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Nonlinear forces and response of floating platforms in regular and random waves

Chitrapu, Srinivasamurthy Achuta, Ph.D.

University of Hawaii, 1992
NONLINEAR FORCES AND RESPONSE OF FLOATING PLATFORMS
IN REGULAR AND RANDOM WAVES

A DISSERTATION SUBMITTED TO THE GRADUATE DIVISION OF
THE UNIVERSITY OF HAWAII IN PARTIAL FULFILLMENT
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DOCTOR OF PHILOSOPHY
IN OCEAN ENGINEERING

DECEMBER 1992

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Abstract

Floating platforms in the ocean are subjected to mean and low-frequency drift forces, and high-frequency springing forces in addition to the first-order forces in the wave-frequency range. These forces, which are caused by both potential and viscous effects, can induce large amplitude, resonant response of platforms due to the near absence of damping at such frequencies.

In this study, a frequency-domain method based on the relative velocity formulation of Morison’s equation is presented to compute viscous drift forces and moments in regular and random waves. The method, which is applied to a semisubmersible and tension-leg platforms, indicates significant mean forces and moments in surge, pitch and yaw modes of the platform motion. The combined effect of waves and current, variable submergence of platform members and computation of forces in the displaced location of the platform appear to have a pronounced effect on the computed drift forces and moments. It is shown that the viscous drift forces are important in the long-period range and hence must be considered under design wave conditions.

A time-domain model which uses Morison’s equation method for force computations is developed to simulate platform motions. This model can include most of the nonlinearities such as the nonlinear drag force, effect of finite wave elevation, nonlinear restoration of the positioning system and nonlinearities in the equations of motion of the platform. The viscous drift forces and response obtained
from the frequency-domain method are compared with results from time-domain simulations. Good agreement has been found for the forces and response, both in regular and random waves. This frequency-domain method can be used to predict viscous drift forces and response in the preliminary design stage and for parametric studies due to its superior computational efficiency as compared to the time-domain simulations.

The effect of the nonlinear drag force, in inducing higher-harmonic forces and tether-tension response, has been studied using the time-domain simulation results together with power spectral methods. For the wave and current conditions used in this study, second- and higher-harmonic drag force and tether-tension response are observed in regular, bi-chromatic and random waves and current. Inclusion of current is shown to affect the nonlinear response of the platform.

Another theoretical model, based on the application of linear potential theory in the time domain, to simulate large amplitude nonlinear motions of platforms is also presented. The theory is based on the combination of potential and viscous flow effects in the time domain to determine forces acting on the platform. Hydrodynamic coefficients and wave excitation forces, obtained a priori from the linear, three-dimensional potential theory, are included in the nonlinear, large amplitude simulation model for platform motions. First-order memory effects are included through velocity based convolution integrals. The results obtained from this simulation model are compared with those obtained using Morison's equation model and the agreement is found to be good. It is believed that this method, due to its
ability to model both potential and viscous-flow effects accurately in a large amplitude motion simulation model, will give better predictions to the various nonlinear effects mentioned above.
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Chapter 1
Introduction

Floating platforms in the open ocean are subject to a wide variety of forces due to winds, waves and currents. These forces include mean and low-frequency drift forces and high-frequency springing forces, in addition to the first-order or primary wave forces. The first-order wave forces occur at the same frequencies as those present in the wave spectrum and they do not yield any mean force on the platform. The higher-order forces occur due to both potential and viscous effects, and are caused by several nonlinear effects in the fluid-body interaction problem. These include nonlinearities in the fluid forces, such as the quadratic drag force, system nonlinearities, such as the nonlinearities in the equations of motion, nonlinear effects of the positioning system and nonlinear effects to the free-surface-force integration.

The effect of these higher-order forces on offshore platforms varies depending on the type of platform and the nature of the mooring system used. There are two types of platforms which are commonly used in offshore industry, namely the semisubmersible platform and the tension-leg platform (TLP). Both platforms have a similar underwater structure which is made up of a number of cylindrical members constituting the columns, pontoons and braces of the platform. The main difference in the dynamic behavior of these two platforms comes from the mooring system used. A semisubmersible platform is moored by a catenary system and a TLP is moored
by a taut mooring system in which the tethers do not go slack (by design) at any
time. These platforms are usually designed such that their natural frequencies, in
various modes of platform motion, lie outside the frequency range of maximum wave
energy. Table 1.1 shows typical natural periods of these platforms.

Table 1.1 Typical natural periods of TLP and Semisubmersible (moored) platforms

<table>
<thead>
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<th>Mode of motion</th>
<th>TLP</th>
<th>Semisubmersible</th>
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<tr>
<td>Surge, Sway</td>
<td>70-120 s</td>
<td>very long</td>
</tr>
<tr>
<td>Heave</td>
<td>2 s</td>
<td>20-25 s</td>
</tr>
<tr>
<td>Pitch, Roll</td>
<td>2-4 s</td>
<td>20-30 s</td>
</tr>
<tr>
<td>Yaw</td>
<td>40-50 s</td>
<td>very long</td>
</tr>
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We can see from this table that surge, sway and yaw motions of a TLP and
the heave, pitch and roll motions of a semisubmersible can be excited by the low-
frequency drift forces, while the high-frequency springing forces can influence the
heave, roll and pitch motions of a TLP. In irregular waves, the interaction among
wave components with different frequencies gives rise to forces at the sum- and
difference-frequencies of the waves. The mean and low-frequency drift forces cause
large-amplitude resonant surge motions of a TLP. Due to the very high axial
stiffness of TLP tethers, the heave, roll and pitch motions are very small. However,
high-frequency springing forces can cause large-amplitude, high-frequency tether tension variations. This is an important design factor. The sub-harmonic roll response of a semisubmersible platform is also attributed to the effect of drift forces (see Lundgren and Berg, 1982).

1.1 **Review of previous work**

As mentioned before, both potential- and viscous-flow effects are important to cause mean and low-frequency drift forces. The potential drift force is important for ships and very large floating structures, cases in which diffraction effects dominate the total forces, especially at moderate and small wave periods. But for many semisubmersible platforms and some tension-leg platforms which do not always fall into this category, the nature of second-order forces and the relative importance of potential and viscous effects are not very clear. The potential-drift problem deals with the second-order effects of the flow of an inviscid fluid around a body, including the effect of second-order potential. This problem has been studied extensively and approximate methods to determine the mean and slowly-varying potential drift forces are now available. The potential drift forces are computed by the application of the principle of conservation of fluid momentum surrounding the body (see Maruo, 1960 and Newman, 1967) or by the direct integration of pressure on the instantaneous wetted surface of the body (Pinkster, 1979). Recent investigations are based on the later approach and include, among others, the works by Standing *et al.* (1981) and Matsui (1988). Pinkster (1980) gives the total second-order potential force as a sum
of several terms involving the product of first-order quantities. Matsui (1988) presented an "exact" second-order formulation for computing the slowly-varying potential drift force in irregular waves.

Viscous drift forces originate mainly due to the existence of the drag force over the instantaneous submerged length of platform members, drag force due to the presence of current, and due to wave-current interaction effects. Traditionally, these forces have been computed using the drag force term of Morison's equation (Morison et al., 1950) to determine the mean force and moment.

Most of the previous studies on viscous drift forces concentrated on analytical and/or experimental results for a single circular cylinder (see for instance Ferretti and Berta, 1980, Chakrabarti, 1984 and Hodges, 1985). Chakrabarti (1984) presented analytical expressions for viscous and potential drift forces on a vertical cylinder and compared the predictions with some experimental data. The relative importance of the viscous and potential drift contributions has also been discussed. Ferretti and Berta (1980) compared the viscous drift contribution to a vertically moored cylinder with potential drift computed by Newman's (1967) formulation. Comparisons between floating and fixed cylinders and the interaction between waves and current have also been studied in this paper. Lundgren et al. (1982) discussed various contributions to potential and viscous drift forces and damping. Pijfers and Brink (1977) presented a method to compute mean and slowly-varying drift forces on semisubmersible platforms. The effect of wind and current are also included in this study. Burns (1983) used the relative velocity model of Morison's equation
where the platform motion in surge is considered in determining the total relative velocity between the body and fluid. A method to obtain nonlinear viscous drift force transfer functions which can be used to determine the mean and slowly-varying surge drift forces in irregular seas has also been presented. Other studies of viscous drift forces on TLP (see for example, Botelho et al., 1984 and Finnigan et al., 1984) are also limited to the analysis of surge drift forces and motions. Ertekin and Chitrapu (1987), Chitrapu (1988), and Ertekin and Chitrapu (1990) computed the wave- and current-induced viscous drift forces and moments in all six degrees-of-freedom of a tension-leg platform. In this study, first-order motions of the platform in all six degrees-of-freedom, given by the frequency-domain motion transfer functions (see for example, Burke, 1969, and Paulling, 1970) are used to calculate the body velocity, and the resulting relative velocity is used to calculate the drift forces and moments. Chitrapu et al. (1992) computed the wave- and current-induced viscous drift forces in regular and irregular waves on a tension-leg platform. The frequency-domain results obtained were also compared with time-domain simulations in that study.

The evaluation of viscous drift forces in irregular waves is complicated due to the fact that the drift force is a nonlinear function of the wave height. From the results obtained in regular wave analysis, it appears that the viscous drift force, in the presence of current, is proportional to the third power of the wave amplitude as compared to the potential drift force which is proportional to the wave amplitude squared. Suitable methods have been developed to obtain potential drift forces in
irregular waves using the regular wave results (see Pinkster, 1979 and Standing et al., 1981) in conjunction with spectral methods. The alternative is to use a time-domain method in which all nonlinearities can approximately be included. However, frequency-domain methods for the computation of viscous drift forces in irregular waves are preferred in the preliminary design stage since they are computationally more efficient. The time-domain analysis is considered indispensable at the final design stage. For this reason, several investigations are being carried out to develop frequency-domain methods for calculating viscous drift forces in irregular waves. Olagnon et al. (1987) studied the nonlinear spectral computation of drag force on a fixed cylinder using the Volterra series expansion method. In this method expressions were derived for the spectral density of drag force and cylinder response. It may be noted that equivalent linearization methods, traditionally used for fixed platforms, do not yield satisfactory results for the prediction of sub-harmonic response of compliant structures since the sum and difference frequency effects are lost in this process. Spanos and Donley (1990) investigated the stochastic response of a TLP due to viscous drift force using a similar method and using an equivalent quadratization method for the drag force nonlinearity. Comparison of response statistics obtained from time-domain analysis was also made. However the drag force was computed only over the mean wetted surface of the columns. Li and Kareem (1990) proposed a method based on multi-variate Hermite polynomial expansion for the relative velocity drag force. This method of solution also
estimates the mean and slowly-varying viscous drift forces in irregular waves using a frequency-domain formulation.

The occurrence of high-frequency forces (at the sum- and higher-harmonic frequencies of the wave frequency) has been observed experimentally. de Boom et al. (1983) studied the second-harmonics of tether forces in regular waves. Herfjord and Nielsen (1986) presented a method to calculate the second-order sum-frequency forces on a cylinder in irregular waves. Comparison with model tests were also made in these cases. Petrauskas and Liu (1987) presented experimental results to confirm the presence of springing forces on a large diameter cylinder and on a TLP model. These studies are based on potential-theory assumptions. Other studies based on potential theory include Salvesen et al. (1984) and Kim and Yue (1991). Salvesen et al. (1984) computed the springing tendon loads in regular, bi-chromatic and random waves. These forces, occurring at such frequencies, are very important in inducing large tendon loads due to the small natural periods of TLP in heave and pitch modes. It is not clear whether springing forces and tendon responses observed experimentally can be fully explained by potential theory computations. It is known that the nonlinear drag term of Morison's equation can give wave forces at higher-harmonic frequencies (at odd multiples of the exciting frequency). Dello Stritto and Horton (1984) studied the harmonics of wave forces predicted by Morison's equation in which it was observed that the forces predicted by Morison's equation contain a mixture of steady, sub- and super-harmonic frequencies, depending on the frequency and phase of the input waves. It was also observed that the simultaneous presence
of current and waves gives rise to forces at even harmonics of the exciting frequency (Gudmestad and Connor, 1983). The effect of such forces on the high-frequency and large-amplitude variation of tendon tension of a tension-leg platform has not been studied before. Springing tendon loads are mainly caused by the pitch motions resulting from the horizontal springing forces. The tendon tension also depends, to a considerable extent, on the horizontal excursion of the platform and on vertical forces. The presence of current causes large horizontal excursions in both regular and irregular waves due to the effect of viscous drift forces. Hence, it appears that viscous forces due to wave and wave-current interactions can induce springing tendon response. Chitrapu and Ertekin (1992) presented numerical results of higher harmonic drag force and tether-tension response of a tension-leg platform. Numerical simulation of platform motions together with spectral methods are used to obtain the results presented in that study.

Other nonlinearities, such as the integration of drag force over the instantaneous wetted length of platform members, can also give rise to force components outside the wave frequency range. Gudmestad and Poumbouras (1988) showed that different methods of computing wave kinematics near the free surface yields different values of forces at higher-harmonic frequencies of the exciting wave frequency. It may be noted that the effect of drag force at small wave periods may be negligible as far as the wave-frequency response of the platform is concerned, but the same can not be said of the low- and high-frequency effects of these forces, and
therefore, they must be included in order to accurately predict the platform response in all frequency ranges.

The resonant response of a dynamic system, such as the motion response of a floating platform, to low/high-frequency wave excitation is critically dependent on the amount of damping present in the system, besides the wave excitation at such frequencies. It is known that wave radiation damping is negligible at these frequencies. Hence the low-frequency response is almost entirely determined by wave drift damping and viscous damping. Several theoretical and experimental studies have been made in the past to determine wave drift damping (see Wichers and Huijsmans, 1984, Hearn and Tong, 1986, Cotter and Chakrabarti, 1989 and Hearn and Tong, 1989). The wave drift damping is associated with the change in mean potential drift force with the low-frequency velocity of the structure in waves. Experimental studies indicate that the wave drift damping is linearly proportional to the low-frequency velocity of the structure and to the square of the wave amplitude. Theoretical predictions of drift damping are in development stages now and several uncertainties still remain with regard to comparison with experiments. Also, the role of wave drift damping and viscous damping in the low-frequency damping is not very well understood. Standing (1991) gave a detailed review of several uncertainties involved in the estimation of low-frequency excitation and damping. It was observed that comparisons with model tests showed good agreement only after the viscous drag forces were included. Similar observation was made by Kobayashi et al. (1987). Other experimental studies (see for example, Brendling and Wilson,
indicate that wave drift damping is important for predicting the low-frequency motions of ships. More studies are needed to determine the relative importance of various damping contributions to total low-frequency damping. This will enable us to use both contributions of damping in motion simulation models.

The situation with high-frequency damping is much less clear. The high-frequency tendon tensions of a TLP are very much influenced by this. Studies in this respect are being made only recently (see Huse, 1990 and Chakrabarti and Hanna, 1991). The effect of skin friction drag and pressure drag on the lower edge of a heaving cylindrical column were experimentally investigated in these studies.

The preceding discussion shows that viscous effect contributions are important in predicting mean and low-frequency drift forces and high-frequency springing forces. Also, viscous damping at such frequencies may play an important role in determining the resonant response of floating platforms due to excitation by drift and springing forces.

1.2 Objectives and methodology

In this study, the effect of the nonlinear drag force, as predicted by Morison's equation, on the wave force and response of floating platforms is studied, with particular emphasis on the low- and high frequency components of forces generated by the use of the nonlinear drag term of Morison's equation. Both frequency- and time-domain methods are employed. First, a frequency-domain method, used to
compute viscous drift forces and moments in regular and random waves, is presented. The relative velocity drag term of Morison's equation is used in conjunction with frequency domain first-order motion transfer functions to compute drift forces and moments. No assumption of linearity, with regard to drag force, is made in this procedure except that the first-order motion transfer functions are obtained, separately, using Morison's equation formulation which linearizes the drag force to determine the frequency response of the platform in regular waves. In fact, these motion transfer functions can as well be obtained by employing the linear potential theory. Mean viscous drift forces and moments in regular waves in all six degrees-of-freedom of a TLP and a semisubmersible platform are computed. The relative importance of steady current, wave-current interaction and platform motions on the computed drift forces and moments are discussed. The frequency-domain results obtained in regular waves are used to compute the mean and low-frequency drift forces and response of a platform in irregular waves by using existing methods developed for potential drift computations. Here, it is assumed that the drift force in regular waves is proportional to the square of the wave amplitude. The theory is presented in Chapter 2.

Chapter 3 deals with the formulation of the complete nonlinear rigid body equations of motion in time domain. The force and response time histories are obtained by a time-domain simulation method which uses Morison's equation. This model can include most of the nonlinearities such as the integration of forces over the instantaneous wetted length of platform members, nonlinear mooring
characteristics and nonlinear terms in the equations of motion, in addition to the nonlinear drag force. The effect of various force contributions to the low-frequency drift forces and high-frequency springing forces is studied with the help of time simulations and spectral methods giving particular emphasis to the nonlinear drag force.

As mentioned before, nonlinear response of floating platforms is affected by potential and viscous effects, both in terms of wave excitation and damping. Such response can be predicted only by theoretical models which can account for potential and viscous effects, simultaneously. No such model which can be used to predict large-amplitude resonant response of floating platforms in extreme waves is presently available. Several studies have been made in the past with regard to the intact and damaged semisubmersible platform in large-amplitude waves. Paulling (1977) presented a time-domain model for the simulation of nonlinear platform motions. Huang et al. (1982) presented experimental results for forces and motions in regular waves of a semisubmersible having a large permanent list angle. In this study, it was observed that the platform response is asymmetric with respect to the wave direction and motion responses showed additional mean displacements in heave and pitch modes. Also, the roll response was found to be nonsinusoidal, with response containing two frequencies, one at the input wave frequency and the other at half of the input wave frequency. Takarada et al. (1986) studied the large-amplitude motion behavior of a semisubmersible, including capsizing, using nonlinear time-domain simulations and experiments. Soylemez and Incecik (1989) presented a nonlinear
time-domain simulation method to predict the large-amplitude motions of a semisubmersible platform. Results presented in that study indicate that integration of forces acting on the complete immersed length of platform members will give additional steady tilt of the platform. The steady tilt angles predicted by the nonlinear simulations showed partially satisfactory agreement with experimental results. From this discussion, it appears that several aspects of the large-amplitude nonlinear response of floating platforms can not be satisfactorily explained by the existing computational procedures. In this study, an attempt has been made to develop a theoretical model to simulate large-amplitude nonlinear motions of platforms. The model is based on the combination of potential and viscous effects in the time domain. Hydrodynamic coefficients and wave exciting forces, obtained a priori using three-dimensional potential theory, are used to determine the forces acting on a platform which makes large motions. First-order memory effects are included through the velocity-based convolution integrals. The procedure followed here is largely based on the early works of Cummins (1962), Ogilvie (1964) and Wehausen (1967, 1971) in the application of linear potential theory in time-domain, and on the recent work of de Kat and Paulling (1989). Viscous effects are included through the nonlinear drag term of Morison's equation.

The rigid-body equations of motion of the platform are derived in Chapter 3. The description of the random seaway and simulation of particle kinematics are also described in this chapter. The effect of large-amplitude waves are considered
through the use of 'stretching' and 'extrapolation' methods to determine the particle kinematics in random waves.

Chapter 4 deals with the details of computing the forces acting on a platform. Both approaches to force calculations, that is Morison's equation method and the potential theory method, are discussed in detail. Computation of nonlinear restoration forces due to the positioning system is also discussed in this chapter. The forces computed by the two methods are then utilized in the equations of motion to determine the platform response. This is described in Chapter 5. Detailed derivation of the equations of motion, suitable for time-domain integration, are presented there for both methods of force computation. The evaluation of the kernel functions and constant added mass coefficients from the frequency-dependent hydrodynamic coefficients is also described in Chapter 5. The numerical method used and solution details are also presented there. Chapter 6 presents the discussion of various results obtained. Conclusions and recommendations made are presented in Chapter 7.
Chapter 2

Frequency-Domain Computation of Viscous Drift

Forces and Moments

2.1 Introduction

In this chapter, a frequency-domain method to determine the viscous drift forces and moments in regular waves is presented. The procedure to obtain the body, wave-particle and current kinematics is discussed. First-order motion transfer functions of the platform, which are obtained separately, are used to determine the position and velocity of any point on the body. Determination of drift forces and response in irregular waves is discussed next. Viscous drift force transfer functions are obtained from the regular wave results and these are used to obtain the drift forces and response of the platform in irregular waves and current.

2.2 Viscous Drift Forces and Moments in Regular Waves

The following coordinate systems are used for the determination of wave- and platform dynamics (see Fig. 2.1).

(i) Auxiliary system (a-system): This system is used to define the body geometry in terms of the starting and ending coordinates of each member.

(ii) Body-fixed system (b-system): This is fixed in the body (at its center of gravity) and translates and rotates with the body with respect to an inertial system which is defined below.
(iii) Global system (g-system): This is an inertial (or earth-bound) system located at the mean position of the body. The origin of this system coincides with the center of gravity of the body when the body is at rest.

(iv) Local member system (l-system): This is fixed in each member such that the local x-axis of this system is along the axis of the member. This system is used for computing local member forces and for integration of these forces along the member length.

(v) Wave system (w-system): This system is located on the still water level with its own x-axis placed along the direction of wave propagation. All wave particle kinematics are computed in this system.

Details of the transformations between these systems are given in Appendix A.

2.2.1 Body and Wave Kinematics

Since the body is assumed to have six-degrees-of-freedom, the unit base vectors in the b-system are time-dependent in the g-system. Coordinates of any point m on the nth member of the body can be expressed as follows (see Fig. 2.1):

$$\mathbf{r}_{gm}^n(t) = \mathbf{r}_{gs}^n(t) + x_{m1} e_{l1}.$$  \hfill (2.1)

In Eq. (2.1), and throughout this paper, the following notation is used for vector quantities. Any vector $\mathbf{r}_{gm}^n$, which is the position vector of the point m on the nth
member, referred to the g-coordinate system can be written as \( \vec{r}_{gm}^n = x_{gm1} \vec{e}_g^i \) using the indicial notation and Einstein's summation convention. In this notation, the superscript \( n \) refers to the \( n \)th member, the subscript \( g \) (or \( b \) or \( l \)) refers to the coordinate system in which the vector is being specified, the subscript \( m \) (or \( s \) or \( e \)) refers to the point considered. \( \vec{e}_g^i \) is the unit base vector in the g-system and the subscript \( i \) refers to its direction.

From Fig. 2.1, and using the coordinate transformation procedure described in Appendix A, we can write Eq. (2.1) as follows:

\[
\vec{r}_{gm}^n(t) = x_{gb1}(t) \vec{e}_g^i + P_{ji}(t) x_{baj} \vec{e}_g^i + x_{lmi} \gamma_i^j(t) \vec{e}_g^i ,
\]

(2.2)

where \( P_{ij}(t) \) are the elements of the orthogonal transformation matrix \([P] \) used for coordinate transformation between the b- and g-systems (see Eq. (A.8) in Appendix A).

The angular velocities can be expressed in terms of the derivatives of the Euler angles using the following relation:

\[
\omega_{bi}(t) \vec{e}_b^i = B_{ij}(t) \frac{d\theta_j}{dt} \vec{e}_b^i ,
\]

(2.3)

where \( \omega = \omega_{bi}(t) \vec{e}_b^i \) is the angular velocity of the body with its components \( \omega_{bi} \) along the body axes, and \( d\theta_j/dt \) is the time-derivative of the Euler angle in the \( j \)th
mode of motion. $B_i(t)$ is the transformation matrix relating the angular velocities to the time-derivatives of the Euler angles (see Chitrapu, 1988), and is given by

$$
[B] = \begin{bmatrix}
\cos \theta_2 \cos \theta_3 & \sin \theta_3 & 0 \\
-sin \theta_2 \cos \theta_2 & \cos \theta_2 & 0 \\
\sin \theta_2 & 0 & 1
\end{bmatrix}.
$$

(2.4)

In the frequency-domain computation of viscous drift forces and moments, the first-order body motions are assumed to have been given by the frequency-domain motion transfer functions, i.e., the amplitude and phase angle of platform motions are specified as functions of wave frequency. Denoting the amplitude transfer function (motion amplitude/wave amplitude) by $\xi_i$, and the corresponding phase angles by $\omega_i$, we can write the body displacements as follows.

$$
x_i(\sigma) = \frac{H}{2} \xi_i \cos(\sigma t + \omega_i), \quad i = 1,2,3, \quad (2.5)
$$

and

$$
\theta_i(\sigma) = \frac{H}{2} \xi_i \cos(\sigma t + \omega_i), \quad i = 4,5,6, \quad (2.6)
$$

where $x_i, i=1,2,3$, are the translatory displacements (surge, sway and heave) and $\theta_i, i=4,5,6$, are the rotational displacements (roll, pitch and yaw) of the platform, $\sigma$ is the radian wave frequency, $H$ is the wave height and $\omega_i$ is the phase angle with
respect to the crest of the wave at the origin. The translational velocity of the
center of gravity of the body, $\bar{\zeta}$, is given by

$$\bar{\zeta} = \dot{x}_i \bar{e}_{g_i},$$

(2.7)

and the angular velocity is given by Eq. (2.3). It may be noted that the superposed
dot indicates the time-derivative of the function involved. Using Eqs. (2.3) and
(2.7), we can write the velocity of any point m on the nth member of the platform
as

$$\bar{v}_{sm}^n = \bar{\zeta} + [P]^T (\bar{\omega} \times \bar{r}_{bm}^n),$$

(2.8)

where $\bar{r}_{bm}^n$ is the position vector of the point m in the b-system. The term $[P]^T$ is
required as the terms enclosed in the parenthesis in Eq. (2.8) are given in the b-
system. The matrix [P] is the previously defined coordinate transformation matrix.

It is assumed that regular waves make an arbitrary angle $\psi_w$ with the $x_{g1}$-axis,
measured positive in the counterclockwise direction. The surface elevation of linear
waves, at any point m, is given by

$$\eta(x_{g1m}, x_{g2m}, t) = \frac{H}{2} \cos(k_i x_{gmi} - \omega t), \; i = 1, 2,$$

(2.9)

where $H$ is the wave height, and $\bar{k}$ is the wave number vector whose components
in the g-system are $k_i$, $i = 1, 2$. The particle velocity components in deep water, and
in a coordinate system which coincides with the direction of wave propagation, are given by

\[ u = \frac{H}{2} \sigma \exp(\|\vec{k}\| x_{gm3}) \cos(k x_{gm3} - \sigma t), \]  

(2.10)

and

\[ u_{wm3} = \frac{H}{2} \sigma \exp(\|\vec{k}\| x_{gm3}) \sin(k x_{gm3} - \sigma t), \]  

(2.11)

where \( u \) and \( u_{wm3} \) are the horizontal and vertical components of particle velocity.

The particle velocities resolved along the axes of the \( g \)-system are given by

\[ \vec{v}_{wm}^a = u_{wm} \vec{e}_{gi}, \]  

(2.12)

where \( u_{wm1} = u \cos \psi_w \), \( u_{wm2} = u \sin \psi_w \) and \( u_{wm3} \) is given by Eq. (2.11).

