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TAYLOR, Ronald Charles, 1932-
SOME COMPUTATIONS OF SURFACE AIR TRAJECTORIES OVER THE OCEANIC TROPICS AND ACCOMPANYING WEATHER.

University of Hawaii, Ph.D., 1968
Physics, meteorology

University Microfilms, Inc., Ann Arbor, Michigan
SOME COMPUTATIONS OF SURFACE AIR TRAJECTORIES
OVER THE OCEANIC TROPICS AND ACCOMPANYING WEATHER

A DISSERTATION SUBMITTED TO THE GRADUATE DIVISION
OF THE UNIVERSITY OF HAWAII IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
IN GEOSCIENCES (METEOROLOGY)
SEPTEMBER 1968

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ABSTRACT

An attempt is made to compute some near surface air trajectories from an initial specification of the (steady) pressure field over selected regions of the oceanic tropics. Friction is assumed to oppose the motion and is proportional to the speed, while the percentage change of horizontal areas is computed to obtain the horizontal velocity divergence along the parcel paths. The latter results are related to observed distributions of low-level cloudiness and rainfall in the Indian Ocean and selected regions of the Pacific. Further interpretative comment is offered through an analysis of the divergence equation; the latter indicates that both the divergence of the pressure-gradient force and Coriolis forces are significant in determining the temporal change of horizontal velocity divergence along a trajectory.
PROLOGUE

"To put the point in a crude form, I do not know whether, in practice, the winds have to adjust themselves to the pressure conditions, or the pressure distribution is the result of the motion of the air."

Sir Napier Shaw

Introduction to Gold's *Barometric Gradient and Wind Force*, 1908

"I am still uncertain what to do with the winds that appear to cross the equator . . . I always find my pen sticks in the paper and refuses to move when I try to draw an isobar across the equator."

Sir Napier Shaw

The *Air and its Ways*, 1923
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LIST OF MATHEMATICAL SYMBOLS

D  - the horizontal velocity divergence (sec⁻¹)

f  - the vertical component of the earth's rotation;  
    the Coriolis parameter, 2 Ω sin φ (sec⁻¹)

ζ  - the vertical component of the relative vorticity  
    (sec⁻¹)

Def - total deformation of the horizontal wind (sec⁻¹)

β  - latitudinal variation of the Coriolis parameter  
    (f) (cm sec⁻¹)

K  - coefficient of friction (sec⁻¹)

ρ  - density (gm cm⁻³)

Pₓ,Py - horizontal pressure gradient per unit mass  
    (cm sec⁻²)

u,v,w - wind speeds directed eastward (u), northwards (v),  
    and upwards (w)

φ  - latitude

d/dt - substantial derivative "following the motion"

θ  - angle between isobar and wind vector

Ω  - the earth's rotational speed (7.292 x 10⁻⁵ sec⁻¹)
I. Introduction

Can we account for the following observation? During the Indian summer monsoon satellite observations show striking variations of cloudiness over comparatively short distances, with a tendency for the clouds to be arranged in nearly east-west bands. For example, massive cloudiness between the equator and five degrees north; clear between the equator and five south. Further these patterns persist for many days in the steady flow over the mid-Indian Ocean. At the same time, an examination of the surface synoptic chart shows the air drifting across the isobars towards the lower pressure north of the equator with, again, the isobars directed more-or-less east and west.

Equally striking features in the low-level distribution of cloudiness are observed over the tropical Pacific extending, as well, to the distribution of rainfall over the atolls of the Gilbert-Ellice, and Line Islands. Though we may lack confidence, on a particular occasion, as to the relevant details of either the fields of pressure or motion, we often find rather "bland" looking pressure fields apparently producing rather striking features in the field of motion.
Leaving aside for the moment the perennial question of data deficiencies, how are we to account for these features of the motion field in terms of the pressure field, from which the air derives its kinetic energy?

The question, of course, is not a new one; it was a central theme of meteorological inquiry nearly a century ago. One empirical solution was offered by Abercromby (1) depicting the distribution of clouds and weather around a composite mid-latitude depression and became the basis of successful forecasting in the United Kingdom around the turn of the century. This approach, of course, is successful since in middle latitudes, the pressure field is a reasonably good first approximation to the field of motion.

The quality of this approximation degrades rather rapidly with decreasing latitude, though it is not impossible to expect it to hold on some occasions. The general validity of such reasoning, however, has yet to be demonstrated with conviction. Indeed, rather than persevere in this dynamical direction of pressure wind tropical investigators more and more focused their attention on just the field of motion, attempting to relate the distribution of clouds and weather to the directly computed horizontal divergence.
This they did with some success, but only with confidence when the observational network was considerably expanded, during selected periods, viz., nuclear weapons testing.

With the modest expansion of observational networks in low latitudes in recent years, the habit of analysing the field of motion, to the near exclusion of the pressure field has been widely adopted. Further, within the last few years tropical forecasters in Asia and Africa have expressed growing doubts over the usefulness of just the field of motion as a forecasting tool and have again directed their attention to the pressure field.

This is the approach we adopt here in an effort to understand the questions outlined in the first few paragraphs. We offer a first attempt to compute the field of motion in the near surface layer and its divergence from an initially specified (steady) pressure field, a frictional constraint, and, implicitly, a variable Coriolis acceleration. We compute the paths of individual parcels, treating them as steady streamlines under suitable assumptions, and examine the results diagnostically.

It appears that many of the puzzling features of cloud and weather distribution can be accounted for by the
divergence of the pressure field and its implications in how moving parcels of air change their horizontal divergence as they move about.
II. **Historical**

One of the lasting achievements of mathematicians and physicists during the last half of the eighteenth century was the introduction of the motion of a continuum and two basically different ways of describing it dynamically. Previously, several useful results—fluid pressure, for example—had been attained by considering fluid media to be a collection of Newtonian particles undergoing elastic collisions between themselves and their surroundings. Such a conceptual scheme, in the hands of Jean and Damiell Bernoulli, for example, failed to give additional results of general interest, and not until the work of Euler, d'Alembert, and Lagrange were results of wide application available in the form of the general field equations of hydrodynamics, the equation of continuity, and the so-called Lagrangian equations of fluid media.

The description of continuous media in terms of field-equations has made an enormous impact upon physical theory over the past two hundred years. This technique, of specifying, say, the velocity at fixed locations throughout a region of space at a specified time, is almost solely the contribution of Leonhard Euler (2), undoubtedly the
greatest analyst of his generation. Earlier, however, d'Alembert made use of the field description for a particular problem, but did not generalize his technique. Further, Euler provided an alternative description to that of his field equations in a form of what mistakenly is generally referred to as the Lagrenian description! Euler did not perfect this technique of describing some dynamical feature of one particle and following its history for subsequent time, but he did originate the method, including the elegant formula describing parcel mass continuity,

\[ \text{div} = \frac{1}{J} \frac{dJ}{dt} \quad (1) \]

where \( J \) is the Jacobean expressing the point transformation between initial and subsequent co-ordinates, though this refinement was the contribution of Jacoby some fifty years later. The particle interpretation was, of course, very much in the spirit of Newtonian mechanics, though little was gained mathematically since both descriptions, the field and parcel techniques, are essentially non-linear. The parcel technique, through a long-standing neglect of Euler's work, is generally associated with the name of Lagrange, who perfected it in his famous book,
'Mechanique Analytique' (3), though it contains no reference to the earlier work of Euler. In fact, this error was not corrected during Lagrange's lifetime, and only in the third edition (1853) edited by Joseph Bertrand is a footnote finally added to correct the historical record. By this time the precedent had been established and subsequent investigations entailing the dynamics of fluid parcels were henceforth said to apply Lagrange's technique. Clearly, it would be more appropriate to identify the parcel method as the Euler-Lagrange technique. Nevertheless, with few exceptions; viz., Sommerfeld (4)—Euler's early development of the parcel equations is almost universally identified with the name of Lagrange.

One of the first applications of the "Lagrangian technique", which held considerable physical interest, was offered by Franz Gerstner (5) who investigated steady-state water waves. Gerstner's solution was criticized because his waves were rotational, though he was one of the first to show that his solutions correspond to a possible fluid motion since they satisfied the parcel equation of continuity originally derived by Euler. A modern discussion of Gerstner's troncoidal waves is provided by Milne-Thomson (6).
The application of Lagrangian techniques to the study of atmospheric motions is quite recent; nevertheless, it is useful to recall that the earliest synoptic investigations by Brandes (7), Espy (8) and others—all using a field description—generally expressed their results in terms of air parcels flowing along particular "trajectories." Indeed, the distinction between a streamline and a trajectory in meteorological discussions was, at best, obscure; it remained so until after the turn of the century.

The first, and still memorable, application of Lagrangian techniques was the extensive investigation of surface air trajectories by Shaw and Lempfert (9), but before considering their results it is fitting to consider a simpler dynamical problem: the motion of a free particle on a rotating sphere.

This problem has had a decidedly checkered career beginning with its basic formulation in the hands of Carl Gauss (10), who applied it to the problem of a freely falling body. Later, Coriolis (11), through his interest in waterwheels (!), contributed three fundamental papers of wide interest concerning rotating co-ordinate systems, but never considered any geophysical applications. A year
later, Poisson (12), being interested in projectile motion, contributed two papers which included a detailed discussion of the motion of a free particle relative to a rotating sphere, though again no particular geophysical interpretations were offered.

In the light of the results published between 1803 and 1838 by Gauss, Coriolis and Poisson, it is indeed surprising to find three investigations by Von Baeyer (13), Ohlert (14), and Mousson (15) who compute inertial trajectories of air parcels through an appeal to the discredited Hadleian idea of angular momentum conservation! Though Von Baeyer expressed some skepticism about Hadley's principle his dynamical courage finally deserted him and he ultimately returned to Hadley's formulation; both Ohlert and Mousson, on the other hand, did not express any doubts concerning the basic validity of this principle. These dynamical discussions were finally corrected by Sprung (16) who appealed to the results of Coriolis, but confined his discussion to inertial motion near the pole where the normal elliptic integrals are greatly simplified; regarding the wider problem, where the elliptic integrals must be evaluated, Sprung notes that ". . . the solution of this problem . . .
does not seem to be worth the while . . . because . . . the careful determination of the path has only a subordinate interest in meteorology, since the motion . . . that the particles of air actually follow, the 'inertia path,' has been completely refuted by the synoptic weather charts." Apparently, Sprung did not think that a knowledge of the actual inertial paths would lead to a deeper understanding of the forces producing actual motions. This attitude, however, was later adopted by Whipple, but before discussing his results, it is worth remembering a nearly forgotten book on atmospheric dynamics published by Cordeiro (17). In his chapter on "Motion Relative to the Earth" the author presents several results of inertial motion derived from a simple physical understanding of the salient forces, along with some reasonably direct mathematical description. He obtains correct results for inertial trajectories in both high and low latitudes without resorting to the evaluation of elliptic integrals. The author does not include references to previous work, and he does not appear to have published any other meteorological investigations; his book was reviewed by Gold (18).
In contrast to Cordeiro's approach, Whipple (19) provided a reasonably complete mathematical discussion of inertial motion on a smooth rotating sphere, and took Sprung to task for not completing the problem the latter set out to solve. Whipple includes seven distinct cases of inertial trajectories, depending upon values of various arbitrary constants, and notes that "the study of motion under no forces should precede the theoretical study of motion governed by pressure." Here, there is a distinct ring of Napier Shaw's views, who, incidentally, submitted Whipple's paper to the Philosophical Magazine.

As noted above, the distinction between a streamline and a trajectory was not widely appreciated in meteorological discussions of air motions, say, 70 years ago. Though such expressions as "paths of air" and "the path of air motions" were crudely related to trajectories, it is clear from the context of many discussions that the latter were often confused with streamlines. It is therefore not surprising that Napier Shaw (20) thought sufficiently of this state of affairs to point out this distinction explicitly—about the time he and Lempfert were beginning their investigation of "The Life History of Surface Air
Currents"—and to demonstrate some of its mathematical consequences. Moreover, Shaw encouraged Hudson (21) to investigate additional mathematical consequences of the combined effect of a moving circular depression and air motions relative to its center; the resulting trajectories Hudson called "anemoids."

The rather prosaic frontispiece in Shaw and Lempfert's (9) memorable paper is an immediate indication that something new is being offered in the matter of examining the motion of the atmosphere; here we see eight graphs, depicting various classes of trajectories, that display changes of temperature, pressure, and speed as a function of time; in short, the "life history of surface air currents."

Shaw and Lempfert constructed their trajectories from hourly observations of wind and pressure made at stations throughout the British Isles and the adjacent regions of western Europe; several trajectories, covering the width of the Atlantic, were constructed from 1882 synoptic charts.

The major dynamical conclusion of this investigation, was expressed by Shaw in his "Introduction" to Gold's (22) memoir two years later: "... one of the most obvious conclusions from that research in the kinematics of the
atmosphere, is that, as a matter of fact, air in its motion is subject to very little and very slow change of velocity. The meteorological time unit is the hour or the day, and in these intervals the changes in the velocity of the moving air are comparatively small . . . " Five years later Shaw (23) provided a formal discussion of this conclusion in the form of his "strophic balance," and coined a new term for the first edition of the Meteorological Glossary (24), the geostrophic wind. This firm dynamical formulation, drawn from Lagrangian studies of the basic wind-pressure relationship for large scale air motions, did not arise full grown from Shaw and Lempfert. Indeed, the familiar geostrophic formula appears in the literature nearly forty years earlier, though its full dynamical significance was hardly appreciated; rather, it appears, in the hands of Mohn (25) and Peslin (26), as little more than an empirical formula of uncertain value. Further, these latter investigations were published in rather obscure places, and neither appears to have attracted much notice; indeed, Shaw and Lempfert do not refer to either paper, nor does Sprung (27), in his otherwise thorough book. Finally, Shaw (9) suggested the next
step and asserted that subsequent investigations of wind and pressure ought to start from the "strophic-balance," and that departures from such a balance "are likely to be productive as a subject of meteorological study."

