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SURVIVABLE OVERLAY LAYOUT OF IP OVER WDM

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To my Mother

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ABSTRACT

Wavelength Division Multiplexing (WDM) technology's capacity of providing very wide bandwidths in optical transport network makes it a good choice to meet the exponential growth of Internet traffic. We consider the problem of routing the Internet Protocol (IP) network over the WDM network in such a way that the IP network is still connected under physical failures. We call such a routing *survivable*. We formulate the survivable routing problem dealing with any single fiber cut as a *Mixed Integer Linear Program* (MILP), which is a modified version of the *Integer Linear Program* (ILP) in [1]. We route various IP networks over a number of WDM networks to show the dramatic run-time improvement of this MILP compared to the ILP in [1]. We also consider the survivable routing problem dealing with multiple link failures that are referred to as a *shared risk link group*. Finally, we study a scenario where we design a IP network based on a traffic matrix, and then overlay the IP network over the WDM network.

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Chapter 1

Introduction

Wavelength Division Multiplexing (WDM) is essentially frequency division multiplexing of optical signals onto fibers. With WDM technology, we are able to transmit data simultaneously at multiple carrier *wavelengths* (which are the inverse of carrier frequencies) over a fiber. WDM technology has dramatically increased the capacity of the optical transport network by using existing fiber links. For example, as shown in Figure 1.1, the WDM multiplexer takes in N data streams, each of B b/s, and then multiplexes them into a single fiber with a total aggregate rate of NB b/s. Today, the WDM channels are 2.5 Gbps (OC-48), 10 Gbps (OC-192), or higher. IP (Internet Protocol) over WDM, where IP packets are carried over WDM networks, is expected to meet the increasing bandwidth demands of Internet traffic.

Some important concepts are clarified in Figure 1.2. The WDM network is defined by its *physical topology*, which is a set of *optical cross-connects* (OXCs) and the fiber links connecting them. A *lightpath* is an optical connection that may span multiple fiber links. We assume that wavelength converters are available in the WDM network so that a lightpath can possibly use a different wavelength along each of the links in its path. An IP network is defined by its *logical topology*, which in turn is a set of nodes (i.e., IP routers) and the IP links connecting them. The IP links are realized by the lightpaths of the WDM network. In this thesis, we overlay the IP network over the WDM network by finding the routes of the lightpaths that realize the IP links. We want the lightpaths to be routed in such a way that the IP network is still connected when the WDM network has failures. We call

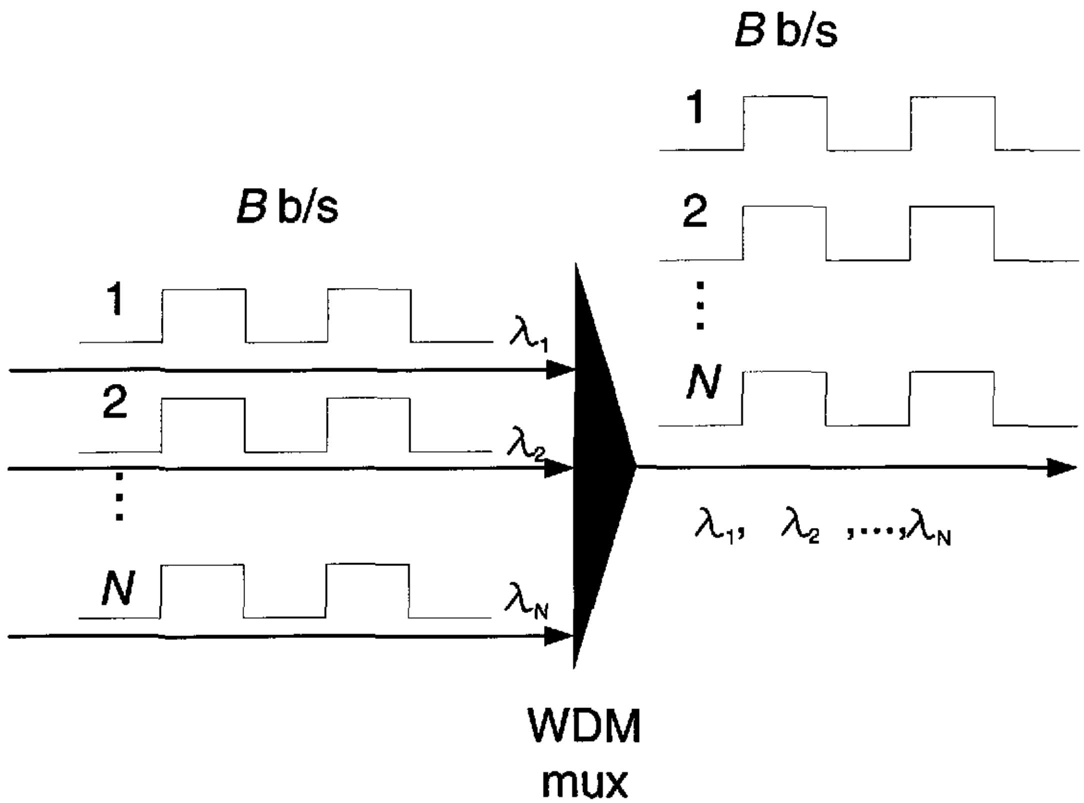

Figure 1.1: WDM technology

such a routing *survivable*. The routing problem we consider is a static version, i.e., we are not considering a dynamic reconfiguration of the network following failures.

The example in Figure 1.2 shows a survivable routing of IP over WDM dealing with a single fiber link cut. Here, the WDM network only provides unprotected lightpath service where only one connection is set up for each lightpath. The logical topology is *2-connected*, which means the topology remains connected under any single logical link failure. The routes of the lightpaths, which can be referred to as a *layout*, are shown over the physical topology. We can see that the lightpaths are routed such that, under any single fiber link cut, the logical topology remains connected.

Alternatively, if we route the lightpath for logical link (C2, B2) on fiber link (C1, D1) and (D1, B1) as shown in Figure 1.3, two logical links are routed on one fiber link. As a result, fiber cut on (C1, D1) will disconnect the logical topology. This example illustrates that the 2-connected property of the logical topology does not ensure survivability because with WDM technology two or more lightpaths (as many as the wavelengths on the fiber link) can share a fiber link. The failure of a single fiber link will result in the failure of multiple links in the logical topology.

Many researchers have done work on the survivability of logical topologies in [1], [2], and [3]. Most of the work is focused on the event of any single fiber link failure. In [2], the problem of laying out a survivable logical topology, which was referred to as the "design protection " problem, was formulated as an optimization problem. This is difficult to solve. Thus, in [2] a heuristic based on tabu search was proposed. In [3], other simple layout heuristics that have lower time complexity were discussed. In [1], the survivable routing problem was formulated as an *integer linear programming* (ILP) problem, which is a variation of the *linear programming* problem. In an ILP, all the variables are restricted to take integer values only. It was proven in [1] that the survivable routing problem is NP-complete. Therefore, it is not scalable with the size of the network. It should be noted that the objective of the problem is minimizing the total cost of the network. The cost could be the total number of wavelengths required, fiber links used, or the total number of connections set up, etc..

The major disadvantage of the ILP formulation in [1] is that the number of survivability constraints increases *exponentially* with the number of nodes of the logical topology.

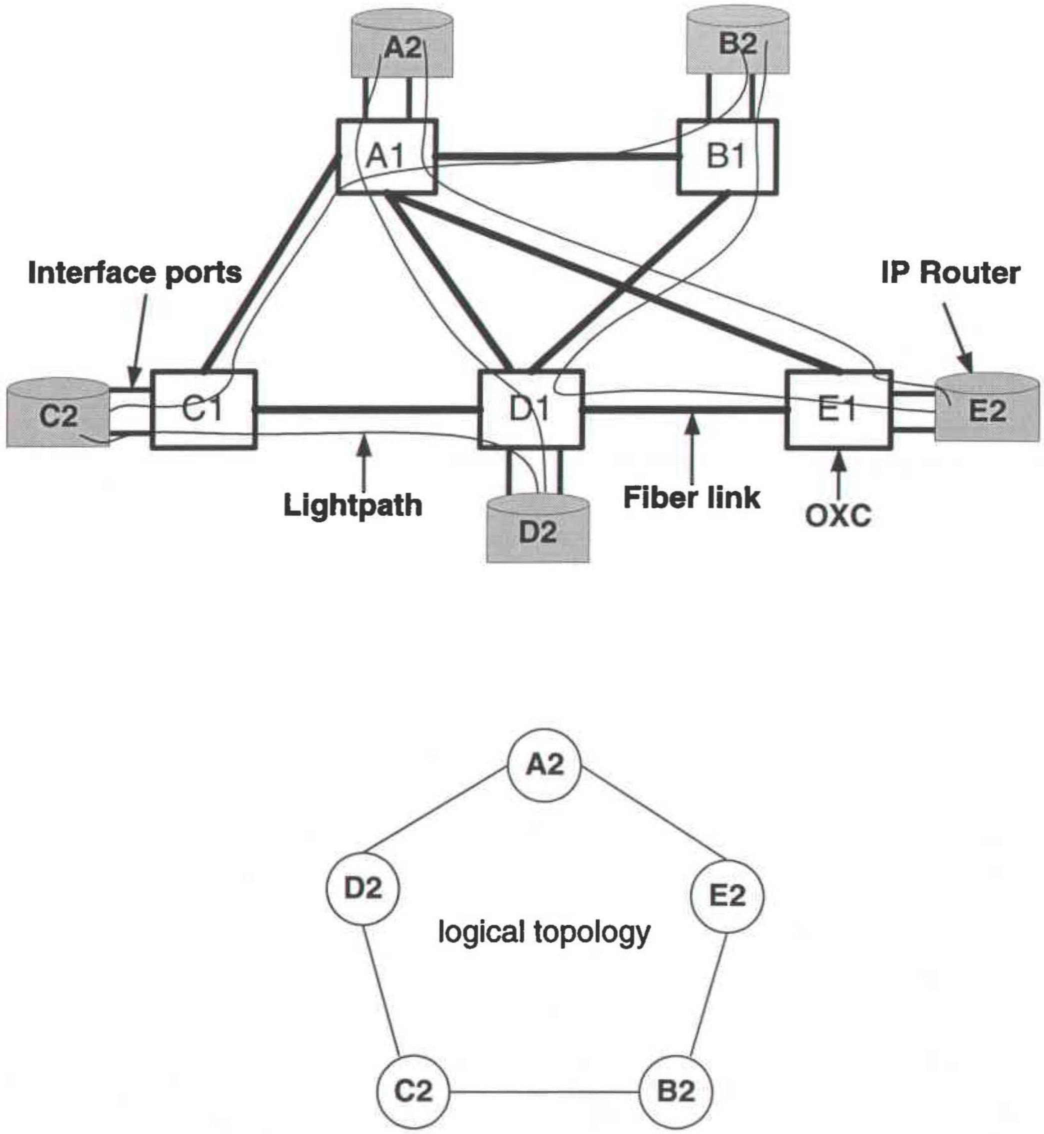

Figure 1.2: Survivable layout of an IP network over a WDM network

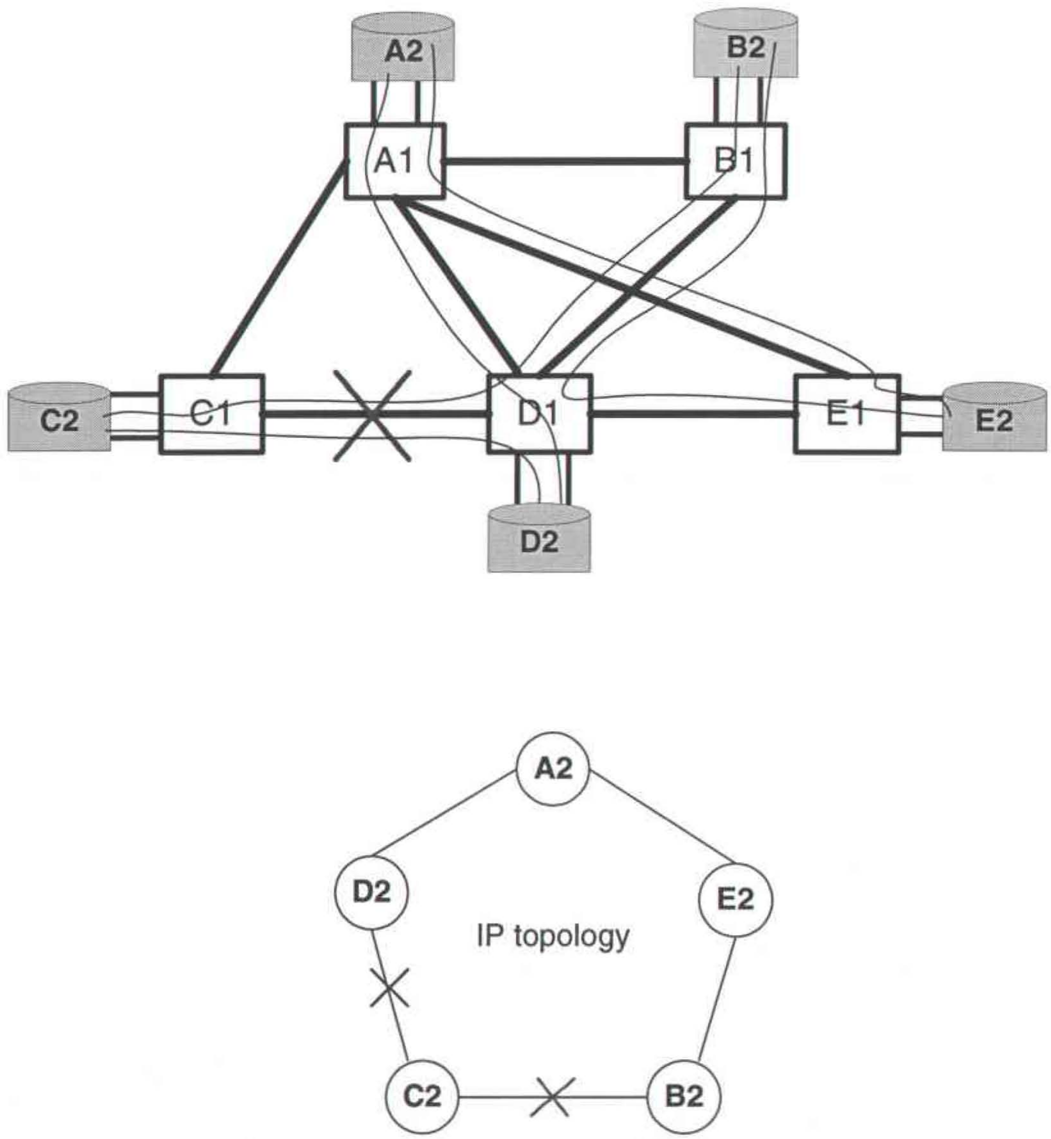

Figure 1.3: An unprotected layout of IP over WDM

Due to the large number of constraints, solving this ILP will consume much memory space and processing time of the computer, and would be very difficult to get results for large scale networks. To resolve this difficulty, two kinds of relaxations were explored in [1]. These relaxations have much less constraints than the original one, which improves the run-time of the ILP. However,their solutions are not guaranteed to be survivable. The authors of [1] also simplified the ILP formulation for ring logical topologies, which leads to a good time-complexity. In this thesis, we propose a modified version of the ILP (without any relaxation) in [1] that has a smaller number of constraints, which is not restrict to ring logical topologies. The modified problem is a *mixed integer linear programming* (MILP) where some variables are integer values but not necessarily all. This MILP differs from the ILP in [1] because its total number of constraints that guarantee the survivable routing grows as a *polynomial* with the number of nodes. Our simulations show that this improvement reduces the actual computation time considerably.

In this thesis, we formulate survivable routing problems under different scenarios as ILPs or MILPs. In Chapter 2, we first review the ILP formulation in [1], which only considered the unprotected lightpath. Then modify the ILP to our new MILP formulation that improves the run-times dramatically. Note that previous work of [1], [2], [3] only considered the cases when the unprotected lightpath service is available in the underlying WDM network. In the rest of Chapter 2, we study another scenario that appears to be new in the literature, where the WDM network provides unprotected and 1+1 protected lightpath services. As illustrated in Figure 1.4, in the 1+1 protected lightpath service, two connections between the source-destination pair called the *working path* and *protection path* are set up. The source sends copies of the signal on both paths. The destination normally accepts the signal from the working path, but if there is a failure on the working path, the destination will accept the signal from the protection path. The protection path has no fiber link in common with the working path, and is dedicated to back up the corresponding working path. We formulate an ILP and then modify it to a MILP. The MILP shows a significant improvement in run-time, which is consistent with the first scenario. In this scenario, we can always find a survivable layout of the logical topology.

