AIRCRAFT HEALTH MONITORING SYSTEM USING INTERACTING MULTIPLE MODEL ESTIMATION

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IN

ELECTRICAL ENGINEERING

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by

Mark A. Saewong
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ABSTRACT

An effective approach for modeling and simulating failure detection and identification (FDI) of aircraft components is presented. A telescoping main landing gear and pressure control system is modeled and simulated subject to various fault conditions. Systems subject to faults cannot be modeled well by a single set of equations. A more appropriate model would be a model whose state not only varies continuously, but may also jump from one state to another, which is the so-called stochastic hybrid system. The use of multiple models for FDI where each model represents a fault or the nominal mode fits well into such a system. The interacting multiple-model (IMM) estimation algorithm is one of the most effective approaches for FDI. It is able to detect and identify multiple faults more quickly and reliably than many existing approaches. It runs a bank of Kalman filters in parallel and switches from one model to another in a probabilistic manner. In this thesis, the dynamics of the landing gear and pressure control system are modeled as a stochastic hybrid system where FDI based on the IMM estimation algorithm is simulated and performance is evaluated. Simulation results show that FDI based on IMM can detect and identify sensor failures and failures in system components.
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List of Symbols

\[(C_dA)_{\text{max}}\] maximum effective area of outflow valve

\[(N_{cs})_c\] Chester Smith factor based on the cabin to ambient pressure ratio

\[(\Delta P_c)_{\text{max}}\] difference between cabin and ambient pressure

\[A_L\] lower chamber cross-sectional area

\[A_R\] annulus area

\[A_o(X_s)\] variable cross-sectional area depending on the stroke of the piston

\[A_s\] area of the snubber orifice

\[A_s^e\] effective area of snubber orifices

\[A_s^c\] effective area of snubber orifices

\[A_u\] upper chamber cross-sectional area

\[C_d\] discharge coefficient

\[C_dA\] effective area of outflow valve at valve angle \(\beta\)

\[C_t\] tire damping coefficient

\[C_{ds}^c\] discharge coefficient of snubber orifices in compression mode

\[C_{ds}^e\] discharge coefficient of snubber orifices in extension mode

\[D_L\] diameter of lower chamber

\[D_p\] diameter of piston
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{op}$</td>
<td>diameter orifice plate</td>
</tr>
<tr>
<td>$D_{pin}(X_s)$</td>
<td>diameter of orifice pin depending on stroke of piston</td>
</tr>
<tr>
<td>$F_t$</td>
<td>force transmitted through the tire from the ground</td>
</tr>
<tr>
<td>$G$</td>
<td>variable gain element</td>
</tr>
<tr>
<td>$G_k G_i$</td>
<td>controller integrator gain</td>
</tr>
<tr>
<td>$G_k$</td>
<td>controller proportional gain</td>
</tr>
<tr>
<td>$K_t$</td>
<td>tire spring coefficient</td>
</tr>
<tr>
<td>$K_t$</td>
<td>tracking signal gain</td>
</tr>
<tr>
<td>$L$</td>
<td>aerodynamic lift on airplane</td>
</tr>
<tr>
<td>$M_L$</td>
<td>mass of landing gear piston and airplane’s tire</td>
</tr>
<tr>
<td>$M_u$</td>
<td>mass of airplane’s fuselage, lumped with the mass of the main cylinder</td>
</tr>
<tr>
<td>$P_L$</td>
<td>lower chamber pressure</td>
</tr>
<tr>
<td>$P_c$</td>
<td>cabin pressure</td>
</tr>
<tr>
<td>$P_s$</td>
<td>snubber chamber pressure</td>
</tr>
<tr>
<td>$P_u$</td>
<td>upper chamber pressure</td>
</tr>
<tr>
<td>$Pr_{crit}$</td>
<td>critical pressure ratio</td>
</tr>
<tr>
<td>$P_{nt}$</td>
<td>initial nitrogen charge pressure</td>
</tr>
<tr>
<td>$P_{ref}$</td>
<td>cabin pressure reference signal generated by the controller</td>
</tr>
<tr>
<td>$P_{set}$</td>
<td>user-selected cabin pressure altitude</td>
</tr>
<tr>
<td>$P_{tr}$</td>
<td>tracking signal</td>
</tr>
<tr>
<td>$R$</td>
<td>gas constant</td>
</tr>
<tr>
<td>$T_2$</td>
<td>outflow air temperature</td>
</tr>
</tbody>
</table>
$U(t)$  \hspace{1cm} \text{ground input to tire, or runway profile}

$V_c$  \hspace{1cm} \text{cabin volume}

$X_a$  \hspace{1cm} \text{inertial position of lower mass}

$X_s$  \hspace{1cm} \text{stroke of piston}

$X_{ni}$  \hspace{1cm} \text{nitrogen level}

$X_{ni}^0$  \hspace{1cm} \text{initial nitrogen level}

$X_{wg}$  \hspace{1cm} \text{inertial position of upper mass}

$\Delta P$  \hspace{1cm} \text{difference between $P_{ref}$ and $P_{set}$}

$\Delta P_r$  \hspace{1cm} \text{maximum of $\Delta P_{ref}$ and $(\Delta P_c)_{max}$}

$\Delta P_{max}$  \hspace{1cm} \text{maximum allowable cabin pressure differential}

$\Delta P_{ref}$  \hspace{1cm} \text{pressure error signal}

$\beta$  \hspace{1cm} \text{valve angle}

$\beta_{ref}$  \hspace{1cm} \text{reference signal from controller}

$\gamma$  \hspace{1cm} \text{polytropic gas constant (landing gear)}

$\gamma$  \hspace{1cm} \text{ratio of specific heat $C_p/C_v$ (pressure control system)}

$\omega$  \hspace{1cm} \text{actual angular velocity}

$\omega_c$  \hspace{1cm} \text{command angular rate of change}

$\omega_{max}$  \hspace{1cm} \text{maximum valve angle velocity}

$\sigma$  \hspace{1cm} \text{arbitrary small number}

$\tau$  \hspace{1cm} \text{actuator time constant}

$\varepsilon$  \hspace{1cm} \text{arbitrarily small positive number}

$d_o$  \hspace{1cm} \text{effective diameter of main orifice}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_s^c$</td>
<td>diameter of snubber chamber orifice during compression mode</td>
</tr>
<tr>
<td>$d_s^e$</td>
<td>diameter of snubber chamber orifice during extension mode</td>
</tr>
<tr>
<td>$d_s^e$</td>
<td>diameter of snubber orifice</td>
</tr>
<tr>
<td>$d_s^e$</td>
<td>diameter of snubber orifice</td>
</tr>
<tr>
<td>$f$</td>
<td>friction force between piston and the cylinder wall</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration</td>
</tr>
<tr>
<td>$r_a$</td>
<td>maximum rate of change of pressure during ascent</td>
</tr>
<tr>
<td>$r_d$</td>
<td>maximum rate of change of pressure during descent</td>
</tr>
<tr>
<td>$s$</td>
<td>Laplace operator</td>
</tr>
<tr>
<td>$w_1$</td>
<td>airflow rate in cabin</td>
</tr>
<tr>
<td>$w_2$</td>
<td>airflow rate out of cabin</td>
</tr>
<tr>
<td>$w_{max}$</td>
<td>maximum flow through outflow valve at sea level</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

An integral element of a highly reliable aircraft health monitoring system is an efficient fault detection and identification (FDI) scheme that can detect and isolate sensor, actuator, or system component failures so that appropriate actions can be undertaken. A fault is defined to be any deviation of a system from its normal or intended behavior, but the system may still be functional. Whereas a failure is defined to be caused by a fault where now the system is no longer functional. In this thesis, the use of the terms fault and failure will be used interchangeably. Diagnosis is the process of detecting an abnormality in the system behavior and isolating the cause or the source. Hard failure can be rapidly detected by on-line built-in-testing (BIT) whereas the more subtle or "soft" drifting-type failures can only be detected by the use of more sophisticated techniques, based on modern estimation/decision theory [1]. Towards this, many methods have been developed for fault detection and identification of dynamic systems [2]–[5].

