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AN EXPERIMENTAL INVESTIGATION OF TRANSITION RADIATION IN THE X-RAY REGION.

University of Hawaii, Ph.D., 1976
Physics, general

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AN EXPERIMENTAL INVESTIGATION OF
TRANSITION RADIATION IN THE X-RAY REGION

A DISSERTATION SUBMITTED TO THE GRADUATE DIVISION OF THE
UNIVERSITY OF HAWAII IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
IN PHYSICS
MAY 1976

By
TOMOTARO KATSURA

Dissertation Committee
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  W. R. Steiger
  V. J. Stenger
To my parents
ABSTRACT

Transition Radiation (TR) in the x-ray region was detected at the Stanford Linear Accelerator Center (SLAC) with electrons and pions at momenta 3, 9, and 15 GeV/c. The x-ray detectors were multiwire proportional chambers (MWPC) filled with either a krypton-methane or xenon-methane mixture. The transition radiators were made of 100-180 Mylar or polyethylene foils of thicknesses 1/2, 1, and 2 mils equally spaced with 100-125 mil air gaps.

Two types of arrays (radiator and chamber combination) were used: (1) a sandwich array (SW) consisting of 8 modules of radiator-and-"thin" chamber combinations. The chambers were made thin so that a major competing process, i.e., ionization loss, could be reduced to a minimum; and (2) the magnetic separation array (MS) being composed of a radiator, a weak magnetic field, and a "thick" chamber in series. The weak field swept away the incident charged particles from TR x-rays which could be detected without being mixed in the large signals of ionization loss. The MS array measured successfully a "pure" spectrum of transition radiation.

The chambers were calibrated not only with standard x-ray sources (Fe$^{55}$, Cd$^{109}$, and Pm$^{145}$) but also with monochromatic x-ray beams. The calibration of the chambers were cross-checked with the constancy of ionization loss of the penetrating charged particles.
The results are in good agreement with the predictions for the shape of transition radiation spectra, the linear dependence of the total yield on the Lorentz factor $\gamma$ of particles up to 10,000, and the "saturation" in the higher $\gamma$ region. The chamber detection efficiency was precisely studied and corrections to the predicted yield were made for absorption, range escape, fluorescence yield, etc.

A clear discrimination between pions and electrons was obtained in the SW array. The results were compared with some other groups' results for the resolution of the TR detector system in the measurement of $\gamma$.

The MS array is very promising as an efficient TR detector system because of its good signal-to-background ratio. Two possible directions in the development of particle identification are discussed for particles in several hundred GeV region such as pions and kaons.
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1. INTRODUCTION

The possible use of transition radiation (TR) as a means of identifying ultra-high energy particles has recently become a subject of experimental investigations. Our intention in the present work is first to study the physics of these phenomena and as a goal to find a way of building a detector which can efficiently identify the individual particles.

The existence of transition radiation was predicted in 1946 by V. L. Ginzburg and I. M. Frank. The radiation is emitted when a particle moves uniformly across the interface of two media with different dielectric constants. The radiation which is predicted directly from the equations of electrodynamics, however, was not studied experimentally for a long time because the expected number of quanta was exceedingly small. After the first observation of the transition radiation in the optical region, many early studies indicated that the application of the optical transition radiation for the detection and identification of individual particles seemed to be severely limited owing to the inherent low intensity of the radiation.

Interests in transition radiation was renewed when G. M. Garibian showed that the radiation also should appear in the x-ray region for ultra-relativistic particles. His theory predicted some remarkable features for transition radiation in the x-ray region. First, the total energy loss
by a charged particle should be proportional to the Lorentz factor of the particle, $\gamma (\gamma = E/m$, where $E$ is the energy of the particle and $m$ its rest mass in units in which $c=1$). This is quite attractive for determining the energy of the particles because some of the known methods of detection becomes marginal at high values of $\gamma$, e.g., the Cerenkov radiation detector. Second, as in any other source of radiation from relativistic particles, the radiation is extremely forward peaked within the angle of order $1/\gamma$. This fact could make the detection of transition radiation complicated by the necessity of separating the charged particles from the radiation. Third, the electrodynamic nature of the radiation indicates that in an interface between two different dielectric media, the number of photons per particle is of order $\alpha = 1/137$. Nevertheless, by using a multilayered dielectric radiator spaced in vacuum, it is possible to cumulatively increase the radiation yield. The required minimum space between layers is proportional to the radiation wave-length and is correspondingly shorter for radiation in the x-ray region than in the optical region.

Immediately, the prediction of the x-ray transition radiation attracted attention of the experimentalists. The works of the Soviet physicists\textsuperscript{7-13} and of Yuan and collaborators\textsuperscript{14,15,16} using beams of high energy electrons provided the evidence for the existence of transition radiation in both the optical and x-ray regions. The purpose
of these experiments was to determine $\gamma$ of a single particle by using a combination of multilayered radiators and specifically designed x-ray detectors. The detectors were mainly solid state counters, streamer chambers, and sodium iodide scintillators.\textsuperscript{12,13,14} In due course, proportional chambers were introduced as more convenient x-ray detectors.\textsuperscript{16,17} In spite of these very extensive works, it was still a marginal radiation and not convincing to physicists that transition radiation would be useful for the identification of ultra-relativistic particles.

Calculations made by Professor Yodh predicted that the Hawaii set of multiwire proportional chambers (MWPC) should be able to detect a transition radiation signal from 3 GeV/c electron well above normal ionization pulses. Our existing set of MWPC's, built for our quantameter test, had the desirable properties of thin windows to detect the ionization and transition radiation with sufficient separation. Clearly, if such an effect were readily detectable with reasonably simple apparatus, it would make a useful detector to distinguish pions from kaons in the 100-500 GeV/c momentum range. It was therefore felt that an independent confirmation of the existence and properties of transition radiation, especially due to a single particle, was highly desirable.

At the Berkeley Bevatron, where the proportional chambers were available from the quantameter test, a preliminary experiment was made by the collaborators from Hawaii,
Maryland, and Oxford. Eleven MWPC's, each preceded by transition radiator stacks (sandwich array), were exposed to the beam of electrons. The electron beam intensity was as small as $10^{-4}$ per pion because the electrons were decay products of pions of a secondary beam from the proton accelerator. In the sandwich array, the same detector observed the transition radiation x-rays as well as the ionization of the primary particle. The results were obtained as several kinds of pulse height distributions of transition radiation energy deposited in the MWPC's. For example, a pair of histograms show the separation between pions and electrons at the same momentum. The average pulse height observed in the eleven chambers filled with krypton gas were 64 keV for pions and 122 keV for electrons at 3 GeV/c momentum. These results confirmed that transition radiation was readily observable with an array of MWPC's and foil radiators and helped to put transition radiation on a firm basis.\textsuperscript{18}

Encouraged by the results of the Bevatron experiment, we proceeded to conduct a more extensive experiment under a better beam condition at SLAC, where well-focused and intense beams of nearly pure electrons (pions) were available. The objectives of the SLAC run were to study the experimental properties of the transition radiation phenomena (e.g., spectra and gamma dependence of the energy loss) and to search for possible methods for developing a highly efficient device for the identification
of individual particles. Some important features of the experiment, which will be discussed in the main text, are the following: (a) A new system in which the primary particle was separated from the radiation x-rays by means of a magnetic field (magnetic separation array); (b) Well-sealed chambers were filled with xenon gas for providing better detection efficiency of high energy x-rays; (c) Polyethylene as well as Mylar foils were used for a radiator stack as the former is less absorptive than the latter.

Some of the results from the SLAC experiment are given below. First, the transition radiation signal detected from ultra-relativistic particles is as large as the ionization signal in the same thickness of a gas-filled MWPC, i.e., a transition radiation signal is easily detectable. Second, the magnetic separation method works as predicted. Complete separation of transition radiation photons from high energy (3-15 GeV) electrons has been achieved. The magnetic separation method requires a long lever arm in order to avoid background from synchrotron radiation of electrons. It is easier to apply this method to heavy particles (pions, kaons, protons) since synchrotron radiation is then not a problem. Furthermore, the results of the SLAC experiment are in good agreement with theory. The gamma-dependence of the transition radiation signal has been measured well beyond the linear region and saturation effects have been noted.
We are convinced that the present work will provide information necessary for physicists to design transition radiation detectors in different experimental situations and that transition radiation detectors should become a practical device in the several hundred GeV energy region (Fermilab energies).

Section 2 reviews the fundamental formulas of transition radiation theory primarily established by Soviet physicists. Both qualitative and quantitative analysis of the theoretical formulas are given. Some computational methods to determine the radiation yields are developed. Section 3 discusses the experimental methods on how to detect transition x-rays and how to identify particles with the detected signals. The Bevatron experiment is briefly reviewed as a preliminary to the SLAC experiment. Section 4 presents the devices and procedures in the SLAC experiment. Section 5 gives the data analysis. The criteria to choose good events are listed and the calibration procedure of the chambers is shown. Various corrections for both experimental and theoretical results are discussed. Section 6 summarizes the results and gives conclusions.
2. THEORETICAL CALCULATION

Transition Radiation is the term used to describe a radiation emitted when a charged particle crosses a boundary between two different dielectric media.

The first theoretical description of this phenomena was given by Ginzburg and Frank\(^1\) in 1946 long before any experimental studies had been started. The radiation from a uniformly moving charge in a medium with variable dielectric constant is analogous to the radiation from a particle moving non-uniformly in vacuum. In both cases the radiation is related to the phase velocity of the electromagnetic waves in the given medium and the velocity of the charged particle. The difference is that in the former case the phase velocity of the wave changes, while in the latter the particle velocity changes. (In this sense, TR and bremsstrahlung have the same origin.)

In Section 2.1, first the TR intensity for non-relativistic particle is derived directly from the analogy of the phenomenon to bremsstrahlung by taking into account the optical properties of the medium. This will give some idea about the radiation mechanism. Next, we review the general formulation of TR obtained by direct application of the Maxwell's equations to the process. The derivations of the formulas have been intensively studied by many authors. Garibian\(^{19}\) has written an
excellent review of the subject. Also, Ter-Mikaelyan's text\textsuperscript{20} gives a complete derivation. A more educational and quicker method is shown in Jackson's new textbook.\textsuperscript{21} In quoting those results, we focus on the properties of radiation produced by relativistic particles, especially in the x-ray region, where the formulas will be greatly simplified. The summation of the yield from many layers, which are necessary to strengthen the feeble radiation, also becomes simpler and is easily obtained only by considering the phase shift along the particle path through the multifoil radiator.

In Section 2.2, we study the TR quantitatively. First, we will review the spectrum of TR produced by a particle entering a boundary from vacuum to the foil medium and show that the total energy loss of the particle is proportional to the Lorentz factor of the particle. Secondly, the yield from a single foil of dielectric will be shown to be the coherent summation of TR from both surfaces of the foil placed in a gas. Finally, the yield from a radiator stack consisting of N foils will be formulated.

In Section 2.3, we will discuss the modification of the formulas obtained above to include the absorption of TR photons by the radiator material itself. As the absorption coefficients are strong functions of energy, the shape of the TR spectrum may be modified greatly. The total yield emerging from a multifoil stack will be reduced significantly due to the self-absorption.
2.1 Derivation of Radiation Flux

a. Non-Relativistic Derivation

**On the Boundary of Ideal Conductors**

As an extreme case of the phenomena, assume that a non-relativistic charged particle enters a metal from vacuum. The metal is assumed to be perfectly conductive. The TR is identical with the optical portion of the bremsstrahlung spectrum because the particle is brought to a complete stop at the surface of the metal. For a charge moving at a constant velocity \( v \) normal to the surface, the energy radiated into solid angle \( d\Omega \) at an angle \( \theta \) to the trajectory of the particle is given by\(^\text{23}\)

\[
\frac{dW}{d\omega d\Omega} = \frac{e^2 v^2}{\pi c^3} \sin^2 \theta
\]  \hspace{1cm} (1)

in a bandwidth of frequency \( d\omega \) (corresponding to a sudden deceleration \( v \rightarrow 0 \)). The angular distribution of the radiation is the same as for a dipole placed on the surface. The radiation intensity is proportional to \( v^2 \). Therefore, the radiation is the same when particle the goes into vacuum from a metal.

**On the Boundary of Dielectrics**

For the dielectric material, we can extend this method to calculate the radiation intensity.\(^\text{24}\) The amplitude of the radiation field may be represented by three terms:
(1) The field of the electron moving in vacuum and stopping suddenly at a surface of medium, point A in Fig. 1.

(2) The field of the electron which begins at the same moment as its motion from the same point into the medium, point B.

(3) The field of electrical image (point B') of the particle moving in the medium. This is equivalent to the field of some virtual particle moving suddenly at a velocity $v$ from the medium towards point B'.

The first term (1) represents the bremsstrahlung field which appears when the particle is stopped. The second term (2) gives the bremsstrahlung when the particle is suddenly accelerated. Together, they represent the field of a particle in uniform motion. When a material transition occurs, we must allow also for the presence of the third term (3). We thus obtain a formula for the spectral density of the radiation per unit solid angle in vacuum along a direction making an angle $\theta$ with the normal line to the surface, i.e.,

$$\frac{dW}{d\omega d\Omega} = \frac{e^2 v_z}{4\pi^2 c^3} \sin^2 \theta n |1 + r - nf|^2.$$  \hspace{1cm} (2)

Here $v_z$ is the velocity of the particle directed along the normal to the surface of the medium. The unity in the modulus represents the field of a particle starting at the boundary. The third term gives the field of an incident particle discontinuing its motion in the medium. The symbol
Fig. 1. Schematic Explanation of the Theoretical Calculation in Eq. (2). AB is the path of a particle having a charge e and a velocity v and moving from vacuum into a medium. B' is the electrical image of the particle B moving inside the medium away from the surface. The radiation is emitted at an angle θ with the normal to the surface.

Fig. 2. Coordinate of the Wave Number k for Eq. (5) on the surface Between the Two Medium ε₁ and ε₂. The angle θ of the wave number is measured from the normal, i.e., the direction of the particle flight.
f is Fresnel's coefficient for the refracted wave. n is the complex refractive index of the medium. The middle term gives the field of the electrical image of the particle. The quantity r is Fresnel's coefficient for the reflected wave. The Fresnel coefficients r and f are given by

\[ r = \frac{\cos \theta - n \sqrt{1 - n^2 \sin^2 \theta}}{\cos \theta + n \sqrt{1 - n^2 \sin^2 \theta}} : \text{reflected wave} \quad (3) \]

\[ f = \frac{2n \cos \theta}{\cos \theta + n \sqrt{1 - n^2 \sin^2 \theta}} : \text{refracted wave} \quad (4) \]

The angle $\theta$ is measured in the medium for forward radiation (Fig. 1). In an optical isotropic medium, the electric vector of the light wave lies in a plane defined by the normal to the surface and the direction of the beam, so that the transition radiation is completely polarized. It follows that for $\varepsilon = n^2 + \infty$, i.e., an ideal conductor, radiation density (2) reduces to (1).

b. Relativistic Treatment

In the relativistic case, a more general formulation is obtained by solving directly the Maxwell equations with the appropriate continuity conditions imposed at the boundary of the two media. The results of this formulation are given in many theoretical works.\(^6,20,25\) Hence we simply quote the results and discuss the properties. At the boundary of two different dielectrics $\varepsilon_1$ and $\varepsilon_2$ ($\mu_1 = \mu_2 = 0$ is assumed), a charged particle with
constant velocity \( v \) emits the TR into the forward hemisphere with its intensity (Fig. 2)

\[
\frac{d^2W}{d\omega d\Omega} = \frac{e^2v^2 \sin^2 \theta \cos^2 \theta \sqrt{\varepsilon_2}}{\pi^2 c^3 (1 - \beta^2 \varepsilon_2 \cos^2 \theta)}
\]

\[
k \left[ \frac{(\varepsilon_1 - \varepsilon_2)(1 - \beta^2 \varepsilon_2 - \beta \sqrt{\varepsilon_1 - \varepsilon_2 \sin^2 \theta})}{(1 - \beta \sqrt{\varepsilon_1 - \varepsilon_2 \sin^2 \theta})(\varepsilon_1 \cos \theta + \sqrt{\varepsilon_1 \varepsilon_2 - \varepsilon_2^2 \sin^2 \theta})} \right]^2.
\]

It can be seen that in the limit \( \beta \to 0 \), i.e., for non-relativistic particles, (5) reduces to (2) by setting \( \varepsilon_1 = 1 \) and \( \varepsilon_2 = \varepsilon \).

For the relativistic case, many new features of TR will be obtained. For instance, for the opposite direction of velocity, i.e., when the particle moves from medium to vacuum, the sign in front of \( \beta \) in all terms must be reversed. Thus, in contrast to a non-relativistic particle, the radiation in the present case depends on the sign of the particle velocity with respect to the medium, i.e., a characteristic directionality appears in this case.

The most attractive feature is that TR by relativistic particles also appears in the x-ray region. For its simplicity, let us assume that \( \varepsilon_1 = \varepsilon \) and \( \varepsilon_2 = 1 \) corresponding to the case when the incident particle passes the boundary from medium to vacuum. The energy flux in the vacuum because of transition radiation is found as
For relativistic particles ($\gamma \gg 1$), it can be seen that the radiation has a sharp maximum at a small angle. The term $(1 - \beta^2 \cos^2 \theta)$ in the denominator becomes minimum at the angle of order $1/\gamma$ because the term can be approximated as $1 - \beta^2 \cos^2 \theta = 1 - (1 - \gamma^2) \cos^2 \theta \approx \gamma^{-2} + \theta^2$ for large $\gamma$ and small angle ($\gamma = 1/\sqrt{1 - \beta^2}$). The greatest contribution to the spectrum results when the deviation of $\varepsilon$ from unity becomes small. The expression for $\varepsilon$ at high frequency is given by

$$\varepsilon = 1 - \omega_p^2/\omega^2 = 1 - \xi^2,$$  \hspace{1cm} (7)

where $\omega_p$ is plasma frequency and is given by

$$\omega_p = \sqrt{\frac{4n_e^2}{m}} = 28.8 \sqrt[3]{\frac{\rho Z}{A}} \text{ eV},$$

with

- $n$: volume density of electrons (cm$^{-3}$)
- $m$: electron mass (gm)
- $\rho$: density of material (gm cm$^{-3}$)
- $Z$: atomic number
- $A$: atomic weight
The deviation $\xi^2$ from unity at the high frequency can become the same order of $1/\gamma$. Taking into account this fact, the main contribution to the integral in (6) is at frequencies larger than the optical frequencies because of the existence of the small factor in the denominator $1 - \beta \sqrt{\varepsilon - \sin^2 \theta}$. This factor is of order of $\gamma^{-2}$ as

$$1 - (1 - \gamma^{-2})^{\frac{1}{2}}(1 - \xi^2 - \sin^2 \theta)^{\frac{1}{4}} \sim \frac{1}{2} (\gamma^{-2} + \xi^2 + \theta^2)$$

for $\beta \lesssim 1$, $\varepsilon \lesssim 1$, and $\theta \ll 1$. The range of frequencies emitted in the relativistic case will therefore extend toward the x-rays.

The TR in the first medium (backward radiation) can be obtained in essentially the same way by replacing $\beta$ with $-\varepsilon$ in (5). The small factor in the denominator disappears and therefore the backward TR is comprised only of the optical part of the spectrum.

The expressions (5) and (6) describe the transition radiation under the conditions $(\omega/c)R\sin^2 \theta \gg 1, 6, 26$ where $R$ is the distance from the point of emergence of the particle from the medium to the point of observation, and $\theta$ is the angle between the $z$-axis and $R$. In the region of space $R \sim (c/\omega) \sin^{-2} \theta$, redistribution of the wave field of transition radiation takes place. As shown in (6), most of the radiation is emitted at an angle $\sin \theta \sim 1/\gamma$ for relativistic particles so that we get $R \sim \gamma^2 c/\omega$, which is called the "formation zone" in vacuum. In vacuum the
formation zone becomes macroscopic in size; e.g., for \( \gamma \approx 5000 \) and \( \omega \approx 10 \) KeV, \( R \approx 1 \) mm.

A formation zone also is obtained in the medium. After Ter-Mikaelyan,\(^{26}\) as a consequence of the interaction with the electron plasma, the longitudinal momentum transferred by the particle to the medium is given by

\[
q_{\parallel} = \hbar \omega (1 - \beta \sqrt{\epsilon} \cos \theta).
\]

(It is obtained by satisfying (1) energy conservation of the incident particle and its radiation where the energy transfer to the medium is ignored; (2) momentum conservation of particle, radiation photon and medium along the direction of incident particle. Refer to Ellsworth et al.\(^{17}\) for its derivation.) As the incident energy increases \( (\beta \rightarrow 1) \), \( q_{\parallel} \) becomes very small. Therefore, to emit a quantum \( \hbar \omega \), it is necessary for the particle to travel a long distance \( \hbar / q_{\parallel} \).

This quantum mechanical consequence corresponds to the result obtained from a wave mechanical consideration as below.

Appreciable coherent superposition from different points in the medium will occur when the product of the driving fields of the particle \( e^{i(\omega/v)z} \) and the generated wave \( e^{-i\mathbf{k} \cdot \mathbf{z}} \) does not change its phase significantly over the region.\(^{21}\) In other words, the phase difference of photons radiated from two points along the passage of the
particle with a distance \( z \) must be of order of unity, i.e.,
\[
|\frac{\omega}{v}z - \frac{k}{v}z| \sim 1,
\]
which gives
\[
z \sim \left| \frac{\omega}{v} - k \right|^{-1}
\]
where \( v \) is the velocity of the incident particle and \( k \) is the wave number of the radiation in the medium. In the relativistic case, its order of magnitude in vacuum is
\[
z \sim 2c \left( \frac{\gamma}{\gamma^2 + \frac{\theta^2}{\omega}} \right)
\]
(which reduces to \( R \sim \gamma^2 \frac{c}{\omega} \) for the angle \( \theta \sim 1/\gamma \)) and in the medium at frequencies larger than optical
\[
z \sim \frac{2c}{\omega(\gamma^2 + \frac{\theta^2}{\omega} + \xi^2)}.
\]

An appreciable intensity is obtainable only when a charged particle can traverse a distance of the order of the formation zone in a medium. In Fig. 3, formation zones of polyethylene and air are shown as functions of \( \omega \) for different values of \( \gamma \). The formation zone of air is roughly 100 times larger than that of polyethylene.

2.2 Radiation from Multilayered Media

a. Simplification for Ultrarelativistic Particles

Interface Yield

Substituting (7) into (5) and making approximations for small angles, the intensity reduces to a simple form
\[ Z = \frac{2c}{\omega} \left[ \gamma^{-2} + \left( \frac{\omega_p}{\omega} \right)^2 + \Theta^2 \right]^{-1} \]

**AIR**  \( \hbar \omega_p = 0.70 \text{ eV} \)

**POLYETHYLENE**  \( \hbar \omega_p = 20.0 \text{ eV} \)

**Fig. 3**
Formation Zones of Polyethylene and Air
\[
\frac{dW}{d\omega d\theta} = \frac{2\alpha}{\pi} f_0(\theta)
\]  
(8)

with

\[
f_0(\theta) = \theta^3 \left( \frac{1}{\gamma^{-2} + \theta^2 + \xi_g^2} - \frac{1}{\gamma^{-2} + \theta^2 + \xi_f^2} \right)^2,
\]  
(9)

where

\[\alpha : \text{fine structure constant } 1/137.\]

\[\omega : \text{energy of radiated photon (unit keV will be used throughout the text. It is equivalent to setting } \hbar c = 1.97 \times 10^{-8} \text{ keV cm}.\]

\[\omega_i : \text{plasma frequency of medium and vacuum. Suffix } f \text{ and } g \text{ are for later use.}\]

\[\xi_i : \omega_i/\omega.\]

In most practical situation, the lower density medium is a gas rather than vacuum, and the \(\omega_g\) term appears as above. The medium is usually foil form, and will be indicated with suffix \(f\). Figure 4 shows the angular distribution of the single surface yield (9) for \(\gamma = 5870\) and \(\omega = 5-20\) keV. The distribution has a peak at narrow angle \(\theta \sim \gamma^{-1}\) and extends to the angle of order \(\gamma^{-2} + \xi_f^2\).

The integration of expression (8) over angles \(\theta\) gives the energy spectrum

\[
\frac{dW}{d\omega} = \frac{\alpha}{\pi} \left[ \frac{\xi_f^2 + \xi_g^2 + 2\gamma^{-2}}{\xi_f^2 - \xi_g^2} \ln \frac{\gamma^{-2} + \xi_f^2}{\gamma^{-2} + \xi_g^2} - 2 \right].
\]  
(10)
Fig. 4
Angular Distribution of a Single Surface Yield

\[ f_0(\theta) = \theta^3 \left( \frac{1}{\gamma^{-2} + \theta^2 + (\omega_f/\omega)^2} \right) - \frac{1}{\gamma^{-2} + \theta^2 + (\omega_g/\omega)^2} \right)^2 \]
It follows from (9) for large $\xi_f^2/\xi_g^2$ (usually of order of $10^3$) that there exists a cutoff frequency $\omega_c = \omega_f \cdot \gamma$ such that

$$\frac{dW}{d\omega} = \frac{e^2}{6\pi c} \left( \frac{\omega_c}{\omega} \right)^4 \text{ for } \gamma << \xi_f^{-1}, \text{ i.e.}, \omega >> \omega_c,$$

$$\frac{dW}{d\omega} = \frac{2e^2}{\pi} \ln \frac{\omega_c}{\omega} \text{ for } \xi_f^{-1} << \gamma << \xi_g^{-1}, \text{ i.e.}, \omega << \omega_c, \quad (11)$$

and

$$\frac{dW}{d\omega} = \frac{2e^2}{\pi c} \left( \ln \frac{\omega_f}{\omega_g} - 1 \right) \text{ for } \gamma >> \xi_g^{-1}.$$

In the last case the yield is constant. The spectrum from a single surface is a monotonically decreasing function of radiation energy and it drops as rapidly as $\omega^{-4}$ at high frequency. Figure 5 shows a spectrum from a surface of polyethylene for $\gamma = 6000$ and $30000$. The high energy component becomes rich when $\gamma$ increases.

A very important result of TR by relativistic particles is obtained from the integration of the spectrum (10) ($\xi_g = 0$)

$$W_{TR} = \frac{1}{3} \alpha \omega_f \gamma = 2.43 \times 10^{-3} \omega_f \gamma. \quad (12)$$

The main contribution in the integral (11) is due to frequencies which are not too small compared with the limiting frequency. More than 95% of the energy is produced in the range of $0.1 \omega_c < \omega < \omega_c$ (e.g., for $\omega_f = 0.02$ keV and $\gamma = 5000$, $1$ keV < $\omega$ < $100$ keV).
Fig. 5

TR Spectrum from a Polyethylene Surface

\( \gamma = 6.0 \times 10^3 \)

\( \gamma = 3.0 \times 10^4 \)

\( \omega_p = 20 \text{ eV} \)
Finally we consider the number of transition radiation photons. The number of photons with frequencies greater than a lower limit \( \omega \) is given by the expression

\[
N(> \omega) = \frac{\alpha}{\pi} \left\{ \ln \frac{\omega}{\omega_c} \left( \ln \frac{\omega}{\omega_c} - 2 \right) + \frac{\pi^2}{12} + 1 \right\},
\]

where it is apparent that \( N \sim \alpha \) (as \( \omega_c/\omega \approx 10 - 100 \)). Because of this small probability of energetic photon emission per transit of an interface, it is necessary to utilize a large number of boundaries.\(^2,7\) To achieve this, a stack of many foils with gaps between is used in practice.