Many synoptic investigations prior to the work of Shaw and Lempfert attempted to relate the observed weather to particular features of the pressure field, i.e., properties of the field of motion were related to the force field, i.e., properties of the field of motion were related to the force field. The obscure physics of this procedure must have improved Shaw considerably, especially in the light of his admiration for Maxwell's work, but, in any event, he and Lempfert supplied a vital ingredient to a possible solution by relating the weather to a consequence of the field of motion--the "horizontal dilation of areas"--on the Lagrangian plan. Shaw and Lempfert, with the aid of G. T. Bennet's technique of computing divergence from the wind field, were able to present models of the distribution of clouds and precipitation with respect to the center of a typical depression; and, what is more, were able to show why such a distribution ought to be expected.
The provocative results of Shaw and Lempfert appear to have stimulated a wide interest in particle dynamics amongst continental meteorologists as well as those in Britain. Two years after the former investigation was published, Gold (28) presented the results of particle trajectories in both zonal and circular isobaric fields, deriving, respectively, the well known cycloid and cardiod curves. In addition, with Napier Shaw's urging, Gold (22) attempted to compute the time necessary for the geostrophic balance to be achieved by a parcel starting from rest in a pressure field of zonal isobars; in his words "... whether the pressure is likely, in actual circumstances, to continue steady long enough for such a state of affairs to come about (i.e., geostrophic balance). We can get some idea of the time required to lapse by considering the motion of a particle on the earth's surface (starting from rest), moving under a constant force in a constant direction." Gold considered the geostrophic wind to exist when the parcel was moving parallel to the isobars, and computed this time to be about 4 hours at 50°N. Unfortunately, though the parcel does indeed move parallel to the isobars in about this time, it has exactly twice the geostrophic speed; in fact, for time independent pressure fields
the parcel never attains the geostrophic speed unless it is started with it. This is a curious error in a memoir that is otherwise thorough, and clearly not lacking in the relevant mathematical framework. Nevertheless, Gold's memoir is a milestone in the development of dynamic meteorology containing, among other things, the first observational study of a comparison between observed winds at 1000 m and the geostrophic wind deduced from the surface pressure, the first discussion of the physical significance implied by the quadratic nature of the gradient wind equation, and a partial hint of what later became known as the thermal wind equation. Finally, Napier Shaw's "Introduction" to Gold's memoir is a succinct statement of the whole problem of wind and pressure, much of which is hardly outdated 60 years later.

Continental meteorologists contributed to the growing narrative on particle dynamics beginning with Sandstrom's instructive discussion of balanced flow with (surface) friction following the original formulation of Guldberg and Mohn (30). Sandstrom considers some possible frictional trajectories, using data from (Scandanavian) synoptic charts, and develops a plausible physical argument to show that the expected parcel air motions should be "oscillatory paths;"
he then selects some synoptic (winter) situations in which the pressure field is sensibly steady and then depicts the actual field of motion using his recently developed isogon-isotach technique. Appealing to the near steady character of his chosen synoptic charts, he identifies the resulting streamline map with that of actual trajectories, and thereby demonstrates the reasonableness of his initial deduction, namely, that the trajectories do indeed follow "oscillatory paths". Whether or not the Guldberg and Mohn formulation of friction, including Sandstrom's own ingenious modifications, is entirely adequate to the particular geography at hand is perhaps debatable; nevertheless, Sandstrom's formulation undoubtedly contains the salient physics of the situation. Finally, Sandstrom's paper contains no reference to the work of Shaw and Lempfert (9), or Gold (22), nor does Abbe, the translator provide them.

Felix Exner (31) clearly found Sandstrom's results of sufficient interest to warrant a careful theoretical discussion based directly upon the (horizontal) equations of motion. In particular, he presented the two dimensional particle solutions for the case of east-west isobars, constant Coriolis force, and time-independent meridional pressure field,
showing, among other things, how parcel trajectories (without friction) near 50°N crudely resembled those obtained by Sandstrom. Further, Exner also presented the particle solutions for the same case mentioned above with friction, following the Guldberg and Mohn formulation; however, he provided little discussion of these solutions, save a few remarks concerning the inadequacies of the frictional formulation. Finally, Exner (32) provided an extended discussion of particle dynamics in his well known book, though he did not repeat it in his second edition (33).

The lack of any considerable discussion on the part of Exner (31) of the parcel equations with friction was soon supplied by Hesselberg (34), who also included some discussion of time-dependent pressure fields. However, unlike Exner he provided no diagrams of computed trajectories, but only a qualitative discussion. Nevertheless, Hesselberg's results did include the complete particle solutions with a clear acknowledgment of the importance of the initial conditions. If friction is neglected and the pressure force depends only on the time, then his results reduce to a complete particle dynamics of non-viscous flow in a horizontal plane. Thirty years later
Forsythe (35), unaware of Hesselberg's paper, followed the same approach in his discussion of time dependent pressure fields. Lastly, Hesselberg acknowledged the previous work of Gold (22) and Sandstrom (29), as well as Exner (31).

The next twenty-five years (1915-1940) witnessed the first attempts to examine the atmosphere in direct Lagrangian terms, beginning with Meisinger's (36) manned "constant elevation free balloon flights from Fort Omaha." In these experiments, in which he eventually lost his life, he attempted to compare the observed balloon trajectory with features of the surface pressure distribution. On a much smaller scale, Richardson and Proctor (37) examined the Lagrangian results of the dispersal of toy balloons in an attempt to examine some features of atmospheric diffusion in the surface layer. Finally, Ackerman and Piccard (38) reported the first (unmanned) constant-level-balloon observations made in the United States using balloons constructed of cellophane. Subsequent developments along this line are summarized in Angell's (39) extensive review.

The increased attention given to new observational techniques for obtaining Lagrangian data and its interpretation, as well as the equally significant results of
Eulerian perturbation analysis, motivated by the polar front theory, appears to have encouraged Bjerknes' (40) theoretical discussion of both techniques. Among other things he contrasted the theoretical advantages to be gained from a Lagrangian analysis of some simple fluid models, though his paper appears to have gained little notice even amongst theoreticians.

Just prior to World War II German meteorologists took up several aspects of particle dynamics, and, apparently, rediscovered the early work of Shaw and Lempfert. Significant contributions were published by Baur and Philipps (41), Philipps (42), and an interesting, though largely ignored investigation, by Lotz (43); further Koschmieder (44), in his well known book, provided a useful summary of earlier results in the first edition (1933) of his book, though deleted, as Exner did, most of the material in his second edition (1941).

Baur and Philipps (41), though not appealing to the complete solutions of the parcel equations, formulated the problem in terms of balanced flow with friction; they provided numerous graphs giving the pressure gradient force as a function of the angle between isobars and wind vector for various values of wind speed over the sea. Following
Sandstrom's (29) original suggestion for modifying Guldberg and Mohn's (30) formulation of friction, but more importantly following Ekman's (45) results, Baur and Philipp's resultant friction vector is directed $45^\circ$ from the direction of motion rather than just opposing it, as in the original formulation of Guldberg and Mohn. Two years later Lotz (43), in the spirit of Shaw and Lempfert, determined the horizontal velocity divergence from synoptic surface charts by following three sample parcels defining a triangular area. Lotz made a clear distinction between geostrophic trajectories and his "trajekorien des Windes erster Naherung," choosing his first approximation by attempting to evaluate the geostrophic departure, a technique later utilized (independently) by Freeman and Franceschini (46), and Franceschini (47).

In the early 1950's constant-pressure balloons were perfected and flown near the 300 mb level over North America and the Pacific Ocean, providing the first substantial sample of Lagrangian meteorological data on a large scale (39). The success of this new tool stimulated renewed interest in particle dynamics including an attempt to evaluate the (mean) Lagrangian frictional force at 300 mb, and the non-linear oscillations of a particle in a long wave (48).
Finally, Wurtele (49) demonstrated some interesting theoretical advantages to be gained from a Lagrangian approach to some familiar dynamical problems.

Suspecting that certain defects in numerical forecasts resulted from the Eulerian technique itself, Wiin-Nielsen (50) adopted a trajectory approach and applied it to both barotropic and baroclinic models. Other Lagrangian approaches, adapted to numerical prediction, have been advanced by Djuric (51) and Okland (52).

Finally, Lagrangian techniques have been adopted in the experimental and computational study of dispersion; a review of these results would necessarily take us beyond the intended bounds of this brief review.
III. The Equations of Motion and their Integration

As we briefly noted in the introduction, our basic objective is to try to understand the distribution of cloud and weather in the near surface layer over the oceanic tropics through a direct appeal to the pressure forces that ultimately produce the motions. We include a simple frictional constraint, assume the motion applies to the lowest kilometer of the troposphere, valid at, say, 500 m, and, for convenience assume steady pressure fields invariant with height throughout the lowest kilometer.

Specifically, we solve the horizontal momentum equation for individual air parcels and examine the collective motion, particularly the horizontal divergence, and attempt to interpret the results through an appeal to the divergence equation. We note, in passing, that such a Lagrangian approach has the inherent advantage that parcels tend to collect in just those regions that meteorologically are most significant in terms of cloud and weather production. We do not claim that such an exercise leads to facile models of pressure and wind, or weather distribution; rather we confine our attention to those kinematical and dynamical features that appear significant in producing those features
of, say, the divergence that are most striking in reasonably steady conditions.

Finally, we delay our review of air motions in low-latitudes until Section V.

The horizontal momentum equation, with a frictional term proportional to the speed, is*

\[
\frac{dx}{dt} - 2\Omega \sin \psi \frac{dy}{dt} + \frac{d}{dt} \frac{\partial}{\partial x} \frac{1}{\rho} \frac{\partial P}{\partial x} - \kappa \frac{dx}{dt} = -\frac{\partial P}{\partial y} - \kappa \frac{dy}{dt} \tag{1}
\]

\[
\frac{dy}{dt} + 2\Omega \sin \psi \frac{dx}{dt} = -\frac{\partial P}{\partial y} - \kappa \frac{dy}{dt} \tag{2}
\]

If we further assume that, (1) the motion is horizontal \((dz/dt = 0)\), and that (2) the horizontal Coriolis force, and (3) pressure gradient are constant during time \(dt\), then

\[
\frac{dx}{dt} - 2\Omega \sin \psi \frac{dy}{dt} = \rho_x - \kappa \frac{dx}{dt} \tag{3}
\]

\[
\frac{dy}{dt} + 2\Omega \sin \psi \frac{dx}{dt} = \rho_y - \kappa \frac{dy}{dt} \tag{4}
\]

Rewriting eqs. (3) and (4)

\[
\frac{dx}{dt} \left( \frac{dx}{dt} + \kappa \right) - f \frac{dy}{dt} = \rho_x \tag{5}
\]

*A list of symbols is given on page xv.*
It is worth noting that solutions to eqs. (5) and (6) satisfy the requirements of existence, uniqueness, and continuity, i.e., the problem is well set (45).

A little algebra reduces eqs. (5) and (6) to

\[
\frac{d}{dt} \left[ (\frac{d}{dt})^2 + 2k \frac{d}{dt} + (k^2 + f^2) \right] x = kP_x + fP_y \tag{7}
\]

\[
\frac{d}{dt} \left[ (\frac{d}{dt})^2 + 2k \frac{d}{dt} + (k^2 + f^2) \right] y = kP_y - fP_x \tag{8}
\]

The three roots of the characteristic equation are clearly 0, and \(-K \pm if\); thus, the homogeneous equation has solutions

\[
x = c_1 + c_2 e^{-(k+i)f} t + c_3 e^{-(k-i)f} t \tag{9}
\]

\[
y = c_4 + c_5 e^{-(k+i)f} t + c_6 e^{-(k-i)f} t \tag{10}
\]
Particular integrals are, in each case

\[ I(x) = \frac{KP_x + fP_y}{k^2 + f^2} \quad (11) \]

\[ I(y) = \frac{KP_y - fP_x}{k^2 + f^2} \quad (12) \]

Thus, the general solutions become

\[ x = c_1 + c_2 e^{-\frac{(k+i\delta)t}{k^2+f^2}} + c_3 e^{-\frac{(k-i\delta)t}{k^2+f^2}} + \frac{KP_x + fP_y}{k^2+f^2} t \quad (13) \]

\[ y = c_4 + c_5 e^{-\frac{(k+i\delta)t}{k^2+f^2}} + c_6 e^{-\frac{(k-i\delta)t}{k^2+f^2}} + \frac{KP_y - fP_x}{k^2+f^2} t \quad (14) \]

The number of undetermined coefficients in eqs. (13) and (14) are clearly excessive; to reduce the six coefficients to four, substitute eqs. (13) and (14) into eq. (5); the result is

\[ c_2 = -c_6 \quad (15) \]

\[ c_3 = c_5 \quad (16) \]
Making use of Euler's exponential formula, along with eqs. (15) and (16), eqs. (13) and (14) become

\[
x = c_1 + e^{-kt} (c_2 \cos ft + c_3 \sin ft) + \frac{kP_x + fP_y}{k^2 + f^2} t \tag{17}
\]

\[
y = c_4 + e^{-kt} (c_3 \cos ft - c_2 \sin ft) + \frac{kP_y - fP_x}{k^2 + f^2} t \tag{18}
\]

Here the imaginary unit \(i\) has been absorbed into the new constants. The remaining four constants can be evaluated by letting \(x = x_o, \ y = y_o, \ \dot{x} = u_o, \ \dot{y} = v_o\) when \(t = 0\); this leads to

\[
c_1 = x_o + \frac{k u_o + f u_o}{k^2 + f^2} - \frac{2kfP_y + P_x (k^2 - f^2)}{(k^2 + f^2)^2} \tag{19}
\]

\[
c_2 = -\frac{k u_o + f u_o}{k^2 + f^2} + \frac{2kfP_y + P_x (k^2 - f^2)}{(k^2 + f^2)^2} \tag{20}
\]

\[
c_3 = \frac{f u_o - k u_o}{k^2 + f^2} - \frac{2kfP_x - P_y (k^2 - f^2)}{(k^2 + f^2)^2} \tag{21}
\]

\[
c_4 = y_o - \left[ \frac{f u_o - k u_o}{k^2 + f^2} - \frac{2kfP_x - P_y (k^2 - f^2)}{(k^2 + f^2)^2} \right] \tag{22}
\]
Substituting eqs. (19) - (22) into eqs. (17) and (18), and differentiating once with respect to time yields the coordinate and speed solutions for a particle

\[ x = x_0 + (u_0 - u_F) (t \sin \theta - K \cos \theta) \frac{e^{-Kt}}{K^2 + f^2} + (v_0 - v_F) (K \sin \theta - t \cos \theta) \frac{e^{-Kt}}{K^2 + f^2} + u_F t \]  

(23)

\[ y = y_0 + (v_0 - v_F) (t \sin \theta - K \cos \theta) \frac{e^{-Kt}}{K^2 + f^2} - (u_0 - u_F) (K \sin \theta - t \cos \theta) \frac{e^{-Kt}}{K^2 + f^2} + v_F t \]  

(24)

\[ u = (u_0 - u_F) e^{-Kt} \cos \theta + (v_0 - v_F) e^{-Kt} \sin \theta + u_F \]  

(25)

\[ v = (v_0 - v_F) e^{-Kt} \cos \theta - (u_0 - u_F) e^{-Kt} \sin \theta + v_F \]  

(26)

Where, again

\[ u_F = \frac{K P_x + f P_y}{K^2 + f^2} \]  

(27)

\[ v_F = \frac{K P_y - f P_x}{K^2 + f^2} \]  

(28)
The last two expressions (27, 28), originally defined in eqs. (11) and (12), have a simple interpretation: they are the horizontal speeds that an air parcel would achieve in unaccelerated motion—a result that also follows directly from eqs. (3) and (4), and, incidentally, first appeared in a distinguished paper by Guldberg and Mohn (30). Finally, eqs. (25) and (26) were first obtained by Exner (31) and Hesselberg (34).

