at r_w^*. The effect of an increase in the social security tax rate at time $t+1$ on the investment rate at time t is:

$$\frac{\partial(I_t / Y_t)}{\partial \tau_{t+1}} = \frac{g q^* (\partial z_{t+1} / \partial \tau_{t+1})}{\left(1+\frac{q^*}{n} z_t\right)} < 0 . \tag{3.62}$$

An increase in the social security tax at time t decreases labor force participation of the elderly at time t because the elderly receive more non-labor earnings. The investment rate at time $t-1$ decreases if τ_t increases.

The effect of an increase in τ_t on I_t/Y_t is:

$$\frac{\partial(I_t / Y_t)}{\partial \tau_t} = \frac{g\left[\left(1+\frac{q^*}{n} z_t\right) q^* \frac{\partial z_{t+1}}{\partial \tau_t} - \left(1+\frac{q^*}{n} z_{t+1}\right) q^* \frac{\partial z_t}{\partial \tau_t}\right]}{\left(1+\frac{q^*}{n} z_t\right)^2} > 0 . \tag{3.63}$$

As we have seen in the previous subsection, $\frac{\partial z_{t+1}}{\partial \tau_t} > 0$ and $\frac{\partial z_t}{\partial \tau_t} < 0$. Therefore, an increase in τ_t has a positive effect on the investment rate. An increase in τ_t raises the social security benefit to the

elderly at time t, so the elderly reduce labor force participation at time t. Also, the net benefit of social security for prime-age adults at time t decreases, so that they work more at time t+1. Therefore, labor force increases from time t to time t+1, and investors increase investment.

Suppose the payroll tax rate increases at time t form τ^* to τ' ($\tau^*<\tau'$) and stays constant thereafter. If investors anticipate the increase, they realize that the elderly will reduce their labor force participation in the next period. Thus, they will reduce their investment. The labor force participation of the elderly is the least at time t because they paid τ^* when they were of prime-age but their benefit at age 2 increases. The net benefit from social security is greatest for the elderly at time t. At time t+2, the elderly work more compared to at time t+1, so that I_{t+1}/Y_{t+1} increases. At time t+1, the economy reaches the new steady state, where the investment rate is the same as its initial level. Subsequently, the investment rate is constant.

The gross national saving rate is:

$$\frac{S_t}{Y_t}=\frac{s_{1,t}N_{1,t}-s_{1,t-1}N_{1,t-1}}{L_t k^{*\phi}}+\xi k^{*1-\phi}=\frac{(1-\phi)\left(X_t-\dfrac{X_{t-1}}{gn}\right)}{1+\dfrac{q_{t-1}}{n}z_t}+\xi k^{*1-\phi}, \tag{3.64}$$

where X_t is the share of saving of wage income of prime-age adults:

$$X_t=\frac{s_{1,t}}{w^* A_t}=\frac{(1+r_w^*)(q^*\delta+q^*\gamma\delta)(1-\tau_t)-g(\tau_{t+1}n+q^*)}{(1+r_w^*)(1+q^*\delta+q^*\gamma\delta)}. \tag{3.65}$$

Equation (3.65) implies $\dfrac{\partial X_t}{\partial \tau_t}<0$ and $\dfrac{\partial X_t}{\partial \tau_{t+1}}<0$. From equation (3.65), X_t in steady state is:

$$X^*=\frac{(1+r_w^*)(q^*\delta+q^*\gamma\delta)(1-\tau^*)-g(\tau^* n+q^*)}{(1+r_w^*)(1+q^*\delta+q^*\gamma\delta)}. \tag{3.66}$$

Equation (3.66) implies $\frac{\partial X^*}{\partial \tau^*} < 0$. The gross national saving rate in steady state is:

$$\left(\frac{S}{Y}\right)^* = \frac{(1-\phi)X^*}{1+\frac{q^*}{n}z^*}\frac{gn-1}{gn} + \xi k^{*1-\phi}. \tag{3.67}$$

The effect of an increase in the payroll tax rate on the steady state saving rate is:

$$\frac{\partial(S/Y)^*}{\partial \tau^*} = \frac{(1-\phi)\left[\frac{\partial X^*}{\partial \tau^*}\left(1+\frac{q^*}{n}z^*\right) - X^*\frac{q^*}{n}\frac{\partial z^*}{\partial \tau^*}\right]}{\left(1+\frac{q^*}{n}z^*\right)^2}\frac{gn-1}{gn}, \tag{3.68}$$

where $\frac{\partial X^*}{\partial \tau^*} < 0$ for any GDP growth rate because prime-age adults reduce saving when the social security tax rate increases. If $gn>1+r_w^*$, $\frac{\partial z^*}{\partial \tau^*} < 0$. An increase in the social security tax rate decreases saving and decreases labor force participation of the elderly, which increases GDP. Thus, the sign of the effect of an increase in τ^* on the national saving rate is ambiguous. If $1<gn<1+r_w^*$, $\frac{\partial z^*}{\partial \tau^*} > 0$. An increase in the social security tax reduces the saving rate. If $gn<1<1+r_w^*$, an increase in the social security tax rate increases the national saving rate because the decrease in saving of prime-age adults is greater than the decrease in dissaving of the elderly.

Let us discuss the out-of-steady-state effects of an increase in the social security tax. The effect of a one-time increase in τ_t on the national saving rate is:

$$\frac{\partial(S_{t-1}/Y_{t-1})}{\partial \tau_t} = \frac{(1-\phi)\partial X_{t-1}/\partial \tau_t}{1+\frac{q^*}{n}z_{t-1}} < 0. \tag{3.69}$$

Equation (3.69) implies that an increase in τ_t decreases the saving rate at time t-1. Prime-age adults know that they will receive a higher benefit at age 2 inducing them to save less. On the other hand, labor force participation of the elderly does not change. From equation (3.64), we can obtain the effect of an increase in τ_t on S_t/Y_t as:

$$\frac{\partial(S_t/Y_t)}{\partial\tau_t}=\frac{(1-\phi)\left[\left(\frac{\partial X_t}{\partial\tau_t}-\frac{1}{gn}\frac{\partial X_{t-1}}{\partial\tau_t}\right)\left(1+\frac{q_{t-1}}{n}z_t\right)-X_t\frac{q_{t-1}}{n}\frac{\partial z_t}{\partial\tau_t}\right]}{\left(1+\frac{q_{t-1}}{n}z_t\right)^2}, \tag{3.70}$$

where $\frac{\partial X_t}{\partial\tau_t}<0$, $\frac{\partial X_{t-1}}{\partial\tau_t}<0$, and $\frac{\partial z_t}{\partial\tau_t}<0$ hold. If τ_t increases, $s_{1,t}$ decreases because disposable income of prime-age adults decreases. At time t, the elderly dissave what they have saved, and this dissaving is lower under a higher τ_t. The net benefit from social security for the elderly at time t increases, which causes them to work less. This causes a decrease in GDP and thus increases the national saving rate. Whether the total effect of an increase in τ_t is positive or negative is ambiguous.

The effect of an increase in τ_t on S_{t+1}/Y_{t+1} is:

$$\frac{\partial(S_{t+1}/Y_{t+1})}{\partial\tau_t}=\frac{(1-\phi)\left[-\frac{1}{gn}\frac{\partial X_t}{\partial\tau_t}\left(1+\frac{q^*}{n}z_{t+1}\right)-\left(X_{t+1}-\frac{1}{gn}X_t\right)\frac{q_{t-1}}{n}\frac{\partial z_{t+1}}{\partial\tau_t}\right]}{\left(1+\frac{q^*}{n}z_{t+1}\right)^2}, \tag{3.71}$$

where $\frac{\partial X_t}{\partial\tau_t}<0$ and $\frac{\partial z_{t+1}}{\partial\tau_t}>0$ hold. The first term of the numerator of equation (3.71) is the change in the dissaving of the elderly and the second term shows the change in labor force participation of the elderly. An increase in τ_t decreases the dissaving by the elderly because they saved less while working. The elderly work more because they paid more in taxes, and lifetime non-wage income decreases. Whether an increase in τ_t has a positive or negative effect depends on parameters such as the survival rate, the interest rate, the discount rate, the economic growth rate, the population growth rate, and the preference for leisure. An increase in τ_t does not influence the saving rate at time t+2 or thereafter.

The current account balance, *CA*, is saving minus investment. From the results above, if the interest rate is greater than GDP growth, an increase in the social security tax rate decreases the current account balance in steady state. Otherwise, the steady-state and out-of-steady-state effects of an increase in the social security tax on the current account balance are ambiguous. Whether an increase in the social security tax induces capital outflows or inflows is an empirical question.

3.6 Retirement Decision Without an Annuity Market

3.6.1 Consumer's Optimization

We have found that under the perfect annuity market assumption, an increase in longevity can increase the labor force participation of the elderly. However, it is unrealistic to assume that the insurance market is perfect. Here, for the sake of simplicity, we consider the case that insurance is not available.[23] We assume that there is no bequest motive. If a consumer cannot survive until the second period, saving becomes an accidental bequest and is distributed equally to prime-age adults. We assume the same utility function as in equation (3.9). The budget constraint facing a prime-age adult is:

$$c_{1,t} + s_{1,t} = A_t w_t + b_t, \tag{3.72}$$

where b_t represents accidental bequests received by a prime-age adult from those who die at time t. The budget constraint facing the elderly is:

$$c_{2,t+1} = (1 + r_{t+1}) s_{1,t} + A_{t+1} w_{t+1} z_{t+1}. \tag{3.73}$$

Because no annuity is available, the return to saving is not influenced by the survival rate. In equations (3.72) and (3.73), substituting for $s_{1,t}$ in terms of $c_{1,t}$, $c_{2,t+1}$, and z_{t+1} yields:

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[23] Our results of individuals' decision making is similar to Chang's (1990) model. In future research, it is important to consider the degree of imperfection of the insurance market.

$$c_{1,t} + \frac{c_{2,t+1}}{1+r_{t+1}} = A_t w_t + b_t + \frac{A_{t+1} w_{t+1} z_{t+1}}{1+r_{t+1}}. \tag{3.74}$$

Solving the first order condition, we obtain the following results assuming interior solutions.

$$c_{1,t} = \frac{1}{\nu} = \frac{(1+r_{t+1})(A_t w_t + b_t) + A_{t+1} w_{t+1}}{(1+r_{t+1})(1+\delta q_t + \gamma \delta q_t)}, \tag{3.75}$$

$$c_{2,t+1} = \frac{(1+r_{t+1})\delta q_t}{\nu} = \frac{\delta q_t[(1+r_{t+1})(A_t w_t + b_t) + A_{t+1} w_{t+1}]}{(1+\delta q_t + \gamma \delta q_t)}, \tag{3.76}$$

$$z_{t+1} = 1 - \frac{(1+r_{t+1})\delta q_t \gamma}{A_{t+1} w_{t+1} \nu} = \frac{A_{t+1} w_{t+1}(1+\delta q_t) - \delta q_t \gamma (1+r_{t+1})(A_t w_t + b_t)}{A_{t+1} w_{t+1}(1+\delta q_t + \gamma \delta q_t)}, \tag{3.77}$$

where ν is the Lagrangean multiplier. Here, $\frac{\partial(1/\nu)}{\partial q_t} < 0$ and $\frac{\partial(q_t/\nu)}{\partial q_t} > 0$. Therefore, an increase in longevity can increase the saving of prime-age adults and the leisure of the elderly.

From equations (3.17) and (3.18), we can calculate the following age-consumption profile of those who survive until age 2:

$$\frac{c_{2,t+1}}{c_{1,t}} = q_t \delta (1+r_{t+1}). \tag{3.78}$$

Equation (3.78) is different from equation (3.16). An increase in the survival rate increases the share of consumption during age 2. Longevity makes individuals patient and the return to saving does not change. This makes consumption during age 2 more valuable than in the case of lower life expectancy. As in the case of a perfect annuity market, the age-consumption profile is influenced by the discount factor and the interest rate, but is not influenced by labor income. Saving by prime-age adults, $s_{1,t}$, is:

$$\begin{aligned} s_{1,t} &= A_t w_t + b_t - 1/\nu \\ &= \frac{(1+r_{t+1})(\delta q_t + \gamma \delta q_t)(A_t w_t + b_t) - A_{t+1} w_{t+1}}{(1+r_{t+1})(1+\delta q_t + \gamma \delta q_t)}. \end{aligned} \tag{3.79}$$

Taking the partial derivative with respect to q_t on both sides of equation (3.79) gives:

$$\frac{\partial s_{1,t}}{\partial q_t} = -\frac{\partial(1/\nu)}{\partial q_t} > 0 . \tag{3.80}$$

As in the case where insurance is available, an increase in the survival rate increases saving by prime-age adults.

From equation (3.77), the effect of an increase in the survival rate on leisure of the elderly is:

$$\frac{\partial z_{t+1}}{\partial q_t} = -\frac{\gamma\delta(1+r_{t+1})}{A_{t+1}w_{t+1}}\frac{\partial(q_t/\nu)}{\partial q_t} < 0 . \tag{3.81}$$

Contrary to the case of a perfect annuity market, if an annuity is not available, the elderly work less when the survival rate increases. Prime-age adults save more when life expectancy rises. If an annuity is unavailable, the return to saving is not influenced by the survival rate. When they reach age 2, they have more non-wage income, which allows them to enjoy more leisure and consumption. In a perfect annuity market, the return to saving decreases when the survival rate increases. Although a prime-age adult saves more, $\frac{1+r_{t+1}}{q_t}s_t$, non-wage income at age 2, decreases. Therefore, z_{t+1} increases when q_t increases.

Accidental bequests received by a prime-age adult at time t, b_t, is the saving of prime-age adults who died at time t-1. Therefore, $b_t N_{1,t} = (1-q_{t-1})N_{1,t-1}s_{1,t-1}$. This implies:

$$b_t = \frac{1-q_{t-1}}{n}s_{1,t-1} . \tag{3.82}$$

Accidental bequests depend on previous survival rates. An increase in q_{t-1} increases $s_{1,t-1}$ but decreases $(1-q_{t-1})$. Thus whether an increase in q_{t-1} increases or decreases b_t is ambiguous. Therefore, we cannot determine the effect of an increase in q_{t-1} on consumption, saving, and labor force participation.

Also, we assume a small open economy, where the interest rate is constant at r_w^* and the wage per effective worker (w^*) is exogenous. Appendix G presents a detailed discussion of steady state

saving of prime-age adults and labor force participation of the elderly. The share of saving of labor income of prime-age adults in steady state $X^*=s_{1,t}/A_t w_t$ is:

$$X^* = \frac{gn[(1+r_w^*)(q^*\delta + q^*\gamma\delta) - g]}{(1+r_w^*)[gn(1+q^*\delta+q^*\delta\gamma) - (1-q^*)(q^*\delta+q^*\gamma\delta)]}. \tag{3.83}$$

In equation (3.83), the sign of $\frac{\partial X^*}{\partial q^*}$ is ambiguous. Holding bequests constant, an increase in q^* increases saving by prime-age adults. An increase in q^* reduces the number of those who leave accidental bequests, which can decrease saving. Which effect dominates is ambiguous.

Labor force participation of the elderly in steady state, z^* is:

$$z^* = \frac{g(1+q^*\delta) - q^*\gamma\delta(1+r_w^*) - \dfrac{[(1+r_w^*)(q^*\delta+q^*\gamma\delta)-g]q^*(1-q^*)\gamma\delta}{ng(1+q^*\delta+q^*\delta\gamma)-(1-q^*)(q^*\delta+q^*\gamma\delta)}}{g(1+q^*\delta+q^*\gamma\delta)}. \tag{3.84}$$

Whether an increase in q^* increases or decrease z^* is ambiguous. An increase in q^* increases saving and induces earlier retirement. On the other hand, the number of people who leave accidental bequests decreases. This decreases lifetime wealth of the next generation, inducing the elderly to work more. Which effect dominates depends on q^* and other parameters such as r_w^*, γ, and δ.

3.6.2 The Effect of Longevity on the National Saving Rate and Investment Rates

The effect of an increase in the survival rate on the national saving rate depends on saving by prime-age adults, dissaving by elderly, and labor force participation of the elderly. An increase in the survival rate has transitory effects which are different from those in Chapter 2 and Section 3.4. Therefore, the investment rate at time t is $\frac{I_t}{Y_t} = \left[\frac{gn(1+\frac{q_t}{n}z_{t+1})}{1+\frac{q_{t-1}}{n}z_t} - (1-\xi)\right]k^{*1-\phi}$, and the investment rate in steady state is $\left(\frac{I}{Y}\right)^* = [gn-(1-\xi)]k^{*1-\phi}$. In steady state, an increase in the

survival rate does not change the investment rate because labor force participation is constant, as in section 3.4.

The effect of an increase in q_t on the investment rate at time t is:

$$\frac{\partial(I_t / Y_t)}{\partial q_t} = \frac{gn}{1+\frac{q_{t-1}}{n} z_t} k^{*1-\phi} \left(z_t + q_t \frac{\partial z_{t+1}}{\partial q_t} \right), \tag{3.85}$$

where $\frac{\partial z_{t+1}}{\partial q_t} < 0$. Whether $\frac{\partial(I_t / Y_t)}{\partial q_t}$ is positive or negative is ambiguous. An increase in q_t decreases labor force participation of the elderly at time t+1. On the other hand, it increases the number of surviving elderly. It is ambiguous whether total labor force increases or decreases. Because the elderly work less when life expectancy increases, as explained in the preceding subsection, this result is different from that of section 3.4.

The effect of an increase in q_t on I_{t+1}/Y_{t+1} is:

$$\frac{\partial(I_{t+1} / Y_{t+1})}{\partial q_t} = gk^{*1-\phi} \frac{q_{t+1} \frac{\partial z_{t+2}}{\partial q_t} \left(1+\frac{q_t}{n} z_{t+1} \right) - \left(1+\frac{q_{t+1}}{n} z_{t+2} \right) \left(z_{t+1} + q_t \frac{\partial z_{t+1}}{\partial q_t} \right)}{\left(1+\frac{q_t}{n} z_{t+1} \right)^2}. \tag{3.86}$$

$\frac{\partial z_{t+1}}{\partial q_t} < 0$ and $\frac{\partial z_{t+2}}{\partial q_t}$ can be positive or negative, depending on whether an increase in q_t increases accidental bequests. The effect of an increase in q_t on I_{t+2}/Y_{t+2} is:

$$\frac{\partial(I_{t+2} / Y_{t+2})}{\partial q_t} = gk^{*1-\phi} \frac{q_{t+2} \frac{\partial z_{t+3}}{\partial q_t} \left(1+\frac{q_{t+1}}{n} z_{t+2} \right) - \left(1+\frac{q_{t+2}}{n} z_{t+3} \right) q_{t+1} \frac{\partial z_{t+2}}{\partial q_t}}{\left(1+\frac{q_{t+1}}{n} z_{t+2} \right)^2}. \tag{3.87}$$

An increase in q_t affects z_{t+2} and z_{t+3} because it changes accidental bequests received by prime-age adults. Whether an increase in q_t increases or decreases z_{t+2} and z_{t+3} is indeterminate. Therefore, the sign of $\frac{\partial(I_{t+2} / Y_{t+2})}{\partial q_t}$ is ambiguous. An increase in q_t also influences I/Y after time t+3 through

changes in accidental bequests. We would need simulation analysis in order to see the dynamics in detail.

The national saving rate at time t is $\frac{S_t}{Y_t} = \frac{(1-\phi)(X_t - \frac{X_{t-1}}{gn})}{\left(1+\frac{q_{t-1}}{n} z_t\right) k^{*\phi}} + \xi k^{*1-\phi}$, and in steady state, $\left(\frac{S}{Y}\right)^* = \frac{(1-\phi)X^*}{\left(1+\frac{q^*}{n} z^*\right)} \frac{gn-1}{gn} + \xi k^{*1-\phi}$. The steady state effect of an increase in the survival rate on the national saving rate is:

$$\frac{\partial (S/Y)^*}{\partial q^*} = \frac{(1-\phi)\left[\frac{\partial X^*}{\partial q^*}\left(1+\frac{q^*}{n} z^*\right) - X^*\left(z^* + \frac{q^*}{n}\frac{\partial z^*}{\partial q^*}\right)\right]}{\left(1+\frac{q^*}{n} z^*\right)^2 k^{*\phi}} \frac{gn-1}{gn} + \xi k^{*1-\phi}. \tag{3.88}$$

As shown in the previous subsection, the signs of $\frac{\partial X^*}{\partial q^*}$ and $\frac{\partial z^*}{\partial q^*}$ are ambiguous. Whether an increase in the survival rate increases or decreases saving by prime-age adults, the dissaving of the elderly, and labor force participation of the elderly is unclear. Therefore, the effect of an increase in q^* on $(S/Y)^*$ is ambiguous.

An increase in q_t has out-of-steady-state effects on the national saving rate at time t and thereafter. The effect of an increase in q_t on S_t/Y_t is:

$$\frac{\partial (S_t/Y_t)}{\partial q_t} = \frac{(1-\phi)\frac{\partial X_t}{\partial q_t}}{1+\frac{q_{t-1}}{n} z_t} > 0. \tag{3.89}$$

An increase in q_t has a positive effect on the saving rate at time t as Chapter 2, and section 3.4. q_t only affects saving by prime-age adults and does not affect the dissaving or labor force participation of the elderly in the current period.

The effect of an increase in q_t on S_{t+1}/Y_{t+1} is:

$$\frac{\partial(S_{t+1}/Y_{t+1})}{\partial q_t} = (1-\phi)\frac{\left(\frac{\partial X_{t+1}}{\partial q_t} - \frac{1}{gn}\frac{\partial X_t}{\partial q_t}\right)\left(1+\frac{q_t}{n}z_{t+1}\right) - \left(X_{t+1} - \frac{X_t}{gn}\right)\left(\frac{z_{t+1}}{n} + \frac{q_t}{n}\frac{\partial z_{t+1}}{\partial q_t}\right)}{\left(1+\frac{q_t}{n}z_{t+1}\right)^2} \quad (3.90)$$

where $\frac{\partial X_t}{\partial q_t} > 0$ and $\frac{\partial z_{t+1}}{\partial q_t} < 0$. However, the sign of $\frac{\partial X_{t+1}}{\partial q_t}$ is ambiguous because we cannot determine whether an increase in q_t increases or decreases accidental bequests received by prime age adults in the next period. Therefore, we cannot determine whether an increase in q_t increases or decreases S_{t+1}/Y_{t+1}. The effect of an increase in q_t on S_{t+2}/Y_{t+2} is:

$$\frac{\partial(S_{t+2}/Y_{t+2})}{\partial q_t} = (1-\phi)\frac{\left(\frac{\partial X_{t+2}}{\partial q_t} - \frac{1}{gn}\frac{\partial X_{t+1}}{\partial q_t}\right)\left(1+\frac{q_{t+1}}{n}z_{t+2}\right) - \left(X_{t+2} - \frac{X_{t+1}}{gn}\right)\frac{q_{t+1}}{n}\frac{\partial z_{t+2}}{\partial q_t}}{\left(1+\frac{q_t}{n}z_{t+2}\right)^2}. \quad (3.91)$$

X_{t+2}, X_{t+1}, and z_{t+2} are affected by q_t only through accidental bequests received at age 1. As we discussed earlier, the effect of an increase in q_t has either a positive or negative effect on accidental bequests. Therefore, the effect of an increase in q_t on S_{t+2}/Y_{t+2} is ambiguous. After time t+3, q_t influences S/Y, however, whether it has positive or negative effect is ambiguous.

The current account balance (CA) is the national saving rate minus the national investment rate. Because the effect of an increase in the survival rate on the investment rate and the saving rate is ambiguous both in steady state and out of steady state, the effect of an increase in the survival rate on the current account balance is also ambiguous. A one time increase in the survival rate can influence the national saving rate and the investment rate for a long period of time through a change in accidental bequests. Thus, it also affects the current account balance over time.

3.7 The Saving and Investment Rates in Dynamic Simulations

In this section we present simulations of the national saving rate and national investment rate given changes in life expectancy, population growth, technological growth, and the social security tax rate. We simulate the national saving rate of the United States and Japan from 1900 to 2020, as in Chapter 2. Both the simulated national saving rate and investment rate differ from those presented in Chapter 2. The responses and the level of the investment rate is quite similar to those presented in Chapter 2, although the levels are slightly higher. The simulated saving rate is, of course, lower when the elderly are working.

Table 3.1 summarizes parameters used in the simulation. We use the same values for exogenous parameters as in Chapter 2 (listed in Table 2.1) except that we assume that the intertemporal elasticity of substitution ($1/\theta$) is equal to 1.0 in this chapter. Each period lasts 30 years. Prime-age adults are those from ages 30 to 59 years and the elderly are those from ages 60 to 89 years old. For the 30-years period, the discount rate (δ) is 0.522, the depreciation rate (ξ) is 0.785, the share of capital of output (ϕ) is one-third, and the world interest rate (r_w) is 4.644. We set the parameter for the preference for leisure (γ) at 0.3.[24] We assume these parameters are constant over the simulated period.

For the survival rate (q), technological growth (g), and population growth (n), we use the same data described in Chapter 2. The social security tax rate (τ) is obtained from the US Social Security Administration website and the National Institute of Population and Social Security Research for Japan. The social security tax system was introduced after 1930 in both the United States and Japan.

[24] There is no special reason for the value of γ, but a higher γ can cause corner solutions for labor force participation of the elderly (z). To avoid this, we set γ much lower.

Table 3.1 Results of Simulation Analysis

The United States					
Year	q	gn	g	n	τ
1900	0.373	3.6496	1.9437	1.8776	0.0000
1930	0.413	3.1638	2.1389	1.4792	0.0227
1960	0.508	2.5711	1.7602	1.4607	0.0934
1990	0.613	2.2059	1.7095	1.2904	0.1261
2020	0.702	1.9953	1.7095	1.1672	0.1306
2050	0.702				0.1312

	Perfect Annuity Market						No Annuity			Chapter 2	
	No Social Security			With Social Security			No Social Security				
Year	z	S/Y	I/Y	Z	S/Y	I/Y	z	S/Y	I/Y	S/Y	I/Y
1900	0.4292	0.0960	0.1516	0.4292	0.0923	0.1516	0.7553	0.0410	0.1516	0.1173	0.1516
1930	0.4292	0.0975	0.1366	0.4263	0.0779	0.1367	0.7553	0.0284	0.1354	0.1273	0.1302
1960	0.4833	0.1137	0.1045	0.4856	0.0882	0.1058	0.7733	0.0839	0.1080	0.1416	0.1040
1990	0.4028	0.1149	0.0924	0.4401	0.0898	0.0935	0.7227	0.1174	0.0957	0.1464	0.0879
2020	0.4069	0.1090	0.0831	0.4615	0.0875	0.0835	0.6367	0.1091	0.0810	0.1445	0.0786
2050	0.4223			0.4778			0.5563				

Japan					
Year	q	gn	g	n	τ
1900	0.374	2.4308	1.7588	1.3820	0.0000
1930	0.371	4.9827	3.0594	1.6286	0.0075
1960	0.477	7.2787	4.3485	1.6738	0.0502
1990	0.680	1.2626	1.3447	0.9390	0.1076
2020	0.781	0.9440	1.3447	0.7020	0.1270
2050	0.781				0.1270

	Perfect Annuity Market						No Annuity			Chapter 2	
	No Social Security			With Social Security			No Social Security				
Year	z	S/Y	I/Y	z	S/Y	I/Y	z	S/Y	I/Y	S/Y	I/Y
1900	0.3747	0.0863	0.0978	0.3747	0.0856	0.0978	0.7333	0.0462	0.0978	0.1019	0.0978
1930	0.3747	0.0560	0.2183	0.3739	0.0401	0.2183	0.7333	-0.0284	0.2085	0.1010	0.2104
1960	0.6194	0.0556	0.3299	0.6191	0.0110	0.3305	0.8256	-0.0468	0.3294	0.1509	0.3118
1990	0.7208	0.1590	0.0462	0.7257	0.1403	0.0480	0.8877	0.1568	0.0526	0.1975	0.0463
2020	0.2814	0.0754	0.0368	0.3247	0.0646	0.0380	0.5458	0.0896	0.0348	0.0914	0.0322
2050	0.3040			0.3715			0.4336				

Note: q: the survival rate for those aged 30-59 to become 60-89, g: 1+GDP growth rate of coming 30 years, n: 1+growth rate of labor force of coming 30 years, z: labor force participation of the elderly, τ: social security tax rate, S/Y: the national saving rate, I/Y: the national investment rate.

Sources: τ: Social Security Administration (the United States). National Institute of Population and Social Security Research (Japan). For q, g, n, see Table 2.2.