**Single Foil Yield**

First, let us estimate the TR yield produced by a particle passing through a dielectric foil with thickness \( t \) placed in vacuum. The field at the observation point is given as a sum of fields produced at both surfaces of the foil. For relativistic particles, the amplitudes of forward radiation from both surfaces have the same magnitudes. Hence, we consider only the phase difference between the fields. The phase change in the field produced at the first surface is \( kt \cos \theta \) when it reaches the second surface, where \( k \) is the wave number in the foil. The generation of the field from the second surface occurs later in time with the corresponding phase lag \( \omega t/v \) relative to the first field; this field also has a phase difference \( \pi \) compared to the field from the front surface due to the opposite sign of its
polarization. By summing up the two fields, we obtain a multiplying factor to the amplitude

\[ \left| e^{ikt \cos \theta} + e^{i \pi e^{i \omega t/v}} \right|^2 = 4 \sin^2 \left( \frac{kt \cos \theta - \omega t/v}{2} \right) . \]

The yield from one layer of medium in a gas is therefore given:

\[ \frac{d^2 W_{SF}}{d \omega d \Omega} = \frac{2 \alpha}{\pi} f_0(\theta) f_1(\theta) , \quad (14) \]

where

\[ f_1(\theta) = 4 \sin^2 \left( \frac{\phi_f}{2} \right) \quad (15) \]

with

\[ \phi_f = kt \cos \theta - \omega t/v . \]

By using the relations \( k = \frac{\omega}{c} \sqrt{\varepsilon} \) and (7) and approximating \( \cos \theta \sim 1 - \theta^2/2 \) and \( \beta^{-1} \sim 1 + 1/2 \gamma^2 \), we obtain

\[ \phi_f = (\omega t_f/2c)(\gamma^{-2} + \theta^2 + \xi_f^2) . \]

If \( t_f \) is much smaller than the "formation zone" of foil \( z_f = (2c/\omega)(\gamma^{-2} + \theta^2 + \xi_f^2)^{-1} \), the yield is strongly suppressed by the interference factor. If the thickness of foil is much greater than the formation zone of the medium, then the interference term has the value of 2, i.e., the foil radiation is equal to double the radiation on one boundary. If \( \phi_f \gtrsim \pi \), then \( f_1 \gtrsim 4 \) and the radiation is maximum-constructive interference.
Figure 6 shows the angular distribution of transition radiation from a single foil (Eq. (14)). The oscillation of the \( f_l \) term becomes more frequent at a larger angle. At \( \omega = 5 \) keV, the first maximum occurs at \( \theta \sim 2 \) mrad. This is easily seen from the fact that the height of \( f_0 f_l \) in Fig. 6 is four times higher than that of \( f_0 \) (Fig. 4). At higher \( \omega = 20 \) keV, the \( f_0 \) drops faster than the \( f_l \) term oscillates in the same region of angle. Therefore, it is difficult to recognize the bumps in \( f_0 f_l \).

**N-Foil Yield**

Similarly, the amplitude for \( N \) foils equally spaced in gas is just the sum of \( N \) amplitudes each advanced by phase \( \phi \) (see Fig. 7):

\[
a_N = a_1 (1 + e^{i\phi} + e^{2i\phi} + \ldots + e^{(N-1)i\phi}),
\]

where \( a_1 \) is the single foil amplitude and

\[
\phi = (k_f \cos\theta - \omega/v)t_f + (k_g \cos\theta - \omega/v)t_g
\]

with wave numbers \( k_i = (\omega/c)\sqrt{\epsilon_i} \) in each medium. By completing the summation, we find the intensity ratio of \( N \) foils to single foil to be

\[
f_N(\theta) = \frac{|a_n|^2}{|a_1|^2} = \frac{\sin^2(N\phi/2)}{\sin^2(\phi/2)}
\]

which is analogous to multiple-slit diffraction. The yield from a stack of \( N \) foils with equal spacing is given by
Angular Distribution of TR from a Single Mylar Foil
Fig. 7

A Dielectric Stack with N Foils of Thickness $t_f$ and Spacing $t_g$. 
The total phase advance through one cycle of foil and gap is simply written as \( \phi = \phi_f + \phi_g \), including the phase change due to the air gap \( \phi_g = (\omega t_g / 2c)(\gamma^{-2} + \theta^2 + \xi^2) \). (See Refs. (25,27) also.) As the formation zone of air is about 100 times larger than that of the dielectric medium, the appreciable yield from the N-foil stack will be obtained only when the foils are spaced with distance of the order of the air formation zone. Therefore, the ratio \( \tau = t_g / t_f \) is usually kept nearly equal to 100.

Due to multiple scattering of electron, the conditions for constructive interference are seldom met in practice. However, for heavier charged particles (e.g., pions), coherent interference effects may become apparent.

b. Qualitative Study of Spectrum

An interesting feature of intensity \( dW/d\omega \) is its monotonic increase with \( \gamma \). Introducing the variable \( y = \gamma^{-2} = \theta^2 \), the energy spectrum is given by

\[
\frac{d^2W_{NF}}{d\theta d\omega} = \frac{d^2W_{SF}}{d\theta d\omega} f_N(\theta) .
\] (18)

The interference factors depend only on \( y \) (not \( \gamma \) and \( \theta \) separately) and are positive. The above formula defines a monotonically increasing function of \( \gamma \). This equation also
shows that \( \frac{dW}{d\omega} \) reaches a finite limit when \( \gamma \to \infty \), unless one of the medium is vacuum. There is practically no more dependence when \( \gamma > \xi_0^{-1} = \omega/\omega_g \).

Let us now examine in more detail:

**Single Foil Yield**

The difference between the single foil formula (13) and the single surface formula (8) is the interference factor \( 4 \sin^2(\phi_f/2) \). In the relevant region of the angular integration, \( \phi_f \) is of the order of \( (\omega t_f/2c)(\gamma^{-2} + \xi_f^2) \). Hence, if the thickness \( t_f \) is much less than the quantity \( (2c/\omega) \times (\gamma^{-2} + \xi_f^2)^{-1} \), the yield is strongly reduced by the interference effect which varies as \( \phi_f^{-2} \). Before proceeding, let us introduce some convenient notations for later use

\[
\gamma_f = t_f \omega_f / 2c = 6.4 \times 10^4 \omega_f t_f \quad (\gamma_f: \text{keV}, \ t_f: \text{mils}),
\]

\[
\Gamma = \gamma/\gamma_f, \ \eta = \omega/\omega_c,
\]

and

\[
r = \omega_g^2/\omega_f^2 \sim 10^{-3} \quad (\text{for air and dielectrics}).
\]

The \( \phi_f \) is then rewritten in terms of these quantities as

\[
\phi_f = t_f / z_f = (\eta + \eta^{-1} + x)/\Gamma,
\]

where \( x = \eta (\gamma^2 \theta^2) \).

For the general case \( (r \neq 0) \), the single foil yield now is written in a dimensionless expression: \( 28,29 \)
\[
\frac{dW_{SF}}{d\omega} = \frac{2\alpha}{\pi} g(\eta, \Gamma),
\]

where \( g \) is described in terms of \( \Gamma \) and \( \eta \) as

\[
g(\eta, \Gamma) = \int x \, dx \left( \frac{1}{x+\eta+\eta^{-1}r} - \frac{1}{x+\eta^{-1}+\eta} \right)^2 (1 - \cos \phi_f). \tag{21}
\]

(a) **Case of Incoherent Addition**

The interference between the two single interface amplitudes is negligible if the interference factor has many oscillations in the relevant domain of integration in \( \theta \). In the case \( r = 0 \), inspection of formula (21) shows that the incoherent addition occurs when \( \Gamma \ll 1 \).

(b) **Case of Constructive Interference**

When we are interested in the frequencies which contribute to the total yield, we get \( \eta \ll 1 \) as discussed in the integration of spectrum. If \( \eta \ll 1 \) (e.g., for \( \omega = 10 \text{ keV}, \omega_p = 0.02 \text{ keV} \) and \( \gamma = 5000, \eta = 0.1 \)), the single surface factor in (21) has a maximum value (assuming \( r = 0 \)) at \( x = x_{\text{max}} \) with \( x_{\text{max}} \sim \eta \) (corresponding to the angle \( \theta = \gamma^{-1} \)). The interference between the two surfaces causes enhancement if \( \phi_f^{\text{max}} = (x_{\text{max}} + \eta^{-1} + \eta)/\Gamma \) is not far from \((2m + 1)\pi\) and reduction if \( \phi_f^{\text{max}} \sim 2m\pi (m = 0, 1, 2, \ldots) \). In the case \( \eta \ll 1 \), \( \phi_f^{\text{max}} \sim (\eta\Gamma)^{-1} = \omega_f \gamma_f/\omega \). If the maximum yield is sought for a given frequency \( \omega \) and \( \gamma \), we should therefore choose thickness such that \( \omega_f \gamma_f/\omega = \pi \) which gives
\[ \omega = \frac{\omega^2}{2\pi c} t_f^\sim 2 \times 10^4 \omega^2 t_f \quad (22) \]

(for \( m = 1, 2, \ldots \), we have less pronounced interference because of a smaller bandwidth in \( \omega \) and more absorption. We will discuss this point in the next section.)

The peak positions are independent of \( \gamma \) and are determined only by the plasma frequency and thickness of foil.

In the case where \( \eta \) is comparable to unity \( \phi_f^{\text{max}} \) depends less on \( \eta \Gamma \) and more on \( \Gamma \) than in the previous case. Hence, the interference effect can be used to increase the \( \gamma \) dependence of the yield. (More detailed discussion of the use of dimensionless quantities is found in the literature by G. B. Yodh.)

N-Foil Yield

A stack of foils which are regularly spaced presents a new feature. The angular distribution of x-rays is sharply peaked at discrete angles due to N-foil interference. In (17), the angles where the peaks occur satisfy the resonance condition:

\[ \phi = \omega \frac{t_f + t_g}{\nu} - (k_f t_f + k_g t_g) \cos \theta = 2n\pi . \]

The spacing between the peaks is given by

\[ \Delta \cos \theta = \frac{\lambda}{t_f + t_g} \quad (\lambda = \text{x-ray wave length}) , \]

which is a very small quantity, e.g., for \( t_f + t_g \sim 1 \) mm.
= 10^7 \text{ A} and \lambda(\omega = 10 \text{ keV}) = 1.24 \text{ A}, we get \Delta \cos \theta \approx 1.24 \times 10^{-7}. In Fig. 8, f_N(\phi(\theta)) for 100 foils of 1 mil polyethylene spaced 125 mil apart is plotted as a function of angle for \gamma = 5871 and \omega = 5 or 10 keV. This f_N(\theta) in the plot includes the self-absorption of foil and air. Therefore, the heights and widths of peaks are different depending on the absorption. Without absorption, the height must be the same for all \omega and must be equal to N^2 = 10^4. However, the peak positions are independent of the absorption and the values on the plot are correct for both absorptive and non-absorptive cases. More precise behavior of f_N(\theta) will be discussed in Section 2.3 and Appendix A.

There are many theoretical works\(^{19,26,29,30}\) which deal with the detailed nature of the N-foil interference. We describe only a few features which can be derived directly from the given equations.

It has been emphasized that a yield of one photon requires at least about 100 foils. Henceforth, let us discuss the case N >> 1, where the interference factor f_N shows a sharply peaked form. The interference term f_N can then be approximated by a sum of delta functions:

$$\frac{\sin^2 N\phi/2}{\sin^2 \phi/2} = N \sum_{n} \delta(\phi - 2n\pi). \quad (23)$$

The integration of (18) over angle \theta will be obtained by the use of the relation \phi = \phi(\theta^2) as a function of \theta^2. The N-foil yield is then described as follows:
Fig. 8

N-Foil Interference Term $f_N$ as a Function of Angle $\theta$
\[
\frac{dW_N}{d\omega} = N\Delta_N \sum_n f_0(\theta_n^2) F_1(\theta_n^2),
\]

where \(\Delta_N = (4\pi c/\omega t_f)/(1 + \tau)\) as given in Appendix A is the spacing between the peaks of \(f_N\) on \(\theta^2\) axis and \(\theta_n^2\) is the value corresponding to \(\phi(\theta_n^2) = 2n\pi\).

In order to look into this point, let us write (24) more explicitly. By defining \(\phi_0\) as the value of \(\phi\) for \(\theta^2 = 0\), we can write

\[
\phi = \phi_0 + \frac{\omega t_f}{2c}(1 + tg t_f)\theta^2
\]

\[
= \phi_0 + (1 + \tau)x
\]

with a new variable \(x = (\omega t_f/2c)\theta^2\). Substituting \(x_n = (2n\pi - \phi_0)/(1 + \tau)\) into (24), we obtain

\[
I_N = 2\pi \gamma_f^2 \xi_f^2 (1+\tau)^2 \sum_{n=\text{min}}^{n} \frac{2\alpha}{\pi} \frac{(2n\pi - \phi_0)(1 - \cos \frac{2n\pi + \tau/\eta \Gamma}{1 + \tau})}{(2n\pi + \tau/\eta \Gamma)^2(2n\pi - 1/\eta \Gamma)^2},
\]

where \(n\) starts from \(n_{\text{min}} = [\phi_0/2\pi] + 1\) in order for \(x_n\) to be positive-valued, and \(r = 0\) is assumed for simplicity. The \(\phi_0\) can be written in terms of \(\Gamma\) and \(\eta\) as

\[
\phi_0 = \{1 + \eta^2(1 + \tau)/\Gamma \eta\}.
\]

In Appendix A, the same result is obtained as a limit of a more general case which also includes absorption terms.

In the summation (25), the terms with small values of \(n\) contribute the most because of the existence of the term
(2nπ - l/ηΓ) in the denominator. Also, the single foil interference term \(1 - \cos[(2nπ + τ/ηΓ)/(1 + τ)]\) maintains its maximum value for small values of \(n\).

2.3 Computation of Yield from a Stack Including Self-Absorption

a. Modification of Formulas

The radiator materials (foil and air) absorb the x-rays produced by a particle passing through a foil stack. The absorption coefficient is a strong function of x-ray energy, which will cause the shape of spectrum to be largely modified. Let us first see how the yield formulas are to be modified taking into account the self-absorption of the radiator. The following notations will be used to describe absorption by the materials in the stack:

\[
b_{f,g} = (\mu pt)_{f,g}/2,
\]

where \(\mu\), \(\rho\), and \(t\) are the absorption coefficient, the density, and the thickness of the material, respectively (the suffix indicate either a foil or air gap).

Single Foil Yield

We assume the exponential absorption. In the phase relation which is used to obtain (13), the absorption by the foil occurs only in the term \(e^{ikt \cos θ}\). Therefore, the interference term between the two surfaces is modified as

\[
f_1 = \left|e^{-bf_0 e^{ikt \cos θ}} + e^{iπ e^{iωtf/v}}\right|^2.
\]
By using $\phi_f$ defined in (15), we can simplify it:

\[ f_1 = 2 e^{-b_f} (\cosh b_f - \cos \phi_f) . \]  

**N-Foil Yield**

The absorption in N-foil stack occurs both in foils and air gaps. The amplitude is attenuated as $e^{-b_f,g}$ in each medium. We can introduce this effect as the imaginary part of the phase, i.e.,

\[ \phi = \phi + ib , \]

where $\phi = \phi_f + \phi_g$ and $b = b_f + b_g$. Then, substituting $\phi$ in place of $\phi$, (13) is modified as

\[ f_N = e^{-N b} (\cosh Nb - \cos N\phi)/(\cosh b - \cos \phi) . \]

Note that when $b \rightarrow 0$, this result becomes the same as the non-absorptive one (17). When these modified expressions are used, the peak positions of $f_N$ for the angular distribution would not move, but the height of the peak will be decreased and its width becomes wider (Fig. 8).

**b. Computation Methods**

To obtain numerical values of energy spectra produced by a foil stack, we employ the following methods depending on relations between parameters of the radiator.

**Approximation with a Single Foil Formula**

The total flux emerging out of a foil stack is approximated as an incoherent sum of radiations from N foils. It is easily seen that the expression for the total flux
consists of the integration of (18) with respect to $\theta$ and a multiplying factor which describes the incoherent addition of the single foil intensities including the absorption. The latter is expressed as

\[ N_{\text{eff}} = \frac{1 - e^{-\sigma}}{1 - e^{-\sigma_0}}, \]  

(29)

where $\sigma = (\mu_{\text{pt}} f + (\mu_{\text{pt}} g$. This approximation is valid when the condition $\tau \gg 1$ and $\phi_0 \gg 2\pi$, given in Appendix A, are satisfied. In our case, this condition is satisfied in the electron energy $E_e$ less than 3 GeV for a mylar radiator of 1 mil foils spaced by 100 mils. Collecting all the terms, the total flux from a foil stack is given by

\[ \frac{dW}{d\omega} = \frac{2\alpha}{\pi} N_{\text{eff}} \int f_0(\theta)f_1(\theta) \, d\theta. \]  

(30)

The absorption of materials prevent our obtaining an infinite amount of yield from a foil stack. In fact, there exists a certain limit for the number of foils. It is apparent that the yield saturates as $N_{\text{eff}} \rightarrow (1 - e^{-\sigma})^{-1}$ for $N \rightarrow \infty$.

Approximation with Delta Functions

As discussed in Appendix A, the $f_N$ function can be approximated with delta functions under the condition that $N \gg 4\pi\sigma^2/(t_f + t_g)\omega$. This delta function approximation gives the angular integration of the following expression
From Appendix A, we obtain almost the same form as (25) but includes absorption terms \((r \neq 0)\):

\[
\frac{dW}{d\omega} = 2\alpha \frac{f_0}{\pi} \int_0^1 f_N d\theta .
\]

(31)

From Appendix A, we obtain almost the same form as (25) but includes absorption terms \((r \neq 0)\):

\[
\frac{dW}{d\omega} = \sqrt{\pi} a \gamma_f^2 \xi_f^2 e^{-b_f} \frac{(1 - r)^2}{(1 + \tau)}
\]

\[
\times \frac{2\alpha}{\pi} \frac{x_n \left[ \cosh b_f - \cos x_n + \phi_f^0 \right]}{(x_n + \phi_f^0)^2(x_n + \phi_f^0/\tau)^2},
\]

where

\[
a = \frac{1 - e^{-Nb^2}}{1 - e^{-b}} \frac{2 \sinh(b/2) \sinh(Nb/2)}{\sqrt{\sinh^2(Nb/2) - N^2 \sinh^2(b/2)}}
\]

and \(x_n\) is the same. The \(\phi_i^0\) are values of \(\phi_i\), evaluated at \(\theta = 0\).

The results of both approximation methods (i) and (ii) are compared in Fig. 9. The total yield from a stack of 100 polyethylene foils (1 mil thick) spaced 125 mil is plotted vs. \(\gamma\) of electron. The dotted curve is for the single foil approximation and the solid for the delta function approximation.

The saturation at a high value of \(\gamma\) is due to the formation zone effect. The formation zones of polyethylene and air grow larger at higher \(\gamma\) as shown in Fig. 3. In the single foil formula, the formation zone effect is concerned with only the foil thickness. If \(\gamma > 5000\), the
Fig. 9

\( \gamma \) Dependence of Calculated TR Yield from a Foil Stack
formation zone exceeds the given thickness of the polyethylene foil, and the total yield saturates at high $\gamma$. In the N-foil formula, the formation zone effect sets in for both the foil and the gap. If the formation zone of air becomes larger at high $\gamma$ than the given spacing, the growth of the total yield in the N-foil formula will be suppressed more than in the single-foil formula. Some detailed nature of these methods are given in Appendix A.

The choice of the approximation method also depends on the multiple scattering of the electron in the radiator. Because of the multiple scattering, the electron may not travel on a straight line longer than the length required for the coherent addition of radiation from each foil. Therefore, for most of our experimental conditions, the single-foil approximation gives good values. In Section 5.3, the results of both computation methods will be shown and compared with the experimental results.
3. METHODS

We restrict ourselves to the measurement of the spectrum and the total flux of transition radiation from a stack of foils equally spaced. The detector is assumed to be large enough to cover the whole angular distribution of the radiation. The method of observing the transition radiation will be discussed in the following order. First, a choice of radiator material determines the observable spectrum. Second, the x-ray detector must have an appropriate response and good efficiency for the energy spectrum emerging out of the foil stack. Third, among many possibilities, we study two of the most practical arrays of radiator and detector combination with which we can obtain a sufficient number of TR photons and observe the shape of TR spectrum.

3.1 Design of the Detection System

a. Transition Radiator

Due to the small probability of production of TR photons from a single surface of dielectric medium, it is necessary to use a number of interfaces in order to gain enough TR photons. Detection of the TR x-rays is made possible by the use of a multilayered radiator (a stack of foils of dielectrics). As mentioned in Section 2, the use of less absorptive radiator materials allows us to increase the number of interfaces, and increase of the number of radiator-and-detector sets will produce large enough yield by summing the signals from all the sets.
By choosing the materials available rather conveniently, we calculate the TR yield from a certain number of foils with spacing large enough compared to its formation zone at the frequency of the last peak (see Eq. (22)). Figure 10 shows when an electron (the energy $E_e = 3$ GeV corresponding to $\gamma = 6000$) passes through a stack of 100 foils of either polyethylene or Mylar, the flux at the low energy is substantially reduced by the self-absorption of the radiator foils and air gaps (comparison with a single foil yield times 100). To describe a set of radiator, we will use a notation $(N, t_g, t_f)$, where $N$ is the number of foils, $t_f$ the foil thickness, and $t_g$ the gap thickness in mils ($1$ mil $= 10^{-3}$ inch). The name of the radiator material will be placed in front of the parenthesis. We can use only the last peak as an observable spectrum from a foil stack. The last peak position for polyethylene is approximately given by $\omega = 8t_f$ (keV) (refer to Section 2.2b). The spectrum becomes softer if we use materials with smaller plasma frequency and/or thinner foil. Figure 10 shows that the spectrum from a polyethylene radiator peaks at 8 keV, while that from a Mylar radiator peaks at 11.5 keV for the same configuration.

The yield from a foil stack increases up to a certain limit because of the self-absorption of the radiator materials. The effective number of foils at a certain energy is given by Eq. (29). When the number of foil $N$ is large, $N_{eff}$ approaches a value $(1 - e^{-\sigma})^{-1}$. A stack
Fig. 10

Energy Flux Emerging out of a Foil Stack
with more foils than $N_{\text{eff}}$ gives essentially the same amount of yield obtainable from the stack of $N_{\text{eff}}$ foils. The effective number of foils $N_{\text{eff}}$ varies depending on the photon energy (Fig. 11).

In Table 1, the plasma frequency and absorption coefficients of the various materials are listed. The absorption increases as $Z^4/\omega^3$, while the TR total energy is proportional to plasma frequency, which varies as $Z^{1/2}$. Therefore, if we want to increase the number of photons from a stack, it is preferable to use dielectrics of small $Z$ as a radiator material.

Once a maximum value of $\gamma$ is decided as the working range of a TR detector (for a given $\omega_p$), the foil thickness $t_f$ must be kept as thin as possible above its formation zone limit. In this manner, the total yield from the same amount of material (in grams) will be optimized by minimizing the self-absorption (as many interfaces as possible). The spectrum must, however, remain hard enough to pass through the windows of the x-ray detector.

b. X-Ray Detector

The detector observing the spectrum from a stack of foils must be sensitive to the frequency range from 1 keV to nearly 100 keV. But it must be transparent for any high energy photons. The detection system will be exposed to the background of bremsstrahlung from the primary beam going through the radiator stack (this is especially serious when
\[ N_{\text{EFP}} = \frac{1 - e^{-N\sigma}}{1 - e^{-\sigma}} \]

\[ \sigma = (\mu p t)_f + (\mu p t)_g \]

\[ (N, t_f, t_g) = (100, 1, 125) \]

**Fig. 11**

Effective Number of Foils \( N_{\text{EFF}} \) vs. Photon Energy
TABLE 1

RADIATOR MATERIAL PROPERTIES

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (g cm(^{-3}))</th>
<th>Plasma Frequency (eV)</th>
<th>Linear Absorption Coefficient at 10 keV (cm(^{-1}))</th>
<th>Radiation Length (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lithium</td>
<td>0.534</td>
<td>13.8</td>
<td>7.09 x 10(^{-2})</td>
<td>148</td>
</tr>
<tr>
<td>Beryllium</td>
<td>1.84</td>
<td>26.1</td>
<td>7.19 x 10(^{-1})</td>
<td>34.7</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.70</td>
<td>33.1</td>
<td>7.14 x 10(^{1})</td>
<td>8.91</td>
</tr>
<tr>
<td>Polyethylene (CH(_2))</td>
<td>0.925</td>
<td>20.9</td>
<td>1.79 x 10(^{0})</td>
<td>49</td>
</tr>
<tr>
<td>Mylar (C(_5)H(_4)O(_2))</td>
<td>1.38</td>
<td>24.4</td>
<td>8.07 x 10(^{0})</td>
<td>28.7</td>
</tr>
<tr>
<td>Air</td>
<td>2.2 x 10(^{-3})</td>
<td>0.7</td>
<td>9.1 x 10(^{-2})</td>
<td>30870</td>
</tr>
</tbody>
</table>

\(\mu = \text{mass absorption coefficient (cm}^2\text{ g}^{-1}\)\)
we use the electron beam). The background from bremsstrahlung will be calculated in Section 3.4 for the radiator configuration used in the experiments.

The MWPC's which were available from other experiments of our group were readily good x-ray detectors in many specifications. First, the chamber covers a rather large area which is necessary to cover most of the TR angular distribution (a few milliradians). Second, the advantage of an MWPC is that it is sensitive only in the energy range of the x-rays produced by TR. Therefore, the background due to bremsstrahlung of an incident particle can be kept negligibly small. Figure 12 shows the calculated detection efficiency (the fraction of incident flux converted effectively into an MWPC signal; see Eq. (B38)) of 1.5 cm MWPC as a function of photon energy $\omega$. The MWPC is filled with either of rare gases Ar, Kr, or Xe (mixed with 7% methane). The efficient range of energy increases with the atomic number $Z$ of a gas element. A precise argument about the detection efficiency will be given in Section 5 and Appendix B.

c. Combination of Radiator and X-Ray Detectors

It is necessary to use a certain number of modules of radiator and detector in order to obtain sufficient photons from a single particle because the effective number of foils in a foil stack is limited.

**Sandwich Array (SW)**

The simplest setup may be constructed by placing a
Detection Efficiency of 1.5 cm MWPC

Fig. 12
radiator right in front of the chamber and repeating it several times. This is called the "sandwich array" (Fig. 13(a)). The sandwich array has many attractive features for application to many experimental conditions. As it can cover a fairly large solid angle, it will be a desirable detector for secondary reactions or for cosmic-ray research.

