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# THREE ESSAYS ON WEALTH EFFECT

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#### ABSTRACT

We first clarify that changes in fundamental paper wealth are wealth redistributions between current and future asset owners; and increases in fundamental paper wealth tend to make current consumers wealthier and hence have positive impacts on current aggregate consumption. Based on the concept of fundamental paper wealth, we examine three issues related to asset prices.

The first issue is related to the wealth effect of monetary policy. While the wealth and Tobin's q effects are usually treated as two independent monetary policy transmission mechanisms, we show that they are indeed negatively correlated under a general-equilibrium perspective; and their magnitudes depend upon investment elasticity. These insights provide a new perspective to the relationship between asset prices and monetary policy.

Another issue is related to the capital account policy of developing countries. Empirical evidence shows that capital inflows are often used by developing countries to finance excessive consumption. While the existing literature generally explains these phenomena as resulting from institutional imperfections, we show that they can be the results of fundamental paper wealth effect caused by capital inflows. We show that, while risk aversion causes low investment elasticity and hence reduces the total benefit of capital account liberalization for society over time, it nevertheless tends to make current consumers better off and drive consumption booms. We show that a positive yet uncertain future productivity shock is likely to cause consumption booms because of sluggish investment reactions.

The third issue is related to the "asset market meltdown hypothesis", which predicts that baby boomers' prime-time savings will drive up asset prices that will eventually collapse due to their retirement dissavings. While the existing literature generally supports the hypothesis, we find that the meltdown is actually state-contingent and may not necessarily happen because the large capital stock built up by baby boomers' large savings may be able to sustain the asset prices during baby boomers' retirement era. However, we find that, in the case where the meltdown is about to happen, baby boomers as a whole has no escape; and their attempts to escape could push the economy into a liquidity trap.

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### **Chapter One**

# **Conceptual Foundation and Dissertation Overview**

### 1.1 Paper wealth

One academic implication of the impressive stock market booms in the United States (U.S.) during the 1990s is to rekindle the interest in the wealth effect of monetary policy. It seems straightforward that monetary expansion tends to cause asset price appreciation (by lowering the interest rate level) and hence increase the wealth of consumers who will accordingly increase their consumption. However, Gramlich (2001) raised an interesting point by arguing that, if asset price appreciation is merely the result of a decline in the interest rate (as the discount factor), "households are not expecting higher future returns but are simply discounting the same stream of returns at a different rate, so it is less clear that they are truly better off and should increase their consumption."

In other words, Gramlich wonders how changes in "paper wealth" (induced by monetary policy) affect consumption. The term "paper wealth" is often used to describe changes in asset value for reasons other than changes in asset earnings; e.g., asset bubbles will create paper wealth. Since interest-rate-induced changes in asset value are usually not taken as bubbles but as fundamental asset revaluations, we will call them "fundamental paper wealth".

This dissertation was motivated by an attempt to understand the nature of fundamental paper wealth and its effect on consumption. The effort turned out to be fruitful in that the concept of paper wealth provides insights to several issues in different lines of literature. In the following we first clarify the nature of fundamental paper wealth and its effect on consumption in the context of the wealth effect of monetary policy, then provide an overview of three issues related to the concept of paper wealth that will be addressed in the remaining chapters of the dissertation.

#### **1.2.** Fundamental paper wealth and its effect on consumption

The nature of monetary-policy-induced fundamental paper wealth and its effect on consumption have been investigated decades before by the so-called interest-rate-induced wealth effect literature (Leijonhufvud, 1968; Sweeney, 1988).<sup>1</sup> Unfortunately, the insights provided by this unsystematic literature have been largely overlooked by recent discussion on the wealth effect of monetary policy (Mishkin, 1995; Ludvigson et al., 1999), which is based on the lifecycle consumption hypothesis (Modigliani, 1971).

In the following we first refine and articulate the insights provided by the interestrate-induced wealth effect literature, then use them to examine the nature of fundamental paper wealth and its effect on consumption under the framework that is popular in recent studies on the wealth effect of monetary policy.

<sup>&</sup>lt;sup>1</sup> We prefer the term "paper wealth" in that it helps distinguish asset price movements due to earning revaluations; whereas changes in the interest rate are likely to induce changes in earnings through their impacts on investments.

#### **1.2.1** Traditional literature on the wealth effect of monetary policy

The wealth effect of monetary policy refers to the induced changes in spending that occur as a result of an increase in the money supply and/or a reduction in market interest rates. Besides the interest-rate-induced wealth effect, the traditional literature on the wealth effect of monetary policy also includes discussion on the price-induce wealth effect, which is usually referred to as the "real balance effect".

#### 1.2.1.1 Real balance effect

In short, the real balance effect means that a decline in the price level will increase the real value of money and hence have a positive impact on consumption (Haberler, 1941; pp. 388-389). An important implication of the real balance effect is that price variations can help the economy achieve full-employment equilibrium (Pigou, 1943, 1947; Patinkin, 1956, 1965). The real balance effect has sometimes been viewed as theoretical interesting yet empirical irrelevant, because the money balance included in the net wealth of the economy is merely a small amount of "outside" money (Gurley and Shaw, 1960). Yet, some authors argue that bank money can also be part of the net wealth so that the real balance effect is not trivial (Pesek and Saving, 1967; Patinkin, 1969).

