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TRANSPORTATION DEMAND AND CATASTROPHE THEORY: A COMPARATIVE ANALYSIS OF DISAGGREGATED CHOICE MODELS

University of Hawaii PH.D. 1979

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TRANSPORTATION DEMAND AND CATASTROPHE THEORY:

A COMPARATIVE ANALYSIS OF DISAGGREGATED CHOICE MODELS

A DISSERTATION SUBMITTED TO THE GRADUATE DIVISION
OF THE UNIVERSITY OF HAWAII IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

IN AGRICULTURAL ECONOMICS

AUGUST 1979

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ACKNOWLEDGEMENTS

I am indebted to all professors, colleagues and friends who directly or indirectly contributed to my development at the University of Hawaii. Several individuals, however, deserve special mention.

I would like to thank the members of my committee, whose advice and comments undoubtedly enhanced the quality of this work. I especially would like to show my appreciation to my major professor, Dr. Walter Miklius, and to Dr. Peter Garrod, for making themselves available at all times to listen, discuss and criticize my work throughout the research.

I am also grateful to Dr. E. Roy Weintraub, of Duke University, for valuable help in defining, and later criticizing, my dissertation.

I am thankful to the Chairman of the Department of Agricultural and Resource Economics, Dr. Frank S. Scott, Jr., and to Dr. Howard Hogg, of the U.S. Department of Agriculture, for encouraging and largely making possible my studies in Hawaii; and to the former Executive Secretary of ABCAR, Dr. Aloisio Campelo, for authorizing the leave of absence which enabled my pursuit of higher education.

To Ms. Earline Weddle I would like to thank for the many gestures of friendship and for generously spending her time in editing this paper; and to Robert Jurich for high quality typing work.

Finally, and most importantly, I profess my deep gratitude to my parents, Carlos and Léa, and to my brother, Gustavo Adolfo, for unwavering support and encouragement during my studies; and to my wife and daughter, Maria José and Leilani, for their confidence, patience and understanding throughout this period.

With all those that contributed to this study I gratefully share any credits it may receive. Any deficiencies are the sole responsibility of the author.

ABSTRACT

Previous transportation demand studies reveal that, in addition to freight rates, service and commodity characteristics have a significant influence in modal choice and quantity shipped. Most demand models, however, incorporate these characteristics following behavioral and/or quasi-utility arguments rather than a structural form derived from economic theory. One exception is the inventory-theoretic type model which describes transportation demand in a structural manner according to basic tenets of micro-economic and inventory theories.

In this dissertation basic theorems of catastrophe theory were used to re-interpret the transportation problem and develop a generic model which is simpler, but qualitatively equivalent, to the inventory-theoretic model. The resulting model considers simultaneously modal choice and demand, and may be specified for one choice variable (annual demand, order quantity, probability of modal choice, etc.) as determined by any two causal factors in the objective function.

Two versions of this model were prepared: 1) a derived demand model relating quantity shipped by individual modes to cost and revenue parameters; and 2) a choice model, relating the probability of specific choice to direct cost of transportation and mean transit time. This second model was compared with the logit model showing that:

1) The logit model would be likely to misclassify some of the observations; and,

2) Misclassification of a significant number of observations is likely to produce inconsistencies in estimated coefficients, even when the model exhibits a strong (apparent) overall explanatory power. Eventual inconsistencies should be more evident in connection with non-linear variables which affect inventory policy of the shipper, notably transit time.

A subsequent analysis about the effects of product characteristics on demand revealed that perishable products should provide better logit estimations than more durable products, and that a high product value reinforces this effect. A review of published results from previous empirical studies largely confirmed this implication. All estimations involving durable goods provided inconsistent coefficients (insignificant and/or wrong sign) for the variable transit time. The only perishable product under consideration produced highly consistent coefficients, and one study presented inconclusive results due to non-specification of this variable.

Major general conclusions of this study include:

1) The transportation problem may be described through catastrophe theory, and this approach appears to promise a good potential for future modeling efforts in the area. Its main advantages include an ability to describe simultaneously modal choice and demand, and its ability to represent simplified versions of more complex structural models.

2) Inventory considerations do play an important role in transportation decisions and must be considered in empirical investigations.

In particular, linear logit estimations involving non-perishable products are likely to produce unreliable coefficients, which must be used with utmost caution in policy recommendations.

Suggestions for improving future demand estimations included pre-classification and/or pre-treatment of observations prior to their processing through the logit model, modification of this model's formulation, and direct estimation of demand by using the catastrophe theory/inventory-theoretic approach.

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CHAPTER I

INTRODUCTION

In a world of widely separated markets and ever-increasing production specialization, transportation services represent a crucial factor affecting virtually every facet of economic activity. For this reason, a comprehensive understanding of the dynamic forces behind transportation decisions has become an absolute must for management as well as policy-making purposes especially at times, like now, when regulatory/de-regulatory pressures are being almost universally exerted in name of the social good. The extensive volume of literature about the subject, by itself, should reflect the magnitude of the problem and of the ongoing concern with its solution.

However, in spite of the large number of approaches and directions from which this problem has been studied in the recent past, and of considerable advances in the field, no clear scientific consensus has yet emerged about a general methodology to model and estimate freight demand and modal choice simultaneously. This fact alone should be enough justification for further attempts at exploring new (and promising) approaches to a global formulation of the decisory process behind the system. Such an effort constitutes the general objective of this dissertation, which primarily aims to apply catastrophe theory to model and analyze different aspects of demand for freight transportation in a disaggregate framework, that is, at a micro-theoretic level.

Catastrophe theory has recently been received by the scientific world as one of the most promising advances in pure and applied mathematics. Despite its early age, this theory has been of help in explaining phenomena, in different areas of the sciences, which previously could only be described, if at all, by parts instead of as a whole. Most applications of this theory have been made in the areas of theoretical biology, physics and some areas of the social sciences, ranging from brain modeling to an exploratory model of evolution from animal to civilized man (Zeeman, 1976)(Thom, 1975).

In essence, catastrophe theory is a set of differential topology theorems which precisely demonstrate the existing qualitative equivalence among members of individual families of equations. As a consequence, this theory provides the elements for modeling and analyzing complex phenomena by using algebraic and geometric formulations which are simpler, but qualitatively equivalent, to the original forces (energy function) determining an actual system. A following conclusion is that resulting catastrophe theory's pictures of these phenomena may be used not only to expose the global mechanics of the system to a close scrutiny, but also to test quantitative hypotheses about its various aspects, independently of the complexity of the original equation or even of whether it is completely known.

Catastrophe theory applies, in particular, to the description of phenomena which display a discontinuous behavior in response to smooth, continuous causal factors; in other words, to exactly the kind of problem which, although common in nature, has historically resisted the most to attempts of mathematical treatment.

The somewhat exhuberant claims of universality made by early proponents of catastrophe theory have provoked a backlash of criticism against several of its applications as based on faulty theoretical assumptions and reasoning (Zahler and Sussman, 1977). However, these criticisms do not reach the mathematical foundations of the theory itself which, with proper applied theoretical backing, may still offer substantial contributions to future modeling efforts.

In a general way, this dissertation will be geared toward two specific objectives:

1) To prepare an analytical model for transportation demand (and modal choice) using catastrophe theory; and

2) To use this model to draw analytical conclusions about its application to actual demand estimations, and to evaluate its worth in providing additional insights towards more complete understanding of the transportation problem.

We start by reviewing different freight demand models which have been used in previous studies, and by laying down the basic micro-economic foundations behind transportation demand. Both are contained in the next sections. In order to avoid ambiguity in the exposition, the following terms are defined:

Shipper: is the individual or firm ultimately responsible for transportation decisions, which include modal choice and the allocation of different product quantities among available modes or carriers. Shippers may be either buyers or sellers, depending on the circumstances.

Carrier: is an individual firm engaged in the movement of products as a service to shippers.

Transport Demand: is the total quantity of product which is moved in response to parameters such as price differentials between markets, freight rates, service characteristics, etc.

Modal Choice: is the result of a decision process determining which of the available carriers will handle the merchandise. Modal choice and demand are interrelated aspects resulting in modal freight demand.

Direct Cost of Transportation: refers to expenditures made directly in moving merchandise from the source to destination, and includes mainly freight charges (rates), but sometimes also associated costs of packing, insurance, etc.

Inventory Costs: are all costs associated with maintaining inventories, such as depreciation, obsolescence, interest rate, storage facilities, etc.; and include costs of in-transit, working, and/or safety inventories.

Review of Literature on Demand Models

A variety of approaches have been used in the past, with differing degrees of success, for estimating transportation demand and modal choice. Some models, for example, follow a disaggregate approach, while others work with aggregate data. Model formulation, also, follows a wide range of principles and/or theories such as market-share analysis, gravity concepts, spatial-equilibrium, theory of the firm, or may even be based mostly on intuitive reasoning.

Estimation techniques also vary tremendously among authors, going from multiple regression to linear programming, input-output analysis, and choice models such as logit, probit, and discriminant analysis.

Because of the extensiveness of the literature and specific goals of this study, the following review will be considerably less than exhaustive. More comprehensive reviews may be found in Daughety et al. (1976), and Smith (1974).

In a general sense, freight demand models may be roughly classified in two types: 1) Macro-models: those in which aggregate data were used for estimation or which were developed without a microeconomic structural foundation; and 2) Micro-models: those that are based on a micro-theoretic structure such as the theory of the firm and/or use data for estimation originating at the individual shipper's level.

Macro-models

Macro-models tend to conform to three basic classes:

1) Input-Output: models based on Leontief's approach to the interdependence of production systems. These models may be used for determining interregional flows of commodities, and hence to finding transport demand. Examples of works using this type of approach are Moses (1955) and Polenske (1966, 1967). Standard criticism of these models are related to the technique itself: it assumes constant returns to scale for the industry as a whole, and the (assumed) stability of the coefficients in general precludes considerations of technical improvements in the industry.

2) Gravity: these models have shown special appeal to geographers and economists alike. Their attractiveness lies in that they actually predict flows. On the other hand, these models display a serious shortcoming because, by being constructed without a structural link with some fundamental economic theory, they are generally unresponsive to micro-economic parameters such as prices, service and commodity characteristics. Examples of these models are found in Perle (1964), Mathematica I and II (1967), and Little (1974). Basic rationale for gravity models is provided in Wilson (1967, 1968, 1969).

3) Cost-Minimizing: these models generally involve linear programming techniques or other models using linear constraints on flows. Examples are O'Sullivan (1972, 1974), and Kresge and Roberts (1971). Evaluations of linear programming as compared to gravity models may be found in Mera (1971) and O'Sullivan, and demonstrate that the former approach is generally superior to the latter.

Macro-models using a methodology generally associated with analysis of disaggregated data were developed by Miklius (1969) and Kullman (1973). The first author used discriminant analysis for estimating traffic allocation between truck and rail using data from the 1963 Census of Transportation, and concluded that this approach was better than linear regression for the particular situation under study.

Kullman applied a logit model to aggregate data in order to determine shipper's behavior with respect to modal choice. In view of the statistically significant results obtained in several runs, the study indicated a favorable potential for logit models in modal-choice modeling.

A fundamental criticism of macro-models is that in general they are not based on a structural form stemming from the theory of the firm or other theory-based approach. As a consequence, model formulation adheres more closely to the characteristics of the *technique* in use rather than to the theoretical formulation of the underlying demand determinants. Another weakness is that aggregate models generally treat the transportation sector as providing homogeneous services at constant rates. Both problems may lead to unrealistic results which cannot be verified against more established theoretic principles which describe economic behavior.

Micro-models

Micro-models also have traditionally followed a variety of approaches and techniques. Some of them have been based on sound foundations of economic theory while others have applied sophisticated statistical techniques, like logit and discriminant analysis, without much attention to structural relationships among variables. In this sense, the latter are as arbitrary as some of the aggregate models and subject to the same criticisms, even considering the reasonable degree of success in predictive power achieved in particular cases by some of the authors.

Beuthe (1970) prepared a deterministic model to deal simultaneously with mode and destination choice for profit-maximizing shippers, and used discriminant analysis for estimating modal choice for corn shipments in Illinois with disaggregated data.

Stucker (1970) attempted to consider quantity shipped and choice for three modes--rail, truck, and barge. The author defined total transport costs as including both freight rates and associated costs, and performed regressions with data from the 1963 U.S. Census of Transportation.

Allen and Moses (1968) have studied demand for air freight using data on commodities traded in the North Atlantic Route (U.S. and Great Britain). Their theoretical model considered a profit-maximizing firm whose total cost of production included transportation costs. A discriminant model was used for estimating modal choice, which was compared with a regression model. The latter provided poorer results, and this was explained as being a consequence of exclusive modal choice in a majority of the observations. For this reason, the authors proposed a compromise approach, which involved estimating exclusive choice observations by discriminant analysis, the remainder by regression, and drawing together the results of both stages.

Another study using discriminant analysis was put forth by Antle and Haynes (1971), and was based on the works of Allen, Beuthe, Stucker, and Herendeen (1969). Their model developed a demand curve for barge transport of coal in the Upper Ohio River Valley, as compared with rail movements of the same product.

An important article comparing alternative procedures for estimating modal choice was written by Watson (1974). In this study, the author considered four techniques: multiple regression, probit, logit, and discriminant analysis. The results indicated that all four alternatives yielded significant results when applied to disaggregated data. Overall, the most important conclusion of the study was that the logit model presented the best performance, and therefore showed the best potential, for future estimations. Shortcomings associated with the remaining methods were: 1) regression analysis allows for the linear probability function to predict choices outside the unit interval, thus destroying the probability interpretation; 2) discriminant analysis resulted in higher misclassification errors than the other methods; and 3) the probit model requires complex calculations and resulting coefficients are not as easily interpreted as in the logit.

Posterior studies using the logit model are Watson, Hartwig and Linton (1974), Johnson (1976), Miklius, Casavant and Garrod (1976), and Daughety and Inaba (1977).

Daughety and Inaba (1976, three papers)(1977, three papers) have developed a theoretical model based on Samuelson (1952, 1965) and Takayama and Judge (1971). In this model, a spatial price equilibrium framework relates goods and transport sectors, and both shippers and carriers maximize profits. Demand functions include both commodity and service characteristics, and allow for multiple modes and markets. This model has been used to estimate freight demand for grain movements to markets in the Midwest U.S.A. by using alternative formulations of the logit model.

Watson, Hartwig, and Linton used the logit model in demand estimation for large household appliance shipments in inter-city movements. Johnson used the same method on data from grain elevators in Michigan; and Miklius et al. estimated demand for transportation of agricultural commodities (apples and cherries) in the Northwest. The four preceding studies have applied the logit model to disaggregated data and will be described in more detail in Chapter IV.

Inventory-Theoretic Models

A special type of demand models which incorporates not only prices, service and commodity characteristics but also associated inventory costs is the "inventory-theoretic" model, which will be the object of a more detailed discussion in this section.

The use of inventory-related variables in determining modal choice and freight demand goes back to Meyer (1960), and was formally incorporated as an "inventory-theoretic" model in the classic work of Baumol and Vinod (1970). This model became the starting point for most models of the kind. Examples are Townsend (1969), Harvard Business School Model (1970), and Roberts (1971). Although not aiming directly at determining freight demand, models along the same line for choosing transport services were constructed by Constable (1972) and Das (1974, 1975).

At the center of the conceptual argument for considering inventory-related variables in demand models is the fact that firms typically operate in an economic environment over which they have, individually, very little control. These firms are subject to uncertainties and discontinuities in both demand and supply functions facing them, and must, therefore, keep inventories in order to provide their customers with a satisfactory (continuous) level of service. In this sense, decisions such as the purchasing and transporting of merchandise may be viewed as an effort on the part of a particular firm to "equilibrate" its inventories around a level and composition previously decided upon.

The nature of the inventory policy-transportation demand relationship is quite complex, as inventory policies not only determine demand and modal choice, but also are affected in return by commodity and service characteristics of different transport systems. Once a purchase is made, and the merchandise delivered to the carrier, this merchandise may justifiably be considered "inventory-in-transit" and, as such, become an integral part of the buyer's inventory policy. Consequently, costs induced by service characteristics such as transit time and dependability may be seen as inducers of inventory carrying costs which purchasers will attempt to minimize. Who actually pays, buyer or seller, for these costs is irrelevant to the argument, once there are mechanisms which usually transfer these costs to successive buyers.

Freight rates in general are supposed to reflect, to a degree, service quality of individual modes. A slower mode, for example, should provide a lower rate, other things being equal. But from a managerial standpoint, the decision of using a slower mode rather than a faster one will be made by comparing additional inventory costs associated with this choice with the lower fare, which may or may not compensate for each other. In other words, freight rates do not necessarily reflect the *value* placed on service characteristics of a given mode to individuals using it; and therefore freight rates constitute *only one* of several variables to be considered in demand models.

Modal characteristics affect inventory costs in different ways. Long transit-times, for example, increase the required level of working stock. Physical reliability of the mode determines the amount of losses due to damage, theft, and decay, and hence the size of safety inventories. Variations in lead-time due to equipment unavailability, poor scheduling or any other reason also require a larger safety stock. It is evident that the relative importance of any of these factors depends, in addition, on product characteristics such as perishability, weight, vulnerability to damage, and value.

In summary, the preceding discussion suggests that businessmen make shipping decisions simultaneously with inventory decisions. This fact constitutes the basis for formulating inventory-theoretic models of transportation demand based jointly on the theory of the firm and inventory theory. In other words, inventory-theoretic models "derive most of their structure from standard inventory-theory which encompasses, after some modifications, all the basic problems with which the analysis must deal."[1]

In general, inventory-theoretic models are "abstract-mode-abstract-commodity" models, once they describe modes and commodities by a set of measurable attributes. Modes, for example, are described in terms of their speed, reliability, rates, etc., while commodities are represented by qualities such as perishability, weight, volume, value, etc. A slow train, therefore, may rightfully be considered a different

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[1] W.J. Baumol and H.D. Vinod. "An Inventory-Theoretic Model of Freight Transport Demand." Management Science, Vol. 16, No. 7, March 1970. p. 413.

mode from a fast train for analytical purposes. The main advantage of this "abstraction" is that, in principle, the model is valid for all modes and products.

The "abstract mode" approach to modal selection may be demonstrated as in Figure 1.1, which shows an indifference map considering two modal characteristics of importance to the shipper, namely economy and speed. The two indifference curves drawn in this figure (II' and

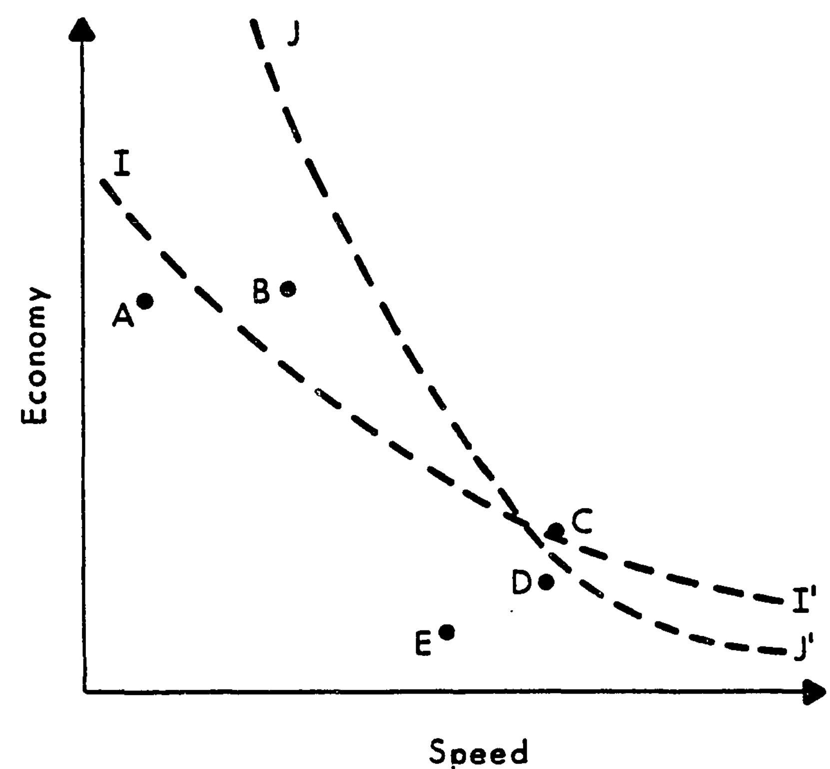

Figure 1.1 -- Illustration of Modal Choice Process through Indifference Curves and Modal Characteristics

Source: W.J. Baumol and H.D. Vinod, "An Inventory-Theoretic Model of Freight Transport Demand," Management Science, Vol. 16, No. 7, March 1970. Figure 1, p. 416.

JJ') may represent iso-cost curves representing either two shippers and one product or one shipper and two products. In any case, the curves indicate point combinations of economy and speed for which the shipper will be indifferent between one or the other quality of the modes. The shape of these indifference curves depends on several factors, such as cost structure of the shipper's operation, product characteristics, or even management's perception about the business opportunities open to their firm at some point in time. Points A, B, C, D, E represent modal "performance" in terms of economy and speed for individual modes.

Shipper's modal choice, in Figure 1.1, is determined by observing the relation between these points and the indifference curves. Modes A, E and D, for example, are less preferable than either B or C. By the same token, it is obvious that mode B is preferable to C for the alternative shown in curve II', while C would be chosen in the situation described by indifference curve JJ'.

Baumol and Vinod's basic formulation of the model included four types of transportation costs: direct shipping cost, total in-transit carrying cost, ordering cost, and consignee's inventory carrying cost. An expanded and modified version of this model is described in Appendix A. Some of the alterations introduced in this model were suggested in the works of Constable (1972) and Das (1974, 1975), and resulted in a reasonably complete description of transportation demand determinants.

The main attractiveness of inventory-theoretic models lies in their completeness in terms of including the most important variables intervening in the process, and theoretical soundness, stemming from their structural formulation of these variables according to basic tenets of economic theory. Their chief limitation as estimation models derives from the large amount of required information, which renders data collection a costly and enervating exercise, and therefore impractical in many cases.

Transportation Demand

This section aims to describe briefly transportation demand in a micro-economic context in order to provide a uniform conceptual foundation for the ensuing discussion.

Joint Demand for Goods and Services

Consumer demand for a certain good is, in effect, a joint demand for that particular good plus demand for a host of associated services such as packing, advertising, transportation, storage and distribution. Although these services are usually provided independently of the product itself, they form with the latter a "package" which enables consumers to purchase the product under the form and at the place of demand. Demand for a service like transportation, therefore, is a *derived demand*, that is, "it is dependent upon and originates from the demand of the product being transported."[2]

As a consequence, demand for freight, unlike that for passenger transportation, depends on the profit function of the shipper rather

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[2] Roy J. Sampson and Martin T. Farris, Domestic Transportation: Practice, Theory and Policy (Boston: Houghton Mifflin, 1966), p. 146.

than in the utility schedule of the passenger, assuming that products are transported to be sold and to make a profit. Therefore, the linking factor between product and freight demand is the shipper's net revenue (profit) function, which may be defined as total revenue minus total cost of production, including those of transportation. This situation may be exemplified by Figure 1.2 below. In a particular market a firm will sell a quantity which equates the demand and supply prices of the product. Considering that the supply curve includes the cost of producing *and transporting* the product, supply and demand for the product will define a residual demand for transportation as a function of transportation costs. The difference

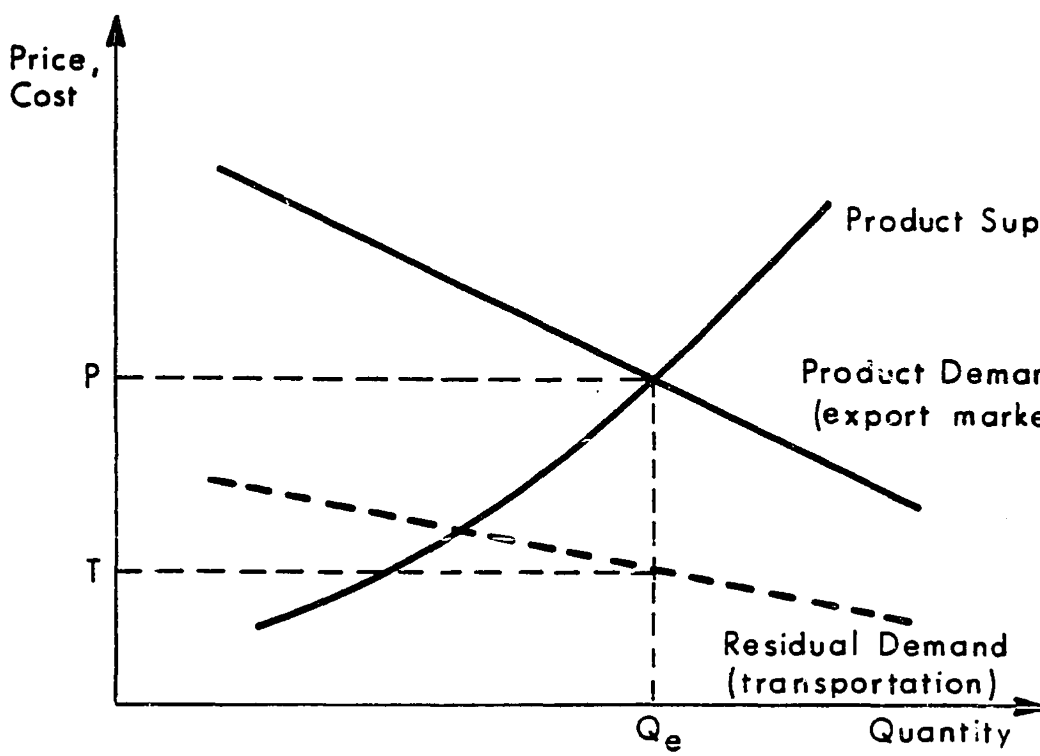


Figure 1.2 -- Product Supply and Demand Equilibrium and Residual Demand for Transportation

between product market price (P) and transportation cost (T) represents the net price of the product in the destination market, which must cover the production cost of the shipper if the good is to move at all to that market.