One effective method is the use of multiple models. For the multiple model approach, each fault condition including the nominal mode is modeled using a bank of
filters in parallel. Each filter is based on a model matching one of the fault conditions or the nominal mode of the system. The overall state estimate is a probabilistically weighted sum of each output of the individual filters. Multiple model algorithms for FDI have been developed for many applications [6]–[9].

The interacting multiple model estimator (IMM) [10] differs from the noninteracting multiple model in that the filters "interact" with each other in a cost-efficient way. The IMM estimator is one of the most cost-effective hybrid estimation techniques [3]. It runs a bank of filters in parallel, each matching to a particular system mode. At each cycle of the algorithm, the initial state estimates to the filters are a mixture of the previous states. The mode probabilities give an indication of the mode in effect. State estimation is then computed as the weighted sum of the single mode filter states. The optimal state estimate of a hybrid system in a switching environment is the conditional mean estimate [11] where computations grow exponentially with time. The IMM algorithm is a suboptimal recursive filter that exploits merging of the hypothesis.

Fuselage and wings of aircraft are designed with aerodynamics and structural loads as the primary consideration, with landing gear placement occurring at a later stage. Long and slender fuselage configurations are very sensitive to vibrations from runways. Due to long term use, the landing gear system is susceptible to system component failure and system structure variations. Therefore, a goal of landing gear design is to reduce disturbance transmission from the ground to the fuselage and diagnose the health of the landing gear system. An embedded active control system was designed to mitigate the transmitted vibration [12], [13]. In this thesis, the focus is on the design of an effective diagnostic algo-
The interacting multiple model-based diagnosis algorithm was developed to monitor the health of the landing gear system.

The landing system is a highly nonlinear dynamical system. The model used in this thesis has two degrees of freedom, both in the vertical direction. As we see in Chapter 3, the system model is highly nonlinear and includes such effects as a polytropic gas model, velocity squared damping term, geometrically governed damping coefficients, stick-slip friction in the gear, and nonlinear tire model. In order to apply the existing multiple model based diagnostic algorithm to the landing gear, the system could be linearized around an equilibrium point. A better way would be to use an extended Kalman filter that linearizes about the current mean and covariance of the state. In Chapter 3, the landing gear equations of motion are developed and simulated to verify the model. Failure detection and identification on the landing gear is then simulated under assumed fault conditions.

Aircrafts are designed to transport passengers safely and comfortably at very high altitudes. Ambient conditions vary greatly during taxiing, takeoff, cruise, and descent. Ambient pressure varies from 15 psi during taxiing to 1.5 psi during cruise; ambient temperature ranges from -65°F to 122°F [14]. Environmental control systems (ECS) on aircraft provide the necessary adjustments to cabin conditions so passengers are comfortable during taxiing and flight. The ECS is the system that maintains cabin air quality such as temperature, pressure, humidity, and pollutants. Volume of air from outside the airplane enters the cabin and then exits the airplane. The ECS is responsible for maintaining cabin pressure and temperature using this outside air and recirculated air through the use of pressure regulators, flow control valves, pressure controllers, and temperature controllers.
Pressure control systems on aircraft are one of many environmental control system components. The function of the pressure controller is to maintain cabin pressure as close to sea level as possible, while not exceeding cabin-to-ambient pressure differential due to structural limits of the airplane. It also prevents rapid changes in pressure to reduce passenger discomfort. The cabin pressure is adjusted by the use of an outflow valve that controls the amount of airflow out of the cabin. The amount of airflow into the cabin is kept constant, thus opening the outflow valve decreases the cabin pressure and closing the valve increases pressure. The electronic cabin pressure control system is modeled and simulated in Chapter 4. FDI performance is then evaluated under assumed fault conditions.

In this thesis, the IMM estimation algorithm for FDI is applied to the landing gear and pressure control systems. The landing gear and pressure control systems are modeled and simulated under various fault conditions and performance is evaluated. The remainder of this thesis is organized as follows. In Chapter 2 the IMM estimator using the extended Kalman filter is introduced. A summary of the IMM estimation algorithm and extended Kalman filter equations are presented. The landing gear equations of motion and pressure control system equations are derived and simulated in Chapter 3 and Chapter 4 respectively. In Chapter 5, FDI for the landing gear and pressure control system is then simulated under various fault conditions using extended Kalman filter as the model-conditional filter. Finally, conclusions and future work are given in Chapter 6.
Chapter 2

Interacting Multiple Model Estimator

The Interacting multiple model based estimator consists of a bank of single-model-based filters running in parallel at each cycle as shown in Figure 2.1. The initial state estimates at the beginning of each cycle for each filter are the mixture of all most recent estimates from the single-model-based filters. The posterior probabilities for each mode is calculated as the indicators of the mode in effect and mode transition at each decision time. The following Table 2.1 summarizes the interacting multiple-model diagnosis scheme with extended Kalman filters as the model-conditional filters.

The main idea of the interacting multiple model (IMM) based diagnosis algorithm is that the actual system is assumed to be a stochastic hybrid system with an uncertain (failure status) parameter vector affecting the matrices defining the structure of the model or depicting the statistics of the measurement or processing noises. Further, the parameters are assumed to take on only discrete values to map the corresponding system models, and each system model is in certain probability drawn from models set designed to represent the all possible system behavior patterns. Then a Kalman filter is designed for each choice
Table 2.1: One cycle of interacting multiple-model based diagnosis scheme

1. Model-conditional filter reinitialization (for $j = 1, \ldots, M$):
- prior mode probability:
  \[ \mu_j(k+1|k) = \sum_i \pi_{ij} \mu_i(k) \]
- mixing state estimate:
  \[ \hat{x}_j^0(k|k) = \sum_i \frac{\hat{x}_i(k) \pi_{ij} \mu_i(k)}{\mu_j(k+1|k)} \]
- mixing covariance:
  \[ P_j^0(k|k) = \sum_i \frac{\pi_{ij} \mu_i(k) P_i(k|k) + \hat{x}_j^0(k|k) \hat{x}_j^0(k|k)^T}{\mu_j(k+1|k)} \]
  where \( \hat{x}_{ij}^0(k|k) = \hat{x}_i^0(k|k) - \hat{x}_j(k|k) \)