In 1930, the social security tax rate was very small, but τ increased substantially from 1960 to 1990 and is expected to increase in the future in both countries.[25]

We simulate the national saving rate and the national investment rate in a small open economy for the following cases: (1) when the annuity market is perfect; (2) when a social security tax is introduced under a perfect annuity market; (3) when an annuity is not available; and (4) when individuals do not work at age 2 (same situation as in Chapter 2 but with a different intertemporal elasticity of substitution) (Table 3.1).[26]

If the annuity market is perfect, labor force participation of the elderly increases from 1930 to 1960, but decreases in 1990 in the United States (Figure 3.1 (a)). The national saving rate increases moderately until 1990 and declines slightly in 2020. The national investment rate decreases throughout the period. In Japan, the labor force participation of the elderly also increases until 1990 and then decreases in 2020 (Figure 3.1 (b)). Contrary to the result of Chapter 2, the national saving rate declines from 1930 to 1960. This results from greater technological growth during this period, and expectations among consumers for higher earnings in the future. Thus, they saved less when they were of prime age. Considering the high saving that occurred in the1960s in Japan, these simulations are not realistic. The simulated national investment rate increases substantially in 1960, similar to the pattern discussed in Chapter 2.

If a social security tax financed by pay-as-you-go system is introduced in a perfect annuity market, the US simulated labor force participation decreases slightly in 1930, increases in 1960, decreases again in 1990, and increases in 2020 and 2050 (Figure 3.2 (a)). The simulated patterns of change of the national saving rate and the investment rate are similar to those in the case of a

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[25] We exclude tax for medical expenditure. We use the average of the 30 years that follow. For example, we use the average social security tax rate from 1930 to 1959 for τ in 1930.

[26] The results are different from those in Table 2.2 because we assume the intertemporal elasticity of substitution is equal to one.

perfect annuity market throughout the simulation period. Comparing this to the case without social security, the national saving rate is lower and the national investment rate is higher here. In Japan, labor force participation of the elderly decreases slightly in 1930, increases in 1960 and 1990, decreases in 2020 and increases slightly in 2050. Just as in the United States, the national saving rate and the national investment rate present similar dynamics as in the case without social security. The national saving rate is lower and the national investment rate is higher than in the case without social security.

If an annuity is unavailable, labor force participation of the elderly is higher than in the previous cases because the return to saving is lower in this case (Figure 3.3(a) and Figure 3.3 (b)). Labor force participation of the elderly increases from 1930 to 1960 and decreases after 1960 in the United States (Figure 3.3 (a)). The pattern of change in the national saving rate is similar to that in Figure 3.1 (a) and 3.2 (a). In Japan, labor force participation of the elderly increases until 1990 and decreases in 2020 and 2050 (Figure 3.3 (b)). The national saving rate is negative in 1930 and 1960, so that the dissaving of the elderly exceeds the sum of the saving of prime-age adults and depreciation.

Throughout the simulation analysis, the national investment rate is higher and the national saving rate is lower when labor force participation is endogenous. Therefore, the current account balance is less than that in Chapter 2. If endogenous retirement is not considered, the effect of longevity on the current account balance is overstated.

Figure 3.1 Simulated Saving and Investment Rates: Perfect Annuity Market

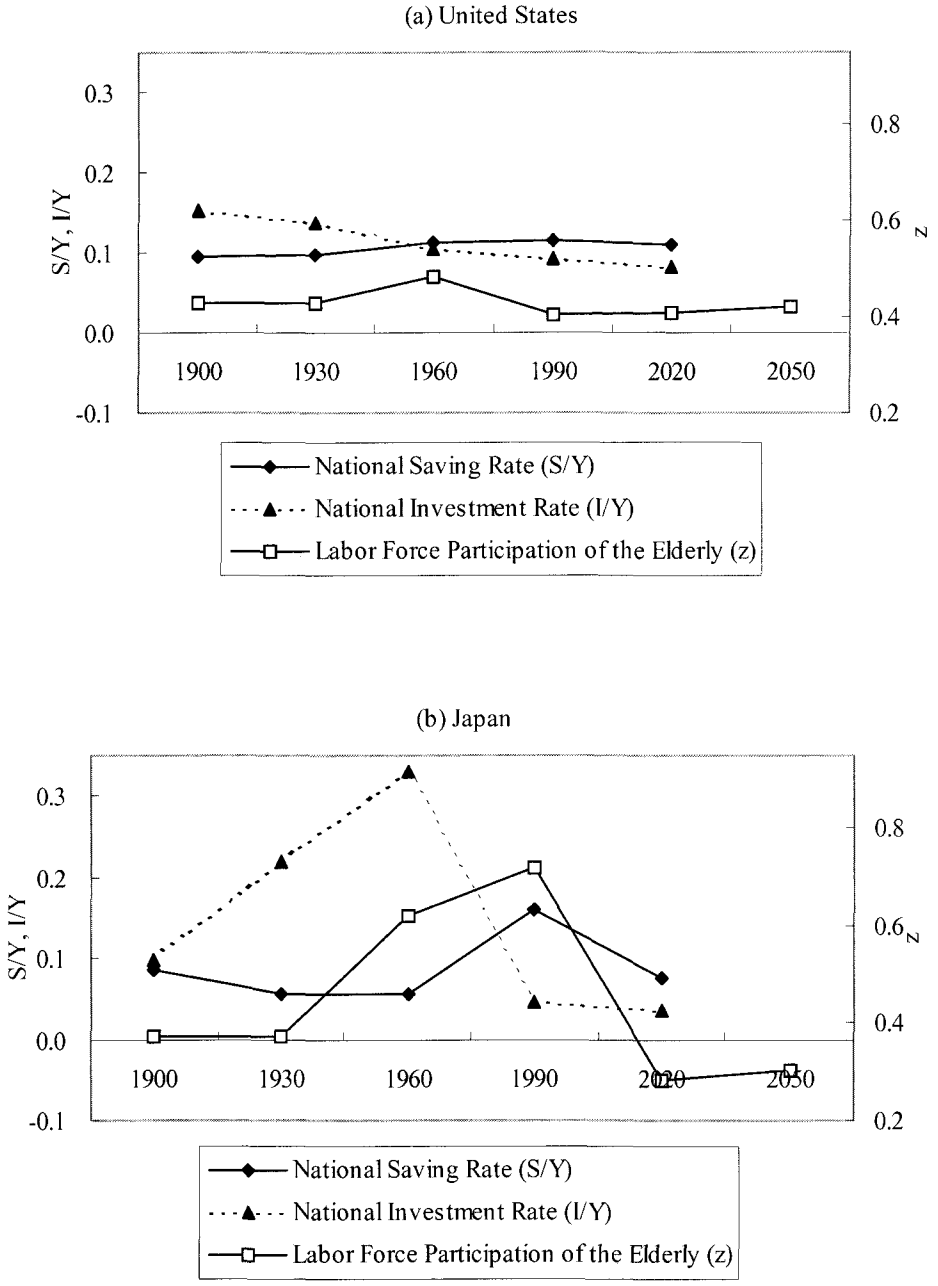
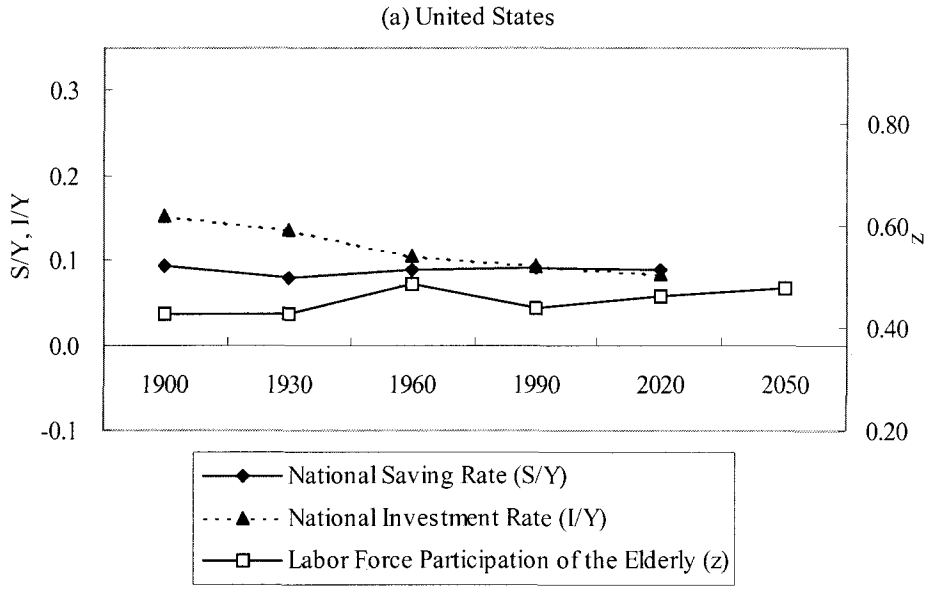


Figure 3.2 Simulated Saving and Investment Rates: Perfect Annuity with Social Security Tax



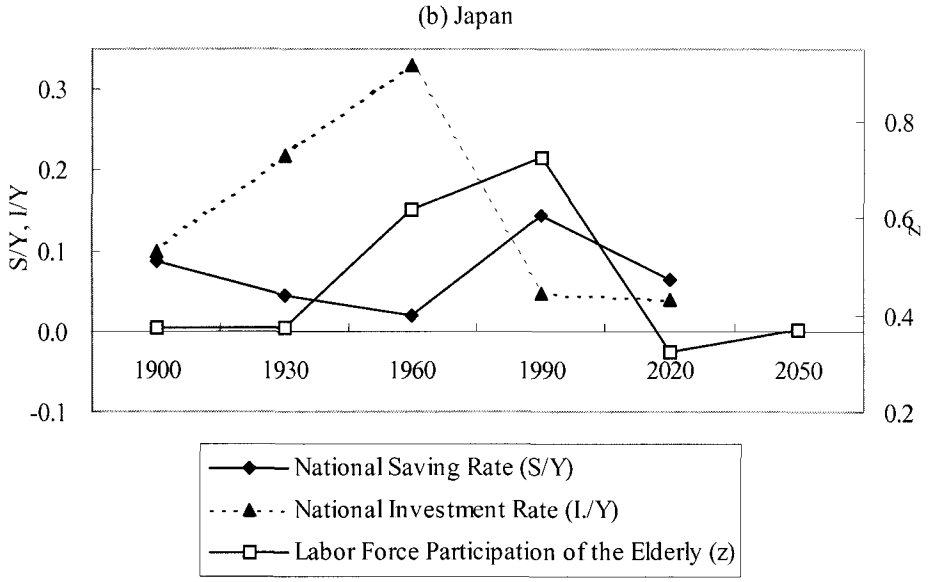
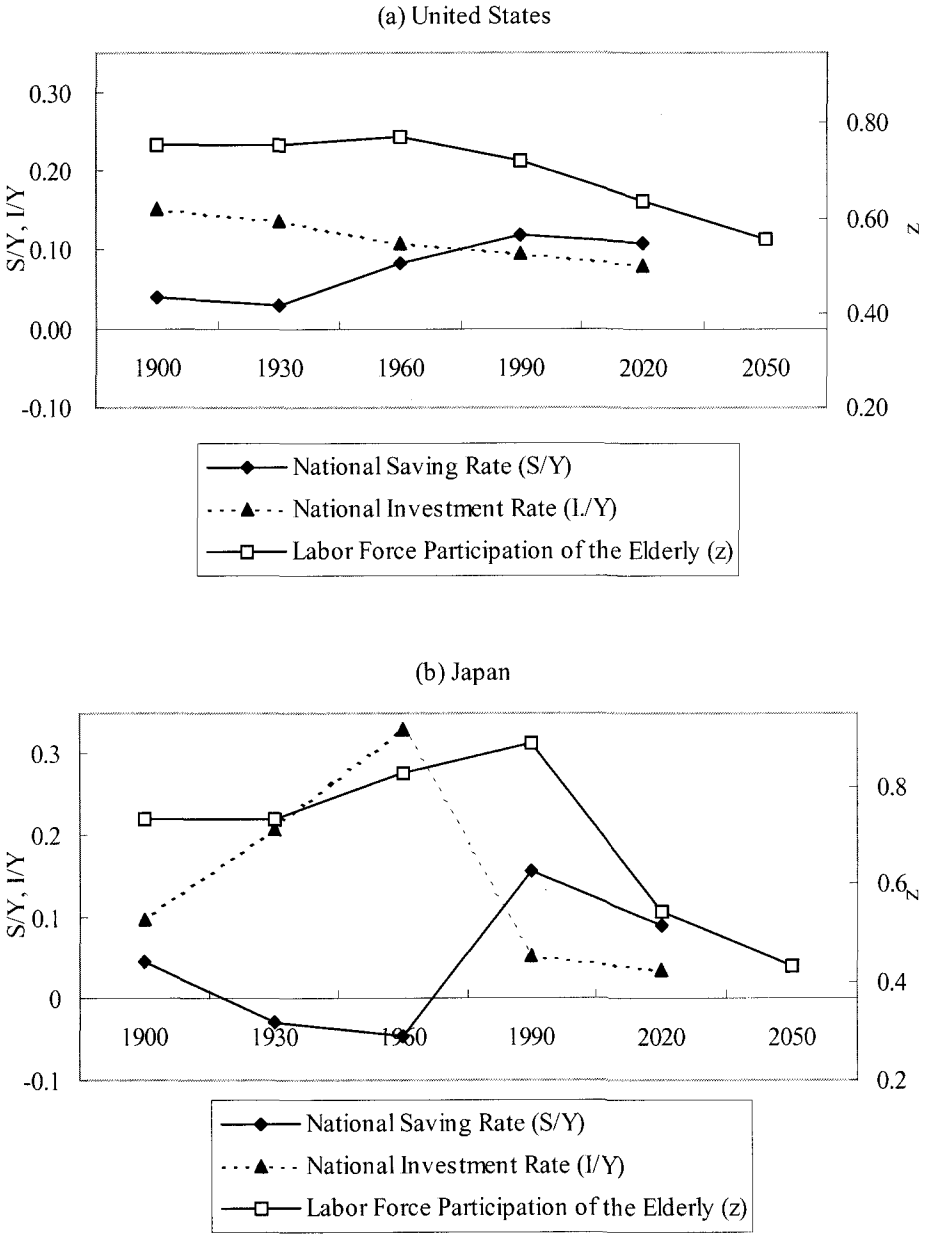


Figure 3.3 Simulated Saving and Investment Rates: No Annuity



3.8 Conclusion

In this chapter, we analyze the effect of an increase in life expectancy on the national saving rate, the investment rate, and the current account balance in a small open economy assuming endogenous labor force participation of the elderly. We develop an overlapping generations model with two periods. Our research is noteworthy in that we deal with endogenous labor force participation, the effect of life expectancy on the national saving rate, national investment rate, and the current account balance in a small open economy, and the dynamic effects of longevity. Findings in this chapter are different from those in Chapter 2, where exogenous retirement is assumed.

The results of this chapter and the previous chapter (only the case of a small open economy) are summarized in Table 3.2. If the annuity market is perfect, an increase in the survival rate decreases the return to saving as Yaari (1965) suggests. In a perfect annuity market, if the survival rate increases at time t, prime-age adults increases saving ((ii)-(3)). In the next period, the elderly work more. ((ii)-(4)).

Investors increase investment at time t, anticipating a larger labor force next period ((ii)-(6)). An increase in the survival rate at time t has a negative effect on the investment rate at time t+1 ((ii)-(7)). Just as in Chapter 2, an increase in the survival rate at time t increases the national saving rate in the same period ((ii)-(10)). An increase in q_t decreases the saving rate at time t+1 due to an increase in dissaving of the elderly and an increase in labor force ((ii)-(11)).

In steady state, the direction of the effect of an increase in the survival rate is ambiguous ((ii)-(15)). If life expectancy rises, prime-age adults increase saving, the elderly work more, the dissaving of the elderly increases, and the number of elderly increases. Considering these effects, the net effect of an increase in life expectancy on the national saving rate is indeterminate.

If an annuity is not available and the survival rate increases at time t, prime-age adults are more inclined to save for old age ((iii)-(3)), which enables the older generation to retire early ((iii)-(4)). It is ambiguous whether longevity increases or decreases total labor force. Out-of-steady-state effects of an increase in the survival rate on the national investment rate depend on an increase in total labor force ((iii)-(6), (7), (8)).

An increase in the survival rate at time t increases the national saving rate at time t ((iii)-(10)). A one-time increase in the survival rate influences the national saving rate forever through a change in accidental bequests. However, whether the net effect of an increase in the survival rate is positive or negative is ambiguous ((iii)-(11), (12)). Also, the effect of an increase in the survival rate on the national saving rate in steady state is ambiguous ((iii)-(16)).

If a social security tax financed by a pay-as-you-go system is introduced once at time t, saving of prime-age adults at times t-1 and t decreases ((iv)-(17), (19)). The elderly at time t work less ((iv)-(18)) and the elderly at time t+1 work more ((iv)-(20)). Investors decrease investment at time t-1, expecting a smaller labor force in the next period ((iv)-(20)), and increase investment at time t, expecting a larger labor force at time t+1 than at time t ((iv)-(21)). The national saving rate decreases at time t-1 ((iv)-(25)). A one-time increase in the social security tax at time t influences the national saving rate at times t and t+1, but whether the net effect is positive or negative is ambiguous.

In steady state, an increase in the social security tax rate decreases saving of prime-age adults ((iv)-(29)). Labor force participation of the elderly is positively (negatively) affected if the interest rate is higher (lower) than the growth rate of GDP ((iv)-(30)). If the interest rate is higher than GDP growth and GDP is growing, an increase in the social security tax rate decreases the national saving rate. If GDP is declining, the effect of an increase in the social security tax on the national saving

rate is positive. If GDP growth is higher than the interest rate, the effect of an increase in the social security tax rate on the national saving rate is ambiguous ((iv)-(31)).

The current account balance is saving minus investment. From Table 3.2, we cannot determine whether an increase in life expectancy increases the current account when we consider endogenous retirement.

We can explain the increasing retirement age in developed countries, where life expectancy is usually high, in the following ways. We find that the effect of an increase in life expectancy decreases retirement under a perfect annuity market and increases retirement under imperfect annuity market. As a matter of fact, annuities are available, but they are far from perfect because of imperfect information. Therefore, an increase in life expectancy induces prime-age adults to save more and retire early, as Chang (1990) and Kalemli-Orzcan and Weil (2002) suggest. Slower economic growth may induce consumers to save more during prime age and they may be discouraged from working after they become old. Also, an expansion of social security benefits may induce earlier retirement.

However, it is uncertain whether earlier retirement will continue in the future. In section 3.5, we discuss that an increase in the social security tax increases retirement if the GDP growth rate is higher than the interest rate. This does not usually hold. Considering lower fertility common in developed countries, it is possible that an increase in the social security tax rate increases the burden on prime-age adults, and may force the elderly of the next generation to work more. Also, in section 3.6, we find that an increase in the survival rate has an ambiguous effect on retirement in steady state if we consider the change in accidental bequests. Therefore, it is ambiguous whether the retirement age will fall in the future.

The effects of an increase in life expectancy on the investment and saving rates depend on the retirement decision of the elderly because domestic investment increases in order to install capital to hold an increase in the labor force. The national saving rate depends on saving by prime-age adults, the dissaving of the elderly, and retirement decision. The effects of an increase in life expectancy on retirement, the national saving rate, the investment rate, and the current account balance are variable under different assumptions. Our simulation analysis shows that the investment rate is higher and the saving rate lower than in Chapter 2 if endogenous retirement is assumed, regardless of the availability of annuity and the existence of a pay-as-you-go social security tax.

We would need a more careful discussion about social security and labor force participation decision. One drawback of our study is the assumption that social security benefits are received immediately after retirement. In many cases, the social security system requires a minimum age to receive benefits. This age would affect the saving and labor force participation decisions. More detailed discussion of the dynamic effect of an increase in the social security tax would be useful for further understanding of this issue.

Another drawback comes from the assumption of log utility. Because log utility is assumed, the substitution and wealth effects of an increase in wage are offset in steady state. Therefore, the wage level does not influence steady state labor force participation of the elderly. However, it is possible that the wealth effect of an increase in wage on labor force participation is substantial. In general, high income regions tend to have a lower labor participation rate. [27]

Furthermore, while social security may be endogenous, we assume it to be exogenous. Social security could be determined politically and might be influenced by age structure. Longevity

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[27] For example, see Clark and Anker (1990).

increases the number of the elderly, and higher social security tax would become politically supported.[28]

[28] In a democratic society, social security is determined through a majority voting process. For details, see Browning (1975), Hu (1979, 1982).

Table 3.2 Summary of the Effects of an Increase in Life Expectancy and the Pay-as-you-go Social Security Tax in a Small Open Economy

	The Effect of an Increase in q_t												The Effect of an Increase in q^*			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
	$s_{1,t-1}$	z_t	$s_{1,t}$	z_{t+1}	$\frac{I_{t-1}}{Y_{t-1}}$	$\frac{I_t}{Y_t}$	$\frac{I_{t+1}}{Y_{t+1}}$	$\frac{I_{t+2}}{Y_{t+2}}$	$\frac{S_{t-1}}{Y_{t-1}}$	$\frac{S_t}{Y_t}$	$\frac{S_{t+1}}{Y_{t+1}}$	$\frac{S_{t+2}}{Y_{t+2}}$	s_1^*	z^*	$\left(\frac{S}{Y}\right)^*$	$\left(\frac{I}{Y}\right)$
(i)Chapter 2 (Exogenous retirement)	0	0	+	0	0	0	0	0	0	+	–	0	+	0	+ if $gn>1$ – if $gn\leq 1$	0
Chapter 3 (ii) Perfect Annuity	0	0	+	+	0	+	–	0	0	+	–	0	+	+	?	0
(iii) No Annuity	0	0	+	–	0	?	?	?	0	+	?	?	+	–	?	0
	The Effect of an Increase in τ_t												The Effect of an Increase in τ^*			
	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)
	$s_{1,t-1}$	z_t	$s_{1,t}$	z_{t+1}	$\frac{I_{t-1}}{Y_{t-1}}$	$\frac{I_t}{Y_t}$	$\frac{I_{t+1}}{Y_{t+1}}$	$\frac{I_{t+2}}{Y_{t+2}}$	$\frac{S_{t-1}}{Y_{t-1}}$	$\frac{S_t}{Y_t}$	$\frac{S_{t+1}}{Y_{t+1}}$	$\frac{S_{t+2}}{Y_{t+2}}$	s_1^*	z^*	$\left(\frac{S}{Y}\right)^*$	$\left(\frac{I}{Y}\right)$
Chapter 3 (iv) Perfect Annuity	–	–	–	+	–	+	0	0	–	?	?	0	–	+ if $1+r_w>gn$, – if $1+r_w\leq gn$	– if $1+r_w>gn>1$, + if $gn<1$, ? if $1+r_w\leq gn$	0

Note: q: survival rate, s_1: saving of prime-age adults, z: labor force participation of the elderly, g: technological growth rate +1, n: population growth rate +1, S/Y: the national saving rate, I/Y: the national investment rate, τ: social security tax rate.

CHAPTER 4. EMPIRICAL ANALYSIS OF LONGEVITY AND SAVING

4.1 Introduction

This chapter presents empirical analyses of the effects of the level and the change in adult survival using world panel data and historical data of selected countries. Historically, life expectancy and adult survival rate were stagnant at low levels before demographic transition commenced. As a country developed, a demographic transition set in. During the demographic transition, which is normally characterized by declining infant mortality and fertility, life expectancy increased remarkably. After demographic transition was completed, infant mortality and fertility stayed at this low level, while adult survival increased moderately.

Life expectancy at birth is not an appropriate measure for adult survival because changes in infant mortality have a large effect on life expectancy at birth. Appropriate measures of adult survival are derived from the data of life expectancy at birth using the model life table of Coale and Demeny (1983).

The findings of the analysis with world panel data are summarized as follows. First, the coefficient of the interaction term of adult survival indices and GDP growth rate is positive and statistically significant in many cases. Therefore, an increase in the level of the adult survival increases the national saving rate if GDP is growing. Second, the analysis with whole world data does not show a significantly positive effect of the change in adult survival on the national saving rate. The results present the significantly positive effect of the change in the adult survival rate on the national saving rate in the regression using only the sample of advanced economies.

One problem of panel analysis is that the time period is not long enough to capture the series of mortality transition. Mortality transition commenced late in the 19^{th} century in many developed

countries. Therefore, it is important to analyze the relationship between adult survival and the national saving rate based on a long-term perspective.

Analyses using the historical data of seven countries – Sweden, Italy, the United Kingdom, the United State, Japan, Taiwan, and India – lead the following conclusions. The historical data show that the mortality transitions of adult survival in the West and Asia are quite distinctive. In the West, there are pre-transition and transition periods. In the pre-transition period, adult survival was low and stagnant. After the transition period began, adult survival increased at a relatively constant rate. In Asia, there are a pre-transition period and a transition period, with an intervening period of rapid increase. In relation to mortality transition with the national saving rate, the following implication of the theoretical model is confirmed using the historical data: (1) saving rates tended to be very low in countries before the onset of the mortality transition; (2) saving rates rose as the level of adult survival increased; (3) a rapid transition in adult mortality brought higher saving rates.

This chapter is organized as follows. In section 4.2 we review previous studies. In section 4.3, new measures of adult survival are established. Section 4.4 describes the empirical specification and data in the empirical analysis with world panel data. In section 4.5, we discuss the estimated results. In section 4.6, the historical trends of adult survival are reviewed and the effects of the level and the change in life expectancy on the national saving rate are analyzed using the historical data. Section 4.7 summarizes our contribution and concludes.

4.2 Literature Reviews

Here, previous studies regarding empirical analysis of demographic effects on saving are introduced. Most research does not investigate the effects of life expectancy and only analyzes the effects of age structure. Age structure could include the effects of life expectancy. It is important to seek an appropriate empirical specification of the effects of age structure. Considering the

specifications of age structure and saving, we would like to set up an empirical specification that can analyze the effect of life expectancy. In this section, empirical specifications of previous research are discussed.

First, Leff (1969) addresses the effects of youth dependency and old dependency on the share of saving in GDP. The author suggests the following empirical specification.

$$\ln(S/Y) = f(\ln(Y/N), \ln Y_{gr}, \ln D1, \ln D2), \tag{4.1}$$

where *S/Y* is the saving rate, *Y/N* is GDP per capita, Y_{gr} is GDP growth, $D1$ is young dependency, and $D2$ is old dependency. Empirical analysis with cross-country data from 74 countries implies that both young and old dependency ratios have significant negative effects on the saving rate. However, the empirical specification is not derived theoretically. That theoretical weakness has been criticized.[29] Here, we should note that the old age dependency ratio captures the effects on age composition of changes in fertility, mortality or life expectancy, and migration. The degree of the effect of each component cannot be calculated from this specification of the model.

Second, Fair and Dominguez (1991) employ an empirical specification with polynomial age effects. The authors establish a method to investigate how much population of each age influences consumption. Their hypothesis is that prime-age adults consume less relative to their income than children or the elderly. Fair and Dominguez assume the following polynomial constraint facing each generation.

$$\alpha_j = \gamma_0 + \gamma_1 j + \gamma_2 j^2, \tag{4.2}$$

where α_j is the coefficient of population of age j. Higgins (1994,1998), Higgins and Williamson (1997), and Williamson and Higgins (2001) also assume a polynomial age effect. The authors

[29] For example, see Mason (1988).

estimate saving and investment equations and find significant results. The authors estimate the demographic influnences on the saving and the investment rates with world panel data considering the effects of shares of population in each age. The analysis with polynomial age effects enables us to estimate how a change in the population age distribution given age-profiles of saving, influences the saving share of GDP. The authors conclude substantial effects of demographic transition.

Analysis with detailed age effects has an advantage in that it is flexible about the way that age distribution is changing. However, the disadvantage is that the authors assume that the age-profile is fixed. Changes in age earning or consumption patterns induced by demographic change are not analyzed.

Third, Mason (1981, 1987) and Fry and Mason (1982) develop a "variable rate of growth (VRG) effect" model. In the life-cycle saving model, the rate of growth in real GDP affects the aggregate saving rate positively. The authors show that the rate of growth effect is not constant and the dependency ratio changes the timing of saving over the life cycle and alters the rate of growth effect. In the estimate of VRG model, Mason (1981, 1987) and Fry and Mason (1982) employ the following equation described in Chapter 1.

$$S/Y \approx \beta_0 + (A_C - A_Y)Y_{gr}, \tag{4.3}$$

where A_C is the mean age of consumption and A_Y is the mean age of earning, and Y_{gr} is the rate of growth in real GDP. From equation (4.3), as an analytically convenient function, the following function is obtained.[30]

$$S/Y \approx f(Y_{gr}, Y_{gr} \cdot D1, D1), \tag{4.4}$$

[30] The derivation of the equation is described in Mason (1981).

where S/Y is the aggregate rate of saving, and, $D1$ is the young dependency ratio. In the empirical analysis, the authors find that the VRG model is plausible. Kelley and Schmidt (1996) also support the VRG model based on their detailed empirical analysis. This specification captures the implication of the theory well because this approach has an advantage in that it allows for changing earning and consumption profiles. The disadvantage of the model is that it relies on steady state assumptions. The model captures the rate of growth effect and the dependency effect caused by an increase in population growth. The authors hypothesize that growth of GDP has a positive effect on the saving rate because it reflects the rate of growth effect suggested by Modigliani and Brumberg (1954) and Modigliani and Ando (1957). $Y_{gr} \cdot D1$ is expected to have a negative effect because children can change the timing of saving toward the early stage of the life. The effect of $D1$ itself is called the level effect. The share of children can change the level of saving, but the direction of the effect is ambiguous. The authors find significant effects of the rate of growth and the timing of an increase in child dependency.

