Asset Transfer Measurement Rules*

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Abstract

We study the design of measurement rules when banks engage in loan transfers to outside investors. Our model incorporates two standard frictions: 1) banks' monitoring incentives decrease in loan transfers, and 2) banks have private information about loan quality. Under only the monitoring friction, we find that the optimal measurement rule sets the same measurement precision regardless of bank characteristics, and strikes a balance between disciplining banks' monitoring efforts vs. facilitating efficient risk sharing. However, under both frictions, uniform measurement rules are no longer optimal but induce excessive retention, thus inhibiting efficient risk sharing. We show that the optimal measurement rule should be contingent on the amount of loan transfers. In particular, measurement decreases in the amount of loan transfers and no measurement should be allowed when banks have transferred most of their loans. We relate our results to current accounting standards for asset transfers.

Keywords: Asset Transfer Measurement Rules; Loan Sale; Securitization; Risk Transfer; Financial Institutions.

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1 Introduction

One of the fundamental accounting issues regarding asset transfers is whether such transfers should be treated as sales or as collateralized borrowings. Current accounting standards generally stipulate that if a transferor gives up control over its assets as a result of such transfers, then it should derecognize those assets and treat the transfers as sales. Otherwise, the transferor should not derecognize the assets. Rather, it should treat any cash inflows from such transfers as borrowings (ASC 860, Financial Accounting Standards Board). However, in practice, the notion of control is difficult to measure unless the incentives of transferors to engage in asset transfers are well understood. Our goal in this paper is to develop a model to capture the basic trade-offs involved in asset transfers. In particular, we seek to understand how measurement rules mediate these trade-offs and affect optimal asset transfer decisions.

We model a representative risk-averse bank that chooses the proportion of its loan portfolio to transfer to a market consisting of a large pool of competitive investors. The bank faces two standard frictions. First, its incentives to monitor borrowers diminish as a result of transferring its loan portfolio. Second, it has private information about the quality of its loan portfolio but such information cannot be credibly disclosed to outsiders. Both costly monitoring and/or higher loan quality stochastically improve the terminal payoffs of the loan portfolio. Given the monitoring and informational frictions, we investigate how a regulator would design measurement rules to maximize social efficiency. We study two types of measurement rules: a *uniform* measurement rule that requires the same measurement precision of the loan's terminal payoffs regardless of banks' characteristics and a *contingent* measurement rule that makes the precision of the measurement rule contingent on banks' observable

characteristics. In practice, a contingent measurement rule may be written on any observable characteristics of banks' environments. We focus on the level of loan retention as it may serve as an indicator of control which is a central criterion for measurement under current accounting standards for asset transfers.

To develop intuition for our main model, we first study a benchmark setting in which the bank's monitoring effort is unobservable but information about its loan quality is publicly known. We show that while, both loan retention and the precision of the measurement rule enhance banks' monitoring incentives, they also impede risk-sharing incentives. In particular, loan retention and precision of the measurement rule are substitutes in providing monitoring incentives but are complements in providing risk-sharing incentives. More importantly, we show that the optimal measurement rule is decoupled from the optimal retention choices so that the standard-setter chooses the uniform measurement rule regardless of optimal retention choices. Such a uniform measurement rule trades off the benefit of providing efficient monitoring incentives vs. facilitating the efficient transfer of risk. Under such a uniform rule, we show that measurement is desirable if and only if the efficiency loss from the reduction in risk-sharing is not too high and/or the bank's monitoring incentives are sufficiently high. The intuition behind this result is straightforward. While measurement inhibits risk-sharing due to the well-known Hirschleifer effect (Hirschleifer, 1971), it also provides efficient monitoring incentives by increasing the sensitivity of prices to banks' fundamentals. Consequently, measurement is more likely in those environments in which gains from efficient monitoring overwhelm losses from inefficient risk-sharing.

When loan quality is unobservable, we show that, besides its monitoring role, the proportion of loan retention acquires an additional informational role. The latter role, in turn,

induces banks to increase loan retention in order to credibly communicate their private information to outsiders. Such excessive retention is inefficient and reduces social efficiency. More interestingly, we show that a uniform measurement rule is no longer optimal because banks' retention policy and the measurement rule are now intertwined. In particular, when risk-sharing considerations are sufficiently strong, measurement exacerbates the over-retention problem arising from the adverse selection problem and reduces efficiency. By tailoring the measurement rule to the proportion of loan retention, the regulator may improve surplus by influencing banks' retention policy.

We show that the optimal measurement rule takes the form of a contingent measurement rule such that measurement precision depends on the bank's retention policy. Under the contingent measurement rule, measurement occurs if and only if the bank retains a sufficiently large proportion of its loan portfolio and whenever measurement occurs, the precision of the measurement rule increases in the proportion retained. Given that the optimal retention policy, in turn, depends on the exogenous parameters of the bank's environment, measurement should only occur when monitoring considerations are relatively more important than risk-sharing considerations. But when risk-sharing considerations are sufficiently important, a no-measurement rule is optimal if and only if the loan quality of the bank's portfolio is sufficiently low.

Our results provide some insights into the current accounting standards for asset transfers. As mentioned previously, the central accounting issue regarding asset transfers is whether such transfers should be treated as borrowings or as sales. Current accounting standards rely on the notion of control as a key guiding principle for the appropriate treatment. We do not explicitly model control issues. However, to the extent that derecognition of assets

implies that the performance of such assets should be measured less precisely, or perhaps not even measured in the financial statements, the contingent measurement rule may provide some insights into when such derecognition may be desirable. In particular, according to the contingent measurement rule, no measurement is optimal if and only if the bank has transferred most of its loans. To the extent that the degree of control is negatively associated with the fraction of assets transferred, our results provide some support for the control framework used under current accounting standards. Furthermore, our comparative statics provide some testable predictions about environments in which measurement is more likely to be optimal.

1.1 Related literature

Our model is closely related to the classical models of Leland and Pyle (1997) and Kanodia and Lee (1998). Similar to Leland and Pyle, we also show that the bank's loan transfer decision conveys information about loan quality inducing the bank to transfer fewer loans. However, our model differs from that of Leland and Pyle in two important ways. First, we consider loan retention decision in the presence of both private information and unobservable monitoring. Second, there are no measurement issues in Leland and Pyle. In our study, we derive the optimal measurement rule plays a key role in influencing a bank's loan retention policy. As in Kanodia and Lee, a more precise measurement rule in our environment disciplines *ex ante* monitoring incentives but also destroys *ex post* risk-sharing. However, Kanodia and Lee do not investigate asset transfer decisions and therefore cannot study the interaction between measurement rules and asset transfer policies which is our main focus.

Our study is also related to the banking literature on credit risk transfers. Early work such as Greenbaum and Thakor (1987) investigate a bank's choice of whether to fund the loans it originates by either issuing deposits or by selling loans to investors. They show that higher quality loans will be sold while lower quality loans will be funded via deposits. Pennacchi (1988) considers a model where banks may improve the returns on loans by monitoring borrowers. He shows that by designing the loan sales contract in a way that gives the bank a disproportionate share of the gains to monitoring, a greater share of the loan can be sold and, hence, a greater level of bank profits can be attained. Gorton and Pennacchi (1995) study a model of incentive-compatible loan sales that allows for implicit contractual features between loan sellers and loan buyers. They theoretically and empirically show that, by maintaining a portion of the loan's risk, banks convince loan buyers of its commitment to evaluate the credit of borrowers. Allen and Carletti (2006) develop a model of how credit risk transfer affects contagion. Using a model with banking and insurance sectors, they show that credit risk transfer is beneficial when banks face uniform demand for liquidity but when they face idiosyncratic liquidity shock, credit risk transfer can increase contagion. Parlour and Plantin (2008) develop a model in which banks receive either proprietary information about loan quality or a shock to their discount rate. Either effect induces banks to transfer credit risk resulting in an adverse selection problem. Parlour and Plantin investigate when such credit risk markets arise and whether this is efficient. Our work is related to these prior studies because either private information, or monitoring or risk-sharing concerns is an important force that affects asset transfers in all these studies. However, none of them study measurement issues which is our main focus. More recently, Goldstein and Leitner (2018) develop a model in which disclosure can destroy risk-sharing opportunities for banks

but some level of disclosure is necessary for risk sharing to occur. However the focus of their study differs significantly from ours. They study the optimal disclosure policy of a regulator who has information about banks as a result of conducting stress tests. We investigate how a regulator should design optimal measurement rules to affect banks' retention decisions in the presence of both moral hazard and adverse selection problems.

Finally, the empirical accounting literature provides evidence on how the accounting for securitization may have economic consequences on firms. Dechow and Shakespeare (2009) investigate whether firms exploit the accounting treatment for securitization to burnish their financial statements. Barth, Omarzabal, and Taylor (2012) show that credit-rating agencies and the bond market differ in their assessments of credit risk transfers in terms of how they evaluate retained vs. non-retained interests of securitized assets. More recently, Dou, Ryan, and Xie (2018) provide evidence on how recent accounting standards that tightened the accounting for securitization and consolidations have real effects on banks' mortgage approval and sale rates. We do not focus on the specifics of the accounting standards for transfers in our model. Instead, we investigate how, given retention policies, measurement rules should be designed to influence *informational* features of accounting reports, and relate the implications of those measurement rules to the current standards for asset transfers.

The rest of the paper proceeds as follows. Section 2 describes the model. Section 3 analyzes the model. Section 4 concludes. An Appendix contains the proofs of the major results.

Figure 1: Timeline of the model

2 The Model

2.1 Timing of events

We examine an environment that consists of a representative bank owner, an accounting regulator (henceforth, regulator), and a large pool of competitive investors. The bank owner (henceforth, bank) has an additive and separable utility function with constant absolute risk aversion $\tau > 0$ whereas the investors are risk neutral. The discount factor is normalized to 1. Figure 1 summarizes the timing of events.

At date $t = 0$, the bank is endowed with a portfolio of loans originated earlier. At the terminal date, $t = 3$, the loan portfolio generates stochastic terminal cash flows, $\pi(\theta, m)$, that depend on both the quality θ of the loan portfolio and the bank's *ex-post* effort m to monitor borrowers. For simplicity, we assume that

$$\pi = \theta + m + \eta. \quad (1)$$

The loan quality θ has a distribution $F(\cdot)$ and a density $f(\cdot)$ with full support on $[\underline{\theta}, \bar{\theta}]$. The random variable η follows a normal distribution with mean 0 and precision h_η . Equation (1) implies that either a higher loan quality or a greater monitoring effort improves the performance of the loan portfolio in the sense of first-order stochastic dominance. The

shock η captures the residual uncertainty regarding the loans' cash flows for a given level of monitoring effort and loan quality. We assume that the bank learns the quality θ of its loan portfolio privately in the process of loan origination and such information cannot be credibly disclosed to outsiders.

After originating the loan portfolio, i.e., at $t = 1$, the bank chooses unobservable effort $m > 0$ to monitor borrowers at a private cost of $\frac{c}{2}m^2$, where $c > 0$. In addition, since the bank is risk averse, it has an incentive to transfer a portion of its loan portfolio to the risk-neutral investors for risk-sharing purposes.¹ In particular, the bank chooses the fraction $\alpha \in [0, 1]$ of the loan portfolio to retain with the remaining fraction to be transferred to investors. We assume that the proportion α is publicly observable.

Before the terminal payoffs π of the loan portfolio are realized, at $t = 2$, the bank issues an accounting report r that measures π . The regulator *ex ante* designs a set of asset transfer measurement rules that govern the measurement process and the informational features of the accounting report r . Specifically, we adopt the following specification of the report

$$r = \pi + \varepsilon. \tag{2}$$

The measurement noise ε is normally distributed with mean 0 and precision h_ε . At $t = 0$, the regulator sets the *ex-ante* measurement rules that determine the precision of the report to maximize total surplus. We consider and compare two types of measurement rules: a *uniform* measurement rule that does not depend on observable bank characteristics and hence is the

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¹To capture risk-sharing, outside investors must be less risk averse than the bank owners. For simplicity, we assume that the outside investors are risk-neutral.

same across all banks and a *contingent* measurement rule that sets measurement precision based on observable bank characteristics. One such bank characteristic in our model is the proportion α of the loan portfolio that the bank retains.

