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LOW FREQUENCY TEMPERATURE FLUCTUATIONS IN THE UPPER 400 METERS OF THE CENTRAL NORTH PACIFIC

University of Hawaii

Ph.D. 1980

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18 Bedford Row, London WC1R 4EJ, England
LOW FREQUENCY TEMPERATURE FLUCTUATIONS IN THE
UPPER 400 METERS OF THE CENTRAL NORTH PACIFIC

A DISSERTATION SUBMITTED TO THE GRADUATE DIVISION OF THE
UNIVERSITY OF HAWAII IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
IN OCEANOGRAPHY
MAY 1980

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ACKNOWLEDGEMENTS

The author wishes to thank Dr. Lorenz Magaard who contributed valuable advice during all phases of this research. The author is also grateful to Drs. Klaus Wyrtki, Brent Gallagher, Edward D. Stroup and Harold G. Loomis for their helpful comments and discussions, and Roger Lukas for his kind help in generating contour plots used in this work.

This research has been supported by the Office of Naval Research under the North Pacific Experiment of the International Decade of Ocean Exploration; this support is gratefully acknowledged.
ABSTRACT

We investigate the low frequency temperature fluctuations, with time scales of months to years, in the upper 400 m of the Central North Pacific by analyzing the TRANSPAC temperature data collected as a part of the North Pacific Experiment and the temperature data at ocean weather stations. The low frequency temperature fluctuations consist of large scale, quasi homogeneous, fluctuations with space scales comparable to the size of the North Pacific basin and smaller scale, wave-like, fluctuations with length scales of a few hundred kilometers. In the upper 100 m of the ocean the temperature fluctuation is predominantly a seasonal variation, the amplitude of which decreases with depth with a typical e-folding depth of 50 m. The non-seasonal temperature anomaly field has a two layer structure; the temperature anomalies in the surface layer, down to about 100 m, have time scales of 2 to 3 years, and those in the lower layer, deeper than 150 m, have time scales of 4 to 7 years and penetrate down to a few hundred meters. More than half of the wave-like temperature fluctuations at the annual frequency consist of a random field of first order baroclinic Rossby waves travelling in a NW direction with wave lengths of about 300 km and phase speeds of about 1 cm/sec. Both the quasi homogeneous and wave-like parts of the low frequency
temperature fluctuations in the western part of the North Pacific Current area are much stronger than those in the eastern part. Along the Subarctic (42°N) and Subtropical (32°N) Fronts the wave-like temperature fluctuations propagate with phase speeds of about 10 cm/sec, and they reverse their direction of propagation with an annual cycle. We estimate the relative contribution of the quasi homogeneous and wave-like fluctuations, and that of the seasonal and non-seasonal fluctuations, to the change of heat content in the upper 400 m.
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I. INTRODUCTION

The sea water temperature both at the surface and in the interior of the ocean is attracting growing interest because of its significant role in climatic changes (Namias, 1973, 1975; Barnett, 1978) and in ecological dynamics (Monin et al., 1977). The sea water temperature is the most easily observed oceanographic factor, and it plays the role of indicating the water motion as well as the thermal state of the ocean. In most previous studies, however, the spatial and temporal structures of the temperature fluctuation fields were not well resolved due to a lack of sufficient oceanographic observations. In the Central North Pacific we are in the fortunate position of having a valuable data set, the TRANSPAC XBT data of the North Pacific Experiment (NORPAX) and the Ocean Weather Stations data, which allows us to investigate the spatial and temporal characteristics of the temperature fluctuation field simultaneously. The aim of this study is to arrive at a comprehensive understanding of the low-frequency temperature fluctuations with time scales of months to years and space scales of hundreds to thousands kilometers. The high-frequency temperature fluctuations associated with the diurnal cycle of insolation and tidal and internal wave motions (e.g., LaFond, 1962) are not included in this study.

The spatial and temporal characteristics, as well as
the underlying physical mechanisms, of the low-frequency temperature fluctuations are investigated through an analysis of data and application of appropriate models to the data. This dissertation is organized as follows: Following a description of the data (Chapter 2) we analyze the spatial distribution of the mean and variance of the temperature fields (Chapter 3). The vertical distributions of the seasonal and non-seasonal fluctuations are discussed in Chapters 4 and 5, respectively. The meso-scale fluctuations with space scales of a few hundred kilometers are analyzed by means of a baroclinic Rossby wave model (Chapters 6, 7 and 8). A propagation of the temperature disturbance along the Subarctic and the Subtropical Fronts is discussed in Chapter 9. The large-scale temperature fluctuations with horizontal scales comparable to the size of the North Pacific basin are described in terms of empirical orthogonal functions (Chapter 10). The heat storage and its rate of change in the upper 400 m of the ocean are analyzed, and they are compared with the heat exchange through the sea surface (Chapter 11). Finally, we present a comprehensive overview of our present knowledge of the low-frequency temperature fluctuation field in the Central North Pacific (Chapter 12).
II. THE DATA

An extensive expendable bathythermograph (XBT) collecting program (TRANSPAC) has been in effect since the beginning of 1975 as part of the "North Pacific Experiment" (NORPAX). The data collected by ships of opportunity are objectively interpolated and mapped (White and Bernstein, 1979). We have monthly values of temperature at every two degrees between $30^\circ$N and $50^\circ$N, every five degrees between $160^\circ$E and $130^\circ$W, and at ten standard depths ($0, 30, 60, 90, 120, 150, 200, 250, 300, 400$ m). The analysis of low-frequency temperature fluctuations in this work is primarily based on 40 months (January 1975 through April 1978) time series of the TRANSPAC data.

Although the TRANSPAC data provide a good spatial coverage of the North Pacific Current area, the length of the time series is not sufficient for an analysis of temperature fluctuations with periods of years. This limitation is partially overcome by means of the bathythermograph data (National Oceanographic Data Center file) at Ocean Weather Stations N($30^\circ$N, $140^\circ$W), P($50^\circ$N, $145^\circ$W) and V($34^\circ$N, $164^\circ$E) (White and Walker, 1974). Observations of temperature at these weather stations were initially limited to a depth of 135 m, but were later extended to 275 m. We use only the latter parts of these time series. The length of time series used in this study is 9 years ($1962 - 1970$) at
Ocean Weather Station N, 12 years (1957 - 1968) at P, and 5 years (1964 - 1968) at V.
III. MEAN AND VARIANCE OF THE TEMPERATURE FIELD

The mean temperatures at depths 0, 90, 200 and 400 m are calculated from the TRANSPAC data for 36 months (May 1975 through April 1978) and mapped in Fig. 3.1. In the Central North Pacific the isotherms of the mean temperature are almost zonal, except in the eastern and western boundary regions. The meridional gradient of the mean temperature decreases as depth increases—about 8°C per 1000 km at the surface and about 3°C per 1000 km at 400 m. The configuration of isotherms in the subsurface layer (200 to 400 m) approximates that of the mean dynamic topography (Wyrtki, 1975). The mean temperature (Fig. 3.1) indicates that the mean current is almost zonal except in the eastern and western boundary regions. The decrease of the meridional gradient of the mean temperature with increasing depth indicates that the magnitude of the zonal flow decreases as depth increases.

The root mean square (RMS) amplitudes of the temperature fluctuations (Fig. 3.2) decrease as depth increases. Fig. 3.2 also shows a contrasting zonal asymmetry of the RMS amplitudes of the temperature fluctuations; the fluctuations in the western part of the Central North Pacific are more energetic than those in the eastern part.

The temperature fluctuations in the Central North Pacific are composed of large-scale fluctuations with
Fig. 3.1 Mean temperature (in °C) at depths 0, 90, 200 and 400 m
Fig. 3.2 RMS amplitudes (in °C) of the temperature fluctuations at depths 0, 90, 200 and 400 m
horizontal scales comparable to the size of the basin and smaller scale fluctuations with length scales of a few hundred kilometers. We call the large-scale portion the quasi homogeneous part and the smaller scale portion the wave-like part of the fluctuations. We separate the temperature fluctuations into the quasi homogeneous and wave-like parts as follows. First, the time average is removed from each time series. Then the horizontal space average is taken for each 10-degree longitude by 4-degree latitude area at each depth and month. The resulting data set is the quasi homogeneous part, and the difference between the original fluctuation and the quasi homogeneous part is regarded as the wave-like part of the fluctuations. The RMS amplitudes of the quasi homogeneous (Fig. 3.3) and the wave-like fluctuations (Fig. 3.4) show that the temperature fluctuations in the western part of the Central North Pacific are more energetic than those in the eastern part both in the large-scale, quasi homogeneous, fluctuations and in the smaller scale, wave-like, fluctuations. The RMS amplitudes of the original, the quasi homogeneous, and the wave-like fluctuations, horizontally averaged over the entire area of the TRANSPAC data, are shown in Fig. 3.5. The quasi homogeneous fluctuations decrease rapidly with depth, and the wave-like fluctuations decrease slowly with depth.
Fig. 3.3 RMS amplitudes (in °C) of the quasi homogeneous part of the temperature fluctuations at depths 0, 90, 200 and 400 m
Fig. 3.4 RMS amplitudes (in °C) of the wave-like part of the temperature fluctuations at depths 0, 90, 200 and 400 m
Fig. 3.5 Depth dependence of the RMS amplitudes (in °C) for the original, the quasi homogeneous, and the wave-like parts of the temperature fluctuations horizontally averaged over the entire area of the TRANSPAC data.
IV. SEASONAL VARIATION OF TEMPERATURE IN THE UPPER LAYER OF THE OCEAN

It is well known that the sea water temperature fluctuations in the surface layer (the upper 100 m) are predominantly seasonal variations. We apply classical methods to our data to have a comprehensive understanding of the vertical and horizontal distributions of the seasonal temperature variations in the Central North Pacific. First, we consider the relationship between the sea surface temperature and the heat exchange through the sea surface. Then, to have a basic understanding of the changes of the amplitude and phase of the seasonal fluctuations with depth, we employ a classical theory of eddy diffusion of heat. Using the vertical distributions of the amplitude and phase of the seasonal variations, determined from the temperature data at Ocean Weather Stations N, P and V, we determine the depth dependence of the eddy diffusion process. Then, we study the horizontal distributions of the seasonal variation in the Central North Pacific by determining the e-folding depth from the TRANSPAC data. And finally, we discuss the role of eddy diffusion of heat in maintaining a large "effective" heat capacity of the ocean.

Figs. 4.1 and 4.2 show the time series of the monthly average temperature and the isotherms at Ocean Weather Stations N, P and V. These figures show that the sea
Fig. 4.1 Monthly sea water temperatures in the upper 240 m of the Ocean Weather Stations N, P and V.
Fig. 4.2 Vertical displacements of isotherms at Ocean Weather Stations N, P and V
surface temperature is highest in August and September. The heat exchange at the sea surface is maximal in June (Wyrtki, 1966). The phase difference between the sea surface temperature $T$ and the heat exchange $Q$ can be understood easily from a simple relation

$$\frac{\partial T}{\partial t} - \frac{Q}{\rho c_p D} = 0,$$  \hspace{1cm} (4.1)

where $\rho$ is the density of seawater, $c_p$ the specific heat, and $D$ the depth of the surface mixed layer. For a harmonic solution of (4.1), there is a phase difference of a quarter cycle between $Q$ and $T$; the corresponding time lag for the annual variation is 3 months.

Fig. 4.1 shows a monotonic decrease of amplitude and an almost linear increase of phase with depth. The highest sea water temperature at 40 m and 80 m occurs about 1 and 2 months, respectively, later than that at the sea surface. These features can be understood as a result of vertical diffusion of heat. When horizontal advection and diffusion of heat are neglected, the temperature fluctuation satisfies

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z} (\kappa \frac{\partial T}{\partial z})$$  \hspace{1cm} (4.2)

with boundary conditions

$$T = T_0 \exp(-i \omega t) \text{ at } z = 0$$  \hspace{1cm} (4.3)

$$T \rightarrow 0 \text{ as } z \rightarrow -\infty,$$
where $K$ is the vertical eddy conductivity, and $T_0$ is the amplitude of the sea water temperature fluctuation. When $K$ is assumed independent of depth, the above equations yield a solution

$$T(z,t) = T_0 \exp\left(\sqrt{\omega/2K} z\right) \exp\{-i(\sqrt{\omega/2K} z + \omega t)\}, \quad (4.4)$$

which shows that the amplitude of the temperature variation decreases exponentially with depth and the phase propagates downward with a phase speed of $-\sqrt{2K\omega}$.

We determine the amplitude and phase of the annual and semiannual temperature variations, at each depth of Ocean Weather Stations N, P and V, by minimizing

$$F = \frac{12}{\sum_{j=1}^{12} \left(T_j - R_1 \cos(\omega_1 t_j + \delta_1) - R_2 \cos(\omega_2 t_j + \delta_2)\right)^2}, \quad (4.5)$$

where $T_j$ is the temperature fluctuation at the $j$-th month, $R_1$ and $R_2$ are amplitudes of the annual and semiannual variations with frequencies $\omega_1$ and $\omega_2$, respectively, and $\delta_1$ and $\delta_2$ are the corresponding phases when $t_j = 0$ (15th of January). Figs. 4.3 and 4.4 show the amplitudes and phases for the annual and semiannual temperature variations in the upper 100 m at Ocean Weather Stations N, P and V. The harmonic solution with the optimum amplitudes and phases thus determined explains more than 90 % of the variances in the upper 100 m.