Finally, some studies examine the determinants of the saving rate by including life expectancy at birth. Doshi (1994) includes life expectancy at birth in the Leff model and finds a significant positive effect of life expectancy at birth on the saving rate. Bloom, Canning and Graham (2003) include life expectancy at birth in the models of Higgins (1998) and Mason (1981, 1987, 1988) and find a positive effect of life expectancy on the saving rate. Kageyama (2003) estimates the effects of the level and the growth rate of life expectancy at birth. The author finds a significant positive effect of an increase in the growth of life expectancy on the saving rate. Regarding the empirical model with life expectancy, more careful discussion about empirical specification is needed in order to be consistent with the implication of the theory. Also, the data of life expectancy at birth is not a proper variable because it is a composite measure of both child mortality and adult mortality.

Throughout the empirical analysis regarding demographic effects on saving, it is controversial what kind of empirical specification captures the implication of the theory. Moreover, empirical analysis with macro data cannot effectively estimate cohort effects. For further understanding of saving behavior, we would need an analysis using both household data and macro data.[31]

4.3 Definition of the Adult Survival Indices

Several studies analyze the determinants of the national saving rate using data on life expectancy at birth. However, life expectancy at birth captures mortality at every age. Lower life expectancy in developing countries results from, in part, high mortality of children, which influences saving differently from the mortality of adults. Life expectancy at birth is not an appropriate measure of the survival of adults. Here, a better measure of adult survival is constructed and discussed.

Adult survival rate is applicable for a measure of the survival rate that is used in our theory in Chapters 2 and 3. The World bank reports the mortality rate of adults. The definition of the adult mortality rate is the probability of dying between the ages of 15 and 60, that is, the probability of a 15 year old dying before reaching age 60. The adult survival rate is calculated by subtracting the adult mortality from 1. From the life table, adult survival rate $q_{15\text{-}60}$ is obtained by l_{60}/l_{15}, where l_x is number of survivors at exact age x.

Coale and Demeny (1983) creates model life tables based on life tables of several countries. Using model life table of Coale and Demeny, two measures of adult survival are calculated using age-specific survival rates. One measure is the ratio of years lived over age 60 to years lived from 30 to 59:

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[31] Deaton and Paxson (2000) use household data in Taiwan and find a remarkable cohort effect. However, this kind of analysis is beyond the scope of this dissertation.

$$q_{UN} = \frac{T_{60}}{T_{30} - T_{60}}, \tag{4.5}$$

where T_x is total number of years lived after age x and can be obtained from the life table. This index does not account for discounting, therefore, we will call q_{UN} the "undiscounted adult survival index" hereafter.

In another measure discounting for future survival is included. The probability of surviving until a higher age is discounted by the interest rate. The present value of person years lived at ages 30 to 59 years old to discounted years lived after 60 years old, q_{PV}, is defined by:

$$q_{PV} = \sum_{x=60}^{M} e^{-rx} L_x \Big/ \sum_{x=30}^{59} e^{-rx} L_x , \tag{4.6}$$

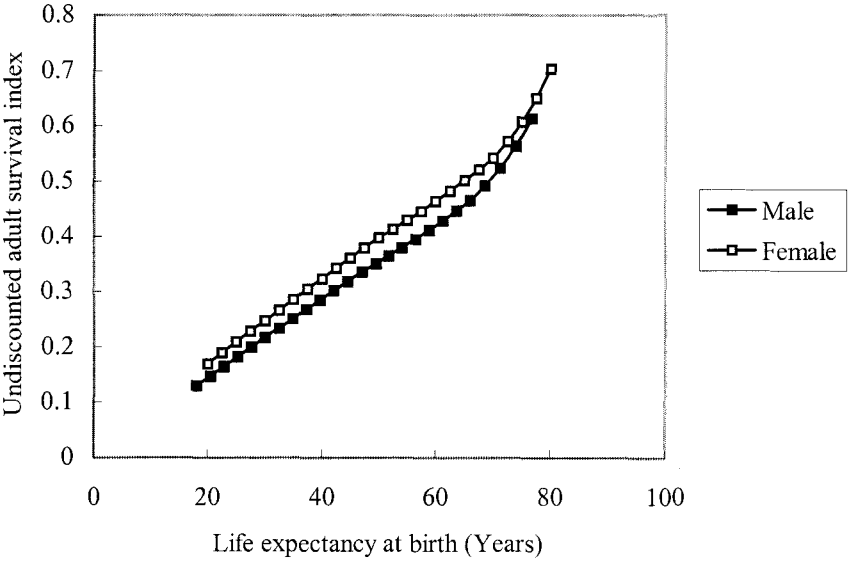
where r is the interest rate, and M is the maximum age. L_x is number of years lived between exact age x and exact age x+1, also obtained from the life table. q_{PV} is called "discounted adult survival index" hereafter.

It is desirable to obtain a life table each year for every country. However, a life table of each year is not available for many countries. Because life expectancy at birth is readily available, it is convenient to formulate the relationship life expectancy and survival rate. Based on West model of Coale and Demeny, the relationships between life expectancy at birth (e_0) and q_{UN} and between e_0 and q_{PV} are estimated. Using the relationship, q_{UN} and q_{PV} are calculated if the data of life expectancy at birth is available. It would also be important to consider the change of the interest rate over time. Moreover, the type of data used for the interest rate would be controversial. However, for simplicity, the interest rate is assumed to be 5% annually.

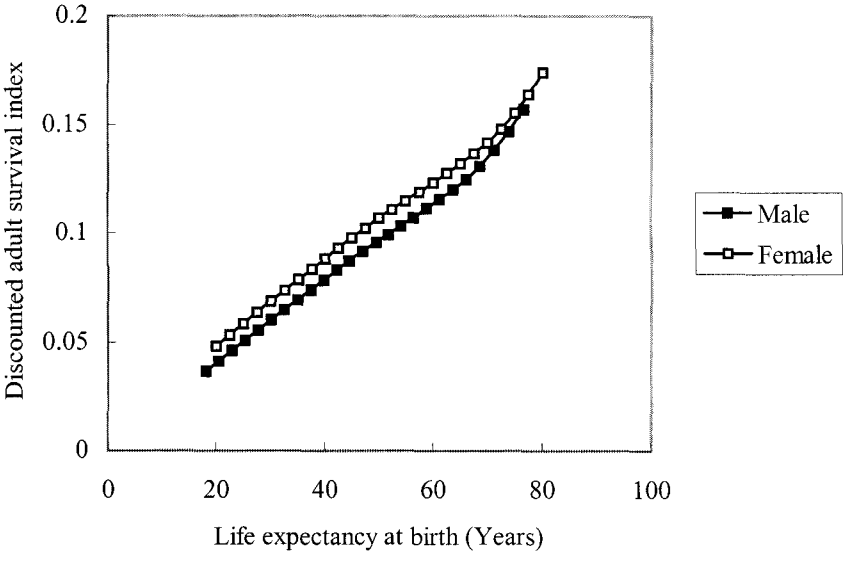
Figure 4.1 (a) presents the relationship between life expectancy at birth and the undiscounted adult survival index, q_{UN}, for females and males. As life expectancy at birth increases, the survival rates of males and females increase. When life expectancy at birth is low, an increase in e_0 raises

Figure 4.1 Relationships Between Life Expectancy at Birth and Adult Survival Indices

(a) Undiscounted Adult Survival Index



(b) Discounted Adult Survival Index



adult survival q_{UN} moderately. If life expectancy is higher than 70, an increase in life expectancy at birth is accompanied with a great increase in q_{UN}. Figure 4.1 (b) presents the relationship between life expectancy at birth and the discounted adult survival index, q_{PV} for females and males, and shows that the discounted adult survival index also increases more rapidly at a higher level of life expectancy. The relationships between e_0 and q_{UN} and e_0 and q_{PV} are estimated using a cubic function, as discussed in Appendix I.

The relationship of the change in life expectancy and the difference between the mean age consumption (A_C) and the mean age of earning (A_Y) based on an age earning and consumption profile is calculated. The implication of our model in Chapter 2 is that the change in the difference A_C and A_Y affects the national saving rate. If a measure of adult survival affects the difference of mean age of consumption and earning proportionately, it is a preferable measure of adult survival. Appendix I describes detailed method. Appendix K shows that that an increase in life expectancy at birth or adult survival rate $q_{15\text{-}60}$ does not increase $A_C - A_Y$ proportionately. An increase in undiscounted or discounted adult survival index increases $A_C - A_Y$ almost proportionately. Therefore, undiscounted adult survival index q_{UN} or q_{PV}, would be an appropriate measure of adult survival.

Undiscounted and discounted adult survival indices of male and female are calculated based on the data of life expectancy at birth male and female in empirical analyses in the following sections. The mean of females and males is used.

4.4 Empirical Specification and Data

Theories in Chapters 2 and 3 imply that the steady state effect of an increase in life expectancy on the national saving rate interacts with GDP growth. If the retirement age is fixed and GDP is

growing, an increase in life expectancy increases the national saving rate. Also, as an out-of-steady-state effect, an increase in life expectancy has a positive transitory effect on the national saving rate that is independent of the growth in GDP.

Based on the discussion in Chapters 2 and 3 and other recent studies, the following empirical specification is used.

$$\frac{S}{Y} = \beta_0 + \beta_1 q \cdot Y_{gr} + \beta_2 \Delta q + \beta_3 Y_{gr} + \beta_4 D1 \cdot Y_{gr} + \beta_5 PRI + \varepsilon , \qquad (4.7)$$

where q is the level of adult survival, Y_{gr} is the growth rate of GDP, Δq is an increase in adult survival, and $D1$ is the youth dependency rate. The effect of adult survival, interacted with the rate of GDP growth ($q \cdot Y_{gr}$), has a positive effect if the retirement age is fixed or relatively unresponsive to changes in life expectancy.[32] The specification of the youth dependency effect follows the variable rate of growth (VRG) model.

$D1 \cdot Y_{gr}$ is expected to influence the national saving rate negatively if an increase in child dependency is associated with a declining child share in aggregate consumption. This would be the case, for example, if the relative cost of children and adults is unchanging. If, however, the number of children is declining due to increases in the price of children (relative to adults) and the price elasticity of demand is inelastic, the share of children in consumption will rise with a decline in child dependency. Under these circumstances, the child dependency ratio could have a positive effect on aggregate saving rates (Mason 1987). Youth dependency is defined as the share of population under 15 years old in the total population.

The estimated rate of growth effect varies with both adult survival and child dependency. The partial effect of an increase in GDP growth is $\beta_1 q + \beta_3 + \beta_4 D1$. The coefficient β_3 has no

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[32] As shown in Chapter 3 an increase in the survival rate can lead to lower saving rates if the age of retirement is sufficiently delayed in response to increased survival. We are aware of no empirical evidence that age of retirement increases with life expectancy.

economic interpretation in isolation and, hence, it may be positive or negative depending on the effects of q and $D1$. However, the steady state rate of growth effect is approximately equal to the difference between the mean age of consumption and the mean age of earning (Mason 1987). This may be either positive or negative.

Following Taylor (1995) and Higgins and Williamson (1996, 1997), the relative price of investment goods (*PRI*) is included in order to control for its possible effect on saving. The *PRI* should have similar effects to the interest rate. An increase in the interest rate may either increase or decrease saving of prime-age adults depending on the strength of the substitution effect and wealth effect. The effect of an increase in the interest rate is ambiguous and it is an empirical question.[33] Considering the transitory effect of longevity, the effect of an increase in adult survival, Δq, is included. It is expected that Δq has a positive effect on the national saving rate. Equation (4.7) is estimated adding dummy variables for years 1980, 1990, and 2000.

As discussed in more detail in section 4.3, the adult survival rate is measured in three ways: (1) the undiscounted ratio of the person-years lived after age 60 to the person-years lived between ages 15 and 60 (q_{UN}); (2) the ratio of the discounted person-years lived after age 60 to the person years lived between ages 15 and 60 (q_{PV}); and (3) the adult survival rate from the World Bank – the proportion surviving from age 15 to age 60 ($q_{15\text{-}60}$). The first and second measures, however, are preferred and they are emphasized in the discussion. The effects of an increase in these measures of the survival rate are estimated separately. For the increase in the survival rate, Δq, an increase in the survival rate in ten years and twenty years are used. Estimates using the first measure are reported below for the purpose of comparison.

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[33] If the economy is perfectly small open, the interest rate is the same for all the countries. The models in Chapter 2 and 3 imply that the effect of the interest rate is also interacted with GDP growth. However, including the interaction term of the interest rate and GDP growth causes multicollinearity, so that the interaction of the interest rate and GDP growth is not considered.

The panel data set is comprised of four periods (1970, 1980, 1990, and 2000) of 149 countries. Data are from international sources such as *Penn World Table* and *World Development Indicator*. A detailed description is found in Appendix L. The equation is estimated using ordinary least squares (OLS) for the entire sample. Also, the equation is estimated separately for the sub-samples of Asia, Africa, Latin America, and the advanced economies. Advanced economies include North America, Europe, Australia, and New Zealand. Countries in each group are listed in Appendix L.

Simultaneity between saving and GDP growth is an important econometric issue. The problem is minimized in the least squares estimates by using growth in the preceding saving equation. As an alternative, we employ two stage least squares (2SLS). We adopt instrumented variables used in Higgins and Williamson (1996, 1997), which are growth of labor force, the lagged values of national investment rate, labor growth, price of investment goods, the price index, real GDP per worker, real GDP per capita, and openness. The Hausman-Wu test is used to assess the endogeneity of GDP growth.

4.5 Econometric Estimates of the National Saving Rate

Appendix M lists the variables used in the analyses. Appendix N summarizes the variables for the full sample, for each period, and for each region. Figure 4.2 graphs the means of selected variables by decade. The national saving rate was high in 1970, decreased in 1980, probably reflecting global recession, and increased thereafter. The share of population under 15 years old has decreased moderately over time. The survival rate of the World Bank, $q_{15\text{-}60}$, increased rapidly in 1980 and 1990 but decreased slightly in 2000. In contrast, the discounted survival index, q_{PV}, increased through out the period, although somewhat more slowly between 1990 and 2000. It is striking that the average national saving rate was highest in 1970 when adult survival and child dependency were presumably least favorable for saving. The global trend for 1980 to 2000 is more

supportive of the thesis that increasing longevity and declining child dependency have favorable effects.

A broad cross-sectional perspective is provided in Figure 4.3, which shows means for major regions of the world. The national saving rates are highest in the advanced economies and Asia, while they are quite low in Africa. The advanced economies have demographics hypothesized to be favorable to high saving rates – a low child dependency ratio and high adult survival, although the rate of change in adult survival (measured over a ten-year period) is quite low for the advanced economies. Likewise, Africa's demographics are presumably quite unfavorable to a high saving rate. The youth share is the highest, adult survival the lowest, and the change in adult survival the lowest. In these respects, the regional cross-sectional saving patterns are consistent with the demographic patterns.

The picture is more mixed with respect to Asia, Latin America, and the Middle East. These three regions have similar child population shares and discounted adult survival rates. The rate of change in the discounted adult survival rate is somewhat faster in Asia and the Middle East than in Latin America. But at this very aggregated level, the effects of demographic variables on national saving rates are far from apparent.[34]

The simple comparisons across decades and regions abstracts from an important feature of the hypothesized relationship between adult survival and national saving rates – that the effect of adult survival should vary with the rate of economic growth. To explore this issue, the full sample is sub-divided into nine groups of equal size based on GDP growth and the level of adult survival.

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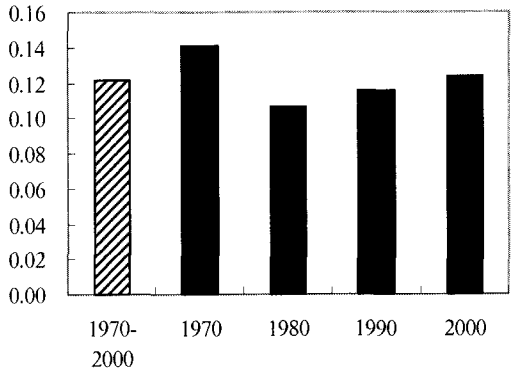
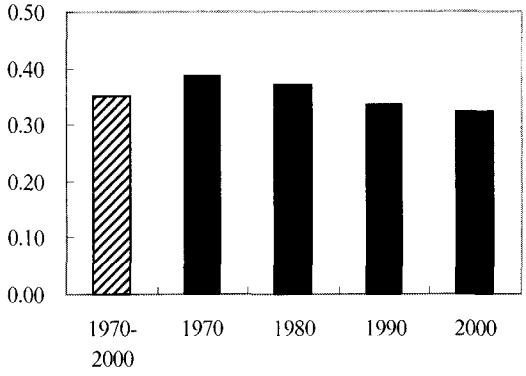
[34] The difference the pattern of increase in these measures of survival rate is probably because model life table is applied when q_{PV} is calculated. If we consider death at each age caused by special reason for each region (for example, disease prevailing in a certain area), the measure of q_{PV} or q_{UN} might be different.

Figure 4.2 Means of Selected Variables in Each Year (1970 – 2000)

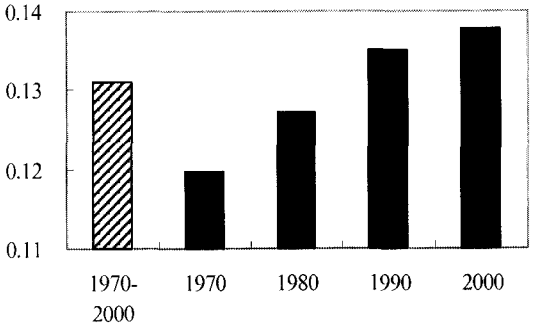
(a) The National Saving Rate



(b) Ratio of Population Aged 0-15



(c) Discounted Adult Survival Index (q_{PV})



(d) Adult Survival Rate ($q_{15\text{-}60}$)

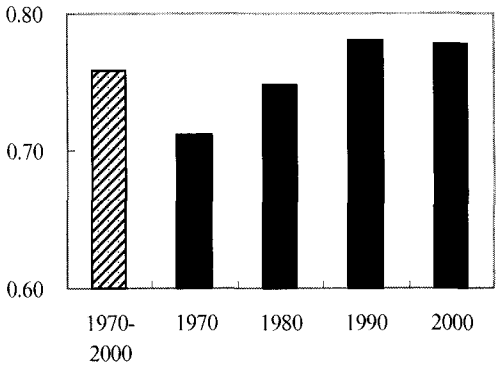
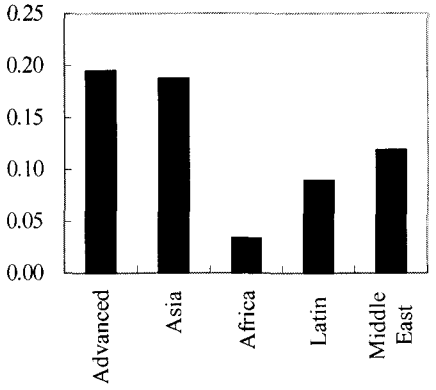
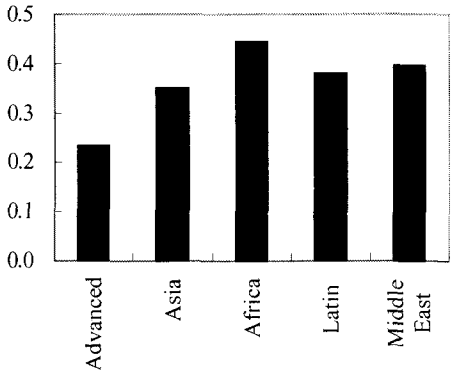


Figure 4.3 Mean of Selected Variables in Each Region (1970 – 2000)

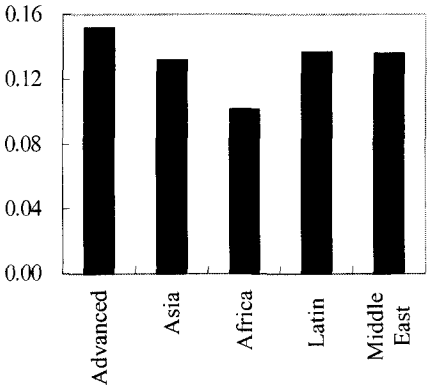
(a) National Saving Rate



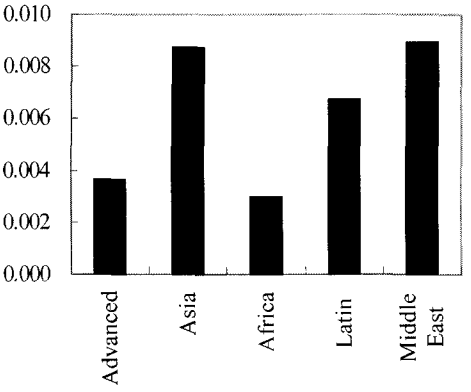
(b) Population Ages 0-15 (% of Total)



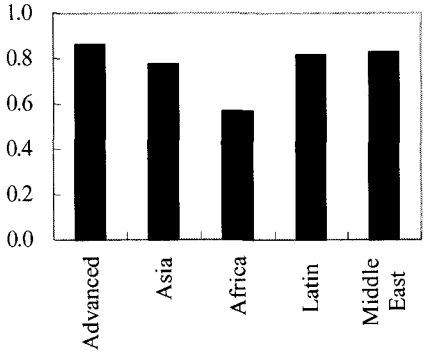
(c) Discounted Adult Survival Rate (q_{PV})



(d) Change in the Discounted Adult Survival Rate (q_{PV}) in 10 years



(e) Adult Survival Rate $q_{15\text{-}60}$



(f) Change in the Adult Survival Rate $q_{15\text{-}60}$ in 10 years

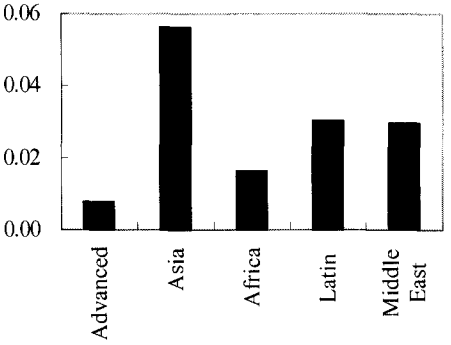


Table 4.1 The National Saving Rate in Each Level of Adult Survival and GDP growth

GDP growth (Y_{gr})	(a) Undiscounted Adult Survival Index (q_{UN})		
	low	mid	high
low	0.0129	0.1441	0.1854
mid	0.0214	0.0884	0.1852
high	0.0763	0.1477	0.2469

GDP growth (Y_{gr})	(b) Discounted Adult Survival Index (q_{PV})		
	low	mid	high
low	0.0129	0.1441	0.1854
mid	0.0214	0.0884	0.1852
high	0.0763	0.1477	0.2469

GDP growth (Y_{gr})	(c) Adult Survival Rate ($q_{15\text{-}60}$)		
	low	mid	high
low	0.0170	0.1341	0.2038
mid	0.0221	0.0910	0.1787
high	0.0704	0.1507	0.2487

(Note) The figures are the national saving rates.

The mean of the national saving rate for each group is reported in Table 4.1. Within the three income growth groups, an increase in adult survival is always associated with a higher saving rate, irrespective of the measure of adult survival employed. The highest saving rate is found in countries with the highest rate of economic growth and the highest rate of adult survival. The relationship between national saving rates and economic growth is less robust in these simple comparisons. For the most part, average saving rates increase with GDP growth given adult survival, but there are important exceptions apparent in Table 4.1.

We now turn from these simple, yet instructive, comparisons to our regression analysis. As will become apparent, the econometric results differ greatly across regions, calling into question the validity of analyzing the pooled sample. Nonetheless, we begin by presenting pooled estimates and turn to regional estimates below.

OLS estimates based on the full sample consistently support the hypothesis that the survival rate, interacted with the GDP growth rate, has a positive effect on the national saving rate. The estimated coefficient is statistically significant for any definition of adult survival (Table 4.2). The estimated effect of the youth share is negative but not statistically significant when the preferred measures of adult survival are employed. Y_{gr}, $q \cdot Y_{gr}$, and $D1 \cdot Y_{gr}$ are jointly significant for all cases, that is, the effect of GDP growth is significant. At the mean values of q and $D1$, the rate of growth effect varies from 0.47 to 0.75, depending on the specification. The price of investment goods is not statistically significant in any specification. Likewise, the effect of a change in the survival rate is not significant in any regression in Table 4.2.

Adding regional dummies (Table 4.3) reduces the estimated effect of the interacted survival rate for any measure of adult survival, however, the coefficients are statistically significant when a preferred measure of adult survival is employed (specifications 1-4). The youth share is no longer statistically significant in specifications 1-4. The change in the survival rate over the preceding decade has a statistically significant *negative* effect on saving contrary to the maintained hypothesis.

Table 4.4 presents the result of 2SLS estimates assuming the GDP growth rate to be endogenous. The Hausman-Wu test implies that GDP growth is endogenous for all equations. The effect of the interacted survival rate is statistically significant and positive for all cases. The youth share is statistically insignificant and *positive* for specifications 1-4. The change in adult survival is *negative* and statistically significant in all specifications. The price of investment goods does not have a significant effect on the national saving rate. The rate of growth effect, calculated at the

Table 4.2 OLS Saving Equation Estimates with Region, No Region Dummies

	(1) $q=q_{UN}$	(2) $q=q_{UN}$	(3) $q=q_{PV}$	(4) $q=q_{PV}$	(5) $q=q_{15\text{-}60}$	(6) $q=q_{15\text{-}60}$
$q \cdot Y_{gr}$	12.3970***	11.4425***	50.8474***	48.4669***	7.4301***	4.3639*
	(2.5196)	(3.3825)	(10.2213)	(13.6915)	(1.5475)	(2.3455)
Δq (10 years)	-0.0358		-0.9536		-0.0866	
	(0.3532)		(1.3742)		(0.0933)	
Δq (20 years)		0.0033		-0.7298		-0.1176
		(0.2570)		(1.0142)		(0.1092)
Y_{gr}	-4.3887**	-3.3941	-4.7556**	-3.9620	-3.0546	0.4417
	(2.1619)	(2.8765)	(2.2267)	(2.9602)	(1.9651)	(2.7973)
$D1 \cdot Y_{gr}$	-3.2404	-5.1706	-3.5135	-5.2534	-6.0312***	-9.8573***
	(2.5872)	(3.4941)	(2.5402)	(3.4326)	(2.3005)	(3.0471)
PRI	-0.0052	-0.0043	-0.0053	-0.0043	-0.0028	-0.0047
	(0.0053)	(0.0066)	(0.0052)	(0.0066)	(0.0051)	(0.0068)
*yr*80	-0.0411**		-0.0391**		-0.0391**	
	(0.0171)		(0.0169)		(0.0168)	
*yr*90	-0.0379**	-0.0025	-0.0359**	-0.0013	-0.0391**	-0.0064
	(0.0172)	(0.0147)	(0.0170)	(0.0147)	(0.0166)	(0.0152)
*yr*100	-0.0555***	-0.0141	-0.0568***	-0.0165	-0.0550***	-0.0219
	(0.0154)	(0.0152)	(0.0155)	(0.0152)	(0.0159)	(0.0161)
Constant	0.1445***	0.1079***	0.1473***	0.1151***	0.1487***	0.1282***
	(0.0183)	(0.0194)	(0.0185)	(0.0193)	(0.0190)	(0.0180)
Adjusted R^2	0.2660	0.2820	0.2640	0.2795	0.2772	0.2578
N	440	340	440	340	413	319
P-values Ygr	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Pvalues, year dummies	0.0044	0.6174	0.0036	0.4934	0.0074	0.3673
Y_{gr} Effect	0.6595	0.6962	0.6815	0.7504	0.4715	0.4593

Note: The dependent variable is the national saving rate.

q: Adult survival rate (Different measures, $q_{15\text{-}60}$, q_{UN}, q_{PV}, are used. For the definition see the text.), Y_{gr}: GDP growth rate, $D1$: young dependency rate, Δq: change in the adult survival rate, *PRI*: price of investment goods, *yr*1980, *yr*1990, and yr2000:dummy variables of year 1980, 1990, 2000 respectively, N: number of observation. "P-value, *Ygr* " is the p-value of F-test of the null hypothesis that the both coefficients of $q \cdot Y_{gr}$ and $D1 \cdot Y_{gr}$ are zero. "Pvalue, year dummies" is the p-value of F-test of the null hypothesis that all the year dummies are zero. " Y_{gr} Effect" is the partial effect of an increase in GDP growth.

*** denotes significant at 1 % level, **denotes significant at 5 % level, and * denotes significant at 10 % level. Figures in parentheses are standard errors.