As the inflation-targeting monetary policy has become increasingly popular, the corresponding price stability makes the real balance effect less relevant. On the other hand, as central banks need to adjust the interest rate to keep price stable, the interest-rate-induced wealth effect is more relevant.

#### **1.2.1.2 Interest-rate-induced wealth effect**

A decline in the interest rate tends to increase asset value through asset price appreciation; the effect of which on consumption has been referred to as the interest-rateinduced wealth effect. Major contributors to the interest-rate-induced wealth effect literature include Keynes (1930, 1936), Hicks (1939), Pesek and Savings (1967), Leijonhufvud (1968), and Sweeney (1988). In the following we summarize the conceptual core of the literature.

Keynes (1931, p. 196) points out that "a country is no richer" from asset price appreciation caused by a decline in the interest rate since the "windfall" capital gain does not increase the country's real resources. Yet he argues that, although consumers become no richer from such "windfall" capital gains, they will still consume more because they "feel" richer.

Perhaps the most important influence, operating through changes in the rate of interest, on the readiness to spend out of a given income, depends on the effect of these changes on the appreciation or depreciation in the price of securities and other assets. For if a man is enjoying a windfall increment in the value of his capital, it is natural that this motives towards current spending should be strengthened, even though in terms of income his capital is worth no more than before; and weakened if he is suffering capital losses." (Keynes 1936, pp. 94; emphasis added) Keynes' explanation of the wealth effect is sometimes called as his "second psychological law of consumption" (Leijonhufvud, 1968). A modern version of this psychological law, which often appears in press discussion on the wealth effect, attributes the wealth effect to consumers' confidence. That is, asset price appreciation will raise consumption through increasing consumers' confidence, even though the appreciation could be paper wealth in nature.

Instead of resorting to consumers' paper wealth illusion, Hicks (1939, pp. 232-235) discusses the impacts of changes in the interest rate on the expenditure behaviors of utility-maximizing consumers. He decomposes the interest-rate effect on current expenditure into the substitution and income effects, with the former accounting for intertemporal expenditure relocations due to the interest-rate effects on the intertemporal prices of expenditures; and the latter accounting for the interest rate effect on consumers' maximum achievable utility (i.e., their "wealthiness").

Accordingly, a decline in the interest rate will have positive substitution effects since consumers tend to substitute future for current expenditures as the decline lowers the costs of current purchases in terms of future purchases. The income effects are ambiguous because the interest rate effects on wealthiness are uncertain, depending on consumers' income and consumption structures. A decline in the interest rate tends to make "borrowers" (whose income horizons are longer than their consumption horizons) wealthier; on the contrary, the decline tends to make "lenders" (whose income horizons are shorter than their consumption horizons) less wealthy. Intuitively, a decline in the interest rate increases both consumers' net worth and (the present value of) their lifetime

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consumption costs; the borrowers become wealthier because, for them, the benefits from higher net worth outweigh the losses from higher consumption costs; vice versa for the lenders.

Hicks argues that, since a lending must correspond to a borrowing, which implies that gains for borrowers have their counterpart losses for lenders (or vice versa), without the consideration of distributional effects, the income effects of the borrowers and lenders tend to cancel out in aggregate so that the aggregate income effect tends to be zero.

As the remaining substitution effects for the borrowers and lenders have the same signs, Hicks (1939, pp. 235) concludes that, "for the market as a whole, a rise in the rate of interest will reduce current expenditure, a fall in the rate of interest increase it".

In summary, Hicks' discussions imply that a country as a whole is no richer from a fundamental paper wealth increase; yet the aggregate consumption tends to increase because of the substitution effect.

As opposed to the "no richer" view of Keynes and Hicks, Pesek and Savings (1967, p362) argue that, since a society as a whole must be a net wealth holder, "a higher (interest) rate must result in a net reduction of society's wealth and thus must result in a reduction in the planned increase in total consumption".

Leijonhufvud (1968; Chapter IV) also argues for a positive aggregate income effect (or wealth effect in his terminology) of a decline in the interest rate. The following excerpt is the core of his argument.

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It is a system wherein the social function of production is eternal and the individual households, in comparison, ephemeral. "In the Long Run we are all dead," but production goes on and the capital stock is maintained and handed down from generation to generation. Ownership is divorced from the function of management of productive resources. Households in the early part of their life cycles consume less than the value of services which they contribute. Their resulting claims on the system's resources they accumulate in the form of "shares" in society's ongoing productive concern. Households in later stages of the life cycle consume more than the value of their concurrent productive contribution and are therefore "impoverishing themselves." This dissaving is to a substantial extent financed through the sale of income sources. Since the ultimate owners of the system's productive resources do not hold their claims on these resources "to maturity," their welfare depends upon the consumption value at which these assets can be resold. The higher the "real value" of these long-term assets the better off is the owner. A fall in the rate of interest means that this value increases and therefore has a positive wealth effect. (Leijonhufvud 1968, pp. 258-259).

In short, Leijonhufvud's arguments imply that the group of current consumers (as a whole) is essentially a "borrower", because the capital stock (as the net wealth of the current consumers as a whole) tend to have a longer income horizon than the consumers' consumption horizons. Therefore, the current consumers in aggregate tend to become wealthier from a decline in the interest rate and hence consume more currently (as well as in the future), i.e., the positive aggregate wealth effect (or income effect in Hicks' terminology).

