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POLARIMETRY OF Be STARS AT 1.25 AND 2.2 MICRONS

A DISSERTATION SUBMITTED TO THE GRADUATE DIVISION OF THE
UNIVERSITY OF HAWAII IN PARTIAL FULFILLMENT
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DOCTOR OF PHILOSOPHY

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By

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ABSTRACT

Presented are observations of the linear polarization of 8 Be stars at 1.25 and 2.2 microns. An equatorial disk model is developed where the polarization due to electron scattering of stellar and disk radiation is treated discretely, allowing two scatters of the stellar radiation and one of the disk radiation. Neutral hydrogen opacity is taken into account and the populations of the levels of hydrogen are allowed to depart from LTE if the dilute and diffuse fields indicate such a departure is necessary. ϕ Per, ζ Tau, γ Cas, and EW Lac all show a decrease in polarization longward of 1 micron. When combined with polarimetry in the visible model calculations indicate an increase in the density of the disk toward the star for these stars. δ Lib, ϵ Cap, and α Aqr all maintain a high polarization into the near-infrared and each requires a different disk structure when modeled. The excess emission longward of 1 micron for these stars can be shown to arise in the disk.

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I INTRODUCTION

The intrinsic polarization of Be stars was first noted to have a wavelength dependence different from interstellar polarization by Serkowski (1968). This was confirmed in a survey of several Be stars by Coyne and Kruszewski (1969). The wavelength dependence of the polarization for high latitude Be stars determined by Coyne (1971) shows the characteristic drop in polarization shortward of the Balmer and Paschen jumps coupled with a general decline in polarization longward of $4000 \text{ \AA}$. This polarization has been attributed to the presence of a circumstellar gas shell where electron scattering provides the polarizing mechanism (Serkowski, 1968). This would be consistent with the geometry of a rapidly rotating extended atmosphere largely confined to the equatorial plane originally suggested by Struve (1931) for Be stars. Recent theoretical studies on the emission line spectra of Be stars (for a review see Marlborough, 1976) use this type of geometry.

Although Harrington and Collins (1968) predicted that rapidly rotating early type stars would show a net polarization due to the distortion of the figure of a star by rotation, a more detailed treatment of the problem by Haisch and Cassinelli (1976) indicates that a rotationally distorted stellar atmosphere alone will not produce polarizations as large as observed in many Be stars (1-2%). Thus, one is forced to invoke a more extensive circumstellar shell. Clearly there must be some asymmetry to the extended atmosphere if a net polarization is to result. A spherically symmetric shell would produce no net polarization. An important question is how

asymmetric must the envelope be to achieve polarizations as high as 2%? Is a geometry as restrictive as the thin equatorial disk proposed by Capps et al. (1973) for ζ Tau necessary? A detailed treatment of scattering in an extended equatorial disk should answer this question.

Presented in this study are new observations of the linear polarization and photometric brightness of eight Be stars and one peculiar emission line A star at 1.25 and 2.2 microns. A model for the extended envelopes of Be stars is developed that treats the scattering of stellar and diffuse flux by electrons in the envelope in greater detail than previous models. When combined with polarimetry and photometry in the visible, the near infrared data prove to be useful in placing constraints on the densities and temperatures of the envelopes of Be stars. The results of these model calculations are in remarkable agreement with the ad hoc density distribution used by investigators of the spectroscopic properties of these stars.

Be stars clearly show an excess flux over normal B stars longward of one micron (Johnson, 1967; Allen, 1973). The most likely source of this excess is free-free and free-bound emission from the hydrogen plasma that makes up the circumstellar shell (Woolf et al., 1970; Gehrz et al., 1974). It would be interesting to find out whether this excess arises in the same region where the scattered polarized flux originates. That is, are the electrons that are responsible for the observed intrinsic polarization of Be stars the same electrons that are producing the emission in the near infrared? This question can be investigated by including in a model of the

circumstellar shell sources of emission that would be produced by the hydrogen plasma.

II INSTRUMENTATION

The linear polarization measurements were made with two different polarimeter systems, systems one and two. System one is illustrated schematically in Figure 1. The data acquisition system is diagramed in Figure 2. A stepper motor is connected through a gear train to a rotating cell containing an analyzer. The stepper motor is run under computer control, and rotates the analyzer through eight positions in 45° increments. At each position the intensity is measured and stored by the computer. At the end of several full rotations of the analyzer, the measured intensities are outputted by the computer in a convenient form. The operation of system one is described in greater detail in Appendix A.

Ideally the analyzer would be rotated as fast as possible to minimize the effects of atmospheric seeing and transmission fluctuations, poor guiding, and variations in the system response (Serkowski 1973). With system one, the speed of rotation is limited by the chopping frequency (10 Hz at the time of use) and the rate at which the motor can be stepped. Since the time spent stepping and pausing is dead time, shortening the integration time to speed up the system will lower the duty cycle. At its fastest, system one could make one revolution in two seconds, but was often run at one half that speed.

System one was used with an Indium Antimonide detector. The detector system was capable of a 3σ detection in 200 seconds of integration time at a K (2.2 micron) magnitude of +13 without the analyzer, at K magnitude of 11.8 with the analyzer in the beam.

Fig. 1 - Mechanical Layout for System 1.

Fig. 2 - Block Diagram of the Data Acquisition System for System 1.

If a polarimetric measurement is considered to be equivalent to two independent photometric measurements, each taking one half of the total integration time, the detector system can in principle measure the polarization of a +11.4 K magnitude object in 200 seconds to a precision of $\pm 33\%$.

The performance of system one is illustrated in Figure 3, where the horizontal axis is the 2.2 micron magnitude of an object being observed and the vertical axis is the accuracy to which the percent polarization can be made in 100 seconds. The quantity δ represents the one sigma error points. The dashed line represents an estimate of the maximum possible performance of the system under detector noise limited operation. For faint objects the observations lie reasonably close to this theoretical limit, but for objects brighter than about 6th magnitude the curve flattens out. There are several reasons for this, primarily the finite speed of the rotation of the analyzer and jitter in the motion of the gear train. These effects introduce errors at the few tenths of a percent level, independent of the brightness of the object. The errors of measurement for systems one and two are discussed in greater detail in Appendix B.

System two is illustrated schematically in Figure 4. Here the analyzer is placed in the beam after the chopping secondary but before any optics in the photometer assembly. To avoid sending 100% polarized light through the optics of the photometer, a fixed Quartz Lyot depolarizer is located directly below the analyzer. The data acquisition system is diagramed in Figure 5, and the operation

Fig. 3 - Performance of System 1. The Quantity δ is the Error of a Polarimetric Measurement for 100 Seconds of Integration Time.

Fig. 4 - Mechanical Layout for System 2.

Fig. 5 - Block Diagram of the Data Acquisition System for System 2.

of system two is discussed in greater detail in Appendix A. The main difference between systems one and two is that for system two a wire grid polarizer is used for the analyzer and the analyzer is rotated continuously. The intensity is integrated over 45° sectors and stored in the computer in a manner similar to system one.

The performance of system two is illustrated in Figure 6, where the axes are the same as in Figure 3. The dashed line also represents an estimate of the detector noise limited performance of the system. Since the detector used with system two had roughly the same performance as the one used with system one, system two's performance for faint objects is about the same. For objects brighter than 6th magnitude, however, δ averages about a factor of two below the values for system one. Thus, for brighter objects (as all the Be stars were) greater precision can be reached in the same amount of time with system two, provided the observations are reproducible from one part of the night to another. This difference is due both to the technique (spinning vs. stepping) and to the difference in efficiency between the two systems. The primary source of technique error in system two is in the poor timing capabilities of the computer.

The efficiency for both systems was less than 1.0. For system one the efficiency was reduced primarily by 1) the time constant of an analog input, 2) the small number of chops at each position of the analyzer, and 3) the imperfect polarizing efficiency of the HR polarizer averaged over the bandpass of the 2.2 (μ) micron filter. The first two of these effects suppress the rate at which the signal

Fig. 6 - Performance of System 2. The Quantity δ is the Error of a Polarimetric Measurement for 100 Seconds of Integration Time.

can respond to a change in the position of the analyzer. Thus, during a significant part of the integration time the signal is still approaching the true value for that position. These effects can be reduced by slowing the system down, but at the risk of increasing sources of error discussed previously.

For system two the analyzer was a wire grid of higher polarizing efficiency at 2.2 microns (~96%). Also a low Z buffer reduced the time constant of the analog input. As a result the efficiency was greatly improved.

The efficiency for both systems was measured by placing a piece of randomly oriented HR polarizer in front of the analyzer and making measurements in a normal manner. The resultant polarization was divided by the efficiency of the HR and then inverted. All subsequent measurements were then multiplied by this factor. Table 1 lists the measured efficiency factors for both systems and the time each system was in use.

Table 1

Efficiency Factors at Indicated λ

System	Dates	1.25 μ	2.2 μ
#1	Aug-Sept, 1976	1.65	2.10
#2	April-Aug, 1977	1.39	1.39

III DATA

The data were all taken between August, 1976, and August, 1977, on the 88" telescope on Mauna Kea with the f 35 chopping secondary. System one was in use for all of the 1976 data; system two was in use for all of the 1977 data. Initial data reduction was performed by the IBM 1800 computer, which provided the first three components of the stokes vector as output. Throughout this section the following definitions for these three quantities will be used:

$$I = \text{Intensity} = (I(0) + I(45) + I(90) + I(135)) / 2$$

$$P_x = Q/I_Q = (I(0) - I(90)) / (I(0) + I(90))$$

$$P_y = U/I_U = (I(45) - I(135)) / (I(45) + I(135))$$

The technique of object-sky chopping was used to provide an AC signal and avoid severe 1/f noise problems when the object and sky are differenced at a slow rate. All observations were through a 9" aperture. The chop was N/S, and separation was typically 16". To eliminate any electrical offsets in the system the object was placed first in the upper beam (A beam) and a measurement made, then placed in the lower beam (B beam) and two measurements made; then the object was returned to the upper beam (A) and measured once more. For the Be stars (which were bright), usually one set of ABBA measurements constituted a single observation, and the computer would be instructed to compute the desired stokes vectors. The intensity at any angle would be the difference (intensity in beam A - intensity in beam B). Typically, one observation would take between two and four minutes.

This sequence would be repeated several times, yielding several observations of each object for the night. The mean of the P_x 's and P_y 's for each object could then be computed, along with an estimate of the error from the calculated standard error of the mean.

During the night many observations were made of several unpolarized standard stars taken from a list compiled by Serkowski (1974). These stars are unpolarized throughout the visible, and I have assumed that they would also show no measurable polarization at 1.2 and 2.2 microns. The P_x 's and P_y 's for all of these observations were averaged as if only a single object had been observed. These final averages were then taken to be the night's value for the instrumental polarization.

The instrumental polarization was removed from the observations vectorially by subtracting the P_x for the instrumental polarization from an object's observed P_x , and similarly for P_y . A more detailed discussion of the validity of this procedure and the errors associated with it is in appendix B.

The polarimetric observations are presented in Table 2. With the exception of ϕ Per, all of the program stars were observed on more than one night. The quality of the polarimetry, although not comparable to polarimetry in the visible, was high.