**Discussion**

The assumption of horizontal motion is a familiar one and finds its justification in the fact that horizontal fluid speeds in the atmosphere are at least an order of magnitude larger than the vertical speeds. Further, if the former can somehow be established, the effects on the latter can be inferred through continuity.

The neglect of the vertical Coriolis force \( (2w_0 \cos \varphi) \) is justified by recognizing that even for vertical speeds of, say, 10 cm sec\(^{-1}\) on the equator, it would still be at least an order of magnitude smaller than typical horizontal pressure forces likely to found near the equator \( (15 \times 10^{-3} \text{ cm sec}^{-2}, \text{ say}) \) (53).
The formulation of friction deserves some comment. Firstly, it follows the original formulation of Guldberg and Mohn, and doubtless is suggested by familiar examples of mechanical damping. Indeed, Rayleigh (54) suggested the same form of damping in his dissipation function, wherein the latter is just twice the dissipation, and leads to a considerable simplification in Lagrange's equations.

However, when applied to fluid motions a frictional force proportional to the speed implies laminar flow a doubtful assumption in the earth's boundary layer. Nevertheless, it is clearly necessary to require some dissipation throughout, say, the lowest 1000 m of the atmosphere; the present formulation is the simplest one in which (3) and (4) can be integrated, and, at the same time contains the essential physics. Moreover, Philipps (55) and Kao (56) have utilized this formulation in the former's well-known numerical experiment, and in the latter's investigation of non-linear particle oscillations in an atmospheric long wave. Again, the motivation was a desire to incorporate some dissipation, not to investigate the mechanics of turbulence.

The position of the present formulation is, then a frankly empirical one: does it lead to sensible speeds?
Further, and perhaps more importantly, with friction opposing the motion, rather than acting at some angle to the motion as implied in the Ekman theory, the present formulation is met if the wind does not turn with height. Judging from Gray's (57) statistics from the tropical oceans this assumption is reasonably good; or, at worst, the small changes in direction through the lowest kilometer (about 10°) are hardly known with sufficient precision to warrant additional refinements.

With a frictional acceleration proportional to the speed, the rate of change of kinetic energy following the motion is proportional to twice the kinetic energy at a particular instant; this is in contrast to the rate of change being directly proportional to the kinetic energy itself when the frictional acceleration is proportional to the square of the speed. Further, K is a measure of the time necessary for the fluid motion to decrease by 1/e of its initial value after the pressure forces are removed. A simple calculation shows that the motion would decrease by about three orders of magnitude in 4 days, which, understandably, is a slightly less than the usual estimates of 7-10 days (58).
The value of $K$

For unaccelerated horizontal motion with the x-axis directed along the isobars ($P_x = 0$), it is easily demonstrated, with the aid of eqs. (3) and (4), that the angle, $\theta$, between the wind vector and the isobar is just

$$\frac{1}{\tan \theta} = \frac{K}{f}$$

Thus, an estimate of $K$ can be obtained from a chart of wind and pressure. Such estimates of $K$ were very numerous in the meteorological literature of the late nineteenth century, frequently including values for every major city of Western Europe! These values were generally of the order $10^{-5}$ sec$^{-1}$; similar estimates over the Indian Ocean within 15 degrees of the equator (50) yield values between 2.0 and $3.0 \times 10^{-5}$ sec$^{-1}$. Such estimates are about the best that can be obtained from the available climatological charts; a somewhat better estimate ($2.8 \times 10^{-5}$ sec$^{-1}$), will be described later.

Since the horizontal pressure gradient is assumed constant with height, this fixes the thickness of the layer to which the computation can apply. Though some evidence has been advanced to suggest an upper limit of about 1500 m over parts of the oceanic tropics, we will arbitrarily assume our
computations apply to the lowest kilometer of the oceanic troposphere, valid at, say, 500 m. The latter assertion is less arbitrary than the assumed layer thickness, since we will try to obtain a value of $K$ valid at this level.

In summary, then our explicit assumptions are:

1. Steady pressure fields.

2. Friction opposing the motion and proportional to the speed (no turning of the wind with height).

3. Horizontal pressure gradient constant with height through the lowest kilometer layer.

Lastly, though we have assumed a constant Coriolis acceleration in obtaining our solutions (23-26), this is only necessary for an interval $dt$, which we may choose as small as we like, and thus incorporate an implicit variation in this significant parameter.

The computations

With a specified pressure field, and initial wind speeds, taken from a meteorological chart we iterate eqs. (23) through (26) to obtain parcel positions, speeds, and, if we wish, accelerations, as functions of space and time. Specifically, the iterations are performed at hourly time
steps, which, as noted above, insures an implicit variation in the Coriolis acceleration; its value was assumed appropriate to the latitude at $t_0$ and was held constant in eqs. (23) - (26) for $t_0$ plus one hour. Further refinements were tried, but none offered significant improvements over that adopted above.

The pressure field was specified by defining the horizontal pressure gradient in "five-degree squares," as deduced from an appropriate chart. Doubtless this procedure smoothed out some of the errors that might otherwise be expected, though it was originally adopted on the plausible assumption that a finer resolution from the available observations did not appear warranted. With the pressure gradients defined this way, they do present some disadvantages; resulting discontinuities lead to too rapid changes in some of the kinematical properties—viz., the divergence. However, these rapid changes can be effectively eliminated by ignoring a few (hourly) computations on either side of the discontinuity. This is substantially the case for those results presented on horizontal maps, where, for the lack of space, only values at, say, five or ten hours, have been entered. However, the hourly computed values presented in
detail later have not been altered.

The Divergence

Since analytical solutions are available describing a relationship between parcel positions and speeds, it would seem natural to use the equation of continuity in the elegant form first proposed by Euler, the so-called Lagrangian equation of continuity

\[
\text{div} \ \nabla = \frac{1}{J} \frac{dJ}{dt}
\]  

(30)

where \( J \) is the Jacobean \( J(u,v,w/x,y,z) \), and \( V \) is the velocity (60).

Rather than use eq. (30) directly, however, a direct appeal to the geometry of the horizontal motion will be adopted, using the familiar identity

\[
\text{div} \ \nabla_h = \frac{1}{A} \frac{dA}{dt}
\]  

(31)

This can be evaluated in several ways; an obvious one, being a direct application of Green's Theorem. Such a computation is easily made by assuming that the area enclosed by four sample parcels can be represented, at hourly intervals,
by straight lines connecting adjacent parcels. For this time interval and the divergence results to be discussed later, this gives quite satisfactory results. However, there is a chance of error if, at sometime during the motion, the sense of traversing the contour is reversed, i.e., if the sample area were to turn "inside-out" between one hour and the next, it is possible that the sign of the divergence could be reversed. Several trials were performed to investigate this possibility; in general, no difficulty was encountered using a wide range of "harmless" looking pressure fields normally encountered in low latitudes, but some problem could arise if the pressure field were rather "exotic." In any event, the computation was subsequently programmed in such a way that a test could be made to examine the situation at any time. Finally, the results presented subsequently are not subject to this difficulty.

If certain symmetries are present or can be assumed in the motion field, then the computation of the divergence can be reduced to an even simpler form. For example, consider four parcels defining a fluid square whose sides are parallel to the conventional cartesian co-ordinate axes. If the two parcels lying along the x-axis are started with the same
initial speed, and are subsequently acted upon by identical forces producing identical motions, then it is easily seen that the percentual change in the x co-ordinate is identically zero, and the divergence reduces to

\[ \text{Div } \nabla H = \frac{1}{y} \frac{dy}{dt} \]  \hspace{1cm} (32)

This simplification arises whenever "x-symmetries" appear in the pressure gradient along with identical initial speeds for two adjacent parcels along the x axis. Such simplifications are often suggested in particular instances, and considerable use will be made of them in subsequent sections.

It is important to recognize that eq. (32) does not assert that the divergence of, say, a field of parcels is the same function of y everywhere. For pairs of adjacent parcels starting with different initial conditions and traversing slightly different regions the divergence, in general, is slightly different function of the y co-ordinate for each pair, i.e., in the general case, we need not expect zonal symmetries in the divergence field.

Finally, the computations were performed on an IBM 7040 computer with the parcel co-ordinate output stored on magnetic tape. The latter product was subsequently used to plot trajectories with the aid of the Benson-Lehner LTE (Large Table Electroplotter).
IV. Interpretative Framework: the Divergence Equation

After computing the horizontal divergence along selected trajectories, we wish to examine the consistence of the results and provide interpretative comment regarding those factors that produce the computed results, i.e., we wish to examine the substantial derivative of the horizontal divergence. This naturally suggests an examination of the divergence equation obtained by taking the horizontal divergence of the vector momentum equation; the direct, but lengthy result is

\[
\frac{dD}{dt} = \frac{1}{2} \left( \Delta \delta - \Delta \beta + \frac{\partial}{\partial \gamma} \right) \omega \frac{\partial D}{\partial \gamma} - \left( \frac{\partial \omega \Delta u + \partial \omega \Delta v}{\partial \gamma + \partial \delta} \right) + \frac{3}{2} \beta u - \Delta \nabla \cdot \mathbf{p} - K
\]  

(33)

Further, it is easily verified that

\[
\nabla_\delta \cdot \frac{d\nabla_\delta}{dt} = \frac{dD}{dt} - \frac{1}{2} \left( \Delta \delta - \Delta \beta + \frac{\partial}{\partial \gamma} \right) \omega \frac{\partial D}{\partial \gamma} + \left( \frac{\partial \omega \Delta u + \partial \omega \Delta v}{\partial \gamma + \partial \delta} \right)
\]  

(34)

that

\[
\nabla_\delta \cdot (2 \nabla_\delta \times \mathbf{K}) = \frac{3}{2} \beta u
\]  

(35)
In other words, in eq. (33), the first term on the left plus the first six terms on the right arise from taking the divergence of the horizontal acceleration; the seventh and eighth terms (\(\frac{3}{2}f + \beta u\)), from taking the divergence of the horizontal Coriolis acceleration; the ninth and tenth \((-\alpha \nabla^2 p, -\mathcal{K}D\)) terms from taking the divergence of the pressure and frictional forces, respectively, or, briefly, that

\[
\nabla_h \cdot \left( \alpha \nabla_h p \right) = \alpha \nabla^2 p \tag{36}
\]

and that

\[
\nabla_h \cdot \left( \mathcal{K} \mathcal{W}_h \right) = \mathcal{K} \mathcal{D} \tag{37}
\]

We may refer to these four major contributions as acceleration, Coriolis, friction, and pressure-force terms respectively.

For horizontal motion eq. (33) reduces to

\[
\frac{dD}{dt} = \frac{1}{2} (D^2 - \frac{3}{2} \mathcal{D}^2) + \frac{3}{2} f - \beta u - \alpha \nabla^2 p - \mathcal{K} \mathcal{D} \tag{39}
\]
Later, we shall consider the likely effect of ignoring the vertical motion terms, but first consider the physical content of eq. (39).

Considering the last two terms on the right hand side of eq. (39), the frictional and pressure-force terms, it seems physically evident that a divergent force field is, in principle, capable of producing divergence in the fluid motion, i.e., of contributing to the left hand side of eq. (39). Moreover, we note that our formulation of friction leads to a term proportional to the divergence itself \((KD)\) in eq. (39). Thus, in the latter equation this term produces changes in the substantial derivative of the horizontal divergence \((\frac{dD}{dt})\) which tends to counteract the sign of the divergence itself--i.e., if \(D\) is positive \(\frac{dD}{dt}\) is negative and conversely. This particular feature is a useful one in the present computations, since it acts effectively to restrain the vertical speed and thus does not aggravate our initial assumption of horizontal motion. Further, it is also useful to consider a more general formulation of friction adopting the normal mixing length theory; if the eddy viscosity is constant with height, then the divergence of the friction term, \(\frac{K}{\rho} \frac{\partial^2}{\partial y^2} \),
is then proportional to the (vertical) curvature of the horizontal divergence profile. When the latter is negative the left hand side of eq. (39) is positive, and conversely, i.e., the sense of the change in the substantial derivative of the horizontal divergence in eq. (39) operates in a similar sense to that of our much simpler frictional term. Of course, if the eddy viscosity varies with height, the latter analysis would not apply.

Secondly, if the last two terms were absent in eq. (39) the motion would be reduced to that in a central force field and suggests the possibility of angular momentum conservation; indeed, the remaining terms in eq. (39) can be deduced from considerations of both linear and angular momentum conservation; more easily if spherical co-ordinates are employed (61). Finally, the neglected terms in eq. (33), the vertical motion terms, represent basically advective contributions and need no elaboration.

Thus, somewhat inexactely, the temporal change of the substantial derivative of the horizontal divergence is the combined result of divergent material and inertial accelerations and divergent force fields. In what follows, we
will attempt to examine our results in the light of both eqs. (38) and (39) providing, on the one hand, a broad view of the four major terms, and on the other, a closer view at the composition of acceleration and Coriolis terms. Specifically, we will attempt to evaluate all terms in eq. (39), save the deformation, which will be obtained from elementary bookkeeping. Additional checks on the evaluation of the acceleration term is available through occasional computations from the parcel accelerations themselves.