### 2.2.2 Finite Wave Amplitude Effects

In order to compute the particle kinematics near the free surface, several empirical models have been proposed (see for example, Wheeler, 1969, Gudmestad and Connor, 1986 and Rodenbusch and Forristall, 1986). Two different methods are utilized to empirically modify the kinematics predicted by the linear wave theory. These methods are discussed below.
Stretching:

In this method, proposed by Wheeler (1969), the depth of submergence of a point at which kinematics are desired is modified using the following equation (see Fig. 2.2):

\[ z_s = \frac{h}{(h + \eta)} (z - \eta) , \] (2.13)

where both \( z \) and \( z_s \) are measured from the still-water plane, positive upwards.

With the notation used in this study, Eq. (2.13) can be written as

\[ x'_{gm3} = \frac{h}{(h + \eta)} (x_{gm3} + Z_{slg} - \eta) , \] (2.14)

where \( x'_{gm3} \) is the stretched vertical coordinate, \( h \) is the constant water depth, \( \eta \) is the surface elevation, \( Z_{slg} \) is the position of the origin of the \( g \)-system with respect to the still-water level such that \( (x_{gm3} + Z_{slg}) \) gives the depth from the still-water level at which kinematics are desired.

Extrapolation:

Another way to compute particle kinematics near the free surface is to extrapolate the kinematics at the still-water level up to the actual free surface (see Rodenbusch and Forristall, 1986). For this, we can write
where $z_1$ is defined as $z_1 = x_{gm3} + z_{slg}$ such that $z_1 = 0$ represents the still-water level, and it is assumed that $\partial u / \partial x_{gm3}$ is constant above this line. A similar equation can also be written for the vertical component of the particle velocity.

Fig. 2.3 shows the vertical distribution of particle horizontal velocities computed in the various methods discussed above. We can see from this figure that stretching method gives smaller velocities under the crest and larger velocities under the trough as compared to the linear wave theory. The extrapolation method gives larger velocities compared to the stretching method in the wave crest.

Let a steady current, whose magnitude and direction can vary with depth, be present along with waves. The magnitude and direction of the current at any depth are specified by $\nu_c(x_{gm3})$ and $\psi_c(x_{gm3})$, respectively. Using a method similar to the stretching method used for wave particle kinematics, the current velocity at any point $m$ on the $n$th member can be written as

$$v_{cm}^n = v_{cml} \tilde{e}_g,$$  

where

$$v_{cml} = |\nu_c(x_{gm3})| \cos \psi_c(x_{gm3}) ,$$

and

$$u(x_{gm1},x_{gm2},z_1) = u(x_{gm1},x_{gm2},0) + z_1 \frac{\partial u}{\partial x_{gm3}}(x_{gm1},x_{gm2},0), \quad z_1 > 0 ,$$ (2.15)
The total relative velocity between the body and fluid is then given by (see Eqs. (2.8), (2.12) and (2.16))

\[ \vec{v}_{rm} = \vec{v}_{wm} + \vec{v}_{cm} - \vec{v}_{sm}. \]  

(2.19)

The normal component (perpendicular to member axis) of this total relative velocity can be written as

\[ \vec{v}_{rmN} = \vec{e}_{il} \times \left( \vec{v}_{rm} \times \vec{e}_{il} \right) = [R] \vec{v}_{rm}, \]

(2.20)

where

\[
[R] = \begin{bmatrix}
1 - (\gamma_1^n)^2 & -\gamma_1^n \gamma_2^n & -\gamma_1^n \gamma_3^n \\
-\gamma_1^n \gamma_2^n & 1 - (\gamma_2^n)^2 & -\gamma_2^n \gamma_3^n \\
-\gamma_1^n \gamma_3^n & -\gamma_2^n \gamma_3^n & 1 - (\gamma_3^n)^2
\end{bmatrix},
\]

(2.21)

in which \( \gamma_i^n \) are the direction cosines of the nth member and Eqs. (A.11), (A.12) and (2.19) are used to determine the matrix \([R]\). The subscript \( N \) in \( \vec{v}_{rmN} \) indicates the normal component.
2.2.3 Viscous Drift Forces and Moments

The drag force per unit length of the member at a point m in the middle of a segment is given by

\[ \frac{dF}{d_y}(m) = \frac{1}{2} \rho C_d^n D^n \bar{v}_{rmN} \| \bar{v}_{rmN} \|, \tag{2.22} \]

where \( \rho \) is the mass density of the fluid, \( C_d^n \) is the drag coefficient and \( D^n \) is the diameter of the \( n \)th member. The elemental moment with respect to the origin of the b-system is then given by

\[ dM_y(m) = \bar{r}_{bn} \times (dF_y(m))_b, \tag{2.23} \]

where the subscript \( b \) outside the parenthesis on the right hand side of the above equation indicates that the components of the moment are resolved in the b-system. This transformation can be written as follows.

\[ (dF_y(m))_b = [P]dF_y(m). \tag{2.24} \]

The instantaneous wetted length of each member of the platform is divided into a number of segments and the forces and moments are determined for each such segment using Eqs. (2.22) and (2.23). The elemental forces and moments are then numerically integrated over the submerged length of the member at each wave phase \( (\sigma t) \). Denoting the forces and moments by \( F_i, i = 1, 2, ..., 6 \), we can write
\[ F_{Vi}(\sigma t) = \int_0^{t^*(\sigma)} dF_{Vi}^n(\sigma t) \; dx_{II} , \; i = 1,2,\ldots,6. \] (2.25)

The mean values of forces and moments on the nth member are then given by

\[ \bar{F}_{Vi}^n(\sigma) = \frac{\sigma}{2\pi} \int_0^{2\pi/\sigma} F_{Vi}^n(\sigma t) \; dt , \; i = 1,2,\ldots,6. \] (2.26)

The total viscous drift forces and moments on the platform are obtained by summing the forces and moments over all K members,

\[ \bar{F}_{Vi}(\sigma) = \sum_{n=1}^{K} \bar{F}_{Vi}^n(\sigma) , \; i = 1,2,\ldots,6. \] (2.27)

2.3 Viscous Drift Force and Response in Irregular Waves

A number of studies have been made in the past to determine mean and low-frequency potential drift forces in irregular waves. Remery and Hermans (1972) computed the slowly-varying drift forces in irregular waves as a function of time using mean drift force values in regular waves. Newman (1974) proposed an approximation which can be used to compute drift forces in irregular waves from the drift force coefficients in regular waves. Pinkster (1975, 1979) derived expressions for mean drift force and spectral density of slowly-varying drift force in irregular waves. This method is used in the present study to compute viscous drift forces in
irregular waves. Complete details of the derivation of these equations can be found in Pinkster (1979, 1980). The derivation of these equations is based on the following general principle.

Let the wave elevation in a regular wave group, consisting of two sinusoidal waves, be given by

\[ \eta(t) = \sum_{i=1}^{2} A_i \cos(\sigma_i t + \epsilon_i), \]  

(2.28)

where \( A_i, \sigma_i \) and \( \epsilon_i \) are the amplitude, circular frequency and phase of the \( i \)th wave component, respectively. The second-order force corresponding to this wave can be written as follows:

\[ F^{(2)}(t) = \sum_{i=1}^{2} \sum_{j=1}^{2} A_i A_j C_{ij} \cos \left( (\sigma_i - \sigma_j) t + (\epsilon_i - \epsilon_j) \right) \]

\[ + \sum_{i=1}^{2} \sum_{j=1}^{2} A_i A_j D_{ij} \sin \left( (\sigma_i - \sigma_j) t + (\epsilon_i - \epsilon_j) \right), \]  

(2.29)

where \( C_{ij} \) and \( D_{ij} \) are the in-phase and out-of-phase components, respectively, of the drift force quadratic transfer function. We can write Eq. (2.29) as follows:

\[ F^{(2)}(t) = A_1^2 C_{11} + A_2^2 C_{22} + A_1 A_2 (C_{12} + C_{21}) \cos \left( (\sigma_1 - \sigma_2) t + (\epsilon_1 - \epsilon_2) \right) \]

\[ + A_1 A_2 (D_{12} - D_{21}) \sin \left( (\sigma_1 - \sigma_2) t + (\epsilon_1 - \epsilon_2) \right). \]  

(2.30)
The first two terms on the right hand side of Eq. (2.30) give the constant drift force, corresponding to the presence of a single regular wave with a frequency $\sigma_1$ and $\sigma_2$ at a time. Thus, $C_{11}$ and $C_{22}$ represent the mean drift force due to two waves of equal frequency $\sigma_1 = \sigma_2$. The remaining two terms in Eq. (2.30) represent the low-frequency force components corresponding to the difference frequency, $\sigma_1 - \sigma_2$, of the two waves. Eq. (2.30) can also be written as

$$F^{(2)}(t) = A_1^2 C_{11} + A_2^2 C_{22} + 2A_1 A_2 G_{12} \cos \left( (\sigma_1 - \sigma_2) t + (e_1 - e_2) + e_{12} \right), \quad (2.31)$$

where $G_{12}$ is the amplitude of the quadratic drift force transfer function which is given by

$$G_{12} = G(\sigma_1, \sigma_2) = \sqrt{C^2(\sigma_1, \sigma_2) + D^2(\sigma_1, \sigma_2)}, \quad (2.32)$$

and $e_{12}$ is a phase angle given by

$$\tan e_{12} = \frac{-D(\sigma_1, \sigma_2)}{C(\sigma_1, \sigma_2)}. \quad (2.33)$$

The above procedure can be generalized for random sea conditions consisting of a number of regular waves with frequencies $\sigma_i$, $i=1,\ldots,N$. Pinkster (1979) derived expressions for the mean and slowly-varying drift forces in irregular waves using Eqs. (2.31), (2.32) and (2.33). It can be shown that the mean drift force in irregular waves is given by (see Pinkster, 1975)
\[ \bar{F}_v = 2 \int_{0}^{\infty} S(f) G(f,f) \, df , \tag{2.34} \]

where \( S(f) \) is the spectral density of incoming wave surface elevation and \( f = \sigma / 2\pi \) is the cyclic wave frequency. This equation can be seen as a summation, over the complete frequency range, of the first two terms in Eq. (2.31). The spectral density of the low-frequency drift force is given by (Pinkster, 1975)

\[ S_v(\mu) = 8 \int_{0}^{\infty} S(f) S(f + \mu) G^2(f,f + \mu) \, df . \tag{2.35} \]

The amplitude of the quadratic drift force transfer function, \( G \), in Eq. (2.35) should, in principle, be evaluated for different combinations of wave component frequencies \( f \) and \( \mu \) (\( f \neq \mu \)). Using Newman's (1974) approximation one can write

\[ G(f,f+\mu) \approx G(f+\frac{\mu}{2},f+\frac{\mu}{2}) , \tag{2.36} \]

which implies that the amplitude of quadratic drift force transfer function representing the force due to two wave components with frequencies \( f \) and \( \mu \) can be replaced by the value of the transfer function evaluated at the mean value of the frequency of the two waves, i.e., \( (f+\mu/2) \). Using Eq. (2.36) we can write Eq. (2.35) as follows:

\[ S_v(\mu) = 8 \int_{0}^{\infty} S(f) S(f + \mu) G^2(f+\frac{\mu}{2},f+\frac{\mu}{2}) \, df . \tag{2.37} \]
Eq. (2.37) implies that the spectral density of the low-frequency drift force in irregular waves can be obtained using the drift force transfer function values obtained in regular waves only.

The above analysis assumes that the drift force in regular waves is proportional to the square of the wave amplitude. Also, the wave spectrum is assumed to be narrow banded. These should be noted for application of these equations to the computation of viscous drift forces in irregular waves, as the viscous drift force in regular waves is only approximately proportional to the square of wave amplitude.

The response due to the low-frequency drift force can be determined by modelling it as a single-degree-of-freedom system as follows:

\[ A \ddot{y} + B \dot{y} + Cy = F_0 e^{i\omega t}, \]

(2.38)

where \( A \) is the total mass (body mass and added mass), \( B \) is the damping coefficient, \( C \) is the stiffness (hydrostatic and anchorline), \( F_0 \) is the amplitude of the low-frequency drift force, \( \omega_e = 2\pi f_e \) is the excitation frequency, and \( y, \dot{y}, \ddot{y} \) are the displacement, velocity and acceleration, respectively, of the platform in a particular mode of motion being considered. We can then write

\[ \left( \frac{y_e(f_0)}{F_0} \right)^2 = \frac{1}{(C-A\omega_e^2)^2 + (B\omega_e)^2}, \]

(2.39)

and the spectral density of slowly-varying motion can be written as
where $y_o$ is the amplitude of the slowly-varying motion.

2.4 Application

The theory presented in the previous sections is applied to a semisubmersible platform and a tension leg platform. Some of the basic features of the two platforms are given in Table 2.1. The details of the semisubmersible platform geometry can be found in Pinkster (1980), and the TLP details are given in Teigen (1983). For regular waves, wave heights ranging from 0 (only current is present) to 15m, with wave periods varying from 4 to 22s are used. Two different current velocities are used. Both are uniform along the depth and their magnitudes are 0.91 m/s and 0.5 m/s. The magnitude and direction of the current can vary with depth, although uniform velocities are used in the results presented here. In the results presented here, wave and current directions are assumed to be the same, though the theory is not limited to such a case. The first-order motions of the platform have been obtained using a frequency-domain method based on Morison's equation. The motion amplitudes and phases thus obtained for the TLP case compared well with the results presented in Teigen (1983) who used the strip theory. A constant drag coefficient of 1.0 has been used for all members and frequencies. It should be noted, however, that one must, in principle, consider the dependency of the drag
coefficient on the Keulegan-Carpenter number, especially with regard to the particular frequency range under consideration. The general procedure followed for the computation can be described as follows.

For a given wave height and period, the viscous drift forces and moments are calculated at a number of wave-phase positions and averaged over the wave cycle using Eq. (2.25). The body motions, specified by the frequency-domain transfer functions, are obtained separately and used to determine the position and velocity of any point of the body at each wave phase. This procedure is repeated for a number of wave periods and heights. The half-amplitude Bretschneider spectrum given by Eq. (2.40) below

\[ S(f) = \frac{5}{16} H_s^2 T_o (fT_o)^{-5} \exp\left( -\frac{5}{4} (fT_o)^{-1} \right), \]  

(2.41)

with a significant wave height \( H_s = 11.4 \text{m} \) and a period \( T_o = 15 \text{s} \), corresponding to peak frequency, is used in the irregular wave analysis.

2.5 Results and Discussion

Regular Waves:

Figs. 2.4 through 2.12 pertain to the tension-leg platform. Fig. 2.4 shows the surge viscous drift force for 0° wave angle and current, and Fig. 2.5 shows similar results without the current. The drift force due to current acting alone is also shown in Fig. 2.4. In these two figures, the influence of current and the nonlinear
interaction between waves and current can be seen. Fig. 2.6 shows the pitch drift moment in waves and current. Fig. 2.7 shows the surge drift force in the presence of waves and current computed by two different methods. In one method, forces are computed at the mean location of the platform, i.e., the platform (while having body velocities as prescribed by the motion transfer functions and phases) is assumed to remain in its mean position for the purpose of computing kinematics and forces. In the second method, the wave and current velocities are computed at the instantaneous position of a platform member at each wave phase. To illustrate this effect further, surge drift force on a single column of the TLP is computed in the two cases discussed above, and is shown in Fig. 2.8. In this case, the forces are computed only up to the still-water level in the absence of current. We can see that there is a marked difference in the computed drift force when forces are computed in the displaced location of the cylinder. It may also be noted that the mean force on the cylinder is not equal to zero even in the absence of current and finite wave elevation effect. The reason for this appears to be due to the phase difference between the wave and cylinder motions.

Fig. 2.9 shows the pitch drift moment on the TLP computed by the two methods described above. Fig. 2.10 shows the roll drift moment for 22.5° waves and current. The yaw drift moment for the same case is shown in Fig. 2.11. Fig. 2.12 shows the surge drift force on the TLP computed based on potential theory, viscous drift based on the method given in this Chapter and model experiments. The potential drift force results and experimental data are taken from Teigen (1983) and
replotted after multiplying the results by \((H/2)^2\) using a wave height of \(H=6\) m. The mean viscous drift force (for \(H=6\) m) plotted in this figure is computed using the extrapolation method for particle kinematics.

Figs. 2.13 through 2.17 pertain to the semisubmersible platform. All theoretical results of the potential drift forces and the experimental results presented in these figures have been taken from Pinkster (1980) and replotted after multiplying by the square of the wave amplitude specified for each figure. Fig. 2.13 shows the surge drift force in head waves and Fig. 2.14 shows the sway drift force in beam waves. The viscous drift force results presented in these two figures have been obtained using the extrapolation method for particle kinematics. We can see that the viscous drift force is very small compared to the potential drift, mainly due to the absence of current.

Figs. 2.15 and 2.16 show the viscous drift force, in the above mentioned cases, after including a steady current of 0.5 m/s, uniform over the depth. Comparison of the two sets of figures show that the current has a pronounced effect on viscous drift force. It may be noted that even with a small current, such as the one used here, the viscous drift force can be large, and in some cases comparable to the potential drift force. It is also noted that viscous drift force is present in the long period range, where the potential drift effects begin to diminish. Hence, it is important to consider the viscous drift contribution to the total drift forces and moments, particularly for design wave conditions. Fig. 2.17 shows the roll drift moment in beam waves including the current.
The mean viscous drift force as a function of the wave amplitude squared is plotted in Fig. 2.18. The surge drift force values obtained from Fig. 2.4 are used to obtain this figure. The force due to current (see Fig. 2.4) is subtracted before obtaining the curves shown in Fig. 2.18. The viscous drift force transfer function \( (F/(H/2)^2) \) has been computed by using the force values at \( H=3m \). Since the drift force does not seem to be proportional to the wave amplitude squared, this method of linearization seems adhoc at first. However, from Fig. 2.18, we can see that at \( H=3m \) the curves are linear and hence this may be justified. Fig. 2.19 shows such a drift force transfer function computed for surge. This transfer function is later used to obtain the viscous drift force in irregular waves. The results for drift forces and response in irregular waves are presented in Chapter 6.

From the results presented so far, we can see that considerably large mean forces and moments are predicted for both types of platform due to viscous effects. The mean surge drift force and pitch moments are very important in determining the surge offset and tether-tension of tension leg platforms. The roll moment on the semisubmersible platform is also important in predicting large-amplitude motions in extreme seas.
Table 2.1 Particulars of TLP and Semisubmersible Platforms

<table>
<thead>
<tr>
<th>Semisubmersible Platform:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of columns</td>
<td>6</td>
</tr>
<tr>
<td>Column dia.</td>
<td>12.6 m</td>
</tr>
<tr>
<td>No. of pontoons</td>
<td>2</td>
</tr>
<tr>
<td>Pontoon dimensions:</td>
<td></td>
</tr>
<tr>
<td>width, breadth</td>
<td>8, 16 m</td>
</tr>
<tr>
<td>length</td>
<td>100 m</td>
</tr>
<tr>
<td>Displacement volume</td>
<td>35,925 m³</td>
</tr>
<tr>
<td>Draft</td>
<td>20 m</td>
</tr>
<tr>
<td>Water depth</td>
<td>40 m</td>
</tr>
<tr>
<td>Center of gravity</td>
<td>13.86 m below SWL</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tension leg platform:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of columns</td>
<td>4</td>
</tr>
<tr>
<td>Column dia.</td>
<td>14.2 m</td>
</tr>
<tr>
<td>No. of pontoons</td>
<td>4</td>
</tr>
<tr>
<td>Pontoon dimensions:</td>
<td></td>
</tr>
<tr>
<td>width,breadth</td>
<td>11.2, 8.3 m</td>
</tr>
<tr>
<td>length</td>
<td>44.1 m</td>
</tr>
<tr>
<td>Displacement volume</td>
<td>33,400 m³</td>
</tr>
<tr>
<td>Draft</td>
<td>26.6 m</td>
</tr>
<tr>
<td>Water depth</td>
<td>500 m</td>
</tr>
<tr>
<td>Tethers</td>
<td>one per column</td>
</tr>
<tr>
<td>diameter</td>
<td>0.4 m</td>
</tr>
<tr>
<td>Center of gravity</td>
<td>9.25 m above SWL</td>
</tr>
</tbody>
</table>
Fig. 2.1 Coordinate system definition
Fig. 2.2 Stretching of particle kinematics
Fig. 2.3 Vertical distribution of particle horizontal velocity
(H=5m, T=7s, water depth = 500m)
Fig. 2.4 Mean surge viscous drift force

(0° wave and current)

Fig. 2.5 Mean surge viscous drift force

(0° wave only)
Fig. 2.6 Mean pitch viscous drift moment

(0° wave and current)

Fig. 2.7 Mean Surge viscous drift force

(0° wave and current, H=12m)
Fig. 2.8 Mean surge viscous drift force (single column)

(0° wave, \(H=12\text{m}\), no current)

Fig. 2.9 Mean pitch viscous drift moment

(0° wave and current, \(H=12\text{m}\))
Fig. 2.10 Mean roll viscous drift moment

(22.5° wave and current)

Fig. 2.11 Mean yaw viscous drift moment

(22.5° wave and current)
Fig. 2.12 Mean surge drift force

(0° wave only)

Fig. 2.13 Mean surge drift force (semisubmersible)

(Head waves, H = 6m, no current)
Fig. 2.14 Mean sway drift force (semisubmersible)

(Beam waves, H=6m, no current)

Fig. 2.15 Mean surge viscous drift force (semisubmersible)

(Head waves and current)
Fig. 2.16 Mean sway viscous drift force (semisubmersible)

(Beam waves and current)

Fig. 2.17 Mean roll viscous drift moment (semisubmersible)

(Beam waves and current)
Fig. 2.18 Mean surge viscous drift force (TLP)

(0° wave and current)

Fig. 2.19 Viscous drift force transfer function

(0° wave and current)
Chapter 3
Description of the Platform Motion and the Sea Surface

3.1 Introduction

In this chapter the equations of motion which describe the motion of a floating platform are derived. The time rate of change of linear and angular momentum of the platform and the external forces and moments acting on the platform are considered in deriving the rigid body equations of motion of the platform. Various nonlinearities present in the equations of motion and in the computation of fluid forces are discussed. The description of the random sea surface and simulation of wave particle kinematics is also discussed.

3.2 Linear and Angular Momentums of the Body

A floating platform is considered as a rigid body, and the equations of motion are formulated by equating the time rate of change of linear and angular momentums of the body to the total external force and moment, respectively. It is convenient to express the linear momentum equation in the g-system. Since the moments and products of inertia of the body remain constant in the body-fixed system, it is convenient to express the angular momentum equation in the b-system.

Consider an arbitrary shaped body shown in Fig. 3.1, in which b indicates the origin of the body-fixed coordinate system and c indicates the center of mass of the body. The center of mass is given by
where \( m_i \) and \( \mathbf{r}_i \) are the mass and position vector of the \( i \)th particle, respectively, and \( M \) is the total mass of the body. If the origin of the body-fixed system (b-system) coincides with the center of mass (as in our case) \( \mathbf{r}_c = 0 \).

Linear momentum \( \mathbf{P} \) of the body is given by

\[
\mathbf{P} = M \mathbf{v}_c = M(\mathbf{v}_b + \bar{\omega} \times \mathbf{r}_c) = M \mathbf{v}_b \quad \text{(if \( c \) and \( b \) coincide)} ,
\]

where \( \mathbf{v}_c \) is the absolute velocity (relative to an inertial system) of the center of mass of the body, \( \mathbf{v}_b \) is the translatory velocity of the origin of the b-system and \( \bar{\omega} \) is the angular velocity of the body.

Angular momentum \( \mathbf{L}_b \) of the body about the origin of b-system can be written as

\[
\mathbf{L}_b = \int \mathbf{r} \times (m \mathbf{v}_c) \, dm = \int \mathbf{r} \times (\mathbf{v}_b + \bar{\omega} \times \mathbf{r}_c) \, dm = -\mathbf{v}_b \times \int \mathbf{r} \, dm + \int \mathbf{r} \times (\bar{\omega} \times \mathbf{r}) \, dm ,
\]

where \( \mathbf{r}, \mathbf{v} \) are the position vector and the absolute velocity of any given point. If 'b' coincides with 'c' the first term in Eq. (3.3) becomes zero and hence

\[
\mathbf{L}_b = \int \mathbf{r} \times (\bar{\omega} \times \mathbf{r}) \, dm
\]
Noting that \( \vec{r} \times (\vec{\omega} \times \vec{r}) = \vec{\omega} (\vec{r} \cdot \vec{r}) - \vec{r} (\vec{\omega} \cdot \vec{r}) \), Eq. (3.4) can be expanded to yield the three components of angular momentum along the body axes i.e.,

\[
\vec{r} \times (\omega \times \vec{r}) = \omega (\vec{r} \cdot \vec{r}) - \vec{r} (\vec{\omega} \cdot \vec{r})
\]

\[
= [\omega_1 r^2 - x(\omega_1 x + \omega_2 y + \omega_3 z)] \hat{i} + [\omega_2 r^2 - y(\omega_1 x + \omega_2 y + \omega_3 z)] \hat{j} + [\omega_3 r^2 - z(\omega_1 x + \omega_2 y + \omega_3 z)] \hat{k}
\]

In the above expression, \((x,y,z)\) and \((\omega_1, \omega_2, \omega_3)\) are, respectively, the components of the position vector \(\vec{r}\) and the angular velocity vector \(\vec{\omega}\) along the body-fixed coordinate system. \(\hat{i}, \hat{j}, \hat{k}\) are the unit base vectors in this system. Considering the first component in the above expression, and using Eq. (3.4), we obtain

\[
L_{b1} = \int m \omega_1 (r^2 - x^2) \, dm - \int m \omega_2 xy \, dm - \int m \omega_3 xz \, dm
\]

We can write the above expression as follows:

\[
L_{b1} = I_{11} \omega_1 - I_{12} \omega_2 - I_{13} \omega_3
\]

where \( I_{11} = \int m (r^2 - x^2) \, dm \); \( I_{12} = \int m xy \, dm \); \( I_{13} = \int m xz \, dm \). \hfill (3.5)

Similarly the second and third components of the angular momentum vector can be written as

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\[ L_{b2} = \int \omega_2 (r^2 - y^2) \, dm - \omega_1 \int xy \, dm - \omega_3 \int yz \, dm = \omega_2 I_{22} - \omega_1 I_{21} - \omega_3 I_{23} , \] (3.6)

where

\[ I_{22} = \int (r^2 - y^2) \, dm ; \quad I_{21} = \int xy \, dm ; \quad I_{23} = \int yz \, dm , \]

and

\[ L_{b3} = \int \omega_3 (r^2 - z^2) \, dm - \omega_1 \int xz \, dm - \omega_2 \int yz \, dm = \omega_3 I_{33} - \omega_1 I_{31} - \omega_2 I_{32} , \] (3.7)

where

\[ I_{31} = \int zx \, dm , \quad I_{33} = \int (r^2 - z^2) , \quad I_{32} = \int zy \, dm . \]

We can write Eq. (3.4) as follows:

\[ \bar{L}_b = \int (\bar{r} \cdot \bar{r}) \, \bar{\omega} - (\bar{\omega} \cdot \bar{r}) \bar{r} \, dm . \] (3.8)

The moments of inertia \((I_{11}, I_{22}, I_{33})\) and products of inertia \((I_{12}, I_{13}, I_{21}, I_{23}, I_{31}, I_{32})\)

can be written as the inertia tensor as follows:

\[ I = \begin{bmatrix} I_{11} & -I_{12} & -I_{13} \\ -I_{21} & I_{22} & -I_{23} \\ -I_{31} & -I_{32} & I_{33} \end{bmatrix} . \] (3.9)
Using Eqs. (3.5), (3.6), (3.7) and (3.9), Eq. (3.8) can be written as

\[
\mathbf{\dot{L}}_b = \begin{bmatrix} L_{b1} \\ L_{b2} \\ L_{b3} \end{bmatrix} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = [I] \mathbf{\dot{\omega}},
\]

(3.10)

or

\[
\mathbf{\dot{L}}_b = [I] \mathbf{\dot{\omega}},
\]

where \( \mathbf{\dot{L}}_b \) and \( \mathbf{\dot{\omega}} \) are the angular momentum and angular velocity vectors, respectively, specified in the b-system. The components of \( \mathbf{\dot{L}}_b \) are given by Eqs. (3.5), (3.6) and (3.7).