A major problem in interpreting the wavelength dependence of the polarization in Be stars is the possibility of contamination by an interstellar component. Generally an interstellar component appears as a rotation of the position angle that is correlated with the strength of the polarization. The higher the intrinsic

Table 2
Polarimetric Observations

HD	Name	JD 244-	$P_J(\%)$	$\theta_J(^{\circ})$	$\epsilon_J(\%)$	$P_K(\%)$	$\theta_K(^{\circ})$	$\epsilon_K(\%)$
144	γ Cas	3044.0	0.43	94.8	0.10	0.27	106.2	0.20
		3045.0	0.54	90.7	0.17	0.23	131.8	0.13
		3380.9	0.46	89.3	0.15	0.38	125.8	0.20
10516	ϕ Per	3019.1	0.96	30.0	0.10	0.61	21.1	0.11
142983	48 Lib	3287.8	0.76	127.6	0.07	0.67	133.3	0.12
		3290.8	0.59	125.3	0.07	0.75	135.6	0.17
148184	χ Oph	3019.8	0.28	171.8	0.10	0.08	-	0.11
		3045.8	0.25	136.1	0.14	0.13	-	0.12
181615	ν Sgr	3019.9	0.26	167.0	0.10	0.06	-	0.13
		3042.9				0.02	-	0.23
		3045.8	0.23	160.2	0.12	0.29	-	0.24
		3048.8	1.60	4.8	0.10	0.19	-	0.22
		3289.0	0.55	131.1	0.10	0.27	36.0	0.13
205637	ϵ Cap	3019.9	0.30	163.2	0.10	0.69	164.3	0.18
		3043.9	1.07	166.7	0.09	0.82	163.6	0.15
		3044.8	0.48	155.1	0.12	0.88	167.5	0.16
		3045.9	0.97	167.8	0.08	0.97	163.1	0.12
209409	$\circ$ Aqr	3019.9	0.40	76.1	0.10	0.76	9.9	0.12
		3043.0	0.61	2.0	0.22			
		3043.9	0.38	175.5	0.09	0.55	7.2	0.13
		3044.9	0.28	61.3	0.14	0.65	19.3	0.11
		3045.9	0.02	-	0.11	0.63	19.2	0.12
		3048.9	1.06	6.9	0.11			
		3382.0	0.46	163.8	0.22	0.39	9.6	0.15
212571	π Aqr	3019.0	0.92	160.5	0.09	0.76	168.4	0.11
		3043.0	2.59	175.7	0.20	0.61	163.0	0.22
		3044.0	3.09	176.2	0.10	0.99	168.6	0.11
		3044.9				2.03	174.9	0.15
		3045.9				2.29	177.3	0.12
		3046.9	0.83	172.5	0.14			
		3048.9	1.11	170.0	0.10			
217050	EW Lac	3380.9	1.20	74.3	0.22	0.83	69.9	0.16
		3381.9	1.30	69.2	0.13	1.04	69.9	0.13

polarization, the greater the contribution from the intrinsic component due to the Be star and the closer the position angle is to the intrinsic position angle. For a weak intrinsic polarization, the contribution of the interstellar component will be greater, and the measured position angle will be closer to the position angle of the interstellar component (Serkowski, 1970; Coyne and Kruszewski, 1969). Since the strength of the interstellar polarization drops rapidly longward of 1 micron (Dyck and Jones, 1978), if the contribution of an interstellar component is small in the visible the measured polarization at 2.2 microns should be predominantly due to the intrinsic polarization of the Be star and should have a position angle close to the intrinsic position angle of the star. Several stars in Table 2 show 2.2 micron position angles significantly different from their position angles in the visible (which, for those stars, undergo some rotation of position angle with wavelength). Presumably, this is due to an interstellar polarization component.

Poeckert (1975) has devised a clever method of determining the interstellar component in the polarimetric measurements of a Be star by observing the rotation in position angle through the $H\alpha$ emission line. If the emission in $H\alpha$ is unpolarized (and Poeckert shows that for several stars it is), it will dilute the continuum polarization by an amount proportional to the emission. This dilution allows the interstellar component to become an increasing contributor to the measured polarization, producing the rotation in position angle. Since the strength of the $H\alpha$ emission can be

measured, the magnitude and direction of the interstellar component can be found. This can then be removed vectorially from the observed polarization using the wavelength dependence of interstellar polarization given by Serkowski (1974) and by assuming that the peak in the interstellar polarization is at 0.55 microns.

Table 3 lists the mean wavelength dependent polarization for six of the stars in the program. The visible polarization data is a vectorial average of the data taken by Poeckert (1977). α Aqr and 48 Lib have had an interstellar component removed that was taken from Figures 10 and 12 of Poeckert (1975). For 48 Lib the resultant intrinsic position angle is fairly constant between wavelengths, with some indication that too large a value of intrinsic polarization was removed, causing the visible position angles to rotate beyond the 2.2 micron position angle. For α Aqr, the removal of the interstellar component leaves the position angles in less than satisfactory agreement, but it does bring the visible polarization closer to the 1.2 and 2.2 micron position angles. For ϕ Per, the interstellar component derived by Coyne and McLean (1975) was used in correcting the infrared polarization. EW Lac and ϵ Cap show no significant rotation of position angle into the infrared and presumably they are unaffected by an interstellar component (although an interstellar component with the same position angle or a position angle different by 90° could be present). The position angle in the near infrared for γ Cas shows a departure from that in the visible, but in an opposite sense at 1.2 microns than at 2.2 microns. Considering the small value of the measured polarization at these

Table 3
Intrinsic Polarization

Object	$\lambda(\mu)$ =	.344	.406	.511	.663	.780	.849	1.25	2.2	Notes
γ Cas	P(%)	0.37	0.74	0.63	0.48	0.41	0.41	0.47	0.27	
	$\theta(^{\circ})$	104.5	108.3	105.8	104.7	103.2	104.5	91.5	120.8	
EW Lac	P(%)	0.81	1.89	-	-	1.33	1.42	1.25	0.94	
	$\theta(^{\circ})$	68.5	70.3	-	-	69.6	70.1	71.7	69.7	
48 Lib	P(%)	0.45	0.76	0.78	0.68	0.62	0.71	0.74	0.74	1
	$\theta(^{\circ})$	145.4	145.1	144.7	144.9	144.4	144.5	141.6	139.0	
ϵ Cap	P(%)	0.55	1.06	1.12	0.94	0.78	0.79	0.69	0.84	
	$\theta(^{\circ})$	157.5	159.3	158.7	157.6	158.5	158.5	164.9	164.6	
o Aqr	P(%)	0.37	0.52	-	-	0.47	0.52	0.34	0.56	1
	$\theta(^{\circ})$	14.1	18.3	-	-	12.4	14.3	18.5	18.7	
ϕ Per	P(%)							1.40	0.78	1,2
	$\theta(^{\circ})$							24.5	19.4	

¹Interstellar Component Removed

²Visual Polarization Will be Taken from Table III of Coyne and McLean (1975)

wavelengths, these differences are within 2 sigma of the position angle in the visible and are probably not significant.

A rotation of position angle into the infrared is also exhibited by π Aqr which was determined to have an interstellar component by (Poeckert, 1975). The 1.2 and 2.2 micron polarizations, however, show a remarkable range of variability, both in magnitude and position angle. In fact, the rotation in position angle is well correlated with the strength of the polarization. In Figure 7 the 2.2 micron polarimetry has been plotted in a P_x, P_y diagram. Since changes in the measured polarization cannot be caused by changes in the interstellar component, they must be due to fluctuations in the intrinsic polarization. The larger the polarization, the more the position angle should rotate toward the intrinsic value. Thus, if the tips of the polarization vectors in Figure 7 are connected, the resultant line will be parallel to the intrinsic polarization vector. For π Aqr this can be done with the 1.2 and 2.2 micron observations reported here, the $H\alpha$ observations of Poeckert, and the visible polarimetry of Coyne and Kruszewski in combination with that of Poeckert. Coyne and Kruszewski observed less polarization in π Aqr than Poeckert and different position angles. Although Coyne and Kruszewski and Poeckert did not use the same filters, the filters were broad or intermediate band and the central wavelengths were close enough for a comparison to be made.

Figure 7 also shows a vectorial plot of the mean polarization and position angle for both sets of data at approximately .41 and .51 microns. The mean values from Coyne and Kruszewski are at about

Fig 7 - Vector Representation of the Polarimetry of π Aqr. The Circles Have a Radius of 1%.

152° . Immediately obvious is a different intrinsic position angle at these wavelengths compared to the 2.2 micron value. This exercise was done at .41, .51, .65, and .83 microns in addition to 1.2 and 2.2 microns and through H α (Poeckert). The resultant position angles are plotted in Figure 8. The error bars are estimates of the errors in this analysis made by rotating the derived intrinsic position angle through the extremes allowed by the data. There appears to be a definite change in the intrinsic position angle for π Aqr between the blue and the near infrared. This will be discussed later.

The photometry is listed in Table 4 and shows no indication of significant variability. The internal errors are larger than normal for J and K photometry because the observing program was geared specifically for measuring polarization, and the accuracy of the photometry suffers accordingly. The photometry at 2.2 microns agrees well with the values at 2.3 microns reported by Gehrz et al. (1974) for all of the program stars except χ Oph. The mean K magnitude for χ Oph from this study is $3.02 \pm .05$ for September, 1977, whereas Gehrz et al. reported $3.60 \pm .05$ for 1973. This constitutes a change of 0.6 magnitudes, clearly a significant brightening of χ Oph at 2 microns.

Be stars clearly show an excess flux over normal B stars longward of 1 micron (Johnson, 1967; Allen, 1973). The most likely source of this excess is free-free and free-bound emission from the plasma that produces the observed intrinsic polarization (Woolf et al., 1970; Gehrz et al., 1974). Calculations performed in the

Fig. 8 - Intrinsic Position Angle of π Aqr as a Function of Wavelength.

Table 4
Photometric Observations

HD	Name	JD 244-	J (mag) *	K (mag) *
144	γ Cas	3044.0	2.36	2.10
		3045.0	2.42	2.04
		3046.0	2.41	2.14
10516	ϕ Per	3019.1	3.78	3.40
142983	48 Lib	3287.8	4.50	4.52
148184	χ Oph	3019.8	3.37	3.00
		3045.8	3.47	3.03
181615	υ Sgr	3019.9	3.89	2.51
		3042.9	3.82	2.53
		3044.8	3.91	2.49
		3045.8	3.97	2.57
		3048.9	3.98	
205637	ε Cap	3019.9	4.89	4.85
		3042.9	4.79	4.82
		3043.9	4.87	4.79
		3044.8	4.91	4.77
		3045.9	4.93	4.88
209409	$\circ$ Aqr	3019.9	4.88	4.78
		3043.0	4.76	4.76
		3043.9	4.86	4.74
		3044.9	4.93	4.73
		3045.9	4.87	4.73
212571	π Aqr	3043.0	4.23	3.74
		3044.0	4.25	3.81
		3044.9	4.32	3.83
		3045.9		3.83
		3046.9	4.31	
217050	EW Lac	3380.9	5.29	5.28
		3381.9	4.91	4.99

*Magnitudes Are Accurate to $\pm 0.05^m$.

next section show this is not only a consistent picture, but a necessary picture. To compute the excess for the Be stars in this program, each was assigned a spectral type. The B-V color for the corresponding Main Sequence B star of the same spectral type (Jonson, 1966) was used to de-redden the observed colors, assuming $R = 3.1$. These de-reddened colors could then be compared to the intrinsic colors from Johnson and the excess relative to the V filter computed. Since the effects of envelope emission on the B-V color have been ignored, this yields only approximate values, and the values are good to only $\pm 0.15^m$. The infrared excess for six of the program stars and ζ Tau are listed in Table 5, where $(V-M)_I$ is the intrinsic color and $(V-M)_O$ is the observed color, corrected for reddening. The photometry used to derive the values in Table 4 are from Gehrz et al. (1974), except for ϵ Cap, where the photometry at 1.2 and 2.2 microns is from this study.