As an extension of the work in Chapter 2, in Chapter 3, we study the survivable routing problem dealing with the simultaneous failure of a set of fiber links, which is re-

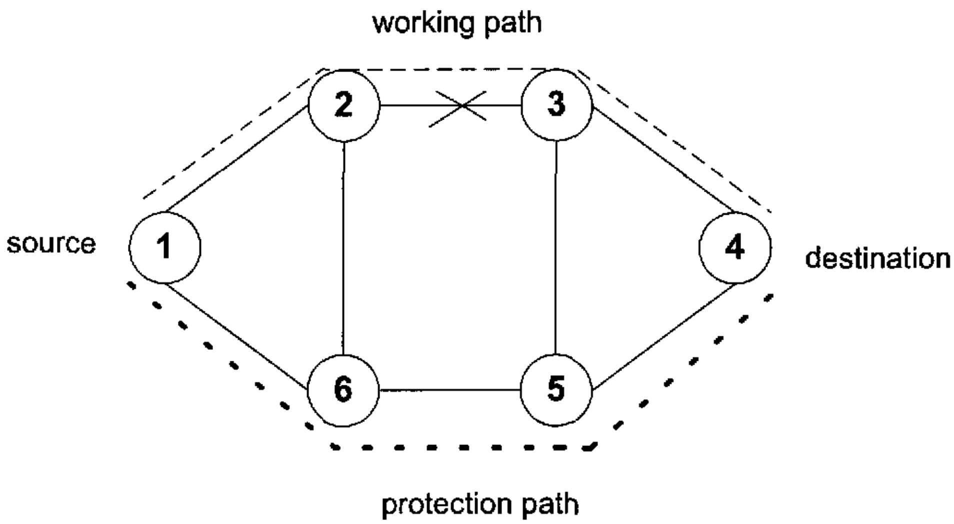

Figure 1.4: Path protection

ferred to as a *shared risk link group* (SRLG). An example of a SRLG is the set of fiber links through a conduit. A single cut on the conduit will lead to a simultaneous failure of all the fiber links through it.

In Chapter 4, we study a realistic scenario where we start with a traffic matrix, which is the traffic between the IP routers when there is no failure. We design the logical topology based on this traffic matrix, and then overlay it over the physical topology in such way that the logical topology can support a surviving traffic matrix under physical failures. We investigate this problem by two methods. One is a two-stage design while the other is a one-stage design. The two-stage design is proposed for the low time-complexity. We design the logical topology in the first stage, and then determine the layout of the logical topology in the second stage. The one-stage design is proposed for the optimal solution. In the one-stage design, we can design the logical topology and overlay it over the physical topology at the same time.

We conclude this thesis in Chapter 5.

Chapter 2

Survivable Layout for A Single Fiber Link Cut

In this chapter, we address the problem of finding a survivable layout of the logical topology over the physical topology that survives any single fiber link cut. In Section 2.1, we state the problem assumptions. In Section 2.2, we study a scenario where the WDM only provides the unprotected lightpath service. We call this scenario "Scenario 1". We review the ILP formulation in [1], and then modify the ILP to an MILP. The run-times of the ILP in [1] and the proposed MILP are compared by solving some samples. In Section 2.3, we study a scenario where the WDM network can provide unprotected and 1+1 protected lightpath services, and then formulate the survivable routing problem as an ILP and an MILP. We call this scenario "Scenario 2".

2.1 Problem assumptions

To describe the physical topology, we define a set $N_P = \{1, 2, \ldots N-1, N\}$ to represent the nodes (i.e., the OXCs), and another set E_P to represent the fiber links of the physical topology. If there is a link from node i to node j, (i, j) belongs to E_P. Note that (i, j) and (j, i) form a bi-directional link between nodes i and j. We assume a single cut on fiber link (i, j) will result in the failure of fiber link (i, j) and (j, i) at the same time.

The logical topology can be described in the same way as the physical topology. The logical topology consists of a set of nodes N_L. Each node in N_L corresponds to an

IP router, and each router accesses the WDM network through an OXC. To simplify the discussion, we assume that there is one IP router connected to an OXC. Thus, we have N_P=N_L. That is, a router is denoted by k if its OXC is denoted by k. Set E_L consists of all of the logical links in the IP topology. If s and t are in N_L and there is a link between them, then (s, t) represents a logical link in E_L. We also take the logical topology as bi-directional, where if (s, t) is in E_L, then so is (t, s). In this chapter, we do not allow multiple links between the same source-destination pair.

2.2 ILP and MILP formulations for unprotected lightpaths

In this section, given the physical and logical topologies, we would like to find the survivable layout of the logical topology over the physical topology. Survivability in this chapter means the logical topology is still connected after any single fiber link failure. It is assumed that the WDM network only provides unprotected lightpath service.

If only unprotected lightpaths are considered, we would like to emphasize that both the physical topology and the logical topology must be 2-connected to help ensure that a survivable layout is possible. If the logical topology or the physical topology is not 2-connected, we can never find a survivable routing of IP over WDM. We illustrate this by Figure 2.1. If the figure is a physical topology, the cut on fiber link (1, 6) will disconnect node 6 from the rest of the network. If the figure is a logical topology, and we implement link $(1, 6)$ with an unprotected lightpath, any single fiber cut that disconnects the lightpath for logical link $(1, 6)$ disconnects node 6 from the logical topology. As mentioned in Chapter 1, the redundancy of the logical topology is a necessary but cannot ensure the survivability of a logical topology. Since one fiber link can carry two or more lightpaths, a single fiber link failure may result in the failure of all of the lightpaths routed over it.

The survivable routing problem dealing with single physical failures was formulated as an ILP in [1]. We review their ILP first, then formulate the survivable routing problem as an MILP. Experimental results based on these two formulations are analyzed at the end of this section.

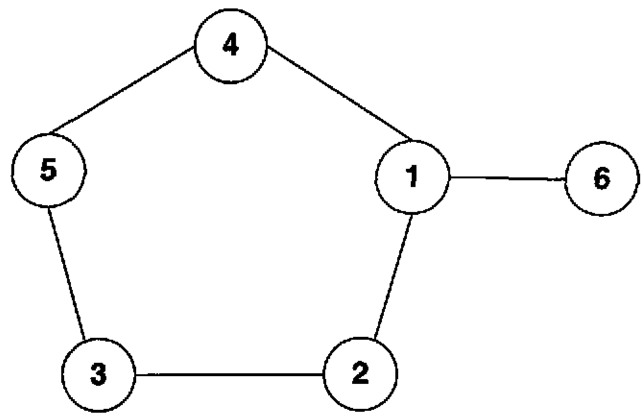

Figure 2.1: A non-redundant topology

2.2.1 Integer linear programming formulation

In this section, we review the ILP formulation for the survivable routing of a logical topology on a physical topology in [1]. In order to find the layout of the logical topology over the physical topology, we must set up a lightpath for every logical link (s,t) in E_L. This lightpath may traverse more than one fiber link connecting node s and t. To find a route for the lightpath from s to t, we route a unit of "flow" from node s to t. Let f_{ij}^{st} be a binary variable. If logical link (s,t) is routed on fiber link (i,j), then $f_{ij}^{st} = 1$, else $f_{ij}^{st} = 0$. It is assumed that the route from s to t and the route from t to s will follow the same physical links in the opposite direction because the lightpaths for the logical links are bi-directional. That is, if the lightpath for logical link (s,t) traverses fiber link (i,j) then the lightpath for (t,s) traverses (j,i). We only need to route logical link (s,t) or (t,s). By imposing $s < t$ to the connectivity constraints, we ensure one lightpath is routed for one bi-directional logical link.

The flow-conservation property of the network says that the total net flow entering a node other than the source or destination must equal the flow leaving that node. The constraints for routing logical link (s,t) can be expressed as follows. For each $(s,t) \in E_L$

such that $s < t$ and $i \in N_P$:

$$\sum_{j:\,(i,j)\in E_P} f_{ij}^{st} - \sum_{j:\,(j,i)\in E_P} f_{ji}^{st} = \begin{cases} 1 & \text{if } s = i \\ -1 & \text{if } t = i \\ 0 & otherwise \end{cases}$$

There are many feasible solutions from the above constraints. Any feasible solution specifies a layout of all lightpaths over the physical topology. However, the layout may or may not be survivable. The layout is survivable if and only if it satisfies a survivability constraint, which is an immediate result from Theorem 1 in [1]. The theorem gives a necessary and sufficient condition for a survivable routing of the logical topology. We will restate it after preliminary definitions.

For a directed topology consisting of a set of nodes $N = \{1, 2, \ldots, N-1, N\}$ and a set of links E, a *cut-set* is defined as follows. For any nonempty proper subset S of N:

$CS(S, N \setminus S) = \{(s,t) \in E : s \in S \text{ and } t \in N \setminus S, \text{ or } t \in S \text{ and } s \in N \setminus S\}$

A cut-set consists of the links with one endnode in S and the other in $N \setminus S$. For a bi-directional topology, we only include links (s,t) which satisfy $s < t$. The size of a cut-set is the total number of links in it, which is denoted by $|CS(S, N \setminus S)|$. Notice that the total number of nonempty subsets of N is $2^N - 2$, which grows exponentially with N. The total number of cut-sets is $2^{N-1} - 1$, i.e., half of $2^N - 2$. We illustrate this by the example in Figure 2.2. Here, $S = \{1, 4\}$ and $N \setminus S = \{2, 3, 5\}$. Then $CS(S, N \setminus S) = \{(1,2)(4,5)\}$ and $|CS(S, N \setminus S)| = 2$. When we consider $S = \{2, 3, 5\}$, we will get the same links in the corresponding cut-set.

Theorem 1 [1]: A routing is survivable *if and only if* for every cut-set $CS(S, N_L \setminus S)$ of the logical topology the following holds. Let $E(s,t)$ be the set of physical links used by logical link (s,t), i.e., Let $E(s,t) = \{(i,j) \in E_P \text{ for } f_{ij}^{st} = 1\}$. Then, for every cut-set $CS(S, N_L \setminus S)$,

$$\bigcap_{(s,t)\in CS(S,N_L\setminus S)} E(s,t) = \emptyset.$$

The condition in this theorem requires that all the logical links belonging to a cut-set cannot share the same fiber link.

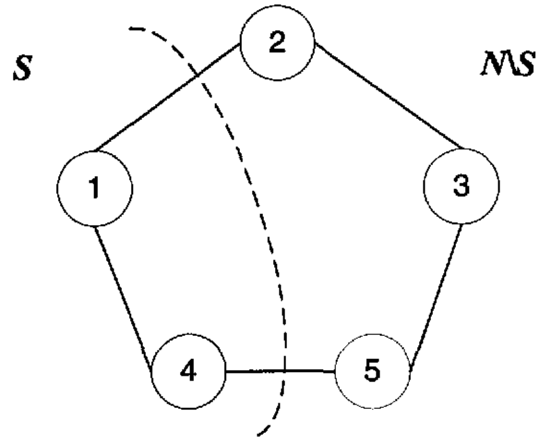

Figure 2.2: Definition of the cut-set

Using the above theorem, the survivability constraint can be expressed as follows. For each $(i, j) \in E_P$ such that $i < j$ and nonempty $S \subset N_L$:

$$\sum_{(s,t) \in CS(S, N_L \setminus S)} (f_{ij}^{st} + f_{ji}^{st}) < | CS(S, N_L \setminus S) |$$

This survivability constraint states that for any cut-set of the logical topology, the total number of links in a cut-set must be larger than the number of cut-set links routed over any single bi-directional fiber link. In other words, not all the lightpaths for the logical links in a cut-set can be carried on one fiber link.

The capacity constraint, which limits the total number of wavelengths on a bi-directional fiber link to W can be expressed as follows. For each $(i, j) \in E_P$ such that $i < j$:

$$\sum_{\substack{(s,t) \in E_L: \\ s<t}} (f_{ij}^{st} + f_{ji}^{st}) \leq W$$

For the rest of the thesis, we ignore this constraint to simplify the formulation. If we want this constraint, it could be added easily. By ignoring this constraint we assume that each fiber link has enough capacity to support all lightpaths on it.

Our objective is to minimize the total number of wavelengths occupied. We assume that one wavelength occupies one optical channel. Thus, the total wavelengths are also the total bandwidths occupied in the WDM network. The optimal survivable routing problem in [1] is restated below as an integer linear program.

$$\min \sum_{(i,j)\in E_P} \sum_{\substack{(s,t)\in E_L:\\ s<t}} f_{ij}^{st}$$

Subject to :

1. *Integer flow constraints: for each* $(i,j) \in E_P$ and each (s,t) in E_L such that $s < t$:

$$f_{ij}^{st} \in \{0,1\}$$

2. Connectivity constraints: for each $(s,t) \in E_L$ such that $s < t$ and $i \in N_P$:

$$\sum_{j:\,(i,j)\in E_P} f_{ij}^{st} - \sum_{j:\,(j,i)\in E_P} f_{ji}^{st} = \begin{cases} 1 & \text{if } s = i \\ -1 & \text{if } t = i \\ 0 & otherwise \end{cases}$$

3. Survivability constraints: for each $(i,j)\in E_P$ such that $i < j$ and each nonempty $S \subset N_L$:

$$\sum_{(s,t)\in CS(S,N_L\setminus S)} (f_{ij}^{st} + f_{ji}^{st}) <\mid CS(S, N_L \setminus S) \mid$$

If we overlay a logical topology over a physical topology successfully by using the above ILP, the solution obtained is the total number of wavelengths used. Meanwhile, we can get the routing of each lightpath for the logical link from the values of $\{f_{ij}^{st} : (s,t) \in E_L$ and $(i,j) \in E_P\}$.

In [1], it was stated that the large number of constraints stemming from the survivability constraints lead to difficulty solving large networks. The following example will illustrate that the survivability constraints dominate the total number of constraints in above ILP.Suppos e the physical topology has 12 nodes and 15 bi-directional links and the IP topology has 12 nodes and 12 bi-directional links (like a ring topology). We have

$$|E_L| = 2 \times 12 = 24 \qquad |E_P| = 2 \times 15 = 30 \qquad |N_P| = |N_L| = 12.$$

The number of integer variable constraints is

$$|E_P| \times \frac{|E_L|}{2} = 30 \times 12 = 360.$$

The number of constraints stemming from connectivity constraints is

$$\frac{|E_L|}{2} \times |N_P| = 12 \times 12 = 144.$$

The number of constraints stemming from survivability constraints is

$$\frac{|E_P|}{2} \times (2^{|N_L|-1} - 1) = 15 \times 2^{12-1} - 1 = 30705.$$

The sum of all constraints is

$$|E_P| \times \frac{|E_L|}{2} + \frac{|E_L|}{2} \times |N_P| + \frac{|E_P|}{2} \times (2^{|N_L|-1} - 1) = 360 + 144 + 30705 = 31209.$$

We can see that the number of survivability constraints dominates the total number of constraints, and it grows exponentially with $|N_L|$.

A simple relaxation was studied in [1] to reduce the total number of constraints. In this relaxation, the survivability constraints are discarded except for those where $|S| = 1$, i.e., S is a single node set. Thus, for a logical topology with $|N_L|$ nodes, there will be only $|N_L|$ survivability constraints. This relaxation formulation speeds up the run-time significantly and can be solved easily for large networks. However, since the relaxed ILP only considers the subsets of N_L including only a single node, the survivability constraints only ensure that not all of the logical links to a certain node share the same fiber link. Thus, the survivability constraints become necessary but not sufficient, and not all of the solutions ensure survivability. We illustrate this by the example in Figure 2.3. After solving the relaxed ILP, the layout of the logical topology over the physical topology is shown in the figure. The thick lines and nodes represent the underlying physical topology, and the thin lines are the lightpaths for the logical links. We can see that fiber link (3, 4) failure will disconnect logical link (3, 4), (3, 5) and (0, 4). As a result, the logical topology will be disconnected. In other words, this layout is not survivable.

2.2.2 Mixed integer linear program formulation

In this section, we will develop an MILP formulation which is a modified version of the ILP in [1]. As analyzed in Section 2.2.1, the disadvantage of the ILP in [1] is the

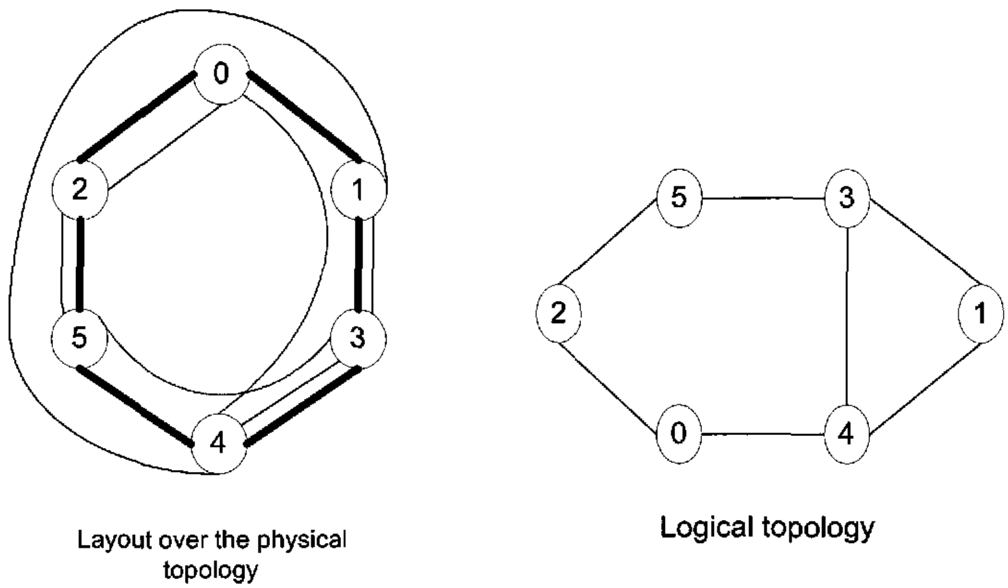

Figure 2.3: An unprotected solution by the relaxed ILP

exponential growth of the number of the survivability constraints. We want to modify the ILP by replacing the survivability constraints. The new survivability constraints ensure that, under any possible fiber link fault, the logical topology remains connected.