3.3 Rigid Body Equations of Motion

The equations of motion expressing the motion of the body are written in terms of the external forces and moments and the mass and inertia properties of the body using Newton’s and Euler’s equations of motion, as follows. The force equation can be written by equating the total external force \( \mathbf{\bar{F}} \) acting on the body to the time-rate of change of linear momentum, that is;

\[
\mathbf{\bar{F}} = \frac{d\mathbf{\bar{P}}}{dt} = M \frac{d\mathbf{\bar{v}}_b}{dt}.
\]

(3.11)

The momentum equation is obtained by equating the total external moment acting
on the body to the time-rate of change of angular momentum, as follows:

\[
\mathbf{\dot{M}}_b = \frac{d\mathbf{F}_b}{dt},
\]

(3.12)

where

\[\mathbf{F} = \text{total external force in the g-system},\]

\[\mathbf{\dot{M}}_b = \text{total external moment about the origin of the b-system},\]

\[\mathbf{P}, \mathbf{L}_b = \text{linear momentum in the g-system and angular momentum about the origin of the b-system}.,\]

Eq. (3.11) is written in the g-system in which \(\mathbf{v}_b\) is the absolute velocity of the origin of the b-system relative to the inertial system. As mentioned before, it is convenient to evaluate Eq. (3.12) in the b-system since the inertia tensor (Eq. 3.9) is constant in this system. The time rate of change, with respect to an inertial system, of a vector which is defined in a rotating coordinate system is given by (see for example, Meirovitch, 1970)

\[\frac{d\mathbf{\bar{r}}}{dt} = \frac{d\mathbf{\bar{r}}'}{dt} + \mathbf{\bar{\omega}} \times \mathbf{\bar{r}}'.\]

(3.13)

In Eq. (3.13), \(\mathbf{\bar{r}}'\) is a vector fixed in the b-system and it rotates relative to the inertial system. \(d\mathbf{\bar{r}}'/dt\) is the rate of change of \(\mathbf{\bar{r}}'\) relative to the b-system, \(\mathbf{\bar{\omega}} \times \mathbf{\bar{r}}'\) is the rate of change of \(\mathbf{\bar{r}}'\) due to the rotation of the coordinate system and \(d\mathbf{\bar{r}}_g/dt\) is the
rate of change of \( \mathbf{r}_e \) with respect to the inertial system. The right hand side of Eq. (3.13) is the rate of change of \( \mathbf{r}' \) due to the rotation of the body-fixed system relative to an inertial system, expressed in the b-system. Eq. (3.13) is valid for any vector, defined in a rotating frame of reference, such as the angular velocity vector \( \mathbf{L}_b \).

Using Eq. (3.13), Eq. (3.12) can be written as

\[
\vec{M}_b = \frac{d\mathbf{L}_b}{dt} = \left\{ \frac{d\mathbf{L}_b}{dt} \right\}_b + \vec{\omega} \times \mathbf{L}_b.
\]

In the above equation the subscript \( b \) outside the parenthesis of the first term on the right hand side indicates that the derivative within the parenthesis is evaluated in the b-system.

Using Eq. (3.10) for \( \mathbf{L}_b \), we can write the above equation as

\[
\vec{M}_b = [I] \frac{d\vec{\omega}}{dt} + \vec{\omega} \times ([I] \vec{\omega}).
\]  

(3.14)

Note that \( (d\mathbf{L}_b/dt)_b \) is the rate of change of \( \mathbf{L}_b \) relative to the rotating system itself.

\( \vec{\omega} \) is the angular velocity vector with components along the body set of axes.

Eqs. (3.11) and (3.14) are the force and moment equations governing the motion of the platform. The angular velocity \( \vec{\omega} \) of the body is expressed in terms of time-derivatives of the Euler angles (see Eq. (2.31)). These equations can be summarized as follows.
\[
\frac{d\vec{v}}{dt} = [M]^{-1} \vec{F}, \\
\frac{d\vec{x}}{dt} = \vec{v}, \\
\frac{d\vec{\omega}}{dt} = [I]^{-1} \{\vec{M}_b - \vec{\omega} \times ([I] \vec{\omega})\}, \\
\frac{d\vec{\theta}}{dt} = [B]^{-1} \vec{\phi},
\]

(3.15)

where

\(\vec{x}, \vec{v} = \) translational displacement and velocity vectors of the origin of the \(b\)-system,

\(\vec{\theta}, \vec{\omega} = \) angular displacement and angular velocity vectors,

\(\vec{F}, \vec{M}_b = \) external force in the \(g\)-system and external moment about the origin of the \(b\)-system, respectively,

\([M], [I] = \) body mass and inertia matrices,

\([B] = \) transformation matrix relating angular velocity to time-derivatives of the Euler angles.

It should also be noted that the subscript \(b\) for \(\vec{v}_b\) has been dropped with the understanding that all displacements and velocities refer to the center of mass of the platform, which has been taken to coincide with the origin of the \(b\)-system. It is also noted that the platform mass \(M\) is now written in a matrix form for convenience. \([M]\) is a diagonal matrix with platform mass as its elements.

Eqs. (3.15) represent a system of 12 first-order, coupled, nonlinear ordinary
differential equations with $\mathbf{x}$, $\mathbf{v}$, $\mathbf{\dot{\theta}}$ and $\mathbf{\ddot{\theta}}$ being the 12 state variables. The system of equations is in a form suitable for time-domain integration using any of a number of standard methods, such as the 4th-order Runge-Kutta method. The force and moment that appears in Eq. (3.15) depend on platform position, velocities, accelerations and time. The dependence of forces and moments on body accelerations introduces difficulties in the numerical integration of the equations of motion. The specific method of solution to be used depends on the method in which external forces and moments are computed. These are discussed in the subsequent Chapters. However, it is worthwhile, at this point, to consider the various nonlinearities present in the equations of motion.

Nonlinearities in the equations of motion:

As mentioned before, Eq. (3.15) is a system of nonlinear ordinary differential equations. The nonlinearities can be summarized as follows:

(i) Transformation matrices for large angles:

The coordinate transformation matrices $[P]$ and $[B]$ contain terms which are products of trigonometric functions of the rotational displacements (see Eqs. (A.8) and (2.4)). The transformation matrix $[P]$ is used to transform the forces computed in the g-system into the b-system to obtain $[M_b]$ in Eq. (3.15). The forces and moments which use these matrices are thus nonlinear functions of the rotational displacements. Since no assumption with regard to the magnitude of the Euler angles is made (such as 'small rotational displacements'), the terms containing the
products of sine and cosine terms of the rotational displacements in the transformation matrices cannot be neglected.

(ii) **Nonlinear terms in the equations of motion:**

The equations of motion, Eq. (3.15), are nonlinear due to the presence of the $\vec{\omega} \times \vec{I} \vec{\omega}$ term.

(iii) **Nonlinearities in the computation of external forces and moments:**

The fluid forces and moments may depend, in a nonlinear manner, on the wave elevation and particle velocities. As will be discussed in subsequent Chapters, computation of fluid forces over the instantaneous wetted length of platform members introduces a nonlinear dependence of fluid forces on the wave surface elevation. Viscous forces, such as the relative velocity drag force, depend on the square of the particle and body velocities. The forces due to the positioning system may also include nonlinear terms containing particle and body kinematic variables.

### 3.4 Description of the Sea Surface

In this study, the response of floating platforms is simulated under different types of wave excitation, such as regular monochromatic waves, bi-chromatic waves, and random waves. It is necessary to predict the wave surface and particle kinematics at different locations on the platform. There are a number of different methods in use for the simulation of random waves corresponding to a given theoretical wave spectrum. A method which is commonly used involves the summation of a finite number of sinusoidal waves. The amplitudes of the
component waves are computed based on the energy density spectrum and the phase angles are assumed to be random variables, uniformly distributed between \((0, 2\pi)\) (see for example, Borgman, 1969). In this procedure, the random nature of the simulated quantities is assumed to come from the random phase angles. This method has been found to be satisfactory if sufficiently large number of wave components are used in the summation. Tucker et al. (1984) observed that this method introduces errors in the wave group statistics due to the assumption that the amplitudes of the component waves can be obtained deterministically from the target wave spectrum. Some methods of choosing both the amplitudes and phases in a random manner are also described in this study.

Another method for wave surface simulation is to choose unequally spaced frequency intervals to generate the amplitudes of the component waves. The amplitudes of the component waves in this case can be generated either deterministically or from a random distribution. This method appears to improve the randomness of the simulated time series. Several other methods are also proposed to improve the simulation process (see for example, Samii and Vandiver, 1984 and Stansberg, 1989). In the present study, the method of summation of sinusoids at unequal frequency intervals with random phase angles is used, mainly due to its simplicity.

3.4.1 Simulation of Wave Surface Elevation

Let \(S(\sigma)\) be a one-sided theoretical spectral density function to be simulated
and defined between the frequency limits 0 and $\sigma_M$, where $\sigma_M$ is the maximum frequency (rad/s) above which $S(\sigma)$ is zero. The wave elevation $\eta(x,t)$ can be written as

$$\eta(x,t) = \sum_{i=1}^{N} A_i \cos(k_i x - \sigma_i t + \epsilon_i),$$

where

$N = \text{number of wave components},$

$A_i = \text{amplitude of the ith wave component}; \quad = \sqrt{2S(\sigma_i) \Delta \sigma_i},$

$k_i = \text{wave number of the ith component},$

$x = x$-coordinate of the point, along the wave direction,

$\sigma_i = 2\pi f_i = \text{radian frequency of the ith component},$

$\epsilon_i = \text{phase angle of the ith component}.$

We can then write Eq. (3.16) as

$$\eta(x,t) = \sum_{i=1}^{N} \sqrt{2S(\sigma_i) \Delta \sigma_i} \cos(k_i x - 2\pi f_i t + \epsilon_i). \quad (3.17)$$

The phase angles $\epsilon_i$ are random variables uniformly distributed between $(0,2\pi)$. $k_i$ are given by the dispersion relation:
\( \sigma_i^2 = (2\pi f_i)^2 = g k_i \tanh(k_i h), \quad (3.18) \)

where \( h \) is the water depth and \( g \) is the acceleration due to gravity.

Eq. (3.17) indicates that the target spectrum \( S(\sigma) \) should be specified at frequency intervals \( \Delta \sigma_i = \sigma_i - \sigma_{i-1} \). These frequency intervals, \( \Delta \sigma_i \), can be made equal i.e., \( \Delta \sigma = \Delta \sigma_i = \sigma_i - \sigma_{i-1} = \sigma_M/N \). In such a case, however, the series repeats itself with a period \( 1/\Delta \sigma \). To avoid such a periodicity in the generated time series, we could choose unequally spaced frequency intervals. One way of choosing such intervals is by the use of cumulative spectrum (see Borgman, 1969). The cumulative spectrum, \( S_c(\sigma) \) is defined as

\[
S_c(\sigma) = \int_0^\sigma S(\sigma) \, d\sigma . \quad (3.19)
\]

Let \( \Delta S_c = S_c(\sigma_i) - S_c(\sigma_{i-1}) = \) constant for all \( i \). Since \( S_c(\sigma_i) - S_c(\sigma_{i-1}) = S(\sigma_i) \Delta \sigma_i \), selection of \( N \) frequencies in the above manner will result in unequally spaced intervals, with closely spaced intervals near the peak of the wave spectrum and wide intervals near the tail section of the spectrum. The constant can be determined by solving the following equation:

\[
S_c(\sigma_i) = \frac{i}{N} S_c(\infty) . \quad (3.20)
\]

We can solve Eq. (3.20) for a specified theoretical wave spectra such as the
Bretschneider or Pierson-Moskowitz (P-M) spectra as follows (see Borgman, 1969).

The P-M or Bretschneider spectra can be represented by

\[ S(\sigma) = \frac{A}{\sigma^5} \exp \left( -\frac{B}{\sigma^4} \right), \]  

(3.21)

where the constants A and B differ for the two spectra. The cumulative spectrum \( S_c(\sigma) \) is written as

\[ S_c(\sigma) = A \int_0^\sigma \sigma^{-5} \exp(-B \sigma^{-4}) \, d\sigma. \]

Evaluating the above integral and applying the limits one obtains

\[ S_c(\sigma) = \frac{A}{4B} \exp(-B \sigma^{-4}), \]  

(3.22)

and hence,

\[ S_c(\infty) = \frac{A}{4B}. \]  

(3.23)

We can then write Eq. (3.20) as follows:

\[ S_c(\sigma_i) = \frac{i}{N} \frac{A}{4B}. \]

We also have
Equating the two expressions for $S_e(\sigma)$, we obtain

$$\sigma_i = \left[ \frac{B}{\ln(N/i)} \right]^{1/4}. \quad (3.24)$$

The frequency components $\sigma_i$, $i = 1, 2, ..., N$ can be determined using Eq. (3.24). Once $\sigma_i$ are determined, Eq. (3.17) can be used to generate the time history of wave surface elevation at a given location. The values of the constants $A$ and $B$ will now be determined for both spectra. One-sided energy-density spectrum with circular frequency as the abscissa will be considered. The P-M spectrum is given by (see Pierson and Moskowitz, 1964)

$$S(\sigma) = \alpha g^2 \sigma^{-5} \exp \left[ -0.74 \left( \frac{\sigma U_w}{g} \right)^{-4} \right], \quad (3.25)$$

where $\alpha = 0.0081$ (Phillips constant) and $U_w$ is wind speed. In terms of the frequency of spectral peak $\sigma_o$, the spectrum can be written as (see Chakrabarti, 1987)
Noting that the root mean square value of water surface elevation is \( \sigma_s = \sqrt{\sigma_0^2 \ g / \sigma_o^2} \), and the significant wave height is \( H_s = 4 \sigma_s \), we can write Eq. (3.26) as follows:

\[
S(\sigma) = A \sigma^{-5} \exp[-B \sigma^{-4}],
\]

where \( A = B H_s^2/4 \), \( B = 5 \sigma_0^4/4 \), and \( \sigma_o^2 = 0.161 \ g / \sigma_s \).

The two-parameter Bretschneider spectrum (Bretschneider, 1979) is given by

\[
S(f) = \frac{5}{16} \frac{H_s^2}{f_o} \left( \frac{f}{f_o} \right)^{-5} \exp \left( -\frac{5}{4} \left( \frac{f}{f_o} \right)^{-4} \right),
\]

where \( H_s \) = significant wave height and \( f_o \) is the modal frequency of the spectrum.

Noting that \( s(f) \, df = s(\sigma) \, d\sigma \), and substituting \( \sigma = 2\pi f \), the spectrum in terms of circular frequency can be written as

\[
S(\sigma) = A \sigma^{-5} \exp(-B \sigma^{-4}),
\]

where \( B = 5 \sigma_0^4/4 \) and \( A = B H_s^2/4 \).
3.4.2 Simulation of Particle Kinematics

The wave elevation and particle kinematics are first simulated in the w-system (for a given point in the g-system) and then transformed into the g-system. During the simulation, particle kinematics at any point \( m (x_{gm1}, x_{gm2}, x_{gm3}) \) will be required to compute the forces acting on the platform. Using Eq. (A.17), the coordinates of this point in the w-system can be written as

\[
\begin{align*}
x_{wm1} &= x_{gm1} \cos \theta_w + x_{gm2} \sin \theta_w, \\
x_{wm2} &= -x_{gm1} \sin \theta_w + x_{gm2} \cos \theta_w, \\
x_{wm3} &= x_{gm3} - z_{gw}.
\end{align*}
\]  

(3.30)

Using Eq. (3.16), the wave elevation at this point is given by

\[
\eta(x_{wm1}, t) = \sum_{i=1}^{N} A_i \cos(k_i x_{wm1} - \sigma_i t + \epsilon_i).
\]

(3.31)

Other quantities, such as the particle velocities, acceleration, etc., can be obtained by the principle of linear superposition, in which the properties of component waves are summed, to obtain the kinematics in irregular waves. These are given as follows:

Velocity potential \( \phi_{wm}(x_{wm1}, x_{wm2}, x_{wm3}, t) \):

\[
\phi_{wm} = \sum_{i=1}^{N} \phi_{wi} = \sum_{i=1}^{N} gA_i \frac{\cosh k_i (x_{wm3} + h)}{\cosh k_i h} \sin(k_i x_{wm1} - \sigma_i t + \epsilon_i),
\]

(3.32)
where $\phi_{wi}$ is the velocity potential due to the $i$th component of the incident wave system at point $m$.

Particle horizontal velocity ($u_{wm1}$):

$$u_{wm1}(x_{wm1}, x_{wm3}, t) = \sum_{i=1}^{N} \frac{\partial \phi_{wi}}{\partial x_{wm1}}$$

$$= \sum_{i=1}^{N} g A_i k_i \frac{\cosh k_i(x_{wm3} + h)}{\sinh (k_i h)} \cos (k_i x_{wm1} - \sigma_i t + \epsilon_i)$$

Particle vertical velocity ($u_{wm3}$):

$$u_{wm3}(x_{wm1}, x_{wm3}, t) = \sum_{i=1}^{N} \frac{\partial \phi_{wi}}{\partial x_{wm3}}$$

$$= \sum_{i=1}^{N} g A_i k_i \frac{\sinh k_i(x_{wm3} + h)}{\cosh (k_i h)} \sin (k_i x_{wm1} - \sigma_i t + \epsilon_i)$$

Particle horizontal acceleration ($a_{wm1}$):

$$a_{wm1}(x_{wm1}, x_{wm3}, t) = \sum_{i=1}^{N} \frac{\partial u_{wm1}}{\partial t}$$

$$= \sum_{i=1}^{N} g A_i k_i \frac{\cosh k_i(x_{wm3} + h)}{\cosh (k_i h)} \sin (k_i x_{wm1} - \sigma_i t + \epsilon_i)$$
Particle vertical acceleration \( (a_{wm3}) \):

\[
a_{wm3}(x_{wm1}, x_{wm3}, t) = \sum_{i=1}^{N} \frac{\partial u_{wm3}}{\partial t} = -gA_i k_i \frac{\sinh k_i (x_{wm3} + h)}{\cosh (k_i h)} \cos (k_i x_{wm1} - \sigma_i t + \epsilon_i) \tag{3.36}
\]

Dynamic pressure, \( p_d \):

\[
p_d(x_{wm1}, x_{wm2}, t) = -\rho \sum_{i=1}^{N} \frac{\partial \phi_{wi}}{\partial t} = \rho \sum_{i=1}^{N} gA_i \frac{\cosh k_i (x_{wm3} + h)}{\cosh k_i h} \cos (k_i x_{wm1} - \sigma_i t + \epsilon_i). \tag{3.37}
\]

It may be noted that all the above expressions are given in the w-system, along the \( x_{w1} \)-axis which is in the direction of wave propagation. These quantities will have to be transformed to the g-system. Particle horizontal velocity in the g-system is then given by

\[
u_{wgm1} = u_{wm1} \cos \theta_w , \quad u_{wgm2} = u_{wm1} \sin \theta_w , \tag{3.38}
\]

and the particle vertical velocity is given by

\[
u_{wgm3} = u_{wm3} . \tag{3.39}
\]

Particle horizontal acceleration can be written as
and the particle vertical acceleration is given by

\[ a_{\text{wgm}3} = a_{\text{wm}3} \cdot \] (3.41)

In order to compute the Froude-Krylov force on the platform members, it is necessary to compute the pressure gradients of the incident wave pressure. These gradients need to be computed in a coordinate system in which the forces are computed. Eq. (3.29) is used in Eq. (3.36) to obtain the pressure gradients which are given below (in the g-system).

**Pressure gradients:**

\[
\frac{\partial p}{\partial x_{\text{gm}1}} = 
\begin{align*}
-pg \sum_{i=1}^{N} A_i \frac{\cosh k_i(x_{\text{gm}3} - z_{gw} + h)}{\cosh k_i h} \sin [k_i(x_{\text{gm}1} \cos \theta_w + x_{\text{gm}2} \sin \theta_w) - \sigma_i t + \epsilon_i] k_i \cos \theta_w 
\end{align*}
\] (3.42)

\[
\frac{\partial p}{\partial x_{\text{gm}2}} = 
\begin{align*}
-pg \sum_{i=1}^{N} A_i \frac{\cosh k_i(x_{\text{gm}3} - z_{gw} + h)}{\cosh k_i h} \sin [k_i(x_{\text{gm}1} \cos \theta_w + x_{\text{gm}2} \sin \theta_w) - \sigma_i t + \epsilon_i] k_i \sin \theta_w 
\end{align*}
\] (3.43)
If a steady current is present along with the waves, the following procedure is used to give velocity and direction of current at any given point. If $\vec{v}_c$ is the velocity of current and $\theta_c$ is the angle that direction of current makes with the $x_{g1}$-axis (this need not be equal to $\theta_w$, direction of wave approach), we can write the following equations (see Eq. (3.30)).

\[
x_{wm1} = x_{gm1} \cos \theta_c + x_{gm2} \sin \theta_c,
\]
\[
x_{wm2} = -x_{gm2} \sin \theta_c + x_{gm2} \cos \theta_c,
\]
\[
x_{wm3} = x_{gm3} - z_{gw}.
\]

The current velocity at this point $(x_{wm1}, x_{wm2}, x_{wm3})$ can now be obtained from the current data. The current velocity components along the global coordinate axes are given by

\[
u_{cgm1} = u_{cm1} \cos \theta_c, \quad u_{cgm2} = u_{cm1} \sin \theta_c, \quad u_{cgm3} = u_{cm3} = 0.
\]

Eq. (3.46) gives the components of the current velocity at any point m in the $g$-
system, for a current which makes an angle $\theta_e$ with the g-system.

The effect of large amplitude waves is treated in a similar manner as done in the regular wave case. Here, only the stretching method (see Eqs. (2.14)) is used. The surface elevation as obtained by Eq. (3.16) is used in determining the stretched particle kinematics in irregular waves.

In the previous sections the general method for the simulation of random wave surface elevation and particle kinematics is described. During the simulation of platform motions, the wave elevation and particle kinematics are required to be simulated at different points on the body, at each time step. The procedure followed to compute these quantities can be outlined as follows.

1. First a series of random numbers is generated from a Gaussian distribution. The number corresponds to the total number of component waves to be used in the simulation. These represent the random phase angles for each of the component waves used in the simulation.

2. For the theoretical wave spectra to be simulated, the constants $A$ and $B$ are then determined using Eqs. (3.27) or (3.29). The frequencies of the component waves are then generated using Eq. (3.24). The amplitude and wave number of the component waves are then determined using the relations given in Eqs. (3.16) and (3.18).

3. Thus the quantities $\sigma_i, k_i, e_i$, and $A_i$, which do not depend on time are first determined, before the start of the time-stepping procedure. At each time step the kinematics are required to be computed at any point $(x_{g1}, x_{g2}, x_{g3})$. The
coordinates of this point in the w-system are first determined using Eq. (3.30). The wave surface elevation is then determined using Eq. (3.31) and other particle kinematics using Eqs. (3.33) through (3.37). These quantities are given in the w-system. These are then transformed into the g-system using Eqs. (3.38) through (3.41). The same procedure is repeated for other points at this time step and for other time steps.
Fig. 3.1 Description of the rigid body motion
Chapter 4
Hydrodynamic Forces and Moments

4.1 Introduction

The equations governing the motion of a floating platform are described in Chapter 3 (see Eq. (3.15)). From this equation it is seen that the total external force $\bar{F}$ and moment $\bar{M}_b$ acting on the platform are to be evaluated at each time-step in order to solve for platform motions. This chapter describes two different methods commonly used to calculate these forces and moments. Expressions are derived for various components of forces which constitute the total force and moment. Modifications made in these approaches to accommodate large-amplitude platform motions are also discussed.

Wave forces on offshore structures can be computed in several different ways. All these methods make certain simplifying assumptions with regard to the nature of the flow, fluid properties and body geometry. There are two basic approaches to computing fluid forces on structures: i) potential-flow theory in which viscous effects are neglected, and ii) slender member method in which potential and viscous effects are included approximately.

In the potential theory approach, fluid forces are computed by first solving a boundary-value problem for the velocity potential representing the incident waves, diffracted waves by fixed body and radiated waves due to the prescribed motions of the body in an otherwise calm water. The decomposition of the total potential as described above will yield the Froude-Krylov force and the scattering force, which
together are called the wave exciting forces, and the added mass and wave damping
coefficients representing, respectively, the forces due to body acceleration and
velocity. For implementation of this method to different structures two approaches
are in use. The first one is the "strip theory" (see Salvesen et al., 1970) method
which is valid for slender three-dimensional shapes such as thin ships and lower hulls
of semisubmersible type of platforms. In this method, the wave- and motion induced
forces are first computed for two-dimensional sections, along the length of the body,
and then integrated over the length to obtain the total forces. This method has
been widely used, although, it can not include 3-dimensional end effects such as at
the ends of cylindrical members, and interference effects due to closely spaced
members whose geometry can not be represented accurately in the 2-dimensional
plane.

The second method is to use the complete 3-dimensional potential theory
using the source distribution technique or the boundary element method. Detailed
description of such procedures can be found, for example, in Garrison (1977). It
should be noted that in both of the methods used in potential theory, viscous effects
can not be included or can only be included in an approximate way by using some
kind of linearization of the nonlinear viscous forces. For many offshore platforms
consisting of cylindrical members, drag forces are important and should be
considered, particularly at resonant frequencies. In such situations, the slender
member approach is preferred in which the total forces are given by a modified form
of Morison's equation. However, in this method, wave diffraction effects,
frequency-dependence of the hydrodynamic coefficients and interference effects due to closely spaced members cannot be included. Both methods have advantages in some situations and prove to be inadequate in some other applications. Paulling (1981) discussed various issues relating to the use of these methods. In particular, it was pointed out in this paper that due to the under estimation of damping by the slender-member method, the tendon tensions of a TLP are over estimated as compared to the peak tension values predicted by the potential theory. On the other hand, the heave motion of a semisubmersible platform is predicted better by the slender-member method as compared to the potential theory method, particularly near resonance. This is mainly due to the inclusion of viscous damping in the slender-member method, whereas it is totally neglected in the potential theory method. Moreover, the radiation damping predicted by the potential theory is very small at resonant heave frequency of semisubmersible platforms.