In summary, the interest-rate-induced wealth effect literature provides the following insights. First, a decrease in the interest rate tends to increase consumption through the substitution effect. Second, an increase in fundamental paper wealth (due to a decline in the interest rate) does not necessarily make individual consumers wealthier, because the interest-rate decline also tends to increase the (present value of) consumers' lifetime consumption costs. Third, a fundamental paper wealth increase nevertheless tends to make the current consumers as a whole wealthier, because they are in aggregate a borrower from future consumers.

Unfortunately, these insights have not been appreciated adequately by recent discussion on the wealth effect of monetary policy, which usually takes the wealth effect of monetary policy for granted. The unsystematic nature of the interest-rate-induced wealth effect literature may be one factor responsible for this situation.<sup>2</sup> Also, since the positive wealth effect of monetary expansion is both intuitive and supported by lifecycle consumption models based on consumers' intertemporal optimizing behaviors, the "paper" nature of asset price appreciation due to monetary expansion has not been an issue in the modern literature on the wealth effect of monetary policy until recently brought up by Gramlich (2001).

 $<sup>^2</sup>$  The interest-rate-induced wealth effect literature is insightful yet unsystematic: The authors approach the issue from different perspectives; they use different conceptual frameworks and terminologies; most of them use narrative approach to impart insights without the support of formal analysis; most of their discussions are embedded in books with broader agendas. We do not find a single journal article that focuses on the issue. To our knowledge, the only attempt to formalize the interest-rate-induced wealth effect literature is in Sweeney's (1988) book, which nevertheless has little impact on the recent mainstream literature on the wealth effect of monetary policy.

In the following, we will use the insights articulated in the above to clarify the nature of fundamental paper wealth and its effect on consumption in the context of the modern literature on the wealth effect of monetary policy.

#### 1.2.2 Modern literature on the wealth effect of monetary policy

Most of the recent research on the wealth effect of monetary policy is conceptually based on the lifecycle consumption hypothesis (Modigliani, 1971 and 1986; Modigliani and Ando, 1963; Modigliani and Brumberg, 1954). A life cycle consumption function suggested by Modigliani (1971) is

$$C_0 = \xi W_0 , \qquad (1)$$

which implies that current consumption  $(C_0)$  is positively (consumption propensity  $\xi > 0$ ) related to the net worth  $(W_0)$  measured by the present value of lifetime incomes. Note that equation (1) is a standard lifecycle consumption function that can be derived from household intertemporal optimization models (e.g. Obstfeld and Rogoff, 1996, p. 182).

Under the convenient and common assumption of unit elasticity of intertemporal substitution, a fall in the interest rate (due to monetary expansion) has no effect on  $\xi$  but increases  $W_0$ , which according to equation (1) implies a *proportional* increase in consumption, despite the paper wealth nature.

For clarification, we call such a *proportional* effect on consumption the fundamental paper wealth effect of monetary policy.<sup>3</sup> Indeed, increases in fundamental paper wealth are often casually referred to as increases in consumers' wealth or lifetime resources; and the fundamental paper wealth effects are accordingly called as the "wealth effect" (Mishkin, 1995; Modigliani, 1971, among many others).

In short, as the positive wealth effect of monetary policy on consumption is both intuitive and supported by the lifecycle consumption function that has a solid microfoundation, the paper nature of the monetary impact on asset price has not been taken as an issue by the modern wealth effect literature, until recently being questioned by Gramlich (2001) and currently being analyzed here.

#### 1.1.3 Two-effect vs. three-effect decomposition

To clarify the fundamental paper wealth effect of changes in the interest rate on consumption, we first compare two methods of decomposing the interest-rate effect on consumption.

Denote a Marshallian (current) consumption demand and the corresponding Hicksian demand as  $C_0(i, W_0)$  and  $C_0^h(i, u^*)$  respectively, where the interest rate *i* measures the price of current consumption in terms of future consumption; and  $u^*(i, W_0)$  is indirect utility that measures the maximum achievable utility under *i* and  $W_0$ .

 $<sup>^{3}</sup>$  According to equation (1), consumption will increase proportionally to a rise in net worth, no matter whether the rise is caused by increases in future earnings or a decline in the interest rate. While the effect of the former is straightforward since consumers are richer from the increase in future earnings, the proportional consumption increase due to paper wealth increase caused by the interest rate fall needs clarification.

A total differentiation of  $C_0(i, W_0)$  gives

$$\frac{dC_0(i, W_0)}{di} = \frac{\partial C_0}{\partial i} + \frac{\partial C_0}{\partial W_0} \frac{dW_0}{di},$$
(2)

in which the first term on the right-hand side can be further decomposed via the wellknown Slutsky decomposition:

$$\frac{\partial C_0(i, W_0)}{\partial i} = \frac{\partial C_0^h}{\partial i} - \frac{\partial C_0}{\partial W_0} \frac{\partial e_0(i, u^*)}{\partial i}, \qquad (3)$$

where  $e_0(i, u^*)$  is the expenditure budget in terms of current consumption. Substituting equation (3) into (2) we obtain a three-effect decomposition of the interest-rate effect on consumption:

$$\frac{dC_0}{di} = \left(\frac{\partial \overline{C_0}^h}{\partial i}\right) + \left(-\frac{\partial \overline{C_0}}{\partial W_0}\frac{\partial e}{\partial i}\right) + \left(\frac{\partial \overline{C_0}}{\partial W_0}\frac{dW_0}{di}\right),\tag{4}$$

where the first, second and third terms on the RHS are usually termed respectively as substitution, income and wealth effects—theirs signs on their tops.<sup>4</sup>

Intuitively, the substitution effect measures the interest-rate effect on consumption given the original indirect utility level  $(u^*)$ ; the income effect measures the interest-rate effect on consumption through the lifetime consumption cost  $(e_0)$  that is measured by the present value of future consumption expenditures; and the wealth effect measures the interest rate effect on consumption through the net worth  $(W_0)$ .