Table 5
 $(V-M_\lambda)_I - (V-M_\lambda)_O$

Object	1.25 μ	2.3 μ	3.4 μ	4.9 μ
ϕ Per	-.38	-.99	-1.47	-1.85
ζ Tau	-.27	-.64	-0.98	-1.24
γ Cas	-.23	-.71	-1.08	-1.39
EW Lac	-.32	-.71	-1.10	-1.48
48 Lib	-.58	-.68	-0.95	-1.34
ϵ Cap	-.19	-.42	-	-
o Aqr	+.09	-.10	-0.27	-1.04

IV MODEL

Coyne and Kruszcwski (1969) suggested that the characteristic wavelength dependence of the intrinsic polarization for most Be stars they had studied was due to Hydrogen bound-free opacity in an extended equatorial disk. Although no quantitative calculations were done, the dramatic drop in polarization at .33 and .64 microns corresponded nicely to the Hydrogen absorption edges at the Balmer and Paschen jumps. Caps et al. (1973) later considered the problem from a different point of view. They suggested that for an optically thin extended equatorial disk of Hydrogen plasma, bound-free and free-free emission would be radiated by the disk and dilute the otherwise wavelength independent polarization of the scattered stellar radiation. The crucial assumption is that the disk emission is unpolarized by scattering and, to a lesser extent, that the disk is truly optically thin. This model explains the observations of ζ Tau fairly well and the electron density and disk dimensions are consistent with the optically thin case (assuming the levels of neutral hydrogen are populated in LTE). But is it valid to assume that the disk emission is not polarized by scattering? Is the optically thin disk assumption consistent with the hydrogen populations that would be obtained from statistical equilibrium calculations?

The first question can be investigated in a simple way. Consider two identical cubes with uniform density, temperature, and volume emission coefficient, and consider them to be optically thin at some wavelength. Place them side by side with the long face of the

resultant rectangle in the plane of the sky (see Figure 9). Radiation from one cube will scatter off of the free electrons in the adjacent cube into the line of sight to the observer and will be polarized due to Thompson scattering. For those paths from one cube to the other that result in a 90° scattering angle, the scattered radiation will be 100% polarized. For angles other than 90° the scattered radiation will be less than 100% polarized. The exact polarization of the scattered radiation can be computed for this simple case by dividing each cube up into many smaller cubes and considering the scattering angle to be determined by the ray between the centers of these small cells. This was done for the orientation illustrated in Figure 1, and the scattered flux was found to have 56% polarization. If the cubes are moved farther apart but kept the same distance from the observer, this value would increase. At distances between the cubes of greater than seven times the size of a cube side the polarization of the scattered flux is nearly 100%. The net polarization for the adjacent cubes will depend on the relative contributions of the polarized scattered flux and the unpolarized emission from the cube. Since approximately 25% (i.e. $\sim \pi$ steradians) of the emission from one cell passes through the adjacent cell, the net polarization will be approximately

$$P \sim .56 \cdot .25 \cdot \tau$$

where τ is the optical depth due to electron scattering of one cube. For $a\tau$ of 0.1, the net polarization is 1.5%, a value comparable to the polarization observed in Be stars. Using the parameters for ζ Tau determined by the model of Capps et al., the optical depth due

Fig. 9 - Geometry for the Adjacent Cube Problem.

to electron scattering from the center to the edge of the disk is 0.6. The disk in their model had a radius 3 times its thickness, and the assumption that the disk emission is unpolarized is not valid. Calculations performed using a more detailed Be star model discussed later show this to be the case.

The problem of opacity in the disks of Be stars is more complex. Capps et al., in their analysis of ζ Tau, and Gehrz et al., in their photometric study of several Be stars, have been able to fit the infrared excess exhibited by the stars with simple uniform density disks. As long as the density of the disk is nowhere too large ($N_e < 3 \times 10^{11}$) and the dimensions of the disk not too large ($R_g < 4 \times 10^{12}$), the disk will be optically thin at temperatures around 12,000 K in the Balmer and Paschen continua and in the near infrared if the disk is in LTE. If the role of the disk emission in reducing the polarization at the Balmer and Paschen jumps and longward of one micron is inadequate, then an increased opacity in the Balmer and Paschen continua and longward of one micron must be present in order to reduce the amount of scattered flux at those wavelengths. Detailed model calculations show this to be the case.

To model a Be star envelope effectively the following requirements should be met:

- 1) The star must not be treated as a point source but rather as a finite sphere.
- 2) The scattering of disk emission must be explicitly treated.
- 3) The details of the calculation of the net polarization must allow for a non-uniform density distribution in the disk, the finite

thickness of the disk, and the possibility of a viewing angle other than edge-on.

4) The neutral hydrogen populations must be allowed to depart from LTE if the diffuse and dilute radiation fields indicate such a departure is necessary.

The model described below incorporates the first three requirements in an accurate manner. The fourth requirement is very difficult to treat accurately and is incorporated in an approximate manner and only when necessary.

The geometry of the extended equatorial disk model used in this paper is illustrated in Figure 10. The disk is uniform in thickness except near the star. Theoretical work on the density structure of the envelopes of Be stars (Limber, 1964) suggests that near the star the material in the envelope is more highly concentrated toward the equatorial plane. Theoretical work by other authors on the spectroscopic properties of Be stars use similar density contours (Marlbrough, 1969; Hutchings, 1970). For this reason the boundary of the disk is allowed to taper toward a point near the star. The Z axis is the rotational axis of the star. ω is the ratio of the radial distance from the center of the star to the radius of the star:

$$\omega = r/R_{\star} \quad \text{where } r = (X^2 + Y^2)^{\frac{1}{2}} \quad 1$$

The X and Y axes are in the equatorial plane and oriented so that the line of sight to the observer always lies in the Y-Z plane.

The central illuminating star is taken to be a sphere with no limb or gravity darkening. The envelope density is constant in the

Fig. 10 - Geometry of the Equatorial Disk Model.

Z direction but is allowed to vary in the ω direction. The model computations are performed by dividing up the disk into a grid pattern of many cells in the X-Y plane. The disk is considered to be one cell thick, i.e. there are no divisions along the Z axis.

The scattering of the stellar flux and disk emission is treated discretely by allowing for two scatters of the stellar radiation and a single scatter of the radiation emitted by the envelope. The paths for a typical first and second scatter are shown in the polar view in Figure 10 and from a different perspective in Figure 11. The origin of the coordinate system in Figure 11 has been placed at the center of the cell that is the site of the second scatter, and the X' axis is placed along the line of sight between the first and second cells for simplicity. Radiation at any point in the envelope is represented by the familiar stokes vector notation;

$$\mathbf{S} = (I, Q, U) \quad 2$$

where I is the mean intensity. Q is the difference of the intensity with an electric vector perpendicular to the plane of scattering and the intensity with an electric vector in the plane of scattering. In Chandrasekar's $I_{\perp}$ $I_{\parallel}$ notation, $Q = I_{\perp} - I_{\parallel}$ (Chandrasekar, 1960). U is the difference of intensities with electric vectors rotated 45° from those for Q in a clockwise direction as seen by the observer. The circular polarization component has been dropped for this study.

Consider the cell in Figure 3 that is the site of the first scatter. The incident stokes vector from the star is:

Fig. 11 - Geometry of Two Representative Scatters in the Disk Model.

$$\mathbf{S}'_1 = (\pi F_\nu W e^{-\tau'_1}, 0, 0) \quad 3$$

where: F_ν is the astrophysical flux from a blanketed model atmosphere (Kuruz, Peytremann, and Averett, 1974).

W is the dilution factor

τ'_1 is the optical depth from the nearest point on the star.

The emergent scattered flux in the line of sight to the observer is:

$$\mathbf{S}_1 = (\mathbf{S}'_1 \{\Phi\} \{R\} \{\bar{Q}\}_1 \sigma N_e + \mathbf{J}_\nu) V_1 e^{-\tau_1/D^2} * \quad 4$$

where $\{\Phi\}$

is the dipole scattering matrix -

$$\frac{3}{16\pi} \begin{Bmatrix} 1+\cos^2\theta & 1-\cos^2\theta & 0 \\ 1-\cos^2\theta & 1+\cos^2\theta & 0 \\ 0 & 0 & 2\cos\theta \end{Bmatrix}$$

* All stokes vectors will be in bold face letters.

$\{R\}$

is the rotation matrix -

$$\begin{Bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\phi & \sin 2\phi \\ 0 & -\sin 2\phi & \cos 2\phi \end{Bmatrix}$$

$\{\bar{Q}\}_1$ is a matrix factor which accounts for the finite size of the central star (see Appendix C).

σ is the electron scattering cross section $6.65 \times 10^{-25} \text{ cm}^2$

N_e is the electron density at the value of ω corresponding to the center of the cell

V_1 is the volume of the cell

D is the distance to the observer assuming $D \gg R_*$

τ_1 is the optical depth from the center of the cell to the outside edge of the disk in the line of sight to the observer

and $\mathbf{J}_v$ is the volume emission stokes vector.

In Figure 11 the plane of scattering is the plane formed by $\vec{S}_1$ and $\vec{S}_1'$. The angle between these rays, θ , is the angle of scattering in the matrix $\{\Phi\}$. After scattering by $\{\Phi\}$ the resultant stokes vector must be rotated into the plane of observation by $\{R\}$. The plane of observation is the plane perpendicular to the exit rays $\vec{S}_1$ or $\vec{S}_2$ (i.e. the line of sight to the observer). The angle of rotation ϕ is the angle between the two rays $\perp_0$ and $\perp_1$, where:

$$\perp_1 = (\vec{S}_1' \times \vec{S}_1) / |\vec{S}_1' \times \vec{S}_1| \quad 5$$

and $\perp_0$ is a ray in the plane of observation with no X component (X as defined in Figure 10). If the disk is being viewed edge-on, $Z = \perp_0 = \perp_1$, $\phi = 0.0$, and $\{R\}$ becomes the identity matrix.

For an optically thin cell, the amount of flux scattered along the total path length through the cell ($V^{1/3}$) is $\sigma N_e V^{1/3}$. The cell presents a solid angle $\Omega = V^{2/3} / D^2$ to the observer, hence the total observed flux is obtained by the factor $\sigma N_e V / D^2$ in equation 4.

Now consider radiation traveling in the direction of the second cell from the first cell where it can be scattered again. (Actually this scatter represents the second scatter of the stellar radiation but only the first scatter of the disk radiation. However, to avoid confusion it will always be referred to as the second scatter.) The

specific intensity of this radiation is:

$$\mathbf{I} = (\mathbf{S}'_1 \{\Phi\} \sigma N_e + \mathbf{J}_\nu) V^{-1} e^{-\tau'_1} \quad 6$$

where τ'_1 is the optical depth between cell centers. The scattering angle in $\{\Phi\}$ is the angle between the rays $\vec{S}'_1$ and $\vec{S}_1$. Thus the flux intercepted by the second cell is:

$$\mathbf{S}' = \mathbf{I} \Omega \quad 7$$

where Ω is the mean solid angle that the first cell presents to the second. An approximate formula for this is derived in Appendix E.

