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Tsunami runup in coastal regions

Yücel, Feyza Ayse, Ph.D.
University of Hawaii, 1990
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My appreciation and thanks are extended to Freda Hellinger for typing the final copy of my dissertation.
In this study wave runup from tsunamis in coastal regions is investigated. The objective of this study is to develop a computational method to determine the runup of tsunami waves using the method of characteristics. This study limits itself to waves arriving at the coastline perpendicularly, and depth contours are assumed parallel to the shoreline. In the runup calculation for a bore-type tsunami energy losses due to bottom friction and to breaking are considered. The runup results obtained from the method of characteristics in which energy losses due to bottom friction are considered, compare favorably with those of Bretschneider and Wybro (1976) and with Amein's (1964) numerical results. When energy losses due to both friction and wave breaking are considered the results of the calculations compare well with experimental results by Miller (1968). The runup calculations are applied to a few characteristic situations on Oahu, taking into consideration historical tsunami data.
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CHAPTER I

INTRODUCTION

The term 'tsunami' is of Japanese origin, meaning 'harbor wave'. It is derived from 'tsu' (a small bay, harbor) and 'nami' (a wave). The tsunami is of seismic origin and has no reference to tidal phenomena.

The tsunami runup problem is of great importance to the coastal areas of the Hawaiian Islands and other coasts around the Pacific Ocean, because of the destructive forces caused by tsunamis. It is very important to know the runup expected from tsunamis of various magnitude for coastal engineering design and planning in the coastal zone. Therefore, there is a need to develop reliable runup prediction methods, not only for planning purposes but also to provide safe evacuation guidelines.

Information is scarce both regarding tsunami wave heights in deep water and tsunami runup on land. The effects of tsunamis on the coastal zone not only depend on the topography of the coastline and the bathymetry but also on the steepness and type of the incoming waves approaching the coastline.

This paper attempts to develop a computational method for tsunami runup in the coastal zone based on available information, however scarce, in deep water as well as information on tsunami runup both from field and laboratory experiments. Information on the behavior of tsunami waves in deep water is obtained from the historical tsunami study conducted by Okada and Tada (1983) at Miyako, Japan and by Wigen
(1977) at Tofino, Canada, which give information on the size of the tsunami waves for extreme events in specific areas. It was found that the frequency distribution of maximum amplitude of tsunamis for two locations, Tofino and Miyako, show similar characteristics. This information has been used to estimate the maximum amplitude of extreme tsunamis for Hawaii's North Shore.

Various methods in the literature which are available to calculate tsunami runup have been used for comparison with the author's method. These are Bretschneider and Wybro's (1976) study, Amein's (1964) method of characteristics with no bottom friction and Miller's (1968) experimental runup study on undular and fully developed bores.

In Bretschneider and Wybro's study the methodology followed is similar to the method of characteristics, using the maximum surge height envelope to calculate the dry bore runup on beds of various roughnesses and slopes. The present study uses the method of characteristics and includes energy losses due to breaking in addition to energy losses due to bottom friction in calculating runup. The methodology applied in this study differs from Amein's (1964) method by inclusion of bottom friction both in the wet-bore part and in the runup phase.

In the author's dry bore formulation both energy losses due to bottom friction and to breaking are included. Whereas Amein uses Friedrich's (1948) second asymptotic theory in deep water this study employs linear wave theory in deep water in intermediate depth. Also, Amein (1964) assumes that when the bore reaches the shoreline, the bore height vanishes, whereas in this study this assumption is not made. The latter is in agreement with Miller's hydraulic experiments.
The runup results of the present study are compared with runup data from Miller's (1968) hydraulic experiments of undular and fully developed bores. In Miller's laboratory model, the desired waves were generated by a rigid vertical plate or piston. In his laboratory experiments the undisturbed water depth ranged from .061m to .122m and Froude numbers ranged from 1.25 to 2.80. The comparison of the present results with Miller's runup data (Figures 4.24 and 4.25) is satisfactory.

The results of the present study are applied to two characteristic locations on the North Shore of Oahu: Waimea Bay and Mokulea Beach for which the runup height and inundation limits are calculated. The results are compared with runup data of various tsunamis. It is believed that these results may be useful for planning in the coastal zone in this area and for the civil defense authority to identify tsunami inundation zones.

This study is subdivided in the following chapters: Chapter II, Literature survey; Chapter III, Theoretical developments; Chapter IV, Application of method of characteristics to tsunami runup problems; and Chapter V, Applications to Waimea Bay and Mokulea Beach. Conclusions are given in Chapter VI.

A mathematical derivation of the method of characteristics is presented in Appendix A, whereas in Appendix B additional figures are presented. References are listed at the end of the study.
CHAPTER II
LITERATURE STUDY

The main topic of this study is the runup of tsunamis in coastal regions. This topic is of historical interest to many. In 1977 the International Tsunami Information Center initiated a program to access and make available to researchers the tsunami information contained in tide records. Data on tsunamis at Tofino, Canada, a fishing community located on the Pacific shore of Vancouver Island have been collected in a study by Wigen (1983) using a list of 1500 known and possible tsunamigenic events compiled by the Information Center. The marigrams from the Tofino tide station have been systematically searched for each possible event. A total of forty-three tsunamis have been identified in a 75-year period of records.

Another historical study of tsunamis was conducted by Okada and Tada (1983) at Miyako, along the Sanriku coast of Japan, in which the characteristics of small tsunamis were studied by inspecting all tidal records on strip charts kept by the Miyako Weather Station at around the estimated time of tsunami arrival expected from seismic and volcanic events compiled in the catalog by Wigen (1977). Fifty-eight events since 1937 were regarded as records of tsunami or possible tsunami. It was found that the frequency distribution of maximum amplitude of tsunamis of both locations, Tofino and Miyako, show similar characteristics.

Some of the most destructive tsunamis have occurred in Japan. A historical study carried out by Imamura (1934) lists 15 tsunamis that
occurred in Japan prior to 1934 (from 869 to 1933). The destructive tsunami following the Sanriku earthquake on the Pacific coast of northeastern Japan on June 15, 1896 destroyed 12,000 houses and killed at least 27,000 people. There were three main waves and the greatest amplitude was 15.2 meters. Since the epicenter of the Sanriku earthquake of March 3, 1933 was offshore, destruction by the earthquake itself on land was small. Most of the destruction from the earthquake was caused by the tsunami such that about 5,000 houses were destroyed and 2,986 people were killed. Another tsunami of considerable amplitude was generated by a severe Sanriku earthquake on December 7, 1944. Wave amplitudes as large as 8 meters were recorded. Following a Chilean earthquake of magnitude 8.5 on May 22, 1960, a tsunami invaded Japan on May 24 and achieved amplitudes to 9 meters whereby 180 people were killed.

Wilson and Tørum (1968) studied the tsunami of the 1964 Alaskan earthquake which damaged areas in Alaska, Canada, Washington, Oregon and California. The authors conclude that one of the places with great damage, Crescent City, California, is susceptible to large wave response from major tsunamis, due to its crescent-shaped and bowl-shaped Continental Shelf. There were large amplitude oscillations occurring on top of the 13.4 feet high tsunami waves with a 1.77 hour period at Crescent City. The 1964 tsunami caused no loss of life and no serious structural damage in the Hawaiian Islands. The highest water levels reported were generally about 10 feet above mean lower low water on the northern shores of Maui and Oahu and in Hilo Bay.

Tsunamis occur predominantly in the Pacific Ocean and can produce massive inundation and destruction. The catastrophic earthquake which
shook Chile late in May of 1960 triggered a series of seismic sea waves which traversed the Pacific and struck the Hawaiian Islands as a destructive tsunami on the morning of May 23, 1960. Hilo, with a 1965 resident population estimated at approximately 26,000, and situated on the northeast coast of the island of Hawaii was severely damaged. Hilo Bay has long been susceptible to damaging waves generated by seismic disturbances along the Western Coast of North and South America. Hilo Bay has experienced varying degrees of damage from tsunamis in 1946, 1952, 1957, and 1960. The most severe of these were the 1946 and 1960 tsunamis, both of which caused extensive damage along the Hilo waterfront. Shepard et al. (1950) have studied the April 1, 1946 tsunami and its destructive effects on the Hawaiian Islands. Observers at Hilo reported a marked recession of the water shortly before 7 a.m., and the arrival of the first wave crest soon after 7 o'clock. At the Honolulu Harbor tide gauge the third wave was recorded the highest, about 2.5 feet. The automatic water-stage recorder maintained on the Thomas Square artesian well in Honolulu recorded the tsunami, the well water was disturbed by an oscillation of more than 0.1 foot, which was probably caused by the arrival of the earthquake waves. Shepard states that a mean wave period of 15 minutes was reported between the wave crests at Honolulu. The wave heights in the head of Hilo Bay mostly ranged from 21 to 26 feet above lower low water. At the mouth of Wailuku River the water rose about 17 feet. The railroad bridge was destroyed by this wave and one of the steel spans was carried 750 feet upstream (Shepard, 1950). Most of the frame structures on the seaward side of Kamehameha Avenue were destroyed and the few that
remained were shifted and badly damaged. Although the Hilo breakwater suffered severely from the wave attack, the breakwater probably played a part in lessening the severity of the waves in the head of Hilo Bay. A considerable part of the damage to buildings was due to light types of construction customary in the islands. None of the reinforced concrete buildings were seriously damaged even at places where the attack was severe. A total of 488 houses were demolished and 936 houses were damaged by the 1946 tsunami (Shepard, 1950). A total of 159 people were killed in that tsunami but the heaviest loss of life was at Hilo, where 36 lives were lost. Reese and Matlock (1968) conducted a study on the structural damage from the 1960 tsunami at Hilo, Hawaii. All of the evidence gathered in this survey of structural damage indicated that the third wave of the tsunami which approached the shoreline of Hilo Harbor as a bore approximately 15 feet above normal sea level swept inland and it was this mass of high-velocity water moving laterally which inflicted the heavy damage to structures on shore. The general evaluation of Reese and Matlock (1968) was that the elevation of the water surface varied from 14 ft. to 17 ft. which was significant in causing structural damage. Most buildings in Hilo were constructed of a light wood frame and they were completely demolished by the full force of the bore. With only one exception, no damage was found to heavy structural steel-frame buildings during the survey. Concrete lightpoles were all broken and no undamaged parking-meter stanchions were found in the area across which the bore passed. Reese and Matlock (1968) have evaluated the 1960 tsunami and suggested that the structure be oriented with its
long dimension parallel with the anticipated direction of the wave, such that shear walls could be included parallel to the anticipated wave direction in order to avoid damage.

The U.S. Army Corps of Engineers have conducted a tsunami hydraulic model study for the Hilo Harbor which includes the possibility of tsunami bore formation in Hilo Bay and the various possible schemes of protection (Palmer and Funasaki, 1965). The tsunami model investigation was conducted primarily to determine the feasibility of protecting the city and harbor of Hilo from future tsunami wave attacks. The use of continuous protective barriers to control tsunami action in Hilo Bay by preventing or limiting the inundation of Hilo was found to be technically feasible, but costly. The heights of these continuous barriers necessary to withstand the design tsunami (the 1960 tsunami was selected as the design tsunami) were recommended by Palmer and Funasaki (1965) between 30 and 35 feet above mean lower low water. However, the construction of the suggested protective barriers was never carried out.

Since the Hawaiian Islands have been subject to severe tsunami destruction the determination and delineation of evacuation zones at times of tsunami warnings is of extreme significance. Cox D. (1961) has described in his study the shoreline areas in Hawaii that are subject to risk of inundation by tsunamis sufficient to justify their evacuation during tsunami warning periods. The Geophysical Society of Hawaii Tsunami Warning System Review Committee (1960) recommended that conservative limits be explicitly outlined with the best geophysical advice available for the guidance of police. It was
in response to this recommendation that Cox's study was undertaken. Cox (1961) found by trial and error that for exposed areas an assumed 50 foot wave height at an arbitrary starting point of 10 foot depth below mean lower low water, with a one percent decline with distance traversed, would provide an estimate of potential width of inundation area and runup height that appeared reasonable when compared with the records (Figure 2.1).

![Figure 2.1. Typical inundation pattern in a valley fronted by a dune ridge and reef, showing means of construction of limit of potential inundation. (after Cox, 1961)](image-url)
A flood insurance study (1980) was conducted to investigate the existence and severity of flood hazards in the City and County of Honolulu, Hawaii, and to aid in the administration of the National Flood Insurance Act of 1968 and the Flood Disaster Protection Act of 1973. Flood insurance maps provide guidelines for the inundation and flooding levels for the Hawaiian Islands. In the flood insurance study the runup heights used are in general those estimated by Houston et al. (1977).

Loomis's (1976) report presents maps of the shorelines of the Hawaiian Islands on which are recorded the wave heights of tsunamis in 1946, 1952, 1957, 1960, 1964 and 1975. Cox (1978) states that in the National Flood Insurance Program, 100-year tsunami inundation zones have been plotted assuming that the tsunami runup frequency distributions on which they are based apply uniformly 200 feet inland from the shoreline. The assumption that the runup height frequency distributions apply 200 feet inland from the shoreline is based on a suggestion by Cox (1977). Cox's statement was that "a runup height of a historic tsunami that was not measured at or near the limit of inundation may be assumed to have been measured within 100 feet inland of the vegetation line and hence, on the average about 200 feet from the mean sea level line." Cox in subsequent repetitions has emphasized the qualification that the 200 feet inland locus be used only in the absence of contrary information. Five alternative courses of action in the use of the frequency distributions information in the Flood Insurance Program are recommended by Cox (1978), each of which may be appropriate in some
coastal areas. Cox states that the simplest course of action would be to assume the 100-year runup elevation applies 200 feet from the shoreline as is now generally done in the National Flood Insurance Program. In order to determine the frequency distribution of runup heights of tsunamis Cox (1964) and Houston et al. (1977) used a negative linear correlation between the runup heights of the larger tsunamis and the logarithms of their recurrence frequencies such that

\[ H = -B - A \log_{10} F \]  

(2.1)

where, 

- \( H \) = runup height
- \( F \) = expectable recurrence frequency of tsunamis of height equal to or greater than \( H \).
- \( A, B \) = coefficients determinable by least-squares regression.

The historic tsunamis used in determining the \( A \) and \( B \) coefficients for each site by the above equation are listed in Table 2.1.

Table 2.1 Historic tsunamis

<table>
<thead>
<tr>
<th>Year</th>
<th>Month</th>
<th>Location</th>
<th>Year</th>
<th>Month</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1837</td>
<td>Nov 7</td>
<td>Chile</td>
<td>1906</td>
<td>Aug 16</td>
<td>Chile</td>
</tr>
<tr>
<td>1841</td>
<td>May 17</td>
<td>Kamchatka</td>
<td>1919</td>
<td>Apr 30</td>
<td>Tonga ?</td>
</tr>
<tr>
<td>1868</td>
<td>Apr 2</td>
<td>Hawaii</td>
<td>1923</td>
<td>Feb 3</td>
<td>Kamchatka</td>
</tr>
<tr>
<td>1868</td>
<td>Aug 13</td>
<td>Peru-Chile</td>
<td>1933</td>
<td>Mar 2</td>
<td>Japan</td>
</tr>
<tr>
<td>1868</td>
<td>Oct 1</td>
<td>Hawaii ?</td>
<td>1946</td>
<td>Apr 1</td>
<td>Aleutians</td>
</tr>
<tr>
<td>1869</td>
<td>Jul 24</td>
<td>Hawaii ?</td>
<td>1952</td>
<td>Nov 4</td>
<td>Kamchatka</td>
</tr>
<tr>
<td>1877</td>
<td>May 10</td>
<td>Chile</td>
<td>1957</td>
<td>Mar 9</td>
<td>Aleutians</td>
</tr>
<tr>
<td>1878</td>
<td>Jan 20</td>
<td>Hawaii</td>
<td>1960</td>
<td>May 23</td>
<td>Chile</td>
</tr>
<tr>
<td>1896</td>
<td>Jun 15</td>
<td>Japan</td>
<td>1964</td>
<td>Mar 27</td>
<td>Alaska</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1975</td>
<td>Nov 29</td>
<td>Hawaii</td>
</tr>
</tbody>
</table>
In addition the 1946, 1952, 1957, 1960, 1964, 1975 historic tsunamis for which runup data are available are given in Loomis (1976). For most of the other tsunamis in Table 2.1 because of the nonexistence or inadequacy of measurements runup was estimated by Houston et al. (1977). Therefore most of the runup data used for determining A, B coefficients were estimated.

The U.S. Corps of Engineers Pacific Ocean Division has prepared a one-dimensional tsunami runup program based on Houston et al. (1977) to determine the tsunami runup for various frequencies. After a certain wave runup is obtained at the assumed 200 foot locus inland by Houston's coefficients the inland wave travel and maximum inundation limits were determined utilizing a study by Bretschneider and Wybro (1976). The tsunami inundation limit is then obtained from Bretschneider and Wybro's study in which the wave runup elevations are dependent upon initial tsunami elevations, ground elevations, roughness factors, terrain slope and Froude number.

Cox (1979) stated that the influence of wave period on inundation and the influence of the combination of wave period and wave height on bore formation is not taken into account in any method then available. Generally, a study of the wave runup problem induced by tsunamis contains two different aspects: a statistical aspect and a deterministic aspect. In the statistical analysis observed data from tsunami runup are used from which predictions are made for expectations on recurrence intervals in the future. The deterministic approach deals with the analysis of the tsunami runup as a hydrodynamic problem.
Bretschnieder and Wybro's (1976) study, referred to above, determines the runup and inundation characteristics of tsunami surges. They predict the tsunami runup and inundation that are essential for a reliable tsunami alert system. In the method of Bretschnieder and Wybro the tsunami runup and inundation can be determined provided that one knows

1. the wave elevation at the coastline or other suitable datum seaward of coastline,

2. the average value of the roughness of the terrain and the slope of it over which the wave travels.

In the theoretical dam break problem, which shows much similarity with the runup problem, Keulegan (1949), considering frictionless flow, arrives at a Froude number value of 2. Experiments by Fukui et al. (1963) found that Froude number is equal to 1.73 for a bore propagating over a dry bed with Manning's roughness of n= .013. Experiments by Cross (1966) for tsunami surges results in a Froude number of 1.41 for a Manning's roughness value of .02. Bretschnieder and Wybro (1976) recommend the usage of a Froude number value equal to 2 to predict runup data (until more exact information is known). The Manning's roughness coefficients used in this study for various terrain are taken from an unpublished report by Bretschnieder et al. A detailed review of the Bretschnieder and Wybro (1976) approach is given in section 3.2.2 of Chapter III.