The standard errors of the amplitudes and phases are
Fig. 4.3 Amplitudes of the annual (full lines) and the semiannual (dotted lines) temperature variations at Ocean Weather Stations N, P and V.
Fig. 4.4 Phases of the annual (full lines) and the semi-annual (dotted lines) temperature variations at Ocean Weather Stations N, P and V.
estimated by (cf. Bevington, 1969)

\[ \Delta R_1 = \left( \sigma^2 / \sum_j \cos^2(\omega_1 t_j + \delta_1) \right)^{1/2} \]

\[ \Delta R_2 = \left( \sigma^2 / \sum_j \cos^2(\omega_2 t_j + \delta_2) \right)^{1/2} \]

\[ \Delta \delta_1 = \left( \sigma^2 / R_1 \sum_j \sin^2(\omega_1 t_j + \delta_1) \right)^{1/2} \]

\[ \Delta \delta_2 = \left( \sigma^2 / R_2 \sum_j \sin^2(\omega_2 t_j + \delta_2) \right)^{1/2}, \]

where

\[ \sigma^2 = \frac{F_{\text{min}}}{N - M} \]  

is the residual variance of the least squares fit, \( F_{\text{min}} \) the minimum value of \( F \), \( N \) the number of data points (\( N = 12 \)), and \( M \) the number of parameters (\( M = 4 \)). Fig. 4.3 shows that the annual temperature variation at Ocean Weather Stations N and P penetrates to only 100 m, but at V, which is located in the proximity of the Kuroshio, it penetrates deeper than 250 m. This reflects the fact that eddy conductivity in the strong current region is larger than that in the weak current region.

Now, we investigate the depth dependence of the eddy diffusion process. The vertical distributions of the amplitude (Fig. 4.3) and phase (Fig. 4.4) show that the amplitude of the seasonal variation does not follow an exponential law exactly, and the phase change is not
exactly linear with respect to depth. These facts suggest that the eddy conductivity depends on depth. With allowance for depth dependence of eddy conductivity we seek a solution of (4.2) and (4.3) in a form

$$T(z,t) = R(z) \exp\{-i(\omega t + \delta(z))\}, \quad (4.8)$$

and get

$$\left\{ \frac{d}{dz}(KR\frac{dR}{dz}) - KR\left(\frac{d\delta}{dz}\right)^2 \right\} - i\left(\frac{d}{dz}(KR\frac{d\delta}{dz}) + K\frac{dR}{dz} \frac{d\delta}{dz} - \omega R \right) = 0. \quad (4.9)$$

The imaginary part becomes

$$\frac{d}{dz}(KR^2 \frac{d\delta}{dz}) - \omega R^2 = 0, \quad (4.10)$$

and by integrating this with respect to z we get

$$K(z) = \frac{\omega}{R^2 (d\delta/dz) - d} \int R^2(z')dz', \quad (4.11)$$

where d is the depth at which the influence of the seasonal variation vanishes. This solution was first obtained by Fjeldstad (1933). The eddy conductivities at Ocean Weather Stations N, P and V, calculated numerically by (4.11), are shown in Fig. 4.5. The values of d at Ocean Weather Stations N, P and V are chosen to be 200, 150 and 250 m, respectively. The large magnitude of eddy conductivity in the surface indicates a strong mixing of sea water in the
Fig. 4.5  Eddy conductivity for the annual (full lines) and the semiannual (dotted lines) temperature variations at Ocean Weather Stations N, P and V
upper 30 m. The eddy conductivity at 100 m is one order of magnitude smaller than that at the surface.

We have so far described the vertical distributions of the amplitude and phase of the seasonal temperature variation and corresponding eddy conductivity. In what follows, we discuss the horizontal distribution of the seasonal variation by calculating the e-folding depth from the quasi homogeneous part of temperature fluctuations in the Central North Pacific. With an assumption that the amplitude of the seasonal variation decreases exponentially with depth in the surface layer, we estimate the e-folding depth, or the depth of penetration, from the vertical distribution of RMS amplitude of the quasi homogeneous fluctuation at depths 30, 60 and 90 m by a formula

\[
\begin{align*}
    z_e &= \frac{1}{3} \left( \frac{z_2 - z_1}{\ln(R_1/R_2)} + \frac{z_3 - z_1}{\ln(R_1/R_3)} + \frac{z_3 - z_1}{\ln(R_1/R_2)} \right),
\end{align*}
\]

where \(R_1, R_2\) and \(R_3\) are RMS amplitudes at depths \(z_1(30\text{ m}), z_2(60\text{ m})\) and \(z_3(90\text{ m})\), respectively. The e-folding depths in the Central North Pacific (Fig. 4.6) indicate that the amplitude of the seasonal variation of temperature in the surface layer decreases to about one third with an increase of depth of about 50 m.

Before we end this chapter, we discuss the importance of the eddy diffusion of heat in maintaining a large "effective" heat capacity of the ocean. If the whole
Fig. 4.6 e-folding depths (in meters) for the seasonal variation of temperature in the area of the TRANSPAC data
surface of the earth were covered by water of uniform depth, the mean depth would be 2.7 km, which corresponds to a pressure of 270 kg/cm². Recalling that the atmospheric pressure is approximately 1 kg/cm², the mass of the ocean is about 270 times larger than that of the atmosphere. Since the specific heat of sea water, 1 cal (°C)⁻¹ gm⁻¹, is about 4 times larger than that of air, 0.242 cal (°C)⁻¹ gm⁻¹, the total heat capacity of the ocean is about 1000 times larger than that of the atmosphere. The amount of heat stored in a vertical column of atmosphere extending from the earth's surface to the fringe of space is approximately the same as the heat contained in a water column of only 2.5 m. If the entire vertical column of the ocean is thermally "in a direct communication" with an atmosphere lying over the ocean, then 0.1°C change of sea water temperature would result about 100°C change in atmospheric temperature. It should be noted, however, that only the upper layer of the ocean is in a direct thermal communication with the atmosphere. As shown before, a typical e-folding depth of a seasonal variation of temperature is about 50 m, and an "effective" heat capacity of the ocean is about 10 times larger than the heat capacity of the atmosphere.

If there were no eddy diffusion of heat, then the effective heat capacity of the ocean would be much smaller. To see the role of eddy diffusion of heat more clearly,
let's consider a hypothetical ocean without eddy diffusion of heat. The sea water temperature in this hypothetical ocean will be changed by a direct absorption of solar energy and also by molecular diffusion of heat. The heating of the ocean by an absorption of solar energy takes place within a layer about 10 m or less, because about 50 % and 75 % of the solar energy flux is absorbed in the first 1 and 5 m, respectively (Ivanoff, 1977). Besides, the penetration of heat by molecular diffusion would be limited to the upper few meters only, because the corresponding e-folding depth, \( \sqrt{2K'}/\omega \), where \( K' \) is the molecular conductivity, is only about 2 m. Hence, the seasonal variation of temperature in that hypothetical ocean would be noticeable only within a top layer of about 10 m or less. In this case the amplitude of the sea surface temperature fluctuation would be about 10 times larger than that in the present ocean, and consequently the climate in the atmosphere would be much different from the present one we have. This simple argument demonstrates that the eddy diffusion of heat plays a key role in making the effective heat capacity of the ocean large, and consequently in maintaining our climate as the "moderate" one we actually have.
V. TEMPERATURE ANOMALIES AT OCEAN WEATHER STATIONS N, P AND V

In the previous chapter we described the seasonal variation of the temperature in the upper 400 m. In this chapter we discuss the non-seasonal fluctuations of the temperature, namely, the anomalous temperature fluctuations. A temperature anomaly is a deviation of the temperature from the long term average at the same time of the year and at the same position. In other words, it is a deviation of the temperature fluctuation from the seasonal variation. The temperature anomalies at the sea surface are understood to have a close relationship with climatic changes, and climatic variability cannot be fully understood without corresponding studies of the oceans (Namias, 1973, 1975; Barnett, 1978).

In order to get time series of temperature anomalies we need long records, of many years, of the temperature fluctuations. The time series of the temperature fluctuations at Ocean Weather Stations N, P and V in the Central North Pacific provide an opportunity to study the vertical distributions of the temperature anomaly field. White and Walker (1974) showed that the low-frequency thermal variability at Ocean Weather Stations N, P and V has time scales of 3 to 6 years and propagates downward from the surface at an approximate rate of 100 m per year. We
re-examine the temperature anomalies at the Ocean Weather Stations to understand the depth dependence of the temperature anomaly field.

The time series of temperature anomalies in the upper 240 m at Ocean Weather Stations N, P and V are shown in Figs. 5.1, 5.2 and 5.3. As can be seen from these figures, the temperature anomalies are composed of high-frequency fluctuations with time scales of 2 to 3 months and low-frequency fluctuations with time scales of an order of years. To visualize the high and the low frequency parts of the temperature anomaly field more clearly we apply high-pass (periods less than 6 months) and low-pass (periods longer than 2 years) symmetric convolution digital filters (Goldon, 1973; Ulrych et al, 1973). The filters have a length of 51 points. In addition, we use Hamming windowing to reduce Gibbs's phenomena. The high-passed and the low-passed time series of temperature anomalies at the Ocean Weather Stations are shown in Figs. 5.4 through 5.9.

The high-frequency temperature anomalies (Figs. 5.4, 5.5 and 5.6) have time scales of 2 to 3 months and they are vertically coherent. This high-frequency fluctuation seems to be associated with quasi-geostrophic eddies (Gill, 1975).

The low-frequency temperature anomalies at Ocean Weather Stations N (Fig. 5.7) and P (Fig. 5.8) show a two-layered structure. The low-frequency anomalies in the
surface layer down to about 100 m have time scales of 2 to 3 years, and those in the subsurface layer deeper than 150 m have time scales of 4 to 7 years. This two-layered structure suggests that the physical mechanisms responsible for the low-frequency temperature anomaly field in the surface layer may be different from those in the subsurface layer. Kort (1970) argued that the temperature anomalies in the surface layer are closely related with heat exchange at the sea surface, and those in the subsurface layer are associated with an advection of heat by large-scale oceanic circulation. At Ocean Weather Station V (Fig. 5.9) the surface layer reaches deeper than 240 m, and the low-frequency anomaly field is characterized by a quasi-biennial cycle, at least during the period of 1964 to 1968. The quasi-biennial cycle of the temperature anomaly field in the surface layer may exert a significant influence on air-sea interaction process (Zverev, 1977). The quasi-biennial cycle with periods of 2 to 3 years penetrates more than 240 m at Ocean Weather Station V which is located near the Kuroshio.

The length of time series we have used are not long enough to resolve the periods of the low-frequency temperature anomalies. However, the available time series show a two-layered structure of the temperature anomaly field. The depth of the surface layer for the low-frequency temperature anomaly field is about 100 m at Ocean Weather
Stations N and P, and deeper than 240 m at V. In the previous chapter we showed that the eddy conductivity at Ocean Weather Station V is larger than that at N and P. These observations suggest that a direct thermal interaction of the ocean with the atmosphere is limited to the upper 100 m in the central and eastern parts of the Central North Pacific, but that at the western part near the Kuroshio involves a thick layer deeper than 250 m.
Fig. 5.1 Temperature anomalies at Ocean Weather Station N (30°N, 140°W)
Fig. 5.2 Temperature anomalies at Ocean Weather Station P (50°N, 145°W)
Fig. 5.3 Temperature anomalies at Ocean Weather Station V (34°N, 164°E)
Fig. 5.4 High-passed temperature anomalies at Ocean Weather Station N
Fig. 5.5 High-passed temperature anomalies at Ocean Weather Station P
Fig. 5.6 High-passed temperature anomalies at Ocean Weather Station V
Fig. 5.7 Low-passed temperature anomalies at Ocean Weather Station N
Fig. 5.8 Low-passed temperature anomalies at Ocean Weather Station P
Fig. 5.9 Low-passed temperature anomalies at Ocean Weather Station V
VI. THEORY OF ROSSBY WAVES IN AN
OCEAN WITH MEAN SHEAR FLOW

As mentioned before in Chapter 3, the temperature
fluctuations in the Central North Pacific are composed of
the large-scale, quasi homogeneous, fluctuations with
horizontal scales comparable to the size of the basin and
the smaller scale, wave-like, fluctuations with length
scales of a few hundred kilometers. In the last two
chapters we discussed the seasonal variations and the non­
seasonal temperature anomaly field. The wave-like part of
the temperature fluctuations is investigated by means of a
Rossby wave model in the following three chapters. After
we introduce a theory of Rossby waves in an ocean with mean
shear flow (Chapter 6), we discuss a method of analyzing
Rossby waves from the temperature fluctuations (Chapter 7).
The analyzed Rossby wave field will be presented in Chapter
8.