The flux scattered by the second cell in the direction of the observer is:

$$\mathbf{S}_2 = \mathbf{S}'_2 \{\mathbf{R}\}_1 \{\Phi\} \{\mathbf{R}\}_2 \sigma N_e V_2 e^{-\tau_2} / D^2 \{\bar{\mathbf{Q}}\}_2 \quad 8$$

$\{\mathbf{R}\}_1$ rotates the stokes vector $\mathbf{S}'_2$ into the plane of the second scatter formed by the rays $\vec{S}'_2$ and $\vec{S}_2$. The angle of rotation is the angle between $\perp_2$ and the Z axis, where:

$$\perp_2 = (\vec{S}'_2 \times \vec{S}_2) / |\vec{S}'_2 \times \vec{S}_2| \quad 9$$

The angle of scattering in $\{\Phi\}$ is the angle between $\vec{S}'_2$ and $\vec{S}_2$ (θ_2 in Figure 10). $\{\mathbf{R}\}_2$ rotates the resultant scattered Stokes vector from the plane of scattering into the plane of observation. This angle is the angle between $\perp_2$ and $\perp_0$ where $\perp_0$ is defined previously. $\{\bar{\mathbf{Q}}\}_2$ is a matrix that accounts for the finite thickness of the disk. The computations are performed in the following manner:

1. $\mathbf{S}_1$ is computed for all cells visible by the observer and

summed. That is, cells which are blocked from view by the star are excluded.

2a. For a given cell, designated C, S_2 is computed for all other cells that C can see and summed. That is, flux from all other cells in the envelope that have a line of sight between them and C, not blocked by the star, is scattered at C and added to the net emergent flux from C.

2b. Step 2a is repeated for all cells visible by the observer, each cell in turn becoming C, and the result summed.

Thus if there are N cells, it takes N sets of computations to do step 1, and N^2 sets of computations to do step 2. Due to symmetry in the disk, the computations need be performed for only half of the disk. Typically 128 cells would be used for model fitting, then the results of that fit checked with finer grids. For 128 cells the computing time is about one minute per frequency point on a PDP 11-40 computer.

The volume emission stokes vector contains contributions from free-bound and free-free transitions (Aller, 1963).

$$j_{\nu f-b} = \sum_{n^*}^{\infty} 1.716 \times 10^{-33} \frac{N_e^2}{T_e^{3/2} n^3} \exp(-h(\nu - \nu_0/n^2)/kT_e)$$

where $\nu_0 = 3.288 \times 10^{15}$ Hz and $n^* > (\nu_0/\nu)^{1/2}$ 10

$$j_{\nu f-f} = 5.443 \times 10^{-39} \frac{N_e}{T_e^2} \exp(-h\nu/kT_e) \text{ ergs-sec}^{-1}\text{-cm}^{-2}\text{-Hz}^{-1}\text{-ster}^{-1}$$

Thus

$$\mathbf{J}_\nu = (j_{\nu f-b} + j_{\nu f-f}, 0, 0)$$

The optical depth due to hydrogen bound-free and free-free opacity is computed using a modified version of the subroutine HOP from the code by Kuruz (1970). This subroutine allows for the inclusion of departure coefficients for the level populations of neutral hydrogen. In this model the disk is pure hydrogen, so the proton density is equal to the electron density. The final opacity is obtained by adding the opacity due to electron scattering.

The final value for the net polarization is given by

$$P = (Q^2 + U^2)^{\frac{1}{2}}/I \quad 11$$

where Q is the second component of $\Sigma S_1 + \Sigma S_2$

U is the third component of $\Sigma S_1 + \Sigma S_2$

I is the first component of $\Sigma S_1 + \Sigma S_2$

$$\text{plus } \eta_\nu \pi F_\nu R_*^2/D^2$$

where η_ν is a factor to account for the partial obscuration of the star by the disk.

The electron density is represented by a formula of the form

$$N_e = A + B e^{-M\omega} \quad 12$$

where A , B , and M are input variables. This form for the density distribution was chosen because of its flexibility and has no direct physical significance. Because of symmetry in the envelope, the U component of the net emergent flux from the disk is zero, although I have performed calculations to provide a check on the extent of round off errors in the computations. Typically U is down by

10^{-3} from Q and makes no contributions to the polarization.

Computing the excitation equilibrium for a multilevel hydrogen atom within a disk shaped envelope is difficult. At electron densities and temperatures typical of Be stars ($N_e > 10^{11}$, $T_e \sim 10^4$) the LTE recombination rate is sufficiently high to cause the Lyman continuum to become optically thick beyond a short distance from the star. Marlborough (1969) dealt with this problem by allowing the approaching edge of the rotating central star to doppler shift Balmer continuum photons just longward of the Lyman limit into the Lyman continuum as seen by a point in the envelope. Thus, a small fraction of these photons could reach any point in the envelope unabsorbed by the intervening material. Since Marlborough ignored any contribution from the diffuse field, these photons were the only means of keeping the envelope from becoming neutral. At sufficiently large ω , this ionizing flux is far less than the black body flux appropriate for a thick plasma at T_e . Hence recombination dominates over photoionization and for $\omega > 1.5$, $z = 0$ the lower levels are overpopulated relative to the LTE values in Marlborough's model.

The diffuse field at any point is

$$4\pi J_\nu = \oint B_\nu (1 - e^{-\tau_\nu(\Omega)}) d\Omega \quad 13$$

If the envelope is thick in the Lyman continuum (at least in the line of sight to the star) then some fraction of the 4π steradians visible to a grid point will have the Planck function (B_ν) as the specific intensity. This radiation can make a substantial difference in the population of the lower levels. Unfortunately, since each point in the disk contributes to $4\pi J$ at every other point, and hence the

populations there, a rigorous treatment would be beyond the scope of this work. Instead, the populations were assumed to be in LTE at the T_e of the disk. A best fit model was then computed. Then various grid points were investigated for their excitation equilibrium by making a rough estimate of the solid angle that the thicker parts of the disk subtend. If departures from LTE were significant, the process was repeated, incorporating results from the statistical equilibrium calculations, until it converged to an acceptable fit. This was necessary for only two of the stars modeled.

The statistical equilibrium equations were set up with seven levels for the hydrogen atom. The Einstein A values were taken from Wiese et al. (1966). The B values are related by

$$A_{ij} = \frac{2h\nu^3}{c^2} B_{ij} \quad \text{and} \quad B_{ij} = g_i/g_j B_{ji}$$

where B_{ij} is the coefficient for stimulated emission. The collisional cross section for ionization C_{ik} , and the collisional cross section for excitation C_{ij} are from Mihalas (1967). The photoionization rate is given by:

$$R_{PI_i} = N_i \frac{4.249 \times 10^{55}}{i^5} \int_{\nu_0/i^2}^{\infty} \frac{4\pi J_{\nu} + F_{\nu*}}{\nu^4} d\nu \quad 14$$

where $4\pi J$ is from equation 13 with a rough estimate of the solid angle. The contribution from the star is treated in a manner similar to that used by Marlborough. The recombination rate is:

$$R_{RE_i} = N_e \frac{3.26 \times 10^{-6}}{T_e^{3/2} i^3} \exp(h\nu_0/i^2 k T_e) E_1(h\nu_0/i^2 k T_e) \quad 15$$

and the three body recombination rate is:

$$R_{3B_i} = N_e \frac{i^2 N_e 4.147 \times 10^{-16}}{T_e^{3/2}} \exp(h\nu_0/i^2 k T_e) C_{ik} \quad 16$$

At the densities used in the model fitting, the envelope can be considered to be optically thick to all line radiation.

For most of the stars being modeled, the parts of the envelope where significant electron scattering takes place are optically thick in the Lyman continuum in all directions and the populations are near LTE. Any departure is determined primarily by how much flux is present in the Balmer and higher continua. If the disk is thick in the Lyman continuum but thin in the higher continua, regions of the disk far from the star will lose the small but significant photoionizations from levels 2 and above and become mildly overpopulated in the lower levels. In regions very near the star where the optical depth in the Lyman continuum is small and only the Lyman lines are optically thick the lower levels become underpopulated, principally levels 1, 2, and 3. At the outer edge of the disk where the optical depth to the star is large but only one hemisphere is filled with significant diffuse radiation, recombination dominates and overpopulates levels 1, 2, 3, and, to some extent, level 4. Except for the extreme cases of low densities near the star or very high densities ($N_e \sim 10^{12}$) with little

ionizing flux from either the star or the diffuse field (a difficult configuration to create), there is no significant departure from LTE for levels 4 and above. Thus, for the majority of the envelopes of most of the stars modeled the LTE populations suffice.

Since the LTE population of levels 4 and above vary weakly with temperature over the range 8×10^3 to 2×10^4 K, the opacity in the envelope due to neutral hydrogen longward of 1.5 microns is largely independent of temperature. The free-free opacity at these wavelengths is also relatively independent of temperature ($\propto T^{-\frac{1}{2}}$); thus the 2.2 micron polarization is only mildly dependent on temperature and is determined largely by $N_e(\omega)$. This fact is very useful for determining the model parameters that best fit the data for a particular star.

The major constraints provided by the polarization and photometry data are listed in Table 6, along with their dependence on the model input variables. Increasing the electron density increases the neutral hydrogen density rapidly ($\propto N_e^2$). This raises the bound-free opacity and increases the contrast across a jump. The Balmer jump is more sensitive to a change in the temperature than the Paschen jump because the population of the second level is more sensitive to the temperature than the population of the third level. Free-free emission, the major contributor to the infrared excess, is proportional to the square of the electron density. Thus, small changes in the electron density can have large effects on the infrared excess.

To find the general behavior of the model, a constant density

Table 6
Model Dependence on Input Variables

Parameter	Strong	Moderated	Weak
$P(.37)/P(.36)^*$	$N_e, T_e, DC^\dagger$		R_s, H
$P(.90)/P(.70)$	N_e	T_e, DC	
$P(.36)/P(.70)$	T_e	N_e, DC	
$P(.37)/P(2.2)$	N_e		T_e
IR excess	N_e, R_s, H		T_e
$P_{\max}$	H, β	N_e	R_s

*Numbers in Parentheses are Wavelength in Microns.

† Departure Coefficients for Non-LTE.

model was computed for an edge-on disk in LTE at 10^4 K. The disk radius is $5R_*$, and the thickness is $1R_*$. The central star is assumed to be a B1 V star with $T_{\text{eff}} = 25,000$ K and $R_* = 7R_{\odot}$. In order to achieve a maximum polarization of about 2%, the value observed for the more highly polarized Be stars, the disk must have an electron density of about 5.5×10^{11} . If the model is run with no second scatter and no hydrogen bound-free opacity (only free-free opacity), the wavelength dependence of the polarization is curve "a" in Figure 12. The dip in the curve shortward of the Balmer and Paschen jumps is due to dilution by free-bound emission. The general decline longward of 1 micron is due to a combination of diluting free-free emission and free-free opacity that reduces the amount of polarized scattered flux. If bound-free opacity is included, curve "b" results. In curve "b" the contrast across the Balmer and Paschen jumps is greater due to the additional opacity at those wavelengths. In curve "c" the second scatter is included, but bound-free opacity is not. The contrast across the jumps is nearly gone due to the scattering (and hence polarizing) of free-bound emission from the disk off of the disk itself. The polarization still drops longward of 1 micron due to the free-free opacity at those wavelengths.