The major deficiency of this array is that the primary particles are also observed in the same detector. The ionization of the primary particle in the gas gives a signal proportional to the thickness in gram cm$^{-2}$ and has a broad Landau distribution. Therefore, this ionization (dE/dx) distribution is always superimposed onto the TR x-ray spectrum. By measuring a single pulse, we cannot be sure whether the signal is caused by the TR x-rays or not. The MWPC has a different response in the detection of x-rays and dE/dx. The ionization loss in the chamber depends mainly on the amount of gas to be passed through by charged particles, while the x-ray absorption increases as $Z^4$ of gas atoms. It is necessary to minimize the ionization loss of the incident particle. The chamber thickness must therefore be kept as thin as possible but not too thin as to lose TR x-rays.

**Magnetic Separation Array (MS)**

The pure spectrum of TR can be observed when the incident particle is removed from the x-ray trajectory
(a) SANDWICH (SW) ARRAY

(b) MAGNETIC SEPARATION (MS) ARRAY

Fig. 13

TR Radiator-Detector Arrays (Schematic only)

*Counters $S_1$ and $S_2$ define the beam.
and the detector measures only the straight-ahead x-rays (Fig. 13(b)). The particle can be swept away from the direction of an x-ray by a deflecting magnetic field whose strength must be kept so weak that the incident particles does not produce any sizable background due to synchrotron radiation in the B field. The evaluation of the synchrotron radiation for the experimental conditions is given at the end of this section.

In both the SW and MS setups, there exists an important background mainly due to bremsstrahlung by the incident particle passing through the radiator material. In order to measure such a background, we ran a background run in pairs with each radiator run. In the background run, a slab of an equal amount of material (equivalent absorber) was used in place of a radiator. The measured background was subtracted from the data taken with the radiator stack.

3.2 Preliminary Experiment at the Bevatron

During the quantameter experiment at the Bevatron, a prediction showed that a sizable separation of a pulse height distribution of a transition signal from that of a background signal would be obtainable in a sandwich array with the MWPC's. As the MWPC's were readily available from the quantameter test, we prepared to run the preliminary experiment. Several more chambers and Mylar radiator stacks were built.
At the Bevatron, high γ electrons were available even though they were very scarce. We made a short run by using the sandwich setup consisting of 11 modules of radiator and MWPC detectors. The radiator was made of aluminized\textsuperscript{31} Mylar with thicknesses of 1/6 and 1/2 mil. Each radiator stack had 100 foils. The MWPC's were filled with either a Ar-methane or Krypton-methane mixture. The signals from the chambers were fed to a sample-and-hold unit which sent 11 signals consecutively onto an oscilloscope, which displayed 11 square-shaped analogue pulses. Data were recorded on film by a movie camera and analyzed by PEPR (Precision Encoding and Pattern Recognition) at Oxford University.

Some results of the Bevatron experiment are shown in Figs. 14 and 15. We can observe the pulse height distributions taken with radiators in the SW array (shaded curve) clearly displaced from the other two distributions. The dotted curve was taken with equivalent absorbers in place of the radiators. The solid-line histogram was obtained with pions at the same momentum. In the equivalent absorber run, the chambers measured ionization loss of electrons and background mainly due to bremsstrahlung in the absorbers. The krypton chambers separated the two distributions, one with radiators and the other with equivalent absorbers, more clearly than the argon chambers.
EQUAL AREA SPECTRA OF ENERGY LOSS IN II CHAMBERS (ARGON + METHANE) (WITHOUT SATURATION CORRECTION)

3 GeV/c PIONS

3 GeV/c ELECTRONS (Ionization Loss Only)

3 GeV/c ELECTRONS With 100 x 1/2 Mil Radiators

Fig. 14

Results of the Bevatron Experiment with Argon MWPC's
Fig. 15

Results of the Bevatron Experiment with Krypton MWPC's
The data taken at 3 GeV/c gave the total number of TR photons detected in 11 chambers, 3.6 per electron with argon filling and 5.0 with krypton filling.

The pions at 3 GeV/c gave a very low average energy compared to that of electrons because the TR effect from pions was negligible at this momentum. The electrons \( (\gamma = 6000) \) showed greater ionization in the chambers than pions \( (\gamma = 21) \) because of the relativistic rise in the high \( \gamma \) region (compare the two histograms on the left in Fig. 14 or 15).

The following were the problems recognized in the Bevatron experiment: (a) the system of MWPC's and their electronics circuits did not accept large pulses from energetic TR photons. The system saturated at a rather low energy between 12 and 21 keV. This fact hindered us from comparing the experimental results and the predictions precisely; (b) the gases used in the chambers were argon and krypton which were not very efficient to absorb high energy photons; (c) the electron beam intensity from the Proton Machine was too weak to give a good amount of data.

In spite of these problems, the remarkable displacement due to the TR effect observed in the Bevatron experiment, encouraged us to refine the experiment. More details of the experiment can be found in Ref. 18.

### 3.3 Present Experiment at SLAC

At the Stanford Linear Accelerator Center, a pure electron beam is available at high repetition rates. The
energy can go up much higher than that from the Bevatron.

We prepared a fast data-taking system consisting of minicomputer NOVA 800 and its devices: the teletype, CRT, and tape unit.

The MWPC's were leak-tight and sealed off so that expensive gas could be used in the chambers, such as krypton and xenon. The high Z gas was expected to have a much better efficiency because the absorption increases at $Z^4$ but the ionization depends upon only the chamber thickness.

a. **Sandwich Array**

First, the sandwich array setup was re-used as in the Bevatron experiment. The radiators were made of 1/2 and 1 mil Mylar foils. MWPC's were sealed off with a krypton-methane mixture. Eight modules of radiators and chambers were aligned along the well-focused beam of electrons or pions.

b. **Magnetic Separation Array**

A new array called magnetic separation (MS) was employed in this experiment. An application of a weak magnetic field separated the incident beam from the TR x-rays which made it possible to eliminate the ionization loss by the incident beam from the x-ray detector. The magnetic field strength must be low enough so that the synchrotron radiation by the incident particle would not
cause any significant background. In this array, the x-ray detector could be as thick as necessary to absorb the x-ray flux efficiently. The "thick" detector was composed by combining all eight chambers closely together. In addition, the high Z gas xenon filled the chambers which increased the detection efficiency of x-rays. This array enabled us to observe the pure TR spectrum as well as to improve the resolution of a $\gamma$ value because no significant background exists in the detection of the x-rays as compared with the SW array.

c. Radiator Material and Detection Efficiency of MWPC Polyethylene and Mylar Radiators

In order to maximize the TR detected yield for certain combinations of a radiator and detector, one must consider the following parameters: plasma frequency and absorption coefficients of radiator materials, and the efficiency of the detector for the specific spectrum to be seen.

The plasma frequencies of polyethylene and Mylar decide the peak height (refer to the total yield in Eq. (12) which is proportional to $\omega_p$) and position of the last peak of the TR spectrum, as has been shown in Section 3.1.

The absorption coefficients for both Mylar and polyethylene are compared in Fig. 16. In the energy region where the last peak of the spectrum appears, the ratio of the absorption coefficients for polyethylene to Mylar is about 0.63. The total energy from a foil stack still
Fig. 16

Absorption Coefficients $\mu$ of Mylar and Polyethylene.
remains greater for Mylar than for polyethylene because of the hardness of the spectrum (Table 1).

Nevertheless, in the case when the MWPC is used as an x-ray detector, the efficiency for a softer spectrum is much better. The harder spectrum from the Mylar radiator is detected less efficiently by the MWPC than the softer spectrum. As shown in Table 2, the yield from the polyethylene radiator will be detected more efficiently with the gas in the chamber (whichever gas Kr or Xe is used) than that from the Mylar radiator. In columns 4 and 6, the overall detection efficiencies (= total detected yield in a 1.5 cm chamber/emerging flux out of a stack) for Mylar and polyethylene radiators are compared with each other. Hence, the radiator material producing softer spectrum gives more gain when it is combined with gas-filled MWPC's. The most important caution in this case is to use thin windows for the chamber which preserves a soft component of the TR spectrum.

Furthermore, the less absorptive material gives a higher saturation limit to the number of foils as indicated in Section 3.1. The number of polyethylene foils can be increased by 30% over that of Mylar foils.

Kr and Xe Gas Fillings for the MWPC's

Comparing again column 4 with column 6 in Table 2, we can tell that the Xe chamber is much more efficient
### TABLE 2

**CALCULATED OVERALL DETECTION EFFICIENCY \( \langle e \rangle \) OF 1.5 cm MWPC WITH Kr OR Xe RADIATOR CONFIGURATION (100, 1, 125)**

**3 GeV ELECTRON**

<table>
<thead>
<tr>
<th>Radiator Material</th>
<th>Emerging Yield Out of Stack S keV</th>
<th>Detected Yield by a Single 1.5 cm MWPC Krypton W keV ( \langle e \rangle ) %</th>
<th>Xenon W keV ( \langle e \rangle ) %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mylar</td>
<td>41.6</td>
<td>4.52</td>
<td>11</td>
</tr>
<tr>
<td>Polyethylene</td>
<td>33.5</td>
<td>4.64</td>
<td>14</td>
</tr>
</tbody>
</table>

\[ \langle e \rangle = \frac{W}{S} = \frac{\text{Total Detected Yield in Chamber}}{\text{Emerging Flux out of Stack}} \]
than the Kr one. We chose Kr filling for the chambers in the SW array to cross-check it with the Bevatron experiment.

Being limited to use only one set of radiator and detector in the MS array because of the necessity of gradual bending, we chose the best combination (polyethylene radiator/Xe MWPC). With all eight chambers, we expected to have an overall detection efficiency of 53%.

3.4 Photon Background from a Bremsstrahlung and Synchrotron Radiation

The backgrounds are associated with a well-defined beam; namely, gamma rays produced in the bremsstrahlung of the particle going through the radiator materials and the synchrotron radiation emitted by the particle passing through the weak magnetic field. The following computation will provide information about the background.

a. Bremsstrahlung

Since the frequency distribution of bremsstrahlung is nearly flat, the radiation spectrum by the particle going through the material of thickness \( t \) may be approximated by

\[
\frac{dW}{dE} = \frac{t}{X_0}
\]

(33)

for the radiated energy \( E < E_e = \) electron energy, \( t \) the total thickness of the foil radiator, and \( X_0 \) the radiation length of the material. The values of \( X_0 \) for the different radiator materials are listed in the last column of Table 1.
The number of photons emitted in $E$ and $E + dE$ is obtained

$$\frac{dN}{dE} = \frac{dW}{dE} \cdot \frac{1}{E} \cdot . \quad (34)$$

The total number of photons having energies greater than $E_{\text{min}}$ is obtained by integrating it

$$N(E_{\text{min}} < E < E_{\text{max}}) = \int_{E_{\text{min}}}^{E_{\text{max}}} \frac{dN}{dE} \, dE = \frac{t}{X_0} \ln \frac{E_{\text{max}}}{E_{\text{min}}} \cdot \quad (35)$$

By considering $E_e = 15$ GeV $= 15 \times 10^6$ keV electrons and setting the maximum observable energy of the x-rays $E_{\text{max}}$, the number of photons $N$ is given in Table 3. The Mylar $(100, 1, 125)$ radiator is chosen for the material.

However, most of the energetic photons from the bremsstrahlung will not be detected by the proportional chambers. The upper most energy to be detected by a xenon-filled MWPC is around $E_{\text{max}} \sim 100$ keV. In other words, the very energetic x-rays will not be absorbed by the 1.5 cm chamber. What is relevant is the detection efficiency of transition x-rays relative to that of bremsstrahlung. The second column of Table 3 gives maximum boundary $E_{\text{max}}$, where the chamber becomes insensitive to the high energy x-rays. The total amount of bremsstrahlung produced between the threshold of the chamber $E_{\text{min}}$ and $E_{\text{max}}$ is listed in terms of energy and number of photons in columns 3 and 4. When we take this
TABLE 3

PHOTON BACKGROUNDS FROM BREMSSTRAHLUNG PRODUCED BY AN ELECTRON PASSING THROUGH A MYLAR RADIATOR (100, 1, 125)

<table>
<thead>
<tr>
<th>Gas in 1.5 cm MWPC</th>
<th>MWPC Cutoff E&lt;sub&gt;max&lt;/sub&gt;</th>
<th>Energy Produced W,E&lt;sub&gt;min&lt;/sub&gt;&lt;sup&gt;+&lt;/sup&gt;&lt;E&lt;E&lt;sub&gt;max&lt;/sub&gt;</th>
<th>No. of photons N,E&lt;sub&gt;min&lt;/sub&gt;&lt;sup&gt;+&lt;/sup&gt;&lt;E&lt;E&lt;sub&gt;max&lt;/sub&gt;</th>
<th>Energy detected by MWPC E&lt;sub&gt;max&lt;/sub&gt; m&lt;sub&gt;1&lt;/sub&gt; m&lt;sub&gt;n&lt;/sub&gt; max keV keV keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar-CH&lt;sub&gt;4&lt;/sub&gt;</td>
<td>24 0.183</td>
<td>1.81 x 10&lt;sup&gt;-2&lt;/sup&gt;</td>
<td>0.055</td>
<td></td>
</tr>
<tr>
<td>Kr-CH&lt;sub&gt;4&lt;/sub&gt;</td>
<td>59 0.487</td>
<td>2.60 x 10&lt;sup&gt;-2&lt;/sup&gt;</td>
<td>0.114</td>
<td></td>
</tr>
<tr>
<td>Xe-CH&lt;sub&gt;4&lt;/sub&gt;</td>
<td>96 0.809</td>
<td>3.01 x 10&lt;sup&gt;-2&lt;/sup&gt;</td>
<td>0.203</td>
<td></td>
</tr>
</tbody>
</table>

<sup>+</sup> E<sub>min</sub> = 3 keV (Detection Threshold of MWPC)
small range of energy, the energy produced by bremsstrahlung is independent of the electron energy. By taking into account the detection efficiency of the chamber, we obtain the detected energy from bremsstrahlung in the last column. For this comparison, the number of bremsstrahlung x-rays will be about 5 per cent of the TR x-rays.

b. Synchrotron Radiation

The synchrotron radiation is given by the formula

\[
\delta E_{\text{synch}}^{\text{(1 revolution)}} = 8.85 \times 10^6 \frac{E^4}{\rho} (\text{keV}), \tag{36}
\]

where \( E \) is the energy of the particle in GeV and \( \rho \) is the orbit radius in cm. The critical frequency \( \omega_{\text{crit}} \) beyond which there is negligible radiation at any angle, is defined as

\[
\omega_{\text{crit}} = 3\gamma^3 \frac{c}{\rho} = 1.33 \times 10^{-4} \frac{E^2 B}{(\text{keV})}, \tag{37}
\]

where \( B \) is the magnetic field in gauss. For a particle having an arc trajectory of length \( L \) in the \( B \) field, a term \( L/2\pi \) must be multiplied to (36). The total radiation is

\[
\delta E_{\text{synch}}^{\text{(arc)}} = 1.268 \times 10^{-7} \frac{p^2 B^2 L}{(\text{keV})}, \tag{38}
\]

where \( L \) is in cm and \( p \) in GeV.

In this experiment, we varied \( L \) in order to have the same amount of deflection at the detector for different momenta. Columns 2 and 4 of Table 4 gives the critical
### TABLE 4

PHOTON BACKGROUNDS FROM SYNCHROTRON RADIATION

**B = 0.25 kilogauss**

<table>
<thead>
<tr>
<th>Electron Energy $E_e$ (GeV)</th>
<th>Critical Frequency $\omega_{crit}$ (keV)</th>
<th>Path in B field $L$ (cm)</th>
<th>Total Synchrotron Radiation $\delta E_{synch}$ (keV)</th>
<th>Detected by 1.5 cm Xe MWPC $^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.30</td>
<td>180</td>
<td>0.013</td>
<td>≈ 0</td>
</tr>
<tr>
<td>9</td>
<td>2.69</td>
<td>600</td>
<td>0.39</td>
<td>0.17</td>
</tr>
<tr>
<td>15</td>
<td>7.54</td>
<td>600</td>
<td>1.07</td>
<td>0.81</td>
</tr>
</tbody>
</table>

$^+$ Detection Threshold of MWPC = 3 keV
energy and total synchrotron radiation at three electron momenta: 3, 9, and 15 GeV/c. The synchrotron radiation peaks around $0.1 \omega_{\text{crit}}$ which is below the detection threshold of the chamber (a few keV). Therefore, the detector sees only a portion of the synchrotron spectrum. Column 5 of Table 4 gives the total synchrotron radiation detected by 1.5 cm Xe-filled MWPC. The synchrotron radiation flux increases rapidly with the value of $\gamma$.

The calculated photon background will be compared with the background measured in equivalent absorber runs which were made for every radiator run (see Section 5.3).
4. EXPERIMENTAL PROCEDURES

4.1 General

At the Stanford Linear Accelerator Center, experiments were undertaken with beams of electrons and pions. Figure 17 shows the beamline appearing from End Station C of the experimental yard at SLAC. The beam is heading almost toward the east.

Upstream (not shown in Fig. 17) the central beamline \((C_E)\) was shared with the streamer chamber experiment, which required a high intensity beam. The beamline was branched into two: one beamline (number 23) to the north for the streamer chamber and the other beamline (number 6) to the south for the 82" bubble chamber. Part of the bubble chamber beam 6 was trimmed by a target and a bending magnet down to the intensity desired for use by the bubble chamber. The trimmed portion of the beam was put into a beam dump on the north side of beamline 6.

Beamline 6 was directed to the bubble chamber, which was located about 100 feet away from the last bending magnet 6D4.

The counters and detectors for the experiment were assembled in a special trailer (TR trailer) in front of the bubble chamber hut. The first beam-defining counter \(T_1\) was located upstream in BLDG 110 (Streamer Chamber Building) and the remaining setup was put in the TR trailer (Fig. 17).
TC: TARGET CHANGER
PR: BEAM PROFILE MONITOR
I: BEAM CURRENT INTENSITY MONITOR
Q: MAGNETIC QUADRUPOLE
D: D.C. BENDING MAGNET

Fig. 17
Beamline Layout
The beam is steered by a number of quadrupoles and bending magnets (numbered, starting with 6). The last focus of the beam was adjusted to be at counters T\textsubscript{2} and T\textsubscript{3}, placed at the rear end of the TR trailer (Fig. 18).

The intensity of the beam is monitored by ionization chamber I. The energies of the beam were ranged from 3 to 15 GeV/c. The pulse rep rate available at this beamline was 10 to 30 pps, and the pulse width of the beam was 1.6 \(\mu\)sec.

The beam specifications required for our experiment were as follows:

(a) low intensity; the intensity of the beam was trimmed low enough so that no more than one incident particle went through the detection system during its data acquisition period;
(b) selection of electrons or pions at will;
(c) a small angular divergence of beam especially in the MS array;
(d) a small size of beam spot at the radiator.

In the following sections, we will discuss each of these points in detail.

4.2 Beam Definition

a. Counters

The beam of electrons or pions was defined by scintillators T\textsubscript{1}, T\textsubscript{2}, T\textsubscript{3}, and identified by a shower counter (SH).
Experimental Setups
The counters A₁ through A₄ surrounding T₁, except for the hole in the center for counter T₁, and counter Z placed behind counter T₃, are used in fast coincidence (Z·ΣA) to veto the coincidence of counters T₁, T₂, and T₃ in order to reject non-collimated particles in accidental T₁ coincidence (Fig. 18).

The collimator covering mostly the A's consisted of 8" thick lead bricks and a hole 2.5" x 2" in the center. This collimator absorbed some backgrounds.

The lead-lucite shower counter at the end of the beam-line identifies either the electron or pion. The shower counter was made of a total of 8 radiation lengths of lead, segmented into 21 thin slabs and sandwiched with lucite.

b. Fast Logic

The fast logic system for these counters are shown in Fig. 19. The signals from beam-defining counters T₁, T₂, and T₃ go into a triple coincidence circuit of which output is supplied to the first CHRONETICS 103 coincidence unit. The output from the triple coincidence unit (T₁·T₂·T₃) 103 was put with the SH signal in the next stage. The output of the latter 103 unit gave a prompt signal of coincidence (T₁·T₂·T₃·SH) among T₁, T₂, T₃, and SH.

The T₁ signal going into the first 103 unit also came out as a feed-through signal going into a third 103 unit. A delayed signal from counter T₁ was connected to the third 103 unit (T₁·T₁ DELAY).
Fig. 19

Fast Logic Diagram
On the top portion of the diagram (Fig. 19), all A signals were fed to a fan-in and went to a 103 unit (Z·ΣA) to meet a signal from the Z-counter.

To reject accidentals, we made a vetoing signal to the main coincidence from two kinds of signals. First, a 101 unit (101 DELAYED) was kept on once it was signaled by a first particle coming into the system during a beam pulse. If any particle came after the first one in the same pulse, it would produce a signal from unit 103 (T₁·T₁ DELAY). Another vetoing signal was the output of the fan in the A counters. The accidental events were thus removed by vetoing the trigger T₁·T₂·T₃ in unit 103 (MASTER), which were delayed by 1.7 μsec due to the need for waiting out the beam pulse (1.6 μsec). Refer to the timing chart in Fig. 20. The A counter veto was placed in coincidence with the Z-counter in order to increase beam efficiency (i.e., veto only directional background).

c. Adjustment of the System

The beam was fundamentally controlled as follows.
(a) Intensity: by varying the "takeoff" angle from the upstream target (tungsten or beryllium), the scattered beam intensity going through the steering system was controlled.
(b) Location: the bending magnet 6D2 was used mainly for energy-defining. 6D2-1 bends the beam vertically. 6D3 and 6D4 translate the beam to north-south, i.e., horizontally.
Timing Chart for Fast Logic

Fig. 20h

MACHINE R.F. PULSE

BEAM PIP

GATE FOR
COINCIDENCE UNITS

PROMPT MASTER
TRIGGER

DELAYED MASTER

2.2 µs

1.6 µs

1.8 µs

2 µs

1.7 µs

Timing Chart for Fast Logic
(c) Focusing: 6Q5 and 6Q6 focused the beam onto counters $T_2$ and $T_3$.

**Background**

At the beginning when we were receiving the beam pulses in phase with the streamer chamber beam, the large muon background coming from the direction of the beam dump kept our counting system too busy (as beam 6 was bent back to the north. See Fig. 17). This was found by the fact that the background did not decrease the intensity even with an absorber of 40 radiation lengths. The SLAC people arranged for us to receive the electron beam in a different pulse. The muon background was reduced low enough so that our detector system could recognize electrons (pions).

**Beam Efficiency**

The background correlated with the beam was also difficult to be removed. The correlation was recognized by the fact that the anti-counter registered 4 counts per pulse when the beam was on and 0 when the beam was off. Furthermore, it was found that 50% of the anti-counts were going through the chambers, which were observed by triggering the chambers with the signal from the anti-counters. The beam efficiency in a beam survey (Table 5a) reached up to 6.5% at $T_1$/pulse = 1 when the beam intensity was controlled by the focusing magnet $Q_{1,2}$.

The Poisson statistics, however, predicts the maximum
TABLE 5
ADJUSTMENT OF COUNTING RATE ON T₁ WHICH MAXIMIZES BEAM EFFICIENCY
WITH 15 GeV ELECTRONS

<table>
<thead>
<tr>
<th>Magnet Setting</th>
<th>Counting Rates</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q₁,₂ mV</td>
<td>T₁</td>
<td>T.SH</td>
</tr>
<tr>
<td>(a) Before Z-Counter added to veto with 2000 beam pulses</td>
<td></td>
<td></td>
</tr>
<tr>
<td>See below*</td>
<td>2283</td>
<td>493</td>
</tr>
<tr>
<td></td>
<td>564</td>
<td>107</td>
</tr>
<tr>
<td></td>
<td>5120</td>
<td>1085</td>
</tr>
<tr>
<td>(b) After Z-Counter added to veto with 300 beam pulses</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50.05</td>
<td>1142</td>
<td>604</td>
</tr>
<tr>
<td>44.00</td>
<td>351</td>
<td>212</td>
</tr>
<tr>
<td>37.00</td>
<td>141</td>
<td>83</td>
</tr>
</tbody>
</table>

* The beam intensity on T₁-counter was controlled by the focussing magnet Q₁,₂.
* The values of Q₁,₂ setting during this beam survey are not available.
possible efficiency as follows. Let the probability of counting \( n \) electrons with the mean counting rate \( m \):

\[
P_m(n) = e^{-m} \frac{m^n}{n!} \quad (n = 0, 1, 2, \ldots)
\]

We do not want to have \( n \geq 2 \) cases too often. Therefore, we must look for the average counting \( m \) which maximizes the probability of counting one electron, \( P_m(1) \). By taking the derivative of \( P_m \) with respect to \( m \), we obtain the average counting rate \( m = n \), i.e., \( P_m(1) \) can be maximized with average \( m = 1 \). The maximum probability attainable with the average counting \( m = 1 \) is given by

\[
P_1(n) = e^{-1}/n!
\]

Then we obtain \( P(0) = 0.368 \), \( P(1) = 0.368 \), and \( P(n \geq 2) = 0.264 \) with \( P(0) + P(1) + P(n \geq 2) = 1 \). The maximum efficiency attainable is therefore 36.8%. In addition, it was found that the anti-counting rate \( T_1/\Sigma_A \) was almost independent of \( T_1/\text{pulse} \) as shown in Fig. 21. This fact shows that the accidentals were beam-associated.

In order to improve the beam efficiency, we added another anti counter \( Z \) behind the chambers and scintillation counters \( T_2, T_3 \) downstream, and the signal from the \( Z \)-counter was added into the vetoing signal as \( \Sigma A \cdot Z \). Table 5b shows how the efficiency was improved. The anti \( \Sigma A \cdot Z \) now became strongly dependent on the \( T_1 \) counting rate. Peaking around \( T_1 = 1 \), the efficiency increased up to 20%. In Table 6
Fig. 21

Rate of Beam Associated Accidental

15 GeV/c e⁻
### TABLE 6

**TYPICAL BEAM EFFICIENCY DURING RUN**

<table>
<thead>
<tr>
<th>Type of Setup</th>
<th>Beam Energy $E_e$ GeV/c</th>
<th>Counting Rates $T_1$/pulse</th>
<th>$T$/pulse</th>
<th>Ratios $T/T_1$</th>
<th>$\Sigma A\cdot Z/T$</th>
<th>Efficiency Master pulse $\times 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Setup</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EAo</td>
<td>3</td>
<td>0.572</td>
<td>0.141</td>
<td>0.247</td>
<td>0.201</td>
<td>5.7</td>
</tr>
<tr>
<td></td>
<td>SW 9</td>
<td>0.906</td>
<td>0.241</td>
<td>0.266</td>
<td>0.223</td>
<td>6.7</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>1.31</td>
<td>0.564</td>
<td>0.431</td>
<td>0.213</td>
<td>14.8</td>
</tr>
<tr>
<td></td>
<td>MS 3</td>
<td>1.085</td>
<td>0.110</td>
<td>0.101</td>
<td>0.183</td>
<td>3.8</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>1.228</td>
<td>0.354</td>
<td>0.288</td>
<td>0.083</td>
<td>11.6</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>1.655</td>
<td>0.647</td>
<td>0.391</td>
<td>0.121</td>
<td>13.4</td>
</tr>
</tbody>
</table>
the typical beam efficiency during the run is listed. The efficiency at 3 GeV was small mainly because the intensity of the beam was low. The beam condition remained mostly stable and the efficiency varied about ±15%.