After releasing the report, the bank transfers a fraction $1 - \alpha$ of its loan portfolio to the investors at a per-unit transfer price p . Since the risk-neutral investors are perfectly competitive, they offer a break-even price that equals the expected terminal cash flows of the loan portfolio conditional on the report

$$p = E[\pi|r]. \quad (3)$$

At $t = 3$, the terminal payoffs π of the loan portfolio are realized and distributed to the bank and the investors. The investors receive the proportion $1 - \alpha$ of the terminal payoffs whereas the bank receives the proportion α .

2.2 Payoffs

As a preliminary analysis, we specify the payoffs and optimization programs of the players in our model, i.e., the investors, the bank, and the regulator. First, since the investors set the price p to break even in expectation, their expected payoffs are always 0.

The bank obtains payoffs at three dates. At $t = 1$, the bank incurs a private monitoring cost of $\frac{\varepsilon}{2}m^2$. At $t = 2$, the bank receives a transfer price of $(1 - \alpha)p$ from the investors. We will verify that, given the loan quality θ , the equilibrium price p is normally distributed. Therefore, using standard results in finance, the bank's expected utility of the date-2

consumption can be represented as

$$(1 - \alpha) E[p|\theta] - \frac{\tau}{2} (1 - \alpha)^2 \text{Var}(p|\theta). \quad (4)$$

Note that since the bank is risk averse, higher price volatility reduces the bank's payoffs.

At $t = 3$, the bank receives a terminal cash flow from the loans it retains, $\alpha\pi$. Since π is normally distributed given θ , the bank's expected utility of the date-3 consumption can be represented as

$$\alpha E[\pi|\theta] - \frac{\tau}{2} \alpha^2 \text{Var}(\pi|\theta). \quad (5)$$

In sum, since the bank's utility is additive and separable, its total payoff is the sum of its expected payoffs at the three dates $t \in \{1, 2, 3\}$, i.e., for a given θ , the bank chooses its monitoring effort m and the asset transfer decision α to maximize

$$U(m, \alpha; \theta) = E[(1 - \alpha)p + \alpha\pi|\theta] - \frac{\tau}{2} [\alpha^2 \text{Var}(\pi|\theta) + (1 - \alpha)^2 \text{Var}(p|\theta)] - \frac{c}{2} m^2. \quad (6)$$

Lastly, the regulator sets the measurement rules to maximize the total surplus which equals the *ex-ante* payoffs of all players. Since the investors always break even, the total surplus equals the bank's *ex-ante* expected utility

$$W = E_{\theta}[U(m, \alpha; \theta)] = \int_{\underline{\theta}}^{\bar{\theta}} U(m, \alpha; \theta) f(\theta) d\theta. \quad (7)$$

2.3 Assumptions

We now motivate some key ingredients of our model. First, we assume that the bank is risk averse whereas outside investors are risk neutral. This assumption creates a demand for the bank to engage in loan transfers with investors for risk-sharing benefits. We believe risk transfer plays a key economic motive for banks' activities of loan sales and securitization in practice. For instance, Pozar et al (2010), in discussing the efficiency gain of securitization, argue that "securitization involving real credit risk transfer is an important way for an issuer to limit concentrations to certain borrowers, loan types and geographies on its balance sheet." In this light, outside investors may be better suited than loan originators in bearing loan risk, because investors often hold a broadly diversified portfolio of assets. Similarly, Stein (2010) argues "When banks sell their loans into the securitization market, they distribute the risks associated with these loans across a wider range of end investors, including pension funds, endowments, insurance companies, and hedge funds, rather than taking on the risks entirely themselves. This improved risk-sharing represents a real economic efficiency and lowers the ultimate cost of making the loans."

Second, our main focus is to capture how asset transfer measurement rules affect the informational features (i.e., measurement precision) of accounting reports. In practice, asset measurement rules affect the informativeness of accounting reports because, for example, they determine the criteria regarding whether an asset transfer should be accounted for as a sale or collateralized borrowing (ASC 860). If a transaction of asset transfer meets the sale criteria, the seller of the asset can derecognize the assets transferred ("off-balance-sheet"); otherwise, the seller must continue reporting the asset on its balance sheet. To the

extent that “on-balance-sheet” treatment results in more accounting measurements than “off-balance-sheet” treatment, the asset transfer measurement rules affect the informativeness of accounting reports through setting the derecognition threshold. Furthermore, when a transfer is accounted for as a sale, the measurement rules also regulate how the seller reports various aspects of the sale, e.g., the value of servicing assets and liabilities, the recognition of gains or losses on the sale, the carrying value of the retained interests, etc., all of which have significant impacts on the informational properties of accounting reports.

Lastly, our model allows the regulator to set either a uniform measurement rule or a contingent measurement rule based on the amount of assets banks transfer. In practice, the accounting standards for asset transfer appear to be contingent and, importantly, depend on whether the transferor surrenders control over the assets transferred. For example, under ASC 860 *Transfers and Servicing*, “A transfer of financial assets (or a component of a financial asset) is recognized as a sale if the transferor surrenders control over those assets in exchange for consideration.” However, the detailed conditions that determine whether control has been surrendered are stated, at best, vaguely and require substantial amounts of subjective judgement, for instance, judgement about whether the transferred assets have been legally isolated from the transferor (ASC 860-10-40-5). We seek clarity on the issue of how the proportion of loans banks have transferred should be factored into judgements about how the transferred loans should be measured. To the extent that a bank that has transferred most of its loans would have relatively little control over the loans, our examination of measurement should be made contingent on the amounts of asset transfer may generate policy implications for adopting “control” as the key guiding principle under current asset transfer measurement rules.

3 Analysis

3.1 Observable loan quality

We start the analysis by assuming that loan quality θ is publicly observable. We solve the model using backward induction. At $t = 2$, the investors offer a price that equals the expected terminal cash flows π of the loan portfolio conditional on the report r . Since terminal cash flows π depend on the bank's unobservable monitoring effort, rational investors form a conjecture $\hat{m}$ about the bank's monitoring effort in order to use the report to update their beliefs about π . Of course, this conjecture must be correct in equilibrium. Given $\hat{m}$, the distribution of π conditional on r is normal so that the price for loans is given by

$$p^* = E[\pi|r, \hat{m}] = \beta r + (1 - \beta)(\theta + \hat{m}). \quad (8)$$

The price is a weighted average of the accounting report r and the investors' prior expectation about π , given their conjecture about the monitoring effort. The weight $\beta \equiv \frac{h_\varepsilon}{h_\varepsilon + h_\eta}$ placed on the report r is strictly increasing in and is isomorphic to the measurement precision h_ε . Note that $\beta = 0$ corresponds to the case of no measurement whereas $\beta = 1$ corresponds to the case of perfect measurement. For expositional convenience, we hereafter refer to β as the regulator's choice of measurement precision.

We next solve for the bank's choice of monitoring effort m at $t = 1$. Substituting the

transfer price (8) into the bank's payoff (6) and rearranging terms yields

$$U(m, \alpha; \theta) = \theta + \underbrace{[\alpha + (1 - \alpha)\beta]m - \frac{c}{2}m^2}_{\text{gain from monitoring}} + (1 - \alpha)(1 - \beta)\hat{m} - \underbrace{\frac{\tau}{2h_\eta}(\alpha^2 + (1 - \alpha)^2\beta)}_{\text{loss from inefficient risk sharing}}. \quad (9)$$

From (9), it follows that choosing both the proportion of loan retention and the precision of measurement of loan performance result in a trade-off: they both provide monitoring incentives that increase the bank's payoffs but they simultaneously inhibit efficient risk-sharing that reduces the bank's payoffs.

To see the monitoring roles of loan retention and measurement, differentiate (9) with respect to m yields the *equilibrium* monitoring effort m^* that satisfies

$$\alpha + (1 - \alpha)\beta = cm^*. \quad (10)$$

The right-hand side of (10) represents the marginal cost of monitoring, while the left-hand side represents the marginal benefit of monitoring. The left-hand side captures the often-debated incentive problem when banks transfer loans (e.g., Keys et al, 2010): their incentives to monitor their loans are lower whenever they have “less skin in the game.” To see this more clearly, note that when $\beta = 0$, m^* decreases as the proportion of loan retention α decreases. However, in the presence of measurement, i.e., when $\beta > 0$, (10) also suggests that higher precision of measuring loan performance (i.e., a larger β) is a *substitute* for loan retention in incentivizing the bank to monitor. More precise measurement improves investors' ability to price the loans in alignment with their underlying cash flows, thereby disciplining the bank.

Similarly, to see their risk-sharing roles, we next derive the bank's loan transfer decision

α and the regulator's choice of measurement rule β . Note first that, when the loan quality θ is observable, implementing either a uniform measurement rule or a rule contingent on α does not make a difference. This is because, in this case a uniform rule essentially allows the regulator to set the measurement precision β regardless of α whereas a contingent rule allows the regulator to choose β after observing α . However, the order regarding the choices of α and β does not matter because, absent any private information about θ , the bank's objective function coincides with that of the regulator, thereby making the uniform and the contingent rules equivalent. To see the algebraic equivalence, substitute the equilibrium monitoring effort specified in (10) into (9) to get

$$U(m^*, \alpha; \theta) = \theta + m^* - \frac{c}{2}(m^*)^2 - \frac{\tau}{2h_\eta}(\alpha^2 + (1 - \alpha)^2\beta). \quad (11)$$

Note that we have imposed the rational expectation requirement that the investors' conjecture is consistent with the equilibrium, i.e., $\hat{m} = m^*$.² In addition, from (7), the total surplus

$$W \equiv \int_{\underline{\theta}}^{\bar{\theta}} U(m, \alpha; \theta) f(\theta) d\theta = E[\theta] + m^* - \frac{c}{2}(m^*)^2 - \frac{\tau}{2h_\eta}(\alpha^2 + (1 - \alpha)^2\beta). \quad (12)$$

Note that W differs from $U(m^*, \alpha; \theta)$ only by the constants θ and $E[\theta]$. Therefore, the pair of $\{\alpha, \beta\}$ that maximizes the bank's payoff $U(m^*, \alpha; \theta)$ also maximizes the total surplus W . In the next lemma, we formally state the equivalence result between the uniform and the contingent measurement rules when private information is absent.

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²Note that given that α , β and c are common knowledge, investors can perfectly conjecture m from (10).

Lemma 1 *When the loan quality θ is publicly observable, the equilibrium outcomes under the uniform measurement rule coincide with those under the contingent measurement rule.*

A direct implication of Lemma 1 is that, since the regulator shares the same objective function with the bank, we only need to solve for the pair of $\{\alpha, \beta\}$ that maximizes (12) to completely characterize the equilibrium. Differentiating (12) with respect to β and rearranging terms yields

$$(1 - cm^*) \frac{\partial m^*}{\partial \beta} = \frac{\tau}{2h_\eta} (1 - \alpha)^2. \quad (13)$$

Equation (13) characterizes the regulator's trade-off in setting the measurement precision. The left-hand side of (13) represents the marginal benefit of improving measurement precision in disciplining the bank's monitoring effort.³ The right-hand side represents the marginal cost of more precise measurement in inhibiting the efficient transfer of risk. Recall that transferring the loans from the bank to the investors improves efficiency because the risk-neutral investors are better at bearing risk than the risk-averse bank. Such risk transfer is best achieved if the transfer price is insensitive to the performance of the loan portfolio so that the risk-averse bank bears no risk after the transfer. However, more precise measurement makes the price more sensitive to the terminal payoffs so that the bank faces higher price volatility even after off-loading its loans. Such volatility constitutes a cost of measurement.

Interestingly, the right-hand side of (13) decreases in α which, in turn, suggests that—unlike their monitoring roles discussed above where they are substitutes—higher precision of measurement and loan retention are *complements* when it comes to risk-sharing in the sense

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³The left-hand side of (13) is strictly positive because, from (10), $\frac{\partial m^*}{\partial \beta} = \frac{1-\alpha}{c} > 0$ and $1 - cm^* = (1 - \alpha)(1 - \beta) > 0$. Thus the left-hand side of (13) is given by $\frac{(1-\alpha)^2}{c} (1 - \beta)$.

that the risk-sharing loss from measurement is lower when the bank has retained more loans, and *vice versa*. Intuitively, the price volatility induced by measurement is lower the larger the proportion of the loan portfolio that the bank retains. Accordingly, measurement is least costly to the bank from a risk-sharing perspective if the bank has retained a large proportion of its loan portfolio.