The details of how the second scatter eliminates the contrast across the jumps can be investigated by a detailed analysis of the various contributors to the scattered flux. The flux in the first scatter is about 6% of the total stellar flux and is 34% polarized, yielding a net polarization of about 2%. The flux in the second

Fig. 12 - Wavelength Dependence of the Polarization for the Disk Model. The Lettered Curves are Discussed in the Text.

scatter arising from scattered disk emission is about 11% of the total disk emission, but is only 20% polarized, yielding a net polarization for the disk alone of about 2%. Consequently, even though the geometry for the scattering of disk emission is less efficient at polarizing than the star disk geometry, a greater fraction of the emitted disk flux is scattered than the stellar flux. Thus, the net polarization for the disk emission is comparable to the net polarization of the stellar flux for these model parameters. The amount of stellar flux in the second scatter is only about one sixth the flux in the first scatter and makes only a small contribution to the final net polarization.

Finally, curve "d" in Figure 12 is the result when bound-free opacity is included and represents the most physically realistic case. Once again, the increased opacity at various wavelengths reduces the polarization at those wavelengths.

The dependence of this test model on the tilt of the disk (the angle β) is illustrated in Figure 13. The fat solid curve is for the polarization at 0.37 microns, just longward of the Balmer jump where the opacity is relatively small. The thin solid line is for 0.36 microns, just shortward of the Balmer jump, where the opacity is higher. The dotted line is from a relationship derived by Poeckert and Marlborough (1977), here after referred to as PM, for a much simpler model. The parameters for the model being considered here have been used in equation A31 of PM where $i = \pi/2 - \beta$. As expected the general tendency is for the polarization to drop as the disk is seen from more above the plane. The moderate rise from

Fig. 13 - Dependence of the Polarization on the Tilt of the Model Disk (β).

$\beta = 0$ to $\beta = 10$ occurs because the near edge of the envelope blocks the bottom pole of the star and decreases the contribution of the direct (unpolarized) stellar flux. The polarization for the model for tilts greater than 20° remains higher than the relation derived by PM because as the envelope is tilted, the line of sight from the observer to the inner regions of the disk passes through less intervening envelope material. That is, the inner regions no longer must be viewed through the entire edge-on disk but can be seen more directly from above the equatorial plane. The polarization ratio at 0.36 microns remains higher than at 0.37 microns because the opacity is greater at that wavelength and the effect of reducing the exit pathlength through the disk is greater. By a tilt of about 60° , all three curves converge. At these angles the star no longer blocks the portion of the disk behind it (as is the case for $\beta = 0$), and the model geometry approaches the simple geometry used by PM.

The maximum polarization for this model is 2.6%. This is somewhat higher than the maximum polarizations observed in Be stars but clearly indicates that a significant number of electrons must be in the disk for polarizations approaching 2% to be achieved. The drop in polarization at the Balmer and Paschen jumps and longward of 1 micron cannot be due to dilution by emission from material distributed about the star outside the equatorial disk because the net infrared excess for the entire configuration would be too high. Dilution alone simply cannot explain the wavelength dependence of the polarization in Be stars.

The infrared excess for this model is (in magnitudes) $-.41$ at

1.25 microns, $-.84$ at 2.2, -1.2 at 3.4, and -1.6 at 5.0 microns. This excess for the star-plus-disk is comparable to the observed infrared excess in Be stars (see Table 5). Thus, the electrons that are doing the polarizing are also providing the infrared excess.

This model has a few undesirable features. The optical depth between a cell and the star is taken from the center of the cell to the nearest point on the star. Ideally, the stellar flux at any point in the envelope would be determined by dividing up the visible stellar disk into many smaller solid angles and computing the optical depth for each one individually. For instance, a point in the disk near the star will see the pole of the star through less intervening material than the equator. This is compensated for somewhat by using a pre-computed averaging factor to obtain a mean optical depth for the stellar radiation seen by the innermost ring of cells. This approach works fairly well for small optical depths, but breaks down at larger optical depths.

The problem of large optical depths in the disk is the major drawback of the model. All of the equations developed in this section require that $e^{-\tau} = 1-\tau$ to within 10% ($\tau < .4$) for acceptable accuracy. If the densities in the disk for some cells become too large, the optical depth becomes too large and the grid size must be reduced. The computing time is proportional to the square of the number of cells, and halving the cell size requires sixteen times the computing time. For this reason the model was run with more than 128 cells only as a check on the accuracy of the coarser grid. Only in the case of γ Cas was the fine grid version significantly

different than the coarse one. In the case of γ Cas a few very fine grid versions (1028 cells) were run at only a few critical wavelength points.

Another problem with the model is the lack of any change in the neutral hydrogen populations in the Z direction. Material near the top or bottom boundaries of the disk sees a portion of the star (one of the poles) unobscured and sees the diffuse field over 2π steradians only. Thus, the populations may differ significantly from those more toward the equatorial plane. In the statistical equilibrium calculations, a crude approximation was made to try to compensate for this. A distance of $H/4$ (H = thickness; see Figure 10) above the equatorial plane (or below; the model is symmetric in the Z direction) was taken to represent an average or "representative" point for a cell, and the diffuse and dilute fields for that point were used in the calculations. Considering the approximate manner in which the statistical equilibrium equations are applied, a more detailed treatment would not be useful. Clearly, the problem of determining the excitation equilibrium of difficult geometries is a major obstacle in modeling Be stars.

V DISCUSSION

The six stars with sufficiently well determined polarization to be modeled with the formulation developed in the preceeding section are listed in Table 7. The choice of effective temperatures and stellar radii are discussed for each star individually. The results of the best model fit for these stars are listed in Table 8. ϕ Per, ζ Tau, γ Cas, and EW Lac all show a general decline in polarization longward of one micron, suggesting similarities in their envelope structure and model calculations show this to be true. 48 Lib, ϵ Cap, and α Aqr differ from the previous stars by maintaining a relatively constant polarization longward of one micron. This is a case that clearly cannot be described by the simple dilution model, for both 48 Lib and ϵ Cap have significant infrared excess at those wavelengths. Unfortunately there is no simple unifying characteristic that will explain the high near infrared polarization for all three of these stars.

ϕ Per

Iesh (1968) has classified ϕ Per as B2Ve; Peters (1976) used B0, $T_{\text{eff}} = 28,000$ K to obtain $\log g = 4.0$; and Schild (1976) was able to fit the visible spectral energy distribution with a T_{eff} of 25,000 K and $\log g = 3.5$. For this study ϕ Per was modeled as a main sequence star with $T_{\text{eff}} = 25,000$ K and a radius of $7 R_{\odot}$. The visible polarization is from Table III of Coyne and McLean (1975) where the interstellar component has been removed. Coyne and McLean also report 1.25 and 2.2 micron polarimetric observations and the values

Table 7
Stellar Parameters

Object	Sp	<u>vsini</u>	T_{eff} (K)	$R_{\star} (R_{\odot})$
ϕ Per	B1 V	450	25,000	7
ζ Tau	B2 IV	310	20,000	7
γ Cas	B0 IV	230	30,000	10
EW Lac	B4 III-IV	300	16,000	7
48 Lib	B4 IV	400	16,000	7
ϵ Cap	B3 III	280	18,000	10
σ Aqr	B8 V	320	12,000	3.5

Table 8
Results of Model Calculations

Object	T_e	R_S/R_*	H/R_*	$\max N_e$ (10^{12} cm^{-3})	$\min N_e$ (10^{11} cm^{-3})	β (deg)
ϕ Per	11,000	10	.75	1.4	3.2	0
ζ Tau	11,000	5	.60	2.9	1.4	0
γ Cas	14,000	3.6	.45	4.5	4.4	45
EW Lac	9,700	5	.80	1.8	1.5	0
48 Lib	9,000	4.5	.60	3.1*	1.3	0
ϵ Cap	12,000	5	.60	0.4	4.0	0
o Aqr	7,500	3.1	.85	0.2	2.0	0

*Disk Begins at $\omega = 2$.

at these wavelengths agree well with the values reported here.

The high intrinsic polarization of $\sim 1.9\%$ just longward of the Balmer jump and the high vsini (>400 km/s) suggest that ϕ Per is nearly edge-on ($\beta = 0$). A constant density, edge-on model similar to the one described in the previous section is capable of reproducing the maximum in the polarization but has too little contrast across the Balmer and Paschen jumps and yields too high a polarization in the near-infrared. By concentrating the density of the disk more toward the star, where the first scatter has its major contribution, the bound-free and free-free opacities can be increased to provide the correct polarization shortward of the Balmer and Paschen jumps and in the near-infrared. To maintain the proper values for the polarization at both jumps, a shell temperature of $11,000 \pm 1,000$ -500 K was necessary. This density distribution and temperature also yield the correct infrared excess. The fact that part of this excess comes from the outer regions of the disk ($\omega > 5$), where the geometry for a highly polarized second scatter is poor, helps further to reduce the 2.2 micron polarization to the observed value. The best fit for ϕ Per is shown in Figure 14. The fit is good at all wavelengths except 1.25 microns, where the datum is 1.5 sigma above the model value. Considering the good agreement at 2.2 and 1.65 microns (the 1.65 micron value is from Coyne and McLean, 1975), this discrepancy is not significant.

It is important to consider how much the data constrain the values of the input variables listed in Table 7. If the temperature of the envelope is raised, the LTE population of neutral hydrogen

Fig. 14 - Wavelength Dependence of the Polarization of the Best Fit Model for ϕ Per. The Open Circles Are from Coyne and McLean (1975).

is in the disk is reduced, and the contrast across the Balmer and Paschen jumps is correspondingly reduced. If the temperature is raised to 12,000 K and all other parameters are left unchanged, the polarization at 0.36 microns changes from .95% to 1.20%, an increase of one quarter. The polarization at 0.37 microns changes from 1.93% to 1.96%. If the electron density is raised to restore the hydrogen opacity, the infrared excess becomes too large and the 2.2 micron polarization too small. The higher temperature may cause the populations to depart from LTE and overpopulate the second and third levels of hydrogen, restoring the contrast without affecting the infrared data very strongly. Rough statistical equilibrium calculations for the disk at a higher temperature show that the Balmer continuum is thinner than at 11,000 K, but this results in only a slight overpopulation of the second and third levels and cannot restore the proper polarization. A combination of a moderate increase in temperature with a moderate increase in the electron density can produce an acceptable fit, and the upper limit for this change is about 1000 K.

If the temperature is lowered, the disk approaches LTE, and the increased neutral hydrogen density increases the contrast across the jumps such that the model fit is no longer acceptable. The electron density can be reduced in order to lower the neutral hydrogen density, but this lowers the infrared excess and causes the 2.2 micron polarization to increase above the observed datum. Also, at a lower temperature, the contrast across the Balmer and Paschen jumps cannot be fitted simultaneously. If the Balmer jump is fitted, then the

polarization shortward of the Paschen jump is too small. If the Paschen jump is fitted, then the polarization shortward of the Balmer jump is too low. Since these effects are more sensitive to the temperature when it is decreased, a limit of 500 K can be placed on the allowable change.

The dependence of the model on the size and thickness of the disk is less of a constraint. The thickness of the disk is varied to change the magnitude of the polarization and the infrared excess. The radius of the disk primarily effects the infrared excess and for ω greater than 5 only weakly effects the magnitude of the polarization. For an edge-on model the radius of the disk will influence the exit optical depth for the inner regions of the disk. For a density distribution with a concentration of material in the inner regions of the disk most of the opacity is in the inner regions, and a change in the disk radius will have only a small effect on the wavelength dependence of the polarization.