The Demand Function

From the knowledge that freight demand is a derived demand dependent on the shipper's profit function, the former may be defined as follows:

The profit function may be specified:

$$\pi = (PQ - TQ) - f(Q) \tag{1.1}$$

where P : product price; T : transportation cost; Q : quantity sold (shipped); π : profit; and $f(Q) = \sum_{i=1}^{n} r_i q_i$: cost of production as a function of Q .

In order to maximize profit a derivative of π with respect to Q is taken and made equal to zero. Assuming that second-order requirements are met, we have:

$$\frac{d\pi}{dQ} = (P - T) - f'(Q) = 0 \tag{1.2}$$

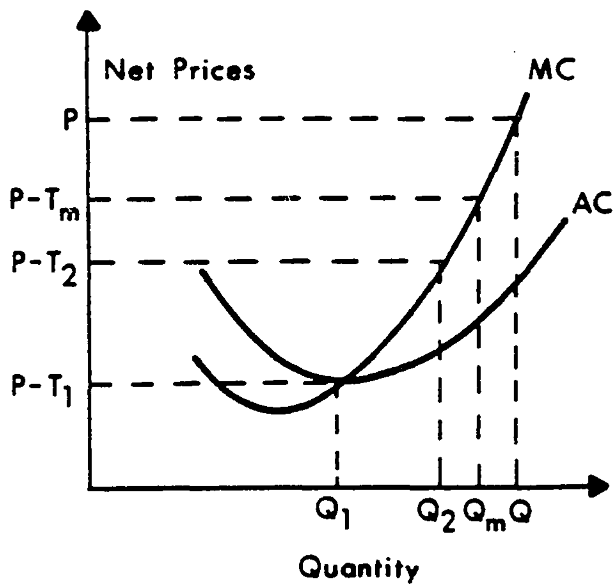
and

$$f'(Q) = P - T \tag{1.3}$$

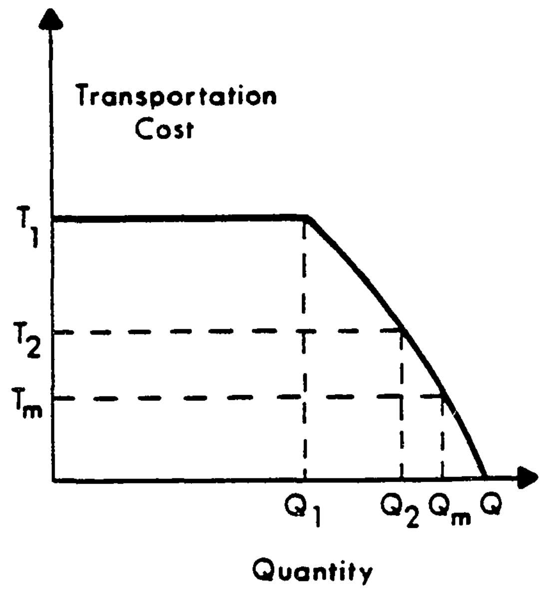
which is the conventional expression for marginal cost equating marginal revenue which characterizes profit maximization. Equation (1.3), when made explicit for Q will result in a freight demand schedule, as exemplified in Figures 1.3(a) and (b) on the following page.

These figures show how the marginal and average cost curves of the shipper define transportation demand as a function of transportation cost. In Figure 1.3(a) shippers' marginal and average costs are represented by MC and AC, and $P - T_1$, $P - T_2$, $P - T_m$ represent different net prices this shipper would receive if transportation costs were, respectively, T_1, T_2, T_m $(T_1 > T_2 > T_m)$. If these costs were, for example, equal to T_m, net price will be $P - T_m$ and quantity shipped equal to Q_m. If the cost increases to T_2, quantity shipped will decrease to Q_2 and so on. However, if cost of transportation becomes higher than T_1, then net price will drop below average cost, and no merchandise will be shipped. Figure 1.3(b) shows a schedule of different quantities shipped as a response to corresponding transportation costs as defined by the curves in Figure 1.3(a). This is the demand curve for transportation.

The demand equation specified through equation (1.3) may be modified to accommodate additional features of the underlying functions. First, "cost of production" in a spatial context may be interpreted as an "opportunity cost" of selling the merchandise locally as compared to the export market under consideration. To allow for this interpretation, $f'(Q)$ may be re-labeled P_L and P re-named P_E, meaning prices prevailing in the local and export markets, respectively. Movement of goods may be expected to occur as long as the price differential between markets is greater than transportation cost, and maximum profit will occur at the point where $\Delta P = T$,



(a) Shipper's Marginal and Average Cost Curves



(b) Shipper's Demand Curve

Figure 1.3 -- Determination of Transportation Demand Schedule

Source: W. Bruce Allen and Leon N. Moses. "Choice of Mode in U.S. Overseas Trade: A Study of Air Cargo Demand" in Papers, T.R.F., September 1968. Diagrams 1-a and 1-b, p. 237.

as shown in equation (1.4).

$$f'(Q) = P - T \quad \therefore \quad P_E = P_L - T \quad \therefore \tag{1.3}$$

$$(P_E - P_L) = \Delta P = T \tag{1.4}$$

The second, and perhaps more important, modification involves expanding the variable T (in equation (1.4)) into a more complex expression describing the structure of transportation costs, which include not only direct transportation costs but also associated inventory costs. This may be accomplished, in generic terms, by making T a function of Q in equation (1.1). Following the same procedure will result in equation (1.5). Then, by

$$\Delta P = T(Q) + QT'(Q) \tag{1.5}$$

making this expression explicit for Q , we have a freight demand equation for profit-maximizing shippers, as a function of product prices and transportation costs--direct and indirect.

A final comment is necessary about the *shape* of the total transportation cost curve. Joint inventory-transportation costs have increasing and diminishing components. Therefore it is possible, if these components are measurable, to find a point which will yield a minimum total cost--or, depending on the total revenue function, maximum profit. In other words, a typical transportation cost curve is downward convex, as shown in <u>Figure 1.4</u> on the following page. This point has been shown formally in textbooks of operations analysis

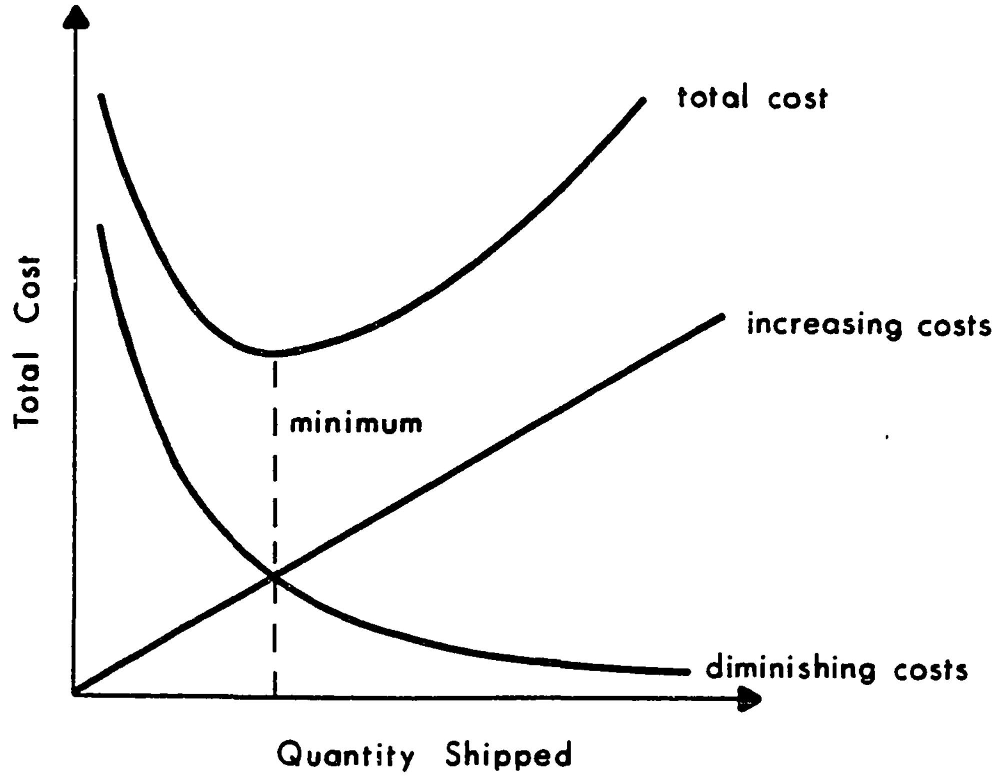


Figure 1.4 -- Joint Transport-Inventory Costs by Components

and inventory systems.[3]

It is important to notice also that in this section transportation systems were presented as offering, on the whole, a uniform service similar to a situation where there is only one mode available. Although the economic principles involved remain the same, the existence of additional modes introduces additional complications to the analysis, because modal choice decisions will have to be made simultaneously

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[3] See, for example: G. Hadley and T.M. Whitin, Analysis of Inventory Systems (Englewood Cliffs, N.J.: Prentice-Hall, 1963), p. 188.

with quantity decisions. This problem will be addressed in future chapters.

Conclusion and Outline of Subsequent Chapters

This chapter was oriented towards providing background for the following analysis. For this reason, the general purposes of the study were first outlined, and a brief review of literature on the subject was performed. In addition, basic micro-economic elements involved in transportation demand were described in general terms so as to provide the theoretical foundations for subsequent modeling efforts. These elements provided the basis for formulating the inventory-theoretic model described in Appendix A, which will constitute the guideline for the proposed re-interpretation of transportation demand through catastrophe theory.

Chapter II describes the essentials of catastrophe theory in general and the cusp castastrophe in particular.

Chapter III applies the elements of preceding discussions to the formulation of two demand models based on the catastrophe theory-inventory theoretic approach. The first model presents a global picture of the transportation problem, including demand and modal choice. The second aims at describing modal choice in the same terms as the most common specification of the logit model, in order to permit a subsequent analytical comparison between the two approaches. Implications stemming from this comparison provide the rationale for an attempt to explain some inconsistencies found in previous studies which used the logit model on disaggregated data.

In Chapter IV the conditions for occurrence of inconsistent results were explained with help of the proposed model, and then a review was made of empirical estimations published by different authors in order to verify whether those conditions were associated with obtained inconsistent results. Conclusions and implications from available evidence were then utilized in suggesting possible areas of improvement for future demand estimations.

The last chapter of the dissertation outlines a summary of the study and enumerates the conclusions which were reached about the use of catastrophe theory in transportation studies; and finally offers some comments on new research directions applying this theory to other fields in the general area of economics.

Appendix A contains a reasonably complete inventory-theoretic model of freight demand, which is used as a guideline for model-building in this dissertation; and Appendix B presents an exploratory study about the effects which may be inferred from conclusions and implications of the present study for future transport demand estimation efforts in the State of Hawaii.

CHAPTER II

CATASTROPHE THEORY AND THE CUSP CATASTROPHE

General Review of Catastrophe Theory

The discontinuous nature of the behavior of some systems has been a difficult problem to comprehend and to model for many scientists. Originally conceived and developed by the French mathematician René Thom, catastrophe theory brought considerable help for approaching this type of problem in a scientifically accurate manner.

Being a purely mathematical theory, for other fields of the sciences catastrophe theory represents only a framework for describing, and a methodology for analyzing, particular classes of previously hard-to-treat problems. In this sense, the contributions of this theory are similar to those of other mathematical theories, like the theory of differential equations, which have found ample application in other sciences, such as in optimization problems in economics and business.

Theoretical foundations and basic philosophy behind catastrophe theory, as well as some areas of application, may be found in the English version of Thom's seminal book "Structural Stability and Morphogenesis" (1975). Mathematical introduction to the subject may be found in a didactic form in Lu (1976) and Poston and Stewart (1976). Zeeman's "Selected Papers" (1977) offers a widely diversified set of articles, ranging from a non-mathematical introduction to several applications, including proofs of basic theorems.

This chapter's description of catastrophe theory is by no means comprehensive, since it is limited to a sketch of the fundamental concepts and formulations necessary for following the ensuing discussion. Additional information is available in previously cited works as well as in the Bibliography.

Divergent Phenomena and Catastrophic Change

A common characteristic of events in many areas of natural and behavioral sciences is the phenomenon of divergence. It is not unusual to observe, in these areas, circumstances where very small differences in causal factors are accountable for large dichotomies in behavior. One conceivable example, well known to marketing experts, is that a nearly insignificant price difference between two closely substitutable products in a supermarket shelf may result in disproportionately high sales for the cheaper product, even though satisfaction and prices associated with both are virtually the same for consumers. Another example is that two very similar sets of circumstances may provoke an animal to either attack or to flee, depending on its prior mood. A third is that "sharp divisions of opinion can emerge in a population even though the opinion of each individual may have evolved gradually and smoothly."[1] All these are examples

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[1] C.A. Isnard and E.C. Zeeman. "Some Models from Catastrophe Theory in the Social Sciences" in Catastrophe Theory: Selected Papers. Addison-Wesley: Reading, Mass. (1977). p. 304.

of divergent phenomena.

Divergence may obtain for variables other than time, and brings about the associated concept of "catastrophic change"; that is, in divergent phenomena there must be points where small changes in causal variables may be responsible for sudden "jumps" in behavior. Divergence and behavior discontinuity are not limited to biological or social sciences: in physics, "shock waves" is a phenomenon of virtual universality, and it may be described as sudden changes in a physical system from one stable state to another, usually caused by relatively continuous factors.

Catastrophe theory's manner of describing the dynamics behind divergent phenomena is to consider that:

> "Many phenomena may be thought of as governed by a potential function of some form. The stable states of the system, i.e. those states which are actually observed to occur, may then be regarded as states for which some (potential) function is minimized.... If a function has multiple minima, then more than one stable state may be accessible to the system. Changing the control parameters in an experiment may alter the form of the governing potential in such a way as to change positions, relative heights, or even total number of local minima. Thus, the stable states accessible to the system may change in a discontinuous way as controls change smoothly. Observed discontinuous changes of state have been called 'catastrophes.'"[2]

A more specific view of the same approach is:

> "Suppose, in general, a system can be described by internal variables, x, and external (perhaps control) variables, a, together with an energy function of these variables, $E(x,a)$. Then for a given a the possible equilibrium values of x are the minima of E, and hence will be solutions of
>
> $$\frac{\partial E}{\partial x} = 0 \qquad (1)$$

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[2] Yung-Chen Lu. Singularity Theory and an Introduction to Catastrophe Theory. Springer-Verlag: New York, 1977. pp. 95/96.

> As a varies, the values of x, which are the minima given by equation (1), determine the surface in (x,a) space. Commonly, such surfaces are single sheets: that is, there is a unique x for each given a. However, there may also be values of a which give rise to multiple values of x, in which case the surface is folded. Catastrophe Theory takes its name from the sudden jumps which can occur from one minimum of E, or equilibrium value of x, to another as a changes smoothly near the boundary of some critical region."[3,4]

This approach to phenomena description, which is intuitively applicable to most systems, does not produce great insight by itself: energy functions behind a great many of them are unknown or not precisely defined. Others are so complex, involving so many variables and dimensions that their internal dynamics are completely beyond scientific descriptive and computational abilities, in some cases even of significant comprehension. On the other hand, while some studies aim to describe and analyze a system starting from a (known, but complicated) energy function, others aim exactly at the opposite: to find out more about the underlying dynamics causing an observable behavior. Catastrophe theory offers the elements for overcoming these difficulties in both types of problems through the enunciation of the "qualitative equivalence" concept deriving from Thom's Classification Theorem.

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[3] A.G. Wilson. "Catastrophe Theory and Urban Modelling: An Application to Modal Choice." Environment and Planning A, Vol. 8, 1976.

[4] Both Wilson and Lu's use of the term "minimum (a)" may be substituted, without loss, for "optimum (a)."

Qualitative Equivalence and the Classification Theorem

The essence of catastrophe theory rests with the idea of qualitative equivalence within families of equations, defined as a precise (qualitative) mathematical concept by the Classification Theorem.

A simplified version of the Classification Theorem (for the cusp catastrophe) may be enunciated:

> "Let C be a 2-dimensional control (or parameter) space, let X be a 1-dimensional behavior (or state) space, and let f be a smooth generic function on X parametrized by C. Let M be the set of stationary values of f (given by $\partial f / \partial x = 0$, where x is a coordinate for X). Then M is a smooth surface in C$\times x$, and the only singularities of the projection of M onto C are fold curves and cusp-catastrophes."[5,6]

In effect, what the theorem asserts is that if characteristics of a system fit those of the cusp catastrophe, this system may be described accurately (in a qualitative sense) by the graph and canonical formula of this catastrophe, *independently of the quantitative properties of the function f determining behavior*. In other words, the cusp is the most complicated thing that can happen locally. Other qualitative features of the cusp catastrophe, such as the fold curve, bimodality, catastrophe, hysteresis, divergence, and inaccessibility may be inferred in the model because equivalence will preserve them all.

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[5] E.C. Zeeman. "Catastrophe Theory: Draft for a Scientific American Article in Selected Papers. Addison-Wesley: Reading, Mass. (1977), p. 23.

[6] Definitions and implications of the terms "smooth," "generic," "equivalence," and "singularity" are offered in the same paper, pp. 23/25.

The major implication of this theorem is that a catastrophe theory model of a conforming phenomenon is strictly equivalent qualitatively to the original dynamics of the phenomenon, and therefore may be used to model and quantitatively test hypotheses about the system. This statement is helpful when one considers that f may assume a variety of forms, and x a dimension anywhere from 1 to n, while the same simple model still would implicitly apply.

In what probably amounts to a somewhat overconfident statement about the applicability of catastrophe theory, Isnard and Zeeman argue that qualitative equivalence may be used in describing and modeling different phenomena because:

> "The statement itself is a synthesis of many ideas, with the following aspects:
>
> Profundity due to the mathematical uniqueness and stability, depending on deep theorems.
>
> Universality. In any aspect of nature, or any scientific experiment, where two factors influence behavior, where splitting and discontinuous effects are observed, and where smooth genericity may be assumed, the graph must contain a cusp catastrophe.
>
> Insight. From the model one can explain, predict, and relate a variety of phenomena that previously may not have appeared to be related."[7]

Elementary Catastrophes

By using the Classification Theorem and associated principles, Thom has classified the ways discontinuities can occur in seven

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[7] Isnard and Zeeman, op. cit., p. 333.

"elementary catastrophes," which may be used in modeling different problems according to their peculiarities. It should be noticed that higher dimensional catastrophes are "always made up of lower dimensional ones, together with a new singularity at the origin."[8] Each of these catastrophes has a particular geometry, and their standard canonical formulas are shown in Table 2.1 below. Parameters of C are denoted by a , b , c , d ; and x , y are variables of X .

Table 2.1 -- Standard Formulations and Dimensions of the Seven Elementary Catastrophes

Name		Dimensions of X	Dimensions of C	Function f
Fold		1	1	$\frac{1}{3}x^3 - ax$
Cusp		1	2	$\frac{1}{4}x^4 - ax - \frac{1}{2}bx^2$
Swallowtail		1	3	$\frac{1}{5}x^5 - ax - \frac{1}{2}bx^2 - \frac{1}{3}cx^3$
Butterfly		1	4	$\frac{1}{6}x^6 - ax - \frac{1}{2}bx^2 - \frac{1}{3}cx^3 - \frac{1}{4}dx^4$
Umbilics	Hyperbolic	2	3	$x^3 + y^2 + ax + by + cxy$
	Elliptic	2	3	$x^3 - xy^2 + ax + by + c(x^2 + y^2)$
	Parabolic	2	4	$x^2y + y^4 + ax + by + cx^2 + dy^2$

Source: E.C. Zeeman. "Catastrophe Theory: A Draft for a Scientific American Article." Catastrophe Theory: Selected Papers (1972-1977). Addison-Wesley: Reading, Mass., 1977. Table 3, p. 27.

[8] E.C. Zeeman, op. cit., p. 25.

Applications of Catastrophe Theory in Economics

In spite of an obvious potential for applications in some areas, the use of catastrophe theory in economics has not kept pace with its applications in other sciences such as physics, biology, and even other areas of the social sciences (see Thom (1975), Zeeman (1977)).

Isnard and Zeeman (1976) are due credit for an introductory paper on the use of catastrophe theory in social sciences in general. Although few articles have gone so far as to actual estimations, modeling efforts in economics include applications to development strategies (Ribeill, 1975), the behavior of stock exchanges (Zeeman, 1974), equilibrium analysis of discontinuous consumer choice (Brown, 1977), consumer attitude studies (Chidley, 1976), study of business cycles (Varian, 1978), and Pareto optimum (Smale, 1973). Articles on applications of catastrophe theory to urban growth modeling were prepared by Amson (1972, 1975), Mees (1975), and Wilson (1976). The latter study is an exploratory article on the possibility of using catastrophe theory in travel modal choice for urban modeling. A critique on "claims and accomplishments of applied catastrophe theory" was made by Zahler and Sussman (1977).

The Cusp Catastrophe

Most applications of catastrophe theory have used the cusp catastrophe. For describing the transportation problem in a bimodal context the same catastrophe is applicable, and therefore will be described in more detail in this section. This catastrophe applies to situations where the energy function causes a bimodal nature in some observable behavior, and describes this behavior in response to two causal factors, independently of the fact that the objective function may require its quantitative representation through a polynomial equation of higher degree.

Energy Function and Decision Rule

In order to describe the cusp catastrophe it is convenient to start at the energy function determining a bimodal behavior. This function may be defined alternatively as a structural equation or a probability distribution, depending on the particular case. The function itself may not be completely defined (or known), but at least some of its parameters must be known, and it must be possible to assume that the relationship between the energy function's variables and behavior is one of cause and effect. This situation may be depicted as in Figure 2.1, which relates some function $E(x,a)$ to behavior x .

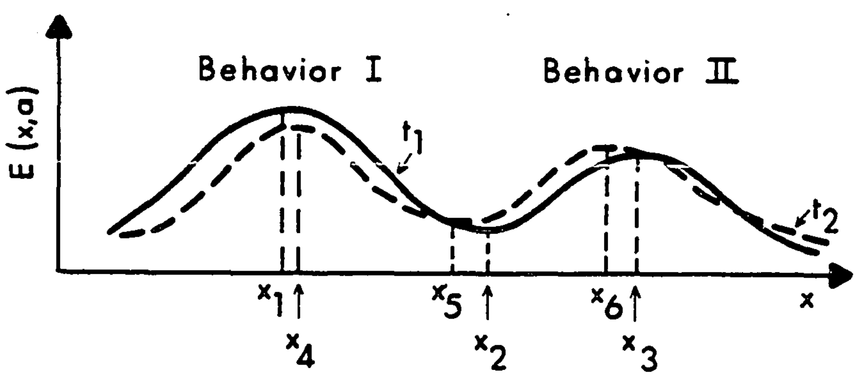

Figure 2.1 -- Relation Between an Energy Function and a Bimodal Behavior

A criterion that implies in maximization of this objective function would result, in static sense (curve t_1), in the choice of Behavior I at point x_1 , which is the global maximum for the function. However, function $E(x,a)$ may be expected to change with the evolution of its external parameters a , say, as in the interrupted line in the same figure, denoting the situation in t_2 . In this case, maximizing criterion would still determine Behavior I, because the "peak" at x_4 is still higher than at x_6 . But in the behavior scale, the position would have changed smoothly to x_4 .

