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GEOTHERMAL RESERVOIR
AND
WELL TEST ANALYSIS:
A LITERATURE SURVEY

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GEOHERMAL RESERVOIR AND WELL TEST ANALYSIS: A LITERATURE SURVEY

INTRODUCTION

In order to evaluate the geothermal reservoir, be it at the drilling, development or production stage, the geothermal engineer must have data on the various parameters vital to its analysis. These data include formation thickness, permeability, porosity, viscosity, compressibility, thermal conductivity, fluid and rock density, temperature and most of all formation average pressure.

Broadly speaking, average reservoir pressures are used for characterizing a reservoir, computing its geothermal liquids in place and predicting future behavior. In characterizing a reservoir, pressures are used to relate the amount of production in a given interval of time to the pressure drop. For example, if the pressure drop is large for a given amount of production, this may indicate drainage from a small reservoir or with poor permeability.

In addition to this semi-quantitative use, pressures find a quantitative use in materials - heat balance calculations of geothermal liquid in place. Thus the characteristics of the reservoir - be it compressed liquid, saturated liquid and steam or superheated steam - can be determined. Extrapolation into the future is also made by using the above method which relates future production to future formation average pressure.

The primary product of a petroleum reservoir is oil and gas. It is, therefore, imperative that we measure the reservoir average pressure as it is vital to the fluid flow. However, the primary product of a geothermal reservoir is best. We need not only the pressure measurements to monitor the flow, but also temperature measurements to monitor the heat content.

From the above discussion, we see that pressure, temperature and flow measurements are vital to the evaluation and prediction of a geothermal reservoir. Therefore, every single well must have a complete and comprehensive program to secure and analyze all these measurements.

This report deals primarily with the analysis of the required parameters, focusing on the various pressure measurements and analysis techniques. It also surveys the material - heat balance equations essential for the establishment of a reservoir model for performance matching prediction.

This report also assumes that we have a geothermal reservoir with little or no net heat transfer. With this assumption one can regard the geothermal reservoir as a reservoir with isothermal fluid flow. This is the same assumption used in developing pressure analysis techniques in a petroleum reservoir. Therefore, petroleum well test analysis techniques, with very little modification, can be utilized for geothermal reservoir analysis.

FUNDAMENTAL EQUATIONS (1)*

Fluid flow through porous media is generally considered to be laminar with the exception of flow near a well. The basic physical principles used to describe the flows are: 1) the law of conservation of mass; 2) Darcy's law; and 3) the equation(s) of state.

In general, we can write a material balance equation over a differential element of reservoir volume as

$$(\text{mass rate in}) - (\text{mass rate out}) = (\text{mass storage rate})$$

If we consider radial flow of uniform thickness then we have the continuity equation

$$\frac{1}{r} \frac{\partial}{\partial r} (\rho r u_r) = - \frac{\partial}{\partial t} (\phi \rho) \quad (1)$$

where r = distance in the radial direction
 ρ = fluid density
 u_r = radial fluid velocity
 ϕ = formation porosity
 t = time

For the same radial flow and neglecting gravitational forces, one can write the Darcy's law as

$$u_r = - \frac{k}{\mu} \frac{dp}{dr} \quad (2)$$

where k = formation permeability
 μ = fluid viscosity

Substituting equation (2) into (1), the continuity equation becomes

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{rk\rho}{\mu} \frac{\partial p}{\partial r} \right) = \frac{\partial}{\partial t} (\phi \rho) \quad (3)$$

*References are given at the end of this report.

Next, consider isothermal single-phase liquid flow through porous media. The density can be considered as a function of pressure only. The isothermal compressibility is defined as

$$c = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_t = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial p} \right)_t \quad (4)$$

If compressibility, c , is constant, then the above relationship can be integrated to yield

$$\rho = \rho_0 e^{c(p-p_0)} \quad (5)$$

where ρ_0 is the value of ρ at some reference pressure, p_0 .

From equations (3) and (5), one can either eliminate ρ or p from the equations. Let us now eliminate the density ρ to obtain an equation with p only. Assuming the permeability and viscosity are constant, we get

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + c \left(\frac{\partial p}{\partial r} \right)^2 = \frac{\phi \mu}{k} \left(c + \frac{1}{\phi} \frac{\partial \phi}{\partial p} \right) \frac{\partial p}{\partial t} \quad (6)$$

The rock compressibility, c_r , may be defined as an equivalent pore space compressibility: $c_r = \frac{1}{\phi} \frac{\partial \phi}{\partial p}$

Thus equation (6) becomes

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) + c \left(\frac{\partial p}{\partial r} \right)^2 = \frac{\phi \mu}{k} (c + c_r) \frac{\partial p}{\partial t} \quad (7)$$

or

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) + c \left(\frac{\partial p}{\partial r} \right)^2 = \frac{\phi \mu}{k} c_t \frac{\partial p}{\partial t} \quad (8)$$

where $c_t = c + c_r$ is the total system effective compressibility.

Equation (8) is clearly a non-linear partial differential equation in p . If the pressure gradient is assumed to be small everywhere in the flow system, then equation (8) becomes

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) = \frac{\phi \mu}{k} c_t \frac{\partial p}{\partial t} \quad (9)$$

This equation is commonly used in the analysis of flow through porous media, and its solutions form the foundation of well test analysis. As a word of

caution, it should be remembered that equation (9) is obtained through the assumptions of: a) small pressure gradient; b) fluid of small and constant compressibility; and c) rock properties not a function of the angular direction or vertical coordinates.

DIMENSIONLESS EQUATIONS

It is customary to transform equation (9) into dimensionless form. Let us define

$$t_D = \frac{kt}{\phi\mu c_t r_w^2} \quad (10)$$

$$r_D = \frac{r}{r_w} \quad (11)$$

and
$$p_D = \frac{2\pi kh}{q\mu} (p_i - p_{r,t}) \quad (12)$$

where r_w = the radius of the producing well

p_i = the initial formation pressure.

Substituting equations (10), (11) and (12) into (9), we have

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_D}{\partial r_D} = \frac{\partial p_D}{\partial t_D} \quad (13)$$

LINE SOURCE SOLUTION WITH INFINITE RESERVOIR

In order to solve equation (13) we need one initial condition and two boundary conditions. One may assume the initial condition to be:

$$p = p_i \quad \text{at} \quad t = 0 \quad \text{for all } r \quad (14)$$

or
$$p = p_D \quad \text{at} \quad t_D = 0 \quad \text{for all } r_D \quad (15)$$

At the outer boundary, we assume an infinitely large reservoir, therefore the pressure at the outer boundary is equal to the initial pressure.

$$\lim_{r \rightarrow \infty} p(r,t) = p_i \quad (16)$$

or

$$\lim_{r_D \rightarrow \infty} p(r_D, t_D) = 0 \quad (17)$$

At the inner boundary, we assume the well is producing at a constant rate, therefore,

$$\frac{q}{A} = \frac{k}{\mu} \left(\frac{dp}{dr} \right)_{r_w} \quad (18)$$

$$q = \frac{k(2\pi r_w h)}{\mu} \left(\frac{dp}{dr} \right)_{r_w} \quad (19)$$

or

$$\left(r \frac{dp}{dr} \right)_{r_w} = \frac{q\mu}{2\pi kh} \quad (20)$$

where h is the formation thickness. For a line source approximation, we rewrite equation (20) to be

$$\lim_{r \rightarrow 0} \left(r \frac{dp}{dr} \right) = \frac{q\mu}{2\pi kh}, \text{ a constant} \quad (21)$$

or

$$\lim_{r_D \rightarrow 0} \left(r_D \frac{dp_D}{dr_D} \right) = -1 \quad (22)$$

Now we are ready to solve equation (13), with initial condition (15) and boundary conditions (17) and (22). The actual solving will not be presented here. See Ref. 1. Only the solution is listed:

$$p_D(r_D, t_D) = -\frac{1}{2} \text{Ei} \left[-\frac{r_D^2}{4t_D} \right] \quad (23)$$

where

$$-\text{Ei}(-x) = \int_x^\infty \frac{e^{-u}}{u} du$$

or

$$p_D(r_D, t_D) \approx \frac{1}{2} \left[\ln \frac{t_D}{r_D^2} + 0.80907 \right] \quad (24)$$

where

$$\frac{t_D}{r_D^2} > 70$$

Figure 1 shows the graph of p_D vs. t_D/r_D^2 on log-log paper. Mueller and Witherspoon [2] explored the solution of equation (13) by assuming finite well radius rather than the line source approximation. Figure 2 shows their results. Note that the curve corresponding to $r_D = \infty$ is the line source solution.