Rossby waves in an ocean with vanishing mean flow
were investigated by many authors (e.g., Longuet-Higgins,
1964, 1965; Emery and Magaard, 1976). In the North Pacific
Current area, however, we cannot make the simplifying
assumption of vanishing mean flow. In this chapter we
discuss the hydrodynamics of the meso-scale quasi­
geostrophic wave motions in an ocean with zonal shear flow.
We consider an inviscid and incompressible ocean with zonal
mean flow:

\[ \mathbf{\bar{u}} \neq 0, \mathbf{\bar{v}} = 0, \mathbf{\bar{w}} = 0, \frac{\partial \mathbf{\bar{u}}}{\partial x} = 0, \frac{\partial \mathbf{\bar{v}}}{\partial x} = 0, \frac{\partial \mathbf{\bar{w}}}{\partial x} = 0, \frac{\partial \mathbf{\bar{p}}}{\partial x} = 0, \] (6.1)

where \( \mathbf{\bar{u}}, \mathbf{\bar{v}} \) and \( \mathbf{\bar{w}} \) are the mean currents in the x (eastward), y (northward) and z (upward) directions, respectively, \( \mathbf{\bar{\rho}} \) the mean density, and \( \mathbf{\bar{p}} \) the mean pressure. Using the Boussinesq, the traditional, and the hydrostatic approximations, the governing equations of motion are:

\[ \frac{\partial \mathbf{\bar{u}}}{\partial t} + (\mathbf{\bar{u}} + \mathbf{u}) \frac{\partial \mathbf{\bar{u}}}{\partial x} + \mathbf{v} \frac{\partial (\mathbf{\bar{u}} + \mathbf{u})}{\partial y} + \mathbf{w} \frac{\partial (\mathbf{\bar{u}} + \mathbf{u})}{\partial z} - f\mathbf{v} + \frac{1}{\mathbf{\rho}_0} \frac{\partial \mathbf{p}}{\partial x} = 0 \] (6.2)

\[ \frac{\partial \mathbf{\bar{v}}}{\partial t} + (\mathbf{\bar{u}} + \mathbf{u}) \frac{\partial \mathbf{\bar{v}}}{\partial x} + \mathbf{v} \frac{\partial (\mathbf{\bar{v}} + \mathbf{v})}{\partial y} + \mathbf{w} \frac{\partial (\mathbf{\bar{v}} + \mathbf{v})}{\partial z} + f(\mathbf{\bar{u}} + \mathbf{u}) + \frac{1}{\mathbf{\rho}_0} \frac{\partial (\mathbf{\bar{v}} + \mathbf{v})}{\partial y} = 0 \] (6.3)

\[ (\mathbf{\bar{\rho}} + \mathbf{\rho}) g + \frac{\partial}{\partial z} (\mathbf{\bar{p}} + \mathbf{p}) = 0 \] (6.4)

\[ \frac{\partial \mathbf{\bar{u}}}{\partial x} + \frac{\partial \mathbf{\bar{v}}}{\partial y} + \frac{\partial \mathbf{\bar{w}}}{\partial z} = 0 \] (6.5)

\[ \frac{\partial \mathbf{\bar{p}}}{\partial t} + (\mathbf{\bar{u}} + \mathbf{u}) \frac{\partial \mathbf{\bar{p}}}{\partial x} + \mathbf{v} \frac{\partial (\mathbf{\bar{p}} + \mathbf{p})}{\partial y} + \mathbf{w} \frac{\partial (\mathbf{\bar{p}} + \mathbf{p})}{\partial z} = 0, \] (6.6)

where \( \mathbf{u}, \mathbf{v} \) and \( \mathbf{w} \) are fluctuations of current in the x-, y- and z-directions, respectively, \( f = 2\Omega \sin \phi \) the Coriolis parameter, \( \Omega \) the angular velocity of the earth's rotation, \( \mathbf{\rho}_0 \) a reference density, \( \mathbf{\rho} \) the density fluctuation, and \( p \) the pressure fluctuation.
We consider the time independent terms and the time
dependent terms of the above equations separately. The
time independent terms in (6.3) and (6.4)

\[ \frac{f\bar{u}}{\rho_0} + \frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial y} = 0 \quad (6.7) \]

\[ \frac{\rho g}{\rho_0} + \frac{\partial \bar{p}}{\partial z} = 0 \quad (6.8) \]
yield the thermal wind equation for the mean field

\[ \frac{f^2 \bar{u}}{\partial z^2} - \frac{g}{\rho_0} \frac{\partial \bar{p}}{\partial y} = 0. \quad (6.9) \]

The time dependent terms in the governing equations are:

\[ \frac{\partial u}{\partial t} + (\bar{u}+u) \frac{\partial u}{\partial x} + v \frac{\partial}{\partial y} (\bar{u}+u) + w \frac{\partial}{\partial z} (\bar{u}+u) - fu + \frac{1}{\rho_0} \frac{\partial p}{\partial x} = 0 \quad (6.10) \]

\[ \frac{\partial v}{\partial t} + (\bar{u}+u) \frac{\partial v}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} + fu + \frac{1}{\rho_0} \frac{\partial p}{\partial y} = 0 \quad (6.11) \]

\[ \rho g + \frac{\partial \bar{p}}{\partial z} = 0 \quad (6.12) \]

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (6.13) \]

\[ \frac{\partial p}{\partial t} + (\bar{u}+u) \frac{\partial p}{\partial x} + v \frac{\partial}{\partial y} (\bar{\rho}+\rho) + w \frac{\partial}{\partial z} (\bar{\rho}+\rho) = 0. \quad (6.14) \]

Note that these equations include nonlinear advection terms.
Cross differentiating the horizontal momentum equations (6.10) and (6.11) and using the continuity equation (6.13) yields the vorticity equation

\[
\left( \frac{\partial}{\partial t} + \frac{u}{\partial x} \right) \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + v \left( \frac{\partial^2 u}{\partial y^2} - \beta \right) + f \frac{\partial w}{\partial z} \\
+ \left( \frac{\partial v}{\partial x} - \frac{\partial (u+u)}{\partial z} \right) \frac{\partial w}{\partial x} + \left( \frac{\partial u+u}{\partial y} \right) \frac{\partial w}{\partial y} + \frac{\partial^2 (u+u)}{\partial y \partial z} - \frac{\partial v}{\partial z} \frac{\partial w}{\partial x} - \frac{\partial^2 v}{\partial x \partial z} w
\]

= 0, \quad (6.15)

where \( \beta = \partial f / \partial y \).

Let \( L, H, U \) and \( L/U \) be the scales of the horizontal length, depth, horizontal velocity, and time, respectively, for the mesoscale fluctuations. We assume that the scales of velocity of the mean zonal flow and that of the horizontal fluctuations are about the same. Since the time scale of the mesoscale motion, \( L/U \), is much larger than the inertial period, \( 2\pi/f \), we have

\[
Ro = \frac{U}{fL} \ll 1, \quad (6.16)
\]

where \( Ro \) is the Rossby number \( (Ro = 10^{-2}) \). The length scale \( L \) is much smaller than the radius \( a \) of the earth, and we assume

\[
\frac{L}{a} = O(Ro). \quad (6.17)
\]

In the vorticity equation (6.15), we can neglect all
terms involving \( w \) except one term, \( \frac{f^2 w}{\partial z} \), because they are smaller by a factor \( \text{Ro} \). The simplified vorticity equation is then

\[
\frac{\partial}{\partial t} \left( \frac{u^2}{\partial x} + \frac{v^2}{\partial y} + \frac{u \partial u}{\partial x} + \frac{v \partial v}{\partial y} \right) \left( \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) + v \left( \frac{\partial^2 u}{\partial y^2} - g \right) + \frac{f^2 w}{\partial z} = 0. \tag{6.18}
\]

By applying a scale analysis to this vorticity equation we obtain the result that the appropriate scale \( W \) for the vertical velocity associated with the mesoscale motion should be

\[
W = \text{Ro} U H / L. \tag{6.19}
\]

By applying this scale to the continuity equation (6.13) we see immediately that the mesoscale motion is nearly non-divergent. Furthermore, from the scale analysis of the horizontal momentum equations (6.10) and (6.11), we see a quasi-geostrophic balance between the Coriolis force and pressure gradient:

\[
u = \frac{1}{f \rho_0} \frac{\partial p}{\partial y}, \tag{6.20}
\]

\[
v = \frac{1}{f \rho_0} \frac{\partial p}{\partial x}. \tag{6.21}
\]

On the other hand, using the thermal wind equation (6.9) and the hydrostatic balance (6.12) in the equation of state
(6.14), we get

$$\left( \frac{\partial}{\partial t} + \frac{u}{\partial x} + \frac{v}{\partial y} + \frac{w}{\partial z} \right) \frac{\partial p}{\partial z} - \rho_0 f v \frac{\partial u}{\partial z} + \rho_0 N^2 w = 0, \quad (6.22)$$

where

$$N^2 = - \frac{g}{\rho_0} \frac{\partial \rho}{\partial z} \quad (6.23)$$

is the square of the Brunt-Väisälä frequency. The vertical advection term is by a factor Ro smaller than the horizontal ones, and therefore can be neglected.

By means of the quasi-geostrophic relations (6.20) and (6.21) we can express the horizontal velocity components $u$ and $v$ in (6.18) and (6.22) in terms of pressure. It should be noted, however, that it is not adequate to apply quasi-geostrophic approximations to the horizontal momentum equations (6.10) and (6.11) before we get a vorticity equation, because the vorticity equation relates balances among "non-geostrophic" terms of the horizontal momentum equations. From (6.18) and (6.22), by eliminating $w$, we can get an equation in terms of pressure only.

Using a plane wave solution of the form

$$p(x,y,z,t) = \tilde{p}(z) \exp\{i(\kappa x + \eta y - \omega t)\}, \quad (6.24)$$

it can be easily shown, by using the quasi-geostrophic relations (6.20) and (6.21), that the nonlinear horizontal
advection terms in (6.18) and (6.22) cancel each other:

\[
(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y}) (\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}) = 0 \tag{6.25}
\]

\[
(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y}) \frac{\partial p}{\partial z} = 0. \tag{6.26}
\]

The cancellation justifies the conclusion that the Rossby wave dynamics is essentially linear even though the wave-associated particle speed is much larger than the phase speed of the wave. Now (6.18) and (6.22) becomes, by using the quasi-geostrophic relations (6.20) and (6.21),

\[
\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial x}\right) v \frac{h^2}{\rho} \frac{\partial p}{\partial t} + (\beta - \frac{\partial^2 u}{\partial y^2}) \frac{\partial p}{\partial x} - f^2 \rho_o \frac{\partial w}{\partial z} = 0 \tag{6.27}
\]

\[
w = \frac{1}{\rho_o N^2} \left\{-\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial x}\right) \frac{\partial p}{\partial x} + \frac{\partial u}{\partial x} \frac{\partial p}{\partial z}\right\}, \tag{6.28}
\]

where \( v_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \). Eliminating \( w \) in these two equations, we get

\[
\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial x}\right) v \frac{h^2}{\rho} \frac{\partial p}{\partial t} + \frac{\partial}{\partial z} \left(\frac{f^2}{N^2} \frac{\partial p}{\partial z}\right) + (\beta - \frac{\partial^2 u}{\partial y^2}) - \frac{\partial}{\partial z} \left(\frac{f^2}{N^2} \frac{\partial u}{\partial z}\right) \frac{\partial p}{\partial x} = 0,
\]

or, by using the plane wave assumption (6.24),

\[
(-\bar{u} - c) \left\{\frac{d}{dz} \left(\frac{f^2}{N^2} \frac{dp}{dz}\right) - (\kappa^2 + \eta^2) \bar{p}\right\} + Q_y \tilde{p} = 0, \tag{6.30}
\]

where
\[ C = \omega / \kappa \quad (6.31) \]

\[ Q_y = \beta - \frac{\partial^2 u}{\partial y^2} - \frac{\partial}{\partial z} \left( \frac{N^2}{f^2} \frac{\partial u}{\partial z} \right). \quad (6.32) \]

The general surface and bottom boundary conditions are

\[ w = 0 \quad \text{at} \quad z = 0 \quad (6.33) \]

\[ w + u \frac{\partial H}{\partial x} + v \frac{\partial H}{\partial y} = 0 \quad \text{at} \quad z = -H, \]

where \( H \) is the depth of the ocean. These boundary conditions can be represented, by means of (6.20) and (6.21), in terms of pressure alone:

\[ -\left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} \right) \frac{\partial p}{\partial z} + \frac{\partial u}{\partial z} \frac{\partial p}{\partial x} = 0 \quad \text{at} \quad z = 0 \quad (6.34) \]

\[ -\left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} \right) \frac{\partial p}{\partial z} + \left( \frac{\partial u}{\partial z} + \frac{N^2}{f^2} \frac{\partial H}{\partial y} \frac{\partial p}{\partial x} - \frac{N^2}{f} \frac{\partial H}{\partial x} \frac{\partial p}{\partial y} \right) = 0 \quad \text{at} \quad z = -H. \]

In an ocean with a flat bottom, the boundary conditions for the plane waves are

\[ (\ddot{u} - c) \frac{\partial \ddot{p}}{\partial z} - \frac{\partial u}{\partial z} \ddot{p} = 0 \quad \text{at} \quad z = 0 \quad \text{(rigid surface)} \quad (6.35) \]

\[ \frac{\partial \ddot{p}}{\partial z} = 0 \quad \text{at} \quad z = -H \quad \text{(flat bottom)}. \]
The Rossby wave shear modes can be determined from the eigenvalue problem, (6.30) and (6.35). Kang and Magaard (1979), henceforth referred to as KM, numerically obtained eigenfunctions and corresponding dispersion relations for stable (c real) and unstable (c complex) waves in various subareas of the North Pacific Current area. The necessary information on the mean shear flow and mean stratification in their calculation was obtained from Price and Meyers (1978). Fig. 6.1 shows an example for $\tilde{p}(z)$ of the first order baroclinic shear mode calculated using the average values of the mean shear flow and stratification at a subarea 30 - 40°N, 170 - 150°W when $(\kappa^2 + \eta^2)\kappa = 2\pi/400$ km$^{-1}$.

Once $\tilde{p}(z)$ is known we can calculate the amplitudes of the velocity and density fluctuations associated with the Rossby waves by

$$\tilde{u}(z) = -\frac{in}{f\rho_o} \tilde{p}(z) \quad (6.36)$$

$$\tilde{v}(z) = \frac{i\kappa}{f\rho_o} \tilde{p}(z) \quad (6.37)$$

$$\tilde{w}(z) = -\frac{i\kappa}{\rho_o N^2} \left\{ (\bar{u} - c) \frac{d\tilde{p}}{dz} - \frac{\partial \bar{u}}{\partial z} \tilde{p} \right\} \quad (6.38)$$

$$\tilde{p}(z) = -\frac{1}{g} \frac{d\tilde{p}}{dz} \quad (6.39)$$

The vertical profiles of $\tilde{u}(z)$ and $\tilde{v}(z)$ are essentially the
same as $\tilde{p}(z)$. The amplitude of the vertical velocity $\tilde{w}(z)$ calculated by (6.38) is shown in Fig. 6.2. Since the density fluctuation $\tilde{\rho}(z)$ and the temperature fluctuation $\tilde{T}(z)$ are related by

$$\tilde{\rho} = -\rho_0 \alpha \tilde{T},$$

(6.40)

where $\alpha$ is the thermal expansion coefficient ($\alpha = 2 \times 10^{-4} \degree C^{-1}$), the vertical distribution of the amplitude of the temperature fluctuation can be obtained from

$$\tilde{T} = \frac{1}{\rho_0 \alpha g} \frac{d\tilde{\rho}}{dz},$$

(6.41)

and is shown in Fig. 6.3. This figure shows that the temperature fluctuations associated with the baroclinic Rossby waves affect the sea water temperature as deep as 2000 m.
Fig. 6.1 Amplitude of pressure for the first order baroclinic shear mode (in relative units)
Fig. 6.2 Amplitude of vertical velocity for the first order baroclinic shear mode (in relative units)
Fig. 6.3 Amplitude of temperature for the first order baroclinic shear mode (in relative units)
VII. A METHOD OF ROSSBY WAVE ANALYSIS

In this chapter we present a cross spectral inverse method of analyzing Rossby waves from the time series of temperature fluctuations. The inverse method we present below is, in principle, the same as that used in previous analysis of Rossby waves in the Pacific by Emery and Magaard (1976) and Magaard and Price (1977). These authors applied the method to an area of negligible mean shear flow, where they could convert observed temperature fluctuations to isotherm displacements which they used for their model fittings. In the North Pacific Current area, however, we cannot calculate isotherm displacements from the temperature data because of the mean shear flow. Hence, we use, instead of isotherm displacements, the temperature shear modes to construct a model to be fitted to the temperature fluctuations themselves.