Before stating the new survivability condition, we give several definitions. We assume that, without any physical failure, the capacity of each logical link is 1. Let a binary variable c_{st}^{ij} be the capacity of logical link (s,t) when fiber link (i,j) fails. Then we have $c_{st}^{ij} = 1 - (f_{ij}^{st} + f_{ji}^{st})$. The value of $(f_{ij}^{st} + f_{ji}^{st})$ is determined by the actual routing of (s,t) over the physical topology. Notice that $(f_{ij}^{st} + f_{ji}^{st})$ must be less than or equal to 1 because our objective of the MILP formulation is to minimize the total number of wavelengths used, which implies a lightpath should not traverse a fiber link more than once. We have $(f_{ij}^{st} + f_{ji}^{st}) = 1$, if the lightpath for logical link (s,t) traverses fiber link (i,j). Then when fiber link (i,j) is cut, $c_{st}^{ij} = 1 - 1 = 0$,i.e ., the capacity of logical link (s,t) is 0. We have $(f_{ij}^{st} + f_{ji}^{st}) = 0$, if the lightpath for logical link (s,t) is not routed on fiber link (i,j). Then when fiber link (i,j) is cut, $c_{st}^{ij} = 1 - 0 = 1$, i.e., the capacity of logical link (s,t) is 1. To check if the logical topology is connected when a fiber link (i,j) fails, we check if the set of links $\{(s,t) \in E_L : c_{st}^{ij} = 1\}$ leads to a connected logical topology. This can be done by determining if the link capacities allow a positive flow between all pairs of nodes. An alternate and simpler condition is to determine if there is a positive flow from all nodes to a particular node, say node 1. The survivability constraint we use is to determine if each node (other than node 1) can source a flow of $\frac{1}{|N_L|-1}$ to node 1. The new necessary and sufficient condition for the survivable routing of the logical topology over the physical topology can be stated as the following condition:

Given the capacity of each surviving logical link is 1, a routing is survivable *if and only if* each node s ($s \neq 1$) can source a flow of $\frac{1}{|N_L|-1}$ to node 1 in the surviving logical topology. (Here, we assume that the flow of the logical topology only consists of the flow from other nodes to node 1.)

In order to prove this condition, we must show it is both necessary and sufficient. The condition is sufficient because if all nodes can reach node 1, then all nodes are connected. We need to prove it is necessary. That is, if the logical topology is connected under physical failures, we can always source a flow of $\frac{1}{|N_L|-1}$ from each node s ($s \neq 1$) to node 1. If the logical topology is connected then it has a spanning tree. If all the flows of $\frac{1}{|N_L|-1}$ follow the spanning tree, they will not exceed any link capacity. Necessity is proven.

To formulate the problem of survivable routing as an MILP, we define a real valued variable r_{st}^{ij}, which represents the flow on logical link (s,t), from node s to node t, when there is a cut on fiber link (i,j). We have $0 \leq r_{st}^{ij} \leq c_{st}^{ij}$. Since $c_{st}^{ij} = 1 - (f_{ij}^{st} + f_{ji}^{st})$ the capacity constraints can be simplified as follows. For each $(s,t) \in E_L$ such that $s < t$ and $(i,j) \in E_P$ such that $i < j$:

$$0 \leq r_{st}^{ij} \leq 1 - (f_{ij}^{st} + f_{ji}^{st})$$

$$0 \leq r_{ts}^{ij} \leq 1 - (f_{ij}^{st} + f_{ji}^{st}).$$

In the above constraints, since the cut on fiber link (i,j) will also result in a simultaneous failure of fiber link (j,i), we only consider fiber link (i,j) failure which satisfies $i < j$. The constraint $s < t$ is still needed because we only route one lightpath for a bi-directional logical link (s,t) on the physical topology.

Using the new necessary and sufficient condition of the survivable routing, the survivability constraint can be expressed as: for each $(i,j) \in E_P$ such that $i < j$ and

$s \in N_L$:

$$\sum_{t:\ (s,t)\in E_L} r_{st}^{ij} - \sum_{t:\ (t,s)\in E_L} r_{ts}^{ij} = \begin{cases} -1 & \text{if } s = 1 \\ \frac{1}{|N_L|-1} & otherwise \end{cases}$$

Any feasible solution from the above constraints ensures that each node s ($s \neq 1$) has a flow of $\frac{1}{|N_L|-1}$ to node 1 and the total flow entering node 1 is 1.

The optimal survivable routing problem is formulated as the following mixed integer linear program. Our objective is to minimize the total wavelength cost.

$$\min \sum_{(i,j)\in E_P} \sum_{\substack{(s,t)\in E_L:\\ s<t}} f_{ij}^{st}$$

Subject to :

1. Integer flow constraints: for each $(s,t) \in E_L$ such that $s < t$ and each $(i,j) \in E_P$:

$$f_{ij}^{st} \in \{0,1\}$$

2. Survivable flow constraints: for each (s,t) in E_L and each $(i,j) \in E_P$ such that $i < j$:

$$r_{st}^{ij} \geq 0$$

3. Connectivity constraints: for each (s,t) in E_L such that $s < t$ and $i \in N_P$:

$$\sum_{j:\ (i,j)\in E_P} f_{ij}^{st} - \sum_{j:\ (j,i)\in E_P} f_{ji}^{st} = \begin{cases} 1 & \text{if } s = i \\ -1 & \text{if } t = i \\ 0 & otherwise \end{cases}$$

4. Survivable traffic flow capacity constraints: for each $(s,t) \in E_L$ such that $s < t$ and $(i,j) \in E_P$ such that $i < j$:

$$r_{st}^{ij} \leq 1 - (f_{ij}^{st} + f_{ji}^{st})$$

$$r_{ts}^{ij} \leq 1 - (f_{ij}^{st} + f_{ji}^{st})$$

5. Survivability constraints: for each $(i, j) \in E_P$ such that $i < j$ and $s \in N_L$:

$$\sum_{t:\ (s,t) \in E_L} r_{st}^{ij} - \sum_{t:\ (t,s) \in E_L} r_{ts}^{ij} = \begin{cases} -1 & \text{if } s = 1 \\ \frac{1}{|N_L|-1} & otherwise \end{cases}$$

Our MILP formulation has much less number of constraints compared to the ILP formulation in [1]. We illustrate this by using the same physical topology consisting of 12 nodes and 15 bi-directional links and the IP topology consisting of 12 nodes and 12 bi-directional links in Section 2.2.1. The total number of the integer and nonnegative variable constraints is

$$|E_P| \times \frac{|E_L|}{2} + \frac{|E_P|}{2} \times |E_L| = 30 \times 12 = 720.$$

The number of constraints stemming from connectivity constraints is

$$\frac{|E_L|}{2} \times |N_P| = 12 \times 12 = 144.$$

The number of constraints stemming from capacity constraints is

$$2 \times \frac{|E_L|}{2} \times \frac{|E_P|}{2} \times 2 = 12 \times 15 \times 2 = 360.$$

The number of constraints stemming from the new survivability constraints is

$$\frac{|E_P|}{2} \times |N_L| = 15 \times 12 = 180.$$

the total number of the variables and constraints is

$$(|E_P| \times \frac{|E_L|}{2} + \frac{|E_P|}{2} \times |E_L|) + \frac{|E_L|}{2} \times |N_P| + \frac{|E_L|}{2} \times \frac{|E_P|}{2} \times 2 + \frac{|E_P|}{2} \times |N_L|$$

$$= 360 + 144 + 720 + 180 = 1404.$$

The total number of the constraints is 1404, which is dramatically smaller compared to the ILP formulation, which has 31209 constraints for the same example. Note that the total number of survivability constraints grows as a polynomial with $|N_L|$ for the MILP. The difference in the total number of the constraints between the ILP and the MILP will increase with the size of the network because the total number of survivability constraints in the ILP grows exponentially.

2.2.3 Implementation and experimental results

To compare the run-times of our MILP in Section 2.2.2 with the ILP and the relaxed ILP in [1], we implemented them by the AMPL+CPLEX software package on a Sun blade 100 computer. AMPL is a comprehensive, powerful algebraic modelling language for formulating linear, integer linear, mixed integer and non-linear problems. CPLEX serves as a *solver* using branch and bound techniques to solve ILPs and MILPs. The ILP, the relaxed ILP and the MILP are defined in AMPL as different *model* files, while the parameter values describing the physical topology and the logical topology are given in a separate *data* file. An AMPL script selects the appropriate model and data files, then calls the solver (in our case, CPLEX) and writes the resulting solution to an output file.

There are two kinds run-times that can be checked: the "elapsed run-time" and the "CPU run-time". The "elapsed run-time" is the "wall-clock time" that measures the time from the start of the program until its termination. The "CPU run-time" is the system time plus user time. The "elapsed run-time" may be affected by other processes on the machine, the time for I/O swap, etc., while the "CPU time" is much less affected by unpredictable things. When we were running the ILP or the MILP, we were the only user of the machine. We check both the "elapsed run-time" and "CPU run-time" to compare the run-times of different formulations.

We conducted simulations for networks of different size using the ILP, the MILP and the relaxed ILP in [1]. We generated 9 physical topologies randomly. All random topologies were generated as follows. Initially, we only have a collection of nodes without links. Then links are added between randomly chosen pairs of nodes successively until the topology is 2-connected. For each physical topology, we embedded a random logical topology of similar size on it. The random logical topologies were generated in the same way as the physical topologies. Note that sometimes it may be impossible to get the survivable routing when we try to overlay a logical topology over a physical topology. In our experiment, each logical topology was able to find its survivable routing over the corresponding physical topology. The "elapsed run-times" and "CPU run-times" are summarized in Table 2.1. The label such as "7-node,11-link" corresponds to the number of nodes and links of a physical topology, and the label "Relax-1" corresponds to the relaxed ILP in [1]. As can be seen from the table, the ILP formulation lead to relatively longer run-times than the other two formulations. For "CPU run-times", the ILP took 5 minutes and 32 seconds to solve the 15-node network, while the MILP only required 1.75 seconds and the relaxed ILP took 0.38 seconds.

Table 2.1: Run-times of embedding 9 random logical topologies on random physical topologies for scenario 1

	Elapsed run-times			CPU run-times		
	ILP	MILP	Relax-1	ILP	MILP	Relax-1
7-node,11-link	0.31 sec.	0.21 sec.	0.15 sec.	0.29 sec.	0.20 sec.	0.12 sec.
8-node,15-link	1.03sec.	0.25 sec.	0.17 sec.	1.03 sec.	0.22 sec.	0.14 sec.
9-node,14-link	1.78 sec.	0.31 sec.	0.20sec.	1.63 sec.	0.29 sec.	0.19 sec.
10-node,17-link	4.25 sec.	0.37 sec.	0.21sec.	4.14 sec.	0.37 sec.	0.21 sec.
11-node,19-link	9.00 sec.	0.47sec.	0.23 sec.	8.89 sec.	0.46 sec.	0.22 sec.
12-node,26-link	24.82 sec.	0.51 sec.	0.24 sec.	24.77 sec.	0.50 sec.	0.24 sec.
13-node,25-link	2 min. 5sec.	0.64 sec.	0.28 sec.	64.00 sec.	0.62 sec.	0.26 sec.
14-node,21-link	13 min. 11sec.	0.87 sec.	0.37 sec.	2 min. 29 sec.	0.80 sec.	0.36 sec.
15-node,27-link	80 min. 59sec	1.81 sec.	0.41 sec.	5 min. 32 sec.	1.75 sec.	0.38 sec.

In Figure 2.4 and Figure 2.5, we plot the "elapsed run-times" and the "CPU run-times" of these three formulations based on the results in Table 2.1. In these figures, for the convenience of observation, we use the dashed lines to mark 60 seconds (1 minute), 600 seconds (10 minutes), 3600 seconds (1 hour), and so on. As can be seen from both figures and Table 2.1, with the increase of number of nodes, run-times of the ILP increase dramatically, which is due to the exponential growth of the number of the survivability constraints. For the "CPU run-time", the ILP only took 0.29 seconds to solve the 7-node network while it took more than 5 minutes to solve the 15-node network. This confirms the results in [1] that the ILP is not scalable to large networks. However, the run-times of the MILP and the relaxed ILP increase much more slowly. For the "CPU run-time", the MILP took 0.20 seconds to solve the 7-node network while it took about 2 seconds to solve the 15-node network. The significant run-time improvement of the MILP is due to its polynomial number of survivability constraints. The dramatic run-time improvement of the relaxed ILP is because its survivability constraints only consider the subsets of N_L with only a single node.

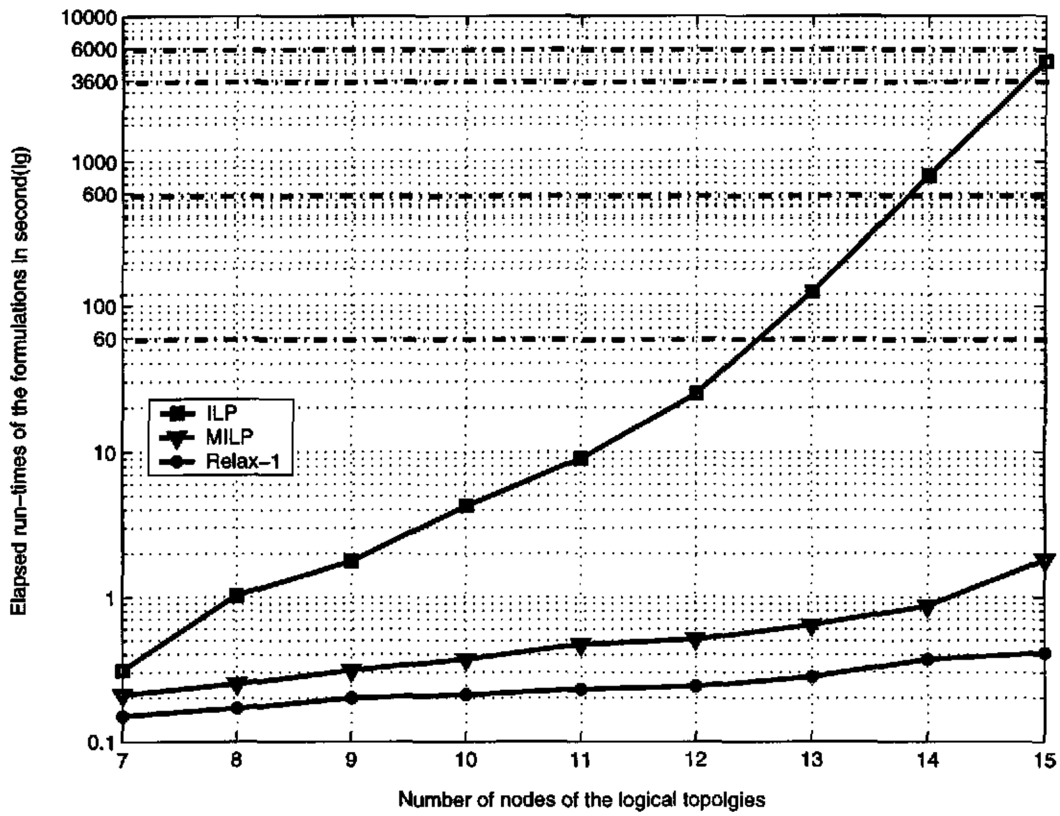

Figure 2.4: Elapsed run-times of embedding 9 logical topologies for scenario 1

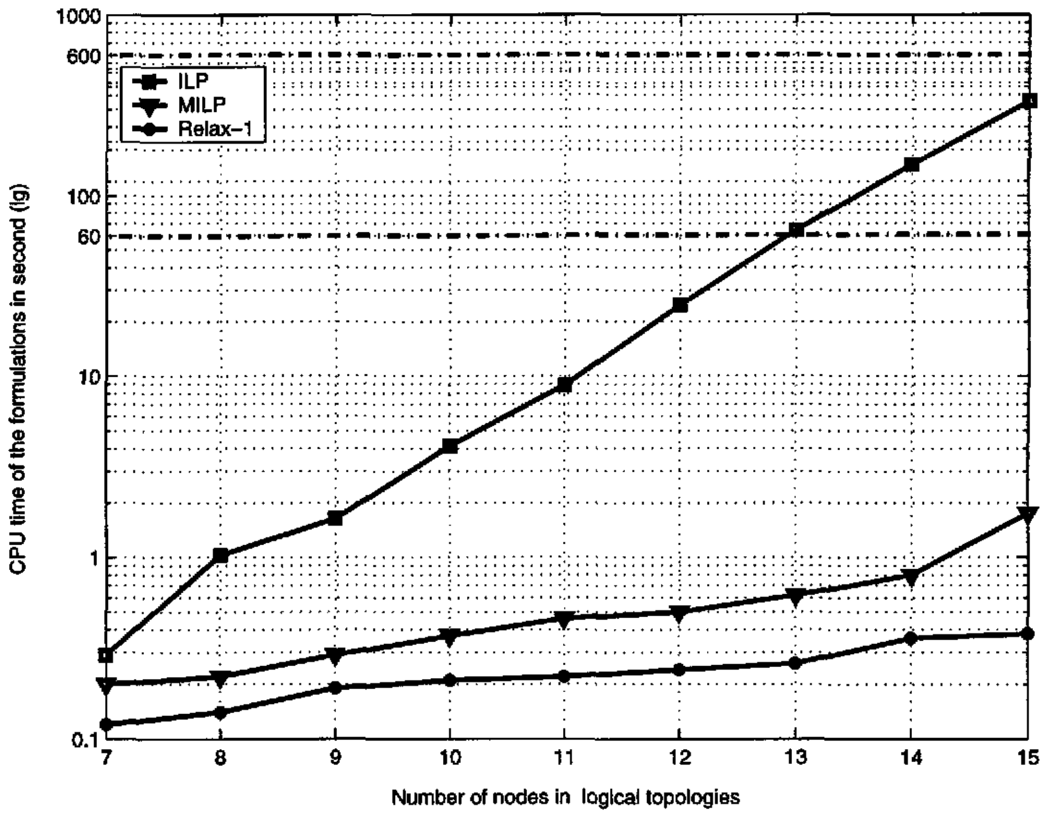

Figure 2.5: CPU times of embedding 9 logical topologies for scenario 1

We can see that there is a difference between the "CPU run-times" and the "elapsed run-times", especially for the ILP for networks with 13 nodes or larger. We speculate this difference is due to the time consumed in I/O access when virtual memory was used. In our SUN workstation, there is 288 MB of RAM for users' programs. As the programs begin to have memory usage close to or exceeding this capacity, we expect virtual memory to be used and the discrepancy between the "CPU run-time" and the "elapsed run-time" to increase. To verify this, we have the memory usage for the experiments in Table 2.2. As can be seen from Tables 2.1 and 2.2, when the ILP used 253 MB memory to solve the 13-node network, the difference between the "elapsed run-time" and the "CPU run-time" is about 1 minute. For the 14-node and 15-node networks, the memory usage becomes larger and the difference between the "CPU run-times" and the "elapsed run-times" also becomes greater. The MILP and the relaxed ILP did not use virtual memory when the total number of nodes is less than 16. Therefore, if there are no other processes on the machine, the "CPU run-times" are close to the "elapsed run-times" for the MILP and the relaxed ILP.

