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ELECTRIFICATION PROCESSES IN WARM RAIN CLOUDS.

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ELECTRIFICATION PROCESSES
IN WARM RAIN CLOUDS

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DOCTOR OF PHILOSOPHY
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By
Lothar H. Ruhnke

Dissertation Committee:
E. J. Workman, Chairman
P. F. Weaver
A. H. Woodcock
C. S. Ramage
E. F. Danielsen
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SYMBOLS

A  factor governing influence of cloud water on electric conductivity
b  ion mobility
D  molecular diffusion constant
E  electric field
e  elementary charge
I  total air-earth current density
i  fair weather current density
i_o  current density at surface
k  Boltzmann constant
ln  natural logarithm
M  drop mass
m  cloud model integer
N  number of drops per unit volume
n  number of molecular ions per unit volume
Q  space charge density due to charges residing on rain drops
Q' effective space charge density carried by wind to and from rain area
Q_p  drop charge
q  space charge density due to charges not residing on rain drops
q_o  space charge density at surface level
q_{oo}  ionization constant
R  drop radius, radius in general
r_1  radius of cloud droplet
r_2  radius of condensation nucleus
S  parameter describing conductivity at cloud boundary
T  absolute temperature
t  time
U  horizontal wind speed
V  terminal velocity of falling raindrops
V'  average velocity difference between raindrops
w  liquid cloud water content
w_0  liquid cloud water parameter
z  distance from upper cloud boundary
z_o  cloud thickness
Z_1  number of cloud droplets per unit volume
Z_2  number of condensation nuclei per unit volume
\alpha  recombination coefficient between small ions
\beta  drop collection parameter
\epsilon  dielectric constant (equal to 1/4 \pi in electrostatic system)
\lambda  electric air conductivity
\lambda_o  air conductivity above clouds
\lambda_{oo}  air conductivity below clouds
\[ \pi \quad 3.14159 \]

\[ \eta \quad \text{efficiency of raindrops to collect cloud droplets} \]

\[ \eta' \quad \text{collection efficiency among raindrops} \]

\[ \rho \quad \text{liquid water density} \]
ABSTRACT

The problem of electrical charges on raindrops from non-thunderstorm clouds is investigated. Measurements of raindrop charges, precipitation currents and space charges made in various parts of the world indicate that on the average positive charges prevail on raindrops. To explain rain charges, a well known hypothesis requires the melting of snow. In Hawaii, however, rain often forms by condensation and coalescence without ice or snow in the upper portions of the cloud. Yet raindrop charges do not differ from charges measured in other types of rain.

Measurements of charges on Hawaiian rain are discussed together with observations of electric field and condensation nuclei profiles. It was found that positive charges are of the order of $10^{-4}$ esu. A correlation of these charges with wind speed was found. Electric fields in rain clouds are generally positive except at the lower cloud boundary where negative fields of up to $-1000 \, V/m$ exist.

Several thunderstorm theories are discussed and their applicability to the warm rain mechanism is examined. A new hypothesis is necessary to explain the observed electrical features of warm rain. A theory based on influence is presented and formulated.
Due to low electrical conductivity inside the cloud, positive space charges form in the upper part of the cloud and negative in the lower, influenced by an impressed conduction current from above the cloud. These space charges diffuse to the raindrops. Considering a coalescence mechanism for drop growth, the residence time of raindrops in the negative space charge area is much smaller than the corresponding time in the positive area, and so the drops usually leave the cloud with a positive charge. Numerical solutions to drop charges and electric field profiles using a realistic cloud model were obtained through the use of digital computer techniques. The theoretical results for charges on raindrops and electric field patterns agree well with observations.
CHAPTER I

INTRODUCTION

Although the general electrical structure of the typical thunderstorm cloud (cumulonimbus) can be regarded as well established, surprisingly little is known about the more frequent clouds that give continuous or shower rain without thunder or lightning. Yet it is certain that there are electrical effects connected with stratus clouds since charges are measured on precipitation particles. Furthermore, the electric field during continuous rain and snow is known to be different from that in fair weather. The effects are, however, much less pronounced than those associated with thunderstorm clouds. Even so, the possibility exists that electrical effects may influence appreciably the mechanism of rain formation in stratus clouds.

A sharp distinction must be made between frozen and liquid precipitation particles, because of the well-known effects of electrification during change of state. All-water clouds also require a different rain-forming mechanism than clouds which simultaneously contain liquid and frozen particles. It seems logical, therefore, to investigate the electrical structure of all-water clouds (warm clouds) separately from other clouds in order to isolate the processes involved in the electrification of rain.
For simplicity the term "warm cloud" will be used in this text to describe clouds whose temperatures are above 0°C at all times and in all parts of the cloud. Drop growth by condensation and coalescence will be assumed in such clouds, and the influence of possible fallout from higher clouds will be neglected. In particular, "Hawaiian warm clouds" and "Hawaiian warm rain" shall denote conditions at the windward side of the island of Hawaii, where orographically enhanced rain clouds frequently form in the trade winds on the slopes of the mountains Mauna Loa and Mauna Kea.

The objective of this study is to evaluate observations of electric charges, fields, and currents of precipitating warm clouds and to discuss possible mechanisms that can lead to electrification. Such a study must first consider the evidence of electrification of warm rain. Second, it seems advisable to discuss the many existing hypotheses of cloud electrification with relation to the experimental data. The attempt can then be made to develop a new hypothesis or modify an existing theory as required by the results of the investigation.
CHAPTER II

HISTORICAL BACKGROUND

The importance of investigating electric charges on precipitation particles was recognized very early (Lord Kelvin, 1860). Elster and Geitel (1888) made measurements of raindrop charges, and Gerdien (1903) measured the total electrical current carried by precipitation. These investigations, however, were not completely satisfactory. The first reliable data were published by Simpson (1909) and later by Gschwend (1922). During the last few decades, a considerable number of raindrop charge measurements have been made. In particular, Chalmers and Pasquil (1938), Scrase (1938), and Gunn (1940) give good statistical samples of charges on rain drops. Until recently, however, no distinction was made as to temperature and the presence of frozen precipitation particles in the clouds.

Explanations for the charge on particles falling from clouds have been suggested by many prominent investigators in the field, but their interest very often shifted to the particular problem of thunderstorm electricity. Early investigators failed to recognize that thunderstorms may involve electrification mechanisms very different from the mechanism inherent in a coalescence process.
In 1937, Simpson and Scrase established that the main charge separation occurs inside a thunderstorm cloud at a level where the temperature is below freezing. Several electrification hypotheses involving ice have been developed since that time, but these theories will not be discussed here.

Hypotheses designed specifically to explain the charges found in continuous rain are relatively few. Chalmers (1958) used an effect investigated by Dinger and Gunn (1946) to suggest that the positive charge on raindrops results from melting snow. More recently Sartor (1961) revived a theory by Elster and Geitel (1885) to explain electromagnetic radiation from clouds as being produced by collisions of charged or uncharged particles in an electric field. Muhleisen (1958) reconsidered an old concept of Volta, in which charges are separated during the evaporation of water. If this concept is substantiated by experimental evidence, it will be important in understanding warm rain electrification.

In warm rain clouds, the growth of raindrops results from collisions and coalescence between numerous small cloud droplets and a few large drops. Langmuir's (1948) solution of the collision problem represents a good first order approximation. Pearcy and Hill (1957) and Hocking (1959) extended Langmuir's theory to collisions between particles of almost equal size. Cochet (1951) demonstrated the influence of electrical charge on the collection efficiency. Sartor (1960), Davis
(1962), and Krasnogorskaja (1963) investigated the role of electric forces in precipitation.

Twomey (1956) developed a technique for determining the charge on individual droplets and found that in warm clouds 80 percent of the droplets were positively charged and that negatively charged droplets appeared to be present only if ice crystals were found in the cloud. Woessner and Gunn (1956) also made contributions toward measuring droplet and drop charges on rain.