Analogously, differentiating (12) with respect to α and rearranging terms yields⁴

$$(1 - cm^*) \frac{\partial m^*}{\partial \alpha} = \frac{\tau [\alpha - (1 - \alpha)\beta]}{h_\eta}. \quad (14)$$

Equation (14) illustrates the trade-off makes in its bank's asset transfer: while transferring more assets allows the bank to achieve better risk sharing with the investors, it also weakens the bank's incentive to monitor. Moreover, the right-hand side of (14) once again illustrates the complementarity between measurement and loan retention in affecting the risk-sharing loss. Given measurement, loan retention has a smaller adverse effect on risk sharing and the adverse effect gets even smaller as the proportion of the loan portfolio transferred increases.⁵ This is because, conditional on measurement, the bank would still incur the risk-sharing loss even if it had transferred most of its loans.

Solving equations (13) and (14) yields the equilibrium levels of loan retention and measurement precision. We formally state the equilibrium in the following proposition.

Proposition 1 *When the loan quality θ is publicly observable, the optimal choices $\{\alpha_0, \beta_0\}$*

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⁴The left-hand side of (14) is strictly positive because, from (10), $\frac{\partial m^*}{\partial \alpha} = \frac{1-\beta}{c} > 0$ and $1 - cm^* = (1 - \alpha)(1 - \beta) > 0$. Thus the left-hand side of (14) is given by $\frac{1-\alpha}{c} (1 - \beta)^2$.

⁵Mathematically, the right-hand side of (14) decreases in β and the decrease is proportional to $(1 - \alpha)$.

of loan retention and measurement precision, respectively are

$$\begin{cases} \alpha_0 = \frac{4h_\eta - c\tau}{8h_\eta - c\tau} \text{ and } \beta_0 = 1 - \frac{c\tau}{2h_\eta} & \text{if } \frac{\tau}{2h_\eta} < \frac{1}{c}; \\ \alpha_0 = \frac{h_\eta}{h_\eta + c\tau} \text{ and } \beta_0 = 0 & \text{if } \frac{\tau}{2h_\eta} \geq \frac{1}{c}. \end{cases} \quad (15)$$

Proposition 1 is intuitive and illustrates that, absent any private information, measurement rules and asset retention decisions are decoupled in the sense that they do not depend on each other. Instead, the optimal measurement rule sets the same measurement precision for all banks regardless of their loan quality θ . Moreover, the measurement rule requires measuring the value of transferred loans if and only if monitoring considerations are more important relative to risk-sharing considerations. In particular, $\beta_0 > 0$ whenever banks have sharp incentives to monitor loans (i.e., c is low) but the bank's risk aversion τ is low and the loan's terminal cash flows are less volatile, i.e., h_η is high. Accordingly, upon measurement, the regulator should require higher precision, i.e., β_0 increases when either c decreases and/or $\frac{\tau}{h_\eta}$ decreases. Conversely, as risk considerations become more important, i.e., $\frac{\tau}{h_\eta}$ increases but monitoring incentives become less sharp, i.e., monitoring cost c increases, the cost of measurement in inhibiting risk transfer increases relative to the benefit of measurement in disciplining the bank. In that case, the optimal measurement rule calls for less precise measurement, i.e., β_0 decreases or even no measurement, i.e., $\beta_0 = 0$.

3.2 Unobservable loan quality

We now analyze the complete model in which the loan quality θ is privately known by the bank. In the presence of the bank's private information, the regulator no longer shares

the same objective function with the bank. Therefore, the equilibrium outcome under the uniform measurement rule may now differ from that under the contingent rule. We examine the equilibrium outcome under each of the two rules separately.

3.2.1 Uniform measurement rule

We start with the uniform measurement rule in which the regulator requires uniform measurement precision β for all banks. We solve the model using backward induction. At $t = 2$, the break-even price p depends on the measurement report r and also on any information about loan quality θ that investors can extract from observing the bank's retention choice α . The loan retention fraction α now acquires an informational role because the bank chooses α after observing θ . Specifically, to infer θ from α , suppose the investors form a conjecture $\alpha(\theta)$ about the bank's loan retention schedule. If the schedule $\alpha(\theta)$ is monotone in θ (as it will turn out to be in equilibrium), investors can infer the exact value of θ . We denote investors' inferred value of θ as $\hat{\theta}(\alpha)$. The break-even price incorporates this inferred value $\hat{\theta}(\alpha)$ rather than the true value θ . Replacing θ with $\hat{\theta}(\alpha)$ in the pricing formula (8) yields

$$p^* = E[\pi|\alpha, r, \hat{m}] = \beta r + (1 - \beta)(\hat{\theta}(\alpha) + \hat{m}) \quad (16)$$

Next, we solve for the bank's choices of monitoring effort m and retention fraction α at $t = 1$. Substituting the transfer price (16) into the bank's payoff (6) and rearranging terms,

we obtain

$$U(m, \alpha; \theta) = [\alpha + (1 - \alpha)\beta](\theta + m) + (1 - \alpha)(1 - \beta)(\hat{\theta}(\alpha) + \hat{m}) - \frac{\tau}{2h_\eta}(\alpha^2 + (1 - \alpha)^2\beta) - \frac{c}{2}m^2. \quad (17)$$

Differentiating (17) with respect to m yields $m^* = \frac{\alpha + (1 - \alpha)\beta}{c}$. Substituting m^* into (17) and imposing the rational expectation requirement that $\hat{m} = m^*$ yields

$$U(m^*, \alpha; \theta) = [\alpha + (1 - \alpha)\beta]\theta + (1 - \alpha)(1 - \beta)\hat{\theta}(\alpha) + m^* - \frac{c}{2}(m^*)^2 - \frac{\tau}{2h_\eta}(\alpha^2 + (1 - \alpha)^2\beta). \quad (18)$$

Expression (18) suggests that the investors' inference $\hat{\theta}(\alpha)$ affects the bank's payoff and hence potentially changes the bank's equilibrium choice of loan retention $\alpha(\theta)$. Rational expectation equilibrium requires that the investors' inference is consistent with the bank's equilibrium choice, i.e., $\hat{\theta}(\alpha(\theta)) = \theta$. As is standard in the literature (e.g., Spence, 1974), such a signaling equilibrium is sustained if the “single-crossing property” holds. In our environment, the single-crossing property requires that the bank with a higher loan quality loan (high θ) is willing to retain a higher fraction of its loan portfolio than the bank with a lower loan quality. To verify this property, note that the marginal rate of substitution between $\hat{\theta}$ and α in the bank's payoff (18) is

$$\frac{\partial \hat{\theta}}{\partial \alpha} = -\frac{U_\alpha}{U_{\hat{\theta}}} = -\frac{\theta - \hat{\theta}(\alpha)}{1 - \alpha} - \frac{1 - \beta}{c} + \frac{\tau(\alpha - (1 - \alpha)\beta)}{(1 - \alpha)(1 - \beta)h_\eta}, \quad (19)$$

which is strictly decreasing in the loan quality θ . In other words, a high θ bank is more willing to retain a higher proportion of the loan portfolio than a low θ bank for the same amount of improvement in the investors' inference. With the single-crossing property established, we now formally construct the fully revealing equilibrium in the following proposition.

Proposition 2 *Given the uniform measurement precision β ,*

1. *the equilibrium loan retention schedule is given by*

$$\alpha_U(\theta; \beta) = 1 + \frac{c\tau}{(1 - \beta)^2 h_\eta + (1 + \beta)c\tau} w \left(-e^{-\left(1 + \frac{(1 - \beta)h_\eta(\theta - \underline{\theta})}{\tau}\right)} \right), \quad (20)$$

where $w(\cdot)$ is the Lambert W function (i.e., the principal solution for y in $x = ye^y$);

2. *the equilibrium loan retention schedule $\alpha_U(\theta; \beta)$ is strictly increasing in loan quality θ .*

Proposition 2 is intuitive and states that a bank with a higher-quality loan portfolio is induced to transfer a smaller proportion of its portfolio to investors in order to obtain a more favorable price.⁶ Furthermore, due to its informational role, such a loan retention schedule exhibits excessive retention which is socially inefficient. More precisely, the following corollary shows that, absent measurement, banks retain larger fractions of loans when there is private information than the optimal fraction under no private information.

Corollary 1 *Under no measurement ($\beta = 0$), banks retain larger fractions of loans when the loan quality θ is unobservable than the optimal fraction when θ is observable, i.e., $\alpha_U(\theta; 0) \geq \alpha_0$, where α_0 is defined in Proposition 1. The inequality is strict if $\theta > \underline{\theta}$.*

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⁶Note that by setting $c = \infty$ (so that $m = 0$) and $\beta = 0$, it is straightforward to verify that $\alpha_U(\theta; 0)$ coincides with the fully revealing retention schedule in Leland and Pyle (1977).

An implication of Corollary 1 is that, given the bank's over-retention incentives under no measurement, and that the loan retention schedule $\alpha_U(\theta; \beta)$ explicitly depends on β , the precision of the measurement rule, the regulator may be able to fine-tune the measurement rules in order to mitigate the over-retention inefficiency, i.e., shifting $\alpha_U(\theta; \beta)$ closer to α_0 . Toward that end, the following proposition sheds light on how a uniform increase of measurement precision over no measurement affects the bank's over-retention incentives.

Proposition 3 *Consider a uniform marginal increase of measurement over no measurement (i.e., $\beta = 0$):*

1. *when $\frac{\tau}{2h_\eta} < \frac{1}{c}$, measurement shrinks over-retention for all banks, i.e.,*

$$\left. \frac{\partial (\alpha_U(\theta; \beta) - \alpha_0)}{\partial \beta} \right|_{\beta=0} < 0 \text{ for all } \theta > \underline{\theta}; \quad (21)$$

2. *but when $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$, measurement exacerbates over-retention if the bank's equilibrium retention fraction is sufficiently small.⁷*

Proposition 3 suggests that measurement rules that mandate a *uniform* increase of measurement may not necessarily be beneficial. A sufficient condition under which uniform measurement diminishes over-retention and thus improves efficiency is that risk-sharing considerations are relatively weak but monitoring incentives are relatively important (i.e., $\frac{\tau}{2h_\eta} < \frac{1}{c}$). But when risk-sharing considerations are significant, uniform measurement could actually worsen the over-retention inefficiency. Proposition 3 thus points to an efficiency gain from

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⁷Note that since $\alpha_U(\theta; 0)$ is strictly increasing in θ , the condition that $\alpha_U(\theta; 0)$ is sufficiently small is equivalent to a condition that the loan quality θ is sufficiently small.

making asset transfer measurement rules contingent on observable bank characteristics. In particular, part 2 of Proposition 3 suggests that the regulator should require less precise, or even no measurement if the bank has transferred most of its loans (i.e., α is small).

To provide some intuition for Proposition 3, it is instructive to investigate how increasing precision affects over-retention incentives. Recall that absent private information, the loan retention schedule α_0 is insensitive to the loan quality θ . But when there is private information, the loan retention schedule α_U is strictly increasing in θ . Therefore, over-retention becomes more severe if the retention schedule rises more steeply in the loan quality (i.e., $\frac{\partial \alpha_U}{\partial \theta}$ is large). To study the behavior of $\frac{\partial \alpha_U}{\partial \theta}$, we reproduce its expression (equation (45) in the appendix) below, i.e.,

$$\frac{\partial \alpha_U}{\partial \theta} = \frac{\overbrace{\tau \frac{(1-\alpha)(1-\beta)}{h_\eta}}^{\text{benefit of over-retention}} - \underbrace{\frac{1-\alpha}{c} (1-\beta)^2}_{\text{loss of over-retention}}}{\tau [\alpha - (1-\alpha)\beta]} \quad (22)$$

Equation (22) illustrates how a uniform measurement rule affects the retention schedule. The numerator of (22) captures the benefit of over-retention stemming from improving the investors' inference $\hat{\theta}(\alpha)$ about the loan quality. This benefit increases as the weight on $\hat{\theta}(\alpha)$ in the bank's payoff (18) increases. Importantly, more precise measurement (i.e., a larger β) diminishes the weight placed on the inference, as the investors rely more on the report and less on the level of retention in their pricing decisions.⁸ Stated differently, measurement reduces the bank's benefit of over-retention and shrinks the amount of excess retention in

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⁸Mathematically, note that, from (18), the weight on $\hat{\theta}$ is exactly $(1-\alpha)(1-\beta)$ (i.e., the numerator of (22)) and strictly decreasing in the measurement precision β .

equilibrium. We call the latter effect, the *inference effect* of measurement.