It is conceivable, though, that if smooth changes in parameters continue in the same direction, the "peak" related to Behavior II would be higher than that of Behavior I in t_3 , and then the same maximization criterion would determine a *sudden jump* from Behavior I to Behavior II, that is, from position x_4 to position x_9 in Figure 2.2 below. This would characterize a "catastrophic change"

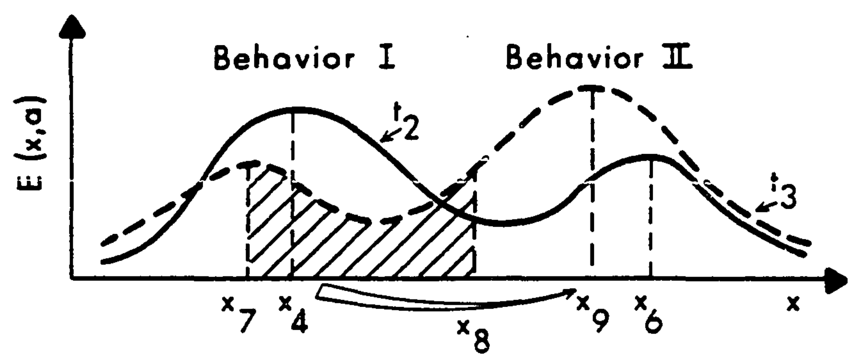

Figure 2.2 -- Catastrophic Change from Behavior I to Behavior II

in behavior, and means that some "borderline" values of a have been crossed, and this was done because no other value of x would provide a global maximum for E in Behavior I. In addition, since this graph relates particular values of E to point values in the behavior scale, it may be shown that there is a range of x values which denotes an area of irrational behavior, such as from x_7 to x_8 . This is so because any values of x between these two points provide the same value of E as corresponding points below x_7 , which require lower values in the parameters of a .

In a situation as that shown in Figure 2.2, it is important to ask a fundamental question: when relative changes in the objective function evolve in the direction of the behavior not presently in use, at which point a decision is made to switch to the alternative behavior? This is essentially a policy decision which depends exclusively on the type of problem being analyzed, and must be justified in terms of hypotheses pertaining to the problem itself. However, catastrophe theory provides alternative criteria for such a choice of rule, and they lead to slightly different models. One is that as soon as the alternative "peak" reaches the same height as the one in use there is an immediate switch to the new behavior (Maxwell Convention). Another is that this shift is delayed for as long as possible, or until the emerging alternative's "advantage," in terms of E , exceeds some pre-determined value (Delay Rules).

Since most behavior shifts studied in the social sciences entail a certain amount of effort (cost, information, or risk-taking), some form of the Delay Rule is usually applicable in this area. In economics, in particular, it is to be expected that behavior will be determined by some form of differential equation (optimization), and Isnard and Zeeman (1977) report that:

> "In fact, whenever the behavior is determined by a differential equation (such as $dx/dt = \partial P/\partial x$) the Delay Rule is a theorem, and so holds automatically (Zeeman, 1973)."

A typical picture of the cusp catastrophe is presented in Figure 2.3. This graph is obtained by maximizing E with regard to behavior x as a function of two variables a and b (parameters, external variables, or control factors), that is, by equating dE/dx to zero. Description of basic characteristics and canonical model of this catastrophe is in following sections. Because of the Classification Theorem, these geometrical and algebraic characteristics are shared by both the canonical (qualitative) model from which this figure was drawn and the underlying (quantitative) model determining the actual system.

Control Space

The control space (C) is shown as the lower portion of the graph in Figure 2.3 and is delineated by perpendicular axes representing the two control factors a and b. These factors are components of the energy function, and therefore each point of the control space is associated with a point in the behavior space.

In this graph, a and b are represented as "normal" and "splitting" factors, respectively. A "normal" factor is defined as any

[9] Isnard and Zeeman, op. cit., p. 309.

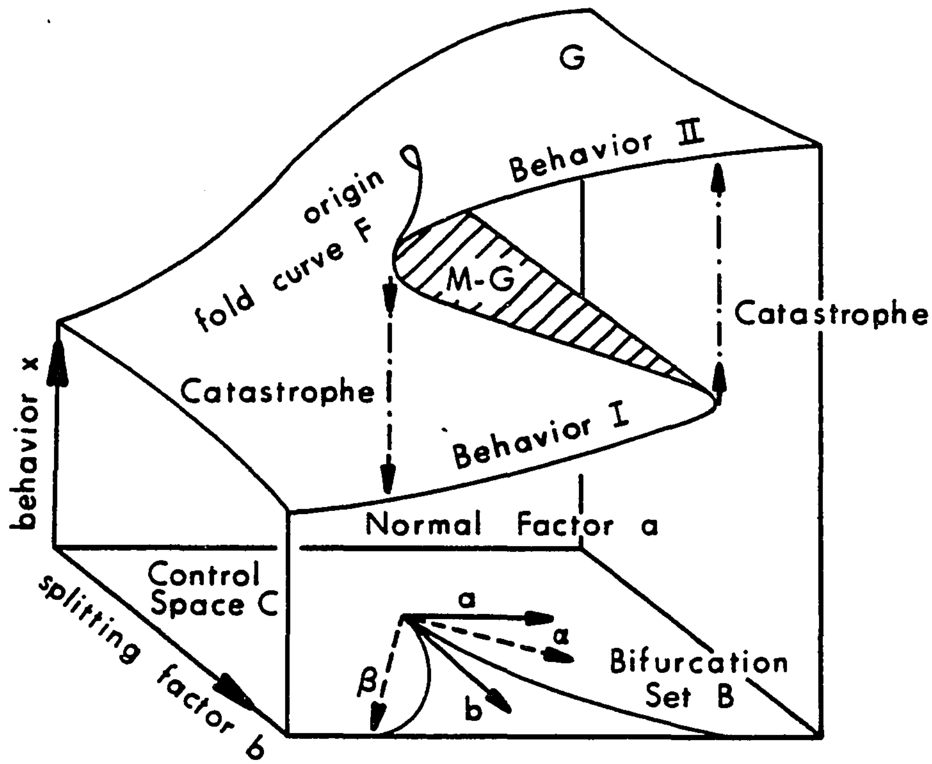

Figure 2.3 -- The Cusp Catastrophe

transverse direction, oriented towards $a > 0$, which will result in a behavior to be smoothly increasing in case of $b < 0$, and to split if $b > 0$. A "splitting" factor is so termed because it is what causes the energy function to produce a divergent behavior, which results in the fold curve F in the behavior space and therefore to catastrophic changes. Essentially, "normal" and "splitting" factors are "competing" for influence in the energy function, but do not "conflict" with each other.

Many problems are not amenable to being described in terms of "normal" and "splitting" factors, because their control factors do not fit these characteristics. Instead, in some cases a better description may be obtained by defining the control factors as "conflicting" factors, that is, factors which display "opposite" forces within the objective function. In the case of "conflicting" factors the splitting is not associated with one of the control variables, but with the joint action of both. A basic characteristic of this type of factor is that while one of them "pushes" behavior in one direction, the other's influence is in exactly the opposite direction. One example of "conflicting" factors is "cost" and "revenue" in a profit function, and they are represented in the cusp catastrophe's graph as two axes passing by the external sides of the "bifurcation set" B , such as α and β in Figure 2.3.

Behavior Space

The behavior space (G) is shown as the upper portion of the graph in Figure 2.3. As behavior results from optimizing the objective function, each point in G represents the "optimum" behavior associated with specific values in parameters a and b (or α and β). This surface is single-sheeted everywhere except in the region in which dichotomous behavior possibilities exist, where it is folded over and creates three overlapping sheets. The internal surface (M - G) represents the region of inaccessible (irrational) behavior given by the minima of the objective function. This area is delineated by the fold curve F whose projection onto C creates the bifurcation set B and a cusp point at the origin.

Existence of this inaccessible behavior area is what impedes a smooth behavior response to smooth changes in parameters in the region, so that when the "threshold" represented by F (and B) is crossed in the opposite direction a catastrophic change in behavior occurs. This is shown by the arrowed lines in G .

Canonical Formulation

Mathematical expression for this figure will result in the canonical formulas for the cusp catastrophe. This canonical representation of the catastrophe is not unique, in the sense that other mathematical expressions may be used to represent the actual model. However, it constitutes the simplest set of formulas capable of describing accurately the system in a qualitative manner, and therefore may be used advantageously in modeling efforts.

The behavior surface (including the rational (G) and irrational (M - G) sheets) is given by the cubic expression:

$$x^3 = a + bx \tag{2.1}$$

where x : behavior scale; a : normal factor; and b : splitting factor.

The fold curve F is defined by the points where vertical lines are tangent to (M - G), and may be found by differentiating (2.1) with respect to x :

$$3x^2 = b \tag{2.2}$$

The projection of F onto the control space provides the bifurcation set B , which is defined by eliminating x from (2.1) and (2.2) above:

$$27a^2 = 4b^2 \tag{2.3}$$

An important observation about this equation is that x does not participate in it, and therefore (2.3) is independent of the quantitative characteristics of the model and thus may be estimated directly.

The surface G is defined by:

$$3x^2 \geq b \tag{2.4}$$

and the inaccessible area (M - G) by:

$$3x^2 < b \tag{2.5}$$

This canonical formulation of the cusp catastrophe is made in terms of "normal" (a) and "splitting" (b) factors. An equivalent formulation for "conflicting" factors may be specified by transforming the latter into the former through the formulas:

$$\alpha = a + b \tag{2.6}$$

and

$$\beta = b - a \tag{2.7}$$

where α , β : conflicting factors; a , b : "normal" and "splitting" factors.

CHAPTER III

THE CUSP CATASTROPHE APPLIED TO THE TRANSPORTATION PROBLEM: MODEL FORMULATION AND COMPARATIVE ANALYSIS

The Transportation Problem Reinterpreted

Preceding chapters have provided micro-economic foundations of transportation demand as well as a brief description of basic concepts behind catastrophe theory and, in particular, of the cusp catastrophe. These elements will be put together in the present chapter as a conceptual foundation for developing a new model describing the various components and basic dynamics behind freight demand and modal choice decisions.

The forthcoming discussion will be limited to the case of two competing transportation modes. It could, however, be expanded to cover situations requiring consideration of additional modes. In addition, for simplicity, the first part of the discussion will concentrate on the most "visible" decision variable of interest, namely *quantity shipped*, although it also would be possible to treat any other decision variable (like order quantity or probability of modal choice) in a similar fashion, as described later in the chapter.

Since inventory-theoretic models of transport demand usually incorporate all basic economic assumptions (such as profit-maximization) which have been presented previously as inherent to a micro-economic view of the transportation problem, the reasonably complete model developed in Appendix A shall be considered as a guideline for the economic part of the discussion.

At the core of this reinterpretation of the transportation problem under catastrophe theory lies the fact that modal shift may be understood as a "catastrophic change" (in a mathematical sense) caused by singularities in the behavior space created by (smooth, continuous) control functions of the model. As mentioned before, the behavior space is occupied, initially, by *quantity shipped* through individual modes, as determined by an objective function--expected profit. The control space is occupied by the objective functions' underlying functions, which are made up by different external parameters, such as expected revenue and cost components.

The Objective Function

In preceding discussions, it has been shown that shippers typically aim at profit-maximization in their decision process. The formula for the profit function may be defined, as in Chapter I, as:

$$\pi = \Delta p Q - g(Q) \tag{3.1}$$

where Δp is product price difference between import and export markets; Q is total quantity shipped in a certain period; and, $g(Q)$ is total cost defined as a function of quantity shipped. In order to maximize profit, a derivative of π with respect to Q is taken and made equal to zero:

$$\frac{\partial \pi}{\partial Q} = \Delta p - g'(Q) = 0 \tag{3.2}$$

Although profit at a certain point is the result of a linear transformation, the optimum profit function is not, certainly, linear, because both of its components are not linear. Even assuming a linear product demand, the total cost function will not be linear because its many parametric functions usually combine into a high degree polynomial expression. In any event, the shape of the expected profit function, *for each mode*, can be shown to be downward concave with respect to quantity shipped.[1]

The existence of two competing modes implies that shippers will be faced with two individual modal profit functions, as depicted in Figure 3.1. As a result of these functions' shape, each mode will have an optimum quantity which maximizes profit, such as points Q_1^*, and Q_3^* in that figure. These two individual curves, viewed globally shipping entrepreneurs, may be represented by a single bimodal distribution as the one in Figure 3.1(b).

The smoothing of the "valley" between the two "peaks" may be justified either by modal split or because at that sub-optimal range relative advantages/disadvantages of either mode become less pronounced or indistinguishable. However, diseconomies related to modal split such as those caused by increases in fixed cost (ordering, order set-up, new structures for dispatch, and changing production schedules, etc.) should be sufficient to warrant maintenance of the curve's shape as long as there are two viable competing modes. Any point in

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[1] See, for example, the discussion in the corresponding section of Chapter I and the section on "Satisfaction of Second-Order Conditions" in Appendix A.

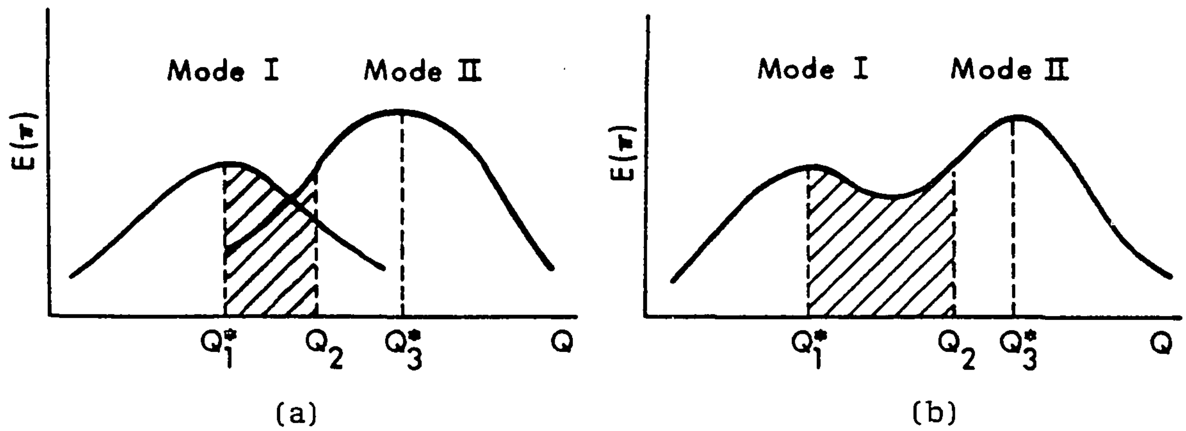

Figure 3.1 -- Bimodal Profit Functions Facing the Shipper

the profit curve represents the profit associated *with a particular quantity shipped*. Points located between points Q_1^* and Q_2 represent an irrational area of choice. In fact, no profit-maximizing entrepreneur would be expected to ship any quantity between Q_1^* and Q_2 since by shipping a smaller quantity (by Mode I) he could produce the same amount of profit with less use of resources.

In a situation such as the one described in Figures 3.1(a) any entrepreneur would strive to ship a quantity which implies in the local maximum profit, that is, either Q_1^*, or Q_3^*, depending on his desired scale of business. When viewing both modes together, however, it is clear that profit-maximizers would ship quantity Q_3^*, by Mode II because that is where the highest overall profit is achievable. The reverse would be true, had the profit "peak" at Q_1^* been higher than at Q_3^*.

A very important observation from the same figure is that a mere increase in the scale of shipments beyond quantity Q_1^* implies automatically in a sudden ("catastrophic") jump from Mode I to Mode II, and from Q_1^* to Q_2, and vice versa. The profit curve generally changes over time due to variations in cost and revenue parameters, which consequently cause changes in relative positions and heights of the "peaks." New shipping decisions, then, are continuously being made in order to adjust to changing parameters. These adjustments will proceed smoothly everywhere along the curve except between quantities Q_1^* and Q_2, which in effect constitute a "threshold" which cannot be crossed without a "catastrophic change" in either direction.

Choice of Decision Rule

Because shipper's behavior is determined by a differential equation (profit maximization), the Delay Rule is applicable in this case. In fact, it makes more economic sense for entrepreneurs to delay a modal shift until there is a clear advantage, in profit terms, of the emerging mode over the old one because of costs and uncertainties connected with any modal shift.

Control Factors

Quantity shipped by either mode is determined by the two most general components of the profit function: revenue and cost. These two functions, therefore, are the controlling factors of the shippers' behavior in the present model.

Revenue and cost represent diametrically opposite forces within the profit function, and have markedly different influences in shippers' behavior. Revenue and cost, therefore, may be termed "conflicting" factors affecting behavior, in the sense that while revenue always "pushes" in direction of higher quantities to be shipped, cost has exactly the opposite effect. By virtue of two different modal cost functions, possible profits show a "divergence" with relation to quantity shipped, that is, there are particular combinations of revenue/cost for which two different quantities may be shipped, through different modes, providing the same amount of overall profit. This situation may be exemplified as in Figure 3.2, which relates profit-maximizing quantities with the revenue obtained from some shipment.

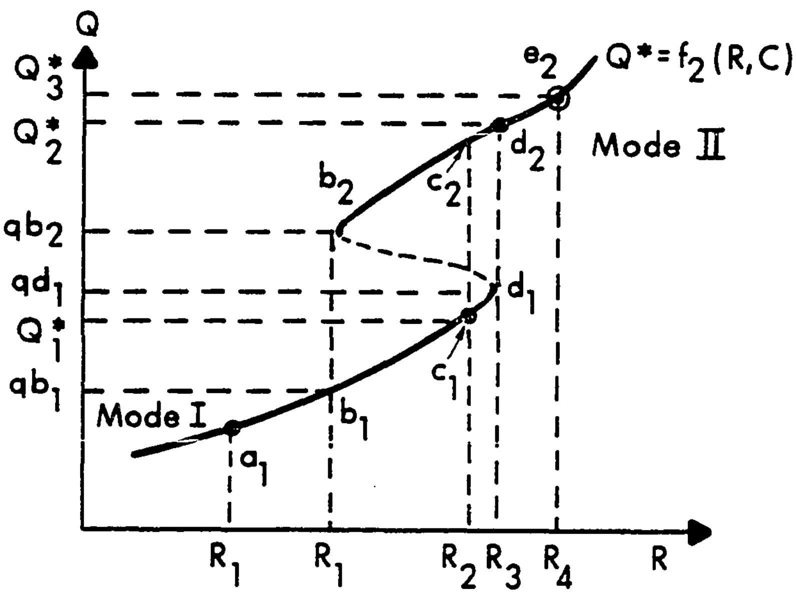

Figure 3.2 -- Quantity Shipped by Individual Modes as a Function of Cost and Revenue

Cost and revenue considerations introduce a dichotomy in shippers' behavior with regard to all decision variables such as total quantity shipped, order quantity, reorder point, and probability of modal choice. As each mode has a particular cost structure which differentiates it from the other, there will be different optimum cost/revenue points for each mode with respect to quantity shipped. This difference implies in the bimodal distribution of Figures 3.1(a) and (b), and in an area of overlapping sheets in the behavior plane (Figure 3.2) in which two quantities could be shipped by each mode providing the same amount of profit. Points d_1 and b_2 in the curve correspond to the "threshold" of the irrational area of choice in the same figure. The extent of sheet overlapping (hysteresis) depends on the choice of rule and on how divergent cost and revenues are for that particular quantity.

The underlying dynamics implied in Figure 3.2 is essentially the same described in Figure 3.1(a) and (b): an entrepreneur sending quantity a_1, for example, would certainly use Mode I because there is a definite profit advantage in using that mode for that quantity. However, if he wishes to expand his operations, at point b_1 he would have two alternative choices providing the same overall return: quantities qb_1 (Mode I) or qb_2 (Mode II). His decision will depend on the decision rule adopted for his response/behavior, but technically either alternative is profit-maximizing. However, the most likely choice would be still to use Mode I because shifting modes would not add anything to profit. This situation will persist up to point d_1, where any further expansion would cause a sudden jump from Mode I to Mode II, and quantity qd_1 to Q_2 . From then on, Mode II would be used in any further expansion in business (or contraction, up to quantity qb_2).

A full understanding of the model's dynamics, however, is not possible without considering the global maximum profit points in Figures 3.1(a) and (b). The existence of these two points implies in two modal stable "equilibrium" points which shippers will aim at when making their decision. Ultimately they will choose the higher of these equilibrium points. This situation may be described with the help of Figure 3.3.

A constantly expanding entrepreneur, for example, if starting operations at point a_1 would strive to reach optimum use of Mode I by moving along the curve to point E_1 , which represents the global

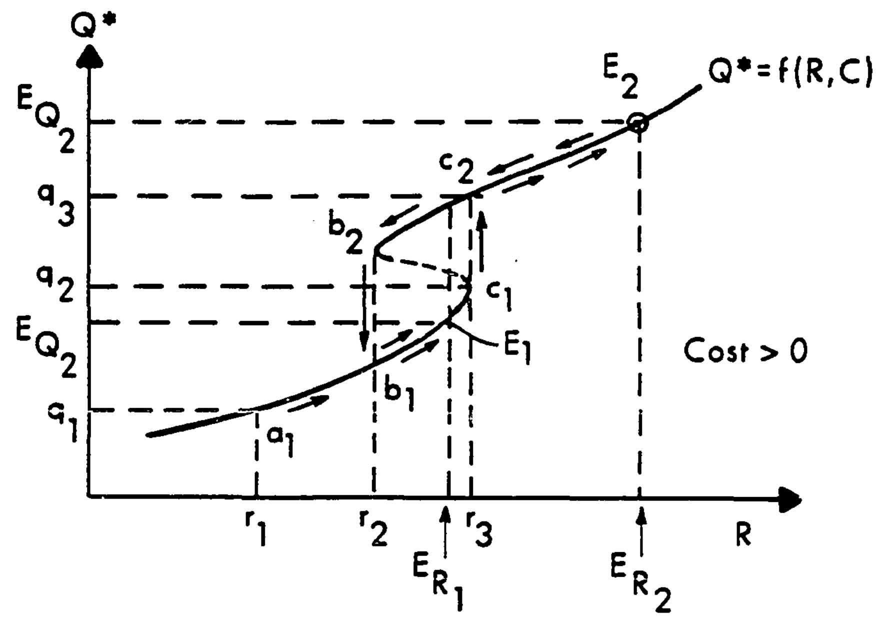

Figure 3.3 -- Quantity Shipped by Individual Modes as a Function of Cost and Revenue--A Dynamic View

optimum for that mode. However, an expanding operator probably will want to maximize his *total profits*, which implies being at point E_2 (Mode II), that is, the most profitable point overall. Then, he would expand his operations in Mode I up to point c_1, when he suddenly switches modes to point c_2 and proceeds on to the next stable equilibrium point at E_2, and will try to stay there as long as the situation described by the parameters of Q^* remains unchanged. If for any reason a contraction in business is desired, he would follow the arrows back up to point E_1 (Mode I), which is the optimum for a "small" operation. It is important to notice, however, that the relative positions of E_1 and E_2 generally change with time, and entrepreneurs adjust their behavior continuously to the new optima of their operations.

The preceding discussion took as an example a case where the decision process dynamics involved business expansion (or contraction) *over time*. Because of factors such as a (high) required level of managerial performance (especially in face of stiff competition), adjustments in behavior must be made in a short period of time once the underlying objective function is perceived to have changed. Thus behavior response to changes in this function may be considered the "fast equation" of the model, obviously depending on the speed at which new information becomes available. The objective function, on the other hand, is expected to change *relatively* much slower because it depends simultaneously on many interrelated parameters. If one--or some--of the parameters are altered abruptly this will trigger a process of internal adjustments which will ultimately change the objective function. However, these internal adjustments themselves take some time to complete, and in effect act like "buffer" against very fast (relative) changes in the overall shape and position of the objective function.