Table 4.3 OLS Saving Equation Estimates with Region Dummies

	(1) $q=q_{UN}$	(2) $q=q_{UN}$	(3) $q=q_{PV}$	(4) $q=q_{PV}$	(5) $q=q_{15\text{-}60}$	(6) $q=q_{15\text{-}60}$
$q \cdot Y_{gr}$	62.2183**	69.6891**	246.4842**	283.5246**	0.0158	0.0041
	(29.0879)	(31.9225)	(120.4250)	(133.2866)	(0.0168)	(0.0227)
Δq(10 years)	-51.7519*		-244.3219**		-0.0106	
	(26.9740)		(105.5942)		(0.0118)	
Δq(20 years)		-15.6637		-105.9229		0.0016
		(24.4252)		(94.8462)		(0.0116)
Y_{gr}	-17.8350	-23.6013	-18.1918	-25.0323	14.8451	32.9699
	(22.6577)	(25.1542)	(23.5549)	(26.3803)		(25.2437)
$D1 \cdot Y_{gr}$	-0.0875	-0.0632	-0.1125	-0.0864	-0.3713*	-0.6526***
	(0.2389)	(0.2726)	(0.2322)	(0.2672)	(0.1924)	(0.2428)
PRI	0.0190	0.0243	0.0196*	0.0248	0.0108	0.0257
	(0.0119)	(0.0164)	(0.0119)	(0.0163)	(0.0113)	(0.0197)
yr1980	-3.0802		-3.1153		-2.1440	
	(1.9752)		(1.9739)		(1.9134)	
yr1990	0.2883	3.4116*	0.2068	3.3762*	2.4377	4.6549***
	(1.9400)	(1.7953)	(1.9364)	(1.7904)	(1.9027)	(1.7631)
yr2000	0.1518	3.9747**	-0.0320	3.7646**	2.3086	5.7085***
	(1.9456)	(1.9077)	(1.9532)	(1.9075)	(1.9155)	(1.8972)
Africa	-20.3127***	-20.5231***	-20.3493***	-20.5823***	-21.1824***	-20.2015***
	(2.1123)	(2.2171)	(2.0995)	(2.2123)	(2.0629)	(2.3193)
Asia	-4.3311*	-2.8917	-4.2082*	-2.6250	-7.8745***	-6.6551**
	(2.3036)	(2.6537)	(2.3117)	(2.6732)	(2.2487)	(2.6810)
Latin	-10.2429***	-11.6920***	-10.1780***	-11.5021***	-10.0355***	-10.1338***
	(1.7872)	(1.9819)	(1.7881)	(1.9865)	(1.7437)	(1.9417)
Mideast	-18.7200***	-20.1910***	-18.5853***	-19.8415***	-19.4798***	-17.7937***
	(3.4107)	(3.1830)	(3.4121)	(3.1980)	(3.1255)	(2.8615)
Constant	15.9664***	12.5892***	16.1503***	13.0342***	14.1391***	10.3534***
	(2.2260)	(2.5634)	(2.2230)	(2.5265)	(2.1093)	(2.7836)
Adjusted R^2	0.4247	0.4670	0.4252	0.4672	0.4622	0.5141
N	424	324	424	324	382	272
P-value Ygr	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Pvalue, year dummies	0.2104	0.0826	0.2222	0.0976	0.0311	0.0067
Pvalue, region dummies	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Y_{gr} Effect	0.5251	0.4806	0.5313	0.5081	0.3918	0.2845

Note: The dependent variable is the national saving rate. Africa, Asia, Latin, Mideast are the dummy variables for Africa, Asia, Latin America, and Middle East. "P-value, region dummies" is the p-value of the F-test for the null hypothesis that all the region dummies are zero. See note to Table 4.2 for other issues.

Table 4.4 2SLS Saving Equation Estimates

	(1) $q=q_{UN}$	(2) $q=q_{UN}$	(3) $q=q_{PV}$	(4) $q=q_{PV}$	(5) $q=q_{15\text{-}60}$	(6) $q=q_{15\text{-}60}$
$q \cdot Y_{gr}$	39.3527***	45.0319***	156.1141***	178.9770***	18.3780***	24.5408***
	(11.5494)	(14.0011)	(46.5645)	(55.5507)	(6.8729)	(9.2275)
Δq(10 years)	-1.9373***		-7.2385***		-0.3844**	
	(0.5729)		(2.1914)		(0.1680)	
Δq(20 years)		-1.7410***		-6.8184***		-0.7897**
		(0.5212)		(1.9443)		(0.3590)
Y_{gr}	-18.1135**	-22.7948*	-18.3888*	-23.1374*	-6.5002	-13.9273
	(10.1743)	(11.9103)	(10.4114)	(12.0594)	(9.0042)	(11.3350)
$D1 \cdot Y_{gr}$	4.3998	8.9063	2.7277	7.3076	-10.2774	-2.0762
	(10.5151)	(12.5643)	(10.1243)	(11.9912)	(8.3901)	(11.4837)
PRI	0.0043	-0.0020	0.0057	-0.0005	0.0138	0.0122
	(0.0091)	(0.0080)	(0.0094)	(0.0083)	(0.0111)	(0.0124)
*yr*1980	-0.0124		-0.0147		-0.0094	
	(0.0317)		(0.0317)		(0.0312)	
*yr*1990	-0.0491	-0.0260	-0.0507	-0.0260	-0.0291	-0.0280
	(0.0379)	(0.0195)	(0.0378)	(0.0194)	(0.0353)	(0.0216)
*yr*2000	-0.0889***	-0.0601***	-0.0870***	-0.0596***	-0.0493**	-0.0469*
	(0.0290)	(0.0202)	(0.0285)	(0.0203)	(0.0247)	(0.0255)
Constant	0.0965	0.1304***	0.0914	0.1202***	0.0039	0.0339
	(0.0650)	(0.0464)	(0.0649)	(0.0458)	(0.0666)	(0.0465)
N	384	294	384	294	363	277
P-value, Ygr	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
P-value, year dummies	0.0009	0.0113	0.0013	0.0122	0.1307	0.1443
P-value, Hausman	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Y_{gr} Effect	3.1099	3.4468	3.0928	3.5327	3.9147	4.4636

Note: The dependent variable is the national saving rate. "P-value, Hausman" is the p-value of Hausman test for the null hypothesis that Y_{gr} is exogenous. Other notations are the same as Table 4.2.

mean q and $D1$, ranges from 3.09 to 4.46 depending on the specification. Thus, controlling for endogeneity leads to a higher rate of growth effect.[35]

The pooled results presented in Tables 4.2 – 4.4 provide no support for a key implication of the theoretical model presented in Chapters 2 and 3: that in transition, the speed with which adult survival is increasing will influence the aggregate saving rate. Yet the positive effect of adult survival on national saving rates is quite robust. These issues are discussed in more detail below, but a serious problem with the pooled results is that the countries in different stages of the mortality transition are heterogeneous. It might be important to estimate the equation within a region that is composed of similar characteristics. The Chow test implies that there are structural changes in the coefficients of q in different regions.[36] The importance, then, of estimating the results for each region separately is undeniable.

Table 4.5 presents OLS and 2SLS estimates using the sub-sample of advanced economies. In all respects, the results confirm the theoretical model presented in Chapters 2 and 3. The estimated effects of interacted adult survival ($q \cdot Y_{gr}$) are significant and positive in all empirical specifications for OLS estimates and in specifications 1 and 3 (the preferred specifications using ten-year changes in survival) for 2SLS estimates. The effect of an increase in the level of the survival rate on the national saving rate is positive if GDP is growing.

In the advanced economies, the change of the survival rate has a positive and statistically significant effect for both the 10-year and 20-year changes in the preferred OLS specifications.

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[35] Number of observation is different in Table 4.2 and Table 4.4 because there are missing observations in instrument variables. We compare the results of OLS and 2SLS within the same observations. The results show that the estimates of 2SLS lead to a higher rate of growth effect. The difference between the coefficients estimated by OLS and 2SLS are not because of the difference of observations. This holds true in the estimation of OLS and 2SLS in sub-samples of each region.

[36] The F statistic tests the null hypothesis that the coefficients of the change in adult survival are the same in all regions.

For the preferred specifications, the 2SLS estimated effects for the 10-year changes are statistically significant while the 20-year changes are not.

The estimated effects of interacted youth dependency are not significant in any of the preferred OLS specifications (1-4), but they are statistically significant in all 2SLS estimates. The coefficients of price of investment goods (*PRI*) are positive and significant in all OLS estimates and one 2SLS estimate. This implies that the substitution effect of the interest rate might be greater than the wealth effect. That is, if the interest rate goes up, future consumption is less discounted Individuals tend to save more expecting high return in the future.

Results for Asia (Table 4.6), Africa (Appendix O) and Latin America (Appendix P) are different than those for the advanced economies and only partially supportive of our theoretical model. The OLS estimates of the effects of the interacted level of adult survival are positive in all preferred specifications in all regions. The effects are not statistically significant in specifications 2 and 4 for Asia. For Africa and Latin America, the effects are statistically significant for almost all preferred specifications. The 2SLS estimates are statistically insignificant for Asia but they are positive and significant in Africa and Latin America.

Estimates from the developing regions provide no support for the transition effects of adult survival. The estimated effects of the change in the survival rate are statistically insignificant in Asia and Latin America. In Africa, the estimated effects are statistically significant but *negative.*

The interaction of GDP growth and young dependency has a positive effect on the national saving rate in 4 out of 6 OLS estimates for Asia, however the coefficients are not significant for 2SLS estimates and almost all estimates for Africa and Asia. The price of investment goods has a significantly positive effect on the national saving rate in OLS estimates but is not significant for

Table 4.5 OLS and 2SLS Saving Equation Estimates of Advanced Economies

	OLS Estimates					
	(1) $q=q_{UN}$	(2) $q=q_{UN}$	(3) $q=q_{PV}$	(4) $q=q_{PV}$	(5) $q=q_{15\text{-}60}$	(6) $q=q_{15\text{-}60}$
$q \cdot Y_{gr}$	17.7077***	14.3661*	80.9625***	67.3042*	13.8141**	11.0557
	(5.8798)	(7.7501)	(25.9426)	(34.6302)	(6.2344)	(7.0676)
Δq(10 years)	1.7138***		7.4885***		0.5518	
	(0.4932)		(2.1753)		(0.4206)	
Δq(20 years)		0.8819***		3.7680***		0.3110
		(0.3264)		(1.4191)		(0.3959)
Y_{gr}	-8.5485**	-6.8726	-10.4470**	-8.6544	-8.7644*	-7.1850
	(3.9568)	(4.9748)	(4.4583)	(5.6995)	(5.2041)	(5.8533)
$D1 \cdot Y_{gr}$	-6.4038	-6.0668	-6.4246	-6.0588	-11.2567**	-9.0379
	(4.3051)	(4.8323)	(4.3349)	(4.8851)	(5.2437)	(5.5198)
PRI	0.0289***	0.0846***	0.0291***	0.0855***	0.0331***	0.0955***
	(0.0082)	(0.0283)	(0.0083)	(0.0285)	(0.0112)	(0.0316)
Constant	0.2040***	0.0970***	0.2038***	0.0970***	0.1984***	0.1095***
	(0.0192)	(0.0274)	(0.0193)	(0.0275)	(0.0224)	(0.0292)
Adjusted R^2	0.4208	0.4369	0.4186	0.4353	0.3429	0.3660
N	133	103	133	103	129	102
P-value Ygr	0.0000	0.0166	0.0000	0.0137	0.0313	0.1745
Pvalue, year dummies	0.0000	0.1112	0.0000	0.1119	0.0047	0.1139
Y_{gr} Effect	0.4357	0.1788	0.3431	0.2848	0.5222	0.3282
	2SLS Estimates					
	(7) $q=q_{UN}$	(8) $q=q_{UN}$	(9) $q=q_{PV}$	(10) $q=q_{PV}$	(11) $q=q_{15\text{-}60}$	(12) $q=q_{15\text{-}60}$
$q \cdot Y_{gr}$	32.8815**	23.9807	143.9634**	107.1307	42.2633	1.1562
	(13.9941)	(22.3290)	(60.8321)	(95.0677)	(26.1279)	(24.9887)
Δq(10 years)	2.0957**		10.1521**		0.2264	
	(0.8554)		(3.9668)		(0.6440)	
Δq(20 years)		0.4582		2.3584		0.3675
		(0.4599)		(2.0920)		(0.5153)
Y_{gr}	-15.4276*	-9.2932	-17.7874*	-11.3875	-27.3296	9.0284
	(9.2970)	(15.2363)	(10.3099)	(16.5100)	(23.8397)	(22.6748)
$D1 \cdot Y_{gr}$	-17.3763**	-20.8307*	-18.2741**	-21.3044*	-42.8604***	-44.1736***
	(8.0573)	(12.4302)	(7.8844)	(12.0936)	(10.0795)	(13.1478)
PRI	-0.0204	0.0342	-0.0173	0.0343	0.0048	0.0927*
	(0.0518)	(0.0612)	(0.0507)	(0.0585)	(0.0472)	(0.0514)
Constant	0.3084***	0.1760***	0.3030***	0.1730***	0.3558***	0.1362**
	(0.0551)	(0.0529)	(0.0549)	(0.0500)	(0.0734)	(0.0599)
N	107	82	107	82	103	79
P-value Ygr	0.0003	0.0070	0.0002	0.0048	0.0000	0.0024
P-value, year dummies	0.0000	0.1284	0.0000	0.1052	0.0002	0.3043
P-value, Hausman	0.0000	0.0252	0.0000	0.0218	0.0010	0.1316
Y_{gr} Effect	-0.0222	0.0717	-0.1965	0.2207	-0.8792	0.0989

Note: Dependent variable is the national saving rate. Year dummies are included in the estimation. (Each coefficient of year dummy is not reported.) See the notes of Tables 4.2 and 4.4.

Table 4.6 OLS and 2SLS Saving Equation Estimates of Asia

	OLS Estimates					
	(1) $q=q_{UN}$	(2) $q=q_{UN}$	(3) $q=q_{PV}$	(4) $q=q_{PV}$	(5) $q=q_{15\text{-}60}$	(6) $q=q_{15\text{-}60}$
$q \cdot Y_{gr}$	11.8507*	5.6163	46.8221*	27.7756	2.8082	-0.3653
	(6.2188)	(9.0747)	(25.7227)	(36.3195)	(4.6573)	(6.2467)
Δq(10 years)	-0.6851		-3.3887		-0.1580	
	(0.9187)		(3.4110)		(0.2261)	
Δq(20 years)		0.3220		0.3426		-0.0283
		(0.9006)		(3.4017)		(0.2065)
Y_{gr}	-1.7673	3.4191	-1.8348	2.3375	4.8210	8.8855
	(5.2323)	(7.3562)	(5.4298)	(7.5078)	(5.3727)	(7.1618)
$D1 \cdot Y_{gr}$	-7.7243	-15.5205*	-8.1028	-14.8000*	-15.8191***	-22.0027***
	(6.5734)	(8.9364)	(6.4107)	(8.6823)	(5.7337)	(7.5848)
PRI	0.1649**	0.1958***	0.1708**	0.2020***	0.1431***	0.1320***
	(0.0738)	(0.0637)	(0.0733)	(0.0665)	(0.0465)	(0.0480)
Constant	0.0654	0.0080	0.0645	0.0209	0.0928	0.0785
	(0.0485)	(0.0918)	(0.0486)	(0.0914)	(0.0592)	(0.0629)
Adjusted R^2	0.4848	0.5564	0.4862	0.5555	0.4869	0.5597
N	61	48	61	48	58	45
P-values Ygr	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Pvalues, year dummies	0.8109	0.9548	0.8102	0.9487	0.2050	0.5978
Y_{gr} Effect	1.4516	1.0516	1.4915	1.0782	1.4360	1.1430
	2SLS Estimates					
	(7) $q=q_{UN}$	(8) $q=q_{UN}$	(9) $q=q_{PV}$	(10) $q=q_{PV}$	(11) $q=q_{15\text{-}60}$	(12) $q=q_{15\text{-}60}$
$q \cdot Y_{gr}$	-16.3988	12.4694	-70.9825	58.1484	-21.6796	10.0351
	(31.5871)	(19.8133)	(115.7769)	(77.8465)	(19.5923)	(14.5647)
Δq(10 years)	-2.0303		-7.4211		-0.0549	
	(1.6767)		(6.2666)		(0.2705)	
Δq(20 years)		0.2003		1.3852		0.0561
		(1.0417)		(4.4716)		(0.3990)
Y_{gr}	25.6551	1.4654	26.6749	-0.1313	36.3666	1.1904
	(27.1931)	(16.9308)	(25.4422)	(16.9155)	(24.3136)	(16.7453)
$D1 \cdot Y_{gr}$	-28.2159	-17.2023	-27.8701	-16.7370	-32.5140	-22.1807
	(26.0223)	(19.6414)	(23.2620)	(18.6384)	(20.1962)	(15.2844)
PRI	0.2570	0.0521	0.2479	0.0502	0.2549	0.0367
	(0.1856)	(0.1311)	(0.1765)	(0.1331)	(0.1722)	(0.1466)
Constant	-0.2991	0.0524	-0.3012	0.0437	-0.4114	0.1058
	(0.2608)	(0.1694)	(0.2530)	(0.1703)	(0.3153)	(0.1992)
N	55	42	55	42	54	41
P-value Ygr	0.0008	0.0013	0.7906	0.0008	0.0098	0.0002
P-value, year dummies	0.8100	0.6901	0.0011	0.6808	0.5938	0.4478
P-value, Hausman	0.0001	0.0137	0.0001	0.0121	0.0000	0.0077
Y_{gr} Effect	7.5204	2.0536	7.5126	2.0579	8.1226	1.6512

Note: Dependent variable is the national saving rate. See the notes of Tables 4.2, 4.4, and 4.6.

2SLS estimates for Asia. In Africa, the coefficient of price of investment goods is not statistically significant in any specification but is significantly negative in most specifications in Latin America.

In summary, empirical analysis with the pooled world data finds that the adult survival indices, interacted with the GDP growth rate, have a positive effect on the national saving rate in many empirical specifications. Youth dependency has a negative effect on the national saving rate if GDP is growing. However, the transitory effect of an increase in adult survival is not found. Estimating the saving equation with the sub-sample of advanced economies, we find that the theory is supported in all respects. We especially find positive effect of the change in adult survival on the national saving rate. It is possible that mortality transition is completed in most countries in advanced economies. Therefore, the change in adult survival is steady. Within such countries, a higher change adult survival is significantly correlated with the national saving rate.

It should be noted that there are several problems in that time periods are not long enough to see the time variation. Also, we do not find strong evidence about the effect of the change in adult survival except in the sub-sample of advanced economies. Because mortality transition happens in different periods in different areas and takes a long time, it is important to discuss the historical relationship between life expectancy and saving. Moreover, because there are three terms that are related to GDP growth in one equation, there may be multicollinearity problems.

4.6 Historical Perspectives on Life Expectancy and the National Saving Rate

In this section, we part company with what has become the standard approach to analyzing the effect of demographic factors on national saving rates by examining the long-term trends in mortality and saving in a limited number of countries for which relatively long-term data are available. This is important because of some of the problems that arise in relying on the international panel data. First, for any country in the sample, we observe only a small portion of

the mortality transition. This is unfortunate, given that the theoretical model addresses the long-term trend in saving that occurs as countries experience a relatively gradual and prolonged transition to modern mortality conditions. Second, estimating the model using time series data for relatively short periods is difficult because of the high degree of multicollinearity and the serial correlation in mortality rates. Countries tend to experience steady increases in life expectancy with little fluctuation from decade to decade. Even when there are fluctuations, these can typically be attributed to short-term effects of war or famine that do not play a role in the theoretical model developed here precisely because they do not have long-lasting effects that influence expectations.

In order to explore these issues, historical data for seven economies – Sweden, Italy, the United Kingdom, the United States, Japan, Taiwan, and India – are presented and analyzed using simple methods. For each of these economies, changes over the entire 20th century are known and in some instances, trends in saving and life expectancy can be extended even further back. More importantly these countries differ in the timing and the speed of their mortality transitions.

Any conclusions that can be reached based on this approach are necessarily tentative, but the historical evidence for these seven countries suggests the following conclusions. First, the transition of adult survival of Asia is distinctive as compared to the West. In the West, adult survival was low and stagnant in the pre-transition period. After the mortality transition began, adult survival has increased at a relatively constant rate. Asian countries experienced a later transition but one that was more rapid. Second, on the whole, the national saving rates tend to increase when adult survival increases. Hence, it appears the level of adult survival influences the national saving rate. Third, at a given level of adult survival, the national saving rate tends to be high if adult survival is increasing rapidly. Thus, long-term historical evidence appears to support the more recent experience of advanced economies with respect to the effect of mortality change.

In the next sub-section, historical trends in life expectancy, adult survival indices, and the national saving rate are reviewed. We also construct the historical survival rates of adults defined in the previous section. Furthermore, changes in the trends in mortality are statistically identified.

4.6.1 Descriptions of the Historical Trend

For most of human history, life expectancy was quite low and there was little sign of improvement. The transition to low mortality began at different times around the world. As will be shown below, life expectancy began to increase in some European countries during the early 19^{th} century. For other countries, mortality conditions began to improve during the early 20^{th} century, while in much of the developing world, rapid mortality decline occurred mostly in the second half of the 20^{th} century.

As countries reach high levels of life expectancy at birth, further gains come more slowly. Among the seven populations that we analyze, we see clear indications of a slowdown. Only India has not yet reached this phase of the mortality transition. As discussed above, however, life expectancy at birth is not the mortality variable that best captures the effect of increased longevity on saving. For the seven countries, the picture is mixed with respect to a slowdown in mortality improvement. Some advanced countries have experienced a slow down while others have not.

A demographic change in one country can be categorized into three stages. The first stage is high fertility and high mortality. High infant mortality is characteristic. If there is no improvement in nutrition or health condition, mortality stays high and life expectancy stays low.

The second stage is characterized by declining mortality and fertility. Infant mortality decreases and life expectancy at birth increases rapidly. Fertility decreases after a while. Also, adult mortality decreases but is slower than the decrease in infant mortality.

The third stage is observed after a country experiences a dramatic demographic change. Both fertility and mortality are low. Infant mortality does not change greatly but adult mortality continues to decrease. These three stages are observed in most countries. Henceforth, we focus on the mortality transition, especially the transition of adult mortality.

Figures 4.4-4.6 and Appendices L-O present the historical trends in life expectancy, adult survival indices, and the national saving rate. Western countries are presented first, followed by Asian countries. In Sweden, life expectancy at birth began to show sustained improvements around the mid-19th century. In the preceding 100 years, life expectancy at birth varied around 35 years with periodic shocks. Famine in 1772-1773 and the Finnish War in 1808-1809 caused enormous increases in the number of deaths and substantial declines in life expectancy. From the late 1870s to the 1940s, life expectancy at birth increased steadily, although the outbreak of the Spanish flu in 1918 had a severe impact. Since the 1940s, life expectancy at birth has increased more slowly. In 2002, life expectancy at birth was 80, which placed Sweden 2nd among the countries of the world.

Both the undiscounted and the discounted survival indices also began to increase during the 18th century. There is an important difference between the adult survival indices and life expectancy at birth. There is no attenuation in the rate of change in the adult survival indices. If anything, the adult survival indices increased more rapidly in the 1980s and 1990s than earlier.

In important respects, the trends in life expectancy in Sweden were similar to those in the United Kingdom, Italy, and the United States in so far as those trends are documented. (Appendices Q, R and S). In the United Kingdom, life expectancy at birth was around 40 in the mid-19th century, well below life expectancy in Sweden, and relatively constant. From the 1840s to 1900, life expectancy at birth increased modestly – by about 4 years. From 1900 to 1950, life

expectancy increased more rapidly – by 4.6 years per decade. After 1950, life expectancy increased more slowly – following a pattern similar to Sweden's.

Data on life expectancy in Italy is available beginning in 1871. At that time, life expectancy at birth was only about 30 years. Thus, the mortality transition could only have just begun even though life expectancy at birth has a relatively steady upward trend, except for 1918 and World War II. Again, there is a clear deceleration in life expectancy at birth during the second half of the 20th century. The life expectancy series is shorter for the United States, covering only the 20th century. Life expectancy at birth was around 45 at the turn of the century, hence the mortality transition was underway. Again we see a deceleration in life expectancy at birth with slower gains after 1950 than before.

The historical trends of the undiscounted and the discounted adult survival indices in the United Kingdom, Italy, and the United States are also similar to those in Sweden. To sum up, in the beginning, the adult survival indices were stagnant at a low level. At some point, the adult survival indices began to rise at a relatively constant rate. There was no deceleration in the adult survival indices in those countries and, for some, we can see a hint at recent acceleration.

The mortality transition in the Asian countries is quite distinctive from that of the West. The transition began later but has been more rapid. Japan's experience illustrates these points (Figure 4.5). Life expectancy at birth was similar in Japan and the US *circa* 1900. But Japanese mortality was stagnant during the first half of the 20th century. After World War II ended, however, life expectancy increased at a remarkable pace. Japan has experienced a considerable deceleration in the rate at which life expectancy is increasing, in part, because it is no longer catching up with the West. In fact, it is leading. By 2000, Japan's life expectancy had reached 81 years – the highest in the world.

Taiwan also experienced a rapid mortality transition (Appendix T). Life expectancy was at a much lower level in Taiwan than in Japan until a substantial increase occurred in the 1920s. After another period of relative stagnation, life expectancy increased quite rapidly, but at a slowing pace.

The trend in Indian life expectancy is similar to Taiwan's in some respects. Life expectancy was around 24 in 1901. Substantial gains were achieved between 1930 and 1950, but life expectancy was still only about 45 at that time. Since the 1950's, life expectancy at birth has increased constantly and at a relatively rapid pace, but in 2001 life expectancy was still lower than in the developed countries (Figure 4.6). A detailed life table of India is not available, so that life expectancy at age 30 is plotted in Figure 4.6 (b). From the 1970s to 1990s, life expectancy at age 30 increased rapidly. It is followed by a period of slower growth after the 1990s.

Increases in adult survival indices during mortality transition were more rapid in Asia than the West and have slowed recently. Current increases in Asia and the West are similar. The trends in national saving rates are summarized as follows. First, with a few exceptions, the national saving rates were low at the beginning of the available time series. Adult survival indices were also low. Second, in Western countries, the saving rates have increased steadily as have adult survival indices. Third, in Asian countries, national saving rates increased rapidly around the same time that adult survival indices accelerated. In these aspects, the saving patterns are consistent with expectations. The experience of one country provides no support for the role of life expectancy, however, US national saving rates are exceptional. They have been almost constant throughout the 20th century.[37] As another exceptional case, the national saving rates in Taiwan were high from the1900s to 1940s

[37] Maddison (1992) also points out the atypical nature of the pattern of the national saving rate in the US.