The denominator of (22) represents the bank's net loss from over-retention. Recall from (14) that, absent information asymmetry, the bank sets the optimal loan retention amount by trading off the marginal gain from improving monitoring incentives against the marginal loss of risk sharing. Excess loan retention causes more risk-sharing loss relative to the monitoring gain, thus resulting in a net loss for the bank. Examining the denominator of (22) suggests that increasing measurement has ambiguous effects on the over-retention loss.⁹ On the one hand, more precise measurement reduces the risk-sharing loss from excess retention due to the complementary roles of retention and measurement on risk-sharing. Over-retention, therefore, is less costly when the bank is already required to measure the loans transferred, compared with under no measurement. This *risk-sharing effect* of measurement, therefore, encourages the bank to retain more loans and exacerbates the over-retention inefficiency. On the other hand, improving measurement precision also decreases the benefits of loan retention on monitoring, because measurement is a substitute for loan retention in incentivizing the bank to monitor loans. This *monitoring effect* of measurement thus increases the net loss from over-retention, which curbs excess retention.

The overall effect of measurement on excess retention therefore depends on the interplay among the risk-sharing effect that increases retention and the inference and the monitoring effects that decrease retention. When risk-sharing considerations are less important than monitoring considerations, the risk-sharing effect is dominated and a uniform increase of measurement always curbs over-retention. This explains the conditions in part 1 of Proposition

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⁹Mathematically, the first term in the denominator of (22) represents the risk-sharing loss from excess retention whereas the second term represents the monitoring gain. Note that both terms are decreasing in the measurement precision β . Hence the overall effect of β on the denominator of (22) is ambiguous.

3. However, when risk-sharing considerations become more important, the risk-sharing effect can sometimes dominate, in which case measurement induces excessive retention, thereby impairing efficiency. The risk-sharing effect of measurement in inducing excessive retention becomes especially important when the bank has transferred a large proportion of its loan portfolio. This explains the conditions in part 2 of Proposition 3.

An implication of Proposition 3 is that the regulator may be able improve efficiency by tailoring asset transfer measurement rules to banks' characteristics. In particular, the regulator should adjust the measurement precision requirement to banks' asset transfer decisions. We next solve for such an optimal contingent measurement rule.

3.2.2 Contingent measurement rule

We denote the regulator's choice of the contingent measurement rule as $\beta(\alpha)$, which is a general function of the retention fraction α that will be determined in equilibrium. We solve the model using backward induction. Note first that given the measurement rule $\beta(\alpha)$, the loan transfer price p and the bank's monitoring effort m^* are the same as those derived under the uniform rule, i.e., equations (16) and (10), respectively.

Next, we derive the bank's equilibrium loan retention schedule $\alpha(\theta)$. Such schedule $\alpha(\theta)$ must satisfy a bank's incentive compatibility (IC) constraints, i.e., a type θ bank must prefer choosing $\alpha(\theta)$ over the retention choice $\alpha(\theta')$ designed for some other type θ' . Without loss of generality, assume that $\theta' > \theta$. Importantly, note that how the regulator sets the contingent rule $\beta(\alpha)$ affects the IC constraints and, through this channel, influences the bank's retention schedule. To illustrate the effect of the contingent measurement rule, we now formally derive the bank's IC constraints. If the type θ bank chooses $\alpha(\theta)$, substituting

$\beta = \beta(\alpha)$ and $\alpha = \alpha(\theta)$ into (18) yields the payoffs

$$\begin{aligned} U(\theta) &\equiv U(m^*(\alpha(\theta)), \alpha(\theta); \theta) \\ &= \theta + m^*(\alpha(\theta)) - \frac{c}{2} (m^*(\alpha(\theta)))^2 - \frac{\tau}{2h_\eta} [\alpha(\theta)^2 + (1 - \alpha(\theta))^2 \beta(\alpha(\theta))], \end{aligned} \quad (23)$$

where the monitoring effort $m^*(\alpha(\theta)) = \frac{\alpha(\theta) + (1 - \alpha(\theta))\beta(\alpha(\theta))}{c}$. Note that we have imposed the rational expectation requirement that the investors' inference is consistent with the bank's equilibrium choice, i.e., $\hat{\theta}(\alpha(\theta)) = \theta$. If the type θ bank deviates from the equilibrium schedule and chooses $\alpha(\theta')$, its payoffs equal

$$\begin{aligned} U(\theta'; \theta) &\equiv U(m^*(\alpha(\theta')), \alpha(\theta'); \theta) \\ &= [\alpha(\theta') + (1 - \alpha(\theta'))\beta(\alpha(\theta'))]\theta + (1 - \alpha(\theta'))(1 - \beta(\alpha(\theta')))\theta' + m^*(\alpha(\theta')) \\ &\quad - \frac{c}{2} (m^*(\alpha(\theta')))^2 - \frac{\tau}{2h_\eta} [\alpha(\theta')^2 + (1 - \alpha(\theta'))^2 \beta(\alpha(\theta'))]. \end{aligned} \quad (24)$$

Note that deviation of type θ to type θ' by choosing $\alpha(\theta')$ instead of $\alpha(\theta)$ results in two differences between $U(\theta)$ and $U(\theta'; \theta)$. First, the investors' inference about the bank's loan quality changes as they infer the bank's type to be θ' upon observing a retention amount of $\alpha(\theta')$. Second, the regulator responds to the change in the bank's asset transfer decision by requiring a different precision level $\beta(\alpha(\theta'))$ regarding how the bank measures the transfer. This feature implies that the regulator may influence the bank's asset transfer decision by imposing different measurement requirements for any type $\theta' \neq \theta$.

To ensure no deviation by the type θ bank, the IC constraint requires that $U(\theta) \geq$

$U(\theta'; \theta)$. Substituting (23) and (24) into the IC constraint and rearranging terms yields

$$\begin{aligned}
U(\theta) &\geq U(\theta'; \theta) \\
&= [\alpha(\theta') + (1 - \alpha(\theta'))\beta(\alpha(\theta'))](\theta - \theta') \\
&\quad + [\alpha(\theta') + (1 - \alpha(\theta'))\beta(\alpha(\theta'))]\theta' + (1 - \alpha(\theta'))(1 - \beta(\alpha(\theta')))\theta' \\
&\quad + m^*(\alpha(\theta')) - \frac{c}{2}(m^*(\alpha(\theta')))^2 - \frac{\tau}{2h_\eta} [\alpha(\theta')^2 + (1 - \alpha(\theta'))^2\beta(\alpha(\theta'))] \\
&= U(\theta') - \underbrace{[\alpha(\theta') + (1 - \alpha(\theta'))\beta(\alpha(\theta'))](\theta' - \theta)}_{\text{deviation loss } > 0}. \tag{25}
\end{aligned}$$

The IC constraint (25) implies the payoffs $U(\theta'; \theta)$ of type θ who chooses the retention allocation of a higher type θ' is strictly lower than $U(\theta')$, the payoffs of type θ' by

$$[\alpha(\theta') + (1 - \alpha(\theta'))\beta(\alpha(\theta'))](\theta' - \theta), \tag{26}$$

which captures the expected loss from deviation. Intuitively, to prevent the type θ bank from choosing the retention of the higher type θ' —the type θ' bank is induced to increase its retention level $\alpha(\theta')$ which reduces $U(\theta'; \theta)$ in equilibrium. Furthermore, the deviation loss term also suggests that by choosing a higher measurement precision, the regulator can reduce the marginal effect of excess loan retention in deterring the type θ bank from mimicking type θ' . In fact, if measurement is perfect, ($\beta(\alpha(\theta')) = 1$), increasing $\alpha(\theta')$ would have no impact on the deviation term. This suggests a disciplinary role of measurement in curbing over-retention. Intuitively, more precise measurement makes the loan transfer price depend more on the measurement report (which reflects the bank's *true* loan quality) and less on the investors' prior expectation about the loan cash flow (which is formed based on the *inferred*

loan quality), thus weakening the bank's incentive to influence the investors' inference via loan retention.¹⁰ Accordingly, an implication from examining the IC constraint (25) is that, when the loan retention is overly high, the regulator could respond by increasing the precision of the measurement in order to curb such over-retention. We will verify that this implication is indeed true when we solve for the optimal measurement rule.

Next, to ensure no deviation by the type θ' bank, the analogous IC constraint requires that $U(\theta') \geq U(\theta; \theta')$, which is given by

$$U(\theta') \geq U(\theta) + [\alpha(\theta) + (1 - \alpha(\theta))\beta(\alpha(\theta))](\theta' - \theta). \quad (27)$$

Combining (25) and (27) yields

$$[\alpha(\theta) + (1 - \alpha(\theta))\beta(\alpha(\theta))](\theta' - \theta) \leq U(\theta') - U(\theta) \leq [\alpha(\theta') + (1 - \alpha(\theta'))\beta(\alpha(\theta'))](\theta' - \theta). \quad (28)$$

Note that (28) implies that for $\theta' > \theta$, $\alpha(\theta') + (1 - \alpha(\theta'))\beta(\alpha(\theta')) > \alpha(\theta) + (1 - \alpha(\theta))\beta(\alpha(\theta))$, i.e., $\alpha(\theta) + (1 - \alpha(\theta))\beta(\alpha(\theta))$ is non-decreasing in θ . Taking the limit of $\theta' \rightarrow \theta$, we obtain the IC constraint as:

$$U'(\theta) = \alpha(\theta) + (1 - \alpha(\theta))\beta(\alpha(\theta)). \quad (29)$$

Formally, we next state the conditions under which the retention schedule $\alpha(\theta)$ is incentive compatible.

Lemma 2 *Given the measurement rule $\beta(\alpha)$, the loan retention schedule $\alpha(\theta)$ is incentive*

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¹⁰The *ex-post* disciplining role of measurement in alleviating over-retention is similar to the disciplinary role of a performance report first developed by Kanodia and Lee (1998).

compatible if and only if:

1. $U'(\theta) = \alpha(\theta) + (1 - \alpha(\theta))\beta(\alpha(\theta))$ and
2. $\alpha(\theta) + (1 - \alpha(\theta))\beta(\alpha(\theta))$ is non-decreasing in θ .

As is standard in the literature, incentive compatibility requires both (29) and a monotonicity condition. In deriving the optimal schedules of $\alpha(\theta)$ and $\beta(\alpha)$, we next ignore the monotonicity condition and verify that the optimal schedules from the relaxed problem do indeed satisfy the monotonicity condition. Therefore, the regulator chooses the contingent measurement rule $\beta(\alpha)$ to solve

$$\begin{aligned} \max_{\beta(\alpha)} W &\equiv \int_{\underline{\theta}}^{\bar{\theta}} U(\theta)f(\theta)d\theta, \\ s.t., \quad U'(\theta) &= \alpha(\theta) + (1 - \alpha(\theta))\beta(\alpha(\theta)). \end{aligned} \tag{30}$$

We solve program (30) by solving the following optimal control problem

$$\max_{\alpha(\theta), \beta(\theta)} W \equiv \int_{\underline{\theta}}^{\bar{\theta}} U(\theta)f(\theta)d\theta, \tag{31}$$

$$s.t., \quad U'(\theta) = \alpha(\theta) + (1 - \alpha(\theta))\beta(\theta). \tag{32}$$

We establish in the appendix that programs (30) and (31) are equivalent. In other words, to derive the optimal contingent measurement rule, the regulator can first solve for the optimal schedules of measurement precision and loan retention as a function of the bank's loan quality θ , i.e., $\{\alpha^*(\theta), \beta^*(\theta)\}$, although the regulator does not directly observe θ and cannot make the measurement rule contingent on θ . The optimal contingent rule $\beta_C(\alpha)$ is

then given by inverting the optimal schedule of loan retention and substituting it into the optimal measurement schedule, i.e., $\beta_C(\alpha) = \beta^*(\theta^*(\alpha))$, where $\theta^*(\alpha)$ is the inverse function of $\alpha^*(\theta)$. Solving program (31) yields the optimal loan retention and measurement rule schedules that we formally state next.