Williams (1971) conducted an experimental study on the runup of a double-humped wave impinging on a plane, sloping beach.
Carrier and Greenspan (1958), using the inviscid, nonlinear, shallow-water theory and a coordinate transformation following Stoker (1948), obtained explicit solutions for the runup of several different wave forms impinging on a sloping beach. Their results showed that certain wave forms can run up on a beach without breaking and that the initial wave form and the particle velocity distribution govern whether the wave will break or not. Williams (1971) found that his observed runup compared with the predicted runup by Carrier and Greenspan agreed exceptionally well, which implied that frictional effects are of second-order importance for hydrodynamically smooth beaches. Williams noted that for the waves generated for the tests no breaking was observed. It was concluded that additional research is necessary to determine the effects of friction and beach roughness on the runup.

A bore followed by a train of waves is called an undular bore. This phenomenon can be observed in rivers, especially during flood tides, due to the sudden jump of the water surface. The distortion of the tide as it propagates into shallower depth sometimes results in a bore. Similarly, a bore is formed when a special type of tsunami wave enters shallow water.

Miller (1968) determined the runup of an undular surge and bore experimentally for four slopes each with three different bottom roughnesses. Results indicate that dimensionless runup curves of the height of runup versus the height of the initial wave are approximately linear for the undular surge, \( F_r = c/\sqrt{gY_1} \) Froude number \( \leq 1.35 \), and the fully developed bore, Froude number \( \geq 1.55 \), separated by a nonlinear
transition region (reference is made to Chapter IV) (Figure 2.2). $y_1$ is the undisturbed water depth and $c$ the bore velocity. Miller found

![Figure 2.2. Dimensionless graph showing runup data (Miller, 1968)](image)

that the runup is strongly affected by both slope and bottom roughness. Empirical prediction equations are given in the form

$$\frac{h}{y_2} = f_1 (\sin \alpha, f) + f_2 (\sin \alpha, f) \frac{y_2}{y_1}$$  \hspace{1cm} (2.2)$$

where, $h$ is the runup height above undisturbed water level, $\alpha$ is the slope angle, $f$ is a dimensionless friction coefficient, whereas $y_2$ is the height of the wave measured from the channel bottom. In the laboratory model, the desired waves were generated by a rigid vertical plate or piston. In the wave generation the piston is brought rapidly from rest to constant velocity. During an experiment the standard run of the piston was 2.44 meters. The channel was 35.5 cm wide, 19.2 meters long and .91 meters deep with one glass wall covering the full length. The slopes ($2^0, 5^0, 10^0, 15^0$) were erected inside the channel and the generating plate (piston) was run either left or right through the standard 2.44 meter run, depending on which slope was being used. The roughnesses (Nikuradse sand roughness) $k_s = 0.52$ and 3.7 millimeters
were formed by coating the given slope with glass beads of desired size. The smooth surface was formed by several coats of marine enamel. Miller (1968) concluded that the experimental data on runup and changes in the structure and celerity of the wave front during progression up slope disagree with the relevant theory based on the nonlinear long-wave equations. In particular, the prediction equation for runup in the form $U_0^2/2g$ ($U_0$ is the water velocity at the shoreline), independent of beach slope as recommended by nonsaturated breaker theory of Le Méhauté (1963), was shown not to hold true. The (theoretical) conclusion that the bore height collapses to zero at the intersection of undisturbed water level with slope was also shown not to hold true by Miller for the conditions under which the present experiments were made. Miller's experimental results are used in this study to verify the author's equation, using a method of characteristics wave runup model.

Yeh and Ghazali (1988) carried out an experimental study on bore collapse, by using a laser-induced fluorescent method the transition process from bore to runup mode was investigated. Experiments were performed in a wave tank 9 meters long, 1.2 meters wide and .9 meter deep and the beach slope was 7.5 degrees. A single uniform bore is generated by lifting the gate which initially separates the quiescent water on the beach from the deeper water behind the gate. This bore generation scheme was advantageous to Yeh since a theoretical prediction of resulting bore movement can be made without difficulty using the classical dam break theory. For a fully developed bore with Froude number of 1.43 the observed transition process neither was like a
complete bore collapse nor showed a gradual transition as predicted by Hibberd and Peregrine (1979). Yeh and Ghazali (1988) observed that the bore as such never reached the shoreline, but the small wedge-shaped water body along the shore ahead of the front was suddenly pushed forward by the bore. A substantial momentum exchange occurs between the bore and the water along the shoreline. During the momentum exchange it was observed that the bore height decreases with steepening of the front face to almost vertical. The bore front first steepens but becomes less turbulent as it reduces its height while the water in front of it is extremely turbulent. Yeh and Ghazali concluded that the turbulence generated at the front near the shore is advected forward onto the beach instead of being left behind. Furthermore, the behavior of a weak bore with a Froude number of 1.18 was examined by Yeh and Ghazali (1988). The transition process from a weak bore to runup mode was found to be different from that for a developed bore. Instead of the occurrence of momentum exchange, the process is characterized by the overturning of the bore front face directly onto the dry beach surface.

A great deal of information is available for runup of short period waves. A compilation of results is presented in a report by the Technical Advisory Committee to the Dutch Government (Schijf, 1974). These results are of significant interest because under certain conditions long waves may break up into waves of much shorter period, e.g., 15-30 seconds, for which short wave behavior is applicable.

An example is the transformation of a solitary wave into a number of solitary waves on a shelf. Johnson (1972) shows that the
solitary waves (at depth \( d \)) moves into a shelf area (with depth \( d_0 \)) the wave breaks up into \( n \) waves on the shelf provided

\[
d_0 = \left[ \frac{1}{2} n(n+1) \right]^{-4/9} \quad \text{for} \quad (d_0 < d)
\]  

where, \( d_0 \) (\( d_0 < 1 \)) is the depth of the shelf. The shelf must be shallower than the uniform depth.

A distinction may be made between two categories of wave runup theory: theories for waves which do not break and theories for breaking waves. Mathematical description of the propagation of a breaking wave is still only possible in the context of the nonlinear long-wave theory in which the breaker is treated as a progressive (shock wave) bore. Stoker (1957) drew attention to the application of the method of characteristics to the problem of runup of a breaking wave. The method of characteristics is mainly used to solve hyperbolic equations to which the classical long wave equation belongs. (Detailed review in section 3.3, Appendix A.) Stoker and various other authors studied how a bore with given characteristics is propagated through initially still water of decreasing depth. Greenspan (1958) used the characteristic method to determine the point at which a given oncoming wave will break. Whitham (1958) gives an approximation method to calculate the nonuniform propagation of a shock wave. Keller, Levine, and Whitham (1960) carried out a number of calculations which appear to confirm the validity of Whitham's approximation. They developed the method for the transformation of a bore over a sloping bed. Ho and Meyer (1962), Shen and Meyer (1963) and Ho, Meyer, and Shen (1963) have given mathematically based qualitative description of the behavior of a bore as it runs up a slope. They examined a number of properties of possible solutions to the long wave equations with a plane slope.
In the conventional long wave theory a non-breaking positive wave may become a bore under certain initial conditions because the front becomes steeper. This theory does not allow for an intermediate phase in the form of a spilling breaker. Le Méhauté (1963) attempted to overcome this drawback by giving a semi-theoretical account of the energy balance for a spilling breaker partly based on the solitary wave theory. Due to the spilling breaker a given amount of energy is dissipated in such a way that the wave crest follows the breaking index curve defined by

\[ H = 0.78d \] (2.4)

where, \( d \) is the undisturbed water depth.

Then the spilling breaker is transformed into a bore when the slope becomes steeper. Le Méhauté (1963) also included a stabilizing term in the equation of motion related to the curvature of the streamlines (Boussinesq term) and a frictional dissipation (resistance) term.

Theory that incorporates vertical acceleration effects arising from streamline curvature in approximations to the horizontal momentum equation is called Boussinesq theory after Boussinesq (1872). (Reference is made to Chapter III.) Results of solitary waves and laboratory scale flood waves propagating in one-dimensional channels are compared by finite difference analog equations to the de Saint Venant and Boussinesq equation systems (Basco, 1989). The Saint Venant equation is the basic long wave equation without taking into account the streamline curvature effect on the vertical pressure distribution. Basco (1989) found that both the de Saint Venant and Boussinesq equations gave very similar results, except for wave periods below about 100 sec.
Basco concluded that since prototype scale flood waves are all of much longer duration, the de Saint Venant equations are more than adequate to capture the physics of these events.

Freeman and Le Méhauté (1964) have given a detailed mathematical treatment of the general problem of wave breakers on a beach and surges on a dry bed. The general problem of a wave of arbitrary shape moving over a gently sloping beach is treated by the method of characteristics. One important result of the work of Freeman and Le Méhauté (1964) is their clarification of the hydrodynamic nature of the water surge on a dry bed. They reason that the height of the bore front becomes zero when it reaches the shoreline which fact however is disputed by other authors and by experiments. The assumption of a simple frictional effect results in a bore-like wave cutting short the leading edge. The leading edge is defined as the waterfront of the surge over a dry bed. It was concluded that friction makes the runup sensitive to the shape of the wave rather than just the value of the water velocity when the bore moves into water of decreasing depth.

Dronkers (1964) described the characteristics of a bore in a tidal river. From a hydraulic point of view the bore can be considered as a moving hydraulic jump which propagates with the velocity c (section 3.2). Dronkers (1964) uses the finite difference method to solve the conventional long wave equation (equation of motion) which includes the frictional dissipation term. Dronkers discussed the addition of an artificial viscosity term to the equation of motion such that the bore is no longer a discontinuity, but a narrow region across which pressure and density vary rapidly, but continuously. The artificial
viscosity term (reference is made to Chapter III) is described as the apparent shear stress per unit width and represents the dissipation loss due to turbulence.

Schönfeld (1955) considered both the propagation and the shape of the bore theoretically and these considerations were checked by experimental evidence obtained on a tide with a bore in a laboratory canal. The propagation is treated by considering the bore as a mobile hydraulic jump. The theory is checked twice, first by combining the data on the bore where it passed the observation points, and secondly by computing the entire tide with the bore by the method of characteristics. Schönfeld considered the shape of the bore and discussed the influence of capillarity (detailed review in section 3.2).

In Amein's (1966) study, the propagation of long-period waves is determined by the first-order linear small amplitude surface wave theory for regions away from the shore and by the first-order nonlinear shallow water theory for regions near the shore. Amein discards the Boussinesq streamline curvature effect on the wave transformation. Calculations by the linear theory were made by using Friedrich's second asymptotic representation, which is suitable for small beach slopes. Amein (1966) states that since the linear theory predicts an infinite wave amplitude at the shoreline, the assumption on which the theory is based is not valid for the area near the shore, conversely, the first order nonlinear shallow-water theory does not present this difficulty. In shallow water, the calculations were made using the nonlinear theory on a digital computer by a finite difference technique based on the method of characteristics. In the linear and first-order
nonlinear shallow water theories, the pressure distribution is assumed to be hydrostatic. Amein (1966) states that in the application of the shallow water theory to long waves, this assumption is valid because these waves have negligibly small surface curvatures. The bore equations are coupled to the equations of the nonlinear theory, and a procedure for the calculation of the wave runup on the dry sloping beach is given. Amein does not consider frictional resistance because he believes that unrealistic results are obtained if a resistance term is introduced according to existing formulae. A number of calculations show that runup increases with the wave period.
CHAPTER III
THEORETICAL DEVELOPMENTS

3.1 Long Wave Equations

In the calculation of runup use will be made of the one-dimensional long wave equation representing the equation of motion in the x-direction.

The equation of motion for an inviscid fluid in which the shear stresses are zero is given by Euler's equation:

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + X$$  \hspace{1cm} (3.1.1)

where,

\(x\) = the horizontal coordinate axis.
\(t\) = time.
\(p\) = pressure under the waves.
\(X\) = the body force (external force) per unit mass acting in the x direction.
\(U\) = the horizontal particle velocity in the x direction (uniform on z).

The particle velocities in the y and z directions respectively, \((V,W)\) are zero for the one-dimensional long wave equation.

In many real flow situations, stresses are due to viscous effects and turbulence effects in the fluid. In this section, viscous stresses governed by the Newtonian shear stress relationship will provide an expression for the bottom shear stress.
External forces of interest are the x-component of gravity and the coriolis force. The former is zero when the x-axis is taken in the horizontal direction. The coriolis force is an apparent force which needs to be introduced when the coordinate system in which we formulate the problem, is subject to a rotating movement. However, since we are assuming the problem to be two-dimensional and runup distances are relatively short the coriolis force is also neglected. It is then assumed that the external body force X is zero.

If internal stresses other than pressure are included, the equation of motion in the x-direction can be formulated by writing the surface forces which can be obtained using the Taylor series expansion (Figure 3.1).

\[
\sigma_{xx} + \frac{\partial \sigma_{xz}}{\partial z} dz + \tau_{xy} + \frac{\partial \tau_{xy}}{\partial z} dz + \tau_{xz} + \frac{\partial \tau_{xz}}{\partial z} dz = 0
\]

\[
t_{xx} = \frac{\partial t_{xx}}{\partial x} dx + \frac{\partial t_{xy}}{\partial y} dy + \frac{\partial t_{xz}}{\partial z} dz
\]

\[
t_{xy} = \frac{\partial t_{xy}}{\partial x} dx + \frac{\partial t_{xy}}{\partial y} dy + \frac{\partial t_{xz}}{\partial z} dz
\]

\[
t_{xz} = \frac{\partial t_{xz}}{\partial x} dx + \frac{\partial t_{xz}}{\partial y} dy + \frac{\partial t_{xz}}{\partial z} dz
\]

Figure 3.1. Shear and normal stresses on a fluid cube
The equation of motion in the x-direction then can be written as

$$\rho \frac{DU}{Dt} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$  \hspace{2cm} (3.1.2)$$

where,

$$\rho = \text{sea water density.}$$

By writing

$$\sigma_{xx} = - p + \tau_{xx}$$  \hspace{2cm} (3.1.3)$$
equation (3.1.2) develops into:

$$\frac{DU}{Dt} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right)$$  \hspace{2cm} (3.1.4)$$

The pressure $p$, under long waves is found to be hydrostatic since the vertical accelerations in long waves can be shown to be very small. Taking z vertically upward from the mean sea level, $p$ can be expressed by

$$p = -\rho g z + \rho g \eta$$  \hspace{2cm} (3.1.5)$$
or

$$p = -\rho g (z - \eta)$$  \hspace{2cm} (3.1.6)$$

where,

$$\eta = \text{wave elevation above mean sea level (Figure 3.2).}$$

$$g = \text{gravitational acceleration.}$$

The first term on the right-hand side of equation (3.1.4) then becomes
\[-\frac{1}{\rho} \frac{\partial p}{\partial x} = -g \frac{\partial h}{\partial x}\]  (3.1.7)

which gives

\[\frac{\partial U}{\partial t} = -g \frac{\partial h}{\partial x} + \frac{1}{\rho} \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right)\]  (3.1.8)

The bottom slope \(S\) is defined by

\[S = -\frac{\partial d}{\partial x}\]  (3.1.9)

where,

\(d\) = depth below mean sea level (Figure 3.2).

Figure 3.2. Schematic representation of the parameters

By considering the zero quantity (Le Méhauté, 1976)

\[-g \frac{\partial d}{\partial x} - gS = 0\]

and adding this quantity to the right-hand side of the equation (3.1.8), the result is

\[\frac{\partial U}{\partial t} = -g \frac{\partial h}{\partial x} - g \frac{\partial d}{\partial x} + gS + \frac{1}{\rho} \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right)\]  (3.1.10)

which can be written as
\[
\frac{DU}{Dt} = - g \frac{\partial h}{\partial x} - gS + \frac{1}{\rho} \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) \tag{3.1.11}
\]

where the total water depth \( h \) is defined by Figure 3.2

\[ h = d + \eta \tag{3.1.12} \]

In a purely two-dimensional situation the term \( \frac{\partial \tau_{yx}}{\partial y} \) vanishes. Consider the case where \( \frac{\partial \tau_{xx}}{\partial x} \) is small compared to \( \frac{\partial \tau_{zx}}{\partial z} \). The equation then reduces to

\[
\frac{DU}{Dt} = - g \frac{\partial h}{\partial x} - gS + \frac{1}{\rho} \frac{\partial \tau_{zx}}{\partial z} \tag{3.1.13}
\]

Integration of equation (3.1.13) with respect to \( z \) from bottom to surface and dividing by the depth \( h \) while using the boundary conditions at the surface and at the bottom the horizontal momentum equation becomes

\[
\frac{Du}{Dt} = - g \frac{\partial h}{\partial x} - gS + \frac{1}{(\rho h)} \left( \tau(\eta) - \tau(-d) \right) \tag{3.1.14}
\]

where \( u \) is the horizontal particle velocity averaged over depth. The term \( \frac{1}{\rho h} \tau(\eta) \) on the right represents the tangential stresses at the sea surface and is neglected in this study. The term \( \frac{1}{\rho h} (\tau(-d)) \) represents the tangential bottom stresses which are significant. The tangential bottom stresses in uniform flow may be expressed by

\[ \tau(-d) = \tau_b = \rho g h S \tag{3.1.15} \]

The Chezy formula for uniform flow is (Rouse, 1938)

\[ u = C_h \sqrt{hS} \tag{3.1.16} \]
for a wide channel \((b \gg h)\), where
\[ C_h = \text{Chezy coefficient}. \]

Assuming that the flow in a long wave varies so slowly that the formula for uniform flow may be applied to a long wave situation we can write
\[ \tau_b = \rho g \frac{u^2}{(C_h)^2} \quad (3.1.17) \]
Furthermore, inserting the above equation into the horizontal momentum equation \((3.1.14)\) gives
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -g \frac{\partial h}{\partial x} - gS - \frac{u^2}{C_h^2 h} \quad (3.1.18) \]

For an oscillatory flow, it is clear that as the fluid reverses direction, so also must the bottom friction. Therefore an absolute value sign for the velocity is introduced to the last term of the above equation which represents bottom friction in its quadratic form. In this way the acting friction force is always acting in a direction opposite to the direction of \(u\).
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -g \frac{\partial h}{\partial x} - gS - \frac{|u| u}{C_h^2 h} \quad (3.1.19) \]

In certain types of problems the assumption of neglected vertical accelerations leads to results that do not fully agree with experimental values.