Figure 4.4 Life Expectancy, Adult Survival, and the Saving Rate in Sweden

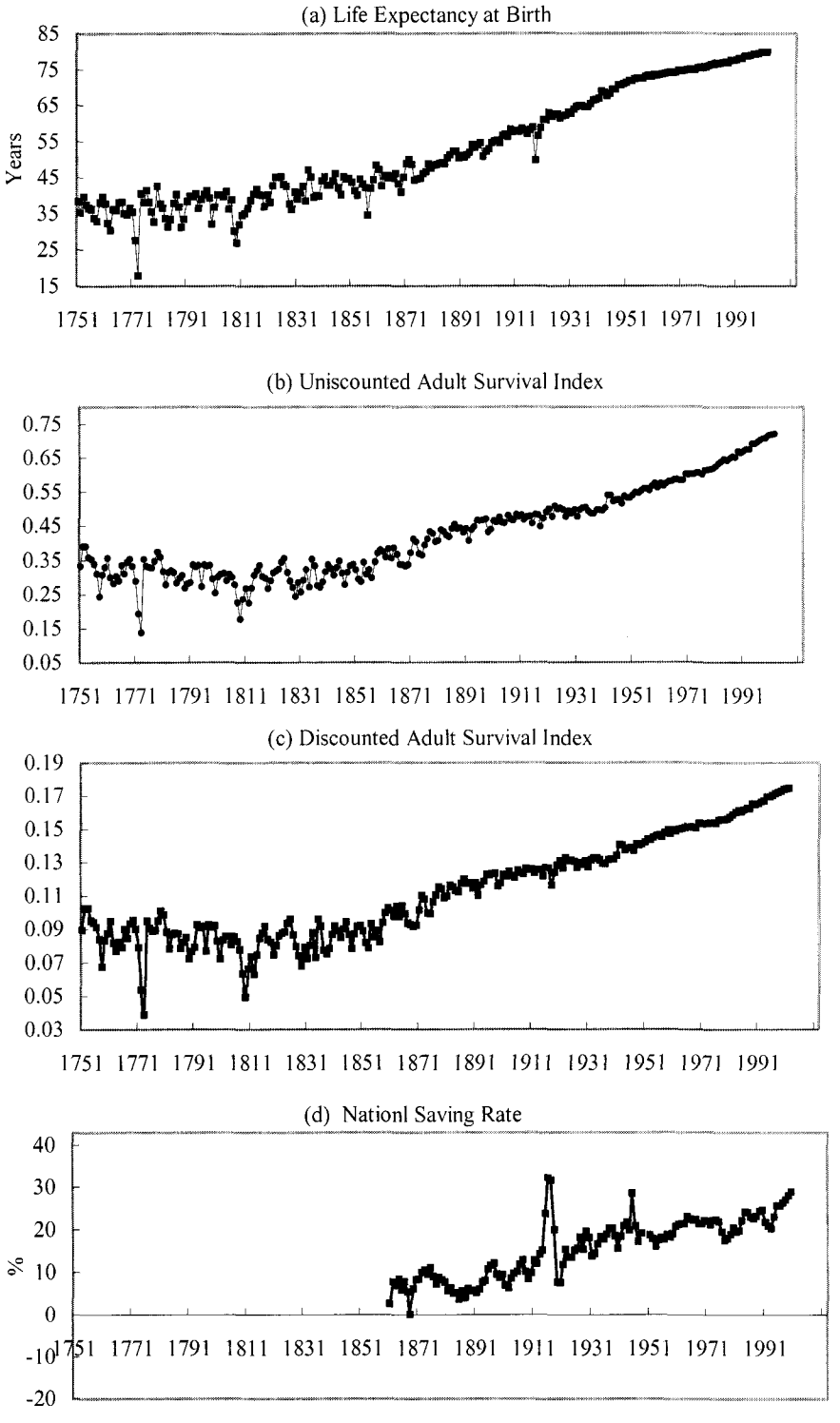


Figure 4.5 Life Expectancy, Adult Survival, and the Saving Rate in Japan

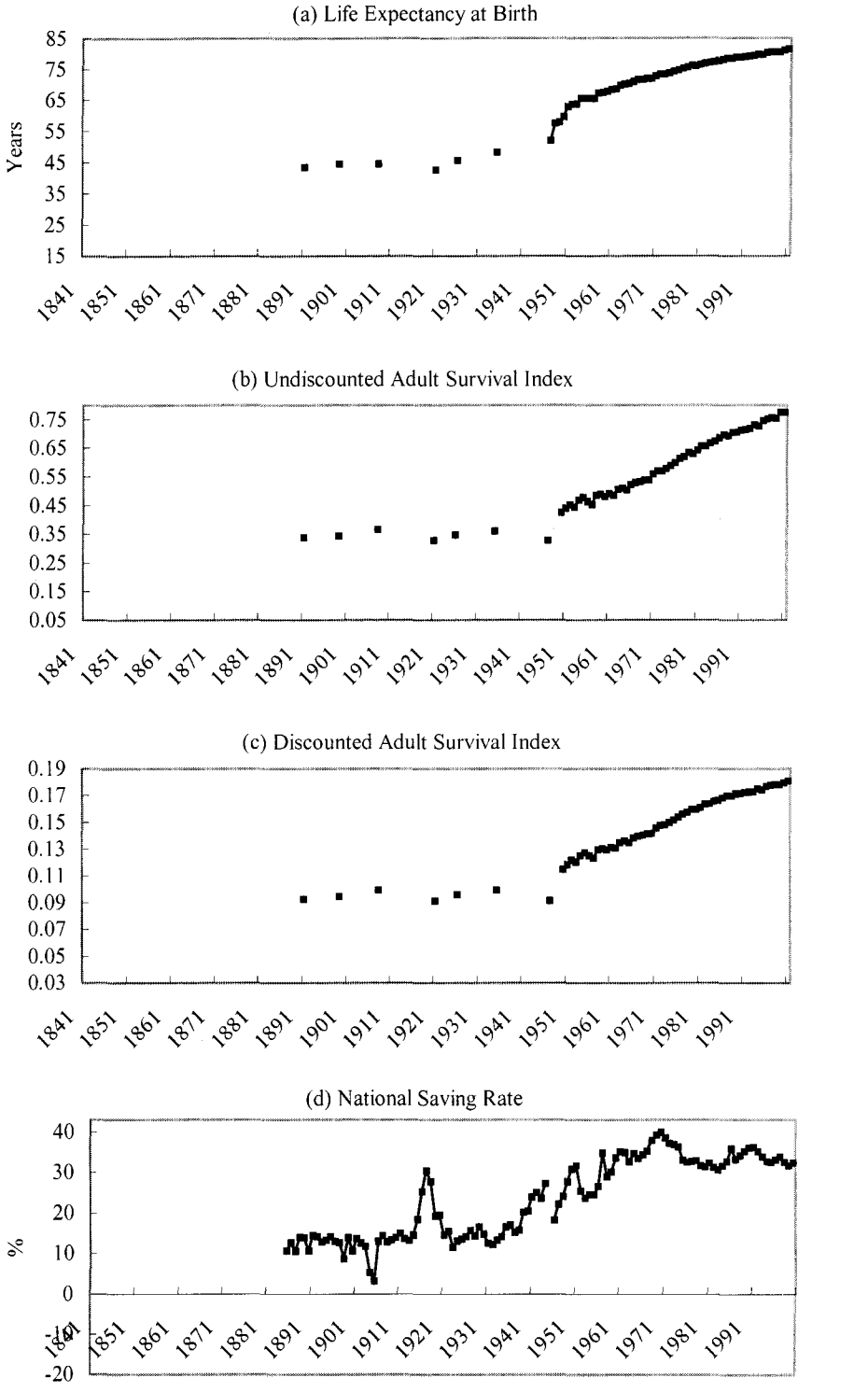
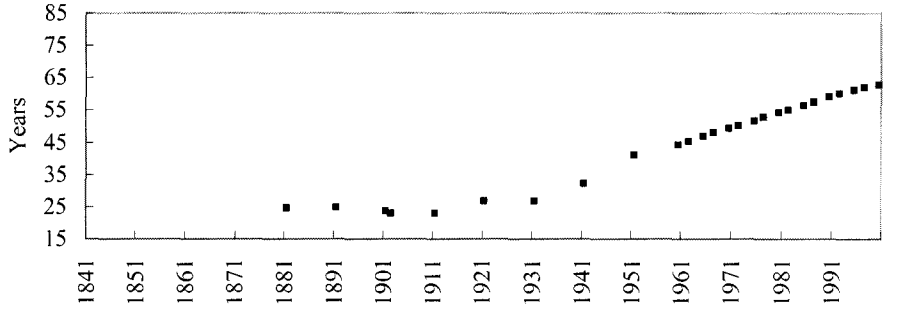
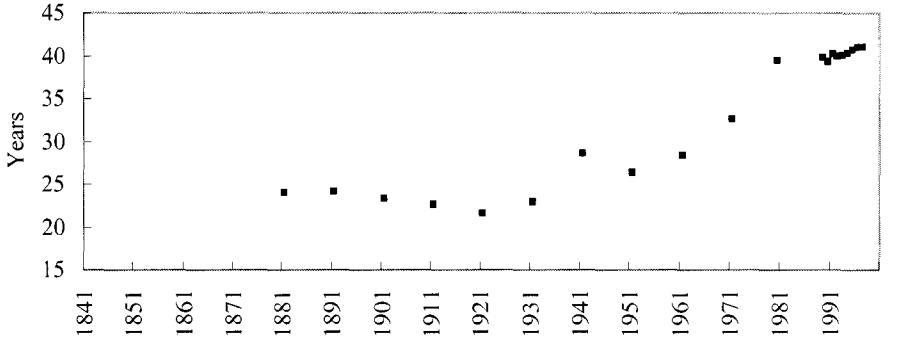


Figure 4.6 Life Expectancy at Birth and Age 30 and the Saving Rate in India

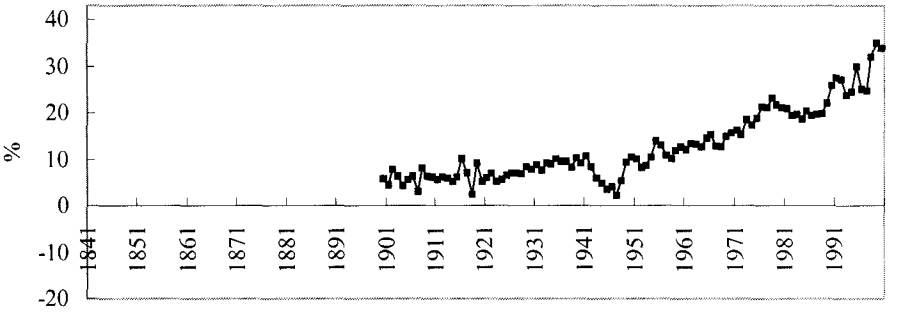
(a) LifeExpectancy at Birth



(b) Life Expectancy at Age 30



(c) National Saving Rate



in the colonial period because exports to Japan were great. However, after the colonial period, the national saving rates show a great upward trend in the1970s and 1980s, as in other Asian countries.

4.6.2 Structural Change of Adult Survival

The historical trends of adult survival support several generalizations. For the Western countries, transition in adult survival has only two phases: (1) pre-transition period when adult survival is low and stagnant and (2) a transition period when adult survival is increasing steadily. In Asia, we have three phases: pre-transition and transition, as in the West, with an intervening period of rapid increase or catch-up. Here, we formalize the historical analysis and examine whether there is a significant change in the trends at a certain period in each country. Empirical results show that the increases in the undiscounted survival index and the discounted survival index have been subject to statistically significant structural changes in certain periods.

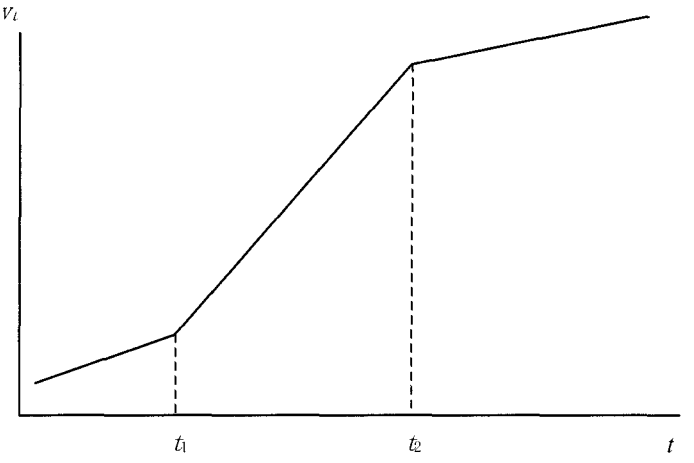
The time trends of adult survival indices are characterized using spline functions similar to that shown in Figure 4.7. The function is piece-wise linear and continuous. Structural change is represented by changes in the slope at the specified break point.

The Western mortality transition is captured allowing for one break point using the following equation.

$$v_t = \beta_0 + \beta_1 t + \beta_2 d_1 (t - t_1) + \varepsilon_t, \qquad d_1 = 1 \text{ if } t \geq t_1 \text{ and } 0 \text{ otherwise}, \tag{4.8}$$

where v_t is the dependent variable at time t, either the undiscounted survival index or the discounted survival index are considered. The variable t is time trend and d_1 is the dummy variable which marks the beginning of a structural change. Positive values for β_1 and β_2 are expected. In Italy and the United States, only the second stage is observed. Thus, the equation is estimated using a single time trend.

Figure 4.7 Spline Function



In the Asian countries, there appear to be two structural changes in a pattern captured using the following spline function:

$$v_t = \beta_0 + \beta_1 t + \beta_2 d_1(t - t_1) + \beta_3 d_2(t - t_2) + \varepsilon_t,$$
$$d_1 = 1 \text{ if } t \geq t_1 \text{ and } 0 \text{ otherwise,} \qquad (4.9)$$
$$d_2 = 1 \text{ if } t \geq t_2 \text{ and } 0 \text{ otherwise.}$$

The two dummy variables mark the break points in the trend. The coefficients β_1 and β_2 should be greater than zero, while β_3 will be less than zero if the rate of increase for $t \geq t_2$ is less than the rate of increase for $t_1 \leq t \leq t_2$. Dummy variables are also introduced to capture some outliers caused by epidemics or war.[38]

The results are presented in Table 4.7.[39] The results imply that there are structural shifts in the trends of adult survival. The coefficients of $d_1(t - t_1)$ are all significantly positive for both

[38] It is assumed that breaking point is known. For further analysis, it would be better to estimate when the structural change occurs. In this dissertation, we depend on a simple method for simplicity.

[39] Low Durbin-Watson statistics implies serial correlation. We check the trend stationarity of the variables by Augmented Dickey-Fuller test and find that some variables are not trend stationary. Further consideration of this issue would be necessary.

undiscounted and discounted adult survival indices in Sweden and the United Kingdom. The results imply that adult survival indices has been increasing more rapidly than before since around 1876 in Sweden and around 1920 in the United Kingdom.

Table 4.8 presents the rates of change in undiscounted and discounted adult survival indices calculated from the estimates in Table 4.8. In pre-transition period, the changes in discounted and undiscounted adult survival indices are not different so much in the West and Asia. Asian countries have distinguishable catch-up periods, when adult survival increased remarkably. In transition period, undiscounted and discounted adult survival indices increased more rapidly than in pre-transition period in all countries. During the transition period, the increase in adult survival indices are almost the same in all seven countries except undiscounted adult survival index in Japan.

In summary, the empirical analysis confirms the different adult mortality transitions in the West and Asia. In the West, adult survival was low and there was no substantial increase in adult survival in pre-transitional period. After the mortality transition commenced, adult survival increased steadily. In Asia, similarly to the West, adult survival was stagnant at a low level in the pre-transitional period. Post transitional period is distinctive from the Western countries in that adult survival increased quite rapidly in the beginning of the demographic transition.

4.6.3 The Change and the Level of Adult Survival and the National Saving Rate

Then, how has the mortality transition influenced the national saving rate historically? Our theoretical models in Chapters 2 and 3 imply that both the level and change in adult survival affect the national saving rate. How does the historical data compare with our theory? Three implications of the theoretical model can be addressed using the historical data: (1) saving rates will be very low in countries before the onset of the mortality transition; (2) saving rates will rise as the proportion of

Table 4.7 Estimated Results of Structural Changes

	Sweden		United Kingdom		Italy		United States	
Dependent Variables	q_{UN}	q_{PV}	q_{UN}	q_{PV}	q_{UN}	q_{PV}	q_{UN}	q_{PV}
years	1751-2002	1751-2002	1841-1998	1841-1998	1872-2000	1872-2000	1900-2001	1900-2001
t_1	1876	1876	1900	1900				
t	0.0006***	0.0002***	0.0009***	0.0003***	0.0029***	0.0006***	0.0033***	0.0007***
	(0.0001)	(2.05E-05)	(0.0001)	(2.24E-05)	(0.0001)	(1.02E-05)	(0.0002)	(2.47E-05)
$(t-t_1)d_1$	0.0020***	0.0004***	0.0024***	0.0004***				
	(0.0001)	(3.08E-05)	(0.0001)	(3.58E-05)				
Dm1773	-0.0231***	-0.0073***	0.0004***	-0.0029***	-0.1231***	-0.0318***		
	(0.0027)	(0.0006)		(0.0008)	(0.0023)	(0.0004)		
Dm1808	-0.1625***	-0.0434***						
	(0.0063)	(0.0016)						
Dm1918	-0.0960***	-0.0246***						
	(0.0040)	(0.0010)						
Constant	0.2869***	0.0785***	0.3205***	0.0864***	0.2518***	0.0760***	0.2245***	0.0735***
	(0.0080)	(0.0020)	(0.0049)	(0.0013)	(0.0053)	(0.0009)	(0.0195)	(0.0028)
Adjusted R^2	0.9251	0.9202	0.9587	0.9525	0.9471	0.9714	0.9585	0.9760
N	252	252	158	158	129	129	48	48
P-value	0.0000	0.0000	0.0000	0.0000				
D.W.	0.5765	0.6203	0.7026	0.5399	0.3970	0.6849	0.0958	0.1264

	Japan		Taiwan		India
Dependent Variables	q_{UN}	q_{PV}	q_{UN}	q_{PV}	e_{30}
years	1891-2001	1891-2001	1906-2002	1906-2002	1881-1997
t_1	1947	1947	1940	1940	1951
t_2	1989	1989	1983	1983	1989
t	0.2507***	0.0003***	0.0004	0.0005*	0.0392
	(0.0384)	(0.0001)	(0.0010)	(0.0003)	(0.0232)
$(t-t_1)d_1$	0.9665***	0.0012***	0.0056***	0.0009***	0.3457***
	(0.1281)	(0.0001)	(0.0011)	(0.0003)	(0.0599)
$(t-t_2)d_2$	-0.9091***	-0.0010***	-0.0024***	-0.0008***	-0.3075*
	(0.1085)	(0.0002)	(0.0003)	(0.0001)	(0.1568)
Constant	38.7400***	0.0874***	0.2860***	0.0684***	22.7955***
	(2.0024)	(0.0029)	(0.0322)	(0.0092)	(0.8454)
Adjusted R^2	0.9795	0.9825	0.9944	0.9898	0.9628
N	61	59	43	43	20
P-value	0.0000	0.0000	0.0000	0.0000	0.0000
D.W.	0.3897	0.2019	0.3651	0.1157	0.0338

Note: q_{UN}: undiscounted adult survival index, q_{PV}: discounted adult survival index, t time trend, e_{30}: life expectancy at age 30. The equation $v_t = \beta_0 + \beta_1 t + \beta_2 d_1 (t - t_1) + \beta_3 d_2 (t - t_2) + \varepsilon_t$ is estimated, where $d_1 = 1$ if $t \geq t_1$ and 0 otherwise, and d_2=1 if $t \geq t_2$ and 0 otherwise. Dm1773 is dummy variable of year 1773=1. In the same way, Dm1808, Dm1918 are defined. N is number of observations. "P-value" is the p-value of F-test for the null hypothesis that β_1, β_2, and β_3 are all zero. D.W. is Durbin-Watson statistics.

Table 4.8 Rate of Change Per Year in Adult Survival Indices

Undiscounted (q_{UN})	Pre-transition	Catch-up	Transition
Sweden	0.0006		0.0026
United Kingdom	0.0009		0.0032
Italy			0.0029
United States			0.0033
Japan	0.0010	0.0075	0.0061
Taiwan	0.0004	0.0061	0.0036
Dndiscounted (q_{PV})	Pre-transition	Catch-up	Transition
Sweden	0.0002		0.0006
United Kingdom	0.0003		0.0007
Italy			0.0006
United States			0.0007
Japan	0.0003	0.0015	0.0005
Taiwan	0.0005	0.0014	0.0006

adult years spent in old-age increases; (3) a rapid transition in adult mortality leads to higher saving rates.

Table 4.9 shows that the means of the national saving rates are higher in the transition period than in the pre-transition period in all seven countries. As discussed in the previous subsection, adult survival indices increase more during the transition period. Also, because adult survival trends upward, the higher level of adult survival indices contribute to the national saving rate.

In order to see the relationship between the change in adult survival and the national saving rate, we compare the national saving rate and the change in the undiscounted adult survival index when the level of the undiscounted adult survival index is almost the same. Table 4.8 presents the change of the undiscounted adult survival index in 10 years and the national saving rate. When undiscounted is approximately 0.4, the national saving rate caries across countries. Within Western countries, except the United States, the more the change in the undiscounted adult survival index, the higher is the national saving rate. The adult survival index increased much more rapidly in

Table 4.9 Means of the National Saving Rates in Pre-Transition and Transition Periods

	Sweden	United Kingdom	Italy	United States	Japan	Taiwan	India
Years, whole sample	1861-2000	1850-1998	1872-2000	1900-2001	1891-2000	1906-1999	1900-1997
Years, transition	1876-2000	1900-1998	1872-2000	1900-2001	1947-2000	1940-1999	1951-1997
Pre-transition	7.305	11.630			15.657	22.120	6.880
Transition	16.621	13.024	17.562	17.650	32.579	26.064	17.682

Japan than in other countries in 1950, and the national saving rate was much higher than inWestern countries. In the same way, we see some evidence that the national saving rate tends to be high if the adult survival index increases rapidly when we set the undiscounted adult survival index to 0.5 and 0.6.

As described above, Table 4.10 shows distinguishable difference between the change in adult survival in the West and in Asia. On the whole, at a certain level of the adult survival index, adult survival increased more rapidly in Asia than in the West. Also, the national saving rate is high except at the beginning stage of the mortality transition.

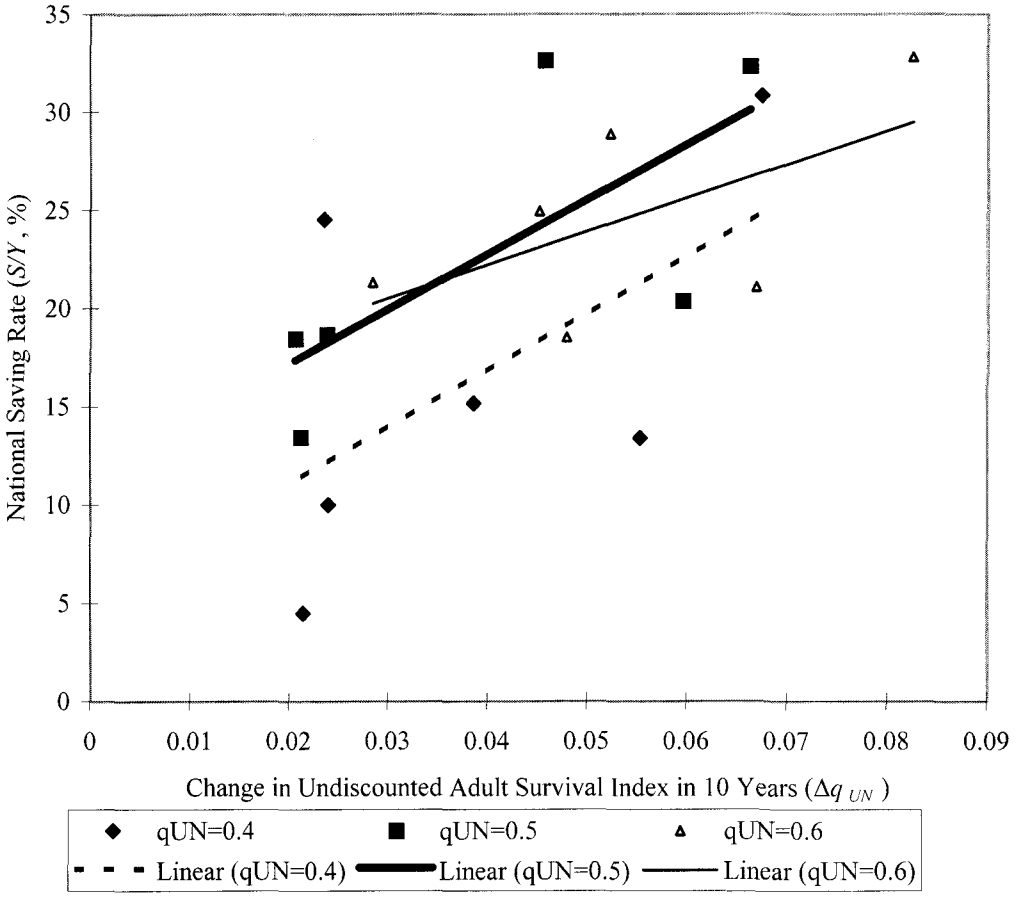
The positive relationship between the change in adult survival and the national saving rate is expressed more clearly in Figure 4.8. Figure 4.8 plots the national saving rates and the changes in undiscounted adult survival index when the levels of the adult survival indices are around 0.4, 0.5, and 0.6 in Table 4.9. For each level of the adult survival index, the approximated linear curve is drawn. The approximated curves show that the transitory effect of an increase in adult survival index is positive. Therefore, a rapid increase in adult survival might cause a higher national saving rate.

Table 4.10 Changes of Adult Survival Index and the National Saving Rate

		$q_{UN} \approx 0.4$		
Country	Year	q_{UN}	Δq_{UN} (10 years)	S/Y
Sweden	1872	0.4120	0.0240	9.9820
United Kingdom	1919	0.4373	0.0387	15.1525
Italy	1920	0.4035	0.0215	4.4707
United States	1920	0.4094	0.0236	24.5325
Japan	1950	0.4243	0.0675	30.8260
Taiwan	1956	0.4167	0.0554	13.3931
		$q_{UN} \approx 0.5$		
Country	Year	q_{UN}	Δq_{UN} (10 years)	S/Y
Sweden	1941	0.5036	0.0206	18.4440
United Kingdom	1952	0.5052	0.0213	13.4318
Italy	1950	0.5191	0.0596	20.3677
United States	1959	0.5220	0.0238	18.6357
Japan	1963	0.5066	0.0458	32.6310
Taiwan	1972	0.5041	0.0663	32.3228
		$q_{UN} \approx 0.6$		
Country	Year	q_{UN}	Δq_{UN} (10 years)	S/Y
Sweden	1972	0.6016	0.0285	21.3352
United Kingdom	1987	0.6047	0.0479	18.5231
Italy	1984	0.6175	0.0452	24.9634
United States	1979	0.6056	0.0669	21.0907
Japan	1977	0.6131	0.0826	32.8150
Taiwan	1993	0.6024	0.0523	28.8949

(Note) q_{UN} is the undiscounted adult survival index. "Δq_{UN} (10 years)" is the change in the adult survival index in 10 years. To avoid a fluctuation caused by exogenous shocks, the centered average of 5 years is calculated. S/Y is the national saving rate.

Figure 4.8 A Change in Undiscounted Survival Index and the National Saving Rate



$(q_{UN}=0.4)$:S/Y = $293.8\Delta q_{UN} + 5.0966$

$R^2 = 0.3401$

$(q_{UN}=0.5)$ $S/Y = 279.87\Delta q_{UN} + 11.563$

$R^2 = 0.5177$

$(q_{UN}=0.6)$ $S/Y = 170.73\Delta q_{UN} + 15.4$

$R^2 = 0.3502$

Historical data also present some evidences that the level of the adult survival positively influences the national saving rate. Figure 4.9 (a) plots the undiscounted adult survival rates of Sweden and Japan on the horizontal axis and the national saving rate on the vertical axis. The undiscounted adult survival indices and the national saving rates in Sweden, the United Kingdom, Italy, the United States, Japan, Taiwan, and India are plotted in Appendix U.[40] In Sweden, the saving rate is almost constant when undiscounted adult survival varies around 0.3 to 0.5. If the undiscounted survival index is more than 0.5, it seems that the national saving rate and the adult survival index is correlated linearly. In Japan, the national saving rate is higher than Sweden on the whole. In Japan, the national saving rate was low when undiscounted adult survival index was below 0.4 with only one outlier around World War I. When the adult survival index exceeded 0.4, the national saving rate increased to a quite high level. The national saving rate is highest when the undiscounted adult survival index is around 0.5. If the undiscounted adult survival index was more than 0.5, the national saving rate ranged from 30 % to 40 %. The discounted adult survival index has a similar relationship with the nationals saving rate (Appendix V).

The pattern of Sweden is similar to that of other Western countries in the following ways.[41] The national saving rate was stagnant if the undiscounted or discounted adult survival index was lower than a certain level. If the adult survival index exceeded that level, the national saving rate increased moderately with a higher increase in the adult survival rate.

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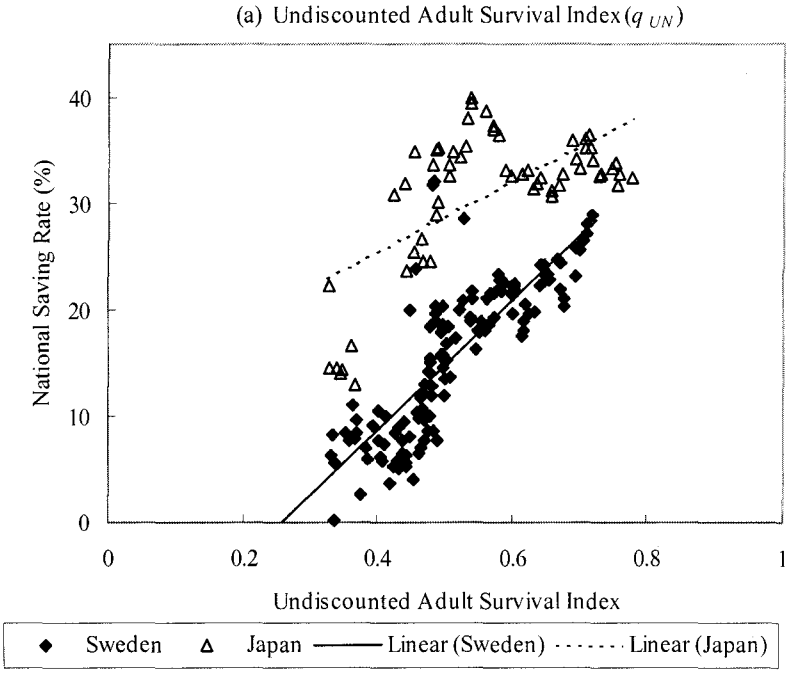
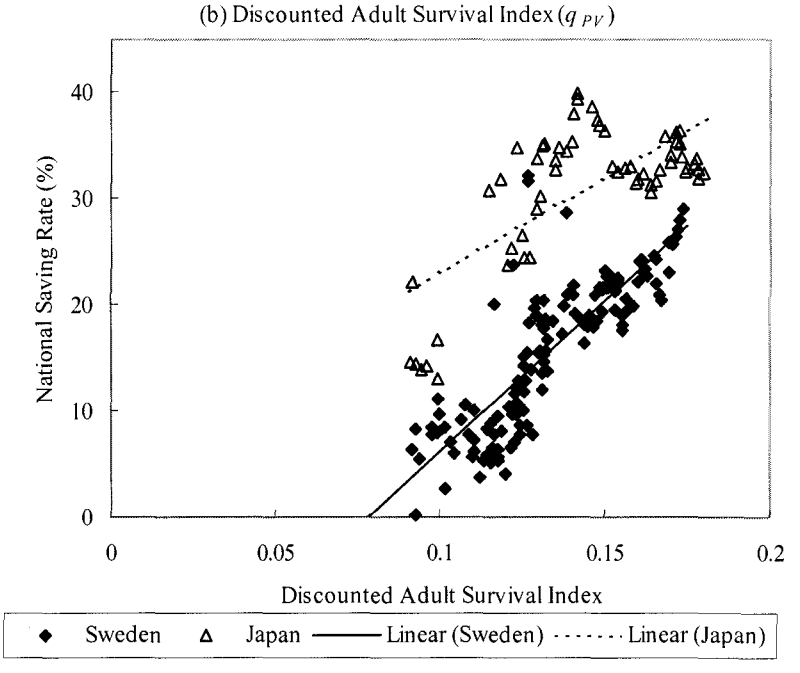
[40] In India, life expectancy at age 30 and the national saving rate are plotted.