Given relatively *slow functional* and *fast behavior* adjustments, at *any point in time* an observation is expected to show entrepreneurs in the position which maximizes overall profits, such as point E_2 in Figure 3.3. This position naturally will depend on existing parameters at the moment of decision, and is therefore specific with regard to scale of business, type of merchandise, modal characteristics, market and cost situation, etc., *for that shipper at that particular moment*. But not only this, it has been shown in the range from r_2 to r_3 (hysteresis area, in Figure 3.3) where two profit-maximizing alternatives are available, the choice of mode also will be determined by *where the previous equilibrium point for the shipper was located.*

Global Overview and Canonical Formulation of the Model

A general overview of the transportation problem may be obtained by combining all elements discussed in preceding sections in a tri-dimensional graph, similar to Figure 2.3 in Chapter II, such as the one depicted in Figure 3.4 ahead. The control space (C) contains two axes representing revenue and cost. The vertical axis represents profit-maximizing quantities shipped by individual modes. For each combination of revenue/cost there will be an optimum quantity to be shipped by either mode, and the collection of these points configurate the behavior surface (G) in the graph. The surface (M - G) located

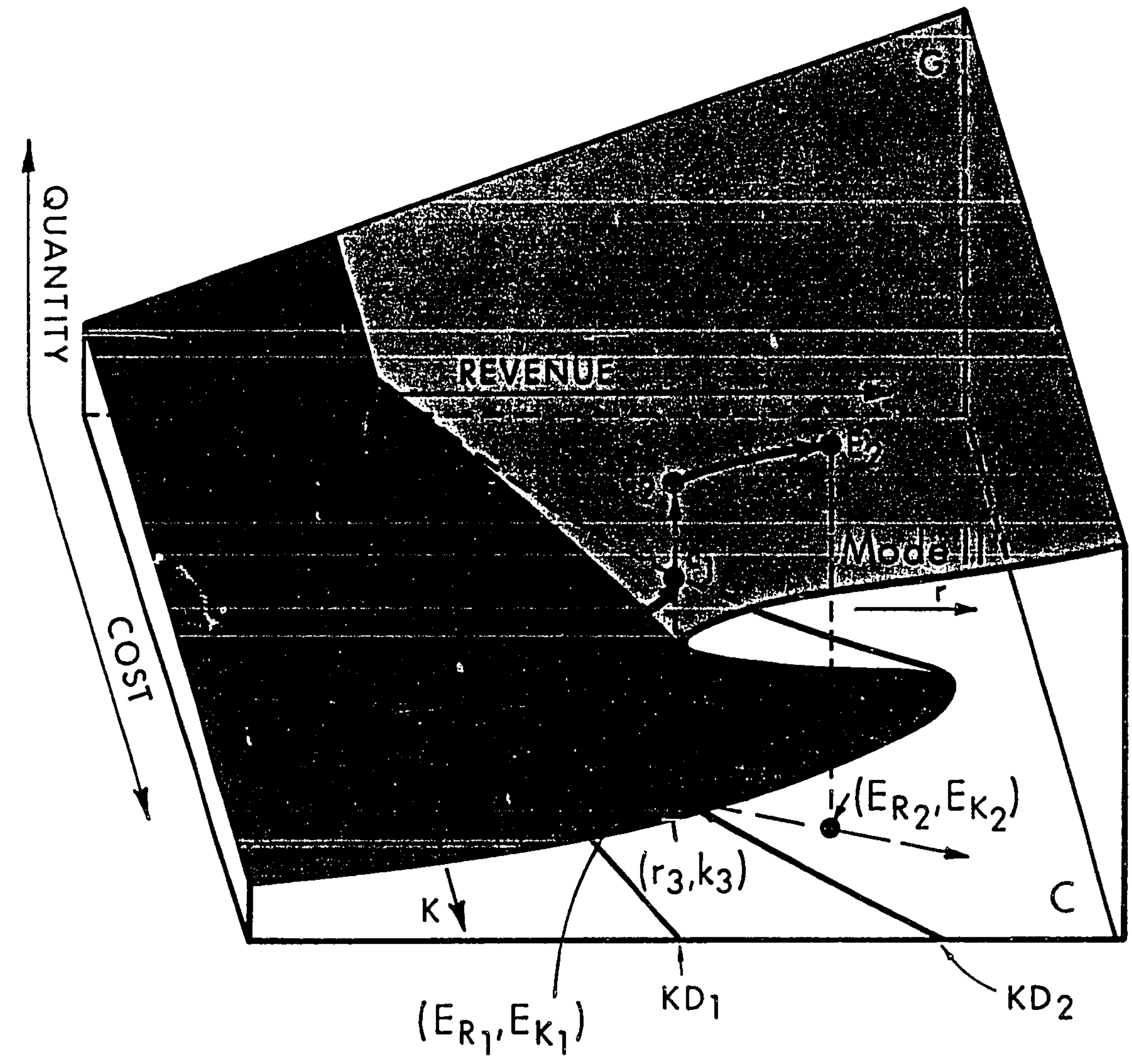

Figure 3.4 -- Global Overview of Transportation Demand and Modal Choice

between the two overlapping surfaces of G is the region of inaccessible (economically irrational) behavior. The projection of the folding lines of the behavior surface (G) on the control plane (C) forms the bifurcation set from KD_1, to CP, to KD_2, which contain, in effect, the threshold points where sudden modal shifts occur.

The same example developed in describing the dynamics behind Figure 3.3 may be used again to illustrate what the more complete graph in Figure 3.4 represents. In fact, the earlier graph may be considered a vertical "cut" in Figure 3.4, perpendicular to the cost/revenue path line $(r_1, k_1) / \left({}^{E}R_2, {}^{E}K_2 \right)$ in the control space C. Each point in this path is determined by a pair of coordinates originating at the cost (K) and revenue (R) axes, and the resulting point defines the quantity shipped by each mode in the behavior space G. Therefore, if the cost/revenue situation of an expanding operation follows the path $(r_1, k_1) \longrightarrow \left({}^{E}R_2, {}^{E}K_2 \right)$ in the control space, the response of the shipper will be to follow the concomitant path from a_1 to the overall maximum point E_2 in the behavior space. All the implications discussed previously also apply to this expanded version of Figure 3.3.

Mathematical expression for this model may be developed through the use of the canonical formulas of the cusp catastrophe presented in Chapter II. As the canonical formulas for the cusp catastrophe are defined in terms of "normal" (a) and "splitting" (b) factors, the first step will be to define their expressions from the "conflicting" factors revenue (r) and cost (c). This can be done by making:

$$r = a + b$$

$$c = b - a$$

resulting in the "normal" and "splitting" factors:

$$a = \frac{r - c}{2} \tag{3.3}$$

$$b = \frac{c + r}{2} \tag{3.4}$$

The behavior surface, then, is provided by the expression:

$$q^3 = \frac{r - c}{2} + \frac{c + r}{2} q \tag{3.5}$$

where q : quantity shipped by a particular mode; r : revenue; and c : cost.

The fold curve is defined by:

$$3q^2 = \frac{c + r}{2} \tag{3.6}$$

The equation for the bifurcation set is:

$$27\left(\frac{r - c}{2}\right)^2 = 4\left(\frac{c + r}{2}\right)^3 \tag{3.7}$$

As before, this equation is independent of the quantitative characteristics of the model. The surface G is given by:

$$3q^2 \geq \frac{c + r}{2} \tag{3.8}$$

and the inaccessible area (M - G) is defined by:

$$3q^2 < \frac{c + r}{2} \tag{3.9}$$

The foregoing discussion is a straightforward application of catastrophe theory to the analysis of the transportation problem. Before proceeding to the formulation of a choice model it is convenient to recapitulate briefly some of the most important properties of this general model so far:

1) The model, as formulated, is entirely coherent with the basic tenets of micro-economic theory, and therefore constitutes a powerful new way of considering the transportation problem, including some of its underlying dynamics. In particular, any of the variables in use may be defined as in the structurally more complete inventory-theoretic model so as to analyze almost any desired angle of the problem, including those related with inventory policy. The single additional assumption made in this model stems from the use of the Delay Rule which, besides having been previously justified on economic behavior grounds, in this case is a *theorem* of catastrophe theory.

2) Although this model is qualitative in nature, it represents a faithful qualitative picture of the quantitative phenomena behind transportation decisions by force of Thom's Classification Theorem.

3) Due to the general nature of the model, any two control and one individual behavior variables participating in the objective (or "energy") function of the problem may be singled out for analysis without loss of generality. This quality provides an enormous

advantage in studying specific policy options in the transportation field.

Formulation and Analysis of the Choice Model

One of the stated objectives of this research was to verify possible contributions of the catastrophe theory/inventory theoretic approach in providing additional insights to the analysis of the transportation problem. This may be accomplished by comparing the proposed model with other estimation models which have been most successful, or displayed the greatest potential, for estimating transportation demand. Judging from recent literature, it appears that the most favored methods at present include multiple regression, discriminant analysis, probit and logit models applied to disaggregated data, and that the greatest potential seems to be connected with the use of logit models.[2]

Therefore, the proposed model of transportation demand will now be reformulated so as to tackle the modal choice problem in the same terms as the most common formulation of the logit model, in order to permit an analytical comparison of the two models. The modifications will involve:

1) Redefining behavior as the odds-ratio in favor of a particular mode being chosen over the other; and

2) Specifying as control variables the same variables, in the same scale, as those which have been consistently considered the

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[2] See discussion on the subject in Chapter I.

most important variables in previous studies.

Probability of Choice as a Behavior

Once profit-maximization (or cost minimization) has been assumed implicitly (as in the logit model) or explicitly (as in the inventory-theoretic model) as the objective function, all decision variables (which represent behavior) will originate from the optimization of this objective function. Conceptually, therefore, there is no difficulty in defining the probability of choosing a particular mode as a behavior which is analogous to, say, order quantity (Q) or total annual quantity shipped (A). In order to emphasize this point, the trivial graph of Figure 3.5(a) may be used to demonstrate the interrelationship between those three behavior variables. As all three variables are determined endogenously and are maximized simultaneously with the objective function, the exact relation of $P(x)^*$ and A^* or Q^* is just a matter of defining appropriate scales for the vertical and horizontal axes. Conceptually at least, the values of A^* and Q^* may go from 0 to ∞ , depending on the underlying parameter values. However, in order to be meaningful, the probability function must have values between 0 and 1. This constraint may be satisfied by a logarithmic transformation of the curve in the graph of Figure 3.5(a), which will result in a curve similar to that in Figure 3.5(b). The basic nature of the relationship, however, remains unaltered: it is logical to assume that for a particular observation, the farther away the optimal shipping quantity is from the origin of the graph in one direction, the higher will be the probability of that particular mode

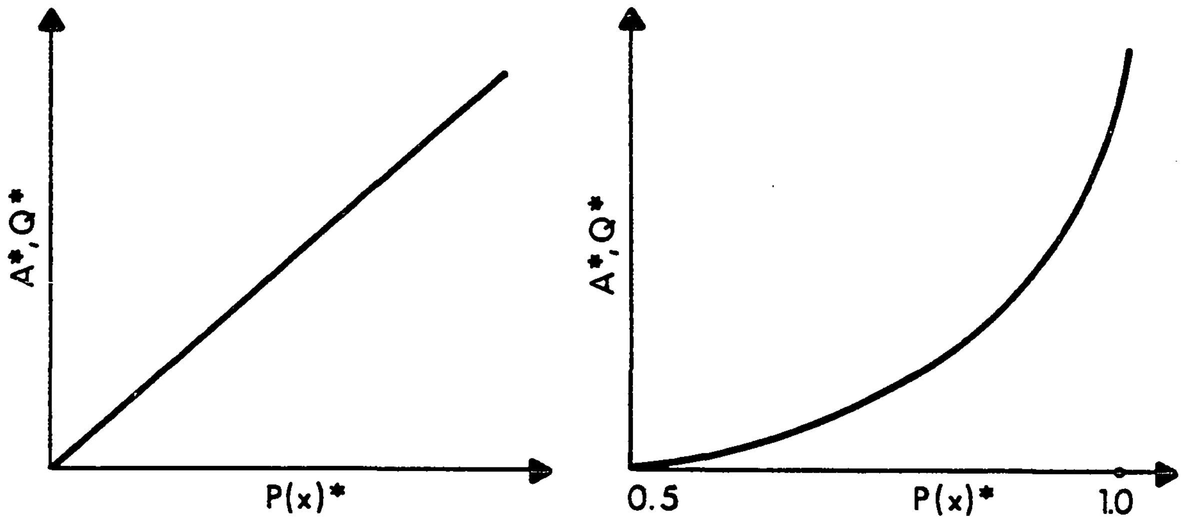

Figure 3.5 -- Relation Between Quantity Shipped and Probability of Modal Choice

being selected.

One problem of defining "probability of a specific choice" as a behavior is that, although the probability function represents an actual function, it cannot be observed in real life. Therefore, this function must be estimated statistically from actual choices (revealed preference) by an "either/or" method such as the logit method. Individual probability functions, however, must resemble the "expected profit/quantity shipped" function described in Figures 3.1 (a) and (b). Consequently, when a large number of observations are available, their frequency distribution must also resemble that profit function through an "envelope curve" type of effect. By defining the probability of choice, like in the logit model, so as to have values ranging from zero to one (Mode I being chosen at $P(x) < 5$ and Mode II at $P(x) > 5$), Figure 3.1(b) may be relabelled as the upper portion of Figure 3.6 on the following page.

Since in absence of revenue considerations profit is the inverse of cost, the lower portion of the same figure shows that the points of minimum total cost in each mode correspond to the highest number of observations in a particular sample. The shaded area in the graph shows the area of irrational choice.

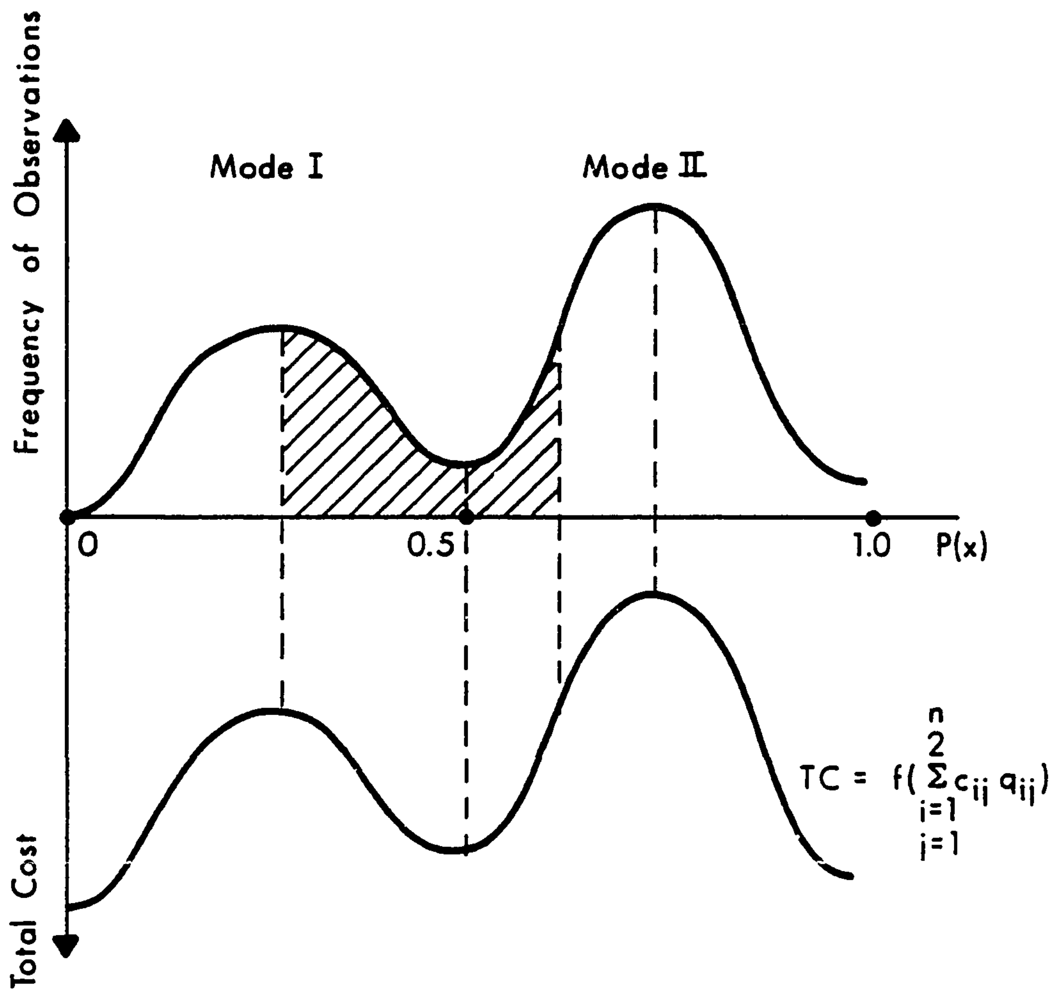

Figure 3.6 -- Probability of Choice Frequency Distribution

Control Variables

Among the independent variables figuring most prominently in previous demand estimations, the two which bear major interest for this analysis are "direct transportation cost" and "transit-time." The first, because it has been shown to be consistently significant in previous works both theoretically and practically; and, the second because, while having been exhaustively pointed out as a theoretically important demand determinant, in actual (logit) estimations it has been plagued by inconsistent findings such as wrong sign and/or insignificant coefficients.[3]

Direct shipping cost (Cs) and *transit time* (M) are both components of *total shipping cost*, and thus "compete" for a proportion of total costs. Transit time, in fact, must be understood as a proxy for "costs stemming from transit time" which, as may be seen in the inventory-theoretic model, also include other variables.[4] Cs and M, in this case, must be treated differently than "revenue" and "cost" in the previous example because, while "revenue" and "cost" have opposite signs in the profit function (they are "conflicting factors"), Cs and M have the same sign as components of the cost function. For this reason, these variables must be treated in terms of "normal" and "splitting" factors.

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[3] Some of these studies are described in Chapter IV.

[4] In order to have a uniform notation throughout this analysis, from now on symbols for all variables shall follow the convention adopted in Appendix A.

The specification of Cs as a *normal factor* is straightforward: *in absence of any other cost*, total cost is determined exclusively by the formula $TC = (r \cdot Q) = Cs$, that is, freight rate per unit, *times* quantity shipped. This implies in a linear relationship between A and r [Equation (8), Appendix A]. In terms of rate differences, this relation may be written $Q = K(r_1 - r_2)$. By making the choice of mode to depend on whether Δr is positive (Mode II) or negative (Mode I), graph (a) of Figure 3.7 may be constructed. Considering that $Cs = r \cdot Q$, and that $P(x)_{CT}$ is a logarithmic transformation of Q because of constraints in values of $P(x)_{CT}$ between 1 and 0, the same figure may be modified as in (b). This figure shows the probability of choice function as function of ΔCs, *provided that all other costs have zero or negative value.*

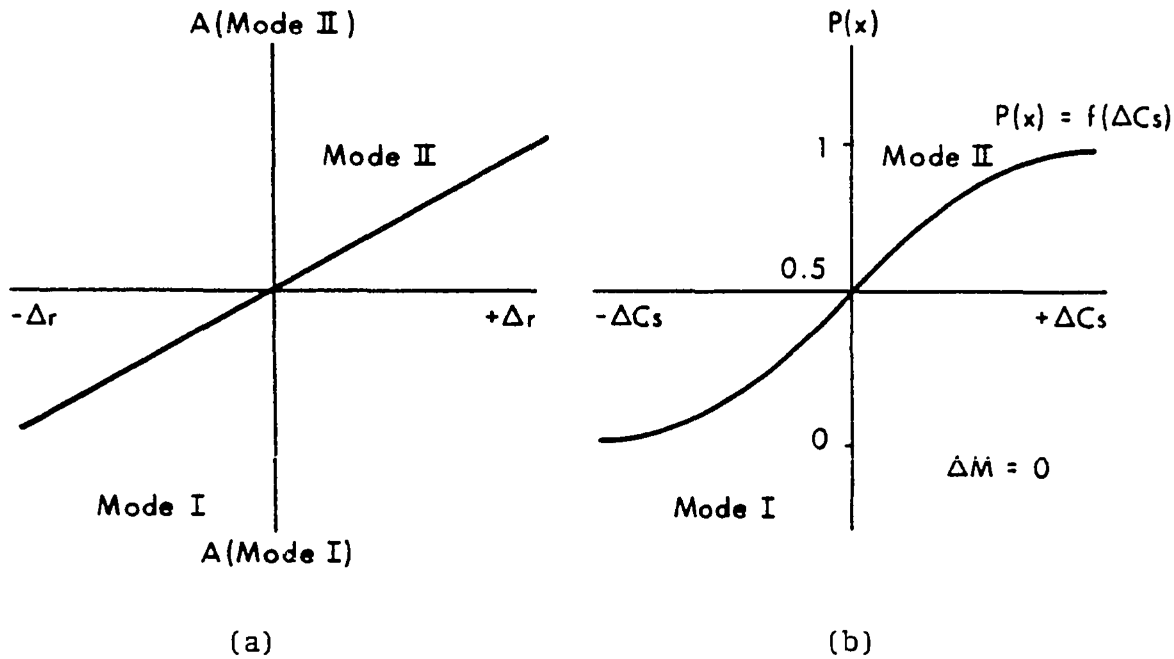

Figure 3.7 -- Relation Between Choice Variables and Direct Cost of Transportation-Linear Variables

Mean transit-time (M) may be defined as the "splitting factor." In order to justify this decision, it must be remembered that transit-time is important to shipping decisions *exclusively* to the extent of the costs it induces in the total cost function. Therefore, the value of transit-time to a shipper stems from the influence of this variable in the two cost components it affects most directly: *Cost of Inventory In-Transit* (C_t), and *Consignee's Inventory Carrying Cost* (C_I).

a) Inventory In-Transit

The expression for *cost of inventory in-transit* in the demand equation of the inventory-theoretic model [Equation (8), Appendix A] is:

$$C_t^* = zM \tag{3.10}$$

where M : mean lead-time, and z : in-transit inventory carrying cost. This formula characterizes a *direct linear* (but negative) association of transit-time (M) with total quantity shipped (A). Through analogy with the reasoning behind Figure 3.7, the association between $P(x)$ and ΔM may be shown to be equivalent to that pictured in Figure 3.7(b) (with ΔM figuring in the horizontal axis), but only *so long as* C_t *is the only inventory cost under consideration.*

b) Consignee's Inventory Cost

However, ΔM also affects fundamentally the Consignee's Inventory Cost (C_I) by helping to define the levels (and cost) of "working inventory," "safety inventory," and the total cost of "orders set-up." The influence of ΔM derives from its being a basic determinant of the expressions for "order quantity" (Q), "reorder point" (R), and "expected shortage" (S), which together define C_I. The role of ΔM in C_I and ultimately in the demand function may then be traced as follows:

The demand function [Equation (8), Appendix A] is:

$$A^*(Q^*, R^*) = 1/b[\Delta p - r - zM - K/Q^* - \xi S^*/Q^*] \qquad (3.11)$$

Therefore, the expression for C_I^* is:

$$C_I^* = 1/b[-K/Q^* - \xi S^*/Q^*] \qquad (3.12)$$

By definition, R [Equation (11), Appendix A] is:

$$R = u_L + G(q)\sigma_L \qquad (3.13)$$

The effect of transit time (M) on reorder point (R) is defined by the fact that mean lead-time (u_L) is given by $u_L = M \cdot u$; and its variance (σ_L) by $\sigma_L = M\sigma^2 + Vu^2$. The expression for R enters the shortage definition [Equation (6), Appendix A] as:

$$S = \int_{L=R}^{\infty} (L - R)\phi(L)\,dL \qquad (3.14)$$

And, ultimately, the relationship between transit time and order quantity becomes clear, once the expression for the latter is given [Equation (9), Appendix A] by:

$$Q^* = [2A^*(K + \xi S^*/H)]^{\frac{1}{2}} \qquad (3.15)$$

In spite of their interrelationships being made clear by equations (3.11) to (3.15), an explicit analytical expression connecting M with A* and Q* is difficult to obtain, because this set of equations is complex and cumbersome to work with. However, as equation (3.15) is a quadratic formula, the relation between M and Q* and/or A* will have to be of *at least power two*, that is, non-linear. Therefore, the variable transit-time is one of those higher order variables which cause total cost (profit) curves to have a "bell" shape in bimodal distribution, leading to singularity in the behavior space and to a cusp and bifurcation in the control space.