The theory that incorporates vertical acceleration effects arising from streamline curvature in approximations to the horizontal momentum equation is called Boussinesq theory after Boussinesq (1872). Boussinesq theory takes into account of this path curvature effect
by assuming that the vertical velocity is linearly distributed from
the bottom to the free surface (Le Méhauté, 1976). A correcting term
which results from this can then be added to the right-hand side of
the above momentum equation, as follows
\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -g \frac{\partial h}{\partial x} - gS - gu | u | + \frac{h}{C_h^2} \frac{\partial^3 h}{\partial x^3} \quad (3.1.20)
\]

As the waves become very long (in space and time), the streamline
curvature effects approach zero and the additional correction term
drops out; consequently the equation reduces to that of equation
(3.1.19).

However in the shallow water zone, when waves get shorter and
steeper and may ultimately break, the Boussinesq term may be significant
in the prediction of wave behavior.

The continuity equation can be derived by considering a column
of area dxdy and height (d + η). The continuity equation states that
the sum of all the net fluid flows into the column must be balanced
by an increase of fluid in the column, which since it is an incompressible
fluid, is manifested by the change in height or volume of the
column.

For a two-dimensional flow with free surface (one horizontal and
one vertical direction) the discharge per unit width is expressed by
\[
\begin{align*}
q_x &= uh \\
q_y &= 0
\end{align*} \quad (3.1.21)
\]

The conservation of mass is then expressed as
\[
- \frac{\partial q_x}{\partial x} \, dx \, dy \, dt = \frac{\partial h}{\partial t} \, dx \, dy \, dt \quad (3.1.22)
\]
Inserting equation (3.1.21) into (3.1.22) gives
\[ \frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} = 0 \]  
(3.1.23)

or
\[ \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + h \frac{\partial u}{\partial x} = 0 \]  
(3.1.24)

The continuity equation (3.1.23) for flow in one direction is formulated where the y-term vanishes. This occurs when waves are perpendicular to a straight shoreline which will be the condition of our investigations.

3.2 Bore Equations

In several estuaries the rising tide during its propagation upriver develops into a sudden jump of the water surface. This phenomenon is called a bore. The bore does not exist at the mouth of the estuary where the vertical tide depends on the tides in the sea. When the tide propagates up a river into shallower depth, distortion results often in a shortening of the time interval between low water level and high water level and a corresponding lengthening of the time period between the high water level and the subsequent low water level. Under certain circumstances, the steepness of the tidal curve between low water level and high water level may even become so large that at some specific point, a finite discontinuity in the water level occurs and a bore in the form of a wall of water rushes up the river (Dronkers, 1964). It is such that a part of the front of the bore is vertical, but the main part of the profile will generally have a steep slope. The water behind the bore front is in
violent turbulent motion, but some hundreds of meters farther back, the water surface is again quiet. A similar phenomenon may be experienced when tsunamis enter shallow water. The bore is caused by the distortion of the long wave. In a long wave the vertical accelerations are negligible but in a bore this is no longer the case, therefore the bore needs special treatment. In long wave theory the wave is propagated such that higher parts of the wave seem to have a greater velocity of propagation than the lower parts. The wave elements near the crest have a tendency to catch up with the leading wave elements. A vertical wall of water will soon result, forming a bore. Although this phenomenon may actually occur physically, when it does occur, it happens much later than predicted by the long wave theory. Similarly, the breaking of a long wave on a beach will be predicted sooner than it actually will occur. This deficiency in the long wave theory is the long wave paradox and is due to the fact that the vertical acceleration effects from streamline curvature are not taken into account (reference is made to section 3.2.1).

The amplitude of the long wave is decreased by the influence of bottom friction and the front is somewhat flattened. The development of the long wave is dependent on whether the steepening influence of the distortion or the somewhat flattening influence of friction, is predominant.

From a hydraulic point of view a bore can be treated as a moving hydraulic jump by using a coordinate system that moves with the speed of the waves c. (Figure 3.3). Dronkers (1964) described the characteristics of a bore in a tidal river. If the observer moves with the
velocity of the bore, the phenomenon of a stationary hydraulic jump will be observed. The well-known formulae for the hydraulic jump may then be applied and be transformed into a moving coordinate system.

Figure 3.3. Definition sketch of the bore (Schönfeld, 1955)

With reference to Figure 3.3(b) where \( h_1 \) and \( h_2 \) are the water depths before and after the jump and \( u_1 \) and \( u_2 \) the mean velocities in the bore as shown, the law of conservation of mass gives

\[
\rho b h_1 (u_1 + c) = \rho b h_2 (u_2 + c)
\]  

(3.2.1)

where, in this particular case

\( c = \) the velocity of propagation of the bore.

The derivations are for a channel of width \( b \), for the two-dimensional case \( b \) can be taken equal to 1. Neglecting the frictional losses the law of conservation of momentum gives
\[
\frac{1}{2} \rho g bh_1^2 + \rho (u_1 + c)^2 bh_1 = \frac{1}{2} \rho g bh_2^2 + \rho (u_2 + c)^2 bh_2
\]  \hspace{1cm} (3.2.2)

Introducing equation (3.2.1) into (3.2.2) gives

\[
(u_1 + c)(u_2 + c) = gh_m
\]  \hspace{1cm} (3.2.3)

and

\[
h_m = \frac{h_1 + h_2}{2}
\]  \hspace{1cm} (3.2.4)

Solving \((u_1 + c)\) and \((u_2 + c)\) from equations (3.2.1) and (3.2.3)

\[
\beta = \frac{u_1 - u_2}{h_2 - h_1} = \left( \frac{gh_m}{h_1 h_2} \right)^{1/2}
\]  \hspace{1cm} (3.2.5)

and the velocity of propagation of the bore \(c\) is given as

\[
c = -u_1 + \sqrt{gh_1} \left( \frac{h_2}{h_1} \left( \frac{h_1 + h_2}{2h_1} \right) \right)^{1/2}
\]  \hspace{1cm} (3.2.6)

The difference in energy level \(\Delta H\), for a hydraulic jump is given by

(Rouse, 1938)

\[
\Delta H = \frac{(h_2 - h_1)^3}{4h_1 h_2}
\]  \hspace{1cm} (3.2.7)

where \(h_1\) and \(h_2\) represent depths of water before and after the jump.

The rate of change of energy in a bore per unit width is given by

\[
\frac{dE}{dt} = -\rho g q' \Delta H
\]  \hspace{1cm} (3.2.8)

whereby the discharge \(q'\) per unit width is defined by

\[
q' = (u_1 + c) h_1
\]  \hspace{1cm} (3.2.9)
where \( u_1 \) is mean velocity (Figure 3.3) in the bore at the depth \( h_1 \) relative to the fixed system.

The rate of energy dissipation is equal to the power set free in the jump and this power is obtained by comparing the total head at both sides of the jump. For the entering bore the delivered power due to turbulence is given by (Schönfeld, 1955)

\[
P_g = \rho g q' b \Delta H = \frac{1}{4} \rho g \alpha' b \delta (h_2 - h_1)^3
\]

(3.2.10)

where,

\[ b = \text{channel width (for the two-dimensional case } b \text{ can be taken equal to 1).} \]

\[ \alpha' = \text{the fraction of the total dissipation rate due to turbulence.} \]

Schönfeld considered the shape of the bore such that the bore travels faster than the characteristic wave component in the lower water ahead, but slower than the wave component in the upper water arrear.

Schönfeld considers a wave of which the front is approximated by an exponential solution and the rearward part by a sine wave or a periodic wave.

![Figure 3.4: Features of the bore (from Schönfeld, 1955)](image-url)
According to Schönfeld energy in the bore is dissipated in three different ways.

The group velocity $C_g$ of the short gravity waves is less than the phase velocity. Consequently there is a rearward transport of energy $P_g$, in the trail. The secondary waves formed move in the positive direction but with respect to the bore they move away from the bore. This power is gradually dissipated by internal and bottom friction in the wave train.

The group velocity of the capillary ripples is greater than the phase velocity, there is a forward transmission of energy $P_r$, in the ripple train. The ripple train must be fed from behind and appears in front. This power, $P_r$ is gradually dissipated by viscosity in the ripple train.

Relative to the moving coordinate system the flow appears as decelerated, which induces extra turbulence resulting in loss of energy due to turbulence, $P_d$. Schönfeld states that the delivered power $P$ is the sum of the three dissipations.

$$P = P_g + P_r + P_d \quad (3.2.11)$$

Of the three dissipation mechanisms of equation (3.2.11) the loss due to turbulence is the most significant for breaking waves.

In a bore type wave both bottom friction and energy losses due to turbulence at the bore front must be taken into consideration. Consequently in the long wave equation (3.1.19) an additional term is needed which expresses the energy dissipation at the bore front.

For this two methods will be discussed.
3.2.1 Equation of Motion When Turbulent Dissipation is Included

The equation of motion when dissipation is included is in general terms (Rouse, 1938)

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -g \frac{\partial h}{\partial x} - gS + (\nu + \epsilon) \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)
\]  

(3.2.12)

where,

\( \nu = \) kinematic viscosity.

\( \epsilon = \) apparent kinematic viscosity.

Rewriting equation (3.2.12)

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -g \frac{\partial h}{\partial x} - gS + \nu \nu^2 u + \epsilon \nu^2 u
\]  

(3.2.13)

The third term on the right-hand side of the above equation represents the stress gradients due to viscous effects. Viscosity is a fluid property and plays a role in flow with low Reynolds numbers and in laminar boundary layers. Consequently, it may affect the bottom shear stress, as introduced earlier.

For high Reynolds numbers bottom shear stresses are governed by turbulent boundary layers. In general the bottom shear stress is here expressed in terms of the Chezy coefficient \( C_h \), which is a function of both Reynolds number and relative roughness, and which viscosity may or may not play a role. For the purpose of this analysis it is conceived that the bottom shear stress, introduced earlier in the wave equations (Equation 3.1.17), represents the term \( (\nu \nu^2 u) \) and that losses expressed by \( (\epsilon \nu^2 u) \) are exclusively losses from a different origin, such as in a bore. These losses \( (\epsilon \nu^2 u) \) are due to turbulence. The
coefficient \( \epsilon \), known as apparent kinematic viscosity or as kinematic eddy viscosity, is completely a function of the turbulent fluid motion.

For long wave motion, including a bore the equation of motion will then be of the form

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -g \frac{\partial h}{\partial x} - gS - \frac{\rho u |u|}{C_h^2} + \epsilon v^2 u
\]  

(3.2.14)

The friction term will be applicable over the full wave regime, including the limited area where the bore is developed. In the latter area the last term will dominate. Since it is expected that within the bore regime the gradient of \( u \) with respect to \( y \) is small compared to the gradient with respect to \( x \), the term \( \partial^2 u / \partial y^2 \) becomes negligible. Since we are treating the problem as a two-dimensional problem, the term \( \partial^2 u / \partial z^2 \) also vanishes. This reduces equation (3.2.14) to

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -g \frac{\partial h}{\partial x} - gS - \frac{\rho u |u|}{C_h^2} + \epsilon \frac{\partial^2 u}{\partial x^2}
\]  

(3.2.15)

Reynolds (1895) stated that velocities due to turbulence cause an effective mean shearing stress in turbulent flow. It was shown that Reynolds stresses (apparent stresses) have the following magnitudes (see Figure 3.5). Momentary flow across the line \( a-a \) as the result of the component \( v' \) will exert a normal force in the \( y \) direction and a tangential force in the \( x \)-direction. Stress components in the horizontal \( x \)-direction are:

\[
\tau_{xx}' = \rho u'^2
\]  

(3.2.16)
\[
\tau_{xy} = -\rho u'v' \quad (3.2.17)
\]

\[
\tau_{xz} = -\rho u'w' \quad (3.2.18)
\]

where,

\[u', v', w' \text{ = fluctuating velocities of fluid particles due to turbulence in the } x, y \text{ and } z \text{ directions, respectively.}\]

Figure 3.5. Apparent stresses resulting from momentum transport

In the case of the bore, because of the very rapid change of \(u\) within the short distance of the bore length, the apparent normal stress gradient \(\frac{\partial \tau_{xx}}{\partial x}\) is much higher than the apparent shear stress gradient \(\frac{\partial \tau_{yx}}{\partial y}\) (Rouse, 1938).

Prandtl (1926) related the velocities of turbulence to the general flow characteristics by proposing that small aggregations of fluid
particles are transported by turbulence a certain mean distance \( l \), from regions of one velocity to regions of another and in so doing suffer changes in their general velocities of motion. Prandtl termed the distance \( l \), 'the mixing length' and suggested that the velocity change in the vertical direction incurred by a fluid particle moving through the distance \( l \) was proportional to the fluctuating velocities. In a similar fashion it may be suggested for the bore that

\[
u' = l \frac{du}{dx} \quad (3.2.19)
\]

From this we will have

\[
\tau_{xx} = \rho u' \nu' = \rho l^2 \left( \frac{\partial u}{\partial x} \right)^2 \quad (3.2.20)
\]

Therefore, from a comparison with equation (3.2.15) we find

\[

\frac{1}{\rho} \frac{\partial \tau_{xx}}{\partial x} = \epsilon \frac{\partial^2 u}{\partial x^2} = \frac{1}{\rho} \frac{\partial}{\partial x} \left( \rho l^2 \left( \frac{\partial u}{\partial x} \right)^2 \right) \quad (3.2.21)
\]

Thus inserting the above equation into (3.2.15) gives

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -g \frac{\partial h}{\partial x} - gS - \frac{\partial u |u|}{C_h h} + \frac{1}{\rho} \frac{\partial \tau_{xx}}{\partial x} \quad (3.2.22)
\]

Dronkers (1964) presented an artificial viscosity term, \( q \) defined by

\[
q = \lambda^2 h \left( \frac{\partial u}{\partial x} \right)^2 \quad (3.2.23)
\]
Comparing this with the above analysis it is evident that

\[ q = \frac{\tau_{xx} h}{\rho} \quad (3.2.24) \]

which corresponds to an integrated value of \( \tau_{xx} \) over the depth \( h \), taken per unit of mass. Dronkers proposes that the term should be activated for a bore and be neglected when a bore is not present. Therefore,

\[ q = \kappa^2 h \left( \frac{\partial u}{\partial x} \right)^2 \quad \text{for} \quad \frac{\partial u}{\partial x} < 0 \]
\[ = 0 \quad \text{for} \quad \frac{\partial u}{\partial x} > 0 \quad (3.2.25) \]

Dronkers states that \( \kappa \) in the above equation (3.2.23) has the meaning of a horizontal mixing length and can be expressed by

\[ \kappa = r H_i \quad (3.2.26) \]

where,

\( H_i = \) is the bore height and
\( r = \) a numerical value.

Introducing the artificial viscosity term \( q \) (note that \( q \) is not discharge here) into the equation of motion (3.2.22) gives (Dronkers, 1964)

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -g \frac{\partial h}{\partial x} - gS - qu \frac{|u|}{h c_h^2} - \frac{1}{h} \frac{\partial q}{\partial x} \quad (3.2.27) \]

The idea of introducing an artificial viscosity term originates from Von Neumann and Richtmeyer (1950). This approach has been
developed to avoid the difficulty of having a very fine grid to
determine the precise origin of the bore by the method of character-
istics. With the artificial viscosity term added the discontinuity,
such as a bore is altered in a way that the discontinuous transition
becomes smooth, extending over a small interval $\Delta x$.

Von Neumann and Richtmeyer have added artificial viscosity terms
to the equation of motion for the determination of shock waves in gases
such that the shock is no longer a discontinuity, but a narrow region
across which pressure and density vary rapidly but continuously. Since
the dissipative terms are also added in the regions outside the shock,
their influence must be negligible. This procedure has the advantage
that one does not need to know where the actual shock exists.

According to Preismann and Cunge (1961) introduction of a 'pseudo-
viscosity' or an artificial viscosity term is an approximate representa-
tion of the loss of head due to the travelling surge wave and the
development of the bore.

Numerical experiments carried out by Richtmeyer and Morton (1957)
have shown that when the constant $r$ in the mixing length term, is not
less than .75 there is excellent agreement between theory and experi-
ment. Figure 3.6 shows pressure profiles for which three different
values of $r$ are used. It is observed that for a large value of $r$ the
shock is thick and the pressure profile behind it is smooth, whereas
for a small value of $r$ the shock is sharper and there are oscillations
of pressure behind it.
Figure 3.6. Pressure profiles for three different $r$ values (from Richtmeyer, 1957)

Figure 3.7 shows a case when the artificial viscosity term is omitted. The oscillations behind the shock wave are severe.
3.2.2 Evaluation of Breaking Dissipation Term

In this section we will consider three different approaches to arrive at a formulation for the turbulent energy dissipation that can be used in the application of the characteristic method to solve the propagation of the dry bore in the runup calculations.

It will be shown that the turbulent dissipation term can be used in the form \(-g \frac{\xi H}{2}\), in which \(\xi\) will be defined later, \(H\) is the height of the bore and \(\ell\) is the so-called bore length (Figure 3.8). The
latter is defined as the distance behind the bore front over which the bore loses the major portion of turbulent energy. Using the similarity with the hydraulic jump it can be shown that this distance is typically 5 to 6 times the bore height (Rouse et al., 1958). In the dry bore situation (see Figure 3.8) this term will equal \(-g \frac{\xi h}{x}\).

\[ \ell = \frac{h}{f'} \]

\[ H = h \]

\[ h_1 \]

\[ h_2 \]

Figure 3.8. Bore length definition

The three methods consist of:

a) Further simplification of the artificial roughness formulation.

b) Use of similarity with the hydraulic jump.

c) Comparison with a formulation used for periodic waves and transforming this formulation back to a non-periodic bore.

a) Further simplification of the artificial roughness formulation

The turbulent dissipation in terms of the artificial viscosity \(q\) is (equation 3.2.27) \(-\frac{1}{h} \frac{\partial q}{\partial x}\) and \(q\) is defined by
where,

\[ \lambda_1 \] = horizontal mixing length.