Most of the current discussion on the wealth effect of monetary policy is (explicitly or implicitly) based on this three-effect decomposition. That is, monetary policy will affect

 $<sup>^4</sup>$  See Obstfeld and Rogoff (1996, pp. 28-45) for discussion on the three-effect decomposition in a twoperiod model.

consumption through the substitution, income, and wealth effects. Yet, a key point is that the wealth effect (or more precisely the interest-rate effect on  $W_0$ ) is not a proper measure of the interest rate effect on *wealthiness*, because a fall in the interest rate tends to increase not only the net worth ( $W_0$ ) but also the present discount value of lifetime consumption costs ( $e_0$ ). This is precisely the essence of the aforementioned "no richer" argument by Gramlich (2001) or Keynes (1936).

Indeed, a proper measure of the interest-rate effect on wealthiness is the sum of the wealth and income effects. This can be made clear by another two-effect decomposition, which is well known as the "Hicksian decomposition".

As opposed to the three-effect decomposition totally differentiating the Marshallian demand, the Hicksian decomposition totally differentiates the Hicksian demand:

$$\frac{dC_0(i, W_0)}{di} = \frac{\partial C_0^h}{\partial i} + \frac{\partial C_0^h}{\partial u^*} \frac{du^*}{di}.$$
(5)

Equation (5) implies that the interest-rate effect on consumption can be decomposed into the substitution effect (the first right-hand-side term) and the "Hicksian wealth effect" (the second term).<sup>5</sup> While the substitution effect here is identical to the same effect in the three-effect decomposition, the Hicksian wealth effect measures the interest-rate effect on consumption through the wealthiness level  $(u^*)$ .

<sup>&</sup>lt;sup>5</sup> The "Hicksian wealthiness effect" here is originally called as "income effect" by Hicks (1939) and sometimes also called as "wealth effect" by later users of the Hicksian Apparatus (e.g., Leijonhufvud, 1968; King, 1991; among others). Since the "wealth effect" and "income effect" in the three-effect decomposition have different economic meanings than the same terms used in the Hicksian decomposition, to avoid terminology confusion, we follow the terminologies in the three-effect decomposition but use the "Hicksian wealth effect" to denote the income (or wealth) effect in the Hicksian decomposition.

A comparison between equations (4) and (5) indicates that the sum of the income and wealth effects is equal to the Hicksian wealth effect, which measures the interest rate effect on wealthiness.

#### 1.1.4 Fundamental paper wealth effect: a clarification

With the difference between the wealth and Hicksian wealth effect clarified, we proceed to clarify the fundamental paper wealth effect.

In the consumption function represented by equation (1), the substitution and income effects are captured by the interest rate effect on the propensity to consume out of wealth  $(\xi)$ ; whereas the wealth effect is captured by the interest rate effect on the level of wealth itself ( $W_0$ ).

Given unit elasticity of substitution, the substitution and income effects exactly offset each other (i.e., the interest rate effect on  $\xi$  is zero); thus the interest rate effect on consumption can be conveniently *measured* by the wealth effect. However, to *interpret* it as a wealth effect can be misleading.

First, such an interpretation seems to suggest that an increase in fundamental paper wealth raises consumption because it makes consumers wealthier. However, a consumer who enjoys a fundamental paper wealth appreciation in  $W_0$  could nonetheless become less wealthy because the accompanied increase in  $e_0$  may outweigh the appreciation in  $W_0$ ; i.e., the income effect may dominate the wealth effect so that the Hicksian wealth effect is negative despite the increase in net worth. Second, the exact cancellation between the substitution and income effects is not equivalent to the case of zero substitution as well as income effect—an offset positive substitution effect (implied by the unit elasticity) is still positive. If the income effect and wealth effect happen to cancel out—which is essentially what Hicks (1939, pp. 232-235) argues for in an aggregate sense—what is often called as the wealth effect of monetary policy can also (perhaps should) be called as the substitution effect. <sup>6</sup>

In summary, with the unit elasticity assumption, the fundamental paper wealth effect is attributable (at least) partly to the substitution effect. This explains the aforementioned puzzle that the lifecycle consumption function in equation (1) indicates that consumption will tend to increase proportionally to a rise in net worth, no matter whether the rise is caused by increases in future earnings or a decline in the interest rate. The proportional paper wealth effect on consumption does not imply that paper wealth will make consumers richer in the same way as earning-induced wealth increases do; rather, the paper wealth effect implicitly captures the positive substitution effect on the current consumption caused by a decline in the interest rate.

If the substitution effect is zero,<sup>7</sup> a decline in the interest rate will increase  $W_0$  but decrease  $\xi$  through the income effect. The balance of the two effects (i.e., the Hicksian wealth effect) is ambiguous for individual consumers, depending on their income and consumption structures—the longer a consumer's income horizon relative to her

<sup>&</sup>lt;sup>6</sup> A decline in the interest rate tends to benefit borrowers but cost lenders. Hicks argues that, since a lending must correspond to a borrowing, without considering distributional effects, the benefits of the borrowers and the costs of the lenders tend to cancel out in aggregate. Thus, the interest rate effect on the aggregate wealthiness tends to be zero; i.e., the income and wealth effects cancel out in aggregate.