Research on rain from warm clouds was advanced substantially by investigations carried out on the Island of Hawaii. Warm rains are frequent, and drop sampling is possible along the slopes of Mauna Loa, where orographic clouds often form in the relatively steady trade winds. Woodcock (1952) investigated the effect of sea-salt particles on warm rain formation in Hawaii, Blanchard (1953) determined raindrop-size distributions in Hawaiian rains, and Squires (1958) studied the microstructure and liquid water content of warm clouds. A number of investigators carried out cooperative research on Hawaiian rain during Project Shower in 1954 and during the Warm Rain Project in 1965. As a part of the latter project, Takahashi and Isono (1967) measured the charges on raindrops using balloon-borne equipment. Since 1966 the staff of the Cloud Physics Observatory in Hilo, Hawaii, has sampled charges on warm rain drops at ground level and made measurements of
the electric field, space charge, and pertinent meteorological parameters (Workman, 1968).
CHAPTER III

RESULTS OF EXPERIMENTS

A. Altitude Profile of Electric Fields

1. Measurements Along Slope Roads

In order to arrive at a reasonable description of the process of warm rain electrification, it is necessary to investigate the electric field profile of orographic warm clouds before and during rain. Because it is difficult to use radiosondes to investigate altitude profiles of electric field, an attempt was made to record the electric fields along sloping roads. From such data one can derive an approximate altitude profile. Many earlier measurements of cloud drops and cloud nuclei were successfully made on the windward slope roads of the island.

The influence of the terrain and the horizontal inhomogeneity of the atmosphere make it impossible, of course, to derive more than a rough profile from such data. However, Reiter's (1964) successful derivation of a climatologically valid atmospheric electric altitude profile from a network of ground-based mountain stations suggests the possibility of a similar approach in Hawaii. While driving through the orographic cloud layer on Mauna Loa and Mauna Kea, I measured the electric field using a radioactive probe collector method. A small
solid state electrometer was developed for this purpose. Because of the crude method used, rain and cloud intensity could only be estimated.

Figure 1 shows a typical profile of the electric field through a slightly precipitating orographic cloud on the saddle road. A fair weather field averaging about 130 V/m was measured below nonprecipitating clouds. Inside the cloud, electric fields were variable and larger, often reaching several hundred volts per meter. Above the cloud a fair weather field of 100 V/m was typical.

In light continuous rain, slightly negative values of the electric field were found at the lower cloud boundary (Figure 2). The other parts of the cloud, as well as the field below the cloud, seemed unaltered. In moderate rain a strong negative field appeared within a rather narrow layer at the lower cloud boundary (Figure 3). Values of up to -1000 V/m were typical in this layer. Below cloud, the absolute value of the negative field decreased with distance from the cloud boundary. At sea level, electric fields of between several hundred volts per meter negative up to 100 V/m positive were generally observed. In heavy rain, splashing of rain drops possibly produced negative fields and charges at the surface, thus limiting the method of observation.
2. Radiosonde Measurements

Five successful radiosonde flights were made from the Hilo airport to measure electric fields. Solid-state electrometers were designed, which were used in conjunction with a standard weather radiosonde. All flights were made in continuous light rain. Below the clouds, electric fields were highly variable (Figures 4 and 5). A negative field prevailed on the average; occasional variations were in the range from +100 V/m to -1000 V/m. On all the flights the maximum negative field appeared at the lower cloud boundary. Inside the cloud strong variations occurred. At the upper cloud boundary the electric field decreased very rapidly to about 100 V/m and thereafter continued to decrease steadily.

B. Data on Small Nuclei

In an effort to determine the electrical conductivity of the atmosphere, measurements of Aitken nuclei were made with a Rich-type (1950) nucleus counter. This instrument counts most of the nuclei of diameter $10^{-7}$ cm or larger. From such data a profile of air conductivity and small ion concentration can be estimated, as shown by Ruhnke (1961). On cloudless days, mixing in the layers below the inversion results in a relatively uniform profile up to the inversion height, where the concentration decreases from about 10,000 to a few hundred per cm$^3$ (Figure 6). This profile is altered considerably if clouds are present.
At the condensation level a sharp decrease in nuclei concentration occurs, sometimes to a value even below that found above the tropical inversion.

Similar results were found by Junge (1954) during aerosol sampling in Hawaii. It must be assumed, therefore, that some mechanism operates inside the cloud to remove most of the particles. Facy (1957) observed a similar effect and explained it by a flux of these particles in a field of vapor pressure gradients during the process of condensation.

During light continuous rain the nuclei profile observed in the rain below the cloud did not show a change from no-rain condition. This observation indicates that raindrops do not collect small nuclei.

C. Data on Drop Charges

A substantial amount of unpublished data on raindrop charges is available at the Cloud Physics Observatory in Hilo, Hawaii. Dr. E. J. Workman, Director of the Observatory, allowed the results of measurements made during 1966 and 1967 to be used in the present study. These observations were made during a variety of synoptic conditions, not only in warm rain. For this reason it was necessary to separate warm rain data from measurements that might possibly be influenced by charge generation during the change of phase of water.
A sizable percentage of the total rain in Hawaii may not originate from orographically enhanced clouds, restricted by tropical inversion, but rather from major disturbances, such as Kona storms. Since these disturbances are often associated with the absence of inversion, cloud top growth can reach well above the freezing level. It was considered necessary to establish from statistical data some criteria which would permit the selection of cases of warm rain from the available data.

D. Data Selection Criteria

Cold rain may be defined, by analogy to warm rain, as rain which comes from clouds that reach the 0°C isotherm. This definition does not necessarily imply that ice and snow particles are present, but only that they may be present, and it must be restricted further because of a lack of specific data on cloud top heights. Relative humidity measurements from radiosonde soundings were used to indicate the elevation of the cloud top.

Woodcock and Friedman (1963) correlated values of relative humidity, as indicated by radiosonde soundings, with the maximum altitude of cloud tops and found that at Hilo the level of the tops lies about 500 m above the 50 percent relative humidity level.

Further information was provided by Lee (1967), who listed the observed lightning and thunderstorm activity at Hilo for several years. A comparison of his data with radiosonde records shows that
thunderstorms occur when the indicated relative humidity at the 0°C isotherm is 40 percent or higher (Figure 8). Since the occurrence of thunderstorms is very likely associated with cloud tops that extend well above the freezing level, it is evident how inaccurate humidity sensors are to determine cloud top levels. Nevertheless, radiosonde data on humidity, together with information on temperature inversions, remain the best available indicators of warm rain.

On several occasions cloud top levels were estimated from the slope road on Mauna Loa. Comparison with radiosonde data revealed that cloud tops almost never exceeded the 60 percent relative humidity level.

On the basis of this information, cold rain is defined somewhat arbitrarily as rain that originates from clouds where radiosonde data within a six-hour period indicate 60 percent relative humidity or higher at the 0°C isotherm. Radiosonde soundings were made daily at 0200 hours and 1400 hours LST and each sounding was assumed to represent a twelve-hour period. Hourly rain data were available from recording rain gauges in Hilo.

Because the strong orographic effect on rainfall in Hawaii probably influences the percentage of cold rain, it seemed necessary to compare the results at Hilo with those from a nearby station at a higher elevation but close enough to Hilo so that radiosonde data would be equally applicable. The recording rain gauge at Kaumana (elevation
1000 feet) was chosen for this purpose. It is situated on the saddle road close to the location where studies were carried out during the Warm Rain Project in 1965 (Figure 9). The Cloud Physics Observatory of the University of Hawaii is located between the Hilo airport and Kaumana. The average results, on the ratio of cold to warm rain, therefore, should apply to the site of the Observatory.

The frequency of cold rain conditions is shown in Figure 10. On the average, cold rain conditions existed in Hilo 13 percent of the time. Monthly data on warm and cold rain from May 1965 to April 1967 are shown in Figures 11 and 12. It is evident that an annual variation exists, although some months are exceptional.

In general, the amount of cold rain is highly variable from month to month, whereas warm rain is quite uniformly distributed over the year. The warm rain follows approximately a log normal distribution, with a mean of 5.54 inches per month. Cold rain—although the mean of 5.12 inches per month is not very different—has a distinctly different distribution, with a higher standard deviation.

Figure 13 shows cold rain as a percentage of the total rain. The two-year average indicates that 44 percent of the total rain in Hilo is cold, and the corresponding percentage at Kaumana is 41 percent. The cold rain falls 13 percent of the time.
E. Selection of Six Cases

As noted before, data collected at the Cloud Physics Observatory include both warm and cold rain. Although the so-called cold rain does not necessarily involve the ice and electrification mechanisms that are not found in warm rain, for this study only those cases that were reasonably certain to pertain only to warm rain had to be selected. Five such cases were finally chosen. In each case a strong inversion persisted for a period of at least three days. Cloud development was limited to the inversion height, the 60 percent relative humidity altitude was below the 0°C isotherm, and no high clouds above the inversion were reported. The last criterion was considered necessary to avoid natural seeding by fallout from cirrus and other high clouds.