2. Model-conditional filter updating:
   - time update (from $k$ to $k+1$):
     \[ \hat{x}_j(k+1|k) = f_j(\hat{x}(k|k), u(k), k) \]
     \[ P_j(k+1|k) = A_j P_j(k|k) A_j^T + Q_j \]
   - Kalman filter gain:
     \[ K_j(k+1) = P_j(k+1|k) H_j^T [ H_j P_j(k+1|k) H_j^T + R_j ]^{-1} \]
   - Kalman filter measurement residual:
     \[ r_j(k+1) = z(k+1) - h_j(\hat{x}_j(k+1|k), k+1) \]
   - residual covariance:
     \[ \sigma_j^2(k+1) = H_j P_j(k+1|k) H_j^T + R_j \]
   - measurements update:
     \[ \hat{x}_j(k+1|k+1) = \hat{x}(k+1|k) + K_j(k+1) r_j(k+1) \]
     \[ P_j(k+1|k+1) = P_j(k+1|k) - K_j(k+1) H_j P_j(k+1|k) \]

3. Model-conditional probability update:
   - likelihood function:
     \[ L_j(k+1) = \frac{1}{\sqrt{2\pi\sigma_j^2(k+1)}} \exp\left(-\frac{r_j^2(k+1)+\sigma_j^{-2}(k+1) r_j(k+1)}{2}\right) \]
   - mode probability update:
     \[ \mu_j(k+1) = \frac{\mu_j(k+1|k) L_j(k+1)}{\sum_i \mu_i(k+1|k) L_i(k+1)} \]
   - fault decision:
     if $\mu_j(k+1) = \max_i \mu_i(k+1) > \mu_T \Rightarrow \theta_j$
     if $\mu_j(k+1) = \max_i \mu_i(k+1) < \mu_T \Rightarrow \theta_1$

4. Combination of state and covariance estimate:
   - overall state estimate:
     \[ \hat{x}(k+1) = \sum_j \hat{x}_j(k+1) \mu_j(k+1) \]
   - overall covariance estimate:
     \[ P(k+1) = \sum_j [ P_j(k+1) + \hat{x}_j(k+1) \hat{x}_j(k+1)^T ] \]
     where $\tilde{x}_j(k+1) = \hat{x}(k+1) - \hat{x}_j(k+1)$
of system model, i.e., parameter value, which results in a bank of separate “elemental” filters. The stochastic hybrid systems are described as:

\[
x(k + 1) = A(k, \theta(k))x(k) + B(k, \theta(k))u(k) + w(k) \tag{2.0.1}
\]

\[
z(k + 1) = C(k + 1, \theta(k + 1))x(k + 1) + v(k + 1) \tag{2.0.2}
\]

where \(x \in \mathbb{R}^n\) is the base state vector; \(z \in \mathbb{R}^q\) is the (mode-dependent) measurement vector; \(u \in \mathbb{R}^m\) is the control input vector; \(w \in \mathbb{R}^n\) and \(v \in \mathbb{R}^q\) are processing and measurement noises. \(\theta(k)\) represents the current active system mode, and the set of all possible system modes is \(\theta = (\theta_1, \theta_2, \ldots, \theta_M)\), which is an indirectly observable (hidden) Markov Chain.

The hybrid system (2.0.1) is known as a “jump linear system”: It is linear given the system mode; however, the system may jump from one mode to another at a random time. It can be observed from the system outputs that are in general noisy and mode-dependent. Therefore, the mode information is imbedded (i.e. not directly measured) in

Figure 2.1: IMM algorithm
measurement sequences. It is applicable to filter the measurement sequence to determine which system mode is active, and thus get the health information of the system. IMM-based diagnostic method explicitly emulates the abrupt changes of the system by switching from one model to another in a probabilistic manner. The finite-state-machine technique is employed to represent the switching among various modes. The transition between the different models can be described as a first-order Markov process and is characterized by the transition probability matrix. IMM also consists of a bank of single-model-based filters running in parallel at each cycle. The initial state estimates at the beginning of each cycle for each filter are the mixture of all most recent estimates from the single-model-based filters. It is the mixing that enables the IMM to effectively take into account the history of the mode without the exponential growth in computation and storage [10]. On the other hand, the posterior probabilities for each mode is calculated as the indicators of the mode in effect and mode transition at each decision time. Its main advantage is its reliability and quick detection and identification of simultaneous failures of sensors and system component. The diagnostic performance evaluation shows that the interacting multiple model based estimator for failure detection and identification can detect and identify various faults quickly and reliably.

2.1 Extended Kalman Filter

The Kalman filter [15]–[18] is a recursive algorithm used to estimate the state of a linear dynamical system. It is an optimal filter which minimizes the variance of the estimation error. When the system is non-linear, the system must first be linearized about an
operating point. A better way would be to linearize about the current mean and covariance. The extended Kalman filter is used to estimate the state of a non-linear dynamical system that linearizes about the current mean and covariance of the state.

A non-linear dynamical system state space model associated with a particular hypothesized mode with the subscript $j$ is given as

\begin{align*}
    x_j(k) &= f_j(x_j(k-1), u(k-1), k-1) + w_j(k-1) \tag{2.1.1} \\
    z_j(k) &= h_j(x_j(k), k) + v_j(k) \tag{2.1.2}
\end{align*}

where

- $x_j(k)$ is the state vector for hypothesized mode $j$ at time step $k$,
- $z_j(k)$ is the measurement vector,
- $u(k)$ is the known input,
- $f_j$ is a non-linear transition matrix function relating the previous state at time $k-1$ to the current time $k$,
- $h_j$ is a non-linear measurement matrix function relating the state $x_j(k)$ to the measurement $z_j(k)$,
- $w_j(k)$ is a white Gaussian processing noise with zero mean and covariance defined as
  \[ E[w_j(k)w_j^T(l)] = Q_j \delta_{kl} \tag{2.1.3} \]
  and
- $v_k$ is a white Gaussian measurement noise with zero mean and covariance defined
as

\[ E[v_j(k)v_j^T(l)] = R_j \delta_{kl} \]  \hspace{1cm} (2.1.4)

where \( \delta_{kl} \) is the Kronecker delta function. The measurement noise sequence \( v_j(k) \) and processing noise sequence \( w_j(k) \) are assumed to be independent of each other.

The Kalman filter algorithm uses the above model to define time propagation and measurement update equations of the Kalman filter state estimates and state estimate covariance matrix. The Kalman filter state estimate propagation equation is

\[ \hat{x}_j^-(k) = f(\hat{x}_j(k-1), u(k-1), k-1) \]  \hspace{1cm} (2.1.5)

\[ \hat{z}_j^-(k) = h_j(\hat{x}_j^-(k), k) \]  \hspace{1cm} (2.1.6)

where

\( \hat{x}_j^-(k) \) is the state estimate before the measurement vector is available and

\( \hat{z}_j^-(k) \) is the estimate of the measurement vector before it becomes available.