<sup>&</sup>lt;sup>7</sup> The literature on the magnitude of the elasticity of intertemporal substitution is controversial. In general, empirical results indicate that the elasticity is less than unity; see Elmendorf (1996) for a survey.

consumption horizon, the more likely she becomes wealthier from a decline in the interest rate (Hicks 1939, pp. 232-235).

However, according to Leijonhufvud's (1968, pp. 258-259) argument, a decline in the interest rate tends to make *society* (as a whole) wealthier because capital as the net wealth of society tends to have a longer income horizon than the consumption horizons of its owners.

We clarify in the following that a decline in the interest rate tends to make *current consumers* (as a whole) wealthier at the cost of future unborn consumers; or in general, changes in fundamental paper wealth (induced by interest rate changes) are essentially wealth redistributions between current and future capital owners.

Suppose a monetary expansion lowers the interest rate from *i* to *i'* (*i'*<*i*) and hence raises the price of capital ownership (entitled to constant dividend *r*) from q (=r/i) to q'(=r/i'). Then *q* dollar of current capital ownership will enjoy a (paper) wealth gain of the amount r(i-i')/ii'; it is a gain in the sense that, if sold instantaneously, the ownership can provide r(i-i')/ii' amount of extra consumption. However, the gain is at the cost of future capital owners because the appreciation causes r(i-i')/i amount of earning loss per *q* dollar of future capital ownership per period. The present value of all the losses in all periods is r(i-i')/ii', which is precisely equal to the gain enjoyed by *q* dollar of current ownership.

In summary, changes in fundamental paper wealth are zero-summed wealth redistributions; the paper-wealth gain enjoyed by current capital owners are at the cost of the earning losses of future owners.

With finite-living consumers, capital will eventually be owned by unborn consumers, who will be among the victims of a decrease in the interest rate that drives current fundamental paper wealth appreciation.

Since fundamental paper wealth appreciation is a zero-summed wealth redistribution, *unborn* consumers being victims implies that current *existing* consumers as a whole is a winner. In other words, fundamental paper wealth appreciation tends to make current consumers as a whole wealthier, albeit not in the Pareto but compensational sense.<sup>8</sup>

Setting aside the distributional effects among current consumers, richer current consumers will tend to consume more in aggregate. Therefore, besides the substitution effect, the (aggregate) fundamental paper wealth effect tends to also capture a positive Hicksian wealth effect. In this sense, the casual "wealth effect" interpretation is not a complete misnomer.

In summary, the above discussion clarifies that, notwithstanding being a revaluation of the same amount of underlying incomes, a fundamental paper wealth appreciation does tend to improve the wealthiness of current finite-lived consumers as a whole, and hence have a positive effect on the aggregate consumption. However, without the substitution effect, the increase in the aggregate consumption will be less than proportional to the fundamental paper wealth increase.

<sup>&</sup>lt;sup>8</sup> Some current consumers (as net savers) might lose from assuming future capital ownerships (put plainly, from lower returns to savings); yet the net gain for current existing consumers is positive thanks to the (potential) loss of unborn consumers.

#### **1.3** Three Issues

Based on the concept of paper wealth, this dissertation addresses three issues related to asset prices.

#### 1.3.1 Interplay between the wealth effect and Tobin's q effect of monetary policy

The first issue is the wealth effect of monetary policy under a general-equilibrium perspective. To explain the concept of fundamental paper wealth and its effect on consumption, which serves as a conceptual foundation for the entire dissertation, the above analysis on the wealth effect of monetary policy is based on a partial equilibrium framework under which monetary policy is assumed to have no impacts on assets' earnings. However, since asset price appreciation induced by monetary expansion tends to have a positive impact on investments through Tobin's q effect, the resulting change in the capital stock will tend to affect asset earnings. Therefore, under a general-equilibrium perspective, the wealth effect and Tobin's q effect of monetary policy are interrelated.

While the wealth and Tobin's q effects are usually treated as two independent monetary policy transmission mechanisms, our analysis shows that they are indeed negatively correlated; that is, the greater the Tobin's q effect is, the smaller the wealth effect will be; and the key determinant of the magnitudes of the two effects is investment elasticity. These insights provide a new perspective to the relationship between the role of monetary policy in stabilizing goods market and its role in maintaining financial stability. **1.3.2 Foreign-capital-financed consumption booms in small developing economies** Another issue is related to the capital account policy of developing countries. Empirical evidence shows that capital inflows are often used by developing countries to finance excessive consumption. The existing literature generally explains these phenomena as resulting from institutional imperfections. In contrast, we conjecture that they can be the results of fundamental paper wealth created by asset price appreciation driven by capital inflows. Our analysis shows that, while risk aversion causes low investment elasticity and hence reduces the total benefit of capital account liberalization for society over time, it nevertheless tends to increase the benefit enjoyed by current generations and hence drive consumption booms. We show that the proportion of capital inflows used for financing consumption is negatively correlated with investment elasticity. We show that a positive yet uncertain future productivity shock is likely to cause consumption booms because of sluggish investment reactions. Our analysis shows that, the greater the expected future productivity is; or the greater the uncertainty is, the stronger the consumption booms will be.