4.3 The MWPC and Its Electronics

As has been discussed in Section 3.1, MWPC's were chosen as x-ray detectors mainly because of the following reasons:

(a) sensitivity matches to the TR spectrum;
(b) transparent for high energy photons;
(c) small total mass of the detector preserves beam quality.

a. Structure of the MWPC

The main characteristics of our MWPC must be found in Ref. 32.

The chambers were designed to hold a rare gas mixture inside and be tight enough so that it could be operated without continual flowing of gas. This is important when the chamber gas is expensive as is krypton or xenon.

The schematic drawing in Fig. 22(a) shows the wire planes and windows. The center thin wire plane consists of 100 stainless steel wires (20 μm in diameter). The high voltage wire planes were made of 10 mil copper wires. The spacing of wires in each plane was 2 mm. The high voltage wires were perpendicular to the center of the wires. The chamber frame was made of four plates of a Nema-G10 board.
(a) STRUCTURE OF MWPC

(b) WIRING OF MWPC

Fig. 22

Structure and Wiring of a MWPC
(12" x 12" x 118") layered on top of each other. Each plate
had a 9" x 9" hole in the center. The 100 wires stretching
across the hole of a plate were soldered to two printed-
circuit boards epoxied on the plate. One of the two inner
plates of the frame had two gas holes at the opposite corners
to take 1/8" OD gas tubes.

The window material, the sealant gluing all the plates,
and the gas tubing were selected under a serious requirement
that the chamber must be kept leak-tight and sealed off for
a long run. A preliminary test showed that if there were
a 0.1% contamination of either O₂ or H₂O, it would degrade
the gain of a chamber by about 10 per cent, and the energy
resolution would become worse. The contamination may
increase for many reasons. A pinhole on the window foil
can let air diffuse into the chamber. A small gap between
the Nema board also causes the leakage (greasing them
together did not stop leakage through the small gap between
the plates). An imperfect tubing can absorb contaminating
gases (tygon tubing, swage-locked copper tubes, etc.). All
these possible causes were minimized by the use of well-
selected materials for all the parts:

(i) Laminated window material. The inner surface of
the window must be conductive to avoid the charging and
discharging by the electrostatic field from the high
voltage wire plane. An aluminum window may be chosen for
this purpose. However, it may be full of pinholes. We
chose metalized Mylar sheets manufactured by the G. T. Schjeldahl Company. A sheet (1 mil thick) of metalized Mylar consists of a 1/2 mil aluminum foil and a 1/2 mil Mylar sheet bonded with a thermosetting polyester adhesive.

(ii) Silicon rubber sealant. The Nema boards must be glued to be lead-tight. If no dismounting is necessary, RTV102 silicon rubber can seal off the chamber much better than epoxy glue or vacuum grease. A chamber sealed with vacuum grease often leaked as much as $6 \times 10^{-3}$ cc/sec, which could be detected by a portable gas sniffer, e.g., Bacharach leak detector.

(iii) O-ring vacuum couplings. In order to make tubing to the chamber easy and tight enough, O-ring vacuum couplings (Cajon fittings) were used. The couplings consisted of three parts: nut, gland, and body made of brass. The joints of the chamber were physically rather weak. Hence, these couplings were soldered to flexible-line tubing before they were connected to a more rigid copper tubing. With these demountable couplings, the handling of the chamber also became much easier.

With all of these improvements, the degradation obtained for the closed-off chamber is only 7% for 50 hours. The resolution (FWHM) of Fe-55 peak changes 25% to 40% in the same period. Before being mounted on the array, every chamber was tested for degradation of gain and its gain uniformity over the sensitive area. Only
those chambers with a 10% degradation lifetime greater than a half day and uniformity within 10% over the sensitive area were accepted.

b. Electrical Circuity to the Chamber

The chambers were kept in an electrically-shielded box to decrease background noise in the experimental area. The box was made of copper (17" x 30" x 1.5") and contained a readout print-board, two preamplifiers for the A and B sides, two bus lines of high voltage distributors, and gas plumbings to the chamber. Figure 23 shows the layout inside of the copper box. The chamber is located to the left of the center of the box which has two 8" x 8" holes matched to the sensitive area of the chamber. The center wire plane is hooked to a readout print-board which has two preamplifiers on its opposite ends. Each high voltage wire plane is connected to the bus line. The inner metal surface of the window is also connected to a different high voltage.

The schematic drawing in Fig. 22(b) shows an electrical connection of the chamber to the high voltage line and to the amplifiers. The signal plane wires are connected to the amplifier with a high impedance input stage. The circuit diagram of the preamplifier component is shown in Fig. 24. The FET used in the first stage of the amplifier was to improve signal-to-noise ratio and also to stretch the output pulse.
Fig. 23

Inside Layout of a Copper Box Holding a MWPC
Fig. 24

Schematic Diagram of a Charge Sensitive Amplifier
The outer field between the HV wire and window was necessary because of the following reason. When the MWPC was operated with the same potential on both the window and high voltage wire plane, the resolution degraded as shown in Fig. 25 in 20 hours. With the application of the outer field, the pulse height distribution as good as the first one was resumed. The recombination of free electrons in the space between the window and HV plane was the cause of the decrease in the pulse height. This was also confirmed by the following fact. When we increase the potential on the window relative to the HV wire plane, the counting rate doubled at the window voltage above 2.1 KV as shown in Fig. 26 (i.e., the effective thickness doubles with the application of the outer field). The resolution changed within the window high voltage range 2000 to 2500 volts from 65% to 24% (Xe MWPC as in Fig. 27(a)). The potential value on the window was kept about 100-200 volts greater than that of the wire plane because for a constant gain the resolution became minimum at a certain combination of wire and window HVs as shown in Fig. 27(b).

The high voltage setting for the MWPC was chosen by the following procedures. By varying the high voltage to the chamber, we measured energy resolution for Fe$^{55}$ x-ray source and used the HV setting, with which the best resolution was obtained.
Fig. 25

Degradation of a MWPC Resolution with No Additional Fields Between the HV Wires and Window
Fig. 26

Counting Rate Depending on the Window Voltage Relative to the Wire High Voltage
Window Voltage Dependence of Gain and Resolution of MWPC's
The gain of the MWPC is adjusted so as to give the same pulse height with the same source for different gas fillings by taking the HV setting along a horizontal line in Fig. 28. In this manner we avoid readjustment of the working range of the ADC unit which digitized the chamber signal for the following recording system.

The maximum measurable energy with this amplifier and ADC system was set at 60 keV by using two calibration points of Fe$^{55}$ and 5.9 keV and Cd$^{90}$ at 22.6 keV. The gate width to the ADC was also fixed for the entire run so as to avoid recalibrating the ADC. The gain was adjusted only by changing the high voltage supply to the MWPC's. The difference of gain among the chambers were minimized by adjusting each attenuator so that the Cd$^{109}$ peak for each chamber agreed with a "standard chamber" chosen among the chambers (B4).

The change of MWPC gain during the run was checked regularly with the standard x-ray sources. The gain of all the chambers was adjusted at the beginning of the series of runs. After being closed off, each chamber degraded with a different rate. The correction for this gain change was made by using the data of calibration runs. The details of this point will be discussed in Section 5.1.

4.4 Data Recording System

Figure 29 shows a schematic diagram of the slow logic. The signals from the MWPC's were digitized with the LeCroy
Fig. 28

MWPC GAIN vs. HIGH VOLTAGE

*The gain indicated on the ordinate corresponds to 17 mV output.
Fig. 29

Slow Logic Diagram
2248-S ADC, and the NOVA 800 computer recorded the pulse height onto the magnetic tapes.

The gate pulse for the ADC, required to be less than 200 nsec for accurate position, was positioned by a TEKTRONIX scope 454A which was used to locate the narrow gate right on the peak of chamber signal which had a rise time of 4 μsec and a decay time of 20 μsec or longer. A gate output of 454A went to a pulser which provided the proper width of the gate pulse for the ADC. As the offset (pedestal) of the ADC depends upon the gate width, the width was kept stable and constant in order to avoid recalibration of the ADC. (More detail on the ADC will be discussed in Section 5.1.)

The digitized information was read out in the form of 16-bit words by the NOVA computer which collected data from two ADC units and wrote data in the records onto the tapes at a rate up to 400 per sec. Each record of 120 events had a header for the run number and description on the experimental conditions. The computer also set up histograms which could be displayed by the TEKTRONIX CRT 611 during the run on demand. This enabled the experimenters to check the quality of the data and to detect any trouble. The KENNEDY tape unit wrote each event on magnetic tapes in a group of 9 words. The first word of the group was the event number counted from the beginning of the run. The next 8 words were the compacted data from the 2248-S.
4.5 Sandwich and Magnetic Separation Array Experiments

The apparatus in the experiment was fundamentally common to both SW and MS array setups, except that a weak magnetic field was employed for the latter case. Figure 18 shows the schematic arrangement for the two kinds of arrays.

The SW array setup is shown in the dotted box on the lower right and the MS array on the lower left. For detection of x-rays and beams separated in the MS array, MWPC's were electrically split into two sections: left and right (A and B sides). The experimental runs were made as follows.

a. **Sandwich Array Experiment**

**Setup** - The SW array was tried first. Eight sets of radiator MWPC's were aligned to the direction of the incident electron beam penetrating all the way to the end where a set of beam-defining counters were located. In the SW array, the beam hit the chambers on the A side. In front of each MWPC, a radiator of Mylar (100, 0.5, 100) or (100, 1, 100) was placed. During the SW array experiment, the beamline between counter T₁ and the SW array setup was clear of the MS radiator holder (OXFORD RAD HOLD) and 40 permanent magnets on the top left of Fig. 18.

**Beam** - The beam was defined by two sets of counters T₁ (3" x 3") upstream, and T₂ (1½" x ½") and T₃ (1½" x ½") overlapped by 1" downstream. The beam profile was taken
occasionally during the run. Column 2 in Table 7 shows the measured width with the overlapped counters $T_2$ and $T_3$. With electrons at 3 GeV/c, the computed beam is given below

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st focus image x magnification</td>
<td>.175&quot;</td>
</tr>
<tr>
<td>momentum resolution $\Delta p/p = 2.5%$</td>
<td>.175&quot;</td>
</tr>
<tr>
<td>chromatic aberration</td>
<td>.08&quot;</td>
</tr>
<tr>
<td>multiple scattering in</td>
<td></td>
</tr>
<tr>
<td>6-mil vacuum window</td>
<td></td>
</tr>
<tr>
<td>7' air-following vac window</td>
<td></td>
</tr>
<tr>
<td>0.15&quot; front counter $T_1$</td>
<td></td>
</tr>
</tbody>
</table>

Total size FWHM = .92" for 3 GeV/c e^-

This is consistent with the measured values FWHM = 1" with a 5/16" finger counter and 1.9" with 1" overlapping between $T_2$ and $T_3$.

In Fig. 30 the SW array looking downstream is shown. Eight copper boxes having windows (sheets of aluminized Mylar) were aligned along the beamline for the passage of beams and x-rays. Between chambers, the radiators are sandwiched. On the right-hand side of the copper boxes, the power lines to the amplifiers, the signal cables, the high voltage cables, and the gas tubing are seen.

b. Magnetic Separation Array Experiment

Setup - When the MS runs were made, all 8 chambers were closely packed to each other to form a single "thick"


**TABLE 7**

**BEAM PROFILES**

<table>
<thead>
<tr>
<th>E (GeV)</th>
<th>Sandwich Array</th>
<th>Magnetic Separation Array</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal Size†</td>
<td>With 1&quot; Overlap between T₂ and T₃</td>
</tr>
<tr>
<td>3</td>
<td>1.9&quot;</td>
<td>1.4&quot;</td>
</tr>
<tr>
<td>9</td>
<td>1.5&quot;</td>
<td>0.8&quot;</td>
</tr>
<tr>
<td>15</td>
<td>0.9&quot;</td>
<td>0.6&quot;</td>
</tr>
</tbody>
</table>

† FWHM of Vertical Profile = 1" with T₂ and T₃ of 0.25" vertical size.
Fig. 30. MWPC's in the SW Array (Looking Downstream) (Photograph was taken by L. Shiraishi)
x-ray detector filled with xenon.

Only one radiator set was held in the RAD HOLD. The holder frame for the stack of foils was made of a 12" diameter cardboard which has an 8" hole in the middle. The foil is glued onto the frame of the cardboard. The air spacing was also made of cardboard with the same size hole. The thickness of the foil was varied as 1/2, 1, and 2 mils. The whole stack of foils and spacing cardboards were pressed uniformly with a rigidly-made pipe frame with a rotating rod which also has an 8" hole inside and is lever-operated to press only the rim of the radiator stack. This kind of heavy framing was appropriate only for the MS array in practical reason because the MS array needs only one set of radiator while the SW array needs 8 sets.

A number of permanent magnets were enclosed in helium bags. A long path length for a weak bending field was necessary to minimize the synchrotron radiation. The helium bags (700" long) were used to minimize absorption by air--but we inadvertently made an error in allowing the technicians to use PVC windows. We will come to this problem in Section 5.3b.

Beam - The beam position in the MS array, relative to the chambers, were more difficult than in the SW array. The chambers were shifted 2 cm south (to the right, looking downstream) of the beamline. Figure 31 shows the
Beam and TR X-Ray Locations in the Sensitive Volume of MWPC's in the MS Array
relative positions of TR x-rays and separated electrons coming into the chambers. The electrons went through the B side of all 8 split chambers numbered as B0 - B7.

The necessary deflection of the electron (pion) beam was obtained by changing the path length \( L \) in the permanent magnets. Figure 32(a) sketches the position of the magnets and the detector. The total path \( l \) from the left end of the magnets and to the detector was kept constant, i.e., \( l = L + l' \), where \( l' \) is the free-space length. Only the magnets on the right end were pulled in and out. For a deflection \( d \) and a radius of curvature \( \rho \) of the particle in the magnetic field \( B \approx 250 \) gauss, \( L \) was obtained from the following relation

\[
L = l - \sqrt{l^2 - 2\rho d}.
\]

The permanent magnet was made of barium ferrite in a matrix. A holder of magnets has a structure shown in Fig. 32(b). The 8" x 6" x 1/4" plastiform pole tips were glued to a steel "H" which was made of 1/8" cold rolled steel.

The adjustment of the separation was, for example, made as follows. If a 3 GeV/c electron beam was tuned, the number 10 of magnets necessary for the required amount of center deflection 6 cm from the original undeflected beam (i.e., 4 cm from the center line of the chamber) was read off 10 for \( L = 4.8' \) from Fig. 33(a) and then by shifting \( T_{2,3} \) there, the ratio \( T_{1,2,3}/T_1 \) was maximized
A SERIES OF PERMANENT MAGNETS

(a) MAGNET GEOMETRY

COLD ROLLED STEEL "H"

BARIUM FERRITE POLE TIPS GLUED ON "H"

(b) PERMANENT MAGNET

Fig. 32

Geometry and Structure of Permanent Magnets
(a) B Field Length Necessary for a Given Separation

(b) Width of a 9 GeV Beam Deflected by a Weak B Field

Fig. 33

Deflection of Beam by a Weak Magnetic Field
by readjusting the number of magnets up to 12 from the given number above (Table 5b).

During the MS array, the overlap between $T_2$ and $T_3$ was kept $1/2"$ wide. The beam profile remeasured with moving $T_{2,3}$ with $1/2"$ overlap is shown in Fig. 33(b), and Table 7.

The performance of the run was made with efficiency around 5 to 20 per cent (column 7 in Table 6), and the rate of $T_1$ was kept nearly equal to 1 per pulse in order to maximize the efficiency.

c. Background and Calibration Runs

Background Run

For each radiator run, a background run was made with the equivalent absorber inserted on the beamline instead of the radiator (Fig. 18).

A few background runs with no materials on the beamline were also made. The results will be given in Section 5.2.

Calibration Run

The chambers were continuously calibrated with three standard x-ray sources $Fe^{55}$, $Cd^{109}$, and $Pm^{145}$ between the beam runs.

The calibration run could be made in 10-15 minutes for all the chambers. By switching scope 454A to its self-trigger mode and placing an x-ray source on each chamber, we accumulated enough number of events in less than a minute. The analysis of the calibration data will be given in Section 5.1.
5. DATA ANALYSIS AND RESULTS

Data taken during the SLAC run were analyzed and compared with theoretical predictions. The runs included the SW and MS array experiments with electrons (pions) at momenta 3-15 GeV/c. The calibration data taken with x-ray sources during the experiment were also analyzed to obtain the energy scale to the pulse height distribution of beam runs.

First, we will discuss the energy calibration of MWPC's and make a cross-check with the results of ionization loss data measured as equivalent absorber runs.

Second, the beam runs will be analyzed for the two different setups, sandwich and magnetic separation arrays.

Third, we will calculate the yield by using the formula developed in Section 2, and also include the chamber efficiency and absorption by the materials on the beamline.

Finally, we will compare the results of the experiment and calculation.

5.1 Energy Calibration of the MWPC

The calibration of the chambers are complicated mainly because the ADC unit did not retain a stable pedestal value (an off-setting for the noise level) and because the split chamber "cross-talked." A real pulse from a signal wire could induce a pulse of opposite polarity on the opposite side via the high voltage wire planes.
a. The Calibration of the ADC 2248-8 Unit

LeCroy ADC Model 2248-8

The general characteristics of ADC Model 2248-8 are as follows:

(a) Maximum charge input range 160 picocoulombs (pC) in 256 steps;

(b) One of its bilinear modes used in this experiment had a sensitivity of 0.25 pC up to 128 count and 1 pC above 128 count;

(c) An off-set value (pedestal) was set by supplying a DC current. A residual pedestal was given by \( 4 + 0.015t \) pC, where \( t \) is a gate duration in nanoseconds;

(d) The digitizing time (150 \( \mu \)sec) and the data shifting from memory to buffer (10 \( \mu \)sec) for all 8 chambers of one ADC unit were short enough to transfer data to the computer because the beam pulse was taken to our system every 10 msec.

Method to Convert ADC Counts into the Energy Scale

The x-ray detection by the system of the MWPC and ADC can be described in two steps.

First, the MWPC detected x-rays as a converted kinetic energy of electrons which built a proportional signal to the MWPC wires. The charge collected by the chamber was proportional to the photon energy \( E \). Second, the ADC records the total charge as \( Q = \int Vdt \), where \( V \) is the pulse height
during the integration period of time $t$. During the run, the two steps (chamber amplification and charge integration) were an "overall" step. In the off-line calibration process, we traced from a given count back to the value of x-ray energy. We measured the two steps separately. First, supplying the DC input to the ADC unit (instead of the ADC as in the on-line), we obtained a count to the charge (rather correctly, equivalent volts of the integration $Q = \int V dt$) relation curve (counts vs. Eq. V). Secondly, the equivalent volts vs. the energy relation was obtained. This Eq. V vs. E plot is more convenient to see the linearity of the system than the overall count vs. the energy relation. Therefore, we will use this two-stepped method to obtain energy calibration.

**Analysis of Calibration Data**

**Counts vs. Eq. Volts** - Figure 34 shows the counts vs. eq. volts relation. The slope change at count 128 was due to the 4:1 sensitivity difference of the bilinear mode of the ADC unit. This relation was kept the same throughout the experiment. The pedestal values, however, shifted around the nominal value on the plot. (In other uses of the LeCroy 2248 ADC after this experiment, a double-peaked pedestal had been observed and explained by the AC noise pickup in phase with ADC circuits.) In Fig. 35, an example of the shifting pedestal is shown. The left-hand distribution of
ADC CALIBRATION 'A1'

Fig. 34

Counts vs. Equivalent Volts Relation of LeCroy ADC Unit 2248-S
Fig. 35

Pedestal Value Distribution (Shifting and Stable Ones)
the pedestal is double-peaked while during a stable run, the normal pedestal distribution was like the one on the right. The amount of shifting and frequency was quite irregular in the experiment. Therefore, this shifting pedestal value was used to obtain a new counts vs. Eq. volts plot as is shown in Fig. 36. The amount of pedestal shift is typically about 7 counts, which corresponds to 1 keV in the energy scale.

The pulse height distributions of three x-ray standard sources in a krypton chamber are shown in Fig. 37. The ordinate is in counts of ADC which changes its sensitivity level at count 128. Therefore, we can observe a sharp rise at count 128. In both krypton and xenon, we can also observe the "escape peaks" at the lower energy side. (We will discuss this point in Section 5.3.)

Usually, a calibration run was made successively with three sources, Fe, Cd, and Pm at the end of a series of beam runs. Otherwise, the source Cd was used to make a quick calibration. Also, the resolution and the peak position of pulse height distributions was monitored on the CRT during the run.

Equivalent Volts to the Energy Scale - The three source points were plotted on Eq. volts vs. x-ray energy coordinates. A line fitted to the three points was parametrized in terms of two numbers: the slope and
Fig. 36
Transformation of Counts vs. Equivalent Volts Relation
Fig. 37

Fe\textsuperscript{55}, Cd\textsuperscript{109}, Pm\textsuperscript{145} Pulse Height Distributions in Counts
the intercept on the Eq. volt vs. energy scale (Fig. 38).

Figure 39 shows the gain variation of a chamber in the MS array experiment. On the time scale, the run 128 point shows that the chambers were flushed with xenon gas because of degraded resolutions.

b. Cross-Talk Factor Measurement

In only the MS array, the cross-talk between the A and B sides of the chamber became a problem. This is caused by the fact that the chamber was electrically split into two sections, A and B. The chamber, however, still had two common high voltage wire planes of which wires were spanned perpendicularly to the direction of thin wires on the center plane. The high voltage wire could then propagate the induced signal from one side to the other. Namely, there is a cross-talk between the A and B sides.

The correlation between the A and B sides of the chamber was obtained as a scatter-plot (Fig. 40). When an x-ray source was located on one side of the chamber (A side), an induced signal appeared on the other side B. The B-side output was then less than its pedestal value because the induced signal on the B side had an opposite polarity to that appearing on the A side. Therefore, if a scatter-plot between the two sides of a chamber was made on a plane of A vs. B counts, the slope of the line which fit to a cluster scattered on the plane was negative-valued.
Least-Square Fitting to X-Ray Standard Source Points

Fig. 38
Gain Variation of MWPC in Time

Fig. 39

Gain of Chamber

Equivalent Volts

PM (38 keV)

Cd (22.2 keV)

Fe (5.9 keV)

RUNS IN TIME (mm = 2 hrs)
Fig. 40. Scatter Plot Between the A and B Sides of a Chamber for the Cross-Talk Factor Measurement
In the scatter-plot, the axis for the A side on which the source was placed was linearized by removing the 4:1 ratio of the ADC bilinear mode. (See the vertical scale A.)

The correction to the pulse heights coming out from either side of the chamber is about 12%. Let us define the following quantities:

\( G_{A,B} \) : gain of preamps for the A and B sides;
\( V_{A,B} \) : recorded pulse height by ADC in eq. volts;
\( \eta \) : cross-talk factor which describes characteristics of electric capacity of the chamber;
\( A,B \) : true pulse height on the A and B sides generated by TR and ionization by an electron.

There are relations between these quantities:

\[
\begin{align*}
V_A &= G_A (A - \eta B), \\
V_B &= G_B (B - \eta A).
\end{align*}
\] (40)

Solving these relations for A and B in terms of the recorded values on both the A and B sides, the true TR pulse in eq. volt is obtained as

\[
V_{TR} = G_A \cdot A = \frac{V_A + \eta (G_A/G_B) V_B}{1 - \eta^2}.
\] (41)

Then, \( V_{TR} \) can be converted into energies by using the intercept and slope values obtained from the Eq. volts vs. energy plot. For the derivation and its results, refer to Appendix C. The value of \( \eta \) remain very constant as expected.
because it describes a capacity coupling between planes of the chamber (Fig. 41).

c. Calibration with Monochromatic X-Rays

After this SLAC experiment, a separate measurement of the chamber response was made with a monochromatic x-ray beam from an x-ray generator in order to study the characteristics of the MWPC. The x-rays from radioactive sources were not monochromatic because there were two discrete lines Kα and Kβ.

First, the linearity of the chamber with its preamps was tested by changing the energy of x-rays up to 50 keV. Figure 42 shows the results in which escape peaks are also added besides the photo-peak points. In this case, a PHA was used to record the pulse height instead of the system of the ADC 2248-S and the minicomputer. Hence, we did not have the problem of the shifting pedestal. A pulse height distribution taken with the monochromatic x-ray at 40 keV is shown in Fig. 43. The detail of this distribution is found in Appendix B.

Second, it was found that the resolution of chamber varies depending on the incident x-ray energies as shown in Fig. 44. Data from the measurement with the standard sources (Fe, Cd, and Pm) were also added to be compared with the results of the monochromatic beam experiment. The resolution had a minimum value and varies slowly
Linearity of Krypton Chamber with a Collimated Beam
Pulse Height Distribution of 40 keV X-Rays in a 1.5 cm MWPC
Resolution of Photopeaks vs. the X-Ray Incident Energy for Three Gases
within the region of our interest. The resolution, however, did not change significantly and was about 20% on the average.

The other point was found in scanning over the chamber with a collimated x-ray beam. A "copper peak," e.g., x-ray characteristics K-line of copper (8.3 keV), appeared and disappeared. It is evident that the collimated beam could detect the location of the copper wires on the high voltage plane in the chamber. Figure 45 shows that in the pulse height distribution, a copper peak intensity varies depending on whether the beam hits the copper wire or not. The shadowing of these copper wires was taken into account when the TR yield was calculated in Section 5.3.

5.2 The Beam Run Analysis

When a beam of certain energy was ready, several runs were made with different radiators and equivalent absorbers. For instance, the SW array data with a radiator of 1/2 or 1 mil could be made one after another without changing the beam energy.