For ϕ Per the best fit was obtained for $\beta = 0^\circ$ (edge-on), as expected. Over the range of 0-10 degrees acceptable fits could be made, but for tilts larger than this the polarization begins to drop, and the model wavelength dependence departs from the data. In the region of 0-10 degree tilts, the model is very dependent on the exact geometry of the disk. By changing the dimensions of the disk, the amount of unpolarized stellar flux directly visible to the observer can be changed and the magnitude of the polarization changed accordingly. Since the geometry of this model is largely ad-hoc, these changes cannot be considered to be physically

significant. Thus, the accuracy to which the tilt can be determined is about 10 degrees for ϕ Per.

Marlborough (1969) has modeled the envelopes of Be stars in an effort to fit the hydrogen emission line profiles, principally H α . He used a density distribution developed by Limber (1964) which is more complex than that used here but has similar features, the primary one being an increase in density near the star. Although the envelope used by Marlborough is much more extensive than the one derived for ϕ Per, the outer regions are of sufficiently low density that they would have no effect on either the broad band photometry or polarimetry. Thus, the disk parameters derived for ϕ Per are representative of the regions of an extended envelope that contribute to the broad band polarization and infrared excess only and will not necessarily reproduce all of the details of the spectroscopic observations.

ζ Tau

Although no observations of ζ Tau are reported in this study, Capps et al. (1973) presented linear polarization measurements of ζ Tau at 1.65 and 2.2 microns, in addition to visible polarimetry. It is of interest to fit these observations with the model developed here. The model used by Capps et al. for ζ Tau has been discussed earlier and did not take into account opacity in the envelope or the scattering of envelope emission.

Lesh (1968) classified ζ Tau as B4 III; Peters used $T_{\text{eff}} = 25,000$ K and obtained $\log g = 3.3$; and Schild fit the visible spectral energy

distribution with $T_{\text{eff}} = 20,000$ K and $\log g = 3.5$. For this study the values $T_{\text{eff}} = 20,000$ K and $R_* = 7R_{\odot}$ were used. This corresponds roughly to B2 IV. ζ Tau can be modeled with a density distribution similar to that for ϕ Per. The concentration of material near the star provides the necessary opacity to reproduce the wavelength dependence of the polarization. The disk for ζ Tau is less extensive than that for ϕ Per since the infrared excess is less.

The steep decline in polarization between 1.65 and 2.2 microns (Figure 15) could not be fit exactly either by the simple dilution model of Capps et al. or by the model used in this study. Given that the model curve passes within 1.5 sigma of the data at 1.65 and 2.2 microns, the fit is acceptable. The best fit envelope temperature is 11,000 K and is constrained in the same manner as for ϕ Per. The electron density near ζ Tau is larger than for ϕ Per because the contrast in polarization across the Balmer jump is larger and the polarization longward of the Paschen jump is low. This density enhancement reduces the polarization shortward of the Balmer jump due to greater bound-free opacity and smooths out the Paschen jump by increasing the free-free opacity in the disk to the point where it is significant by 0.9 microns. It is this free-free opacity combined with the less (but nonetheless significant) polarized free-free emission that causes the 0.9 micron polarization for both ϕ Per and ζ Tau to be less than at .37 microns.

γ Cas

The parameters for γ Cas, $T_{\text{eff}} = 30,000$ K, $R_* = 10 R_{\odot}$, chosen for

Fig. 15 - Wavelength Dependence of the Polarization of the Best Fit Model for ζ Tau. The Open Circles are from Capps et al. (1975).

this study are consistent with other work on the star (Schild, 1976; Hutchings, 1975), including a study of the polarization through the $H \alpha$ line by Poeckert and Marlborough (1977). The model used by Poeckert and Marlborough treated the finite size of the central star with more accuracy than the model being considered here but ignored any contributions from the disk to the polarization or flux.

The wavelength dependence of the polarization in γ Cas is distinctly different from the two stars already considered (Figure 16). The maximum polarization is only 0.8%, and more important, the polarization shortward of the Paschen jump is nearly as low as the polarization shortward of the Balmer jump. If the disk is nearly in LTE, this could be explained by a temperature high enough to bring the populations of the second and third levels of hydrogen closer together. An enhanced electron density is also necessary to provide enough neutral hydrogen for the bound-free opacity as well as the free-free opacity. Although at the temperature used for the best fit ($T_{\text{eff}} = 14,000$ K) the LTE populations of neutral hydrogen are much lower than at 11,000 K, and the volume absorption coefficient is correspondingly lower, the larger path lengths in the disk tend to compensate. The path lengths are larger since the distance to any point in the disk is ωR_* , and R_* is larger for γ Cas. The important point is that the electron density cannot be increased without bound in an effort to obtain more hydrogen opacity without causing problems with the infrared polarization and excess.

The tilt for the γ Cas model had to be larger than about 40° . At $\beta < 40^\circ$ the polarization was always too large for all models that

reproduced the relative polarizations at all wavelengths. That is, for a model that maintains the same ratios of polarization between one part of the spectrum and another as the data, the model polarization would always be too large. The best fit value for the tilt of 45° is in good agreement with the value used to fit the $H \alpha$ line profile by Hutchings (1975) and by Poeckert and Marlborough (1977).

Due to the high electron densities near the star, the model calculations begin to break down because the assumption that the cells are optically thin is no longer valid. To eliminate this problem the model was run at a few critical wavelength points with a total of 1028 cells in the disk. At this fine a grid the optical depth of the most opaque cell is $\tau = 0.44$ near the Balmer jump. For the fine grid model the polarizations were all slightly higher than those computed from the coarse grid model due to the lower opacity to the center of the inner cells. The wavelength dependence matches the observations well at all wavelengths except shortward of the Paschen jump, where the model polarization is too high. This can be corrected by an increase in the temperature and a decrease in the size of the disk (to lower the magnitude of the polarization at all wavelengths). To do this would require more computing time than was available; hence, the model temperature is not very well determined and could easily be as high as 16,000 K. It is not likely that the data could be fitted for a temperature much less than 14,000 K, however.

Statistical equilibrium calculations show that for the

Fig. 16 - Wavelength Dependence of the Polarization of the Best Fit Model for γ Cas. The Open Circles are from Poeckert (1977).

Fig. 17 - Wavelength Dependence of the Polarization of the Best Fit Model for EW Lac. The Open Circles are from Poeckert (1977).

inner regions of the disk with $\omega > 1.25$, the populations of hydrogen are nearly in LTE. Near the star, the lower levels were underpopulated, but the statistical equilibrium calculations may be inaccurate for the densities and radiation fields present near the star. Consequently, the region between $\omega = 1.0$ and $\omega = 1.25$ has been eliminated from the model. Since the model disk tapers sharply close to the star, the effects of removing the contributions from this region are small.

EW Lac

EW Lac was modeled as a B4 III-IV central star. The data for this star exhibit many similarities to the three previous stars, and the model parameters follow the basic pattern which requires the density to increase near the star. The LTE temperature for the best fit is relatively low at $9700 \text{ K} \pm 1000 \text{ K}$. At this envelope temperature (with $T_{\text{eff}} = 16,000 \text{ K}$ for the central star) the disk is thick enough in the Balmer continuum to ensure nearly LTE populations. The high polarization for EW Lac necessitated keeping $\beta < 10^\circ$, essentially edge-on.

EW Lac maintains a higher polarization from about 0.5 microns into the red (relative to the maximum polarization) than any of the other stars discussed previously (Figure 17). The lower temperature combined with a lower density in the model reproduces this nicely. The contrast across the Balmer jump is still quite high because at a lower temperature the relative populations of the second and third levels of neutral hydrogen are significantly

different. Thus there can be substantial opacity in the Balmer continuum but considerably less in the Paschen continuum.

48 Lib

Iesh (1968) classified 48 Lib as B5 III, and Geuverink (1970) used B3 V to analyse the emission spectrum. For this study a star midway between these two spectral and luminosity types was chosen (B4 IV). The most striking feature of 48 Lib is the nearly constant value of polarization longward of the Balmer jump with only a small change in polarization across the Paschen jump (Figure 18). At first glance a rather thin, cool envelope with the lower levels of hydrogen overpopulated to increase the opacity near the jumps would explain the data. The low density would thin the Balmer continuum considerably and thin the Lyman continuum enough to overpopulate the lower levels. An appropriate low temperature would allow the opacity in the Balmer continuum to be significant while allowing the Paschen continuum to be relatively thin. At other wavelengths the low density would allow the opacity to be determined primarily by electron scattering, and, hence, the polarization would be roughly constant. Unfortunately, the infrared excess for such a disk would be far too small to fit the observations. If the excess were allowed to arise in a more extensive envelope, the infrared polarization would be diluted to values well below the observations. If the infrared excess must come from the model disk, the necessary densities destroy the non-LTE population distribution and cause an increase in the free-free opacity to the point where the model

Fig. 18 - Wavelength Dependence of the Polarization of the Best Fit Model for 48 Lib. The Open Circles are from Poeckert (1977).

predictions in the infrared are too low relative to the maximum polarization at .37 microns.

This dilemma can be avoided by moving the inner edge of the disk away from the star. With this geometry the infrared polarization is due primarily to scattered envelope emission, which can now compete with the polarization in the blue (which is primarily scattered stellar radiation). The scattered stellar radiation is reduced because the scattering electrons are further away from the star where the stellar radiation field is weaker. This geometry is similar to the "ring" model developed by Huang (1972) and Hazlehurst (1966). The best fit for 48 Lib is shown in Figure 18, where the disk begins at $\omega = 2$, $R_g/R_\star = 4.5$, and $\beta = 0$.

Geuverink (1970) has made a study of the shell spectrum of 48 Lib. Although the spectrum of 48 Lib is highly variable, red plates taken at Mauna Kea Observatory within a few months of the polarimetric observations show that the hydrogen emission line profiles are essentially the same as when Geuverink made his observations; hence, Geuverink's analysis should be applicable. Two of his conclusions are that the disk is nearly edge-on and that a rough curve of growth analysis yields a disk temperature of 9100 ± 900 K. From the velocities of the lines in the spectrum Geuverink determined that the inner regions of the shell are moving away from the star at a greater velocity than the outer regions. The upper members of the Balmer series are formed at about $\omega = 3$; the radius of the shell is approximately $5 R_\star$; and the electron density in the shell is about 6×10^{11} in the inner regions. Finally, the double

NaI D, CaII K and FeII lines suggest that a shell has been ejected between 1967 and 1968. These conclusions are entirely consistent with the model parameters determined here if this shell is now centrifugally supported between $\omega = 2$ and $\omega = 5$.

ϵ Cap

ϵ Cap also maintains a high polarization at 2.2 microns, but shows the more characteristic decline in polarization longward of the Balmer jump toward the Paschen jump. All attempts to model ϵ Cap in LTE with a ring geometry similar to that for 48 Lib failed. This is due to the fact that (in order to maintain a high polarization at 2.2 microns in a ring geometry) the electron density must be high enough to produce the necessary combination of free-free emission, electron scattering opacity, and free-free opacity to have the net polarization arise from scattered disk radiation. Unfortunately the observed infrared excess for ϵ Cap is too small (-0.42^m at 2.2 microns compared to -0.95^m for 48 Lib) to allow such densities unless the thickness of the disk is made very small. For thin disks the model breaks down because it requires a cell to view the star through the intervening shell material. In the limit of an infinitely thin disk the entire star would be visible without obscuration. It can be concluded, however, that for a thin ring-like disk to produce the lower polarization shortward of the Balmer and Paschen jumps, the opacity due to hydrogen bound-free absorption must be high everywhere in the disk within $\omega \approx 5$. The electron densities required for this are too high and produce too large an

infrared excess.