Table 2.2: Memory usage of embedding 9 random logical topologies on random physical topologies for Scenario 1

	7-node	8-node	9-node	10-node	11-node	12-node	13-node	14-node	15-node
ILP	1.6 M	3.1 M	8.4 M	20.8 M	41.6 M	79.8 M	253 M	523 M	979 M
MILP	1.1 M	1.4M	2.1 M	2.4 M	2.5 M	3.5 M	3.9 M	4.1 M	5.6 M
Relax-1	786 K	868 K	1.13 M	1.3 M	1.4 M	1.9 M	2.2 M	2.3 M	2.4 M

The limitation of the virtual memory (1.5 GB) available in our machine makes it impossible to solve the ILP for the network with more than 15 nodes. However, using the MILP and the relaxed ILP, it is possible to solve survivable routing problems for large networks (between 20 and 100 nodes) with 1 GB memory. In Table 2.3 we compare the run-times of the MILP and the relaxed ILP for networks with a wide range of sizes. The physical topologies and logical topologies were generated randomly as before. The "elapsed run-times" and "CPU run-times" of embedding a logical topology on a physical topology are listed in Table 2.3. The "CPU run-time" of solving the MILP is 10 minutes 17 seconds for a 100-node network, which is not too prohibitive. We plot the "elapsed run-times" in Figure 2.6 and the "CPU run-times" in Figure 2.7. We can see that the relaxed

ILP shows a better time-complexity than our MILP. However, the MILP ensures the routing to be survivable while the relaxed ILP does not.

The memory usage while running the MILP and the relaxed ILP is summarized in Table 2.4. We notice again that the difference between the "elapsed run-time" and the "CPU run-time" becomes significant when the MILP and the relaxed ILP use virtual memory. For the MILP, the memory usage approaches 288 MB for the 70-node network and it exceeds 288 MB for the networks with more than 70 nodes. For the relaxed ILP, memory usage is 373 MB for the network with 100 nodes, and the difference between the "elapsed run-time" and the "CPU run-time" is about 3 minutes.

Table 2.3: Run-times of embedding 9 large random logical topologies on random physical topologies for scenario 1

	Elapsed run-times		CPU run-times	
	MILP	Relax-1	MILP	Relax-1
20-node,65-link	2.02 sec.	0.51 sec.	2.01 sec.	0.47 sec.
30-node,83-link	10.91 sec.	2.54 sec.	10.91 sec.	2.39 sec.
40-node,98-link	12.84 sec.	2.87 sec.	12.83 sec.	2.81 sec.
50-node,129-link	17.00 sec.	5.15 sec.	16.80 sec.	5 sec.
60-node,175-link	2 min. 1 sec.	7.60 sec.	2 min. 1 sec.	7.50 sec.
70-node,199-link	4 min. 47 sec.	16.30 sec.	2 min. 51 sec.	16.10 sec.
80-node,243-link	13 min. 37 sec.	22.00 sec.	4 min. 57sec.	21.75 sec.
90-node,306-link	14 min. 12 sec.	27.30 sec.	6 min. 7 sec.	26.71 sec.
100-node,320-link	67 min.	3 min 49 sec.	10 min. 17 sec.	64.80 sec.

Table 2.4: Memory usage of embedding 9 large random logical topologies on random physical topologies for scenario 1

	20-node	30-node	40-node	50-node	60-node	70-node	80-node	90-node	100-node
MILP	7.1 M	33.6 M	46.4 M	82 M	113 M	231 M	275 M	393 M	807 M
Relax-1	3.7 M	16.3 M	21.9 M	41.8 M	54.7 M	112 M	130.6 M	164 M	373 M

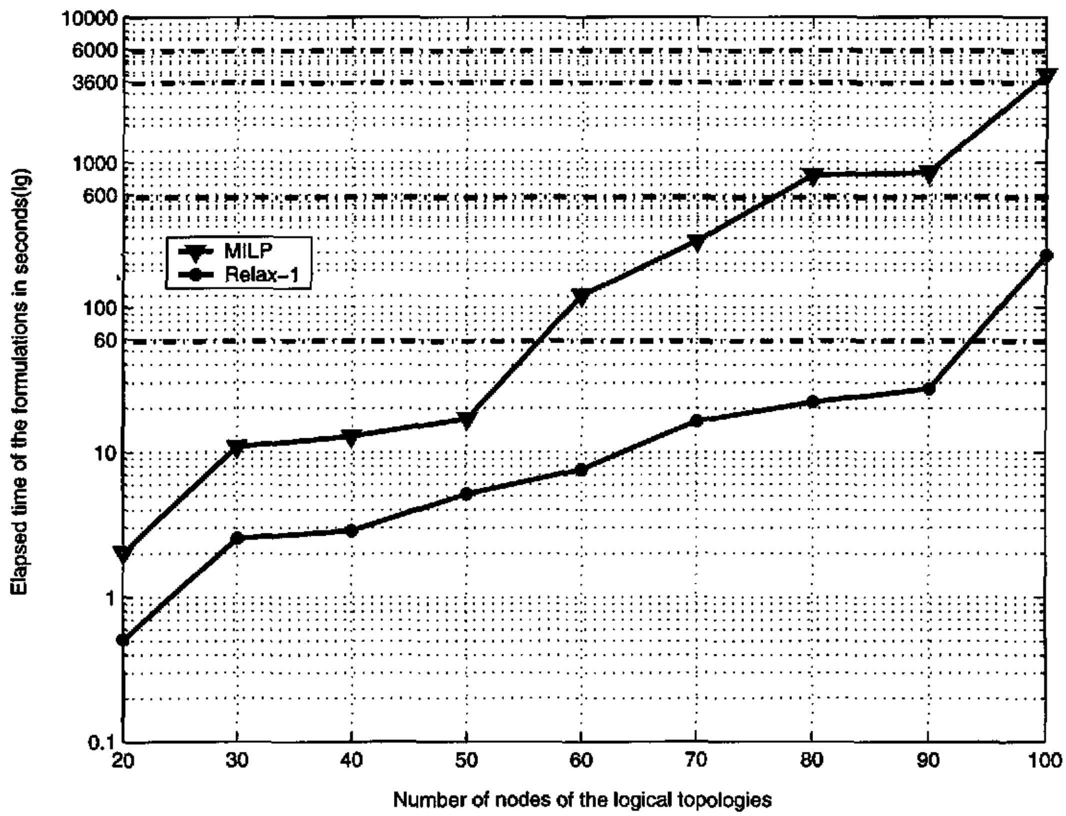

Figure 2.6: Elapsed run-times of embedding 9 large logical topologies for scenario 1

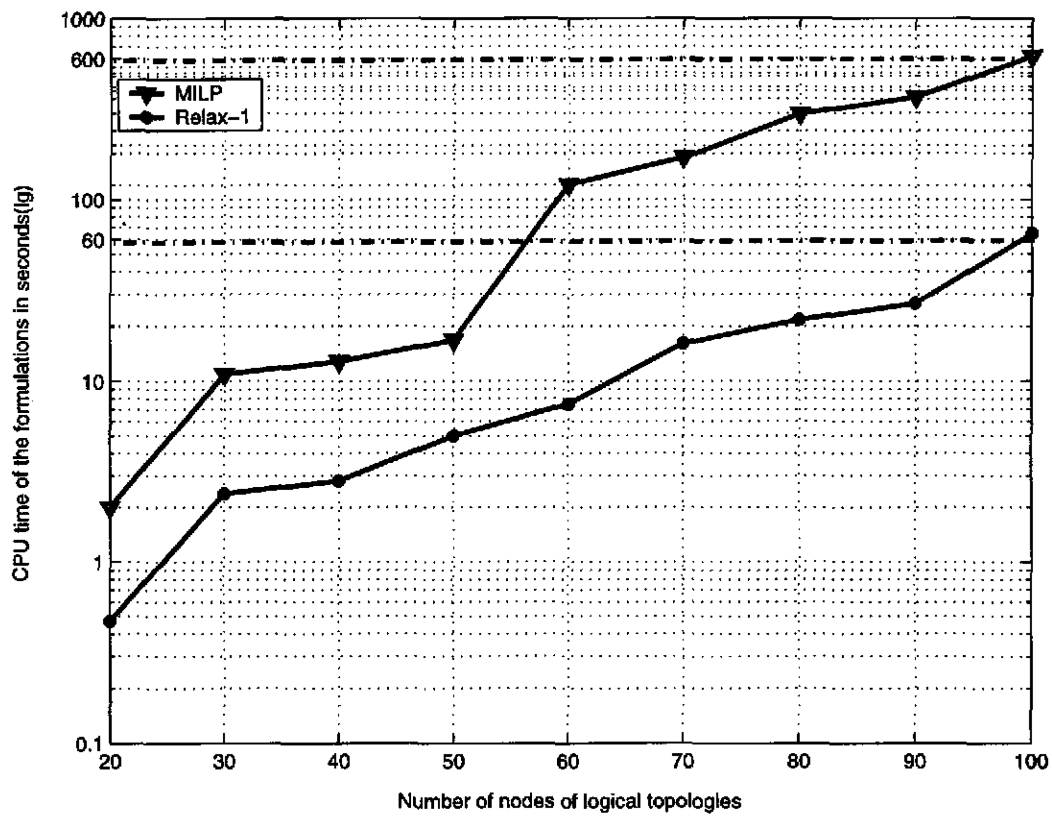

Figure 2.7: CPU times of embedding 9 large logical topologies for scenario 1

2.3 ILP and MILP formulations when the 1+1 protected lightpath service is available

In [1], the ILP formulation only considers the unprotected lightpaths. In this section, we discuss a more realistic scenario where the WDM network provides the 1+1 protected lightpath service in addition to the unprotected lightpath service. As mentioned in Chapter 1, in the 1+1 protected lightpath service the working path and the protection path must be link disjoint. We formulate the survivable routing problem as an ILP and an MILP, and then compare their time-complexity.

2.3.1 ILP formulation

Given the physical and logical topologies, we need to find a survivable routing of the logical topology over the physical topology while determining the route and the protection type of each lightpath for the logical link. To formulate this problem as an ILP, we define a integer variable F_{st}, which is the number of connections set up between s and t. If the lightpath for logical link (s,t) is unprotected, $F_{st} = 1$. If the lightpath for logical link (s,t) is protected, $F_{st} = 2$, where one connection is the working path and the other is the protection path. The connectivity constraints can be expressed as follows. For each (s,t) in E_L such that $s < t$ and $i \in N_P$:

$$\sum_{j:\ (i,j)\in E_P} f_{ij}^{st} - \sum_{j:\ (j,i)\in E_P} f_{ji}^{st} = \begin{cases} F_{st} & \text{if } s = i \\ -F_{st} & \text{if } t = i \\ 0 & otherwise \end{cases}$$

In this ILP, we formulate the survivability constraints based on Theorem 1 in [1]. For each $(i,j)\in E_P$ such that $i < j$ and each nonempty $S \subset N_L$:

$$\sum_{(s,t)\in CS(S,N_L\setminus S)} (f_{ij}^{st} + f_{ji}^{st}) < \sum_{(s,t)\in CS(S,N_L\setminus S)} F_{st}$$

The right hand side is the total number of working and protection paths for all the logical links in the $CS(S, N_L \setminus S)$. The left hand side is the number of such connections that cross fiber link (i,j). Then this constraint ensures that, under a fiber link failure, there exists at least one connection that goes between S and $N \setminus S$. Notice that the number of the survivability constraints increases exponentially with the total number of nodes.

The disjoint routing constraint for the protected lightpath can be expressed by the following constraint. For each (s,t) in E_L such that $s < t$ and each $(i,j) \in E_P$ such that $i < j$:

$$f_{ij}^{st} + f_{ji}^{st} \leq 1$$

This constraint ensures that the working path and the protection path for logical link (s,t) do not occupy the same fiber link. The objective of this ILP is to minimize the total wavelengths used. The complete integer linear program is expressed as follows.

$$\min \sum_{(i,j)\in E_P} \sum_{\substack{(s,t)\in E_L: \\ s<t}} f_{ij}^{st}$$

Subject to :

1. Integer flow constraints: for each (s,t) in E_L such that $s < t$ and each (i,j) in E_P:

$$f_{ij}^{st} \in \{0,1\}$$

2. IP topology integer connection constraints: for each (s,t) in E_L such that $s < t$:

$$F_{st} \in \{1,2\}$$

3. Connectivity constraints: for each (s,t) in E_L such that $s < t$ and each $i \in N_P$:

$$\sum_{j:\ (i,j)\in E_P} f_{ij}^{st} - \sum_{j:\ (j,i)\in E_P} f_{ji}^{st} = \begin{cases} F_{st} & \text{if } s = i \\ -F_{st} & \text{if } t = i \\ 0 & otherwise \end{cases}$$

4. Survivability constraints: for each $(i,j)\in E_P$ such that $i < j$ and each nonempty $S \subset N_L$:

$$\sum_{(s,t)\in CS(S,N_L\setminus S)} (f_{ij}^{st} + f_{ji}^{st}) < \sum_{(s,t)\in CS(S,N_L\setminus S)} F_{st}$$

5. Disjoint routing constraints for the protected lightpaths: for each (s,t) in E_L such that $s < t$ and each $(i,j) \in E_P$ such that $i < j$:

$$f_{ij}^{st} + f_{ji}^{st} \leq 1$$

Finally, the value of F_{st} will specify the protection type (protected or unprotected) of the lightpath for logical link (s,t). We can always find survivable layout of the logical topology over the physical topology as long as the physical topology is 2-connected. In particular, one solution that guarantees this is having all logical links implemented by protected lightpaths.

2.3.2 MILP formulation

It can be seen that the ILP formulation in Section 2.3.1 has an exponential number of survivability constraints. In this section, we formulate the survivable routing problem as a mixed integer linear program by replacing the survivability constraints of the ILP in Section 2.3.1.

We still assume that the original capacity of the logical link (s,t) equals to 1 while there is no physical failure. Variable r_{st}^{ij} represents the flow on logical link (s,t), from s to t, when fiber link (i,j) fails. Variable F_{st} has the same definition as described in Section 2.3.1. The value of $F_{st} - (f_{ij}^{st} + f_{ji}^{st})$ is an upper bound of r_{st}^{ij}. Therefore, the value of r_{st}^{ij} is restricted by the following constraints. For each (i,j) in E_P such that $i < j$ and each (s,t) in E_L such that $s < t$:

$$r_{st}^{ij} \leq F_{st} - (f_{ij}^{st} + f_{ji}^{st})$$

$$r_{ts}^{ij} \leq F_{st} - (f_{ij}^{st} + f_{ji}^{st})$$

$$0 \leq r_{st}^{ij} \leq 1$$

We have the disjoint routing constraint for the working path and the protection path to restrict the value of $(f_{ij}^{st} + f_{ji}^{st})$ to be less than or equal to 1. Clearly, if logical link (s,t) is implemented on the physical topology with an unprotected lightpath, $F_{st} = 1$. Then $0 \leq r_{st}^{ij} \leq 1 - (f_{ij}^{st} + f_{ji}^{st})$. If the unprotected lightpath is routed over fiber link

(i,j), $f_{ij}^{st} + f_{ji}^{st} = 1$. Then r_{st}^{ij} will be 0 after fiber link (i,j) cut, which means (s,t) will be disconnected after (i,j) fails. If logical link (s,t) is implemented by a 1+1 protected lightpath, $F_{st} = 2$. This upper bound becomes $2 - (f_{ij}^{st} + f_{ji}^{st})$. The single fiber link cut cannot be on the working path and the protection path at the same time. Then the upper bound is at least one, and r_{st}^{ij} can have value from 0 to 1.

The survivability constraint based on the necessary and sufficient condition in Section 2.2.2 can be expressed as follows. For each (i,j) in E_P such that $i < j$ and $s \in N_L$:

$$\sum_{t:\ (s,t)\in E_L} r_{st}^{ij} - \sum_{t:\ (t,s)\in E_L} r_{ts}^{ij} = \begin{cases} -1 & \text{if } s = 1 \\ \frac{1}{|N_L|-1} & otherwise \end{cases}$$

Notice that the total number of the survivability constraints is a polynomial of the number of nodes.