The beam intensity in the earlier part of the SW array experiment was too intense so that data taken with the chambers filled with argon became unanalyzable. We therefore had SW array data only with krypton. In the MS array, we had the MWPC's filled with only xenon as mentioned in Section 4.5b. In the analysis of events, the following tests were made
Fig. 45

Wire Profile Measured with a Laterally Sweeping Beam
(a) if the particle is an electron or pion by the signal from the shower counter;
(b) if the particle is penetrating through all the chambers or not;
(c) only for the MS array, if any particle is going through the x-ray detection side of the chamber or not.

a. SW Array Data

Selection of Events
The beam penetrated directly through both radiators and chambers on the A side only. "Pion events" and "electron events" were selected depending on the shower counter pulse height. The shower counter pulse height distributions (SHOWER COUNTER) taken with electrons at 3, 9, and 15 GeV are shown in the top left frames of Figs. 46-48 with a 1/2 mil Mylar radiator, and in Figs. 49-51 with a 1 mil radiator. The number of electrons attached indicates the number of electrons to be selected (before the shower cut was applied). The position of the shower counter cut is indicated with an arrow. A pion data analysis was obtained by selecting events lower than the cut level applied to the electrons (Fig. 52). The average pulse height of electron runs in the shower counter is plotted with respect to the incident beam energy in Fig. 53(a). The ratio of the number of pions to the total number of particles is shown in Fig. 53(b). At 9 GeV, about 60% of the beam was pions.
Fig. 46

Pulse Height Distributions in the SW Array with Mylar Radiator (100, 0.5, 100) and Equivalent Absorber at Electron Beam Momentum 3 GeV/c
Fig. 46
(continued)
Pulse Height Distributions in the SW Array with Mylar Radiator (100,0.5,100) and Equivalent Absorber at Electron Beam Momentum 9 GeV/c
Fig. 47 (continued)
Fig. 48

Pulse Height Distributions in the SW Array with Mylar Radiator (100, 0.5, 100) and Equivalent Absorber at Electron Beam Momentum 15 GeV/c
SINGLE PULSE HEIGHTS OF CHAMBER A2

15GEV, E, KR, NYLAR (1.00 - 5.100) SW

SINGLE PULSE HEIGHTS OF CHAMBER A3

2575 ELECTRONS

SINGLE PULSE HEIGHTS OF CHAMBER A4

2576 ELECTRONS

SINGLE PULSE HEIGHTS OF CHAMBER A5

2576 ELECTRONS

SINGLE PULSE HEIGHTS OF CHAMBER A6

2576 ELECTRONS

SINGLE PULSE HEIGHTS OF CHAMBER A7

2571 ELECTRONS

Fig. 48

(continued)
Fig. 49

Pulse Height Distributions in the SW Array with Mylar Radiator (100,1,100) and Equivalent Absorber at Electron Beam Momentum 15 GeV/c
Fig. 49
(continued)
Fig. 50

Pulse Height Distributions in the SW Array with Mylar Radiator (100,1,100) and Equivalent Absorber at Electron Beam Momentum 9 GeV/c
9GEV.E.KR.MYLAR (100,1,00) SW

SINGLE PULSE HEIGHTS OF CHAMBER A2

1109 ELECTRONS

SINGLE PULSE HEIGHTS OF CHAMBER A3

1112 ELECTRONS

SINGLE PULSE HEIGHTS OF CHAMBER A4

1112 ELECTRONS

SINGLE PULSE HEIGHTS OF CHAMBER A5

1112 ELECTRONS

SINGLE PULSE HEIGHTS OF CHAMBER A6

1111 ELECTRONS

SINGLE PULSE HEIGHTS OF CHAMBER A7

1112 ELECTRONS

PULSE HEIGHT (KEV)

PULSE HEIGHT (KEV)

Fig. 50

(continued)
Pulse Height Distributions in the SW Array with Mylar Radiator (100,1,100) and Equivalent Absorber at Electron Beam Momentum 9 GeV/c
Fig. 51  
(continued)
Fig. 52
Pulse Height Distribution in the SW Array with Mylar Radiator (100,1,100) at Pion Beam Momentum 15 GeV/c
Fig. 52
(continued)
Average Shower Counter Energy and Component of Beam Obtained from Pulse Height Discrimination in Shower Counter
Pulse Height Distributions of Chambers

All the frames in a figure shows two kinds of distributions: one taken with the radiator (solid-line histogram) and the other with the equivalent absorber (asterisk histogram). The equivalent absorber run measured mainly the ionization loss of electrons in the chamber. Both histograms have the same number of electrons in each plot.

The second frame of each figure shows the plot of the arithmetic mean of signals over all 8 chambers. Instead, as we have 8 outputs from all the chambers for a single electron event, we can plot all 8 points in the same histogram as in the third frame (Single Pulse Heights of All Chambers). All the rest of the 8 frames show histograms of individual chamber pulse heights.

The number attached to each chamber distribution (the number of selected electrons after the shower cut was applied) indicates the total number of electrons seen by the chamber, which fluctuates by 0.1% from chamber to chamber. This is consistent with the fact that most of the particle were passing through all the chambers.

Detected Yield

The TR spectrum detected is difficult to obtain from the sandwich array, since it required an "unfolding" of
Fig. 54

TR Yield in SW Chambers with Mylar Radiator (90, 0.5, 100) at Electron Momenta 3, 9, and 15 GeV/c
Fig. 55

TR Yield in SW Chambers with Mylar Radiator
(90,1,1,100) at Electron Momenta 3, 9, and 15 GeV/c
The total detected yield by the whole SW array is evaluated by either of two methods:

(i) By summing up yields from all 8 chambers from Figs. 54 and 55.

(ii) By applying the method of integrating the spectrum either to the ARITHMETIC MEAN OF PULSE HEIGHT or to the SINGLE PULSE HEIGHTS OF ALL CHAMBERS in Figs. 46-52.

Both of them give the same result.

Table 8 shows all of the results from the SW array. In column 5, the average values of <RAD>, <EA>, and <TR> = <RAD> - <EA> are given in keV per particle chamber. The values of <TR> are plotted in Fig. 56 with respect to γ of electrons. The <EA> values are the average dE/dx in 1.5 cm MWPC. We will obtain the most probable values of dE/dx as a cross-check of energy calibration in section 5.2c. In Table 8, a run with no absorber on the beamline is compared with an equivalent absorber run. This result shows that the bremsstrahlung background from the equivalent radiator was about 0.67 keV per chamber electron at 15 GeV. The pion data are also given in the last line. It is not expected for pions to produce much TR energy at this momentum. The measured average value is totally dE/dx by pions in the chamber. The value is lower than that of electrons at the same momentum because of the relativistic rise effect.
TABLE 8
TRANSITION RADIATION YIELD IN SANDWICH ARRAY OF EIGHT SETS OF MYLAR RADIATOR \( (N, t_x, t_y) \) AND MWPC WITH KR-\( \text{CH}_4 \) MIXTURE

<table>
<thead>
<tr>
<th>Particle Type</th>
<th>Particle Type of Analyzed Events</th>
<th>Energy per Photon, particle-MWPC (keV)</th>
<th>Photons* per particle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positron ( e^- ) or ( \pi^- )</td>
<td>Energy run ( (100, 1/2, 100) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( e^- )</td>
<td>RAD 3185 18.45 7.97</td>
<td>RAD 2100 21.14 7.97</td>
<td>RAD 2773 24.13 7.98</td>
</tr>
<tr>
<td></td>
<td>EA 415 12.48 4.68</td>
<td>EA 2037 13.88 4.26</td>
<td>EA 3034 13.69 7.97</td>
</tr>
<tr>
<td></td>
<td>TR 5.97 3.29</td>
<td>TR 7.26 3.71</td>
<td>TR 10.28 3.12</td>
</tr>
<tr>
<td></td>
<td>EA 3034 13.02 7.70</td>
<td>EA 2003 13.02 7.70</td>
<td>EA-NA 0.67 0.27</td>
</tr>
<tr>
<td>Positron ( e^- ) or ( \pi^- )</td>
<td>Energy run ( (100, 1, 100) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( e^- )</td>
<td>RAD 2736 20.27 7.88</td>
<td>RAD 2629 25.35 7.98</td>
<td>RAD 3018 27.24 7.98</td>
</tr>
<tr>
<td></td>
<td>EA 644 12.55 4.25</td>
<td>EA 2037 14.08 4.16</td>
<td>EA 2061 13.75 3.08</td>
</tr>
<tr>
<td></td>
<td>TR 7.72 3.63</td>
<td>TR 11.27 4.82</td>
<td>TR 13.49 4.90</td>
</tr>
<tr>
<td>( \pi^- )</td>
<td>15 RAD 2011 9.26 7.96</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The number of photons listed on the first line of each energy entry is essentially 8 corresponding to the number of chambers.
Fig. 56

TR Detected Yield vs. γ in the SW Array
**Number of TR Photons**

The number of TR photons produced per electron was obtained by the method in Ref. 18. First, for the lower portion of the SINGLE PULSE HEIGHT DISTRIBUTIONS (Figs. 46-52), the portions of the curves for the EA run and the RAD run should be identical, since most of the pulses are rarely due to ionization loss. Typically, the region of this normalization was chosen between 3 and 9 keV, where the pulse height in both RAD and EA histograms were lower than the peak value. It can be assumed that most of these pulses were produced purely by the ionization process in the chamber, not by TR. The area bounded between the RAD and these normalized EA curves gave a lower limit for the number of photons produced by TR. This can be regarded as a lower limit to the number of TR photons since sometimes 2 or more photons are absorbed in one chamber. In the last column of Table 8, the number of TR photons detected is listed. At 15 GeV, we obtained 3 photons from the Mylar (100, 0.5, 100) radiator and 5 photons from the 1 mil radiator.

b. **MS Array Data**

**Selection of Events**

In the MS case, the following cuts have been applied in order to restrict our sample to good events. We reject events with

(a) Charged particles going through an x-ray detection side (A side) - By counting the number of chambers
in which the pulse was higher than the preset background level (b.g. level = 3 keV, for instance), the event was thrown away if the number was equal or greater than 5. Symbolically, \[ A \geq 3 \text{ keV} \geq 5. \] This condition will be examined for different values of the b.g. level below.

(b) No evidence of a charged particle on the B side - By counting the number of B chambers of which pulse height was less than zero keV, the event was thrown away if the number was greater than 3. Symbolically, \[ B \leq 0 \text{ keV} \geq 3. \]

(c) Unwanted incident particle - By looking at the shower counter distribution, the event was thrown away if the pulse height was less than the preset threshold for the electron, vice versa for the pion. The cut applied for each run has been shown on the shower counter pulse height histogram with an arrow.

(d) Too large cross-talk from the B side: a large dE/dx pulse pulls down (TR THRESHOLD + PED) to the lowest count of ADC, i.e., to count 0 - If the value of dE/dx was greater than (TR THRESHOLD + PED) keV, the event was thrown away.

The raw value of the A-side signal for this type of event tended to pile up on the ADC count 0 because the TR spectrum had most of its pulses above the threshold and a few tens
keV. Therefore, the correction to this type of event with the large signal on the B-side may place it back to a higher value than necessary. Actually, there were not many events to be thrown away by this criterion.

All the cuts were applied successively in the following order: (a), (b), (c), and (d), and are shown in Table 9. Each line entry gives the "cumulative" effect of cuts. The 9 GeV run has lost about 50\% events by the shower counter cut as mentioned earlier.

The two types of cuts (a) and (b) were checked for its validity by varying the b.g. level and the number of chambers. It is observed that once a certain combination of the two values was obtained, no further change around this combination affected the result. Some of the examined combinations are listed below with the symbolic expressions.

(i)  (ii)  (iii)  (iv)
(a) \[ A > 0 \] \geq 3 \quad [A > 3] \geq 3 \quad [A > 3] \geq 5 \quad [A > 3] \geq 7
(b) \[ B < 0 \] > 1 \quad [B < 0] > 1 \quad [B < 3] \geq 3 \quad [B < 0] > 3

From left to right, the condition becomes less severely restricting for the events. From (i) to (ii), the total yield from all the chambers increased by 12\%. From (ii) to (iii), another 12\% increase of the yield was observed. After this point, any further change from (iii) to (iv) did not affect the result at all. The combination (iii), therefore, was chosen for all the cases.
### TABLE 9

CUTS APPLIED TO THE EVENTS IN MAGNETIC SEPARATION ARRAY

8 Xe MWPC's, POLYETHYLENE (100, 1, 125), 9 GEV ELECTRONS

<table>
<thead>
<tr>
<th>Cut Applied</th>
<th>Type of Cut Applied</th>
<th>Type of Run</th>
<th>Total No. of Events Analyzed with Each Cut Applied</th>
<th>No. of Events Lost with Each Cut Applied</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>No cuts applied.</td>
<td>RAD</td>
<td>4002</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>EA</td>
<td>6858</td>
<td></td>
</tr>
<tr>
<td>Cut A</td>
<td>Cut for charged</td>
<td>RAD</td>
<td>3885</td>
<td>117</td>
</tr>
<tr>
<td></td>
<td>particles on A-side.</td>
<td>EA</td>
<td>6671</td>
<td>187</td>
</tr>
<tr>
<td></td>
<td>[ A&gt;3 ]&gt;5†</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cut B</td>
<td>Cut A and cut for no particles on B-side.</td>
<td>RAD</td>
<td>3853</td>
<td>.32</td>
</tr>
<tr>
<td></td>
<td>[ B&lt;3 ]&gt;3†</td>
<td>EA</td>
<td>6627</td>
<td>.44</td>
</tr>
<tr>
<td>Cut C</td>
<td>Cut B and cut on shower counter.</td>
<td>RAD</td>
<td>1924</td>
<td>1929</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EA</td>
<td>3280</td>
<td>3347</td>
</tr>
<tr>
<td>Cut D</td>
<td>Cut C and cut for large dE/dx pulse which pulls TR + PED down to count 1.</td>
<td>RAD</td>
<td>1906</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EA</td>
<td>3244</td>
<td>36</td>
</tr>
</tbody>
</table>

† See Section 5.3b for the notation.
Pulse Height Distributions

In the MS array, it is straightforward to obtain the TR spectra. First, we plotted the pulse height distributions with the radiator-in and equivalent absorber for the same number of electrons. On the left-half of Fig. 57, the pulse height distributions for the first two chambers A0 and A1 are shown. (All A0-A7 distributions will be given later.) The top-right frame shows the pulse height distributions of the signals counted in all 8 individual chambers. The number attached, therefore, is 8 times as great as the number of real electrons going through all the chambers in the array. This type of distribution will be denoted as COMBINED. The bottom-right frame gives the distribution of the pulse heights summed over all 8 chambers for each electron. This will be named as SEPARATE. In all plots, those events in the 0 keV bin are not plotted because the number of counts is too large for the same scale.

In these plots, we notice that (a) a single chamber distribution (A0 or A1) shows a shoulder around 30 to 40 keV; (b) the SEPARATE distribution tails off much more smoothly than the single chamber on the high energy side; and (c) the COMBINED distribution shows a sizable shoulder on the high energy side. From these facts, it is apparent that to observe the "pure" TR spectrum, we must obtain not the summed pulse height distributions but the pulse height distributions in single chambers individually. The single
Fig. 57. Pulse Height Distribution in the MS Array with Polyethylene Radiator (100,2,125) and Equivalent Absorber at Electron Momentum 15 GeV/c
SINGLE PULSE HEIGHT OF CHAMBER AD

11201 ELECTRONS

RADIATOR

EQ. ABSORBER

SINGLE PULSE HEIGHT OF CHAMBER AI

11201 ELECTRONS

SUM OF PULSE HEIGHT FROM 8 CHAMBERS

11201 X 8 = 89608 COUNTS

COUNTS/2KEV

PULSE HEIGHT (KEV)

COUNTS/2KEV

PULSE HEIGHT (KEV)

Fig. 57
chambers can preserve the original shape of the TR spectrum better than the COMBINED ones.

The pure TR spectrum was obtained by directly sub-structing the EA distribution from the RAD distribution. Figures 58-66 give the results for polyethylene radiators of 1/2, 1, and 2 mil at electron energies 3, 9, 15 GeV. The top histogram is the TR spectrum and the background run (EA) is shown below it. The ordinate is dN/dω in count per 5 keV, and the abscissa is the photon energy in keV. It is easily observed that the spectrum becomes hardened in the succeeding chambers. This fact indicates that the soft components are absorbed in the front chambers. The smooth curve is the calculation which will be given in section 5.3b, where we will find that the shoulder around 30 keV is caused by the K-edge jump of the xenon absorption coefficients.

**Detected Yield**

The energy detected in each chamber was obtained by integrating \( W = \int E(dN/dE)dE \) over the whole spectrum. To obtain the energy detected per particle, we must divide \( W \) by the total number of electrons. Figures 67-69 show that the yields of all 8 chambers plotted vs. chamber number A0-A7. In contrast to the SW-array "buildup" along the chambers, we now see the attenuation of the characteristic TR spectrum. About 70% of the total energy was observed in the first 3 chambers (3 x 1.5 cm xenon chambers).
TR Spectrum in the MS Chambers with Polyethylene Radiator (180, 0.5, 125) at Electron Momentum 3 GeV/c
Fig. 59

TR Spectrum in the MS Chambers with Polyethylene Radiator (180, 0.5, 125) at Electron Momentum 9 GeV/c
Fig. 60

TR Spectrum in the MS Chambers with Polyethylene Radiator (180, 0.5, 125) at Electron Momentum 15 GeV/c
Fig. 61

TR Spectrum in the MS Chambers with Polyethylene Radiator (100,1,125) at Electron Momentum 3 GeV/c
Fig. 62

TR Spectrum in the MS Chambers with Polyethylene Radiator (100,1,125) at Electron Momentum 9 GeV/c
Fig. 63
TR Spectrum in the MS Chambers with Polyethylene Radiator (100,1,125) at Electron Momentum 3 GeV/c
Fig. 63
(continued)
Fig. 64

TR Spectrum in the MS Chambers with Polyethylene Radiator (100,1,125) at Electron Momentum 3 GeV/c
Fig. 65

TR Spectrum in the MS Chambers with Polyethylene Radiator (100,2,125) at Electron Momentum 9 GeV/c
Fig. 65
(continued)
Fig. 66

TR Spectrum in the MS Chambers with Polyethylene Radiator (100,2,125) at Electron Momentum 15 GeV/c
Fig. 66
(continued)
Absorption of TR X-Rays in the MS Chambers with Polyethylene (180, 0.5, 125) at Electron Momenta 3, 9, and 15 GeV/c

Fig. 67
Absorption of TR X-Rays in the MS Chambers with Polyethylene (100,1,125) at Electron Momenta 3, 9, and 15 GeV/c

Fig. 68
Fig. 69

Absorption of TR X-Rays in the MS Chambers with Polyethylene (100, 2, 125) at Electron Momenta 3, 9, and 15 GeV/c
The total energy detected by all 8 chambers was obtained by summing the energy deposited in all chambers. Table 10 gives the results for the three kinds of radiators. In column 5, the total energy detected per electron is shown. Pion data are also given with a 1 mil radiator at 9 GeV and resulted in negligible TR radiation. Figures 70 and 71 give the $\gamma$ dependence of the results. The calculation compared with the experiment will be given in section 5.3b.

The error bars indicated are only statistical ones. They are estimated by using several subsets of a run as samples. Each sample was analyzed in the same manner as the complete run, and the standard deviation of the results were estimated among the samples. The errors in both RAD and EA runs were added statistically to obtain an error bar for TR results.

**Photon Background**

The EA runs gave the measurement of background caused mainly by bremsstrahlung and synchrotron radiation by the incident particle. The bremsstrahlung background does not depend on the electron energy, while the synchrotron radiation varies largely depending on the electron energy. The background caused by these processes were only measurable directly in the MS array in which the charged particle was not giving $dE/dx$ signals on the x-ray detecting side of the chamber. The photon background measured in the
<table>
<thead>
<tr>
<th>Particle e⁻ or τ⁻</th>
<th>Particle energy (GeV)</th>
<th>Type of run</th>
<th>Analyzed events</th>
<th>Energy per particle (keV)</th>
<th>Total Detected No. of photons per 1000 particles</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>RAD</td>
<td>1841</td>
<td>6.81</td>
<td>284</td>
</tr>
<tr>
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<td>3058</td>
<td>3.10</td>
<td>98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TR</td>
<td>3.71</td>
<td>166</td>
<td>220</td>
</tr>
<tr>
<td>e⁻</td>
<td>(180, 1/2, 125)</td>
<td>RAD</td>
<td>4093</td>
<td>9.19</td>
<td>346</td>
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<td></td>
<td></td>
<td>EA</td>
<td>1853</td>
<td>5.04</td>
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<td></td>
<td></td>
<td>TR</td>
<td>4.15</td>
<td>199</td>
<td>245</td>
</tr>
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<td></td>
<td></td>
<td>RAD</td>
<td>7595</td>
<td>8.66</td>
<td>301</td>
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<td></td>
<td></td>
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<td>3789</td>
<td>4.27</td>
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<td>TR</td>
<td>4.40</td>
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<td></td>
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<td>3819</td>
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<td>9.37</td>
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<tr>
<td>e⁻</td>
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<td>4.45</td>
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<td>RAD</td>
<td>7611</td>
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<td>4.56</td>
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<td></td>
<td>TR</td>
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<td>322</td>
<td>484</td>
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<tr>
<td>τ⁻</td>
<td>(100, 2, 125)</td>
<td>RAD</td>
<td>1913</td>
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<td>100</td>
</tr>
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<td></td>
<td></td>
<td>EA</td>
<td>3316</td>
<td>1.31</td>
<td>87</td>
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<td></td>
<td>TR</td>
<td>0.05</td>
<td>13</td>
<td>17</td>
</tr>
<tr>
<td>e⁻</td>
<td>(100, 2, 125)</td>
<td>RAD</td>
<td>3100</td>
<td>13.88</td>
<td>467</td>
</tr>
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<td></td>
<td>EA</td>
<td>3056</td>
<td>3.11</td>
<td>100</td>
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<td>10.77</td>
<td>367</td>
<td>529</td>
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<td>24.30</td>
<td>673</td>
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<td></td>
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<td></td>
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<td>20.03</td>
<td>544</td>
<td>936</td>
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<td>3795</td>
<td>4.38</td>
<td>133</td>
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<td></td>
<td></td>
<td>TR</td>
<td>22.45</td>
<td>511</td>
<td>911</td>
</tr>
</tbody>
</table>
TR Detected Yield in the MS Array vs. $\gamma$ Compared with the Single Foil Formula

Fig. 70

SINGLE FOIL FORMULA

$\{100, 2, 125\}$

$\{100, 1, 125\}$

$\{180, 0.5, 125\}$

$\gamma (E/m_e)$

0 10000 20000 30000

3 GeV/c 9 GeV/c 15 GeV/c

DETECTED ENERGY/GeV (KEV)
Fig. 71

TR Detected Yield in the MS Array vs. $\gamma$ Compared with the N-Foil $\delta$ Function Method
experiment is consistent with the calculated values as shown in Table 11.

**Number of TR Photons**

The number of photons produced per electron is obtained by dividing the total number of events with the positive definite pulse by the total number of electrons. In Table 10, columns 6 and 7 show the number of photons per 1000 electrons detected by all 8 chambers. The number of photons detected for 1/2, 1, and 2 mil radiators at 15 GeV/c are 0.2, 0.5, and 0.9 per electron, respectively.

The columns COMBINED and SEPARATE give some difference for the number of photons detected. The COMBINED is obtained by summing up outputs of all 8 chambers, while the SEPARATE gives the pulse height distribution of all individual chambers. The latter allows a multiphoton event to be plotted with the same frequency as the number of fired chambers. The multiphoton events happen much less frequently than the single photon events. For example, the COMBINED and SEPARATE give 322 and 482 photons per 1000 electrons at 15 GeV/c with a 1 mil radiator. Therefore, if we assume that all the multiphotons consist of only two photon events, then we obtain \((482-322)/2 = 80\) events in 322 events as the double photon events, i.e., 25% as an upper limit.

c. **dE/dx Measurement**

During the EA runs, we could measure the dE/dx distribution because most of the photon background were much
**TABLE 11**

PHOTON BACKGROUND PRODUCED BY AN ELECTRON GOING THROUGH THE POLYETHYLENE RADIATOR AND OBSERVED IN A 1.5 CM XENON CHAMBER IN THE MAGNETIC SEPARATION ARRAY EXPERIMENT

<table>
<thead>
<tr>
<th>Electron Energy $E_e$ GeV</th>
<th>Calculated Background ($\text{Synchroton} + \text{Bremsstrahlung}$) keV</th>
<th>Experimental Background (EA Run) keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.22</td>
<td>0.34</td>
</tr>
<tr>
<td>9</td>
<td>0.61</td>
<td>0.68</td>
</tr>
<tr>
<td>15</td>
<td>1.01</td>
<td>0.79</td>
</tr>
</tbody>
</table>
smaller than the dE/dx values as shown above. The stable values of dE/dx guarantees that the calibration of the MWPC's was made consistently. Data taken in both the Kr and Xe chambers are shown in Figs. 72 and 73. The dE/dx fluctuation stayed small within its standard deviations: 0.3 keV both for dE/dx = 6.3 keV for krypton (SW array) and dE/dx = 9.6 keV for xenon (MS array). The variation of dE/dx between the chambers were about 0.5 keV. Figure 74 shows the γ dependence of the dE/dx measured in the 1.5 cm MWPC filled with either krypton or xenon. The calculations are obtained from Sternheimer's formula.36

There are some works on the use of the ionization loss for particle identification in the relativistic region where the most probable energy loss of the particle increases logarithmically.37,38,39 The MWPC is a desirable detector for the ionization measurement because the rare gases in the chamber give the largest relativistic rise in the ionization loss. The detailed discussion can be found in the literatures.

5.3 Comparison of Results with Theoretical Prediction

We have obtained the emerging flux of transition radiation from a foil stack in Sections 2.3 and 3.1. To predict the detected yield in the chambers, we trace the TR x-ray flux from the radiator to the chamber output. The materials along the path of the x-rays are absorptive. The photoelectric absorption coefficients of materials are
Fig. 72

dE/dx in the Krypton Chambers
Fig. 73

deE/dx in the Xenon Chambers.
SOLID CURVES FROM STERNHEIMER'S FORMULA
(1 ATM, 293°K, 1.5 cm)

- XENON
- KRYPTON

Ionization Loss in 1.5 cm MWPC

Fig. 74
strong functions of energy in the range of interest. In the calculation of the effective energy absorbed by the chambers, we include a series of absorption processes. Furthermore, the chambers have a complex characteristic in the range of TR energy. The energy of the x-rays absorbed by the chambers do not fully convert into the chamber signal because a large portion of the absorbed energy leaks out of the "thin" chambers as fluorescence x-rays or energetic electrons.

a. Detection Efficiency of Chambers

Since the chambers were thin, the loss of energy due to secondary processes such as fluorescence escape of photons and escape of long-ranged electrons must be calculated. For the energy range of TR photons, the absorption by high Z elements (i.e., krypton or xenon) therefore cannot be described by a monotonic function like light elements. The method of escape corrections is well known for both krypton and xenon gases. The details of the calculation must be found in Appendix B. Only a brief description of the method is given below. When the incident photons are absorbed by the gas in the chamber, we must know the efficiency of the conversion of the photon energy into the electronic energy.