Considering the gradient of \( h \) small compared to the gradient of \( u \) in the bore, we have

\[
\frac{1}{h} \frac{\partial q}{\partial x} = \frac{2}{h} \left( \lambda_1 \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial x^2} \right) \quad (3.2.29)
\]

We will now simplify this equation for the dry bore situation. Further expressing the above equation in terms of a characteristic length \( \lambda_2 \), over which the gradient of \( \frac{\partial u}{\partial x} \) develops.

\[
\frac{1}{h} \frac{\partial q}{\partial x} = \frac{1}{h} \left( 2 \lambda_1 h \frac{\partial u}{\partial x} \left( \frac{\partial^2 u}{\partial x^2} \right) \right) = 2 \frac{\lambda_1}{\lambda_2} \left( \frac{\partial u}{\partial x} \right)^2 \quad (3.2.30)
\]

The term \( \frac{\partial u}{\partial x} \) is the gradient of \( u \) with respect to \( x \) and it can be approximated by \( \frac{u}{\lambda_2} \).

\[
\frac{1}{h} \frac{\partial q}{\partial x} = 2 \frac{\lambda_1}{\lambda_2} \left( \frac{u}{\lambda_2} \right)^2 \quad (3.2.31)
\]

For a dry bore, the velocity \( u \) is assumed to be related to the dry bore height \( h \) (Bretschneider and Wybro, 1976) by the following relationship.

\[
u = c = \frac{F_r}{\sqrt{gh_d}} \quad (3.2.32)\]
where,

\[ c = \text{dry bore velocity} \]
\[ F_r = \text{Froude number} \]
\[ h = h_d = \text{dry bore height}. \]

For the dry bore case, the water depth \( h \) is equal to the dry bore height.

In the above equation the Froude number is considered to have a constant value. Equation (3.2.32) then replaces the continuity equation for a dry bore.

Inserting equation (3.2.32) into equation (3.2.31) gives

\[ \frac{1}{h} \frac{\partial q}{\partial x} = 2 \frac{\lambda_1^2}{\lambda_2^2} (F_r^2 g h) \]  

(3.2.33)

\( \lambda_1 \) and \( \lambda_2 \) are lengths that are related to the bore length and can be expressed as

\[ \lambda_1 = \alpha' \lambda \]  

(3.2.34)

\[ \lambda_2 = \beta' \lambda \]  

(3.2.35)

where \( \alpha' \) and \( \beta' \) are constants

\[ \frac{1}{h} \frac{\partial q}{\partial x} = \left( \frac{2\alpha'^2 F_r^2}{\beta' 3} \right) g \frac{h}{\lambda} \]  

(3.2.36)

Therefore, it is shown that the above equation is in the following form
\[
\frac{1}{h} \frac{\partial q}{\partial x} = g \xi \frac{h}{x}
\]

(3.2.37)

where \( \xi \) is a constant equal to \( 2\alpha' \beta'^2 P' / \beta'^3 \).

b) Use of similarity with the hydraulic jump

The rate of change of energy in a bore per unit width is given by (Dronkers, 1964)

\[
\frac{dE'}{dt} = - \rho qq' \Delta H
\]

(3.2.38)

whereby, \( q' \) is the discharge per unit width.

\( E' \) = energy of bore per unit of width.

The difference in energy level \( \Delta H \), for a hydraulic jump is given by (Dronkers, 1964)

\[
\Delta H = \left( \frac{h_2 - h_1}{4h_1 h_2} \right)^3
\]

(3.2.39)

where \( h_1 \) and \( h_2 \) represent depths of water before and after the jump.

The discharge \( q' \) per unit width is defined by

\[
q' = (u_1 + c)h_1 = (u_2 + c)h_2 = \tilde{h}c
\]

where, \( u_1 \) is mean velocity (Figure 3.3) in the bore at the depth \( h_1 \) relative to the fixed system and \( \tilde{h} \) is a representative water depth.

Furthermore,

\[
\frac{dE'}{dt} = c \frac{dE'}{dx} \quad \frac{dE'}{dx} = \frac{1}{c} \frac{dE'}{dt}
\]

(3.2.40)
The energy gradient per unit width \( \frac{dE'}{dx} \) is given by

\[
\frac{dE'}{dx} = -\frac{\alpha'' \rho g \Delta H}{c} = -\frac{\alpha'' \rho g h^3}{4h_1(h_1 + H)}
\]  

(3.2.41)

We assume that major portion, \( \alpha'' \), of the energy is dissipated over the bore length \( \lambda \).

Energy of the bore per unit width \( E' \) can be expressed by

\[
E' = \rho g \Delta H' + \text{constant}
\]  

(3.2.42)

where, \( H' \) = energy per unit of weight.

The turbulent dissipation gradient is then expressed by

\[
\frac{dH'}{dx} = \frac{1}{\rho g h} \frac{dE'}{dx} = -\frac{\alpha' \xi}{\lambda} \frac{H^3}{4h_1(h_1 + H)}
\]  

(3.2.43)

We set

\[
h_1(h_1 + H) = \beta'' H^2
\]

where, \( \beta'' \) is a constant.

Then, for a wet bore the turbulent dissipation gradient is

\[
\frac{dH'}{dx} = -\frac{1}{4} \frac{\alpha'' H}{\beta'' \lambda} = -\frac{\xi}{\lambda} \frac{H}{\lambda}
\]  

(3.2.44)

where, \( \xi \) is a constant equal to \( \frac{1}{4} \alpha'' / \beta''' \).

For a dry bore where \( h = H \)

\[
\frac{dH'}{dx} = -\frac{\xi}{\lambda} \frac{h}{\lambda}
\]  

(3.2.45)
Furthermore, the turbulent dissipation for a dry bore is
\[ g \frac{dH'}{dx} = - g \xi \frac{h}{L} \]  
(3.2.46)

c) Comparison with a formulation used for periodic waves and transforming this formulation back to a non-periodic bore

For periodic waves (swell) on a reef, under stationary conditions, the gradient of energy flux of the waves \( dF/dx \) in the \( x \)-direction is expressed by
\[ \frac{dF}{dx} = - \varepsilon_b \]  
(3.2.47)

Gerritsen (1981) using Schonfeld's (1955) analysis of energy losses in a breaking wave related \( \varepsilon_b \) to a dimensionless breaking loss coefficient \( \xi \) by the relationship
\[ \varepsilon_b = \frac{\xi}{8\pi(2)(1/2)} \rho g \omega H^2 \]  
(3.2.48)

where the angular frequency is represented by
\[ \omega = 2 \frac{\pi}{T} \]  
(3.2.49)

\( H = \) breaking wave height

\[ \begin{align*}
\text{Figure 3.9. Breaking wave} \\
\text{We would like to compare this formulation with the previous approaches where we consider the bore (non-periodic wave). The breaking loss coefficient \( \xi \) has been calculated experimentally on shallow reef with swell type breakers.}
\end{align*} \]
Furthermore, for horizontal bottom and under stationary conditions.

when moving with the wave

\[
\frac{dF}{dx} = c \frac{dE}{dx}
\]  

(3.2.50)

so that

\[
\frac{dE}{dx} = - \frac{6b}{c} = - \frac{c}{8\pi \nu^2} \rho g \frac{2\pi}{c^2} H^2 = - \frac{c}{4\nu^2} \rho g H^2
\]  

(3.2.51)

in which, \(E\) is energy per unit of area and \(L\) is the length of a wave in the breaking zone.

To get back from a periodic wave to a bore type wave we have to multiply with the wavelength:

\[
\frac{dE'}{dx} = L \frac{dE}{dx} = - \frac{c}{4\nu^2} \rho g H^2
\]  

(3.2.52)

where \(E'\) is the energy per unit width and \(dE'/dx\) is the energy gradient in the bore (moving with the wave).

Again we assume that the dissipation of turbulent energy in the bore takes place over a length \(L\) (bore length).

We may write:

\[
E' = \rho g \bar{h} H' + \text{constant}
\]  

(3.2.53)

where, \(\bar{h}\) is a representative depth and \(H'\) is the equivalent energy head.
Then the energy gradient in the bore is given by

\[ \frac{dE'}{dx} = \rho g h \frac{dH'}{dx} \]  

(3.2.54)

Inserting equation (3.2.52) into equation (3.2.54) we get

\[ \frac{dH'}{dx} = -\frac{\zeta H^2}{4\sqrt{2} \lambda h} \]

(3.2.55)

For a dry bore, where \( H = h \) and \( \bar{h} = h \) we find

\[ \frac{dH'}{dx} = -\frac{\zeta h}{4\sqrt{2} \lambda} \]

(3.2.56)

Furthermore, the turbulent dissipation term for a dry bore which is in the form \(-g \varepsilon \frac{h}{\lambda}\), is given by

\[ g \frac{dH'}{dx} = -g \frac{\zeta h}{4\sqrt{2} \lambda} \]

(3.2.57)

In this study, we would like to use this approach since the loss coefficient \( \zeta \) has been evaluated experimentally on shallow reef with swell type breakers (Gerritsen, 1981) and can be used for comparison purposes.

The equation of motion (3.2.14) then becomes

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -g \frac{\partial h}{\partial x} - gS - gu \sum \frac{|u|}{hC_h^2} - \frac{ghH}{hC_h^2 \lambda} \]

(3.2.58)

where the last term is to be activated at the bore.
For a dry bore $H$ in equation (3.2.58) will become $h$, which represents the height of the front of the wave above the bottom. Then, the velocity $u$ is assumed to be related to $h$ by (Bretschneider and Wybro, 1976).

$$u = c = F_r \sqrt{gh_d} \quad (3.2.59)$$

where,

$c = $ dry bore velocity
$F_r = $ Froude number
$h = h_d = $ dry bore height.

For the dry bore case, the water depth $h$ is equal to the dry bore height.

In the above equation the Froude number is considered to have a constant value. Equation (3.2.59) then replaces the continuity equation for a dry bore.

Bretschneider and Wybro (1976) describe the long wave equation of motion in the form

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -g \frac{\partial h}{\partial x} - gS - \frac{guu}{C_h^2 h} \quad (3.2.60)$$

In Bretschneider and Wybro's equation Manning's $n$ is used for the calculation of energy losses due to bottom friction. The replacement of the Chezy formulation by a Manning's $n$ formulation is justified for rough turbulent flow conditions which are always experienced in the field. The relationship between Chezy coefficient and $n$ is then given by the expression
The friction term in Bretschneider's above equation is then expressed in Manning's formulation in English units by

\[ C_h = \frac{h^{(1/6)}}{n} \]  

(3.2.61)

In the Bretschneider and Wybro approach, the energy dissipation due to breaking (turbulence) is not taken into consideration. In the author's approach, the energy losses due to breaking are included by introducing a breaking energy dissipation term in the equation of motion (3.2.58).

Consequently there is a difference in the results between the author's approach and Bretschneider and Wybro's (1976) approach.

In the author's approach, the inclusion of energy losses due to turbulence result in a reduction of runup values compared to those obtained by Bretschneider and Wybro.

Bretschneider and Wybro's formulation of the dry bore (equation 3.2.62) and its solution on a horizontal bottom is equivalent to the method of characteristics approach where one moves with the bore.

(Reference is made to Appendix A (part b), p. 146.)

3.3 Method of Characteristics

Possible methods that are most appropriate to solve the above equations are: the finite difference method and the method of characteristics.
The method of characteristics is mainly used to solve hyperbolic equations. This powerful method has been in common use in hydraulics for studying flood routing and tidal waves in estuaries. Stoker (1957) was the first to propose the application of the method of characteristics to the problem of wave breaking on a beach.

Schönfeld (1951) and Le Méhauté (1976) define the propagation velocity of a disturbance, $c$, as follows:

$$ c = \sqrt{g(d + \eta)} = \sqrt{gh} \quad (3.3.1) $$

Differentiating the above equation with respect to $x$

$$ 2c \frac{\partial c}{\partial x} = g \frac{\partial \eta}{\partial x} + g \frac{\partial d}{\partial x} \quad (3.3.2) $$

Inserting the above relationship into equation (3.1.19) gives

$$ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + c \frac{\partial c}{\partial x} = - gS - g \frac{|u|}{hc} = W \quad (3.3.3) $$

Assuming the Chezy coefficient is constant within the bore or for a short distance travelled by the bore (the right hand side of equation 3.3.3), $W$ is a constant. $W$ can include the breaking dissipation term given in equation (3.2.58) although it is not included in the above equation.

The continuity equation expressed in terms of $u$ and $c$, by substituting equation (3.3.1) into equation (3.1.23) is:

$$ \frac{\partial (uc^2)}{\partial x} + \frac{\partial c^2}{\partial t} = 0 \quad (3.3.4) $$
Expanding equation (3.3.4) yields

\[
\frac{a(2c)}{\partial t} + u \frac{a(2c)}{\partial x} + c \frac{\partial u}{\partial x} = 0
\]  \hspace{1cm} (3.3.5)

Adding and subtracting equations (3.3.3) and (3.3.5) we obtain the following two equations:

\[
\frac{3(u + 2c)}{\partial t} + (u + c) \frac{3(u + 2c)}{\partial x} = W = d(u + 2c)
\]  \hspace{1cm} (3.3.6)

\[
\frac{3(u - 2c)}{\partial t} + (u - c) \frac{3(u - 2c)}{\partial x} = W = d(u - 2c)
\]  \hspace{1cm} (3.3.7)

It is clear from equations (3.3.6) and (3.3.7) respectively that

\[
\frac{dx}{dt} = u \pm c
\]  \hspace{1cm} (3.3.8)

Equation (3.3.6) is true along the characteristic whose slope is \( u + c \) and equation (3.3.7) is true along the characteristic whose slope is \( u - c \) as shown in Figure 3.10.

The lines of slope \( u + c \) are called the advancing or positive characteristics. The lines of slope \( u - c \) are called the receding or negative characteristics.

The time history of the wave evolution can then be determined step by step as follows:

The values of \( u(x,t_1) \) and \( \eta(x,t_1) \) are given at time \( t=t_1 \). In the particular case of Figure 3.10 \( t_2=t_1 \), however it need not be so. Then the values of \( u(x,t_2) \) and \( \eta(x,t_2) \) are given for a wave at a time
Figure 3.10. Advancing and receding characteristics

t=t_2. The characteristic line of slope \( u_1 + c_1 \) is drawn from point 1 and the characteristic line of slope \( u_2 - c_2 \) is drawn from point 2. The intersection at point 3 defines \( x_3, t_3 \) graphically. By applying the characteristic equation along these lines, the values of \( u_3 \) and \( c_3 \) (and consequently \( \eta_3 \)) are found from the two equations:

\[
\begin{align*}
u_3 + 2c_3 &= u_1 + 2c_1 + W_1(t_3 - t_1) \\
u_3 - 2c_3 &= u_2 - 2c_2 + W_2(t_3 - t_2)
\end{align*}
\] (3.3.9) (3.3.10)

Similarly, \( u_5, c_5 \) are found from the points 2 and 4, \( u_6, c_6 \) from the points 3 and 5 and so on. Repeating this procedure of calculation for each point of the x-t diagram permits calculation of the complete wave evolution as a function of space and time.
Another approach to the method of characteristics is given in Appendix A.

3.4 Solution of Equation (3.2.58) for a Dry Bore

In Appendix A (part b) we will show that the characteristic solution of equation (3.2.58) for the bore front corresponds with the solution of the following equation.

\[
\frac{3}{2} \left( h + \frac{u^2}{2g} \right) = -u \frac{|u|}{hC_h} - \frac{\zeta h}{2(\sqrt{2}x)} - S
\]  

(3.4.1)

The author's approach for the solution of the dry bore problem (equation 3.2.60) is given in the following four cases:

a) Solution for horizontal bottom including both friction and breaking dissipation terms in the dry bore equation.

b) Solution for horizontal bottom including only the friction dissipation terms in the dry bore equation.

c) Solution for a sloping bottom including both friction and breaking dissipation terms in the dry bore equation.

d) Solution for a sloping bottom including only the friction dissipation term in the dry bore equation.

Solutions are presented in section (4.1.3).

3.5 Tsunami Characteristics

Tsunamis are generally caused by a sudden violent displacement of the sea floor resulting from an earthquake. The direction of the vertical movement determines whether a 'crest' or 'trough' is the initial sea surface disturbance. The exact mechanism of the movement is, however, open to debate. The range of theories as to the mechanism
of the movement is widespread, from block movement along a well-defined fault or fault system (Richter, 1958) to submarine landslides (Gutenberg, 1939). Volcanic explosions and coastal landslides are both tsunami generators, although their effects are generally localized. Not all earthquakes produce tsunamis; whether they do or not depends on their magnitude and depth of epicenter below the surface of the earth. The generation mechanism differs from event to event.

Tsunamis principally occur following undersea earthquakes; they are waves of low height in deep water and are usually not detectable visually until they gain height on approaching coastlines where they may reach damaging proportions as bores or surges (Wilson, 1963). The speed of travel of the tsunami waves in deep water can reach 223 m/sec (500 miles per hour), but speed is greatly retarded as the waves approach the coast. Convergence of wave energy by refraction together with shoaling and resonance effects greatly increase the destructive capacity (Wilson, 1963).

At the source waves are generated with different frequencies. The long waves, traveling with the speed \( \sqrt{gh} \), travel the fastest and are often followed by a dispersive wave train (Ward, 1980). The waves are of varying periods, generally ranging from a few minutes to one hour but sometimes attaining periods of several hours. Deep water wave heights are of the order of one foot or less; the longer periods have extremely long wavelengths, and these waves travel as shallow water waves. Thus, their celerity \( c \) is governed by total water depth as given in equation (3.3.1). Given an average Pacific Ocean depth of 4,267 m (14,000 ft) the average celerity of the tsunami is 205 m/sec
(458 mph). Upon crossing the Continental Shelf, tsunamis undergo transformations due to reflection, refraction and shoaling. As the tsunami travels into shallow water it undergoes distortion and under certain conditions may transform into a bore.

Wave runup is the elevation a wave reaches above the reference water level upon propagation over the land surface. Tsunami runup calculations are of vital importance in determining the tsunami elevations and inundation limits, which depend on the initial tsunami, and on the slope of the offshore bottom and of the land, and on the roughness of the coastal terrain at the time and place of the tsunami occurrence.

The Hawaiian Islands have a long history of destruction due to tsunamis. The historical data have been compiled from historical accounts and other reports, newspaper archives and recent mareographic data. The earliest recorded tsunami occurred on April 11, 1819 and was comprised of a wave originating in Chile which reached a height of 2 m along the west coast of the island of Hawaii. Though this tsunami was observed in Honolulu it was of very small magnitude and merited little consideration. There are no known records of tsunamis during the time of ancient Hawaii.