Thus, it appears that no single theoretical model is satisfactory to predict the resonant response of platforms, and methods which can include both viscous and potential effects should be developed. The two methods of computing the forces acting on platforms are discussed in more detail in the following sections. Both methods are used, later on, to simulate platform motions. It should be noted that inclusion of the quadratic viscous damping in the equations of motion requires that the equations be solved in time domain. On the other hand, linearized drag forces can be handled in a frequency-domain procedure. In this case, however, some of
the nonlinear effects, such as the sub- and super-harmonic responses are lost due to the linearization process.

4.2 Hydrodynamic Forces Using Morison's Equation

In this approach the total force on the platform can be written as follows.

\[ \bar{F} = \bar{F}_S + \bar{F}_P + \bar{F}_A + \bar{F}_V + \bar{F}_M, \]  

(4.1)

where \( \bar{F}_S \) is the hydrostatic (buoyancy) force, \( \bar{F}_P \) is the Froude-Krylov force due to the dynamic pressure force, \( \bar{F}_A \) is the force due to body- and wave-particle accelerations, \( \bar{F}_V \) is the velocity (viscous drag) force, and \( \bar{F}_M \) is the force due to the positioning system. Other forces such as due to wind can be added to the right hand side of the above equation. The force due to platform weight is assumed to have been included in the static force term ( \( \bar{F}_S \) ). The first four terms on the right hand side of Eq. (4.1) constitute the hydrodynamic forces. The determination of these forces is discussed in the following sections.

When the platform consists of several cylindrical members, whose dimensions are 'small' compared to the wave length, a semi-empirical model commonly known as Morison's equation (see Morison et al., 1950) is used to compute wave forces on platforms. One of the basic assumptions of this method is that the wave kinematics are not affected by the presence of the body due to its small member dimensions.
It is also assumed that the members of the platform are deeply submerged so that the free surface effects on the hydrodynamic coefficients can be neglected. With these assumptions, the force per unit length of a member at any point \( m \) on the member can be written as

\[
\mathbf{dF} = \int_{s} p \mathbf{n} \, ds + \rho \pi D^2 C_m \mathbf{r}_{mn}^n + \frac{1}{2} \rho C_d D \mathbf{v}_{mn}^n \left| \mathbf{v}_{mn}^n \right|, \quad (4.2)
\]

where

\( p \) = hydrodynamic pressure due to the incident wave,

\( \mathbf{n} \) = unit normal vector pointing out of the body,

\( s \) = surface of the element,

\( D \) = diameter of the member,

\( C_m, C_d \) = added mass and drag coefficients respectively,

\( \mathbf{v}_{mn}^n, \mathbf{a}_{mn}^n \) = relative velocity and acceleration, at any point \( m \), normal to the \( n \)th member axis,

\( \rho \) = mass density of the fluid.

Eq. (4.2) is a modified form of Morison's equation. The first term on the right hand side of the above equation is the force due to the undisturbed incident wave, which is also known as the Froude-Krylov force. The second term is the acceleration force, which is considered proportional to the relative acceleration.
normal to the member axis, and the last term is the relative velocity drag force. Computation of these forces are discussed in detail in the following sections. It should be noted that in Eq. (4.2), and throughout this study, the superscript \( n \) refers to the \( n \)th member and the subscript \( N \) refers to the normal component to the member axis of the quantity involved.

4.2.1 Froude-Krylov Force

The first term in Eq. (4.2) gives the pressure force due to the incident wave, also known as the Froude-Krylov force. For convenience, the surface integral can be converted into a volume integral using Gauss's theorem. That is

\[
\mathbf{d} \mathbf{F}_p = - \int_{s} p \mathbf{n} ds = - \int_{v} \mathbf{v}_p dv ,
\]

where \( v \) is the volume enclosed by the closed surface \( s \). If \( A \) is the cross-sectional area of the cylinder, then the elemental volume \( dv = A dx_{II} \), where \( dx_{II} \) is the length of the element along the member axis. Then the Froude-Krylov force on the element can be written as

\[
\mathbf{d} \mathbf{F}_p = -A \mathbf{v}_p dx_{II} ,
\]

where it is assumed that, due to the assumption of small cross-sectional area of the cylinder, the pressure gradient around the cylinder can be approximated by its value at the centerline of the member axis. This will allow us to replace the volume
integral in Eq. (4.3) by a line integral along the member length. The total Froude-Krylov force on a member is then obtained by integrating the force given by Eq. (4.4) along the instantaneous submerged length, $l^a(t)$, of the member

$$\vec{F}_p = -A \int_{l^a(t)} \vec{v}_p \, dx_{11},$$  \hspace{1cm} (4.5)$$

It should be noted that $\vec{F}_p$ can be computed in any coordinate system, depending on the system in which the pressure gradients $\vec{v}_p$ are evaluated. For example, to obtain the forces in the g-system, Eq. (4.4) will be of the form

$$d\vec{F}_p = -A \begin{bmatrix} \frac{\partial p}{\partial x_{gm1}} \\ \frac{\partial p}{\partial x_{gm2}} \\ \frac{\partial p}{\partial x_{gm3}} \end{bmatrix} \, dx_{11},$$  \hspace{1cm} (4.6)$$

in which the pressure gradients at point m are evaluated in the g-system. Due to the 'stretching' method used for particle kinematics, the expressions for pressure and its gradients will be slightly modified from those given in Section 3 (see Eqs. (3.42), (3.43) and (3.44)). Detailed derivations of these equations is given in Appendix B. It should be noted that Eq. (4.6) gives the three components of the Froude-Krylov force on a elemental segment of the member, including the force along the member length. This component, when integrated over the submerged length of the member, yields the end-plane pressure force. Determination of moment due to this force is
discussed in subsequent paragraphs, along with moments due to other force components.

**Hydrostatic pressure forces**

In this study, hydrostatic pressure forces are also computed over the instantaneous wetted length of each of the platform members and hence this force is also a nonlinear function of time. The same procedure as followed for dynamic pressure force is used here. That is, the hydrostatic force on an elemental segment is given by

\[
d\bar{F}_s = -\int_P n \, ds = -\int_V \nabla P_s \, dv = -A \nabla P_s \, dx_{ii}
\]

where \( P_s = -\rho g (x_{gm} - z_{gw}) \) is the hydrostatic pressure. The total force on the member is then obtained by integrating the elemental force along the immersed length of the member using an equation similar to Eq. (4.5). Thus the hydrostatic forces and moments are treated as external forces instead of considering them as restoration forces, as done in the linear analysis, in which a hydrostatic stiffness matrix, proportional to body displacements, is derived. It can be shown that Eq. (4.7) can be reduced to the form of the stiffness matrix if we make use of the small
angle assumption in the coordinate transformation matrix. However, the advantage of the current method is that variable submergence, which can occur due to body motion as well as change in wave surface elevation, can be taken into account in a precise manner.

The submerged length of each platform member is determined at each time step and divided into a number of segments. The hydrostatic and Froude-Krylov pressure forces are determined for each such segment and then numerically integrated over the submerged length of the member using Eq. (4.5). The forces corresponding to the first and second terms of Eq. (4.1) are now determined.

4.2.2 Velocity and Acceleration Forces

It is necessary, first, to determined the rigid body velocity and acceleration at any point on the platform member. These can then be used in Eq. (4.2) to compute the relative velocity and acceleration forces.

Rigid body velocity and acceleration:

Referring to Fig. (2.1), we can write the velocity of the body at point \( m \) on the \( n \)-th member as follows:

\[
\bar{v}_{bm}^n(t) = \bar{u}_r + (\bar{\omega} \times \bar{r}_{bm}^n),
\]  

where \( \bar{u}_r \) is the translational velocity of the origin of the \( b \)-system with respect to the \( g \)-system, \( \bar{\omega} \) is the angular velocity of the body in the \( b \)-system and \( \bar{r}_{bm}^n \) is the position...
vector of point m on the nth member in the b-system. To express \( \vec{v}_{bm}(t) \) in the g-

system, one can write

\[
\vec{v}_{bm}^n(t) = \vec{u}_T + [P]^T(\vec{\omega} \times \vec{r}_{bm}^n),
\]

(4.9)

where \([P]\) is the transformation matrix (see Eq. (A.8)) used to express the quantity

enclosed in parenthesis in the g-system. The acceleration of point m is obtained by

taking the time-derivative of Eq. (4.8), that is;

\[
\frac{d}{dt} \left( \vec{v}_{bm}^n(t) \right) = \frac{d}{dt}(\vec{u}_T) + \frac{d\vec{\omega}}{dt} \times \vec{r}_{bm}^n + \vec{\omega} \times \frac{d}{dt}(\vec{r}_{bm}^n).
\]

(4.10)

All time-derivatives in the above equation should be taken in the g-system. The
time rate of change of any vector \( \vec{r} \), which is fixed in a rotating frame of reference

is given by

\[
\left\{ \frac{d}{dt}(\vec{r}) \right\}_g = \left\{ \frac{d}{dt}(\vec{r}) \right\}_b + (\vec{\omega} \times \vec{r}).
\]

(4.11)

In Eq. (4.11), the subscripts outside the parenthesis indicate the coordinate system

in which the derivative is taken. This notation is also used in this study to represent

forces and moments resolved along a particular coordinate system. Defining

\[
\left\{ \frac{d}{dt}(\vec{a}_T) \right\}_g = \vec{a}_T, \text{ the absolute translational acceleration of the origin of the b-system,}
\]

we can write Eq. (4.10) as follows:
\[
\ddot{a}_{bn}(t) = \ddot{a}_r + \left\{ \frac{d\dot{\omega}}{dt} \times \dot{r}_{bn} \right\}_g + \left\{ \dot{\omega} \times (\dot{\omega} \times \ddot{r}_{bn}) \right\}_g \\
= \ddot{a}_r + [P]^T \left[ \frac{d\dot{\omega}}{dt} \times \dot{r}_{bn} \right] + [P]^T \left[ \dot{\omega} \times (\dot{\omega} \times \ddot{r}_{bn}) \right],
\]

(4.12)

where \( \ddot{a}_{bn}(t) \) is the absolute acceleration of the point \( m \) in the \( g \)-system. The last term in Eq. (4.12) is the centripetal acceleration which is a nonlinear function of the angular velocity. Eqs. (4.9) and (4.12) thus give the absolute velocity and acceleration of any point \( m \) on the \( n \)th member of the body.

**Acceleration force:**

The acceleration force per unit length of the platform member is given by (see Eq. (4.2))

\[
d\bar{F}_A = \rho \frac{\pi}{4} D^2 C_m \bar{a}_{rmN},
\]

(4.13)

The relative acceleration between the body and fluid is given by

\[
\ddot{a}_{rm} = \ddot{a}_{wm} - \ddot{a}_{bn},
\]

(4.14)

where \( \ddot{a}_{wm} \) and \( \ddot{a}_{bn} \) are the wave particle and rigid body accelerations, at any point \( m \) on the \( n \)th member of the body. The particle accelerations (Eqs. (3.39) and (3.40)) are represented here as \( \ddot{a}_{wm} \). Eq. (4.12) is used to obtain \( \ddot{a}_{bn} \). The normal component to the member axis of the relative acceleration can be written as (see Chitrapu, 1988)
where the elements of the matrix \([\mathbf{R}]\), containing the direction cosines of the member \(n\), are given by Eq. (2.21).

The elemental acceleration force as given by Eq. (4.13) is then numerically integrated over the submerged length of the member \(n\) to obtain total acceleration force on the member, that is,

\[
\bar{F}^a_A = \int_{t^0} d\bar{F}_A dx_{11}.
\]

The computation of relative velocity drag force is already discussed in Chapter 2. A similar procedure is followed here to obtain the drag force \(\bar{F}_V^a\), and therefore this force will not be discussed here.

**Evaluation of moments:**

The moments acting on the platform should be computed in the body-fixed system, as discussed in Section 3.2. Also, in order to be able to know the sensitivity of response to different force components, we need to compute different forces and moments separately. The elemental forces (due to pressure, velocity and acceleration) are first transformed into the b-system, that is,

\[
(d\bar{F})_b = [P] d\bar{F},
\]
where \( \mathbf{dF} \) is any one of the elemental forces given by Eqs. (4.4), (4.7) or (4.13), and \([P]\) is the previously defined transformation matrix (Eq. A8). The elemental moment, about the origin of the \( \mathbf{b} \)-system, due to this force is then obtained by

\[
\mathbf{dM}_b = \mathbf{r}_m \times (\mathbf{dF}_b),
\]

(4.18)

where \( \mathbf{r}_m \) is the position vector of the point \( \mathbf{m} \) at which the elemental force is computed. Eq. (4.17) is used to compute the moments due to pressure, velocity and acceleration forces.

The elemental forces and moments obtained in this manner are then integrated over the immersed length of platform members using equations similar to Eq. (4.16). Simpson's \( \frac{1}{3} \)-rule is used for numerical integration. The member-wise forces and moments are then summed over all members of the platform to obtain the total hydrodynamic force and moment acting on the platform. Details of this procedure and inclusion of these forces in the equations of motion are discussed in Chapter 5.

4.2.3 End-plane Forces

In the previous sections, we have determined the velocity- and acceleration-dependent force components normal to member axis. We also need to compute the forces on the end-planes of the members to obtain the axial forces. These forces are computed for all members at each submerged end of the member. For
members with free ends, these forces will provide the additional axial forces and for members whose ends intersect other members, these forces will compensate for the erroneous force computed on the member without considering the reduction in exposed area due to the intersection. The end-plane forces due to dynamic pressure are already included as explained in Section 4.2.1. Determination of the end-plane forces due to fluid particle velocity and acceleration are described below.

End-plane velocity force

The end-plane drag force at point \( s \) of the member is given by

\[
\vec{F}_{VE}^s(t) = \frac{1}{2} \rho C_d A \vec{V}_r \| \vec{V}_r \| ,
\]

(4.19)

where \( \vec{V}_r \) is the relative velocity along the member axis. Relative velocity at point \( s \) in the g-system is given by

\[
\vec{V}_rs(t) = \vec{V}_{w}(t) + \vec{V}_{c}(t) - \vec{V}_{b}(t),
\]

(4.20)

where the three terms on the right hand side represent the velocities due to wave particle, current and body motion, respectively. The relative velocity in the \( l \)-system is given by (see Eq. (A.19))

\[
\{ \vec{V}_rs(t) \}_l = [Q_L] \vec{V}_rs(t),
\]

(4.21)
where \([Q_L]\) is the transformation matrix given in Appendix A. If we denote, by

\(\bar{F}^s_{VEI}(t)\), the end-plane velocity force in the \(I\)-system, its component in the \(x_{II}\)-direction in the local system is thus given by

\[
F^s_{VEI}(t) = \frac{1}{2} \rho C_A v_{rsl} |v_{rsl}|,
\]

(4.22)

where \(v_{rsl}\) is the component of the local velocity \(\tilde{v}_s(t)\) along the \(x_{II}\)-axis. This force is transformed into the \(g\)-system as follows (see Eq. (A.20)).

\[
\bar{F}^s_{VE}(t) = [Q_L]^T \bar{F}^s_{VEI}(t),
\]

(4.23)

which gives three components of the end-plane force in the \(g\)-system. The end-plane force on the submerged end, \(e\), of the member is also determined in a similar fashion, if this end is submerged.

**End-plane acceleration force:**

The acceleration force component in the \(x_{II}\)-direction due to fluid acceleration at point \(s\) of the member end-plane (in the \(I\)-system) is given by (see Garrison, 1982)

\[
F^s_{AEI} = \rho \left( \frac{D^3}{2} \right) C_{MII} \omega_{II},
\]

(4.24)
where $a_{w11}$ is the local wave particle acceleration along the $x_{11}$-axis, $C_{M11}$ is the added mass coefficient for a truncated cylinder in axial motion. For a truncated cylinder whose end is deeply submerged (a tension-leg platform column) Garrison's (1982) results based on the potential theory show that the added mass coefficient is constant and equal to 2.05 for long wave periods. This value is used in Eq. (4.24) to compute the end-plane acceleration force. The force, thus obtained, is then transformed into the g-system, as done for end-plane velocity forces (see Eq. (4.23)).

The fluid acceleration, which is first obtained in the g-system, should be resolved along the $x_{11}$-axis, to obtain $a_{w11}$ (see Eq. (4.21)).

It is to be noted that if the end point $e$ of the member pierces the free surface the end-plane force at this point is not computed. Also, it is assumed that the vertical coordinate of point $s$ is equal to or below the point $e$. This condition is checked in the program and the coordinates of the two points are interchanged if necessary. This is also required to be done in order for the force computations to proceed in the correct direction along the member length.

The moments due to these end-plane forces are obtained using similar procedure as used for elemental forces normal to the member axis (see Eqs. (4.17) and (4.18)). The end-plane forces and moments, as described in this section, are computed at each time-step and then included in Eq. (4.1) to obtain total force and moment acting on the platform.
4.3 Hydrodynamic Forces Using the Potential Theory

In this approach the total force acting on the platform can be written in the following manner:

\[ \mathbf{F} = \mathbf{F}_s + \mathbf{F}_p + \mathbf{F}_R + \mathbf{F}_D + \mathbf{F}_v + \mathbf{F}_M, \]  

(4.25)

where \( \mathbf{F}_R \) is the radiation force due to body motion and \( \mathbf{F}_D \) is the diffraction or scattering force due to the diffraction of waves by a fixed body. The remaining terms in Eq. (4.25) have the same definitions as those given in Eq. (4.1). We can see from Eqs. (4.1) and (4.25) that the main difference in the computation of forces in the two methods comes in the way the wave excitation and hydrodynamic forces are computed. As mentioned before, hydrodynamic forces using the potential theory are obtained by solving the boundary value problem described in Section 5.3.1. In this procedure, it is assumed that wave and body motions are sinusoidal with 'small' amplitudes. With this assumption, the boundary value problem is solved for a number of wave frequencies. However, in order to solve the equations of motion in time, we need to obtain the radiation forces and wave exciting forces as a function of time. Moreover, the body may be making arbitrary motions in irregular waves. We then have to use alternate approaches suitable for time-domain computations. These are discussed in the following sections.

The approach followed here is to combine the 3-dimensional potential theory results in the time-domain in which the effects of large angle platform motions are included. Viscous effects are also included through the nonlinear drag term of
Morison's equation. The method used is similar to that of de Kat (1988) who used 2-dimensional strip theory results in time domain to simulate the large-amplitude motions of ships.

4.3.1 Radiation Forces

The initial value problem of forced motion of a body has been studied by a number of researchers. Cummins (1962) proposed the method of impulse response function and a decomposition for the velocity potential to represent the forced, arbitrary, but 'small' motions of a ship. Using this method of decomposition, he derived the equations of motion of the ship subjected to arbitrary exciting forces. Extensive theoretical investigations of this problem were also carried out by Ogilvie (1964) and Wehausen (1967, 1971). In these studies, the force on the body at any time t is given by the conditions at t=0 and a kernel function representing the hydrodynamic properties of the body. This kernel function is integrated from t=0 to the present time, thus taking into account the "memory" effect of the waves that have already been generated prior to the time t. Ogilvie (1964) derived expressions showing the relationship between the frequency-dependent added mass and damping coefficients and the kernel functions in the time domain.

Application of this method to the prediction of ship and platform motions in the time domain has been undertaken in several studies. Van Oortmerssen (1976) used Cummins' decomposition for velocity potential to derive the expression for radiation forces acting on the ship. In that study, the kernel functions are obtained
from the frequency-dependent hydrodynamic coefficients using the relationships derived by Ogilvie (1964). The equations of motion are then solved in the time domain. A similar approach has been used by Chou et al. (1983) to determine the motions of a tension leg platform. Van Oortmerssen et al. (1986) and more recently de Kat and Paulling (1989) also used this approach for ship motion problems. It should be noted that all the above studies use the indirect formulation, that is, the kernel functions are determined using the hydrodynamic coefficients obtained from the solution of the boundary-value problem in frequency domain. Recent studies in this area attempt to solve the problem using a direct formulation, i.e., in time domain. In this method, the kernel functions are evaluated by using the time-dependent Green function in the solution of the boundary-value problem. Some examples of this approach include the works of Liapis and Beck (1985), Beck and Liapis (1987) and Beck and Magee (1991).

In this study, the theory based on Cummins' decomposition is used to compute the radiation forces. Since the theoretical details are given in Cummins (1962) and Wehausen (1967, 1971), they are only briefly discussed here. The velocity potential $\phi$ due to an arbitrary small motion of a body in the $k$th mode of motion can be written as

$$\phi(t) = \dot{x}_k \psi_k + \int_{-\infty}^{t} \varphi_k(t-\tau) \dot{x}_k(\tau) d\tau,$$

(4.26)
where $\psi_k$ is the velocity potential representing the instantaneous reaction of the body to the impulsive displacement and $\varphi_k$ is the velocity potential of the free-surface disturbance generated by the body motion, $\phi$ is the total velocity potential and $\dot{x}_j$ is the velocity of the body. The subscript $k$ in all the quantities in Eq. (4.26) refers to the mode of body motion. The various boundary conditions that should be satisfied by these potentials are discussed in Cummins (1962). Noting that the dynamic pressure $p = -\rho \partial \phi / \partial t$, we can write the following expressions for force (and moment):

\begin{equation}
F_{x_j}(t) = -\sum_{k=1}^{6} \left[ m_{jk} \ddot{x}_k(t) + \int_{-\infty}^{t} L_{jk}(t-\tau) \dot{x}_k(\tau) d\tau \right], \quad j = 1, 2, \ldots, 6, \tag{4.27}
\end{equation}

where

\begin{equation}
m_{jk} = \rho \int \int s \psi_k n_j \, ds, \tag{4.28}
\end{equation}

\begin{equation}
L_{jk}(\tau) = \rho \int \int s \frac{\partial \phi_k(\tau)}{\partial t} n_j \, ds, \tag{4.29}
\end{equation}

in which $n_j$ is the $j$th component of the unit normal vector pointing out of the body, $s$ is the area of the mean wetted surface. The first term on the right hand side of Eq. (4.27) is analogous to the added mass force, and the second term is the memory-
effect integral based on the velocity of the body. Wehausen (1967) used the following decomposition for the forced motion potential:

$$\phi_p(p;t) = \sum_{k=1}^{6} \int_{0}^{t} \phi_k(p; t-\tau) \ddot{x}_k(\tau) \, d\tau,$$  \hspace{1cm} (4.30)

where $\phi_k(p; \tau)$ is the velocity potential at point $p$ due to the impulse in the $k$th mode of motion at time $\tau$. The force corresponding to this potential is given as

$$F_{RJ}(t) = -\sum_{k=1}^{6} \left[ \mu_{jk} \ddot{x}_k(t) + \int_{0}^{t} K_{jk}(t-\tau) \ddot{x}_k(\tau) \, d\tau \right],$$  \hspace{1cm} (4.31)

where

$$\mu_{jk} = \int \phi_k(p; 0) n_j \, ds,$$  \hspace{1cm} (4.32)

$$K_{jk} = \int \phi_{kt}(p; t) n_j \, ds,$$  \hspace{1cm} (4.33)

in which $\phi_{kt}$ denotes $\partial \phi_k / \partial t$. Noting the changes in definition of the potentials and the quantities $L_{jk}(t)$, and $K_{jk}(t)$, it can be shown that the two expressions for force, Eqs. (4.27) and (4.31) are equivalent. Integrating by parts once, Eq. (4.30) can be written as follows:
The force equation corresponding to this potential can be written as

\[
\phi_p(p; t) = \sum_{k=1}^{6} \left[ \phi_k(p; 0) \dot{x}_k(t) - \phi_k(p; t) \dot{x}_k(0) + \int_0^t \dot{x}_k(\tau) \phi_{kt}(p; t-\tau) \, d\tau \right].
\] (4.34)

Using Eq. (4.32) and (4.33), we can write Eq. (4.35) as follows:

\[
F_{Rj(t)} = -\sum_{k=1}^{6} \left[ \rho \int_s \phi_k(p; 0) n_j \, ds \dot{x}_k(t) - \rho \int_s \phi_{kt}(p; t) n_j \, ds \dot{x}_k(0) \right.
\]

\[
+ \int_0^t \dot{x}_k(\tau) \, d\tau \rho \int_s \phi_{kt}(p; t-\tau) n_j \, ds \right].
\] (4.35)

Assuming that initial velocity of the body \( \dot{x}_k(0) = 0 \), the two expressions for force Eqs. (4.36) and (4.27) are identical with the following definitions.

\[
\psi_k = \phi_k(p; 0), \quad \varphi_k(\tau) = \phi_{kt}(p; \tau).
\] (4.37)

Also from Eqs. (4.28), (4.29), (4.32), (4.33) and (4.37) we can write

\[
\mu_{jk} = \rho \int_s \phi_k(p; 0) n_j \, ds = \rho \int_s \psi_k n_j \, ds = m_{jk},
\]

\[
K_{jk}(t) = \rho \int_s \phi_{kt}(p; t) n_j \, ds = \rho \int_s \phi_k(t) n_j \, ds = L_{jk}(t).
\] (4.38)
Therefore, Eqs. (4.27) and (4.31) are equivalent.

In this study, for convenience, velocity based memory-effect integrals are used to compute radiation forces. It may be noted that Eq. (4.27) is applicable to arbitrary motions of the body, including sinusoidal motions. Ogilvie (1964) derived expressions similar to Eq. (4.27) for sinusoidal body motions and showed that such equations are in the form generally used in the frequency-domain formulation of the equations of motion and that the frequency-dependent hydrodynamic coefficients are related to the time-domain counterparts in the following manner:

\[ L_{jk}(\tau) = \frac{2}{\pi} \int_{0}^{\infty} B_{jk}(\sigma) \cos \sigma \tau \, d\sigma, \]

\[ A_{jk} = A_{jk}(\sigma') + \frac{1}{\sigma'} \int_{0}^{\infty} L_{jk}(\tau) \sin \sigma' \tau \, d\tau, \]  

(4.39)

where \( A_{jk} \) are the constant (genuine) added mass coefficients (which are previously defined as \( m_{jk} \) and \( \mu_{jk} \)), \( \sigma' \) is any arbitrary frequency, \( A_{jk}(\sigma) \) and \( B_{jk}(\sigma) \) are the frequency-dependent added mass and damping coefficients, \( \sigma \) is the wave frequency and \( \tau \) is the time lag, respectively. Eqs. (4.27) and (4.39) provide all the necessary information to calculate the radiation force and moment acting on the platform due to arbitrary body motions. The force can be evaluated in the time domain using the frequency-domain hydrodynamic coefficients. It should be noted that these equations are valid for arbitrary, but small motions of a body. These linear radiation forces are to be incorporated into a large-amplitude motion simulation.
model, despite the apparent inconsistency. de Kat and Paulling (1989) proposed two methods to do this for the problem of ship motions with forward speed. The same approach is used here, with slight modifications due to the differences in coordinate systems, which are necessitated due to the absence of forward speed.