We consider a random field of temperature fluctuations $T(x,z,t)$ which are composed of a superposition of uncorrelated plane wave modes:

$$T(x,z,t) = \sum_{n,\omega,\kappa} \theta_n(\kappa,\omega) \tilde{T}_n(\kappa,z) e^{i(\kappa \cdot x - \omega t)}, \quad (7.1)$$

where $x = (x,y)$ the horizontal position vector, $\kappa = (\kappa,n)$ the wavenumber vector, $n$ the mode number, $\theta_n$ the complex amplitude of the $n$-th temperature mode, and $\tilde{T}_n$ the vertical
structure of the n-th mode determined by the methods explained in the previous chapter. The correlation function \( K \) between two time series \( T(x, z, t) \) and \( T(x+r, z', t+\tau) \), where \( r = (r_x, r_y) \) is the separation vector and \( \tau \) is the time lag, will be

\[
K = \langle T(x, z, t) \, T(x+r, z', t+\tau) \rangle
\]

\[
= \sum_{n, \kappa, \omega, n', K', \omega'} \langle \Theta_n(\kappa, \omega) \, \Theta_{n'}(\kappa', \omega') \, \tilde{T}_n(\kappa, z) \, \tilde{T}_{n'}(\kappa', z') \rangle \cdot e^{i(\kappa + K') \cdot x - (\omega + \omega') t} \cdot e^{i(\kappa' \cdot r - \kappa \cdot r - \omega' \tau)} , \tag{7.2}
\]

where \( \langle \ldots \rangle \) means ensemble average. We assume \( T(x, z, t) \) is a stationary and horizontally homogeneous process, and thus obtain

\[
\langle \Theta_n(\kappa, \omega) \, \Theta_{n'}(\kappa', \omega') \, \tilde{T}_n(\kappa, z) \, \tilde{T}_{n'}(\kappa', z') \rangle = 0
\]

if \( \kappa + K' \neq 0 \) or \( \omega + \omega' \neq 0 \). \tag{7.3}

Since \( T_n(x, z, t) \), the temperature fluctuation associated with the n-th mode, should be real for any \( \kappa \) and \( \omega \) we have

\[
\Theta_n(-\kappa, -\omega) \, \tilde{T}_n(-\kappa, z) = \Theta^*_n(\kappa, \omega) \, \tilde{T}^*_n(\kappa, z) , \tag{7.4}
\]

where the asterisk means complex conjugate. Using (7.3) and (7.4), the correlation function becomes
\[
K(\mathbf{r}, z, z', \tau) = \sum_{\kappa, \omega} \int_{\Omega_n} \theta_n^{\ast}(\kappa, \omega) \theta_n(\kappa, \omega) \tilde{T}_n(\kappa, z) \tilde{T}^\ast_n(\kappa, z') e^{-i(\kappa \cdot \mathbf{r} - \omega \tau)},
\]

(7.5)

or, by introducing a continuous representation,

\[
K(\mathbf{r}, z, z', \tau) = \int_{\kappa} \int_{\omega} E_n(\kappa, \omega) \tilde{T}_n(\kappa, z) \tilde{T}^\ast_n(\kappa, z') e^{-i(\kappa \cdot \mathbf{r} - \omega \tau)} d\kappa d\omega,
\]

(7.6)

where \(E_n(\kappa, \omega)\) is the temperature frequency-wavenumber spectrum of the \(n\)-th mode.

The cross spectrum \(A\) is related to the correlation function \(K\),

\[
K(\mathbf{r}, z, z', \tau) = \int_{\Omega_n} A(\mathbf{r}, z, z', \omega) e^{i\omega \tau} d\omega,
\]

(7.7)

and by inspection of (7.6) and (7.7) we have

\[
A(\mathbf{r}, z, z', \omega) = \int_{\Omega_n} E_n(\kappa, \omega) \tilde{T}_n(\kappa, z) \tilde{T}^\ast_n(\kappa, z') e^{-i\kappa \cdot \mathbf{r}} d\kappa.
\]

(7.8)

For our model we assume that for each mode we have only one direction of propagation, described by wavenumber vector \(\kappa_n\). Then

\[
E_n(\kappa, \omega) = \varepsilon_n(\omega) \delta(\kappa - \kappa_n),
\]

(7.9)
where $\epsilon_n(\omega)$ is the temperature frequency spectrum of the $n$-th mode, and $\delta$ is Dirac's delta function. Using (7.9) and the integral

$$\int f(x) \delta(x-x_n) \, dx = f(x_n), \quad (7.10)$$

we get from (7.8)

$$A(x, z, z', \omega) = \sum_n \epsilon_n(\omega) \tilde{T}_n(z) \tilde{T}_n^*(z') e^{-i\kappa_n \cdot \vec{r}}. \quad (7.11)$$

For the free annual Rossby waves we are considering in the North Pacific Current area, only the first mode is possible, because higher modes in that area have periods larger than one year (KM). We fit our model cross spectra, $A = C - iQ$, to the observed cross spectra, $a = c - iq$, by minimizing

$$F = \sum_{j=1}^{L} (c_j - C_j)^2 + (q_j - Q_j)^2$$

$$= \sum_j \left( c_j - \epsilon_1 D_{1j} \cos \kappa_1 \cdot \vec{r}_j \right)^2 + \left( q_j - \epsilon_1 D_{1j} \sin \kappa_1 \cdot \vec{r}_j \right)^2, \quad (7.12)$$

where the index $j$ counts the pairs of points at which the co- and quadrature spectra, $c_j$ and $q_j$, are calculated, $L$ is the total number of pairs, and

$$D_{1j} = \tilde{T}_1(\kappa_1, z_j) \tilde{T}_1^*(\kappa_1, z'_j). \quad (7.13)$$
In the North Pacific Current area the dependence of $\tilde{T}_1$ on $\kappa$ is weak; we can treat $\tilde{T}_1(z)$ as being independent of $\kappa$ for practical applications (KM). The function $F$ in (7.12) depends on three parameters, $\kappa$, $\eta$, and $\epsilon$ (The index 1 is dropped). From the necessary condition $\partial F/\partial \epsilon = 0$ we get

$$\epsilon(\kappa) = \sum_j D_j (c_j \cos \kappa r_j + q_j \sin \kappa r_j)/\sum_j D_j^2$$  (7.14)

$$F(\kappa) = F_0 - \sum_j D_j (c_j \cos \kappa r_j + q_j \sin \kappa r_j)^2/\sum_j D_j^2,$$  (7.15)

where

$$F_0 = \sum_j (c_j^2 + q_j^2).$$  (7.16)

When $\partial F/\partial \epsilon = 0$ is satisfied, the function $F$ is minimized with respect to $\epsilon$. This can be shown easily by substituting (7.14) into (7.12) and expressing the resultant equation in a quadratic form with respect to $\epsilon$. Now the function $F$ in (7.15) depends on two free parameters: $\kappa$ and $\epsilon$. We have not assumed any dispersion relation so far. We want to determine the best fitting wavenumber vector independently and then compare it with the dispersion relation of Rossby waves. The only Rossby wave features in our model so far is the depth dependence of $\tilde{T}(z)$. If a dispersion relation is applied to (7.15), then the problem becomes a minimization with respect to one parameter $\kappa$ or $\eta$, as was done by
Emery and Magaard (1976). Instead of using a particular dispersion relation, we determine the two parameters $\kappa$ and $\eta$ that minimize $F(\kappa)$, as was done by Magaard and Price (1977). The wavenumber vector that minimizes $F(\kappa)$ can be found by an iterative method of minimization developed by Fletcher and Powell (1963). Using the values of $\kappa$ thus obtained we can easily calculate $\varepsilon$ from (7.14).

It should be noted that the wavenumber vector determined by the minimization procedure discussed above is not unique. When the grid points of the time series are horizontally separated by regular intervals, "folding" or "repetition" patterns of the wavenumber vectors that minimize $F(\kappa)$ appear. If $\kappa$ is a wavenumber vector that minimizes $F(\kappa)$, then any other wavenumber vector $\kappa'$ which satisfies

$$\kappa' \cdot \mathbf{r}_j = \kappa \cdot \mathbf{r}_j + 2\pi n,$$

where $n$ is an integer, will also minimize $F(\kappa)$. These patterns are analogous to the repetition patterns in conventional directional wavenumber spectra (e.g., Barber, 1961). However, we can determine the best wavenumber vector by using the known dispersion relations of the Rossby waves.

Standard errors of parameters, $\kappa$, $\eta$ and $\varepsilon$, determined by the least squares fit described above, can be estimated as follows. We write $F$ in a matrix notation as

$$F = (\mathbf{y} - \mathbf{Y})(\mathbf{y} - \mathbf{Y}).$$

(7.18)
where $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_{2L}) = (c_1, c_2, \ldots, c_L, q_1, q_2, \ldots, q_L)$ is the observed cross spectra, $\mathbf{Y}$ is the corresponding model spectra, and a prime indicates a transposed matrix. We represent the parameters by a matrix, $\mathbf{z} = (z_1, z_2, z_3) = (\kappa, \eta, \varepsilon)$, and linearize the model spectra $\mathbf{Y}$ in the vicinity of the minimum point $\mathbf{z}^0$ by

$$\mathbf{Y}(\mathbf{z}) = \mathbf{Y}^0 + \mathbf{A} (\mathbf{z} - \mathbf{z}^0),$$

(7.19)

where $\mathbf{Y}^0 = \mathbf{Y}(\mathbf{z}^0)$, and

$$\mathbf{A} = \{A_{ij}\} = \{\partial Y_i / \partial z_j\}. \quad (7.20)$$

This linearization allows us to utilize a standard technique of error estimation for least squares fit (e.g., Bevington, 1969). Since $\partial F / \partial \mathbf{z} = 0$ at the minimum point $\mathbf{z}^0$, we have the normal equation

$$\mathbf{C} (\mathbf{z} - \mathbf{z}^0) - \mathbf{A}' (\mathbf{Y} - \mathbf{Y}^0) = 0,$$

(7.21)

where $\mathbf{C} = \mathbf{A}' \mathbf{A}$ is the curvature matrix. For the $m$-th parameter $z_m$, we obtain from (7.21) the relation

$$z_m - z_m^0 = \sum_j \sum_k (C^{-1})_{mk} A_{jk} (Y_j - Y_j^0).$$

(7.22)

We assume that the difference between the data point and the model point for the $i$-th pair, $y_i - Y_i^0$, is uncorrelated
with that for another j-th pair, \( y_j - Y_j^0 \), i.e.,

\[
(y_i - Y_i^0)(y_j - Y_j^0) = \sigma^2 \delta_{ij} ,
\]

where \( \sigma^2 \) is the residual variance of the least squares fit. Then

\[
<(z_m - z_m^0)(z_n - z_n^0)> = \sigma^2 (C^{-1})_{mn} ,
\]

and the standard error \( \Delta z_m \) of the m-th parameter \( z_m \) is

\[
\Delta z_m = \{\sigma^2 (C^{-1})_{mm}\}^{1/2} ,
\]

where the m-th diagonal element of the curvature matrix \( C_{mm} \) is

\[
C_{mm} = \sum_j \left( \frac{\partial Y_j}{\partial z_m} \right)^2 .
\]

The residual variance \( \sigma^2 \) of the least squares fit in our case is

\[
\sigma^2 = \frac{F_{\text{min}}}{L_{\text{eff}} - M} ,
\]

where \( L_{\text{eff}} \) is the "effective" degree of freedom of the cross spectral least squares fit, \( F_{\text{min}} \) is the minimum value of \( F \), and \( M \) is the number of parameters (\( M = 3 \)). Following Emery and Magaard (1976) we estimate \( L_{\text{eff}} \) by
\[ L_{\text{eff}} = \left( \sum_i E_i \right)^2 \frac{d}{(\sum_{i,j} \gamma_{ij}^2 E_i E_j)}, \]  

where \( \gamma_{ij} \) is the coherence between two time series \( T_i \) and \( T_j \), and \( d \) is the number of degree of freedom for the individual power spectra \( E_i \). Since

\[ Y_j(z) = \varepsilon D_j \cos \kappa \cdot r_j \quad \text{for } j = 1, L, \]

and

\[ Y_j(z) = \varepsilon D_j \sin \kappa \cdot r_j \quad \text{for } j = L+1, 2L, \]

we have

\[ C_{11} = \sum_{j=1}^{2L} \left( \frac{\partial Y_j}{\partial \kappa} \right)^2 = \varepsilon^2 \sum_{j=1}^{L} D_j^2 r_{xj}^2 \]

\[ C_{22} = \sum_{j=1}^{2L} \left( \frac{\partial Y_j}{\partial \eta} \right)^2 = \varepsilon^2 \sum_{j=1}^{L} D_j^2 r_{yj}^2 \]  

\[ C_{33} = \sum_{j=1}^{2L} \left( \frac{\partial Y_j}{\partial \varepsilon} \right)^2 = \sum_{j=1}^{L} D_j^2 \]

and the standard errors of the parameters are, from (7.25),

\[ \Delta \kappa = \left| \frac{\sigma^2}{\varepsilon^2} \sum_j D_j^2 r_{xj}^2 \right|^{1/2} \]

\[ \Delta \eta = \left| \frac{\sigma^2}{\varepsilon^2} \sum_j D_j^2 r_{yj}^2 \right|^{1/2} \]  