[41] The pattern is not satisfied in the United States. As discussed before, the United States is the exceptional case.

Figure 4.9 Undiscounted and Discounted Adult Survival Indices and the National Saving Rate in Sweden and Japan





The pattern of Japan is characteristic of Asian countries such as Taiwan and India in that the slope is quite steep over a certain range of adult survival indices.[42] The national saving rate did not necessarily increase when adult survival increased if the level of the adult survival index was at a certain level. We also found that the adult survival increased rapidly around the period when the national saving rate was highest.

The figures depicting the relationship between the adult survival indices and the national saving rate imply that the national saving rate is positively correlated with the adult survival in some sense. The slopes of approximated linear functions in Figures 4.9 and Appendices U and V are all positive except for the United States. This implies that the national saving rate tends to increase as adult survival increases. However, as discussed above, the national saving rate is not influenced by only the level of adult survival.

4.7 Conclusion

The theory in Chapters 2 and 3 suggests that a rapid life expectancy increase is as important a determinant of the national saving rate as is the level of life expectancy. Also, the steady state effect of an increase in life expectancy is interacted with the effect of GDP growth. Then, how do the level and the change in life expectancy explain the change in the national saving rate empirically? A series of empirical analyses conclude that a higher adult survival increases the national saving rate and also, a rapid increase in adult survival increases the national saving rate. The contributions of the analyses in this chapter can be summarized as follows.

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[42] In Taiwan, the national saving rate is high when the undiscounted adult survival index is less than 0.4 and the discounted adult survival index is less than 0.1. However, these reflects special situation of colonial period, so we do not discuss it here.

First, we established an appropriate measure of adult survival using model life tables. Previous studies that deal with demographic change on saving rates do not consider the effect of life expectancy or employ data on life expectancy at birth. Life expectancy at birth is not a proper variable to estimate our model because it is influenced by child mortality. Also, the relationship between life expectancy at birth and adult survival is not linear. When life expectancy at birth is higher, the increase in adult survival is much greater. Therefore, using the data of life expectancy at birth will skew the effect of adult survival on the national saving rate.

Second, empirical analysis with world panel data estimates the effect of both the level and change in adult survival on the national saving rate. As the theories in Chapters 2 and 3 suggest, the effect of an increase in the level in adult survival are interacted with GDP growth. Furthermore, the effect of the change in adult survival is estimated. The results show that an increase in adult survival increases the national saving rate if GDP is growing. The estimated results with whole world data do not show significantly positive effects of the change in adult survival on the saving rate. However, within the sub-sample of advanced economies, the transitory effect of an increase in adult survival is positive and significant.

Third, reviewing the historical data, we formalize the pattern of the transition in adult survival by estimating the structural change of the time trend for the adult survival indices. Historical data show that most countries experienced a significant mortality transition. Mortality transitions distinctive in the West are compared with Asia. In the West, there are two phases of transition: pre-transition and transition. In the pre-transition period, adult survival was low and stagnant. In the transition period, adult survival increased steadily. In Asia, mortality transition began later than the West. The transition period in Asia was interrupted by a catch-up period, when adult survival increased rapidly.

Fourth, the historical data confirm that a higher level of adult survival and a more rapid increase in adult survival increases the national saving rate. In relation to mortality transition, the historical change of the national saving rate can be characterized as: (1) the national saving rate was low in the pre-transition period; (2) the national saving rate rose as the ratio of adult years spent in old age increased; (3) a rapid transition in adult mortality brought a higher saving rate. The first issue is confirmed by comparing the national saving rates during pre-transition and transition periods. The second issue is inferred by observing the relationship between adult survival and the national saving rate. To analyze the third issue, we compare the national saving rate and the change in adult survival when the levels of adult survival are similar. The results imply that the national saving rate tends to be high when adult survival increases rapidly.

CHAPTER 5. CONCLUSION

Life expectancy at birth exceeded 60 years only recently. Life expectancy at birth was low, varying from 40 to 50 years in the early 19th century even in developed European countries. Most developing countries experienced a rapid decrease in mortality in the 20th century. Infant mortality decreased first and after a while, adult mortality decreased remarkably. Life expectancy is still increasing slowly in developed countries, but some demographers stress that there are binding biological limits of human life. The national saving rate also increased remarkably in the 20th century in many countries. This dissertation investigates how the changes in the life expectancy of adults influence saving.

The mortality transition may bring dramatic change in the saving rate judging from the swing in life expectancy and the national saving rate. This dissertation deals with the following research questions. (1) How does a higher life expectancy influence the national saving rate? (2) How does a rapid increase in life expectancy affect the national saving rate? We summarize the findings in the context of these two questions.

Chapter 2 presents an overlapping generations model with two generations: prime-age adults and the elderly. We assume exogenous retirement. During the first period individuals work and during the second period they are retired. Changes in adult survival influence the expected duration of retirement. An increase in adult survival rate will cause prime-age adults to save more for retirement. An increase in adult survival increases both saving of prime-age adults and dissaving of the elderly in steady state. The national saving rate increases (decreases) in steady state if GDP is growing (decreasing) because saving of prime-age adults is more (less) than dissaving of the elderly in the economy. A transitory effect of an increase in life expectancy on the national saving

rate is positive because it only increases saving of prime-age adults. As a result, more rapid increase in life expectancy usually causes a higher national saving rate.

Chapter 3 relaxes the assumption that individuals cannot respond to increases in the expected duration of their adult life by working longer. The two-period OLG framework is retained but individuals may choose to work during "old age". An increase in life expectancy delays retirement in the presence of a perfect annuity market. An increase in life expectancy induces individuals to retire early in the absence of an annuity market. The changes in adult survival influence saving of prime-age adults, dissaving of the elderly, labor force participation of the elderly, and the number of elderly who consume without saving. The change in accidental bequests received by prime-age adults also changes if adult survival changes in the absence of annuities. An increase in life expectancy does not necessarily increase the national saving rate in steady state even if GDP is growing. However, an increase in life expectancy brings a transitory increase in the national saving rate.

In Chapter 4, the implications of our theory are examined empirically using world panel data and longer-term historical data. The analysis of world panel data shows that the level of adult survival has a positive effect on the national saving rate if GDP is growing. A significant effect of the rate of change of adult survival is not found in analysis based on the world data nor the developing countries data. The rate of change in adult survival has a significant positive effect on the national saving rate within sub-sample of advanced economies.

Historical data of Sweden, the United Kingdom, Italy, the United States, Japan, and Taiwan show that transition of adult survival in the West is distinctive from Asia. In the West, mortality trends consisted of a pre-transition period and a transition period. In the pre-transition period, adult survival was low and stagnant. After the transition period began, adult survival increased at a

relatively constant rate. In Asia, the mortality transition began later than in the West, but it increased quite rapidly after World War II.

Three implications of theoretical model are confirmed using the historical data. First, the saving rates stayed low in countries before the mortality transition began. Second, saving rates rose as the proportion of adult years spent in old-age increased. Third, the rapid transition in adult mortality led to higher saving rates. In Asian countries a period of rapid change in adult survival was accompanied by an accelerated increase in the saving rate.

In conclusion, the main contribution of this dissertation is the inclusion of both steady-state and out-of-steady-state effects in the theoretical and empirical analysis. The effects of an increase in adult survival depend on the labor force responses. In the simplest case, a fixed age at retirement, an increase in life expectancy unambiguously leads to an increase in saving in a growing economy. If the age of retirement increases in response to an increase in life expectancy, however, the response of saving to higher life expectancy is an empirical issue. Under any conditions, however, a rapid increase in life expectancy leads to higher saving. The empirical evidence consistently supports the conclusion that an increase in life expectancy leads to higher saving rates. This is not entirely surprising given that, as an empirical matter, the age at retirement has been declining rather than rising around the world. The evidence regarding the rate of change of life expectancy is mixed. Analysis of recent data for the developing world does not support this key hypothesis. Recent experience in the developed world and longer-term historical developments in the few countries for which data are available support the hypothesis that rapid mortality transitions lead to elevated saving rates and more rapid accumulation of wealth.

There is much room for further research in this area. Empirical analysis investigating the change in retirement is important. Not only retirement decision but also labor force participation

decisions by women may be influencing the national saving rate. Intergenerational transfer including familial transfer and social security system may have a considerable influence on the saving rate. Further consideration of both the theoretical and empirical aspects of intergenerational transfer is important.

APPENDIX A. DETAILS OF THE CONSUMER'S OPTIMIZATION, CHAPTER 2

Here, we will give a detailed explanation about consumer's optimization without labor force participation decision. Individuals are assumed to maximize the lifetime utility in equation (2.1) under the budget constraint (2.4). We can set up a Lagrangian expression to assess the household maximization of utility: .

$$L = \frac{c_{1,t}^{1-\theta}}{1-\theta} + \delta q_t \frac{c_{2,t+1}^{1-\theta}}{1-\theta} - \lambda\left(c_{1,t} + \frac{q_t}{1+r_{t+1}} c_{2,t+1} - A_t w_t \right) \tag{A.1}$$

where λ is a Lagrangian multiplier. First order conditions are given as follows.

$$\frac{\partial L}{\partial c_{1,t}} = c_{1,t}^{-\theta} - \lambda = 0 \tag{A.2}$$

$$\frac{\partial L}{\partial c_{2,t+1}} = \delta q_t c_{2,t+1}^{-\theta} - \frac{q_t}{1+r_{t+1}} \lambda = 0 \tag{A.3}$$

$$\frac{\partial L}{\partial \lambda} = -c_{1,t} - \frac{q_t}{1+r_{t+1}} c_{2,t+1} + A_t w_t = 0 \tag{A.4}$$

Substituting (A.2) and (A.3) into equation yields:

$$\lambda = \frac{[1 + q_t \delta^{\frac{1}{\theta}} (1+r_{t+1})^{\frac{1-\theta}{\theta}}]^{\theta}}{A_t^{\theta} w_t^{\theta}} \tag{A.5}$$

Substituting back for λ yields:

$$c_{1,t} = \frac{(1+r_{t+1})^{\frac{1-\theta}{\theta}} A_t w_t}{q_t \delta^{\frac{1}{\theta}} + (1+r_{t+1})^{\frac{1-\theta}{\theta}}} \tag{A.6}$$

$$c_{2,t+1} = \frac{\delta^{\frac{1}{\theta}} (1+r_{t+1}) A_t w_t}{q_t \delta^{\frac{1}{\theta}} + (1+r_{t+1})^{\frac{1-\theta}{\theta}}} \tag{A.7}$$

Equations (A.6) and (A.7) correspond to equations (2.3) and (2.4).

APPENDIX B. SAVING OF PRIME-AGE ADULTS AND THE AGGREGATE WEALTH

Substituting (2.2) and (2.3) into equation (2.16) yields:

$$\begin{aligned}(K_{t+1}+F_{t+1})-(K_t+F_t) &= w_t A_t N_{1,t} + r_t(K_t+F_t) - c_{1,t}N_{1,t} - c_{2,t}N_{2,t} \\ &= w_t A_t N_{1,t} + r_t(K_t+F_t) - (w_t - s_{1,t})N_{1,t} - \frac{(1+r_t)}{q_{t-1}} s_{1,t-1}N_{2,t} \\ &= s_{1,t}N_{1,t} + r_t K_t - (1+r_t)s_{1,t-1}N_{1,t-1}. \quad (\because N_{2,t} = q_{t-1}N_{1,t-1})\end{aligned}$$

Therefore,

$$K_{t+1} + F_{t+1} = s_{1,t}N_{1,t} + (1+r_t)(K_t + F_t - s_{1,t-1}N_{1,t-1}). \tag{B.1}$$

From equation (2.16),

$$(K_2 + F_2) - (K_1 + F_1) = w_1 A_1 N_{1,1} + r_1(K_1 + F_1) - c_{1,1}N_{1,1} - c_{2,1}N_{2,0}. \tag{B.2}$$

Suppose $c_{2,1}N_{2,0} = (1+r_1)(K_1 + F_1)$ holds,

$$\begin{aligned}K_2 + F_2 &= w_1 A_1 N_{1,1} - c_{1,1}N_{1,1} \\ &= s_{1,1}N_{1,1}.\end{aligned} \tag{B.3}$$

Equation (B.3) implies equation (2.17) holds for $t = 1$. Equation (B.1) implies that equation (2.17) holds for all $t \geq 2$.

APPENDIX C. SIMULATED SAVING RATETES OF SAVING RATES OF THE UNITED SATAES AND JAPAN (NO CHANGE IN THE SURVIVAL RATE)

The United States

					Closed Economy			Small Open Economy	
Time	*q*	*g*	*n*	*gn*	*k*	*K/Y*	*S/Y*	*S/Y*	*I/Y*
1900	0.373	1.944	1.878	3.650	0.009	0.0441	0.152	0.152	0.152
1930	0.373	2.139	1.479	3.164	0.009	0.0441	0.149	0.152	0.130
1960	0.373	1.760	1.461	2.571	0.011	0.0480	0.143	0.145	0.104
1990	0.373	1.709	1.290	2.206	0.013	0.0554	0.137	0.133	0.088
2020	0.373	1.709	1.167	1.995	0.016	0.0632	0.132	0.123	0.079
2050					0.018	0.0695			

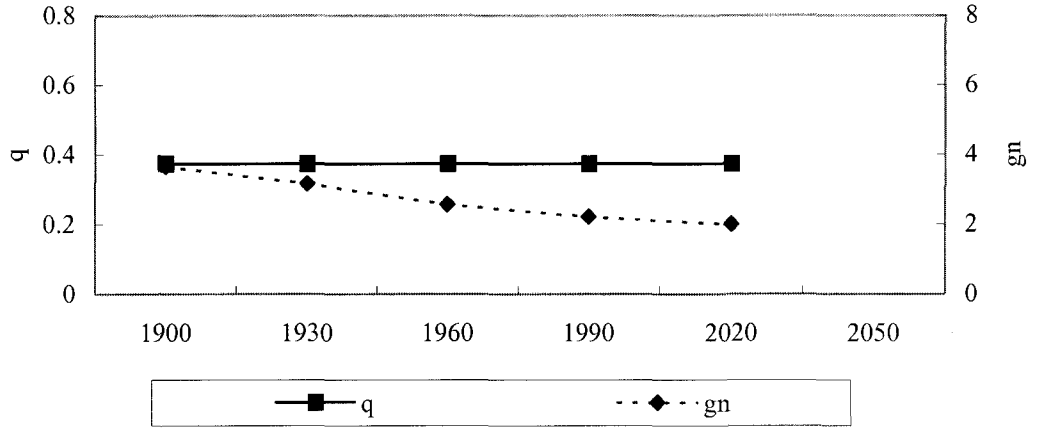
Japan

					Closed Economy			Small Open Economies	
Time	*q*	*g*	*n*	*gn*	*k*	*K/Y*	*S/Y*	*S/Y*	*I/Y*
1900	0.374	1.759	1.382	2.431	0.015	0.0616	0.137	0.130	0.098
1930	0.374	3.059	1.629	4.983	0.015	0.0616	0.151	0.130	0.210
1960	0.374	4.349	1.674	7.279	0.008	0.0406	0.168	0.164	0.312
1990	0.374	1.345	0.939	1.263	0.005	0.0289	-0.006	0.174	0.046
2020	0.374	1.345	0.702	0.944	$8.96*10^{-5}$	0.0020	0.165	0.068	0.032
2050					0.008	0.0395			

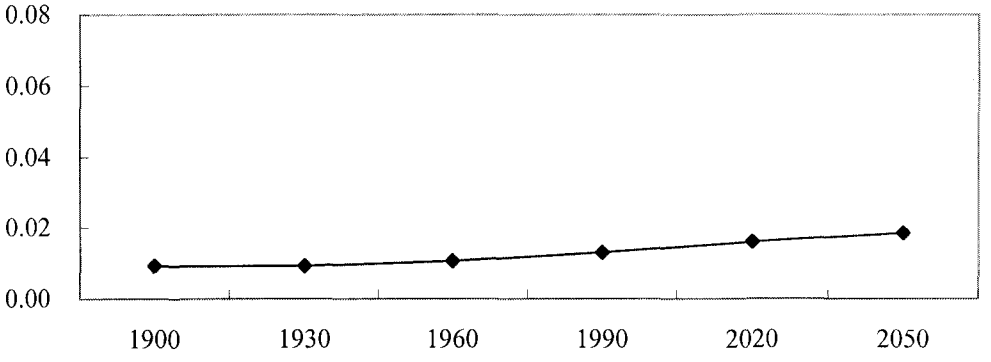
Note: For definition and the sources of the variables, see the note of Table 2.2.

APPENDIX D. SIMULATION ANALYSIS FOR THE UNITED STATES (NO CHANGE IN THE SURVIVAL RATE)

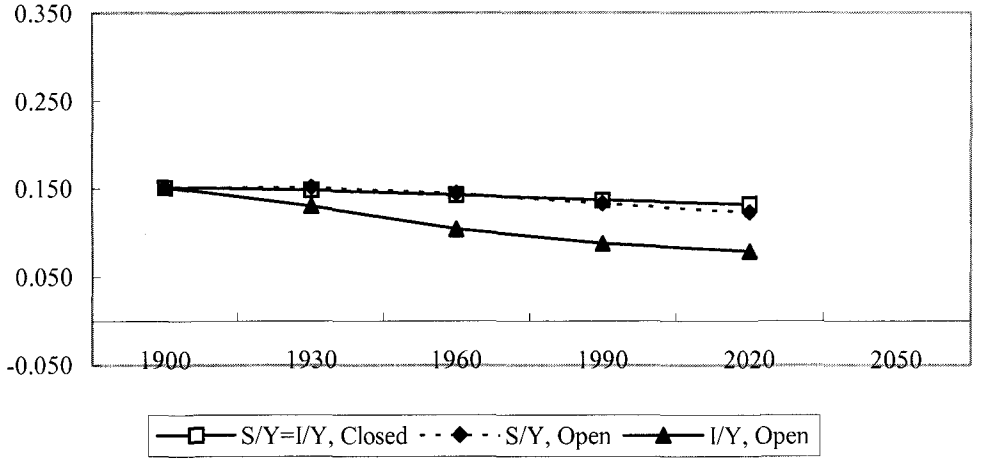
(a) Survival Rate (q) and GDP Growth (gn)



(b) Capital per Effective Labor

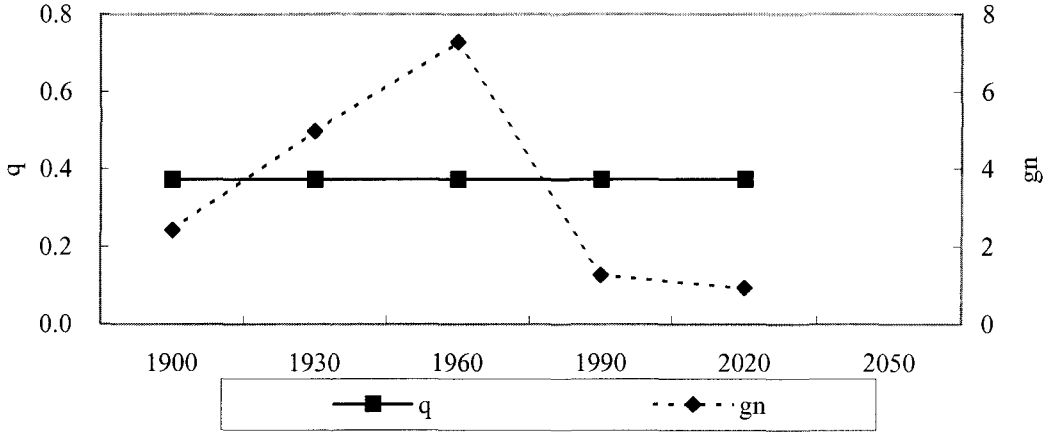


(c) Saving Rate (S/Y) and Investment Rate (I/Y)

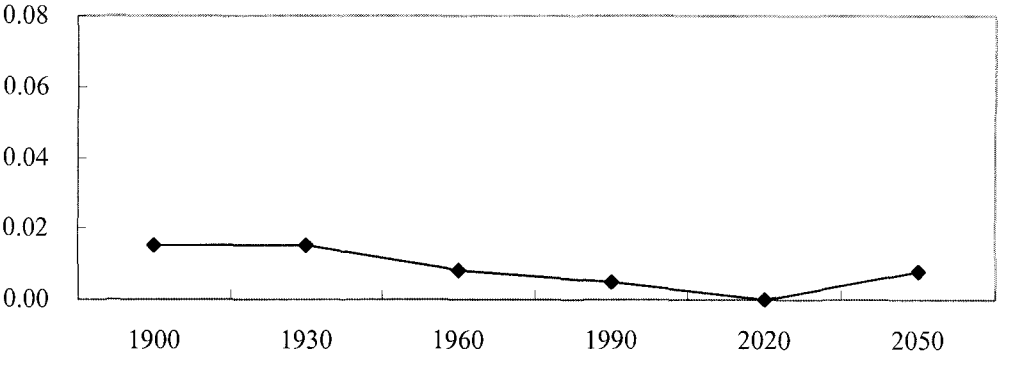


APPENDIX E. SIMULATION ANALYSIS FOR JAPAN (NO CHANGE IN THE SURVIVAL RATE)

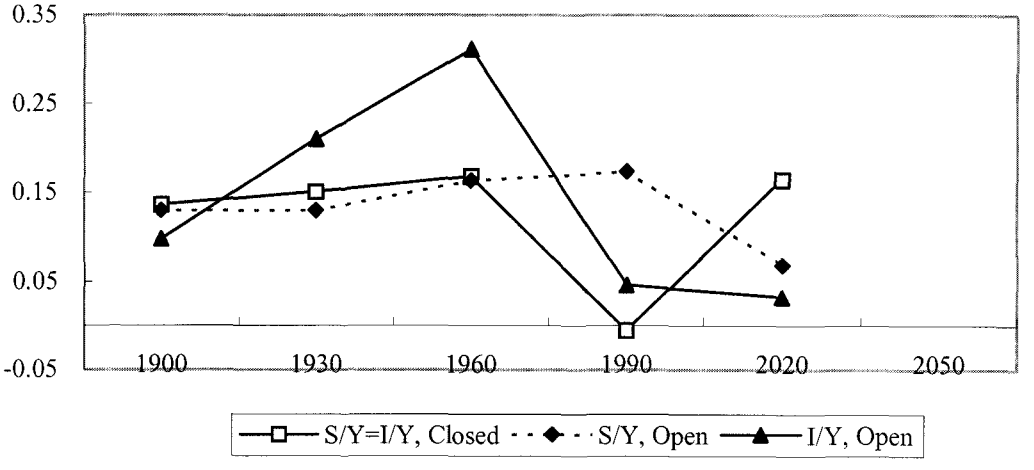
(a) Survival Rate (q) and GDP Growth (gn)



(b) Capital per Effective Labor



(c) Saving Rate (S/Y) and Investment Rate (I/Y)



APPENDIX F. DETAILS OF THE CONSUMER'S OPTIMIZATION UNDER PERFECT ANNUITY MARKET, CHAPTER 3

Here, we will give a detailed explanation about consumers' optimization with labor force participation decision. Individuals are assumed to maximize lifetime utility in equation (3.9) under the budget constraint (3.12). We can set up a Lagrangian expression to assess household maximization of utility:

$$L = \ln c_{1,t} + \delta q_t [\ln c_{2,t+1} + \gamma \ln(1 - z_{t+1})] \\ -\lambda\left(c_{1,t} + \frac{q_t}{1+r_{t+1}} c_{2,t+1} - A_t w_t - \frac{q_t}{1+r_{t+1}} A_{t+1} w_{t+1} z_{t+1} \right), \tag{F.4}$$

where λ is the Lagrangian multiplier. First order conditions are:

$$\frac{\partial L}{\partial c_{1,t}} = \frac{1}{c_{1,t}} - \lambda = 0, \tag{F.5}$$

$$\frac{\partial L}{\partial c_{2,t+1}} = \frac{\delta q_t}{c_{2,t+1}} - \frac{q_t}{1+r_{t+1}} \lambda = 0, \tag{F.6}$$

$$\frac{\partial L}{\partial z_{t+1}} = -\frac{\delta q_t \gamma}{1 - z_{t+1}} + \frac{q_t}{1+r_{t+1}} A_{t+1} w_{t+1} \lambda \le 0, \tag{F.7}$$

$$\frac{\partial L}{\partial \lambda} = -c_{1,t} - \frac{q_t}{1+r_{t+1}} c_{2,t+1} + A_t w_t + \frac{q_t}{1+r_{t+1}} A_{t+1} w_{t+1} z_{t+1} = 0. \tag{F.8}$$

In the case of an interior solution, $z_{t+1} > 0$, equality holds in equation (F.6). Substituting (F.5), (F.6), and (F.7) into (F.8) yields $\lambda = \frac{(1+r_{t+1})(1+q_t\delta+q_t\gamma\delta)}{(1+r_{t+1})A_t w_t + q_t A_{t+1} w_{t+1}}$. Substituting back for λ yields equations (3.13), (3.14), and (3.15). Equations (3.13), (3.14), (3.15), and (3.17) imply that the effects of an increase in q_t on $c_{1,t}$, $c_{2,t+1}$, z_{t+1}, and $s_{1,t}$ depend on $\frac{\partial(1/\lambda)}{\partial q_t}$.

$$\frac{1}{\lambda} = \frac{(1+r_{t+1})A_t w_t + q_t A_{t+1} w_{t+1}}{(1+r_{t+1})(1+q_t\delta+q_t\gamma\delta)}. \tag{F.9}$$

Differentiating 1/λ with respect to q_t yields:

$$\frac{\partial(1/\lambda)}{\partial q_t} = \frac{A_{t+1} w_{t+1} - \left[(1+r_{t+1})A_t w_t + q_t A_{t+1} w_{t+1}\right](\delta+\gamma\delta)}{(1+r_{t+1})(1+q_t\delta+q_t\gamma\delta)^2}. \tag{F.10}$$

The denominator of the right hand side of equation (F.10) is positive, and the numerator is positive for plausible parameter values. Therefore, $\frac{\partial(1/\lambda)}{\partial q_t} > 0$.

In the case of a corner solution, $z_{t+1} = 0$, $c_{1,t}$ and $c_{2,t+1}$ are obtained as equations (3.21) and (3.22) from equations (F.5), (F.6), and (F.7).

Consumption during ages 1 and 2, and labor force participation at age 2 in sections 3.5 and 3.6 are obtained in the same way.

APPENDIX G. STEADY STATE SAVING AND LABOR FORCE PARTICIPATION WITHOUT ANNUITY, CHAPTER 3

Let $q_t=q_{t-1}=q^*$, $r_{t+1}=r_w^*$, and $w_t=w_{t+1}=w^*$. From equations (3.77) and (3.82), labor force participation during age 2, z_{t+1}, is:

$$z_{t+1}=\frac{g(1+\delta q^*)-\delta q^*\gamma(1+r_w^*)\left(1+\frac{1-q^*}{ng}X_{t-1}\right)}{g(1+\delta q^*+\gamma\delta q^*)}. \tag{G.1}$$

From equations (3.79) and (3.82), the share of saving of labor income at age 1, X_t, is:

$$X_t=\frac{(1+r_w^*)(\delta q^*+\gamma\delta q^*)\left(1+\frac{1-q^*}{ng}X_{t-1}\right)-g}{(1+r_w^*)(1+\delta q^*+\gamma\delta q^*)}. \tag{G.2}$$

Equation (G.1) implies that labor force participation of the elderly is constant if X is constant. If labor force participation is constant, the economy is in steady state. When X is constant, that is, $X_t=X_{t-1}=X^*$, X^* and z^* are obtained as equations (3.83) and (3.84), respectively.