Once we have the relationship between p_D and t_D/r_D^2 , we can go back to equation (12) and obtain the real pressure term $p_{r,t}$ as:

$$p_{r,t} = p_i - \frac{q\mu}{2\pi kh} p_D \quad (25)$$

at any location and time.

INTERFERENCE TEST

The line source solution is quite useful for interference test data analysis. "Interference" means the production of one well causes a detectable pressure drop at an adjacent well. The most simple case of interference is when pressure drop is measured at a shut-in well. From Figure 2 we can see that the line source solution will work as long as the distance between the shut-in and producing wells is greater than approximately 25 times the radius of the producing well. This can be easily achieved as wells seldom have a radius larger than a foot. To perform the interference test data analysis, it is necessary for us to develop a useful technique -- type curve matching. If we take the logarithm of both sides of equation (12), we get

$$\log_{10} p_D = \log_{10} \frac{2\pi kh}{q\mu} (p_i - p_{r,t}) = \log_{10} \left(\frac{2\pi kh}{q\mu} \right) + \log_{10} (p_i - p_{r,t}) \quad (26)$$

Thus, the plot of p_D and $(p_i - p_{r,t})$ will be the same on log-log type paper with the difference of a constant term. A plot of the logarithm of the real pressure differences must look exactly like a graph of the logarithm of p_D as long as the

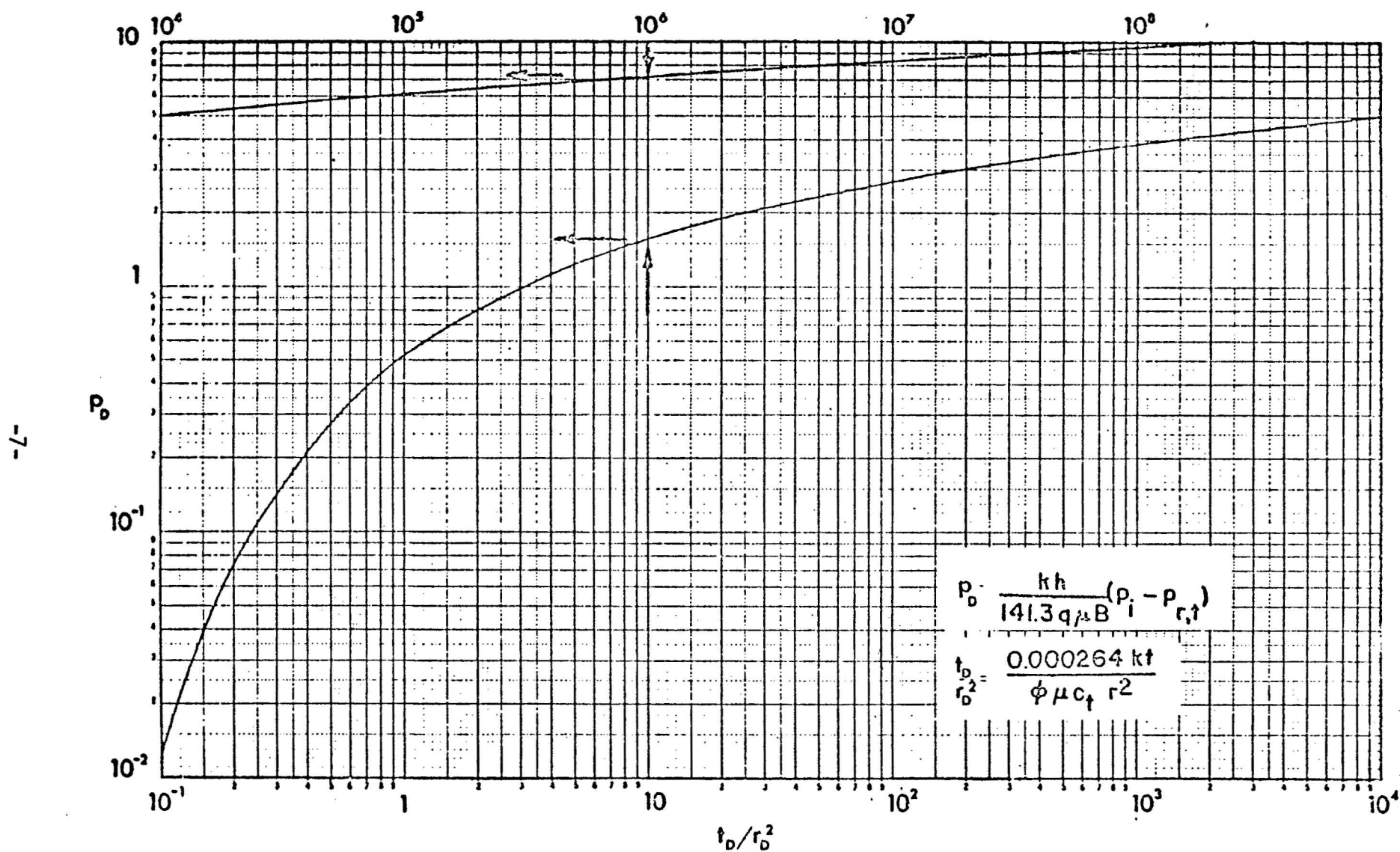


FIG. 1 THE LINE SOURCE SOLUTION TYPE CURVE (After Ref. 12)

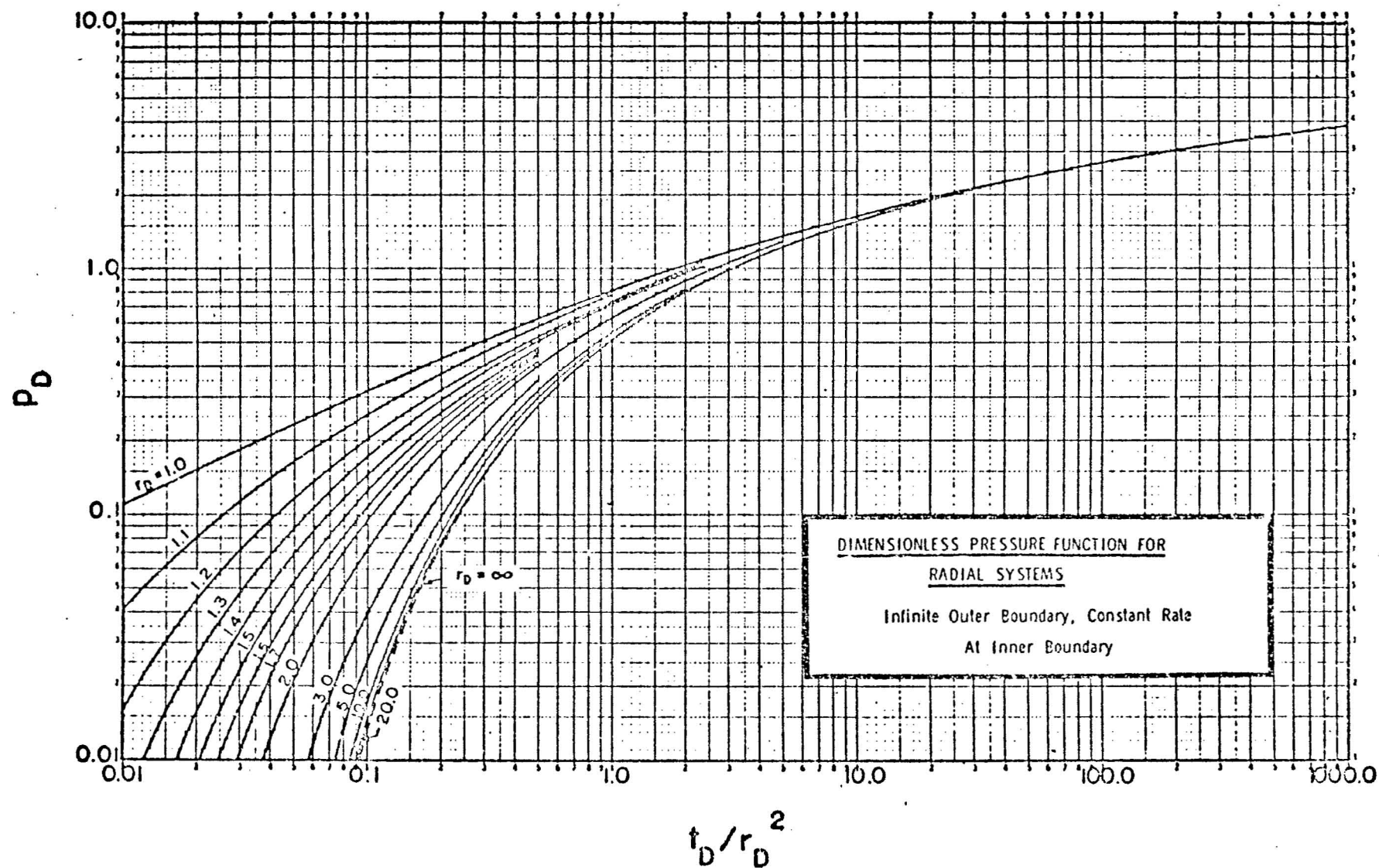


FIG. 2 P_D Vs t_D/r_D^2 FOR A LINE SOURCE AND A FINITE RADIUS WELL (After Ref. 12)

same size log cycles are used. The same is true if we take the log of equations (10) and (11).