Since the effects of M in the demand function are due to the sum of a linear and a non-linear relationship, the resulting effect must necessarily be non-linear, and this variable may consequently be characterized as the splitting factor. After the logarithmic transformation to assure asymptoticity of the curve and allow for the Delay Rule, when the absolute value of ΔM is larger than zero, Figure 3.7(b) must be modified as in Figure 3.8 on the following page. The range of overlapping behavior sheets will be determined by the Delay Rule and, more importantly, by the absolute value of ΔM and *the extent by which the non-linear component of transit-time-induced cost (consignee's inventory cost) overpowers the linear component (cost of inventory in-transit).*

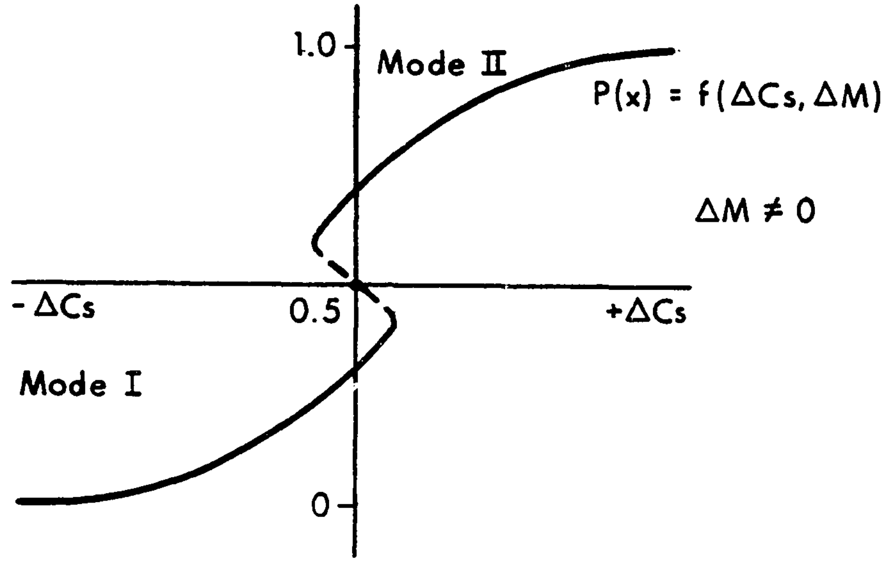

Figure 3.8 -- Probability of Modal Choice with the Inclusion of Transit Time in the Objective Function

Global Overview and Canonical Formulation

A global overview of the model is described in Figure 3.9 ahead. In analogy with the canonical formulation of the general transportation demand model discussed in earlier sections of this paper, a mathematical formulation for the choice model, then, may be specified:

a) Normal Factor (a):

$$a = K_1 \Delta Cs \tag{3.16}$$

b) Splitting Factor (b):

$$b = K_2(\Delta M)^2 + K_3 \tag{3.17}$$

where K_1 , K_2 , K_3 are constants to be estimated statistically.

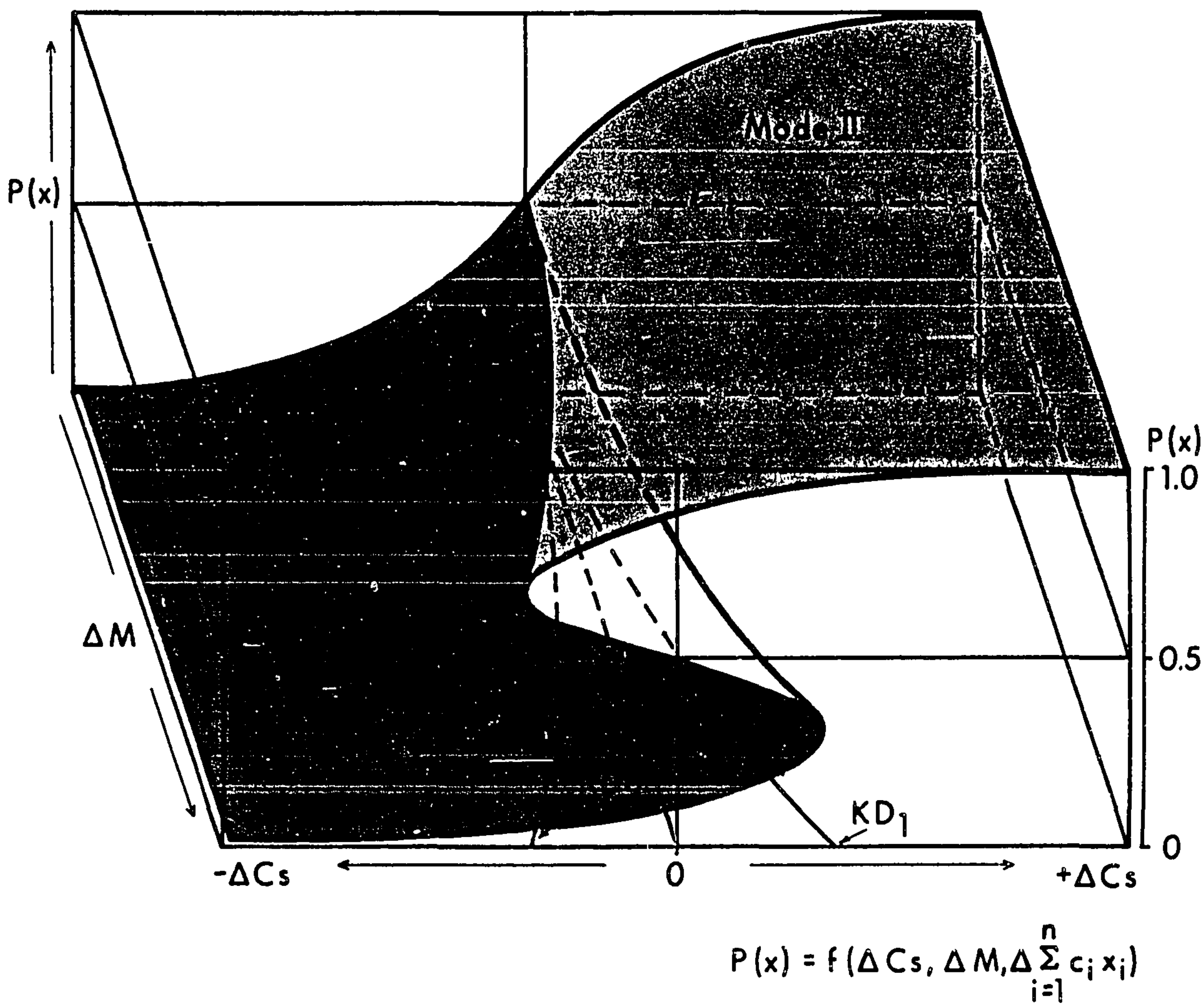

Figure 3.9 -- Global Overview of a Modal Choice Model Specified Through Catastrophe Theory

c) Probability of Choice Surface $[P(x)]^3_{CT}$:

$$[P(x)]^3_{CT} = [K_1 \Delta Cs] + [K_2(\Delta M)^2 + K_3][P(x)] \quad (3.18)$$

d) Fold Curve in $[P(x)]$:

$$3[P(x)]^2_{CT} = K_2(\Delta M)^2 + K_3 \quad (3.19)$$

e) Bifurcation set in the control space (CP to KD_1, and KD_2) is defined by the solutions to equation:

$$27(K_1 \Delta Cs)^2 = 4[K_2(\Delta M)^2 + K_3]^3 \quad (3.20)$$

As before, this equation is independent of the quantitative properties of the model, except for the constants K_1, K_2, K_3.

Since most of the practical conclusions and implications of the present model must be drawn by comparing some of its characteristics with those of the logit model, a brief review of the latter is provided in the next section.

The Logit Model in Freight Demand Estimations

Logit models take advantage of some properties of odds ratio in favor of a certain event occurring, and its logarithm. This type of model had been originally proposed and used in the transportation field to define a function relating components of passenger utility function to observed modal choice so as to estimate a function for travel demand transportation.[5] From estimating passenger travel demand, the same method has been subsequently adopted to estimate a function akin to freight demand for transportation.[6] A formal discussion of a modified version of the logit model to handle choices among unranked alternatives may be seen in McFadden (1968).[7]

In short, this model may be described as follows: assume, for example, that an objective function (Total Cost, Net Price, etc.) to be optimized for a certain problem may be defined in terms of modal

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[5] See for example:

Charles Rivers Associates, Inc., A Disaggregated Behavioral Model of Urban Travel Demand. Prepared under contract for the Federal Highway Administration, Final Report, March 1972.

Stopher, D.R.; and T.E. Lisco. Modelling Travel Demand: A Disaggregate Behavioral Approach. Proceedings, Eleventh Annual Meeting, T.R.F., Indiana, 1970.

Quarmby, D.A. "Choice of Travel Mode for the Journey to Work: Some Findings." J.T.E.P., Vol. 1, No. 3, September 1967, pp. 273-314.

[6] Some examples are:

Watson, P.L.; Hartwig, J.C.; and Linton, W.E. "Factors Influencing Shipping Mode Choice for Intercity Freight: A Disaggregate Approach." In: Proceedings, Fifth Annual Meeting, T.R.F., Vol. XV, No. 1, October 1974, pp. 137/144.

Daughety, A.F.; and Inaba, F.S. "Estimation of Service-Differentiated Transport Demand Functions." Working Paper #601-77-16, The Transportation Center, Northwestern University, 1977.

Miklius, W.; Casavant, K.L.; and Garrod, P.V. "Estimation of Demand for Transportation of Agricultural Commodities." A.J.A.E., Vol. 58, No. 2, May 1976.

[7] McFadden, D. "Conditional Logit Analysis of Qualitative Choice Behavior," in Frontiers in Econometrics, ed. Paul Zarembka, pp. 105/42. New York: Academic Press, 1974.

characteristics:

$$(OF)_1 = f_1\left(\sum_{i=1}^{n} x_{i1}\beta_{i1}\right), \text{ and } (OF)_2 = f_2\left(\sum_{i=1}^{n} x_{i2}\beta_{i2}\right) \quad (3.21)$$

where x_{ij} are modal characteristics, and β_{ij} their coefficients. These functions may be combined into a single objective function in terms of modal differences:

$$OF = [(OF)_1 - (OF)_2] = w\left[\sum_{i=1}^{n} x_{i1}\beta_{i1} - \sum_{i=1}^{n} x_{i2}\beta_{i2}\right] \quad (3.22)$$

In the case of modal choice, considering that not only modal characteristics but also shipment, buyer or seller characteristics affect choice decisions, this overall objective function (3.22) may be rewritten:

$$OF = g(XB) = \alpha + \left[\sum_{i=1}^{n} x_{i1}\beta_{i1} - \sum_{i=1}^{n} x_{i2}\beta_{i2}\right] + \sum_{k=1}^{n} s_k y_k \quad (3.23)$$

where s_k stands for non-modal characteristics affecting choice, and y_k their coefficients. The probability that one of the modes will be chosen may be written:

$$P(M_1) = \frac{1}{1 + e^{g(XB)}} \quad (3.24)$$

and, consequently, the probability $[1 - P(M_1)]$ that the other mode will be chosen is:

$$[1 - P(M_1)] = \frac{e^{g(XB)}}{1 + e^{g(XB)}} \tag{3.25}$$

then, by properties of logarithms:

$$P(x)_L = \log \frac{P(M_1)}{1 - P(M_1)} = -g(XB) \tag{3.26}$$

where $P(x)_L$ is, by definition, the log of positive odds favoring the first mode, which has a value of 0 to 1. At $P(x)$ higher than 0.5, the first mode will be chosen, while at $P(x)$ values of < 0.5 the second mode is chosen.

The two major assumptions in studies applying the logit model are:

1) The probability function $P(x)_L = -g(XB)$ is linear; and

2) The function $g(XB)$ contains stochastic terms ε_i and ε_j which are assumed to be statistically independent and jointly distributed with an identical reciprocal distribution.[8]

Under these circumstances, the function $P(x)_L = -g(XB)$ may be plotted in a graph against one of the components of $g(XB)$ such as, say, Cs , as described in Figure 3.10 on the following page.

[8] See Charles Rivers Associates, Inc., op. cit., pp. 5/16.

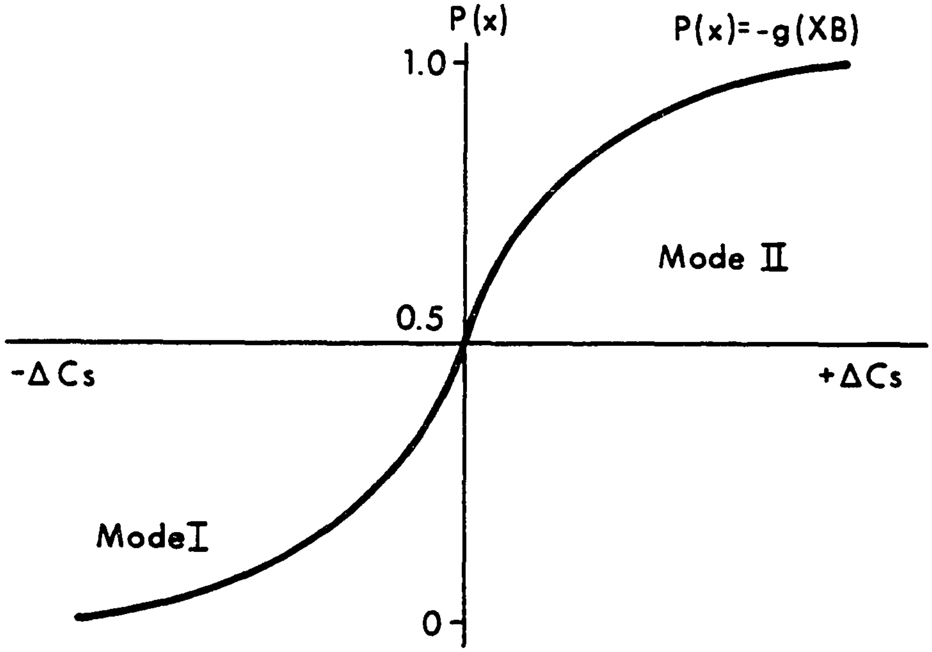

Figure 3.10 -- Choice Function of a Logit Model

Analytical Comparison of the Two Models

At this point it is important to call attention to the similarity between Figure 3.10 (logit model) and our formulation of the transportation problem in Figure 3.7(b) (inventory-theoretic/catastrophe theory). This similarity is not coincidental: in both models the variables have been defined in the same way, and they purport to describe the same underlying phenomena. However, in Figure 3.7(b) $P(x)_{CT}$ was defined as a function of the single (linear) variable ΔCs, while the logit $P(x)_L$ is a function of *all* variables in the objective function (which are assumed linear).

Basic principles of the theory of the firm, and of inventory theory assert that the profit function is non-linear, and that mean transit time (M) is one of the non-linear variables of that function. As a consequence of this fact, plus the Delay Rule in catastrophe theory, whenever ΔM is included as an explanatory variable of $P(x)$, the $P(x)$ function must resemble that of Figure 3.8. An immediate implication is that the inclusion of ΔM will *systematically cause the logit model to misrepresent the probability function* $P(x)$, and that this misrepresentation will be more pronounced the more the value of ΔM in the objective function (profit) departs from a linear relationship with quantity shipped (A^*, Q^*) and, consequently, with $P(x)^*$.

Conclusions and implications of the foregoing discussion may be easily drawn by comparing the catastrophe theory formulation of the inventory-theoretic model with that of the logit model. This may be accomplished visually by superimposing Figures 3.8 and 3.10, as in Figure 3.11 on the following page.

Once the inventory-theoretic model is accepted to be a reasonably complete structural representation of the shipper's true objective function, the function $P(x)_{CT}$ must be accepted as a reasonably accurate description of the true probability function, and $P(x)_L$ its linear approximation. That is to say that principles of economic rationality require the choice of Mode II, *even in presence of negative* ΔCs, up to point λ_1 (or Mode I up to λ_2), and that the logit model will most likely misclassify observations in the range of λ_1 to λ_2.

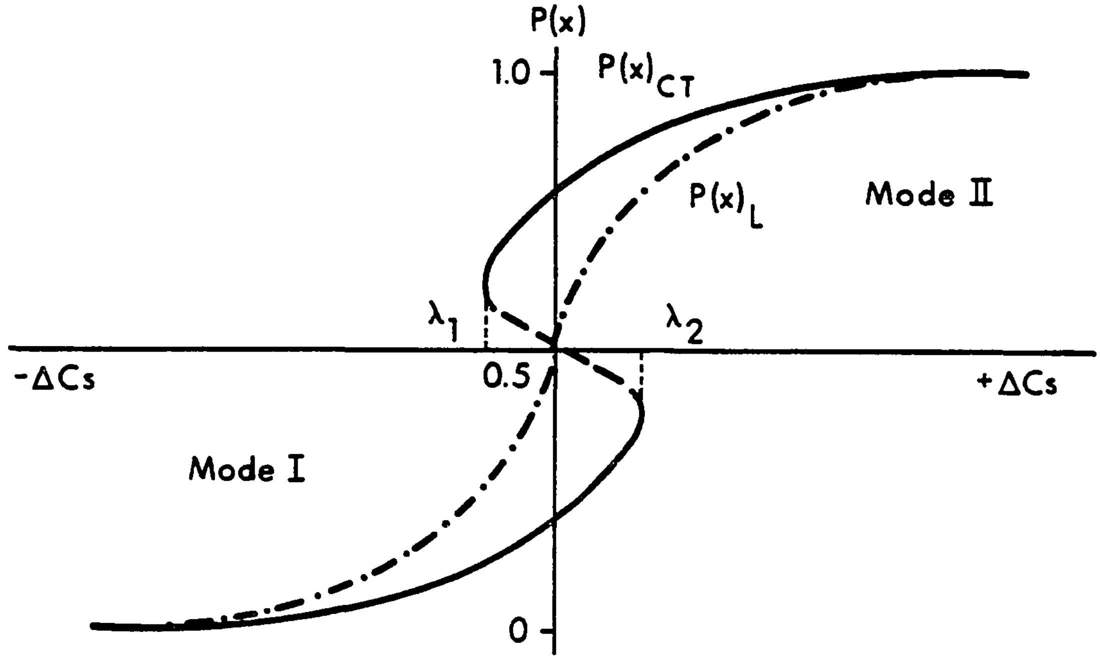

Figure 3.11 -- Comparison Between Logit and Catastrophe Theory Choice Functions with Relation to Transportation Costs

The consequences of such a result in demand studies are manifold, as may be seen by the conclusions reached by Fleiss (1973) about the effects of misclassification errors in statistical estimations:

> "The facts of the matter are that such errors can turn a truly positive association into one that is less strongly positive or even apparently negative; one that is truly strongly negative into one that is less strongly negative or even apparently positive; and one that is nil into one that is apparently strong. These facts contradict the long standing, but erroneous impression that errors of misclassification tend only to reduce the magnitude of association (Newell, 1962)."[9]

These facts indicate that in freight demand estimations by logit models, in at least some of the cases, it must be expected some

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[9] Joseph L. Fleiss. Statistical Methods for Rates and Proportions. New York: John Wiley, 1973, pp. 134/5.

inconsistent results: misclassification errors caused by improper specification of the probability function may very well provide wrong sign and/or insignificant coefficients for some variables, *even in presence of a strong (apparent) explanatory power of the model.* And furthermore, eventual inconsistencies should be expected in connection with non-linear variables, notably "transit-time." Further implications as well as a search for empirical evidence about these conclusions shall be explored in the next chapter.

Summary, Conclusions and Implications

In summary, this chapter has shown:

1) That the transportation problem may be globally described by catastrophe theory in a qualitative but accurate manner, and that this description encompasses a rigorous theoretic treatment of transportation demand, including inventory variables and basic dynamics of the decision process behind modal choice.

2) That this model may be redefined using the same variables, in the same scale, as those in the logit model without loss of generality or accuracy.

3) That the comparison of the resulting (catastrophe theory) model and the logit model results in new insights about the usefulness and limitations of the latter in a wide number of circumstances.

4) That the logit model systematically misrepresents the probability function when non-linear variables such as transit-time are included in the objective function, and the conditions under which the logit approaches or departs from the true probability function

were described; and that the accuracy of logit estimations depends on how close the logit distribution is to the true probability function, because the proportion of misclassified observations will be conditioned by the degree of this approximation.

5) That eventual inconsistent estimated coefficients should be associated with non-linear variables such as transit-time.

6) That the catastrophe theory formulation of the inventory-theoretic model provides the elements for analysis of the above implications.

CHAPTER IV

APPLICATION OF THE MODEL--

CONCLUSIONS, IMPLICATIONS AND SUPPORTING EVIDENCE

An Observable Implication of the Model and Expected Results

If the preceding analysis advances an explanation as to *why* apparently inconsistent results are to be expected in some logit freight demand estimations in connection with transit time, now it is necessary to address the question of *under what circumstances* are inconsistent results more likely to appear. Specific answers to this question may be found by returning to parts of the general model developed in the last chapter. There it has been shown that the logit probability function better approximates the truc probability function when the values of λ_1 and λ_2 (Figure 3.11) tend to zero. This will happen when:

a) The absolute value of ΔM tends to zero, as seen in Figure 3.9; and/or

b) The linear components of "transit-time-induced-costs" substantially exceed their non-linear components in value.

It also has been shown that the basic influence of transit time on demand is due to its effect on inventory carrying cost; and that linear cost components are concentrated in "cost of inventory in-transit," while non-linear components are basic determinants of "consignee's inventory cost." Cost of inventory in-transit is given by:

$$C_t^* = zM \tag{3.10}$$

and consignee's inventory cost by:

$$C_I^* = 1/b\,[-K/Q^* - \xi S^*/Q^*] \tag{3.12}$$

where Q^* is:

$$Q^* = [2A^*(K + \xi S^*)/H]^{\frac{1}{2}} \tag{3.15}$$

The impact of transit time on inventory costs depends directly on product characteristics such as perishability and value/weight. The only variables describing these product characteristics in the inventory-theoretic model are z (equation (3.10)) and H (equation (3.15)), which stand for inventory holding cost (in-transit and consignee's, respectively). These variables may be expanded [Equations (15), (16), Appendix A] as:

$$H = [C_r(\nu + r)] \quad \text{and} \quad z = [C_t(\nu + r)] \tag{4.1}$$

where:

C_r = inventory holding cost per dollar-year

ν = purchase price of merchandise

r = transportation rate

C_t = in-transit inventory carrying cost per dollar of merchandise per day in transit.

Apart from the transformation of annual cost into cost per day, the only major difference between C_r and C_t are due to the inclusion of direct storage cost in the former, and eventual damages (or loss) in the transit in the latter. However, the three most overwhelmingly important determinants of C_r and C_t are depreciation, interest and obsolescence.[1] Depreciation and obsolescence are more intense the shorter the life of the product. The relation between C_r , C_t and product perishability, then, may be written:

$$C_r \; , \; C_t \; = \; \frac{1}{\text{life-time of the product}} \tag{4.2}$$

That is, the shorter the life-span of the product, the higher will be the value of C_r and C_t in the model. Consequently, the more perishable the product, the higher will be the value of H and z , and vice-versa. And H and z will also be higher (lower), the higher (lower) is the value of the product, because interest expenditures are determined by this variable.

From equations (3.10), (3.15) and (3.14) in Chapter III, it may be seen that a high value of H and z will have the following *simultaneous* effects on demand:

1) Increase the influence of the linear components of "transit-time induced costs," because $C_t^* = zM$; and

2) Decrease the influence of non-linear cost components, because as the expression for Q* (3.15) is divided by H , the larger the value of the denominator the smaller will be the value of the resulting expression.

The opposite is also true.

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[1] See section on Inventory-Holding cost in Appendix A.

Because ΔM is only an approximation for both types of cost, it may be generalized, as a consequence of this discussion, that the logit model will provide a better approximation of the true probability function *when the product is highly perishable* than when it is not, and that a high product value will add to this effect. In other words, the more durable the product, the more relevant will be consignee's inventory considerations to modal choice, the worse will be the violation of the logit's assumption of linearity and, consequently, the less accurate estimations obtained through the use of this model.

As a result of this conclusion and following Fleiss' argument (Chapter III), it is possible to advance two propositions whose validity may be verified by a review of previous empirical works dealing with transport demand estimation through the logit model. In short, it is expected that obtained results should demonstrate that:

Proposition 1:

Perishable products should provide consistently better estimates than non-perishable (durable) goods. The (rare) exception is to be expected when the absolute value of ΔM is close to zero, that is, when there is not much difference between modal transit times. Therefore,

Eventual inconsistencies in estimated coefficients (insignificant and/or wrong sign) should be obvious more frequently in estimations involving durable goods as compared to non-durables.