#### 1.3.3 Baby boom and the "asset market meltdown hypothesis"

Another issue that epitomizes the nature of paper wealth is a so-called "asset market meltdown hypothesis", which predicts that baby boomers' prime-time savings will drive up asset prices that will eventually collapse due to their retirement dissavings. Based on the principle of demand and supply, the rationale behind this hypothesis is straightforward: Baby boomers' large savings represent large asset demands that will drive asset prices up; whereas their large dissavings representing large asset supplies will drive the prices down. A footnote to this story may be more revealing: It is the nature of paper wealth. Yet, why does the wealth of baby boomers (or anyone else) have to be of a "paper" nature? Is there a way for baby boomers to protect their hard-earned wealth that represents the goods and services they have helped produce but not yet claimed?

Motivated by these questions, we examine the meltdown hypothesis in Chapter Four. While the existing literature generally supports the hypothesis, we find that the meltdown is actually state-contingent and may not necessarily happen because the large capital stock built up by baby boomers' large savings may be able to sustain the asset prices during baby boomers' retirement era. However, we find that, in the case where the meltdown is about to happen, baby boomers as a whole has no escape; and their attempts to escape from the potential meltdown could drag the economy into a liquidity trap.

#### **1.4** The organization of the dissertation

In the remainder of the dissertation, we address the three issues in Chapter Two, Three, and Four respectively. For clarity, the chapters are written as self-contained essays, with independent notational as well as indexing (for equation, figure, proposition, etc.) systems. Finally, Chapter Five concludes the dissertation with discussion on its practical implications.

#### **Chapter Two**

# Wealth Effect and Tobin's q Effect of Monetary Policy

#### 2.1 Introduction

While most of the literature on monetary policy transmission examines monetary transmission mechanisms separately, in this chapter we investigate the interaction between the wealth effect and Tobin's q effect of monetary policy. We find that as the effect of monetary policy on asset prices depends on investment elasticity, the magnitude of the wealth effect on consumption is negatively related to Tobin's q effect on investment.

This finding has two implications as to monetary policy. First, although monetary policy can help stabilize the goods market in the short run, its transmission mechanism nevertheless tends to allow saving crowd-outs that have detrimental impacts on long-term growth. Second, although monetary policy can help stabilize the goods market, its mechanism nevertheless tends to destabilize the asset market.

In the remainder of this chapter, we first review the literature on monetary policy transmission mechanisms, then formally analyze the interaction between the wealth effect and Tobin's q effect, followed by discussion on its implications. A brief summary is provided in the end.

#### 2.2 Literature on monetary policy transmission mechanism

In general, the literature agrees that expansionary monetary policy will increase the aggregate demand; whereas monetary tightening will decrease it. However, the mechanism by which money affects the aggregate demand has not been completely clear, if not entirely in a "black box" (Bernanke and Gertler, 1995).

According to the literature, major monetary policy transmission mechanisms include the interest rate channel (or equivalently, Tobin's q channel), the credit channel, the exchange rate channel, and the wealth effect channel (Kuttner and Mosser, 2002; Loayza and Schmidt-Hebbel, 2002; Mishkin, 1995; Taylor, 1995). Other monetary policy transmission mechanisms include the monetarist asset price channel by which monetary policy affects relative asset prices (through portfolio balance effects) and hence aggregate demand (Meltzer, 1995), the "expectations" channel by which the central bank uses "open mouth operations" to influence the private sector's expectations (Loayza and Schmidt-Hebbel, 2002), the "cost" channel that provides the puzzle that monetary tightening is associated with higher prices (Rabanal, 2003).

#### 2.2.1 Interest rate channel or Tobin's q channel

Given inflation expectations, monetary expansion will reduce the real interest rate and hence the cost of capital, which will stimulate investments. This mechanism is often called the interest rate channel of monetary policy transmission (Mishkin 1995).

It should be noted that the interest rate channel and another so-called Keynesian asset price (or Tobin's q) channel (Mishkin, 1995) are essentially two sides of one coin.

According to the interest rate channel, monetary expansion stimulates investments by reducing the cost of capital; whereas according to Tobin's q channel, monetary expansion stimulates investments through increasing the demand for capital and hence the capital price q (Mishkin 1995). Nevertheless, the increase in q is in essence a (present-value) measure of the profits from the cost reductions due to the interest rate fall. Thus, the two channels are actually two different views of the same mechanism.

#### 2.2.2 Credit channel

The credit channel is another mechanism by which monetary policy affects investments (Bernanke and Gertler, 1995; Bernanke and Blinder, 1988, 1992). Three key words for this channel are "frictions", "external finance premium" and "financial accelerator".

Imperfect information creates "frictions" in the credit market where entrepreneurs obtain external finance. The frictions will result in "external finance premiums" required by creditors for compensating the risks of their loans to entrepreneurs.

Monetary expansion tends to not only increase investments but also raise asset prices. Asset price appreciation will increase entrepreneurs' net worth, which will strengthen their balance sheets in general, and increase the value of their loan collaterals in particular. These factors will reduce entrepreneurs' default risks, which will in turn reduce entrepreneurs' external finance premiums and hence their borrowing costs.

The reduction in borrowing costs will stimulate investments, which will further raise asset prices, and hence further reduce external finance premiums, and then further stimulate investments, and so on. The mechanism of such "financial accelerators" (Bernanke, Gertler and Gilchrist, 1999) is often called the credit channel of monetary policy.