The procedure for the interference data analysis is as follows:

- 1) Graph the pressure drop at the observation well vs. time.
- 2) With the axes parallel, position the field data curve over the type curve until field data match the line source solution.
- 3) Read a "match point" as the corresponding coordinates of any point common to both graphs, while aligned.
- 4) From the pressure and time match we will be able to determine the values of two reservoir parameters, e.g., permeability (or permeability thickness) and porosity (or porosity thickness).

RESERVOIR UNITS

It is customary for petroleum reservoir engineers to express all parameters in the conventional reservoir units. For this conversion, equations (10), (11), and (12) should be rewritten as:

$$t_D = \frac{0.000264 \text{ kt}}{\phi \mu c_t r_w^2} \quad (27)$$

$$r_D = \frac{r}{r_w} \quad (28)$$

$$p_D = \frac{kh}{141.3 q \mu B} (p_i - p_{r,t}) \quad (29)$$

where

- t_D = dimensionless time
- k = permeability, md
- t = time, hr
- ϕ = fractional porosity
- μ = viscosity, cp
- c_t = total system effective isothermal compressibility, psi^{-1}
- r_w = well radius, ft
- r_D = dimensionless radius
- r = distance from well, ft
- p_D = dimensionless pressure
- h = formation net thickness, ft
- q = production, std bbl/day

B = formation volume factor, res vol/std vol
 p_i = initial pressure, psi
 $p_{r,t}$ = pressure at r ft, time t hrs, psi

In keeping with convention, all future expressions will be in reservoir units as defined above.

SKIN EFFECT

In many cases, the pressures measured in a well do not match the ideal dimensionless pressure computed for the well point -- although field data do seem to parallel the ideal solution. Van Everdingen [3] and Hurst [4] suggested separately that the difference was an additional pressure drop caused by the restriction of flow near the well. They proposed that one can imagine an infinitesimally thin skin on the surface of the sand face at the well. Given the symbol " s ", we write the pressure drop at the bottom hole as

$$\frac{kh}{141.3 q\mu B} (p_i - p_{wf}) = p_D(1, t_D) + s \quad (30)$$

where p_{wf} is the bottom-hole flowing pressure. The real pressure drop due to the skin effect is

$$\Delta p_{\text{skin}} = \frac{141.3 q\mu B}{kh} s \quad (31)$$

Physically, the skin effect is a combination of invasion by drilling fluids, dispersion of clays, presence of a mud cake and of cement, presence of condensation near a steam well, partial well penetration, and limited perforation. Also a factor is stimulation treatment by acidization or hydraulic fracturing.

The skin effect may be positive, negative or zero. If the well is damaged, s will be positive. If the well is stimulated, s will be negative. However, if the permeability in the skin zone is the same as in the rest of the formation, then s will be zero. Also note that skin effect comes into play only if one wants the pressure measured at or near a well.

WELLBORE STORAGE

As the production of a well is changed suddenly, part or all of the production may come from either the expansion of fluids in the wellbore or

from reduction of fluids level in the annulus between casing and tubing. Van Everdingen and Hurst [5] described this effect in their 1949 classic paper. Basically this is a simple materials balance effect.

Assume that we start the production of a well at q standard barrel per day at the surface and all its production comes from the wellbore. Then

$$C = \frac{qB(24t)}{p_i - p_{wf}} \quad (32)$$

where C is the amount of fluids produced per unit of pressure, reservoir barrel per psi,

$$\text{or,} \quad (p_i - p_{wf}) = \frac{24qBt}{C} \quad (33)$$

Expressing equation (33) in terms of p_D and t_D , we have

$$p_D = \frac{t_D}{C_D} \quad (34)$$

where

$$C_D = \frac{C'}{2\pi h \phi C_t r_w^2} \quad (35)$$

and $C' = (5.615)C$, ft^3/psi , where 1 bbl = 5.615 cu.ft.

If we plot equation (34) on log-log type paper, we see that the curve will have a unit slope. This characteristic can be utilized in well test to identify the presence of wellbore storage effects. Ramey et. al. [6, 7, 8, 9] have thoroughly investigated the wellbore storage effect. One result of their study should be mentioned here, i.e., wellbore storage and skin effect do influence the onset of the semi-log straight line used in well test analysis. This time for the start of the semi-log straight line is given by:

$$t = \frac{602.9 C' \mu (60 + 3.5 s)}{kh} \text{ hours} \quad (36)$$

Equation (36) is extremely useful in well test design. It gives the time required to run either a drawdown or buildup test before the conventional semi-log straight line starts. If one log cycle of straight line is desired, then it will be necessary to run the test ten times as long as the time specified by equation (36).

Figure 3 shows a plot of p_D vs. t_D with wellbore storage and skin effects in a well. Notice the unit slope at the beginning of the curve.

p_D vs t_D FOR WELL WITH STORAGE AND SKIN EFFECT

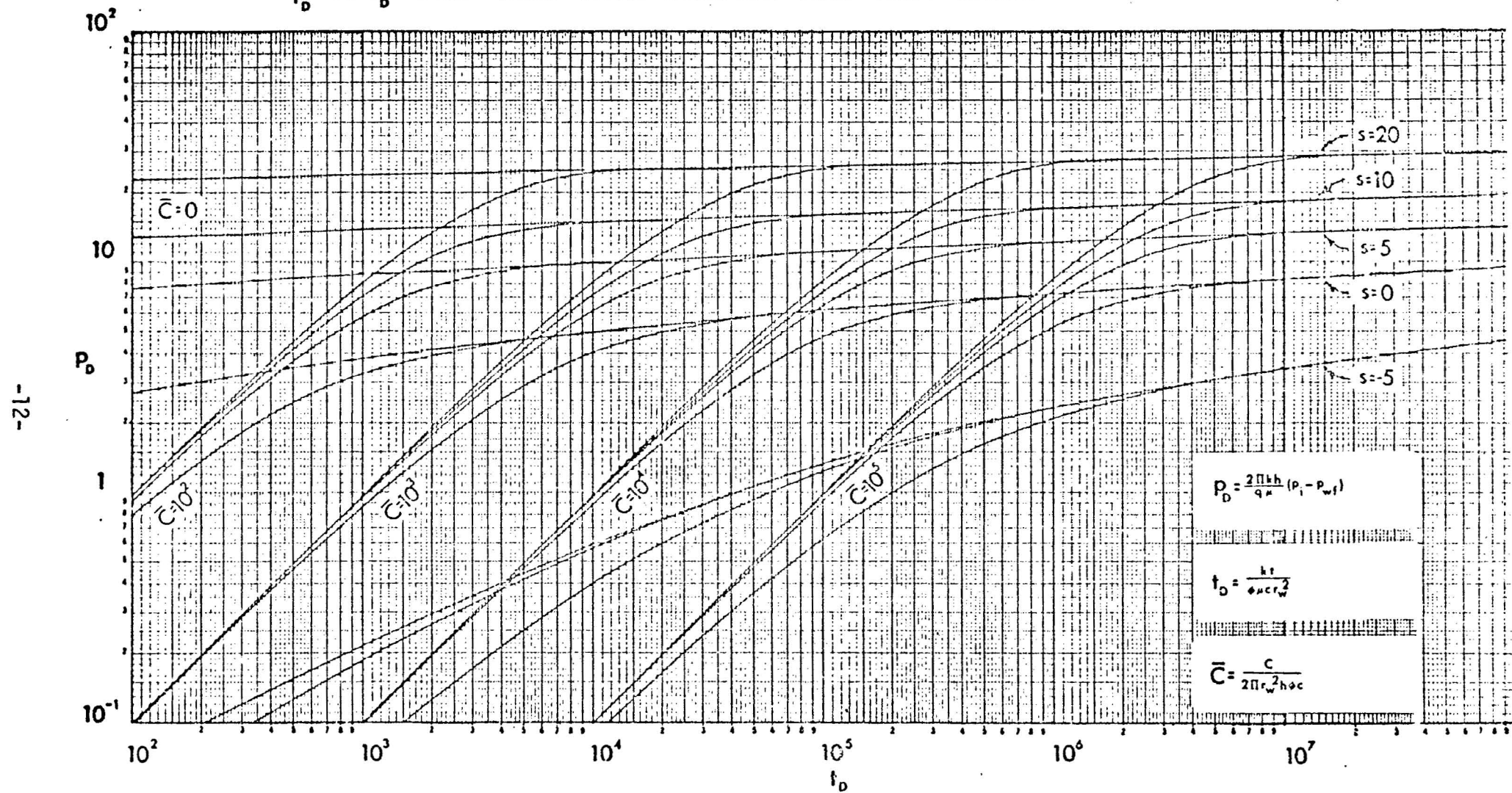


FIG. 3 (After Refs. 7 & 8)

BOUNDED RESERVOIR

Unfortunately, no reservoirs are infinite in size, and most large reservoirs have more than one well. Therefore, all wells more or less have a finite reservoir volume from which fluids are drained. It is imperative that one looks for solutions of equation (9) with different boundary conditions. Van Everdingen and Hurst [5] have solved for two conditions, namely, bounded circular outer boundary and constant pressure outer boundary. For years these solutions were used extensively as the basis of well test analysis. However, the drainage area of many wells seem to be more square or rectangular than circular shape. Earlougher et al. [10] in 1968 utilized a superposition infinite array method to generate solutions for a well at the center of a closed or constant pressure square. We will attempt to show their results in more detail since many characteristics of their results are common to all differently bounded shapes.