The tsunamis which have been the most destructive in the Hawaiian Islands are generated along the coast of South America, the Aleutian Islands, the Kamchatka Peninsula and Japan. More than one-half of all the tsunamis recorded in Hawaii have originated in the Kuril-Kamchatka-Aleutian regions of the north and northwestern America. H. G. Loomis (1976) has compiled tsunami run-up data for the 1946,
1952, 1957, 1960, 1964 tsunamis for the island of Oahu. There seems to be a reliable collection of tsunami run-up data for the 1946 tsunami on the North Shore of Oahu from Kaena point to Mokulea.

For the calculation of wave runup many theories exist based on a variety of assumptions. It can be said that the prediction of wave runup due to short periodic waves is relatively well established because of experimental studies on that subject. These experimental studies are particularly valuable because theories have not provided accurate estimates of wave runup due to breaking waves. In the case of tsunami, the prediction of wave runup is difficult because nonlinear wave behavior in shallow water is involved. Of the many studies on wave runup which were conducted, only one stressed the importance of frictional bottom resistance (paper by Bretschneider and Wybro, 1976).

In the method of Bretschneider and Wybro, the tsunami runup and inundation can be determined provided that one knows the wave elevation at the shoreline, the character of the wave (whether it is a bore or a nonbore surge) and the average value of the roughness of the terrain over which the wave travels. Bretschneider and Wybro (1976) define the bore as a breaking wave as explained in section 3.2 and a nonbore surge as a non-breaking long wave similar to a tidal type wave with a period on the order of 5 minutes to 2 hours. For the nonbore surge the Froude number, \( F_r = \frac{u}{\sqrt{gh}} \) is taken as 1 and for the bore it is taken as 2 by Bretschneider and Wybro.

Up to now theoretical studies have not considered energy losses due to breaking in the analysis. This factor becomes important when a bore is formed in the runup process and therefore should be included.
together with the frictional dissipation in the calculation of tsunami runup in coastal regions. One of the aims of this study is to formulate a breaking dissipation term due to turbulence so as to be able to obtain a more precise method for the prediction of inundation elevations which may be used for planning purposes and for flood insurance analysis. A few examples for Oahu's north shore are presented to illustrate the use of the method. Miller's (1968) and Yeh and Ghazali's (1986) experimental results and observations have been used in this study to verify the author's equation, using the method of characteristics for wave runup calculations.

3.6 Boundary Conditions

Wave data obtained from the buoys installed by NOAA (National Oceanographic and Atmospheric Administration) in 1987-88 include some small tsunamis. (Reference is made to Mader (1988).) The wave sensors at gauge AK8 (located at about 4,000 m depth) near where tsunamis are generated (Figure 3.11) has a recorded period of 5.7 minutes with a maximum amplitude of 3 cm (Figure 3.12). However, after traveling down to gauge WC9 located at about 1,500 m depth (Figure 3.11) the frequency dispersion is seen such that the wave period is 11.9 minutes with a much reduced wave amplitude of 1.5 cm (Figure 3.13). The wave profile at gauge WC9 looks more like a periodic wave than a solitary wave. Historical tsunami data collected by Wigen (1983) and Okada and Tada (1983) show that tsunamis generated in the Pacific Ocean have a period ranging between 10 to 30 minutes. Mader (1988) in his study has used an average of 15 minutes wave period as input for his numerical modeling. Mader has also used a 30 second period wave in his numerical
Figure 3.11. Location of NOAA gauges (1987-1988)
Figure 3.12. Wave recording of NOAA gauge at location AK8 (1987-1988)
Figure 3.13. Wave recording of NOAA gauge at location WC9 (1987-1988)
model as input for Maunala Bay considering the possibility that the long wave is deformed on entering shallow water. The distortion of the long wave and the steepening of the wave front may cause the formation of a bore. Whether or not a bore is formed depends on the initial wave steepness and the slope of the offshore bottom. Based on the above mentioned historical studies and Mader's (1988) study, it seems reasonable to consider the formation of a bore in runup calculations.

Okada and Tada (1983) have attempted to compile all the tsunamis which arrived at the tide station at Miyako, Japan, by examining the original marigrams since 1937 kept at the Miyako Weather Station. They have found using the present data that the relationships between maximum amplitude and frequency of tsunamis at Miyako, Japan and Tofino, Canada (Wigen, 1977) are similar. Figures 3.14 and 3.15 show the frequency distribution of maximum amplitude of tsunamis at Miyako (Figure 3.16) and Tofino, respectively.

Figure 3.15 shows that at Tofino the maximum wave amplitude is 5 meters for a one in two hundred year frequency of occurrence. It can also be seen from Figure 3.14 that for an average depth of 20 meters at Miyako a maximum wave amplitude of 5 meters, for a one in two hundred year tsunami seems a reasonable figure. In this study it is assumed that a wave height of 5 m at a depth of 20 m is a reasonable value for an extreme event affecting the exposed north shore of Oahu, with the characteristics of a bore.
Figure 3.14. Frequency of maximum amplitude of tsunami at Miyako (1937-1981)
\[
\log H_m = -0.965 \log N + 2.3128
\]

Figure 3.15. Tofino tsunami magnitude frequency relationship
(after Wigen, 1977)
Figure 3.16. Location of tide station at Miyako
CHAPTER IV

APPLICATION OF METHOD OF CHARACTERISTICS

TO TSUNAMI RUNUP PROBLEM

4.1 Application of the method of characteristics to the long wave equation (tsunami runup model), including the existence of a bore

In this chapter, the tsunami runup model which consists of two parts, the wet bore and the dry bore, is explained in detail. In the wet bore model the method of characteristics is applied to the long wave equations including the bore equations. The dry bore model is used to calculate runup as the bore travels on dry land. (Reference is made to sections 3.2.2 and 3.4 in Chapter III.)

The equation of continuity and the equation of motion used in the long wave model are respectively given by

\[ \frac{\partial u(d + \eta)}{\partial x} + \frac{\partial \eta}{\partial t} = 0 \]  \hspace{1cm} (4.1.1)

and

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -g \frac{\partial h}{\partial x} - gS - \frac{qu|u|}{C_h^2 h} \]  \hspace{1cm} (4.1.2)

In these equations the x-axis is horizontal and increasing towards shore, \( h \) is the total water depth \( (d + \eta) \), \( u \) is the mean horizontal particle velocity in the x direction.

Le Méhauté (1963) and Schönfeld (1951) have shown that the propagation velocity \( c \) is derived from the long wave characteristic (advancing and receding) wave-component and given by...
\[ c = \sqrt{g(d + \eta)} = \sqrt{gh} \] (4.1.3)

In order to determine the transition depth where waves become shallow water waves linear theory is used in deep water and in intermediate depth whereby

\[ c^2 = \frac{gL}{2\pi} \tanh \frac{2\pi h}{L} \] (4.1.4)

The propagation of a long wave in deep water can therefore be investigated by linear theory and in shallow water by nonlinear theory where\[ c = \sqrt{g(d + \eta)} \]

In the present study, a transition depth is defined for periodic waves as the depth where the linear theory wave celerity

\[ c = \sqrt{gL} \tanh \frac{2\pi h}{L} \] (4.1.5)

and the nonlinear shallow water celerity \[ c = \sqrt{g(d + \eta)} \]

differ by less than five percent. In this study for the calculations it is assumed that the positive wave front arrives at the transition depth at time \( t = 0 \).

Equations (4.1.1) and (4.1.2) are a system of first-order nonlinear hyperbolic partial differential equations. In the present study the method of characteristics is used for the solution of the above equations. This method is based on the principle that the solutions
of the hyperbolic equations can be given along curves, known as characteristics. This method has the advantage that the inception and development of bores can be treated by it. The solution of equations (4.1.1) and (4.1.2) are given by the intersection of the two sets of advancing and receding characteristic curves. The corresponding equations for the advancing and receding characteristics are given in section 3.3, i.e.,

1) \[ \frac{dx}{dt} = u + c \quad \text{(advancing characteristic)} \]  

2) \[ \frac{dx}{dt} = u - c \quad \text{(receding characteristic)} \]  

where, \( c = \sqrt{g(d + \eta)} \)

Furthermore,

3) \[ d(u + 2c - mt) = 0 \quad \text{(4.1.7)} \]

on the advancing characteristic and

4) \[ d(u - 2c - mt) = 0 \quad \text{(4.1.8)} \]

on the receding characteristic where

\[ m = gS - gS_f \quad \text{(4.1.9)} \]

where \( S = \) bottom slope, \( S_f = \frac{u |u|}{C_h} \)

The above four total differential equations replace the system of two partial differential equations. Provided the initial values are known, solutions may be obtained step by step.

The initial values may be taken as (1) the values of \( u \) and \( c \) as functions of \( x \) at time \( t=0 \) and (2) the value of \( c \) as a function of time \( t \) at \( x=x' \). Here \( x' \) is the value of \( x \) at the transition depth
and $t=0$ is the time of arrival of the wave front at $x'$. The point $x=x'$ and $t=0$ is the origin 0 where the calculations are started (Figure 3.2 and Figure 4.1). The first set of values is known from the conditions prior to the arrival of the wave at $x=x'$. The water particle velocity $u$ is zero for all $x \geq x'$ at $t \leq 0$. The second set of values is taken from the values of the water surface elevation at $x=x'$ as determined from the linear theory.

It should be noted from the above that only the values of $c$ are prescribed as initial values at $x=x'$. This is in accordance with Stoker's (1957) observation that either the values of $u$ or $c$ but not both should be used for this purpose.

The numerical integration of the equations of the nonlinear theory is presented in the form of a network of characteristics on the $x,t$ plane. The procedure of obtaining this network suggested by De Prima (Stoker, 1948) consists of first establishing the initial characteristic showing the motion of the wave front, and then issuing the receding characteristics from it. In Figure 4.1 the initial characteristic is drawn from $(x',0)$. The values of $c$ on the $t$-axis, $c(x',t)$, are known from shallow water theory (equation 4.1.3). These values are introduced at appropriate points in the computations and the network is extended.

From $(x',0)$, the initial characteristic is drawn first. The initial characteristic is an advancing one and the values of $u$ and $c$ are known on it from equation (4.1.7). At any point $P$ on the advancing characteristic, since $u(P)$ is zero (Amein, 1964)

$$\left( \frac{dx}{dt} \right)(P) = c(P) \quad (4.1.10)$$
Figure 4.1. Network of characteristics.
and
\[ 2c(0) + g\text{St}(0) = 2c(P) + g\text{St}(P) \]  
(4.1.11)

Therefore,
\[ x(P) = x(0) + c(0)t(P) - \left(\frac{1}{4}\right)g\text{St}^2(P) \]  
(4.1.12)

Let us assume that the calculation has progressed to the line \( P_1J_1 \), with all the characteristics to the left of this line being known. The slopes of the advancing characteristics are known from eqn. (4.1.6) at each of the node points \( P_1, Q_1, R_1, \ldots, J_1 \).

An appropriate time interval is chosen on the characteristic segment \( P_1P_2 \) such that
\[ t(P_2) = t(P_1) + \delta t \]  
(4.1.13)

The values of \( x, u, c \) are known on \( P_2 \) since it lies on the initial characteristic. A receding characteristic can be drawn from \( P_2 \) to intersect an advancing characteristic drawn from \( Q_2 \), the point of intersection being \( Q_2 \). The finite difference forms of equations (4.1.6), (4.1.7), (4.1.8) provide the following set of simultaneous algebraic equations at \( Q_2 \).

\[ x(Q_2) - x(Q_1) = (u(Q_1) + c(Q_1))(t(Q_2) - t(Q_1)) \]  
(4.1.14)

\[ x(Q_2) - x(P_2) = (u(P_2) - c(P_2))(t(Q_2) - t(P_2)) \]  
(4.1.15)

\[ u(Q_2) + 2c(Q_2) - g(-S - S_f(Q_1))t(Q_2) = \]  
\[ u(Q_1) + 2c(Q_1) - g(-S - S_f(Q_1))t(Q_1) \]  
(4.1.16)
\[ u(Q_2) - 2c(Q_2) - g(-S - S_f(P_2))t(Q_2) = \]
\[ u(P_2) - 2c(P_2) - g(-S - S_f(P_2))t(P_2) \]  

(4.1.17)

From the above four equations the four unknowns \( u, c, x, t \) at \( Q_2 \) can be calculated. The computation then proceeds to \( R_2, S_2 \) and on up to \( K_2 \). At the \( K_2 \) node \( x, c, \) and \( t \) are known, since \( K_2 \) lies on the \( t \)-axis. The value of \( u \) at \( K_2 \) will be computed from eqn. (4.1.8) when it is applied between \( K_2 \) and any node point on \( P_2, J_2 \). To advance the computation to the right, storage locations occupied by \( P_1, Q_1, \ldots, J_1 \) will be taken over by \( P_2, Q_2, \ldots, J_2 \) and the storage locations occupied by the latter will be vacated and reserved for the results of the next chain (Amein, 1964). In calculating the interior characteristics, after each chain of calculation the number of known points for evaluating the next chain is reduced by one (Lister, 1960).

4.1.1 Bore propagation

The inception of a bore is predicted by the intersection of adjacent advancing characteristics (Stoker, 1957). The bore is a discontinuity and its propagation cannot be calculated by the shallow water theory since there is a loss of energy in the bore. The bore propagation is obtained by the application of the impulse-momentum principle (Amein, 1964). The velocity of the bore, \( V \) is given by Stoker (1957).

\[ V^2 = \frac{g(d + \eta)(2d + \eta)}{2d} = c^2(1 + \frac{\eta}{2d}) \]  

(4.1.18)

where, \( d \) is the water depth ahead of the bore \( \eta = \) bore height.
The particle velocity behind the bore is given by Amein (1964).

\[ u = (1 - \frac{d}{d+\eta}) V \]  

(4.1.19)

Fig. 4.2 shows a portion of the characteristic network, including the boreline. The line ABC is the boreline, it depicts the position of the bore on the x,t plane. The solid lines are the advancing characteristics and the dashed lines are the receding characteristics. At point B an advancing characteristic intersects the boreline. The bore advancing from A to B is modified by wave elements overtaking it at B. The new bore properties at B are determined from the values on the characteristic terminating at B. The bore is further modified between B and C because it is advancing into water of nonuniform depth (Amein, 1964).

4.1.2 Bore Propagation in Nonuniform Flow

The propagation of a bore in water of varying depth has been investigated by Keller et al. (1960), Ho and Meyer (1962), and Freeman and Le Méhauté (1964). Keller et al. have coupled the above equations (4.1.18) and (4.1.19) with the nonlinear shallow water theory. The flow behind the bore is assumed to be uniform so that wave elements do not overtake the bore from the rear. Keller et al. have defined bores as weak if the ratio of the bore height to the depth ahead of the bore is less than .6262. In Whitham's (1958) paper, if this ratio is much less than 1 the bore is considered weak and if it is much larger than 1 the bore is considered to be a strong one. A bore therefore by this definition starts out as a weak bore away from the shore and becomes a strong bore near the shore. Amein
(1964) states that results of numerical studies have shown that weak bores increase in height while strong bores decrease in height as they advance into water of decreasing depth.

Fig. 4.2 Network of characteristics for bore propagation
In this study the method used for the computation of bore propagation into nonuniform flow is based on the principle that the equations (4.1.7), (4.1.6 advancing characteristic) (Whitham, 1958) valid on an advancing characteristic can be applied to the quantities behind the bore. The values of \( u \) and \( c \) behind the bore and the bore speed \( V \), are expressed in terms of the bore strength \( M \) defined by

\[
M^2 = \frac{V^2}{g(d+n)}
\]  

(4.1.20)

The bore strength, \( M \), is actually representative of the Froude number. The different Froude number definitions used in this study are given in the following section (4.1.4).

In terms of \( M \) (Keller et al., 1960)

\[
u = \frac{(gh)^{1/2}2M(M^2-1)}{(2M^2-1)^{1/2}}
\]  

(4.1.21)

\[
c = (gh)^{1/2}(2M^2-1)^{1/2}
\]  

(4.1.22)

\[
V = (gh)^{1/2}M(2M^2-1)^{1/2}
\]  

(4.1.23)

Obtaining the terms \( du, dc, mdt \) from the above equations and substituting them into equation (4.1.7) we have after some simplification,

\[
\frac{1}{h} \frac{dh}{dM} = \frac{-4(M+1)(M - .5)^2(M^3 + M^2 - M - .5)}{(M - 1)(M^2 - .5)(M^4 + 3M^3 + M^2 - 1.5M - 1)}
\]  

(4.1.24)

The above equation is used to determine bore propagation between the consecutive intersections of the advancing characteristics and the boreline and between the last such intersection and the shoreline.
The numerical computation discussed above ends when the water depth ahead of the bore is less than zero. Beyond this point dry bore calculations (reference is made to sections 3.2.2 and 3.4 of Chapter III) must be used to determine the wave runup as the bore travels on dry land.

The basic difference between Amein's (1964) and the author's approach (amended characteristic model) is that in Amein's model the energy dissipation term due to bottom friction in the equation of motion is not included whereas in this study it is included. Another difference is that Amein uses Friedrich's (1948) second asymptotic theory in deep water whereas this study uses linear theory. Amein (1964) assumes that when the bore reaches the shoreline, the bore height vanishes, however in this study this assumption is not used.

In this study, the author's long wave model requires the input data for bottom slope (S), wave (bore) height (H) and transition depth ($h_{tr}$), and wave period (T) only for a periodic wave. The output from the model gives the water surface elevation above the undisturbed level ($\eta$), the x distance away from the shore (x increasing toward the shore), the horizontal water particle velocity, and the propagation velocity c, for every time step t.

For the purpose of comparison with Amein's (1964) results computations were carried out for a 30 second sinusoidal wave with no bottom friction with the following input data:

$$S = .05 \quad T = 30 \text{ sec} \quad H = 7.86 \text{ m} (25.82 \text{ ft}) \quad h_{tr} = 32.13 \text{ m} (105.41 \text{ ft}) \quad \Delta t = .25 \text{ sec}$$
For the above wave, the first bore forms at approximately 366m (1200 ft) away from the shore when the first two advancing characteristics intersect as can be seen in Figure B-1 (Appendix B). The wave profiles at t=30, 45, 47, 49, 52, 60, 75 seconds are shown in Figure B-2 and agree with the wave profile results of Amein (1964). The initial dry bore height at the shoreline is found to be 2.89m (9.50 ft). The dry bore calculations with both friction and breaking energy dissipation included, for a Manning's roughness coefficient (n) of .02 and a breaking loss coefficient ζ of .5, gave a runup height of 5.22 m (17.15 ft). The runup height obtained from only friction dissipation is found to be 8.69 m (28.54 ft) as shown in Figure B-2 (Appendix B).