The state estimate covariance matrix propagation equation is

\[ P_j^-(k) = F_j(k)P_j(k-1)F_j(k)^T + Q_j(k-1) \]  \hspace{1cm} (2.1.7)

where

\[ F_j(k) = \left. \frac{\partial f_j}{\partial x_j} \right|_{x=\hat{x}_j(k)} \]  \hspace{1cm} (2.1.8)

When the measurement vector at time step \( k \) is available, the state estimates are updated as

\[ \hat{x}_j(k) = \hat{x}_j^-(k) + K_j(k)[z_j(k) - h_j(\hat{x}_j^-(k), k)] \]  \hspace{1cm} (2.1.9)
where the Kalman filter gain is

\[ K_j(k) = P_j^-(k)H_j(k)H_j(k)^T + R_j(k) ]^{-1} \]  

(2.1.10)

and

\[ H_j(k) = \frac{\partial f_j}{\partial x_j} |_{x=x_j^-} \]  

(2.1.11)

The Kalman filter residual vector is defined as

\[ r_j(k) = z(k) - H_j(k)x_j^- (k) \]  

(2.1.12)

which is the difference between the measurements at time \( t_i \) and the Kalman filter measurement estimates based on its model. The residual vector can be assumed to be a set of independent zero-mean Gaussian random variable with covariance as:

\[ S_j(k) = H_j(k)P_j^-(k)H_j(k)^T + R_j(k) \]  

(2.1.13)

Finally, the Kalman filter state estimate covariance matrix is updated using

\[ P_j(k) = P_j^-(k) - K_j(k)H_j(k)P_j^-(k) \]  

(2.1.14)
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Pg. 12
Figure 3.1: Schematic of telescoping main landing gear
Table 3.1: Properties and dimensions for the landing gear system model

<table>
<thead>
<tr>
<th>Landing gear properties</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_u$</td>
<td>0.1524m, (6in.)</td>
</tr>
<tr>
<td>$D_L$</td>
<td>0.1524m, (6in.)</td>
</tr>
<tr>
<td>$D_s$</td>
<td>0.0041m, (0.1614in.)</td>
</tr>
<tr>
<td>$D_p$</td>
<td>0.1397m, (5in.)</td>
</tr>
<tr>
<td>$D_o$</td>
<td>0.02859m, (1.125in.)</td>
</tr>
<tr>
<td>$L$</td>
<td>0.3832m, (15.087in.)</td>
</tr>
<tr>
<td>$M_L$</td>
<td>145.1kg, (319.22lbm)</td>
</tr>
<tr>
<td>$M_u$</td>
<td>4832.7kg, (10609.94lbm)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>5cm, (2in.)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>912kg/m$^3$, (1.22 \times 10^8$lbm/m$^3$)</td>
</tr>
</tbody>
</table>

strokes, the diameter of the orifice plate hole varies due to the metering pin, causing a variable effective orifice diameter, i.e. variable fluid damping. Hydraulic fluid reaches the rebound or snubber chamber through several small snubber orifices, 12 for this landing gear model. The snubber chamber provides damping when the strut extends. The tire adds spring and damping characteristics to the overall performance of the landing gear.

This model has two degrees of freedom both in the vertical direction, the inertial position of the upper mass $X_{wg}$ and position of the lower mass $X_a$. $X_{wg}$ has zero value when the gear is fully extended and tire just touching the ground. $X_a$ has zero value at the tire axle. When the gear is compressed, $X_a$ measures the deflection of the tire to an inertial reference ground input $U(t)$.

The upper mass and main cylinder of the landing gear is shown in Figure 3.2, the lower mass is shown is Figure 3.3. $A_u$ represents the cross-sectional area of the upper chamber with corresponding pressure $P_u$. For the lower chamber, the cross-section area is denoted by $A_L$ with pressure denoted by $P_L$. $D_p$ is the diameter of the piston. The variable
Figure 3.2: Schematic of upper mass and main cylinder

Figure 3.3: Schematic of lower mass
area $A_o(X_s)$ is the main orifice area which is a function of the stroke of the piston. The stroke of the piston is defined as $X_s = X_{wg} - X_a$. $D_{pin}(X_s)$ is the corresponding diameter of the metering pin. Hydraulic fluid reaches the snubber chamber through several orifices of diameter $d^c$ or $d^e$, where the superscripts $c$ and $e$ represent the compression mode and extension mode respectively. The annulus area in the snubber chamber is denoted by $A_R$, with corresponding pressure $P_s$. Some of the actual dimensions are defined in Table 3.1. Tire spring and damping coefficients are denoted by $K_t$ and $C_t$, which are nonlinear and contribute to the calculation of tire force $F_t$.

The forces acting on the upper mass are shown in Figure 3.2. Balancing the forces acting on the upper mass yields the following equation

$$M_u \ddot{X}_{wg} = M_u g - L - P_u A_o - P_L (A_L - A_o) + P_s A_R \pm f$$

(3.1.1)

where $g$ is the gravitational acceleration, and $f$ is the friction force between the piston and the cylinder wall. All other terms were described previously. This equation assumes that the hydraulic fluid pressure in the upper cylinder is identical to the nitrogen pressure. Also, in this development, the variable $A_o$, the main orifice area, is a variable cross-sectional area depending on the stroke of the piston.

Figure 3.3 shows the forces acting on the piston. By adding the forces on the lower mass (piston), the forces balance equation is

$$M_L \ddot{X}_a = M_L g + P_L (A_L - A_s) - P_s (A_R - A_s) - F_t \pm f$$

(3.1.2)
where $A_s$ is the area of the snubber orifice; $F_t$ is the force that is transmitted through the tire from the ground and has the form

$$F_t = K_t(X_a + U) + C_t(\dot{X}_a + \dot{U}) \quad (3.1.3)$$

where the tire force is defined as a linear function of tire stiffness and a damping force.

The unknowns in equations 3.1.1 and 3.1.2 are pressure and friction terms. Pressure terms $P_u$, $P_L$, and $P_s$ need to be related to displacement $X_{wg}$ and $X_a$ or their velocities $\dot{X}_{wg}$ and $\dot{X}_a$. The pressure terms were derived in [19], which are summarized below.

The upper chamber pressure can be expressed as

$$P_u = P_{ni} \left( \frac{X_{ni}^\gamma}{X_{ni}} \right)^\gamma. \quad (3.1.4)$$

The lower chamber pressure $P_L$ and snubber champer pressure $P_s$ depend on the stroke of the piston. For the case of compression, where $\dot{X}_s > 0.0$ and $P_L > P_s$, the lower chamber pressure can be expressed as

$$P_L = P_u + \left( \frac{A_L - A_R}{E_1} \right)^2 \dot{X}_s^2 \quad (3.1.5)$$

and snubber pressure as

$$P_s = P_L - \left( \frac{A_R}{E_2} \right)^2 \dot{X}_s^2 \quad (3.1.6)$$

where

$$E_1 = A_o C_d \sqrt{\frac{2}{\rho \left( 1 - \left( \frac{d_c}{D_L} \right)^4 \right)}}$$

and

$$E_2 = A_o C_{ds} \sqrt{\frac{2}{\rho \left( 1 - \left( \frac{d_s}{D_L} \right)^4 \right)}}$$
Similarly, for the extension case with $\dot{X}_s < 0.0$

$$P_L = P_u - \left( \frac{A_L - A_R}{E_3} \right)^2 \dot{X}_s^2 \quad (3.1.7)$$

and

$$P_s = P_L + \left( \frac{A_R}{E_4} \right)^2 \dot{X}_s^2 \quad (3.1.8)$$

where

$$E_3 = E_1$$

and

$$E_4 = A_s^e C_{ds}^e \sqrt{\frac{2}{\rho \left( 1 - \left( \frac{d_s^e}{D_s} \right)^4 \right)}}$$