#### 2.2.3 Exchange rate channel

The exchange rate channel is a monetary transmission mechanism available for open economies (Mundell, 1963; McCallum, 2001; Obstfeld and Rogoff, 1995). When monetary expansion reduces the domestic interest rate, capital mobility will force the exchange rate to depreciate in order to maintain cross-country interest rate parity. Depreciation in the exchange rate will effectively reduce domestic prices and hence increases net export as one component of the aggregate demand.

The effectiveness of the exchange rate channel depends on the openness of the economy—the channel will not matter too much for countries with relatively small international trade. Another factor is capital mobility. With immobile capital flows a fall in domestic interest rates (due to monetary expansion) will not put much pressure on the exchange rate. Finally, since exchange rate depreciation has beggar-thy-neighbor impacts, the effectiveness of the exchange rate channel depends largely on the reactions of trading partners. If trading partners react to monetary-policy-induced exchange rate depreciation directly via intervening in the foreign exchange market, or indirectly through reducing their interest rates, domestic monetary expansion, despite its effect on reducing domestic interest rates, will not cause exchange rate depreciation and hence cannot stimulate domestic demand through the exchange rate channel.

#### 2.2.4 Wealth effect channel

Monetary expansion tends to increase asset prices and hence the wealth of consumers, who will accordingly increase their consumption. This mechanism is often called the wealth effect channel of monetary policy. In a review of monetary policy transmission mechanisms, Mishkin (1995) describes the wealth effect channel as follows.

An alternative channel for monetary transmission through equity prices occurs through wealth effects on consumption. This channel has been strongly advocated by Franco Modigliani and his MPS model, a version of which is currently in use at the Board of Governors of the Federal Reserve System—see Modigliani (1971). In Modigliani's lifecycle model—explained very clearly in Modigliani (1971)—consumption spending is determined by the lifetime resources of consumers, which are made up of human capital, real capital and financial wealth. A major component of financial wealth is common stocks. When stock prices fall, the value of financial wealth decreases, thus decreasing the lifetime resources of consumption should fall.

Theoretically, based on the lifecycle consumption hypothesis, the wealth effect channel has often been taken for granted and interpreted casually (Modigliani, 1971; Mishkin, 1995). We have provided a clarification in Chapter One; our clarification confirms that, theoretically, monetary expansion tends to make current consumers as a whole wealthier and hence have a positive impact on consumption.<sup>9</sup>

<sup>&</sup>lt;sup>9</sup> Although there has been skepticism about the significance of the wealth effect (see Poterba (2000) for a review), the results of the most recent studies on the issue generally favor the existence of nontrivial wealth

Empirical studies on the wealth effect of monetary policy have produced mixed results: While large-scale econometric models tend to find a strong wealth effect of monetary policy (Modigliani, 1971; Ludvigson et al., 2002), those based on small structural VAR models find little supporting evidence of the wealth effect channel (Ludvigson et al., 2002).

In summary, monetary policy can affect the aggregate demand, which will interact with the aggregate supply and lead to the final impacts on output and inflation. The literature on monetary policy transmission specifically looks into how monetary policy affects the aggregate demand. Such research is important, and deserves more attention, especially in the current situation where monetary policy transmission mechanisms tend to be oversimplified in sophisticated monetary models. For example, in most of currently popular "New Keynesian" models (e.g. Woodford, 2003), the effect of monetary policy on the aggregate demand (captured by an Euler's equation) is via interest-rate-induced consumption intertemporal substitution effect, despite that the elasticity of consumption intertemporal substitution tends to be small (Elmendorf, 1996). Therefore, further research on monetary policy transmission mechanisms is important. The following analysis on the interplay between the wealth and Tobin's q effects is one step towards this direction.

effects (e.g. Case et al., 2001; Davis and Palumbo, 2001; Dynan and Maki, 2001; Funke, 1999; Green, 2002; Lettau and Ludvigson, 2004; Ludvigson and Steindel, 1999; Maki and Palumbo, 2001; Mehra, 2001).

#### 2.3 Interplay between the wealth and Tobin's q effects

Monetary expansion will reduce the interest rate and hence cause asset price appreciation, which can stimulate consumption through the wealth effect and investments through Tobin's q effect. These are two well-known channels of monetary policy transmission through asset prices (Mishkin, 1995).<sup>10</sup> However, being examined separately under partial-equilibrium perspectives, the interplay between them has been overlooked.<sup>11</sup> Intuitively, a large Tobin's q effect on investments will tend to dampen the effect of monetary expansion on asset price appreciation so as to result in a small wealth effect. Therefore, there should exist a negative correlation between the wealth effect and Tobin's q effect. We use a formal model to examine this conjecture in the following.

#### 2.3.1 The Model

To model the wealth effect, an overlapping generations (OLG) framework will be used to capture the features of finite consumption horizon. A parsimonious two-period OLG model will be used; yet the results can be generalized in multi-period models such as Blanchard's (1985) model.

In the model, the private sector is composed of (young and old) consumers, firms and entrepreneurs.

<sup>&</sup>lt;sup>10</sup> The wealth effect and Tobin's q effect are often generally called the "wealth effects" in the central bank's vocabulary.