To anticipate subsequent results, it appears that one serious effect of ignoring the vertical motion terms is to render the deformation negative, i.e., imaginary; the discrepancies, generally of order $1 \times 10^{-11}$ sec$^{-2}$, can usually be accounted for by including these terms, at least in an approximate way. This was the case for the Indian Ocean in July.

Remembering that our parcel is the locus of two or more sample points, we wish to evaluate the terms in eq. (39) consistent with this description. For example, the relative vorticity is determined by evaluating the angular speed at the midpoint of a line connecting two sample points.
defining our parcel. The angular speed is computed by evaluating the (hourly) change in the angle determined by the intersection of two straight lines connecting two sample parcels on adjacent trajectories at two different hours; twice the angular speed is just the relative vorticity. Since we know the hourly coordinates of each parcel, a direct appeal to the simple geometry gives the angular speed. We must, of course, adopt some sign convention for the relative vorticity; here we follow the familiar one and assign local clockwise rotation a negative sign. Again, the Coriolis parameter ($f$) appearing in the vorticity term ($\beta f$) is also evaluated at the same midpoint, as well as the remaining part ($\Theta u$) of the Coriolis term.

On the other hand, the evaluation of the pressure-force term is not quite so obvious. In the first place, this term ($\nabla^2 p$) as it stands, is basically a Eulerian property of the pressure field, and would normally be evaluated by, say, finite difference techniques applied directly to a pressure map. Though the question of a suitable horizontal distance might be bothersome, this too could be decided in various plausible ways. The real question, however, is whether we wish to evaluate
some terms of eq. (39) "following the motion" and use a
field description in the case of the pressure-force term.
We feel that greater consistency is served by adopting
the former Lagrangian view and appeal to the following
intuitive argument: since we know the pressure distri-
bution in space, and the distance between sample points
(s) defining our fluid parcel, we may compute the Lap-
lacian using the familiar finite difference expression

$$\nabla^2 p = \frac{P_1 + P_2 - 2P_0}{s^2} \quad (40)$$

knowing the value of the pressure \((P_1, P_2)\) at the extre-
mities of the fluid parcel, and at the midpoint \((P_0)\).
This procedure assures us that the pressure-force term
will be evaluated in a manner consistent with that adopted
for the other terms, and avoids the nagging doubts entailed
in attempting to use a single estimate of the distance, \(s\),
throughout the entire trajectory.

As a sample test, we evaluated the Laplacian using
both methods for the mean pressure distribution prevailing
over the Indian Ocean during July. In the direct field
evaluation, we took \(s\) to be 2.5 degrees of latitude everywhere
while in the Lagrangian evaluation, the latter varied between 2 and 4 degrees of latitude along one sample trajectory. The resulting patterns were quite similar, though because $s$ necessarily varied along the trajectory in the Lagrangian case, actual values differed by as much as two in some cases.

Some appeal could be made to Gerber's (62) development of the Laplacian, though the advantages to the present investigation are not obvious.
V. Some Computations from the Oceanic Tropics

1. The motion of air in low latitudes

Serious investigation of pressure and wind in low latitudes has a reasonably long though very broken history. Gulberg and Mohn (30) first published a (graphical) solution based on the horizontal equations of motion through a specification of the pressure field; though rather crude, their trajectories did depict some major features of the summer circulation over the Indian Ocean. However, the experimental evidence was meager, and, not surprisingly, their results lacked general conviction amongst a wider audience of dynamical meteorologists.

After Shaw and Gold offered some convincing proof of the validity of the gradient and geostrophic wind formulations, Shaw was of the opinion that if any pressure-wind relationship was valid for low latitudes it could only be the gradient wind. Such an assertion necessarily remained speculative without convincing observational evidence; and this was not available until World War II.

Grimes (63), appealing to particle dynamics, offered some theoretical speculations concerning wind and pressure, assuming, among other things, non-divergent flow.
With the dramatic expansion of observations in the tropics, particularly the Pacific, during World War II, Treloar, Gibbs, and later Crossley (64), continued the examination of air motions in low latitudes. The latter two investigators again appealed to particle dynamics and attempted to incorporate friction (Gibbs) and the likely effects of accelerations close to the equator. Gibbs employed the Gulberg-Mohn formulation of friction and attempted to evaluate the frictional constant, $K$, through a direct appeal to measured pressure gradients in low latitudes. These and other developments were summarized by Grimes (65). Later, Gordon and Shaw (66), again appealing to particle dynamics, attempted to derive a low-latitude general circulation model.

With the exception of the early attempt by Gulberg and Mohn (30) to compute the field of motion directly from the observed pressure field, only two recent attempts appear to have sustained this dynamical approach: one by Wyrtki (67), valid at the surface over the oceanic tropics, and another by Johnson & Morth (68) working in tropical Africa.
Wyrtki attempts to compute the field of (horizontal) motion directly from the pressure field using the same formulations embodied in eqs. (3) and (4), but assuming the horizontal accelerations vanish; with this assumption, a simple computation shows that the scalar wind speed and direction are given by

\[ v = \left\{ \frac{p_x^2 + p_y^2}{k^2 + \frac{j^2}{f^2}} \right\}^{\frac{1}{2}}, \quad \tan \theta = \frac{h}{j} \]  

(42)

After specifying three simple pressure fields (... "equidistant isobars parallel to the equator ... isobars crossing the equator under 45° ... a pressure trough at 8 degrees north ...") the speed and direction of the wind is computed from (42) assuming "that the movements are stationary." Further, the pressure fields, it must be assumed, are fixed in space, and not dependent.

The results are presented in his figures 3, 4, and 5, from which it is easily seen that the computed speeds vary along the trajectory, and thus contradict the basic assumption of vanishing accelerations embodied in eq. (42). Such a result, clearly, is not acceptable.
Johnson and Morth (68), and Morth and Johnson (69), working in tropical Africa, have summarized their experience in the form of several synoptic models relating wind and pressure in the lower troposphere. Later, Morth (70) offered minor amendments to two of the four basic models, but has not apparently altered his interpretive framework.

Briefly, the above investigators have provided several models of wind and pressure, together with inferred divergence patterns, which have, in their judgment, provided some insight into forecasting over East Africa. Their synoptic results of particular flow patterns and observed weather are encouraging, and apparently have led to improved forecasts. Nevertheless, their results warrant comment on two grounds: (1) their ability to analyze the pressure field with requisite detail in equatorial latitudes, and (2) their appeal to particle dynamics as an interpretive framework.

Firstly, to obtain accurate measurements of the surface pressure in low latitudes, even on isolated atolls, is difficult; further, to measure pressure-heights with conventional radiosondes from the same atolls to heights above the 700 mb surface is more difficult still. To
add to this a continental station network, wherein the
surface elevation itself is uncertain, to say nothing of
using radiosondes of various manufacture, is to increase
the overall uncertainty by a considerable amount. A dis-
cussion of these factors would have been a welcome addi-
tion to the synoptic charts presented by the authors, and
would have contributed in some measure to understanding
their subjective treatment of some of the observations.

To fix ideas, if the 200 mb surface is analyzed in
height intervals of 40 gpm, as the above authors have done,
then the mean vertical temperature of the air column
between 850 and 200 mb must be known to within 1°C. Simi-
larly, the 500 mb surface could be analyzed in intervals
of 20 gpm if the mean virtual temperature were known
within about 1.3°C. In low latitudes, the latter require-
ment can probably be met; the former is marginal. Adding
to this, a synoptic network where surface elevations
themselves are not known with confidence, then computed
pressure-heights cannot be accepted without some reserva-
vations. Unfortunately, these nagging doubts have not
been removed by Thompson (71) in an otherwise extensive
work on the climatology of Africa. In fact, the analysis
of mean pressure-heights throughout a sea-level network of uniform instrumentation entails serious limitations. In this regard, Ballif's (59) experience in the Marshall Islands is instructive: pressure-height "values consistent with those at the surface were obtained at the 850 and 700 mb levels; above this, no significant pattern could be drawn."

Secondly, Morth and Johnson have attempted to furnish a theoretical interpretation of their empirical results within the framework of non-viscous particle dynamics. They have presented three distinct pressure distributions, with attendant wind fields: the equatorial "Duct" (low pressure on the equator, high to the north and south), the "Drift" (high to the north, low to the south of the equator), the "Bridge" (high pressure on the equator, low to the north and south), and a "composite model." Numerically computed trajectories are presented for the "Duct," the "Drift," and a "trajectory illustrating deflection into westerly downgradient flow;" the "Bridge" and "Composite Model" are schematic only; lastly, in one case (the "Drift") an analytical solution is provided for comparison with the numerical one.
After the pressure field is specified the problem, clearly, is one of initial values; the authors, unfortunately have failed to provide any discussion to demonstrate that their initial states are, in some way, preferred ones. This is not a trivial point; indeed, it can be demonstrated quite easily (72), that with a constant pressure gradient force and Coriolis parameter, solutions of the horizontal, inviscid, equations of motion admit of three different kinds of trajectories "depending, respectively, on whether the magnitude of the geostrophic velocity is greater than, equal to, or smaller than that of the initial velocity of geostrophic deviation." That is, whether

\[ |u_q| \lessgtr |u_0 - u_g| \]  

(43)

In particular, the three cases are wave shaped, cycloidal, and a series of loops, respectively.

The fact that Morth and Johnson have incorporated a variable Coriolis parameter, and that the above criteria are for a constant one, does not alter the basic objection; for example, the above analysis could easily be applied to their Figure 7.1.23--where the parcel amplitude is about 2 degrees of latitude--obtaining a very different
result using different initial speeds. Thus, Morth and Johnson's assertion that "... for a given pressure gradient in the ideal drift, there is a critical latitude equatorwards of which geostrophic flow is only in neutral stability and, for all subgeostrophic velocities the air will flow southwards across the equator. As one proceeds polewards of the critical latitude the flow is stable for increasingly subgeostrophic winds," is not, in the general case, a valid conclusion. In fact, a "stable" trajectory can be achieved at any latitude provided the initial state is managed properly.

Though it is, in general, true that such a "critical latitude" exists under the conditions noted, it is also true that all parcels would be "stable" if the initial speeds were supergeostrophic. Moreover, it is instructive to realize how sensitive this critical latitude is; for example, with a constant meridional gradient of 1.4 mb/° latitude \((23.0 \times 10^{-3} \text{ cm sec}^{-2})\), and an initial easterly speed of \(-13.6 \times 10^2 \text{ cm sec}^{-1}\) at 5°N, and \(-6.8 \times 10^2 \text{ cm sec}^{-1}\) at 10°N, the parcel at 5°N will cross the equator, while the one at 10°N will not. On the other hand, with a similar gradient of 1.6 mb/° latitude \((25.0 \times 10^{-3} \text{ cm sec}^{-2})\), and an initial westerly speed of \(-12.8 \times 10^2 \text{ cm sec}^{-1}\) at 5°N, and \(-6.8 \times 10^2 \text{ cm sec}^{-1}\) at 10°N, the parcel at 5°N will remain north of the equator, while the one at 10°N will cross it.
sec^2), and the same initial speeds both parcels now cross the equator. The initial speeds are both subgeostrophic: in the first case by 4.50 \times 10^2 \text{ cm sec}^{-1} and 2.26 \times 10^2 \text{ cm sec}^{-1} respectively; in the second case by 6.07 \times 10^2 \text{ cm sec}^{-1}, and 3.05 \times 10^2 \text{ cm sec}^{-1}, respectively. Thus, a difference of 79.0 \text{ cm sec}^{-1} in the two geostrophic deviations at 10N is sufficient to determine whether or not the parcel at 10N would, or would not cross the equator! Such distinctions are vital in determining the character of the divergence; the latter is very sensitive to the initial state, and can easily be demonstrated (73) that the change of the divergence with respect to small changes in the initial speed is inversely proportional to both the geostrophic deviation \((u_o - u_g)\) and the change of initial speed with respect to initial latitude. Is it to be inferred that the necessary discrimination of possible initial speeds has been obtained from the author's synoptic experience?

Again, consider the 7 trajectories in Fig. 1 computed from eqs. (23-26), with \(K = 0\), and adopting the pressure gradient \((12.7 \times 10^{-3} \text{ cm sec}^{-2})\) provided by the authors in their Figure 7.1.24; the other initial speeds were
determined by arbitrarily adding 10 cm sec\(^{-1}\) to the u-component equatorwards of 5\(^\circ\)N, and subtracting the same amount from the v-component over the same distance. The parcel leaving 5\(^\circ\)N displays good agreement in both position (within the limitations of the authors' Figure) and speed with that obtained by the authors. On the other hand, the parcel leaving 6\(^\circ\)N travels northward performing a "stable oscillation," while the rest cross the equator. Under these conditions the "critical latitude" is somewhere between 5 and 6\(^\circ\)N, whereas in the example discussed previously it was north of 10\(^\circ\)N. Since the number of air parcels actually crossing the equator is proportional to the longitudinal extent of the divergence zone to the south, the precise location of the "critical latitude" is of considerable synoptic interest. Again, this can be assured if the initial states are carefully selected.

By way of contrasting and comparing particle trajectories for the "drift" and "duct" Morth and Johnson present "a comparison between the trajectories of the two particles which initially have identical velocities in identical gradients at 5\(^\circ\)N . . . In one case the pressure gradient is constant, in the other the meridional profile is
bowl-shaped as appropriate to a "Duct". The "Duct" case is stable; the drift is not. This result is due to the decrease of the "Duct, and in very low latitudes air which has only a slightly subgeostrophic speed will attempt to cross the equator . . ." These comparisons are presented in their Figure 7.1.24, while the "bowl-shaped" pressure profiles are depicted in their Figure 7.1.20. Unfortunately, it is not possible to provide a comparison similar to that in Figure 1 since the authors have not identified which "bowl-shaped" profile they have used. Nevertheless, their "stable" trajectory could easily be altered to something dramatically different by changing the initial speeds by as little as a few tenths of a meter per second.