Our objective is to minimize the total wavelength cost and the complete MILP formulation is as follows.

$$\min \sum_{(i,j)\in E_P} \sum_{\substack{(s,t)\in E_L: \\ s<t}} f_{ij}^{st}$$

Subject to :

1. Integer flow constraints: for each (s,t) in E_L such that $s < t$ and each $(i,j) \in E_P$:

$$f_{ij}^{st} \in \{0,1\}$$

2. Survivable flow constraints: for each (s,t) in E_L and each $(i,j) \in E_P$ such that $i < j$:

$$0 \le r_{st}^{ij} \le 1$$

3. IP topology integer connection constraints: for each (s,t) in E_L such that $s < t$:

$$F_{st} \in \{1,2\}$$

4. Connectivity constraints: for each (s,t) in E_L such that $s < t$ and each $i \in N_P$:

$$\sum_{j:\ (i,j)\in E_P} f_{ij}^{st} - \sum_{j:\ (j,i)\in E_P} f_{ji}^{st} = \begin{cases} F_{st} & \text{if } s = i \\ -F_{st} & \text{if } t = i \\ 0 & otherwise \end{cases}$$

5. Disjoint routing constraints for the protected lightpaths: for each (s,t) in E_L such that $s < t$ and each pair $(i,j) \in E_P$ such that $i < j$:

$$f_{ij}^{st} + f_{ji}^{st} \leq 1$$

6. Survivable traffic flow capacity constraints: for each (i,j) in E_P such that $i < j$ and each (s,t) in E_L such that $s < t$:

$$r_{st}^{ij} \leq F_{st} - (f_{ij}^{st} + f_{ji}^{st})$$

$$r_{ts}^{ij} \leq F_{st} - (f_{ij}^{st} + f_{ji}^{st})$$

7. Survivability constraints: for each (i,j) in E_P such that $i < j$ and $s \in N_L$:

$$\sum_{t:\ (s,t)\in E_L} r_{st}^{ij} - \sum_{t:\ (t,s)\in E_L} r_{ts}^{ij} = \begin{cases} -1 & \text{if } s = 1 \\ \frac{1}{|N_L|-1} & otherwise \end{cases}$$

In this MILP formulation, by replacing the exponential number of survivability constraints of the ILP in Section 2.3.1, the total number of constraints is reduced significantly, which contributes to a lower time-complexity than the ILP formulation. The run-times of the ILP and the MILP will be compared in the next section.

2.3.3 Experimental results

To compare the run-times of the ILP and the MILP, we used the same network topologies as those in Table 2.1 except the 15-node network, because it takes up too much memory. We also check the run-times of the relaxed ILP that only considers the subsets of N_L including only a single node. We plot the "elapsed run-times" and the "CPU run-times" in Figure 2.8 and Figure 2.9 based on the results in Table 2.5. As expected, the run-times of the ILP increases dramatically with the number of nodes. Our MILP shows a dramatic improvement in run-times, especially for larger networks with more than 12 nodes. For the "CPU run-times", the MILP only took only about 1 second while the ILP took more 4 minutes to solve a 14-node network problem. The difference between the "elapsed run-times" and the "CPU run-times" is mainly due to the I/O swap delay as explained in Section 2.2.3. We summarized the memory usage of the three formulations in Table 2.6. It can be seen from Table 2.5, the difference between the "elapsed run-time" and "CPU run-time" becomes significant when the ILP solve networks with 13 node or larger.

Table 2.5: Run-times of embedding 8 random logical topologies on random physical topologies for scenario 2

	Elapsed run-times			CPU run-times		
	ILP	MILP	Relax-1	ILP	MILP	Relax-1
7-node,11-link	0.37 sec.	0.23 sec.	0.19 sec.	0.36 sec.	0.21 sec.	0.16 sec.
8-node,15-link	1.44 sec.	0.33 sec.	0.21 sec.	1.41 sec.	0.28 sec.	0.18 sec.
9-node,14-link	2.98 sec.	0.37 sec.	0.25 sec.	2.92 sec.	0.34 sec.	0.23 sec.
10-node,17-link	7.89 sec.	0.44 sec.	0.29 sec.	7.81 sec.	0.44 sec.	0.26 sec.
11-node,19-link	20.49 sec.	0.66 sec.	0.31 sec.	20.35 sec.	0.57 sec.	0.27 sec.
12-node,26-link	45.08 sec.	0.74 sec.	0.37 sec.	42.75 sec.	0.73 sec.	0.34 sec.
13-node,25-link	5 min. 6 sec.	0.82 sec.	0.42 sec.	2 min 26 sec.	0.81 sec.	0.39 sec.
14-node,21-link	47 min.	1.03 sec.	0.51 sec.	4 min. 28 sec.	0.96 sec.	0.44 sec.

Table 2.6: Memory usage of embedding 8 random logical topologies on random physical topologies for scenario 2

	7-node	8-node	9-node	10-node	11-node	12-node	13-node	14-node
ILP	2.0 M	4.1 M	11.8 M	36.1 M	58.1 M	172.3 M	357.0 M	750.8 M
MILP	1.3 M	1.5 M	2.4 M	2.8 M	2.9 M	4.2 M	4.6 M	4.8 M
Relax-1	885 K	983 K	1.5 M	1.7 M	1.8 M	2.3 M	2.6 M	2.8 M

The MILP and the relaxed ILP were implemented using the same large network topologies in Table 2.3. The run-times are summarized in Table 2.7. It can be seen from the table, for the "CPU run-time", the MILP was able to solve the 100-node network problem within 16 minutes. We plot the "elapsed run-times" and the "CPU run-times" listed in Table 2.7 in Figures 2.10 and 2.11. As can be seen from these figures, the relaxed ILP

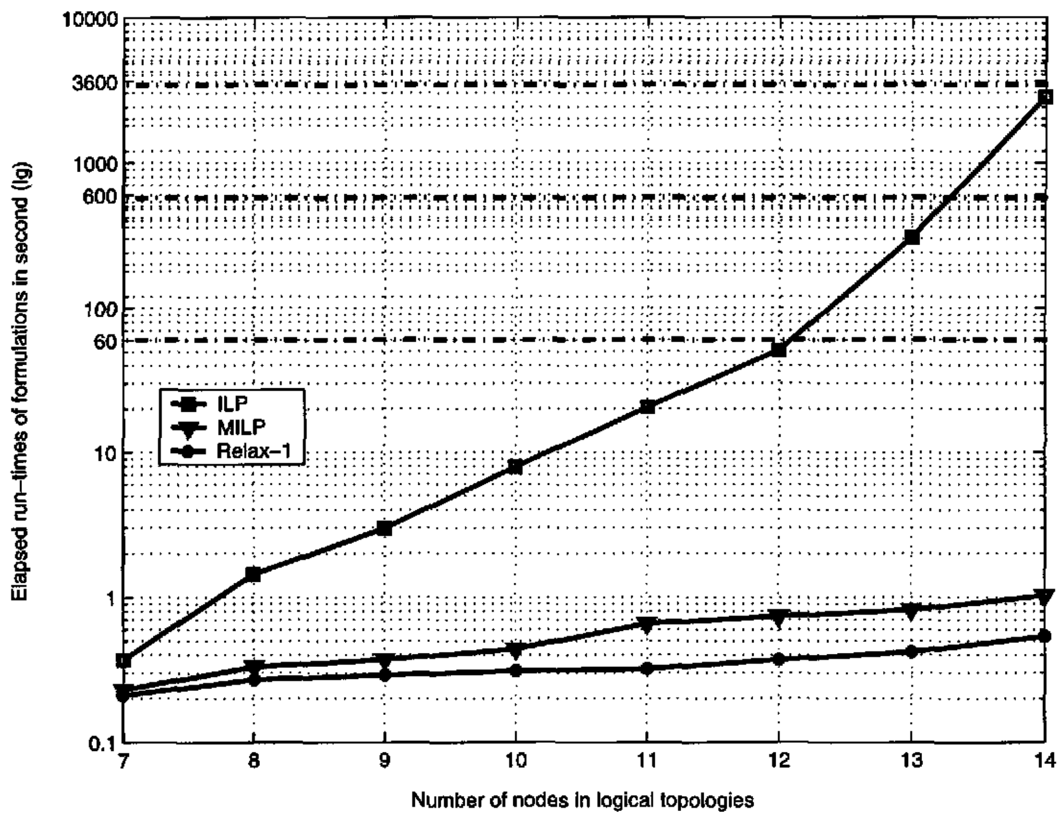

Figure 2.8: Elapsed run-times of embedding 8 logical topologies for scenario 2

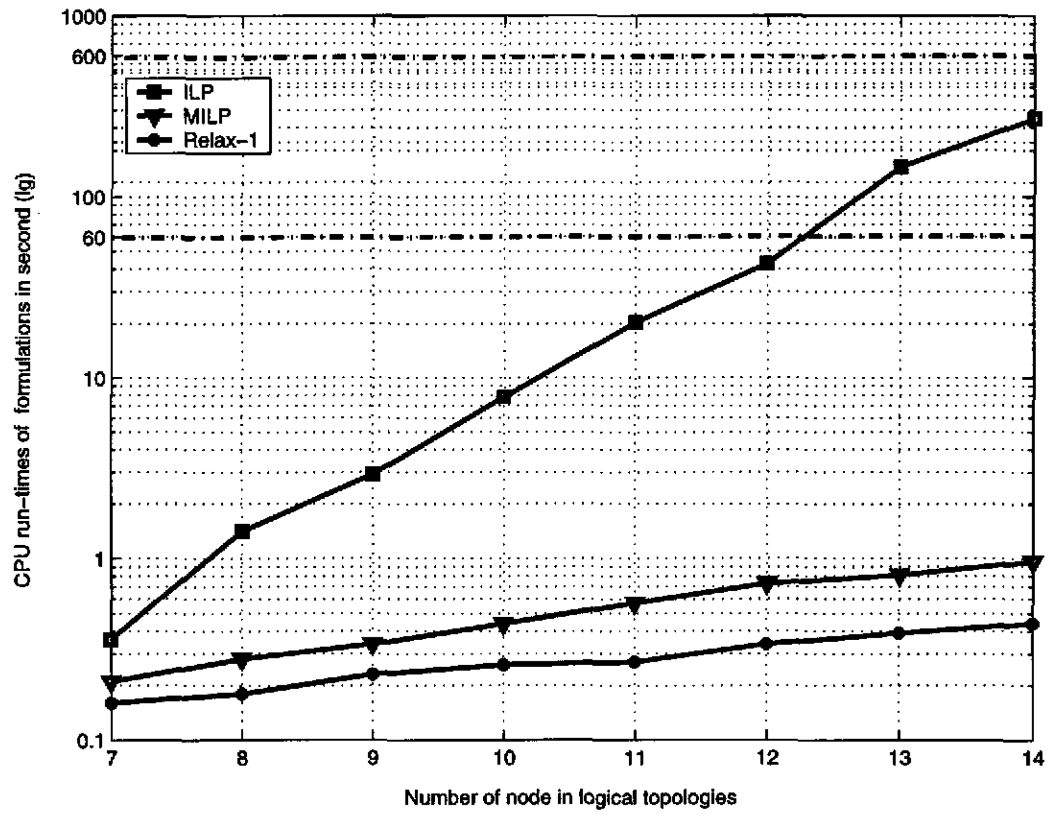

Figure 2.9: CPU times of embedding 8 logical topologies for scenario 2

shows a better time-complexity than the MILP. However, the MILP ensures the routing to be survivable while the relaxed ILP does not. We summarized the memory usage of the MILP and the relaxed ILP in Table 2.8. As analyzed in Section 2.2.3, the difference between the "elapsed run-times" and the "CPU run-times" becomes great when the MILP began to use virtual memory to solve networks with 70 nodes or more. For the relaxed ILP, the memory usage is close to or more than 288 MB for the network with 80 nodes or larger, and the difference between the "elapsed run-time" and "CPU run-time" is *significant*.

Table 2.7: Run-times of embedding 9 large random logical topologies on random physical topologies for Scenario 2

	Elapsed run-times		CPU run-times	
	MILP	Relax-1	MILP	Relax-1
20-node,65-link	2.15 sec.	0.78 sec.	2.08 sec.	0.73 sec.
30-node,83-link	14.85 sec.	3.70 sec.	14.71 sec.	3.62 sec.
40-node,98-link	17.36 sec.	4.87 sec.	16.82 sec.	4.78 sec.
50-node,129-link	20.80 sec.	7.93 sec.	20.56 sec.	7.83 sec.
60-node,175-link	2 min. 15 sec.	11.78 sec.	2 min. 12 sec.	11.61 sec.
70-node,199-link	6 min. 51 sec.	26.50 sec.	3 min. 3 sec.	26.23 sec.
80-node,243-link	25 min. 25 sec.	48.26 sec.	8 min .1 sec.	35.90 sec.
90-node,306-link	27 min. 46 sec.	1 min. 34 sec.	9 min. 56 sec.	44.81 sec.
100-node,320-link	1 h. 40 m.20 sec.	10 min 52 sec.	15 min. 40 sec.	2 min. 9 sec.

Table 2.8: Memory usage of embedding 9 large random logical topologies on random physical topologies for Scenario 2

	20-node	30-node	40-node	50-node	60-node	70-node	80-node	90-node	100-node
MILP	8.5 M	35.5 M	53.6 M	86.6 M	131.4 M	271.8 M	369 M	449 M	963 M
Relax-1	4.7 M	21.2 M	29.6 M	54.5 M	68.3 M	141.3 M	213 M	236 M	486 M

We have discussed two survivable routing scenarios so far. Scenario 1 where the WDM network only provides unprotected lightpath service was studied in Section 2.2. Scenario 2 where the WDM network can provide unprotected and 1+1 protected lightpath services was addressed in this section. In Scenario 1, it may be impossible to find a survivable layout, while in Scenario 2, we can always find a survivable layout of the logical topology over the physical topology. We show this fact by solving some samples using

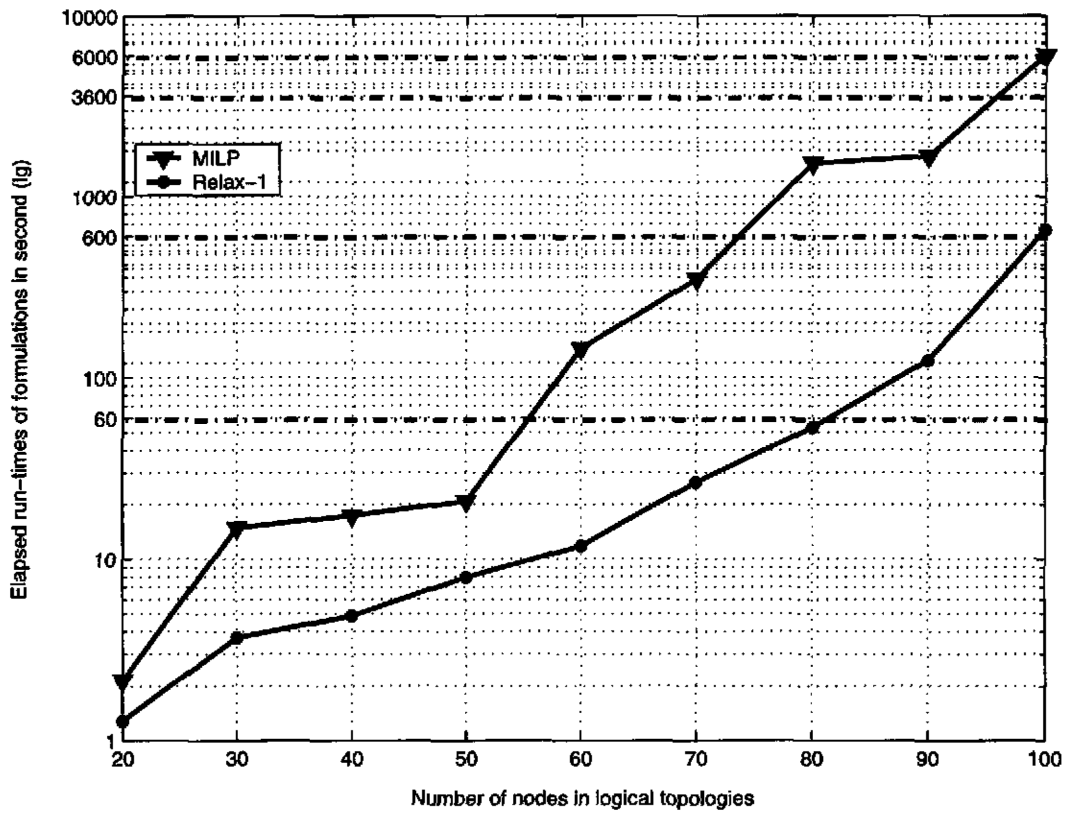

Figure 2.10: Elapsed run-times of embedding 9 large logical topologies for scenario 2