Equation (G.2) can be rewritten as:

$$X_t=hX_{t-1}+j, \tag{G.3}$$

where $h=\frac{(1+r_w^*)(\delta q^*+\gamma\delta q^*)(1-q^*)}{(1+r_w^*)(1+\delta q^*+\gamma\delta q^*)ng}$ and $j=\frac{(1+r_w^*)(\delta q^*+\gamma\delta q^*)-g}{(1+r_w^*)(1+\delta q^*+\gamma\delta q^*)}$. Because $0\le h\le 1$ holds for plausible parameter values, the steady state is stable and nonoscillatory.

APPENDIX H. RELATIONSHIP BETWEEN LIFE EXPECTANCY AT BIRTH AND THE NATIONAL SAVING RATE

The relationship between e_0 and q_{UN} is estimated using a cubic function. For male,

$$\underset{(0.0015)}{q_{UN} = 0.0184e_0} \underset{(2.99\cdot10^{-5})}{- 0.0003e_0^{\,2}} \underset{(1.96\cdot10^{-7})}{+ 1.94\cdot10^{-6}e_0^{\,3}} \underset{(0.0193)}{- 0.1209} \qquad \overline{R}^2 = 0.9984, \tag{H.4}$$

and for female,

$$\underset{(0.0023)}{q_{UN} = 0.0213e_0} \underset{(4.97\cdot10^{-5})}{- 0.0003e_0^{\,2}} \underset{(3.30\cdot10^{-7})}{+ 2.47\cdot10^{-6}e_0^{\,3}} \underset{(0.0338)}{- 0.1517} \qquad \overline{R}^2 = 0.9974, \tag{H.5}$$

where figures in parentheses are standard errors.

Life expectancy at birth and discounted adult survival index, q_{PV}, are shown in Figure 4.2. However, similarly to q_{UN}, q_{PV} increases more when life expectancy at birth increases if life expectancy at birth is higher than 70. It is also inferred that q_{PV} is a cubic function of e_0. The relationship between e_0 and q_{PV} for male is estimated as:

$$\underset{(2.71\cdot10^{-4})}{q_{PV} = 0.0043e_0} \underset{(5.46\cdot10^{-6})}{- 0.0001e_0^{\,2}} \underset{(3.57\cdot10^{-8})}{+ 3.82\cdot10^{-7}e_0^{\,3}} \underset{(0.0035)}{- 0.0240} \qquad \overline{R}^2 = 0.9992. \tag{H.6}$$

The relationship between e_0 and q_{PV} for female is:

$$\underset{(4.09\cdot10^{-4})}{q_{PV} = 0.0048e_0} \underset{(8.68\cdot10^{-6})}{- 0.0001e_0^{\,2}} \underset{(5.76\cdot10^{-8})}{+ 4.73\cdot10^{-7}e_0^{\,3}} \underset{(0.0059)}{- 0.0269} \qquad \overline{R}^2 = 0.9945. \tag{H.7}$$

Equations (H.4), (H.5), (H.6), and (H.7) are used when q_{UN} and q_{PV} are estimated based on the data of life expectancy at birth.

APPENDIX I. THE EFFECT OF ADULT SURVIVAL ON MEAN AGES OF EARNING AND CONSUMPTION

Age earning and consumption profiles are drawn in Appendix J[43] based on Family Income and Expenditure Survey for Taiwan 1998. Consumption profile is estimated using Engel's method.[44]

The data include information on age earning and consumption profiles under 30. In order to focus on the effect of a change in adult mortality, consumption and earning profiles of over 30 iare considered. We investigate how an increase in life expectancy at birth and the measures of survival rate above influence the mean age of consumption and mean age of earning over 30. Hereafter, mean age of consumption or earning considers consumption or earning over 30 without notice.

Mean age of consumption, A_C, is:

$$A_C = \sum_{x=30}^{M} xl(x)c(x) \Big/ \sum_{x=30}^{M} l(x)c(x), \tag{I.1}$$

where x is age, $l(x)$ is probability to survive at age x, and $c(x)$ is consumption at age x. Mean age of earning, A_Y, is:

$$A_Y = \sum_{x=30}^{M} xl(x)Y(x) \Big/ \sum_{x=30}^{M} l(x)Y(x). \tag{I.2}$$

Appendix K demonstrates the relationship between the difference between mean age of consumption and mean age of earning and four measures of longevity, life expectancy at birth, adult survival rate of World Development Indicator, $q_{15\text{-}60}$, undiscounted adult survival rate, q_{UN}, and discounted survival index, q_{PV}. Plotting life expectancy at birth and $A_C - A_Y$ indicates that an increase in life expectancy at birth increases the difference between mean age of consumption and mean age of earning (Appendix K (a)). It increases both A_C and A_Y, but an increase in A_C is greater

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[43] Age earning and consumption profile could be changed if life expectancy changes. Further discussion about this issue would be necessary.

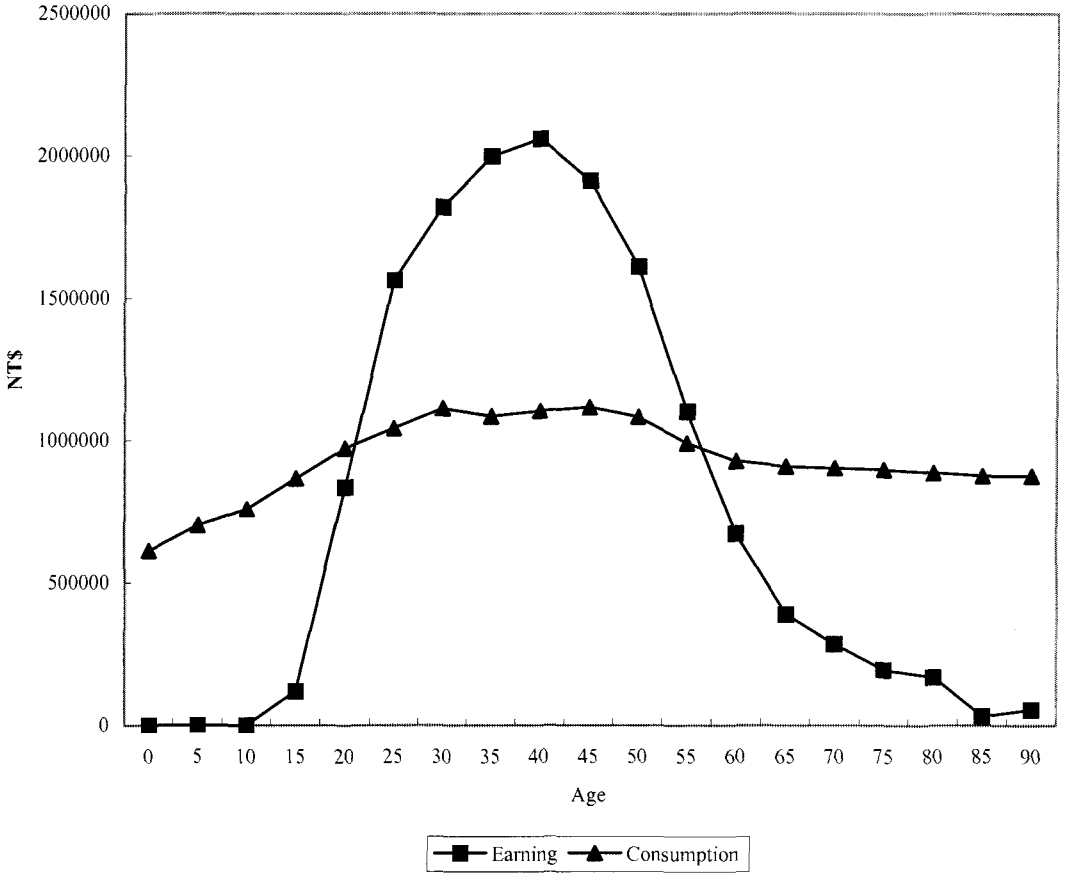
[44] The data is obtained by personal communication from Mun Sim Lai.

at any time. In both cases of male and female, an increase in life expectancy at birth increases $A_C - A_Y$ moderately when e_0 is less than 50. When e_0 exceeds 50, an increase in e_0 increases greatly.

Appendix K (b) suggests that $A_C - A_Y$ and $q_{15\text{-}60}$ have an almost linear relationship if $q_{15\text{-}60}$ is less than 0.8 for both female and male. If $q_{15\text{-}60}$ exceeds 0.8, an increase in $q_{15\text{-}60}$ has a great effect on $A_C - A_Y$.

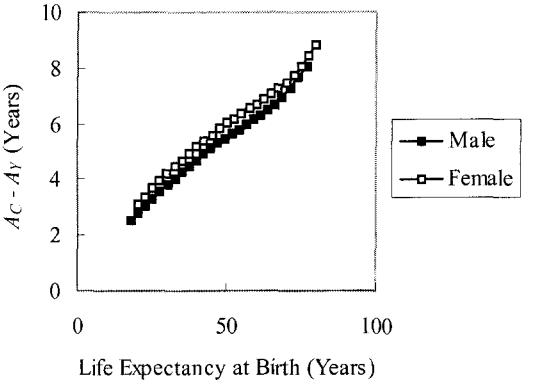
Undiscounted adult survival index (q_{UN}) and $A_C - A_Y$ have an almost linear relationship for both male and female (Appendix K (c)). Discounted adult survival index (q_{PV}) of both female and male are linearly correlated with $A_C - A_Y$ (Appendix K (d)). Appendix K indicates that using e_0 or $q_{15\text{-}60}$ as a measure of the survival rate may bias the effect of an increase in the level of the survival rate on the national saving rate.

APPENDIX J. AGE EARNING AND CONSUMPTION PROFILES IN TAIWAN

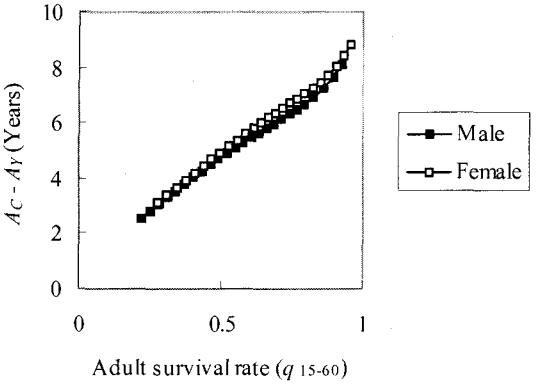


APPENDIX K. RELATIONSHIPS BETWEEN DIFFERENCE IN MEAN AGE CONSUMPTION AND MEAN AGE EARNING AND VARIOUS MEASURES OF LONGEVITY

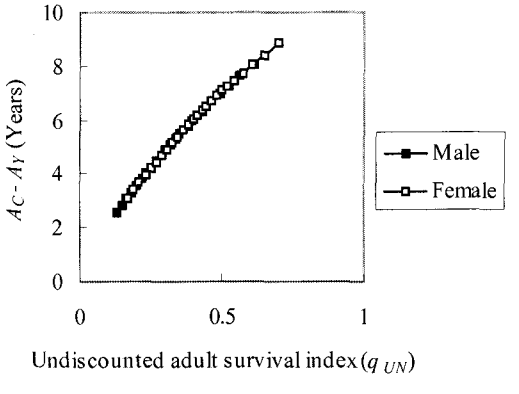
(a) Life Expectancy at Birth



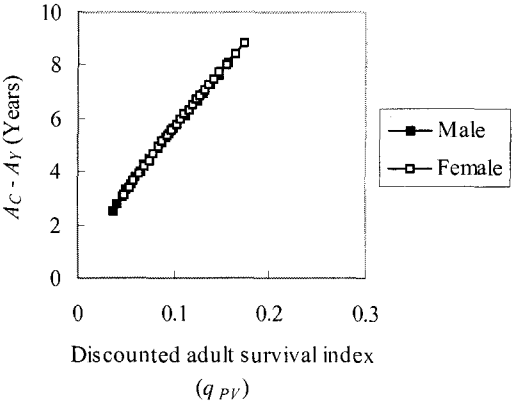
(b) Adult Survival Rate ($q_{15\text{-}60}$)



(c) Undiscounted Adult Survival Index (q_{UN})



(d) Discounted Adult Survival Index (q_{PV})



APPENDIX L. DESCRIPTION OF DATA

L.1 World Panel Data

(1) Data Sources

All variables are calculated as centered average of 10 years. For example, the saving rate in 1970 means the mean of the saving rate from 1965 to 1975. (The variable in 2000 is calculated as the mean from 1995 to 2000). The national saving rate (S/Y) is calculated as (100 – kc – kg)/100, where kc is consumption share of real GDP and kg is government share of real GDP. kc and kg are from *Penn World Table* (PWT, hereafter). The observations with the national saving rate less than –1 are excluded. Adult survival rate ($q_{15\text{-}60}$) is (1000 - adult mortality)/1000. Adult mortality per 1000 is from *World Development Indicator* (WDI, hereafter). Undiscounted and discounted adult survival rate, q_{UN} and q_{PV} are calculated from life expectancy at birth using the model life table as described in the text. Life expectancy at birth is from WDI. GDP growth rate (Y_{gr}) is the growth rate of GDP in 10 years. GDP is defined by rgdpch * pop, where rgdpch is real GDP per capita (constant prices, chain series) and pop is population. rgdpch and pop are from PWT. Price of investment goods (*PRI*, ppp/xrate) is price level of investment from PWT (US=1). Young dependency rate ($D1$) is the share of population under 15 and from WDI. Labor force is from WDI. Price index (price level of gross domestic product, US = 100), GDP per capita, and openness are obtained from PWT.

(2) Sample Countries

The analysis divides the whole sample into five regional groups: advanced economies, Asia, Latin America, Africa, and Middle East. Advanced economies contains Albania, Armenia, Australia, Austria, Azerbaijan, Belgium, Bulgaria, Belarus, Canada, Switzerland, Czech Republic, Denmark, Spain, Estonia, Finland, France, United Kingdom, Georgia, Greece, Croatia, Hungary,

Ireland, Iceland, Italy, Kazakhstan, Kyrgyz Republic, Lithuania, Luxembourg, Latvia, Moldova, Macedonia, Netherlands, Norway, New Zealand, Poland, Portugal, Romania, Russian Federation, Slovak Republic, Slovenia, Sweden, Tajikistan, Turkey, Ukraine, and United States.

Africa consists Angola, Burundi, Benin, Burkina Faso, Botswana, Central African Republic, Cote d'Ivoire, Cameroon, Congo Republic, Comoros, Cape Verde, Ethiopia, Gabon, Ghana, Guinea, Gambia, Guinea Bissau, Equatorial Guinea, Kenya, Madagascar, Mali, Mozambique, Mauritania, Mauritius, Malawi, Namibia, Niger, Nigeria, Rwanda, Senegal, Sierra Leone, Seychelles, Chad, Togo, Tanzania, Uganda, South Africa, Democratic Republic of the Congo, Zambia and Zimbabwe.

Asia contains Bangladesh, China, Hong Kong, Indonesia, India, Japan, Republic of Korea, Sri Lanka, Macao, Malaysia, Nepal, Pakistan, Philippines, and Thailand.

Latin America includes Argentina, Antigua and Barbuda, Belize, Bolivia, Brazil, Barbados, Chile, Colombia, Costa Rica, Cuba, Dominica, Dominican Republic, Ecuador, Grenada, Guatemala, Guyana, Honduras, Haiti, Jamaica, St. Kitts and Nevis, St. Lucia, Mexico, Nicaragua, Panama, Peru, Paraguay, El Salvador, Trinidad and Tobago, Uruguay, St. Vincent and the Grenadine, Venezuela.

Middle East consists Algeria, Egypt, Iran, Israel, Jordan, Lebanon, Morocco, Syrian Arab Republic, Tunisia, and Yemen.

L.2 Historical Data

If the data of national saving rate is not available, the national saving rate is calculated from the identity: National saving = Investment + Current account balance.

(1) Sweden

Life expectancy at birth and adult survival indices are from *The Human Mortality Database.* The national saving rate is from Jones and Obstfeld (2001) (1860 – 1899), *European Historical Statistics 1750-1970* (1900 – 1948) and *Penn World Table* (1950 – 2000).

(2) United Kingdom

Life expectancy at birth and adult survival indces are from *The Human Mortality Database.* The national saving rate is from Jones and Obstfeld (2001) (1850 – 1945), Maddison (1992) (1946 – 1991), and *Penn World Table* (1992 – 2000).

(3) Italy

Life expectancy at birth and adult survival indices are from *The Human Mortality Database.* The national saving rate is from Jones and Obstfeld (2001) (1906 – 1945) and *Penn World Table* (1950 – 2000).

(4) United States

Life expectancy at birth is from *The Statistical History of the United States from Colonial Times to the Present* (1900 – 1959) and *World Development Indicator* (1960 – 2000). The national saving rate is from Maddison (1992) (1870 – 1988) and *Penn World Table* (1989 – 2000).

(5) Japan

Life expectancy at birth is from *Life Table*, various years and The Statistics and Information Department, Minister's Secretariat at the Ministry of Health, Labour and Welfare (1947 – 2000). The national saving rate is from Maddison (1992) (1885 – 1988) and *Penn World Table* (1989 – 2000).

(6) Taiwan

Life expectancy at birth and adult survival indices are from Mizaee (1979) (1906 – 1965) and the Department of Statistics, Ministry of the Interior, ROC (1970 – 2002). The national saving rate is from Maddison (1992). Death rate is from *International Historical Statistics: Africa and Asia* (1906 – 1975) and *Statistical Yearbook of the Republic of China* (1976 – 2001).

(7) India

Life expectancy at birth is from *Statistical Abstract India* (1881, 1991), United Nations, *Demographic Yearbook* 1967 (1901 – 1951) and *World Development Indicator* (1960 – 2000). Life expectancy at age 30 is from *Statistical Abstract India*. The national saving rate is from Maddison (1992) (1900 – 1987) and *Penn World Table* (1988 – 2000).

APPENDIX M. LIST OF THE VARIABLES

Variables	Description
S/Y	National saving rate
Y_{gr}	GDP growth
PRI	Price of investment goods
$D1$	Young dependency
q_{UN}	Undiscounted adult survival index
q_{PV}	Discounted adult survival index
$q_{15\text{-}60}$	Adult survival rate
Δq_{UN}(10 years)	Change in undiscounted adult survival index in 10 years
Δq_{UN}(20 years)	Change in undiscounted adult survival index in 20 years
Δq_{PV}(10 years)	Change in discounted adult survival index in 10 years
Δq_{PV}(20 years)	Change in discounted adult survival index in 20 years
$\Delta q_{15\text{-}60}$(10 years)	Change in adult survival rate in 10 years
$\Delta q_{15\text{-}60}$(20 years)	Change in adult survival rate in 20 years
$yr80$	Dummy variable for year 1980
$yr90$	Dummy variable for year 1990
$yr100$	Dummy variable for year 2000
Africa	Region dummy for Africa
Asia	Region dummy for Asia
Latin	Region dummy for Latin America
Mideast	Region dummy for Middle East

APPENDIX N. SUMMARY OF THE VARIABLES

	Total			1970			1980		
Variable	N	Mean	Standard Error	N	Mean	Standard Error	N	Mean	Standard Error
S/Y	475	0.1211	0.1304	98	0.1411	0.1323	103	0.1064	0.1256
Y_{gr}	459	0.0360	0.0302	98	0.0505	0.0221	103	0.0355	0.0282
PRI	475	0.8456	0.6691	98	0.8309	0.8800	103	0.9567	0.9503
$D1$	458	0.3501	0.1018	97	0.3869	0.0858	100	0.3710	0.0956
$D1 \cdot Y_{gr}$	443	0.0130	0.0118	97	0.0195	0.0103	100	0.0133	0.0122
q_{UN}	467	0.4983	0.0993	96	0.4511	0.0950	101	0.4819	0.0931
q_{PV}	467	0.1310	0.0238	96	0.1197	0.0233	101	0.1272	0.0225
$q_{15\text{-}60}$	451	0.7583	0.1402	95	0.7111	0.1456	95	0.7479	0.1323
Δq_{UN}(10 years)	463	0.0220	0.0198	96	0.0211	0.0127	99	0.0301	0.0126
Δq_{UN}(20 years)	357	0.0488	0.0316	0			99	0.0514	0.0219
Δq_{PV}(10 years)	463	0.0052	0.0049	96	0.0053	0.0032	99	0.0073	0.0031
Δq_{PV}(20 years)	357	0.0116	0.0077	0			99	0.0126	0.0055
$\Delta q_{15\text{-}60}$(10 years)	434	0.0234	0.0503	92	0.0388	0.0548	95	0.0343	0.0321
$\Delta q_{15\text{-}60}$(20 years)	336	0.0492	0.0720	0			93	0.0739	0.0722
$q_{UN} \cdot Y_{gr}$	452	0.0178	0.0154	96	0.0233	0.0115	101	0.0172	0.0136
$q_{PV} \cdot Y_{gr}$	452	0.0047	0.0040	96	0.0062	0.0030	101	0.0045	0.0036
$q_{15\text{-}60} \cdot Y_{gr}$	437	0.0274	0.0234	95	0.0370	0.0183	95	0.0265	0.0203

	1990			2000		
	N	Mean	Standard Error	N	Mean	Standard Error
S/Y	129	0.1151	0.1269	145	0.1234	0.1349
Y_{gr}	125	0.0248	0.0369	133	0.0363	0.0250
PRI	129	0.8039	0.3864	145	0.8138	0.4099
$D1$	123	0.3363	0.1032	138	0.3215	0.1051
$D1 \cdot Y_{gr}$	119	0.0088	0.0125	127	0.0117	0.0098
q_{UN}	127	0.5146	0.0919	143	0.5270	0.0998
q_{PV}	127	0.1350	0.0218	143	0.1377	0.0237
$q_{15\text{-}60}$	120	0.7809	0.1246	141	0.7778	0.1469
Δq_{UN}(10 years)	126	0.0268	0.0165	142	0.0128	0.0258
Δq_{UN}(20 years)	118	0.0559	0.0282	140	0.0411	0.0379
Δq_{PV}(10 years)	126	0.0063	0.0040	142	0.0028	0.0063
Δq_{PV}(20 years)	118	0.0134	0.0067	140	0.0095	0.0091
$\Delta q_{15\text{-}60}$(10 years)	114	0.0269	0.0287	133	0.0020	0.0638
$\Delta q_{15\text{-}60}$(20 years)	113	0.0542	0.0519	130	0.0270	0.0803
$q_{UN} \cdot Y_{gr}$	123	0.0127	0.0194	132	0.0192	0.0135
$q_{PV} \cdot Y_{gr}$	123	0.0033	0.0051	132	0.0050	0.0035
$q_{15\text{-}60} \cdot Y_{gr}$	116	0.0190	0.0294	131	0.0284	0.0201

Note: N is the number of observations.

APPENDIX N. SUMMARY OF THE VARIABLES (CONTINUED)

	Advanced Economies			Asia			Africa		
Variable	N	Mean	Standard Error	N	Mean	Standard Error	N	Mean	Standard Error
S/Y	139	0.1953	0.0955	64	0.1878	0.1580	125	0.0334	0.0988
Y_{gr}	134	0.0276	0.0335	61	0.0586	0.0273	124	0.0322	0.0298
PRI	139	0.8064	0.5821	64	0.5497	0.2291	125	1.0340	1.0252
$D1$	139	0.2329	0.0547	64	0.3516	0.0809	121	0.4451	0.0383
$D1 \cdot Y_{gr}$	134	0.0068	0.0085	61	0.0205	0.0110	120	0.0142	0.0138
$q_{15\text{-}60}$	136	0.8620	0.0415	62	0.7754	0.1004	119	0.5655	0.0950
q_{UN}	139	0.5857	0.0428	64	0.5008	0.0852	124	0.3765	0.0536
q_{PV}	139	0.1518	0.0096	64	0.1319	0.0200	124	0.1015	0.0135
$\Delta q_{15\text{-}60}$(10 years)	134	0.0077	0.0192	60	0.0562	0.0603	114	0.0164	0.0754
$\Delta q_{15\text{-}60}$(20 years)	107	0.0167	0.0278	47	0.1115	0.0735	88	0.0401	0.1034
Δq_{UN}(10 years)	138	0.0166	0.0137	64	0.0363	0.0147	123	0.0114	0.0250
Δq_{UN}(20 years)	106	0.0354	0.0238	50	0.0753	0.0211	94	0.0308	0.0326
Δq_{PV}(10 years)	138	0.0037	0.0031	64	0.0087	0.0037	123	0.0030	0.0065
Δq_{PV}(20 years)	106	0.0079	0.0053	50	0.0181	0.0051	94	0.0080	0.0085
$q_{15\text{-}60} \cdot Y_{gr}$	131	0.0238	0.0284	60	0.0454	0.0220	118	0.0190	0.0175
$q_{UN} \cdot Y_{gr}$	134	0.0163	0.0189	61	0.0295	0.0146	123	0.0124	0.0117
$q_{PV} \cdot Y_{gr}$	134	0.0042	0.0049	61	0.0078	0.0038	123	0.0033	0.0031

	Latin America			Middle East		
	N	Mean	Standard Error	N	Mean	Standard Error
S/Y	110	0.0896	0.1107	30	0.1188	0.1309
Y_{gr}	109	0.0354	0.0207	25	0.0499	0.0237
PRI	110	0.8223	0.3153	30	0.9750	0.5635
$D1$	98	0.3811	0.0674	30	0.3963	0.0621
$D1 \cdot Y_{gr}$	97	0.0136	0.0092	25	0.0205	0.0115
$q_{15\text{-}60}$	100	0.8167	0.0649	28	0.8269	0.0518
q_{UN}	105	0.5207	0.0626	29	0.5174	0.0722
q_{PV}	105	0.1368	0.0147	29	0.1359	0.0168
$\Delta q_{15\text{-}60}$(10 years)	92	0.0303	0.0278	28	0.0296	0.0168
$\Delta q_{15\text{-}60}$(20 years)	67	0.0608	0.0400	22	0.0659	0.0460
Δq_{UN}(10 years)	104	0.0286	0.0122	28	0.0375	0.0158
Δq_{UN}(20 years)	79	0.0607	0.0202	23	0.0849	0.0274
Δq_{PV}(10 years)	104	0.0067	0.0029	28	0.0089	0.0038
Δq_{PV}(20 years)	79	0.0143	0.0049	23	0.0202	0.0066
$q_{15\text{-}60} \cdot Y_{gr}$	99	0.0288	0.0172	23	0.0387	0.0162
$q_{UN} \cdot Y_{gr}$	104	0.0181	0.0109	24	0.0249	0.0115
$q_{PV} \cdot Y_{gr}$	104	0.0048	0.0029	24	0.0066	0.0030

APPENDIX O. OLS AND 2SLS SAVING ESTIMATES OF AFRICA

	OLS Estimates					
	(1) $q=q_{UN}$	(2) $q=q_{UN}$	(3) $q=q_{PV}$	(4) $q=q_{PV}$	(5) $q=q_{15\text{-}60}$	(6) $q=q_{15\text{-}60}$
$q \cdot Y_{gr}$	14.4532***	19.8914***	57.5550***	78.8202***	6.7463**	1.7424
	(5.3071)	(5.8444)	(20.6319)	(22.6233)	(2.7257)	(4.5950)
Δq(10 years)	-1.1108**		-4.2974**		-0.2054	
	(0.4758)		(1.8292)		(0.1269)	
Δq(20 years)		-0.8237*		-3.1726*		-0.0703
		(0.4760)		(1.7997)		(0.1578)
Y_{gr}	-3.5600	-9.1007**	-3.9185	-9.5837**	-2.4273	1.4575
	(3.5101)	(4.4012)	(3.5483)	(4.4166)	(3.1884)	(5.3894)
$D1 \cdot Y_{gr}$	-3.1405	4.3331	-3.2594	4.2438	-2.6356	-4.8438
	(5.0252)	(5.9802)	(4.9867)	(5.8384)	(4.4674)	(6.4493)
PRI	0.0026	0.0056	0.0026	0.0055	0.0045	0.0075
	(0.0089)	(0.0126)	(0.0089)	(0.0126)	(0.0082)	(0.0119)
Constant	0.0557***	0.0263	0.0569***	0.0268	0.0387*	-0.0045
	(0.0216)	(0.0263)	(0.0216)	(0.0263)	(0.0226)	(0.0228)
Adjusted R^2	0.0991	0.0926	0.1028	0.0980	0.0340	-0.0293
N	120	92	120	92	109	84
P-values Ygr	0.0128	0.0036	0.0113	0.0029	0.0080	0.0347
P-value, year dummies	0.1789	0.9972	0.1617	0.9978	0.2780	0.7565
Y_{gr} Effect	0.4842	0.5040	0.4718	0.4954	0.2145	0.3011
	2SLS Estimates					
	(7) $q=q_{UN}$	(8) $q=q_{UN}$	(9) $q=q_{PV}$	(10) $q=q_{PV}$	(11) $q=q_{15\text{-}60}$	(12) $q=q_{15\text{-}60}$
$q \cdot Y_{gr}$	52.7313**	71.1869*	207.8891**	285.0031*	32.6990	-27.9698
	(24.0027)	(42.5131)	(92.2748)	(162.1702)	(21.3307)	(21.8711)
Δq(10 years)	-2.2736***		-8.4416***		-0.7954**	
	(0.6554)		(2.5474)		(0.3879)	
Δq(20 years)		-2.2616**		-8.2643**		0.8377
		(0.9199)		(3.3346)		(0.6619)
Y_{gr}	-23.1727	-36.6757	-24.2889	-39.0724	-23.6650	38.7744
	(17.5865)	(29.9757)	(17.6598)	(30.0588)	(23.7059)	(23.6481)
$D1 \cdot Y_{gr}$	9.6215	23.2928	8.9388	23.1263	11.7997	-40.2208*
	(19.4277)	(29.7684)	(18.9110)	(28.9607)	(23.0428)	(23.1488)
PRI	0.0048	0.0077	0.0049	0.0078	0.0083	-0.0018
	(0.0134)	(0.0183)	(0.0132)	(0.0180)	(0.0129)	(0.0116)
Constant	0.0804	0.0553	0.0875	0.0615	0.0823	-0.1881*
	(0.1079)	(0.1096)	(0.1075)	(0.1086)	(0.1535)	(0.1050)
N	116	89	116	89	105	81
P-value Ygr	0.0001	0.0194	0.0001	0.0189	0.0105	0.1066
P-value, year dummies	0.0688	0.7081	0.0777	0.7843	0.324	0.4082
P-value, Hausman	0.0000	0.0000	0.0000	0.0000	0.0003	0.0000
Y_{gr} Effect	0.9660	1.1720	0.7888	0.8448	0.0789	4.6921

Note: Dependent variable is the national saving rate. See the notes of Tables 4.2, 4.4, and 4.6.