4.1.3 Dry Bore Calculations

The characteristic solution of equation (3.2.58) for a dry bore is given in Chapter III (equation 3.4.1).

The bore length, ℓ, in the turbulent dissipation term (last term) in equation (3.2.58) can be defined in two ways, in which either ℓ or r will be constant. In case of constant ℓ, ℓ = rH_i, where H_i is the initial bore height (bore height at the shoreline) and r is a constant value (r ~ 6). (Rouse et al. (1958) experimentally determined that energy dissipation in a hydraulic jump occurs over approximately six times the bore height, r=6.) In the case of constant r, ℓ = rH, where H is the bore height and r is a constant value r ~ 6. In the latter case the bore length will change with bore height H. There is no experimental evidence to show which one of the above two cases is closer to physical reality of the bore length. Probably, the physical reality of the bore length will be in between the above two
cases. In this study, the dry bore calculations have been carried out for constant $\lambda$ but for comparison purposes two examples are given for constant $r$ (Figures 4.3, 4.4 and 4.5). In these examples for comparison purposes (to be consistent with the graphs in the Manual for Determining Tsunami Runup Profiles on Coastal Areas of Hawaii (1978) based on the study by Bretschneider and Wybro (1976)), an initial bore height of 15 meters and a constant Froude number of 2 are used. In these calculations, an average value of .5 is used for the breaking loss coefficient, $\zeta$. It is shown from these examples that there is not much difference between the results for constant $\lambda$ and constant $r$ when $r \geq 5$.

a) When the friction and breaking dissipation terms are added the dry bore equation for a horizontal bottom becomes

$$\frac{\partial}{\partial x} (h + \frac{u^2}{2g}) = -u \frac{|u|}{h_{\text{ch}}} - \frac{\zeta h}{4\sqrt{2\lambda}}$$

(4.1.25)

where $H = h$ for the dry bore (Figure 4.6).

Substituting equation (3.2.61) and (3.4.1) we obtain

$$\frac{\partial}{\partial x} h(1 + \frac{F^2}{2}) = -g \frac{F^2}{C_h} - \frac{\zeta h}{4\sqrt{2\lambda}}$$

(4.1.26)

$C_h$, Chezy coefficient can be assumed as constant within the bore or for a short distance travelled by the bore. The Froude number is taken as constant, equal to 2 for the dry bore calculations (Bretschneider and Wybro, 1976).

The Chezy coefficient is actually a function of the water depth and the bottom roughness. It can be expressed in terms of the depth...
This study, friction only, corresponds with Bretschneider and Wybro.

This study, friction and breaking losses included, borelength 2 H.

This study, friction and breaking losses included, borelength 3 H.

This study, friction and breaking losses included, borelength 5 H.

*This study, friction and breaking losses included, for borelengths 30, 45 and 75 meters

Figure 4.3. Bore dissipation for a slope of S=0%, n=0.1 and F=2 for constant χ and constant r
This study, friction only, corresponds with Bretschneider and Wybro.

This study, friction and breaking losses included, borelength 2 H.

This study, friction and breaking losses included, borelength 8 H.

*This study, friction and breaking losses included, for borelengths 30, and 120 meters

Figure 4.4. Bore dissipation for a slope of S=5%, n=.03 and F=2 for constant \( \ell \) and constant \( r \)
This study, friction only, corresponds with Bretschneider and Wybro.

This study, friction and breaking losses included, borelength 5 H.

*This study, friction and breaking losses included, for borelength 75m

Figure 4.5. Bore dissipation for a slope of S=5%, n=.03 and F=2 for constant θ=75° and constant r
and of the Manning's roughness $n$, which is a function of only the bottom roughness as in equation (3.2.61) (Bretschneider and Wybro, 1976). Roughness parameters as established by Bretschneider et al. (unpublished report) will be used to quantify terrain. According to Bretschneider et al., it should be noted that the roughness factors may change with respect to time, from when and where the observations were made. Coastal areas can change due to severe storms and hurricanes. Bretschneider et al. state that perhaps the worst changes would be due to development along the coastal areas such as the bull-dozing of bushy areas, trees, rocky terrain, replacing it with grass for a golf course. For example, in Bretschneider et al.'s unpublished report Manning's roughness $n$ for lawn (golf course), beach rock and boulders is given as .030, .036 and .041, respectively.

Inserting equation (3.2.61) into the above equation (4.1.26) gives

$$\frac{2}{3} \frac{a}{h} \left( 1 + \frac{F_{r}^{2}}{2} \right) = -g \frac{r_{h}^{2}}{h^{1/3}} - \frac{zh}{4\sqrt{2x}}$$

(4.1.27)
If constants $\alpha$, $\beta$, $\gamma$ are defined as follows:

$$\alpha = 1 + \frac{Fr^2}{2} \quad (4.1.28)$$

$$\beta = \frac{\zeta}{4\sqrt{2}\kappa} \quad (4.1.29)$$

$$\gamma = gFr^2n^2 \quad (4.1.30)$$

Equation (4.1.27) reduces to

$$\frac{\partial h}{\partial x} = -(\gamma h^{-1/3} + \beta h) \quad (4.1.31)$$

$$\frac{h^{1/3}dh}{\beta h^{4/3} + \gamma} = -\frac{1}{\alpha} dx \quad (4.1.32)$$

Integrating the above equation gives

$$\ln \left(\frac{\beta h_1^{4/3} + \gamma}{\beta h_2^{4/3} + \gamma}\right) = \frac{4}{3}\frac{\beta}{\alpha} (x_2 - x_1) \quad (4.1.33)$$

where,

$h_1$ = bore height at location $x_1$

$h_2$ = bore height at location $x_2$

In a previous section, $h_1$ and $h_2$ terminology has been used to identify the height before and after the hydraulic jump respectively. However, in the dry bore calculations there is only one bore height, $h$. In this section, the terminology $h_1$ and $h_2$ is used as the bore height at location $x_1$ and $x_2$ respectively, in order to define the envelope curve of the tsunami wave elevation.
When $h_2 = 0$ the above equation becomes

$$\ln \left( \frac{\beta h_1^{4/3} + \gamma}{\gamma} \right) = \frac{4}{3} \beta x_r$$

(4.1.34)

where $x_r$ is the distance the bore travels for complete energy dissipation.

The envelope curve of the maximum surge height for horizontal bottom is defined in Figure 4.7.

![Envelope curve of maximum surge height for horizontal bottom](image)

Figure 4.7. Envelope curve of maximum surge height for horizontal bottom

Therefore for a known bore height $h_1$ at location $x_1$ (Figure 4.7) solving equation (4.1.32) for $h_2$ gives

$$h_2 = \left( \frac{\beta h_1^{4/3} + \gamma}{\beta} \right) e^{B 3/4} - \frac{\gamma}{\beta}$$

(4.1.35)

where,

$$B = - \frac{4}{3} \frac{\beta \alpha x}{\alpha}$$

(4.1.36)

In this way using the above procedure the envelope curve for the maximum dry bore surge height will be defined.
b) When only bottom friction is considered, the dry bore equation for a horizontal bottom becomes

\[ \frac{3}{\alpha x} h \left( 1 + \frac{F_r^2}{2} \right) = -g \frac{F_r n^2}{h^{1/3}} \]  

(4.1.37)

Expressing equation (4.1.37) in terms of \( \alpha \) gives

\[ \frac{3 \alpha h}{\alpha x} = - \frac{g F_r n^2}{h^{1/3}} \]  

(4.1.38)

Then,

\[ \int_{h_1}^{h_2} \alpha h^{1/3} \, dh = -g \int_{x_1}^{x_2} (n F_r)^2 \, dx \]  

(4.1.39)

Integrating the above equation gives

\[ \frac{3 \alpha h^{4/3}}{4} \left. \frac{h^2}{h_1} \right| = -g (n F_r)^2 x \left. \right|_{x_1}^{x_2} \]  

(4.1.40)

Inserting the integration limits gives

\[ \frac{3 \alpha}{4} (h_2^{4/3} - h_1^{4/3}) = -g (n F_r)^2 (x_2 - x_1) \]  

(4.1.41)

Solving for \( h_2 \) gives

\[ h_2 = \left\{ - \frac{4}{3} \frac{g (n F_r)^2}{\alpha} (x_2 - x_1) + h_1^{4/3} \right\}^{3/4} \]  

(4.1.42)
If $\gamma_f$ is defined as

$$\gamma_f = -\frac{4}{3} \frac{g(nF_r)^2}{\alpha} (x_2 - x_1) \quad (4.1.43)$$

Equation (4.1.40) then reduces to

$$h_2 = \{\gamma_f + h_1^{4/3}\}^{3/4} \quad (4.1.44)$$

c) When the friction and breaking dissipation terms are included, the dry bore equation for a sloping bottom becomes

$$\frac{\partial h}{\partial x} \left(1 + \frac{F_r^2}{2}\right) = -S - \frac{gF_r^2}{c_h^2} - \frac{\varsigma h}{4\sqrt{2} \ell} \quad (4.1.45)$$

Inserting equation (3.2.61) into the above equation (4.1.45) gives

$$\frac{\partial h}{\partial x} \left(1 + \frac{F_r^2}{2}\right) = -S - g \frac{F_r^2 n^2}{h^{1/3}} - \frac{\varsigma h}{4\sqrt{2} \ell} \quad (4.1.46)$$

If $\alpha$, $\beta$ and $\gamma$ are defined as in equations (4.1.28), (4.1.29) and (4.1.30) respectively.

Inserting $\alpha$, $\beta$ and $\gamma$ into equation (4.1.46) gives

$$\alpha \frac{\partial h}{\partial x} = -(S + \frac{\gamma}{h^{1/3}} + \beta h) \quad (4.1.47)$$

The above equation is numerically integrated due to the difficulty in giving an analytic solution for this particular equation.
We then obtain

\[ \Delta h = \frac{1}{\alpha} \left( -S - \frac{\gamma}{h^{1/3}} - \phi h_1 \right) \Delta x \]  \hspace{1cm} (4.1.48)

Then, for a known bore height \( h_1 \) at location \( x_1 \) Figure (4.7) and known \( \Delta x \), the bore height \( h_2 \) at location \( x_2 \) is given by

\[ h_2 = h_1 - |\Delta h| \]  \hspace{1cm} (4.1.49)

Using the above procedure the envelope curve for the maximum dry bore surge height will be defined.

d) When only friction dissipation is included the dry bore equation for a sloping bottom becomes

\[ \left( 1 + \frac{Fr^2}{2} \right) \frac{\partial h}{\partial x} = -S - \frac{g(nF)^2}{h^{1/3}} \]  \hspace{1cm} (4.1.50)

Inserting \( \alpha \) and \( \gamma \) into the above equation gives

\[ \alpha \frac{\partial h}{\partial x} = -S - \frac{\gamma}{h^{1/3}} \]  \hspace{1cm} (4.1.51)

The above equation (4.1.51) is numerically integrated due to the difficulty in giving an analytic solution for this particular equation.

We then obtain

\[ \Delta h = \frac{1}{\alpha} \left( -S - \frac{\gamma}{h^{1/3}} \right) \Delta x \]  \hspace{1cm} (4.1.52)
Then, for a known bore height $h_1$ at location $x_1$ and known $\Delta x$, the bore height $h_2$ at location $x_2$ is given by

$$h_2 = h_1 - |\Delta h|$$  \hspace{1cm} (4.1.53)

Using the above procedure the envelope curve for the maximum dry bore surge height will be defined.

The following formulation is used to obtain the appropriate value of Manning's roughness, $n$, to be used in the dry bore solutions (Bretschneider et al., unpublished report).

$$n = 0.056 Z_0^{1/6} \quad (Z_0 \text{ in feet})$$  \hspace{1cm} (4.1.54)

where,

$Z_0$ = the distance from the boundary where the velocity is zero.

For turbulent rough boundaries $Z_0$ is approximated by

$$Z_0 = \frac{k_s}{30}$$  \hspace{1cm} (4.1.55)

where,

$k_s$ = bottom roughness parameter (Nikuradse sand roughness).

The values of the Manning's roughness $n$, the constant Froude number, $F_r$, the slope $S$ and the initial bore height $h_i$ (which is substituted for $h_1$ in equations (4.1.35), (4.1.44), (4.1.49) and (4.1.53) are used as input in the dry bore equation solutions. The initial bore height, $h_i$ (initial $h_1$ value) is taken as the average bore height of the first bore which reaches the shoreline. When the breaking dissipation is included in the dry bore equation the breaking loss coefficient, $\zeta$, is used as input also.
4.1.4 Definition of Froude Number

In bore problems the Froude number plays a very important role. In the literature the Froude number has been defined in different ways. The Froude number definitions used in this study are given as follows:

In the case of a hydraulic jump the Froude number is defined by (with reference to Figure 3.3).

\[ F_r = \frac{u_{1+c}}{\sqrt{gh_1}} \]  

(4.1.56)

where, \( h_1 \) = depth ahead of the jump, and \( u_{1+c} \) the velocity at that location.

For the wet bore Amein (1964) defines a Froude number as

\[ F_r = \frac{V}{\sqrt{gh_2}} = M \]  

(4.1.57)

where,

\( V \) = bore velocity

\( h_2 \) = total water depth behind the bore \( (h_2 = d + \eta) \).

Miller's (1968) definition of Froude number for the wet bore is

\[ F_r = \frac{V}{\sqrt{gh_1}} \]  

(4.1.58)

where, \( V \) is the bore velocity.

In case of the dry bore, where the bore travels on dry land Froude number is defined as
\[ F_r = \frac{c}{\sqrt{gh_d}} \]  
(4.1.59)

where, \( h_d \) = dry bore height

\( c \) = propagation velocity \((c = u)\) of bore.

There exists the following relationship between Miller's Froude number definition and Amein's Froude number definition.

\[
\frac{(F_r)_{\text{Miller}}}{(F_r)_{\text{Amein}}} = \frac{V/\sqrt{gh_1}}{V/\sqrt{gh_2}} = \sqrt{\frac{h_2}{h_1}} \]  
(4.1.60)

4.2 Comparison of the Dry Bore Calculations with Results of Bretschneider and Wybro (1976)

The Manual for Determining Tsunami Runup Profiles on Coastal Areas of Hawaii (1978) which is based on the study by Bretschneider and Wybro (1976) contains graphs which show the dry bore tsunami wave elevations versus the distance the bore travels inland for various slopes and Manning's \( n \), values. The tsunami wave elevations from these graphs have been used for comparison with the dry bore calculation results of this study. In the comparison a constant Froude number value of 2 is used to be consistent with Bretschneider and Wybro's (1976) method. The dry bore calculations in this study were carried out for horizontal bottom and for sloping bottoms \((S = 2\% \text{ and } 5\%)\) to allow comparison with Bretschneider and Wybro's results.

The initial dry bore height in the dry bore calculations has been taken as 15 meters for comparison purposes since Bretschneider and Wybro (1976) used the same value (Manual for Determining Tsunami Runup
Profiles on Coastal Areas of Hawaii (1978), reference is made to plates number 81, 88, 94).

For horizontal bottom the dry bore calculations have been carried out (using equations (4.1.35) and (4.1.44)) for selected values of Manning's roughness, $n$, to determine the dry bore dissipation distances inland. (An average value of $\zeta = 0.5$, for the breaking loss coefficient is used throughout the dry bore calculations in this section when breaking dissipation is included.) For a horizontal bottom the envelope curve of the tsunami (bore) elevation which shows the dry bore dissipation is given for Manning's roughness ($n$) values of $0.015, 0.03, 0.05, 0.07$ and $0.1$ in Figures 4.8, 4.9, 4.10, 4.11, 4.12, respectively. In these graphs the tsunami elevation curves are calculated when breaking losses are included for various bore constant lengths of two, three, five and sometimes eight times the initial bore height.

Similarly, for a slope of 2% the dry bore calculations have been carried out using equations (4.1.49), (4.1.53) and the envelope curve of the tsunami elevation is given for Manning's roughness ($n$) values of $0.015, 0.03, 0.05, 0.07, 0.08$ and $0.1$ in Figures 4.13, 4.14, 4.15, 4.16, 4.17, 4.18 respectively. It appears that as the friction increases the difference between the author's results and those of Bretschneider and Wybro decreases (Figure 4.19). It is also evident that the assumption of a greater bore length reduces the effect of the breaking losses. From experiments by Rouse et al. (1958) it appears that energy dissipation in a hydraulic jump occurs over approximately six times the bore height. Therefore, the results of a bore length of 75 meters which is five times the bore height seem to be the most acceptable.
This study, friction only, corresponds with Bretschneider and Wybro.

This study, friction and breaking losses included, borelength 30m.

This study, friction and breaking losses included, borelength 45m.

This study, friction and breaking losses included, borelength 75m.

Figure 4.8. Bore dissipation for a horizontal bottom ($S=0\%, n=.015, F_r = 2$)
This study, friction only, corresponds with Bretschneider and Wybro.

- - - - This study, friction and breaking losses included, bore length 30m.
- - - - - - This study, friction and breaking losses included, bore length 45m.
- - - - - - - - This study, friction and breaking losses included, bore length 75m.
- - - - - - - - - - This study, friction and breaking losses included, bore length 120m.

Figure 4.9. Bore dissipation for a horizontal bottom (S=0%, n=0.03, F_r=2)
This study, friction only, corresponds with Bretschneider and Wybro.

This study, friction and breaking losses included, bore length 30m.

This study, friction and breaking losses included, bore length 45m.

This study, friction and breaking losses included, bore length 75m.

This study, friction and breaking losses included, bore length 120m.

Figure 4.10. Bore dissipation for a horizontal bottom ($S=0\%, n=0.05, F_r=2$)
This study, friction only, corresponds with Bretschneider and Wybro.
- This study, friction and breaking losses included, bore length 30m.
- - This study, friction and breaking losses included, bore length 45m.
- - - This study, friction and breaking losses included, bore length 75m.

Figure 4.11. Bore dissipation for a horizontal bottom ($S=0\%, n= .07, F_r=2$)
This study, friction only, corresponds with Bretschneider and Wybro.

This study, friction and breaking losses included, bore length 30m.

This study, friction and breaking losses included, bore length 45m.

This study, friction and breaking losses included, bore length 75m.

This study, friction and breaking losses included, bore length 120m.

Figure 4.12. Bore dissipation for a horizontal bottom ($S=0\%, n= .1, F_r=2$)
This study, friction only, corresponds with Bretschneider and Wybro.