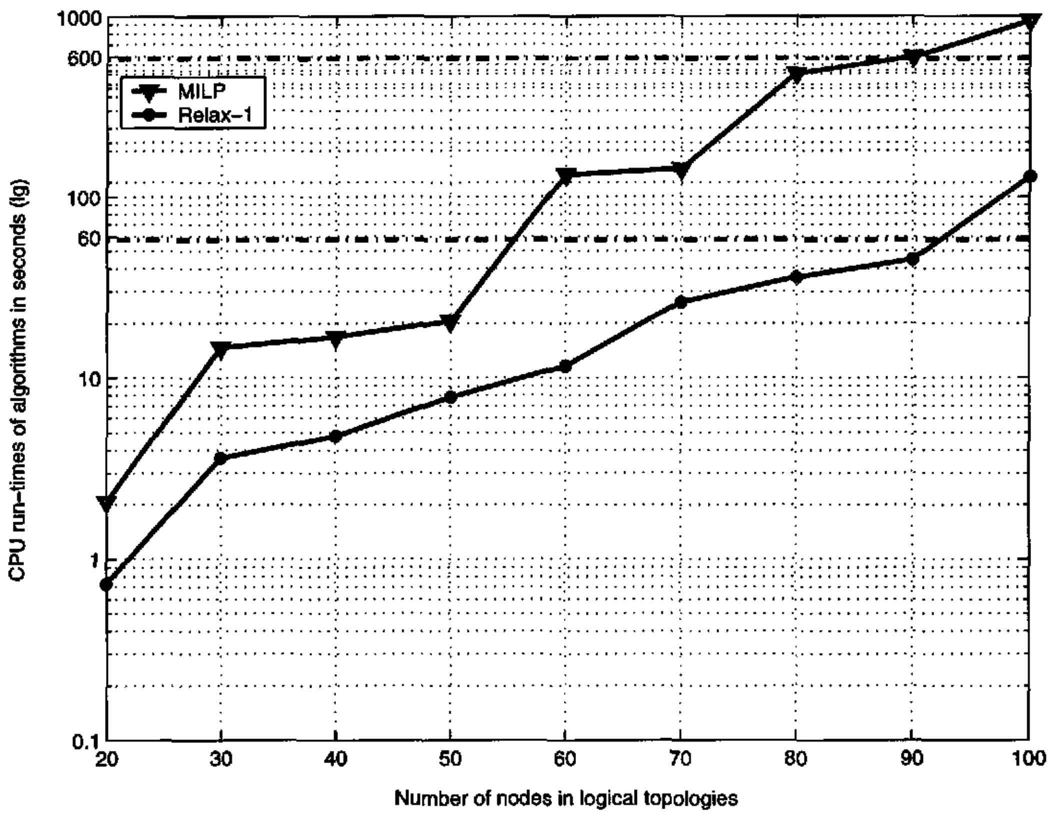

Figure 2.11: CPU times of embedding 9 large logical topologies for scenario 2

the MILP formulations for different scenarios. For the underlying physical topologies, we choose the 12-node ring, the 3Cycle topology in Figure 2.12 and the 14-node NSFNET in Figure 2.13. For each physical topology, we routed 100 random logical topologies over it. In Table 2.9, the label "MILP-U" represents the MILP for Scenario 1 and the label "MILP-U+P" corresponds to the MILP formulation for Scenario 2. We can see that, using "MILP-U", some logical topologies did not lead to survivable layouts while using "MILP-U+P", we obtained a survivable layout for every logical topology. By allocating some lightpaths as protected lightpaths, all the infeasible cases in Scenario 1 had survivable routings using "MILP-U+P".

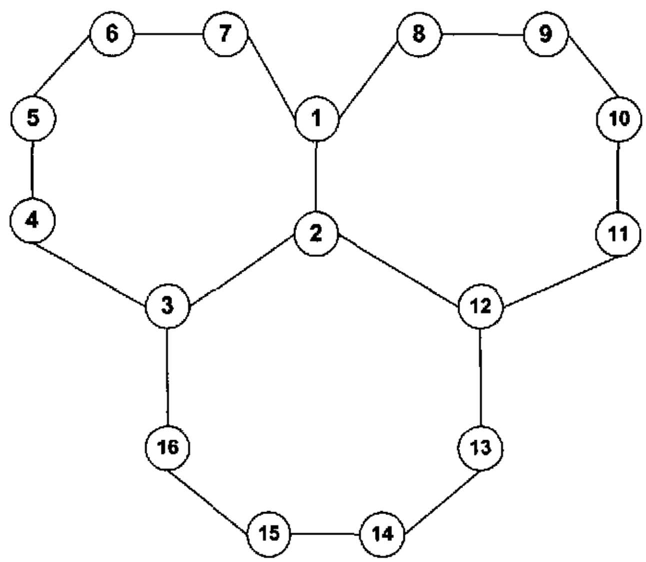

Figure 2.12: 3Cycle physical topology

Table 2.9: Embedding topologies on three physical topologies

	Ring 12		3Cycle		14-node NSFNET	
	Infeasible cases	Feasible cases	Infeasible cases	Feasible cases	Infeasible cases	Feasible cases
MILP-U	21	79	8	92	1	99
MILP-U+P	0	100	0	100	0	100

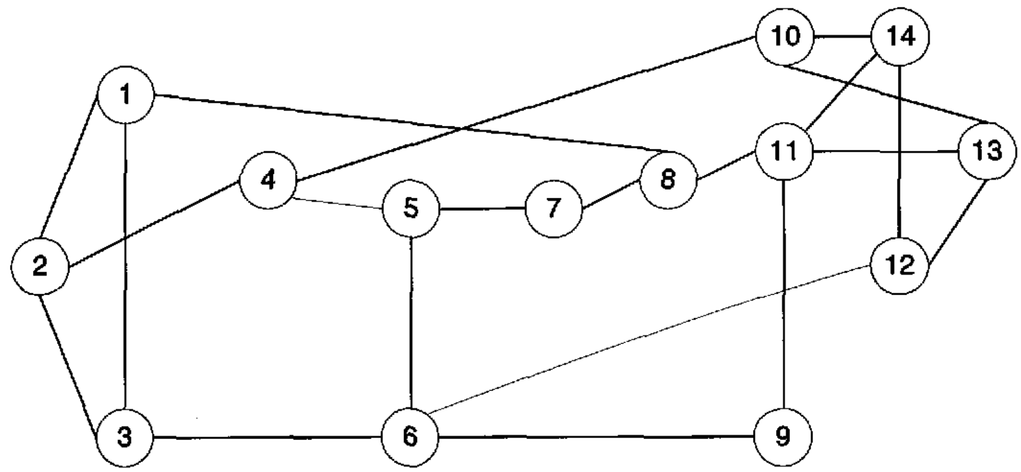

Figure 2.13: The 14-node 21-link NSFNET

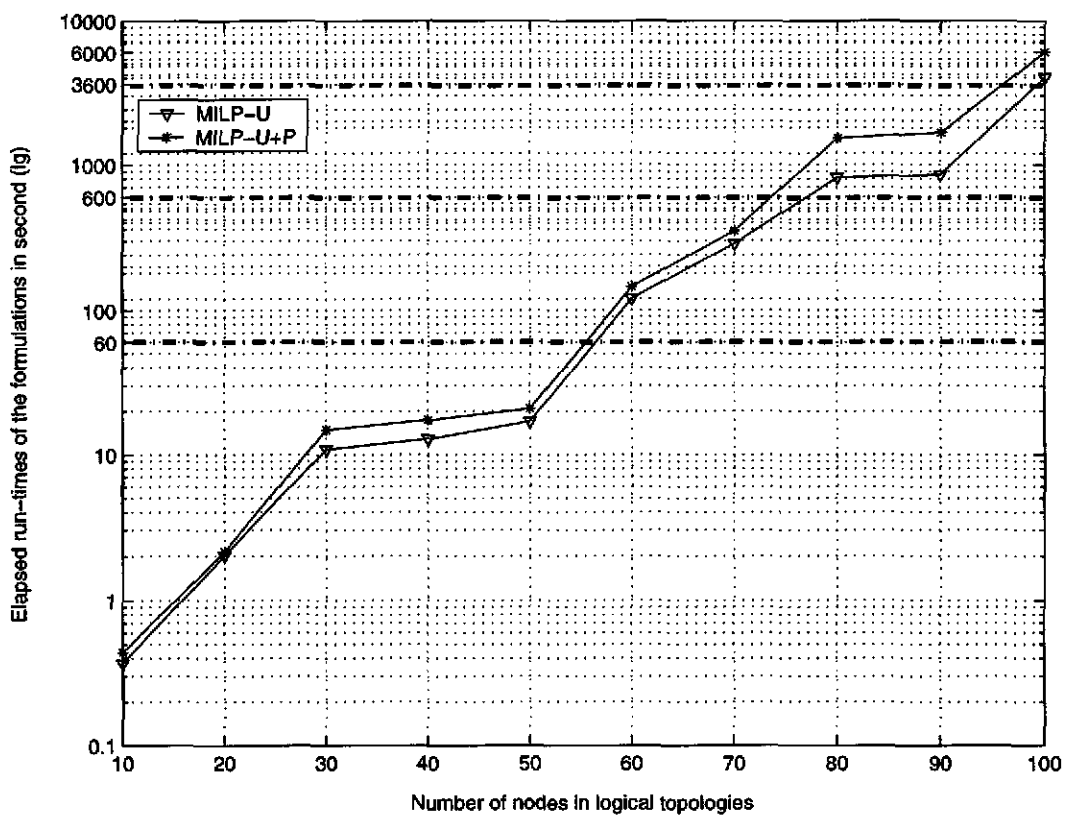

Figure 2.14: Elapsed run-times of MILPs for scenario 1 and scenario 2

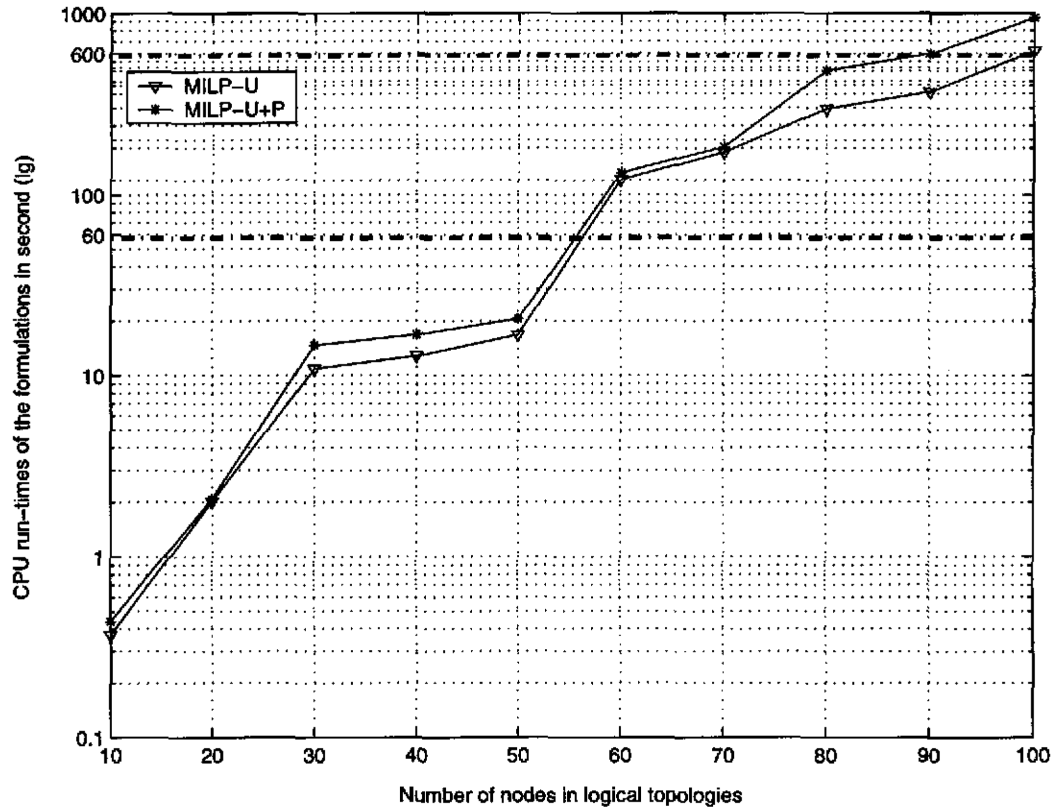

Figure 2.15: CPU run-times of MILPs for scenario 1 and scenario 2

In Figure 2.14 and 2.15, we plot the "elapsed run-times" and "CPU run-times" of "MILP-U" and "MILP-U+P" based on various network topologies. "MILP-U+P" takes longer time than MILP-U to solve a problem. It is because in Scenario 2, we need to decide the protection type as well as the routing of each lightpath.

Chapter 3

Survivable Layout for the Shared Risk Link Group

In the previous chapter,we discussed survivable routing problem for single fiber link failures because they are the most common failures in the WDM network. However, there are other failures in the WDM network, such as OXC failures and conduit cuts, which result in multiple fiber link failures at the same time. Multiple fiber link failures due to a failure event (such as a conduit failure) is known as a *shared risk link group* (SRLG). A shared risk link group (SRLG) defines the group of links that share a component whose failure causes the failure of all links of the group. A typical SRLG is all the fiber links in a conduit, as shown in Figure 3.1. If this conduit fails, the fiber links (OXC1, OXC2) and (OXC3, OXC4) will fail together.

In this chapter, we develop a survivable layout problem that requires the logical topology remains connected after an SRLG failure occurs. In Section 3.1, we provide an MILP formulation to route logical links over the physical topology in order to survive any SRLG failure.

3.1 Problem formulation

We assume that the WDM network can provide unprotected and 1+1 protected lightpath services. The physical topology, the logical topology and the SRLGs are given.

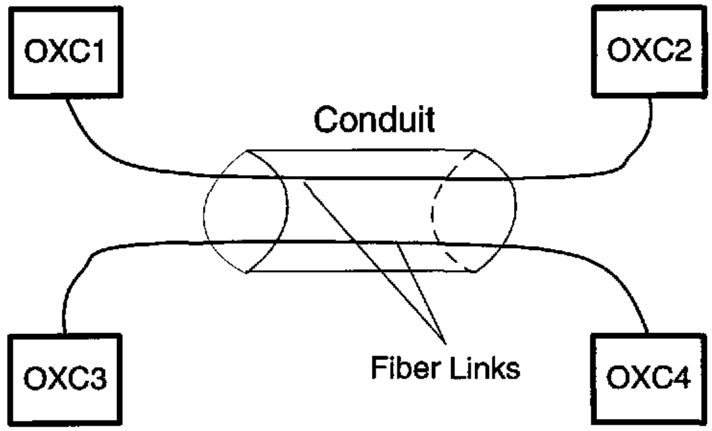

Figure 3.1: An example of a conduit

The problem is to determine the protection type and route of each lightpath so that the logical topology remains connected under any SRLG failure. Using the proposed survivable routing condition, we are able to formulate the survivable routing problem for the SRLG failure as an MILP by modifying the MILP in Section 2.3.2.

Let m denote the total number of SRLGs. SRLG$[k]$ represents the kth SRLG where $k = 1, 2, \ldots, m$. Since fiber link (i, j) and (j, i) fail simultaneously, the SRLGs only include fiber links (i, j) that satisfies $i < j$.

The lightpath for logical link (s, t) always has a working path. The route of the working path will be found by routing one unit of flow from s to t. Variable f_{ij}^{st} has the same definition as Section 2.2.1 and the route of the working path for logical link (s, t) is determined by the following constraints. For each (s, t) in E_L such that $s < t$ and each $i \in N_P$:

$$\sum_{j:\,(i,j)\in E_P} f_{ij}^{st} - \sum_{j:\,(j,i)\in E_P} f_{ji}^{st} = \begin{cases} 1 & \text{if } s = i \\ -1 & \text{if } t = i \\ 0 & otherwise \end{cases}$$

We define a binary variable P_{st} as follows. Let $P_{st} = 1$, if the lightpath for logical link (s, t) has a protection path, and $P_{st} = 0$ otherwise. That means, if the lightpath is protected, one unit of flow from s to t is routed for the working path, and another unit of flow from s to t is routed for the protection path. Let p_{ij}^{st} be a binary variable such that $p_{ij}^{st} = 1$, if the protection path from s to t is routed over fiber link (i, j), and $p_{ij}^{st} = 0$

otherwise. The protection path connectivity constraints can be expressed as follows. For each (s,t) in E_L such that $s < t$ and each $i \in N_P$:

$$\sum_{j:\ (i,j)\in E_P} p_{ij}^{st} - \sum_{j:\ (j,i)\in E_P} p_{ji}^{st} = \begin{cases} P_{st} & \text{if } s = i \\ -P_{st} & \text{if } t = i \\ 0 & otherwise \end{cases}$$

The working path and the protection path for logical link (s,t) must be link disjoint. The following constraint ensures this. For each (s,t) in E_L such that $s < t$ and each (i,j) in E_P such that $i < j$:

$$f_{ij}^{st} + f_{ji}^{st} + p_{ij}^{st} + p_{ji}^{st} \leq 1$$

The survivability constraints are based on our new necessary and sufficient condition: The routing is survivable if and only if each node s ($s \neq 1$) can source a flow of $\frac{1}{|N_L|-1}$ to node 1 in the logical topology. We define two binary variables, h_{st}^k and g_{st}^k, which reflect the capacities of the working and protection paths, respectively, of logical link (s,t) when the kth SRLG fails. The value $1 - (f_{ij}^{st} + f_{ji}^{st})$ is the upper bound of h_{st}^k. For $k = 1, 2, \ldots, m$ and (s,t) in E_L such that $s < t$, we have the following constraint. For all (i,j) in $SRLG[k]$:

$$h_{st}^k \leq 1 - (f_{ij}^{st} + f_{ji}^{st})$$

This constraint states that if the working path for logical link (s,t) goes through any fiber link (i,j) in SRLG[k], then h_{st}^k equals 0.