Substituting pressure equations 3.1.4, 3.1.5, and 3.1.6 into the balancing equation for the upper mass 3.1.1, we get for compression mode

$$M_u \ddot{X}_w = M_u g - L + (A_R - A_L) P_{ni} \left( \frac{X_{ni}}{X_s} \right)^\gamma \nabla + \left\{ \left[ \left( \frac{A_L - A_R}{E_1} \right)^2 - \left( \frac{A_R}{E_2} \right)^2 \right] A_R - \left( \frac{A_L - A_R}{E_1} \right)^2 (A_L - A_o) \right\} \dot{X}_s^2 + f \quad (3.1.9)$$

and

$$M_L \ddot{X}_a = M_L g - F_i + (A_L - A_R) P_{ni} \left( \frac{X_{ni}}{X_s} \right)^\gamma \nabla + \left\{ \left[ \left( \frac{A_R}{E_2} \right)^2 - \left( \frac{A_L - A_R}{E_1} \right)^2 \right] (A_R - A_s^e) - \left( \frac{A_L - A_R}{E_1} \right)^2 (A_L - A_s^e) \right\} \dot{X}_s^2 - f \quad (3.1.10)$$
For the extension mode case, substituting equations 3.1.4, 3.1.7, and 3.1.8 into the lower mass balancing equation 3.1.2 we get

\[
M_u \ddot{X}_{wg} = M_u g - L + (A_R - A_L) P_{ni} \left( \frac{X_{ni}}{X_s} \right)^\gamma \\
\left\{ \left( \frac{A_L - A_R}{E_3} \right)^2 (A_L - A_o) \\
- \left[ \left( \frac{A_L - A_R}{E_3} \right)^2 - \left( \frac{A_R}{E_4} \right)^2 \right] A_R \right\} \dot{X}_s^2 - f
\] (3.1.11)

and

\[
M_L \ddot{X}_a = M_L g - F_t + (A_L - A_R) P_{ni} \left( \frac{X_{ni}}{X_s} \right)^\gamma \\
+ \left\{ \left( \frac{A_L - A_R}{E_3} \right)^2 - \left( \frac{A_R}{E_4} \right)^2 \right\} (A_R - A_s) \\
- \left( \frac{A_L - A_R}{E_3} \right)^2 (A_L - A_s) \right\} \dot{X}_s^2 + f.
\] (3.1.12)

By introducing a new notation using subscripts: “1” and “2” will be associated with compression, and “3” and “4” with extension, the above balancing equation can be simplified as

\[
M_u \ddot{X}_{wg} = M_u g - L + C_{1/3} \dot{X}_s^2 + K_{1/3} X_s^{-\gamma} \pm f
\] (3.1.13)

\[
M_L \ddot{X}_a = M_L g + C_{2/4} \dot{X}_s^2 + K_{2/4} X_s^{-\gamma} - F_t \mp f
\] (3.1.14)

which the coefficients of the stroke rate squared term are assigned the \( C_i \)'s; and the coefficient of the stroke position term are the \( K_i \)'s.

The stiction friction model used is based on the Karnopp friction model [20]. Friction in this landing gear comes from mainly two sources, the tightness of the seal and offset wheel (moment). Stick friction can be expressed as

\[
F_{stick} = \frac{M_L}{M_u + M_L} F_1 - \frac{M_u}{M_u + M_L} F_2
\] (3.1.15)
where $F_1$ and $F_2$ will be defined in the next section. When slipping, the friction can be any arbitrary function. Here the hyperbolic tangent was used as a continuous function to cause friction to change sign when slipping. For more details on the pressure and friction models see [13], [19].

### 3.2 Summary of Equations

In summary, the landing gear model equation in state variable form are as follows:

when piston sticks in cylinder:

$$\ddot{X}_{wg} = \frac{F_1}{M_u} - \frac{F_{stick}}{M_u}$$  \hspace{1cm} (3.2.1)

$$\ddot{X}_u = \frac{F_2}{M_L} + \frac{F_{stick}}{M_L}$$  \hspace{1cm} (3.2.2)

when there is relative motion between piston and cylinder, with friction present

$$\ddot{X}_{wg} = \frac{F_1}{M_u} + \frac{F_{frict}}{M_u}$$  \hspace{1cm} (3.2.3)

$$\ddot{X}_u = \frac{F_2}{M_L} - \frac{F_{frict}}{M_L}$$  \hspace{1cm} (3.2.4)

where

$$F_1 = M_u g - L + C_{1/3} \dot{X}_s^3 + K_{1/3} X_s^{-\gamma}$$  \hspace{1cm} (3.2.5)

$$F_2 = M_L g + C_{2/4} \dot{X}_s^2 + K_{2/4} X_s^{-\gamma} - F_i$$  \hspace{1cm} (3.2.6)

$$F_{stick} = \frac{M_L}{M_u + m_L} F_1 - \frac{M_u}{M_u + m_L} F_2$$  \hspace{1cm} (3.2.7)

$$F_{frict} = -\tanh(\dot{X} s).$$  \hspace{1cm} (3.2.8)
Under compression,

\[ C_1 = \left[ \left( \frac{A_L - A_R}{E_1} \right)^2 - \left( \frac{A_R}{E_2} \right)^2 \right] A_R - \left( \frac{A_L - A_R}{E_1} \right)^2 (A_L - A_o) \]  \hspace{2cm} (3.2.9)

\[ K_1 = (A_R - A_L)P_{ni}X_{ni}^\gamma \]  \hspace{2cm} (3.2.10)

\[ C_2 = \left[ \left( \frac{A_R}{E_2} \right)^2 - \left( \frac{A_L - A_R}{E_1} \right)^2 \right] (A_R - A_s) - \left( \frac{A_L - A_R}{E_1} \right)^2 (A_L - A_s^\gamma) \]  \hspace{2cm} (3.2.11)

\[ K_2 = (A_L - A_R)P_{ni}X_{ni}^\gamma \]  \hspace{2cm} (3.2.12)

Under extension,

\[ C_3 = \left( \frac{A_L - A_R}{E_3} \right)^2 (A_L - A_o) - \left[ \left( \frac{A_L - A_R}{E_3} \right)^2 - \left( \frac{A_R}{E_4} \right)^2 \right] A_R \]  \hspace{2cm} (3.2.13)

\[ K_3 = (A_R - A_L)P_{ni}X_{ni}^\gamma \]  \hspace{2cm} (3.2.14)

\[ C_4 = \left[ \left( \frac{A_L - A_R}{E_3} \right)^2 - \left( \frac{A_R}{E_4} \right)^2 \right] (A_R - A_s^e) - \left( \frac{A_L - A_R}{E_3} \right)^2 (A_L - A_s^e) \]  \hspace{2cm} (3.2.15)

\[ K_4 = (A_L - A_R)P_{ni}X_{ni}^\gamma \]  \hspace{2cm} (3.2.16)

where

\[ E_1 = A_o C_d \sqrt{\frac{2}{\rho \left( 1 - \left( \frac{d_o}{D_L} \right)^4 \right)}} \]

\[ E_2 = A_s^e C_{ds} \sqrt{\frac{2}{\rho \left( 1 - \left( \frac{d_s^e}{D_L} \right)^4 \right)}} \]

\[ E_3 = E_1 \]

and

\[ E_4 = A_s^e C_{ds} \sqrt{\frac{2}{\rho \left( 1 - \left( \frac{d_s^e}{D_R} \right)^4 \right)}}. \]
3.3 Simulation of Landing Gear

To verify the equations developed in the previous sections, the landing gear model was simulated. The input (runway profile) used was a sinusoid, $u = A \sin(2\pi ft)$, with amplitude $A = 0.75$ inches, and frequency $f = 1.5$ hertz, and its corresponding derivative, $\dot{u} = A \cos(2\pi ft) 2\pi f$. Figures 3.4 and 3.5 show the simulation results.
Figure 3.5: Simulation of landing gear (velocity)
Environmental control systems on aircraft [14], [21], [22] include controllers that regulate pressure, air flow, temperature, ventilation, humidity, and air contaminants. These includes components such as pressure regulators, flow control valves, cabin pressure controllers, and cabin temperature controllers. Cabin pressure control systems used on current aircraft include pneumatic, electropneumatic, and electronic controllers.