Proposition 2:

Eventual inconsistencies should be connected with non-linear variables, especially transit-time.

In the next section, a review of empirical studies is undertaken in order to verify the validity of these assertions.

Review of Empirical Evidence

Demand estimations using the logit model on disaggregated data are not plentiful. The following review includes some of the most significant published works in this area.

Miklius, Casavant and Garrod (1976)

The authors have used a logit model to estimate elasticities and cross-elasticities of freight transport demand for apples and cherries shipments in three Northwestern states of the U.S.A.[2] The general specification of the logit model in the study was as follows:

$$\log [p/(1-p)] = \alpha + \beta_{1i}C_i - \beta_{1j}C_j + \beta_{2i}T_i - \beta_{2j}T_j + \beta_{3i}VT_i - \beta_{3j}VT_j + \sum_{k=1}^{t} Y_k S_k$$

where $\log [p/(1-p)]$ is the log of the odds-ratio in favor of

[2] Walter Miklius; Kenneth L. Casavant; and Peter V. Garrod, "Estimation of Demand for Transportation of Agricultural Commodities," in A.J.A.E., Vol. 58, No. 2, May 1976, pp. 218-223.

Mode i being chosen; C_i , C_j : direct transportation costs of modes i and j , respectively; T_i , T_j : their transit-times; VT_i , VT_j : their transit-time variances; and $\sum_{k=1}^{t} Y_k S_k$: shipment characteristics considered of importance for modal choice.

Variables included in the cherries estimation were: freight charges, transit-time, age of pack, and gross value of the merchandise. For apples, these variables were: freight charges, transit-time and a proxy for variance in transit-time. For the latter, two alternative formulations were used: expected delay and expected cost of delay. Both provided equivalent results.

Overall performance of the models, for both products, was considered highly satisfactory. In order to assess the overall explanatory power of the model, the likelihood test was used to test the hypothesis that all coefficients were equal to zero. This hypothesis was rejected in both runs, with χ^2 values that were way above the critical value of 18.55 for six degrees of freedom. The χ^2 for cherries was 301.42 and, for apples, 290.19. Estimated coefficients and elasticities obtained in this study are reported in Tables 4.1 and 4.2 ahead.

A summary of the results reveal that:

1) In the cherries example, all coefficients had "a priori" expected signs and were significant at the 1 percent level using the normal approximation; and

2) Results for apples shipments were not as uniformly consistent: freight charge coefficients had expected signs and were highly

Table 4.1 -- Elasticities and Cross-Elasticities of Demand for Rail and Truck Transportation of Cherries

Variable	Coefficient	*t*-Ratio	Elasticity or Cross-Elasticity for Rail Service	Elasticity or Cross-Elasticity for Truck Service
Rail Freight Charges	-0.0161	5.6701	-2.8696	4.1466
Truck Freight Charges	0.0040	2.8060	1.1484	-1.6594
Rail Transit Time	-2.3438	6.6594	-5.5678	8.0454
Truck Transit Time	2.7822	4.9945	4.1194	-5.9524
Age of Pack	-0.3775	5.5102	-0.4660	0.6732
Gross Value	-0.0006	5.0270	-1.3133	1.9008
Constant	6.4788	6.6264	--	--

Source: Walter Miklius; K.L. Casavant; and P. Garrod, "Estimation of Demand for Transportation of Agricultural Commodoties," A.J.A.E., Vol. 58, No. 2, May 1976. Table 1, p. 220.

Table 4.2 -- Elasticities and Cross-Elasticities of Demand for Rail and Truck Transportation of Apples

Variable	Coefficient	*t*-Ratio	Elasticity or Cross-Elasticity for Rail Service	Elasticity or Cross-Elasticity for Truck Service
Rail Freight Charges	-5.9088	6.7364	-12.5702	3.5455
Truck Freight Charges	4.7388	6.7960	10.0454	-6.4396
Rail Transit Time	0.7106	2.1924	2.7252	-0.7686
Truck Transit Time	-0.4670	0.7559	-1.1759	0.3317
Expected Delay-Rail	-0.7682	2.7636	-0.5272	0.1811
Expected Delay-Truck	9.9354	1.6899	0.1778	-0.0501
Constant	0.4460	0.2637	--	--

Source: Ibid., p. 221.

significant; variability of transit time had expected sign and were (less) significant at the 5 percent level; but rail transit time had a *positive (unexpected) sign* and was significant at 5 percent level, and truck transit time was *insignificant statistically*.

The authors concluded that:

> "The results for cherry shipments are rather encouraging. All estimated coefficients had a priori expected signs and were highly significant. Overall, performance of the model in explaining modal choices was highly satisfactory. The results for apple shipments are less satisfactory. It is difficult to provide a satisfactory explanation for the positive sign of the rail transit time coefficient and statistically insignificant truck transit time coefficient. Incomplete specification of the model is one possible explanation. The missing interrelationship between inventory and transport mode decisions is another. Inventory considerations as well as the decision where to buy may be closely interrelated with the choice of the transport mode." [3]

With regard to the propositions advanced in the last section, the results obtained in this study are exactly as expected. The only major differences between cherries and apples as "abstract commodities" are perishability and value/weight, and it was shown that:

1) Highly perishable cherry shipments *did* provide better estimates than the more durable apple shipments (Proposition 1). Although both products resulted in models with high overall explanatory power, in the cherries example, *all* variables under study were significant at the 1 percent level, while apple shipments produced inconsistent and insignificant coefficients in theoretically important variables.

2) The inconsistent coefficients were found in connection with the variable transit-time (Proposition 2).

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[3] Ibid., p. 221.

Watson, Hartwig and Linton (1974)

The authors also used a logit model to estimate freight demand of a *large household appliance* for truck and rail shipments on inter-city movements.[4] Their objectives in this study were: a) estimation, through logit models, of the most influential factors in shippers' modal choice decisions; b) evaluation of the predictive ability of these models; and c) definition of coefficient values and elasticities connected with major factors. Their data were obtained from copies of freight bills, and the total sample included 485 rail and 134 truck shipments. Their general model was specified exactly as in the study of Miklius, Casavant and Garrod described in the preceding section. Three alternative formulations (hypotheses) of the model were used: the first, including only direct cost differences (ΔC) and transit-time differences (ΔT); the second, including ΔC , ΔT , and relative mode reliability (ΔR); and finally, including all three variables *plus* value of the commodity (V). Estimated coefficients and measures of association obtained from the three formulations are reported in Table 4.3 ahead.

These results indicate that the best model was the one including all variables (Hypothesis 3), but that transit-time was never statistically significant in any of the model formulations, while all other variables were significant at the 99 percent level. In order to verify the

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[4] P.L. Watson; J.C. Hartwig; and W.E. Linton, "Factors Influencing Shipping Mode Choice for Intercity Freight: A Disaggregate Approach," Proceedings, Fifth Annual Meeting, T.R.F., Vol. XV, No. 1, October 1974, pp. 138-144.

Table 4.3 -- Estimated Coefficients and Measures of Association Between Different Variables and the Odds-Ratio in Favor of Rail Choice

Hypotheses	Constant	ΔC	ΔT	ΔR	V	Likelihood Ratio
<u>Hypothesis 1</u>	-0.715	-0.010	-0.053			169.10
(Standard Error)	(0.232)	(0.001)	(0.032)			
"t"	[-3.08]	[-7.6]	[-1.68]			
<u>Hypothesis 2</u>	-0.808	-0.010	-0.025	-0.47		181.473
(Standard Error)	(0.236)	(0.001)	(0.033)	(0.128)		
"t"	[-3.42]	[-8.03]	[-0.75]			
<u>Hypothesis 3</u>	-8.571	-0.013	-0.020	-0.446	0.065	334.744
(Standard Error)	(0.894)	(0.002)	(0.040)	(0.168)	(0.007)	
"t"	[-9.59]	[-6.10]	[-0.497]	[-2.65]	[9.146]	

Source: P.L. Watson; J.C. Hartwig; and W.E. Linton, "Factors Influencing Shipping Mode Choice for Intercity Freight: A Disaggregate Approach," in <u>Proceedings</u>, Fifth Annual Meeting, T.R.F., Vol. XV, No. 1, October 1974. Table 3.1.1, p. 141.

impact of the correlation between ΔT and ΔR (-0.40) in the model, additional runs were made in which alternatively either variable was deleted. The results indicate that:

> "the equations with either ΔT or ΔR deleted are indistinguishable from the equation including both of these variables. Comparing the formulations with either ΔT or ΔR deleted, ΔT is still statistically insignificant and has a large standard error term while ΔR remains significant."[5]

A test designed to verify the accuracy of the "best" model (Hypothesis 3) as internal predictor revealed an excellent internal predictor. The expected number of truck shipments predicted in the test was:

Actual:	67.0
Predicted:	66.91
Error:	-0.09

About the insignificance of transit time in the decision process for modal choice, the authors revealed that:

> "The statistical insignificance of ΔT is peculiar inasmuch as we had hypothesized it to be important and also since other researchers have found it to be important. The reason for its lack of significance would appear to be due to the activities of the firm from which we obtained our data and to the commodity types under our examination."[6]

and, about the effects of commodity type on ΔT (considered an important reason):

> "The relatively low commodity value per hundred weight as compared to other consumer durables, e.g., electronic products, makes the inventory carrying costs low enough to make extensive regional warehousing practical when weighted against the alternative of the high transportation costs which would have to be incurred in order to facilitate the frequent shipping of small orders which would be

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[5] Ibid., p. 142.

[6] Ibid., pp. 140-141.

> necessary to maintain a small inventory at the manufacturing plant site."[7]

While this argument is certainly true, it is unlikely that it would amount to a major factor in reducing the significance of transit time in the demand equation. Although interest rate (the major value-related cost in inventory holding) is indeed an important cost of inventory holding, the major portion of expenses in an average inventory are related to combined cost of obsolescence, depreciation, handling and distribution, insurance and storage facilities.[8] All these factors tend to *increase*, not decrease, the value of transit time. Besides, the usually high cost associated with shortages (loss of business opportunities, of customer's goodwill towards the firm, etc.), especially with wholesalers, would tend to reinforce this effect.

In essence, for our particular interest, this study has shown that:

1) Unexpected results were found in freight demand estimation of a durable good through the logit model (Proposition 1); and

2) The insignificant coefficient *was* connected with transit-time (Proposition 2).

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[7] Ibid., p. 141.

[8] Within reports average interest expenditures in typical inventories in the U.S.A. to be approximately 24 percent of total carrying costs. The bulk of the remaining expenses are due to the remaining (mentioned) factors. Thomas M. Whitin, The Theory of Inventory Management, Princeton University Press, Princeton, N.J., 1957, p. 221.

Johnson (1976)

The author estimated freight demand for grain shipments with data accumulated from grain elevators in Michigan during the year of 1973 in an attempt to associate service quality characteristics with modal demand for transport services.[9] His objective was to identify the most important service characteristics affecting levels of modal (rail) utilization, and infer usage implications from changes in these factors. In the study, transportation demand was treated as a derived (product) demand and two alternative regression models were formulated: 1) an ordinary derived demand model relating quantity of services demanded to explanatory variables; and, 2) a logit formulation of the same function aiming at defining the probability in favor of using rail services. Ordinary least squares were used, then, to estimate regression coefficients under an index (log-linear) and logit formulations. These formulations were:

$$x_r \ , \ 1_n\left(\frac{Pr}{1 - Pr}\right) = dr(S, T, R, D, Sp_r, A_r, A_m, V_r, V_m, L_r, L_m, B)$$

where:

x_r = quantity of rail service demanded (tons)

P_r = proportion of grain shipped by railroad during the year

S = an indicator of firm size (1,000 bushels storage

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[9] Marc A. Johnson, "Estimating the Influence of Service Quality on Transportation Demand," A.J.A.E., Vol. 58, No. 3, August 1976, pp. 496-503.

units)

T = 1 if the firm owns a truck, and = 0 if not

R = 1 if the firm owns or leases a rail car, and = 0 if not

D = average distance of rail shipments (miles)

Sp_r = average railroad speed (miles per day)

A_r = availability of rail cars (average days of equipment delay)

A_m = availability of motor equipment (average days of equipment delay)

V_r = number of promotional visits by railroad firms

V_m = number of promotional visits by trucking firms

L_r = average value of damage in rail transit per $1,000 value

L_m = average value of damage in truck transit per $1,000 value

B = proportion of total shipments intended for railroad but diverted to trucks for lack of railroad cars.

Estimated results of the two models are summarized in Table 4.4 ahead.

An analysis of the results depicted in Table 4.4 demonstrates that just like with the two previously mentioned works, some of the variables' estimated coefficients were inconsistent with what should be expected from the theoretical formulation of both models. In the

Table 4.4 -- Regression Results for Railroad Demand and Railroad Selection Probability Estimators

Demand Determinant	Railroad Demand	Railroad Selection Probability
Constant	11.0332 (1.5893)[a]	2.1350 (0.9127)
Market Distance	-0.0020 (0.0023)	0.0029 (0.0014)
Storage Capacity	0.0013 (0.0005)	-0.0037 (0.0009)
Truck Ownership	-0.3917 (0.8206)	-1.2669 (0.5653)
Rail Car Leasing	0.7434 (0.7577)	-0.3925 (0.4692)
Tonnage Diverted to Truck (%)	0.0029 (0.0035)	0.0016 (0.0041)
Truck Equipment Delay		-0.4129 (0.1565)
Rail Equipment Delay	-0.0562 (0.0298)	-0.0007 (0.0201)
Damage by Truck		-0.1894 (0.1628)
Damage by Railroad	-0.0125 (0.0054)	0.0252 (0.0157)
Truck Promotional Effort		-0.0671 (0.0197)
Railroad Promotional Effort	-0.0145 (0.0360)	0.5510 (0.1342)
Railroad Speed	-0.0043 (0.0079)	-0.0294 (0.0054)
Standard Error	0.9751	0.4664
$\bar{R}^2$	0.451	0.744
F	$F_{9,10} = 3.163$	$F_{12,7} = 6.626$

[a] Standard Errors of coefficients are given in parenthesis.

Source: Marc A. Johnson, "Estimating the Influence of Service Quality on Transportation Demand, A.J.A.E., Vol. 58, No. 3, August 1976. Table 1, p. 501.

author's words:

> "Other service quality results are less intuitively appealing. The faster is railroad service or the less reliable is delivery of trucking equipment, the smaller the relative amount of railroad service used, *ceteris paribus*. The relative preference for slower service may indicate that the value of railroad cars as storage containers for shipments of undetermined destination exceeds the loss of inventory charges on grain owned during transit. Reasons for inverse relationship between delay in motor equipment delivery and the ratio of rail to motor carriage used are unclear but may be related to multicollinearity."[10]

Without entering the merit of the advanced explanation for the inverse relationship between delay in motor equipment and rail usage, again it must be insisted that because direct storage costs are such a small proportion of inventory holding costs it is unlikely that the "inventory-in-transit" argument would justify the wrong sign in the variable "railroad speed."[11] On the other hand, transit time is, by definition, equal to (distance/speed), and estimated coefficients for these two variables were (Table 4.4) 0.0029 and -0.0294, respectively. As the denominator of the fraction is negative, it follows that if the variable transit-time had been estimated directly in the model it would present also an (inconsistent) negative sign. Thus the expected results enunciated in our Propositions 1 and 2 are once more obtained in an applied study.

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10 *Ibid.*, p. 502.

11 Whitin, *op. cit.*, p. 221.

Daughety and Inaba (1977)

Two very interesting recent works directed at estimating freight demand by logit models on disaggregated data were prepared by Daughety and Inaba (1977) in order to evaluate grain movements (corn) to markets in Midwest U.S.A.[12,13] These studies are result of an (still ongoing) intense effort in the area by the authors as members of The Transportation Center at Northwestern University, which introduced a number of interesting conceptual and methodological approaches to demand models.[14] The authors developed a model based on the theory of the firm which places freight demand as a derived demand for goods and as function of spatial distribution of production centers and markets. Therefore, this model aims at explaining "not only the interrelationship among goods prices in geographically separated markets, but also to explain the interdependence between spatially separated markets and the transport sector" (p. 1).

The general model is specified in terms of a "typical" shipper's profit function:

$$\Pi(q_{11}, \ldots, q_{jm}) = \sum_{j}\sum_{m} [P_j q_{jm} - t_{jm} q_{jm} - H_{jm}(q_{jm})] - C(q)$$

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[12] A.F. Daughety and F.S. Inaba, "Empirical Aspects of Service-Differentiated Transport Demand," Working Paper No. 601-77-11, The Transportation Center, Northwestern University, Evanston, Ill., 1977.

[13] A.F. Daughety and F.S. Inaba, "Estimation of Service-Differentiated Transport Demand Functions," Working Paper No. 601-77-16, The Transportation Center, Northwestern University, Evanston, Ill., 1977.

[14] For other (previous) studies by the authors, consult the Bibliography and the literature reviewed in Chapter I.

where:

$\Pi(q_{11}, \ldots, q_{jm})$ = profit associated with quantities shipped by market-mode pairs

P_j = price of the product in market $j = 1, \ldots, J$

t_{jm} = transport rate to market j by mode m, $j = 1, \ldots, J$; and $m = 1, \ldots, M$

q_{jm} = quantity shipped to market j by mode m

$H_{jm}(q_{jm})$ = service-induced transport cost of shipping q_{jm}

$C(q)$ = cost of producing $q = \sum_j \sum_m q_{jm}$

As the shipper is expected to choose non-negative q_{jm}'s so as to maximize $\Pi(q_{11}, \ldots, q_{jm})$, the profit function is maximized:

$$P_j - t_{jm} - H'_{jm}(q_{jm}) - C'(q) = 0$$

where:

$$q = \sum_j \sum_m q_{jm}$$

Although recognizing that the service-induced cost function (H_{jm}) is in general strictly convex (because it includes a strictly convex risk function), the authors argue that because elevators under study typically ship only a fraction of their holdings at a time, to assume a linear approximation for the risk function is a reasonable assumption. This assumption, therefore, in their contention, makes demand-estimation through the logit model adequate for the particular case under study. After this justification, two alternative formulations of the logit model (net-price and revenue) were estimated through the general specification:

$$P_r\{y = (j,m)/P(n),\ t(n),\ A(n)\} =$$

$$\frac{e^{\alpha_{jl}P_j(n)+\alpha_{jm2}t_{jm}(n)+\alpha_{m3}A_m(n)}}{\sum_j\sum_m e^{\alpha_{jl}P_j(n)+\alpha_{jm2}t_{jm}(n)+\alpha_{m3}A_m(n)}}$$

where:

$P_r\{y = (j,m)/P(n),\ t(n),\ A(n)\}$ = the odds-ratio in favor of choosing the (j,m) market-mode pair;

$P_j(n)$ = price or revenue (i.e., price × quantity) at the j-th market;

$t_{jm}(n)$ = transport rate or cost (i.e., rate × quantity) of shipping to the j-th market by the m-th mode; and,

$A_m(n)$ = perceived availability cost per bushel or per shipment (i.e., per bushel cost × quantity) of shipping by the m-th mode.

A summary of results obtained from estimating the logit model in the first study (Footnote 12) is reported in Table 4.5 ahead. In this study, two alternative specifications of the model were used in independent runs: the first computed only data from truck and single-car-rail (SCR) users, while the second run computed data from the users of these two alternatives, plus of multiple-car-rail (MCR)

Table 4.5 -- Estimated Logit (Probability) Coefficients and Summary Statistics for Corn Shipments to Markets in Midwest U.S.A.-- (First Paper)

Run, Summary Statistics	PRICE		TRUCK		SCR		MCR		
	River	Local	River	Local	River	Local	River	Local	Availability
RUN #1	0.0007764857 (2.35)*		-0.00960727 (-4.3)		-0.005135358 (-3.45)		----		-0.07392352 (-2.52)
		0.0006308472 (1.79)		-0.009930996 (-2.62)		-0.0006584478 (-1.81)		----	
LRI: 0.29 DF: 227 %: 78.27									
RUN #2	0.0006665327 (2.41)		-0.01052284 (-4.67)		-0.005925606 (-3.99)		-0.01332888 (-4.21)	----	-0.06042358 (-2.24)
		0.000438601 (1.49)		-0.008076278 (-2.39)		-0.0005457282 (-1.73)		-1 (-0.001)	
LRI: 0.41 DF: 416 %: 80									

LRI: Likelihood Ratio Index
DF: Degrees of Freedom
%: Percentage of Right Choice
* t-values for the coefficients are shown in parentheses

Source: A.F. Daughety and F.S. Inaba, "Empirical Aspects of Service-Differentiated Transport Demand." Working Paper No. 601-77-11, The Transportation Center, Northwestern University, Evanston, Ill.: 1977. Table 2, p. 20.

service. Asymptotic t values for the variables are placed between parentheses, and summary statistics are reported in the first column.

These results show that all variables had expected signs as long as a type I error of 0.07 ($t \approx 1.79$) is acceptable. The first model provided the right choice 78% of the time, while the second model predicted the right choice 80% of the time.

In the second paper (Footnote 13), both net-price and net-profit formulations were estimated, allowing for two modes (truck and single-car-rail) and two markets (River and Local). Estimated coefficients and measures of association are described in Table 4.6 ahead.

Results reported in this study are noticeably better than those described in the preceding table. Run #1 (net-price formulation) predicted the correct choice in 90% of the cases, while Run #2 (net-profit formulation) predicted the right choice 82% of the time. The Likelihood Ratio Index for both runs were 0.6865 and 0.4028, respectively. In spite of an apparently higher overall explanatory power for the first run, by comparing the t values of the variable "product price" in both runs the authors have detected an extensive market imperfection in terms of bid negotiation, which invalidated the use of results reported in Run #1. Therefore, the analysis presented in the rest of the paper was based in the results of Run #2.

A closer look at these results reveals that all variables had the right sign and were highly significant. This fact by itself indicates their relevance to the modal choice decision process. Two facts, however, must be pointed out:

Table 4.6 -- Estimated Logit (Probability) Coefficients and Summary Statistics for Corn Shipments to Markets in the Midwest U.S.A.--(Second Paper)

Run, Summary Statistics	PRICE		TRUCK		RAIL		
	River	Local	River	Local	River	Local	Availability
Run #1 NET-PRICE	2.626 (1.046)*	3.176 (1.193)	-33.21 (-3.889)	-64.63 (-4.491)	-16.74 (-3.547)	-25.29 (-3.410)	-457.5 (-2.394)
%: 90 LRI: 0.6865							
Run #2 NET-PROFIT	0.00141 (3,412)	0.00131 (2.945)	-0.009604 (-3.925)	-0.01282 (-3.297)	-0.004848 (-3.635)	-0.001574 (-3.060)	-0.06695 (-1.951)
%: 82 LRI: 0.4028							

%: Percentage of Right Choice
LRI: Likelihood Ratio Index
* t-values for the coefficients are shown in parentheses

Source: A.F. Daughety and F.S. Inaba, "Estimation of Service-Differentiated Transport Demand Functions." Working Paper No. 66-77-16, The Transportation Center, Northwestern University, Evanston, Ill.: 1977. Table 2, p. 19.

a) While all variables related with product price and direct cost of transportation were significant at at least the 99.5% level, the sole representative of service quality (cost of equipment availability) fared less well: it is significant at 95% but insignificant at the 97.5% level; and

b) the relatively high proportion of misclassifications (18%) of the model suggests that specification of additional service-quality variables could improve significantly the explanatory power of the model, and consequently, its predictive ability.

The authors then proceed to estimate demand functions and approximate equilibrium quantities of grain which were expected to move by different market-mode alternatives. For this, an ingenious technique involving the generation of a posterior on shipment size conditioned on alternative choice from a prior shipment size and the estimated choice model was developed and used. The resulting expected posterior combined with industry supply functions provided demand equations. This methodology and results, while interesting as a procedure of potential value for future estimations, exceed our present analytical needs, and therefore will not be further reproduced here.[15]

Among the valuable contributions of these working papers are:

1) The general model is firmly anchored in spatial equilibrium and the theory of the firm, a fact which permits introducing specific

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[15] A.F. Daughety and F.S. Inaba, "Estimation of Service-Differentiated Transport Demand Functions," Working Paper No. 601-77-16, The Transportation Center, Northwestern University, Evanston, Ill., 1977, pp. 20-27.

variable modifications (or additions) without re-thinking the whole model;

2) The proposed technique for obtaining demand estimations from probability coefficients may prove very useful in studies concerned with the prediction of the flow of goods to various markets through different modes;

3) Although justifying the use of the logit model for this particular application, the authors recognize the potentially serious effects which convexity in service-induced transport costs would have in freight demand estimations through techniques such as the logit model;[16] and

4) Estimated results confirm the importance of commodity price, transport cost, and equipment availability as factors which deeply influence market-modal choice decisions.