The value of $P_{st} - (p_{ij}^{st} + p_{ji}^{st})$ is the upper bound of g_{st}^k. For $k = 1, 2, \ldots, m$ and (s,t) in E_L such that $s < t$, we have the following constraints. For all (i,j) in $SRLG[k]$:

$$g_{st}^k \leq P_{st} - (p_{ij}^{st} + p_{ji}^{st})$$

Since logical link (s,t) may be realized by an unprotected or 1+1 protected lightpath, this upper bound of g_{st}^k is meaningful only when the lightpath of logical link (s,t) has a protection path. After SRLG[k] fails, if the protection path for logical link (s,t) is routed on any fiber link (i,j) belonging to the failed SRLG, then g_{st}^k equals 0.

Variable r_{st}^k is denoted as a nonnegative variable to represent the flow on logical link (s,t) from s to t after SRLG[k] fails. $h_{st}^k + g_{st}^k$ is the upper bound of r_{st}^k. That is, for each $k = 1, 2, \ldots, m$ and (s,t) in E_L such that $s < t$:

$$r_{st}^k \leq g_{st}^k + h_{st}^k$$

$$r_{ts}^k \leq g_{st}^k + h_{st}^k$$

We assume that the capacity of a logical link is 1 when there is no physical failure. Therefore, $0 \leq r_{st}^k \leq 1$. These constraints relating to r_{st}^k say that if SRLG[k] failure disables both the working and the protection paths for logical link (s,t), r_{st}^k must equal 0, which means the logical link is disconnected. Otherwise, the flow on logical link (s,t) is between 0 and 1.

The constraints ensuring the connectivity of the logical topology under any SRLG failure can be expressed as follows. For each $k = 1, 2, \ldots, m$ and s in N_L:

$$\sum_{t:\ (s,t)\in E_L} r_{st}^k - \sum_{t:\ (t,s)\in E_L} r_{ts}^k = \begin{cases} -1 & \text{if } s = 1 \\ \frac{1}{|N_L|-1} & otherwise \end{cases}$$

Our objective is to minimize the total wavelength links occupied by the working and protection paths. The MILP formulation for the survivable routing dealing with the SRLG failure can be expressed as follows.

$$\min \sum_{(i,j)\in E_P} \sum_{\substack{(s,t)\in E_L: \\ s<t}} (f_{ij}^{st} + p_{ij}^{st})$$

Subject to :

1. Integer flow constraints: for each (s,t) in E_L such that $s < t$ and each (i,j) in E_P:

$$f_{ij}^{st} \in \{0,1\} \qquad p_{ij}^{st} \in \{0,1\}$$

2. IP topology integer connection constraints: for each (s,t) in E_L such that $s < t$:

$$P_{st} \in \{0,1\}$$

3. IP topology nonnegative flow constraints: for each (s,t) in E_L and $k = 1, 2, \ldots, m$:

$$0 \le r_{st}^k \le 1$$

4. Integer variable constraints: for each (s,t) in E_L such that $s < t$ and $k = 1, 2, \ldots, m$:

$$h_{st}^k \in \{0,1\} \qquad g_{st}^k \in \{0,1\}$$

5. Working path connectivity constraints: for each (s,t) in E_L such that $s < t$ and each $i \in N_P$:

$$\sum_{j:\ (i,j)\in E_P} f_{ij}^{st} - \sum_{j:\ (j,i)\in E_P} f_{ji}^{st} = \begin{cases} 1 & \text{if } s = i \\ -1 & \text{if } t = i \\ 0 & otherwise \end{cases}$$

6. Protection connectivity constraints: for each (s,t) in E_L such that $s < t$ and each $i \in N_P$:

$$\sum_{j:\ (i,j)\in E_P} p_{ij}^{st} - \sum_{j:\ (j,i)\in E_P} p_{ji}^{st} = \begin{cases} P_{st} & \text{if } s = i \\ -P_{st} & \text{if } t = i \\ 0 & otherwise \end{cases}$$

7. Disjoint routing constraints for the protected lightpaths: for each (s,t) in E_L such that $s < t$ and each (i,j) in E_P such that $i < j$:

$$f_{ij}^{st} + f_{ji}^{st} + p_{ij}^{st} + p_{ji}^{st} \le 1$$

8. Survivable traffic flow capacity constraints: for $k = 1, 2, \ldots, m$, (i,j) in SRLG[k] and (s,t) in E_L such that $s < t$:

$$h_{st}^k \le 1 - (f_{ij}^{st} + f_{ji}^{st})$$
$$g_{st}^k \le P_{st} - (p_{ij}^{st} + p_{ji}^{st})$$

9. Survivable flow constraints: for $k = 1, 2, \ldots, m$ and (s,t)in E_L such that $s < t$:

$$r_{st}^k \le g_{st}^k + h_{st}^k$$
$$r_{ts}^k \le g_{st}^k + h_{st}^k$$

10. Survivability constraints: for $k = 1, 2, \ldots, m$ and s in N_L:

$$\sum_{t:\ (s,t)\in E_L} r_{st}^k - \sum_{t:\ (t,s)\in E_L} r_{ts}^k = \begin{cases} -1 & \text{if } s = 1 \\ \frac{1}{|N_L|-1} & otherwise \end{cases}$$

The solution obtained from the above MILP gives us the survivable routing of the logical topology over the physical topology while withstanding any SRLG failure. The values of $\{f_{ij}^{st} : (s,t) \in E_L$ and $(i,j) \in E_P\}$ specify the routes of the working paths for logical links and the values of $\{p_{ij}^{st} : (s,t) \in E_L$ and $(i,j) \in E_P\}$ specify the routes of the protection paths for logical links.

Chapter 4

Logical Topology Design and Survivable Routing

In previous chapters, we discussed the survivable routing problems, which starts with a logical topology. Then this logical topology is overlaid over a given physical topology so that it is survivable. In practice, we may need to design the logical topology too. In this chapter, we study a realistic scenario where we start with a traffic matrix T, which specifies the traffic rates between all pairs of IP routers when there are no failures. We are given another traffic matrix R, which represents the traffic rates between all pairs of IP routers that the network must support even under physical failures. We assume the traffic is full duplex, so the traffic matrices are symmetric about their diagonal. In addition, it is assumed that for each node pair (s, t), $R[s, t] \leq T[s, t]$.

We design a logical topology that can support traffic matrix T, and then overlay the logical topology over a given physical topology in such a way that the logical topology can support traffic matrix R under physical failures. Note that now we require the IP network to support the traffic matrix R even under physical failures. This is a stronger requirement than simply requiring the "connectivity" of the logical topology under physical failure. However, this may occur in practice since the IP network will support many users.

To solve the logical design and survivable routing problems, we can follow two different approaches. One is a two-stage design while the other is a one-stage design. The two-stage design has a lower time-complexity than the one-stage design. However, the two-stage design may only obtain a sub-optimal solution while the one-stage design can obtain a optimal solution. In Section 4.1, we describe the two-stage procedure to solve the problem. In the first stage, we design a logical topology that supports traffic matrix T, and in the second stage, we determine its survivable layout over a physical topology. In Section 4.2, we discuss the one-stage design approach. Finally, after solving some sample problems, we compare the results of these two design approaches in Section 4.3. From the results, we can see that the two-stage approach may only obtain a sub-optimal solution while the one-stage approach obtains an optimal solution. However, the two-stage approach has a lower time-complexity than the one-stage approach.

4.1 Two-stage design

In this section, we discuss the two-stage design procedure. In Section 4.1.1, we discuss the first stage design. Given the traffic matrix T and the nodes in the IP network, we try to design a logical topology by specifying the logical links. There are no constraints imposed by the underlying physical topology. In Section 4.1.2, we discuss the second stage design. We overlay the logical topology obtained from the first stage over the physical topology while determining the routing and the protection type of each lightpath. We want the logical topology to support a surviving traffic R under physical failures. We state the MILP formulations for single fiber link failures and SRLG failures as well.

4.1.1 Logical topology design

In this section, we will study the problem of designing a logical topology that supports the traffic matrix T. We assume that all lightpaths to be established are bi-directional.

Before addressing the mathematical formulation, we give some definitions. At this stage, we are given a set $N_L = \{1, 2, 3...N\}$ including all of the nodes in the logical topology. We are also given traffic matrix T, which is an $|N_L| \times |N_L|$ matrix and is symmetric about the diagonal. $T[s,t]$ is the traffic rate from node s to t. Under normal working conditions, that is, when there are no failures, the network must support this traffic rate. We refer to it as the "*nominal traffic*". Each entry of T is between 0 and 1. The diagonal entries of T are zeros, which indicates the traffic from a node to itself is 0.

Let t_{ij}^{st} be a nonnegative real valued variable which represents the traffic from s to t that goes through logical link (i, j) (if it exists). Let b_{ij} be a binary variable, where $b_{ij} = 1$, if there is a logical link between node i and j, and $b_{ij} = 0$ otherwise.

We assume that it is possible to set up a logical link between any pair of nodes, and we can arbitrarily split the traffic between each source-destination pair over different paths through the logical topology. The connectivity constraints can be expressed as follows. For each s, t, i in N_L such that $s < t$:

$$\sum_{j \in N_L} t_{ij}^{st} - \sum_{j \in N_L} t_{ji}^{st} = \begin{cases} T[s,t] & \text{if } s = i \\ -T[s,t] & \text{if } t = i \\ 0 & otherwise \end{cases}$$

The left hand side of the connectivity constraint is the difference between the total outgoing flow from node i and the total incoming flow to node i for the traffic from s to t. Since the traffic matrix is symmetric about the diagonal, we only need to consider the upper diagonal traffic matrix where $s < t$. When we find the route from s to t, it is implicit in that the traffic from t to s will follow the same logical links in the opposite direction.

We assume that the capacity of a logical link (i, j) is 1 if there exists a logical link between i and j. We have the following capacity constraint to restrict the total amount of traffic on a logical link. for each i, j in N_L such that $i < j$:

$$\sum_{\substack{s,t \in N_L: \\ s<t}} (t_{ij}^{st} + t_{ji}^{st}) \leq b_{ij}$$

Our objective is to minimize the total cost. We assume that each possible logical link between each pair of nodes has the same cost. Therefore, our objective is to minimize the total number of links in the logical topology. The MILP is expressed as follows.

$$\min \sum_{\substack{i \in N_L, j \in N_L: \\ i<j}} b_{ij}$$

Subject to :

1. Nonnegative variable constraints: for each i, j, s, t in N_L such that $s < t$:

$$t_{ij}^{st} \geq 0$$

2. Integer variable constraints: for each i, j in N_L such that $i < j$:

$$b_{ij} \in \{0, 1\}$$

3. Connectivity constraints: for each s, t, i in N_L such that $s < t$:

$$\sum_{j \in N_L} t_{ij}^{st} - \sum_{j \in N_L} t_{ji}^{st} = \begin{cases} T[s,t] & \text{if } s = i \\ -T[s,t] & \text{if } t = i \\ 0 & otherwise \end{cases}$$

4. Capacity constraints: for each i, j in N_L such that $i < j$:

$$\sum_{\substack{s,t \in N_L: \\ s<t}} (t_{ij}^{st} + t_{ji}^{st}) \leq b_{ij}$$

The result of this mixed integer linear program will give us a logical topology, which can support the traffic matrix T. Note that this MILP does not allow multiple links between the same pair of nodes. What we get is a logical topology consisting of bi-directional links.

As an example, we solved the above MILP for a 6-node network. The traffic matrix T for 6-node network was generated randomly by Matlab from a uniform distribution in (0,1).

$$\mathbf{T} = \begin{bmatrix} 0.00 & 0.53 & 0.52 & 0.71 & 0.80 & 0.79 \\ 0.53 & 0.00 & 0.42 & 0.23 & 0.85 & 0.14 \\ 0.52 & 0.42 & 0.00 & 0.64 & 0.33 & 0.51 \\ 0.71 & 0.23 & 0.64 & 0.00 & 0.61 & 0.68 \\ 0.80 & 0.85 & 0.33 & 0.61 & 0.00 & 0.25 \\ 0.79 & 0.14 & 0.51 & 0.68 & 0.25 & 0.00 \end{bmatrix}$$

The resulted logical network is shown in Figure 4.1.

4.1.2 Survivable routing

In this section, we overlay the logical topology we got from the first stage over the physical topology in a survivable manner, and we require the logical topology to support

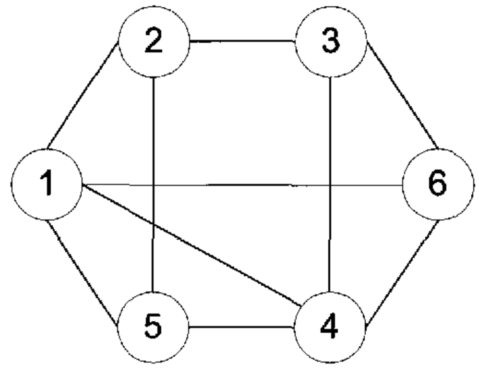

Figure 4.1: Designed logical topology

a surviving traffic matrix under physical failures. We assume that the WDM network can provide unprotected and 1+1 protected lightpath services.

First, we discuss the survivable routing problem dealing with a single fiber link cut. We modify the MILP in Section 2.3.2 by replacing its survivability constraints. We are given the logical topology obtained from the first stage, the physical topology and another $|N_L| \times |N_L|$ traffic matrix R. R represents the traffic matrix that must be supported when there is a single fiber link cut, and we refer to it as the *"surviving traffic"*. It is symmetric about the diagonal and its diagonal entries are zeros. Obviously, each entry of R should be less than or equal to the corresponding entry of matrix T. That is, for each node pair (s,t), $R[s,t] \leq T[s,t]$. Let $r_{st}^{(i,j)p.q}$ be a nonnegative real valued variable which represents the traffic from node p to q going through logical link (s,t) when fiber link (i,j) fails. Let variables F_{st} and f_{ij}^{st} be defined as in Section 2.3. We assume that the capacity of a logical link is 1. Then $F_{st} - (f_{ij}^{st} + f_{ji}^{st})$ is an upper bound of the flow on logical link (s,t) when fiber link (i,j) fails.

We assume that we can split the traffic from p to q arbitrarily over different paths through the logical topology. The survivability constraint can be expressed as, for each (i,j) in E_P such that $i < j$ and each s, p, q in N_L such that $p < q$:

$$\sum_{t:(s,t)\in E_L} r_{st}^{(i,j)p.q} - \sum_{t:(t,s)\in E_L} r_{ts}^{(i,j)p.q} = \begin{cases} R[p,q] & \text{if } s = p \\ -R[p,q] & \text{if } s = q \\ 0 & \textit{otherwise} \end{cases}$$

The solution satisfying the above survivability constraints will give us the routes from node p to node q in the logical topology after fiber link (i,j) is cut, which supports traffic rate $R[p,q]$.

We assume that the capacity of each surviving logical link equals 1. Thus, for each (i,j) in E_P such that $i < j$ and (s,t) in E_L such that $s < t$:

$$\sum_{\substack{p,q \in N_L: \\ p<q}} \left(r_{st}^{(i,j)p.q} + r_{ts}^{(i,j)p.q}\right) \leq 1$$

Our objective is to minimize the total wavelength cost. The complete MILP formulation can be expressed as follows.

$$\min \sum_{(i,j) \in E_P} \sum_{\substack{(s,t) \in E_L: \\ s<t}} f_{ij}^{st}$$

Subject to :

1. Integer flow constraints: for each (s,t) in E_L such that $s < t$ and each (i,j) in E_P:

$$f_{ij}^{st} \in \{0, 1\}$$

2. Survivable traffic flow capacity constraints: for each (i,j) in E_P such that $i < j$, each p, q in N_L such that $p < q$, and each (s,t) in E_L:

$$r_{st}^{(i,j)p.q} \geq 0$$

3. IP topology integer connection constraints: for each (s,t) in E_L such that $s < t$:

$$F_{st} \in \{1, 2\}$$

4. Connectivity constraints: for each (s,t) in E_L such that $s < t$ and each i in N_P:

$$\sum_{j:\, (i,j) \in E_P} f_{ij}^{st} - \sum_{j:\, (j,i) \in E_P} f_{ji}^{st} = \begin{cases} F_{st} & \text{if } s = i \\ -F_{st} & \text{if } t = i \\ 0 & \mathit{otherwise} \end{cases}$$

5. Disjoint routing constraints for the protected lightpaths: for each (s, t) in E_L such that $s < t$ and each (i, j) in E_P such that $i < j$:

$$f_{ij}^{st} + f_{ji}^{st} \leq 1$$

6. Survivable IP link capacity constraints: for each (i, j) in E_P such that $i < j$, each p, q in N_L such that $p < q$ and each (s, t) in E_L such that $s < t$:

$$r_{st}^{(i,j)p.q} \leq F_{st} - (f_{ij}^{st} + f_{ji}^{st})$$
$$r_{ts}^{(i,j)p.q} \leq F_{st} - (f_{ij}^{st} + f_{ji}^{st})$$

7. Survivability traffic flow conservation constraints: for each (i, j) in E_P such that $i < j$ and each s, p, q in N_L such that $p < q$:

$$\sum_{t:(s,t)\in E_L} r_{st}^{(i,j)p.q} - \sum_{t:(t,s)\in E_L} r_{ts}^{(i,j)p.q} = \begin{cases} R[p,q] & \text{if } s = p \\ -R[p,q] & \text{if } s = q \\ 0 & otherwise \end{cases}$$

8. IP topology link original capacity constraints : for each (i, j) in E_P such that $i < j$ and each (s, t) in E_L such that $s < t$:

$$\sum_{\substack{p,q\in N_L: \\ p<q}} (r_{st}^{(i,j)p.q} + r_{ts}^{(i,j)p.q}) \leq 1$$

Since we are not given any information about the physical topology at the first stage, we have to assume that the costs of all the possible logical links are the same. In fact, logical links may have different cost since their lightpaths may have different fiber hops. Thus, we can see this two-stage design may not give us the final optimal solution (total wavelength cost).