Proposition 4 *When the loan quality θ is unobservable, the optimal schedules of loan retention and measurement rule, $\{\alpha^*(\theta), \beta_C(\alpha)\}$ satisfy:*

1. *the regulator imposes a contingent measurement rule $\beta_C(\alpha) = \frac{3\alpha-1}{1-\alpha}$ if $\alpha > \frac{1}{3}$ and requires no measurement if $\alpha \leq \frac{1}{3}$;*
2. *if $\frac{\tau}{2h_\eta} < \frac{1}{c}$, the equilibrium loan retention schedule $\alpha^*(\theta) \in (\frac{1}{3}, \frac{1}{2})$ satisfies*

$$\frac{\partial \alpha^*(\theta)}{\partial \theta} = H(\alpha^*(\theta)), \quad (33)$$

where the function $H(x) \equiv \frac{c(1-2x)}{\frac{c\tau}{h_\eta}(1-x)-4(1-2x)}$;

3. *but if $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$, there exists a cutoff $\theta_c \in [\underline{\theta}, \bar{\theta}]$, where θ_c solves $\alpha_U(\theta_c; 0) = \frac{1}{3}$ such that, for $\theta > \theta_c$, $\alpha^*(\theta) \in (\frac{1}{3}, \frac{1}{2})$ solves (33), while for $\theta \leq \theta_c$, $\alpha^*(\theta) = \alpha_U(\theta; 0) \in (0, \frac{1}{3}]$.¹¹*

Proposition 4 states that, when banks have private information about the quality of their loan portfolios, the optimal measurement rule for loan transfers should be made contingent on the amount of loans transferred. In particular, the regulator should require measurement if and only if banks retain a sufficiently large proportion of their loan portfolios *and* the precision of such measurement should increase in the proportion of loan retention. Figure 2 provides a graphic illustration of the optimal contingent measurement rule. The intuition for

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¹¹Recall that the expression of $\alpha_U(\theta; 0)$ is as given in Proposition 2.

Figure 2: Optimal contingent measurement precision $\beta_C(\alpha)$ as a function of loan retention fraction α . Note that, from Proposition 4, the equilibrium level of α cannot exceed 0.5.

Proposition 4 follows from the impact of measurement on the over-retention inefficiency discussed in Proposition 3. Recall that measurement can worsen the over-retention inefficiency if the bank has transferred most of its loans (i.e., α is small). Accordingly, the optimal rule should require no measurement under those circumstances.

We now relate the implications of Proposition 4 to current accounting standards on asset transfers. As discussed previously, in practice, the central issue at debate is whether and when transferred assets should be recognized on the balance sheet (i.e., treated as collateralized borrowing) or derecognized (i.e., treated as a sale). Under current standards, the key guiding principle for derecognition is whether the transferor has surrendered control. To the extent that derecognition implies less measurement or even no measurement of the performance of the loans, the contingent measurement rule derived in Proposition 4 provides conditions under which derecognition is desirable. In this light, Proposition 4 states that no

measurement should be allowed when the bank has transferred most of the loans. Moreover, to the extent that the degree of control is negatively associated with the fraction of assets transferred, Proposition 4 lends some support for adopting the control principle in the current accounting standards.

Furthermore, because the contingent measurement rule depends on the bank's asset transfer decision, the exogenous parameters that drive the bank's equilibrium choice of asset transfer provides some insights into environments when measurement is more likely to be useful. In particular, when monitoring considerations are more important relative to risk-sharing considerations (i.e., $\frac{\tau}{2h_\eta} < \frac{1}{c}$), measurement always occurs in order to provide efficient monitoring incentives. But when risk-sharing considerations become sufficiently more important (i.e., $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$), Proposition 4 suggests that there exists a measurement cutoff θ_c on the bank's loan quality below which measurement never occurs. Interestingly, such equilibrium measurement schedule resembles the threshold disclosure strategy derived in the voluntary disclosure literature; yet, the mechanisms under which the two equilibria are sustained are completely different. In our model, the measurement cutoff arises due to the regulator's optimal design of the *ex ante* mandatory measurement rule, whereas, in the voluntary disclosure literature, the disclosure threshold prevails as a consequence of firms' own *ex post* voluntary disclosure choice. To generate additional implications, we next provides some comparative statics on the equilibrium measurement cutoff θ_c .

Corollary 2 *If $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$, the measurement cutoff θ_c is strictly increasing in the degree of risk aversion τ , the monitoring cost c and the residual variance of loan cash flows $\frac{1}{h_\eta}$.*

Corollary 2 is intuitive given the preceding discussion. Under the optimal measurement

rule, measurement is less likely to occur (i.e., a higher measurement cutoff θ_c) when risk-sharing considerations overwhelm monitoring considerations—measurement is less likely when the bank is more risk averse, loan cash flows are more volatile, and/or it is more costly to induce monitoring.

Proposition 4 also sheds light on how the contingent measurement rule affects the bank’s loan retention schedule. It suggests that, interestingly, the measurement rule results in a “kink” point in the loan retention schedule. When the loan quality θ is lower than the measurement cutoff θ_c , the loan retention schedule under the contingent measurement rule overlaps with that under no-measurement. However, the loan retention schedule becomes less steep and falls below the schedule under no-measurement, as the retention amount passes the cutoff that triggers measurement (i.e., $\theta > \theta_c$). We summarize this result in the following corollary.

Corollary 3 *The comparison between the loan retention fraction under the optimal contingent measurement rule $\alpha^*(\theta)$ and that under no-measurement $\alpha_U(\theta; 0)$ is as follows:*

1. *if $\frac{\tau}{2h_\eta} < \frac{1}{c}$, $\alpha^*(\theta) < \alpha_U(\theta; 0)$ for all θ ;*
2. *if $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$, $\alpha^*(\theta) = \alpha_U(\theta; 0)$ if $\theta \leq \theta_c$, whereas $\alpha^*(\theta) < \alpha_U(\theta; 0)$ if $\theta > \theta_c$.*

Figure 3 provides a graphic illustration of Corollary 3 in the case of $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$. Intuitively, when the loan quality is unobservable, the bank over-retains loans and retains even more when the loans are of higher quality. As the loan quality improves above some level, the bank retains a sufficient amount of loans that triggers measurement. The measurement, in turn, curbs the bank’s over-retention motive, shifts the retention amount downward, and thus results in a kink in the equilibrium loan retention schedule.

Figure 3: Equilibrium loan retention schedule $\alpha^*(\theta)$ under optimal measurement rule as a function of loan quality θ . The following parameter values are used in this plot: $c = 2$, $\tau = 2$ and $h_\eta = 1$.

Finally, Proposition 4 generates insights into the real effects of asset transfer measurement rule and describes how the measurement rule affects the bank's monitoring incentives, which determines the performance of the bank's loan portfolio. We find that the optimal contingent measurement rule results in higher monitoring effort and thus improves the loan performance, relative to no measurement. We summarize this result in the following corollary.

Corollary 4 *The comparison between the monitoring effort under the optimal contingent measurement rule $m(\theta) \equiv m^*(\alpha^*(\theta))$ and that under no-measurement $m_U(\theta) \equiv m^*(\alpha_U(\theta; 0))$ is as follows:*

1. if $\frac{\tau}{2h_\eta} < \frac{1}{c}$, $m(\theta) > m_U(\theta)$ for all θ ;
2. if $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$, $m(\theta) = m_U(\theta)$ if $\theta \leq \theta_c$, whereas $m(\theta) > m_U(\theta)$ if $\theta > \theta_c$.

Figure 4 provides a graphic illustration of Corollary 4 in the case of $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$. It suggests

Figure 4: Equilibrium monitoring effort under optimal measurement rule, $m(\theta) \equiv m^*(\alpha^*(\theta))$, as a function of loan quality θ . The following parameter values are used in this plot: $c = 2$, $\tau = 2$ and $h_\eta = 1$.

that the optimal measurement rule plays a beneficial role in mitigating the problem of banks' reduced monitoring incentives after loan transfers. As previously explained, measurement substitutes for loan retention in incentivizing the bank to monitor, which helps to maintain an adequate level of monitoring effort even after the bank off-loads some of its loans.

4 Conclusion

We provide a simple model of a representative bank to study the trade-offs that banks face in engaging in asset transfers. Given those trade-offs, we study how *ex ante* measurement rules affect asset transfer policies. Our main result is that, in the presence of monitoring and informational frictions, a contingent measurement rule is optimal: banks should report the performance of the transferred loans if and only if the amount of loans retained is sufficiently high. We relate our results to the current accounting standards for asset transfers.

To focus on measurement rules, we have ignored some important features of loan transfers. In practice, asset transfers can take various forms such as securitization, factoring, transfers with recourse, etc. To focus on the role of measurement, we have considered the simplest form of asset transfer in which banks transfer their loans proportionally to outside investors. Future research may expand our simple framework of asset transfer and measurement to incorporate various institutional features of asset transfer and examine how these features interact with measurement rules.

We also do not model regulatory capital that plays an important role in affecting banks' incentives to engage in asset transfers. As we have shown in prior work, accounting measurements play a crucial role in the design of regulatory capital (Mahieux, Sapra, and Zhang, 2020). It would therefore be useful to investigate how such measurements interact with regulatory capital to affect loan transfer decisions. We leave this important and interesting issue for future research.

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Appendix: proofs

Proof. of Lemma 1: See the main text. ■

Proof. of Proposition 1: Substituting the equilibrium monitoring effort in (10) into (13) and (14) yields:

$$\frac{1 - \beta}{c} = \frac{\tau}{2h_\eta}, \quad (34)$$

$$\frac{(1 - \alpha)(1 - \beta)^2}{c} = \frac{\tau[\alpha - (1 - \alpha)\beta]}{h_\eta}. \quad (35)$$

If $\frac{\tau}{2h_\eta} < \frac{1}{c}$, (34) gives $\beta_0 = 1 - \frac{c\tau}{2h_\eta} > 0$. Solving (35) gives:

$$\alpha_0(\beta) = \frac{(1 - \beta)^2 h_\eta + \beta c \tau}{(1 - \beta)^2 h_\eta + (1 + \beta) c \tau}. \quad (36)$$

Substituting β_0 into (36) gives $\alpha_0 = \frac{4h_\eta - c\tau}{8h_\eta - c\tau}$. If $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$, the left-hand side of (34) is always smaller than the right-hand side, i.e.,

$$\frac{1 - \beta}{c} \leq \frac{1}{c} \leq \frac{\tau}{2h_\eta}.$$

Therefore, $\beta_0 = 0$. Substituting $\beta_0 = 0$ into (36) gives $\alpha_0 = \frac{h_\eta}{h_\eta + c\tau}$. ■

Proof. of Proposition 2: In equilibrium, since $\hat{\theta}(\alpha) = \theta$, replacing $\hat{\theta}$ with θ in (18) yields

the bank's equilibrium payoff for a given θ :

$$\begin{aligned}
U(\theta) &\equiv U(m^*(\alpha(\theta)), \alpha(\theta); \theta) \\
&= \theta + \frac{\alpha(\theta) + (1 - \alpha(\theta))\beta}{c} - \frac{[\alpha(\theta) + (1 - \alpha(\theta))\beta]^2}{2c} \\
&\quad - \frac{\tau}{2} \left(\frac{\alpha(\theta)^2 + (1 - \alpha(\theta))^2 \beta}{h_\eta} \right).
\end{aligned} \tag{37}$$

Consider a deviation in which the bank chooses a different retention fraction $\alpha(\theta')$ rather than $\alpha(\theta)$. Without loss of generality, let $\theta' > \theta$. The bank's payoff is then given by:

$$\begin{aligned}
U(\theta'; \theta) &\equiv U(m^*(\alpha(\theta')), \alpha(\theta'); \theta) \\
&= [\alpha(\theta') + (1 - \alpha(\theta'))\beta] \theta + (1 - \alpha(\theta'))(1 - \beta) \theta' \\
&\quad + (\alpha(\theta') + (1 - \alpha(\theta'))\beta) \frac{\alpha(\theta') + (1 - \alpha(\theta'))\beta}{c} - \frac{[\alpha(\theta') + (1 - \alpha(\theta'))\beta]^2}{2c}.
\end{aligned} \tag{38}$$

The incentive-compatible (IC) constraint requires that

$$U(\theta) \geq U(\theta'; \theta), \tag{39}$$

which can be simplified into

$$U(\theta') - U(\theta) \leq [\alpha(\theta') + (1 - \alpha(\theta'))\beta] (\theta' - \theta). \tag{40}$$

Analogously, the IC for the bank with θ' requires that

$$U(\theta') \geq U(\theta; \theta'), \tag{41}$$

which can be simplified into

$$U(\theta') - U(\theta) \geq [\alpha(\theta) + (1 - \alpha(\theta))\beta](\theta' - \theta). \quad (42)$$

Combining (40) and (42) yields:

$$[\alpha(\theta) + (1 - \alpha(\theta))\beta](\theta' - \theta) \leq U(\theta') - U(\theta) \leq [\alpha(\theta') + (1 - \alpha(\theta'))\beta](\theta' - \theta). \quad (43)$$

Taking the limit of $\theta' \rightarrow \theta$ gives:

$$U'(\theta) = \alpha(\theta) + (1 - \alpha(\theta))\beta. \quad (44)$$

As similarly shown in Lemma 2, (44) and a monotonicity condition that $\alpha(\theta)$ is strictly increasing in θ are sufficient and necessary for the IC constraints. We next ignore the monotonicity condition and derive the optimal retention schedule $\alpha(\theta)$. Later we verify the equilibrium $\alpha(\theta)$ indeed satisfies the monotonicity condition.