Lastly, similar comments could be directed at the results presented in the author's Figure 7.1.27 ("Trajectory illustrating deflection into westerly downgradient flow"); for example, if the particle were directed either towards the south or due north the time and location of the particle approaching the equator would be vastly different. Again, is the chosen initial state a preferred one drawn from either synoptic or climatological experience?
FIG. 1. Seven sample trajectories computed from Morth and Johnson's (69) Fig. 7.1.24. They give initial conditions for only the parcels at 5N ($U_o = -9.80$, $V_o = +.50$ m sec$^{-1}$); the remaining initial speeds were assigned by arbitrarily adding 10 cm sec$^{-1}$ to the $u$-components equatorwards of 5N, and subtracting the same amount from the $v$-component over the same distance.
In summary, if the relationships between pressure and wind, as suggested by Morth and Johnson (69), are to be accepted on the basis of inferences drawn from particle dynamics, then this acceptance must await some demonstration that the tropical atmosphere does, indeed, select some preferred initial state; otherwise, their theoretical discussion lacks conviction.

We turn now to a consideration of some computations drawn from actual pressure fields, assigned initial wind speeds, and compute a selection of trajectories, the accompanying divergence, and attempt to interpret the results diagnostically. Again, the pressure field is assumed steady, friction opposes the motion, and the results are assumed to apply to the lowest kilometer of the troposphere valid at, say, 500 meters.

We offer examples from the Marshall Islands, the Indian Ocean, a few cases of theoretical interest, and the Gilbert-Ellice Islands.

2. **The Marshall Islands—April 1958.**

We noted above that with the aid of eq. (29) and a proper meteorological chart, it was possible to obtain a value of $K$, our frictional coefficient. Such an exercise,
we noted, leads to values of about $2 \times 10^{-5}$ sec$^{-1}$ over various regions of the oceanic tropics. We now propose to seek a more exact value of $K$ by attempting to compute the speed field at 500 m over the Marshall Islands, and compare the results with the observed (mean) speed field. The relevant observations are taken from (74), as published in extenso by Joint Task Force Seven, during operation Hardtack.

From the daily observations of surface pressure and upper winds, we computed the monthly mean surface pressure (75), and mean 500 m vector winds for the 10 available stations; the results are depicted in Fig. 2. Next, we released 20 parcels over a field of 5-degree squares (5-25N, 160-175E), and computed their subsequent motion according to the scheme outlined above. With a preliminary value of $K$ we compared the computed speeds with the observed ones, preferring to take our comparisons, in a particular square, from a parcel that has arrived from outside the square--preferably some distance upstream. We varied the value of $K$ until we obtained a tolerable agreement of $10^0$ in direction and 5 kt in speed. Three or four trials gave a value of $2.8 \times 10^{-5}$ sec$^{-2}$; some
FIG. 2. Observed mean sea-level pressure (+1000 mb), and 500 m vector winds (knots) over the Marshall Islands, April 1958, together with computed sample trajectories. The observed pressure and mean wind speed are plotted above each station location for Wake (W), Eniwetok (E), Bikini (B), Utirik (U), Kwajalein (KJ), Majuro (M), Ponape (P), Kusaie (K), Kapingamarangi (K), Nauru (N), and Tarawa (TW).
sample trajectories are included in Fig. 2, while the overall speed comparison is depicted in Fig. 3. The agreement appears reasonably good; indeed, perhaps surprising, for it is by no means clear that a single value of $K$ is sufficient (76).

By computing the percentage change of area of our 5-degree squares, determined by the motion of 4-corner parcels, we obtained the horizontal divergence, which, we analyzed as a continuous field using the successive positions of individual centroids of each square as datum points for the divergence. The results are depicted in Fig. 4 along with the observed rainfall. Though we must be cautious in making such correlations, these results in the light of other investigations (77) seem reasonable. Moreover, the actual divergence values are in good agreement with those computed from the winds by Sinclair (78).

We cannot, of course, claim that this particular value of $K$ is in any way universal. Indeed, it seems plausible that $K$ is related to the static stability, the wind speed, and the latter's variation with height, and thus likely to vary from case to case. It is impossible to appeal to other oceanic regions (where these factors are
FIG. 3. A comparison between the observed mean wind speed (solid lines), and computed speeds (dashed lines), in knots, as taken from the trajectories of Fig. 2.
FIG. 4. Observed monthly rainfall (inches) for April 1958 and computed horizontal divergence ($x10^{-6}$ sec$^{-1}$) determined by the displacement of 5x5 degree squares. The small dots are the locus of individual centroids for each 5x5 degree square.
likely to be different) for similar comparisons. We only claim that this particular value leads to a reasonable comparison in this instance, and that, for shear lack of definitive observations elsewhere, we assume it to apply in subsequent cases.

Though the comparison between rainfall and divergence is physically satisfying—the region of maximum rainfall had from 13 to 18 days of precipitation equal to or greater than 0.25 inches—we must retain some reservations about the region equatorwards of 5N, as noted above (75).

3. **Indian Ocean—January**

Turning now to the mean conditions over the Indian Ocean in January, we relaxed our specification of the pressure field and included just the meridional variation—certainly a plausible assumption near 60E (79). In addition, though the (Beaufort) speed field is rather coarse, the available observations show the greatest variation of (initial) speeds to be in the meridional rather than the zonal direction. We made practical use of these assumptions to adapt our original "grid" description of the Marshall Islands to a one dimensional analogue of several parcels
released from a common meridian. We may still think of moving fluid squares, but since we assumed two parcels originally lying along the same latitude circle to have the same initial speeds, and acted upon by the same pressure force, we economized this description and examined the results of just two parcels originally starting at two different latitudes. Again, our computational requirements demanded that the x-pressure gradient be the same for all y, and that the y pressure gradient be the same for all x; though restrictive, the latter is sufficiently elastic to include several cases of general interest.

Thus, we released seven parcels along 60E between 15N and 15S, computed their individual trajectories, the horizontal divergence between mutual pairs and analyzed the results as a continuous field. Again, the pressure gradients and initial speeds were taken from the Dutch Atlas (79) and displayed in Fig. 5; the trajectories and accompanying divergence in Fig. 6. Further, for comparison, a few sample surface wind vectors taken from (79) have been added in Fig. 6, while Fig. 7 compares our computed speeds, assumed valid at 500 m, and the Beaufort
FIG. 5. Observed meridional profile of sea-level pressure (mb) near 60E over the Indian Ocean in January, together with the profile of observed wind speed (m sec$^{-1}$). A few wind flags have been entered on the latter profile to indicate direction (north at top of diagram); each full flag represents two units in the Beaufort code. All data taken from (79).
FIG. 6. Computed trajectories and divergence ($10^{-6}$ sec$^{-1}$) using the initial conditions of Fig. 5, for the Indian Ocean, January.
FIG. 7. A comparison between the observed and computed speeds (m sec$^{-1}$) along the trajectory released at 15N. The two solid lines represent the speed interval corresponding to the Beaufort code number along the trajectory. The region between 5N and 3S had insufficient data. Observed winds taken from (79).
surface speeds of (79). Finally, we propose to compare the divergence results of Fig. 7 with the observed mean cloudiness, Fig. 8, as deduced from averaging some 40 satellite orbits during January of 1966 and 1967 (80).

Before discussing the results we must again acknowledge the shortcoming of comparing the computation of divergence based upon long-term climatological averages, and the resulting distribution of cloudiness taken from two particular years. However, the available pressure data for January 1966 and 1967 were insufficient to provide reliable average maps, while data on mean cloudiness drawn from surface observations are too crude to provide a convincing comparison. Again, the diagnostic character of the investigation ought to be borne in mind, with some regard to the kind of detail the present technique is capable of depicting, against the kind of detail we actually observe on at least some occasions. Again, we chose the highly steady regimes of the Indian Ocean, in both summer and winter, with the hope that if some interpretative success were not possible here then little would be likely in less steady, or non-steady regimes elsewhere.
FIG. 8. Observed satellite cloud cover (oktas) for January 1966 and 1967 (80).
Returning to the observed cloudiness of Figure 8 and the computed divergence of Figure 6, we see a broad agreement to the extent that both patterns display maximum cloudiness and convergence near 5 and 10N with a secondary maximum in the convergence near 3N. The observed cloudiness also shows a slight maximum near 5N decreasing westward. Though, the distinction to be drawn from a difference of 0.3 okta in the cloud cover is, understandably, open to question, the individual daily cloud cover displays this feature quite conspicuously.

Next, what are the factors responsible for producing the computed divergence along the trajectories of Fig. 6? Since the distribution of divergence is basically zonal, it will be sufficient to exhibit the results for just two sample parcels; we chose the two released from 10 and 15N (indicated by asterisks in Fig. 5) and examined the results in the light of eq. (38) and (39), i.e., how do these results satisfy the divergence equation?

First, we considered our results in the context of eq. (38) displaying the four major contributions to the divergence of acceleration, pressure-force, friction and Coriolis terms (Fig. 9); then in terms of eq. (39) wherein
FIG. 9. The divergence of the acceleration (A), Coriolis forces (B), pressure gradient force (C), and frictional force (D) for the two sample parcel indicated by the asterisks in Fig. 6. The values are taken from Table 1; the plotted curves satisfy eq. (38).
the acceleration and Coriolis terms are displayed explicitly (Fig. 10). That is, we wish to understand what factors determine curve D (Fig. 9) and the left hand side of eq. (39), plotted as squares in Fig. 10.

We mention again that we are trying to understand what dynamical-kinematical factors are responsible for producing the computed divergence along some selected trajectory. Since the divergence of our frictional force is just the frictional coefficient, K, times the divergence itself, it will be sufficient to understand the general shape of the curves of KD in subsequent figures. Specifically, these are labeled "D" in Figs. 9, 16, 24, and 25.

Crudely speaking, between about 9.0N and 0.3S (Fig. 9) the divergence of the acceleration is nearly equal but opposite to the divergence of the Coriolis terms. On the other hand, the divergence of the pressure force term is positive between about 4.0N and 8.0N; thus, by eq. (38), the remaining term, KD, must be negative and must decrease (8.0N-4.0N) and increase (3.0N-1.0N) as depicted in Fig. 9. i.e., since K is a constant, the divergence itself must be negative and follows the same spatial changes as noted above. Further, there is a lag between the divergence of
FIG. 10. An expanded form of Fig. 9 showing the explicit contributions to the divergence of the Coriolis terms: $u$, Curve F, and $f$, Curve E; two of the four terms comprising the divergence of the acceleration: $\frac{dD}{dt}$, solid squares, $\frac{1}{2}Def^2$, Curve G. The remaining curves, C and D, are the same ones appearing in Fig. 9. Units are $10^{-11}$ sec$^{-2}$; the curves have been plotted with due regard for the algebraic signs in eq. (39).
the pressure-force term and K times the divergence; this is understandable if we recognize that some finite time (about 10 hours in the present case) is required for an air parcel to adjust its motion to a previous change in the pressure gradient. As a result, such changes are displaced downstream relative to changes in the pressure gradient. (Changes in the latter quantity near $5^\circ N$ amounted to $0.4 \text{ mb in } 5.1^\circ \text{ latitude}$. ) Further, the small area of positive divergence near $8^\circ N$ is essentially the consequence of the divergence of the Coriolis terms since this quantity remained negative even after subtracting the acceleration and pressure-force terms, i.e., the divergence must remain positive.

Southward from $3^\circ N$ the divergence decreases rather rapidly—largely the result of the rapid decrease in the divergence of the pressure-force term. Between the equator and $3^\circ S$ the divergence itself is still negative but small (since the other terms have small positive values and all ordinates must add to zero). Again, south of $3^\circ S$ all terms, save the divergence increase thus demanding a decrease in the latter to a value of about $-40 \times 10^{-6} \text{ sec}^{-1}$ near $5.5^\circ S$. 
In contrast to eq. (38), we exhibit the same result in terms of eq. (39). Again, all terms of eq. (39) have been evaluated save the deformation term, which was deduced from elementary bookkeeping. The results for the same two parcels are displayed in Fig. 10. The two curves D and C also appear in Fig. 9 with the addition of two explicit curves (E and F) for the Coriolis terms and, to avoid clutter, the time derivative of the horizontal divergence plotted as discreet squares rather than a continuous curve. All values, it should be noted, were computed in hourly time-steps with Table 1 containing the numerical values at five-hour intervals.

Recalling that Fig. 10 is plotted with due regard for the algebraic signs appearing in eq. (39), we see that the two terms comprising the divergence of the Coriolis forces are negatively correlated north and south of the equator, and positively correlated within the narrow region 0.28 - 1.38. Broadly speaking, both terms are about the same order of magnitude through u is slightly larger within a degree or so of the equator.

Firstly, we recognize that the divergence squared \((-\frac{1}{2}D^2\)), deformation \((-\frac{1}{2}Def^2\)), and friction \((-KD\)) terms
Table 1. Computed values (x10^{-11} \text{sec}^{-2}) of terms in eq. (39) for the two parcels indicated by asterisks in Fig. 6, plus the divergences of the acceleration (A), Coriolis force (B), pressure-gradient force (C), and friction (D); the former are plotted in Fig. 10, the latter in Fig. 9. Indian Ocean, January.

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always decrease the time derivative of the horizontal divergence; on the other hand, the vorticity term \( +\frac{1}{2} \xi^2 \) increases it, while the Coriolis \( 3f - 3u \) and pressure-force \( -dV^2p \) terms may act in either sense. Generally speaking, both the divergence squared \( -\frac{1}{2}D^2 \) and vorticity \( (\xi^2) \) terms are usually smaller than the other terms mentioned above.

Again, near 10N the positive contribution to the time derivative of the horizontal divergence, and the resulting increase in the divergence itself, is largely the effect of the Coriolis terms; specifically the \( \beta u \) term up to about 10N when this is reinforced by the \( 3f \) term. The large oscillations in the latter term between 11 and 12N are not entirely clear; perhaps this represents the initial adjustment to the coarse initial state, and, later, to the rapid change in the divergence of the pressure-force term. In any event, both Coriolis terms sustain their positive contribution to given a local maximum to the divergence (Curve D) near 7.5N; by contrast, the pressure-force term acts in the opposite direction. Near 7.5N the derivative of the divergence changes sign and remains negative producing a minimum in the divergence.
itself near 3N; during this time, the divergence of the pressure-force term has increased to a maximum (of $14.0 \times 10^{-11}$ sec$^{-2}$), while the deformation is nearly, but not quite, balanced by the Coriolis terms. Thus, near 4.5N, where the pressure-force term is a maximum, it makes the largest single contribution to decreasing the parcel divergence along the motion, while the deformation is second. As the divergence of the pressure-force term decreases to zero near 2.5N, the deformation becomes the largest single contributor to decreasing the divergence itself, though because we have neglected the vertical motion terms in eq. (39) the deformation is likely to be an overestimate. By the time we reach 1.5N, the divergence of the pressure-force has vanished and the major positive contributions to the time derivative of the divergence arise largely from the vorticity ($\frac{1}{2} \mathbf{S}^2$) and friction ($-KD$) terms, since again the Coriolis and deformation terms nearly cancel. Near 1.5S the divergence attains a small relative minimum, again, largely the result of the pressure-force term, as before, displaced downstream by about two degrees of latitude (equivalent to 10 or 12 hours). Finally, near 3.5S all terms, save
the frictional term \((-kD)\) produce negative contributions to the time derivative of the divergence producing a rapid decrease in the divergence itself reaching about \(-40\times 10^{-6} \text{ sec}^{-1}\) near 5.5S.