As an example, we embedded the 6-node logical topology in Section 4.1.1 on a random 6-node physical topology shown in Figure 4.2. The following matrix is the traffic matrix R that the 6-node network needs to support after any single fiber link failure. Note that $R = \frac{1}{2}T$.

$$\mathbf{R} = \begin{bmatrix} 0.00 & 0.27 & 0.26 & 0.35 & 0.40 & 0.40 \\ 0.27 & 0.00 & 0.21 & 0.12 & 0.42 & 0.07 \\ 0.26 & 0.21 & 0.00 & 0.32 & 0.17 & 0.26 \\ 0.35 & 0.12 & 0.32 & 0.00 & 0.30 & 0.34 \\ 0.40 & 0.42 & 0.17 & 0.30 & 0.00 & 0.13 \\ 0.40 & 0.07 & 0.26 & 0.34 & 0.13 & 0.00 \end{bmatrix}$$

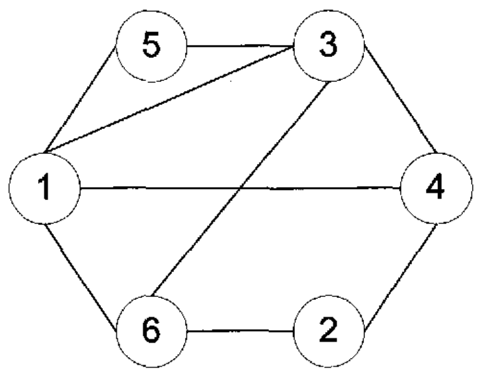

Figure 4.2: A 6-node physical topology

In Table 4.1, we summarize results of some examples. For each example, we first design a logical topology based on a randomly generated matrix T. Then route it over a random physical topology. The physical topologies are shown in Figure 4.3. The logical topology must support traffic matrix R under any singlelfiber nk cut. For simplicity, we take $R = \frac{1}{2}T$. The 6-node network is labelled as 6-node, and so forth. The run-times have always been our concern when we formulate MILPs. As expected, this problem becomes computationally difficult for larger networks. In each stage design, the run-times increased dramatically with the size of the network. For the physical topology with nodes less than 10, we could get the final result after the second stage within 8 minutes.

For the survivable routing problem for SRLGs, we can use a modification of the MILP in Chapter 3. The modification replaces the survivability constraint with the following one. We let $r_{st}^{k;(p,q)}$ be a nonnegative real valued variable between 0 and 1 which represents the traffic from node p to q routed on logical link (s,t) when SRLG$[k]$ fails. The survivability constraints are given below.

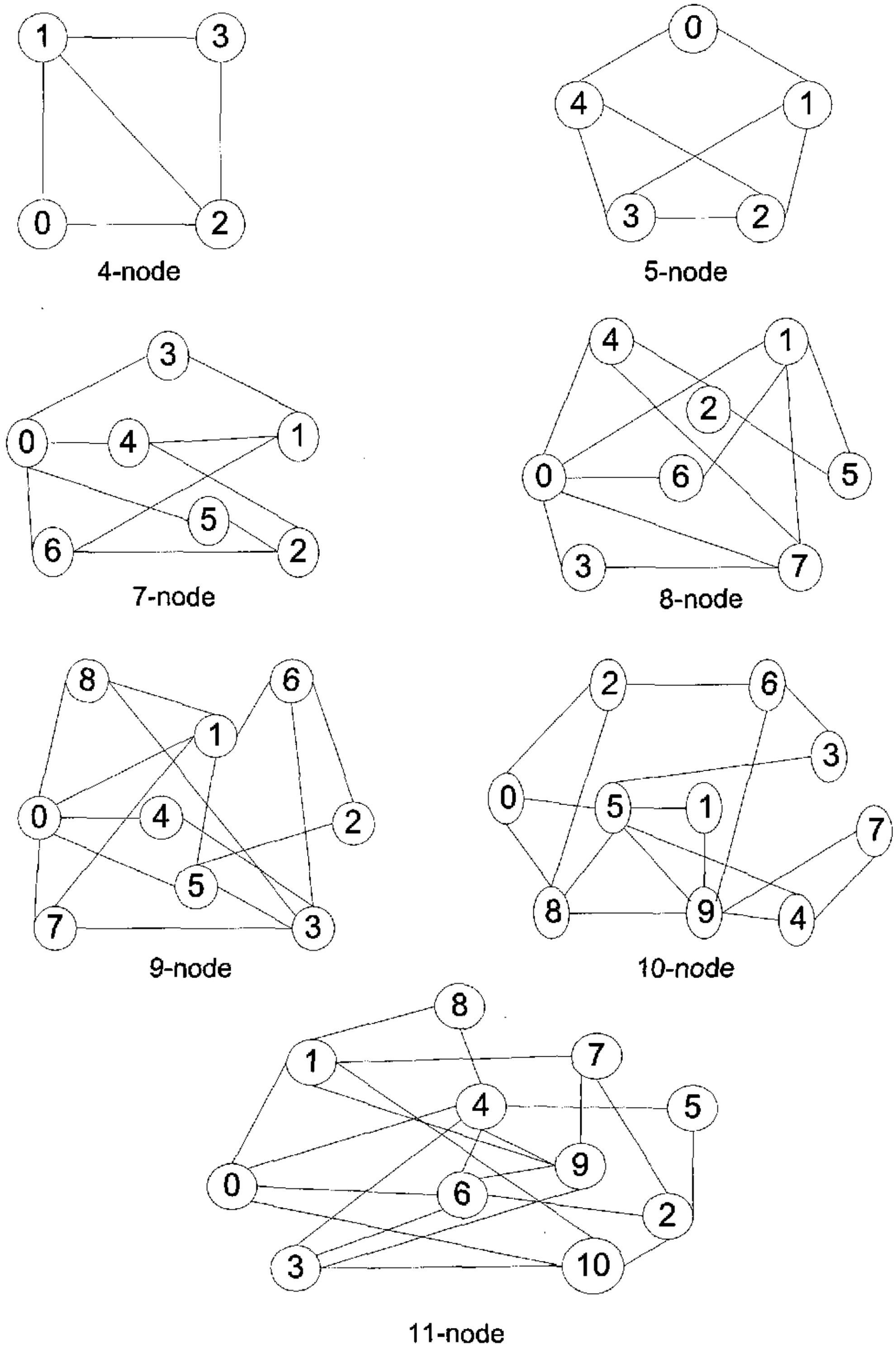

Figure 4.3: Underlying physical topologies

Table 4.1: Results of the two-stage design

	Stage one			Stage two		
	Elapsed run-times	CPU run-times	Logical links	Elapsed run-times	CPU run-times	Total λ links
4-node	0.53 sec.	0.36 sec.	4	0.61 sec.	0.50 sec.	9
5-node	0.54 sec.	0.38 sec.	8	1.49 sec.	1.34 sec.	12
6-node	0.58 sec.	0.45 sec.	10	8.86 sec.	8.77 sec.	20
7-node	11.76 sec.	11.48 sec.	8	22.50 sec.	22.37 sec.	21
8-node	40.23 sec.	40.08 sec.	12	37.50 sec.	37.37 sec.	18
9-node	2 min. 42 sec.	2 min. 42 sec.	15	1 min. 56 sec.	1 min. 55 sec.	23
10-node	4 min. 23 sec.	4 min. 22 sec.	18	9 min. 44 sec.	9 min. 45 sec.	29
11-node	5 min. 13 sec.	5 min. 12 sec.	21	29 min. 29 sec.	29 min. 14 sec.	41

For $k = 1, 2, \ldots, m$ and each $s \in N_L$

$$\sum_{t:\ (s,t)\in E_L} r_{st}^{k.(p,q)} - \sum_{t:\ (t,s)\in E_L} r_{ts}^{k(p,q)} = \begin{cases} R[p,q] & \text{if } s = p \\ -R[p,q] & \text{if } s = q \\ 0 & otherwise \end{cases}$$

The total flow on logical link (s,t) is restricted by the following constraint. For $k = 1, 2, \ldots, m$ and each (s,t) in E_L such that $s < t$:

$$\sum_{\substack{p,q \in N_L: \\ p<q}} (r_{st}^{(i,j)p.q} + r_{ts}^{(i,j)p.q}) \leq 1$$

All the other variables and constraints are similar to Section 3.2. Each feasible solution resulted from the modified MILP gives us a layout that can support traffic matrix R when a SRLG fails. From the value of $r_{st}^{k.(p,q)}$, we will know the traffic route from p to q in the logical topology after the kth SRLG fails.

4.2 One-stage design

We now consider a one-stage design procedure. Given the underlying physical topology, the nominal traffic T and surviving traffic R between each pair of logical nodes, we can design the logical topology and find its survivable routing at the same time. This one-stage design leads to an optimal solution. However, it leads to a much larger time-complexity compared to the two-stage design. We assume that the WDM network provides unprotected and protected lightpath services. We formulate the logical topology design and survivable routing problem as an MILP. The MILP formulation discussed in this section is for a single fiber link cut.

4.2.1 Problem formulation

The nominal traffic matrix T, the surviving traffic matrix R, and the physical topology are given. To give the MILP formulation, we would like to redefine some variables in Section 4.1. Let F_{st} be the number of connections set up between s and t. If there does not exist a logical link between s and t, $F_{st} = 0$. If logical link (s, t) is realized by an unprotected lightpath, $F_{st} = 1$, and if logical link (s, t) is realized by 1+1 protected lightpath, $F_{st} = 2$. We can see that the value of F_{st} is related to the logical topology. The logical topology is unknown, and will be specified by variables $\{b_{ij}\}$. The variable b_{ij} is binary valued such that $b_{ij} = 1$, if there is a logical link between i and j, and $b_{ij} = 0$ otherwise. The following constraint states the relationship between F_{st} and b_{ij}. For each s, t in N_L and i, j in N_L such that $s < t$, $i < j$, $i = s$ and $j = t$:

$$b_{ij} \leq F_{st} \leq 2b_{ij}$$

This constraint ensures that if there is no logical link between s and t, $F_{st} = 0$ and if there exists a logical link between s and t, F_{st} could be 1 or 2. Variable $r_{st}^{(i,j)p,q}$ is the traffic flow from node p to q going through logical link (s, t) (if it exists) after fiber link (i, j) fails.

Our objective is to minimize the total wavelength cost. The MILP formulation for the logical topology design and its survivable layout can be expressed as follows.

$$\min \sum_{(i,j)\in E_P} \sum_{\substack{s,t\in N_L:\\ s<t}} f_{ij}^{st}$$

Subject to :

1. Nonnegative variable constraints: for all i, j, s, t in N_L such that $s < t$:

$$t_{ij}^{st} \geq 0$$

2. Integer variable constraints: for each i, j in N_L such that $i < j$:

$$b_{ij} \in \{0, 1\}$$

3. Integer connection constraints: for each s, t in N_L such that $s < t$:

$$F_{st} \in \{0, 1, 2\}$$

4. Integer flow constraints: for each s, t in N_L such that $s < t$ and each $(i, j) \in E_P$:

$$f_{ij}^{st} \in \{0, 1\}$$

5. IP topology flow constraints: for each (i, j) in E_P such that $i < j$, and each s, t, p, q in N_L such that $p < q$:

$$r_{st}^{(i,j)p,q} \geq 0$$

6. Logical topology connectivity constraints: for each s, t, i in N_L such that $s < t$:

$$\sum_{j \in N_L} t_{ij}^{st} - \sum_{j \in N_L} t_{ji}^{st} = \begin{cases} T[s,t] & \text{if } s = i \\ -T[s,t] & \text{if } t = i \\ 0 & otherwise \end{cases}$$

7. Link capacity constraints: for each i, j in N_L such that $i < j$:

$$\sum_{\substack{s,t \in N_L: \\ s<t}} (t_{ij}^{st} + t_{ji}^{st}) \leq b_{ij}$$

8. IP topology integer connection constraints: for each s, t in N_L and i, j in N_L such that $s < t$, $i < j$, $i = s$ and $j = t$:

$$b_{ij} \leq F_{st} \leq 2b_{ij}$$

9. Connectivity constraints: for each s, t in N_L such that $s < t$ and each i in N_P:

$$\sum_{j:\ (i,j) \in E_P} f_{ij}^{st} - \sum_{j:\ (j,i) \in E_P} f_{ji}^{st} = \begin{cases} F_{st} & \text{if } s = i \\ -F_{st} & \text{if } t = i \\ 0 & otherwise \end{cases}$$

10. Disjoint routing constraints for the protected lightpath: for each s in N_L, t in N_L such that $s < t$ and each (i, j) in E_P such that $i < j$:

$$f_{ij}^{st} + f_{ji}^{st} \leq 1$$

11. Survivable traffic constraints: for each (i, j) in E_P such that $i < j$ and each s, t, p, q in N_L such that $s < t$ and $p < q$:

$$r_{st}^{(i,j)p.q} \leq F_{st} - (f_{ij}^{st} + f_{ji}^{st})$$
$$r_{ts}^{(i,j)p.q} \leq F_{st} - (f_{ij}^{st} + f_{ji}^{st})$$

12. Survivability constraints: for each (i, j) in E_P such that $i < j$ and each s, p, q in N_L such that $p < q$:

$$\sum_{t \in N_L} r_{st}^{(i,j)p.q} - \sum_{t \in N_L} r_{ts}^{(i,j)p.q} = \begin{cases} R[p, q] & \text{if } s = p \\ -R[p, q] & \text{if } s = q \\ 0 & \textit{otherwise} \end{cases}$$

13. IP topology link original capacity constraints: for each (i, j) in E_P and each s, t in N_L such that $s < t$:

$$\sum_{\substack{p,q \in N_L: \\ p<q}} r_{st}^{(i,j)p.q} + r_{ts}^{(i,j)p.q} \leq 1$$

The solution of this MILP gives us a logical topology and its survivable layout over the physical topology. The values of $\{b_{ij}$: for each i, j in $N_L\}$ will specify the designed logical topology, and the value of F_{st} will specify the protection type of the lightpath for logical link (s, t). The resulting $r_{st}^{(i,j)p.q}$ determines the flow on logical link (s, t) carrying the traffic from node p to node q after fiber link (i, j) fails.

The MILP for the SRLG could be formulated easily based on the same idea of the MILP for the single fiber link cut.

4.2.2 Experimental results

We solved the MILP in Section 4.2.1 for the 4-node, 5-node and 6-node networks. For each of them, we used the same traffic matrices T, R and physical topologies as the examples in Table 4.1. We tabulate the results in Table 4.2. We can see that, for the total wavelength link cost, the two-stage design got a sub-optimal solution while one-stage design obtained an optimal solution. The run-times are also compared. As expected, the one-stage approach consumed much more time than the two-stage approach. As can be seen from Table 4.2, for the 6-node network, the one-stage procedure took about 20 minutes while the two-stage procedure only took about 9 second. In practice, since one-stage approach takes much more time than the two-stage approach, we may choose the two-stage design approach although the solution is sub-optimal.

Table 4.2: One-stage vs. two-stage

	4-node			5-node			6-node		
	Total CPU times	Total IP links	Total λ links	Total CPU times	Total IP links	Total λ links	Total CPU times	Total IP links	Total λ links
Two-stage	0.86 sec.	4	9	1.72 sec.	8	12	9.22 sec.	10	20
One-stage	2.05 sec.	5	5	58.70 sec.	8	11	20 min. 23 sec.	11	15

Chapter 5

Conclusion

In this thesis, we considered the problem of overlaying the IP network over the WDM network so that the IP network is still connected under physical failures. In Chapter 2, we reviewed the ILP formulation in [1] dealing with any single fiber link failure. Due to the exponential growth of the number of the survivability constraints, to solve this ILP becomes very difficult for larger networks. We modified this ILP to an MILP based on a new necessary and sufficient condition for the survivability of the IP network. The proposed MILP has a polynomial number of survivability constraints. We conducted many network examples to illustrate that our MILP improves the run-time dramatically compared to the ILP in [1] and it could be solved for large networks. We also formulated an ILP and an MILP for a scenario where the WDM network can provide unprotected and 1+1 protected lightpath services. In this scenario, we can always find the survivable routing of the IP network over the WDM network.

The proposed survivable routing condition is not restricted to dealing with any single fiber link failure, so we formulated an MILP to embed the logical topology on the physical topology while withstanding the shared risk link group failure in Chapter 3.

In Chapter 4, we addressed a realistic scenario where we design the logical topology based upon a nominal traffic matrix, and then find its survivable routing over a physical topology. The logical topology must support a surviving traffic matrix under physical failures. We proposed two different approaches to solve this problem. One is the two-stage design while the other is the one-stage design. The one-stage design results in an optimal solution while the two-stage design may only get a sub-optimal final solution. However, the two-stage design has a lower time-complexity than the one-stage design.

Many extensions are possible for this work. In Chapter 3, while we focused on the share risk link group consisting of the fiber links in a conduit, all the fiber links to an OXC is another kind of SRLG that can be considered. For the problem in Chapter 4, one-stage design leads to a large time-complexity even for a 7-node network. Some new ideas may come up to solve this difficulty by formulating the MILP in a different way. The fiber link capacity constraints and wavelength continuity constraints may be incorporated to the problem formulations. While our final objective of the ILP/MILP is to minimize the total number of wavelength links occupied by the lightpaths, some other kinds of costs such as the total number of physical links used, the total number of connections set up, etc., can be minimized for different application interests.

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