Figure 4 (after Ref. 11) shows the dimensionless pressure vs. dimensionless time for a well in the center of a square drainage area. The straight line in the middle is the line source solution for an infinite reservoir. For both cases, in early times they behave just as the line source solution. This period is identified as the initial transient period, where the boundary effect has not been felt by the system. Dimensionless pressure and time have the relationship as being expressed by equations (23) and (24). This relationship can also be expressed as:

$$p_D(t_{DA}) = \frac{1}{2} \left[\ln \frac{4At_{DA}}{\gamma r_w^2} \right] \quad (37)$$

where

$$t_{DA} = \frac{0.000264kt}{\phi\mu C_t A} = t_D \frac{r_w^2}{A} \quad (38)$$

and

$$\gamma = \text{exponential of Euler's constant} = 1.781$$

From Figure 4 and Ref. 10 and 11, it has been shown that equation (37) is valid up to $t_{DA} = 0.05$. In other words, at t_{DA} less than 0.05, the effect of the boundaries is not felt at the well, which behaves like a well in an infinite reservoir.

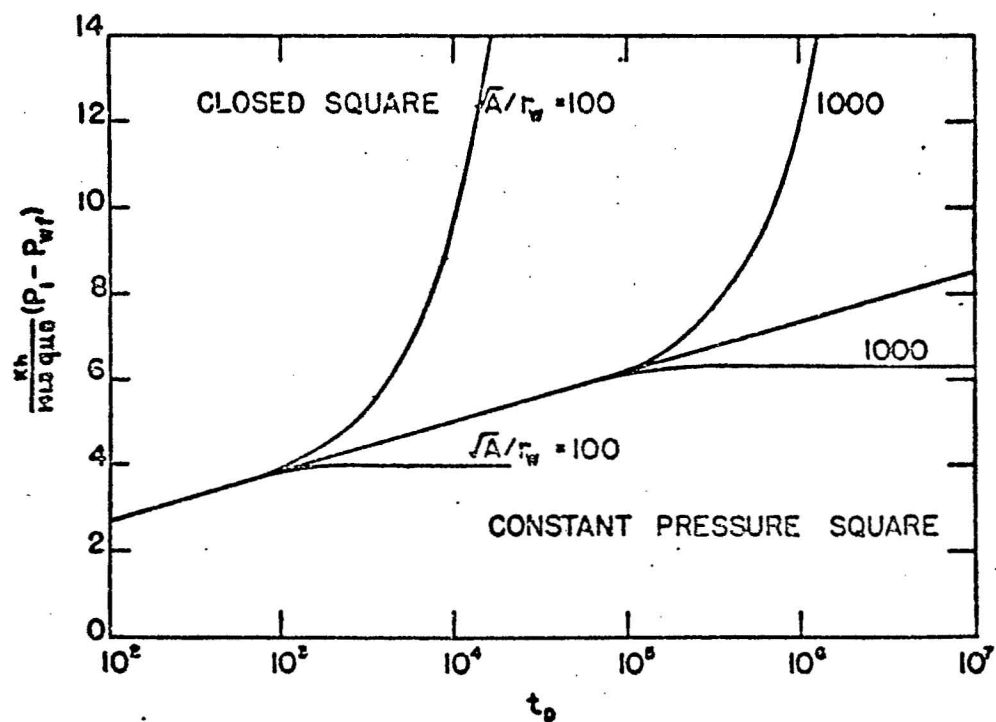


FIG. 4 DIMENSIONLESS PRESSURE VS. DIMENSIONLESS TIME FOR A WELL IN THE CENTER OF A SQUARE DRAINAGE REGION (After Ref. 12)

One also notices that for a bounded reservoir, i.e., where no fluids cross the boundary, the dimensionless pressure increases rapidly as $t_{DA} \geq 0.1$. This obviously is due to the depletion of fluids throughout the reservoir. On the other hand for a constant pressure reservoir, the dimensionless pressure approaches a constant which is due to the replenishment of fluids through the boundary. Figure 5 shows the graph relating p_D and t_{DA} . It is shown that the onset of steady-state pressure for constant pressure square occurs at $t_{DA} = 0.25$; whereas the onset of pseudo-steady-state for bounded reservoir occurs at $t_{DA} = 0.1$. At steady-state, the dimensionless pressure at the well in the center of a constant pressure square is given by the equation (12):

$$p_D = \frac{1}{2} \ln \frac{16 A}{\gamma C_A r_w^2}, \quad t_{DA} \geq 0.25 \quad (39)$$

where $C_A = 30.88$ is the shape factor for a well in a closed square. (See Ref. 10). At pseudo-steady-state, the linear equation for dimensionless pressure at the well in the center of a bounded square is

$$p_D = \frac{1}{2} \ln \left[\frac{4A}{\gamma C_A r_w^2} \right] + 2\pi t_{DA}, \quad t_{DA} \geq 0.1 \quad (40)$$

Equations (39) and (40) are perfectly general equations for all different shapes of reservoir and well locations, if the shape factor C_A can be determined. See Ref. 10 and 11 for the discussion on shape factors.

From the above discussion, it is quite clear that there are gaps between the initial transient period and the final steady or pseudo-steady state. This period is generally called the late transient period. It should be noted that this period may be broad or narrow depending on the location of the well in the reservoir and the nature of the reservoir.

PRESSURE DRAWDOWN TEST

A pressure drawdown test is a series of bottom-hole pressure measurements made during a period of constant producing rate flow. Prior to the flow test, the well is usually shut-in to allow the pressure to be equalized throughout

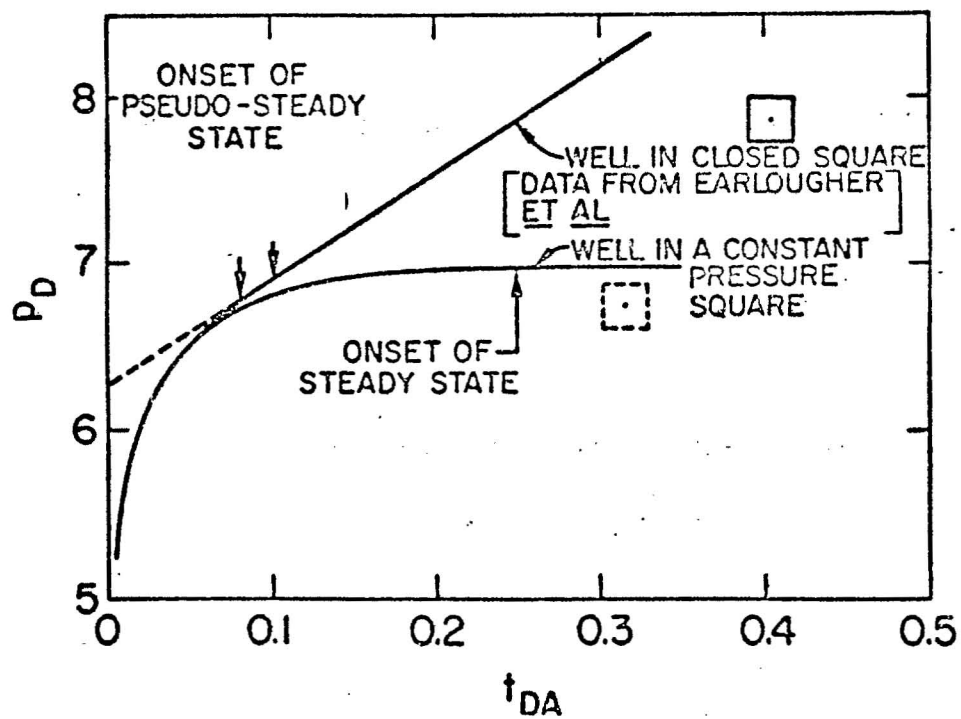


FIG. 5 DIMENSIONLESS PRESSURE VS. DIMENSIONLESS TIME
FOR A WELL IN THE CENTER OF A SQUARE
(After Ref. 12)

the formation. Drawdown tests are done in new wells or after a long period of shut-in time. In some instances, extended periods of drawdown tests are done to estimate the reservoir limits.