The equations of motion (see Eq. (3.15)) require that the total external forces should be evaluated in the g-system, while the moments should be expressed in the b-system. Let \( \vec{v} \) and \( \vec{a} \) be the absolute velocity (translational) and acceleration vectors of the center of gravity of the body in the g-system. Also let \( \vec{\omega} \) and \( \frac{d\vec{\omega}}{dt} \) be the rotational velocity and accelerations in the b-system. The transformation of rotational velocities and accelerations between the b- and g-systems can be written as follows.

\[
\{\vec{\omega}\}_g = \frac{d\vec{\theta}}{dt} = [B]^{-1} \{\vec{\omega}\}_b, \quad \left\{\frac{d\vec{\omega}}{dt}\right\}_g = [P]^T \left\{\frac{d\vec{\omega}}{dt}\right\}_b
\]  

(4.40)

where \([B]\) is the transformation matrix relating the time-derivative of the Euler angles \( \vec{\theta} \) with angular velocity vector \( \vec{\omega} \) in the b-system, \([P]\) is the previously defined transformation matrix consisting of the Euler angles for transformation of any vector from the b- to the g-system. Two different methods are used to determine the radiation forces and moments. These methods differ in the way the velocities and accelerations used in Eq. (4.27) are computed.
Method (1):

In this method, it is assumed that the rotational angles are small so that no distinction is made between the rotated position of the body and its equilibrium position. This method is more consistent with the linear theory assumption. The procedure is outlined in the following steps.

(i) The linear and angular velocities (and accelerations) are first determined in the space-fixed (g-) system.

(ii) It is assumed that the platform is upright in the g-system (with the center of gravity located at the origin of the g-system) with the velocities and accelerations determined in (i).

(iii) Radiation forces and moments are computed in the g-system. The moments (which need to be computed in the b-system) in the b-system are assumed to be the same as those in the g-system (a consequence of assuming the body to be upright in the g-system).

Denoting the velocity \( \mathbf{v} \) and acceleration \( \mathbf{a} \) by \( \mathbf{v}^* \) and \( \mathbf{a}^* \), respectively, the radiation force and moment in the jth mode of motion (in the g-system) can be written as

\[
F_{Rj}(t) = -A_{jk}^* \mathbf{a}^* - \int_0^t L_{jk}(\tau) \mathbf{v}^*_k(t-\tau) d\tau, \quad j,k = 1,2,\ldots,6, \tag{4.41}
\]

where \( \mathbf{a}^* \) and \( \mathbf{v}^* \) are components of vectors \( \mathbf{a}^* \) and \( \mathbf{v}^* \), as given below:
\[ \ddot{a}^* = \{a_k^*\} = \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{v}_3 \\ \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix}, \quad \ddot{\theta}^* = \{\theta_k^*\} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \]

The forces, as computed above, are given in the g-system. It is to be noted that the hydrodynamic coefficients \((A_{jk} \text{ and } B_{jk})\) are obtained from the 3-D potential theory and these coefficients and exciting forces are also given in the g-system. The moments in the b-system are assumed to be the same as those given in Eq. (4.41).

**Method (2):**

In this method, the translational velocities and accelerations are first determined in the g-system. These are then resolved into the b-system and then used to calculate forces and moments in the g-system. The velocities and accelerations are given by

\[ \ddot{v}^* = \begin{bmatrix} [P] \ddot{v} \\ [B]^{-1} \ddot{\omega} \end{bmatrix}, \quad \ddot{\theta}^* = \begin{bmatrix} [P] \frac{dv}{dt} \\ \frac{d\omega}{dt} \end{bmatrix}. \] (4.42)

The forces and moments are calculated using Eq. (4.41). The moments are then transformed back to the b-system, where the large angles are retained in the transformation process, that is,
In Eq. (4.45) the subscript b outside the parenthesis on the left hand side indicates that the components of force are taken in the b-system.

4.3.2 Scattering Forces

The scattering force in the time domain is computed using the wave exciting forces obtained in the frequency domain by solving for the diffraction potential. Modifications similar to those made for radiation force are made here to obtain the scattering force for a platform making large-amplitude motions. Here, it is assumed that scattering potential and force components could be obtained separately from the Froude-Krylov forces in the frequency-domain solution of the wave exciting forces.

Let us assume that the wave surface elevation (for a regular wave) is given in the following form:

\[ \eta(x,t) = A e^{i(kx - \sigma t + \epsilon)} \quad i = \sqrt{-1}, \]  

where \( A, \sigma, k \) and \( \epsilon \) are the amplitude, circular frequency, wave number and phase angle, respectively, of the incident wave. The wave elevation, as given by Eq. (4.44), at the center of gravity of the platform is considered for the evaluation of diffraction forces. The diffraction forces and moments, as obtained from linear diffraction theory, are also given in the g-system.
It is assumed that diffraction forces are given in terms of frequency-domain transfer functions, from linear diffraction theory, as follows:

\[
H_j^D(\sigma) = \frac{F_j^D(\sigma)}{A}, \quad j = 1, 2, ..., 6, \quad (4.45)
\]

where \(F_j^D(\sigma)\) is the complex diffraction force for the frequency \(\sigma\). The diffraction force in the time-domain, with the correct phase can be given as (see de Kat and Paulling, 1989).

\[
F_{Dj}(t) = AH_j^D(\sigma)e^{i\theta}, \quad \theta = kx - \sigma t + \epsilon, \quad j = 1, 2, ..., 6, \quad i = \sqrt{-1}. \quad (4.46)
\]

Eq. (4.46) is obtained by taking the Fourier transform of \(F_j^D(\sigma)\) and then using the convolution theorem. Eq. (4.47) gives the diffraction force for a regular wave of amplitude \(A\), where the phase information is contained in \(H_j^D(\sigma)\). Writing

\[
H_j^D(\sigma) = HR_j(\sigma) + iHI_j(\sigma), \quad (4.47)
\]

where \(HR_j(\sigma)\) and \(HI_j(\sigma)\) are the real and imaginary parts, we can write Eq. (4.46) as follows:

\[
F_{Dj}(t) = A\{HR_j(\sigma) + iHI_j(\sigma)\}(\cos \theta + i \sin \theta)
\]

\[
= A\{HR_j(\sigma)\cos \theta - HI_j(\sigma)\sin \theta\}, \quad j = 1, 2, ..., 6, \quad (4.48)
\]

where only the real part of Eq. (4.48), which has a physical meaning, is considered.
For long-crested random waves, the diffraction force can be obtained in a similar manner.

Wave elevation at time t and at a given location x can be written as

$$\eta(x,t) = \sum_{n=1}^{N} A_n e^{i(k_n x - \sigma_n t + \epsilon_n)} ,$$

(4.49)

where $A_n$, $k_n$, $\sigma_n$, $\epsilon_n$ are, respectively, the amplitude, wave number, frequency and phase angles of the nth wave component. Diffraction force at time t is then given by

$$F_{D_j}(t) = \text{Re}\left\{ \sum_{n=1}^{N} H_j^D(\sigma_n) A_n e^{i(k_n x - \sigma_n t + \epsilon_n)} \right\} .$$

(4.50)

Using Eq. (4.47) we can write Eq. (4.50) as follows:

$$F_{D_j}(t) = \sum_{n=1}^{N} A_n \{ H_R(\sigma_n) \cos(k_n x - \sigma_n t + \epsilon_n) - H_I(\sigma_n) \sin(k_n x - \sigma_n t + \epsilon_n) \} .$$

(4.51)

In Eqs. (4.49) and (4.50), the wave elevation simulated at the center of gravity of the body is considered.

The force and moment components given in Eq. (4.51) are expressed in the g-system. Using similar arguments as given for the radiation forces for large angle motions of a platform, the diffraction moments in b-system are assumed to be the same as those given by Eq. (4.51) in Method (1). The moments are transformed to the b-system when Method (2) is used.
It may be noted that the Froude-Krylov force, due to the incident wave pressure, can also be treated in a manner similar to the scattering force described above. However, Froude-Krylov forces in this method (potential-theory approach) are treated in the same way as was done in Morison's equation method (see Eq. (4.2)). It is believed that this force is very important to the total force and, thus, should be considered over the complete submerged portion of the platform. The hydrostatic force is also computed over the instantaneous wetted length of the platform members, as done in Morison's equation method. Viscous forces are also included using the relative velocity drag term of Morison's equation. All force components are computed over the instantaneous wetted length of platform members. As discussed before, the linear radiation and scattering forces are included with approximations made to include the effect of large platform motions.

4.4 Discussion on the Force Calculations Using Morison's Equation and Potential Theory

In the previous section, two different methods to calculate the hydrodynamic forces acting on a platform are presented (see Eqs. (4.1) and (4.25)). As mentioned before, both methods include certain fluid-flow effects and neglect some. Since these methods are being used to simulate large-amplitude, nonlinear motions of platforms, it is important to know the limitations of these methods in order to understand the effect of various force contributions on the platform response. Each
of the force components is considered, and the difference in the method of computation in both methods is discussed.

The computational methods for the Froude-Krylov and hydrostatic pressure forces are already discussed in the previous section. In both formulations, these forces are computed over the instantaneous wetted length of platform members as explained in Section 4.2.1. The acceleration force may be considered as that due to fluid and body accelerations. In Morison’s equation method, the fluid inertia force is in the form of uniform inviscid flow past a fixed body. The added mass coefficient for uniform potential flow past a circular cylinder is equal to 1.0. It can be shown that, from the closed-form solution of diffraction force for a pile (see MacCamy and Fuchs, 1954), the inertia coefficient defined through Morison’s equation inertia term approaches the value of 2.0, as the ratio of pile diameter to wave length \((D/L)\) becomes very small. Thus, it is seen that the diffraction theory results for the added mass approaches the inertia coefficient of Morison’s equation for \(D/L < 0.2\). For \(D/L\) values greater than this value, use of Morison’s equation to compute fluid inertia force will lead to errors. Thus scattering force computation in the potential theory method is analogous to the fluid acceleration force in Morison’s equation, with the limitations as discussed above.

The body inertia force is also considered to be proportional to the acceleration normal to the member axis in Morison’s equation. A constant value of added-mass coefficient \((1.0)\) is used (see Eq. (4.13)). In the potential theory method, the forces due to body motions are given by the radiation forces, which give
rise to added-mass forces proportional to the body acceleration and potential
damping forces proportional to the body velocity. The wave radiation damping,
which is frequency-dependent, is not accounted for in Morison's equation. The body
velocity force is included in Morison's equation through the quadratic viscous
damping term. This is the only damping present in Morison's equation. Due to the
inclusion of the viscous drag term in the potential theory presented in this study, this
method, as presented in this study, includes both the wave radiation and viscous
damping contributions, besides considering the potential effects accurately.

Another aspect that should be considered is the inclusion of drift forces. Due
to the inclusion of quadratic viscous drag force over the instantaneous wetted length
of members, and inclusion of the effect of a steady current, the viscous drift forces
are included in both methods of force computation. The potential drift forces are
caused by the second-order effects of inviscid flow past the body. Pinkster (1980)
discussed the various contributions to the mean potential drift force based on the
near-field method of computation. From this it may be noted that the effect of the
relative wave elevation and the $|\nabla \phi|^2$ term in Euler's integral for the dynamic
pressure contribute most significantly to the total drift force. The body translational
and rotational displacement terms give smaller contributions. Burns and Liu (1983)
discussed approximate methods to compute potential drift force by using modified
inertia coefficient in Morison's equation inertia term, and integrating the fluid and
body acceleration forces from the still-water level up to the actual wave elevation.
An approximate method to include the interaction effects between closely spaced
columns is also discussed. In the present study, the Froude-Krylov force terms are integrated over the complete wetted length of platform members in both methods of force computation. In Morison's equation, the body and fluid acceleration forces are also computed in a similar manner. Thus, the relative wave elevation effect of the near-field potential drift computation is included in both methods of force computation. The other potential drift effects (such as the quadratic term in Euler's integral, 2nd order velocity potential term) are not included in either method of force computations described in the previous sections.

4.5 Forces and Moments Due to the Positioning System

Two types of mooring systems are usually employed in order to keep floating platforms in position. One is the vertical or slanted taut cable configuration used for tension-leg platforms and the other is the catenary mooring system. When the platform is displaced from its equilibrium position, the mooring lines exert forces on the platform to restore the platform to its original position. These restoration forces are, in general, nonlinear functions of platform displacements. Computation of these forces and moments is discussed in the following sections. It should be noted that only the nonlinear statics of the mooring line is considered. That is, the forces acting on the mooring lines themselves are neglected, as also the dynamic interaction between the platform motion and mooring lines.
4.5.1 Tension-leg Mooring System

The procedure described here is applicable to both vertical- and slanted-taut cable configurations. Fig. 4.2 shows the layout of one such cable. In this figure s is the anchor point, e is the top end of the cable which is attached to the platform at fairlead. Due to arbitrary platform motions, the point e is displaced to a new position f. Let \( L_o \) and \( L \) represent, respectively, the initial and stretched lengths of the cable. The force in the tendon at any time \( t \) is given by

\[
\tilde{F}_m(t) = T(t)\bar{\tau},
\]

where \( T(t) \) is the tendon tension at time \( t \) and \( \bar{\tau} \) is a unit base vector along the tendon, expressed in the \( g \)-system. The tension in the tendon at any time \( t \) is given by

\[
T(t) = T_0 + k\Delta L,
\]

where \( T_0 \) is the initial tension in the tendon, \( k \) is the axial stiffness of the tendon and \( \Delta L \) is the elongation of the cable. The unit base vector \( \bar{\tau} \) can be obtained as (see Fig. 4.2)

\[
\bar{\tau} = \frac{\bar{r}_{sf}}{L},
\]

where

\[
L = \| \bar{r}_{sf} \| = \sqrt{(x_{g1} - x_{g2})^2 + (x_{g2} - x_{g3})^2 + (x_{g3} - x_{g4})^2},
\]
where \( x_{gfi} \) and \( x_{gsi} \) are the coordinates of the cable end points \( s \) and \( f \) in the displaced position of the cable. The elongation of the cable \( \Delta L \) is obtained as

\[
\Delta L = L - L_0.
\] (4.56)

It is, therefore, necessary to determine the coordinate of the top end of the tendon at each time step in order to compute the tendon length using Eq. (4.55). We can write

\[
\vec{r}_{sf}(t) = \vec{r}_{gf}(t) - \vec{r}_{gs} = \vec{r}_{gb}(t) + [P]^T \vec{r}_{bf} - \vec{r}_{gs} = \vec{r}_{gb}(t) + [P]^T \vec{r}_{be} - \vec{r}_{bs},
\] (4.57)

where \([P]\) is the previously defined (Eq. A.8) coordinate transformation matrix. It is also noted that \( \vec{r}_{gs} = \vec{r}_{bs} \), since the point \( s \) is fixed and \( \vec{r}_{bf} = \vec{r}_{be} \), since the point \( e \) is fixed in the \( b \)-system. As noted earlier, \([P]\) is a nonlinear function of the rotational displacements of the platform. Hence, the restoration force, given by Eq. (4.52) depends on the body motions in a nonlinear manner.

The computational procedure can be outlined as follows. At each time step, the coordinate of the top end of the cable is determined using the complete coordinate transformation matrix (see Eq. (A.8)). The elongation of the cable is then determined using Eq. (4.56), and finally, the tension in the cable and the restoration force are determined using Eqs. (4.53) and (4.52) respectively. The moments due to the cable force are obtained by first transforming the restoration forces into the \( b \)-system, that is,
\[
\{ \vec{R}_M(t) \}_b = [P] \vec{R}_M(t),
\]

and

\[
\{ \vec{M}_M(t) \}_b = \vec{r}_{be} \times \{ \vec{R}_M(t) \}_b,
\]

where \( \{ \vec{M}_M(t) \}_b \) is the restoration moment, in the b-system, about the origin of the b-system and \( \vec{r}_{be} \) is the position vector of the top end of the tendon in the b-system.

The above procedure is repeated for each of the mooring tendons and the forces and moments are summed over all cables.

If we assume that the transformation matrix \([P]\) can be linearized under the 'small angle' assumption, and if we neglect all the higher order terms involving the product of terms such as \( \Delta L \) and motion variables, we can derive a 'stiffness' matrix, which relates the restoration forces as a linear function of body motion variables. This is the usual method followed (see for example, Chou et al., 1983). Such a procedure can be used if the tension variations, about the mean tension, are small. However, as we will see later on, large tendon tension variations occur, and these can be considered satisfactorily, only by using the procedure described in this section.

In this study, only the tension-leg mooring, as described here, is implemented in the computer programs. However, the analysis of catenary cable configuration is discussed in Appendix C for future studies of the subject.
In the previous sections, a discussion is given of the various force components and their computations. Other forces on the platform include those due to body weight, wind, etc. Forces due to body weight will yield only a vertical component in the g-system as will be discussed in the next chapter. Wind forces are not considered in this study.
Fig. 4.1 End-plane forces
Fig. 4.2 Tension-leg mooring system
Chapter 5
Formulation of the Equations of Motion in Time Domain

5.1 Introduction

The equations of motion of the platform have already been derived in Chapter 3. These are given by twelve first-order nonlinear ordinary differential equations (see Eq. (3.15)). As mentioned in Chapter 3, the force and moment terms appearing on the right hand side of Eq. (3.15) depend on the displacement, velocity and acceleration of the platform. In such a case, the system of equations can be written in the following form:

\[ \dot{x}_i = f(x_j, \dot{x}_j, t), \quad i, j = 1, 2, \ldots, 12. \quad (5.1) \]

There are several methods available for solving a system of 1st order ordinary differential equations if they can be written in the following form:

\[ \dot{x}_i = f(x_j, t), \quad i, j = 1, 2, \ldots, 12. \quad (5.2) \]

If the initial conditions for all the state variables are known, the system of equations given by Eq. (5.2) can be solved by any of a number of standard numerical methods, such as the 4th order Runge-Kutta method. Depending on the method used for numerical integration, the general procedure involves the evaluation of the right hand side of Eq. (5.2) one or more times at each time step and extrapolating the solution using the function values. The procedure becomes computationally
intensive if the right hand side is a complicated function of the state variables and if more than one evaluation is needed at each time-step.

We can see from Eq. (5.1) that the right hand side contains not only the unknown but also their derivatives. In this case, the function evaluations can not be made without first estimating the derivatives, even if approximately. One way to deal with such a problem is to use a finite difference scheme to estimate the value of the derivative in terms of, for example, the known values of the variables at previous time steps (forward stepping). An iterative scheme or a predictor-corrector method can be used to correct the values of the derivative in subsequent iterations for the same time-step until convergence is achieved. Liu et al. (1980) discussed this problem in a similar context. They used the average value of the derivative at the previous intervals and found the method to work satisfactorily. Sen et al. (1989) also reported numerical instabilities in the integration of equations of motion due to the method used for estimating the $d\phi/dt$ term in the force evaluation. They have used a backward difference scheme to compute the $d\phi/dt$ term. An explicit method (Adams-Bashforth predictor) is then used to advance the solution. For the second and subsequent iterations, a central difference scheme is used to compute the $d\phi/dt$ term. This method appeared to have yielded satisfactory results although a smoothing procedure for force was used to prevent the 'saw-tooth' type instabilities which were observed later in the simulation process.
An attempt has been made in this study to solve the system of Eq. (3.15) using the procedure described in the preceding paragraphs. The general procedure followed can be outlined as follows (see Fig. 5.1)

\[ t-1 \quad t \quad t+1 \]

Fig. 5.1 Time-stepping procedure

At any time-step \( t \), the values of linear and angular displacements and velocities are known. In order to advance the solution to the \( t+1 \)th step, it is necessary to evaluate the right hand side of Eq. (3.15) at the current step, \( t \). First, a backward difference scheme based on velocities is used to estimate the accelerations at \( t \). An explicit method is then used to integrate the equations of motion. For the second and subsequent iterations, the predicted velocities at the \( t+1 \)th step are used in a central difference scheme to correct the accelerations at the time step \( t \). The corrected accelerations are used to correct the solution at the \( t+1 \)th time step, again using an explicit method. The procedure can be repeated until convergence is achieved. Two types of integration methods, namely, the Modified Euler method and the 4-step Adams-Bashforth predictor method were used and both methods were found to be unstable. The solution could not be obtained even for few time steps. Next, an implicit scheme (Adams-Moulton Corrector method) was used for the
second and higher iterations, using the predicted variables at the $t+1$ time step. Even this method was found to be unstable. Finally, a 4th order Runge-Kutta method was tried, without success. The stability problem could not be remedied by reducing the time-step value or by increasing the number of iterations. At that stage, it was decided to reformulate the equations of motion such that the right hand side of the equations does not contain the acceleration terms. This is discussed in the following sections. The general procedure followed here is similar to that described in Paulling (1977) and Paulling and Shin (1985). Since the forces which depend on the body accelerations are computed differently in the two methods discussed in Chapter 3 (see Eqs. (4.1) and (4.25)), the procedure to formulate the equations of motion is also different. These two methods are discussed below.

5.2 Morison's Equation Method

The elemental force on the segment of unit length of the member is given by (see Eq. 4.2)

$$ d\tilde{F}^n(t) = -\int_0^L \rho \tilde{F} ds + C_M (\tilde{a}_{wmN}^n - \tilde{a}_{bmN}^n) + C_D \tilde{v}_{rnN}^n \| \tilde{v}_{rnN}^n \|, $$

(5.3)

where $C_M = C_m \rho \pi D^2/4$ and $C_D = \rho C_d D/2$ have been introduced for convenience. The remaining terms are defined in Chapter 4.

Eq. (5.3) can be written as follows:
where all the terms on the right hand side of Eq. (5.3), except the body acceleration term, are included in \( \mathbf{dF} \) in Eq. (5.4).

The acceleration of any point \( m \) on the body due to rigid body motions is given by (see Eq. (4.12))

\[
\mathbf{a}_{bnm}^a = \frac{d\bar{v}^a}{dt} + [P]^T \left( \frac{d\bar{\omega}}{dt} \times \bar{r}_{bnm}^a \right) + [P]^T \left( \bar{\omega} \times (\bar{\omega} \times \bar{r}_{bnm}^a) \right),
\]

where

\[
\frac{d\bar{v}}{dt}, \frac{d\bar{\omega}}{dt} = \text{translational and angular acceleration vectors of the center of gravity of the body},
\]

\( \bar{v}, \bar{\omega} = \text{translational and angular velocities} \)

\( \bar{r}_{bnm}^a = \text{position vector of point } m \text{ on the } n \text{th member in the body-fixed coordinate system} \)

\( [P] = \text{transformation matrix for coordinate transformation between } g- \text{ and } b- \text{systems}. \)

Note that \( \bar{v} \) and \( d\bar{v}/dt \), the translational velocity and acceleration of the center of gravity of the body, are previously defined as \( \bar{u}_T \) and \( \bar{a}_T \) (see Eqs. (4.8) and (4.12)).

Normal component of this acceleration can be written as (see Eq. (4.15)),
\[ a_{bn}^n = [R] a_{bm}^n, \]  

(5.6)

where \([R]\) is the previously defined transformation matrix consisting of the time-dependent direction cosines of the member \(n\) (see Eq. (2.21)). Using Eqs. (5.5) and (5.6) we can write Eq. (5.4) as follows:

\[ dF_n^a(t) = -C_M [R] a_{bn}^n + dF_o^n(t). \]  

(5.7)

We can also write Eq. (5.5) as follows:

\[ a_{bn}^n = \frac{dv}{dt} + [P]^T [Q] \frac{d\omega}{dt} + [P]^T (\omega \times (\omega \times r_{bn}^n)), \]  

(5.8)

where

\[ [Q] = \begin{bmatrix} 0 & x_{bm3} & -x_{bm2} \\ -x_{bm3} & 0 & x_{bm1} \\ x_{bm2} & -x_{bm1} & 0 \end{bmatrix} \]

has been introduced to represent the term \(d\omega/dt \times r_{bm}^n\) and \(x_{bm1}, x_{bm2}\) and \(x_{bm3}\) are the components of the position vector \(r_{bm}^n\).

In Eq. (5.8), the first two terms depend on the translational and angular acceleration of the center of gravity of the body (which are unknown during the
simulation). The third term, the centripetal acceleration, depends only on the angular velocity of the body. This term can be evaluated without difficulty.

Using Eqs. (5.6) and (5.8), we have

\[
\tilde{\mathbf{a}}^n_{bm} = [\mathbf{R}] \tilde{\mathbf{a}}^n_{bm}, \quad (5.9)
\]

Substituting Eq. (5.9) into Eq. (5.4), we obtain

\[
\dot{\mathbf{\bar{F}}}^n(t) = -C_M[R] \frac{d\mathbf{v}}{dt} + [P]^T[Q] \frac{d\tilde{\omega}}{dt} + [P]^T(\tilde{\omega} \times (\tilde{\omega} \times \bar{r}^n_{bm})) + \mathbf{d\bar{F}}^n_o(t).
\]

If we include the \((\tilde{\omega} \times (\tilde{\omega} \times \bar{r}^n_{bm}))\) term in the above equation in \(\mathbf{d\bar{F}}^n_o(t)\), the above equation can be written as

\[
\dot{\mathbf{\bar{F}}}^n(t) = -C_M[R] \frac{d\mathbf{v}}{dt} - C_M[R][P]^T[Q] \frac{d\tilde{\omega}}{dt} + \mathbf{d\bar{F}}^n_o(t).
\]

We can then write

\[
\dot{\mathbf{\bar{F}}}^n(t) = \mathbf{d\bar{F}}^n_a(t) + \mathbf{d\bar{F}}^n_o(t). \quad (5.10)
\]

where

\[
\mathbf{d\bar{F}}^n_a(t) = -C_M[R] \frac{d\mathbf{v}}{dt} - C_M[R][P]^T[Q] \frac{d\tilde{\omega}}{dt}. \quad (5.11)
\]
The total elemental force, Eq. (5.10), transformed to the b-system, can be written as

\[
\{d\bar{F}^n(t)\}_b = [P]\left[d\bar{F}^n_a(t) + d\bar{F}^n_o(t)\right],
\]  

(5.12)

where, as before, the subscript b outside the parenthesis on the left hand side of the above equation indicates that the force is given the b-system.