(7.31)
Next, we consider to what extent the observed data are explained by the plane wave model. The percentage of the observed cross spectra explained by the model spectra is measured by

\[
R = \frac{F_o - F_{\text{min}}}{F_o} .
\]  

(7.32)

\(R = 0\) means that nothing is explained, and \(R = 100\%\) means a perfect fit. We also examine the consistency of the model by checking whether the model cross spectra, \(C_j\) and \(Q_j\), fall into the confidence range of the observed cross spectra, \(c_j\) and \(q_j\) (cf., Olbers et al., 1976; Müller et al., 1978). We can regard the model as a consistent representation of the data if

\[
S = \frac{F_{\text{min}}}{\sum_j \{\text{var}(c_j) + \text{var}(q_j)\}} < 1. 
\]

(7.33)

Since the variance of the co- and quadrature spectra are (Henkins and Watts, 1968, p. 378)

\[
\text{var}(c_j) = \frac{1}{d} (E_j E_j' + c_j^2 - q_j^2) 
\]

(7.34)

\[
\text{var}(q_j) = \frac{1}{d} (E_j E_j' - c_j^2 + q_j^2) ,
\]
where $E_j$ and $E'_j$ are the power spectra of two time series of the $j$-th pair, we have

$$\sum_j \{\text{var}(c_j) + \text{var}(q_j)\} = \frac{2}{d} \sum_j E_j E'_j .$$  \hfill (7.35)

Appropriate configurations of the grid points for an application of our method were determined by the following considerations. The temperature fluctuations in a surface layer down to about 100 m are dominated by the direct influence of the seasonal variation of insolation (Chapter 4). Besides that, atmospheric disturbances add substantial changes to the thermal structure of the upper ocean. During extreme weather conditions the depth of the wind-stirred surface mixed layer reaches more than 200 m. Thus we consider temperature data at depths larger than 200 m for our wave analysis. The horizontal extension of our configuration of points for an individual model fit should be small enough to satisfy our assumption of horizontal homogeneity, and at the same time, should be large enough to allow good resolution of the wave parameters and enhance the statistical significance of our results. With that in mind we use the configuration of the grid points shown in Fig. 7.1. Horizontally the configuration covers a 10- by 4-degree area. The depths of grid points are 250, 300 and 400 m. This scheme contains 15 grid points with time series of temperature, and the number of pairs is
accordingly 105.

The cross spectral inverse method described above is applied to the wave-like part of the temperature fluctuations. A reasonable separation of the temperature fluctuations into the quasi homogeneous and wave-like parts, by the method of space averaging described in Chapter 2, turns out to be possible only south of 40°N. The quasi homogeneous part of the temperature fluctuations, obtained by our space averaging technique, south of 40°N manifest the characteristics of the large scale fluctuations well; those north of 40°N do not. The 40°N is also the critical latitude for the annual baroclinic Rossby wave (KM). Hence we restrict our wave analysis to the area 30°N - 40°N, 160°E - 130°W. This area covers 35 subareas in which we have the grid point configuration described above (Fig. 7.1).
Fig. 7.1 The configuration of grid points used for wave analysis in each of 35 subareas (This figure shows, for example, the subarea centered at 34°N, 150°W).
In this chapter we discuss the annual baroclinic Rossby wave fields in the Central North Pacific (30° - 40°N, 160°E - 130°W) analyzed by means of the cross spectral inverse method described in Chapter 7. First, the wavenumber vectors determined from the wave-like part of the temperature fluctuations at the annual frequency are compared with the theoretical dispersion relations of the Rossby waves. Then we describe the spatial distribution of the amplitudes of the fluctuations of the temperature, the pressure, the horizontal and vertical velocities, and the corresponding particle excursions associated with the analyzed annual baroclinic Rossby waves.

Tab. 8.1 shows the best-fitting wavenumber vectors and the corresponding ε values, (7.14), at the annual frequency in our 35 subareas. For each value the standard error, calculated by (7.31), is also indicated. Tab. 8.1 further shows the corresponding R values, (7.32), and S values, (7.33). The values of R range from 10 to 80 %; R exceeds 50 % in more than half of the subareas. The condition S < 1 is satisfied in 30 subareas; in the remaining 5 subareas the condition is violated only slightly. The best-fitting wavenumber vectors together with their error ellipses are shown in Fig. 8.1. We see a consistent pattern of waves travelling in the NW.
<table>
<thead>
<tr>
<th>Location</th>
<th>$\kappa(10^{-5} \text{m}^{-1})$</th>
<th>$\eta(10^{-5} \text{m}^{-1})$</th>
<th>$\epsilon(\text{C}^2/\text{cpm})$</th>
<th>$R(%)$</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40N 165E</td>
<td>-1.48±0.15</td>
<td>1.91±0.28</td>
<td>0.154±0.090</td>
<td>64.0</td>
<td>0.50</td>
</tr>
<tr>
<td>40N 175E</td>
<td>-1.50±0.09</td>
<td>1.66±0.17</td>
<td>0.168±0.060</td>
<td>82.2</td>
<td>0.33</td>
</tr>
<tr>
<td>40N 175W</td>
<td>-1.54±0.13</td>
<td>1.52±0.24</td>
<td>0.119±0.060</td>
<td>74.3</td>
<td>0.50</td>
</tr>
<tr>
<td>40N 165W</td>
<td>-1.42±0.29</td>
<td>1.43±0.56</td>
<td>0.029±0.033</td>
<td>33.8</td>
<td>1.17</td>
</tr>
<tr>
<td>40N 155W</td>
<td>-1.51±0.29</td>
<td>1.19±0.56</td>
<td>0.007±0.008</td>
<td>24.9</td>
<td>1.09</td>
</tr>
<tr>
<td>40N 145W</td>
<td>-1.45±0.22</td>
<td>1.59±0.41</td>
<td>0.020±0.017</td>
<td>34.9</td>
<td>0.88</td>
</tr>
<tr>
<td>40N 135W</td>
<td>-1.70±0.18</td>
<td>1.34±0.35</td>
<td>0.024±0.017</td>
<td>44.7</td>
<td>0.91</td>
</tr>
<tr>
<td>38N 165E</td>
<td>-1.54±0.16</td>
<td>2.03±0.32</td>
<td>0.122±0.081</td>
<td>46.0</td>
<td>0.74</td>
</tr>
<tr>
<td>38N 175E</td>
<td>-1.34±0.13</td>
<td>1.69±0.25</td>
<td>0.089±0.045</td>
<td>57.4</td>
<td>0.59</td>
</tr>
<tr>
<td>38N 175W</td>
<td>-1.48±0.19</td>
<td>1.82±0.37</td>
<td>0.048±0.036</td>
<td>45.7</td>
<td>0.83</td>
</tr>
<tr>
<td>38N 165W</td>
<td>-1.13±0.23</td>
<td>1.88±0.46</td>
<td>0.021±0.019</td>
<td>37.7</td>
<td>1.01</td>
</tr>
<tr>
<td>38N 155W</td>
<td>-0.84±0.28</td>
<td>0.62±0.55</td>
<td>0.010±0.012</td>
<td>25.1</td>
<td>0.98</td>
</tr>
<tr>
<td>38N 145W</td>
<td>-1.28±0.46</td>
<td>1.02±0.91</td>
<td>0.007±0.013</td>
<td>10.3</td>
<td>1.25</td>
</tr>
<tr>
<td>38N 135W</td>
<td>-1.33±0.15</td>
<td>1.20±0.30</td>
<td>0.012±0.007</td>
<td>54.1</td>
<td>0.69</td>
</tr>
<tr>
<td>36N 165E</td>
<td>-1.48±0.16</td>
<td>1.80±0.33</td>
<td>0.269±0.184</td>
<td>52.9</td>
<td>0.75</td>
</tr>
<tr>
<td>36N 175E</td>
<td>-1.51±0.16</td>
<td>1.82±0.33</td>
<td>0.134±0.092</td>
<td>49.1</td>
<td>0.76</td>
</tr>
<tr>
<td>36N 175W</td>
<td>-1.44±0.18</td>
<td>1.75±0.37</td>
<td>0.048±0.036</td>
<td>44.0</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Tab. 8.1 Wave parameters for the analyzed annual baroclinic Rossby waves in 35 subareas.
<table>
<thead>
<tr>
<th>Location</th>
<th>$\kappa(10^{-5}\text{m}^{-1})$</th>
<th>$\mu(10^{-5}\text{m}^{-1})$</th>
<th>$\epsilon(\text{C}^2/\text{cpm})$</th>
<th>$R(%)$</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>36N 165W</td>
<td>-1.29±0.16</td>
<td>1.38±0.32</td>
<td>0.015±0.010</td>
<td>54.6</td>
<td>0.73</td>
</tr>
<tr>
<td>36N 155W</td>
<td>-1.41±0.21</td>
<td>1.52±0.42</td>
<td>0.015±0.013</td>
<td>32.1</td>
<td>0.84</td>
</tr>
<tr>
<td>36N 145W</td>
<td>-1.35±0.21</td>
<td>1.64±0.43</td>
<td>0.013±0.012</td>
<td>30.6</td>
<td>0.89</td>
</tr>
<tr>
<td>36N 135W</td>
<td>-1.11±0.11</td>
<td>1.27±0.24</td>
<td>0.012±0.006</td>
<td>58.0</td>
<td>0.54</td>
</tr>
<tr>
<td>34N 165E</td>
<td>-1.40±0.11</td>
<td>1.46±0.24</td>
<td>0.375±0.183</td>
<td>71.7</td>
<td>0.50</td>
</tr>
<tr>
<td>34N 175E</td>
<td>-1.46±0.08</td>
<td>1.92±0.18</td>
<td>0.160±0.059</td>
<td>75.3</td>
<td>0.36</td>
</tr>
<tr>
<td>34N 175W</td>
<td>-1.46±0.13</td>
<td>1.80±0.26</td>
<td>0.055±0.030</td>
<td>67.0</td>
<td>0.56</td>
</tr>
<tr>
<td>34N 165W</td>
<td>-1.25±0.15</td>
<td>1.62±0.31</td>
<td>0.019±0.012</td>
<td>54.8</td>
<td>0.71</td>
</tr>
<tr>
<td>34N 155W</td>
<td>-1.22±0.22</td>
<td>1.85±0.46</td>
<td>0.013±0.013</td>
<td>34.8</td>
<td>0.93</td>
</tr>
<tr>
<td>34N 145W</td>
<td>-1.31±0.12</td>
<td>1.80±0.25</td>
<td>0.021±0.011</td>
<td>59.6</td>
<td>0.57</td>
</tr>
<tr>
<td>34N 135W</td>
<td>-1.41±0.13</td>
<td>1.47±0.28</td>
<td>0.021±0.012</td>
<td>62.7</td>
<td>0.64</td>
</tr>
<tr>
<td>32N 165E</td>
<td>-1.41±0.19</td>
<td>1.49±0.40</td>
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<td>32N 175E</td>
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<td>0.141±0.108</td>
<td>52.5</td>
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<td>32N 175W</td>
<td>-1.55±0.17</td>
<td>2.00±0.36</td>
<td>0.034±0.025</td>
<td>52.3</td>
<td>0.82</td>
</tr>
<tr>
<td>32N 165W</td>
<td>-1.02±0.15</td>
<td>2.14±0.31</td>
<td>0.015±0.009</td>
<td>55.4</td>
<td>0.77</td>
</tr>
<tr>
<td>32N 155W</td>
<td>-1.33±0.20</td>
<td>1.66±0.41</td>
<td>0.022±0.019</td>
<td>52.0</td>
<td>0.92</td>
</tr>
<tr>
<td>32N 145W</td>
<td>-1.40±0.19</td>
<td>1.78±0.41</td>
<td>0.016±0.014</td>
<td>49.5</td>
<td>0.92</td>
</tr>
<tr>
<td>32N 135W</td>
<td>-1.47±0.26</td>
<td>1.43±0.55</td>
<td>0.003±0.003</td>
<td>22.3</td>
<td>1.01</td>
</tr>
</tbody>
</table>

Tab. 8.1 (Continued)
Fig. 8.1 The best-fitting wavenumber vectors with error ellipses in the 35 subareas. Each square in this figure represents a wavenumber vector space of size $2 \times 10^{-5} \text{ m}^{-1}$ by $2 \times 10^{-5} \text{ m}^{-1}$ with the origin of the coordinates at the lower-right corner.
direction. The wave length, $\lambda$, and phase speed, $c_p$, are calculated by

$$\lambda = \frac{2\pi}{(\kappa^2 + \eta^2)^{\frac{1}{2}}} \quad (8.1)$$

$$c_p = \frac{\omega}{(\kappa^2 + \eta^2)^{\frac{1}{2}}} \quad (8.2)$$

The typical wave length is about 300 km, and the corresponding phase speed is about 1 cm/sec. Hence the apparent zonal and meridional wave lengths are about 400 km. This is consistent with the length scales of fluctuations of several oceanographic patterns determined by Roden (1977, 1979) for a portion of the North Pacific including our area of investigation.

As mentioned before in Chapter 7 these wavenumber vectors are determined without assuming any dispersion relation. A comparison between the theoretical locus of wavenumber vectors (slowness curves) of the first order baroclinic Rossby waves (KM) and our best-fitting wavenumber vectors is given in Fig. 8.2 for the area 30 - 40°N, 170°W - 150°W (subarea 4 of KM) and in Fig. 8.3 for the area 30 - 40°N, 170°E - 170°W (subarea 3 of KM).