A sixth case was selected for comparison. In this case the inversion was very weak, and cloud development reached the freezing level at about 13,500 feet. Such conditions are typical of a tropical disturbance (Kona storm) as far as rainfall and cloud cover are concerned.

F. Results of Data Evaluation on Drop Charges

Table I summarizes the results of the six cases and indicates the mean electric charge and drop size, together with the prevailing weather on these days. Figure 14 shows the relative drop size spectra.
<table>
<thead>
<tr>
<th>Case No.</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
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<tr>
<td>Number of sampled drops</td>
<td>114</td>
<td>67</td>
<td>65</td>
<td>105</td>
<td>133</td>
<td>137</td>
</tr>
<tr>
<td>Sampling duration (min)</td>
<td>39</td>
<td>2</td>
<td>2</td>
<td>7</td>
<td>9</td>
<td>48</td>
</tr>
<tr>
<td>Average radius (mm)</td>
<td>0.54</td>
<td>0.78</td>
<td>1.33</td>
<td>1.04</td>
<td>0.65</td>
<td>0.57</td>
</tr>
<tr>
<td>Average drop mass (mgm)</td>
<td>0.78</td>
<td>2.28</td>
<td>11.1</td>
<td>6.49</td>
<td>1.13</td>
<td>0.70</td>
</tr>
<tr>
<td>Average charge (esu)</td>
<td>$3.3 \times 10^{-4}$</td>
<td>$1.6 \times 10^{-4}$</td>
<td>$1.4 \times 10^{-4}$</td>
<td>$3.6 \times 10^{-4}$</td>
<td>$1.0 \times 10^{-5}$</td>
<td>$-2.5 \times 10^{-6}$</td>
</tr>
<tr>
<td>Average drop potential (esu)</td>
<td>$5.9 \times 10^{-3}$</td>
<td>$2.2 \times 10^{-3}$</td>
<td>$1.3 \times 10^{-3}$</td>
<td>$4.1 \times 10^{-3}$</td>
<td>$2.0 \times 10^{-4}$</td>
<td>$-1.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>Average charge per average mass (esu/gm)</td>
<td>$4.3 \times 10^{-1}$</td>
<td>$7.0 \times 10^{-2}$</td>
<td>$1.2 \times 10^{-2}$</td>
<td>$5.6 \times 10^{-2}$</td>
<td>$9.0 \times 10^{-3}$</td>
<td>$-3.6 \times 10^{-3}$</td>
</tr>
<tr>
<td>Average charge per mass (esu/gm)</td>
<td>$7.1 \times 10^{-1}$</td>
<td>$1.2 \times 10^{-1}$</td>
<td>$2.2 \times 10^{-2}$</td>
<td>$1.1 \times 10^{-1}$</td>
<td>$1.2 \times 10^{-2}$</td>
<td>$-1.9 \times 10^{-2}$</td>
</tr>
<tr>
<td>Surface wind direction (degree)</td>
<td>160</td>
<td>130</td>
<td>270</td>
<td>240</td>
<td>140</td>
<td>250</td>
</tr>
<tr>
<td>Surface wind speed (m/sec)</td>
<td>3.8</td>
<td>3.0</td>
<td>3.0</td>
<td>3.2</td>
<td>2.9</td>
<td>1.9</td>
</tr>
<tr>
<td>Cloud ceiling (m)</td>
<td>1500</td>
<td>610</td>
<td>1370</td>
<td>610</td>
<td>610</td>
<td>610</td>
</tr>
<tr>
<td>Inversion height (m)</td>
<td>2000</td>
<td>2200</td>
<td>1800</td>
<td>2200</td>
<td>2300</td>
<td>--</td>
</tr>
</tbody>
</table>
The fact that the maximum of some of the cases appears at diameters above 1 mm seems to contradict Blanchard's (1953) results, but it should be kept in mind that these cases are not averages over long periods but merely samples of few minutes of rain. Furthermore, the observation site is located some 600 m below the cloud base, with the inherent possibility that windshear and evaporation processes considerably modify the spectrum measured at the cloud base. Blanchard (1953) showed a dependence of average drop size on the rainfall rate. It is possible, and probable, that the average drop diameter can considerably exceed 1 mm during brief periods of high intensity rainfall.

1. Charge-Size Relationship

Although a rather narrow size distribution exists for all cases, it is profitable to investigate the charge-size relationship for each case. Figure 15 shows the case of July 10, 1967. The drop charge increases approximately in proportion to the drop radius. The ratio of charge to radius therefore can be assumed to be constant. This ratio is the potential of the drop.

2. Drop Potentials

Figures 16, 17, 18, 19, and 20 show histograms of the distribution of drop potential. The method of measurement does not allow a determination of the number of uncharged drops or of drops whose charge is below the minimum charge sensitivity of the instrument. A
gap in the distribution thus exists around zero charge. The error made in the average potential by neglecting uncharged particles is estimated to be less than 5 percent. Case 6 shows a different distribution (Figure 21). Many particles with small negative charges are balanced by a relatively small number of positively charged drops.

The average drop charge per unit mass, charge per unit particle surface, or charge per unit diameter varies considerably from one rainstorm to another. I tried, therefore, to correlate the average drop potential of each storm with other meteorological characteristics. Using the hourly surface wind observations at the Hilo airport, I found that the wind velocity has a good correlation with the average drop potential. The results are shown in Figure 22 where the average drop potential versus wind speed is plotted.
CHAPTER IV

DISCUSSION OF EXPERIMENTAL DATA

A. Summary of Results

Most of the workers in the field agree with the evidence presented here that shower rain and continuous rain from warm clouds are positively charged (Krasnogorskaja, 1963). This polarity persists through the cloud. However, cases have been observed of rain that is negatively charged, and under any conditions both polarities may be present simultaneously. Therefore, the polarity of charge on liquid precipitation is not an absolute one.

The electric field below the cloud is lowered by the fallout of positively charged particles and often reversed to negative values (opposite in polarity to the fair weather electric field).

Space charge measurements inside the cloud indicate the existence of positive space charge in the upper portion and negative space charges in the lower part (Krasnogorskaja, 1963).

The electrification process is continuous and not abrupt. Electric field conditions inside and outside the cloud change in relation to the rate at which charged particles fall out.
There is a correlation between the average charge on rain drops and the velocity of wind. It is possible that the charges which rain out have been carried to the rain area by advection.

A consistent picture of charges on drops in Hawaiian rain is obtained only under conditions where cloud development is restricted to the region below the inversion layer. In thick clouds, reaching to and above the freezing level, a wider charge distribution with smaller average charge exists.

That a melting ice phase is not necessary for rain to be positively charged is demonstrated clearly by the charges found on drops of warm rain. This fact negates Chalmers' (1958) concept that the main process of rain electrification is due to charge production during the melting of ice and snow, as observed by Dinger and Gunn (1945).

A requirement of appreciable convection inside the cloud also seems unnecessary. Direct observations of updrafts in Hawaiian rain are not available, however, Lavoie (1967) observed the movements of floating balloons in nonraining orographic clouds in Hawaii and found updrafts of the order of a few centimeters per second. This is the same order of magnitude as the drift velocity of atmospheric ions in the fair weather electric field.

The magnitude of electric fields measured in raining clouds indicates that the governing charging mechanism does not increase the
electric field present before the rain. This precludes the use of many of the old thunderstorm theories in explaining the electrification of rain.

The electrification process must operate within all clouds under the same physical conditions. One can expect, therefore, that if the characteristics of rain clouds are known, the measured charges will be predictable. Such characteristics might include liquid water content, drop and droplet size, updraft, windshear, fair weather electric field, and other features as yet unknown.

From the data presented it seems possible to derive a valid estimate of the electrical structure of rain clouds. The average magnitude and the range of variations of electric fields and charges can be estimated with some degree of reliability. Based on these estimates and on actual observations an average model of electric charges and fields associated with a rain cloud may be derived. The model cloud will not depend on solid precipitation particles for the rain process.

Typical conditions in Hawaiian clouds, from which charge data have been obtained, can be summarized as follows: Cloud base at 600 m, cloud top at 3000 m; maximum liquid water content of 1 g/m³ at about 1000 m altitude (Squires, 1958). At the altitude of 1000 m the liquid water content is made up of 70 droplets/cm³, each droplet 30 μ in diameter. From such a cloud a 1 mm/hr rain composed of 1-mm diameter drops is falling. A wind of 3 m/sec prevails at the cloud base with very little wind at the surface. An updraft of 5 cm/sec may
be assumed from Lavoie's (1967) constant-level balloon soundings. Each particle is carrying a charge of $+10^{-4}$ esu, the total rain current being about $5 \cdot 10^{-6}$ esu/cm$^2$. The electric field at ground level is negative or positive and a few hundred volts per meter. The electric field at the cloud base is $-500$ V/m and positive in the rest of the cloud. Above the cloud the field decreases to about 100 V/m. Clear skies prevail above the cloud layer. An inversion with a temperature increase of 3°C between 2800 m and 3000 m restricts cloud development, vertical convection, and mixing. The lowest temperature is 15°C. The cloud is larger in horizontal dimension than it is thick, and is quasi-stationary.