Differentiating $U(\theta)$ in (37) with respect to θ and substituting into (44) gives:

$$\frac{\partial \alpha(\theta)}{\partial \theta} = \frac{(\alpha(\theta) - 1)(1 - \beta)}{\frac{1 - \alpha(\theta)}{c}(1 - \beta)^2 - \frac{\tau(\alpha(\theta) - (1 - \alpha(\theta))\beta)}{h_\eta}}. \quad (45)$$

Solving the differential equation yields

$$\alpha_U(\theta; \beta) = 1 + \frac{c\tau}{(1 - \beta)^2 h_\eta + (1 + \beta)c\tau} w \left(e^{\frac{-(1 - \beta)(1 - \beta + c\theta)h_\eta + K}{c\tau}} \frac{(1 - \beta)^2 h_\eta + (1 + \beta)c\tau}{c\tau} \right), \quad (46)$$

where K is a constant and $w(\cdot)$ is the Lambert W function (i.e., the principal solution for y in $x = ye^y$).

The initial condition for (46) is given by type $\underline{\theta}$ bank's choice. Type $\underline{\theta}$ bank chooses α to maximize $U(\underline{\theta})$ in (37) and taking the first-order condition gives:

$$\alpha_U(\underline{\theta}; \beta) = \frac{(1 - \beta)^2 h_\eta + \beta c\tau}{(1 - \beta)^2 h_\eta + (1 + \beta)c\tau}. \quad (47)$$

Plugging the initial condition (47) into (46) gives:

$$\alpha_U(\theta; \beta) = 1 + \frac{c\tau}{(1 - \beta)^2 h_\eta + (1 + \beta)c\tau} w \left(-e^{-\left(1 + \frac{(1 - \beta)h_\eta(\theta - \underline{\theta})}{\tau}\right)} \right). \quad (48)$$

This proves part 1 of the proposition.

Note that, by observing the expression of $\alpha_U(\theta; \beta)$, it is strictly increasing in θ . This proves part 2 of the proposition. This also verifies the monotonicity requirement of the IC constraints. ■

Proof. of Corollary 1: Note that

$$\alpha_U(\theta; 0) - \alpha_0 \geq \alpha_U(\theta; 0) - \alpha_U(\underline{\theta}; 0) = \int_{\underline{\theta}}^{\theta} \frac{\partial \alpha_U(t; 0)}{\partial t} dt \geq 0. \quad (49)$$

The first inequality uses that, from (47), $\alpha_U(\underline{\theta}; 0) = \frac{h_\eta}{h_\eta + c\tau}$, which equals α_0 if $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$. If $\frac{\tau}{2h_\eta} < \frac{1}{c}$, $\alpha_0 = \frac{4h_\eta - c\tau}{8h_\eta - c\tau} < \frac{h_\eta}{h_\eta + c\tau} = \alpha_U(\underline{\theta}; 0)$. The last inequality is strict if $\theta > \underline{\theta}$, and it holds because $\frac{\partial \alpha_U(t; 0)}{\partial t} > 0$ for any $t > \underline{\theta}$ (Part 2 of Proposition 2). ■

Proof. of Proposition 3: Note that since α_0 is independent of β , $\frac{\partial(\alpha_U(\theta; \beta) - \alpha_0)}{\partial \beta} = \frac{\partial \alpha_U(\theta; \beta)}{\partial \beta}$.

Taking the derivative of (48) with respect to β at $\beta = 0$ gives:

$$\left. \frac{\partial \alpha_U(\theta; \beta)}{\partial \beta} \right|_{\beta=0} = \frac{cw \left(-e^{-\left(1 + \frac{h_\eta(\theta - \underline{\theta})}{\tau}\right)} \right)}{(h_\eta + c\tau)^2} \left(\tau(2h_\eta - c\tau) + \frac{h_\eta(h_\eta + c\tau)(\theta - \underline{\theta})}{1 + w \left(-e^{-\left(1 + \frac{h_\eta(\theta - \underline{\theta})}{\tau}\right)} \right)} \right). \quad (50)$$

Note that since $-e^{-\left(1 + \frac{h_\eta(\theta - \underline{\theta})}{\tau}\right)} \in [-\frac{1}{e}, 0)$, the value of the Lambert W function $w \left(-e^{-\left(1 + \frac{h_\eta(\theta - \underline{\theta})}{\tau}\right)} \right) \in [-1, 0)$, i.e.,

$$-1 \equiv w \left(-\frac{1}{e} \right) \leq w \left(-e^{-\left(1 + \frac{h_\eta(\theta - \underline{\theta})}{\tau}\right)} \right) < w(0) \equiv 0. \quad (51)$$

Therefore, $\left. \frac{\partial \alpha_U(\theta; \beta)}{\partial \beta} \right|_{\beta=0}$ has the same sign as the following expression:

$$\tau(c\tau - 2h_\eta) - \frac{h_\eta(h_\eta + c\tau)(\theta - \underline{\theta})}{1 + w \left(-e^{-\left(1 + \frac{h_\eta(\theta - \underline{\theta})}{\tau}\right)} \right)}. \quad (52)$$

If $\frac{\tau}{2h_\eta} < \frac{1}{c}$, (52) is always negative because both its first term and second term are negative. The latter is true because $\theta \geq \underline{\theta}$ and $w \left(-e^{-\left(1 + \frac{h_\eta(\theta - \underline{\theta})}{\tau}\right)} \right) \geq -1$. Therefore, $\left. \frac{\partial \alpha_U(\theta; \beta)}{\partial \beta} \right|_{\beta=0} < 0$. This proves part 1 of the proposition.

If $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$, the first term of (52) is positive while the second term is negative. As a result,

the sign of (52) can be ambiguous. Consider a limiting case of $\theta \rightarrow \underline{\theta}$:

$$\begin{aligned}
& \lim_{\theta \rightarrow \underline{\theta}} \tau(c\tau - 2h_\eta) - \frac{h_\eta(h_\eta + c\tau)(\theta - \underline{\theta})}{1 + w\left(-e^{-\left(1 + \frac{h_\eta(\theta - \underline{\theta})}{\tau}\right)}\right)} \tag{53} \\
&= \tau(c\tau - 2h_\eta) - \lim_{\theta \rightarrow \underline{\theta}} \frac{h_\eta(h_\eta + c\tau)(\theta - \underline{\theta})}{1 + w\left(-e^{-\left(1 + \frac{h_\eta(\theta - \underline{\theta})}{\tau}\right)}\right)} \\
&= \tau(c\tau - 2h_\eta) - \lim_{\theta \rightarrow \underline{\theta}} \frac{h_\eta(h_\eta + c\tau) \left[-e^{-\left(1 + \frac{h_\eta(\theta - \underline{\theta})}{\tau}\right)} + e^{w\left(-e^{-\left(1 + \frac{h_\eta(\theta - \underline{\theta})}{\tau}\right)}\right)} \right]}{e^{-\left(1 + \frac{h_\eta(\theta - \underline{\theta})}{\tau}\right)} \frac{h_\eta}{\tau}} \\
&= \tau(c\tau - 2h_\eta) \\
&\geq 0.
\end{aligned}$$

The second step uses the L' Hospital's Rule. The third step uses that

$$\lim_{\theta \rightarrow \underline{\theta}} -e^{-\left(1 + \frac{h_\eta(\theta - \underline{\theta})}{\tau}\right)} + e^{w\left(-e^{-\left(1 + \frac{h_\eta(\theta - \underline{\theta})}{\tau}\right)}\right)} = -\frac{1}{e} + \frac{1}{e} = 0, \tag{54}$$

where the second equality uses that $w\left(-\frac{1}{e}\right) \equiv -1$. By continuity, $\frac{\partial \alpha_U(\theta; \beta)}{\partial \beta} \Big|_{\beta=0} > 0$ if θ is sufficiently low. Recall that from Proposition 2, $\alpha_U(\theta; 0)$ is strictly increasing in θ . Hence the condition that θ is sufficiently low is equivalent to the condition that $\alpha_U(\theta; 0)$ is sufficiently low. This proves part 2 of the proposition. ■

Proof. of Lemma 2: We have proved the part of necessity in the main text of the paper.

To prove sufficiency, we show that any retention schedule that satisfies conditions (1) and

(2) must be incentive compatible. Consider $\theta' > \theta$. From condition (1),

$$\int_{\theta}^{\theta'} U'(t) dt = \int_{\theta}^{\theta'} [\alpha(t) + (1 - \alpha(t)) \beta(\alpha(t))] dt, \quad (55)$$

and from condition (2),

$$\int_{\theta}^{\theta'} [\alpha(t) + (1 - \alpha(t)) \beta(\alpha(t))] dt \leq \int_{\theta}^{\theta'} [\alpha(\theta') + (1 - \alpha(\theta')) \beta(\alpha(\theta'))] dt. \quad (56)$$

Therefore,

$$\int_{\theta}^{\theta'} U'(t) dt = U(\theta') - U(\theta) \leq [\alpha(\theta') + (1 - \alpha(\theta')) \beta(\alpha(\theta'))] (\theta' - \theta). \quad (57)$$

This proves that conditions (1) and (2) yield incentive compatibility. ■

Proof. of Proposition 4: We first prove that the solutions $\{\alpha^*(\theta), \beta^*(\theta)\}$ to program (31) also solve program (30). Note first that $\alpha^*(\theta)$ and $\beta^*(\theta^*(\alpha))$ also satisfy the IC constraint in program (30). This is because,

$$U'(\theta) = \alpha^*(\theta) + (1 - \alpha^*(\theta)) \beta^*(\theta) = \alpha^*(\theta) + (1 - \alpha^*(\theta)) \beta^*(\theta^*(\alpha^*(\theta))). \quad (58)$$

The first equality uses the IC constraint in program (31) and the second equality uses $\theta^*(\alpha^*(\theta)) = \theta$. Next, we prove that given $\alpha^*(\theta)$, the regulator's choice of $\beta^*(\theta^*(\alpha))$ also maximizes the objective in program (30). Assume by contradiction, that there exists some $\{\alpha'(\theta), \beta'(\alpha)\}$ that produces higher total surplus than $\{\alpha^*(\theta), \beta^*(\theta^*(\alpha))\}$ and satisfies the IC constraint in program (30). Define $\beta'(\theta) \equiv \beta'(\alpha(\theta))$. As proved previously,

$\{\alpha'(\theta), \beta'(\theta)\}$ also satisfy the IC constraint in program (31). Note that $\{\alpha'(\theta), \beta'(\theta)\}$ would achieve the same total surplus as $\{\alpha'(\theta), \beta'(\alpha)\}$ because at every θ , $\beta'(\theta) = \beta'(\alpha(\theta))$. In addition, $\{\alpha^*(\theta), \beta^*(\theta)\}$ would achieve the same total surplus as $\{\alpha^*(\theta), \beta^*(\theta^*(\alpha))\}$ because at every θ , $\beta^*(\theta^*(\alpha)) = \beta^*(\theta)$, where the equality uses $\theta^*(\alpha^*(\theta)) = \theta$. Note that this implies a contradiction because at $\{\alpha'(\theta), \beta'(\theta)\}$, the total surplus is lower than that at $\{\alpha^*(\theta), \beta^*(\theta)\}$, whereas at $\{\alpha'(\theta), \beta'(\alpha)\}$, the total surplus is higher than that at $\{\alpha^*(\theta), \beta^*(\theta^*(\alpha))\}$.