We cannot expect, in general, an exact agreement between the best-fitting wavenumber vector and the theoretical slowness curve, because we cannot avoid some errors in obtaining the temperature data themselves, in interpolating the data to grid points, in separating the
Fig. 8.2 The slowness curves of the first baroclinic shear mode (full line) and the ordinary mode without mean shear flow (dotted line) at the annual frequency in subarea 4 of KM. The best-fitting wavenumber vectors for our plane wave model from 8 our subareas of subarea 4 of KM are shown by dots.
Fig. 8.3 The same as in Fig. 8.2, however, for subarea 3 of KM.
temperature fluctuations into the quasi homogeneous and wave-like parts by our space averaging technique, and in estimating cross spectra from time series of finite length. Besides that, the theoretical slowness curve may not be accurate due to the influences of the local inhomogeneity of the mean stratification with respect to time and space, and the influence of the irregular bottom topography. However, by using the error ellipses, we have determined whether each wavenumber vector is significantly different from the slowness curves for the Rossby shear mode (full lines) and the Rossby mode calculated neglecting the mean shear flow (dotted lines) in Figs. 8.2 and 8.3. For subarea 4 of KM the result is that 7 out of 8 best-fitting wavenumber vectors are not significantly different from the slowness curve of the shear mode; 4 of these best-fitting wavenumber vectors are also not significantly different from the non-shear slowness curve. Only one out of the 8 is significantly different from both curves. For subarea 3 of KM only one best-fitting wavenumber vector is not significantly different from the slowness curve of the shear mode; 5 other of the 8 best-fitting vectors are not significantly different from the slowness curve of the ordinary mode. Two of the 8 best-fitting wavenumber vectors are significantly different from both curves.

Calculating the slowness curves for the shear mode, KM neglected the meridional component of the mean shear
flow. This is justified in subarea 4 of KM much better than in subarea 3 of KM. Therefore we consider subarea 4 of KM as more important for our comparison. We conclude that the shear mode leads to the better model. However, the degree of improvement by including the shear flow appears limited, because in the wavenumber region in question the two slowness curves are close together. We consider it demonstrated, however, that the annual fluctuations we have analyzed are indeed Rossby waves.

The group velocity of these waves is perpendicular to the corresponding slowness curves. For subarea 4 of KM the group velocity found from the slowness curve for the shear mode at the annual frequency (KM) points to the SW and has a magnitude of about 1.3 cm/sec.

From the $\varepsilon$ values we can determine the amplitude of the temperature fluctuations associated with the analyzed annual baroclinic Rossby waves. Assuming $D_j$ of (7.13) is the same for each pair, the amplitude spectrum $A_m$ becomes, using (7.11),

$$A_m(\omega) = \left(c_j^2(\omega) + q_j^2(\omega)\right)^{1/2}$$

$$= \left(\varepsilon^2 D_j^2 \cos^2(\kappa \cdot r_j) + \varepsilon^2 D_j^2 \sin^2(\kappa \cdot r_j)\right)^{1/2}$$

$$= \varepsilon D_j . \quad (8.3)$$
The amplitude, $T_a$, of the temperature fluctuation associated with the analyzed Rossby wave is calculated by

$$T_a(-300\text{m}) = (2A_m\Delta f)^{1/2} = (2\varepsilon D_j\Delta f)^{1/2},$$

(8.4)

where $\Delta f$ is the bandwidth, or spectral resolution. Fig. 8.4 shows the horizontal distribution of the amplitude of the temperature fluctuations at the sea surface. The vertical distribution of that amplitude is described by the function $\tilde{T}(z)$ and is displayed in Fig. 6.3. Absolute figures for the surface amplitudes can be obtained by adjusting the relative units of temperature in Fig. 6.3 to the absolute values at the sea surface as given in Fig. 8.4. The amplitude of the pressure fluctuation associated with the analyzed annual Rossby wave is obtained from (6.41). The surface values are shown in Fig. 8.5. The vertical distribution can be obtained from Fig. 6.1 by means of the corresponding adjustment of the relative units of pressure. Fig. 8.5 can also be interpreted as a description of the amplitude of sea level fluctuations associated with our Rossby waves. The maximum wave height is found to be about 6 cm.

The amplitude of the wave-induced vertical velocity, $w_a$, at 300 m is calculated by (6.38), and its horizontal distribution is shown in Fig. 8.6. The amplitude, $\zeta_a$, of the vertical displacement is calculated by
Fig. 8.4 Amplitudes of temperature fluctuations (in °C) associated with the analyzed annual baroclinic Rossby waves at the sea surface.

Fig. 8.5 Amplitudes of pressure fluctuations (in $10^3$ gm cm$^{-1}$ sec$^{-2}$) associated with the analyzed annual baroclinic Rossby waves at the sea surface.
\[ \zeta_a(-300m) = \frac{\tau}{4} \int_0^1 w_a \cos \omega t \, dt = \frac{w_a}{\omega}, \tag{8.5} \]

where \( \tau \) is the period, and is shown in Fig. 8.7. The maximum wave height at 300 m depth is 12 m. The corresponding wave heights for other depths can be obtained from Fig. 6.2. These results are in good agreement with White (1978). White's larger figures in the western part of our area are accounted for by the quasi homogeneous portion of the data.

The horizontal velocity components of water particles associated with the analyzed annual baroclinic Rossby waves are calculated by (6.36) and (6.37). The particle moves along a straight line perpendicular to the wavenumber vector (Fig. 8.1). The amplitudes of horizontal velocity, \((u_a^2 + v_a^2)^{1/2}\), and the corresponding particle excursion, \((u_a^2 + v_a^2)^{1/2}/\omega\), at the sea surface are shown in Figs. 8.8 and 8.9, respectively. The vertical distribution of these fields can be inferred from Fig. 6.1 because, according to (6.36) and (6.37), the horizontal velocity components and pressure have the same vertical profile. The wave-induced particle speed is up to one order of magnitude larger than the phase speed of the wave. In spite of the large ratio of the particle velocity to phase speed, the waves are still essentially linear, because the only non-negligible nonlinear terms (horizontal advections) associated with
Fig. 8.6 Amplitudes of the vertical velocity (in $10^{-5}$ cm/sec) associated with the analyzed annual baroclinic Rossby waves at 300 m

Fig. 8.7 Amplitudes of the vertical displacements (in meters) associated with the analyzed annual baroclinic Rossby waves at 300 m
Fig. 8.8 Amplitudes of the horizontal particle velocity (in cm/sec) associated with the analyzed annual baroclinic Rossby waves at the sea surface.

Fig. 8.9 Amplitudes of the horizontal excursion (in kilometers) associated with the analyzed annual baroclinic Rossby waves at the sea surface.
the wave motion cancel each other, as was shown in Chapter 6.

In this chapter we have shown that the wave-like annual fluctuations of temperature in the area $30^\circ N - 40^\circ N$, $160^\circ E - 130^\circ W$ consist of first order baroclinic Rossby waves travelling NW with wave lengths of about 300 km. The amplitudes of temperature, pressure, sea level elevation, velocity, and the displacement associated with the analyzed annual baroclinic Rossby waves are shown. From these results we learn that the intensities of the analyzed annual Rossby waves in the western part (west of the Emperor Sea Mount Chain) are much stronger than those in the eastern part.
IX. PROPAGATION OF TEMPERATURE DISTURBANCES ALONG THE SUBARCTIC AND SUBTROPICAL FRONTS

In this chapter we discuss the propagation of the wave-like part of the temperature fluctuations at special locations: the Subarctic and the Subtropical Fronts in the North Pacific. The Subarctic and the Subtropical Fronts are approximately located along about 42°N and 32°N, respectively, and are characterized by sharp gradients of density, temperature, and salinity (Roden, 1975).

Figs. 9.1 and 9.2 show the contours of the wave-like part of temperature fluctuations at 300 m, in a plane of time and longitude, along 42°N and 32°N, respectively. These figures show zonal propagation of the temperature disturbances at those latitudes. At the Subarctic Front along 42°N, especially in the region from 170°W to 145°W, the temperature disturbance propagates westward during the spring and eastward during the fall with a typical speed of about 12 cm/sec (Fig. 9.1). At the Subtropical Front along 32°N, in particular in the region from 180 to 150°W, the temperature disturbance propagates westward during the spring and eastward during the fall and winter with a typical speed of about 8 cm/sec (Fig. 9.2). In Figs. 9.1 and 9.2 we show the temperature disturbances at 300 m, as an example, along 42°N and 32°N. Similar patterns are observed at other depths (e.g., 150, 200, 250, 400 m)
Fig. 9.1 Contours of the wave-like part of the temperature fluctuations (in °C) at 300 m in the plane of time and longitude along 42°N.
Fig. 9.2 The same as in Fig. 9.1, however, along 32°N.
along those oceanic fronts. However, we cannot observe similar propagation patterns in the area of the TRANSPAC data at latitudes other than 32°N and 42°N.

These observed "unusual" propagations of the temperature disturbances along the oceanic fronts can be understood from the vorticity equation with mean shear flow, (6.30),

\[
(\overline{u} - c) \left( \frac{d}{dz} \left( \frac{f^2}{N^2} \frac{d\tilde{p}}{dz} \right) - (\kappa^2 + n^2) \tilde{p} \right) + Q_y \tilde{p} = 0, \quad (9.1)
\]

where \( c = \omega / \kappa \), and

\[
Q_y = \beta - \frac{\partial^2 \overline{u}}{\partial y^2} - \frac{3}{\partial z} \left( \frac{f^2}{N^2} \frac{\partial \overline{u}}{\partial z} \right), \quad (9.2)
\]

which can be regarded as an "effective-\( \beta \)" for the vorticity balance in an ocean with mean shear flow. In a region far away from the oceanic fronts, typical orders of magnitude for each term of \( Q_y \) are:

\[
\beta \approx 2 \cdot 10^{-13} \text{ cm}^{-1} \text{ sec}^{-1}
\]

\[
\frac{\partial^2 \overline{u}}{\partial y^2} \approx 10^{-16} \text{ cm}^{-1} \text{ sec}^{-1} \quad (9.3)
\]

\[
\frac{3}{\partial z} \left( \frac{f^2}{N^2} \frac{\partial \overline{u}}{\partial z} \right) \approx 10^{-14} \sim 10^{-13} \text{ cm}^{-1} \text{ sec}^{-1}.
\]

Hence we can neglect the term \( \frac{\partial^2 \overline{u}}{\partial y^2} \) compared to the two
other terms. However, the term $\frac{\partial}{\partial z} \left( \frac{f^2}{N^2} \frac{\partial u}{\partial z} \right)$ gives a "moderate" modification to the planetary $\beta$-term, and we expect that the westward propagation of the disturbances will have different speeds due to the mean shear flow. The wave-like part of the temperature fluctuations corresponding to this situation is analyzed in previous chapters.

At the oceanic front, however, the magnitudes of the meridional and the vertical shear terms are much larger than those in a region far away from the front. At the Subarctic and the Subtropical Fronts, using an estimate for the temperature gradient of $5^\circ$C and for the current shear of 40 cm/sec per 60 km (Roden, 1975), we obtain the following orders of magnitudes:

\[
\frac{\partial^2 u}{\partial y^2} = 10^{-12} \text{ cm}^{-1} \text{ sec}^{-1} \tag{9.4}
\]

\[
\frac{\partial}{\partial z} \left( \frac{f^2}{N^2} \frac{\partial u}{\partial z} \right) = - \frac{\partial}{\partial z} \left( \frac{f \alpha}{N^2} \frac{\partial T}{\partial y} \right) = 10^{-12} \text{ cm}^{-1} \text{ sec}^{-1},
\]

where $\alpha$ is the thermal expansion coefficient. These estimates indicate that the meridional and the vertical shear terms are up to one order of magnitude larger than the planetary $\beta$-term, and therefore $Q_Y$ at the oceanic fronts is determined mainly by the mean shear flow. Depending on the signs of the shear terms, $Q_Y$ can be either positive or negative, and consequently the temperature disturbances can propagate either westward or
eastward. The seasonal migration of the fronts or the
seasonal intensification of the shear flows at the fronts
can change the direction of propagation.

An approximate order of the phase speed can be
estimated by

\[ c_p = \frac{-Q_y}{(\kappa^2 + \eta^2) + \frac{f^2}{gh}}, \]

(9.5)

where \( h \) is the equivalent depth for the baroclinic mode
(h is about 1 m for the first order baroclinic mode).
When \( Q_y \), which is up to one order of magnitude larger than
\( \beta \), changes its sign seasonally at fixed latitudes along
32°N or 42°N, the temperature disturbance may change its
direction of propagation accordingly with a typical propa-
gation speed of 10 cm/sec, which is about 10 times larger
than the phase speed of the analyzed Rossby waves in a
region away from the fronts (Chapter 8). These expected
situations are actually observed along the latitudes 42°N
(Subarctic Front) and 32°N (Subtropical Front).
X. EMPIRICAL ORTHOGONAL FUNCTION ANALYSIS OF THE LARGE-SCALE TEMPERATURE FLUCTUATIONS

As mentioned above, the temperature fluctuations are composed of large-scale fluctuations with space scales comparable to the size of the basin and smaller scale fluctuations with length scales of a few hundred kilometers. We call them the quasi homogeneous part and the wave-like part, respectively. The wave-like part of the temperature fluctuations was analyzed in the previous chapters. In this chapter the quasi homogeneous part of the temperature fluctuations is investigated by means of an empirical orthogonal function analysis (eigenfunction analysis), which is particularly useful in representing both the temporal and the spatial characteristics simultaneously. Previous applications of the empirical orthogonal function analysis (e.g., Weare et al, 1976; Davis, 1976) were limited to the temperature fluctuations at the sea surface. We analyze the quasi homogeneous part of the temperature fluctuations in the interior of the ocean as well as at the sea surface by means of the empirical orthogonal function analysis. First, we discuss the underlying idea of the empirical orthogonal function analysis in order to make our results more easily understood.