Such an idealization of an orographic cloud was proposed by Wexler (1954), but he did not consider the electrical aspects. The purpose of the cloud model is not to describe average conditions but rather to serve as a means of testing different electrification processes. Whatever the process may be, it must reasonably explain the charges of this model.
CHAPTER V

DISCUSSION OF ELECTRIFICATION THEORIES

A. Relation of the Problem to Thunderstorm Theories

Much effort has been expended during the last eighty years to prove and disprove many theories of the production of charges and large electric fields in the atmosphere. Today, for many different reasons, most of these theories have been discarded as inadequate to explain thunderstorm electrification. Many were abandoned because the predicted magnitude and polarity of charges and electric fields did not agree with measured data. It seems advisable, however, to re-examine these theories in the light of the warm-rain electrification process. Electrification theories that deal with ice, hail and snow, as well as freezing and melting, as basic requirements automatically can be eliminated from the present discussion. The remaining theories can be divided into two groups: 1) theories that postulate an external field to influence charges (influence theories) and 2) those that do not (non-influence theories).
B. Non-influence Theories

Based on the known fact that condensation on ions is polarity dependent, J. J. Thompson suggested in 1898 that charge separation in clouds might take place during the process of condensation. Wilson (1899) showed, however, that 400 percent supersaturation is necessary before condensation on airborne ions becomes significant. This condition is never reached in the lower troposphere, where condensation nuclei, active at a few percent supersaturation, are always plentiful.

Gunn (1954) presented a similar idea according to which ions selectively diffuse to cloud drops because of difference in their diffusion properties. For the drop charge $Q_p$ he gives

$$Q_p = \frac{kT}{e} R \ln \frac{\lambda_+}{\lambda_-}. \quad (1)$$

For values of $\lambda_+ / \lambda_-$ of 1.2, the average found by Gish (1955) from data over a number of years, Gunn determined the equilibrium charge to be

$$Q_p = 1.5 \times 10^{-4} \cdot R. \quad (2)$$

Gish's value of the conductivity ratio is based on surface data, where the electrode effect might be responsible for the increased positive polar conductivity. Sagalyn (1958a) measured a mean value of 0.95 at the cloud level in maritime air. Her data agree well with Gunn's later (1956) calculations and are consistent with her own determination of the influence of pollution particles on this ratio (Sagalyn 1958b).
These data make Gunn's earlier estimates, which lead to Equation (2), very questionable. From the mechanism proposed by Gunn, one should expect, on the average, a small negative charge on raindrops. It is difficult, therefore, to explain the electrification of warm rain with this concept.

Frenkel (1944) considered a preferred diffusion of negative ions to water surfaces due to the dipole structure of the water molecule. He proposed that the electric double layer extended somewhat into the gaseous regime above the water surface and preferred the accretion of negative ions until the electrokinetic potential was compensated for by the potential of the charged drop. Such a theory leads to negatively charged drops.

Shishkin (1963) suggested a charging mechanism for coalescence rain based on Frenkel's ideas. He assumed a constant potential for all cloud droplets of $-10^{-4}$ esu and calculated the resulting charge after coalescence. He also calculated the electric field inside the cloud by assuming a constant supply of small charged droplets. For a rain current of 10 mm/hr the electric field reaches breakdown conditions inside the cloud. Shishkin states that this model gives thunderstorm conditions. Unexplained, however, are his assumptions for the charge-up of small droplets, the influence of the opposite polarity of charges that
must stay somewhere in the cloud, and the influence of conduction and corona currents in such very high fields.

C. Evaporation of Charges

1. Muhleisen's Theory

Muhleisen (1958) revived Volta's concept that charges may be freed by evaporation. Assuming that water molecules leave the surface as single molecules it is highly improbable that a measurable number of charged molecules will have sufficient energy to overcome the electrical image force (Brook, 1958). It seems possible, however, that large clusters of associated molecules may play a role in the evaporation process. Such clusters could carry an elementary charge to a distance where the image force is neglectable and then disintegrate into molecules.

2. The Experiment

Muhleisen (1958) carried out laboratory experiments but he failed to describe fully all of the experimental conditions. For the purpose of repeating his experiments on a quantitative basis, a laboratory arrangement was set up as shown in Figure 23. A water dish 30 cm by 50 cm and 6 cm deep was placed in a container of 70,000 cm³. The container was ventilated with air of variable humidity and temperature, from which small and intermediate ions were removed with an electrostatic
filter. The dish could be heated with an infrared lamp from the outside through a thin plastic window. The temperature of the surface was measured with an infrared thermometer. The air leaving the container passed through a cylindrical arrangement that acted as an ion collector and conductivity sensor. A sensitive current meter was used to detect ions of either polarity in the air. Several experiments were performed with this apparatus at the Mauna Loa Observatory at an altitude of 3400 m. This level is well above the tropical inversion. The ambient air at this location has an extremely low concentration of large ions and pollution particles.

In his experiment, Muhleisen produced about $8 \times 10^{-13}$ amp/m$^2$ during evaporation at high relative humidities. The experiment required, therefore, a sensitivity of at least $10^{-13}$ amp to recognize the effect observed by Muhleisen. We could easily measure $10^{-14}$ amp with good reliability. At a flow rate of 75 liter/sec the limit sensitivity was one elementary charge per cubic centimeter. This was the background level of ion production by cosmic rays during the time required for the air to move from the electrostatic filter to the ion counter.

3. Experimental Results

The results of this investigation were negative if "aged" water was used. "Aged" water is water that has been at rest with the environment for at least five hours. Such water did not show any noticeable
electrification at water temperatures from 0°C to 80°C and at environmental air temperatures from 10°C to 50°C, with relative humidities ranging from 20 percent to 90 percent. The strong temperature gradient produced by infrared heating at the water surface also failed to produce any noticeable effect.

Some electrification was noted if "fresh" tap water or rainwater was used. Figure 24 shows the charge production in a sample of rainwater that had been in contact for several days with air at an over-pressure of two atmospheres. The ion collector registered a negative current six times larger than the noise level for about 100 sec before decaying to an insignificant level. Careful observations revealed that, during the time of charge production, low concentrations of microbubbles were visible in the liquid. This test was easily reproducible and was insensitive to moderate contamination. The test also was insensitive to the state of the ambient air and the rate at which water evaporated. It is probable that the charge separation observed is similar to that observed in bubbling water by Blanchard (1961). The magnitude of the charge separation was rather small. The sample illustrated in Figure 24 produced $3 \times 10^{-4}$ esu/g of water. An estimate of the change in air-solubility for droplet water during the lifetime of a drop indicates that this mechanism is too small to account for even the smallest observed charges found on warm rain drops.
D. Charge Breakup Theory

Simpson (1927) proposed a theory of drop charging using an effect discovered by Lenard (1892), who showed that the splashing of water drops separates charges.

It is obvious that drop breakup occurs when large drops are subjected to rapid changes in air speed. Zeleny (1933) carried out laboratory experiments on the breakup of drops, using 6 mm diameter drops injected into a 20 m/sec air jet. The minimum speed required to break up such a drop was 16 m/sec. By carefully avoiding water contamination he obtained a charge per mass ratio of +0.03 esu/g.

Although this is the magnitude of the charge found on warm drops, it is highly improbable that any drop in Hawaiian orographic rain ever experienced a sudden velocity change of 16 m/sec. Blanchard's (1953) data, among others, show that large drops (larger than 3 mm diameter) are very rare in Hawaii. It is, therefore, unlikely that drops reach the critical size (6 mm diameter) necessary for breakup at terminal velocity. Yet the raindrop charges found in Hawaii are comparable to those on continuous rain at locations where large drops possibly may exist in sufficient quantity. From this fact it can be deduced that drop breakup is not necessary for rain electrification.
E. Influence Theories

1. Definition

These theories are based on the observation that charges develop on inhomogeneities of conductivity in the presence of an impressed field or current. Electrostatic energy is produced when these charges are moved by gravity and convection against Coulomb forces.