The basis of drawdown analysis is equations (24) and (30), or

$$\frac{kh}{141.3q\mu B} (p_i - p_{wf}) = \frac{1}{2} \left[\ln \frac{0.000264kt}{\phi\mu C_t r_w^2} + 0.80907 + 2s \right] \quad (41)$$

If we change the basis of the logarithm from e to 10, we have

$$p_{wf} = p_i - \frac{162.6q\mu B}{kh} \left[\log_{10} t + \log_{10} \frac{k}{\phi\mu C_t r_w^2} - 3.23 + 0.87s \right] \quad (42)$$

Therefore, a plot of p_{wf} vs. $\log_{10}$ will yield a straight line with a slope m psi/cycle:

$$|m| = 162.6 \frac{q\mu B}{kh} \quad (43)$$

From the above equation one can calculate the permeability, k , or the permeability thickness, kh .

Equation (42) can also be used to calculate the skin effect, s . Letting $p_{1 \text{ hr}}$ be the value of p_{wf} at the one hour flowing time on the correct semi-log straight line, we may rearrange equation (42) to yield:

$$s = 1.15 \left[\frac{p_i - p_{1 \text{ hr}}}{|m|} - \log_{10} \frac{k}{\phi\mu C_t r_w^2} + 3.23 \right] \quad (44)$$

From equation (31) we can calculate the pressure drop due to the skin:

$$\Delta p_{\text{skin}} = 0.87 |m|s \quad (45)$$

Perhaps a better relative index than skin effect for deciding well efficiency is the "flow efficiency." This is defined as the ratio of the actual productivity index of a well to its productivity index if there was no skin.

$$\begin{aligned}
 FE &= \frac{PI \text{ actual}}{PI \text{ ideal}} = \frac{q/p_i - p_{wf}}{q/p_i - p_{wf} - \Delta p_{skin}} \\
 &= \frac{p_i - p_{wf} - \Delta p_{skin}}{p_i - p_{wf}}
 \end{aligned}
 \tag{46}$$

If $FE = 0.5$, we know that the well will produce half the rate it would were there no skin.

Figure 6 shows a plot of a typical drawdown test. Notice that the semi-log straight line does not start immediately after the initial flowing period. This is because equation (41) uses the logarithm approximation of the line source solution and the well may have wellbore or storage stimulation effects. The start of the semi-log straight line will be given by equation (36). Figure 6 also shows that as time passes, wells with bounded or constant pressure outer boundaries will eventually deviate from the line source solution. As discussed in the last section, when t_{DA} reaches 0.05, the plot should start to deviate from the straight line. Therefore, if the drawdown test is run long enough to observe this effect, one may speculate on the nature of the boundary, and it may be possible to estimate the size of the drainage area by equation (38):

$$A = \frac{0.000264kt}{\phi\mu C_t (0.05)} = \frac{0.00528kt}{\phi\mu C_t} \text{ sq. ft.}
 \tag{47}$$

However, constant flow rate may be difficult to maintain for a long period of time and in turn the boundary effects could be very difficult to detect.

The following summarizes the procedure of drawdown analysis:

- 1) Plot p_{wf} vs. $\log_{10} t$.
2. Find correct semi-log straight line, slope, m and $p_{1 \text{ hr}}$. It may at times appear to have more than one possibility for a straight line. Suggested procedure is to plot $(p_i - p_{wf})$ vs. t on log-log type paper and compare with Figures 1 and 3 to determine the onset of correct straight line.
- 3) Find k , s , p_{skin} and FE by equations (43), (44), (45), and (46).

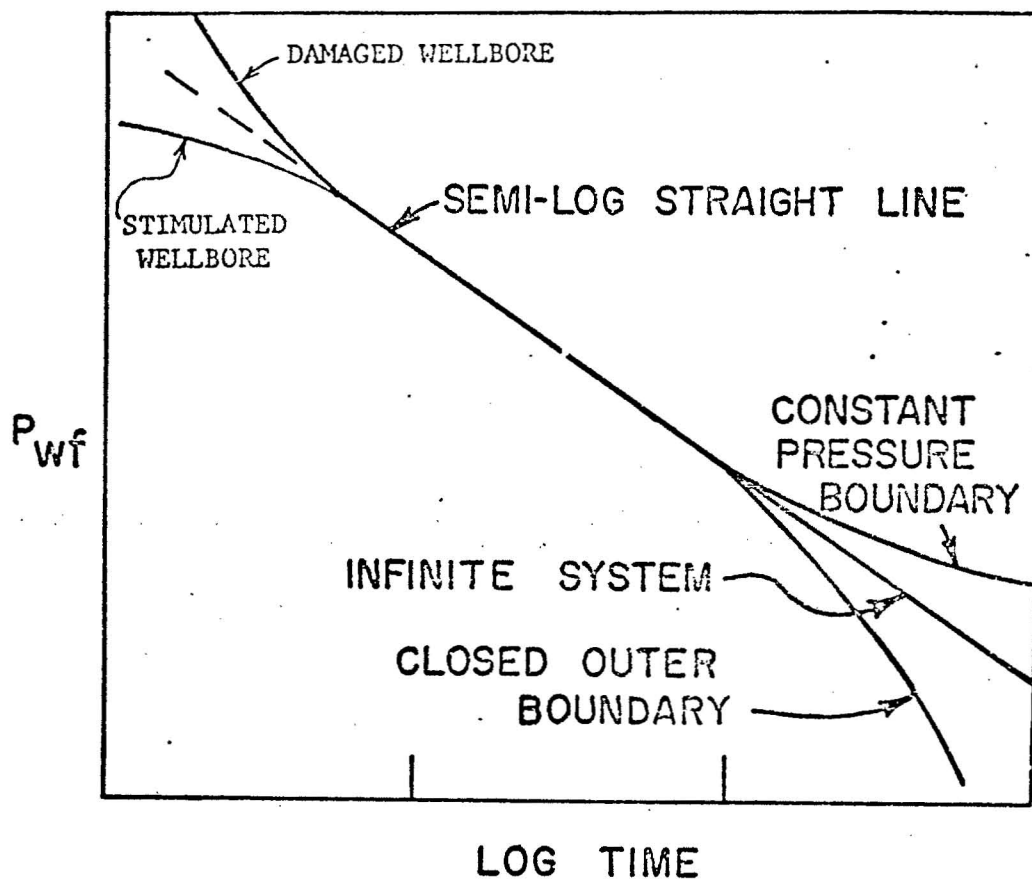


FIG. 6 DRAW-DOWN TEST SHOWING DRAINAGE LIMITS
(After Ref. 12)

PRESSURE BUILDUP TEST

The pressure buildup test is the most important well test in reservoir engineering because it yields the greatest amount of information. Permeability skin effect and, perhaps most important of all, the static average pressure in the drainage area are obtained.

The basis of buildup test is the powerful superposition methods. Equations (9) or (12) are linear equations in p or p_D ; therefore, any linear combinations of p 's or p_D 's are also solutions of their various governing equations.

Assume a well is producing at a continuous rate q for a period $(t + \Delta t)$. At time t , a second well is started at the exact same position as the first well, but it is producing at a rate of $-q$ or equivalent to injecting at rate q . The second well continues for a total time Δt . The net effect of the two wells is a net production of zero after time t at the well location. Buildup pressures can be expressed by:

$$\begin{aligned} \frac{kh}{141.3q\mu B} (p_i - p_{ws}) &= p_D [(t + \Delta t)_D] + s - p_D (\Delta t)_D - s \\ &= p_D [(t + \Delta t)_D] - p_D (\Delta t)_D \end{aligned} \quad (48)$$

If we add and subtract $1/2 \ln (t + \Delta t)_D$ to the right side of equation (48) and substitute equation (24) for $p_D (\Delta t)_D$ we have

$$\begin{aligned} \frac{kh}{141.3q\mu B} (p_i - p_{ws}) &= p_D [(t + \Delta t)_D] - \frac{1}{2} [\ln (\Delta t)_D + 0.80907] \\ &\quad + \frac{1}{2} \ln (t + \Delta t)_D - \frac{1}{2} \ln (t + \Delta t)_D \end{aligned} \quad (49)$$

Rearranging equation (49) we have:

$$\begin{aligned} \frac{kh}{141.3q\mu B} (p_i - p_{ws}) &= \frac{1}{2} \ln \left(\frac{t + \Delta t}{\Delta t} \right) + p_D [(t + \Delta t)_D] \\ &\quad - \frac{1}{2} [\ln (t + \Delta t)_D + 0.80907] \end{aligned} \quad (50)$$

If we are testing a well with an infinite reservoir, then the last two terms on the right hand side of equation (50) cancel out and we have

$$p_{ws} = p_i - \frac{162.6q\mu B}{kh} \log_{10} \left(\frac{t + \Delta t}{\Delta t} \right) \quad (51)$$

By plotting p_{ws} vs. $\log_{10} \left(\frac{t + \Delta t}{\Delta t} \right)$, we should have a straight line with a slope, m :

$$|m| = \frac{162.6q\mu B}{kh} \quad (52)$$

Extrapolating the straight-line to infinite shut-in time, $\frac{t + \Delta t}{\Delta t} = 1$, then we have $p_{ws} = p_i$, the initial formation pressure.