The elemental moment about the origin of the b-system is given by

\[
d\bar{M}^n(t) = \bar{r}^n \times \{d\bar{F}^n(t)\}_b = \bar{r}^n \times [P] d\bar{F}^n_a(t) + \bar{r}^n \times [P] d\bar{F}^n_o(t),
\]  

(5.13)

where Eq. (5.12) has been used for \{d\bar{F}^n(t)\}_b. We can write Eq. (5.13) as follows

\[
d\bar{M}^n(t) = d\bar{M}^n_a(t) + d\bar{M}^n_o(t),
\]  

(5.14)

where

\[
d\bar{M}^n_a(t) = \bar{r}^n \times [P] d\bar{F}^n_a(t) = -[Q][P] d\bar{F}^n_a(t).
\]  

(5.15)

Similarly, we can write

\[
d\bar{M}^n_o(t) = -[Q][P] d\bar{F}^n_o(t),
\]  

(5.16)

where \([Q]\) is as defined before (see Eq. (5.8)). Using Eq. (5.11), we can write Eq. (5.15) as follows:
Eqs. (5.11) and (5.17) give the elemental force and moment which depend on the translational and angular accelerations of the body. All the remaining forces and moments are included in \( d\vec{F}_n(t) \) and \( d\vec{M}_n(t) \) terms. The total force and moment on the \( n \)th member is obtained by integrating Eqs. (5.11) and (5.17) over the instantaneous submerged length of the member, \( l_n(t) \), i.e.,

\[
\vec{F}_n(t) = \int_{l_n(t)} d\vec{F}_n(t) \, dx = -C_M [R] I_n(t) \frac{dv}{dt} - C_M [R] \int_{l_n(t)} [Q] \frac{d\phi}{dt} \, dx_n. \tag{5.18}
\]

We can write Eq. (5.18) as

\[
\vec{F}_n(t) = [A_1^n(t)] \frac{dv}{dt} + [A_2^n(t)] \frac{d\phi}{dt}, \tag{5.19}
\]

where

\[
[A_1^n(t)] = -C_M [R] I_n(t)
\]

\[
[A_2^n(t)] = -C_M [R][P]^T \int_{l_n(t)} [Q] \, dx_n. \tag{5.20}
\]

The moment (due to acceleration-dependent force) on the member about the origin of the \( b \)-system can be written as
\[ \tilde{M}_s(t) = \int_{l^s(t)} d\tilde{M}_s(t) dx_{l1}, \]
\[ = \int_{l^s(t)} [Q][P] C_M[R] \frac{d\tilde{V}}{dt} dx_{l1} + \int_{l^s(t)} [Q][P] C_M[R][P]^T[Q] \frac{d\tilde{\omega}}{dt} dx_{l1}. \] (5.21)

Eq. (5.21) can be written as follows:

\[ \tilde{M}_s(t) = [B_1(t)] \frac{d\tilde{V}}{dt} + [B_2(t)] \frac{d\tilde{\omega}}{dt}, \] (5.22)

where

\[ [B_1(t)] = \int_{l^s(t)} [Q][P] C_M[R] dx_{l1}, \] (5.23)
\[ [B_2(t)] = \int_{l^s(t)} [Q][P] C_M[R][P]^T[Q] dx_{l1}. \]

The total force and moment (which depend on \(\frac{d\tilde{V}}{dt}\) and \(\frac{d\tilde{\omega}}{dt}\)) on the platform are then obtained by summing over all K members of the platform.

\[ \bar{F}_s(t) = \sum_{n=1}^{K} \bar{F}_s^n(t) = \sum_{n=1}^{K} [A_1^n(t)] \frac{d\tilde{V}}{dt} + \sum_{n=1}^{K} [A_2^n(t)] \frac{d\tilde{\omega}}{dt}, \] (5.24)

which can be rewritten as follows:

\[ \bar{F}_s(t) = [a_1(t)] \frac{d\tilde{V}}{dt} + [a_2(t)] \frac{d\tilde{\omega}}{dt}, \] (5.25)

where
\[ [a_1(t)] = \sum_{n=1}^{K} [A_1^n(t)] , \quad [a_2(t)] = \sum_{n=1}^{K} [A_2^n(t)]. \tag{5.26} \]

The total moment on the platform, about the origin of the b-system, can similarly be written as

\[
\tilde{M}_a(t) = \sum_{n=1}^{K} \tilde{M}_a^n(t) = \sum_{n=1}^{K} [B_1^n(t)] \frac{dv}{dt} + \sum_{n=1}^{K} [B_2^n(t)] \frac{d\phi}{dt}. \tag{5.27} \]

Eq. (5.27) can be rewritten in the following form, similar to Eq. (5.25):

\[
\tilde{M}_a(t) = [b_1(t)] \frac{dv}{dt} + [b_2(t)] \frac{d\phi}{dt}, \tag{5.28} \]

where

\[
[b_1(t)] = \sum_{n=1}^{K} [B_1^n(t)], \quad [b_2(t)] = \sum_{n=1}^{K} [B_2^n(t)]. \tag{5.29} \]

The total force and moment acting on the platform at any time \( t \) is then given by

\[
\bar{F}(t) = \bar{F}_a(t) + \bar{F}_o(t),
\]

\[
\bar{M}_b(t) = \bar{M}_a(t) + \bar{M}_o(t),
\tag{5.30} \]

where \( \bar{F}_o(t) \) and \( \bar{M}_o(t) \) are the quantities resulting from integrating \( d\bar{F}_o^n(t) \) and \( d\bar{M}_o^n(t) \) along the member length and then summing over all members of the platform (similar to the procedure followed for acceleration dependent forces).
and \( \ddot{M}_o \) includes all other forces such as due to body weight, mooring restoration, drag force, etc. Using Eqs. (5.25) and (5.28), we can write Eq. (5.30) as follows:

\[
\ddot{F}(t) = [a_1(t)] \frac{dv}{dt} + [a_2(t)] \frac{d\omega}{dt} + \ddot{F}_o(t),
\]

\[
\ddot{M}_b(t) = [b_1(t)] \frac{dv}{dt} + [b_2(t)] \frac{d\omega}{dt} + \ddot{M}_o(t).
\]

Eq. (5.31) gives the total force and moment acting on the platform at any time \( t \). The forces and moments which depend on the translational and rotational accelerations of the center of gravity of the body have been separated out from all other force components. The added mass and inertia matrices \([a_1(t)], [a_2(t)], [b_1(t)], [b_2(t)]\) are time-dependent due to the integration of elemental forces and moments over the instantaneous wetted length of the platform members. The equations of motion of the body can be written as (see Eq. (3.15))

\[
[M] \frac{dv}{dt} = \ddot{F}(t),
\]

\[
[I] \frac{d\omega}{dt} = \ddot{M}_b(t) - (\dot{\omega} \times [I] \ddot{\omega}),
\]

\[
\frac{dx}{dt} = \dot{v},
\]

\[
\frac{d\ddot{\omega}}{dt} = [B]^{-1} \ddot{\omega},
\]

Using Eq. (5.31), the first two equations of Eq. (5.32) can be written as follows:
\[ [M] \frac{dv}{dt} = [a_1(t)] \frac{dv}{dt} + [a_2(t)] \frac{d\omega}{dt} + \vec{F}_o(t), \]
\[ [I] \frac{d\omega}{dt} = [b_1(t)] \frac{dv}{dt} + [b_2(t)] \frac{d\omega}{dt} + \vec{M}_o(t) - (\omega \times [I] \ddot{\omega}). \]

Rearranging, we can write Eq. (5.33) as
\[
([M] - [a_1(t)]) \frac{dv}{dt} - [a_2(t)] \frac{d\omega}{dt} = \vec{F}_o(t),
\]
\[-[b_1(t)] \frac{dv}{dt} + ([I] - [b_2(t)]) \frac{d\omega}{dt} = \vec{M}_o(t) - (\omega \times [I] \ddot{\omega}),
\]

which can be written as
\[
\begin{bmatrix}
([M] - [a_1(t)]) & -[a_2(t)] \\
-[b_1(t)] & ([I] - [b_2(t)])
\end{bmatrix}
\begin{bmatrix}
\frac{dv}{dt} \\
\frac{d\omega}{dt}
\end{bmatrix}
= \begin{bmatrix}
\vec{F}_o(t) \\
\vec{M}_o(t) - (\omega \times [I] \ddot{\omega})
\end{bmatrix}.
\]

Multiplying both sides of the above equation by the inverse of the time-dependent coefficient matrix, \([ ]\), we obtain
\[
\begin{bmatrix}
\frac{dv}{dt} \\
\frac{d\omega}{dt}
\end{bmatrix}
= \begin{bmatrix}
([M] - [a_1(t)]) & -[a_2(t)] \\
-[b_1(t)] & ([I] - [b_2(t)])
\end{bmatrix}^{-1}
\begin{bmatrix}
\vec{F}_o(t) \\
\vec{M}_o(t) - (\omega \times [I] \ddot{\omega})
\end{bmatrix}
= \begin{bmatrix}
[a_{11}(t)] & [a_{12}(t)] \\
[a_{21}(t)] & [a_{22}(t)]
\end{bmatrix}
\begin{bmatrix}
\vec{F}_o(t) \\
\vec{M}_o(t) - (\omega \times [I] \ddot{\omega})
\end{bmatrix},
\]

where
Using Eqs. (5.34) and (5.35), we can finally write Eq. (5.32) as follows:

\[
\frac{dv}{dt} = [a_{11}(t)] \ddot{F}_o(t) + [a_{12}(t)] (M_o(t) - \ddot{\omega} \times [I] \ddot{\omega}) ,
\]

\[
\frac{d\ddot{\omega}}{dt} = [a_{21}(t)] \ddot{F}_o(t) + [a_{22}(t)] (M_o(t) - \ddot{\omega} \times [I] \ddot{\omega}) ,
\]

\[
\frac{dx}{dt} = \ddot{v} ,
\]

\[
\frac{d\ddot{\omega}}{dt} = [B]^{-1} \ddot{\omega} .
\]

The right hand side of Eq. (5.36) does not have any terms which depend on accelerations and hence can be evaluated at any time. Eq. (5.36) can be integrated using any of the standard methods.

5.3 Potential Theory Approach

The development here is very similar to the one used for Morison’s equation method discussed in Section 5.2. Some differences arise due to the method in which the radiation forces are computed here.

Following the development in the previous section, the total force and moment acting on the platform can be written as (see Eq. (5.30))

\[
\ddot{F}(t) = \ddot{F}_R(t) + \ddot{F}_o(t) ,
\] (5.37)
where $\bar{F}_R(t)$ is the radiation force (moment) and $\bar{F}_o(t)$ is the sum of all other forces.

The radiation force and moment on the platform are given by (see Eq. (4.27))

$$\bar{F}_R(t) = -A_{ij} a_j^* - \int_0^\infty L_{ij}(\tau)v_j^*(t-\tau)\,d\tau, \quad i,j = 1,2,\ldots,6.$$  \hspace{1cm} (5.38)

where $A_{ij}$ are the constant (genuine) added mass coefficients and $v_j^*$ and $a_j^*$ are the velocity and acceleration of the platform center of gravity in the jth mode of platform motion. These velocities and accelerations are computed by two different methods which are discussed in Chapter 4. Hence, the formulation of the equations of motion will also differ accordingly. We will first formulate the equations for Method (1) discussed in Section 4.3.

Using Eq. (5.38), we can write Eq. (5.37) as follows:

$$\bar{F}(t) = -A_{ij} a_j^* - \int_0^\infty L_{ij}(\tau)v_j^*(t-\tau)\,d\tau + \bar{F}_o(t), \quad i,j = 1,2,\ldots,6.$$  \hspace{1cm} (5.39)

Noting that the velocities and accelerations are given by Eq. (4.41) for Method (i), Eq. (5.39) becomes

$$\bar{F}(t) = -A_{ij} \left\{ \frac{dv}{dt} + \frac{d\dot{v}}{dt} \right\} - \int_0^\infty L_{ij}(\tau)v_j^*(t-\tau)\,d\tau + \bar{F}_o(t), \quad i,j = 1,2,\ldots,6.$$  \hspace{1cm} (5.40)
Denoting the forces by \( F \) and \( F_o \) and the moments by \( M \) and \( M_o \), we can write Eq. (5.39) in the following manner

\[
\begin{align*}
\ddot{F}(t) &= [a_1] \frac{d\dot{v}}{dt} + [a_2] \frac{d\ddot{\phi}}{dt} + F_o(t), \\
\ddot{M}(t) &= [b_1] \frac{d\dot{v}}{dt} + [b_2] \frac{d\ddot{\phi}}{dt} + M_o(t).
\end{align*}
\] (5.41)

In Eq. (5.41), \( \dot{d}/dt \) and \( d\ddot{\phi}/dt \) are the translational and angular accelerations, and together they constitute the components of acceleration \( \ddot{a}^* \) used in Eq. (5.39).

It is also noted that the memory effect integral terms for \( i = 1, 2 \) and \( 3 \) (second term in Eq. (5.39)) are now included in \( F_o(t) \), along with other force contributions from diffraction, Froude-Krylov, etc. The corresponding terms for \( i = 4, 5 \) and \( 6 \) are included in \( M_o(t) \). \([a_1], [a_2], [b_1], [b_2] \) are the \( 3 \times 3 \) partitions of the \( 6 \times 6 \) constant added-mass coefficient matrix \( A_{ij} \) with the negative sign included (see Eq. (5.39)), i.e.,

\[
[a_1] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} ; \quad [a_2] = [-A_{ij}], \quad i = 1, 2, 3, j = 4, 5, 6
\]

\[
[b_1] = [-A_{ij}], \quad i = 4, 5, 6, j = 1, 2, 3 ; \quad [b_2] = [-A_{ij}], \quad i = 4, 5, 6, j = 4, 5, 6.
\] (5.42)

Eq. (5.41) gives the total force and moment acting on the platform at any time \( t \), where the forces and moments that depend on the body accelerations are separated.
out so that they can be added to the structural mass terms on the left hand side of the equations of motion, Eq. (3.15). It should be noted that the added mass matrices \([a_1], [a_2], [b_1] \text{ and } [b_2]\) are independent of time, as a consequence of the assumptions made in deriving the radiation force and moment in Method (1). On the other hand, if we use Method (2), the velocities and accelerations are given by Eq. (4.42) which makes the added mass matrices time-dependent, as shown below.

The radiation force and moment in this case are given by

\[
\bar{F}_R(t) = -A_{ij} \begin{pmatrix} [P] \frac{d\gamma}{dt} \\ -\int_{0}^{\infty} L_{ij}(\tau) v_j^* d\tau \end{pmatrix} \quad i,j = 1,2,\ldots,6. \tag{5.43}
\]

and the total force and moment are written as (see Eq. (5.40))

\[
\bar{F}(t) = -A_{ij} \begin{pmatrix} [P] \frac{d\gamma}{dt} \\ -\int_{0}^{\infty} L_{ij}(\tau) v_j^* d\tau + \bar{F}_o(t) \end{pmatrix}. \tag{5.44}
\]

The force and moment are now written in the form of Eq. (5.41), i.e.,

\[
\bar{F}(t) = [a_1(t)] \frac{d\gamma}{dt} + [a_2(t)] \frac{d\bar{o}}{dt} + \bar{F}_o(t), \tag{5.45}
\]

\[
\bar{M}(t) = [b_1(t)] \frac{d\gamma}{dt} + [b_2(t)] \frac{d\bar{o}}{dt} + \bar{M}_o(t),
\]

where the time-dependent added mass matrices are given by
\[ a_1(t) = [-A_{ij}] P, \quad i, j = 1, 2, 3, \]
\[ a_2(t) = [-A_{ij}], \quad i = 1, 2, 3, \quad j = 4, 5, 6, \]
\[ b_1(t) = [-A_{ij}] P, \quad i = 4, 5, 6, \quad j = 1, 2, 3, \]
\[ b_2(t) = [-A_{ij}], \quad i, j = 4, 5, 6. \]

Thus, the two methods of computing the radiation forces results in the change in the added mass matrices. Here the matrices \[ a_1(t) \] and \[ b_1(t) \] become time-dependent due to the consideration of instantaneous body orientation in determining the radiation forces (see Eq. (4.42)). The remaining procedure is the same for both methods. The equations of motion can then be written as follows:

\[ [M] \frac{dv}{dt} = [a_1(t)] \frac{dv}{dt} + [a_2(t)] \frac{d\tilde{\omega}}{dt} + \tilde{F}_o(t), \]
\[ [I] \frac{d\tilde{\omega}}{dt} = [b_1(t)] \frac{dv}{dt} + [b_2(t)] \frac{d\tilde{\omega}}{dt} + (\tilde{M}_o(t) - (\tilde{\omega} \times [I] \tilde{\omega})), \]
\[ \frac{dx}{dt} = v, \]
\[ \frac{d\tilde{\omega}}{dt} = [B]^{-1} \tilde{\omega}. \]

These equations are now in the same form as derived for Morison's equation approach (see Eqs. (5.32) and (5.33)). Using the same procedure for algebraic manipulation, we obtain equations similar to Eq. (5.36), i.e.,
\[
\frac{d\vec{v}}{dt} = [a_{11}(t)] \vec{F}_o(t) + [a_{12}(t)] (\vec{M}_o(t) - \vec{\omega} \times [I] \vec{\omega}),
\]
\[
\frac{d\vec{\omega}}{dt} = [a_{21}(t)] \vec{F}_o(t) + [a_{22}(t)] (\vec{M}_o(t) - \vec{\omega} \times [I] \vec{\omega}),
\]
\[
\frac{d\vec{x}}{dt} = \vec{v},
\]
\[
\frac{d\vec{\theta}}{dt} = [B]^{-1} \vec{\omega},
\]

where

\[
\begin{bmatrix}
[a_{11}(t)] & [a_{12}(t)] \\
[a_{21}(t)] & [a_{22}(t)]
\end{bmatrix} = \begin{bmatrix}
([M] - [a_1(t)]) & -[a_2(t)] \\
-[b_1(t)] & ([I] - [b_2(t)])
\end{bmatrix}^{-1}.
\]

Eq. (5.48) is now in a form suitable for numerical integration.

In the present study both Methods (1) and (2) have been implemented in the computer programs. Both methods gave very similar results. This should be expected, since the numerical model has been applied to a tension-leg platform, which has very small motions. For this reason, further simulations are carried out using Method 1 only. The two methods will presumably yield different results in the case of platforms undergoing larger motions, such as a semisubmersible platform.

It is thus seen that the form of the equations of motion is same in Morison's equation and potential theory methods (see Eqs. (5.36) and (5.48)). The calculation of the total external forces and moments in the two methods is different (see Eqs. (4.1) and (4.25)). Due to the difference in the method of computing the forces in
the two approaches, the coefficient matrices in Eqs. (5.36) and (5.48) are also different. These are summarized below.

Morison’s equation:

- Force and moment: Eq. (4.1)
- Equations of motion: Eq. (5.36)
- Coefficient matrices: Eqs. (5.26), (5.29) and (5.35)

Potential Theory:

- Force and moment: Eq. (4.25)
- Equations of motion: Eq. (5.48)
- Coefficient matrices: Eqs. (5.42) and (5.49) for Method (1)
  Eqs. (5.46) and (5.49) for Method (2).

5.3.1 Hydrodynamic Coefficients Using 3-D Potential Theory

As discussed in Chapter 4, linear potential theory results are used in the time domain to calculate forces and moments on the platform. The hydrodynamic coefficients and wave exciting forces can, in principle, be obtained using the strip theory method (see Salvesen et al., 1970) or by using a complete three dimensional potential theory (see, for example, Garrison, 1977). The 3-D potential theory is preferred due to its ability to consider the interference effects due to closely spaced members, scattering due to 'large' members, and the three dimensional effects at the ends of members. For this reason, it was decided to use 3-D potential theory to obtain the hydrodynamic coefficients and wave exciting forces. The hydrodynamic
coefficients and wave exciting forces, to the first order, can be obtained by solving
the following boundary value problem. Only a very brief description of the problem
is given here for completeness. The complete details can be found in many
references such as Yeung (1973), Newman (1977), Wu (1984) and also in Chakrabarti

It is assumed that the fluid is inviscid and incompressible and the flow is
irrotational. Assuming small amplitude progressive waves and corresponding small
motions of the body, the total velocity potential $\phi$ can be written in the following
form:

$$\phi = \phi_\lambda + \sum_{j=1}^{6} \phi_j,$$

where $\phi_\lambda$ represents the velocity potential due to incident waves and its interaction
with a fixed body, $\phi_j$, $j = 1, 2, \ldots, 6$ is the velocity potential representing the rigid body
motions in the jth mode in the absence of waves. $\phi_\lambda$ can further be decomposed
into two potentials, $\phi_0$ and $\phi_\pi$, representing, respectively, the potentials due to the
undisturbed incident wave and the scattering or diffraction effect due to the
disturbance of the incident wave by a fixed body. The incident wave potential $\phi_0$ is
known from linear wave theory. The radiation potentials $\phi_j$, $j = 1, 2, \ldots, 6$, satisfy the
following boundary conditions on the body surface:
\[
\frac{\partial \phi_j}{\partial \mathbf{n}} = -n_j, \quad j = 1, 2, 3, \quad \frac{\partial \phi_j}{\partial \mathbf{n}} = -\mathbf{r} \times \mathbf{n}, \quad j = 4, 5, 6,
\]

where \( \mathbf{n} \) is a unit normal vector directed out of the body, and \( \mathbf{r} \) is the position vector of a point on the body. The potentials \( \phi_A \) and \( \phi_7 \) satisfy the following boundary conditions:

\[
\frac{\partial \phi_A}{\partial \mathbf{n}} = 0, \\
\frac{\partial \phi_7}{\partial \mathbf{n}} = -\frac{\partial \phi_0}{\partial \mathbf{n}}, \quad \text{on the body surface}.
\]

All potentials must satisfy Laplace's equation

\[
\nabla^2 \phi_j = 0, \quad j = 0, 1, \ldots, 7,
\]

and the linearized free surface boundary condition

\[
\frac{\partial^2 \phi_j}{\partial t^2} + g \frac{\partial \phi_j}{\partial z} = 0, \quad \text{on} \quad z = 0, \quad j = 0, 1, \ldots, 7,
\]

where \( g \) is the acceleration due to gravity. Also, for finite water depth, \( h \),

\[
\frac{\partial \phi_j}{\partial \mathbf{n}} = 0 \quad \text{at} \quad z = -h, \quad j = 0, 1, \ldots, 7.
\]

For infinite water depth,

\[
\phi_j = 0, \quad z \to -\infty, \quad j = 0, 1, \ldots, 7
\]

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In addition to the conditions on the body, free surface, and sea floor, the radiation potentials and the diffraction potential, $\phi_j$, $j=1,2,\ldots,7$, must satisfy a radiation condition at infinity. For a three-dimensional body such a condition, called the Sommerfeld condition, can be specified as follows:

$$\lim_{R \to \infty} R^{\frac{1}{2}} \left( \frac{\partial}{\partial R} \mp i k \right) \phi_j = 0, \quad j = 1,2,\ldots,7,$$

where $R$ is the radial distance, i.e., $R = \sqrt{x^2+y^2}$, from the center of the structure, and $k$ is the wave number.

The above-described boundary-value problem can be solved using different methods such as the Green function, boundary element or finite-element methods. In this study, a computer program (THAFTS) based on the Green function method has been used (Wu, 1984).

This analysis is applied to the tension-leg platform geometry described in Section 2.5. Fig. 5.2 shows the discretization of one quarter of the body surface. A total of 600 panels have been used with 150 panels per quarter. Fig. 5.3 shows the added mass coefficients and Fig. 5.4 shows the damping coefficients obtained by the Green function method. These coefficients and wave exciting forces are used in time domain to obtain forces and response of the platform, as already described in earlier Sections.
5.3.2 Evaluation of Kernel Functions and Constant Added Mass Coefficients

Eqs. (4.39) give the kernel function and constant (genuine) added mass coefficients based on frequency-dependent hydrodynamic coefficients. The 3-dimensional added mass and damping coefficients are obtained for a number of frequencies as explained in the previous sections. These results can not be obtained for very high frequencies (due to irregular frequencies) and hence the results are obtained up to a frequency $\sigma_L$. As can be seen from Eq. (4.39), the behavior of damping coefficients, in principle, should be known up to a large value of frequency. Therefore, it is necessary to predict high-frequency damping coefficients by means of some form of decay curves. The high-frequency behavior of damping coefficients for semisubmersible and tension-leg platforms is not clearly known. Van Oortmerssen (1976) used higher-order decay curves to predict damping at high frequencies. This method is suitable for ship-like structures, as the damping curve shows a smooth behavior over the complete frequency range. However for semisubmersibles and TLPs the damping curve shows hump-hollow behavior due to the effect of column spacing and force cancellation effects. For this reason, it is not possible to use higher-order decay curves as given in Van Oortmerssen (1976). Instead, a cubic-spline interpolation scheme is used to extrapolate the damping coefficients from the arbitrarily chosen cut-off frequency, $\sigma_L$, as described below.

First a suitable frequency $\sigma_p$ is chosen at which $B_{ij}$ becomes zero. The cubic-spline coefficients are then obtained for a frequency range from 0 to $\sigma_p$. For
frequencies between $\sigma_L$ and $\sigma_F$, the damping values are obtained using the spline coefficients. A number of test computations have been made with different values for both $\sigma_L$ and $\sigma_F$. Results indicated no significant differences in the computed kernel functions or in the constant added mass coefficients. From this it appeared that the method is satisfactory.

Another difficulty arises in the numerical integration of Eqs. (4.39) due to the oscillatory nature of the integrand for high values of $\tau$. Numerical integration methods such as the trapezoidal and Simpson's rule will require very closely spaced intervals in order to consider the oscillatory nature of the function. A method, due to Filon (see Hildebrand, 1974), developed for integrations of the form

$$\int_{a}^{b} f(x) \sin kx, \quad \int_{a}^{b} f(x) \cos kx,$$

has been used in the present study. Details of this method are given in de Kat (1988). The basic idea is that if the function $f(x)$ is smooth in the interval $(a,b)$, it can be approximated by a polynomial and the integration over a given sub-interval can be performed analytically. The entire integration, between the limits $a$ and $b$, can then be approximated by a summation of discrete integrals over the sub-intervals. For example, the integral

$$\int_{x_1}^{x_N} f(x) \cos kx$$
can be integrated as follows. In this $x_1, x_2, \ldots, x_N$ are the $N$ sub-intervals over which the function is to be integrated. Let us assume that $f(x)$ varies linearly over any sub-interval. We can write

$$
\int_{x_1}^{x_N} f(x) \cos kx \, dx = \sum_{i=2}^{N} I_i,
$$

(5.50)

where

$$
I_i = \int_{x_{i-1}}^{x_i} (a_i + b_i x) \cos kx \, dx,
$$

(5.51)

in which the coefficients $a_i$ and $b_i$ are given by

$$
a_i = \frac{f(x_{i-1}) x_i - f(x_i) x_{i-1}}{x_i - x_{i-1}},
$$

(5.52)

$$
b_i = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}.
$$

The constants for any interval $i$ can be determined using Eq. (5.52) and the value of the function at these discrete points. We can then integrate Eq. (5.51) analytically (integrating by parts) to obtain the following expression:

$$
I_i = a_i \left[ \frac{\sin kx_i - \sin kx_{i-1}}{k} \right] + b_i \left[ \frac{\cos kx_i - \cos kx_{i-1}}{k^2} \right] + b_i \left[ \frac{x_i \sin kx_i - x_{i-1} \sin kx_{i-1}}{k} \right].
$$

(5.53)
The integration over the entire range, from $x_i$ to $x_N$, can be considered as a sum of $N-1$ discrete integrals, such as the one given by Eq. (5.53).

Eq. (4.39) is integrated using Filon's method to obtain the kernel functions, $L_{ij}(\tau)$. The integration is carried out only up to the value of $x_P$, above which the integrand becomes zero. The results for $L_{11}(\tau)$ are plotted in Fig. 5.5. For comparison, the result obtained using Simpson's $1/3$-rule has also been plotted in this figure. We can see that the kernel functions reaches a constant value with Filon's method, while the results obtained using Simpson's method shows large fluctuations, particularly for large values of $\tau$. For small values of $\tau$ both methods seem to be in good agreement. The kernel functions obtained in this manner for surge and heave motions are shown in Figs. 5.6 and 5.7 respectively.