Let \( f(x,t) \) be time series of \( m \) space dimensions.
In other words, $f(x,t)$ represents a collection of all time series at $m$ locations. The length of an individual time series at each location is taken to be $n \Delta t$, where $\Delta t$ is the sampling interval. In general, we can express $f(x,t)$ by

$$f(x,t) = \sum_{i=1}^{m} e_i(x) c_i(t),$$  \hspace{1cm} (10.1)$$

where $e_i(x)$ is the $i$-th eigenfunction that satisfies the orthonormality condition

$$e_i(x) \cdot e_j(x) = \delta_{ij},$$  \hspace{1cm} (10.2)$$

and $c_i(t)$ is the time-dependent coefficient, which is also orthogonal (see (10.9)). That is, the multi-dimensional time series $f(x,t)$ is represented as a superposition of spatially and temporally orthogonal functions. In matrix form we can write

$$F = E \, C$$  \hspace{1cm} (10.3)$$

$$E' \, E = I,$$

where $F$ is an $m$ by $n$ matrix that represents the multi-dimensional time series, $E$ is a space-dependent $m$ by $m$ matrix, the $i$-th column of which represents the $i$-th eigenvector, $C$ is an $m$ by $n$ matrix that represents the temporal characteristics, $I$ is the unit matrix, and prime
denotes a transposed matrix. The eigenmatrix $E$ can be
found as follows. An $m$-component unit vector $e$ that has
the highest resemblance to $F$ can be found by maximizing

$$
\frac{1}{n} \frac{(e'F)^2}{e'e} = \frac{e'R e}{e'e},
$$

(10.4)

where

$$
R = \frac{1}{n} F'F
$$

(10.5)

is the $m$ by $m$ covariance matrix of the data (Kutzbach,
1967). The unit vector $e$ that maximizes (10.4) can be
found by solving the eigenvalue problem,

$$
R e = \lambda e
$$

(10.6)

$$
e'e = 1
$$

(Mathews and Walker, 1964, p. 320). There are $m$ eigen
values $\lambda_1, \lambda_2, \ldots, \lambda_m$, and associated eigenvectors $e_1$, 
$e_2, \ldots, e_m$. Since the covariance matrix $R$ is symmetric,
the eigenvalues are real. It is convenient to rearrange
the eigenvalues in a descending order, $\lambda_1 > \lambda_2 > \ldots > \lambda_m$, 
and also the eigenfunctions accordingly. Considering all
eigenvalues and eigenvectors simultaneously, we can write
(10.6) as

$$
R E = E L
$$

(10.7)
\[ E'E = I, \]

where \( I \) is an \( m \) by \( m \) diagonal matrix in which the diagonal elements are eigenvalues, and \( E \) is an \( m \) by \( m \) matrix in which the \( i \)-th column is the \( i \)-th eigenvector \( e_i \). From a given multi-dimensional time series \( F \) we can calculate \( E \) and \( L \) by a standard numerical method for eigenvalue problem (10.7). The time-dependent coefficient matrix \( C \) can easily be calculated by

\[ C = E'F, \]  

(10.8)

and the time series \( F \) can be decomposed into orthogonal functions by \( F = E'C \). It can be shown, using (10.5), (10.7) and (10.8), that

\[ C'C = n L. \]  

(10.9)

Hence, not only the eigenmatrix \( E \) but also the time-dependent matrix \( C \) is orthogonal. Furthermore, (10.9) shows that the eigenvalues, \( \lambda \)'s, are real and positive. Since the variance of the \( i \)-th eigenfunction is

\[ \text{var}(c_i(t)) = \frac{1}{n} \sum_{t=1}^{n} c_i^2(t) = \lambda_i, \]  

(10.10)

the fraction of the total variance explained by the \( i \)-th eigenfunction is
The time-dependent coefficient $c_i(t)$ represents the temporal characteristics associated with the $i$-th eigenfunction, but its magnitude is not the same as the physical magnitude. The actual RMS amplitude $A_i$ of the time series associated with the $i$-th eigenfunction at location $x$ can be calculated by

$$A_i^2(x) = \frac{1}{n} \sum_{t=1}^{n} \{ e_i(x) \cdot c_i(t) \}^2$$

$$= e_i^2(x) \cdot \text{var}\{c_i(t)\}$$

$$= e_i^2(x) \cdot \lambda_i.$$

(10.12)

An interpretation of the eigenfunctions is simple and easy. The time series, $f_i$, represented by the $i$-th eigenfunction is

$$f_i(x,t) = e_i(x) \cdot c_i(t),$$

(10.13)

where $e_i(x)$ and $c_i(t)$ represent the spatial and temporal characteristics, respectively, of the $i$-th eigenfunction. One of the main advantages of the eigenfunction analysis is its ability to describe both the temporal and spatial characteristics of a data set at many locations by

$$\text{(Frac)}_i = \lambda_i / \sum_{j=1}^{m} \lambda_j.$$
considering only a few eigenfunctions.

In what follows we present the results of the eigenfunction analysis of the quasi homogeneous part of the temperature fluctuations at the surface and in the subsurface layer (150 - 400 m). The quasi homogeneous temperature fluctuations in 35 subareas (10 by 4 degree rectangles) are used for eigenfunction analysis at each depth.

The first eigenfunction of the quasi homogeneous part of the sea surface temperature fluctuations, which explains 96.5% of the total variance, is shown in Fig. 10.1. The corresponding time coefficient and the RMS amplitude are shown in Figs. 10.2 and 10.3, respectively. The time-dependent coefficient shows that the quasi homogeneous part of the sea surface temperature fluctuations is mainly a seasonal variation (Fig. 10.2). The space-dependent eigenfunction shown in Fig. 10.1 does not change its sign; this indicates that the seasonal temperature fluctuation at the sea surface is almost in phase everywhere in the North Pacific Current area. The corresponding RMS amplitude of the temperature fluctuation is about 3.5°C in the western part and about 2.5°C in the eastern part (Fig. 10.3). These results agree quantitatively with the seasonal variation of the sea surface temperature analyzed by means of the harmonic method (Wyrtki, 1965). Note that the RMS amplitude for a sinusoidal fluctuation is only 0.707 times
Fig. 10.1 Spatial characteristics, $e_1(x)$, of the first eigenfunction for the quasi homogeneous part of the temperature fluctuations at the sea surface.

Fig. 10.2 Time-dependent coefficient, $c_1(t)$, associated with the first eigenfunction for the quasi homogeneous part of the temperature fluctuations at the sea surface.

Fig. 10.3 RMS amplitudes (in °C) associated with the first eigenfunction of the quasi homogeneous part of the temperature fluctuations at the sea surface.
of the "sinusoidal" amplitude.

The seasonal variation of the sea water temperatures associated with the seasonal change of insolation decreases in magnitude with increasing depth, and is practically restricted to the upper 100 to 200 m of the ocean (Chapter 4). Hence an analysis of the quasi homogeneous part of the temperature fluctuations in a layer deeper than 150 m will show non-seasonal large-scale features of the temperature fluctuations. The eigenfunctions of the quasi homogeneous temperature fluctuations in the subsurface layer (150 to 400 m) are shown in Fig. 10.4. The percentages of the variance explained by the first eigenfunctions at 150, 200, 250, 300 and 400 m are 66.1, 61.0, 53.0, 45.1 and 38.1 %, respectively. The corresponding time coefficients and the RMS amplitudes are shown in Figs. 10.5 and 10.6. The second and higher order eigenfunctions each explain only 13 % or less of the total variances, and our discussion is limited to the first eigenfunctions, which explain about 53 % of the total variance. The spatial characteristics of the first eigenfunction at each depth from 150 to 400 m are remarkably similar (Fig. 10.4). Moreover, the temporal characteristics are also similar (Fig. 10.5). The limited time series of only 40 months is not sufficient to permit any definite conclusion on time scales. However, a relatively higher temperature in early 1975 and a relatively lower temperature in early
Fig. 10.4 Spatial characteristics, $e_1(x)$, of the first eigenfunctions for the quasi homogeneous temperature fluctuations at 150, 200, 250, 300 and 400 m.
Fig. 10.5  Time-dependent coefficients, $c_1(t)$, associated with the first eigenfunctions for the quasi homogeneous temperature fluctuations at 150, 200, 250, 300 and 400 m.
Fig. 10.6 RMS amplitudes (in °C) associated with the first eigenfunctions of the quasi homogeneous temperature fluctuations at 150, 200, 250, 300 and 400 m.
1978 suggest a probable time scale of 5 to 7 years, which is in agreement with the analysis of the temperature anomalies at Ocean Weather Stations N and P (Chapter 5). The spatial characteristics of the quasi homogeneous temperature fluctuations in the subsurface layer (Fig. 10.4) are quite similar to the corresponding characteristics of the sea surface temperature anomalies analyzed by Davis (1976, Fig. 4) and Weare et al (1976, Fig. 7). The RMS amplitudes associated with the first eigenfunctions decrease slowly with increasing depth (Fig. 10.6). The RMS amplitudes, horizontally averaged over the whole TRANSPAC data area, at depths 150, 200, 250, 300 and 400 m are 0.26, 0.19, 0.15, 0.12 and 0.10°C, respectively. Horizontally the intensity of this low-frequency and large-scale temperature anomaly field is not uniform. Maximum variability of temperature at subsurface depths is found at about 40°N, 175°E, where the Kuroshio and the Oyashio extensions meet (Roden, 1975). It seems that the large-scale temperature anomaly field in the subsurface layer is closely related to an interannual variability of the large-scale oceanic circulation, which is a manifestation of the large-scale air-sea interaction (Kort, 1970).

From an empirical orthogonal function analysis of the quasi homogeneous temperature fluctuations in the subsurface layer discussed above, we can conclude that the temperature anomaly field, which has space scales
comparable to the size of the basin and time scales of perhaps 5 to 7 years, penetrates more than 400 m, and its intensity is maximal in the western part of the North Pacific Current area where the Kuroshio and the Oyashio extensions meet. This temperature anomaly field may be closely related to the global air-sea interaction.
XI. HEAT STORAGE AND ITS CHANGES
IN THE UPPER 400 METERS

In foregoing chapters we discussed the temporal and spatial characteristics of the temperature fluctuations in the Central North Pacific. In particular, we presented the vertical distributions of the seasonal variation of the temperature, the temperature anomaly field, and the temperature fluctuations associated with the Rossby waves. In this chapter we discuss the vertically integrated temperature, or the heat content, in the upper 400 m. First we present the horizontal distributions of the heat content and the rate of heat change. Then we discuss various physical mechanisms responsible for the quasi homogeneous and wave-like parts of the heat content fluctuations. The e-folding depths calculated from the heat content fluctuations and the sea surface temperature fluctuations are compared to our previous estimates obtained for the seasonal variation. After we present the time series of the heat content and its rate of change in ten-degree-squares, we calculate the horizontal distributions of the mean monthly rate of heat change. These are compared to the heat exchange at the sea surface (Wyrtki, 1966). We compare the rates of heat change in the same month of different years. Finally, we estimate the relative importance of the seasonal and the non-seasonal parts
of the heat content fluctuations.

The heat content, $H_t$, in the upper 400 m is calculated by

$$H_t(x,y,t) = \rho_o c_p \int_{-400m}^{0} T(x,y,z,t) \, dz,$$  \hspace{1cm} (11.1)

where $\rho_o$ is the density, $c_p$ the specific heat, and $T$ the temperature (in °C). The horizontal distribution of the average heat content in our area, shown in Fig. 11.1, is similar to the horizontal distribution of the mean dynamic topography (Wyrtki, 1975). The RMS amplitudes of the heat content fluctuation in our area are shown in Fig. 11.2. The RMS amplitudes in the western part of the North Pacific Current area are much larger than those in the eastern part, and the main stream of the North Pacific Current (about $35^\circ$N to $45^\circ$N) has larger amplitudes of fluctuations than the border area (Fig. 11.2).

In previous chapters we considered the large scale and the smaller scale fluctuations of temperature separately. Similarly, we separate the large scale fluctuations of the heat content and the smaller ones by our space averaging technique described in Chapter 2. The RMS amplitudes of the large scale, quasi homogeneous part, and the smaller scale, wave-like part, of the heat content fluctuations in the upper 400 m are shown in Figs. 11.3 and 11.4, respectively. The RMS amplitudes of both the quasi
Fig. 11.1 Average heat content (in Kcal cm\(^{-2}\)) in the upper 400 m.

Fig. 11.2 RMS amplitudes of heat content fluctuations (in Kcal cm\(^{-2}\)) in the upper 400 m.
Fig. 11.3 RMS amplitudes (in Kcal cm$^{-2}$) of the quasi homogeneous part of the heat content fluctuations in the upper 400 m.

Fig. 11.4 RMS amplitudes (in Kcal cm$^{-2}$) of the wave-like part of the heat content fluctuations in the upper 400 m.
homogeneous and the wave-like parts of fluctuations in the western part of the North Pacific Current area are larger than those in the eastern part. The RMS amplitude associated with the wave-like part of the heat content fluctuation in the upper 400 m is about one third of that of the quasi homogeneous part.

The amplitudes of the heat content fluctuations in the upper 400 m associated with the analyzed annual baroclinic Rossby waves (Chapter 8) can be estimated from Figs. 6.3 and 8.4. The approximate amplitudes of the heat content fluctuation associated with the analyzed annual Rossby waves are about 10 Kcal/cm² in the western part and about 3 Kcal/cm² in the eastern part of the analyzed region (30° - 40°N), and their horizontal distribution reasonably agrees with that of the wave-like part of the heat content fluctuations shown in Fig. 11.4. This suggests that the annual Rossby wave motion is one of the main mechanisms responsible for the wave-like part of the heat content fluctuations.