Second, we solve for $\alpha^*(\theta)$ and $\beta^*(\theta)$. To economize on notation, we often omit the superscript “*” in the remaining proof of Proposition 4 whenever no confusion arises. To solve program (31), let $L(\theta)$ be the Lagrange multiplier associated with constraint (32). Differentiating the Hamiltonian with respect to $\alpha(\theta)$ and $\beta(\theta)$, respectively, yields:

$$\left[\frac{\tau [\alpha(\theta) - (1 - \alpha(\theta))\beta(\theta)]}{h_\eta} - \frac{1 - \alpha(\theta)}{c} (1 - \beta(\theta))^2 \right] f(\theta) = L(\theta) (1 - \beta(\theta)), \quad (59)$$

$$(1 - \alpha(\theta)) \left[\frac{1 - \beta(\theta)}{c} - \frac{\tau}{2h_\eta} \right] f(\theta) = L(\theta) (1 - \alpha(\theta)). \quad (60)$$

Dividing (59) by (60) gives:

$$\beta(\theta) = \frac{3\alpha(\theta) - 1}{1 - \alpha(\theta)}, \quad (61)$$

if $\alpha(\theta) \in (\frac{1}{3}, \frac{1}{2})$. If $\alpha(\theta) \leq \frac{1}{3}$, $\beta(\theta) = 0$ and if $\alpha(\theta) \geq \frac{1}{2}$, $\beta(\theta) = 1$. As we will verify later, the bank in equilibrium always sets $\alpha(\theta) < \frac{1}{2}$ so the last case never prevails in equilibrium.

This proves part 1 of the proposition.

Next we derive the equilibrium loan retention schedule $\alpha(\theta)$. Consider the first case that $\frac{\tau}{2h_\eta} < \frac{1}{c}$. At $\theta = \underline{\theta}$, the bank does not distort its choice, i.e., $\beta(\underline{\theta}) = \beta_0 = 1 - \frac{c\tau}{2h_\eta}$ and

$\alpha(\underline{\theta}) = \frac{4h_\eta - c\tau}{8h_\eta - c\tau} \in (\frac{1}{3}, \frac{1}{2})$. Suppose that for $\theta > \underline{\theta}$, $\alpha(\theta) \in (\frac{1}{3}, \frac{1}{2})$ and we will verify this conjecture after solving the equilibrium. Substituting the expression (23) of $U(\theta)$ into the IC constraint (32) gives that

$$\begin{aligned} & 1 + \frac{\alpha'(\theta)(1 - \beta(\theta)) + (1 - \alpha(\theta))\beta'(\theta)}{c} \\ &= \frac{\tau}{2h_\eta} \frac{2[\alpha(\theta) - (1 - \alpha(\theta))\beta(\theta)]\alpha'(\theta) + (1 - \alpha(\theta))^2\beta'(\theta)}{(1 - \alpha(\theta))(1 - \beta(\theta))}. \end{aligned} \quad (62)$$

Given the conjecture that $\alpha(\theta) \in (\frac{1}{3}, \frac{1}{2})$, $\beta(\theta) = \frac{3\alpha(\theta)-1}{1-\alpha(\theta)}$ and plugging this equality into (62) gives

$$\alpha'(\theta) = H(\alpha(\theta)), \quad (63)$$

where

$$H(x) = \frac{c(1 - 2x)}{\frac{c\tau}{h_\eta}(1 - x) - 4(1 - 2x)}. \quad (64)$$

Note that the numerator of $H(x)$ is positive if and only if $x < \frac{1}{2}$. The sign of the denominator is as follows. If $\frac{\tau}{h_\eta} < \frac{8}{c}$, the denominator of $H(x)$ is positive if and only if $x > \frac{4h_\eta - c\tau}{8h_\eta - c\tau}$. If $\frac{\tau}{h_\eta} \geq \frac{8}{c}$, the denominator of $H(x)$ is positive for any $x \in [0, 1]$.

We now prove that $\alpha(\theta) \in (\frac{1}{3}, \frac{1}{2})$. First, consider the case that $\frac{\tau}{2h_\eta} < \frac{1}{c}$. At the initial point of $\alpha(\underline{\theta}) = \frac{4h_\eta - c\tau}{8h_\eta - c\tau} \in (\frac{1}{3}, \frac{1}{2})$, $\alpha'(\underline{\theta}) = H(\alpha(\underline{\theta})) = +\infty > 0$. The schedule $\alpha(\theta)$ thus increases. As long as $\alpha(\theta)$ is below $\frac{1}{2}$ and above $\frac{4h_\eta - c\tau}{8h_\eta - c\tau}$, $H(\alpha(\theta)) > 0$ and $\alpha(\theta)$ keeps rising. However, $\alpha(\theta)$ can never go above $\frac{1}{2}$, because if $\alpha(\theta) = \frac{1}{2}$, $\alpha'(\theta) = H(\frac{1}{2}) = 0$ and $\alpha(\theta)$ will remain at $\frac{1}{2}$. Therefore, $\alpha(\theta) < \frac{1}{2}$. In addition, $\alpha(\theta) \geq \alpha(\underline{\theta}) = \frac{4h_\eta - c\tau}{8h_\eta - c\tau} > \frac{1}{3}$. This proves that $\alpha(\theta) \in (\frac{1}{3}, \frac{1}{2})$ and completes the proof for part 2 of the proposition.

Second, consider the case that $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$. At $\theta = \underline{\theta}$, the bank does not distort its choice,

i.e., $\beta(\underline{\theta}) = \beta_0 = 0$ and $\alpha(\underline{\theta}) = \frac{h_\eta}{h_\eta + c\tau} \in (0, \frac{1}{3})$. By continuity, there exists a cutoff θ_c such that for $\theta < \theta_c$, $\alpha(\theta) < \frac{1}{3}$ and $\beta(\theta) = 0$. Under $\theta < \theta_c$, plugging $\beta(\theta) = 0$ into (62) gives

$$\alpha'(\theta) = G(\alpha(\theta)), \quad (65)$$

where

$$G(x) = \frac{c(1-x)}{x\left(1 + \frac{c\tau}{h_\eta}\right) - 1}. \quad (66)$$

Note that upon $\beta = 0$, $\alpha(\theta) \equiv \alpha_U(\theta; 0)$ given in Proposition 2. In addition, $G(x) > 0$ if and only if $x > \alpha(\underline{\theta}) = \frac{h_\eta}{h_\eta + c\tau}$. Now consider the dynamics regarding $\alpha(\theta)$ for $\theta < \theta_c$. At the initial point of $\alpha(\underline{\theta}) = \frac{h_\eta}{h_\eta + c\tau}$, $\alpha'(\underline{\theta}) = G(\alpha(\underline{\theta})) > 0$. The schedule $\alpha(\theta)$ thus increases. As long as $\alpha(\theta) < 1$, $\alpha'(\theta) = G(\alpha(\theta)) > 0$ and the schedule $\alpha(\theta)$ keeps increasing towards 1. By the intermediate value theorem, there exists a unique cutoff θ_c such that $\alpha(\theta_c) = \frac{1}{3}$. This proves that for $\theta \leq \theta_c$, $\alpha(\theta) \leq \frac{1}{3}$.

Next, consider the case that $\theta > \theta_c$. Suppose that for $\theta > \theta_c$, $\alpha(\theta) \in (\frac{1}{3}, \frac{1}{2})$ and we will verify this conjecture after solving the equilibrium. Under this conjecture, $\beta(\theta) = \frac{3\alpha(\theta)-1}{1-\alpha(\theta)}$, and thus $\alpha'(\theta) = H(\alpha(\theta))$. Note that the numerator of $H(x)$ is positive if and only if $x < \frac{1}{2}$. However, under $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$, the sign of $H(x)$'s denominator may be different from that under $\frac{\tau}{2h_\eta} < \frac{1}{c}$. More specifically, if $\frac{\tau}{h_\eta} < \frac{8}{c}$, $H(x)$ is positive if and only if $x \in \left(\frac{4h_\eta - c\tau}{8h_\eta - c\tau}, \frac{1}{2}\right)$, whereas if $\frac{\tau}{h_\eta} \geq \frac{8}{c}$, $H(x)$ is positive if and only if $x < \frac{1}{2}$. We thus discuss the cases of $\frac{\tau}{h_\eta} < \frac{8}{c}$ and $\frac{\tau}{h_\eta} \geq \frac{8}{c}$ separately.

Consider first the dynamics regarding $\alpha(\theta)$ if $\frac{\tau}{h_\eta} < \frac{8}{c}$. At the initial point of $\alpha(\theta_c) = \frac{1}{3}$, since $\frac{1}{3} \geq \frac{4h_\eta - c\tau}{8h_\eta - c\tau}$ under $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$, $\alpha'(\theta_c) = H(\alpha(\theta_c)) = H\left(\frac{1}{3}\right) > 0$. The schedule $\alpha(\theta)$ thus

increases. As long as $\alpha(\theta)$ is below $\frac{1}{2}$, $H(\alpha(\theta)) > 0$ and $\alpha(\theta)$ keeps rising. However, $\alpha(\theta)$ can never go above $\frac{1}{2}$, because if $\alpha(\theta) = \frac{1}{2}$, $\alpha'(\theta) = H(\frac{1}{2}) = 0$ and $\alpha(\theta)$ will remain at $\frac{1}{2}$. Therefore, $\alpha(\theta) < \frac{1}{2}$. In addition, $\alpha(\theta) \geq \alpha(\theta_c) = \frac{1}{3}$. This proves that $\alpha(\theta) \in (\frac{1}{3}, \frac{1}{2})$ for $\theta > \theta_c$ if $\frac{\tau}{h_\eta} < \frac{8}{c}$ and $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$.

If $\frac{\tau}{h_\eta} \geq \frac{8}{c}$ and at the initial point of $\alpha(\theta_c) = \frac{1}{3}$, since $\alpha(\theta_c) = \frac{1}{3} < \frac{1}{2}$, $\alpha'(\theta_c) = H(\alpha(\theta_c)) = H(\frac{1}{3}) > 0$. The schedule $\alpha(\theta)$ thus increases. As long as $\alpha(\theta)$ is below $\frac{1}{2}$, $H(\alpha(\theta)) > 0$ and $\alpha(\theta)$ keeps rising. However, $\alpha(\theta)$ can never go above $\frac{1}{2}$, because if $\alpha(\theta) = \frac{1}{2}$, $\alpha'(\theta) = H(\frac{1}{2}) = 0$ and $\alpha(\theta)$ will remain at $\frac{1}{2}$. Therefore, $\alpha(\theta) < \frac{1}{2}$. In addition, $\alpha(\theta) \geq \alpha(\theta_c) = \frac{1}{3}$. This proves that $\alpha(\theta) \in (\frac{1}{3}, \frac{1}{2})$ for $\theta > \theta_c$ if $\frac{\tau}{h_\eta} < \frac{8}{c}$ and $\frac{\tau}{h_\eta} \geq \frac{8}{c}$. Combining all the cases completes the proof for part 3 of the proposition.