In terms of collecting evidence regarding our propositions about expected inconsistencies in transit-time, unfortunately the absence of this variable in the model's specification results in inconclusive evidence in either direction. This exclusion is particularly unsettling when the authors recognize early in the paper that transit-time-induced costs are "central to the theory of transport demand as a derived demand" and also that collected data "contained information on quantity shipped, mode, contract price, transit time, transport rate, who paid the transport, destination, expected travel time, date

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[16] Ibid., pp. 6-11.

of contract commitment and shipment due date," among others.[17] This, however, may be due to their apparent (primary) interest in estimating the equilibrium grain quantities shipped to different markets, by different modes, rather than to address the more specific question of the influence of service characteristics in modal choice and demand.

In any case, the possibility that inclusion of expected transit-time-induced costs (of which equipment availability cost is a component) into the model's specification under a linear form would provide inconsistent results cannot be dismissed at this point, in spite of the justification for the risk linearity assumption made by the authors, especially in face of the results obtained in other studies described previously in this section.

Conclusions and Implications from the Evidence

Considering the relative paucity of published empirical works employing the logit model to estimate freight demand using disaggregate data, the studies reviewed in the preceding section might well constitute not only the most significant works in this area but a good part of the literature available on the subject as well. The products involved were cherries (seasonal, perishable), apples (seasonal, semi-perishable), grains (seasonal, semi-durable), and large household appliances (durable). As far as one can tell, these studies were conducted carefully and made use of the best data available for this

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[17] Ibid., pp. 4 and 16, respectively.

type of work. Their results, as should be expected, are not strictly comparable since their data referred to different products, were collected in different parts of the United States at different times, and different model specifications were followed according to individual researchers' priority objectives. However, with respect to expected results enunciated in Propositions 1 and 2, the preceding review clearly suggests a pattern in the findings connecting inconsistent logit coefficients (insignificant and/or wrong sign) with product perishability and that, also as expected, these inconsistencies were found associated with the variable transit-time.

The best evidence in support of the advanced propositions were found in Miklius, Casavant, and Garrod (1976), because these authors worked simultaneously with a perishable (cherries) and semi-perishable (apples) product. As predicted, the "cherries" estimation produced a model with high explanatory power and all variables presented correct sign and were significant, while the "apples" example, although displaying good explanatory power, resulted in wrong sign (rail) and insignificant (truck) coefficients for the variable transit-time.

Estimating freight demand for a durable (manufactured) good, Watson, Hartwig, and Linton (1974) confirm these results. In spite of being an excellent internal predictor, their model resulted in insignificant coefficients for transit time in three different formulations, in spite of the authors having hypothesized "a priori" this variable to be important.

Further evidence in support of the advanced propositions were obtained in the work of Johnson (1976), who estimated freight demand for a semi-durable (grain) commodity in Michigan. Using a model specification which was different from that of the two preceding works, the author found the coefficient for railroad speed to be negative, which led to the conclusion that transit time also would present the wrong sign had it been estimated directly.

Finally, Daughety and Inaba (1977) estimated freight demand for corn shipments to markets in the Midwest U.S.A. Because the variable transit-time was not included in their model's specification, no conclusion could be made with respect to either of the two propositions. However, the possibility that inconsistent results would be obtained in case of the inclusion of transit time also could not be discarded "a priori."

In summary, of the five products studied in the four reviewed articles, evidence was found in four of them supporting the argument that the logit function misrepresents systematically the true probability function in case of non-perishable products, and that this misrepresentation is relevant to transportation demand estimations. No evidence against this argument was found in any of them. The one inconclusive result might have been due to non-specification of transit time in the model.

Although such a pattern does not constitute proof of correctness of the advanced propositions, it certainly raises reasonably credible (albeit circumstantial) evidence in their favor. This fact may justify careful acceptance of the model which led to their formulation, at least until further studies are conducted to verify these findings.

In any case, acceptance of these conclusions implies that estimating transport demand through the present (linear) formulation of the logit model will probabily produce theoretically inconsistent coefficients, which may give origin to wrong or misleading policy recommendations.

Suggested Model Improvement and Ideas for Testing

A number of different approaches are open for improving currently used models of transportation demand and/or developing new ones. Conclusions of the preceding analysis suggest that the catastrophe theory-inventory theoretic approach may be of substantial help in defining and testing some of these possibilities, some of the most intuitively appealing of which, in our opinion, will briefly be explored in this section.

While pursuing more extensively any of these possibilities clearly exceeds stated goals of this analysis, the suggestions are offered in the expectation of their being useful for future works dealing with the subject. Some of them imply in improving the logit model by introducing modifications in its specification and/or general formulation, while others purport to use the properties of catastrophe theory to simplify complex (but structurally complete) theoretical demand models to be estimated by other statistical techniques.

A note of caution is in order at this point: because these suggestions have not as yet been developed to the point where they could be applied to practical estimations, it is not possible to foretell whether or not they would encounter conceptual and/or estimation problems further along the line.

Pre-Classification of Observations

One of the most interesting possibilities for improving the logit model stems from the precise delimitation of the singularity in the behavior space (the probability function) provided by the catastrophe theory-inventory theoretic approach. This fact may be inferred by observing again Figures 3.9 and 3.11 in Chapter III. Figure 3.9 shows that as the absolute value of ΔM increases, so will the cost divergence increase between the modes (curve $KD_1/CP/KD_2$ in the graph) with a consequent increase in the overlapping sheets of $P(x)$, that is, of the area where either mode could be chosen rationally in the economic sense.

Figure 3.11 which may be considered as a two-dimensional cut parallel to the ΔCS axis in Figure 3.9, demonstrates the manner by which the logit probability function $[P(x)_L]$ approximates the "true" probability function $[P(x)_{CT}]$. As may be seen in this figure, for any ΔM different than zero, points λ_1 and λ_2 delimit the range of ΔCs within which the logit model is most likely to misclassify the observations, while outside this range the logit probability curve approximates reasonably well $[P(x)_{CT}]$. As it has been advanced in previous sections that this logit misclassification of observations is the most likely source for inconsistent coefficients found in previous estimations, it is possible, at least conceptually, that a pre-classification of cases according to this criterion might be used to eliminate the inconsistencies and obtain more reliable coefficients for the non-linear variables without modifying the logit model "per se."

Points λ_1 and λ_2 may be defined through the formula for the bifurcation set (curve $KD_1/CP/KD_2$ in Fig. 3.9), which *is independent of quantitative properties of the model*, and thus provides the boundaries for the pre-classification of observations. The bifurcation set is defined by Equation (3.23), Chapter III:

$$27(K_1 \Delta C_s)^2 = 4[K_2(\Delta M)^2 + K_3]^3 \tag{3.23}$$

where K_1, K_2, and K_3 are constants defined statistically from the sample for any observed value of ΔM. Therefore, λ_1 and λ_2 will be the roots of the quadratic expression (ΔM is given):

$$(\Delta C_s)^2 = \frac{4[K_2(\Delta M)^2 + K_3]^3}{27K_1^2} \tag{4.3}$$

or,

$$\Delta Cs_2 = \lambda_2 = \sqrt{\frac{4[K_2(\Delta M)^2 + K_3]^3}{27K_1^2}} \tag{4.4}$$

$$\Delta Cs_1 = \lambda_1 = -\sqrt{\frac{4[K_2(\Delta M)^2 + K_3]^3}{27K_1^2}} \tag{4.5}$$

An implication of this definition of λ_1 and λ_2 is that the mere elimination of observations falling between the two boundaries should provide a better approximation of actual variable coefficients than those estimated with the available observations. Depending on the objectives of the study, it is conceivable that this approach may offer an opportunity to test specific hypotheses about freight demand, including some aiming to test the validity of some of the catastrophe theory-inventory theoretic model's assertions.

However, as an operational model, the shortcomings of this approach are obvious: 1) all thusly estimated coefficients will refer only to the part of the cases outside of λ_1/λ_2 . Although it is expected that in general those will be the majority of cases, estimated coefficients will not be applicable to the universe of observations, because those located within this range should present dramatically different coefficients, and 2) in a sense, the effect of this shortcoming is more severe in the case of predictions, since observations within this range are supposedly more sensitive to parameter changes in terms of modal shift than the remaining cases.

Therefore, before this pre-classification criterion may be used in actual estimations, a system must be devised for the treatment of observations with the λ_1/λ_2 range, if the major interest of the study is to provide policy recommendations.

Pre-Treatment of Variables

Another possibility open to improve logit model's estimations lies on weighting observed variables according to their functional form in structural models (such as the inventory-theoretic model) prior to their processing. In this case, the functional definition of the logit model would better approximate that of a more precise theoretical model, with a likely improvement in both global explanatory power and significance of estimated coefficients. Although this modification still would not account for the effects of the Delay Rule the results might be considerably better than using the variables without previous treatment.

In practice, what could be done is to trace the approximate functional form of the variables under consideration with respect to the dependent variable in a theoretically complete model, and then multiply observed variables by a factor which approximates this relationship. After this step, proceed with the logit analysis as usual.

Direct Modification of the Logit Model

One of the more promising angles for improving the logit model would be to explore the possibility of relaxing its assumptions to allow for modifications demanded by economic theory. In essence, these modifications would comprise:

1) Allowing for non-linear variables; and

2) Modifying the classification criteria so as to allow, as correct choice, alternative model selection up to λ_1 or λ_2 whichever is the mode actually selected by the shipper in that case.

If these modifications are proven to be feasible, it would be possible to specify the logit model according to the formulation of more complex models, such as the inventory-theoretic, a fact which would most likely improve the estimation accuracy of the model. Although conceptually possible, this expansion of the logit model might take considerable effort to develop, particularly because the assumptions about the stochastic error terms would have to be re-examined, in order to verify if some bias would be introduced in the model through the proposed modifications.

Catastrophe Theory Model

A possibly more rewarding, but also more challenging, alternative would be to estimate freight demand directly through a model such as the ones described in Chapter III. The concept of qualitative equivalence derived from deep mathematical theorems of catastrophe theory asserts that *locally* the phenomena described in this analysis must be equivalent to the general description of Figure 3.9, and therefore the canonical formulation of the cusp catastrophe may be fitted to actual data for analysis and prediction purposes with appropriate statistical techniques.

A major advantage of this type of approach is that because such a model would be equivalent to more complex structural models, analyses and conclusions based on them would be automatically backed by fundamental tenets of economic theory as well as by statistical evidence. This fact should provide a firmer ground for policy recommendations in the transportation field, especially those dealing with service characteristics' modifications. In a more practical sense, because of the generic nature of the model, its formulation may be defined so as to provide specific answers to the particular problem at hand. Behavior, for example, may be identified by the (observable) decision variable of major interest for the study--such as annual demand, order quantity, or probability of modal choice, while control parameters may be defined as prescribed by theory according to particular policy questions to be answered. In any formulation, this model is expected to provide more information than alternatives now in use, because it estimates not only association between variables, but also the points where modal shifts are bound to occur.

The most straightforward application of the model is probably to use a (non-linear) regression technique to estimate the behavior space equation. The coefficients thus estimated would then be used to calculate all other borderline values through additional formulas provided in the models of Chapter III.

The behavior surface is determined by a cubic equation on the behavior variable, and therefore will have in this problem, by the theory of equations, either one or three real roots, depending on parameter values. As a consequence, the behavior sheet will have one value outside of λ_1 and λ_2 and three values within that range.[18] The middle-valued root represents the irrational economic option for the shipper, as a point in middle sheet of the behavior space.

One of the most demanding estimation problems that may be anticipated by use of this model is how to define exactly a methodology to estimate a non-explicit cubic equation. However, this should not amount to a very serious obstacle once there is evidence in the literature of existing computer routines to tackle this type of problem, such as the one prepared by R.E. Quandt and others at Mathematica,

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[18] For proof and general discussion of this statement, see A.G. Wilson, "Catastrophe Theory and Urban Modelling: An Application to Model Choice," in Environment and Planning A, Vol. 8, 1976. Appendix, pp. 355-356.

reported by Baumol and Vinod.[19]

[19] W.J. Baumol and H.D. Vinod, "An Inventory-Theoretic Model of Freight Transport Demand," Management Sciences, Vol. 16, No. 7, March 1970, Footnote 10, p. 421.

CHAPTER V

GENERAL SUMMARY, CONCLUSIONS, AND NEW RESEARCH DIRECTIONS

Summary of the Study and Conclusions

This analysis was undertaken in the expectation that catastrophe theory could be used to describe globally the transportation problem and to explore the possibility that a simultaneous treatment of freight demand and modal choice would provide new insights leading to a more comprehensive understanding of the dynamic forces behind transportation decisions. An additional goal of this study was to evaluate what contributions (if any) this new approach could provide to better understand the shortcomings of, and improvement possibilities for, currently used practices of freight demand estimations.

Considering that catastrophe theory is only a mathematical set of concepts devoid of any economic meaning, the first part of the analysis concentrated in laying down the micro-economic foundations of transportation demand as derived product demand. This effort culminated with the development of a theoretically sound and reasonably complete inventory-theoretic model of transportation demand, based on the works of Baumol and Vinod (1970), Das (1974, 1975), and Constable (1972), which is described in Appendix A of this dissertation. This model incorporates all essential demand determinants in a structural manner, including inventory-policy-related variables which are important to transportation decisions.

Taking advantage of the economic assumptions and postulates of the inventory-theoretic model, the transportation problem, then, was reinterpreted through catastrophe theory, resulting in a simplified model permitting simultaneous analysis of modal choice and freight demand in a generic way. Because of the properties of catastrophe theory, this model is expected to be rigorously equivalent (qualitatively) to more complex structural models.

As the logit model has been considered in earlier chapters to be one of the most promising new approaches to estimating freight demand with disaggregated data, the general catastrophe theory model was redefined into a choice model which could be compared analytically with the logit model in the same terms. This comparison resulted in a number of theoretical conclusions and implications about the suitability of the logit model for freight demand estimations, such as:

1) The logit function systematically misrepresents the true probability function for modal choice;

2) The reason for this misrepresentation lies mainly (but not only) on the logit's inability, because of its fundamental assumptions, to describe properly non-linear variables affecting inventory policy of the shipper;

3) The major non-linear variable affecting inventory policy in the model is transit time, and therefore the inclusion of this variable in model specification was a cause for this misrepresentation of the probability function.

A review of the consequences of an improper specification of the objective function of choice models in statistical estimations revealed that:

1) The model would be likely to classify erroneously some of the observations; and

2) Misclassification of a significant number of observations would introduce a number of inconsistencies in estimated coefficients and their measures of association, even in presence of a strong (apparent) overall explanatory power estimated for the model.

In view of these results, the next step of the analysis was to determine under which conditions the logit function would better approximate or depart from the "true" probability function. From the preceding analysis it was made clear that those conditions would depend mostly on whether the variable transit time could be described functionally as a linear variable or not. By tracing back the role of this variable in the profit function of the inventory-theoretic model in Appendix A it was shown that transit time affects the profit function mainly through its influence on inventory-related costs; and that this influence manifested itself through two independent (but simultaneous) components:

1) Cost of inventory in-transit, which is essentially a *linear* component of total transportation cost in the profit function; and

2) Consignee's inventory cost, which was shown to be a *non-linear* component of transportation cost, therefore representing a non-linear influence in the profit-function.

A natural conclusion is that the logit function will get closer to the true probability function the more the *value* of transit-time-induced costs in inventory in-transit exceeds the *value* of these costs on total consignee's inventory cost, and vice versa. As a result of this conclusion, an effort was undertaken to verify what are the major factors influencing the actual value of transit-time-induced costs in both types of inventory. The result of this query indicated that the most important variables in this sense were product characteristics, particularly product perishability, and that product value magnifies the effect of product perishability in the equation. With the help of the inventory-theoretic model it was shown that a high product perishability (and value) has two simultaneous effects in the demand equation:

1) It *increases* the influence of the linear component of transit-time-induced cost; *and*

2) *Decreases* the influence of non-linear components of these costs in the equation.

An implication of these findings is that logit estimations of non-perishable products' transportation demand should provide less reliable (and consistent) estimates when compared with estimations regarding perishable products. In order to corroborate this implication in actual circumstances, two verifiable propositions concerning expected results from actual empirical estimations using the logit model were advanced:

1) "Eventual inconsistencies in estimated coefficients (insignificant or wrong sign) should be more frequently obvious in estimations involving durable goods as compared to non-durables;" and

2) "Eventual inconsistencies should be connected with non-linear variables, especially transit-time."

A review of available literature on empirical studies involving the application of the logit model on disaggregated data was then undertaken, and the results of each study checked against the two propositions. Although none of the results of the studies were strictly comparable, they largely confirmed the predictions contained in the propositions. Of the four studies reviewed, three displayed inconsistent coefficients connected with the variable transit time for semi-perishable and durable goods. One study presented inconclusive results regarding the propositions because of exclusion of this variable; and the only estimation concerning a perishable commodity resulted in consistent and highly significant coefficients for all variables, including transit time.

Considering the results of the present analysis, several exploratory suggestions oriented towards future improvement of transportation demand estimation models were briefly described. These included perfecting the logit model through a pre-classification of observations, model estimation using pre-treated variables, and/or direct modification of the logit's formulation. A fourth suggestion dealt with directly estimating demand functions through the use of a model developed using catastrophe theory.

Among the major general conclusions which may be abstracted from this study are:

1) The transportation problem may be described through catastrophe theory, and this approach promises great potential for

future works in this area. Most of the advantages of this approach stem from its being able to describe simultaneously modal choice and demand for transportation, and from its ability to represent simplified versions of structural models where cause and effect between variables are clearly explicit.

2) Inventory considerations do play an important role in transportation decisions, and transportation demand models must take into account this factor in empirical estimations. In particular, applying the common specification of the logit model to freight demand estimations for non-perishable products will most likely result in unreliable (and probably conspicuously inconsistent) coefficients, which must be used in policy recommendations with utmost caution.

New Research Directions

Crucial decisions in the transportation area must be continuously made by users and policy-makers alike in order to insure fast, cheap, and reliable delivery of merchandise to their intended markets. Conscientious decision-making in this field, as in any other, depends heavily on reasonably complete understanding of the underlying dynamics responsible for transportation service operations.

Policy information needs are not limited, in this area, to the description of short-term interrelationships between modal choice (or transportation demand) and a score of variables intervening in the process. Instead, modeling efforts should be able, in addition, to predict flows of merchandise to different markets by individual modes, allow for discontinuous changes in modal use, permit analysis at the level of aggregation required by the particular need and, perhaps above all, describe the dynamics of the decision process so as to predict (and mold) structural changes in transportation services over time. Despite an impressive volume of work in this area, no single model developed so far provides answers to all these questions as interdependent aspects of the same general problem in a practical manner. Although a number of approaches have been met with considerable success in describing specific situations, the global picture is still to be completed, and policy recommendations based on some of the previous models have been incomplete, unreliable or even frankly erroneous.

Although it is not possible to claim, at this point, that a transportation demand model based on catastrophe theory would be able to answer all the necessary questions, or even to surpass the performance of some of the existing models in practical applications, there are clear indications that some of its characteristics are indeed encouraging in terms of potential development and use. First, this type of model is a *global* model, and therefore considers simultaneously the concomitant problems of demand estimation and model choice, which by necessity are treated only partially or in a step-by-step fashion by other models. The advantages of the former over the latter types of models are that the study of only one side of the problem yields incomplete information, and the step-by-step approach requires complicated lexiographic solution methods which are not only cumbersome to work with but also generally imply in a degree of loss of estimation accuracy.

In the second place, a catastrophe theory model may be defined as a qualitatively accurate simplified version of a *structural model*, that is, it may have its variables specified as *cause and effect* in a functional form prescribed by basic elements of economic theory. This quality should provide for relatively simple statistical manipulation of actual data while still counting with full theoretical backing from already proven principles governing economic decision-making by entrepreneurs in general. And finally, a transportation demand model based on catastrophe theory is essentially a *dynamic model*, that is, it tries to emulate shippers' decision-making as a process in continuous evolution over time as a function of changing parameters. In this sense, catastrophe theory may provide the elements for analyzing transportation demand not only as a result of one optimizing set of decisions but also offer an explanation about the adjustment process between such optimum points. This property not only facilitates aggregate studies in this area but also may be used to explain the process of structural change through which the transportation industry evolves as a whole, which at times occurs with sudden abruptness even though in response to a smooth change in causal variables. It is a certainty that this sort of information would be of great value for policy-makers, especially regulatory agencies in the field.

In conclusion, it is apparent that further studies on application of catastrophe theory to transportation economics merits serious consideration by researchers dedicated to this area. The prospective payoff in terms of benefits from better analyses and forecasts necessary for sound policy decisions seems at this point to be very attractive in view of the amount of research effort needed for constructing and testing an operational model.

As for application of catastrophe theory to other areas of the economic science, the possibilities appear endless. Existing difficulties in adequately describing many types of economic problems could benefit from a modeling technique capable of describing discontinuous behavior caused by continuous factors. Two outstanding examples are the modeling of utilization of natural renewable (but limited) resources and of the path of adoption of new techniques by producers as part of technological change in economic development.

APPENDIX A

AN INVENTORY-THEORETIC MODEL OF TRANSPORTATION DEMAND

Formulation and solution of this model was made in order to provide formal theoretical backing for describing transportation demand under catastrophe theory--the main objective of this dissertation. Advanced justification for selecting this type of model for this purpose stems from their close adherence to principles of micro-economic as well as of inventory theory. Basic conceptual elements for this approach were outlined in Chapter I, and revolve around the argument that businessmen typically make shipping and inventory decisions simultaneously, as parts of a single overall problem.

Model formulation follows closely the theoretical construct of Baumol and Vinod (1970) and later works of Constable (1972) and Das (1974, 1975) in an expanded and somewhat modified version, and allows for stochastic lead-time demand. Inventory control technique is assumed to be of the "fixed order quantity--variable order point" variety, commonly referred to as the (Q, R) model. This assumption adds to model complexity because it requires optimization of three variables endogenously: total demand (A), order quantity (Q), and reorder point (R) as a function of exogenous variables. However, this formulation has the advantage of being frequently encountered in practice and of being relatively simple to manipulate mathematically.[1]

The (Q, R) system assumes that a certain quantity Q is ordered as soon as inventory level falls to the point R . The convenient (but unnecessary) assumption of continuous reviewing policy was also made in model construction.

Definition of Components

Variable List

Labeling of the variables was made according to the following conventions:

D = demand per day; a strictly positive random variable

μ = mean value of D

σ = standard deviation of D

A = expected annual demand

K = set-up cost per order

H = inventory holding cost per year

M = mean lead-time, in days

V = variance in lead-time

L = total demand during lead-time (lead-time demand)

Δp = product price difference between import and export markets

r = freight rate

Q = order quantity

R = reorder point (quantity)

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[1] This and other inventory control techniques may be found in D.A. Barrett. Automatic Inventory Control Techniques. Business Books: London, 1972. pp. 114/132.