We define the ionization efficiency $\xi$ as the ratio of the amount of energy converted into effective MWPC pulse $I$ to the initial photon energy $k$, i.e., $\xi = I/k$. 
The first step starts in the absorption of x-rays by atoms, which may occur on one of the possible shells of the atom. Then, the photoelectron emitted by the atom will dissipate its energy through collisions with some other atoms in the gas or it will escape from the chamber if it is energetic enough (range escape). The excited atom emits either fluorescence x-rays or Auger electrons. The fluorescence x-rays may or may not be reabsorbed within the chamber. The process in which the de-excitation x-rays are not absorbed is called "fluorescence escape." The calculation of the ionization efficiency therefore consists of a series of photoelectric processes and ionization by electrons.

b. Calculation of Yield Including Absorption

We introduce a notation to describe the absorption of material:

\[ p_i = e^{-(\mu \rho t) i} , \]

which is the probability that x-rays can remain behind the absorptive i-th material on the path. The materials will be air, aluminum, Mylar, etc. The p's are functions of energy but we leave out the energy variable \( \omega \).

**SW Array**

The TR x-rays produced in each radiator are absorbed mainly by the MWPC immediately following. However, some of them are absorbed in later modules. Hence, a later
chamber detects an additional amount of x-rays besides that from its own radiator.

When the TR x-ray flux from the foil stack is written as a function of photon energy $S(\omega)$ (which was calculated in Section 2.3), the first chamber in the SW array yields

$$A_0(\omega) = S(\omega)P_{\text{AIR}} P_{W}(1 - P_{C})\xi(\omega) \quad (44)$$

where the absorption terms by the AIR, Window material (aluminum and Mylar), and the Chamber appear in the expression. For the numerical values of the ionization efficiency, refer to Fig. 75.

The portion of the flux unabsorbed by the first chamber will pass through the rear window of the first chamber, the air, and the second radiator consisting of $N$-foils and $N$-air gaps. After this point, the photons will be absorbed by the second chamber as in the first chamber. Let a quantity $u$ be

$$u = P_{W}^{-2} \cdot P_{\text{AIR}}^{-2} \cdot P_{C}(P_{F} P_{G})^{N} \quad (45)$$

where $P_{F}$ and $P_{G}$ are introduced for the radiator materials. The second chamber then detects

$$A_1(\omega) = A_0(\omega) \cdot u + A_0(\omega) \quad . \quad (46)$$

The corrections made in the calculation of the SW array yield were as follows: The greatest correction is the ionization efficiency correction which reduced the flux
Fig. 75

Ionization Efficiency of Krypton and Xenon Chambers
about 29% on the average for 15 GeV yield with Mylar (100, 1, 100). In this SW array case, the radiator holder was especially made of cardboard so that about 10% of the foils were found touching each other. This fact also is taken into account to calculate the emerging yield out of the stack by reducing the effective number of foils. The copper wires in the chambers shadowed the x-ray flux by 6.4%. Any other corrections were 5% in total (air, aluminum, Mylar).

The results are compared in Fig. 56 with the experimental results. The calculation in Fig. 56 was made with the single foil approximation (Appendix A).

MS Array

There was only one radiator set 60 feet upstream of the point where the x-ray detector is located. The materials in the beamline between the RAD set and the detector are listed in Table 12. There were three He bags, one of which contained 40 permanent magnets.

In the prediction of the emerging yield from RAD, the self-absorption of foil and air gaps were included. Therefore, the flux falling onto the chamber was obtained just by multiplying the absorption factors of materials to the emerging yield.

We introduce a notation \( v \), to write the absorptions by a chamber, two windows, and air in the front and back of the chamber as:
### TABLE 12

**ATTENUATION OF X-RAY FLUX BY MATERIALS IN MAGNETIC SEPARATION ARRAY**

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness</th>
<th>( \mu \rho t ) ( \text{cm}^{-1} )</th>
<th>( e^{-\mu \rho t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PVC</td>
<td>24 mil</td>
<td>( 1.325 \times 10^1 )</td>
<td>0.446</td>
</tr>
<tr>
<td>He</td>
<td>700&quot;</td>
<td>( 3.73 \times 10^{-5} )</td>
<td>0.936</td>
</tr>
<tr>
<td>Air between MWPC's</td>
<td>1.5&quot;</td>
<td>( 1.55 \times 10^{-3} )</td>
<td>0.994</td>
</tr>
<tr>
<td>Mylar in each MWPC</td>
<td>1.5 mil</td>
<td>( 1.46 \times 10^0 )</td>
<td>0.994</td>
</tr>
<tr>
<td>Al in each MWPC</td>
<td>1.0 mil</td>
<td>( 2.13 \times 10^1 )</td>
<td>0.947</td>
</tr>
<tr>
<td>Cu wire in MWPC</td>
<td>5.0 mil φ</td>
<td>( 6.69 \times 10^2 )</td>
<td>( 2.06 \times 10^{-4} )</td>
</tr>
</tbody>
</table>

\[ \mu = \text{mass absorption coefficient (cm}^3/\text{g)} \]

\[ \rho = \text{density (g/cm}^3) \]
\[ v = P_C P_{AIR} p_w^2. \]  

The first chamber in the array detects the yield \( A_0 ', \)

\[ A_0'(\omega) = S(\omega) P_{PVC} P_{HE} P_{AIR} p_w (1 - P_C) \xi(\omega), \]  

where \( S \) is the emerging flux from the radiator. The following chambers observe the yields

\[ A_1 = A_0 v, \quad A_2 = A_1 v, \quad \ldots. \]  

The results are shown in Figs. 70 and 71 for the MS array experiment. Figure 70 shows the calculation made by using the single foil approximation (Appendix A) and Fig. 71 gives the evaluation by using the delta function approximation. With the limited number of experimental points, it is difficult to choose either one of the two approximations.

In addition with the electron beam, the distance in the radiator material which does not change the beam direction, not larger than the TR angular distribution, becomes very short because of the multiple scattering of the electron in the material. This length is called a coherent length. In the Mylar stack of 1 mil foils, the distance corresponds to about 10 foils. Then, the coherence of the N-foil interference term would not be seen. The radiation then shows a nature similar to a single foil.

In Fig. 76, each correction applied in the yield calculation is added one after the other so that reduction
Polyethylene Radiator (100,1,125)

Fig. 76

Decrease of TR Flux by Each Correction in the MS Array
of the total flux by each correction term can be observed. In this MS case, the ionization efficiency correction reduced the yield by 17% in this 1 mil case.

The windows of the helium bags were made of PVC, which was an oversight. These PVC windows absorbed the emerging flux by 56.5%, while all other materials absorbed the flux by only 4.3%; the wire shadow correction was ~ 6.4%.

c. Internal Comparison of Results

SW and MS Results

As the SW and MS arrays were made with different combinations of radiator material and gas filling in the MWPC's, the comparison between them cannot be made directly. Only the first chamber yields in both the SW and MS arrays are compared. The comparison should give the least difference between the two. The detected yields in the first chamber (A0) of both cases are listed in Table 13. In the MS case, about 56% reduction of the flux reaching to the chambers was caused by the PVC windows of the He bags. The predicted overall efficiency for the 1.5 cm xenon chamber is 27% (see Section 3.3, Column 6, Table 2), and 53% for all 8 MWPC's. Because of the PVC windows, we obtained only 7% of the overall efficiency for the 1.5 cm xenon chamber and 23% for all the chambers (calculated values).

Bevatron and SLAC Results

The Bevatron results which were taken with 1 mil Mylar
<table>
<thead>
<tr>
<th>Electron Energy $E_e$ (GeV)</th>
<th>SW (Mylar/Kr)</th>
<th>MS (Polyethylene/Xe)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental keV</td>
<td>Calculated† keV</td>
</tr>
<tr>
<td>3</td>
<td>4.7</td>
<td>3.5</td>
</tr>
<tr>
<td>9</td>
<td>7.4</td>
<td>6.2</td>
</tr>
<tr>
<td>15</td>
<td>8.6</td>
<td>7.2</td>
</tr>
</tbody>
</table>

†Mylar (90,1,100)

*Polyethylene (100,1,125) including PVC absorption
radiators and krypton chambers at the electron momentum 3 GeV/c can be compared with the corresponding one in the SLAC results. From Section 3.2, we review that the 11 krypton chambers (the chamber thickness was the same in both experiments) detected 5 photons, i.e., 5 \times (8/11) = 3.6 photons per electron for 8 chambers, which is consistent with the SLAC result (see Section 5.2a and Column 6 of Table 8).
6. SUMMARY AND CONCLUSIONS

The fundamental agreement between the experimental results and theoretical predictions was attained in this investigation of transition radiation. We are convinced that we have a good understanding of the physics of transition radiation, and that it is now possible to design a transition radiation detector for any specific purpose.

The detection of particles by transition radiation is becoming no problem at all. The development of a better TR detection system for the identification of particles is now the major issue. First, we summarize our results and compare it with some other group's results. Secondly, we discuss the improvement and application of transition radiation technique to the problem of the particle identification. Two different suggestions are presented.

6.1 Summary and Comparison of Results

a. SW Array

When we apply the transition radiation detector for a particle identification, we must determine how efficiently the TR detector can discriminate one kind of particle from another.

Results of the SLAC Experiment

Figure 77 shows a result from the SW array (sandwich array) consisting of 8 modules of the Mylar radiator and MWPC. The pulse height distributions for pions and electrons at the same momentum are compared. The bottom
Fig. 77
PROBABILITY OF DISCRIMINATING ELECTRONS FROM PIONS
frame shows the probabilities of detecting electrons and pions as a function energy. The probability of detecting electrons (pions) of pulse heights greater than a certain preset value is defined as $P_{e, \pi}$, i.e., the ratio of the area on the right-hand side of the distribution above the certain pulse height to the total area. It is easily seen that the best ratio of $p_e/p_{\pi}$ will be obtained between 15 keV and 18 keV without losing many electrons. This kind of good separation is possible at the relativistic energy with many modules in an SW array. It would be very useful to discriminate pions and electrons.

Next, let us look at the absolute separation of the peaks between a radiator run and an equivalent absorber run. A "resolving power" (R.P.) for the separation of the pulse height distributions for the SW array is defined as

$$\text{Resolving power of SW array} = \frac{<\text{RAD}> - <\text{EA}>}{<\text{EA}>}.$$ 

The results are shown in Table 14. The sandwich array experiment always has the large background from dE/dx. Therefore, by looking at a single pulse, we cannot be sure whether the signal is from the transition radiation or ionization.

**Comparison with Other Groups' Results**

There are many publications on the application of transition radiation. We discuss only those data which
**TABLE 14**

RESOLVING POWER OF THE SANDWICH ARRAY OF 8 MODULES OF

A MYLAR RADIATOR (100;1,100) AND KRYPTON MWPC

<table>
<thead>
<tr>
<th>Electron Energy E_e (GeV)</th>
<th>Average Total Energy Detected With Radiator &lt;RAD&gt; (keV)</th>
<th>With Equivalent Absorber &lt;EA&gt; (keV)</th>
<th>Resolving Power &lt;RAD&gt; - &lt;EA&gt;</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>20.27</td>
<td>12.55</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>25.35</td>
<td>14.08</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>27.24</td>
<td>13.75</td>
<td>0.98</td>
<td></td>
</tr>
</tbody>
</table>
have used setups similar to this experiment and compare them on the basis of resolving power. Table 15 lists the data used for the discussion.

(1) Yuan et al., measured TR with 10-30 modules of radiator-chamber. Results (b) and (c) of Table 15 were taken with electrons at momenta 2.7-3.5 GeV/c and with pions at momenta 100-250 GeV/c. The same group also used a light radiator material lithium with 2 GeV electrons (result (d)). The pulse in the 0.02" thick NaI detector included rather large ionization loss of 250 keV compared to the measured 80 keV TR.

(2) From the works by Alikhanian et al., we discuss the latest one, which is classified here by the fact that the detector observed the ionization of the particle as well as the TR x-rays. The radiation and particle were registered by means of a gaseous xenon scintillator (4 cm xenon at a pressure of 1.6 atm). A phototube was inserted from the lateral side. The scintillation signal was comprised of both the x-rays and particle ionization. The total number of photons observed in the interval 75-200 keV was 0.87 per electrons with a Mylar radiator and the background caused mainly by the ionization was 0.11 photons. The number of TR photons detected was 0.76 (result (a)).

(3) Cherry et al., used very similar setups to ours. The results are listed in (f) of Table 15. Both krypton and xenon chambers (total of 7 chambers) were used in
TABLE 15

COMPARISON OF SANDWICH ARRAY EXPERIMENTS ON THE BASIS OF RESOLVING POWER

<table>
<thead>
<tr>
<th>Authors</th>
<th>Radiator*</th>
<th>Detector**</th>
<th>Modules</th>
<th>Beam Particle</th>
<th>Beam energy (GeV)</th>
<th>Resolving†</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) This work</td>
<td>Mylar(90,1,100)</td>
<td>MWPC 1.5 cm Kr</td>
<td>8</td>
<td>e⁻</td>
<td>3</td>
<td>0.62</td>
</tr>
<tr>
<td>(b) Bemberger et al.⁴²</td>
<td>Mylar(100,0.5,60)</td>
<td>MWPC 2.2 cm Kr</td>
<td>10</td>
<td>e⁻</td>
<td>3.5</td>
<td>0.55</td>
</tr>
<tr>
<td>(c) Yuan et al.⁴³</td>
<td>Mylar(100,0.5,30)</td>
<td>MWPC 2.2 cm Ar</td>
<td>30</td>
<td>π⁻</td>
<td>250</td>
<td>0.40</td>
</tr>
<tr>
<td>(d) Yuan et al.⁴⁴</td>
<td>Li(500,0.5,30)</td>
<td>NaI 20 mil</td>
<td>1</td>
<td>e⁻</td>
<td>2</td>
<td>0.31</td>
</tr>
<tr>
<td>(e) Alikhanian et al.¹²</td>
<td>Mylar(1000,0.4,28)</td>
<td>Xe scintillator 4 cm</td>
<td>1</td>
<td>e⁻</td>
<td>31</td>
<td>1.6</td>
</tr>
<tr>
<td>(f) Cherry et al.⁴⁵</td>
<td>Mylar(188,1,60)</td>
<td>MWPC 4.3 cm Kr/Xe††</td>
<td>7</td>
<td>e⁻</td>
<td>3</td>
<td>0.64</td>
</tr>
<tr>
<td>(g) Comps et al.⁴⁶</td>
<td>Mylar(5000,0.4,4)</td>
<td>MWPC 0.8 cm Xe</td>
<td>1</td>
<td>e⁻</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>(h) Fischer et al.⁴⁷</td>
<td>Li(1000,2,20)</td>
<td>MWPC 1.27 cm Xe</td>
<td>1</td>
<td>e⁻</td>
<td>2.5</td>
<td>3.3</td>
</tr>
<tr>
<td>(i) Bosshard et al.⁴⁸</td>
<td>Li(500,2,20)</td>
<td>MWPC 1.27 cm Ar</td>
<td>4</td>
<td>π⁻</td>
<td>250</td>
<td>0.85</td>
</tr>
</tbody>
</table>

* To describe the radiator, the notation Material(N,tf,tg) where N is the number of foils and t is the foil and gap thicknesses in mils is used.

** The specifications are the thickness of the detector and the filling gas if it is MWPC.

† Resolving power = (<RAD> - <EA>)/<EA>. See Section 6.1.

†† They used 4 Kr MWPC’s and 3 Xe ones in series.
an SW array. These results are comparable to ours (a).

(4) Camps et al., measured TR to seek the lowest practical values of \( \gamma \) (400-2000) at which TR can still be measured. Above \( \gamma = 1000 \) (R.P. = 0.5), the TR method will be workable for the particle identification. One of their results is listed in (g).

(5) Fischer et al., showed that very good resolution may be obtained by the use of a lithium foil radiator with a xenon MWPC detector. The group also tested their system with electrons and pions at 100-250 GeV/c (result (i)).

In conclusion, we summarize our study. First, this comparison confirms that to obtain a good resolution, we must prepare (i) low Z radiators (compare (c) with result (i)). (ii) thin detectors, i.e., MWPC's (compare (d) with (h) and (g)); and (iii) a high Z gas for the MWPC (it is not directly observable from this comparison but as we have seen in Section 5.3, xenon gives much better detection efficiency than Kr which has its K-edge too close to the last peak region of radiators).

Secondly, the resolving power is restricted to be the order of unity because of the dominating background, i.e., \( \text{d}E/\text{dx} \). However, it is rather easy to produce as many photons in the SW array by increasing the number of modules. Thus, the detection efficiency can be made very high in the SW array.
b. Separation Array

MS Array Results of the SLAC Experiment

In the MS array, the measurements are essentially free from any dominating backgrounds. The background from bremsstrahlung and synchrotron radiation is about 10% of the transition radiation signal. Hence, if we use the term resolving power, it is an order of 10 instead 1. This feature is a great advantage in identifying particles such as pions and kaons. With these heavy particles, both types of backgrounds will become less noticeable than with electrons.

Comparison with Other Group's Results

(1) Wang et al., employed a magnetic field and a NaI detector to observe the TR spectrum. The results have rather large background in the wide sensitive range of 0-300 keV. The "detection efficiency" of positron was 80% at 2 GeV. It is difficult to make further comparison.

(2) Mack et al., used a unique technique to separate some portion of the TR photons from the charged background by Compton scattering. The NaI crystals were mounted around a Compton scatterer, covering 65% of the $4\pi$ solid angle. Only the multiplicity of detected photons in 8 NaI counters were recorded. It is possible to compare the results with others only in terms of the number of photons per electron. The multiplicities from 1000 foils of 1 mil Mylar for electron momenta 3 and 15 GeV/c are 0.92 and
1.28 per electron after subtracting the background (about 0.47 photons). The data with 2 mil foils are 30% higher at 15 GeV than the 1 mil data.

The latter results can be compared with our MS results (the last column of Table 10). The detection efficiency of an electron is about the same in both results. However, no "pure" TR spectrum can be obtained by their method because of the angular dependence of the Compton scattering \((1 + \cos^2 \theta)\), where \(\theta\) is the scattering angle of photons.

Both Mack et al., and our results have almost the same amount of background, which is about 27% of the total photons detected from the 1 mil radiator with 15 GeV electrons. The MS array will be used most effectively for identification of heavy particles \((\pi/K)\) when the signal-to-noise ratio is kept as high as possible by using low Z radiators.

6.2 Further Development

We will consider two different possible directions to pursue beyond this study before we can consider an effective system for particle identification.

a. Efficient MA Array for \(\pi/K\) Identification

Because of its good signal-to-background ratio, the MS array could be made to perform with greater efficient arrangement than the present system. This may be possible by making use of the following points.
(a) Low Z material with very small absorption in order to be able to increase the number of photons per electron, for example, beryllium or lithium.

(b) High Z gas to fill a thick chamber for better detection efficiency.

These improvements will enable us to give better resolution in $\gamma$, which depends on the statistics of the number of TR photons ($N_\gamma \sim 5$ photons, $\Delta\gamma/\gamma \sim 1$, where $\Delta\gamma =$ FWHM of the pulse height distribution). The detection of x-rays with energies around 10 keV or higher requires a larger volume of an x-ray MWPC with one single unit. The high energy photo- and Auger-electrons and fluorescent x-rays should be captured within the chamber to become an effective pulse. This type of deficiency in detecting high energy TR x-rays also degrades the resolution in $\gamma$.

Figure 78 shows the pulse height distribution from the 1000 Be radiator with kaon or pion. We predict that one can identify pions with 96% efficiency by setting the discrimination level at 30 keV and only 4% kaon will count.

b. **Use of N-Foil Angular Distribution**

It is also proposed to measure the sharp angular distribution of the $f_N$ function, which is strongly angle-dependent for a given x-ray energy. If x-ray energy $E$ is measured accurately, then the angular cone $\theta(E)$ will also have about the same resolution as the energy detection.
Fig. 78

π/K Discrimination in the MS Array with a Be Radiator
Then, the uncertainty in $\gamma$ due to angular spread can be made as good as $\Delta\gamma/\gamma \sim 0.2$. The resolution of $\gamma$ may be improved over that obtained from the total pulse height alone. More detailed discussions on this subject are beyond the scope of this dissertation. They are found in FNAL Proposal 206.\textsuperscript{51}

Finally, we obtained quite useful information on the development of TR detectors. A TR detector system can be constructed rather economically compared to many other equipments in high energy physics. Therefore, the possibility for further extensive applications of TR to the development of particle identification is encouraging.
APPENDIX A

ANGULAR INTEGRATION OF THE N-FOIL FORMULA

We discuss the evaluation of the yield from a foil stack including the N-foil interference term $f_N$ and give a calculation method in aid of delta functions.

A.1 Characteristics of the N-Foil Interference Term

Let us now look into the integration of (16):

$$\frac{dW_{NF}}{d\omega} = \int \frac{d^2W_{NF}}{d\theta d\omega} d\theta = \frac{2e^2}{\pi c} \int f_0(\theta) f_L(\theta) f_N(\theta) d\theta,$$

where the $f$'s are given in (9), (15), and (17). First of all, when $f_1 f_N \gg f_2$, then the sharp peaks of $f_N$ disappear because the $f_1$ term cancels the denominator of the $f_N$ term and $f_1 f_N \sim \sin^2 N\phi_f$. This fact causes a destructive interference and the yield results in a single foil yield of thickness $Nt_f$. In order to avoid this, $\phi_f$ must be at least the same order of magnitude as $\phi_f$ for the relevant values of $\omega$ and $\gamma$. This require $t_g$ to be of the order of the formation zone of the air gap, $z_g = (2c/\omega) (\gamma^{-2} + \xi_g^{-2})^{-1}$. In the practical cases, the ratio $\tau = t_g/t_f$ then becomes much greater than unity.

Let us define a quantity

$$I_N = \frac{1}{N} \frac{dW_{NF}}{d\omega},$$

which will be compared with the yield from a single foil $I_S$ in the following discussion.
In most of our cases, the $f_N$ peaks appear several times within the range of $f_0 f_1$ as observed in Fig. 6. The peak positions and the spacings of function $f_0$, $f_1$, and $f_N$ are easily obtained as follows:

\[ f_0 : \text{The peak angle is given by } \theta^2 \approx \gamma^{-2} \text{ for any value of } \omega \text{ and its range is of order } \gamma^{-2} + \frac{\theta^2}{\ell_f^2}, \]

\[ f_1 : \theta^2_s = (2c/\omega t_f)[(2s+1)\pi - \phi^0_f] \text{ for } s = 0,1,2,... \]
\[ \text{and } \Delta_S = 4\pi c/\omega t_f, \]

\[ f_N : \theta^2_n = (2c/\omega t_f)(\frac{2n\pi - \phi^0_0}{1 + \tau}) \text{ for } n = 1,2,... \text{ and } \]
\[ \Delta_N = \Delta_S/(1+\tau). \]

The value of $I_N$ can depend on two facts, first on the step $\Delta_N$ in the sum (24) and second on the peak positions $\theta^2_n$ relative to those of $f_0 f_1$.

Let us imagine the case when one of the values of $\theta^2_n$ equals to $\theta^2_s = \gamma^{-2}$ which corresponds to the maximum of the function $f_0(\theta^2) f_1(\theta^2)$ and the step $\Delta_N$ at the same time is much greater than the spacing $\Delta_S$ between the $f_1$ peaks. It is clear that in this case we will obtain $I_N >> I_S$.

On the contrary to the imaginative situation as previewed above, we obtain $\Delta_N << \Delta_S$ for the case when $\tau >> 1$. Furthermore, if $\phi_0$ is large compared to $2\pi$, the terms within the summation of (25) varies slowly with $n$. Then, the sum can be replaced with an integral and $I_N = I_S$. 

is obtained. Namely, the N-foil yield is obtained simply as \( N \) times the single-foil yield.

The comparison of the results between the single foil and N-foil formulas is given in Fig. 9. All the features mentioned above are observed:

(a) At a low value of \( \gamma \), the yield is exactly the same for both cases--\( I_N = I_S \).

(b) Around \( \gamma = 6000 \), the N-foil formula pronounces its constructive interference effect--\( I_N \approx I_S \).

(c) At higher \( \gamma \), for the N-foil formula, the saturation sets in earlier than for the single-foil formula--\( I_N \approx I_S \).

A.2 Approximation with Delta Functions

To approximate \( f_N \) with a sum of delta functions, two conditions must be satisfied. From the argument in Section 2.2, the width of the \( f_N \) peak \( \Delta_N/\sqrt{N} \) must be much smaller than the width of the peak of the combined function \( f_0 f_1 \). This requires that \( N \gg 4\pi c \gamma^2/(t_f + t_g) \).\(^4\) When the absorption of radiator materials is included in the \( f_N \), it is also necessary to keep a condition \((\mu\rho t)N_f \ll 1\) in order to obtain the narrow peaks of \( f_N \).

As the angular integration of (A1) with no absorption has been given in (25), we consider here a more general case when the \( f_N \) function includes the absorption of the radiator.
The \( f_N(\phi) \) has sharp peaks at \( \phi = 2k\pi \) \((k = 0,1,2\ldots)\). Because of periodic behavior of this function in \( \phi \), all the peaks have the same amplitude and shape. Therefore, let us consider the peak at \( \phi = 0 \). By introducing a function \( g_N = \ln f_N \), we expand \( g_N(\phi) \) around \( \phi \approx 0 \).

\[
g_N(\phi) = g_N(0) + g_N'(0)\phi + \frac{1}{2} g_N''(0)\phi^2 + \ldots. \tag{A2}
\]

By taking the derivation of \( g_N(\phi) \) as a function of \( \phi \), we obtain

\[
g_N'(\phi) = \frac{N\sin N\phi}{\cosh Nb - \cos N\phi} - \frac{\sin\phi}{\cosh b - \cos\phi} \tag{A3}
\]

and it is required that \( g_N'(0) = 0 \) so that \( f_N(\phi) \) has maximum at \( \phi = 0 \).

Let us define \( g_N''(0) = -2/\beta^2 \) and then \( g_N(\phi) \) will be rewritten with terms up to the second order,

\[
g_N(\phi) \approx g_N(0) - \phi^2/\beta^2. \tag{A4}
\]

By the definition of \( g_N \), we resume \( f_N(\phi) \):

\[
f_N(\phi) = f_N(0) e^{-\phi^2/\beta^2}. \tag{A5}
\]

By setting \( \phi = 0 \) in (A5), we denote \( f_N(0) \) as \( \alpha \):

\[
\alpha = e^{-(N-1)B} \frac{\cosh Nb - 1}{\cosh b - 1}. \tag{A6}
\]

If the peak is sharp enough, the area under the peak will be approximated by integrating the right-hand side of (5) over all the region of \( \phi(-\infty,\infty) \):
Thus, the sharp peak at φ = 0 is approximated by \( \sqrt{\pi} \alpha \beta \delta(\phi) \).