Once the kernel functions are determined, we can compute the constant added mass coefficients using Eq. (4.39)$_2$. Alternatively, the added mass coefficients corresponding to the infinite frequency value can be used. This procedure is usually followed in practice (see for example, Chou et al., 1983, de Kat and Paulling, 1989). Van Oortmerssen (1976) computed the constant added mass coefficients using the kernel functions and frequency-dependent added mass coefficients. The computation using Eq. (4.39)$_2$ can be carried out for any arbitrary frequency $\omega'$, as the value of $A_{ij}$ is independent of $\omega'$. However, due to a number of assumptions used in the determination of kernel functions, some scatter in the values of computed
$A_{ij}$ is to be expected. Van Oortmerssen (1976) presented results for the mean and variance of 'constant' added mass coefficients obtained in this manner, which exhibited considerable scatter. Published results for TLPs and semisubmersible platforms are scarce. Table 5.1 shows the 'constant' added mass coefficients ($A_{11}$) computed for four different values of $\sigma'$. The coefficients are non-dimensionalized by $\rho V$, where $\rho$ is the mass density and $V$ is the volume of the platform.

Table 5.1 Constant added mass coefficients

<table>
<thead>
<tr>
<th>Frequency (rad/s)</th>
<th>$A_{11}(\sigma)$</th>
<th>$A_{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.06</td>
<td>0.5590</td>
<td>0.8593</td>
</tr>
<tr>
<td>1.33</td>
<td>0.6398</td>
<td>0.7816</td>
</tr>
<tr>
<td>1.69</td>
<td>0.62</td>
<td>0.8272</td>
</tr>
<tr>
<td>1.96</td>
<td>0.62</td>
<td>0.8391</td>
</tr>
</tbody>
</table>

The results seem to be satisfactory. In fact, surge and sway added mass coefficients ($A_{11}$ and $A_{22}$) showed the most scatter compared to other coefficients. As mentioned before, a number of computations have been made by adjusting the cut-off frequency limits ($\sigma_L$ and $\sigma_F$) in the determination of kernel functions and the
subsequent computation of added mass coefficients. The computed $A_{ij}$ did not show significant differences, even though the high-frequency behavior of damping coefficients has been treated rather arbitrarily. This is mainly due to the small value of damping coefficients for the TLP case considered here.

5.4 Application and Numerical Procedures

The system of equations represented by Eqs. (5.36) or (5.48) can be solved using any of the standard methods available for time-domain integration. For reasons mentioned before, 4th-order Runge-Kutta method is selected. Some of the details of this method for implementation to the system of equations given by Eq. (5.36) are presented in the next Section. However, we still need to evaluate the memory effect integral (see Eq. (5.38)) in order to attempt to solve Eq. (5.48). The memory effect integral can be written as follows:

$$M_i(t) = \int_0^\tau L_{ij}(\tau)v_j^*(t-\tau)\,d\tau, \quad i,j = 1, 2, \ldots, 6,$$

(5.54)

where $L_{ij}(\tau)$ is the kernel function and $v_j^*$ is the velocity of the center of gravity of the platform. This velocity vector is given by Eqs. (4.41) or (4.42) depending on the method used. The upper limit of the integration in Eq. (5.54) is replaced by a constant value, $\tau_{\text{max}}$, above which the kernel function $L_{ij}(\tau)$ reaches a constant value. This value has been chosen to be the same for all the kernel functions.
$L_{ij}(\tau)$ based on the function which takes the longest time to reach a constant value.

A constant value of 30s is thus selected for $\tau_{\text{max}}$ to evaluate all memory effect integrals. The time interval $d\tau$ has been chosen to be same as $dt$, the time step value used for numerical integration of the equations of motion. In this way, the time history of velocities and kernel functions are specified at the same discrete time intervals. If $dt$ and $d\tau$ are not equal, an interpolation scheme may be required to obtain velocities at points at which kernel functions are specified. A value of $\Delta t = \Delta \tau = 0.2s$ has been used in this study. Since the Runge-Kutta method used in this study is very stable, we can use a reasonably large time step, such as the one used here. The memory effect integrals are evaluated using the trapezoidal scheme.

5.4.1 Numerical Integration of the Equations of Motion

The system of equations given by Eqs. (5.36) or (5.48) can be written in the following form:

$$\frac{du_i}{dt} = f_i(t, u_j), \quad i, j = 1, 2, \ldots, 12, \quad (5.55)$$

where $u_i$ is the $i$th state variable which could be a function of all the 12 state variables $u_j$ and time. We assume that the initial conditions for all variables are specified. If we denote the solution of the variable $u_i$ at any time $t_k$ as $w(i, k)$ and
the time step as $\Delta t$, then the solution to the above system of equations can be written as follows:

$$w(i,k+1) = w(i,k) + \frac{1}{6} \left[ s(1,i) + 2s(2,i) + 2s(3,i) + s(4,i) \right],$$  \hspace{1cm} (5.56)

where $s(1,i), s(2,i), \ldots$ are given by the following equations:

$$s(1,i) = \Delta t f_1(t_k, w(1,k), w(2,k), \ldots, w(12,k))$$

$$s(2,i) = \Delta t f_1(t_k + \frac{\Delta t}{2}, w(1,k) + \frac{1}{2} s(1,1), \ldots, w(12,k) + \frac{1}{2} s(1,12)),$$

$$s(3,i) = \Delta t f_1(t_k + \frac{\Delta t}{2}, w(1,k) + \frac{1}{2} s(2,1), \ldots, w(12,k) + \frac{1}{2} s(2,12)),$$

$$s(4,i) = \Delta t f_1(t_k + \Delta t, w(1,k) + s(3,1), \ldots, w(12,k) + s(3,12)), \quad i = 1, 2, \ldots, 12.$$  \hspace{1cm} (5.57)

As can be seen from Eqs. (5.56) and (5.57), it is necessary to evaluate the right hand side of the system of equations four times for each time step. The computational procedure followed can be outlined as follows.

Starting from a known position of the platform, specified by the initial conditions, first the elemental forces and moments are determined (see Eq. (5.3)) at different sections of a given member. These forces (moments) are then integrated along the instantaneous submerged length of the member which is determined at each time step for all members of the platform. The forces thus obtained are summed over all members of the platform to obtain the total force (moment) acting on the platform. Thus, the coefficient matrices $[a_1(t)], [a_2(t)], [b_1(t)]$ and $[b_2(t)]$
and the quantities $\mathbf{F}_o(t)$ and $\mathbf{M}_o(t)$ (see Eq. (5.31)) are now determined. The right hand side of Eq. (5.36) or (5.48) can be evaluated to give the functions $s(1,i)$ in Eq. (5.57). The body position is now updated with the modified values of the state variables. The forces and moments on the platform are once again computed at the updated body position (and at $t+\Delta t/2$) to obtain $s(2,i)$. The above procedure is repeated to obtain $s(3,i)$ and $s(4,i)$. The solution for the next time step is then obtained using Eq. (5.56). In the case of the potential theory, the memory effect integrals, Eq. (5.54), should also be evaluated four times for each time step. The different components of forces (moments), and the responses at each time step are written in separate output files. This will enable us to determine the influence of any specific force component (say, for example, the Froude-Krylov force) on the response of the platform. This will particularly be useful when nonlinear response of the platform is being studied in which case one is interested in finding out the sensitivity of the response to changes in any specific force component. It may be noted that the model(s), as developed in this study, are suitable to study the nonlinear, large-amplitude responses of semisubmersible and tension-leg platforms. Resonant motions of the semisubmersible platform up to the point of capsizing, high-frequency tendon tension response of the TLP are good examples of studies that use these numerical models.

In order to reduce the effect of transient response at the beginning of the simulation, a cosine taper function is used to multiply the total force and moment
terms appearing in Eqs. (5.36) and (5.48). This will enable the exciting force to build up gradually, form 0 to its appropriate value over a period of time, specified by the taper function. This function is given by (see Paulling, 1977)

\[
Z(t) = \begin{cases} 
\frac{1}{2} \left( 1 - \cos \frac{\pi t}{t_{\text{max}}} \right), & t \leq t_{\text{max}} \\
1, & t > t_{\text{max}}
\end{cases}
\]  

(5.58)  

where \( t_{\text{max}} \) is the time up to which the taper function is applied effectively.
Fig. 5.2 Surface discretization scheme for the tension-leg platform
Fig. 5.3 Added-mass coefficients in surge and heave

(Non-dimensionalized by \( \rho v \))
Fig. 5.4 Surge and Heave damping coefficients

(Non-dimensionalized by $\rho v \sqrt{g/L}$)
Fig. 5.5 Kernel function computed in two different methods

Fig. 5.6 Kernel function in surge
Fig. 5.7 Kernel function in heave
Chapter 6

Results and Discussion

6.1 Introduction

The theory presented in the previous chapters is applied to the tension-leg platform described in Chapter 2. First the results obtained from the frequency-domain method discussed in Chapter 2 are compared with time-domain simulations. This is to establish the validity of the frequency-domain method. The nonlinear motions and tether-tension response of the platform are then studied using time-domain simulations and power spectral methods. Results from the motion-simulation model based on Morison's equation are presented first. The low- and high-frequency response of the platform is studied in different environmental conditions, such as regular waves, bi-chromatic waves and random waves in the presence of current. Finally, results obtained using the potential-theory method are presented and compared with those obtained from Morison's equation model.

6.2 Comparison of Frequency-domain and Time-domain Results of Viscous Drift Forces

As mentioned before, the frequency domain results presented in Chapter 2 need to be validated by comparison with either experimental or with time-domain results. In this study, the frequency domain results are compared with results obtained from the time-domain simulation model based on Morison's equation,
presented in Chapter 5. The mean drift forces and response are compared both in regular and random wave cases. These comparisons have been made for the tension-leg platform described in Chapter 2. A steady current of 0.91 m/s is used as mentioned in different figures. The same wave conditions, as used in Chapter 2, are used for the case of random waves.

Fig. 6.1 shows the time history of surge motion obtained through time-domain simulation method for the case of regular wave along with uniform current. Fig. 6.2 shows the mean surge displacements obtained from frequency-domain (FD) and time-domain (TD) methods. In the FD method, the mean displacement of the platform is obtained by dividing the mean surge drift force (for a given wave height and period) by the surge stiffness. The mean viscous drift force, as computed by Eq. (2.27), for a given wave height and period is used in the determination of mean offset of the platform. The horizontal stiffness of all the tethers of the platform is summed to obtain the total stiffness. The mean of the time-history of surge response is used for the TD results. The results seem to be in good agreement for $H=5m$. For larger wave height ($H=10m$) the discrepancy is possibly due to the increased importance of potential effects. Note that the mean displacements in the time-domain results contain some contribution due to the integration of the inertia force up to the actual free surface. Also, for longer wave periods, the response amplitude operators are higher, and the phase difference between the wave and surge motion of the platform contributes to the Froude-Krylov drift (see de Kat and Paulling, 1989).
In this study, viscous drift forces and the resulting responses in irregular waves are computed using the theory presented in Section 2.3. Fig. 6.3 shows the theoretical Bretschneider wave spectrum and the spectrum of the simulated time-history of wave surface elevation. The time-history of wave surface elevation is simulated using the method presented in Section 3.4.1. Fifty wave components are used in the summation with unequally spaced frequency intervals. The mean surge drift force in irregular waves is obtained using Eq. (2.34) and spectral density of the slowly-varying drift force using Eq. (2.37). The procedure followed to obtain these quantities can be explained as follows: First, the mean viscous drift force in regular waves is obtained for a series of wave heights and wave periods. The viscous drift force transfer function $G(f,f)$ is then obtained as explained in Chapter 2.2. This function, shown in Fig. 2.19, is then used in Eq. (2.34) to obtain the mean drift force in irregular waves. A cubic-spline interpolation scheme is used to obtain the drift force transfer function values at $f + \mu/2$ in Eq. (2.37). The spectral density is evaluated at about 15 to 20 frequencies around the natural frequency in surge. The integrations in Eqs. (2.34) and (2.37) are carried out using the Trapezoidal rule. Computation of mean surge displacement in the case of random waves and current and comparison with time-domain results are shown in Table 6.1. The mean surge displacement from the time-domain analysis (shown in Table 6.1) is obtained from Fig. 6.20, which shows the surge response in random waves and current. From the results presented in this table, it is seen that the mean displacements are in good agreement, considering that time-domain displacements contain some contribution.
from the inertia force. It is also noted that the mean offset of the TLP is very much influenced by viscous forces.

Table 6.1 Mean viscous drift force and surge offset in random waves and current

| Mean viscous drift force due to current only | 1052.23 KN |
| Surge stiffness | 265.47 KN/m |
| Mean surge offset due to current | 3.96m |
| Mean viscous drift force in irregular waves | 656.814 KN |
| Mean surge offset due to this force | 2.47m |
| Total mean offset | 6.43m |
| Mean surge offset from time-domain simulation | 7.63m |

The spectral density of slowly-varying surge motion is computed by using Eq. (2.40). It is to be noted that low-frequency response depends to a good extent on the damping, particularly near resonance. The evaluation of low-frequency damping and the importance of wave drift- and viscous-damping are discussed in Chapter 1. In the present study, only viscous damping is considered to determine the platform response. It should be noted that the wave drift damping should also be considered, in principle, to determine the low-frequency response, although its effect may be
small in the present context. Due to the presence of current, damping will be influenced by viscous effects. Damping ratio in the absence of current is determined here using the motion decay curves obtained from time-domain analysis. The damping ratio is found to be about 6%. The current damping is determined using a method in which the current force is linearized (see Huijsmans and Dekker, 1989) as follows:

\[ B_c = \frac{2F_c}{v_c} \]  \hspace{1cm} (6.1)

and

\[ \tau_c = \frac{B_c}{B_{crit}} \]  \hspace{1cm} (6.2)

where \( \tau_c \) is the damping ratio due to current, \( B_c \) is the damping due to current, \( F_c \) is the current force, \( v_c \) is the current velocity and \( B_{crit} \) is the critical damping. Using the value of \( F_c \) obtained from time-domain analysis, \( \tau_c \) is computed to be 35%, and hence, the total damping is found to be 41%. Fig. 6.4 shows the spectral density of low-frequency surge motion obtained as explained above. A damping coefficient of 0.4 has been used to obtain the frequency-domain result. The surge motion spectrum obtained from the spectral analysis of the time-history of surge motion is also plotted in this figure. The low-frequency spectra appear to be in good agreement. Three different values of damping coefficients (\( \tau = 0.3, 0.35 \) and

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0.4) were used to see the effect of damping on the low-frequency spectrum and the results are shown in Fig. 6.5.

6.3 Platform Response in Regular, Bi-chromatic and Random Waves

The nonlinear response of the platform will now be studied in more detail with the help of the numerical simulation models developed in this study. Use will also be made of power spectral analysis methods to process the time-records of various responses obtained in these simulations.

Regular waves:

For regular wave studies wave height of \( H = 5 \text{m} \) and wave period \( T = 7\text{s} \) are used with a steady current of 0.91m/s. Figs. 6.6 and 6.7 show the heave and pitch motions, respectively, of the TLP in regular wave and current. The surge motion of the platform is already shown in Fig. 6.1. Fig. 6.8 shows the tether tension variation of the forward tether in the case of regular wave and Fig. 6.9 shows the tether-tension including the steady current. The purpose here is to see the effect of current on tether tension of the platform. It may be noted that tether tension of a TLP depends on the horizontal excursion of the platform as well as the vertical forces acting on the platform. The presence of current causes large mean offset of the platform as already discussed and hence this may cause additional tension in the tethers. One can see, from Figs. 6.8 and 6.9, that the mean tension is slightly higher when current is included. The effect of current can also be seen from Fig. 6.10
which shows the spectra of tether-tensions plotted in Figs. 6.8 and 6.9. In this study, all results of the spectral analysis are obtained using a Fast Fourier Transform (FFT) method which uses an overlapping window. Comparison with results obtained from a method based on covariance method showed good agreement. It is to be noted that the tether tension shows response at higher harmonic frequencies, both in the case of a regular wave acting alone and in the case of wave and current. The spectral peaks appear to be at the input wave frequency (0.14 Hz) and at second- and third- harmonics of the input wave-frequency. The current seems to reduce the spectral peaks at the input wave frequency and the third harmonics. The reason for this could be the additional damping provided by the presence of current. In the time-domain simulation, the elemental forces (such as the Froude-Krylov, velocity and acceleration) forces are computed at each time step, at several points along the member length and then integrated over the instantaneous wetted length of the member. The forces computed in such a manner are then summed over all platform members to obtain the total force acting on the platform. Thus, during the simulation process, different force components (such as Froude-Krylov, drag etc.) are available separately (instead of the total force and moment acting on the platform). The relative velocity surge drag force obtained in this manner is plotted in Fig. 6.11. The increase in the mean surge force due to current can be clearly seen from this figure. The spectra of this force in the two cases mentioned above is shown in Fig. 6.12. We can see from this figure that current introduces a strong second-harmonic while reducing the peak at the input frequency. As already
mentioned, the nonlinear term in Morison’s equation gives forces at higher-harmonics depending on the input frequency (see Dello Stritto and Horton, 1984). It was also noted, in the above mentioned study, that the presence of steady components along with sinusoidal waves in the input causes the occurrence of peaks at even-multiples of the input frequency. However, the occurrence of similar peaks in tether tension is significant in the present case.

**Bi-chromatic Waves:**

The response of the platform in regular wave groups is studied next. Two regular waves with equal wave heights of \( H_1 = H_2 = 5 \) m and wave periods equal to \( T_1 = 7 \) s and \( T_2 = 7.5 \) s are used. These wave periods are such that their difference frequency corresponds to a period of about 10.5 s which is the surge natural period of the TLP. Similarly the sum frequency of these two waves correspond to a period of 3.5 s, close to the natural period in pitch of the TLP which is 2.5 s. Wave periods smaller than 7s can not be selected due to the limitation of the application of Morison’s equation for short waves. The wave surface elevation generated for the combined waves is shown in Fig. 6.13. The surge motion obtained from time-domain simulation is shown in Fig. 6.14. The surge motion shows long period oscillation with a period corresponding to the difference-frequency of the two waves. The spectra of surge motion in bi-chromatic waves and current is shown in Fig. 6.15. We can see that low-frequency peaks appear in both cases, that is in presence of current and in the case of no current. Current seems to magnify the low-frequency
response. From this, it appears that two sinusoidal waves can cause resonant surge response of the TLP under some conditions. While such a tuned combination of waves may not occur in practice, this example gives an idea of the possibility of such an event. A similar analysis has been carried out for the surge drag force. The drag force spectrum in the case of the two waves acting without current is shown in Fig. 6.16. The low-frequency force peak is to be noted in this figure. Fig. 6.17 shows the time-history of pitch motion and the tether tension is shown in Fig. 6.18. The tether-tension spectrum in this case is shown in Fig. 6.19. The second- and third-harmonic responses of tether-tension are significant and appear to be equally important as the response occurring at the wave-frequency.

Irregular waves:

Fig. (6.20) shows the wave surface elevation and the surge response in random waves and uniform current. The results correspond to the Bretschneider wave spectrum described in Chapter 2. All random wave simulations are carried out for a period of 1200 s, with 0.2s as the time step value used for numerical integration. Fig. (6.21) shows the time-histories of heave and pitch motions. The heave and pitch motions are very small due to the tension-leg mooring system. The time-history of the tether-tension in the forward tether is shown in Fig. 6.22. The pretension in each of the mooring tendons is $0.305 \times 10^8$N. We can see that the tether-tension shows considerable deviation from the mean value. The surge drag force in random waves and current along with the time-history of wave surface
elevation is shown in Fig. 6.23. The mean surge force obtained from this time record is 1621 KN. This value is in good agreement with the mean drift force computed using the frequency-domain method (see Table. 6.1). The total mean viscous drift force (mean force due to current and mean drift force in irregular waves) obtained from this table amounts to 1709 KN. The mean surge displacements also showed satisfactory agreement, as already discussed. From this, it appears that the frequency-domain method presented in this study can be used to determine the mean viscous drift force and response in regular and irregular waves. This is particularly useful to determine the extreme surge response of a TLP. In such cases this model can be used in conjunction with other methods to include the effect of wind and potential drift forces to determine the total surge response of the platform. Fig. 6.24 shows the spectrum of the drag force time-history discussed above (Fig. 6.23). The low- and high-frequency peaks outside the wave-frequency range should be noted. The tether-tension spectrum obtained in a similar manner is shown in Fig. 6.25. The high-frequency peak near the frequency corresponding to the pitch natural frequency is to be noted. Similar result has been reported experimentally (see Marthinsen, 1991).

6.4 Response Prediction using the Potential Theory Approach

In the previous section the platform motion and tether-tension responses obtained using the simulation model based on Morison's equation method are presented. As discussed in Chapters 4 and 5, use of the potential theory method in
the time-domain simulation model is preferable in order to consider the three dimensional fluid flow-effects in a better fashion than that possible using Morison's equation. Also, the frequency-dependency of the hydrodynamic coefficients, wave diffraction-and interference-effects due to closely-spaced members can be considered in this approach. This section will present the results obtained using a simulation model developed based on the theory presented in Chapters 4 and 5. These results will be evaluated by comparing them with results obtained from Morison's equation model. This will provide a preliminary validation of the potential-theory simulation model developed in this study. Comparisons are made for regular and random wave cases including current.

Fig. 6.26 shows the surge motion obtained using the two methods in regular wave and steady current conditions. Similar results for heave and pitch motions are shown in Figs. 6.27 and 6.28, respectively. In general, the results seem to be in reasonably good agreement. The potential-theory method appears to give slightly smaller magnitudes of response in all motion modes. This may be due to the fact that the diffraction effects are accurately modelled in the potential theory method. Considering the fact that the wave period used (7 s) is in the vicinity of limiting value for the application of Morison's equation, it appears that the diffraction effects begin to affect the platform response and hence the potential theory method is expected to predict the response more accurately. It should also be noted that the added-mass values in Morison's equation method are time-dependent, due to the variable submergence of platform members with time. As mentioned before, the
added-mass matrices (see Eq. 5.26) are computed at each time step by integrating the elemental acceleration-dependent forces over the complete immersed length of each platform member. However, in the potential-theory method the constant added-mass coefficients (see Eqs. 4.39 and 5.42) are used in calculating the response. The surge added mass, as computed in the two methods, is shown in Fig. 6.29. The potential-theory method seems to give a higher value as compared to Morison's equation method. This could be another reason for the smaller response predicted by the potential-theory method. The comparisons shown in Figs. 6.26 through 6.28 also show that Morison's equation can be used to predict platform response fairly accurately, even for small wave periods.

A similar situation is observed in the case of response in random waves. The surge response obtained in the two methods is plotted in Fig. 6.30. Heave and pitch motions are compared in Figs. 6.31 and 6.32 respectively. Once again, it is seen that the agreement between the two methods appears to be quite good. The variation of added mass with time in irregular waves in the case of Morison's equation is shown in Fig. 6.33. The constant surge added-mass value in the case of potential-theory method is already shown in Fig. 6.29.

From the preceding discussion, it appears that the simulation model based on the application of potential theory in the time domain is giving correct results. As described in the earlier chapters, this model can include potential and viscous effects in a large-amplitude platform motion simulation model and hence will be very useful in studying the extreme response of floating platforms. Though the model has not
been applied to such problems, the validity of the numerical model has been established. The comparisons made in Figs. 6.26 through 6.32 also establish the validity of the numerical simulation model developed based on Morison's equation.
Fig. 6.1 Time-history of surge response

\((H=5\text{m}, T=7\text{s}, \text{uniform current})\)

Fig. 6.2 Comparison of mean surge displacements
Fig. 6.3 Bretschneider wave spectrum

\((H_s = 11.4\,m, T_0 = 15\,s)\)

Fig. 6.4 Surge motion spectrum

\((H_s = 11.4\,m, T_0 = 15\,s, \text{ uniform current})\)
Fig. 6.5 Low-frequency surge motion spectrum

Fig. 6.6 Heave response of the tension-leg platform

\( (H=5\text{m}, T=7\text{s}, \text{uniform current}) \)
Fig. 6.7 Time-history of pitch response

\((H=5m,T=7s, \text{uniform current})\)

Fig. 6.8 Time-history of tether tension

\((H=5m,T=7s)\)
Fig. 6.9 Time-history of tether tension

\((H=5\text{m}, T=7\text{s}, \text{uniform current})\)

Fig. 6.10 Tether tension spectra

\((H=5\text{m}, T=7\text{s})\)
Fig. 6.11 Time-history of surge drag force

\[ (H=5m, T=7s) \]

Fig. 6.12 Surge drag force spectra

\[ (H=5m, T=7s) \]
Fig. 6.13 Wave surface elevation

(Bi-chromatic waves)

Fig. 6.14 Time-history of surge response

(Bi-chromatic waves)
Fig. 6.15 Surge motion spectra

(Bi-chromatic waves)

Fig. 6.16 Spectra of surge drag force

(Bi-chromatic waves)
Fig. 6.17 Time-history of pitch motion
(Bi-chromatic waves)

Fig. 6.18 Time-history of tether tension
(Bi-chromatic waves)
Fig. 6.19 Tether-tension spectrum

(Bi-chromatic waves)
Fig. 6.20 Wave surface elevation and surge response
(random waves and current)
Fig. 6.21 Time-histories of heave and pitch motions (random waves and current)
Fig. 6.22 Time-history of tether tension (random waves and current)
Fig. 6.23 Time-history of wave elevation and surge drag force (random waves and current)
Fig. 6.24 Surge drag force spectrum
(Random waves and current)

Fig. 6.25 Tether-tension spectrum
(Random waves and current)
Fig. 6.26 Surge response prediction using Morison's equation and potential theory ($H=5m, T=7s$, uniform current)

Fig. 6.27 Heave response prediction using Morison's equation and potential theory ($H=5m, T=7s$, uniform current)
Fig. 6.28 Pitch response prediction using Morison's equation and potential theory (H=5m, T=7s, uniform current)

Fig. 6.29 Surge added mass in Morison's equation and potential theory methods (H=5m, T=7s)
Fig. 6.30 Surge response prediction using Morison's equation and potential theory (random waves and current)
Fig. 6.31 Heave motion prediction using Morison's equation and potential theory (random waves and current)
Fig. 6.32 Pitch response prediction using Morison’s equation and potential theory (random waves and current)
Fig. 6.33 Surge added mass using Morison’s equation (random waves)
Chapter 7
Conclusions and Recommendations

In this study the nonlinear forces and response of floating platforms is studied using frequency- and time-domain methods. Particular emphasis is given to the influence of nonlinear drag force in predicting the mean- and low-frequency viscous drift forces and high-frequency springing response of floating platforms.

First, a frequency-domain method to compute viscous drift forces in regular and random waves is presented. The method, which is applied to a semisubmersible and tension-leg platforms predicted significant forces in surge, pitch and yaw modes of platform motion. The combined effect of waves and current, variable submergence of platform members and computation of forces in the displaced position of the platform members appears to have pronounced effect on the computed drift forces and moments. Comparison of the viscous drift forces predicted in this study with potential drift forces and experiments indicate that inclusion of current can have a significant effect on the total drift forces on a platform. The viscous drift forces seem to dominate the long-period range in which the potential drift effects begin to diminish and hence the viscous drift forces must be considered under design-wave conditions. It is also noted that viscous effects are important in determining the low-frequency damping and response of the platform.