We estimate the e-folding depth, or the depth of penetration, of the temperature fluctuations with the simplifying assumption that the amplitude of the temperature fluctuation decreases exponentially with depth. In this case the heat content in the upper 400 m can be expressed by
where \( z_e \) is the e-folding depth, \( \overline{T} \) the mean temperature, \( T_0 \) the temperature fluctuation at the sea surface, and \( \overline{H}_t \) the mean heat content in the upper 400 m. Figs. 11.5 shows the e-folding depth calculated by

\[
\begin{align*}
  z_e &= \frac{\text{Amp}(H_t)}{\rho_o c_p \text{Amp}(T_0)}, \\
  &= \frac{\text{Amp}(H_t)}{\rho_o c_p \text{Amp}(T_0)},
\end{align*}
\]

where \( \text{Amp}(H_t) \) is the RMS amplitude of the heat content fluctuation in the upper 400 m (Fig. 11.4) and \( \text{Amp}(T_0) \) is the RMS amplitude of the sea surface temperature fluctuation (Fig. 3.1). The e-folding depths calculated by (11.3) are shown in Fig. 11.5. A comparison of Figs. 11.5 and 4.6 shows that the e-folding depths calculated from the heat content fluctuations are slightly larger than our previous estimates obtained from the vertical distribution of the amplitudes of the temperature fluctuations at 30, 60 and 90 m. These differences are probably due to the temperature anomalies, which penetrate more than 400 m, and the Rossby wave fields, which involve the temperature
Fig. 11.5 e-folding depths (in meters) calculated by (11.3).
fluctuations up to about 2000 m (see Fig. 6.3).

Figs. 11.6 to 11.9 show the time series of the heat content fluctuation (full lines) and the rate of heat change $\dot{H}_t/\dot{t}$ (dotted lines) in the upper 400 m in 8 ten-degree-squares. The position of the center of each ten-degree square is shown in each figure. The rate of heat change is calculated by the central finite difference method. The Figs. 11.6 to 11.9 show that the heat content fluctuation and the rate of heat change are dominated by a seasonal cycle; however, non-seasonal fluctuations are also appreciable. During the period 1975 to 1978 the heat content in the upper 400 m of the Central North Pacific was generally decreasing, especially in the western and central parts of the North Pacific Current area.

In what follows we consider the seasonal variation of the heat content fluctuation and its rate of change. The relative importance of the non-seasonal fluctuations will be discussed later. Figs. 11.10 to 11.12 show the horizontal distributions of the rate of heat change for every month from January to December. These are calculated by an average over the same month (e.g., every January) from the time series of 40 months. During April to September the heat content generally increases, whereas it decreases during November to February. The western part of the North Pacific Current area near the Kuroshio extension has larger amplitude in the monthly rate of heat change than the
Fig. 11.6  The heat content fluctuation (full lines) and the rate of heat change (dotted lines) in ten-degree-squares centered at (45°N, 165°E) and (35°N, 165°E).
Fig. 11.7 The same as in Fig. 11.6, however, in ten-degree-squares centered at (45°N, 175°W) and (35°N, 175°W).
Fig. 11.8 The same as in Fig. 11.6, however, in ten-degrees-squares centered at (45°N, 155°W) and (35°N, 155°W).
Fig. 11.9 The same as in Fig. 11.6, however, in ten-degree-squares centered at (45°N, 135°W) and (35°N, 135°W).
Fig. 11.10 The rates of heat change (in cal cm$^{-2}$ day$^{-1}$) in the upper 400 m for January, February, March and April.
Fig. 11.11 The same as in Fig. 11.10, however, for May, June, July and August.
Fig. 11.12 The same as in Fig. 11.10, however, for September, October, November and December.
eastern part. The geographical distribution of the intensity of the monthly rate of heat change agrees reasonably well with that of the heat exchange at the sea surface calculated from the climatic data of ships' weather reports (Wyrtki, 1966). However, the amplitude of the rate of heat change in the upper 400 m is larger than the heat exchange at the sea surface. This difference is probably due to the advection and the diffusion of heat in the upper 400 m (Bathen, 1971).

The horizontal distribution of the rate of heat change in June and December of different years, 1975, 1976 and 1977, are shown in Figs. 11.13 and 11.14. In order to measure an overall "resemblance" of the horizontal distribution of the rate of heat change in the same month of different years (e.g., January 1976 versus January 1977), we introduce a "map correlation", or a "spatial correlation", for the rate of heat change defined by

\[ M = 2 \sum_{i,j} p_{ij} q_{ij} / \sum_{i,j} (p_{ij}^2 + q_{ij}^2), \]  

(11.4)

where \( p_{ij} \) is the rate of heat change of a given month at location \( (x_i, y_j) \) and \( q_{ij} \) is the corresponding one for the same month in another year. If the rate of heat change everywhere is only a seasonal one, then we will have \( M = 1 \). On the other hand, if there is no seasonal repetition at all, we will have \( M = 0 \). The map correlations of
Fig. 11.13 The rates of heat change (in cal cm\(^{-2}\) day\(^{-1}\)) for June in three different years, 1975, 1976 and 1977.
Fig. 11.14 The rates of heat change (in cal cm\(^{-2}\) day\(^{-1}\)) for December in three different years, 1975, 1976 and 1977.
<table>
<thead>
<tr>
<th>Month</th>
<th>Mean/75</th>
<th>Mean/76</th>
<th>Mean/77</th>
<th>75/76</th>
<th>76/77</th>
<th>75/77</th>
<th>Avg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>0.59</td>
<td>0.92</td>
<td>0.93</td>
<td>0.50</td>
<td>0.80</td>
<td>0.51</td>
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<tr>
<td>Feb</td>
<td>0.81</td>
<td>0.76</td>
<td>0.88</td>
<td>0.80</td>
<td>0.62</td>
<td>0.72</td>
<td>0.76</td>
</tr>
<tr>
<td>Mar</td>
<td>0.73</td>
<td>0.83</td>
<td>0.56</td>
<td>0.70</td>
<td>0.29</td>
<td>0.20</td>
<td>0.55</td>
</tr>
<tr>
<td>Apr</td>
<td>0.81</td>
<td>0.53</td>
<td>0.80</td>
<td>0.54</td>
<td>0.38</td>
<td>0.57</td>
<td>0.60</td>
</tr>
<tr>
<td>May</td>
<td>0.80</td>
<td>0.87</td>
<td>0.77</td>
<td>0.74</td>
<td>0.64</td>
<td>0.50</td>
<td>0.72</td>
</tr>
<tr>
<td>Jun</td>
<td>0.72</td>
<td>0.89</td>
<td>0.82</td>
<td>0.56</td>
<td>0.70</td>
<td>0.36</td>
<td>0.67</td>
</tr>
<tr>
<td>Jul</td>
<td>0.86</td>
<td>0.92</td>
<td>0.90</td>
<td>0.72</td>
<td>0.84</td>
<td>0.67</td>
<td>0.82</td>
</tr>
<tr>
<td>Aug</td>
<td>0.92</td>
<td>0.86</td>
<td>0.78</td>
<td>0.76</td>
<td>0.48</td>
<td>0.65</td>
<td>0.74</td>
</tr>
<tr>
<td>Sep</td>
<td>0.68</td>
<td>0.83</td>
<td>0.83</td>
<td>0.40</td>
<td>0.66</td>
<td>0.38</td>
<td>0.63</td>
</tr>
<tr>
<td>Oct</td>
<td>0.68</td>
<td>0.75</td>
<td>0.62</td>
<td>0.43</td>
<td>0.27</td>
<td>0.10</td>
<td>0.48</td>
</tr>
<tr>
<td>Nov</td>
<td>0.74</td>
<td>0.82</td>
<td>0.80</td>
<td>0.61</td>
<td>0.72</td>
<td>0.53</td>
<td>0.71</td>
</tr>
<tr>
<td>Dec</td>
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<td>0.84</td>
<td>0.79</td>
<td>0.77</td>
<td>0.85</td>
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<tr>
<td>Avg</td>
<td>0.77</td>
<td>0.83</td>
<td>0.80</td>
<td>0.63</td>
<td>0.60</td>
<td>0.59</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Tab. 11.1 Map correlations, (11.4), between two horizontal maps of the rate of heat change in the upper 400 m in the same month of different years.
The relative importance of the seasonal and the non-seasonal fluctuations in the rate of heat change in the upper 400 m is estimated by comparing the RMS amplitudes for the seasonal and non-seasonal fluctuations at each location. We separate the seasonal and the non-seasonal fluctuations as follows. The seasonal fluctuation is calculated by an average over the same month (e.g., over all Januaries) from the time series of 40 months. The deviation of the actual rate of heat change from the seasonal value of that month is regarded as the non-seasonal fluctuation at that specific month. The RMS amplitudes of the seasonal and the non-seasonal parts of the heat content fluctuations in the upper 400 m are shown in Figs. 11.15 and 11.16, respectively. The amplitude of the non-seasonal heat content fluctuation is, on the average, 0.63 times that of seasonal one. The seasonal and the non-seasonal parts of the rate of heat change are calculated by the same method, and their RMS amplitudes are shown in Figs. 11.17 and 11.18. The amplitude of the non-seasonal rate of heat change is, on the average, 0.60 times that of the seasonal one. These estimates show that the non-seasonal fluctuation of heat content and its rate of change
Fig. 11.15  RMS amplitudes (in Kcal cm$^{-2}$) of the seasonal fluctuations of heat content in the upper 400 m.

Fig. 11.16  RMS amplitudes (in Kcal cm$^{-2}$) of the non-seasonal fluctuations of heat content in the upper 400 m.
Fig. 11.17 RMS amplitudes (in cal cm\(^{-2}\) day\(^{-1}\)) of the seasonal fluctuations of the rate of heat change in the upper 400 m.

Fig. 11.18 RMS amplitudes (in cal cm\(^{-2}\) day\(^{-1}\)) of the non-seasonal fluctuations of the rate of heat change in the upper 400 m.
in the upper 400 m ocean are not negligible compared to the seasonal fluctuation. On the whole, about 60% of the heat content fluctuations in the upper 400 m ocean is accounted for by the seasonal variation.
XII. DISCUSSION AND CONCLUSIONS

We have discussed the low frequency temperature fluctuations, with time scales of months to years, in the upper 400 m of the Central North Pacific. In particular, we have discussed in detail the seasonal variations of the sea water temperature in the surface layer, the basin-wide, large-scale temperature anomalies with time scales of a few years, the meso-scale temperature fluctuations associated with the annual baroclinic Rossby waves, and the fluctuations of heat content in the upper 400 m. The low frequency temperature fluctuations consist of large scale, quasi homogeneous, fluctuations with space scales comparable to the size of the basin and the smaller scale, wave-like, fluctuations with length scales of a few hundred kilometers.

The temperature fluctuations in the top 100 m are dominated by the large-scale seasonal variations. The sea surface temperature is maximal in August and September, and the highest temperatures at 40 and 80 m occur about 1 and 2 months later than those at the sea surface. The amplitude of the seasonal temperature variation at the sea surface is about 3 to 5°C. The amplitude of the seasonal temperature fluctuations decreases with increasing depth; a typical e-folding depth is about 50 m. Eddy diffusion of heat plays an important role in the vertical penetration
of heat from the surface down to about 100 m. The seasonal variation of heat storage in the upper 400 m of the Central North Pacific is about 15 to 30 Kcal/cm², which is one order of magnitude larger than the atmospheric heat storage.

The large scale temperature anomaly field in the upper 100 m is quite different from that in the subsurface layer below 150 m. Typical time scales of the temperature anomaly field in the surface layer are about 2 to 3 years, whereas those in the subsurface layer are about 4 to 7 years. These large scale and low frequency temperature anomalies penetrate much deeper than the seasonal variations. The e-folding depth for these low frequency temperature anomalies in the subsurface layer is a few hundred meters while that for the seasonal variation of temperature in the surface layer is only about 50 m. The intensity of this low frequency temperature anomaly field is maximal in the western part of the North Pacific Current area where the Kuroshio and the Oyashio extensions meet.

The annual baroclinic Rossby waves in the North Pacific Current area propagate in a NW direction with phase speeds of about 1 cm/sec and wave lengths of about 300 km. The amplitude of the temperature fluctuations in the sea surface associated with the annual baroclinic Rossby waves is about 0.2°C in the western part and 0.1°C in the eastern part of the North Pacific Current area. In the upper
layer, the temperature fluctuations associated with the Rossby waves are basically meso-scale perturbations superimposed onto the large scale temperature fluctuations. Their contributions to the temperature fluctuations at the sea surface are minor compared to those of the large seasonal fluctuations. In the deeper layer of the ocean, however, the Rossby waves give a major contribution to the temperature fluctuations, because the temperature fluctuations associated with the baroclinic Rossby waves affect the sea water temperature as deep as 2000 m while those of the seasonal variations are restricted to an upper layer of about 100 m.

The particle motions associated with the Rossby waves are in a SW-NE direction, which is perpendicular to the propagation direction of the wave phase. The particle speed associated with the analyzed annual baroclinic Rossby waves at the sea surface is about 2 to 7 cm/sec, and the horizontal range of excursion is about 200 to 700 km. Since the particle motions associated with the Rossby waves are in a region with a meridional gradient of the mean temperature (about 8°C per 1000 km at the sea surface), they can make a significant contribution to the poleward transport of heat. The northward component of flow associated with the Rossby waves carries warmer water to a cooler region and the southward component carries cooler water to a warmer region. Both southward and northward
components of the flow contribute to the net poleward transport of heat. A quantitative estimate of the poleward transport of heat by Rossby waves requires the knowledge of the change of the sea water temperature in the surface mixed layer as the water particle moves to different latitudes. The major contribution to the poleward heat transport will be made in the western part of the North Pacific Current area, where both the wave-associated particle velocity as well as the mean meridional gradient of temperature are larger than those in the eastern part.

The seasonal characteristics as well as the spatial distributions of the rate of heat change in the upper 400 m agree reasonably well with the heat exchange at the sea surface (Wyrtki, 1966). However, the rate of the oceanic heat change is usually larger than the heat exchange at the sea surface. The amplitudes of the non-seasonal fluctuations of the rate of heat change are about 60% of the seasonal ones. The relative contribution of the wave-like temperature fluctuations to the heat storage is about one third of that due to large-scale temperature fluctuations.

Our analysis shows that the low frequency temperature fluctuations in the western part of the Central North Pacific are stronger than those in the eastern part in both the quasi homogeneous and the wave-like parts of the temperature fluctuations. Physical mechanisms responsible for this westward intensification of the low frequency
temperature fluctuations are not well understood yet.


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