The function of the cabin pressure control system is to keep the cabin pressure as close to sea level as practical, while not exceeding the cabin-to-outside pressure differential due to structural limits of the aircraft. The cabin pressure is adjusted by an outflow valve that controls the amount of airflow out of the cabin while the amount of airflow into the cabin is held constant. The pressure rate of change is limited for the comfort of passengers. Normal pressure change rates are limited to 0.26 psi/minute when ascending and 0.16 psi/minute when descending [21].

For fighter aircraft, the cabin is unpressurized until it reaches a certain altitude, which the pressure is then held constant. At higher altitudes, the cabin to ambient pressure
differential is held constant because of structural considerations. These pressure controllers are generally pneumatic.

More sophisticated systems are used for transport or cargo aircraft. The cabin pressure altitude is held constant which is typically at 8,000 ft. The maximum rate of change of pressure altitude is also limited to reduce discomfort for passengers. The maximum cabin to ambient pressure differential is limited because of structural considerations. Such cabin pressure control systems are generally either electronic or electropneumatic.

4.1 Standard Cabin Pressure Controller Model

The following cabin pressure control system follows from [22]. The standard model is a complex electronic cabin pressure control system.

The model analysis requires as inputs the cabin inflow, the sensed cabin pressure, and the cabin temperature. Outputs are the cabin outflow and the outflow valve angle. Additional parametric inputs are

- \( V_c \) cabin volume (ft\(^3\))
- \( P_{sel} \) selected cabin altitude (ft/1,000)
- \( r_a \) maximum rate of change of pressure during ascent (ft/min)
- \( r_d \) maximum rate of change of pressure during descent (ft/min)
- \( \Delta P_{max} \) maximum allowable cabin pressure differential (lbf/in\(^2\))
- \( w_{max} \) maximum flow through outflow valve (lb/min) at sea level
Figure 4.1: Cabin pressure control system
Figure 4.1 illustrates the overall standard cabin pressure control system in block diagram form. Isobaric control operates in the system to maintain cabin pressure $P_c$ at a constant value. During airplane cruise, the cabin pressure is compared to the reference cabin pressure $P_{ref}$. Any difference between the two is referred to as pressure error $\Delta P_{ref}$. The pressure error causes the actuator to position the outflow valve to move the valve in a direction to reduce the pressure error to zero.

During airplane climb, the selected cabin altitude automatically varies from take-off altitude to the selected cabin altitude at the selected ascent rate. During cruise, the selected cabin altitude is unchanged provided the airplane altitude does not change. During descent, the selected cabin altitude varies from the isobaric cruise to the selected landing altitude at the selected descent rate.

### 4.2 Controller Analysis

#### 4.2.1 Reference pressure signal generator

Normal pressure change rates are limited to 0.26 psi/minute when ascending and 0.16 psi/minute when descending. The rate control limits the rate of change of the cabin altitude reference from its initial value to its final value.

The reference pressure signal generator (temporarily disregarding the tracking loop) is shown in Figure 4.2 in block diagram form.

$P_{sel}$ is the user-selected cabin pressure altitude, which may be a function of time and is related to the mission profile. $P_{ref}$ is the cabin pressure reference signal generated
by the controller. From Figure 4.2,

\[ \Delta P = P_{\text{ref}} - P_{\text{sel}} \]  \hspace{1cm} (4.2.1)

\[ \frac{d}{dt}(P_{\text{ref}}) = f(\Delta P) \]  \hspace{1cm} (see equation 7 - 130) (4.2.2)

\[ P_{\text{ref}} = -\frac{1}{s} \frac{d}{dt}(P_{\text{ref}}) \]  \hspace{1cm} (4.2.3)

The rate limiter functions to control the rate of change of \( P_{\text{ref}} \), thereby limiting the cabin pressure rate of change.
From Figure 4.3, the limiting values of \( \frac{d}{dt}(P_{\text{ref}}) \) are given by

\[
\frac{d}{dt}(P_{\text{ref}}) = \begin{cases} 
  r_a + \varepsilon_1(\Delta P - \text{ALT}_2), & \Delta P \geq \text{ALT}_2; \\
  r_a \Delta P/\text{ALT}_2, & 0 \leq \Delta P < \text{ALT}_2; \\
  r_d \Delta P/\text{ALT}_1, & \text{ALT}_1 \leq \Delta P < 0; \\
  r_d + \varepsilon_1 \Delta P - \text{ALT}_2, & \Delta P < \text{ALT}_1.
\end{cases}
\] (4.2.4)

where \( \varepsilon \) = arbitrarily small positive number

Solving for \( P_{\text{ref}} \) yields

\[
P_{\text{ref}} = \frac{P_{\text{set}}}{\frac{\text{ALT}}{r} + 1}, \quad (\text{ALT}_1 < \Delta P < \text{ALT}_2) \] (4.2.5)

where

\[
\Delta P > 0 \quad r = r_a, \quad \text{ALT} = \text{ALT}_2
\] (4.2.6)

\[
\Delta P < 0 \quad r = r_d, \quad \text{ALT} = \text{ALT}_1
\] (4.2.7)

\[
\Delta P > \text{ALT}_2 \quad \frac{dP_{\text{ref}}}{dt} \approx r_a
\] (4.2.8)

\[
\Delta P > \text{ALT}_1 \quad \frac{dP_{\text{ref}}}{dt} \approx r_d
\] (4.2.9)

In the above equations, \( \text{ALT}_1 \) and \( \text{ALT}_2 \) are in feet of air, and \( r_a \) and \( r_d \) are in feet per minute. Thus \( r_a \) is the maximum rate of change of pressure during ascent, and \( r_d \) the maximum rate of change during descent.

### 4.2.2 Tracking loop feedback

Automatic tracking is included in the feedback loop around the rate limiter integrator to keep the cabin pressure reference signal and the sensed cabin pressure within
Figure 4.4: Tracking loop feedback

approximately ± 0.003 psig (60 ft). This control ensures that no rapid changes in cabin pressure can occur.

The tracking loop feedback is activated when the pressure error signal exceeds the 0.03-lbf/in² deadband, shown in Figure 4.4.

When the absolute value of $\Delta P_{ref}$ exceeds 0.03 lbf/in², a tracking signal $P_{tr}$ is inputted to the reference signal integrator. The signal $P_{tr}$ can be defined as

$$P_{tr} = \begin{cases} 
K_t \Delta P_{ref} - 0.03, & \Delta P_{ref} > 0.03; \\
0, & -0.03 \leq \Delta P < 0.03; \\
K_t \Delta P_{ref} + 0.03, & \Delta P_{ref} < 0.03.
\end{cases} \quad (4.2.10)$$

where $K_t$ is a large gain such that when $\Delta P_{ref} > 0.03$, the signal $P_{tr}$ is sufficiently large to overpower the other inputs to the reference signal integrator. This large feedback signal drives the pressure error signal $\Delta P_{ref}$ within the range of ±0.03 lbf/in².
4.2.3 Most positive selector

The cabin to ambient pressure differential must be maintained below the structural limits of the airplane. The maximum cabin to ambient pressure control overrides the isobaric control to keep the maximum pressure difference user selected limit.