The frequency-domain results are then compared with results obtained from a numerical motion-simulation model based on Morison's equation. Comparisons
of drift force and response values showed good agreement, both in regular and random waves. This suggests that the frequency-domain model can be used to predict the viscous drift forces and response in the preliminary design stage and for repetitive parametric studies due to its superior computational efficiency as compared to the time-domain simulations.

The nonlinear response of platforms is studied in different environmental conditions using the simulation model based on Morison's equation. The response in regular, bi-chromatic and random waves including steady current is studied using the simulation results and spectral analysis method. Preliminary results from these studies indicate that second- and higher-harmonic surge forces and tether-tension responses can occur in regular wave, bi-chromatic and random wave and current conditions. It is shown, through numerical simulations, that low- and high-frequency resonant response of platforms can occur even in regular and bi-chromatic waves under some conditions, due to several nonlinearities such as the nonlinear drag force and consideration of variable submergence of platform members for force calculations. Inclusion of current appears to affect the nonlinear response significantly, both in terms of providing viscous drift force excitation and damping.

A theoretical model based on the application of linear 3-D potential theory in time domain is also developed in this study. The large-amplitude nonlinear motions of floating platforms in extreme waves can be studied using this model which can include potential- and viscous-flow effects. The results obtained from this model are compared with those obtained using Morison's equation model and the
agreement is found to be good. Thus, the validity of the potential-theory model is established. However, more extensive comparisons, preferably with experiments, should be undertaken so as to test the validity of this model. In the present study, the tension-leg platform response only is studied using the potential-theory simulation model. Since the motions of the TLP are, by design, small it is difficult to validate this numerical model to predict large-amplitude motions. It may be noted that this model can be used to study nonlinear motions, such as the steady tilt and large-amplitude motions leading to the capsizing of semisubmersible platforms. It is recommended that such studies be undertaken in order to test the validity of the model in predicting large platform motions. One of the unique features of the simulation models developed in this study is the ability to have any force component during the simulation process. This will be very useful to understand the relative importance of different force contributions to the total force and response of the platform. Also, the sensitivity of response to different force components can be studied easily, which is not possible with experimental studies.

It is also noted that motion response results obtained using the simulation model based on Morison's equation gives very good results, in spite of its limitations and its empirical nature. It is suggested that such a model can be used for most semisubmersible and tension-leg platforms, which can be considered as 'slender' for most wave conditions and definitely for design wave conditions. The 3-D potential theory can be used in the final design stage for verification. On the other hand, if one is interested in the nonlinear, large-amplitude motions and response of platforms
in extreme waves, the simulation model based on the potential-theory is preferred. As discussed earlier, the three-dimensional potential theory results obtained in frequency-domain are used in the simulation model to predict large amplitude motions of the platform. More research is needed to correctly map the linear quantities into a nonlinear simulation model. These type of models can be computationally advantageous compared to solving the potential-flow problem in time-domain.

The accurate prediction of low- and high-frequency damping is critical to predict the nonlinear platform response. The role of wave-drift and viscous-damping contributions to the low-frequency damping needs to be studied in detail, particularly for semisubmersible and tension-leg platforms. The high-frequency damping, which is very important for the tether-tensions of the TLP, also need to be studied.
Appendix A

Coordinate System Transformations

A.1 Transformation between the a- and b-systems:

Let $\boldsymbol{e}_b^i$ and $\boldsymbol{e}_a^j$ be the unit base vectors in the b- and a-systems, respectively. The orientation of the b-system with respect to the a-system can be specified if the direction cosines of the angles made by $\boldsymbol{e}_b^1$ with $x_a^1$, $x_a^2$, $x_a^3$ and $\boldsymbol{e}_b^2$ with $x_a^1$, $x_a^2$, $x_a^3$, etc., are known. If we define $T_{ij} = \cos(\theta_{bi}, \theta_{aj})$, $i, j = 1, 2, 3$, then we can write

$$\boldsymbol{e}_b^i = T_{ij} \boldsymbol{e}_a^j, \quad i, j = 1, 2, 3, \quad (A.1)$$

where $T_{ij}$ are the direction cosines as defined above. Note that Einstein's summation convention has been used. Similarly any position vector $\boldsymbol{r}$, with components $x_a^i$, can be expressed in the b-system as follows.

$$\boldsymbol{r} = x_a^i \boldsymbol{e}_a^i = x_b^i \boldsymbol{e}_b^i, \quad x_b^i = T_{ij} x_a^j \quad \text{or} \quad \{x_b^i\} = [T] \{x_a^i\}. \quad (A.2)$$

Eq. (A.2) expresses the transformation between the a- and b-systems when both systems have the same origin. The generalized transformation (including translation of the origin of the b-system) can be given as follows.

$$x_{b1} = (x_{a1} - x_{b1}) T_{ik} \quad \text{or} \quad \{x_b^i\} = [T] \{x_a^i - x_{b1}\}, \quad (A.3)$$
where $\mathbf{r}_{ab} = \mathbf{x}_{abl} \mathbf{e}_{ai}$ is the translation of the origin of the b-system with respect to the a-system and $\mathbf{r}_{bs}$, $\mathbf{r}_{as}$ are the position vectors of point s in b- and a-systems, respectively.

A.2 Transformation between the b- and g-systems:

The position vector of any point m on the body varies with time as the body translates and rotates with respect to an inertial system. It is necessary to determine the position of any point on a member, when the platform makes arbitrary motion, in order to compute body- and wave-kinematics at this point. The position vector of a point m on the nth member can be written as (see Fig. 2.1):

$$\mathbf{r}_{gm}^n(t) = \mathbf{r}_{gb}(t) + \mathbf{r}_{bn}^n(t) = \mathbf{r}_{gs}(t) + \mathbf{x}_{fml} \mathbf{e}_{fi}^n(t). \quad (A.4)$$

Noting that

$$\mathbf{r}_{gs}(t) = \mathbf{r}_{gb}(t) + \mathbf{r}_{bs}(t), \quad (A.5)$$

we can write

$$\mathbf{r}_{gm}^n(t) = \mathbf{r}_{gb}(t) + \mathbf{r}_{bs}(t) + \mathbf{x}_{fml} \mathbf{e}_{fi}^n(t). \quad (A.6)$$

It should be noted that unit base vectors in the b-system, when expressed in the g-system, become time-dependent. $\mathbf{r}_{gb}(t)$ in Eq. (A.5) is the vector of translational
displacement of the origin of the b-system. $\vec{e}_{n1}(t)$ is the unit base vector in the $l$-system expressed in the g-system. We can express $\vec{r}_{bs}^n$ in the g-system as follows:

$$\vec{r}_{bs}^n = x_{bsi}^n \vec{e}_{bi} = x_{gsl}^n \vec{e}_{gi}.$$  \hfill (A.7)

We can write $\vec{e}_{ij} = P_{ji}(t) \vec{e}_{gi}$, where $P_{ij}(t)$ is the orthogonal transformation matrix for coordinate transformation between the b- and g-systems. This matrix, which is given by Eq. (A.8), can be derived by rotating the body axes successively through the Euler angles $\theta_1$, $\theta_2$, $\theta_3$, representing the roll, pitch and yaw angles respectively (see, for instance, Goldstein, 1981). The derivation of this matrix is also given in Chitrapu (1988). The sequence of rotations followed using the Euler angles is shown in Fig. A.1. The coordinate transformation matrix is given by

$$[P] = \begin{bmatrix}
C_{02}C_{03} & (S_{01}S_{02}C_{03} + C_{01}S_{03}) & (-C_{01}S_{02}C_{03} + S_{01}S_{03}) \\
(-C_{02}S_{03}) & (-S_{01}S_{02}S_{03} + C_{01}C_{03}) & (C_{01}S_{02}S_{03} + S_{01}C_{03}) \\
S_{02} & (-S_{01}C_{02}) & C_{01}C_{02}
\end{bmatrix}, \hfill (A.8)
$$

where $C_{01} = \cos\theta_1$, $S_{01} = \sin\theta_1$ and so forth for the other terms. It is to be noted that the matrix $[P]$ is time-dependent and its components are written as $P_{ij}(t)$. We can then write,

$$\vec{r}_{bs}^n(t) = P_{ji}(t) x_{bsj}^n \vec{e}_{gi},$$  \hfill (A.9)
and hence Eq. (A.5) becomes

\[ \vec{r}_{gs}^n(t) = x_{gbi} \vec{e}_gi + P_{ij}(t)x_{bsj} \vec{e}_gi. \]  \hspace{1cm} (A.10)

We can write

\[ \vec{e}_{li}^n(t) = \frac{\vec{r}_{gs}^n(t) - \vec{r}_{gs}^n(t)}{L^n} = \frac{[x_{gel}(t) - x_{gsl}(t)]}{L^n} \vec{e}_gi, \]

or

\[ \vec{e}_{li}^n(t) = \gamma_i^n(t) \vec{e}_gi, \]  \hspace{1cm} (A.11)

where

\[ \gamma_i^n(t) = \frac{[x_{gel}(t) - x_{gsl}(t)]}{L^n} \]  \hspace{1cm} (A.12)

are the time-dependent direction cosines of the nth member in the g-system and \( L^n \) is the length of the nth member. Using Eq. (A.10) and (A.11), we can write Eq. (A.6) as follows:

\[ \vec{r}_{gm}^n(t) = x_{gbi}(t) \vec{e}_gi + P_{ji}(t)x_{bsj} \vec{e}_gi + x_{imi} \gamma_i^n(t) \vec{e}_gi, \]  \hspace{1cm} (A.13)

which gives the coordinate of point m in the g-system, at any time t.

A.3 Transformation between the w- and g-systems:

Let us define the direction cosines \( \alpha_{ij} \) as follows:
\[ \alpha_{ij} = \cos(x_{wi}, x_{gj}), \quad i, j = 1, 2, 3, \quad (A.14) \]

where \( \alpha_{ij} \) are the cosines of the angles made by \( x_{w1} \) with \( x_{g1}, x_{g2}, x_{g3} \) and \( x_{w2} \) with \( x_{g1}, x_{g2}, x_{g3} \), and so on. We can then write

\[
x_{w1} = \alpha_{11} x_{g1} + \alpha_{12} x_{g2} + \alpha_{13} x_{g3},
\]

with similar equations for \( x_{w2} \) and \( x_{w3} \), that is

\[
x_{w1} = \alpha_{ij} x_{gj}, \quad i, j = 1, 2, 3. \quad (A.15)
\]

From Eq. (A.14), and denoting the counterclockwise angles as positive, we obtain

\[
x_{w1} = x_{g1} \cos \theta_w + x_{g2} \sin \theta_w,
\]

\[
x_{w2} = -x_{g1} \sin \theta_w + x_{g2} \cos \theta_w, \quad (A.16)
\]

\[
x_{w3} = x_{g3}.
\]

Considering the translations of the coordinate axes, the complete transformation between the \( g \)- and \( w \)-systems is given by

\[
x_{w1} = x_{g1} \cos \theta_w + x_{g2} \sin \theta_w,
\]

\[
x_{w2} = -x_{g1} \sin \theta_w + x_{g2} \cos \theta_w, \quad (A.17)
\]

\[
x_{w3} = x_{g3} - z_{gw}.
\]

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In Eqs. (A.16) and (A.17), $\theta_w$ is the wave approach angle, the angle between the wave-direction and the $x_{g1}$ axis and $z_{gw}$ is the vertical coordinate of the origin of the $w$-system from $x_{g1}$ axis.

### A.4 Transformation between the $l$- and $g$-systems

Let us denote, by $[Q_L]$, the matrix containing the direction cosines of the angles between the $l$- and $g$-coordinate axes. That is, $Q_{Li,j} = \cos(x_{li}, x_{gj}), i,j = 1,2,3$.

We can then write

$$
\vec{e}_i^n = Q_{Li} \vec{e}_j^g, \quad i,j = 1,2,3. \tag{A.18}
$$

For any arbitrary vector $\vec{r}$, the above equation can be written as

$$
\vec{r}_l = [Q_L] \vec{r}_g, \tag{A.19}
$$

where $\vec{r}_l$ and $\vec{r}_g$ are the position vectors in the $l$- and $g$-systems respectively. The inverse transformation, from the $l$-system to the $g$-system, is then given by

$$
\vec{r}_g = [Q_L]^T \vec{r}_l. \tag{A.20}
$$

The direction cosines can be determined as follows.

$$
Q_{L11} = \vec{e}_i^n \cdot \vec{e}_1^g, \quad Q_{L12} = \vec{e}_i^n \cdot \vec{e}_2^g, \tag{A.21}
$$
and so forth for other terms. From the above equation we see that the unit vectors \( \mathbf{e}_1^n, \mathbf{e}_2^n \) and \( \mathbf{e}_3^n \) are to be determined in order to determine the matrix \([Q_L]\). We can determine \( \mathbf{e}_1^n \) using Eq. A.11. \( \mathbf{e}_2^n \) can be determined as follows.

Let \( m \) be a point on a plane perpendicular to the \( x_{11} \)-axis and passing through the point \( s \). Since \( \mathbf{s}\mathbf{m} \) is perpendicular to \( \mathbf{s}\mathbf{e} \), we can write

\[
\mathbf{s}\mathbf{m} \cdot \mathbf{s}\mathbf{e} = 0. 
\] (A.22)

If we assume any two values (say \( x_m, y_m \)) of the coordinate \( m \), the third component (\( z_m \)) can be determined using Eq. (A.22). Once the coordinates of point \( m \) are determined, we can obtain \( \mathbf{e}_{12}^n \) as follows.

\[
\mathbf{e}_{12}^n = \frac{\mathbf{s}\mathbf{m}}{\|\mathbf{s}\mathbf{m}\|}. 
\] (A.23)

The unit vector \( \mathbf{e}_{13}^n \) is given by

\[
\mathbf{e}_{13}^n = \mathbf{e}_{11}^n \times \mathbf{e}_{12}^n. 
\] (A.24)

We can then determine the components of the transformation matrix \([Q_L]\) using Eq. (A.21).
Initial position \( x_1 \ x_2 \ x_3 \)

(i) Rotation through \( \theta_1 \) about \( x_1 \)

(ii) Rotation through \( \theta_2 \) about \( x'_2 \)

(iii) Rotation through \( \theta_3 \) about \( x''_3 \)

Final position \( x''_1 \ x''_2 \ x''_3 \)

Fig. A.1 Euler Angles
Appendix B

Pressure Gradients for Froude-Krylov Force

B.1 Linear wave theory in regular waves:

The dynamic pressure in the w-system for any point m, whose coordinates in
the g-system are \((x_{gm1}, x_{gm2}, x_{gm3})\), is given by:

\[
p = \rho g A \frac{\cosh k(h + x_{wm3})}{\cosh (kh)} \cos (kx_{wm1} - \sigma t + \epsilon). \tag{B.1}
\]

Noting the relationship for coordinate transformation between the w- and g- systems
given by Eq. (A.17) we can write Eq. (B.1) as follows:

\[
p = \rho g A \left\{ \cosh \left[ k(h + (x_{gm3} - z_{gw})) \right] \right\} \frac{\cosh (kh)}{\cosh (kh)} \cos [k(x_{gm1} \cos \theta_w + x_{gm2} \sin \theta_w) - \sigma t + \epsilon]. \tag{B.2}
\]

For convenience, let us write

\[
B_1 = \frac{\rho g A}{\cosh (kh)},
\]

\[
B_2 = h + (x_{gm3} - z_{gw}),
\]

\[
B_3 = x_{gm1} \cos \theta_w + x_{gm2} \sin \theta_w,
\]

\[
B_4 = k(x_{gm1} \cos \theta_w + x_{gm2} \sin \theta_w) - \sigma t + \epsilon = kB_3 - \sigma t + \epsilon.
\]

We can then write Eq. (B.2) as follows:
\[ p = B_1 \cosh(kB_2) \cos(B_4). \] (B.4)

The pressure gradients in the g-system can then be obtained as follows.

\[ \frac{\partial p}{\partial x_{gm1}} = -B_1 \cosh(kB_2) \sin B_4 k \cos \theta_w, \] (B.5)

\[ \frac{\partial p}{\partial x_{gm2}} = -B_1 \cosh(kB_2) \sin B_4 k \sin \theta_w, \] (B.6)

\[ \frac{\partial p}{\partial x_{gm3}} = B_1 k \sinh(kB_2) \cos B_4. \] (B.7)

**B.2 Pressure gradients in regular waves with stretching:**

The stretching method, as explained in Section 2.2.2 involves the modification of the depth-decay function. If \( x_{gm3} \) is the vertical coordinate at which the particle kinematics are required then the stretched coordinate is given by

\[ x'_{wm3} = \frac{h}{h + \eta} (x_{gm3} - z_{gw} - \eta), \] (B.8)

where \( \eta \) is the wave surface elevation. Substituting this value for \( x_{wm3} \) in Eq. (B.1) and utilizing the coordinate transformation equations (Eq. A.17) we can write Eq. (B.1) as follows:
\[
p = \rho g A \cosh \left( \frac{k h (h + x_{gm1} - z_{gw})}{h + A \cos (k(x_{gm1} \cos \theta_w + x_{gm2} \sin \theta_w) - \sigma t + \epsilon)} \right)
\]

Eq. (B.9)

\[
\cos \left( k(x_{gm1} \cos \theta_w + x_{gm2} \sin \theta_w - \sigma t + \epsilon) \right),
\]

in which \( \eta = A \cos (k(x_{gm1} \cos \theta_w - x_{gm2} \sin \theta_w) - \sigma t + \epsilon) \) has been used. Using the same definitions as given in Eq. (B.3) and denoting by \( B_5 \) the quantity enclosed in \{ \} in Eq. (B.9), we can write Eq. (B.9) as follows:

\[
p = B_1 \cosh (B_5) \cos B_4.
\]

(B.10)

The pressure gradients in the g-system can then be written as follows (noting the definition of various quantities given in Eq. (B.3)):

\[
\frac{\partial p}{\partial x_{gm1}} = B_1 B_2 k^2 a h \sinh (B_5) \frac{1}{(h + A \cos (B_4))^2} \sin (B_4) \cos (B_4) \cos \theta_w - B_1 k \cosh (B_5) \sin (B_4) \cos \theta_w,
\]

(B.11)

\[
\frac{\partial p}{\partial x_{gm1}} = B_1 B_2 k^2 a h \sinh (B_5) \frac{1}{(h + A \cos (B_4))^2} \sin (B_4) \cos (B_4) \sin \theta_w - B_1 k \cosh (B_5) \sin (B_4) \sin \theta_w,
\]

(B.12)

\[
\frac{\partial p}{\partial x_{gm3}} = B_1 \cos (B_4) \sinh (B_5) \frac{kh}{h + A \cos (B_4)}.
\]

(B.13)
B.3. Linear wave theory in random waves:

For random waves the same procedure, as followed for regular waves, is used noting that in this case all quantities will be summed over the number of wave components being used for random wave simulation.

The dynamic pressure in the case of random waves is given by

\[ p = \sum_{i=1}^{N} \rho g A_i \frac{\cosh k_i(x_{gm3} - z_{gw} + h)}{\cosh(k_i)} \cos[k_i(x_{gm1}\cos\theta_w + x_{gm2}\sin\theta_w) - \sigma_i t + \epsilon_i], \]  

where \( N \) is the number of wave components used for summation in the simulation, \( A_i, k_i, \sigma_i, \) and \( \epsilon_i \) are, respectively, the amplitude, wave number, circular frequency and phase of the \( i \)th wave component.

Let us now define the following quantities:

\[ B_{1i} = \frac{\rho g A_i}{\cosh k_i}, \]
\[ B_2 = x_{gm3} - z_{gw} + h, \]  
\[ B_4 = k_i B_3 - \sigma_i t + \epsilon_i. \]

We can then write Eq. (B.14) as follows:

\[ p = \sum_{i=1}^{N} B_{1i} \cosh(k_iB_2) \cos(B_{4i}). \]
The pressure gradients can then be written as follows:

\[ \frac{\partial p}{\partial x_{g_{m1}}} = \sum_{i=1}^{N} -B_{i1} \cosh(k_i B_2) \sin(B_{4i}) k_i \cos \theta_w, \] (B.17)

\[ \frac{\partial p}{\partial x_{g_{m2}}} = \sum_{i=1}^{N} -B_{i1} \cosh(k_i B_2) \sin(B_{4i}) k_i \sin \theta_w, \] (B.18)

\[ \frac{\partial p}{\partial x_{g_{m3}}} = \sum_{i=1}^{N} B_{i1} \sinh(k_i B_2) k_i \cos (B_{4i}). \] (B.19)

In the case of stretching method applied to random waves, the same procedure, as given in Section B.2, is followed to obtain equations similar to (B.11), (B.12) and (B.13) with the definitions of Eq. (B.15).
Appendix C

Restoration Forces due to a Catenary Mooring System

This type of mooring system is often used for semisubmersible platforms. Fig. (C.1) shows the general layout of one such cable. \( m \) is the top end of the cable which is attached to the platform and \( o \) denotes the anchor point. \( oxz \) is a local coordinate system in the vertical plane of the cable. The force and moment exerted by the cable on the platform are described in terms of the local stiffness coefficients of the cable and global displacements of the point \( m \) due to arbitrary platform motions. The local stiffness coefficients give the force exerted due to unit displacements of the cable top in the vertical plane of the cable. For example, \( k_{xz} \) indicates the force exerted by the cable in the \( x \)-direction due to unit displacement of the cable top in the \( z \)-direction. The determination of these coefficients involve the solution of the catenary cable equations. The cable tension is, in general, a nonlinear function of the displacement of the cable. Hence a single value for the stiffness coefficient can not be given. However, in the linearized analysis, an equilibrium position of the platform is first determined, then the linear stiffness of the mooring lines are obtained at this position of the platform.

The solution of the catenary equations to determine stiffness coefficients and other mooring line parameters are given in some other studies (see Nakajima, 1980, Deha Korkut, 1970, Jain, 1980). Some of these studies are valid for taut cables, some make the assumption that the motions are small. Chitrapu and Ertekin (1992)
used the catenary equations to obtain the nonlinear stiffness coefficients. Assuming that the local stiffness coefficients are available, computation of the restoration forces and moments due to arbitrary platform displacements is described below.

The changes in cable tension due to displacements in the x- and z-directions of the mooring line in the local coordinate system are given by

\[ \Delta T_x = k_{xx} \Delta x + k_{xz} \Delta z, \quad \Delta T_z = k_{zx} \Delta x + k_{zz} \Delta z, \]  

(C.1)

where \( \Delta x \) and \( \Delta z \) are the displacements of the cable top in the horizontal and vertical directions, in the vertical plane of the cable. We can write Eq. (C.1) as follows:

\[ \begin{bmatrix} \Delta T_x \\ \Delta T_z \end{bmatrix} = [k] \begin{bmatrix} \Delta x \\ \Delta z \end{bmatrix}, \quad k_{ij} = \frac{\partial T_i}{\partial x_j}, \quad i,j = 1,2, \]  

(C.2)

where \([k]\) is the matrix consisting of the 4 local stiffness coefficients. The restoration force in the g-system can then be written as (see Fig. C.1)

\[ \bar{F}_M = -[T_A]^T \begin{bmatrix} \Delta T_x \\ \Delta T_z \end{bmatrix}, \]  

(C.3)

where

\[ [T_A] = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}, \]
in which $\alpha$ is the angle made by the mooring line (see Fig C.1). It is now necessary to relate the global displacements of point $m$, due to arbitrary body motions, to the local displacements $\Delta x$ and $\Delta z$. Let $\bar{d}_m(t)$ be the displacement of the point $m$ in the $g$-system due to body motions. Then we can write

$$\begin{pmatrix} \Delta x \\ \Delta z \end{pmatrix} = [T_A] \bar{d}_m.$$  \hspace{1cm} (C.4)

Using Eqs. (C.2) and (C.4), we can write Eq. (C.3) as follows:

$$\bar{F}_M = -[T_A]^T [k] [T_A] \bar{d}_m.$$ \hspace{1cm} (C.5)

The restoring moments about the origin of the $b$-system can be obtained by first transforming the forces given by Eq. (C.5) into the $b$-system, that is,

$$\{ \bar{F}_M \}_b = [P] \bar{F}_M,$$ \hspace{1cm} (C.6)

and

$$\bar{M}_M = \bar{r}_{bm} \times \{ \bar{F}_M \}_b.$$ \hspace{1cm} (C.7)

where $\bar{r}_{bm}$ is the position vector of the point $m$ in the $b$-system. Eqs. (C.5) and (C.7) give the restoration forces and moments for a single catenary mooring line. This procedure is repeated for each of the mooring lines. It is to be noted that $[k]$ is a nonlinear function of the local displacements $\Delta x$ and $\Delta z$. For this reason, a table of stiffness coefficients $[k]$ can be setup for different horizontal and vertical
offsets of the cable top from the mean position. During the platform motion simulation, this table can be interpolated to obtain values of the 4 stiffness coefficients for arbitrary platform motions. The global displacement of point m, $\ddot{d}_m(t)$, is also a nonlinear function of platform motions at the center of gravity, due to the nonlinear nature of the transformation matrix $[P]$. If we assume that $[P]$ can be linearized under small angle assumptions and also assume a linear stiffness matrix $[k]$, we can derive a linear global stiffness matrix $[E]$ for the elastic restoration forces and moments as follows.

The displacement $\ddot{d}_m(t)$, under the assumption of small platform motions, can be written as

$$\ddot{d}_m(t) = \ddot{x}_T + (\ddot{\omega} \times \dddot{r}_{bm}),$$

(C.8)

where $\ddot{x}_T$ is the vector of translational displacement of the origin of the b-system with respect to the g-system and $\ddot{\omega}$ is the angular velocity of the body. We can also write Eq. (C.8) as follows:

$$\ddot{d}_m = \ddot{x}_T + [Q_m] \ddot{\omega},$$

(C.9)

where
Using Eq. (C.9), we can write Eq. (C.5) as follows:

\[ \vec{F}_M = -[T_A]^T[k][T_A] \vec{x}_T - [T_A]^T[k][T_A][Q_m] \vec{\omega} . \]  

(C.11)

Using the definition of \([Q_m]\) given in Eq. (C.10) we can write the moment as follows:

\[ \vec{M}_M = \vec{r}_m \times \vec{F}_M = -[Q_m] \vec{F}_M . \]  

(C.12)

Combining Eqs. (C.11) and (C.12) and denoting the restoration forces and moments by \(\vec{F}_M\) and the translational and rotational displacements by \(x_j, j=1,2,\ldots,6\), we can write

\[ F_{Mj} = [E]\{x_j\}, \quad j = 1,2,\ldots,6 , \]  

(C.13)

where

\[ [E] = \begin{bmatrix} -[T_A]^T[k][T_A] & -[T_A]^T[k][T_A][Q_m] \\ [Q_m][T_A]^T[k][T_A] & [Q_m][T_A]^T[k][T_A][Q_m] \end{bmatrix} . \]  

(C.14)
is the \((6 \times 6)\) elastic restoration matrix. \(\mathbf{F}_M\) gives the elastic restoration forces and moments.

The nonlinear stretching of cables is discussed in Chitrapu and Ertekin (1992). As a result, one can incorporate the stretching to determine the new length of the cable and then recalculate the tensions and stiffness coefficients.
Fig. C.1 Catenary mooring system
References


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