This study, friction and breaking losses included, bore length 30m.

This study, friction and breaking losses included, bore length 45m.

This study, friction and breaking losses included, bore length 75m.

This study, friction and breaking losses included, bore length 120m.

Figure 4.13. Bore dissipation for a slope of $S=2\%$, $n=0.015$, $F_r=2$. 
This study, friction only, corresponds with Bretschneider and Wybro.

This study, friction and breaking losses included, bore length 30m.

This study, friction and breaking losses included, bore length 45m.

This study, friction and breaking losses included, bore length 75m.

Figure 4.14. Bore dissipation for a slope of $S=2\%$, $n=0.03$, $F_r=2$
This study, friction only, corresponds with Bretschneider and Wybro.

This study, friction and breaking losses included, bore length 30m.

This study, friction and breaking losses included, bore length 45m.

This study, friction and breaking losses included, bore length 75m.

Figure 4.15. Bore dissipation for a slope of $S=2\%$, $n=0.05$, $F_r=2$
This study, friction only, corresponds with Bretschneider and Wybro.

--- This study, friction and breaking losses included, bore length 30m.

--- This study, friction and breaking losses included, bore length 45m.

--- This study, friction and breaking losses included, bore length 75m.

Figure 4.16. Bore dissipation for a slope of $S=2\%$, $n=0.07$, $Fr=2$. 
This study, friction only, corresponds with Bretschneider and Wybro.

This study, friction and breaking losses included, bore length 30m.

This study, friction and breaking losses included, bore length 45m.

This study, friction and breaking losses included, bore length 75m.

Figure 4.17. Bore dissipation for a slope of S=2%, n=0.08 Fr=2
This study, friction only, corresponds with Bretschneider and Wybro.

This study, friction and breaking losses included, borelength 30m.

This study, friction and breaking losses included, borelength 45m.

This study, friction and breaking losses included, borelength 75m.

Figure 4.18. Bore dissipation for a slope of S=2%, n= .1, F_r=2
This study, friction only, corresponds with Bretschneider and Wybro.
This study, friction and breaking losses included, borelength 75m.
This study, friction and breaking losses included, borelength 120m.

Figure 4.19. Bore dissipation for a slope of $S=2\%$, $n=0.1$, $F_r=2$ (bore length 120m)
For a slope of 5%, the envelope curve of the tsunami elevation showing the bore dissipation distances is given for Manning's roughness values of .015, .03, .05, .1 in Figures 4.20, 4.21, 4.22, 4.23 respectively.

Consequently, from the above dry bore calculation comparison with that of Bretschneider and Wybro (1976) it can be concluded that:

1. The inclusion of energy dissipation due to breaking reduces computed values of runup height and dissipation distance as compared to the case where only bottom friction is considered.

2. The reduction in dissipation distance decreases as Manning's n increases.

3. If only energy losses due to friction are considered, results of this study agree with those of Bretschneider and Wybro (1976).

4. As the assumed bore length increases, the bore dissipation distance increases.

4.3 Comparison of the Author's Method with Miller's (1968)

Experimental Runup Data

Miller (1968) has determined experimentally the runup undular surges and bores for four slopes each with three different bottom roughnesses. Miller's results indicate that dimensionless runup curves of the height of runup (R/h₁) versus the height of the initial wave (hₙ/h₁) (shown as Y₂ in Figures 4.24 and 4.25) are approximately linear for the undular surge, Fₚ ≤ 1.35, and the fully developed bore, Fₚ ≥ 1.55, separated by nonlinear transition region. In the above terminology R is the runup height above undisturbed water level, h₂
This study, friction only, corresponds with Bretschneider and Wybro.

This study, friction and breaking losses included, bore length 30m.

This study, friction and breaking losses included, bore length 45m.

This study, friction and breaking losses included, bore length 75m.

Figure 4.20. Bore dissipation for a slope of $S=5\%$, $n=0.15$, $F_r=2$
This study, friction only, corresponds with Bretschneider and Wybro.

This study, friction and breaking losses included, borelength 30m.

This study, friction and breaking losses included, borelength 45m.

This study, friction and breaking losses included, borelength 75m.

Figure 4.21. Bore dissipation for a slope of S=5%, n=.03, F_r=2
This study, friction only, corresponds with Bretschneider and Wybro.

This study, friction and breaking losses included, bore length 30m.

This study, friction and breaking losses included, bore length 45m.

This study, friction and breaking losses included, bore length 75m.

Figure 4.22. Bore dissipation for a slope of $S=5\%$, $n=0.05$, $F_r=2$
This study, friction only, corresponds with Bretschneider and Wybro.

This study, friction and breaking losses included, bore length 30m.

This study, friction and breaking losses included, bore length 45m.

This study, friction and breaking losses included, bore length 75m.

Figure 4.23. Bore dissipation for a slope of $S=5\%$, $n=.1$, and $F_r=2$.
Figure 4.24. Dimensionless graph showing runup data for slopes $\alpha = 2^\circ$ and $\alpha = 5^\circ$ for three bottom roughnesses (from Miller, 1968)
Figure 4.25. Dimensionless graph showing runup data for slopes $\alpha = 10^\circ$ and $\alpha = 15^\circ$ for three bottom roughnesses (from Miller, 1968)
is the height of the wave measured from the channel bottom and $h_1$ is the undisturbed water depth (Figure 4.26). Miller's (1968) experimental runup data are given in Figure 4.24 (for slopes $2^0$ and $5^0$) and Figure 4.25 (for slopes $10^0$ and $15^0$) for three bottom roughnesses ($\alpha$ in Miller's graphs Figures 4.24 and 4.25 represents slope angle). The three bottom roughnesses are given in terms of Nikuradse sand roughness, $k_s'$, which are smooth, .52 mm, 3.7 mm and correspond to Manning's roughness, $(n)$ values of smooth, .0109 and .015 respectively. Strictly speaking the Manning's $n$ approach is only applicable to a hydraulically rough surface. However, the relationship

$$n = \frac{h^{1/6}}{C_h}$$

is also applied to non-rough surfaces. The corresponding value of $n$ is calculated from the calculated value of $C_h$ for smooth surfaces for the appropriate values of $h$.

Figure 4.26. Schematic of generation of a bore (after Miller, 1968)

In Figures 4.24 and 4.25, the solid triangles indicate model values of $h_1 = 6.1$ cm; open circles $h_1 = 9.1$ cm and open triangles $h_1 = 12.2$ cm.

At first a comparison is made for a smooth bottom.
Comparison with Miller's experimental runup data (Figures 4.24 and 4.25) is carried out with the data points in three regions where the ratio $h_2/h_1$ is 1.7, 2.0-2.5, 3.0-3.5 for three bottom roughnesses. The initial bore height ($H$) according to Miller's (1968) experimental set-up is taken as $(h_2-h_1)$. For the smooth bottom this initial bore height is used as the input wave height ($\eta=H$) for the Amein's (1964) characteristic program (with no bottom friction) and the runup heights ($R$) are (see Figure 4.27) obtained. The runup heights obtained from Amein's characteristic study are given as the ratio $R/h_1$ and are represented by an open triangle in Figure 4.28. The present study runup heights for the smooth bottom are obtained by the dry bore calculations. The initial bore height for the dry bore calculations is taken by using an average bore height of the first bore which reaches the shoreline. Similarly an average Froude number is calculated from the first bore arriving at the shoreline. The runup obtained in this way from the dry bore calculations is the present study result and is represented by a solid square in Figure 4.28. The dotted line in Figures 4.28 and 4.29 represents Miller's experimental runup curve in Figure 4.24. It was found that for smooth bottom the Manning's roughness, $n$, did not change the runup results shown in Table 4.1 much when $n \leq 0.001$.

Table 4.1 summarizes the runup height results for a smooth bottom.

Similarly, comparison with Miller's experimental data is carried out in three regions where the ratio $h_2/h_1$ is 1.7, 2.0-2.5, 3.0-3.5 for $k_s$ bottom roughness values of 0.52 mm and 3.7 mm. The initial bore height $(h_2-h_1)$ is used as the input wave height $(\eta=H=h_2-h_1)$ for the
Figure 4.27. Bore dissipation and runup for smooth bottom ($h_2/h_1 = 1.7$, $t=.9, t=1.1, t=1.2$).
SEE ABOVE

n bottom \( (h_2/h_1 = 1.7, \phi = 2^\circ) \)
Dry Bore $H_i = 4.0 \text{ cm}$

$H = 4.27 \text{ cm}$

$h_r = 6.1 \text{ cm}$

$= .1401 \text{ ft}$

$.2001 \text{ ft}$
Figure 4.28. Comparison of runup height results for a smooth bottom ($\alpha=2^0$).
Figure 4.29. Comparison of runup height results for a smooth bottom ($\alpha=5^\circ$)
Table 4.1. Comparison of runup height results for a smooth bottom
(numbers in brackets are in feet)

<table>
<thead>
<tr>
<th>$h_1$(m)</th>
<th>$h_2$</th>
<th>Amein's characteristic program</th>
<th>Miller's Experimental results</th>
<th>$H_r$(m)</th>
<th>$n$</th>
<th>$F_r$</th>
<th>$\zeta$</th>
<th>Present study runup R(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>For $\alpha = 2^0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.061</td>
<td>2.9</td>
<td>0.308</td>
<td>0.165</td>
<td>0.115</td>
<td>0.0019</td>
<td>1.3</td>
<td>0.15</td>
<td>0.177 (.58 )</td>
</tr>
<tr>
<td>0.091</td>
<td>2.1</td>
<td>0.21</td>
<td>0.183</td>
<td>0.093</td>
<td>0.0011</td>
<td>1.5</td>
<td>0.08</td>
<td>0.183 (.6 )</td>
</tr>
<tr>
<td>0.061</td>
<td>3.5</td>
<td>0.335</td>
<td>0.192</td>
<td>0.128</td>
<td>0.0015</td>
<td>1.5</td>
<td>0.30</td>
<td>0.213 (.70 )</td>
</tr>
<tr>
<td>0.061</td>
<td>1.7</td>
<td>0.095</td>
<td>0.0945</td>
<td>0.04</td>
<td>0.0067</td>
<td>1.8</td>
<td>0.1</td>
<td>0.0949 (.31)</td>
</tr>
<tr>
<td>0.061</td>
<td>2.5</td>
<td>0.186</td>
<td>0.143</td>
<td>0.088</td>
<td>0.0012</td>
<td>1.3</td>
<td>0.15</td>
<td>0.146 (.48)</td>
</tr>
<tr>
<td>For $\alpha = 5^0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.061</td>
<td>1.7</td>
<td>0.0945</td>
<td>0.103</td>
<td>0.042</td>
<td>0.0067</td>
<td>1.6</td>
<td>0.05</td>
<td>0.091 (.30)</td>
</tr>
<tr>
<td>0.091</td>
<td>1.7</td>
<td>0.164</td>
<td>0.162</td>
<td>0.06</td>
<td>0.0005</td>
<td>1.6</td>
<td>0.05</td>
<td>0.134 (.44)</td>
</tr>
<tr>
<td>0.122</td>
<td>2.0</td>
<td>0.3048</td>
<td>0.341</td>
<td>0.118</td>
<td>0.0079</td>
<td>1.8</td>
<td>0.05</td>
<td>0.3048 (1.0)</td>
</tr>
<tr>
<td>0.061</td>
<td>2.5</td>
<td>0.238</td>
<td>0.236</td>
<td>0.089</td>
<td>0.0095</td>
<td>1.7</td>
<td>0.1</td>
<td>0.213 (.7)</td>
</tr>
<tr>
<td>0.061</td>
<td>3.0</td>
<td>0.3048</td>
<td>0.293</td>
<td>0.113</td>
<td>0.0012</td>
<td>1.9</td>
<td>0.2</td>
<td>0.293 (.96)</td>
</tr>
<tr>
<td>0.122</td>
<td>3.0</td>
<td>0.634</td>
<td>0.622</td>
<td>0.231</td>
<td>0.0014</td>
<td>1.94</td>
<td>0.2</td>
<td>0.613 (2.01)</td>
</tr>
</tbody>
</table>
amended characteristic program of this study (with bottom friction, see section 4.1.2) in order to obtain the runup heights, \( R \) for cases where slope is 50° (see Figure 4.30). The first bore which reaches the shoreline is calculated from the amended characteristic program of this study and is shown in Figure 4.30. An initial bore height and an average Froude number representative of the first bore reaching the shoreline is then used as input for the dry bore runup calculations. The runup height obtained using the above as input for the dry bore solutions (see section 3.4) is the present study result and is shown by a solid square in Figures 4.31, 4.32, 4.33, 4.34.

Miller's Froude number ranges corresponding to the \( \zeta \) (breaking loss coefficient) ranges used in the comparison with Miller's runup data in three regions is given in Table 4.2.

Table 4.2. Miller's Froude number ranges corresponding to the \( \zeta \) ranges used for comparison with Miller's (1968) runup data

<table>
<thead>
<tr>
<th>((F_r)_\text{Miller})</th>
<th>(h_2/h_1)</th>
<th>(\zeta)</th>
<th>(n = \text{smooth})</th>
<th>(n = .0109)</th>
<th>(n = .015)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.50</td>
<td>1.7</td>
<td>.05-.1</td>
<td>.01</td>
<td>.01</td>
<td></td>
</tr>
<tr>
<td>1.72-2.08</td>
<td>2.0-2.5</td>
<td>.08-.15</td>
<td>.4</td>
<td>.4</td>
<td></td>
</tr>
<tr>
<td>2.45-2.81</td>
<td>3.0-3.5</td>
<td>.15-.3</td>
<td>.9</td>
<td>.9</td>
<td></td>
</tr>
</tbody>
</table>

Although these Froude numbers and \( \zeta \), breaking loss coefficient ranges are obtained through experimental model study it could also be applicable to field studies.
Figure 4.30. Bore dissipation and runup for $\alpha = 5^\circ$, $n = 0.015$
Figure 4.31. Comparison of runup height results for $\alpha=2^0$, $n=.0109$, $n=.015$
Figure 4.32. Comparison of runup height results for $\alpha = 10^0$ ($n = .015$)
LEGEND

- Present study results
- Amein's (1964) characteristics
- Miller's (1968) experimental results
- Miller's graph (Figs. 4.24 and 4.25)

Figure 4.33. Comparison of runup height results for $\alpha = 5^\circ (n = 0.015)$
Figure 4.34. Comparison of runup height results for $\alpha=15^\circ$, $n=0.015$
4.4 Further Discussion on Runup Behavior

Yeh and Ghazali (1988) investigated experimentally the transition process from bore to runup mode, using a laser-induced fluorescent method. They state that the observed process appears to be different from both previous analytical and numerical predictions. The authors described their visual observations of the transition from bore to runup mode for a weak bore and a fully developed bore.

Yeh and Ghazali observed that for a fully developed bore the transition process is neither like a complete bore collapse nor supports gradual transition as predicted by a numerical simulation (Hibberd and Peregrine, 1979). According to the authors' observations the bore front itself never reaches the shoreline directly, but the small wedge-shaped water body along the shore ahead of the front is suddenly pushed forward by the bore. This pushed water mass initiates the runup process. Yeh and Ghazali (1988) state that during the transition the bore front first steepens to almost vertical but becomes smooth (less turbulent) as it reduces its height, while the water in front of it is extremely turbulent. It is then suggested by the authors that the turbulence generated at the front near the shore is advected forward onto the beach instead of being left behind. Yeh and Ghazali's observations may be viewed as relating to the breaker type definitions in breaking waves, such as plunging, collapsing and surging. It is believed that Yeh and Ghazali's observations for a fully developed bore resemble a surging or a collapsing type of breaker.

Yeh and Ghazali (1988) furthermore observed the transition process from the weak bore to the runup mode, where only the crest part of the
bore front is turbulent when it approaches the shore. The bore front then overturns directly onto the dry beach surface and as the runup process starts, the bore height decreases quickly. Yeh and Ghazali (1988) noted that the shape of the overturning front resembles that of the wave breaking on a steep slope beach. It is believed that this description of Yeh and Ghazali of a weak bore developing into a breaking front is similar to that of a plunging type initial breaker and fully saturated breaker thereafter.

It is worthwhile to note how the present study runup results compare with Yeh and Ghazali's observations. It is believed that Yeh and Ghazali's observations indicate two types of phenomena, which show satisfactory comparison with the present study results:

1) The weak bore situation at the shoreline where a well developed bore does not occur and the run-up occurs in the form of a sheet of water pushed up the slope.

Amein's methods of calculation, modified to the extent that bottom friction is included, may be applicable to this situation.

The type of breaking as the wave runs up the slope resembles more of a surging breaker where the wave slides up the beach with minor production of turbulence (Figure 4.27).

2) The fully developed bore formation at the shoreline, corresponding with a large wave height and a relatively high Froude number, runs up the slope as a dry bore (Figure 4.30).
CHAPTER V
APPLICATIONS TO WAIMEA BAY AND MOKULEA BEACH

The results of this study have been applied to two locations on Oahu, Waimea Bay and Moku1ea Beach (see Figures 5.1 and 5.2).

Tsunamis have hit the north shore of Oahu frequently over the last 50 years.

The following runup observations have been recorded for the coast just east of Waimea Bay (Loomis, 1976):

<table>
<thead>
<tr>
<th>Year</th>
<th>Elevation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1946</td>
<td>4.3 (+14 ft)</td>
</tr>
<tr>
<td>1952</td>
<td>2.1 (+ 7 ft)</td>
</tr>
<tr>
<td>1957</td>
<td>6.7 (+22 ft)</td>
</tr>
<tr>
<td>1960</td>
<td>3.4 (+11 ft)</td>
</tr>
<tr>
<td>1964</td>
<td>4.9 (+16 ft)</td>
</tr>
</tbody>
</table>

These elevations are with respect to M.S.L. which is about 1 foot above M.L.W. The above list does not include many small tsunamis that were not likely to have done any damage of significance.

There is no fixed standard for measurement of tsunami wave heights. What is desired is the highest wave at the shoreline measured above mean sea level, but what is measured is actually the intrusion of the water onto the land (Loomis, 1976). This height could be higher or lower than the wave height on shore.

Some investigators assume that observed elevations represent the water level 200 ft inshore from the shoreline (Cox, 1961).
NOTE:
COASTAL BASE FLOOD ELEVATIONS APPLY ONLY LANDWARD OF THE SHORELINE SHOWN ON THIS MAP.