For a bounded reservoir, the last two terms in equation (50) do not cancel out in general, except when t is short enough such that $t_{DA} < 0.05$, where a semi-log straight line prevails. However, in many instances, Δt is small compared to the producing time, t . Under this condition we can write

$$t + \Delta t \approx t \quad (53)$$

and equation (50) becomes

$$\begin{aligned} \frac{kh}{141.3q\mu B} (p_i - p_{ws}) &= \frac{1}{2} \ln \left(\frac{t + \Delta t}{\Delta t} \right) + p_D (t_D) \\ &\quad - \frac{1}{2} [\ln t_D + 0.80907] \end{aligned} \quad (54)$$

where the last two terms on the right hand side of equation (54) are constants, or:

$$\begin{aligned} p_{ws} &= p_i - \frac{162.6q\mu B}{kh} \log_{10} \left(\frac{t + \Delta t}{\Delta t} \right) \\ &\quad - \frac{141.3q\mu B}{kh} \left[p_D (t_D) - \frac{1}{2} (\ln t_D + 0.80907) \right] \end{aligned} \quad (55)$$

The plot of p_{ws} vs. $\log_{10} \left(\frac{t + \Delta t}{\Delta t} \right)$ will still yield a semi-log straight line where the slope m is

$$|m| = 162.6 \frac{q\mu B}{kh} \quad (56)$$

If we call the pressure extrapolated to infinite shut-in time, p^* , then

$$\frac{kh}{141.3q\mu B} (p_i - p^*) = p_D(t_D) - \frac{1}{2} [\ln t_D + 0.80907] \quad (57)$$

p^* , sometimes called the "false pressure", is neither the initial nor the average pressure. The utilization of p^* will be illustrated later.

The above buildup theory is developed by Horner [13] and the plot is generally called the Horner graph.

To obtain the skin effect, s , we have to utilize equation (30), since s does not appear in equation (48). If we subtract equation (48) from equation (30) we have

$$\begin{aligned} \frac{kh}{141.3q\mu B} [(p_i - p_{wf}) - (p_i - p_{ws})] &= \frac{kh}{141.3q\mu B} (p_{ws} - p_{wf}) \\ &= p_D(t_D) + s - p_D[(t + \Delta t)_D] + p_D(\Delta t_D) \end{aligned} \quad (58)$$

or

$$s = \frac{kh}{141.3q\mu B} (p_{ws} - p_{wf}) + p_D[(t + \Delta t)_D] - p_D(t_D) - p_D(\Delta t_D) \quad (59)$$

If we apply the assumption $\Delta t \ll t$, then

$$s = \frac{kh}{141.3q\mu B} (p_{ws} - p_{wf}) - p_D(\Delta t_D) \quad (60)$$

Let $p_{1 \text{ hr}}$ be the value of p_{ws} at one hour after shut-in on the correct semi-log straight line, and substitute equation (24) for $p_D(\Delta t_D)$,

$$s = 1.15 \left[\frac{p_{1 \text{ hr}} - p_{wf}}{|m|} - \log_{10} \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right] \quad (61)$$

For bounded reservoir, it is interesting to know the volumetric average pressure, $\bar{p}$. If we write a materials balance equation as:

$$\text{res. vol. produced} = \text{expansion of initial fluids} \quad (62)$$

we have:

$$(5.615)qBt = Ah\phi c_t(p_i - \bar{p}) \quad (63)$$

By rearranging equation (63) into dimensionless form:

$$\frac{kh}{141.3q\mu B} (p_i - \bar{p}) = \frac{2\pi(0.00634)kt}{\phi\mu C_t A} = 2\pi t_{DA} \quad (64)$$

Matthews-Brons-Hazebroek [14] related the average pressure, $\bar{p}$, with the false pressure, p^* by subtracting equation (64) from (57):

$$\begin{aligned} \frac{kh}{141.3q\mu B} [(p_i - \bar{p}) - (p_i - p^*)] &= \frac{kh}{141.3q\mu B} (p^* - \bar{p}) \\ &+ 2\pi t_{DA} - p_D(t_D) + \frac{1}{2} (\ln t_D + 0.80907) \end{aligned} \quad (65)$$

or

$$\begin{aligned} p_{D_{MBH}} &= \frac{kh}{70.65q\mu B} (p^* - \bar{p}) \\ &= 4\pi t_{DA} - 2p_D(t_D) + [\ln t_D + 0.80907] \end{aligned} \quad (66)$$

Figures 7 and 8 show two plots of $p_{D_{MBH}}$ vs. t_{DA} for various different drainage shapes. Additional graphs can be found in Ref. 1, pp. 40-45. By these graphs, one can calculate the average pressure by knowing p^* and t_{DA} .

In equation (54) if we solve for $\left(\frac{t + \Delta t}{\Delta t}\right)$ at the time $p_{ws} = \bar{p}$, we have

$$\begin{aligned} \frac{kh}{141.3q\mu B} (p_i - \bar{p}) &= \frac{1}{2} \ln \left(\frac{t + \Delta t}{\Delta t} \right)_{p_{ws}} = \bar{p} + p_D(t_D) \\ &- \frac{1}{2} [\ln t_D + 0.80907] \end{aligned} \quad (67)$$