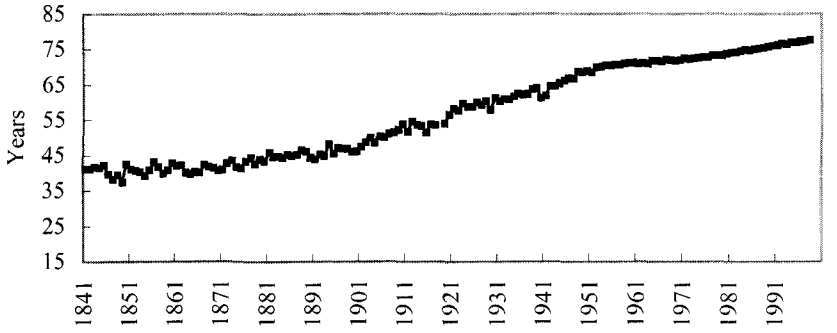
APPENDIX P. OLS AND 2SLS SAVING ESTIMATES OF LATIN AMERICA

	OLS Estimates					
	(1) $q=q_{UN}$	(2) $q=q_{UN}$	(3) $q=q_{PV}$	(4) $q=q_{PV}$	(5) $q=q_{15\text{-}60}$	(6) $q=q_{15\text{-}60}$
Y_{gr}	-10.5805 *	-7.9820	-12.0754 **	-9.5316 *	-14.7548 **	-20.7862 ***
	(5.3977)	(5.2082)	(5.9379)	(5.6140)	(7.1353)	(7.2839)
$q \cdot Y_{gr}$	19.7695 ***	16.0810 **	85.5407 **	71.4458 **	17.1708 ***	22.8745 ***
	(7.5344)	(6.8239)	(33.0332)	(29.2263)	(7.3217)	(7.0772)
$D1 \cdot Y_{gr}$	-0.5661	-1.6604	-0.3457	-1.2794	0.4780	5.2143
	(4.7951)	(5.3884)	(4.7294)	(5.3507)	(5.5944)	(6.0568)
Δq(10 years)	0.1616		0.0209		-0.0537	
	(1.0811)		(4.5000)		(0.5677)	
Δq(20 years)		-0.0767		-0.7883		-0.1921
		(0.5239)		(2.0994)		(0.3189)
PRI	-0.0991 ***	-0.1047 ***	-0.0998 ***	-0.1056 ***	-0.0872 ***	-0.0924 ***
	(0.0289)	(0.0259)	(0.0287)	(0.0260)	(0.0303)	(0.0254)
Constant	0.2741 ***	0.1923 ***	0.2787 ***	0.1994 ***	0.2568 ***	0.1948 ***
	(0.0745)	(0.0622)	(0.0748)	(0.0611)	(0.0805)	(0.0544)
Adjusted R^2	0.1041	0.1248	0.1071	0.1275	0.0795	0.1537
N	97	74	97	74	88	66
P-values Ygr	0.0113	0.0097	0.0151	0.0090	0.0848	0.0087
Pvalues, year dummies	0.4646	0.7808	0.481	0.8012	0.5991	0.6329
Y_{gr} Effect	-0.5025	0.0495	-0.5025	0.0547	-0.5490	0.1687
	2SLS Estimates					
	(7) $q=q_{UN}$	(8) $q=q_{UN}$	(9) $q=q_{PV}$	(10) $q=q_{PV}$	(11) $q=q_{15\text{-}60}$	(12) $q=q_{15\text{-}60}$
$q \cdot Y_{gr}$	67.2560 *	43.5460 *	292.6674 **	203.9623 ***	75.7777 **	76.9025 ***
	(34.1094)	(25.3367)	(137.0070)	(93.3407)	(29.0495)	(19.3787)
Δq(10 years)	0.3212		3.8715		1.3736	
	(1.2935)		(5.7535)		(1.1990)	
Δq(20 years)		-0.2666		-1.5875		-0.1616
		(0.8785)		(2.6486)		(0.5707)
Y_{gr}	-35.8765	-32.0935	-40.1386	-36.3458	-66.9630 *	-78.3239 ***
	(27.3550)	(22.3394)	(27.6489)	(22.1696)	(34.0623)	(26.3404)
$D1 \cdot Y_{gr}$	4.5534	17.2466	2.4744	16.5142	6.8277	33.1161
	(24.3662)	(23.3636)	(23.1032)	(23.0549)	(25.7990)	(27.8133)
PRI	-0.1427 ***	-0.0729	-0.1449 ***	-0.0801	-0.1270 **	-0.0432
	(0.0477)	(0.0719)	(0.0479)	(0.0635)	(0.0592)	(0.0787)
Constant	0.3804 **	0.2656 **	0.3785 **	0.2619 **	0.4690 **	0.2262 *
	(0.1869)	(0.1188)	(0.1824)	(0.1145)	(0.1834)	(0.1255)
N	87	66	87	66	82	62
P-value Ygr	0.0215	0.1033	0.0208	0.0456	0.0147	0.0013
P-value, year dummies	0.1045	0.7925	0.0825	0.7501	0.0357	0.4777
P-value, Hausman	0.0114	0.1908	0.0055	0.1592	0.0004	0.0011
Ygr Effect	0.8783	-2.3814	0.8503	-1.5926	-2.4722	-2.1664

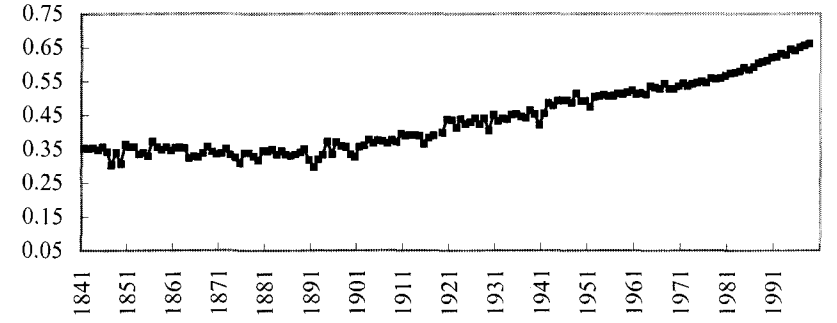
Note: Dependent variable is the national saving rate. See the notes of Tables 4.2, 4.4, and 4.6.

APPENDIX Q. LIFE EXPECTANCY, ADULT SURVIVAL, AND THE SAVING RATE IN THE UNITED KINGDOM

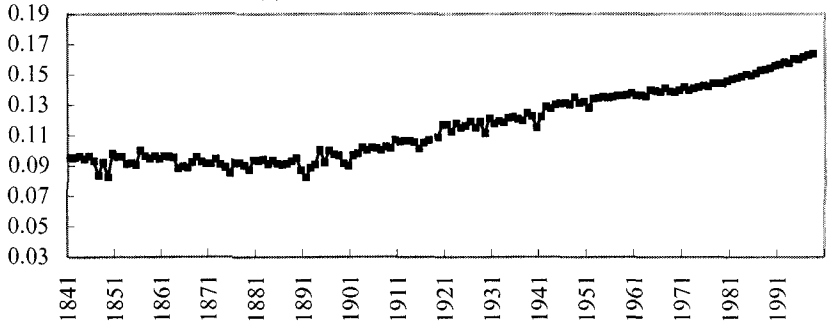
(a) Life Expectancy at Birth



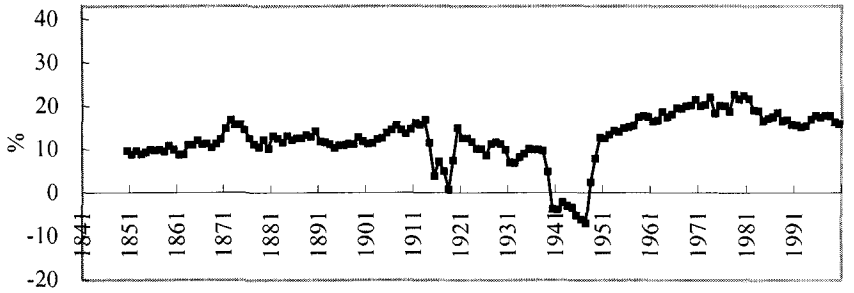
(b) Undiscounted Adult Survival Index



(c) Discounted Adult Survival Index

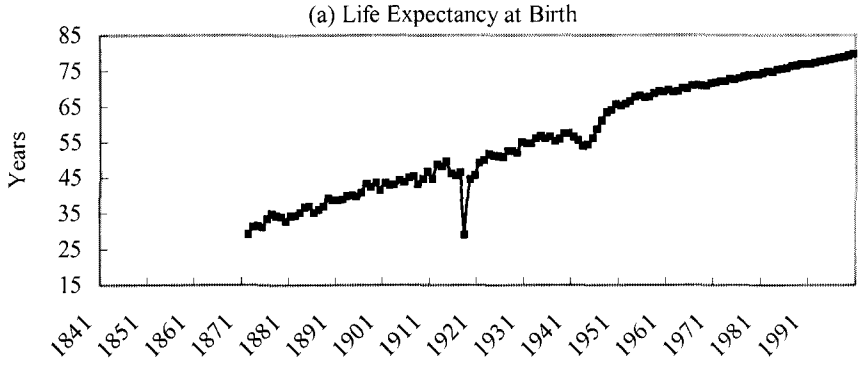


(d) National Saving Rate

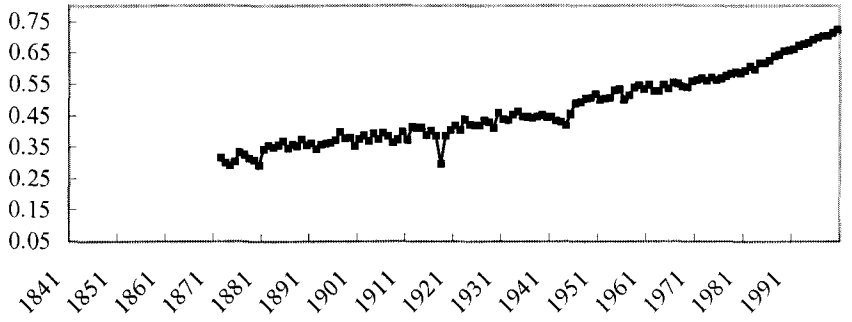


APPENDIX R. LIFE EXPECTANCY, ADULT SURVIVAL, AND THE SAVING RATE IN ITALY

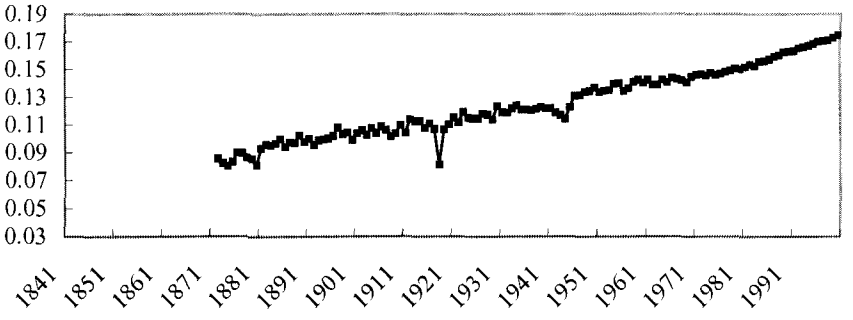
(a) Life Expectancy at Birth



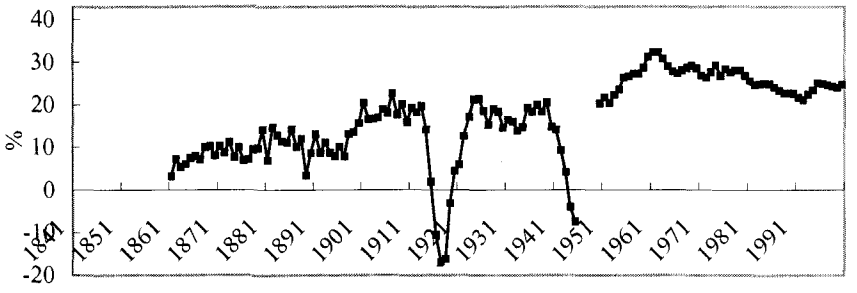
(b) Undiscounted Adult Survival Index



(c) Discounted Adult Survival Index

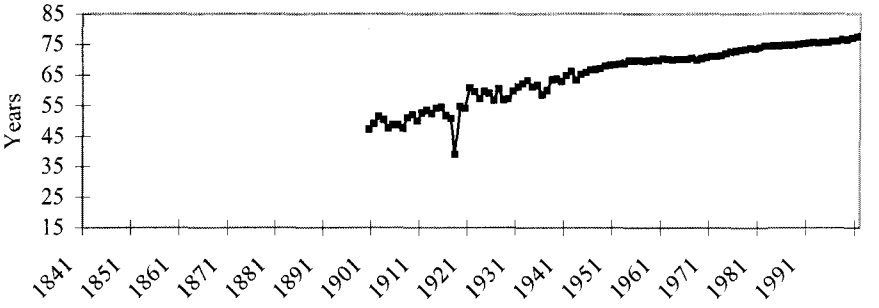


(d) National Saving Rate

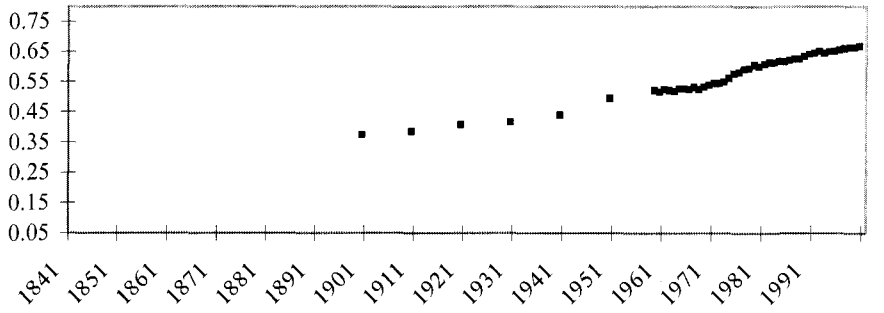


APPENDIX S. LIFE EXPECTANCY, ADULT SURVIVAL, AND THE SAVING RATE IN THE UNITED STATES

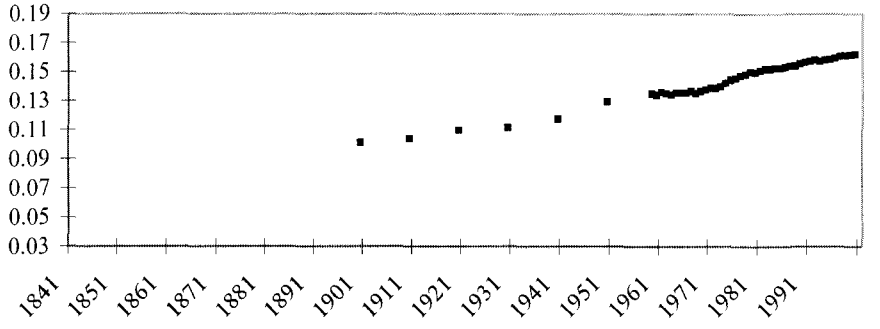
(a) Life Expectancy at Birth



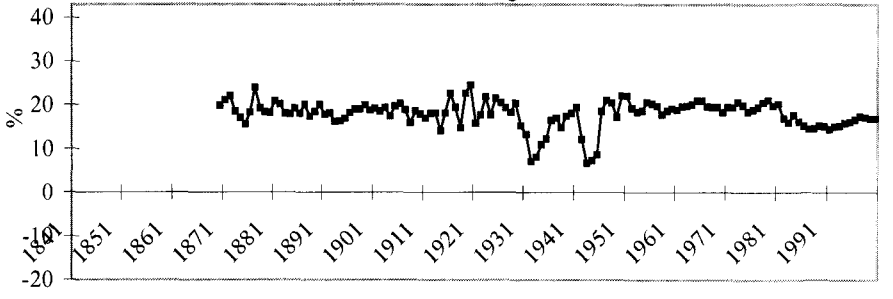
(b) Undiscounted Adult Survival Index



(c) Discounted Adult Survival Index

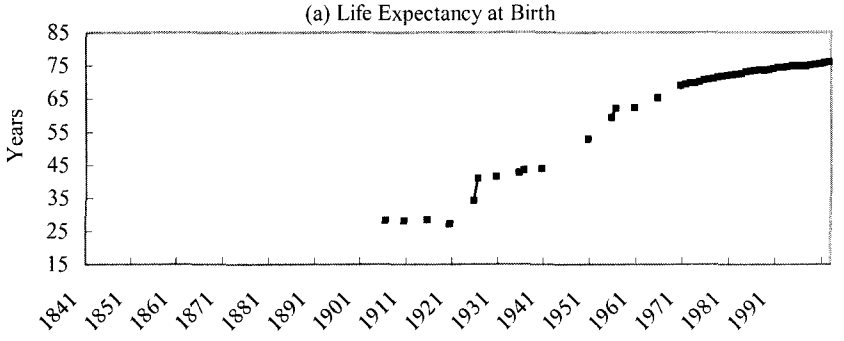


(c) National Saving Rate

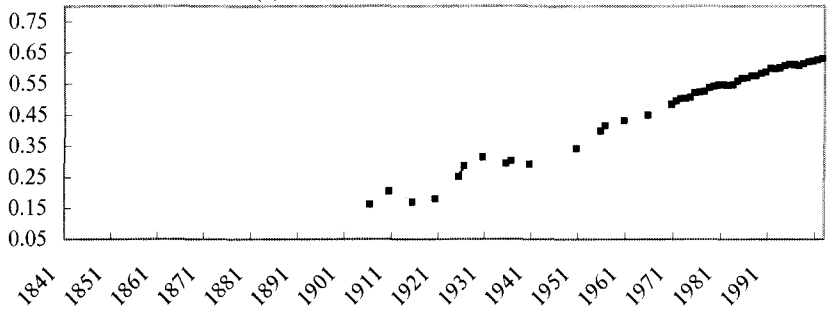


APPENDIX T. LIFE EXPECTANCY, ADULT SURVIVAL, AND THE SAVING RATE IN TAIWAN

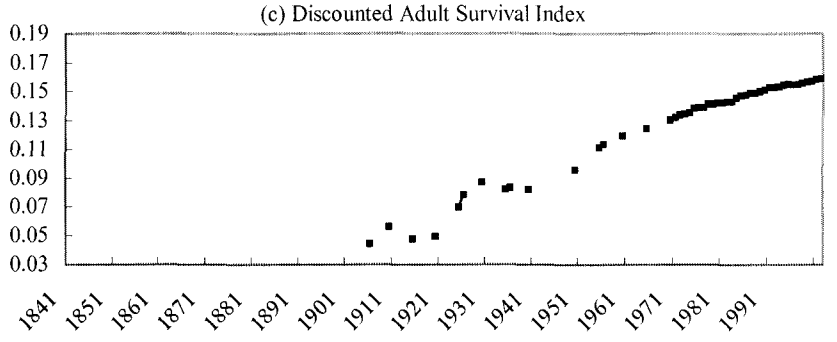
(a) Life Expectancy at Birth



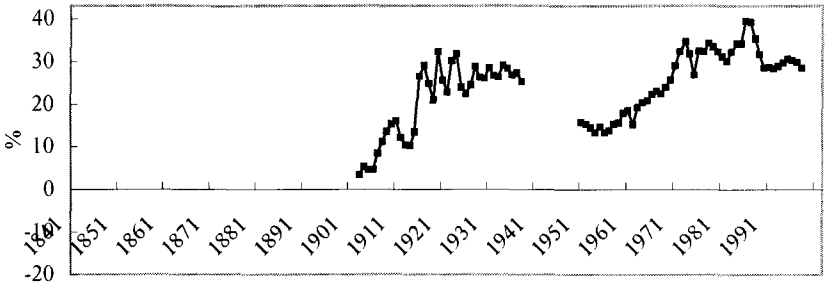
(b) Undiscounted Adult Survival Index



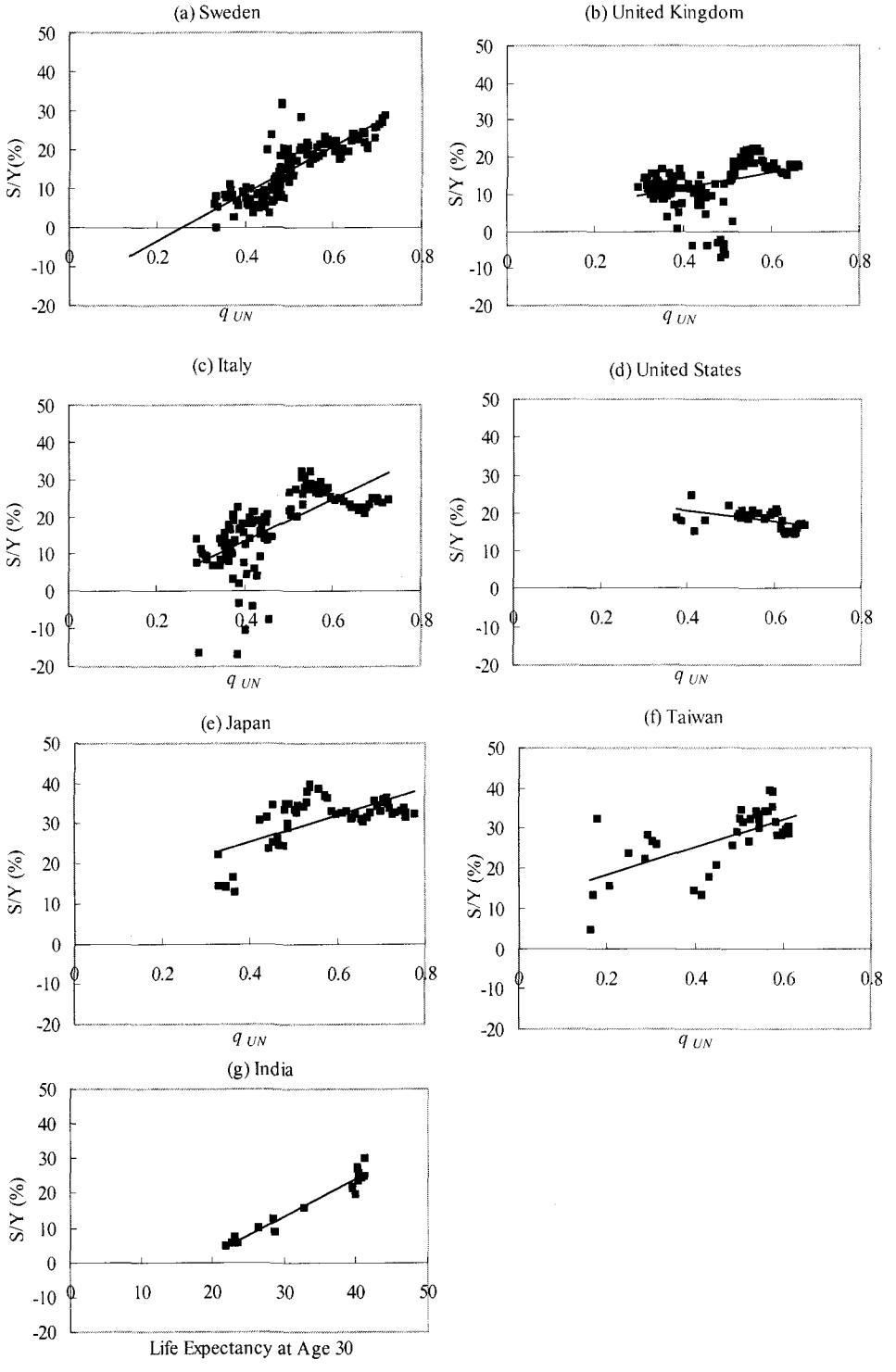
(c) Discounted Adult Survival Index



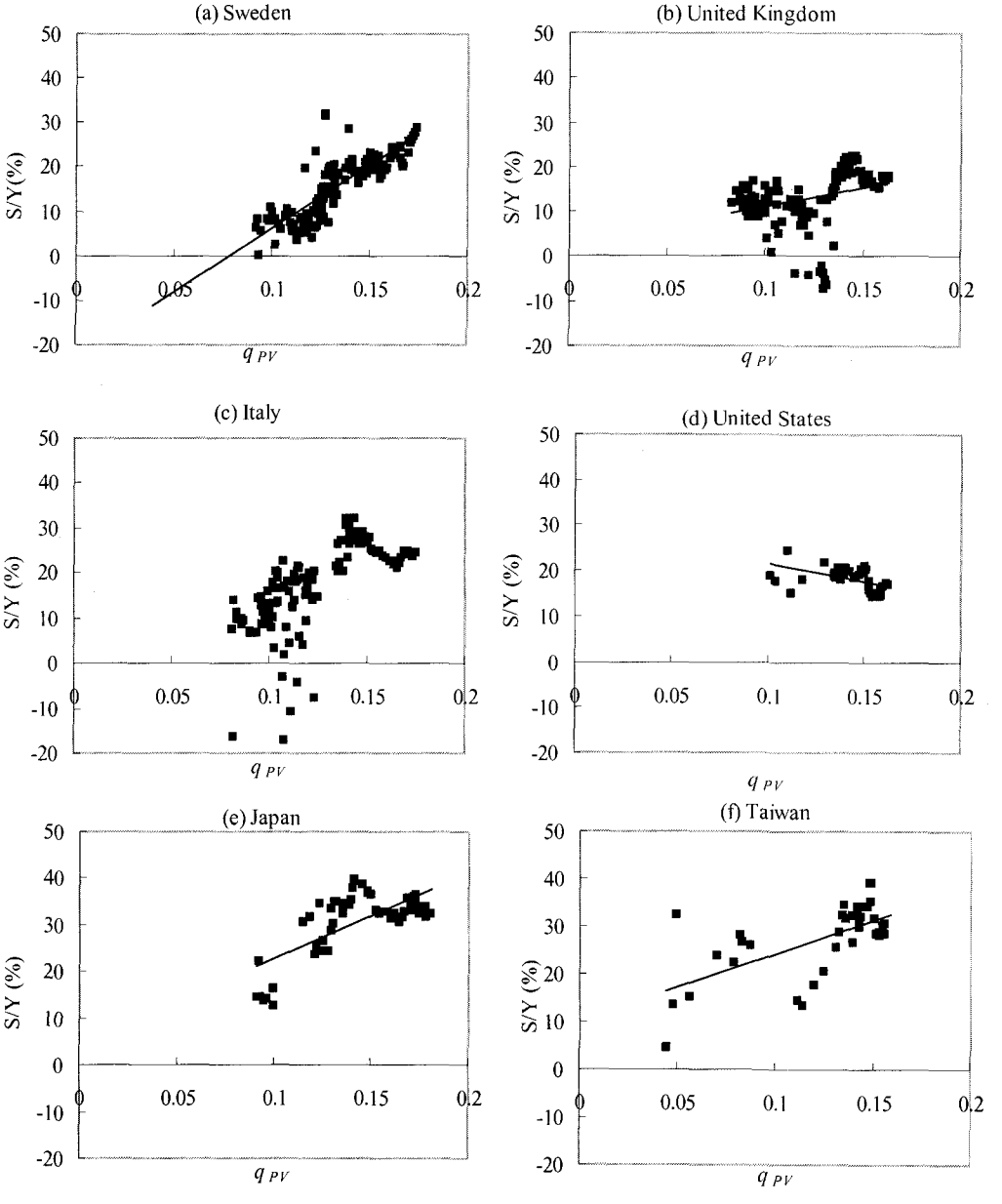
(d) Natioanal Saving Rate



APPENDIX U. UNDISCOUNTED ADULT SURVIVAL INDEX (q_{UN}) AND THE NATIONAL SAVING RATE (S/Y)



APPENDIX V. DISCOUNTED ADULT SURVIVAL INDEX (q_{PV}) AND THE NATIONAL SAVING RATE (S/Y)



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