ξ = shortage cost per unit

z = in-transit inventory carrying cost

π = shipper's profit

S = expected shortage level

In addition, the following definitions were used:

μ_L = mean value of L = $M\mu$

σ^2_L = variance of L = $M\sigma^2 + V\mu^2$

and,

$F(x)$ = cumulative density function of the standardized variable $x = (L - \mu_L)/\sigma_L$, which is assumed independent of other parameters

$f(x)$ = $F'(x)$

$\phi(L)$ = probability density function of L

$G(x)$ = $F^{-1}(x)$

Profit Function (π)

The profit function is equal to net revenue *minus* the costs of inventory and transportation incurred in transporting merchandise between two places (TC). Net revenue is equal to the price differential between the two markets (Δp) times total demand (A); that is, times the total quantity transported. Correct interpretation of this price differential indicates that relevant prices for the model are purchase price at the *source of supply* and *final price charged to* *consumers* of the shippers, because it is from this margin that the shipper will pay for transportation and inventory expenses, in addition to his profit. Therefore, in the case of a local retailer buying from a wholesaler in another area, Δp will be defined as the difference between local retail price minus wholesale price on the original market. Therefore:

$$\pi = \Delta pA - TC \tag{1}$$

Transportation and Inventory Cost (TC)

Total transportation and inventory cost is composed of: direct shipping cost, *plus* in-transit inventory carrying cost, *plus* the consignee's inventory carrying cost. Or:

$$TC = C_s + C_t + C_I$$

where:

C_s = total direct shipping cost

C_t = total in-transit carrying cost

C_I = total consignee's inventory carrying cost

Direct Shipping Cost

Direct shipping cost is equal to the freight rate *times* total quantity transported:

$$C_s = rA \tag{2}$$

In-Transit Inventory Carrying Cost

In-transit inventory carrying cost is equal to in-transit inventory cost per unit/day *times* total quantity transported *times* mean transit time (days):

$$C_t = zAM \tag{3}$$

Inventory Cost

The inventory carrying cost has four components: ordering cost, working inventory cost, safety inventory cost, and costs associated with eventual shortages. The first component is defined as set-up cost per order *times* the number of orders per year (A/Q). Therefore:

$$C_o = KA/Q$$

The average working inventory is $Q/2$, and safety inventory can be defined in terms of the reorder point, which is equal to the mean lead-time demand *plus* safety stock. Or:

$$R = \mu_L + SS$$

Thus, safety inventory is equal to $(R - \mu_L)$, and joint costs of working and safety inventories will be:

$$H \cdot [(Q/2) + R - \mu_L]$$

Annual cost of shortage is equal to the cost of shortage per unit (ξ)

times the expected shortage per order *times* number of orders (A/Q):

$$\xi S \quad A/Q \qquad \text{where} \quad S = \int_{L=R}^{\infty} (L - R)\phi L dL$$

Therefore, total inventory carrying cost is:

$$C_I = AK/Q + H((Q/2) + R - \mu_L) + \xi AS/Q \tag{4}$$

Model Solution

Profit Function

From (1), (2), (3), (4), the complete profit function is formulated:

$$\pi = \Delta pA - rA - zAM - AK/Q - H[(Q/2) + R - \mu_L] - \xi AS/Q \tag{5}$$

where

$$S = \int_{L=R}^{\infty} (L - R)\phi(L) dL \tag{6}$$

The expression for the freight demand function is obtained by determining the values of A , Q , and R for which profit is maximized (A*, Q* and R*) and then preparing an expression for A* as a function of the choice variables and external parameters. In order to find the extreme values of the choice variables which maximize (or minimize) the objective function (profit), first order conditions require that the partial differentials of the profit function with respect to each of the choice variables be equated to zero. Then the demand function may be determined by simultaneous solution of the partially differentiated equations. Evaluation of the second order conditions, however, is necessary to determine whether the extreme values represent a maximum or a minimum.

Determinantal criteria for profit-maximization are verified by examining the second derivatives originating from the profit function. Sufficient condition for a maximum is attained if $d^2\pi$ proves itself to be a negative definite for any (in this case, permissible) variations in dA , dQ , dR (but not all zero). Complete specification of second order conditions for profit maximization with three choice variables may be found in most standard textbooks of quantitative methods in Economics and will not be reproduced here.[2]

The Demand Equation

Differentiating profit (π) with respect to quantity demanded (A), and equating to zero, we have:

$$\frac{\partial \pi}{\partial A} = \Delta p + A\frac{d\Delta p}{dA} - r - zM - K/Q - \xi S/Q = 0 \qquad (7)$$

In order to define the optimum quantity to be transported it is necessary to determine the price coefficient of demand for the final product. This is necessary to indicate that shippers would stop transporting commodities if price of transportation becomes higher than revenues they would obtain by moving these commodities. If it is assumed that demand for the product is linear (that is,

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[2] See, for example, Alpha C. Chiang, Fundamental Methods of Mathematical Economics (Second Edition). New York: McGraw-Hill, 1974, pp. 346-355.

$\Delta p = a' - bA)$, then $d\Delta p/dA = -b$. This value, then, is substituted in (7):

$$\frac{\partial \pi}{\partial A} = \Delta p - bA - r - zM - K/Q - \xi S/Q = 0 \quad \therefore$$

$$bA = \Delta p - r - zM - K/Q - \xi S/Q \quad \therefore$$

$$A = 1/b(\Delta p - r - zM - K/Q - \xi S/Q$$

therefore,

$$A^*(Q^*, R^*) = 1/b(\Delta p - r - zM - K/Q^* - \xi S^*/Q^*) \qquad (8)$$

This is our demand equation determined for profit maximizing choice variables, and dependent on all externally determined parameters. However, this equation is not explicit for A^* , because both Q and R depend on A in turn. Q^* is determined by:

$$\frac{\partial \pi}{\partial Q} = AK/Q^2 - H/2 + \xi AS/Q^2 = 0 \quad \therefore$$

$$AK/Q^2 + \xi AS/Q^2 = H/2 \quad \therefore$$

$$(AK + \xi AS)/Q^2 = H/2 \quad \therefore$$

$$2(AK + \xi AS) = HQ^2 \quad \therefore$$

$$Q^2 = 2A(K + \xi S)/H \quad \therefore$$

$$Q = \pm\{2A(K + \xi S)/H\}^{\frac{1}{2}}$$

but as the model does not accept a negative order quantity, the expression for Q becomes:

$$Q = \{2A(K + \xi S)/H\}^{\frac{1}{2}}$$

so that

$$Q^*(A^*, R^*) = \{2A^*(K + \xi S^*)/H\}^{\frac{1}{2}} \quad (9)$$

Similarly, S^* may be found by:

$$\frac{\partial \pi}{\partial R} = -H + \xi A/Q \int_{L=R}^{\infty} \phi(L)\,dL = 0 \quad \therefore$$

$$\xi A/Q \int_{L=R}^{\infty} \phi(L)\,dL = H \quad \therefore$$

$$\int_{L=R}^{\infty} \phi(L)\,dL = HQ/\xi A \quad (10)$$

This equation provides an opportunity to determine S^*, Q^*, and and A^* independently of R. It is possible to make $q = 1 - (HQ/\xi A)$ and obtaining from (10):

$$R = \mu_L + G(q)\sigma_L \quad (11)$$

Substituting this expression in (6), the shortage expression becomes:

$$S = \int_{L=R}^{\infty} (L - R)\phi(L)\,dL \quad \therefore$$

$$S = \int_{u_L + G(q)\,dL}^{\infty} [L - u_L - G(q)\sigma_L]\,\phi(L)\,dL \quad \therefore$$

$$S = \sigma_L \int_{u_L + G(q)dL}^{\infty} [L - u_L - G(q)]\phi(L)dL \quad \therefore$$

$$S = \sigma_L \int_{G(q)}^{\infty} [x - G(q)]f(x)dx \quad \therefore \tag{12}$$

$$S = \sigma_L \int_{G(q)}^{\infty} xf(x)dx - \sigma_L(1 - q)G(q) \quad \therefore$$

$$S^* = \sigma_L \int_{G(q)}^{\infty} xf(x)dx - \sigma_L HQ^*G(q)/\xi A^* \tag{13}$$

By substituting (12) into (9), the final formula for order quantity becomes:

$$Q^*(A^*, R^*) = \{2A^*(K + \xi S^*)/H\}^{\frac{1}{2}}$$

$$Q^*(A^*, R^*) = \left\{2A^*/H\left[K + \xi\left[\sigma_L \int_{G(q)}^{\infty} [x - G(q)]f(x)dx\right]\right]\right\}^{\frac{1}{2}} \tag{14}$$

Then (14) and (13) may be substituted back into (8) in order to obtain the demand function. Unfortunately, the resulting function cannot be made explicit for A , and therefore it will be very difficult (if not impossible) to find an analytical solution for the problem. Also, obtaining a solution will be impossible before specifying the probability distribution of lead-time demand.[3]

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[3] The probability distribution of lead-time demand is result of the joint distribution of both lead-time and demand. Although for the

Satisfaction of Second-Order Conditions

Evaluation of second-order conditions for a maximum profit cannot be made until values of A* , Q* , and R* are specified. However, mild conditions of regularity suggest that a maximum exists and that it is a global maximum. Constable argues that, *for the cost function*, "there is evidence that a minimum exists, and that it is a global minimum rather than a local minimum for solution vectors in the area of interest."[4] As the profit function is a result of a linear function (ΔpA) minus a *strict downward convex function* (implied from the existence of a global minimum in the cost curve), the result must be a strict downward concave profit function--therefore implying the existence of a global maximum profit point. Thus second-order conditions are satisfied for the area of interest in the study.

Model Extensions

Several important extensions could be incorporated to the previously described model in order to take into account peculiarities of operational situations to be analyzed. Some of these extensions are as follows.

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particular purposes of this study such distribution specification is not required, it is interesting to note that Constable has cited empirical evidence about the adequacy of the gamma distribution to represent both factors' distributions. Constable, op. cit., pp. 87/8 and 90.

[4] G. Constable, op. cit., pp. 60-61. For a more formal treatment, see Hadley and Whitin, op. cit., p. 188.

Inventory-Holding Cost (H)

In the model thus far, inventory-holding cost (H) has been described as a constant. However, in reality, this cost depends on several factors, the most important of which are obsolescence, interest, and depreciation.[5] All of these factors are closely dependent on the value of the product, and therefore this variable should be represented in the model. In addition, transportation rate should rightfully be considered an investment on the value of the commodity. Both problems may be circumvented as follows:

The variable H may be expressed as:

$$H = [C_r(v + r)] \tag{15}$$

where:

C_r = inventory holding cost per dollar/year

v = purchase price of the merchandise, and

r = transportation rate.

Similarly, the in-transit inventory carrying cost (z) may be considered as:

$$z = [C_t(v + r)] \tag{16}$$

where:

C_t = in-transit inventory carrying cost per dollar of merchandise per day in transit.

[5] Thomas M. Whitin. The Theory of Inventory Management. Princeton University Press, Princeton, N.J.: 1968. p. 221.

Freight Rate Structure

Another constant in the model, freight rate (r), also depends on many factors in real-life situations, such as weight, density, value, type of commodity, distance between the two markets, and quantity discounts. Therefore r should be a function of these variables in the model. However, as freight rates are mostly government-regulated and subject to many discontinuities, it would be difficult to approximate a function for it. As a result, in the model, r is determined exogenously by criteria which approximate real-life values placed in each of these influencing factors.

Quantity Discounts in Purchase Price

Another relatively frequent occurrence in business transactions is quantity discounts in purchase price, by which suppliers offer lower unit prices to buyers who purchase larger volumes. In this case, Δp in the model may be made a function of, among other things, these quantity discounts related to the order quantity. Once the structure of these discounts is known, it is possible to find a mathematical expression for it. However, in order not to overburden the already complicated solution process of the model, no attempt was made to define and use any such expression.

Speculative Inventories

Basically, the size of speculative stock depends on how businessmen perceive the combined probabilities of, first, materialization of a strike (or any other disruption), and second, of how long this disruption will last. It is very difficult to estimate these probabilities, and further studies are necessary for clarifying the point before a satisfactory mathematical expression can be formulated.

In any event, there are at least two ways of incorporating speculative motives in the model. One of them is to formulate a separate term for speculative inventory (derived from the joint probabilities mentioned in the previous paragraph) and then adding this term to the total cost equation. The second is to treat speculative motives as a factor of safety inventories. In this case, expected lead-time mean and variance should be considered as including perspective stoppages in delivery schedules, so that the size of safety inventory would increase to accommodate the irregularity, for the same level of expected shortage cost.

Inventory Review Policy

One of the model's assumptions is that inventories would be continuously reviewed and a certain quantity (Q) would be ordered as soon as inventory level reaches a certain quantity: the reorder point (R). Thus, reorder point was defined as:

$$R = F_L + SS_L$$

where:

F_L = forecast of lead-time demand

SS_L = size of safety stock as a function of variations in lead-time demand.

If the continuous review assumption would have to be abandoned because of its inaccuracy in describing inventory control techniques in use, then both reorder point and safety stock would have to take into consideration this fact. In this case, the reorder point formula will be:

$$R = F_L + F_R + SS_{L+R}$$

where the new variables are:

F_R = forecast of review intervals

SS_{L+R} = size of safety stock as a function of variations both in lead-time demand *and* in review intervals.

Review intervals may be, under certain conditions, a very important determinant of the size of inventory holdings, since large periods between inventory reviews may cause reorder point to be considerably surpassed at each time of review. However, in absence of more precise information about policy in use, continuous review is assumed in this model's formulation.

Model Assumptions

Assumptions made during model specification include:

1) An order quantity-reorder point system of inventory control (Q, R model) must be representative of business behavior in inventory management. Similarly, continuous review must be acceptable indication of the reviewing policy in use.

2) Profit-maximization must be acceptable as an objective criterion for transportation decisions.

3) Orders are placed and received in the same order.

4) Demand in one period does not depend on previous or subsequent period demands.

5) Lead-times must be independent from one another and from the demands.

6) Demand and lead-time can be described as a relatively stable probability distribution over time.

7) Product demand must be susceptible to a satisfactory linear approximation.

Theoretical Relations Among Variables in the Model

A preliminary check on model validity may be made by observing if the interrelationships among its variables have intuitive or theoretically correct backing. In the present case, all these relationships adhere to expected results:

1) Expected shortage (S) increases with increases in mean and variance of product demand and lead-time (μ, M, σ and V).

2) Reorder point increases with an increase in the mean and variance of lead-time demand (μ_L and σ_L) and decreases with an increase allowable shortages (S).

3) Order quantity will be larger, the larger the total demand (A), the set-up cost per order (K), the unit cost of shortage (ξ), and the size of expected shortage per order (S); and will be smaller, the larger the freight rate (r), the value of the product (ν), the inventory holding cost (C_r), and the reorder point (R).

4) Total demand for transportation increases with an increase in

shippers' revenues (Δp) and in order quantity (Q), and will diminish with increases in freight rates (r), in-transit inventory carrying cost (z), ordering cost (K), costs associated with shortages (ξ), and size of expected shortages (S).

APPENDIX B

IMPLICATIONS OF THIS ANALYSIS FOR FREIGHT DEMAND ESTIMATIONS IN THE STATE OF HAWAII

The unique geographical location of Hawaii suggests that local implications of the problems discussed in this dissertation may be substantially more severe than elsewhere in the Mainland, both in terms of management and policy-making recommendations.

Geographical isolation logically induces higher levels of overall inventories, and Hawaii is no exception to this rule. First, long lead times imply necessarily in larger working inventories. The same factor usually is connected with higher variance in demand during lead time, requiring more safety stock to avoid stock-out situations. In addition, transportation systems to isolated areas tend to provide a more tenuous link with sources of supply, and the perspective of disruption in this system encourages maintenance of still higher levels of safety and speculative inventories.

The interrelationship between transportation demand and inventory policy, albeit complex, is not in general a negligible one, as previous conclusions of this study indicate. It is only logical that implications of inventory decisions in transportation demand should carry substantially more weight in places, like Hawaii, where inventory keeping must receive central consideration of businessmen in almost every area. An additional complication arises from Hawaii's being served basically by only two transportation modes (airplanes and ships) and one navigation company (Matson) being responsible for around 90% of the cargo originating in the West Coast.[1]

This general inventory situation may be substantiated by a brief review of available data on the subject, such as the one in the next section.

Inventory Situation in Hawaii

All information on actual levels of inventories held in Hawaii indicate local firms maintaining substantially more inventory than the national average.

A comparison of the wholesale/inventory situation prevailing in the United States, the Pacific Division of its Western Region and Hawaii is presented in Table B.1 ahead. From this table, it is possible to verify that Hawaiian wholesalers reported an inventory-sales ratio of 11.23% in 1972, as compared with 6.58% for the U.S. and 6.97% for the Pacific Division; that is, almost double that prevailing in other regions. As a consequence, local firms keep enough stock to last them almost six weeks (5.84), as compared with a national average of about 3.5 weeks (3.42 (U.S.) and 3.62 (P.D.), respectively).

On a historical perspective, the same picture may be obtained by comparing U.S. and Hawaiian wholesale inventory stock-turn with data from the three latest Censuses of Business (1963, 1967, and 1972)

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[1] E.J. McCarthy. Stockpiling as a Solution to Shortages from Maritime Strikes Affecting Hawaii. Economic Research Center, University of Hawaii: February 1964. p. 39.

Table B.1 -- Comparison Between Wholesale Trade Inventories Held in the United States, Pacific Division States, and Hawaii (Census of Business, 1972)

	Sales ($1,000)	Inventories ($1,000)	Inventories Sales (%)	Stockturn (times/year)	Stockturn (weeks)
United States					
Wholesale trade	695,223,644	45,724,605	6.58	15.20	3.42
Durable goods	341,829,504	27,837,362	8.14	12.28	4.23
Non-durable goods	353,394,140	17,887,243	5.06	19.76	2.63
Pacific Division					
Wholesale trade	90,024,301	6,271,835	6.97	14.35	3.62
Durable goods	47,847,682	4,160,967	8.70	11.50	4.52
Non-durable goods	42,176,619	2,110,868	5.00	19.98	2.60
Hawaii					
Wholesale trade	1,561,654	175,370	11.23	8.90	5.84
Durable goods	617,375	96,644	15.65	6.39	8.14
Non-durable goods	944,279	78,726	8.34	11.99	4.34

Source: Census of Business, 1972 (U.S.A.).

as shown in Figure B.1. This figure shows that Hawaiian inventories have largely maintained current relative proportions with the U.S. during the period covered by the Census years.

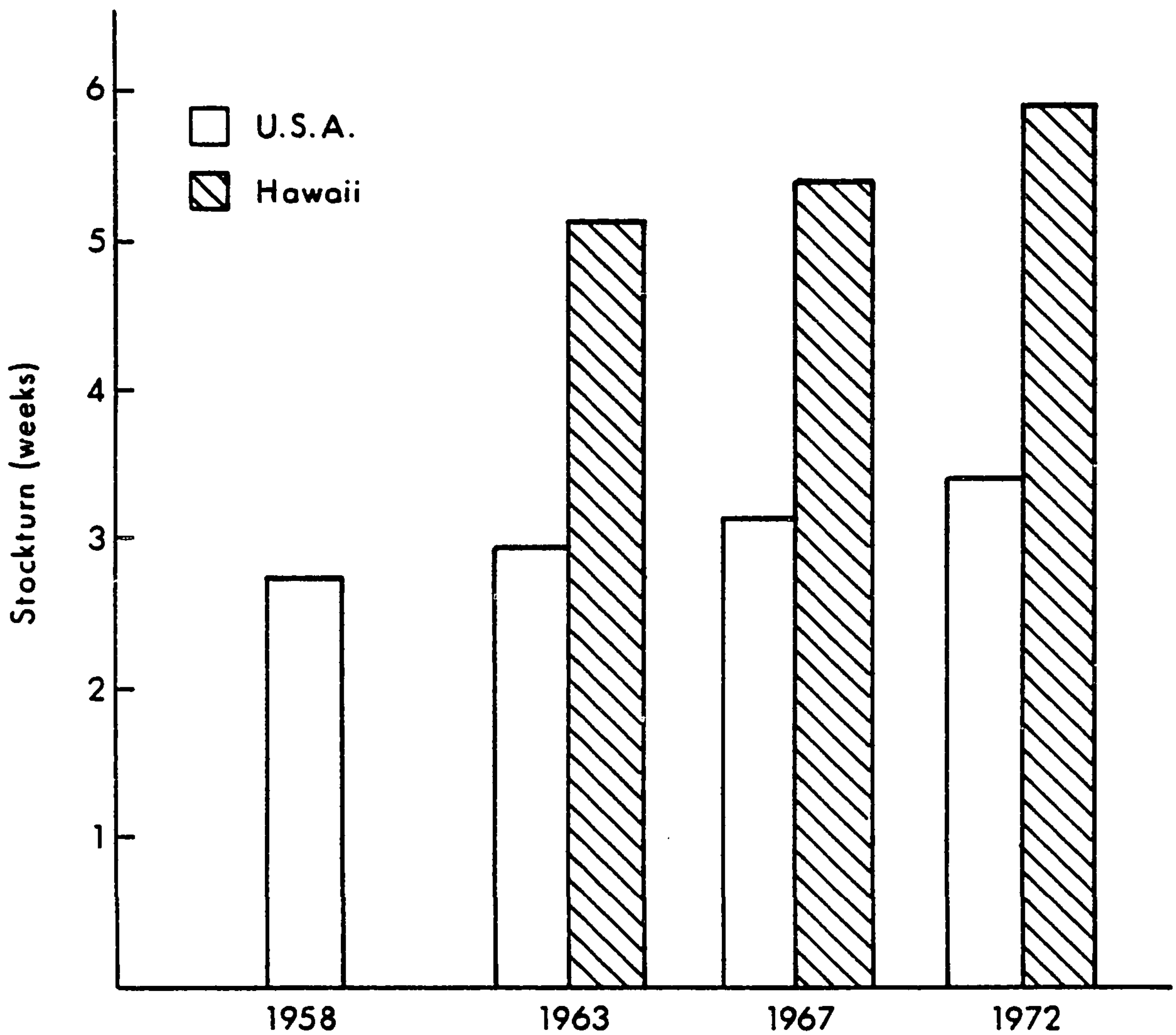

Figure B.1 -- Comparison of Average Wholesale Trade Stockturn (Weeks) Between the U.S. and Hawaii. All Categories, by Census Year.

Source: Department of Commerce, Bureau of Census, U.S. Census of Business--1958, 1963, 1967, 1972.

Wholesale trade comparisons, however, cannot clarify the whole inventory picture because both manufacturers and retailers keep a substantial amount of inventory as well. Although strictly comparable data about these categories are not immediately available for Hawaii, national inventory proportions of the three trading stages may give an idea of the dimensions involved, and are provided in Table B.2 below.

It is clear from this table that both manufacturing and retailing companies hold proportionally more inventories than wholesalers in the country as a whole. There is no evidence that this situation should be the reverse in Hawaii. Rather, when one considers the need for

Table B.2 -- Inventory Turnover in the United States, 1960

	Sales ($\$10^6$)	Inventories ($\$10^6$)	Inventory Turnover (weeks)
Manufacturing			
durable goods	176.2	30.9	9.1
non-durable goods	188.7	22.9	6.3
Wholesale Trade			
durable goods	53.3	6.8	6.6
non-durable goods	94.7	6.4	3.5
Retail Trade			
durable good	70.7	12.3	9.0
non-durable goods	148.8	14.9	5.2

Source: Statistical Abstract of the United States, 1962. p. 501.

imported raw materials in manufacturing, and that many large retailers in Hawaii typically purchase merchandise directly from Mainland wholesalers, it is very likely that these proportions are even more accentuated in the Islands. Results of a study involving direct interviews with the three types of local businessmen largely confirm this assertion: McCarthy concluded that Hawaiian firms, on the whole, maintain a supply of "one or two months *above* mainland inventories."[2]

Another important characteristic of local firms' behavior with regard to inventory-holding has to do with perceived emergency ordering. Any perspective of disruption of supply lines induces local managers to increase substantially the quantity ordered. From direct interviews with 160 retailers, wholesalers, and manufacturers in Hawaii, McCarthy reported that 90% of those who responded indicated that they build up inventories before imminent strikes, some of them by as much as two or three times for certain items, in what one firm labeled "strike insurance." One advanced explanation for this type of behavior was that local firms tended to define their safety stock levels from the "extremes of their past experiences" with relation to shipping dependability."[3,4]

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[2] McCarthy, op. cit., p. 70.

[3] Ibid., pp. 3, 16.

[4] Ibid., p. 41.

Conclusions and Implications

A number of economic effects naturally derive from such a high level of inventories in Hawaii, notably higher prices for imported goods in general, and a stronger dependence of freight demand on prevailing inventory policies. Any attempt to estimate demand for transportation, therefore, must take into account inventory-related variables in a formulation which properly represents management behavior on the subject, including speculatory motives behind emergency ordering, when such a situation occurs. In this sense, special attention must be paid to the specification of non-linear variables, such as transit time, which are essentially inventory cost determinants.

Consequently, the use of linear choice models which include transit-time in Hawaii may well result in highly inconsistent coefficients because of the higher relative weight of inventory cost in total cost of transportation, as compared with the Mainland. The implications of such an event in terms of faulty policy recommendations are particularly serious in view of the high degree of dependence which the State's economy has with relation to its transportation modes.

Support for this argument might well be found in the flagrantly inconsistent results obtained by Sam Herrick (1975) in his attempt at estimating modal choice in Hawaii with a linear logit model applied to disaggregated data of overseas shipments.

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