2. Charging by Collision of Raindrops

Elster and Geitel (1885) postulated that raindrops become polarized in an existing electric field, and they assumed that collision without coalescence occurs often with a charge transfer. In electric field conditions of fair weather polarity this charge transfer resulted in a positive current flow to the larger particles, the negative charge remaining on the smaller particles. They had to assume, however, that the smaller particle, when breaking contact, is on the upper half of the large particle.

Sartor (1967) revived and modified the theory of Elster and Geitel by assuming contact, or near contact, of small particles on the lower half of the larger particles. This mechanism leads to negative charges on large particles and to an increase in the initial field which, in turn, increases the rate at which raindrops are charged. Although the mechanism was designed to explain the fast buildup of electric fields inside
thunderstorms, it is equally applicable in all clouds from which precipitation falls. It cannot, therefore, explain the fact that relatively large drops in coalescence rain are positively charged. Furthermore, this theory cannot explain a net positive current to the ground.

3. Wilson's Theory

Limitations are also evident in the influence mechanism proposed by Wilson (1929) and analytically investigated by Whipple and Chalmers (1944). According to this theory, there is a flux of charge toward rain-drops falling in an electric field in the presence of positive and negative small ions. The process consists essentially in water-drops bringing down some of the negative ions that are moving upward by conduction currents. Positive ions are also collected but at a smaller rate. They effectively limit the negative charge brought down. For warm rain clouds one can calculate that a 1-mm diameter drop can carry a charge of about $-10^{-5}$ esu in the equilibrium state. This charge is very small and of the opposite polarity to that observed. Even to reach this charge a particle has to exist for several multiples of the relaxation time $1/4\pi\tau$ in a fair weather field.

The Wilson mechanism is not effective if fall velocities are small (a few cm/sec). The time a drop spends inside a cloud at sufficiently high fall velocities (a few m/sec) is approximately $1/5$ of the
relaxation time. It seems, therefore, that other effects must overshadow the Wilson process.

4. Convection Theories

In 1955, Vonnegut proposed a charge separation mechanism based on vertical convective motions. A natural positive space charge, produced by the electrode effect or by inhomogeneities in conductivities, occurs in layers below the cloud. This charge is carried by air currents into the cloud, giving rise to negative fields above the cloud and attracting negative ions from farther above. A proper circulation transports these negative charges downward while the positive charges continue to move upward. This process increases the electric field inside the cloud and reverses the polarity below the cloud. Finally, point discharge begins at the ground, further increasing the supply of positive charges. Since this theory makes it necessary that large regions of unipolar ion flow be maintained, it is of limited value as an explanation for thunderstorm electrification.

For the warm-rain process a similar scheme of transporting charges from the ground to the cloud may be assumed. Positive charges, attached to nuclei or in the form of small ions, are carried to the cloud by either updrafts or eddy transport. The supply of ions might come from a space charge formed within regions outside the precipitation area by the electrode effect, or from other sources.
Blanchard (1961) described a mechanism by which bubbles from the sea transport an appreciable positive current into the atmosphere. These charges will decay exponentially, with the relaxation time of the air, but some portion will move into the cloud by updrafts and eddy motion. Such a mechanism cannot explain negative electric fields during precipitation and is limited, therefore, to explaining those cases where the rain current carries positive charges and the electric field at the ground remains positive. This mechanism also is unable to explain rain charges in areas far away from oceans.

In the case of Hawaiian rain, we must expect some influence of bubble charges and estimate the magnitude of this effect. McDonald (1938) determined a mean annual wind of 12 knots for the Hawaiian area, and Blanchard (1961) estimated a current density produced by bubbles from the sea of $2 \times 10^{-8}$ esu/cm$^2$ sec for a wind of 12 knots. Blanchard's estimate is two orders of magnitude less than the average current brought down by precipitation. Let this current constitute a vertical convection current $i$, and assume a space charge density $q$ moving with vertical velocity $V$. Then,

$$q = \frac{i}{v}. \quad (3)$$

A conduction current will flow toward each charged particle according to its surface field and the conductivity $\lambda$ of the atmosphere. This will decrease the convection current density, under steady conditions, according to
where \( i_o \) is the current density at the surface.

At altitude \( z \), where those charges would be captured by raindrops, a precipitation current of \( i \) is possible, provided all charges are captured by the rain. For \( V = 10 \text{ cm/sec} \) a scale height of 100 m is obtained. This shows that at 800-m altitude the original current is reduced to an insignificant amount. It must be concluded, therefore, that in Hawaiian orographic rain, space charge from the ground is rarely carried to cloud levels and hence does not significantly contribute to the charging of warm raindrops. But in convective systems, with updrafts of several m/sec, it is possible that a major portion of the space charge at ground level is carried to altitudes of several thousand meters before flowing back to the ground or upward to even higher elevations as conduction currents.

F. The Need for a New Theory

In summary, no published theory seems to give a reasonable explanation for the charges found in continuous warm rain, in particular, those charges found in Hawaiian warm rain. An attempt is made in this study to use a new concept of charge separation in warm-rain clouds. A model of Hawaiian orographic clouds will be used to test and interpret the concept. It is anticipated that the concept can be
transferred in general to other conditions of continuing or moderate shower rain. This new concept uses only old and basic physical principles, and its "novelty" can therefore, of course, be easily challenged.
CHAPTER VI

THE ELECTRIFICATION PROCESS OF WARM RAIN

A. Basic Assumptions

Assume a cloud layer of infinite extent but finite thickness. Inside the cloud there is a reduced electrical conductivity due to increase in the combination coefficient of small ions in the presence of cloud particles. Assume further an atmospheric electric vertical current density practically uninfluenced by the cloud layer. (A justification for this assumption is given in Appendix A). A higher electric field will develop inside the cloud than outside the cloud. Negative influence charges will appear in the lower part of the cloud and positive in the upper part (Figure 25), but the integral over a vertical column of space charges will be zero if air conductivities above and below the cloud are assumed equal. Assume that these space charges either are residing on or are in the process of diffusing toward cloud particles. A coalescence mechanism of raindrop growth will be assumed. The upper part of the cloud is composed of cloud droplets and many small drops with terminal velocities of a few cm/sec.
B. The Charging Mechanism

During their fall through the cloud the small drops grow appreciably by accretion of cloud droplets and through coalescence of cloud drops. The lower part of the cloud consists therefore of small cloud droplets and relatively few large raindrops. The raindrops will be composed, to an appreciable degree, of small drops from the upper part of the cloud and, to a lesser degree, of small droplets resulting from coalescence in the lower part of the cloud. These drops will fall at a terminal velocity of a few meters per second.

In the upper part, positive space charges find enough time to diffuse to the many small cloud drops. In the lower part, negative charges, because of the small residence time of raindrops in this region, do not compensate, by diffusion to the raindrops, for the positive charge accumulated in the upper region. Thus, raindrops will leave the cloud positively charged.

In intense rainfalls, a limited charge flux and electric field pattern will develop. The maximum precipitation current is determined by the sum of the fair weather current density plus the horizontal flux of space charge by horizontal winds. At low wind speeds, the rain current will be of the same order of magnitude as the fair weather conduction current. The electric field inside the cloud will decrease in the upper portion to the fair weather field outside the cloud and
should become negative in the lower portion. The maximum negative electric field should be found at the lower boundary. This field will not exceed the magnitude of the maximum positive field under no-rain conditions. Below the cloud, the negative field will decrease exponentially in magnitude and can be negative or positive at the surface.

This concept does not require extreme assumptions to satisfy all the observed charge and electric field patterns. If the concept is extended to snowfalls, one can easily see that no appreciable change in terminal velocity will occur during the snow growth process. It must then be expected that snow attains by diffusion the negative charges of the lower part of the cloud. When the snow melts inside the cloud, an abrupt increase in terminal velocity occurs, and the rain stays positive if the melting zone is within the positive region of the cloud. In both rain and snowfall, the maximum fields and currents are limited by horizontal winds and the conductivity profile, thus excluding the possibility that breakdown conditions develop in the cloud.

The basic assumptions of this concept of charge separation are therefore:

(1) Through inhomogenities in air conductivity and with a constant fair weather current density, positive influence charges accumulate by conduction currents in the upper part of the cloud and negative charges accumulate in the lower part of the cloud.
(2) Under steady rain conditions, charge equilibrium is not reached, but space charges diffuse at varying rates to raindrops.

(3) The residence time of individual raindrops is larger in the upper positive space charge region than in the lower parts of the cloud because of increased terminal velocities during drop growth.