Finally, to complete the proof, we verify the monotonicity condition, i.e., $\alpha(\theta) + (1 - \alpha(\theta))\beta_C(\alpha(\theta))$ is non-decreasing in θ . If $\frac{\tau}{2h_\eta} < \frac{1}{c}$, since $\alpha(\theta) \in (\frac{1}{3}, \frac{1}{2})$, then $\beta_C(\alpha(\theta)) = \frac{3\alpha(\theta)-1}{1-\alpha(\theta)}$. This gives

$$\alpha(\theta) + (1 - \alpha(\theta))\beta_C(\alpha(\theta)) = 4\alpha(\theta) - 1, \quad (67)$$

which is non-decreasing in θ because $\alpha(\theta)$ is non-decreasing in θ . If $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$ and $\theta \leq \theta_c$, $\alpha(\theta) = \alpha_U(\theta; 0) \in (0, \frac{1}{3}]$ and $\beta_C(\alpha(\theta)) \equiv 0$. This gives

$$\alpha(\theta) + (1 - \alpha(\theta))\beta_C(\alpha(\theta)) = \alpha(\theta), \quad (68)$$

which is non-decreasing in θ because $\alpha(\theta)$ is non-decreasing in θ . If $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$ and $\theta > \theta_c$,

$\alpha(\theta) \in (\frac{1}{3}, \frac{1}{2})$ and $\beta_C(\alpha(\theta)) = \frac{3\alpha(\theta)-1}{1-\alpha(\theta)}$. This gives

$$\alpha(\theta) + (1 - \alpha(\theta))\beta_C(\alpha(\theta)) = 4\alpha(\theta) - 1, \quad (69)$$

which is non-decreasing in θ because $\alpha(\theta)$ is non-decreasing in θ . ■

Proof. of Corollary 2: From the proof of Proposition 4, when $\frac{\tau}{2h_\eta} > \frac{1}{c}$, θ_c solves $\alpha_U(\theta_c; 0) = \frac{1}{3}$. Substituting the expression for α_U in (48) gives that:

$$\theta_c = -\frac{1}{h_\eta} \left(\tau - \underline{\theta}h_\eta + \tau \log \left[2e^{-\left(\frac{2}{3}\left(1+\frac{h_\eta}{c\tau}\right)\right)} \frac{h_\eta + c\tau}{3c\tau} \right] \right). \quad (70)$$

Differentiating θ_c with respect to τ gives:

$$\frac{\partial \theta_c}{\partial \tau} = \frac{2h_\eta - c\tau + 3(h_\eta + c\tau)(\log[3c\tau] - \log[2(h_\eta + c\tau)])}{3h_\eta(h_\eta + c\tau)}. \quad (71)$$

and

$$\frac{\partial^2 \theta_c}{\partial \tau^2} = \frac{h_\eta}{\tau(h_\eta + c\tau)^2} > 0. \quad (72)$$

Note that, at $\tau = \frac{2h_\eta}{c}$, we have $\frac{\partial \theta_c}{\partial \tau} = 0$. Hence, for any $\tau > \frac{2h_\eta}{c}$, $\frac{\partial \theta_c}{\partial \tau} > \frac{\partial \theta_c}{\partial \tau} \Big|_{\tau=\frac{2h_\eta}{c}} = 0$.

Differentiating θ_c with respect to c gives:

$$\frac{\partial \theta_c}{\partial c} = \frac{c\tau - 2h_\eta}{3c^2(h_\eta + c\tau)} > 0, \quad (73)$$

given that $c > \frac{2h_\eta}{\tau}$.

Differentiating θ_c with respect to h_η gives:

$$\frac{\partial \theta_c}{\partial h_\eta} = \frac{\tau(-2h_\eta + c\tau - 3(h_\eta + c\tau)(\log[3c\tau] - \log[2(h_\eta + c\tau)]))}{3h_\eta^2(h_\eta + c\tau)}. \quad (74)$$

$\frac{\partial \theta_c}{\partial h_\eta} < 0$ is equivalent to

$$\frac{c\tau}{h_\eta + c\tau} - 2\frac{h_\eta}{h_\eta + c\tau} < 3 \log \left[\frac{3}{2} \frac{c\tau}{h_\eta + c\tau} \right]. \quad (75)$$

Denote $x_0 \equiv \frac{c\tau}{h_\eta + c\tau}$. We can verify that the inequality $x_0 - 2(1 - x_0) < 3 \log \left[\frac{3}{2} x_0 \right]$ is always satisfied for any $x_0 \in \left(\frac{2}{3}, 1 \right)$. Thus, for any $h_\eta < \frac{c\tau}{2}$, $\frac{\partial \theta_c}{\partial h_\eta} < 0$. ■

Proof. of Corollary 3: To economize on notation, we often omit the superscript “*” in the proof of Corollary 3 whenever no confusion arises. Consider first the case in which $\frac{\tau}{2h_\eta} < \frac{1}{c}$. Note that under no measurement, the type $\underline{\theta}$ bank chooses a non-distorted retention fraction

$$\alpha_U(\underline{\theta}; 0) = \alpha_0 = \frac{h_\eta}{h_\eta + c\tau}. \quad (76)$$

Therefore, under no measurement, $\alpha_U(\theta) \geq \alpha_U(\underline{\theta}; 0) = \frac{h_\eta}{h_\eta + c\tau}$. Obviously, for the set of θ in which $\alpha(\theta) < \frac{h_\eta}{h_\eta + c\tau}$, $\alpha(\theta) < \alpha_U(\theta)$. This set is non-empty because at $\theta = \underline{\theta}$, $\alpha(\underline{\theta}) = \frac{4h_\eta - c\tau}{8h_\eta - c\tau} < \frac{h_\eta}{h_\eta + c\tau}$ under $\frac{\tau}{2h_\eta} < \frac{1}{c}$. Consider now the set of θ in which $\alpha(\theta) \geq \frac{h_\eta}{h_\eta + c\tau}$. Since $\alpha(\theta)$ is non-decreasing in θ , there exists a cutoff θ_Λ such that $\alpha(\theta) \geq \frac{h_\eta}{h_\eta + c\tau}$ if and only if $\theta \geq \theta_\Lambda$, where $\alpha(\theta_\Lambda) = \frac{h_\eta}{h_\eta + c\tau}$. Note that $\theta_\Lambda > \underline{\theta}$ because $\alpha(\theta_\Lambda) = \frac{h_\eta}{h_\eta + c\tau} > \alpha(\underline{\theta}) = \frac{4h_\eta - c\tau}{8h_\eta - c\tau}$; in addition, $\alpha_U(\theta_\Lambda; 0) > \alpha_U(\underline{\theta}; 0) = \frac{h_\eta}{h_\eta + c\tau} = \alpha(\theta_\Lambda)$. Furthermore, one can verify that for all $\alpha \in \left[\frac{h_\eta}{h_\eta + c\tau}, 1 \right]$, $H(\alpha) < G(\alpha)$. Recall also that $\alpha'(\theta) = H(\alpha(\theta))$ and $\alpha'_U(\theta; 0) = G(\alpha_U(\theta; 0))$.

Thus, for any $\theta \geq \theta_\Lambda$, the slope of $\alpha_U(\theta; 0)$ is higher than the slope of $\alpha(\theta)$. This implies that for any $\theta \geq \theta_\Lambda$, $\alpha_U(\theta; 0) > \alpha(\theta)$ because at the initial point of $\theta = \theta_\Lambda$, $\alpha_U(\theta_\Lambda; 0) > \alpha(\theta_\Lambda)$ and $\alpha_U(\theta; 0)$ increases at a faster speed than $\alpha(\theta)$. This proves part 1 of the corollary.

Next, consider that $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$. From Proposition 4, the initial condition at $\theta = \theta_c$ is $\alpha(\theta_c) = \alpha_U(\theta_c; 0) = \frac{1}{3}$. Furthermore, one can verify that for all $\alpha \in [\frac{1}{3}, \frac{1}{2}]$, $H(\alpha) < G(\alpha)$. This implies that for all $\theta > \theta_c$, $\alpha(\theta) < \alpha_U(\theta; 0)$ because at the initial point of $\theta = \theta_c$, $\alpha_U(\theta_c; 0) = \alpha(\theta_c)$ and $\alpha_U(\theta; 0)$ increases at a faster speed than $\alpha(\theta)$. In addition, from Proposition 4, for all $\theta \leq \theta_c$, $\alpha(\theta) = \alpha_U(\theta; 0)$. This proves part 2 of the corollary. ■

Proof. of Corollary 4: Consider first the case in which $\frac{\tau}{2h_\eta} < \frac{1}{c}$. Note that under no measurement, the type $\underline{\theta}$ bank chooses a monitoring effort that equals:

$$m_U(\underline{\theta}) = \frac{\alpha_0}{c} = \frac{1}{c} \frac{h_\eta}{h_\eta + c\tau}, \quad (77)$$

whereas under optimal measurement, the type $\underline{\theta}$ bank chooses a monitoring effort that equals:

$$m(\underline{\theta}) = \frac{\alpha_0 + (1 - \alpha_0)\beta_0}{c} = \frac{1}{c} \frac{8h_\eta - 3c\tau}{8h_\eta - c\tau}. \quad (78)$$

Note that under $\frac{\tau}{2h_\eta} < \frac{1}{c}$, $m(\underline{\theta}) > m_U(\underline{\theta})$. For $\theta > \underline{\theta}$, $m_U(\theta) = \frac{\alpha_U(\theta; 0)}{c}$, where $\alpha_U(\theta; 0)$ solves $\alpha'_U(\theta; 0) = G(\alpha_U(\theta; 0))$. Replacing $\alpha_U(\theta; 0) = cm_U(\theta)$ gives that

$$m'_U(\theta) = \frac{1}{c} G(cm_U(\theta)). \quad (79)$$

Similarly, for $\theta > \underline{\theta}$,

$$m(\theta) = \frac{\alpha(\theta) + (1 - \alpha(\theta))\beta(\theta)}{c} = \frac{4\alpha(\theta) - 1}{c}. \quad (80)$$

The last equality uses that $\beta(\theta) = \frac{3\alpha(\theta)-1}{1-\alpha(\theta)}$. Note that $m(\theta)$ is non-decreasing in θ since $\alpha(\theta)$ is non-decreasing. Therefore, $m(\theta) > m(\underline{\theta}) = \frac{1}{c} \frac{8h_\eta - 3c\tau}{8h_\eta - c\tau}$. Replacing $\alpha(\theta) = \frac{cm(\theta)+1}{4}$ gives that

$$m'(\theta) = \frac{4}{c} H \left(\frac{cm(\theta) + 1}{4} \right). \quad (81)$$

Furthermore, one can verify that for all $m \in \left[\frac{1}{c} \frac{8h_\eta - 3c\tau}{8h_\eta - c\tau}, \frac{1}{c} \right]$, $\frac{4}{c} H \left(\frac{cm(\theta)+1}{4} \right) > \frac{1}{c} G(cm_U(\theta))$. This implies that for all θ , $m(\theta) > m_U(\theta)$ because at the initial point of $\theta = \underline{\theta}$, $m(\underline{\theta}) > m_U(\underline{\theta})$ and $m(\theta)$ increases at a faster speed than $m_U(\theta)$. This proves part 1 of the corollary.

Next, consider that $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$. From Proposition 4, the initial condition at $\theta = \theta_c$ is $\alpha(\theta_c) = \alpha_U(\theta_c; 0) = \frac{1}{3}$ and $\beta(\theta_c) = 0$. Thus at $\theta = \theta_c$, $m_U(\theta_c) = m(\theta_c) = \frac{1}{3c}$. Furthermore, one can verify that that for all $m \in \left[\frac{1}{3c}, \frac{1}{c} \right]$, $\frac{4}{c} H \left(\frac{cm(\theta)+1}{4} \right) > \frac{1}{c} G(cm_U(\theta))$. This implies that for all $\theta \geq \theta_c$, $m(\theta) > m_U(\theta)$ because at the initial point of $\theta = \underline{\theta}$, $m(\underline{\theta}) > m_U(\underline{\theta})$ and $m(\theta)$ increases at a faster speed than $m_U(\theta)$. In addition, from Proposition 4, for all $\theta \leq \theta_c$, $\alpha(\theta) = \alpha_U(\theta; 0)$ and $\beta(\theta) = 0$. Therefore, $m_U(\theta_c) = m(\theta_c) = \frac{\alpha_U(\theta; 0)}{c}$. This proves part 2 of the corollary. ■