In general, all the peaks are the same, so we can write \( f_N \) as a sum of delta functions:

\[
f_N(\phi) = \sqrt{\pi} \alpha \beta \sum_{k=0}^{\infty} \delta(\phi - 2k\pi).
\]

Now we need to determine the value of \( \beta \). The second derivative of (A3) is given by

\[
g_N''(0) = - \frac{\sinh^2(Nb/2) - N^2 \sinh^2(b/2)}{2 \sinh^2(b/2) \cdot \sinh^2(Nb/2)}.
\]

By using the definition \( \beta^2 = -2/g_N''(0) \), we obtain

\[
\beta = \frac{2 \sinh(b/2) \sinh(Nb/2)}{\sqrt{\sinh^2(Nb/2) - N^2 \sinh^2(b/2)}}.
\]

To integrate an arbitrary function \( f(\theta) \) with respect to \( \theta \), we can now write it with the form of \( f_N \) in (A8):

\[
I = \int_{0}^{\pi/2} f(\theta) f_N(\phi(\theta)) \, d\theta.
\]

Changing the variable from \( \theta \) to \( \phi \) and using \( d\phi = d\phi/\phi'(\theta) \), (A11) is written as

\[
I = \sqrt{\pi} \alpha \beta \sum_{k=0}^{\infty} \int_{\phi_1}^{\phi_2} \frac{f(\theta)}{\phi'(\theta)} \delta(\phi - 2k\pi) \, d\theta.
\]
The integration with respect to $\phi$ gives

$$I = \sqrt{\pi} \alpha \beta \sum_{k=n_1}^{n_2} \frac{f(\theta_k)}{|\phi'(\theta_k)|}, \quad \text{(A13)}$$

where $\theta_k$ satisfies the relation $\phi(\theta_k) = 2k\pi$ and $n_1 = \lambda + 1$ with $\lambda = [\phi_0/2\pi]$, where the brackets indicate that the largest integer whose magnitude does not exceed the magnitude of the argument.

A.3 Explicit Form of $dW_{np}/d\omega$

First let us write down $\phi$ explicitly by using the parameters for a stack of radiator. From the notations given earlier, we can write

$$\phi = \phi_0 + \Gamma \theta^2 \quad \text{(A14)}$$

with $\Gamma = (\omega/2c)(t_f + t_g)$ and $\phi_0 = \phi(\theta = 0)$.

Let us find out $\theta_k$ which satisfies $\phi(\theta_k) = 2k\pi$. Solving it for $\theta$, we obtain

$$\theta_k = \sqrt{(2k\pi - \phi_0)/\Gamma}. \quad \text{(A15)}$$

Now we go back to the summation over $\theta_k$. The form of $f(\theta)$ in this case is given by $f(\theta) = f_0 f_1$, where $f_0$ and $f_1$ are given in (9) and (27). The terms in the summation of (A13) are obtained by the following function at the values of $\theta_k$:

$$F(\theta) = f(\theta)/|\phi'(\theta)|.$$
By using (9), (27), and (A14), \( F(\theta) \) is written as

\[
F(\theta) = \frac{\theta^2 (\xi_f^2 - \xi_g^2)^2 e^{-bf(cosh b_f - \cos \phi_f)}}{\Gamma(\gamma_f^2 + \xi_f^2 + \theta^2)^2 (\gamma_f^2 + \xi_f^2 + \theta^2)^2}. \quad (A16)
\]

Let us introduce some new variables

\[
x = \frac{\omega t_f}{2c} \theta^2, \quad \gamma_f = \frac{t_f \omega_f}{2c}, \quad r = \frac{\omega^2}{\omega_f^2}, \quad \tau = \frac{t_g}{\omega_f},
\]

\[
p = \phi_f^0 = \frac{\omega t_f}{2c} (\gamma_f^2 + \xi_f^2), \quad q = \phi_g^0 / \tau = \frac{\omega t_f}{2c} (\gamma_f^2 + \xi_g^2),
\]

where \( \phi_f^0 \equiv \phi_f (\theta = 0) \). Then we obtain

\[
\phi_f = x + p
\]

and

\[
\phi_g = \tau (x + q).
\]

By simplifying \( F(\theta) \) with these new variables and substituting the result into (A13), we obtain the flux

\[
\frac{dW}{d\omega} = (e^2 / \pi^2 c) I \quad \text{as}
\]

\[
\frac{dW}{d\omega} = I_0 \sum_{k=n_1}^{n_2} G(x_k), \quad (A17)
\]

where all the factors independent of \( x \) are put together as

\[
I_0 = \sqrt{\pi} \gamma_f^2 \xi_f e^{-bf} \frac{(1 - r)^2}{1 + \frac{r}{\tau}} \alpha \beta. \quad (A18)
\]
and the remaining function inside the summation is defined as

\[ G(x) = \frac{2}{137\pi} \frac{x(\cosh b_f - \cos(x + p))}{(x + p)^2 (x + q)^2} . \]  \hspace{1cm} (A19)

By using (A15) and the definition of \( x \), we find

\[ x_k = \frac{(2k\pi - \phi_0)}{(1 + \tau)} . \]

The following facts can be shown:

As \( b \to 0 \),

\[ \alpha = \left( \frac{1 - e^{-Nb}}{1 - e^{-b}} \right)^2 + N^2 \]  \hspace{1cm} (A20)

and

\[ \beta = \frac{2 \sinh(b/2)\sinh(Nb/2)}{\sqrt{\sinh^2(Nb/2) - N^2\sinh^2(b/2)}} \to 2\sqrt{\pi}/N . \]  \hspace{1cm} (A21)

Hence,

\[ \alpha \beta \to 2\sqrt{\pi} N \text{ as } b \to 0 . \]  \hspace{1cm} (A22)

In the above expression (A22), \( \alpha \) is obtained by rearranging terms in (A10). These \( \alpha \) and \( \beta \) describe the self-absorption in the radiator in addition to \( b_f \) appearing in \( G(x) \) and \( \exp(-b_f) \). The results of this approximation have been given at the end of Section A.1.
APPENDIX B

CORRECTIONS FOR THE FLUORESCENCE AND RANGE ESCAPES

B.1 Introduction

The detection efficiency of the MWPC depends on both the geometry (the volume, especially the thickness) of the chamber and the gas filling. The photon flux absorbed from the primary beam (the fraction of an incident beam) will be given by

\[ \alpha = (1 - e^{-\ell/\lambda}) \]  

(B1)

where \( \ell \) is the thickness and \( \lambda \) the mean absorption length as a function of photon energy. The \( \lambda \) changes largely depending on the x-ray energy and the atomic number of the gas element. Figure 79 shows the values of \( \lambda \) for Ar, Kr, and Xe (calculated from the photon cross-section table of Ref. 53). The chamber thickness is shown on the abscissa. Our 1.5 cm thick MWPC was very thin for high energy photons. In the "thin" chamber, the energy of the absorbed photons cannot always be seen as an effective pulse, especially when the beam energy is higher than a certain absorption edge of the gas element.

We have to look into the mechanism of the photo-absorption. Suppose that the energy is sufficiently great to eject photo-electron from the K level of a gas atom (for the photon of energy exceeding the K-edge value, the absorption occurs on the K-shell almost 90\% of the time).
Fig. 79

Mean Absorption Length of Ar, Kr, and Xe
when a K-electron is removed from an atom, a vacancy can be created. This highly excited state will undergo either a radiative or a radiation-less transition. The former is called fluorescence radiation and the latter in which an electron is emitted when the vacancy is filled from a higher shell is called auger emission.

As a result, the energy of the absorbed photon is converted into either electron-kinetic energy (photo-electron and auger electron) or fluorescence x-rays. The energetic electron may not dissipate all its energy within the chamber, and the fluorescence x-rays may not be reabsorbed either to become an effective pulse. In fact, our MWPC was, however, so thin that all fluorescence and energetic electrons produced by the primary photons could not be absorbed before reaching outside the chamber.

First, we calculate the probability that the fluorescence reaches outside in the form of x-rays. The computation is involving an integration over the possible paths to the wall from an element $dl$ in the path of the primary beam. We represent the result by a factor $e^{-\tau d}$ for each fluorescence radiation where $d$ is the effective path length and $\tau$ the true linear absorption coefficient of the gas for the fluorescence lines.

For the escaping electron, we also have to employ an integration in order to evaluate the fraction of electron-kinetic energy deposited in the chamber.
At the end, we will consume all the processes involved to obtain the ionization efficiency of the chamber which is defined as the ratio of the energy seen by the chamber to the incident photon energy.

**B.2 Escape Probability of Fluorescence X-Rays from the MWPC**

To estimate the escape probabilities of the fluorescence x-rays out of the chamber, let us get a crude idea by estimating the mean free paths for each fluorescence x-ray energy. Table 16 shows the mean free paths at K and L fluorescent energies.

The geometry and the coordinates are shown in Fig. 80(b). The sensitive volume of the MWPC is 20 x 20 x 1.5 cm$^3$. X-rays are detected on the half of the volume divided at the center, i.e., volume 20 x 10 x 1.5 cm$^3$ as shown in Fig. 80(a). Fluorescence x-rays are emitted isotropically to any direction at the distance z measured from the entrance to the chamber. The x-axis goes into the paper and azimuthal angle $\phi$ is measured from the x-axis.

Because of the different value of coordinate $\phi$, we simply draw a circle as a first order of approximation. This approximation is applicable for both K and L fluorescence. For K fluorescence, the mean free path is larger than the chamber size and the mean free path is much smaller for the L fluorescence than any distance going outside of the chamber. In either case, the replacement of the chamber shape with a cylindrical one is acceptable.
<table>
<thead>
<tr>
<th>Gas</th>
<th>Density $\rho$ mg/cm$^3$</th>
<th>Fluorescent x-ray energy</th>
<th>Mass Absorption Coefficient $\mu$ mg/cm$^2$</th>
<th>Mean Free Path $1/\mu \rho$ cm</th>
<th>Escape Probabilities $^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kr</td>
<td>3.74</td>
<td>$L_\alpha$ 1.587</td>
<td>1.003</td>
<td>0.2666</td>
<td>0.0968</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L_\beta$ 1.638</td>
<td>0.9335</td>
<td>0.2864</td>
<td>0.1039</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$K_\alpha$ 12.648</td>
<td>0.02618</td>
<td>10.21</td>
<td>0.8094</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$K_\beta$ 14.112</td>
<td>0.01914</td>
<td>13.97</td>
<td>0.8472</td>
</tr>
<tr>
<td>Xe</td>
<td>5.90</td>
<td>$L_\alpha$ 4.111</td>
<td>0.3692</td>
<td>0.4591</td>
<td>0.1657</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L_\beta$ 4.422</td>
<td>0.3087</td>
<td>0.5490</td>
<td>0.1959</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$K_\alpha$ 29.802</td>
<td>0.007971</td>
<td>21.26</td>
<td>0.8893</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$K_\beta$ 33.644</td>
<td>0.005603</td>
<td>30.25</td>
<td>0.9155</td>
</tr>
</tbody>
</table>

$^*$ See equations
For Kr: $\lambda = 1.5$ cm, $R = 7.98$ cm
For Xe: $\lambda = 1.5$ cm, $R_1 = 10.1$ cm, $R_2 = 5.05$ cm
(a) EFFECTIVE SIZE OF MMPC

(b) COORDINATES FOR THE CALCULATION OF ESCAPE PROBABILITY

Fig. 80

Geometry in Escape Probability Calculation
a. Calculation of Average Escape Probabilities

In the course of derivation, the height of the chamber is noted as $h$ in Fig. 80(b) and the radius $R$ for the substituted cylinder must be used in place of $h$ at the end.

The average escape probability for a certain fluorescent x-ray is given by the following integral.

$$\text{Average escape prob. } e^{-\tau d} = \frac{\int_0^{\pi} d\phi \int_0^\xi dz \int_0^{\pi} \sin \theta \ d\theta \ e^{-\tau r}}{\int_0^{\pi} d\phi \int_0^\xi dz \int_0^{\pi} \sin \theta \ d\theta} ,$$

where $\xi$ is the chamber thickness, $\tau = \mu \rho (\text{cm}^{-1})$ with the mass absorption coefficient $\mu$ and the gas density $\rho$, and $r$ the distance to the wall of the chamber from point $z$ along the incident direction of x-rays. The quantity $\exp(-\tau r)$ must be integrated only for the top semidisc. The denominator is the normalization factor.

The numerator in the expression will be divided into three regions when we integrate it with respect to coordinate $\theta$.

$$I_\theta = \int_0^{\theta_1} \sin \theta \ d\theta \ e^{-\tau z/\cos \theta} + \int_{\theta_1}^{\theta_2} \sin \theta \ d\theta \ e^{-\tau h/\sqrt{1 - \cos^2 \theta}}$$

$$+ \int_{\theta_2}^{\pi} \sin \theta \ d\theta \ e^{-\tau(z - \xi)/\cos \theta} ,$$

(B3)
where $\theta_1$ and $\theta_2$ are measured as shown in Fig. 80(b) and suffix $\theta$ indicates integration with respect to $\theta$. By changing variable $\theta$ as $x = \cos \theta$, we obtain

$$I_\theta = \int_{x_1}^{1} dx e^{-\tau z/x} + \int_{x_2}^{x_1} dx e^{-\tau h/\sqrt{1 - x^2}}$$

$$+ \int_{-1}^{x_2} dx e^{-\tau (z - \lambda)/x} = I_\theta^1 + I_\theta^2 + I_\theta^3,$$

where

$$x_1 = \cos \theta_1 = z/\sqrt{h^2 + z^2};$$

$$x_2 = \cos \theta_2 = (z - \lambda)/\sqrt{(\lambda - z)^2 + h^2}.$$}

Under the assumption that $h \gg \lambda$ in our geometry, we can approximate $x_1$ and $x_2 << 1$. Therefore, $I_\theta^2$ will be simplified as

$$I_\theta^2 \approx \int_{x_2}^{x_1} dx e^{-\tau h}.$$}

By using a new variable $t = 1/x$, all three terms of (B4) are integrated as follows. First, $I_\theta^1$ is written

$$I_\theta^1 = \int_{1/x_1}^{1} e^{-\tau z t^2} dt \approx e^{-\tau z} - x_1 e^{-\tau \sqrt{z^2 + h^2}} - \tau z E_1(\tau z),$$

(B5)
where the exponential integral form

\[ E_1(a) = \int_1^\infty e^{-at} \frac{dt}{t} \]

is used under the assumption \( x_1 \), and \( x_2 \ll 1 \) mentioned above.

Similarly, we obtain two other integrals of \( I_\theta \).

\[ I_\theta^2 = (x_1^2 - x_2^2) e^{-\tau h} \quad (B6) \]

and by partially integrating \( I_\theta^3 \), we obtain

\[ I_\theta^3 = x_2 e^{-\tau} \sqrt{(z - \lambda)^2 + h^2} + e^{-\tau}(\lambda - z) \]

\[ + \tau(z - \lambda) \int_1^\infty (dt/t) e^{-\tau(\lambda - z)t} \quad (B7) \]

In the last step of (B7), the third term is obtained by changing variable \( t + -t \) first and noting the fact \( x_2 \ll 0 \).

The integration with respect to the variable \( z \) will be made as follows. Integration of the first term \( I_{\theta z}^1 \) gives

\[ I_{\theta z}^1 = \int_0^\lambda dz (e^{-\tau z} - e^{-\tau} \sqrt{z^2 + h^2} - \tau z E_1(\tau z)) \]

\[ = (1 - e^{-\tau \lambda})/\tau + (e^{-\tau} \sqrt{\lambda^2 + h^2} - e^{-\tau h})/\tau - \int_0^{\tau \lambda} \beta E_1(\beta) d\beta/\tau. \]
A new variable \( \beta = \tau z \) is introduced in the third term. Similarly, the integration of (B6) gives

\[
I_z^2 = \int_0^\ell dz (x_1 - x_2) e^{-\tau h} \overset{\text{new variable}}{=} \ell^2 e^{-\tau h} / h , \tag{B9}
\]

where in the last step, we used the fact that \( h \gg \ell \). By Eq. (B7), we produce

\[
I_z^3 = (-e^{-\tau h} + e^{-\tau \sqrt{\ell^2 + h^2}} + 1 - e^{-\tau \ell})/\tau - \int_0^{\tau \ell} E_1(\beta) d\beta/\tau . \tag{B10}
\]

The third term in (B10) is obtained by using a new variable \( \beta = \tau (\ell - z) \) and \( dz = -dz/\tau \).

Finally, by executing the integration with respect to \( \phi \) coordinate for all terms and dividing by the normalization factor in (B2), we obtain the escape probability averaged over all along the \( z \) axis and in all directions

\[
e^{-\tau d} = \int_0^\pi d\phi (I_z^1 + I_z^2 + I_z^3) / \int_0^\pi d\phi \int_{-\ell}^\ell dx \int_0^\ell dz . \tag{B11}
\]

Integration gives

\[
e^{-\tau d} = (1 - e^{-\tau \ell} + e^{-\tau \sqrt{\ell^2 + h^2}} - e^{-\tau h}
- \int_0^{\tau \ell} E_1(\beta) d\beta)/\ell \tau + \ell e^{-\tau h}/2h . \tag{B12}
\]
b. Application to Each Specific Geometry of SW and MS Arrays

Let us now consider each specific geometry depending on the entrance point of the incident photons onto the chamber.

**SW Array**

We have the beam spot at the center of the A side of the chamber. The disc of thickness 1.5 cm in Fig. 81(a) is obtained by setting its area equal to the sensitive area of the chamber. We then have a symmetry for the top and bottom hemispheres. Therefore, by using the result of Eq. (B2), we evaluate the following quantity with the corresponding value to \( h \), i.e., \( R \). By substituting \( R \) into \( h \), we obtain

\[
e^{-\tau d} = (1 - e^{-\tau l} + e^{-\sqrt{l^2 + R^2}} - e^{-\tau R})
\]

\[
- \int_0^{\tau l} E_1(\beta) d\beta / \tau l + \lambda e^{-\tau R}/2R .
\]

**MS Array**

We draw two semidiscs with different radii as shown in Fig. 81(b). In principle, we apply the same method as the SW array case. Because this case has different radii for two hemispheres, Eq. (B11) must be changed as follows:
Fig. 81

Substitution of Effective Volume by Disc
The integration gives

\[ e^{-\tau d} = \left\{ \int_0^{\pi} d\phi \left( I^1_z + I^2_z + I^3_z \right) \right\}_{h=R_1} + \left\{ \int_0^{2\pi} d\phi \left( I^1_z + I^2_z + I^3_z \right) \right\}_{h=R_2} \]  

\[ \times \left\{ \int_0^{2\pi} d\phi \int_{-1}^{1} dx \int_0^l dz \right\}^{-1}. \]

The integration gives

\[ e^{-\tau d} = \left\{ 1 - e^{-\tau l} - \int_0^{\tau l} \beta E_1(\beta) d\beta \right\} \]

\[ + \left( e^{-\sqrt{l^2 + R_1^2}} + e^{-\sqrt{l^2 + R_2^2}} - e^{-\tau R_1} - e^{-\tau R_2}/2 \right) / \tau l \]

\[ + le^{-\tau R_1/4R_1} + le^{-\tau R_2/4R_2}. \]  

Remarks

The validity for the condition \( x_1, x_2 \ll 1 \) is tested as follows:

In the SW case, \( \min \theta_1 = \theta_1 (z = 0) \) is given by \( \arctan (R/l) = 81.54 \) degrees, so that \( \min (1/x_1) = 6.80. \)

In the MS case, \( \min \theta_1 \) for the bottom semidisc is given by \( \arctan (R_2/l) = 73.46 \) degrees so that \( \min (1/x_1) = 3.51. \)

Therefore, within an acceptable error, we admit that all the approximation made in the estimation of \( \exp(-\tau d) \) is valid.

Table 16 shows the results for both SW (Kr) and MS (Xe) array cases. The K fluorescent x-rays have probabilities as high as 0.9, while the L fluorescent x-rays have so small probability that they hardly escape the chamber.
B.3 Electron Range Escape Correction

The electron has definite range depending on its kinetic energy $E$. If it is long compared to the thickness of the chamber, the probability of such an electron to escape out of the chamber increases significantly.

We calculate a factor $g$ which describes a fraction of electron-kinetic energy deposited within 1.5 cm MWPC. (This fraction of energy becomes an effective pulse out of the chamber.)

a. Calculation of Probabilities

We make assumptions for the emission of electrons:

(a) Flat MWPC of thickness $t$;
(b) Isotropical emission by photo emission;
(c) Uniform creation;
(d) Height of chamber $h >> t$.

We calculate

(i) Probability that a full range of electron with Range $R$ is contained within the chamber:

$$ P(R,x) \text{ for any position } x. $$

$$ <P(R)>_t = \text{averaged over } 0 < x < t. $$

(ii) Probability that an electron will have a path length "$\ell$" ($\ell \leq R$) within the chamber:

$$ p_\ell(x) d\ell dx = \text{probability of having a path length between } \ell \text{ and } (\ell + d\ell) \text{ at position } x. $$

$$ <P(\ell)>_t d\ell = \text{averaged over } 0 < x < t. $$
(iii) Integrated probability that an electron (of potential range \( R \)) will have a path length \( \ell < R \):

\[
I(\ell < R) = \int_{c}^{R} p(\ell) \, d\ell.
\]  

(B15)

Note that \( <p(R)>_t + I(\ell < R)_t = 1 \) holds. The evaluation is divided into two regions and is given below.

\[
\begin{align*}
(A) & \\
0 < R < t & \quad t < R < \infty \\
<p(R)> & = 1 - \frac{R}{2t} & t/2R \\
<p(\ell)> & = \frac{1}{2t} & \begin{cases} 
\frac{1}{2t} & (0 \leq \ell \leq t) \\
\frac{t}{2\ell^2} & (t < \ell \leq R)
\end{cases} \\
I(\ell < R) & = \frac{R}{2t} & 1 - \frac{t}{2R}
\end{align*}
\]

(b. Calculation of the Deposition of Electron Kinetic Energy)

Let the electron range \( R \) and it is given by a relation

\[
R = KE^\gamma \quad \text{or} \quad E = (R/K)^{1/\gamma} = \alpha R^n,
\]  

(B16)

where \( \gamma = 1/n \) and \( \alpha = 1/k^n \). Let \( I \) be the energy deposited as ionization by the electron of range \( R(E) \). We define the ratio of \( I \) to \( E \) as

\[
g = I/E
\]  

(B17)

and

\[
E_0 \quad : \quad \text{initial kinetic energy of electron} \\
R_0 \quad : \quad \text{range of electron with } E_0
\]
Then we calculate the value of $I$.

(A) If $R \leq t$, the path-length distribution is given by column (A) in the list above. We obtain

$$I = (1 - R_0/2t)E_0 + \int_0^{R_0} E(\ell) \, d\ell/2t , \quad (B18)$$

where $E(\ell) = E_0 - E(R_0 - \ell) = \alpha[R_0^n - (R_0 - \ell)^n]$,

and

$$\int_0^{R_0} E(\ell) \, d\ell = E_0 R_0 \, n/(n + 1) .$$

Hence,

$$I(E_0) = E_0(1 - R_0/2t(n + 1)) . \quad (B19)$$

Therefore,

$$g(R_0 < t) = 1 - R_0/2t(n + 1) . \quad (B20)$$

(B) If $R \geq t$, the path-length distribution is given by column (B), which is more complicated

$$I = \int_0^t E(\ell) \, d\ell/2t + \int_t^{R_0} tE(\ell) \, d\ell/2\ell^2 + E_0 t/2R_0 , \quad (B21)$$

where

$$E(\ell) = \alpha[R_0^n - (R_0 - \ell)^n] .$$
In order to evaluate the two integrals, we approximate the probability \( p(\ell) \) for \( 0 < \ell < R_0 \) as
\[
\bar{p}(\ell < R_0) = \frac{(1 - t/2R_0)}{R_0}.
\]
Then,
\[
I = \int_0^{R_0} R_0 \frac{(1 - t/2R_0)}{R_0} E(\ell) \, d\ell + \frac{t}{2R_0} E_0/2R_0.
\]
Since
\[
\int_0^{R_0} E(\ell) \, d\ell = \frac{E_0 R_0}{n(n + 1)},
\]
we obtain
\[
I = E_0 \left[ \frac{t}{2R_0} + \frac{n(1 - t/2R_0)}{(n + 1)} \right].
\]
Therefore,
\[
g(R_0 > t) = \frac{t}{2R_0} + \frac{n(1 - t/2R_0)}{(n + 1)}. \tag{B24}
\]

c. Numerical Example

The electron range relation is given in terms of mean excitation energy \( I \) and the electron energy \( E \) by
\[
R = \frac{(A/z)}{I^2} \frac{\phi(E/I)}{\phi(E/I)}, \tag{B25}
\]
where \( \phi \) is the ionization potential as a function of \( E/I \) and both \( E \) and \( I \) are in keV. For our purposes, the power-law fit is sufficient.
\[ R(\text{mg/cm}^2) = 0.00578(A/Z) \phi^{0.29} E^{1.71}. \]  

The factor is put

\[ K = 0.00578(A/Z) \phi^{0.29}. \]

The \( g(E) \) for Kr and Xe are calculated in the energy region of our interest up to 300 keV. Data used in the computation are listed in Table 17.

Figure 82 shows the \( g \) vs. \( E \) for Kr and Xe.

**B.4 Calculation of the Ionization Efficiency**

The fluorescence yield of the \( i \)-th subshell of an atom, \( \omega_i \), is the probability that a vacancy in that subshell is filled by a radiative transition. The Auger Yield of the same shell, \( a_i \), is defined as the probability that an electron is emitted when the vacancy is filled from a higher shell.

A single vacancy created, for example, in the K-shell is filled in a time of the order of \( 10^{-17} - 10^{-14} \) sec by an electron coming from some higher shell (e.g., the \( L_2 \) subshell), thus shifting the vacancy to the higher shell. The difference in binding energy between the two shells (e.g., the K-L\(_2\) energy difference) either is released as a K x-ray photon or is transferred to another bound electron (e.g., an \( L_3 \) electron) which is ejected (e.g., a K-L\(_2\)L\(_3\) Auger transition). This results in an atom with two vacancies. Continuation of these processes gives rise to the emission
<table>
<thead>
<tr>
<th>Gas in MWPC</th>
<th>Atomic No. (Z)</th>
<th>Atomic Weight (A)</th>
<th>Density (ρ mg cm⁻³)</th>
<th>Ionization Potential (φ keV)</th>
<th>Electron Range for E = 10 keV (mg cm⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kr</td>
<td>36</td>
<td>83.8</td>
<td>3.74</td>
<td>0.381</td>
<td>0.521</td>
</tr>
<tr>
<td>Xe</td>
<td>54</td>
<td>131.3</td>
<td>5.86</td>
<td>0.555</td>
<td>0.608</td>
</tr>
</tbody>
</table>
$g = \text{fraction of electron's kinetic energy deposited within 1.5 cm MWPC}$

**Fig. 82**

Range-Escape Correction
of L, M, N, etc., x-rays and Auger electrons, resulting in an atom with multiple vacancies in its outermost shells, i.e., highly charged ion.