The strong convergence near 5S, also agrees with the seasonal character of the rainfall at the Seychelles (5S, 55E) namely, a maximum. However, we must acknowledge that a similar result is likely with a weak trough between 5-10S, rather than zero gradient as we have used; unfortunately, the pressure data of the Dutch Atlas lacks sufficient detail to depict such a feature if it exists.

Finally, in our discussion of eq. (39) we have neglected the vertical motion terms, i.e., the fourth, fifth, and sixth terms of eq. (33). What effect is this likely to have? Firstly, the fourth term \(\omega \frac{\partial Y}{\partial y}\) we are unable to handle even in an approximate way, since we have assumed that our computations of the horizontal divergence are representative of the lowest kilometer of the troposphere. If this were not true and we permitted some vertical advection, as described by the fourth term in eq. (33), its value would still be an order of magnitude less than the other major terms even if we allowed the
vertical velocity to be 10 cm sec\(^{-1}\), and permitted the horizontal divergence to double (from 12 to \(6 \times 10^{-6}\) sec\(^{-1}\)) through a distance of 1000 m; consequently, we shall ignore the vertical advection term in this and subsequent discussion. The fifth and sixth terms of eq. (33), representing differential vertical motion terms, are likely to be significant, especially in those cases where the horizontal divergence varies rapidly in space. Some assessment of their influence can be obtained by computing the vertical motion from our computations of the divergence, and performing the necessary differentiations in space. We stress the approximate nature of the calculation, and exhibit only the sign of the sixth term of eq. (33) in Table 1; the fifth term can plausibly be neglected in the light of the zonal symmetry in the divergence field (Fig. 6). The results of the last column of Table 1 show that the vertical motion term generally has the same sign as the substantial derivative of the horizontal divergence (column 3); or, the divergence itself is systematically underestimated. Finally, the actual value of the vertical motion term rarely exceeded \(5 \times 10^{-11}\) sec\(^{-2}\).
To summarize both descriptions embodied in Figs. 9 and 10 in terms of the relative extrema in \((K \times \text{the horizontal divergence})\) (Curve D), we recall that both extrema near 11 and 9.5N are the result of the divergence of the Coriolis terms \((\beta u \text{ in the first case; } f\tilde{z} \text{ and } \beta u \text{ in the second})\); the maximum convergence near 2.5N and the modest one near 1.5S result from the divergence of the pressure-force term, while, lastly, the large convergence near 5S is the result of three of the four terms in eq. (38) acting in the same sense. Or, more succinctly, the distribution of horizontal divergence within five degrees of the equator is largely determined by the divergence of the pressure force term, while poleward of this latitude the Coriolis terms are dominant.

4. Indian Ocean—July

Turning to the steady circulation of the summer monsoon, we again extracted mean values of the sea-level pressure and surface wind for July, choosing a meridian (90E) where we may assume zonal isobars, and performed a similar series of calculations as outlined above. Fig. 10 contains the observed profiles of pressure and wind, taken from the Dutch Atlas (79).
As before, we were unable to compare the computed divergence—deduced from a pressure-wind map of some 50 years of observations—with similar time averages of, say, cloudiness; rather, we had to confine our comparisons to the distribution of cloudiness as observed by satellite during July of 1965-66, and averaged, to obtain one-degree averages, by Raman (81). Nevertheless, Raman (personal communication) points out that the July surface pressure distributions during both years were very similar to the climatological average as depicted in the Dutch Atlas. Finally, the standard deviation of surface pressure in the central portions of the Indian Ocean, as deduced from the Dutch data, is about 1.5 mb.

Using the observed pressure distribution of Fig. 11, air parcels were released along 90E at five-degree intervals between 20S and 10N, with initial wind speeds taken from the same figure. Since the Indian subcontinent intervenes near 10N, the computations were terminated there; for clarity only those trajectories originating south of the equator are included in Fig. 12.

Again, for comparison, we include a few sample surface wind vectors (Fig. 12) from (79) together with an
FIG. 11. Observed mean profiles of sea-level pressure (mb) and wind speed (m sec\(^{-1}\)) near 80\(^\circ\)E in the Indian Ocean, July. A few wind flags have been entered on the latter profile to indicate direction (north at top of diagram); each full flag represents two units in the Beaufort code. All data taken from (79).
FIG. 12. Computed trajectories and divergence (x10^-6 sec^-1) using the initial conditions of Fig. 11, for the Indian Ocean, July.
additional comparison of wind speed for the longest trajectory (Fig. 13). The slight discrepancy between observed and computed directions near 10N is likely the result of neglecting a very slight west to east pressure gradient in this region. Further, in the original computations, the parcels leaving 5S and 5N subsequently approached very closely to each other near 7N; the agreement between their respective speeds, though not shown here, was excellent.

In addition to the trajectories, Fig. 12 also contains the resulting divergence pattern. Broadly speaking, we have strong convergence just north of 10S, smaller values between 10S and the equator, a minimum just south of the equator, and a strong convergence in a narrow band just north of the equator followed by smaller values northward.

The comparison with the observed cloud distribution in Fig. 14 is striking. Again, the two convergence zones just north of 10S and the equator, separated by lesser amounts in between are evident. It should be mentioned that Fig. 14 is not just the numerical result of indiscriminate averaging, but represents real distributions observed on many individual days. Fig. 15 is a further example drawn from yet another year (1962).
FIG. 13. A comparison between observed and computed wind speeds (m sec$^{-1}$) along the trajectory released at 20S (Fig. 12). The two solids lines represent the speed interval corresponding to the Beaufort code number along the trajectory. The latter are derived from surface observations; the computed speeds are assumed to apply to a height of 500 m. Observed winds taken from (79).
FIG. 14. Mean cloud cover (per cent) over the central Indian Ocean, as deduced from 1x1 degree averages of satellite observations (81); original data from (80).
FIG. 15. Mosaic of observed cloudiness over the Southwest Indian Ocean, 5 July 1962, 0720 GMT (Tiros V, orbit no. 227).
Again, we emphasize that these are the kinds of divergence-cloud patterns that must be accounted for on many individual days, and that the problem is not likely to be mitigated solely through the necessity of having to use dissimilar time averages of pressure-wind on the one hand, and cloud cover on the other.

Returning to Fig. 12, we note that the distribution of divergence is again nearly zonal, and that the longitudinal variations are the likely result of zonally unsymmetrical initial speeds.

Again, it will be sufficient to examine the results of two sample parcels (indicated by the asterisks in Fig. 12); in particular, those departing 15 and 208, respectively.

Again, we present our analysis in Fig. 16 in the context of eq. (38) and in the expanded form of eq. (39) in Fig. 17. We recall that curves B, C, and D (Fig. 16) are determined independently, while A is the algebraic sum of B, C, and D subject to the additional control that three of the four terms \( \left( \frac{dD}{dt}, \frac{1}{2}D^2, \frac{1}{2}S^2 \right) \) comprising A, the divergence of the acceleration, can also be computed, while the fourth, the deformation \( \left( \frac{1}{2} \text{def}^2 \right) \), can be computed by elementary bookkeeping. Ultimately, curve A is valid
FIG. 16. The divergence of the acceleration (A), Coriolis forces (B), pressure gradient force (C), and frictional force (D) for the two sample parcels indicated by the asterisks in Fig. 12. The numerical values \( x \times 10^{-11} \text{ sec}^{-2} \) are taken from Table 2; the plotted curves satisfy eq. (38).
to the extent that the vertical motion terms in eq. (39) can be neglected. We will offer some comments on this point when discussing the later equation below.

Consider the spatial variations in curve D, the divergence of the frictional term (KD), which again is proportional to the divergence itself. Near 7S the Coriolis and acceleration terms nearly balance each other with the resulting divergence being determined by the pressure-force term by about 1.5 degrees of latitude. At this point the acceleration term (Curve A) decreases rapidly, the pressure-force term (Curve C) becomes positive, and since the Coriolis term (Curve B) has changed little, (K times) the divergence (Curve D), by eq. (38), must decrease. The decrease (Curve D) is sustained until the divergence becomes negative, reaching a relative minimum near 7S. The sustained decrease is, again, largely the result of the pressure force term, since both acceleration and Coriolis terms nearly cancel between 5 and 10S.

Again, we see the familiar lag between (K times) the divergence and the pressure force term (Curves C and D); the latter appearing some 1.5 degrees downstream, which is slightly less than observed in the January case since the
July speeds are slightly higher.

Subsequently (K times) the divergence (Curve D) increases to a relative maximum (but still remaining negative) near 5S since here, though Coriolis and acceleration terms are roughly balanced (Curves A and B), the pressure-force term (Curve C) is decreasing, and again, by eq. (38) (K times) the divergence must increase. Further along (K times) the divergence decreases again reaching a minimum value near 2N; now, however, the pressure-force term is zero, the Coriolis term is increasing throughout, with the acceleration term, for the first time, determining, by eq. (38), the resulting divergence. (It will be noted below that the largest single contribution to the acceleration term comes from the deformation.) Northward of 2N we see that the pressure-force term (Curve C) and the acceleration term (Curve A) nearly balance, and with the Coriolis term being positive and nearly constant (K times) the divergence, by eq. (38), again must be negative, and slowly increases. This feature, the near balance between acceleration and pressure-force terms, was also observed near 16S, and would be expected near 4N in the light of eq. (38) and the near balance, northward of 4N, between
Coriolis and frictional terms. Finally, we note that the July case, unlike January, contained some negative values of the pressure-force terms.

An equivalent analysis in the context of eq. (39) serves to indicate the specific composition of the acceleration and Coriolis terms, and how they contribute to the temporal change of the horizontal divergence, as well as the pressure-force, and frictional terms discussed above.

With reference to Table 2 and Fig. 17 we note, again, that the Coriolis terms \((\frac{2}{3}f, \beta u)\), with the exception of a small region just north of the equator (2-1.8N), are oppositely correlated throughout; south of the equator both terms are about the same order of magnitude—with some obvious exceptions—while north of the equator \(\beta u\) is slightly larger than \(\frac{2}{3}f\).

Between 15 and 4.6S the largest contribution to the divergence of the acceleration comes from the deformation term, and the second largest from the time derivative of the horizontal divergence. Regarding the latter term, we should recall that it has been obtained from our computations of horizontal divergence, and then the deformation derived from elementary bookkeeping through a neglect of
Table 2. Computed values ($x10^{-11}$ sec$^{-2}$) of terms in eq. (39) for the two parcels indicated by asterisks in Fig. 12, plus the divergences of the acceleration (A), Coriolis force (B), pressure-gradient force (C), and friction (D); the former are plotted in Fig. 17, the latter in Fig. 16. Indian Ocean, July.

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FIG. 17. An expanded form of Fig. 16 showing the explicit contributions to the divergence of the Coriolis terms; $u$, Curve F, and $f$, Curve E; two of the four terms comprising the divergence of the accelerations: $\frac{dD}{dt}$, solid squares, $\frac{1}{2}Df^2$, Curve G. The remaining curves, C and D, are the same ones appearing in Fig. 16. Units are $10^{-11}$ sec$^{-2}$; the curves have been plotted with due regard for the algebraic signs in eq. (39).
the vertical motion term in eq. (39). The neglect of the latter term can express itself practically by leading to small negative quantities---i.e., imaginary---for the deformation; this is, in fact, the case for all computations of the deformation between 6.8 and 10N. However, by computing the vertical motion, through a 1 km layer, from the available computations of divergence, and neglecting zonal variations, in the vertical motion, we find that, for a vertical wind shear of 2 m sec\(^{-1}\) per km, the vertical motion term is about +0.5x10\(^{-11}\) sec\(^{-1}\) near 8N. This gives zero deformation at 7.1N; accordingly, the deformation northward of 6.8N was assumed to vanish.

Though no imaginary deformations were computed between 15S and 6.5N, it is useful to see what general effect the neglect of the vertical motion is likely to have. Firstly, by assuming that the zonal variations of vertical motion are smaller than the meridional ones (they tend to compensate anyway), and that, secondly, vertical wind shears are about 1 or 2 m sec\(^{-1}\) per km, then the sixth term of eq. (34) ranges from 1 to 4 between 15S and 6.5N. Specifically, at 15S it is about 2.4x10\(^{-11}\) sec\(^{-2}\), and thus the deformation is overestimated by about 8%; near 7.3N where the
vertical motion term is about $1.3 \times 10^{-11}$ sec$^{-2}$, the deformation is underestimated by about 4%. Since the estimated error never exceeded 10% no corrections were applied to the derived deformations between 15S and 6.5N.

Again, we may also consider what effect the vertical motion terms have upon the temporal change of the horizontal divergence, i.e., the left hand side of eq. (39). As in the January case, we indicate the sign of their contribution in Table 2, and have assumed the vertical shear of the $v$-component to be $2$ m sec$^{-1}$ per km. Again we see that the vertical motion terms are reasonably well correlated with the left hand member of eq. (39), and implies an underestimate in our computations of the horizontal divergence. The actual values never exceeded $5 \times 10^{-11}$ sec$^{-2}$.

The increase in the horizontal divergence near 5S (Fig. 12) is not without some further climatological interest occurring, as it does, in the latitudes of the Seychelles, which, unlike Colombo or Mauritius, experiences minimum rainfall in summer (82). We believe the dynamical cognate of the Seychelles rainfall anomaly is illustrated in Fig. 16 near 5S where the divergence (Curve D) attains a relative maxima. Again, we notice that the Coriolis
and acceleration terms (Curves A and B) are nearly balanced in this region, while at the same time the pressure-force term (Curve C) is decreasing; according to eq. (38) KD must increase—i.e., the divergence must increase.