A mathematical description of this concept will now be developed and numerical tests applied to demonstrate the feasibility of the approach.
CHAPTER VII

MATHEMATICAL FORMULATION OF CHARGING PROCESS

A. Basic Equations

To formulate our theory, we will use for convenience a Cartesian coordinate system with the zero point at the upper cloud boundary and the positive direction increasing downward. We will further assume stratified conditions of all cloud properties to reduce the problem to one dimension. We will use the continuity properties of electrical current flow $I$,

$$\text{div } I = 0,$$

which means that the total current density is continuous and constant in a one-dimensional system. We will assume an impressed fair weather current density $i$, independent of the properties of the cloud. This can be justified by the assumption that the columnar resistance from the ground to the upper cloud boundary is small compared with the columnar resistance between the ground and infinity. This assumption implies that any net charge at cloud level has its image charge at the ground. Another impressed current, $Q'U$, into the cloud system will be any space charge carried by horizontal winds $U$ to or from the precipitating area. The sum of these two currents is balanced by vertical
condudon ($\lambda E$), convection ($QV$), and displacement ($\epsilon \partial E/\partial t$) currents inside a stratified cloud and below. Thus,

$$i + Q'U = \lambda E + QV + \epsilon \partial E/\partial t, \quad (6)$$

where $\lambda$ is the air conductivity produced by positive and negative carriers of electricity, $E$ is the electric field and $Q$ is the space charge density due to charges residing on raindrops that fall with velocity $V$; $\epsilon$ may be considered in this case only a proportionality factor.

We will also use Poisson's equation,

$$\epsilon \text{ div } D = q + Q \quad (7)$$

where $q$ is the space charge density of charges that do not reside on raindrops and which, for a one-dimensional system, reduces to

$$\epsilon \partial E/\partial z = q + Q. \quad (8)$$

The relationship between $q$ and $Q$ is governed by a flux of charge $q$ to the raindrops by diffusion, and by the loss of space charge density by divergence and conduction currents:

$$\frac{\partial Q}{\partial t} = -\frac{\partial}{\partial z} (QV) + 4\pi RDNq - \lambda Q/2\epsilon. \quad (9)$$

Such a diffusion mechanism is a reasonable assumption, particularly since charge equilibrium between droplets, drops and the environment is rarely reached in raining clouds with electrical relaxation times of one to five hours. In Equation (9), $R$ is the drop radius, $D$ the effective molecular diffusion constant for the falling drops, and $N$ the volume concentration of drops.
We will further assume steady-state conditions. This is not to imply that in nature steady rain is the rule. We can, however, assume as a first approximation that no nonlinear relations exist in the rain and charging mechanism, and therefore that steady conditions represent mean conditions. A justification for this assumption is outlined in Appendix B.

Initially, we will neglect horizontal movement of the cloud \((U = 0)\) but may introduce it later as a correction term. The basic equations to be solved then are

\[
i = \lambda E + QV
\]

\[
\epsilon \frac{\partial E}{\partial z} = q + Q
\]

\[
\frac{\partial Q}{\partial z} = 4\pi RDNq/V - \lambda Q/\epsilon 2V - Q \frac{\partial V}{\partial z} \cdot \frac{1}{V}.
\]

B. Related Equations

1. Drop Growth

Because the proposed mechanism is based on the fact that drops grow and fall with increasing speed, we must derive growth relationships as well as a relation for fall velocities.

We will assume drop growth by accretion of cloud droplets of liquid water content \(w\) and also growth by coalescence between drops. Following Langmuir, we can assume for a change of drop mass \(M\),
\[
\frac{dM}{dt} = \pi R^2 \eta V_\text{w} + \pi^2 R^5 \eta' \eta' V' \eta' N \rho \frac{4}{3} \tag{12}
\]

where \( V' \) is the average velocity difference among raindrops, \( \eta' \) is the collection efficiency, \( \rho \) is the density of water, \( \eta \) is the efficiency with which raindrops collect cloud droplets, and \( w \) is the liquid cloud water of particles per unit volume other than raindrops. The efficiency \( \eta \) includes the collision efficiency resulting from differences in fall velocities and from random eddy motions, and the coalescence efficiency produced by any electrical or mechanical contact effects. For steady rain the altitude profile of drop radius is given by

\[
\frac{\partial R}{\partial z} = \frac{V'}{V} \eta' N \pi R^3 / 3 + \frac{wn}{4\rho} . \tag{13}
\]

2. Drop Concentration

We further require a knowledge of drop concentration \( N \), which does change as a result of mass divergence or coalescence.

Assume that the total mass per unit volume \((N \cdot M)\) is changing only by divergence and the collection of cloud water \( w \):

\[
\frac{\partial}{\partial t} (N \cdot M) = -\text{div}(NMV) + N \pi R^2 \eta V_\text{w} . \tag{14}
\]

In the steady state we obtain

\[
NM \frac{\partial V}{\partial z} + NV \frac{\partial M}{\partial z} + MV \frac{\partial N}{\partial z} = N \pi R^2 \eta V_\text{w} \tag{15}
\]

and from Equation (12),
By combining (15) and (16), we obtain

\[ \frac{\partial N}{\partial z} = -N \left( \frac{1}{V} \frac{\partial V}{\partial z} + N\pi R^2 \eta \frac{V'}{V} \right) \]  

(17)

Here it is implicitly assumed that the drop mass would be preserved if the liquid water content from small cloud droplets were zero.

3. Velocity of Drops

For the velocity relation we will use the experimental data of Kinzer and Gunn (1947). To reduce their results to a form applicable to computer programming, we use the following approximation:

\[ V = 1.19 \cdot 10^8 \cdot R^2 \left( 1 + 4.6 \cdot 10^3 \cdot R + 1.195 \cdot 10^7 \cdot R^2 \right) \]  

where \( V \) is in m/sec and \( R \) is in m. From this relation we can also obtain \( \partial V/\partial R \) and \( \partial V/\partial z \).

It is appropriate at this time to estimate the ratio \( V'/V \) for the growth equation. In continuous growth models (where drops grow only on small particles), the size distribution is not affected by the growth mechanism as long as the collection coefficient does not vary with \( R \). Berry (1968) showed that in a stochastic growth model, where coalescence takes place among all particles, the shape of the size distribution is "self-preserving" for drops with diameters larger than 60 \( \mu \). We can assume from his calculations that the standard deviation
is proportional to $R$. We can calculate $V'$ as the difference of velocity $V_1$ at size $R$ and $V_2$ at size $R + \Delta R$, where $\Delta R$ is proportional to $R$.

Therefore,

$$\frac{V'}{V} = \frac{R}{V} \frac{\partial V}{\partial R} \cdot \beta.$$  \hspace{1cm} (19)

The constant $\beta$ must be estimated from the raindrop concentration and size distribution found in the cloud. The term $R/V \partial V/\partial R$ varies from 2 to 0.5 in the range of $R$ from 25$\mu$m to 1 mm.

4. Electrical Conductivity

We will assume a constant value $\lambda_{oo}$ for the air conductivity below cloud level. This simplification is justified in the present approximation because there is sufficient mixing by eddy processes below the cloud. A high number of Aitken nuclei in the size range of $10^{-6}$ cm radius is mainly responsible for the conductivity value. In the lower part of the cloud, most of these very small particles are removed by the cloud, and cloud droplets determine mainly recombination and, hence, the number of small ions available for conduction of electrical currents. Therefore, the number and size of cloud droplets will determine the inside cloud air conductivity. At the upper cloud boundary a value of $\lambda_o$ can be assumed in accordance with the nuclei and ionization conditions above the cloud.

These arguments lead to a conductivity profile inside the cloud of
Using data on nuclei measured in the vicinity of Hawaiian orographic clouds we can assume that

\[ S = \left( \frac{Z}{Z_o} \right)^m, \]  

(21)

where \( Z_o \) is the cloud thickness, and \( m \) is a suitable integer that determines the transition from \( \lambda_{oo} \) to \( \lambda_o \). A reasonable value for \( m \) is 8.