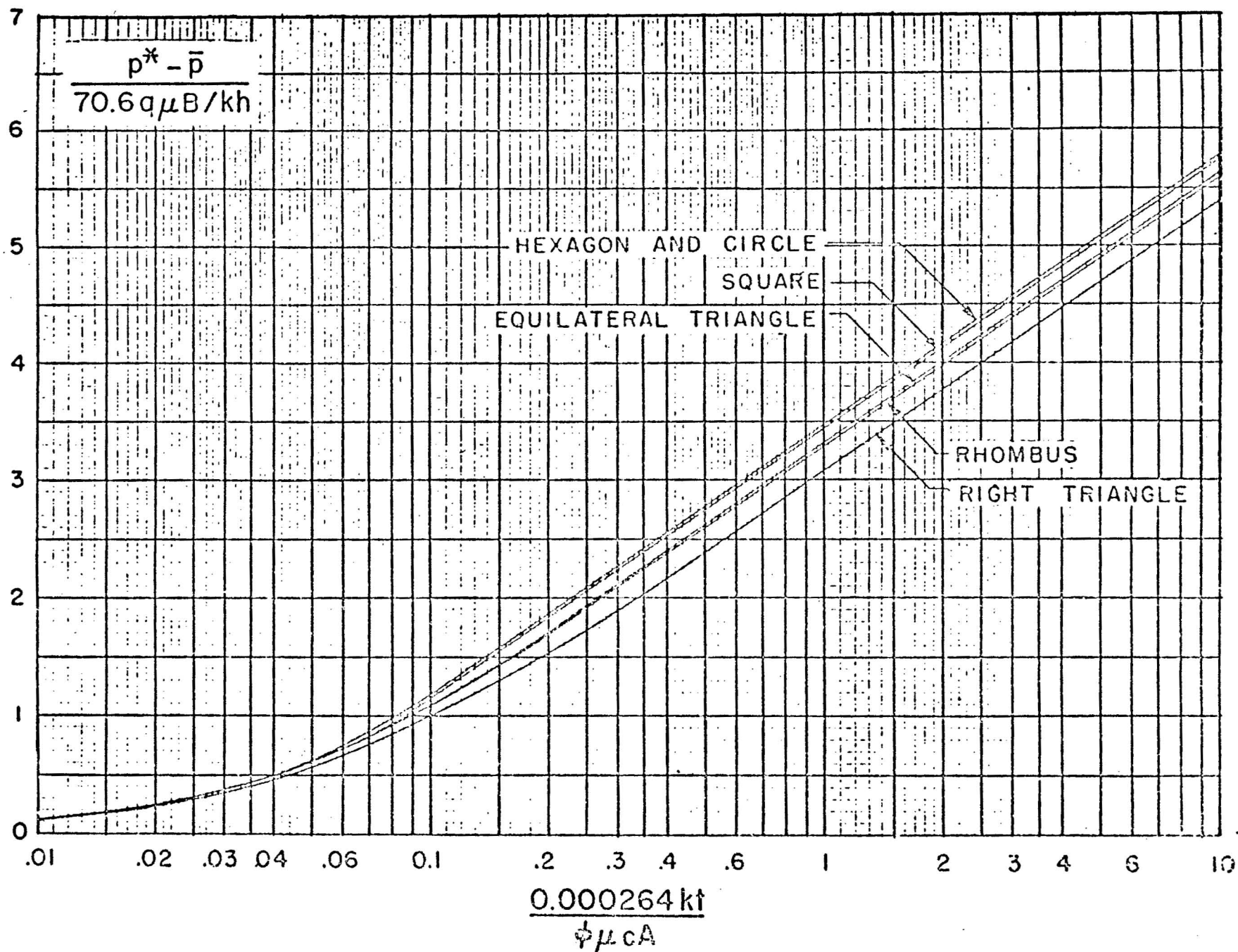


FIG. 7 PRESSURE FUNCTION FOR ONE WELL IN CENTER OF EQUILLATERAL FIGURES
(After Ref. 1)

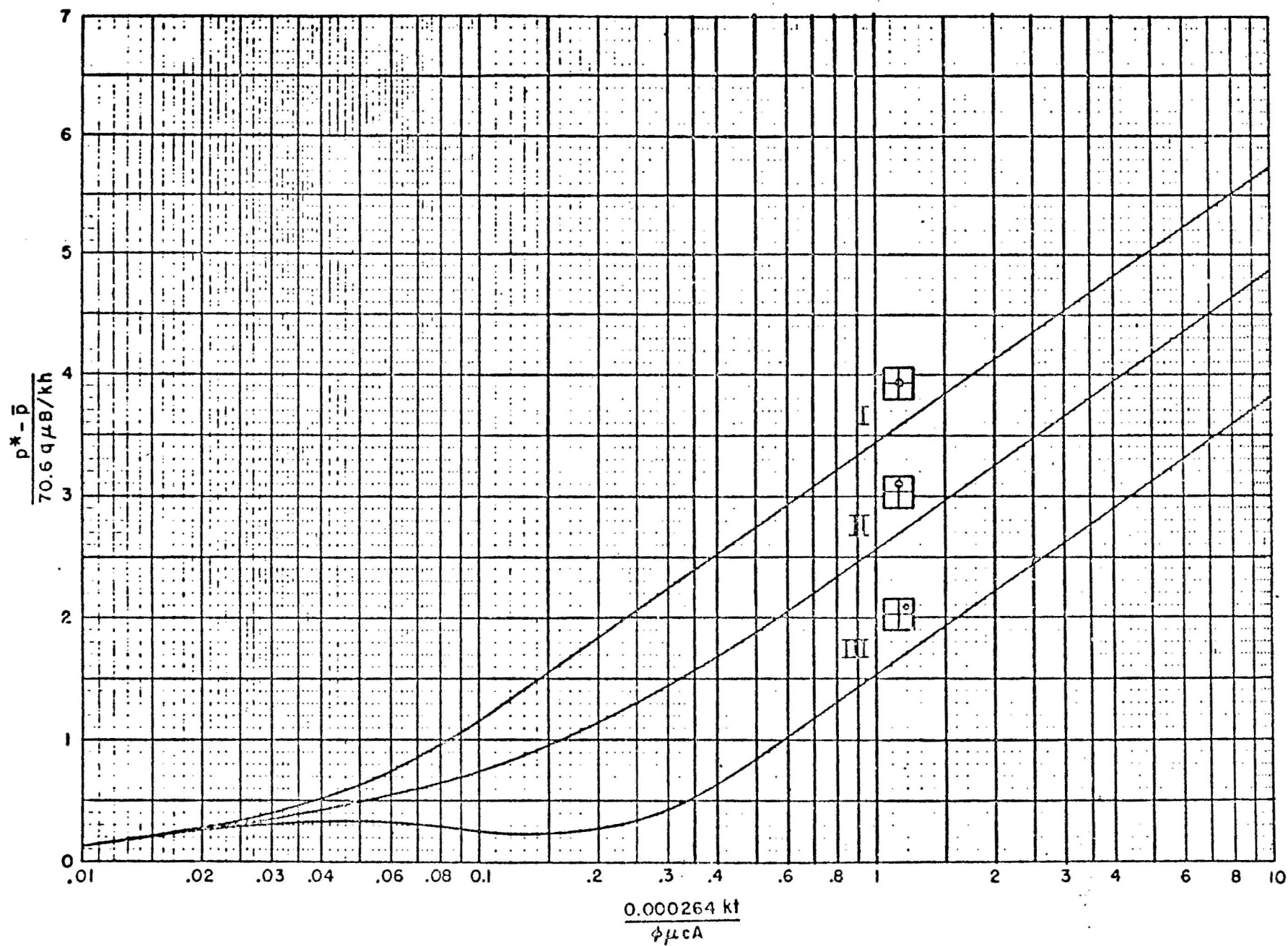


FIG. 8 PRESSURE FUNCTION FOR DIFFERENT WELL LOCATIONS IN A SQUARE BOUNDARY (After Ref. 1)

Equating the right hand sides of equations (64) and (67), we have

$$\ln \left(\frac{t + \Delta t}{\Delta t} \right)_{p_{ws}} = \bar{p} = 4\pi t_{DA} - 2p_D(t_D) + [\ln t_D + 0.80907] \quad (68)$$

Comparing equation (68) to (66), we see

$$p_{D_{MBH}} = \ln \left(\frac{t + \Delta t}{\Delta t} \right)_{p_{ws}} = \bar{p} \quad (69)$$

Thus, a useful interpretation of the Matthew-Brons-Hazebroek function is that it represents a value of $\ln \left(\frac{t + \Delta t}{\Delta t} \right)$ at which extrapolation of the initial straight line provides the proper static pressure.

The following is a summary of pressure buildup analysis by the Horner graph method:

- 1) Plot p_{ws} vs. $\log_{10} \frac{t + \Delta t}{\Delta t}$.
- 2) Find m , $p_{1 \text{ hr}}$, p^* at $\frac{t + \Delta t}{\Delta t} = 1$. At times, it may seem to have more than one possibility for a straight line. The suggested procedure is to plot $(p_{ws} - p_{wf})$ vs. Δt on log-log type paper and compare with Figures 1 and 3 to determine the onset of a correct straight line.
- 3) Use equations (56), (61), (45), and (46) to calculate k , s , Δp_{skin} , and FE.
- 4) For $\bar{p}$:
 - a) Calculate t_{DA} , the dimensionless produced time.
 - b) Determine the drainage shape and well location.
 - c) Find $\bar{p}$ by going to a MBH pressure function plot such as Figures 7 and 8.

GAS VS. LIQUID FLOW

All the theories behind the well test analysis developed so far are based on the solution for a constant and slightly compressible liquid. Strictly speaking, one should obtain solutions to equation (3) with either ideal gas law or real gas law. See for example Ref. 17.

Matthews and Russel [1] discussed this subject in their monograph on well testing. They indicated that if pressure is in excess of 2000 psi

(Wattenbarger and Ramey [18] claimed to be 3000 psi), one may use all the equations developed for liquid for the analysis of gas wells. It is only necessary to convert the gas rate in cubic feet per day to standard barrels per day; and substitute B_g for the formation factor, c_g for the total compressibility, and μ_g for the formation viscosity. B_g , the gas volume factor is defined as

$$B_g = z \frac{T}{T_{sc}} \frac{P_{sc}}{(p^* + p_{ws})/2} \quad (70)$$

where z is the gas deviation factor and subscript sc refers to standard condition. However, p_{ws} changes with time, and it is customary to approximate equation (70) by substituting p_{wf} for p_{ws} .

Matthews and Russel [1] also indicated that if the pressure is below 2000 psi, it is better to plot p_{ws}^2 vs. $\log_{10} \left(\frac{t + \Delta t}{\Delta t} \right)$. The equation looks like:

$$p_{ws}^2 = p^{*2} - 325.2 \frac{q \mu_g z T P_{sc}}{k_g h T_{sc}} \log_{10} \left(\frac{t + \Delta t}{\Delta t} \right) \quad (71)$$

For a discussion of a low-permeability gas well produced at a high rate, see Ref. 19. For a discussion of non-Darcy flow, i.e., higher Reynolds number, see Refs. 19 and 5.