We will consume all the possible transitions and calculate the ionization efficiency which describes conversion of photon energy into an effective output pulse when the incident x-rays are absorbed.

a. **Direct Ionization of Photoelectron**

The first photoelectron emission occurs on either one of three shells, K, L, or M (where higher order is neglected) with probabilities $f_{K,L,M}$ as defined below. We introduce the following notations:

- $k$: incident photon energy;
- $f_i$: fractional contribution of i-th shell to the total cross section; $f_i$ are calculated by using the jump ratio $J_i$ at i-th edge as follows (Table 18):
  \[
  f_K = (1 - 1/J_K),
  \]
  \[
  f_L = (1 - 1/J_L)/J_K, 
  \]
  \[
  f_M = 1/J_K J_L,
  \]
  where $k > E_K$. If $E_L < k < E_K$, set $J_K = 1$ and if $k < E_L$, set $J_K = J_L = 1$;
- $E_i$: i-th edge potential energy;
- $T_i$: kinetic energy of photoelectron out of the i-th shell: $T_i = k - E_i$;
- $g(T)$: fraction of electron-kinetic energy deposited within 1.5 cm MWPC; $g$ is a function of electron
<table>
<thead>
<tr>
<th>Element</th>
<th>Edge</th>
<th>Fluorescence Yields*</th>
<th>Edge Jump§</th>
<th>Weighted Averages of Line Energies</th>
<th>Relative Intensity†</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>E_{K,L}^{\text{keV}} \quad \omega_{K,L} \quad J_{K,L}^{I} \quad K_{L}^{\text{keV}}</td>
<td></td>
</tr>
<tr>
<td>Argon</td>
<td>L</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>3.203</td>
<td>0.122</td>
<td>9.91</td>
<td>\alpha 2.96 \quad \beta 3.19</td>
<td>\alpha 150.7 \quad \beta 12.3</td>
</tr>
<tr>
<td>Krypton</td>
<td>L</td>
<td>1.675</td>
<td>0.075</td>
<td>4.17 \quad 1.63</td>
<td></td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>14.323</td>
<td>0.660</td>
<td>\alpha 12.63 \quad \beta 14.13</td>
<td>\alpha 151.9 \quad \beta 21.5</td>
</tr>
<tr>
<td>Xenon</td>
<td>L</td>
<td>4.781</td>
<td>0.103</td>
<td>2.88 \quad 4.40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>34.561</td>
<td>0.894</td>
<td>\alpha 29.67 \quad \beta 33.76</td>
<td>\alpha 154.1 \quad \beta 27.6</td>
</tr>
</tbody>
</table>

* "Most Reliable" Experimental Values from Reference 55
§ From Reference 53
† "Relative Intensity" of lines with respect to $K_{\alpha}^{I} = 100$ from Reference 56
kinetic energy and will be calculated in Sec. B.3 of this appendix.

The observed energy in the direct ionization process is given by \( D_i = f_i g(T_i) T_i \), which describes that when the incident photon kicks out the electron from the \( i \)-th shell with a probability \( f_i \), a fraction \( g \) of the electron energy can produce a pulse \( D_i \).

Summing the contribution from three shells, we observe the total direct ionization:

\[
D = D_K + D_L + D_M , \tag{B28}
\]

where

\[
D_i = f_i g(T_i) T_i .
\]

There now occurs two associated processes: either Auger emission or fluorescence yield.

b. Ionization by Auger Electrons

The Auger electron ionizes the gas. By collecting all possible cases within the limit of the desired accuracy up to the M-shell. For example, suppose the energy released in a transition from the K to the L state is used up in knocking out another L electron, thus resulting in a doubly ionized atom in the L state. Denoting the kinetic energy of the Auger electron \( T'_K \), the conservation of energy requires that

\[
k_K = \langle E_L \rangle + T'_K , \tag{B29}
\]
where \( <E_L> \) denotes the energy necessary to remove the second L electron, where the brackets indicate the averaged value over different subshells, and the energy of K x-ray, \( k_K \), is given by
\[
k_K = E_K - <E_L>.
\]

Therefore, the kinetic energy of K Auger electrons is given by
\[
T'_K = E_K - 2 <E_L>.
\] (B30)

From now on, \( <E_L> \) is simply denoted as \( E_L \).

For other level Auger electrons, the same formulation is applied to obtain the corresponding kinetic energy.

Each Auger process started from the lowest shell should be traced up to the second order as follows.

\[
A_K = f_K \{ a_K [T'_K g(T'_K) + 2 a_L T'_L g(T'_L)] + \omega_K a_L T'_L g(T'_L) \} ,
\]
\[
A_L = f_L \{ a_L [T'_L g(T'_L) + a_M T'_M g(T'_M)] + \omega_L a_M T'_M g(T'_M) \} , \quad \text{(B31)}
\]
\[
A_M = f_M a_M T'_M g(T'_M),
\]

where \( T'_i \) is estimated as mentioned above. There is a relation \( a_i = 1 - \omega_i \) for the i-th shell. The "most reliable experimental" values of fluorescence yields are listed in column 4 of Table 18.

\( A_K \) includes the following terms. First, the atom having a vacancy in the K-shell will produce an Auger
electron of kinetic energy $T_K'$ with a probability $a_K$. This corresponds to the first term $f_K a_K T_K' g(T_K')$. Second, if otherwise, the atom will radiate a fluorescent x-ray and a vacancy in the L-shell will result in being filled with an upper shell electron and emitting an Auger electron with kinetic energy $T_L'$ with a probability $a_L$. This gives the last term $f_K \omega_L a_L T_L' g(T_L')$. The factor 2 in the middle term comes from the fact that the K-Auger effect creates two vacancies on the L-shell. The Auger effect from the L-shell following the K-Auger emission will then produce two Auger electrons of kinetic energy $T_L'$.

All the terms in a successive chain are arranged from left to right in order of their occurrence. When the absorption of the incident x-ray occurs on the L (or M) shell, the chain of the Auger effect will initiate from the L (or M) shell. $A_L$ and $A_M$ are thus obtained.

c. Reabsorption of Fluorescence X-Rays

Under the assumption that the fluorescence x-rays are emitted isotropically at the point where the initial photo-emission takes place, the escape probability of the fluorescence is averaged over the direction location of radiation in the chamber. The quantities $e^{-\tau_i}$ denote the escape probabilities for i-shell fluorescence x-rays, which are derived in Sec. B.2 of this appendix.

The fluorescent x-ray has a unique energy $k_{K,L}$ depending on the shell from which it is radiated. The kinetic energy
of the emitted electron $T''_i$ in the process of the reabsorption is, therefore, obtained as

- $T''_K = k_K - E_L$ when it is absorbed in the L-shell;
- $T''_M = k_L - E_M$ when it is absorbed in the M-shell.

Here we do not consider the case where this reabsorption takes place in the higher shell because it is negligible.

The ionization by the electron emitted from a secondary atom reabsorbing the fluorescent radiation gives the deposition of energy

$$ F = F_K + F_L $$

with

$$ F_K = f_K \omega_L \alpha_F [T''_K g(T'_K) + \alpha_L T'_L g(T'_L)] , $$

$$ F_L = f_L \omega_L \alpha_L [T''_L g(T'_L) + \alpha_M T'_M g(T'_M)] , $$

where $\alpha_F$ is obtained by using the fluorescence escape probabilities $e^{-\tau_d}$ as $\alpha_F = 1 - e^{-\tau_d}$. The second term in $F_K$ or $F_L$ shows that the Auger yield is associated with the photo emission by the reabsorption of fluorescent radiation in the secondary atom.

All the possible processes by which the energy of incident photon converted into pulse are now consumed. The efficiency to detect the photon of energy $k$ is given by the ratio called "ionization efficiency,"

$$ \xi = I/\lambda , $$

(B33)
where

\[ I = D + A + F . \] (B34)

Figure 75 shows the value of ionization efficiency \( \xi \) for the krypton and xenon chambers of thickness 1.5 cm. In the region of photon energy (5-30 keV) relevant to the specific spectrum of transition radiation in our experiment, the xenon chamber has better efficiency on the average than the krypton chamber.

Beyond the K-edge, the ionization efficiency of the chamber drops drastically as shown in Fig. 75. This is because the K-fluorescence has a high escape probability and the electron ranges become longer at the high energy. In addition, the heavier element xenon, having higher fluorescence yields, causes a greater drop at the K-edge than krypton.

The fluorescence escape is recognized as secondary peaks when the chamber is irradiated to high energy x-rays. Figure 43 shows a pulse height distribution taken with a monochromatic x-ray beam from an x-ray generator. In the xenon chamber, we can observe the photo-peak at 40 keV and two more peaks at the lower energy side (labeled as \( \alpha \) and \( \beta \)). These secondary peaks are the evidence of fluorescence escape. The intensity of these peaks relative to the photo-peak will give the information about the ionization efficiency.
Let \( f_K \) be the K-shell contribution to the total absorption \((f_K = \sigma_K/\sigma \text{ contribution of the K-shell to the total cross-section (see Eq. (B27))})\); then the K-fluorescence from atoms will be \( f_K \omega_K \). Let the K-lines be \( K_\alpha \) and \( K_\beta \) and let the fractional intensities of each line be \( x_\alpha \) and \( x_\beta \). The fractional intensity is calculated from the relative intensities in column 6 of Table 18 as

\[
x_i = \frac{I_i}{\sum_{\alpha, \beta} I_i}.
\]

(B35)

For simplicity, we assume the L,M and higher radiation will be totally absorbed and converted into photoelectron energy before escaping. This assumption is plausible from the discussion given in this subsection. The amount of K-fluorescence reaching outside the chamber will be

\[
f_K \omega_K e^{-\tau K} \mathrm{d}x,
\]

where the average escape probability \( e^{-\tau K} \mathrm{d}x \) for a certain K-line was calculated in Section A.2. We then obtain the intensity of each K-line as

\[
q_{\alpha, \beta} = f_K \omega_K x_\alpha e^{-\tau K} \mathrm{d}x.
\]

(B36)

The fractional intensity of the photo-peak is given by

\[
p = 1 - (q_\alpha + q_\beta).
\]

(B37)

The peak position of each K-line \( E_{\alpha, \beta} \) is given by using the weighted average of K-lines \( \bar{E}_{\alpha, \beta} \) in column 5 of Table 18.
By substituting all the numbers obtained earlier, \( f_K \) from Eq. (B27), \( \omega_K \) in column 3 of Table 18, and \( e^{-\tau d} \) from Table 16, we evaluate \( q_\alpha = 0.562(0.405) \), \( q_\beta = 0.102(0.058) \) and \( p = 0.336(0.538) \) in xenon (krypton). These results are compared with the experimental values in Fig. 43. The smooth curve is calculated by using the fractional intensities q's and p's (for each peak, a Gaussian distribution is assumed). With this observation, it is seen that the more intense the escape will be, the greater the loss of the absorbed energy in the shape of x-rays will result.

The detection efficiency of the chamber for x-rays \( \xi(\omega) \) is the product of the absorption of incident photons (Eq. (B1)) and the ionization efficiency \( \xi(\omega) \):

\[
\xi(\omega) = a(\omega) \xi(\omega) .
\]  

(B38)

Figure 12 shows the detection efficiency as a function of photon energy.
APPENDIX C
ENERGY CALIBRATION FOR THE MS ARRAY
IN CONNECTION WITH THE CROSS-TALK EFFECT

The method to obtain the energy calibration for the MS array data is discussed in connection with the cross-talk effect between the TR and dE/dx detecting sides. The cross-talk causes about 12 per cent corrections on the pulses coming out of either side of a split chamber.

C.1 Derivation

Let us define A and B as the true pulse height generated on TR and the dE/dx detecting sides, respectively. The ADC unit records the output from the amplifier in its count scale. If the gain of the amplifiers are defined as $G_A$ and $G_B$ for the A and B sides of a split chamber, respectively, the recorded pulse heights would be in equivalent volts:

\[ V_A = G_A A_{in} = G_A (A - \eta B) \]  \hspace{1cm} (C1)
\[ V_B = G_B B_{in} = G_B (B - \eta A) \]

where

$A_{in}, B_{in}$: the pulse height going into the amplifiers from the A and B sides of the chamber, respectively;

$\eta$: the quantity describing the cross-talk effect, which is related to the characteristic capacity of the chamber itself (see Section 5.1).
In order to evaluate the quantities A and B, let us solve (1) for A and B:

\[ A = \frac{A_{\text{in}} + \eta B_{\text{in}}}{1 - \eta^2} = \frac{V_A/G_A + \eta V_B/G_B}{1 - \eta^2} \] (C2)

\[ B = \frac{B_{\text{in}} + \eta A_{\text{in}}}{1 - \eta^2} = \frac{V_B/G_B + \eta V_A/G_A}{1 - \eta^2} \]

We must make it clear that the quantities A and B in (C2) are describing true TR and \( dE/dx \) pulses which might have been produced as signals directly from the chamber if there were not any cross-talk. We use the following notations \( A^{\text{TR}} \) and \( B^{dE/dx} \) from now on. Experimentally, we can estimate the quantities \( \eta \) and \( G_A/G_B \) since the calibration runs provide information necessary for determining the cross-talk factor. When the calibrating source is placed on the A side of the chamber, the signal outputs from the A- and B-side amplifiers are

\[ V_A^{\text{SA}} = G_A A^{\text{SA}} \] (C4)

\[ V_B^{\text{SA}} = G_B \eta A^{\text{SA}} , \]

where the superscript \( \text{SA} \) on the quantities indicates that the calibrating source is placed on the A side of the chamber. Then from (C4), we obtain the cross-talk factor

\[ \eta = \frac{G_A V_B^{\text{SA}}}{G_B V_A^{\text{SA}}} . \] (C5)
By moving the same source to the B side, the similar equations can be written with the corresponding superscript SB as

\[ V_{SB}^{A} = G_{A} \eta_{SB}^{B} \quad \text{(C6)} \]
\[ V_{SB}^{B} = G_{B} \eta_{SB}^{B} \]

and

\[ \eta = \frac{G_{B} \ V_{A}^{SB}}{G_{A} \ V_{B}^{SB}} . \quad \text{(C7)} \]

The gains of amplifiers \( G_{A} \) and \( G_{B} \) do not depend on the source position, nor should \( \eta \). Hence, we obtain from (C5) and (C7),

\[ \frac{G_{A} \ V_{SA}^{B}}{G_{B} \ V_{SA}^{B}} = \frac{G_{B} \ V_{A}^{SB}}{G_{A} \ V_{B}^{SB}} . \quad \text{(C8)} \]

Solving (C8) for \( \frac{G_{A}}{G_{B}} \),

\[ \frac{G_{A}}{G_{B}} = \sqrt{\frac{V_{SA}^{A} \ V_{SB}^{A}}{V_{SA}^{A} \ V_{SB}^{A}}} \cdot \sqrt{\frac{V_{SB}^{B}}{V_{B}^{B}}} . \quad \text{(C9)} \]

and substituting (C9) in either (C5) or (C7), we obtain

\[ \eta = \sqrt{\frac{V_{SB}^{A} \ V_{B}^{A}}{V_{SA}^{A} \ V_{SB}^{B}}} . \quad \text{(C10)} \]

As convention, we introduce two new notations
\[ \beta = \frac{V_{SA}^B}{V_{SA}^A} \]
\[ \alpha = \frac{V_{SB}^A}{V_{SB}^A} \]

which are measured as slope by fitting a linear line to the scatter-plot. The scatter-plot was obtained at the same time when the energy calibrating source was placed on either the A(TR) or B(dE/dx) side of the chamber for the energy calibration of the chamber. The pulse height for the A and B sides of the chamber was recorded simultaneously. Then, \( \frac{G_A}{G_B} \) and \( \eta \) can be written

\[ \frac{G_A}{G_B} = \sqrt{\alpha/\beta} \quad (C12) \]
\[ \eta = \sqrt{\alpha \beta} \quad (C13) \]

Finally, we include the energy calibration in order to obtain the energy scale from the equivalent-volt scale of ADC. By rewriting A in (C2), we obtain the pulse height corrected for the cross-talk effect

\[ G_A^{A_{TR}} = \frac{V_A + \eta(G_A/G_B) V_B}{1 - \eta^2} \]

and using the energy value for the x-ray source \( E_{SOURCE} \) and with \( V_{SA}^A \) from (C4), we obtain TR pulse height in energy unit (keV) \( E_{TR} \),

\[ E_{TR} = E_{SOURCE} \frac{G_A^{A_{TR}}}{G_A^{A_{SA}}} \]
\[
E_{\text{SOURCE}} \frac{V_A + \eta(G_A/G_B) V_B}{\frac{V_{SA}}{V_A} \frac{1 - \eta^2}}
\]

or

\[
E_{\text{TR}} = \frac{E_{\text{SOURCE}} \frac{V_A + \alpha V_B}{V_{SA}}}{1 - \eta^2}.
\]  \(\text{(C14)}\)

The first term in the expression is obtained as a slope of a straight line which is fitted to two or three points measured with different calibrating sources (Fig. 38). In the linear-fitting, the value of the slope is given in the unit of equivalent volts/keV.

In the processing of calibration, we first convert counts into equivalent volts, then apply (C14) to obtain the energy scale.

C.2 Evaluation of \(S\) and \(\alpha\)

The scatter-plot between the A and B sides of the chamber was obtained when the x-ray source was placed on either side. A fitting to the scatter-plot is described by the following quantities measured on each axis of A and B.

When the source is placed on the A side, the induced pulse on the B side will pull down the level below its pedestal point. Figure 40 shows that each event with an x-ray source on the A side was scattered in the two dimensional A-B plane. The vertical scale for the A side is linearized by adjusting the bilinear mode of ADC and the
non-linearity near the pedestal (see Fig. 36). The horizontal B axis does not require this consideration because the portion concerning our interest is below the pedestal, where it is linear by itself. This example shows the case in which the source was on the A side, so the pulses appearing on the B side are less than its pedestal value. These measured counts are converted into equivalent volts by using the conversion graph between equivalent volts and counts. This process results in the value of \( \beta \) as given in (C11). For the case when the source is placed on the B side, we get the scatter-plot with the A and B axis exchanged, and this will result in the value of \( \alpha \).

For all calibration runs, the values of \( \beta \) and \( \alpha \) were obtained. In Table 19, the results of calibration run C118 is shown. In the second column, the slope \( \beta \) is given. Reading the coordinates of the line fitted to the scattered events, we convert counts into equivalent volts to obtain the values of \( V_A^{SA} \) and \( V_B^{SA} \). The third column gives the value of \( \beta \). The values of \( G_A/G_B \) and \( \eta \) are calculated from (C12) and (C13) and shown in columns 4 and 5.

The values of \( \eta \) and \( G_A/G_B \) are calculated for all runs. Figure 41 shows the value of \( \eta \) plotted vs. the run number in the linear time scale. As is pointed out that the quantity \( \eta \) describes the electrical capacity of the chamber (which depends on the physical structure of the chamber), its value remains constant throughout the experiment.
<table>
<thead>
<tr>
<th>Chamber</th>
<th>Slopes in Scatter Plots</th>
<th>Amplifier Gain Ratio $G_A/G_B$</th>
<th>Cross-Talk Factor $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Source on A side $\beta$</td>
<td>Source on B side $\alpha$</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-0.128</td>
<td>-0.135</td>
<td>1.025</td>
</tr>
<tr>
<td>1</td>
<td>-0.130</td>
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</tr>
<tr>
<td>2</td>
<td>-0.121</td>
<td>-0.114</td>
<td>0.972</td>
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<td>3</td>
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<td>-0.096</td>
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<td>1.159</td>
</tr>
<tr>
<td>6</td>
<td>-0.094</td>
<td>-0.140</td>
<td>1.220</td>
</tr>
</tbody>
</table>
APPENDIX D

PROOF OF $<\text{TR}> = <\text{RAD}> - <\text{EA}>$

In the analysis of sandwich array data, we used the fact that the average values of two independent frequency distributions are additive. We prove this fact by using probability theory.

By obtaining the two kinds of frequency distribution taken with either the radiator (RAD) or equivalent absorber (EA), we need to calculate the average energy of transition radiation (TR). The "radiator-in" frequency distribution is a superposition of two independent processes: transition radiation and ionization loss by the incident particle penetrating the chamber. It is impracticable to disentangle the frequency distribution of transition radiation from the congruent distribution. In other words, given frequency distributions $f_R(x)$ for the pulse height $x$ in the radiator run and $f_E(y)$ for pulse height $y$ in the equivalent absorber run, we must calculate the average detected energy of transition radiation x-rays without knowing its frequency distribution.

A theorem of probability theory states that we can compute the expectation of a random variable such as $Z = X \pm Y$ where only the frequency distributions of variables $X$ and $Y$ are known and that of variable $Z$ is not known. We review two theorems to show that we can
obtain the expectation of $Z$ directly in terms of two frequency distributions $f_R$ and $f_E$. Before stating the theorems, we introduce the following definitions.

**Definition 1**: The discrete density function $f$ for the random vector $X$ (i.e., the probability of an event $\{X = x\}$) is defined by

$$f(x) = P(X = x).$$

(D1)

This function $f$ has the following three properties:

(i) $f(x) \geq 0$;

(ii) $\{x: f(x) \neq 0\}$ is a finite or countably infinite subset of real numbers $\mathbb{R}^r$, which is denoted by $\{x_1, x_2, \ldots\}$;

(iii) $\sum f(x_i) = 1$.

The function $f$ is usually called the joint density of random variables $X_1, X_2, \ldots, X_r$, which are the components of $\underline{X} = (X, \ldots, X_r)$. Let $X$ and $Y$ be two discrete random variables. For any real numbers $x$ and $y$, the set $\{X = x$ and $Y = y\}$ is an event. Suppose that the distinct possible values of $X$ are $x_1, x_2, \ldots$, and that the distinct possible values of $Y$ are $y_1, y_2, \ldots$. For each $x$, the events $\{X = x, Y = y_j\}$, $j = 1, 2, \ldots$, are disjoint and their union is the event $\{X = x\}$. Thus

$$P(X = x) = \sum_{y} P(X = x, Y = y).$$
Similarly,

\[ P(Y = y) = \sum_x P(X = x, Y = y) . \]

**Definition 2:** Suppose that \( X \) is any random variable having possible values \( x, x_2, \ldots \). We define the expectation of \( X \) as

\[
EX = \sum_{j=1}^{\infty} x_j f(x_j) . \tag{D2}
\]

**Theorem 1:** Let \( X \) be a discrete random vector having density \( f \) and let \( \phi \) be a real-valued function on \( \mathbb{R}^r \). Then the random variable \( Z = \phi(X) \) holds

\[
EZ = \sum_x \phi(x) f(x) = \sum_j \phi(x_j) f(x_j) . \tag{D3}
\]

We will use the notational convention as the first equality in place of the last.

**Proof:** Let \( z_1, z_2, \ldots \) denote the distinct possible values of \( Z \) and let \( x_1, x_2, \ldots \) denote the distinct possible values of \( X \). For any \( z_j \) there is at least one \( x_i \) such that \( z_j = \phi(x_j) \), but there may be more than one such \( x_i \). Let \( A_j \) denote the collection of such \( x_i \), that is,

\[
A_j = \{ x_i | \phi(x_i) = z_j \} .
\]

Then \( \{ X \in A_j \} \) and \( \{ Z = z_j \} \) denote exactly the same events. Thus

\[
P(Z = z_j) = P(X \in A_j) = \sum_{x \in A_j} f_x(x) .
\]
Consequently,

\[ \sum z_j f_Z(z_j) = \sum z_j P(Z = z_j) \]

\[ = \sum z_j \sum_{x \in A_j} f_x(x) \]

\[ = \sum_{j} \sum_{x \in A_j} f_x(x). \]

Since \( \phi(x) = z_j \) for \( x \) in \( A_j \), it follows that

\[ \sum z_j f_Z(z_j) = \sum_{j} \sum_{x \in A_j} \phi(x) f_x(x). \]

By definition, the sets \( A_j \) are disjoint for distinct values of \( j \), and their union is the set of all possible values of \( X \). Therefore,

\[ \sum z_j f_Z(z_j) = \sum_{x} \phi(x) f_x(x). \]

We shall use Theorem 1 to establish the following property of expectation.

Theorem 2: Let \( X \) and \( Y \) be two random variables having finite expectation. Then, \( X + Y \) has expectation

\[ E(X + Y) = EX + EY. \] (D4)

Proof: Let \( \phi(x,y) = x + y \) and let \( f \) denote the joint density of \( X \) and \( Y \). Then, applying Theorem 1

\[ E(X + Y) = \sum_{x,y} (x + y) f(x,y). \]
\[ \sum \sum xf(x,y) \pm \sum \sum yf(x,y) \]
\[ = EX \pm EY . \]

This result will be used in the following notation: \[ <z> = <x> \pm <y>, \]
where the brackets <> denote the expectation of the corresponding variable inside. As the pulse height of an event with a radiator is a sum of two independent processes, transition x-ray energy and ionization loss, the theorem confirms that the average energy of transition radiation is given by
\[ <TR> = <RAD> - <EA> . \] (D5)

These results also prove that when we add two or more random variables, then the average value of the added random variable is obtained as the sum of all average values of individual distributions. This applies to the SW array in which we added pulses from eight chambers to obtain a clear separation of RAD distribution peak from that of dE/dx distribution.
REFERENCES

12. A. I. Alikhanian, K. A. Ispirian, A. G. Oganessian, and A. G. Tamanian, Nucl. Instr. and Meth. 89, 147 (1970);
18. S. I. Parker, invited paper in the Instr. Session at the XVIth Int. Conf. on High Energy Physics, Chicago, 1972;


23. I. M. Frank, Usp. Fiz. Nauk 75, 231 (1961);

24. I. M. Frank, Usp. Fiz. Nauk 87, 189 (1965);
[Sov. Phys. Uspekii 8, 729 (1966)].


F. G. Bass and V. M. Yakovenko, Usp. Fiz. Nauk 86, 189 (1965);
[Sov. Phys. Uspekii 8, 3 (1965)].


[Sov. Phys. JETP 33, 23 (1971)].

31. The aluminum coating was very thin (~200Å) and was required to prevent electrostatic attraction between adjacent foils.

33. G. T. Schjeldahl Company, Northfield, Minnesota.

34. Cajon Company, 32550 Old South Miles Road, Cleveland, Ohio.

35. The Oxford group made an independent analysis for the calibration data for the MS array (J. N. Bunch, Nuclear Physics Laboratory, Oxford University, 1973).


40. D. West, Prog. Nucl. Phys. 3, 18 (1953);
41. Preliminary results of SLAC experiment were reported at the April 1973 Washington, D. C., APS meeting, APS Bull. 18, 596 (1973);
V. Z. Peterson, Proc. of Int. Conf. on Instr. for High Energy Physics, Frascati, 1973, p. 442;
42. A. Bamberger, G. F. Dell, Jr., H. Uto, L. C. L. Yuan, and P. W. Alley, Phys. Letters 43B, 153 (1973);
45. M. L. Cherry, D. Muller, and T. A. Prince, Nucl. Instr. and Meth. 115, 141 (1974);
    T. A. Prince, D. Muller, G. Hartmann, and M. L. Cherry, Nucl. Instr. and Meth. 123, 321 (1975);


47. J. Fischer, S. Iwata, V. Radeka, C. L. Wang, and W. J. Willis, Phys. Letters 49B, 393 (1974);


51. F. Harris, T. Katsura, S. Parker, V. Peterson, R. Ellsworth, G. B. Yodh, S. Pruss, "Development of a Transition Radiation Detector for Discriminating

52. P. N. Dobson, Jr., private communication.


55. R. W. Fink, R. C. Jopson, H. Mark, and C. D. Swift, Rev. Mod. Phys. 38, 513 (1966);

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