The most positive selector functions to permit the maximum $\Delta P$ control loop to limit the cabin to ambient pressure differential to $\Delta P_{\text{max}}$.

From Figure 4.5

$$\Delta P_{rr} = \max(\Delta P_{\text{ref}}, (\Delta P_c)_{\text{max}})$$ (4.2.11)

as $\Delta P_{\text{ref}}$ is limited by the tracking loop to within $\pm 0.03$ lbf/in$^2$. A positive $\Delta P_{rr}$ signal results in a signal to open the outflow valves and thus reduce the cabin pressure $P_c$. 

Figure 4.5: Maximum differential pressure control
4.2.4 Controller

The controller functions to reduce the error signal $\Delta P_{rr}$ to zero by initiating a command signal $\beta_{ref}$ to the outflow valve. The controller incorporates proportional plus integral control that results in adequate response characteristics and zero drop of error. The controller also includes a gain shaping network which attempts to maintain the overall pressure loop gain at a constant level. When the outflow valve is near the closed position, the outflow valve flow gain $\frac{\Delta(\text{flow})}{\Delta(\text{position})}$ is large, and the gain in the gain shaping network is made small. When the outflow valve is near the full open position, the flow gain is small, and the gain shaping network is made large. The output from the controller is a reference position signal to the outflow valve.

The controller output $\beta_{ref}$ is defined by

$$\beta_{ref} = GG_k(1 + \frac{G_i}{s})\Delta P_{rr} \quad (4.2.12)$$

where

$G_k = \text{controller proportional gain}$

$G_kG_i = \text{controller integrator gain}$

$G = \text{variable gain element}$

and

$s = \text{Laplace operator.}$

Equation 4.2.12 can be expressed in state variable format in the time domain by appropriate substitution of the variable $E_r$, where

$$\frac{dE_r}{dt} = G_i\Delta P_{rr} \quad (4.2.13)$$

32
Figure 4.6: Actuator position control

Equation 4.2.12 then becomes

\[ \beta_{ref} = GG_k(\Delta P_{tr} + E_r). \]  \( (4.2.14) \)

### 4.2.5 Outflow valve position

The position of the outflow valve is defined by the actuator position control loop diagram, Figure 4.6, where

- \( \beta_{ref} \) = reference signal from controller
- \( \omega_c \) = command angular rate of change
- \( \omega \) = actual angular velocity
- \( \tau \) = actuator time constant

and

- \( \beta \) = valve angle.
The valve angular velocity $\omega$ is limited in reality by friction and load effects. As shown in Figure 4.6, the valve angular velocity is given by

$$\omega = \begin{cases} 
\omega_{max} + \sigma(\omega_c - \omega_{max}), & \omega_c > \omega_{max} \\
\omega_c, & \omega_{max} \leq \omega_c \leq \omega_{max} \\
-\omega_{max} + \sigma(\omega_c + \omega_{max}), & \omega_c < \omega_{max}
\end{cases}$$

(4.2.15)

where $\sigma = \text{small number}$ and $\omega_{max}$ is the maximum valve angle velocity.

### 4.2.6 Outflow valve flow rate

Equations 4.2.1 through 4.2.15 define the response of the cabin pressure control system by calculating the outflow valve angle as a function of time for given values of selected and actual cabin pressure. The control loop can now be closed by calculating the flow rate through the outflow valve, and hence the rate of change of cabin pressure, as shown in the following paragraphs.

In the standard cabin pressure controller, the outflow valve is sized automatically by calculating the required effective area to pass the maximum flow rate (a user input) at sea level with a cabin-to-ambient differential of 0.1 lbf/in$^2$. The effective area at any angle is then calculated as

$$C_dA = (C_dA)_{max}(1 - \cos \beta)$$

(4.2.16)

where

$(C_dA)_{max} = \text{maximum effective area}$

and

$C_dA = \text{effective area at a valve angle } \beta.$
The type of valve most commonly used in aircraft ECSs is the butterfly valve [23] as shown in Figure 4.7. The outflow through the valve can be calculated as

\[ w_2 = \frac{0.532(C_d A)(N_{cs})_c}{\sqrt{T_2}} P_e \]  

(4.2.17)

where

\[(N_{cs})_c = \text{Chester Smith factor based on the cabin to ambient pressure ratio}\]

and

\[ T_2 = \text{temperature of the outlet air}. \]

The Chester Smith function [23] is the ratio of actual flow to maximum (choke) flow.

\[ N_{cs} = \begin{cases} \left[ \frac{\left( \frac{P_1}{P_2} \right)^{\frac{2}{2-1}} - \left( \frac{P_1}{P_2} \right)^{\frac{2+1}{2-1}}} {\left( \frac{2-1}{2} \right) \left( \frac{2+1}{2-1} \right)} \right]^{\frac{1}{2}}, & \frac{P_1}{P_2} < P_{crit} \\ 1, & \frac{P_1}{P_2} > P_{crit} \end{cases} \]  

(4.2.18)
4.2.7 Cabin pressure change rate

The cabin pressure change rate is given by the continuity equation

$$\frac{dP_c}{dt} = \frac{RT_2}{V_c}(w_1 - w_2)$$  \hspace{1cm} (4.2.19)

where

- $w_1$ = airflow rate into cabin
- $w_2$ = airflow rate out of cabin
- $T_2$ = outflow air temperature
- $V_c$ = cabin volume

and

- $R$ = gas constant

4.3 Summary of Equations

In summary, the cabin pressure controller model equation in state variable form ignoring the friction effects of the valve are as follows:

$$\dot{P}_{ref} = f(\Delta P)$$  \hspace{1cm} (4.3.1)

$$\dot{E}_r = G_i \Delta P_{rr}$$  \hspace{1cm} (4.3.2)

$$\dot{\beta} = \beta_{ref} - \beta$$  \hspace{1cm} (4.3.3)

$$\dot{P}_c = \frac{RT_2}{V_c}(w_1 - w_2)$$  \hspace{1cm} (4.3.4)
where

\[
f(\Delta P) = \begin{cases} 
    r_a + \varepsilon_1(\Delta P - ALT_2), & \Delta P \geq ALT_2 \\
    r_a\Delta P/ALT_2, & 0 \leq \Delta P < ALT_2 \\
    r_d\Delta P/ALT_1, & ALT_1 \leq \Delta P < 0 \\
    r_d + \varepsilon_1\Delta P - ALT_2, & \Delta P < ALT_1 
\end{cases}
\]

\[\Delta P_{rr} = \max(\Delta P_{ref}, (\Delta P_c)_{max})\]

\[\beta_{ref} = GG_k(\Delta P_{rr} + E_r)\]

\[w_2 = \frac{0.532(C_dA)(N_{cs})_c}{\sqrt{T_2}}P_c\]

### 4.4 Simulation of Pressure Controller

To verify the equations developed in the previous sections, the cabin pressure controller model was simulated. The input used was a sinusoid, \(P_{sel} = B + \sin(2\pi f t)\), with bias \(B=14.7\) psi, amplitude \(A = 1\) psi, and frequency \(f = 1/1800\) hertz. Figure 4.8 shows the simulations results. \(P_{sel}\) is the user selected cabin pressure, \(P_c\) is the corresponding cabin pressure, and \(beta\) is the valve angel of the outflow valve. The cabin pressure \(P_c\) correctly follow the input \(P_{sel}\).
Figure 4.8: Simulation of pressure controller
Chapter 5