Figure 5.1. Map showing Waimea Bay location (Oahu Flood insurance rate map, 1987)
Figure 5.2. Map showing locations on Mokulea Beach (Kaena Pt. quadrangle, bathymetric chart) and runup elevations for the 1946 tsunami (above M.S.L)
The following table represents tsunami elevations at Waimea according to different prediction methods (M.S.L.).

Table 5.1 Tsunami wave elevations by different prediction methods for Waimea Bay

<table>
<thead>
<tr>
<th>Prediction Method</th>
<th>Elevation (+ft)</th>
<th>Meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oahu Flood Insurance Map recommended flood level</td>
<td>(+18 ft)</td>
<td>5.48 m</td>
</tr>
<tr>
<td>Recommended flood level one in 100 year frequency</td>
<td>(+21 ft)</td>
<td>6.9 m</td>
</tr>
<tr>
<td>(U.S. Army Corps of Engineers, Pacific Ocean Division, 1978)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recommended flood level one in 200 year frequency</td>
<td>(+25.5 ft)</td>
<td>7.8 m</td>
</tr>
<tr>
<td>(U.S. Army Corps of Engineers, Pacific Ocean Division, 1978)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loomis (1987)--one in 100 year frequency</td>
<td>(+26.2 ft)</td>
<td>7.9 m</td>
</tr>
<tr>
<td>(Gumbel distribution)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loomis (1987)--one in 200 year frequency</td>
<td>(+30.8 ft)</td>
<td>9.4 m</td>
</tr>
<tr>
<td>(Gumbel distribution)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In order to make an independent evaluation, computations have been carried out in this study using the amended method of characteristics program and the dry bore runup solutions including energy dissipation both by friction and breaking in the hydrodynamic equations (equations (4.1.35), (4.1.44), (4.1.49), (4.1.53)).

As input for a 200 year tsunami a wave height of 5m (16.4 ft) at a depth of 20m (65.6 ft) is used for the calculations. This input is in corresponding with historical data (Okada and Tada, 1983; Wigen, 1983; Figure 3.14) and is assumed to represent a one in two
hundred year frequency. It is also assumed that a bore forms under these conditions (n=H) as input. This assumption is one that is difficult to verify, however, the results obtained near the shore and the runup that results from it seem to compare realistically with the observed and predicted levels. Furthermore, with reference to the calculations carried out in section 4.1.2 using Amein's characteristics (with no bottom friction) with a sinusoidal wave of H=7.86m (25.87 ft) at a transition depth, h_tr = 32.13m (105.41 ft) and S= .05, T=30 sec it is found that the first bore forms at 18m (60 ft) of depth (1201 ft away from the shore) with an average bore height of 4.2m (13.7 ft). The above numerical experiment supports the assumption that it is possible for a bore to occur at 20m (65.6 ft) depth with a height of 5m (16.4 ft) (for a 200 year tsunami), although they are not exactly the same situations.

For Waimea Bay, for the wet bore calculations using the amended characteristics method the following input is used:

H=5m  h_tr =20m  S=.043  n=.028  Δt=.25 sec

From these input values, an average initial bore height of 5.42m (17.8 ft) and an average Froude number of 1.5 is obtained from the first bore that arrives at the shoreline.

For the dry bore runup calculations (Figure 5.3), the following input values are used; initial dry bore height, h_i=5.42m, F_r=1.5, n=.04, breaking loss coefficient ζ=.1, and the slope values are shown in Figure 5.3. The bore height, at the end of 26.82m horizontal distance with a slope of 1:4, is calculated as 1.95m (Figure 5.3). This bore height of 1.95m is used as initial bore height for the 1:20
sloping section. These calculations result in a water level 200 feet inland (Cox, 1961), of 29.5 ft (8.99m) which agrees favorably with the 200 year prediction made by Loomis (Table 5.1).

An inundation level of 30 feet (rounded figure) is considered possible for such extreme event.

Figure 5.3. Dry bore sketch for Waimea Bay (not to scale)

In view of the lack of adequate field data for tsunami related runup, an inundation level for Waimea Bay between 7.9m (26 ft) and 10m (33 ft), ±10% of the calculated 9.1m (30 ft), can be considered possible, as a realistic estimate for an extreme event.

For Mokulea Beach location A (Figure 5.2), for the wet bore calculations using the amended characteristics method the following data are used as input:

\[
H = 5 \text{m} \quad h_{tr} = 20 \text{m} \quad S = 0.02 \quad n = 0.028 \quad \Delta t = 0.4 \text{ sec}
\]

From these input values, an average initial bore height of 5.55m (18.2 feet) and an average Froude number of about 1.5 are obtained from the first bore that arrives at the shoreline.
For the dry bore runup calculations (Figure 5.4) the following input values were used; initial dry bore height of 5.55m, $F_r=1.5$, $n=.04$, breaking loss coefficient $\zeta=.1$ and dry slope $S=.07$. This input yields a water level of about 7.3m (24 ft) 61m (200 feet) inland and a runup $R$ of 9.3m (30.6 ft) which is 133.5 meters inland.

![Figure 5.4. Dry Bore sketch for location A Mokulea Beach (not to scale)](image)

For location B at Mokulea Beach since the wet bore slope is .02 also, the wet bore results for initial bore height for location A is used with the same input values. For the dry bore runup calculations the following input values are used; initial dry bore height of 5.55m, $F_r=1.5$, $n=.04$, breaking loss coefficient of .08, and dry bore slope $S=.045$. This input yields a runup of 8.5m (28 ft) which is 190m (623 ft) inland.

The observed runup data obtained from the 1946 tsunami for location A is 6.1m (20 ft) and for location B is 7.9m (26 ft) as shown in Figure 5.2. It is not known exactly how far away inland
from the shore these runup heights occurred. However, whether it is assumed that these runup heights occur 200 feet inland (Cox, 1961) or more, the runup results of this study are higher than the observed runup heights. This is to be expected because the one in two hundred year assumed tsunami is more severe than the 1946 event.

Locations A and B at Moku1ea Beach are shown on Figure 5.5 which gives the tsunami wave elevations for the 1946, 1952, 1957, 1960, 1964 tsunamis (Loomis, 1976). From this information at Location A at Moku1ea Beach the water level is (14 ft) 4.3 m and at location B the water level is (20 ft) 6.1 m for the 1946 tsunami.

Cox (1961) found by trial and error that at an arbitrary starting point chosen as 3 m (10 ft) depth with an assumed 15.2 m (50 ft) wave height, a one-percent decline with distance traversed, would provide an estimate of inundation areas and runup heights. Applying Cox's construction line to locations A and B at Moku1ea Beach runup heights of 12.2 m (40 ft) and 10.9 m (36 ft) are obtained for locations A and B respectively (Figures 5.6 and 5.7). These values are considerably higher than the results of the calculations.

Table 5.2 represents tsunami wave elevations by different prediction methods for Moku1ea Beach (locations A and B) (Figure 5.8).

Based on the above evaluations, an inundation level for Moku1ea Beach between 7.6 m (25 ft) and 10 m (33 ft) can be considered possible, as a realistic estimate for an extreme event.
Figure 5.5. Tsunami wave elevations (after Loomis, 1976)
Cox's Runup = 12.19 m (40 ft.)
Present Study Runup = 9.32 m (30.6 ft.)
1946 Tsunami Data = 6.09 m (20 ft.)

Figure 5.6. Mokulea Beach location A, Cox's construction line
Cox's Runup = 10.97 m (36 ft.)
Present Study Runup = 8.53 m (28 ft.)
1946 Tsunami Data = 7.92 m (26 ft.)

Figure 5.7. Mokulea Beach location B, Cox's construction line
Table 5.2. Tsunami wave elevations by different prediction methods for Mokulea Beach

<table>
<thead>
<tr>
<th>Method</th>
<th>Elevation (ft)</th>
<th>Runup (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oahu Flood Insurance Rate Map recommended flood level (Figure 5.8)</td>
<td>(+11)</td>
<td>3.4</td>
</tr>
<tr>
<td>Cox's prediction of flood level location A (Figure 5.6)</td>
<td>(+40)</td>
<td>12.2</td>
</tr>
<tr>
<td>Present study runup for location A (Figure 5.4)</td>
<td>(+30.6)</td>
<td>9.3</td>
</tr>
<tr>
<td>1946 observed tsunami runup for location A</td>
<td>(20)</td>
<td>6.1</td>
</tr>
<tr>
<td>Loomis (1976) reported tsunami data for location A (Figure 5.5)</td>
<td>(14)</td>
<td>4.3</td>
</tr>
<tr>
<td>Cox's prediction of flood level location B (Figure 5.7)</td>
<td>(36)</td>
<td>10.9</td>
</tr>
<tr>
<td>Present study runup for location B</td>
<td>(28)</td>
<td>8.5</td>
</tr>
<tr>
<td>1946 observed tsunami runup for location B</td>
<td>(26)</td>
<td>7.9</td>
</tr>
<tr>
<td>Loomis (1976) reported tsunami data for location B (Figure 5.5)</td>
<td>(20)</td>
<td>6.1</td>
</tr>
</tbody>
</table>
Figure 5.8. Mokulea Beach Oahu flood insurance rate map (1987)
CHAPTER VI
CONCLUSIONS

In this study a computational method is developed to determine the runup of tsunami waves using the method of characteristics. In the runup calculations energy losses both due to friction and to breaking are considered.

If only bottom friction is considered the results of this study compare favorably with those of Bretschneider and Wybro (1976). The inclusion of breaking losses in the calculations leads to lower runup values compared to those obtained by Bretschneider and Wybro (1976).

The methodology applied in this study differs from Amein's method by inclusion of bottom friction both in the wet-bore part and in the runup phase. The results agree with Amein's results when bottom friction is neglected.

The results of this study compare satisfactorily with Miller's (1968) experimental results in the laboratory considering a wet-bore approach before it reaches the shoreline and a dry bore approach after that point.

The method developed in this study is considered to be an improvement above presently available methods for the calculation of tsunami runup in coastal zone and is applied to two locations on Oahu's North Shore (Waimea Bay and Mokulea Beach).
APPENDIX A
DERIVATION OF METHOD OF CHARACTERISTICS

a) A second approach to method of characteristics involves a hyperbolic system of linear equations. Many physical problems include the formulation of a quasilinear system of first order equations; such equations are linear in the first derivatives of the dependent variables, but the coefficients may be functions of the dependent variables. The two independent variables are often the time and one space variable, \( t \) and \( x \). If the dependent variables are \( u_i(x,t) \), \( i=1, \ldots, n \) the general quasi-linear first order system is

\[
A_{ij} \frac{\partial u_j}{\partial t} + a_{ij} \frac{\partial u_j}{\partial x} + b_i = 0 \quad i = \ldots, n \quad (A.1)
\]

where the matrices \( A, a \) and the vector \( b \) may be functions of \( u_1, u_n \) as well as \( x \) and \( t \).

In this appendix the conditions for equation A.1 are established to be hyperbolic. Any one of the equations in A.1 provides information on the rates of change of the different directions and has different combinations of

\[
\frac{\partial u_j}{\partial t}, \frac{\partial u_j}{\partial x}
\]

for each \( u_j \).

Let us consider the linear combination

\[
\varepsilon_i (A_{ij} \frac{\partial u_j}{\partial t} + a_{ij} \frac{\partial u_j}{\partial x}) + \varepsilon_i b_i = 0 \quad (A.2)
\]
where the vector \( \ell \) is a function of \( x, t, u \) and investigate whether \( \ell \)
can be chosen so that equation A.2 takes the form

\[
m_j \left( \beta \frac{\partial u_j}{\partial t} + \alpha \frac{\partial u_j}{\partial x} \right) + \ell_j b_j = 0
\]  

(A.3)

Equation A.3 provides a relation between the directional derivatives of all the \( u_j \) in the single direction \((\alpha, \beta)\). If \( x = X(n) \) and \( t = T(n) \) is the parametric representation of a typical number of this family, the total derivative of \( u_j \) on the curve in the \( x-t \) plane is

\[
\frac{du_j}{dn} = T' \frac{\partial u_j}{\partial t} + X' \frac{\partial u_j}{\partial x}
\]  

(A.4)

Let us take \( \alpha = X'(n), \beta = T'(n) \) and write equation A.3 as

\[
m_j \frac{du_j}{dn} + \ell_j b_j = 0
\]  

(A.5)

The conditions for equation A.1 to be in the form equation A.5 are:

\[
l_i A_{ij} = m_j T', \quad l_i a_{ij} = m_j X'
\]

and \( m_j \) is eliminated to give

\[
l_i (A_{ij} X' - a_{ij} T') = 0
\]  

(A.6)

These are \( n \) equations for the multipliers \( l_i \) and direction \((X'T')\).

Since they are homogeneous in the \( l_i \), a necessary and sufficient condition for a nontrivial solution is that the determinant

\[
|A_{ij} X' - a_{ij} T'| = 0
\]  

(A.7)
This is a condition on the direction of the curve. Such a curve is said to be a characteristic and the corresponding equation (A.5) is said to be in characteristic form.

Let us now consider the linear system of equations to solve the wave problem. There are four unknowns and four equations to solve:

a. Equation of motion (rearranging equation (3.1.19))

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial x} = -gS - gu \frac{|u|}{h^2} = W
\]  
(A.8)

where \( W \) is a constant

b. Equation of continuity

\[
\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + h \frac{\partial u}{\partial x} = 0
\]  
(A.9)

c.

\[
du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial t} dt
\]  
(A.10)

d.

\[
dh = \frac{\partial h}{\partial x} dx + \frac{\partial h}{\partial t} dt
\]  
(A.11)

The above equations form the following matrix:

\[
\begin{bmatrix}
1 & u & 0 & g \\
0 & h & 1 & u \\
dt & dx & 0 & 0 \\
0 & 0 & dt & dx
\end{bmatrix}
\begin{bmatrix}
\frac{\partial u}{\partial t} \\
\frac{\partial u}{\partial x} \\
\frac{\partial h}{\partial t} \\
\frac{\partial h}{\partial x}
\end{bmatrix}
= \begin{bmatrix}
W \\
0 \\
\frac{\partial u}{\partial t} \\
\frac{\partial h}{\partial t}
\end{bmatrix}
\]  
(A.12)
Satisfying the condition of equation (A.7) yields the following

\[
\begin{bmatrix}
1 & u & 0 & g \\
0 & h & 1 & u \\
dt & dx & 0 & 0 \\
0 & 0 & dt & dx \\
\end{bmatrix} = 0 \tag{A.13}
\]

which results in the equation

\[ dx^2 - 2udxdt - (gh - u^2)dt^2 = 0 \tag{A.14} \]

Equation (A.14) is then reduced to

\[ c = \frac{dx}{dt} = u \pm (gh)^{1/2} \tag{A.15} \]

Another important relation is obtained from the matrix

\[
\begin{bmatrix}
1 & u & 0 & W \\
0 & h & 1 & 0 \\
dt & dx & 0 & du \\
0 & 0 & dt & dh \\
\end{bmatrix} \tag{A.16}
\]

whose determinant is equal to zero

\[ hdu + (\frac{dx}{dt} - u)dh - Whdt = 0 \tag{A.17} \]

Inserting equation (A.15) into (A.17)

\[ du \pm \left(\frac{g}{h}\right)^{1/2} dh = Wdt \tag{A.18} \]

Considering the frictionless case, \( W = 0 \)

\[ du \pm \left(\frac{g}{h}\right)^{1/2} dh = 0 \tag{A.19} \]
Integrating the above equation gives

\[ u + 2(gh)^{1/2} = \text{Constant} \] (A.20)

which holds along the characteristic curves given by equation (A.15) as shown in Figure A.1.

![Figure A.1. Characteristic curves](image)

b) The characteristic method evolves into a single wave in the case of the dry bore, where the bore moves on dry land. Bretschneider, Wybro (1976) assumes \( u = c \) (equation (3.2.59)) for dry bore on horizontal bottom. This is equivalent to the method of characteristics approach where one moves with the crest of the wave (bore).

In this section we will show that the characteristic solution of equation (3.2.58) for the bore front corresponds with the solution of the following equation

\[
\frac{\partial}{\partial x} \left( h + \frac{u^2}{2g} \right) = -\frac{u|u|}{c_2^2h} - \frac{ch}{4\sqrt{g} \lambda} - S \] (A.21)
Proof that the characteristic solution of equation (3.2.60) corresponds with equation (A.21):

Along the advancing characteristic we have

$$\frac{d(u + 2c)}{dt} = W = -g \frac{u + u}{c h} - \frac{g c h}{4 \sqrt{2} L} - g S$$  \hspace{1cm} (A.22)

At the bore front $dx = u \, dt$ and $dt = dx / u$

Then,

$$\frac{ud(u + 2c)}{dx} = W$$  \hspace{1cm} (A.23)

$$\frac{udu + 2udc}{dx} = W$$  \hspace{1cm} (A.24)

At the crest of the bore $u = c \sqrt{gh}$. Therefore,

$$\frac{udu + 2\sqrt{gh}(d\sqrt{gh})}{dx} = W$$  \hspace{1cm} (A.25)

Rewriting the above equation

$$\frac{d\left(\frac{u^2}{2}ight) + gdh}{dx} = W$$  \hspace{1cm} (A.26)

$$\frac{d\left(\frac{u^2}{2g} + h\right)}{dx} = W$$  \hspace{1cm} (A.27)

Then,
\[ \frac{d(u^2 + h)}{dx} = \frac{W}{g} = - \frac{u|u|}{C_h^2} - \frac{\zeta h}{4\sqrt{2} \ell} - S \]  \hspace{1cm} (A.28)

Following the wave, \[ \frac{d()}{dx} = \frac{d()}{dx} \]

Therefore, the above equation becomes equal to equation (A.21)

\[ \frac{\partial}{\partial x} (h + \frac{u^2}{2g}) = - \frac{u|u|}{C_h^2} - \frac{\zeta h}{4\sqrt{2} \ell} - S \]  \hspace{1cm} (A.29)
APPENDIX B

ADDITIONAL FIGURES
Figure B.1. Characteristic curves
Figure B.2. Wave profiles at t=30, 45, 47, 49, 52, 60, 75 seconds
DRY BORE HEIGHT = 9.50185 FT.
VELOCITY AT SHORE = 13.219 FT/SEC
FROUDE = 1.899
APPROXIMATE RUNUP = 17.149 FT.

seconds

X DISTANCE (FT)
DRY BORE HEIGHT = 9.50185 FT.
VELOCITY AT SHORE = 33.219 FT/SEC
FROUDE = 1.899
APPROXIMATE RUNUP = 17.149 FT.
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