TWO PHASE FLOW

It is also possible that the reservoir condition is initially in two phases. Therefore, it is imperative that we have equations to handle this situation. One major difference between a steam well and an oil well is that the pressure and temperature are the same across the water-steam interface, whereas the pressure is different across the oil-gas interface.

Perrine [20] and Weller [21] have successfully demonstrated that if one substitutes

$$\left(\frac{k}{\mu} \right)_t = \left(\frac{k}{\mu} \right)_w + \left(\frac{k}{\mu} \right)_s \quad (72)$$

for $\frac{k}{\mu}$ and

$$c_t = S_s c_s + S_w c_w + c_r \quad (73)$$

where

Subscripts s and w represent steam and water

S = Saturation of steam or water

$$S_s + S_w = 1$$

for c_t in equation (9), we have successfully converted two phase flow to a single phase flow situation. See Refs. [1], [20] and [21].

FRACTURED WELLS

Traditionally, if a well is not producing at a high enough rate due to well damaging, it can be stimulated by either acidizing or hydraulic fracturing. It is also possible in some cases that natural fractures exist around the wells. These phenomena significantly influence the onset of the semi-log straight line, which is vital to the well analysis.

Gringarten et al. [22] recently studied this behavior and Figure 9 shows one of their p_D vs. t_{DL} curves. It shows that when t_{DL} is small, the slope of $\log_{10} p_D$ vs. $\log_{10} t_{DL}$ is $\frac{1}{2}$ and the dimensionless pressure at the onset of semi-log straight line is approximately twice that at the end of the half-slope period.

MISCELLANEOUS CONSIDERATIONS

In some instances, it is not possible to shut a well in completely to perform buildup tests. This could be due to condensation or production reasons. In this case, a two-rate test is in order. The basis for performing a two-rate test is presented in Ref. 1. In general, the production rate change should be at least 50%.

In all pressure measurements that we have mentioned thus far, we are measuring the bottom-hole pressure. This is done by lowering pressure gauges with either self-contained recorders or surface recorders. In some cases where the holes are not lined, it may be difficult to lower the gauges to the the right location. Surface or wellhead pressure measurements must be used to substitute for the bottom-hole measurements. Before these surface pressure measurements can be used, conversion must be made to bottom-hole conditions. See Ref. 15.

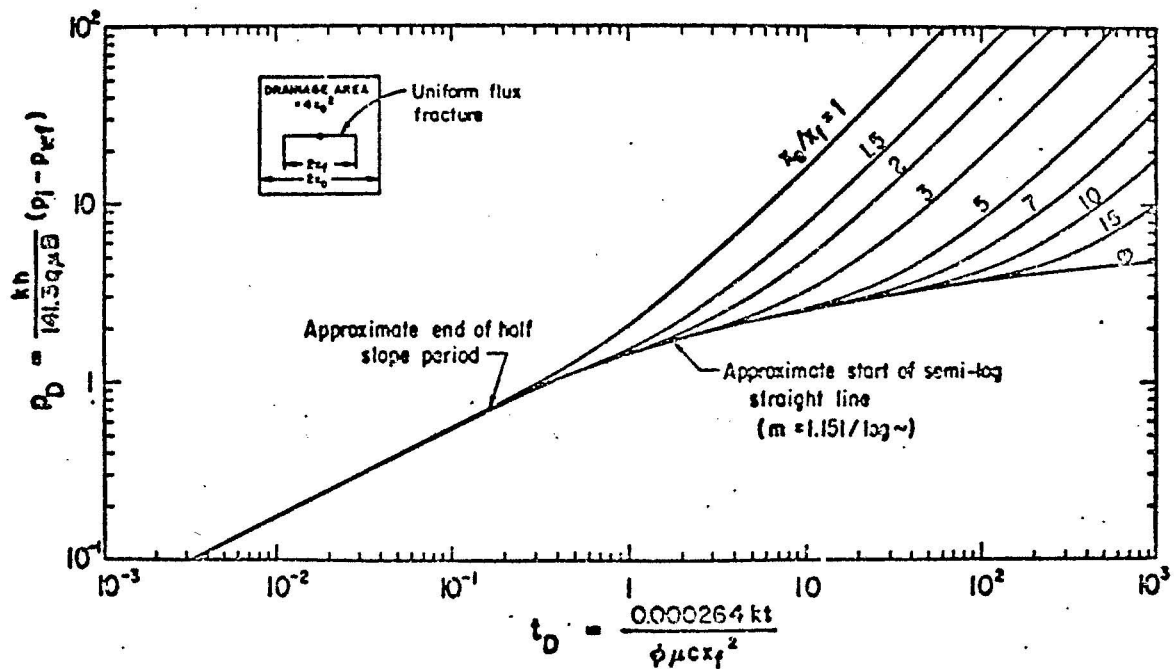


FIG. 9 $P_D - t_{DL}$ FOR A UNIFORM FLUX VERTICAL FRACTURE
(After Ref. 22)

GEOHERMAL RESERVOIR

A powerful tool that is being used by petroleum engineers to estimate the nature and volume of an oil or gas reservoir is the materials balance equations. Heat exchanges between rocks and fluids or water influx and fluids are not considered. Only mass and volume balance equations are utilized.

Heat exchange is important in a geothermal reservoir since heat is our major interest rather than the fluids themselves. Whiting and Ramey [16] have developed a general set of materials and heat balance equations for geothermal fields and have applied them to the Geyser and Wairakei fields. The following is a brief summary of their presentation.

A. Mass Balance

At any time, the current mass of fluids will equal the initial mass minus the cumulative mass produced and lost via wild wells, springs, etc., plus the cumulative mass recharge from a contiguous aquifer.

$$W_C = W - W_P - W_L + W_e \quad (74)$$

where W_C = current mass in reservoir, lbs
 W = initial mass in reservoir at the start of production, lbs
 W_P = mass produced, lbs
 W_L = mass lost via springs, wild wells, etc., lbs
 W_e = mass influx from aquifer, lbs

B. Volumetric Balance

At any time the mass of the fluids in the reservoir must fill the pore space.

$$V\phi = (W - W_P - W_L + W_e) (x (v_g - v_f) + v_f) \quad (75)$$

where

V = reservoir bulk volume, ft^3
 ϕ = porosity, fraction of bulk volume
 x = steam quality in reservoir, mass fraction of fluid which is steam
 v_g = specific volume of steam ft^3/lb
 v_f = specific volume of liquid water, ft^3/lb

C. Heat Balance

$$W_C h_C + (1 - \phi) V \rho_r C_r (T - T_0) = W h_i + (1 - \phi) V \rho_r C_r (T_i - T_0) \quad (76)$$

$$- W_p h_p - W_L h_L + W_e h_e + Q_s$$

where

- h_C = average enthalpy of total fluids in reservoir, btu/bl
- h_i = average enthalpy of initial fluids in reservoir, btu/lb
- h_p = average enthalpy of produced fluids, btu/lb
- h_L = average enthalpy of lost fluids, btu/lb
- h_e = average enthalpy of liquid water influx, btu/lb
- ρ_r = formation density lb/ft³
- C_r = specific heat of formation, btu/lb-F
- T = current reservoir temperature, F
- T_i = initial reservoir temperature, F
- T_0 = some reference temperature, F
- Q_s = net heat conducted into reservoir, btu

The average enthalpy of any liquid-steam combination can be expressed by:

$$h = x (h_g - h_f) + h_f \quad (77)$$

where

- h = enthalpy of steam quality x , btu/lb
- h_g = enthalpy of saturated steam, btu/lb
- h_f = enthalpy of saturated liquid, btu/lb

Equations (74), (75) and (76) can be used to describe the balance of any geothermal field, be it liquid, liquid-steam or steam. The initial condition of the reservoir could be compressed liquid, saturated liquid and steam or superheated steam. The production of the fluids can also be any of the above combinations. Temperature and pressure are related either on the normal vapor pressure curve or other related curves.

In general, available will be continuous records of the average pressure of the reservoir vs. its cumulative production. With this information, it is

possible to use equations (74), (75) and (76) to optimize some reservoir parameters, e.g., initial volume, temperature and pressure. It is also possible to conjecture about the initial fluid condition. Once this condition and initial fluid parameters are known, it is possible to make performance predictions into the future. A "good" match of field performance and a postulated aquifer model may or may not be the real reservoir condition. However, it is not imperative that we have the correct model. The important criterion is that the performance of the selected model coincides with the real reservoir in the time frame considered.

SUMMARY

With the assumption that the geothermal reservoir has very little or no net heat transfer, one can use the petroleum well test analysis techniques on geothermal reservoir with very little modification.

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