Failure Detection and Identification

Simulation

5.1 Failure Detection and Identification Scheme

Failure detection and identification can be achieved using the mode probabilities of the IMM estimator. The mode probabilities give an indication of the mode in effect at each time step. Detection of faults can be made by the following rule

$$
\mu_j(k+1) = \max_i \mu_i(k+1) \begin{cases} > \mu_T, & H_j: \text{fault j occurred;} \\ < \mu_T, & H_0: \text{no fault occurred} \end{cases} \quad (5.1.1)
$$

where $\mu_T$ is the detection threshold. In most cases FDI performance varies little with the choice of this threshold. The mode probabilities by itself gives a meaningful measure of the likelihood of each fault mode without the need of this threshold [3]. This fault detection rule not only provides fault detection, but also diagnosis or identification.
In this thesis, system failure modes are focused on the $f$ and $h$ variations because they are the most common failure scenarios. The non-linear transition matrix $f$ relates the states at the previous time to the current time. A variation in $f$ indicates structural changes of the system. For the landing gear and pressure control system models, the measurement function $h$ is a linear function of the states. A variation in $h$ indicates sensor failure. For sensor failure, we have two situations:

1. **Partial sensor failure**: Partial sensor failure can be modeled by increasing the measurement noise covariance matrix $R$.

2. **Total sensor failure**: Total sensor failure can be modeled by annihilating the appropriate column of the measurement matrix $H$ as:

   $$ z(k) = (H + L_j)x(k) + v(k) \quad (5.1.2) $$

   That is to choose the matrix $L_j$ with all zero elements except that the $j^{th}$ column is taken to the negative of the $j^{th}$ element of measurement matrix $H$.

For the landing gear and pressure control system, failures in system components are simulated by varying the transition matrix $f$, total sensor failures are simulated by annihilating the appropriate column of the measurement matrix $H$, and partial sensor failures are simulated by increasing the measurement noise covariance matrix $R$. When we design the Kalman filter bank, we assume the Kalman filter model and the true model are of the same dimension.
5.2 Landing Gear FDI simulation results

The landing gear is assumed to experience three possible fault conditions. The failure scenario is depicted in Figure 5.1. Each failure situation will alter the system state space model by changing the nonlinear state transition matrix $f$ or by increasing the measurement noise $R$. The description of each fault condition is explained in detailed below.

- $H_0$ Nominal operation: The landing gear system is operating and functioning properly.

- $H_1$ Noisy position sensor: The landing gear system except the sensor for the position of the upper mass $X_{ug}$ is functioning properly. Soft failure of the sensor is simulated as the increase in strength of the measurement noise.

- $H_2$ Decreased tire force: All components are function properly except there is now a decrease in tire force.

- $H_3$ Noisy position sensor: Similar to scenario $H_1$, but the lower mass $X_a$ position sensor noise is increased.

![Figure 5.1: Assumed fault sequence for landing gear](image)
Figure 5.2: Landing gear system dynamic response under faults and measurement noise

Figure 5.3: IMM-based diagnosis simulation for landing gear
It is assumed the sensor for the upper mass position $X_{wg}$ will suffer failure during time 3 to 6 seconds. There will be a decrease in tire force $F_t$ from time 9 to 12 seconds. Finally, the sensor for the lower mass $X_a$ will suffer failure during time 15 to 18 seconds.

The landing gear dynamic response under the fault scenarios depicted above with is shown in Figure 5.2. The fault detection and identification simulation is illustrated in Figure 5.3. As shown in Figure 5.3, by picking the maximum mode-based probability, the system operation mode at each time can be identified correctly. The IMM state estimate of the system response to the runway input is demonstrated in Figure 5.4 under the assumed fault scenarios.
5.3 Pressure Controller FDI simulation results

The pressure control system is assumed to experience four possible fault conditions. The failure scenario is depicted in Figure 5.5. Similar to the landing gear system, each failure situation will alter the system state space model by changing the nonlinear state transition matrix $f$ or by increasing the measurement noise. The description of each fault condition is explained in detailed below.

- $H_0$ Nominal operation: All pressure control components are functioning properly. The system is operating at its intended behavior.

- $H_1$ Noisy valve angle sensor: All pressure control components except the sensor of the valve angle $\beta$ are functioning properly. Soft failure of the sensor is simulated as the increase in strength of the sensor measurement noise.

- $H_2$ Noisy cabin pressure sensor: Similar to scenario $H_1$, but the cabin pressure sensor $P_c$ is increased.

- $H_3$ Increased valve time constant: All components are functioning properly except the time constant $\tau$ of the outflow valve angle is increased.

- $H_4$ Decreased inflow air: Similar to scenario $H_3$, but now the amount of constant airflow into the cabin is decreased.

It is assumed the sensor for the valve angle position $\beta$ will suffer failure during time 30 to 60 minutes. The sensor for the cabin pressure $P_c$ will suffer failure during time 90 to 120 minutes. For time 150 to 180 minutes, the valve angle time constant $\tau$ will
Figure 5.5: Assumed fault sequence for pressure controller

increase. Finally, from time 210 to 240 minutes, the amount of constant inflow air into the cabin will decrease.

The dynamic response of the pressure control system under the assume fault conditions and measurement noise is illustrated in Figure 5.6. The fault detection and identification simulation is illustrated in Figure 5.7. Similar to the pressure controller case, by picking the maximum mode-based probability the system operation mode at each time can be identified instantaneously and correctly. The IMM state estimate of the system under the fault conditions previously described is shown in Figure 5.8.
Figure 5.6: Pressure controller system dynamic response under faults and measurement noise

Figure 5.7: IMM-based diagnosis simulation for pressure controller
Figure 5.8: IMM-based diagnosis simulation
Chapter 6

Conclusions

6.1 Summary

This thesis presented an effective approach for failure detection and identification of aircraft components using the interacting multiple model estimator. The telescoping landing gear and pressure control system were two models used for FDI simulation, which both were nonlinear and discontinuous. The systems were simulated subject to various known fault conditions, and FDI based on the IMM estimator was applied. Both systems were modeled as stochastic hybrid system where the state may jump and as well as vary continuously, which fits well with FDI where multiple models represent the nominal mode as well as the fault modes. The interacting multiple model estimation algorithm was the basis for failure detection and identification. It is one of the most effective approaches for FDI, as it is able to detect and identify multiple faults more quickly and reliably than many existing approaches. It runs a bank of filters in parallel and switches from one model
to another in a probabilistic manner. Since both systems were nonlinear, the extended Kalman filter was used as the model-conditional filter.

Simulations were performed on the landing gear and pressure controller where FDI based on the IMM estimation algorithm was simulated and performance evaluated. Simulation results showed that the diagnostic algorithm can accurately and instantaneously detect and identify landing gear and pressure control system component failure, such as system structure variations and sensor failures.

6.2 Future work

There are many systems on aircrafts. This thesis presented only two such systems, the landing gear and pressure control system. Future work would be to model and simulate other system components such as the temperature control system, pressure regulator, and flow control valve for environment control systems, engines, or wing flaps. This thesis only dealt with the diagnosis of faults. Future work would be to develop prognostic reasoners that would allow prediction such as life-time remaining.
Bibliography