In summary then, the distribution of (K times) the divergence along our sample trajectory—the shape of Curve D in Fig. 15—between 30°S and 10°N can be ascribed to the following: between 30°S and the equator, the divergence of the pressure-force term; north of the equator, the combined effect on the divergence of both acceleration (0-4°N) and Coriolis terms (north of 4°N).

5. Double Equatorial Troughs

Rather incomplete data have often suggested that two troughs might exist in the equatorial pressure field nearly symmetrical with the equator. Such a suggestion has sometimes been prompted by synoptic charts of the Eastern Pacific, and at other times from the Line Islands. At the same time relatively higher pressure prevails on the equator (83). Unfortunately, the available observations are too scanty to provide sufficient data for a convincing
test; nevertheless, we offer two computations with double troughs symmetrically located at 10N and 10S with higher pressure on the equator. The gradients equatorward of the troughs are 0.5 mb per 5 degrees of latitude; the zonal gradient is assumed zero; again $K$ is $2.8 \times 10^{-5}$ sec$^{-1}$.

Two initial states are used: (1) 5 m sec$^{-1}$ easterly winds everywhere, and (2) 5 m sec$^{-1}$ westerly winds on the equatorward side of both troughs, and the same easterly winds prevailing poleward of both troughs.

The initial state, trajectories, and divergence, appear in Figs. 18 and 19. The profile of horizontal divergence, taken from a common meridian to allow the motion to sustain some adjustment after some 20 hours, understandably is symmetrical with respect to the equator. Both initial states lead to substantially similar profiles, though divergence exists everywhere (8N-8S) in the case of easterly winds, while weak convergence zones are found near 3N and 3S in the case of westerly winds; further, in both cases westerly winds are found on the equatorward side of the trough, a feature Sadler and others have called attention to, though the position of the trough cannot always be determined with confidence.
FIG. 18. (a) Assumed profiles of pressure and wind; the latter are easterly everywhere, and no pressure gradient along the equator.

(b) Resulting trajectories using the initial conditions of (a).

(c) Mean profile of horizontal divergence derived from (b). Frictional constant, \( K = 2.8 \times 10^{-5} \) sec\(^{-1}\).
FIG. 19. (a) Same profile of pressure as Fig. 18a, but (zonal) winds are now westerly equatorwards of 10N and 10S, but remain easterly everywhere polewards of 10N and 10S.

(b) Resulting trajectories using the initial conditions of (a).

(c) Mean profile of horizontal divergence taken near 2 degrees relative longitude. No pressure gradient along the equator; $K = 2.8 \times 10^{-5}$ sec$^{-1}$. 
Lastly, a search of Sadler's cloud data (80) shows symmetrical cloud distribution only in the Eastern Pacific during both 1966 and 1967, and only in November; unfortunately, neither pressure nor wind data exist for the same period, and, besides, such symmetries could easily result from other distributions of pressure and wind than the one considered here.

The above results, admittedly, are more or less of academic interest, but do provide a useful exercise in the hazards of facile portraits of pressure and wind in low latitudes, even when friction is present.

A less ambiguous case is suggested by observations from the Gilbert-Ellice Islands.

6. Gilbert-Ellice Islands--July 1964

The Gilbert-Ellice chain of islands consist of 25 atolls extending in a near north-south direction from 3N to 10S near 175E; their total land area is about 125 squares miles, with a current population of about 40,000. All atolls take rainfall observations while 4 (Tarawa, Arorae, Funafuti, Nurakita) observe sea-level pressure (kw barometers) four times per day. In addition to these stations we may add Majuro (7N) and Rotuma (12.58) giving
six stations between 7N and 12.5S, certainly the best
distribution of pressure measurements in the equatorial
oceanic tropics. Lastly, Nauru and Ocean Island, on the
equator, provide two additional pressure measurements
some 5 degrees of longitude west of the major Gilbert-
Ellice chain. Indeed, with these stations and others
some confidence can be attached to the surface pressure
field in the region of 10N-10S, 170W-160E.

One of the striking climatological features of the
Gilbert-Ellice chain is the dramatic latitudinal variation
of rainfall, a feature that has been the object of some
comment for over a century. Indeed, this feature was
related to scientists of the Wilkes Expedition during their
visit to the Islands in 1859. It is also remarked upon
in the early reports by the British Colonial Office (viz.
1910) and was subsequently documented from actual obser-
vations beginning, for the most part, in the 1920's.
Seelye's maps (84) call attention to the general features,
while Fig. 20 shows a profile of the observed rainfall for
a particular month (July 1964), which is quite typical of
other months and years.
FIG. 20. Observed rainfall (inches), July 1964, cloud cover (oktas), and computed divergence near 173E; the latter was extracted from Fig. 22. The rainfall profile is taken from 26 stations between 10N and 9S in the Gilbert-Ellice islands (85) near 173E.
Returning to the question of high pressure on the equator raised in the last section, we undertook a latitude-time section analysis of the monthly mean sea-level pressure for the six stations (7N-12.5S) mentioned above. The mean values were drawn from four observations per day at all stations (85) for the period 1957-1965 (an additional station, Nui, 7.3S was added in 1965). This analysis, not reproduced here, showed that relatively high pressure on the equator usually occurs between July and October, but not every year. For the period at hand, such a distribution did occur in 1957, 1960, 1962, 1964, and 1965. Of these periods 1964 presented the best example of sustained high pressure lasting from June through October; the actual differences in sea-level pressure between the equator (Tarawa, 1N) and 7N (Majuro), on the one hand, and between the equator and 3S (Arorea) on the other, ranged between 0.4 and 1.0 mb. One of these six months (July, 1964) was chosen for further analysis.

The observed meridional distribution of sea-level pressure for July 1964 is presented in Fig. 21; in addition an east to west gradient along the equator is indicated by the mean values from Ocean island. Without
FIG. 21. Observed meridional profile of sea-level pressure (mb) in the Gilbert-Ellice islands for July 1964. The profile has been adjusted slightly to conform to the computational requirements of defining the gradient over 5-degree belts of latitude. In addition, a pressure gradient along the equator of 1 mb/5 degrees of longitude was indicated by the observations at Ocean Island.
the latter station we would have committed a serious error in the assessment of this zonal gradient.

An assessment of the initial state is less satisfactory; we only have frequency distributions of (Beaufort) wind speed and direction (from 2 observations per day) as provided in the summaries of the New Zealand Meteorological Service (85); these are available for the same stations observing pressure, and reveal only easterly winds during the entire month--no westerly wind being reported at any station. Since about two-thirds of the wind speeds fall in the (Beaufort) interval of 0-5.4 m sec$^{-1}$ a uniform easterly wind of 5 m sec$^{-1}$ was chosen for all parcels. The approximate character of the initial speeds is not likely to be a serious problem providing we examine the results only after several iterations (86).

The actual computation entailed releasing 13 parcels every two degrees of latitude between 12N and 12S, with a zonal easterly wind of 5.0 m sec$^{-1}$, which, incidentally, ensured that all parcels started with zero divergence. The meridional pressure field, Fig. 21, was adjusted slightly (the dotted line) to conform to the computational requirement of defining the pressure gradients over five-degree
latitudinal strips. Since this adjustment near 10S could lead to a rather complicated interpretation, we shall not press our observational comparisons much beyond 6S. In addition, we assumed a zonal (east to west) gradient of 1 mb/5 degrees of latitude as suggested by the Ocean Island observations, and was applied at all latitudes. Further, we recognize that this latter feature can only be defined with confidence over five, perhaps ten degrees of longitude, and, again we do not propose to extend our observational comparisons much beyond five degrees of longitude.

The resulting trajectories and divergence field are presented in Fig. 22 and, for general interest, have been extended over some 15-20 degrees of longitude, though, as noted above, we propose to examine only the first five-degree strip.

Unfortunately, no streamline analysis is available for comparison, though observed vector winds are available (87) for both Majuro (7N, 171E) and Funafuti (8.5S, 179E). At 950 mb (537 mb) at Majuro the observed wind was 101° 9.5 kt compared with the computed one of 102° 11.2 kt (­); at Funafuti the observed wind at 300 m (450 m was not
FIG. 22. Computed trajectories and divergence \((x10^{-6} \text{ sec}^{-1})\) using the initial conditions of Fig. 21. Initial wind speeds were assumed easterly everywhere at 5.0 m sec\(^{-1}\). The results are assumed valid between 175 and 167°E; elsewhere the pressure gradient of Fig. 21 may not apply, though the trajectories were extended over several degrees of longitude for general interest.
available) was 85° 11.9 kt compared with the computed value of 75° 11.2 kt; though we suspect that the pressure gradients near 9S have not been approximated as well as they might have been, the comparison is quite acceptable.

The meridional variation of the horizontal divergence (Fig. 22) is quite striking, and displays similar features in its rapid variation with latitude as does that of rainfall. To see this more clearly, we have constructed a meridional profile of the computed divergence taken from the meridian indicated by the large asterisk in the lower right hand corner of Fig. 22; the result, along with the observed rainfall is displayed in Fig. 20. Again, the comparison between gradients of observed rainfall and cloudiness and computed divergence are striking.

Lastly, as additional evidence that the rainfall curve roughly mirrors the distribution of cloudiness, in reasonably steady conditions, we include a sample of the latter (Fig. 23) taken from 25 days of satellite data (80) during February 1965.

Returning to Fig. 22, how can we account for the rapid change in horizontal divergence between, say, the equator and 5N? Specifically, what factors are responsible
FIG. 23. Observed distribution of satellite cloud cover (80) in oktas, and rainfall (inches), July 1965, for the Gilbert-Ellice islands.
for altering the divergence along the trajectories formed by parcels released at the equator, 2N, and 4N?

Again, we base our analysis upon eq. (38) and present separate results for parcels released at the equator and 2N (Fig. 24) on the one hand, and parcels released at 2N and 4N (Fig. 25) on the other. The three parcels are identified by asterisks in Fig. 22, while Tables 3 and 4 contain the relevant (hourly) observations. How then are we to understand the relative shapes of the "D" curves in Figs. 24 and 25?

We recall that the divergence of the acceleration (eq. 34) consists of four separate terms when the vertical motion terms are ignored. According to Table 3, the divergence squared and the vorticity squared nearly cancel each other, while the deformation squared is nearly an order of magnitude smaller than the remaining terms. Further, the largest positive contribution to the substantial derivative of the divergence arises from the Coriolis terms; not surprisingly the $\beta u$ portion. Since the pressure-force term is zero, the $\beta u$ term largely determines the entire variation of the divergence along the trajectory leading to the general shape of
Table 3. Computed values ($10^{-11}$ sec$^{-2}$) of terms in eq. (39) for the two parcels leaving 0 and 2N in Fig. 22, plus the divergences of the acceleration (A), Coriolis force (B), pressure-gradient force (C), and friction (D); the latter are plotted in Fig. 23. Gilbert-Ellice Islands, July 1964.

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Table 4. Computed values ($x10^{-11}$ sec$^{-2}$) of terms in eq. (39) for the two parcels leaving 2 and 4N in Fig. 22, plus the divergences of the acceleration (A), Coriolis force (B), pressure-gradient force (C), and friction (D); the latter are plotted in Fig. 24. Gilbert-Ellice Islands, July 1964.

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### Table 4 (Continued)

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Curve D in Fig. 24. Stated somewhat differently, given Curve A (the acceleration) and B (the Coriolis terms) (Fig. 24) then Curve D, by eq. (38), follows, i.e., the divergence must increase along the trajectory.

By contrast, the next pair of parcels released at 2 and 4N determine quite a different pattern, largely the result of a non-zero pressure-force term. Indeed between 172.5 and 175.0E the general shape of Curves A, B, and D are very similar in both Figs. 24 and 25 (notice the difference in the vertical scale). However, between 170 and 172E the divergence of the pressure-force term for the parcels at 2 and 4N increases sharply; this feature largely determines the character of Curve D between 170 and 172E. This is easily deduced if we let the acceleration term (Curve A) between 172.5 and 170.5 be zero. Then appealing to eq. (38) and the independently determined Curves B and C, then Curve D (K times) the divergence, must decrease. Beyond 170.5E the acceleration can no longer be assumed zero, but increases largely in response to the substantial derivative of the horizontal divergence (Table 4) and eq. (34). Lastly, if Fig. 25 were extended to the right, i.e., further downstream (K times) the divergence soon
FIG. 24. The divergence of the acceleration (A), Coriolis forces (B), pressure gradient force (C), and frictional force (D) for the two parcels released at 0 and 2N and indicated by asterisks on the right hand margin of Fig. 22. The numerical values are taken from Table 3 and satisfy eq. (38).
FIG. 25. Same analysis as that presented in Fig. 24, but for the two parcels released at 2 and 4N. The numerical values are taken from Table 4 and satisfy eq. (38).
reaches a minimum and begins to increase again, thus dis-
play the spatial lag between the pressure-force term
and the divergence already noticed in cases from the Indian
Ocean when the divergence of the pressure-force term was
also positive.

How is the above analysis altered if we attempt to
include the fifth and sixth terms of eq. (33), the vertical
motion terms? For each pair of parcels we performed five
sample computations—roughly every degree of longitude—
using our values of the divergence to compute the vertical
motion at 500 m, assuming a vertical wind shear of 1 m sec⁻¹.
In the case of the parcels released at the equator and 2N,
the complete vertical motion term averaged \(-1.5 \times 10^{-11}\) sec⁻²
and thus gave a positive contribution (Table 3) to the
left hand side of eq. (39); a similar result was also noted
for both Indian Ocean cases, and, again, the neglect of
the vertical motion terms leads to slight underestimates
of the divergence itself.

In the case of the second pair of parcels (2 and 4N),
a similar result was obtained (Table 4) though here, under-
standably, the computed values were slightly larger ranging
from 0 to \(10 \times 10^{-11}\) sec⁻².
Thus, the significant difference in the horizontal divergence between the above pair of adjacent parcels results from a divergence pressure field; or more familiarly, the curvature of the pressure profile is capable of producing significant differences in the horizontal divergence along adjacent trajectories.

7. Pressure gradient along the equator

As a further exercise of our curiosity, we examined some consequences of equatorial motions produced solely by gradients along the equator. We are, of course, under no illusion as to their likely meteorological application; rather, we wondered what would happen if . . . ?