A disk with a density distribution similar to ϕ Per, ζ Tau, and γ Cas can reproduce the polarization out to one micron but fails to achieve the high polarization at 1.2 and 2.2 microns. This brings one back to the possibility of a relatively thin disk (but no ring) with the lower levels of neutral hydrogen overpopulated so that the opacity shortward of the jumps will be significant in most of the disk. The problem with this approach is that a best fit can be found only by iterating between the polarimetry model and the statistical equilibrium calculations, a very time consuming exercise. To avoid this problem, ϵ Cap was modeled using a constant density disk with dimensions well constrained by the infrared excess. This allowed for an easier estimate of the departure from LTE. Given some estimate of the departure coefficients for hydrogen, the polarimetry model could be run to find the best fit temperature and density. Then the statistical equilibrium calculations could be performed again with these new parameters and the process repeated until a reasonable fit was obtained and there was a consistency between the departure coefficients and the parameters of the model. This procedure is very time consuming and does not allow for much experimentation. Consequently, only a few promising computations could be made. This will not result in as accurate a fit as for the previous stars, but it should determine whether or not the basic idea is valid.

Figure 19 shows the best fit to ϵ Cap for a moderately non-LTE constant density disk (level three is overpopulated by a factor of

Fig. 19 - Wavelength Dependence of the Polarization of the Best Fit Model for ϵ Cap. The Open Circles are from Poeckert (1977).

Fig. 20 - Wavelength Dependence of the Polarization of the Best Fit Model for o Aqr. The Open Circles are from Poeckert (1977).

six most everywhere in the disk). The primary source of the overpopulation of the lower levels of hydrogen in the disk is the very thin Balmer and higher continua for an envelope temperature of 12,000 K. Beyond $\omega=1.2$ the dilute radiation field in the Balmer continuum also falls well below the flux in an LTE case ($4\pi B$). The overpopulation in levels two and three cause the necessary increased opacity shortward of the jumps without requiring too large an electron density. The polarization in the near infrared remains high because the density is low enough for the free-free opacity not to become too large and reduce the scattered flux. The fit is quite good everywhere except at 0.9 microns. Here, as would be expected, the model predicts a high polarization since there is little free-free opacity (or emission) at that wavelength. It is likely that if more time could be spent on the model, this discrepancy could be eliminated.

o Aqr

o Aqr is the coolest star of all the stars being modeled (B8). This star shows very little infrared excess at 2.2 microns; hence, the high polarization at 2.2 microns relative to the maximum polarization cannot be due to scattered envelope emission as in 48 Lib. At the low densities necessary to keep the infrared excess down the change in polarization across the Balmer and Paschen jumps can be fit simultaneously only with a relatively low temperature where the neutral hydrogen density is sufficient to produce the observed contrast at the Balmer jump but not too large a contrast at

the Paschen jump. At these low densities the statistical equilibrium calculations indicate a mild overpopulation in the lower levels. A similar approach to the one used with ϵ Cap is taken with $\omicron$ Aqr. With $\omicron$ Aqr, however, the choice of a constant density disk is not too bad since the very low infrared excess precludes any high density regions.

Figure 20 shows the best fit for $\omicron$ Aqr with a small, low density disk. At the model temperature (which is consistent with the data and the excitation equilibrium), level three of hydrogen is overpopulated by a factor of about 3.5 and the optical depth from the equatorial plane to the upper surface boundary of the disk is about $\tau \sim .15$ in the Balmer continuum shortward of the Balmer jump. The fit is remarkably good; the infrared polarization is almost completely scattered stellar radiation due to the virtually non-existent free-free opacity or emission. Since the disk is already thin in the Balmer continuum (but very thick in the Lyman continuum), higher temperatures do not substantially alter the departure coefficients. Thus, an increase in the disk temperature will cause the opacity in the Balmer continuum to decrease and the polarization to become too high. The picture one draws of the shell around $\omicron$ Aqr is one of a small, low density, low temperature disk.

The free-free emission from the model disk reproduces the infrared excess for $\omicron$ Aqr at 1.25, 2.2, and 3.4 microns, but not at 4.9 microns. At 4.9 microns the model excess is $-.38^m$ and the observed value is -1.04^m , a very large discrepancy. The very steep rise in excess emission exhibited by $\omicron$ Aqr longward of 4 microns

is inconsistent with hydrogen plasma emission. The most likely explanation for the excess at longer wavelengths is a cooler source of emission such as dust or a very cool late-type companion.

χ Oph, π Agr

χ Oph shows some evidence for polarization at 1.25 microns, but is unpolarized at the .11% level at 2.2 microns. If χ Oph is a pole-on star, as suspected (Schild, 1976; Poeckert, 1975), one would expect very little intrinsic polarization at any wavelength. Indeed, the wavelength dependence of the polarization for χ Oph resembles interstellar polarization more than the characteristic polarization of Be stars (Coyne and Kruszewski, 1968). At 2.2 microns an interstellar polarization component would be quite small, given the small polarization shown in the visible ($\sim 0.5\%$). This picture is consistent with the 2.2 micron measurements. At 1.25 microns an interstellar component should be about half the peak value in the visible. The 0.5% or so observed in the visible is mostly interstellar in origin, then the 1.25 micron polarization should be about 0.25% at a position angle of 130° . The 1.25 micron measurements are not inconsistent with this value. The fact that the polarization of χ Oph is variable (Coyne and Kruszewski), is an indication that there is some intrinsic polarization attributable to the star, but there is little hope of extracting much information from this polarimetry.

π Agr presents a problem. If the change in intrinsic position angle with wavelength suggested in section III is real, the model

developed for this study could not be used to model the star since the built-in geometry is the same at all wavelengths. In fact it is difficult to imagine how to cause a rotation in intrinsic position angle with an asymmetric electron scattering envelope alone. Equally bizarre is the large variability of the near infrared polarization of π Aqr, reaching values as high as 3% at 1.25 microns. Also, the strength of the polarization is not exactly correlated between 1.25 and 2.2 microns. On JD2443043 the 1.25 micron polarization was high with a position angle of 176° , but one hour later the polarization at 2.2 microns was low with a position angle of 163° . Clearly π Aqr needs more study before an accurate analysis can be made.

VI CONCLUSIONS

The following conclusions can be drawn from this study:

- 1) A disk geometry for the extended atmosphere of a Be star is consistent with the observed wavelength dependence of the polarization. In fact, a highly assymetric geometry is necessary to obtain the observed maximum polarizations in Be stars.
- 2) The electrons scattering stellar and diffuse flux and creating the observed net polarization are the same electrons that produce the infrared excess. If the free-free emission required to account for the infrared excess is forced to come from material outside the disk (i.e., more symetrically distributed about the star), the density remaining in the equatorial disk is too low to achieve the observed polarization or the wavelength dependence.
- 3) The four stars that showed a drop in polarization longward of one micron all require the disk density to rise near the star. This is consistent with the models of the spectroscopic characteristics of Be stars (Marlborough, 1976), the theoretical work of Limber (1964) and Limber and Marlborough (1968), and with the study of the $H\alpha$ emission and polarization of γ Cas by Poeckert and Marlborough (1977). Thus, it appears that Be stars that show a decline in polarization longward one one micron must have a density gradient in their envelopes increasing toward the star.
- 4) The polarization in the Balmer and Paschen continua is a fair indicator of the disk temperature when considered with the rest of the polarimetric and photometric data. This temperature is indicative only of the regions of the envelope that produce the

scattered polarized flux.

5) The near infrared polarimetry is useful for placing constraints upon the density and temperature of the model. Also, the position angle of the 2.2 micron polarization is a useful indicator of the intrinsic position angle of the star if an interstellar component is present.

6) The hotter stars tend to have hotter envelopes.

7) The analysis of 48 Lib and o Aqr indicate that not all Be stars are similar in their envelope structure. Some may have ring type geometries and others may have density distributions like those discussed by Limber (1964).

8) Although most of the program stars had scattering disks that were near LTE, there is no guarantee that this will be the case. Also, the diffuse field cannot be ignored if realistic equilibrium populations are to be obtained.

VII APPENDIX

A. Operation of Systems One and Two.

For system one a stepper motor capable of 20ms per step with 200 steps per revolution is connected through a gear train to a rotating cell containing an analyzer. The gear train amplifies the motor's rotation by a factor of five. Thus five steps of the motor corresponds to a 45° rotation of the analyzer. The polarimeter is located immediately in front of the dewar window, after two 45° reflections in the photometer assembly that couples the dewar to the telescope.

The data acquisition system is operated as follows. Five evenly spaced holes near the edge of the gear attached to the motor shaft are optically encoded with an LED/phototransistor circuit. The signal from this encoder is coupled to an IBM 1800 computer through an I/O bus and provides a means of starting up the system at the same position each time and a means of checking whether or not the stepper motor has gotten out of step. The motor is stepped by sending a TTL pulse from the I/O bus to the stepper motor controller interface, each pulse causing one step. This setup allows the computer to control and keep track of the angular position of the analyzer. A typical operation would proceed as follows:

Control	Process	Time (ms)
C	Step the motor until the start position is reached.	
C	Signal the observer.	
O	Start the system.	
C	→ Integrate on the lock-in signal and store the value.	500 - 1000
C	→ Step five steps (45°).	100
C	→ Pause for at least one chop cycle.	100 - 200

A C in the first column indicates that process is under computer control, an O means it is under the observers control. The loop is continued until the specified number of full rotations of the analyzer is finished and then the values for the flux at 0° , 45° , 90° , and 135° are outputted by the computer, along with the first three components of the Stokes vector. The telescope's chopping secondary (or the photometer's internal chopper) runs freely with no sync from the computer. For this reason the software pauses for 100 to 200 ms after each set of five steps to allow the chop to catch up. The times listed in the last column represent the trade off between moving rapidly through the analyzer's positions and the need for a high duty cycle for the integration.

For system two a wire grid analyzer (3000 lines per mm) is connected to a motor through a chain belt and two sprockets. The speed of the motor is precisely controlled by the electronics in a separate assembly which has the motor in its feedback loop. Typically the analyzer was spun at 1 Hz.

The speed of the motor is not under computer control but is set by the electronics in the motor controller box. The position of the analyzer is encoded by a disk (see Figure 4) that projects between the jaws of an optical encoder that can be read by the computer. Thus the computer can detect the 0° , 90° , 180° , and 270° crossings, but cannot distinguish between 0° and 180° or 90° and 270° . The operation of the data acquisition system is illustrated in Figure 5. A typical operation would proceed as follows:

Control	Process	Time (ms)
C	Time several 0° and 180° crossings to determine the integration times.	10 sec
C	Signal the observer.	
O	Start the system.	
C	Wait for a 0° or 180° transition.	
C	→ Integrate over 45° and store value.	200
C	→ Integrate over the next 45° and store value.	200
C	→ Check for the 90° or 270° transition.	10
C	→ Integrate over 45° and store value.	200
C	→ Integrate over next 45° and store value.	200
C	→ Check for 0° or 180° transition.	10

The loop is continued until the number of full rotations specified is completed, typically taking about 15 seconds. The data is outputted in a manner similar to system one.