An estimate of a suitable constant, \( A \), which determines the influence of liquid cloud water on conductivity can be derived in the following manner. Ion production \( q_{oo} \) in equilibrium is balanced by the loss of ions by recombination between opposite polar ions and by attachment to cloud particles of radius \( r_1 \) and small condensation nuclei with radius \( r_2 \), with respective densities \( Z_1 \) and \( Z_2 \). Then,

\[ q_{oo} = an^2 + 4\pi r_1 DZ_1 n + 4\pi r_2 DZ_2 n. \]  

(22)

The influence of electrical forces, due to charges on cloud particles or nuclei, on the recombination process may be neglected in a bipolar environment (Ruhnke, 1961). The first term, with \( \alpha \) the recombination coefficient between small ions, may be neglected and we find that

\[ n = \frac{q_{oo}}{4\pi D(r_1 Z_1 + r_2 Z_2)}. \]  

(23)

If \( n_0 \) is the ion density in the absence of cloud particles,
\[
    n = \frac{n_0}{1 + 4\pi D r_1 Z_1 n_0 / q_{oo}}
\]  

With \( \lambda = neb \), where \( e \) is the electronic charge and \( b \) the ion mobility:

\[
    \lambda = \frac{\lambda_0}{(1 + 4\pi D r_1 Z_1 \lambda_0 / q_{oo} eb)}.
\]

The liquid water content of cloud particles can be expressed by

\[
    w = 4/3 \pi r_1^3 Z_1 \rho,
\]

which leads to

\[
    \lambda = \frac{\lambda_0}{(1 + A w)},
\]

where \( A \) is

\[
    A = 3D \lambda_0 / q_{oo} r_1^2 \rho eb.
\]

Assuming a droplet radius of 15 \( \mu \), \( A = 50 \) if \( w \) is measured in g/m\(^3\).

5. Liquid Water Profile

The influence of liquid water on atmospheric electric conductivity makes it necessary to assume a model distribution of water. Squires (1958) determined liquid water profiles in Hawaii and found a maximum value of 1 g/m\(^3\). His records indicate that the maximum liquid water content is located in the lower half of the cloud. We use as a model distribution

\[
    w = w_0 \left( \frac{Z}{Z_0} - \left( \frac{Z}{Z_0} \right)^m \right).
\]

Figure 26 shows the distribution for different values of the parameter \( m \).
The liquid water profile in this form approximates actual conditions, and the influence of small-scale turbulence, molecular diffusion, and localized convection is therefore included in this profile. Such parameterization of the effects of eddy mixing also applies for the conductivity profile, which is derived from the liquid water profile. Not included in parameter form is the effect of turbulence on possible generation of electrical energy. Because of insufficient evidence of their magnitude, such effects by turbulence in warm clouds are neglected.

Finally, cloud depth must be estimated for the model cloud because of the influence of depth on the magnitude of charges and raindrops. Woodcock (1961) found the upper cloud boundary somewhat above the inversion layer when appreciable rainfalls were observed at the ground. We assume, in line with his data, an average height of the cloud top of 3000 m and a lower cloud boundary at 600 m altitude for the Hawaiian cloud investigated.

To solve the set of equations for our model cloud we must know the conditions at the cloud top as far as charges, electric field, drop size and drop concentration are concerned.

C. Initial Conditions

Assuming, on the average, 10 cm/sec updrafts in the cloud, one can reasonably assume that the largest droplets at the cloud top must
have a terminal velocity comparable to the updraft. A drop of 25 \(\mu\) radius may be selected from such an estimate as a reasonable size for initiating raindrops. Blanchard (1953) finds \(10^5\) drops/m\(^3\) of this size range in the upper portion of the cloud.

A reasonable value of the electric field at the upper cloud boundary is 100 V/m, which corresponds to a current density of \(5 \times 10^{-12}\) amp/m\(^2\) and a conductivity \(\lambda_o\) of \(5 \times 10^{-14}\) 1/\(\Omega\)m at the cloud boundary.

An estimate of the initial charge can be made under the assumption that the initial cloud drops travel at updraft speeds of a few centimeters per second from cloud base to cloud top, where they grow by condensation until they reach a terminal velocity sufficient to overcome the updraft speed. During this slow growth the particles acquire charge. Using Equation (9) one can obtain a limit charge by neglecting rain currents at these levels. To determine the effect of the initial charge on the final charge, a numerical test was made. Values from 0 to \(6 \times 10^{-8}\) esu/cm\(^3\) were assumed for the initial charge density. In all cases the drop charge adjusted to a constant value within 100 m from the cloud top. Therefore, the final charge on the falling drop does not depend critically upon the initial charge.
D. Numerical Solutions

1. Drop Charges

Because of the complexity of the problem, even in the simplified form presented above, no attempt was made to develop an analytical solution of the set of equations. A digital computer was programmed to give numerical solutions for the distribution of drop charges, charge per unit mass, rainfall rate, electric field, and other quantities that can be derived easily. Where necessary, the equations were rewritten in finite difference form, with steps of 1 m for Δz. This approach appears to yield results in good agreement with analytical solutions under some simplified assumptions.

Figure 27 gives the computer results for particle radius, particle fall velocity, and liquid water content for a cloud model with $m = 8$. A particle with an initial radius of 25 μm is growing, partially by collecting cloud water and partially by coalescence with other drops, to a size of 750 μm radius. This is about the mean size of drops investigated by us and other investigators. If the drop growth by coalescence with other raindrops is neglected, a radius of 375 μm would result.

Drop charges resulting from this model are shown in Figure 28. It is apparent that the particle charge depends critically on the growth mechanism assumed. The largest charges are obtained by assuming that drops grow only by accretion of other drops ($\eta = 0$). Such a
mechanism leads to charges that are greater than the largest charges observed. The charge per unit mass is approximately preserved during growth. The resulting charge increases with the cube of the particle radius. The charge per unit mass depends mainly on conditions existing when the drop was small.

If the drop is assumed to grow by collection of cloud droplet water only ($\eta' = 0$), the resulting charges are much smaller but still positive above a critical size. For several realistic model distributions of $w$, $\lambda$, and $z_0$, this critical radius was found to be between 0.2 mm and 0.7 mm. Above this radius, charges are positive and increase in magnitude about proportionally with size. The magnitude is about equal to the smallest charges actually observed.

A realistic growth model lies between these two extremes. The curve labeled $\eta = 0.8$ shows the charge distribution when one assumes that half of the growth results from cloud water collection and half is from coalescence between drops. The calculated charge distribution is similar to that measured on July 10, 1967 (Figure 15). It can be assumed, therefore, that the charges found on raindrops can reasonably be explained by the proposed charging mechanism.

The order of magnitude of electric charge is $10^{-4}$ esu for drops of 1-mm diameter. This might appear high when compared with the equilibrium charge such a drop will acquire in the space charge environment of a rain cloud. The value of $10^{-4}$ esu might also appear high if the
gas kinetic energy of an ion (about $4 \times 10^{-2}$ eV) is considered with respect to the energy necessary to overcome the Coulomb forces of a drop with $10^{-4}$ esu charge. To be considered, however, is that the charges are mainly acquired during coalescence when the drops are considerably smaller than at the cloud base. A careful check of the diffusion and conduction current to the drops during the process of coalescence revealed that a net charging current is flowing to the drop only in the first 500 m of the fall through the cloud. From there on the charge per mass decreases due to loss of charge by conduction and by coalescence with less charged raindrops. When the drop has fallen 500 m, it has a charge in the order of magnitude of $10^{-6}$ esu and a diameter of 300 μ. At this magnitude of charge the kinetic energy of an ion can easily overcome the Coulomb forces of the charged raindrop.

The use of a model and a digital computer made it possible to investigate the influence of several parameters on the final charge. Increasing the impressed current density resulted in a proportional increase in the drop charge. Increasing the initial concentration of cloud drops in the upper part of the cloud led to an increase in drop radius and an increase in drop charge, but the charge per drop surface remained approximately constant. A change in initial drop radius resulted in an approximately constant charge per surface ratio. Increasing the initial air conductivity at constant impressed current
density caused a decrease in the particle charge proportional to the square root of the initial conductivity. A change of cloud thickness, from 2400 m to 2000 m, did not noticeably change the drop charge, provided the rainfall rate and drop sizes were kept constant.

Under conditions of heavy rainfall, the total rain current does not appreciably exceed the fair weather current density. Although Chalmers (1958) found a rain current to equal, on the average, the fair weather current, a hypothesis is necessary to explain the higher values of rain currents occasionally measured. During intense rainfall, all available charges are rained out, and the precipitation current reaches a maximum value. One approach to the problem is to consider Hawaiian orographic rain as a series of showers. At the start of each shower, the cloud contains the charge distribution calculated from non-rain conditions. By the time steady-state conditions are reached, the shower is over. This condition can be approximated in the continuous model by a constant horizontal influx of cloud water that influences charges of non-raining clouds. The term $Q'U$ must be added to the fair weather current density. Therefore, the impressed current density may be modified by

$$i' = i + Q'U,$$

which indicates that during high-intensity rain the charges on raindrops will be dependent on the horizontal wind speed at cloud level. The horizontal flux of charge to and from rain areas may possibly explain
precipitation currents that are occasionally higher than the impressed fair weather current.