We considered two general cases: one in which the gradient is directed along the equator from west to east—high pressure to the west, low pressure to the east—and another opposite to this; further, we released ten parcels at one-degree intervals five degrees either side of the equator using three different frictional constants. Lastly, the initial speeds were 5 m sec\(^{-1}\) from the west (first case) and from the east (second case) respectively.

The westerly case is depicted in Fig. 26, the easterly
FIG. 26. Trajectories resulting from parcels released at every degree of latitude between 10N and 10S with an initial speed of 5.0 m sec\(^{-1}\) in a zonal (west-east) pressure gradient of 1 mb/5 degrees of longitude. Three different values of the frictional constant, K, were used, 0.5, 1.0 and 2.0x10\(^{-5}\) sec\(^{-1}\).
one in Fig. 27. The former case leads to massive convergence on the equator, while the latter to divergence. Moreover, in the latter case \( K = 1.0 \) alternating parallel bands of divergence--convergence were produced along latitude circles \( (11.5\text{-con, 13.5\text{-div, 15.5\text{-con, 17.5\text{-div})}} \), symmetrical with respect to the equator, and decreasing in magnitude polewards.

The general kinematics of the trajectories themselves is not difficult to understand, if we recognize the relative contributions of the Coriolis acceleration: towards the equator in one case (westerly case), and away from it in the other (easterly case). The east-west divergence zones were somewhat surprising; indeed even after considerable experience we found it difficult, if not impossible, to guess what divergence patterns would result even from very simple pressure fields.

As we noted above, we do not intend to press any meteorological applications with these particular examples. Nevertheless, the general character of the westerly results is at least suggestive of water trajectories deduced from drogues in the Atlantic equatorial undercurrent (88). The latter results, of course, apply to a very restricted
FIG. 27. Trajectories resulting from the same initial conditions and friction as used in Fig. 26, but here the zonal pressure gradient (1 mb/5 degrees longitude) has been reversed and is directed from east to west.
region on either side of the equator—perhaps 0.5 degrees on either side. Finally, given the assumed pressure gradient, the general character of the oscillation close to the equator seems physically evident: as the particle overshoots the equator Coriolis forces are immediately generated that force the particle back towards the equator; the oscillation damps with time through dissipation.
VI. Some Comments on Errors

Since the momentum equation was solved as an initial value problem, it is vital to ask how changes in the initial conditions are likely to alter the computed speeds and positions as determined by eqs. (23)-(26). (It will be assumed that the initial coordinates are known exactly.) Before offering any specific results, it is useful to consider a simple argument involving just one speed component to see what results might be expected. It is clear from eq. (25) that a small change in $u_0$ leads to a change in $u$ of the form

$$\delta u = e^{-kt} \delta u_0$$

(44)

where, for simplicity, $\cos ft$ has been set equal to one.

Clearly, as time increases the effect of any change ($u$) in the computed speed resulting from a change in the initial speed ($\delta u_0$) rapidly decreases. Specifically if $k = 2 \times 10^{-5}$, and $u_0 = 3\ m\ sec^{-1}$, then the resulting change in the computed speed ($\delta u$) is about one third of the initial change ($\delta u_0$) something like 10 hours. Thus, if the "error" in the initial speed was 3 m/sec, the corresponding "error" in the computed speed is about 1 m/sec in 10 hours.
Five sample parcels were released along an arbitrary meridian between 5N and 25N under a constant pressure force of one millibar per five degrees of latitude. Hourly speeds and positions were determined from eqs. (23)-(26) for a period of 130 hours, which spatially corresponds to distances of the order of 10 degrees of latitude. For convenience, the parcels were given an initial speed equal to that of the geostrophic wind; subsequently, the initial parcel speeds were varied by 1, 3 and 5 m sec\(^{-1}\) above and below the geostrophic value--still under the same constant pressure gradient force--with the result depicted in Fig. 28. The graphs display speed versus time along selected trajectories with the expected result suggested by eq. (42) above, namely, that the computed speeds are tolerably insensitive to the initial speeds after periods of, say 10-15 hours; or, what is the same thing, after distances of the order of 2 or 3 degrees of latitude. Further, it is clear that the format of the above figures is sensible only if the resulting trajectories traverse substantially the same space. This is essentially the case; the largest spatial difference between the examples, for any time, being about a degree of latitude. If, on the other hand, the errors in the initial speeds are
FIG. 28. Parcel speeds as a function of time along three different trajectories (from top to bottom: near 20N, 10N, and 5N) as a function of different initial speeds.
assigned arbitrarily, say $+3 \text{ m sec}^{-1}$ to one parcel $-2 \text{ m sec}^{-1}$ to another, the results are substantially the same as those mentioned above. Because of the close similarity, no diagrams are presented.

The immediate practical consequence of the above results is that the initial state is not terribly critical in terms of our diagnostic approach. Nevertheless we cannot afford to be casual; rather, we must decide what we expect to recover from a particular experiment. If we wish to determine just the sign of the divergence developed by a given family of trajectories, then we can probably sustain some margin of error in the initial speeds. If on the other hand, we require greater detail, then we must know the initial speeds more accurately. For example, almost any initial speeds assigned to the parcels in the examples of the Indian Ocean would have given comparable results at great distances from their initial position; however, features, say, within five degrees of their initial position would have been altered. Even parcels released south of the equator with initial speed of zero recover the large convergent area north of the equator in the Indian Ocean during July.
Appealing to an argument similar to that developed above, it is easy to see that, from eqs. (23) and (25), a small change in the gradient $\delta P_y$ leads to changes in the x-coordinate and speed given approximately by

$$\delta x = \frac{1}{2} (K-f) \frac{e^{-Kt}}{K^2 + f^2} \delta P_y - \frac{f}{K^2 + f^2} \delta P_y$$

(45)

and

$$\delta u = (1+e^{-Kt}) \frac{1}{K^2 + f^2} \delta P_y$$

(46)

where, for convenience, both circular functions have been set equal to one, and $v_o = 0$. For given values of $K$ and $f$, the first term in eq. (45) largely determines the change in $\delta x$ for a given change $\delta P_y$; at later times the second term dominates. Roughly speaking at 10N, with $K = 2.0 \times 10^{-5}$ sec$^{-1}$, and $-P_y = 6 \times 10^{-3}$ cm sec$^{-2}$ (which corresponds to a relative error of about 35% if $P_y = 0.17$ cm sec$^{-2}$), the contribution of the second term doubles between 10 and 20 hours, while that of the first term roughly halves during the same period. In particular, $\delta x$ is about 0.5 and 1.0 degree in 10 and 20 hours, respectively. The analogous
variation in the speed, $\delta u$, is, from eq. (46), about 2.2 m sec$^{-1}$ and 1.9 m sec$^{-1}$ in 10 and 20 hours, respectively.

Since a variable Coriolis term is incorporated into the trajectory computations, further estimates of either speed or position based upon eqs. (45) and (46) may not be too fruitful; indeed, the more pressing question is how pressure gradient errors affect the divergence computations.

It is difficult to provide a single experiment that will cover all potential possibilities—the combinations are endless. Nevertheless, consider the following simple experiment: adopt the January Indian Ocean pressure profile and assume that an observing station is located every five degrees of latitude; let each station have an even chance of having either $\pm 0.2$ mb or zero error; with the new profile (Fig. 29) what divergence will ensue from the resulting trajectories? The comparison is presented in Fig. 30 and displays little significant change; the spatial variations of the "new" trajectories are barely discernible on the scale of the figure.Obviously, we are not suggesting that pressure errors are not "important;" indeed, small errors in regions of weak gradients could be ruinous. Rather, our collective experience suggests that pressure
FIG. 29. Pressure errors assumed in the original January pressure profile for the Indian Ocean, Fig. 5. The errors were assumed to be either 0 or ±0.2 mb at every 5 degrees of latitude between 15N and 5S.
FIG. 30. Comparison between divergence pattern computed with original January pressure profile and the error profile of Fig. 29; values are $10^{-6}$ sec$^{-1}$. 
errors of a few tenths of a millibar (in regions of non-zero gradient) over distances of, say, five degrees of latitude, coupled with speed errors of, say, a few meters per second are likely to preserve our diagnostic results, i.e., we are likely to infer at least the proper sign of the divergence from our computed trajectories.
VII. Summary and Conclusions

Can we now answer the questions raised in the Introduction? Can we understand how air parcels in the surface layer, moving across nearly zonal isobars, are able to undergo such striking changes in their horizontal divergence in comparatively short distances (Fig. 15, for example)?

The results of the foregoing section indicate that the divergence of both the pressure gradient force and the Coriolis forces are significant in determining the temporal change of the horizontal velocity divergence "following the motion" (eq.(39)). Some embroidery is added when the remaining terms of eq. (39) are incorporated, but, in the cases considered here, the latter two, either separately or combined, are never unimportant.

Further, we believe that successful diagnostic results are not possible unless the pressure field is known to within a few tenths of a millibar over distances of five degrees of latitude. On the other hand, the initial wind speeds can be known rather poorly (compared to the pressure) a direct consequence of our frictional constraint and the
particular way it enters our analytical solutions, in eqs. (23) - (28). Without friction these solutions would be extremely sensitive to the initial conditions, as was pointed out above (73).

Lastly, we advance no claims of the likely validity of our formulation of friction over continental regions; our results are deemed valid for oceanic regions only.

Some of our computational procedures are clearly capable of refinement, together with at least one change in the physical assumptions. For example, it would be desirable to have a "smoother" description of the pressure field with the object of eliminating the discontinuities in the pressure gradient employed here. Though, as noted above, the consequences of the latter feature can be effectively surpressed by ignoring a few computed values near such discontinuities, a polynomial representation of the pressure field could eliminate such features ab initio.

One additional computational refinement could entail the direct evaluation of the deformation term in eq. (39), rather than deducing it as a residual as we have done here. Then we could evaluate the vertical motion terms in eq.
(39) with greater confidence, and make the interpretative content of both eqs. (38) and (39) a little more exact.

One of the physical assumptions that could bear further scrutiny is that of friction simply opposing the motion. How would our results be altered, for example, if the frictional force acted at some finite angle to the motion? The angle could be a given constant value, or made to vary along the trajectory.

Finally, we might ask to what heights the effects of the vertical motion, as given by the divergence patterns in Figs. 8 and 14, for example, are likely to extend. Though the observational evidence is meager, some aircraft flights (89) report 5 km as a good average. Thus, if the effects of divergence and convergence, as experienced by near surface air parcels, can extend to say, 5 km on at least some occasions, what effect is this likely to have on the dynamics of horizontal air motions near these levels? Indeed, "in the absence of condensation" is it likely that ... "tropical motions are quasi-horizontal and quasi-nondivergent, having only second order coupling in the vertical and therefore being driven primarily by lateral coupling with extratropical and precipitating tropical motions?" (90).
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73. For $k=0$, eqs. (23)-(29) give

$$
\frac{\partial V}{\partial u_0} = \frac{f}{(u_0 - u_1)} \left( \frac{\partial u_0}{\partial y_0} \right)^{-1}
$$

75. The mean surface pressure was obtained from the daily observations as published by Joint Task Force Seven (74). However, the mean value for Tarawa led to a labored analysis when incorporated with similar mean values from Ocean Island, Arorae, and Funafuti. In fact, the mean pressure reported by the New Zealand Meteorological Service (Annual Meteorological Summary for Fiji, Tonga and Western Pacific High Commission Territories, p. 20, Suva, 1958) from their station on the same island (Betio), using Kew-pattern mercury barometers, was 1.0 mb higher (1009.2) than that deduced from (74). Further, these discrepancies persisted for the remaining part of the test period (May-July 1958); the mean values from (74) were 1008.0, 1008.3, and 1008.6, respectively, while the New Zealand station reported 1008.7, 1009.0, 1009.4 for the same four synoptic observations per day (00, 06, 12, 18 GMT) and taken on the same island of the atoll.
Part of the discrepancy could be the result of errors in coding and checking; perhaps as much as 0.3 mb from faulty sea-level reductions (corresponding to height error of 3 m but it is difficult to account for a whole mb in the mean pressure solely from such sources. Rather, it seems more likely that a major part of the discrepancy is the result of using aneroid barometers (whose scales are rarely graduated to less than 0.5 mb) by personnel of the 6th Weather Squadron (Mobile). A recent note (21 June 1968) from the Squadron asserts that "one of the observers who participated in the project remembers that aneroid barometers were used." Further, the Squadron was unable to provide any specific information regarding calibration procedures employed during the Hardtack operation.

Since the same personnel operated similar stations at both Kusaie and Kapingamarangi, this raises further doubt regarding the quality of the pressure analysis. Though the analysis was extended to include stations south of the equator and found to be consistent, the region is quite open and other
interpretations are entirely possible. It, therefore, follows that the computed trajectories in the region of Kusaie and Kapingamarangi must be accepted with some reservations.

76. We attempted to develop a dimensional argument relating $K$, the frictional coefficient, to the static stability, wind speed, and wind shear in various combinations. Since the undetermined coefficient was nearly as large as some of the other terms, the exercise was not convincing.


83. There is little doubt about the latter feature, no matter how questionable the double trough may be. During 1-8 May 1957 daily sea-level pressure measurements at five stations in the Line Islands (Palmyra--6N, Fanning--4N, Christmas--2N, Malden--3S, Penrhyn--9S) revealed high pressure (1015 mb) on the equator, after sustained raise of 10 mb in seven days.


86. This expedient would not be useful if the meridional distribution of initial speeds contained a pronounced maximum or minimum; none are suggested by the available (Beaufort) data, though, admittedly, the latter are rather approximate.


89. Personal communication, Mr. James R. Nicholson.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge the help, support and assistance of several individuals and organizations during various stages of this investigation.

Our sincere thanks are extended to my colleague, Mr. A. H. Gordon of the Meteorological Office, for countless discussions along with his shared enthusiasm; to Mr. Walter Yee of the Statistical and Computing Center, University of Hawaii, for initial assistance in programming; to Mrs. Charmian Jefferies, Mr. Warren Yogi, and Mr. Lan Li for generous assistance in subsequent programming, and their good humor in making "final" changes in all of the "final" programs; to Prof. George Platzman, Department of Geosciences, University of Chicago, Mr. W. C. Swinbank, and Prof. R. L. Lavoie, Department of Geosciences, University of Hawaii, for very helpful discussions during different stages of the investigation; finally, to the National Science Foundation for their generous support through the International Indian Ocean Expedition, Department of Geosciences, University of Hawaii.