The chopping secondary is run freely at 40 Hz, the maximum possible, to maximize the ability of the preamp signal to keep up with the rotation of the analyzer. Typically 5 to 7 chops occur for every 45° of rotation. The lock-in time constant is set at 30ms to minimize smoothing out the change in signal with rotation. Since the analog input of the computer has a large capacitance associated with it, a low output impedance buffer was inserted after the lock-in to minimize the RC time constant of the connection.

B. Error Analysis.

Although a series of observations taken in succession could show very good internal agreement, with formal errors as low as $\pm .03\%$, a second set of observations of the same object taken later in the night would not always agree with the first to within the accuracy of the formal errors. That is the reproducibility of a set of observations was often worse than the internal agreement. This problem is most

clearly seen in the observations of the unpolarized standards. Figures A1 and A2 are histograms of the deviation from the mean for all of a nights observations of unpolarized standards for the nights of Sept. 10/11 and 13/14 for system one. The quantity ϵ is defined as:

$$\epsilon_x = |P_x - \langle P_x \rangle| \quad A-1$$

with a corresponding definition for ϵ_y . For the night of Sept. 13/14 (Figure A2) the seeing was good, the equipment was working properly, and the errors form a narrow width gaussian distribution from the mean. For such a night the formal error assigned to the determination of the instrumental polarization was taken to be $\delta_x = \sigma_x/\sqrt{N}$ for P_x and similarly for P_y . The combined error is given by:

$$\delta_p = \left[\left(\sqrt{(P_x + \delta_x)^2 + P_y^2} - P \right)^2 + \left(\sqrt{P_x^2 + (P_y + \delta_y)^2} - P \right)^2 \right]^{1/2} \quad A-2$$

For Sept. 13/14 $\delta_p = \pm .04\%$. The fact that the errors scatter away from the mean with roughly a gaussian distribution suggests that this method of assigning errors is appropriate.

For the night of Sept. 10/11 (Figure A1) the situation is quite different. This night was plagued by wandering images and equipment problems, primarily jitter in the gear train. The values plotted correspond to 11 observations taken as three seperate sets of three or four each. While the internal agreement within a set was not particularly good ($\delta_p \approx .12\%$), the agreement among the three sets was worse. This shows up clearly in the Figure as a peak in the distribution displaced from zero. The mean value for each of the sets tended to fall well to one side or the other of the overall mean for the night. On a night such as this all that can be done is to use the overall mean as

Fig. A1 - Histogram of the Deviation from the Mean for all Observations of Unpolarized Standards for the Night of Sept. 10/11, 1976.

Fig. A2 - Histogram of the Deviation from the Mean for all Observations of Unpolarized Standards for the night of Sept. 13/14, 1976.

the value for the instrumental polarization and simply assign a larger error than the formally calculated value. The formal value for this night is $p = .13\%$ but 0.20% is the amount by which a single set of observations of a star is likely to fall away from a nights mean, so this value has been assigned as the error for that night due to uncertainties in the instrumental polarization.

For system two the problem of reproducibility for a single set of observations lies somewhere between the two extremes just discussed. The internal agreement for a single observation was consistently good (as seen in Figure 6), and while the reproducibility was not as good as system one on its best nights, one set of observations was nearly always within two sigma of the nights mean. The difference of one sets mean from another is small enough that all of the observations of unpolarized standards taken as a whole distribute themselves away from the mean in a nearly gaussian manner, as shown in Figure A3 for May 22/23 and Figure A4 for May 24/25 (note the scale change for P_y). More importantly, system two behaved in a more consistent manner from night to night than system one, and was less influenced by bad seeing. No nights showed unusually large deviation in both the individual observations or between sets of observations.

As seen in Figures A3 and A4 the error in P_y was about twice as large as the error in P_x . This was due primarily to the fact that the start of the integrations for $I(0)$ and $I(90)$ were determined by the optical encoder, while the start of the integrations for $I(45)$ and $I(135)$, which determine P_y , were determined by the computer's rather poor timing capabilities. For the nights system two was used, the

Fig. A3 - Histogram of the Deviation from the Mean for all Observations of Unpolarized Standards for the night of May 22/23, 1977.

Fig. A4 - Histogram of the Deviation from the Mean for all Observations of Unpolarized Standards for the night of May 24/25, 1977.

errors in the determination of the instrumental polarization were assigned using equation A-1.

The instrumental polarization was removed from the observations vectorally by subtracting the P_x for the instrumental polarization from an object's observed P_x , and similarly for P_y . For many of the sources of a non-zero instrumental polarization this method works well. For example, if a signal is consistently reduced over a portion of the angular rotation of the analyzer (by moving the image nearer the edge of the aperture or by a change in the transmission of the analyzer) this method will remove such effects to second order. This can be shown in the following way. Note that this effect is not the same as a weak polarizer ahead of the analyzer.

Consider the four portions of the analyzer used for measuring P_x ; 0° , 90° , 180° , and 270° . Say that at 270° the system response is depressed by a factor α where $1-\alpha = \epsilon \ll 1$. For an unpolarized standard the resultant measured value of P_x would be:

$$P_{XI} = \frac{1 - \alpha}{3 + \alpha} = \frac{\epsilon}{4 - \epsilon} \sim \epsilon/4 \quad A-3$$

This value would then be subtracted from the observed P_x for a program star. Suppose a program star has a small intrinsic P_x where $I(90) = I(270) = \beta I(180) = \beta I(0)$, then:

$$P_x = \frac{1 - \beta}{1 + \beta} = \frac{\delta}{2 - \delta} \sim \delta/2 \quad A-4$$

where $\delta = 1-\beta$ and δ is small. Measured through the imperfect system being considered here we have:

$$P_x(\text{measured}) = \frac{2 - \beta - \alpha\beta}{2 + \beta + \alpha\beta} = \frac{2\delta + \epsilon - \epsilon\delta}{4 - 2\delta - \epsilon + \epsilon\delta} \quad \text{A-5}$$

If A-3 is subtracted from this quantity and cross terms are retained for the moment we have:

$$P_x(\text{corrected}) = (8\delta - 4\epsilon\delta)/(16 - 8\delta - 8\epsilon + 6\epsilon\delta + \epsilon^2 - \epsilon^2\delta) \quad \text{A-6}$$

Equation A-6, the value for the program star's P_x after removal of instrumental polarization, does not exactly equal the desired result of equation A-4. By subtracting A-4 from A-6 and retaining only the lower order terms we have an estimate of the error in this method. This is:

$$\text{error} \approx -(P_x P_{XI}^2 + P_{XI} P_x^2) \quad \text{A-7}$$

For $P_x \sim P_{XI} \sim 0.04$, this error is less than 0.0002 (0.02%), far less than the uncertainty for the value of the instrumental polarization for the best night.

For instrumental polarization caused by the combination of a weak polarizer and a weak retarder ahead of the analyzer, vectoral subtraction of the instrumental polarization will remove these effects to the same level of accuracy as well (Serkowski, 1973).

C. $\{\bar{Q}\}_1$

Consider the effect on the Q component of the scattered Stokes vector by the pole to pole variation in Q . To do this, assume the star is an unlimbdarkened disk, and weight the scattered flux by the length of the chord (a in Figure A5) at the angle of view from the point of scattering. From Figure 5 we have:

$$\begin{aligned} I(\omega) &= I(0) \\ Q(\omega) &= Q(0) \cos 2\omega \\ U(\omega) &= -Q(0) \sin 2\omega \end{aligned}$$

$$\text{where: } \omega = a/2R_e$$

$$\bar{Q} = Q(0) \frac{\int_{-R_e}^{+R_e} \gamma \cos 2\theta \, d\theta}{\int_{-R_e}^{+R_e} \gamma \, d\theta} = Q(0) \frac{\int_{-R_e}^{+R_e} a \cos 2\theta \, d\theta}{\int_{-R_e}^{+R_e} a \, d\theta} \quad \text{A-8}$$

$$\bar{U} = -Q(0) \frac{\int_{-R_e}^{+R_e} \gamma \sin 2\theta \, d\theta}{\int_{-R_e}^{+R_e} \gamma \, d\theta} = 0.0 \quad \text{A-9}$$

$\bar{Q}$ has been evaluated numerically and can be approximated by the formula:

$$\bar{Q}/Q \approx 1.0 - \exp\{-(0.207 + 1.114\omega)\} \quad \text{A-10}$$

for $\omega > 1.20$.

Thus

$$\{\bar{Q}\}_1 = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & \bar{Q}/Q & 0 \\ 0 & 0 & 1 \end{Bmatrix} \quad \text{A-11}$$

There is also a reduction in the Q component by the effects of scattering flux from one side of the equator of the star compared to scattering from the nearest point on the star only. This effect is smaller than the pole to pole effect. Also it is compensated for to some extent by the fact that there are many cells in a ring around the star at each increment in ω in the model. Therefore this effect has been ignored.

D. $\{\bar{Q}\}_2$

Since the model disk is only one cell thick, the value for Q

Fig. A5 - Geometry for the Determination of $\{\bar{Q}\}_1$.

Fig. A5 - Geometry for the Determination of $\{\bar{Q}\}_2$.

obtained by considering all of the flux to be scattered along the line connecting cell centers will be too large. This is a problem similar to the two cubes discussed in section IV. Consider two optically thin cells a distance a from center to center (see Figure A6). Since they are optically thin, they may be considered to be cubes. The effect on Q of scatters not in the equatorial plane is:

$$Q(\theta) = Q(0)\cos 2\theta$$

$$\bar{Q}/Q = \frac{\int_{V_1} \int_{V_2} \cos 2\theta \, d\theta}{\int_{V_1} \int_{V_2} d\theta} \quad \text{A-12}$$

where V_1 is the volume of one of the cells and V_2 the volume of the other. $\bar{Q}/Q$ has been computed numerically and can be approximated by the formula:

$$\bar{Q}/Q = 1.0 - \exp\{-(1.11 + 1.37\gamma)\} \quad \text{A-13}$$

where $\gamma = (a-d)/d$. Thus

$$\{\bar{Q}\}_2 = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & \bar{Q}/Q & 0 \\ 0 & 0 & 1 \end{Bmatrix} \quad \text{A-14}$$

The effect of scatters at different angles as seen from above the plane has been ignored because it is smaller than the effect just analyzed and it is partially compensated for by the fact that there are many cells in the plane of the disk. By increasing the number of cells in the disk the effects on Q by non center-to-center scatters is reduced for angles other than center-to-center as seen from above the plane. However, the disk is still only one cell thick and any

increase in the number of cells does not compensate for the effects illustrated in Figures A5 and A6.

E. Ω

Consider an optically thin cube face on a distance d from an observer as illustrated in Figure A7. The solid angle presented by the cube for d determined by Figure A7 can be approximated by:

$$\Omega \sim 2\pi(1 - d/\sqrt{a^2/\pi + d^2}) \quad \text{A-15}$$

The cross sectional area presented by the cube is approximated by a disk whose radius is given by:

$$\pi R^2 = a^2$$

hence

$$R^2 = a^2/\pi$$

in equation A-15. Equation A-15 is derived by integrating over concentric rings:

$$\Omega = \int_0^{\theta_{\max}} d\theta \quad \text{A-16}$$

where $\theta_{\max}$ is given by $\sin^{-1}(d/R)$.

Fig. A7 - Geometry for the Determination of Ω .

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