2. Electric Field

No effect on the electric field is noticeable if only a few drops fall from the cloud. A light rain of only 0.3 mm/hr, however, has a marked effect on the electric field profile. Figure 29 shows the computer results for different rainfall rates. In the lower part of the cloud, fields are negative for rainfall rates between 0.7 mm/hr and 7 mm/hr. More intense rains produce a saturation effect, which reduces the electric field at all levels and prohibits the development of extremely high fields that might otherwise lead to breakdown conditions. All experiments show a very sharp region at the lower cloud boundary where the field is negative. Apparently the conductivity profile at the lower cloud boundary is much more abrupt than that specified in the assumed model. This result also implies that the cloud water profile changes abruptly at the lower cloud boundary. A test with different distributions of cloud droplet concentrations showed that the electric field profile in the lower part of the cloud is very sensitive to changes in the cloud droplet profile. An increase in the model parameter m led to a sharp increase of cloud droplet water at the cloud base and an increase in the magnitude of the negative field. A decrease in the slope of the water distribution in the middle part of the cloud decreased the thickness of the negative field layer.
CHAPTER VIII

SUMMARY AND CONCLUSIONS

Charges on raindrops are, in general, positive for non-thunderstorm rain. A known hypothesis currently used to explain charges on non-thunderstorm rain, namely charge separation by the melting of ice and snow, cannot explain charges on rain from totally warm clouds.

The electric field pattern in moderate rain seems to indicate that positive fields prevail within raining clouds, while a negative field occurs at the lower cloud boundary. The average precipitation current appears to be approximately equal to the fair weather conduction current. With an increasing rainfall rate, the electric field inside the cloud seems to decrease rather than increase in magnitude. This indicates that the governing mechanism, in contrast to the thunderstorm mechanism, depends on a pre-existing current and field. This fact suggests that an "influence mechanism" is responsible for the charge transport by raindrops.

A survey of thunderstorm electrification theories that do not depend on freezing or melting indicates that none can reasonably explain the charge and electric field pattern of warm rain clouds. A
theory was therefore formulated, from which charges and electric field patterns of a typical warm rain cloud could be derived.

The hypothesis presented here is based on raindrops that collect space charge by diffusion. When a drop falls from the cloud top it resides for a comparably long time in the upper part of the cloud which contains positive charge. The fall velocity increases considerably during coalescence growth, and raindrops have a short residence time in the lower part of the cloud where the space charge density is negative. Numerical experiments show that the final charge is generally positive. If drop growth is predominantly due to collection of cloud droplets, drops with radii larger than 0.5 mm are positive, while smaller drops are negatively charged as they leave the cloud. If drop growth is due only to coalescence among raindrops, no negative drops are found below the cloud base for drops with radii larger than 200 μ. Generally, in this case, the charges are found to have larger magnitudes than in the case of coalescence with cloud droplets.

The "influence mechanism" proposed gives charges of the same order of magnitude as those observed. According to this theory, the charge per drop increases approximately with the drop radius and is proportional to the fair weather conduction current. At high rainfall rates, the charger per drop decreases and depends mainly on the horizontal transport of charges to the rain area. The precipitation current
can exceed the total current in the lower part of the cloud, thus making
the electric field negative.

An impression might develop that the complexity of the cloud
structure does not allow the prediction of charges without a detailed
knowledge of cloud parameters. In spite of a large number of assump­
tions for initial conditions, it is found that some electrical features
occur over a wide range of parameter variations. For example, the
computer study showed that drops above a certain size fall out of the
cloud with a positive charge, that the precipitation current is not sig­
nificantly different from the impressed current density, and that a
negative field develops at the lower cloud boundary. These are the
common, basic features that have been observed by many investigators
in studies of rain clouds.

The advantage of the present approach to the complex problem of
raindrop electrification may also be seen in the possibility of deducing
non-electrical parameters, which are difficult to measure, from measure­
ments of drop charges and electric fields. In general, it must be
accepted that the electrical features of rain clouds are related to the
mechanisms of rain formation.

I believe that the calculations on the model cloud prove the
feasibility of the proposed theory as the governing mechanism for the
electrification of raindrops in warm rain.
REFERENCES


44. Thompson, J. J. (1898) On the charge of electricity carried by the ions produced by Rontgen rays, *Phil. Mag.*, 46, pp. 528-545.


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JULY 20, 1966
CASE I

AVERAGE DROP POTENTIAL:
$5.86 \times 10^{-3}$ esu

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NOV. 25, 1966  CASE II
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DROP POTENTIAL HISTOGRAM

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-1.53 \times 10^{-5} \text{ esu}

DROP POTENTIAL HISTOGRAM

\begin{figure}
\centering
\includegraphics[width=\textwidth]{histogram.png}
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APPENDIX A

The assumption of a cloud of infinite horizontal extent is only justified if the actual horizontal extent of the cloud is so large that the current inside the cloud equals approximately the current above or below it. This condition depends strongly on the effective columnar resistance between ground and upper cloud boundary. For the effective columnar resistance, not only does the ohmic conductivity $\lambda$ have to be taken into account, but the property of the raindrops to transport charges by gravitational settling must also be considered. For this problem it is appropriate to calculate the columnar resistance from the potential difference between the ground and upper cloud boundary and the current density that led to these potentials. The columnar resistance so obtained has to be compared with the columnar resistance of the nearby cloud-free area. If the ratio of these two columnar resistances is close to unity, then the assumption of a cloud of infinite extent is justified, even if the actual horizontal extent is small or even equal to the thickness of the cloud.

It must be expected that the rate of rain is a predominant factor in determining the effective columnar resistance of a rain cloud. For the cases shown in Figure 29, an integration of the electric field revealed that for rates of rain between 1 mm/hr and 5 mm/hr the ratio of
effective cloud resistance to free air resistance was unity ±25 percent. This finding also agrees with the observational evidence of electric field soundings through rain clouds, which reveal potentials at cloud top levels to be not much different from potentials at the same altitude in cloud-free areas. A rain cloud therefore will not cause the atmospheric electric fair weather current to flow around it, and the current density inside the cloud will be equal to the current density above it, provided enough raindrops are available to carry this convection current.
APPENDIX B

The assumption of steady-state conditions does not restrict the proposed theory in principle. Rather, this assumption is made to simplify the mechanics of computing charge and field distribution of a fairly realistic model. To consider more closely the problem arising from a steady-state assumption, one must separate small-scale variations in time from large-scale variations. Small-scale turbulence that affects the detailed structure of liquid water content, conductivity, and possibly the collection efficiency of rain drops, may conveniently be accounted for by using empirical formulas based on field data taken under representative conditions. The liquid water content data given by Squires (1958) lead to empirical formulas of liquid water profiles and conductivity. Measured collection efficiencies under conditions of small-scale turbulence are not available. Therefore, all calculations of electric field and drop charges were made on the basis of different values of collection efficiencies. The result showed that in the tested range of 50 percent to 100 percent collection efficiency the principle features of the theory are not affected.

The effect of large-scale variations in time can be estimated from the time function of the drop charge pattern that develops if a sudden change in those parameters is assumed for which the steady-state
assumption is tested. If the steady drop charge pattern follows fast the actual rate of change of the tested parameter, one can reasonably assume steady-state of this parameter. One of the fastest changing parameters in a warm rain cloud is the rate of rain. A numerical test was performed in which the rate of rain changed suddenly from zero to 1 mm/hr. The steady-state value of the raindrop charges at the lower cloud boundary was reached within one minute. After that time, the drop charge continued to increase for ten minutes to a value of about five times the steady state value, before continually decreasing towards the steady-state value. A physical interpretation of this result is that space charges of the non-raining cloud are available and these are readily rained out. At the start of the rain, these space charges attach to the raindrop and precipitate out. After twenty minutes, this source of space charges is approximately exhausted and the externally impressed conduction current maintains the drop charge pattern.

In continuous rain, but with a highly variable rate of rain, the average charge of raindrops after about twenty minutes will equal the charge found on steady rain. The assumption of steady rain, therefore, can reasonably be made for Hawaiian rain, if data for the first twenty minutes are disregarded.

For shower rains, the assumption of steady-state will give an under-estimate of the average charge on raindrops.