<sup>&</sup>lt;sup>11</sup> We are aware of no explicit study on the interplay between the wealth effect and the Tobin's q effect. The two effects are often listed as two monetary policy transmission mechanisms without the interplay being discussed—see Kuttner and Mosser (2002); Loayza and Schmidt-Hebbel (2002); and Mishkin (1995) for surveys of the literature on monetary transmission mechanisms. In general equilibrium models where q plays a major role, the wealth effect is either explicitly dismissed for empirical irrelevancy (Bernanke and Gertler, 1999) or implicitly trivialized by the use of infinite-horizon frameworks.
Young consumers are the owners of human wealth who work and finance consumption by labor incomes, while old consumers are the owners of non-human wealth who retire and finance consumption by asset holdings. Certainly consumers in the real world can own both human and non-human wealth; thus the young and old consumers in this model should be viewed as theoretical abstraction of consumers' characteristics based on their human and non-human wealth respectively.

Firms engage in production that is a process of using capital and labor to produce consumption output; whereas entrepreneurs engage in investment activities that transform consumption goods into new capital.

The public sector is composed of fiscal and monetary authorities. The fiscal authority uses revenues from tax and/or government bond issuance to finance its expenditure. The monetary authority controls the money supply in the private sector through open market operations.

There are three kinds of assets: money, (government) bond and capital. Bond and capital are perfect substitutes. Besides being store of value, money also provides liquidity services, which is modeled by money in utility function.

All of the four markets—(consumption) goods, money, bond and capital—are efficient.

In the following we first model the behaviors of the private and public sectors and then discuss the equilibrium in each market.<sup>12</sup> Our goal is to see the effects of monetary

<sup>&</sup>lt;sup>12</sup>The Walras' Law allows us to discuss only the equilibria in the goods, money and capital market.

policy on asset prices as well as the consumption and investment components of the aggregate demand through the interaction between the wealth effect and the q effect.

### 2.3.1.1 Consumption

A consumer has a two-period life cycle: At the beginning of period t, the t-period young consumer is born; she supplies inelastically one unit of labor during period t and receives real wage income  $(w_t)$  at the end of the period; after paying real tax  $(T_t)$ , she consumes  $C_{1t}$  and saves in capital  $(K_{1t})$ , nominal government bond  $(D_{1t})$ , and/or money  $(M_{1t})$ ; she carries over her assets into and retires during the next period t+1; and at the end of which she finishes her life cycle by cashing in and consuming  $(C_{2t+1})$  the gross return to her savings. Assume no population growth and normalize the number of newborns as one.

The problem of the *t*-period young consumer is given by:

$$Max E_{t} (\log C_{1t} + \frac{1}{1+\theta} \log C_{2t+1} + \beta \log \frac{M_{1t}}{P_{t}}),$$

subject to:

$$C_{1t} + q_t K_{1t} + \frac{M_{1t}}{P_t} + \frac{D_{1t}}{P_t} = w_t - T_t ,$$
  

$$C_{2t+1} = E_t [r_{t+1} K_{1t} + q_{t+1} K_{1t} + \frac{M_{1t}}{P_{t+1}} + \frac{D_{1t}}{P_{t+1}} (1 + i_{t+1})],$$

where  $P_t$  is the consumption price at the end of period t;  $i_t$  is the rate of interest for government bond during period t;  $q_t$  is the real capital price at the end of period t; and  $r_t$ represents real income per unit of capital during period t. Consumers have intertemporally separable utility over consumption with time preference  $\theta$ ; and the real money balance provides liquidity services with  $\beta$  measuring liquidity preference. We assume log utility for analytical convenience.

First order conditions give the period-*t* young consumer's current consumption demand

$$C_{1t} = \xi(w_t - T_t)$$
 (6)

and money demand

$$\frac{M_{1t}}{P_t} = \beta \xi \left( \frac{i_{t+1}}{1+i_{t+1}} + \frac{E_t P_{t+1} - P_t}{P_{t+1}} \right)^{-1} (w_t - T_t),$$
(7)

where  $\xi = (1+\theta)[2+\theta+\beta(1+\theta)]^{-1}$ .

The period-t old consumer will finance her consumption via the gross returns to her assets:

$$C_{2t} = r_t K_{1t-1} + q_t K_{1t-1} + \frac{M_{1t-1}}{P_t} + (1+i_t) \frac{D_{1t-1}}{P_t} \,. \tag{8}$$

### 2.3.1.2 Production

In every period, identical, profit maximizing, and perfectly competitive firms hire capital and labor to produce consumption goods with the standard Cobb-Douglas technology. Given inelastic unit labor supply, the aggregate production function is

$$Y_t = F(K_t) = \lambda_t K_t^a , \qquad (9)$$

where  $K_t$ ,  $Y_t$ , and  $\lambda_t$  are, respectively, capital stock, output and productivity parameter. Profit maximization in a perfectly competitive environment implies that firms will pay factors by their marginal products:

$$r_t = F'(K_t), \tag{10}$$

$$w_{t} = F(K_{t}) - KF'(K_{t}).$$
(11)

## 2.3.1.3 Investment

In every period, identical entrepreneurs engage in investing activities that transform consumption goods into new capital. At the end of period t, an entrepreneur j chooses the amount of investment  $(I_t^j)$  to maximize expected utility:

$$Max E U(\Pi_t^j)$$

where  $\Pi_t^j = q_t I_t^j - c(I_t^j)$  is entrepreneur j's profit from investment—c(I) is the investment cost in terms of consumption. If any, entrepreneurs will hold investment profits earned at the end of period t in form of capital and sell them at the end of period t+1 for consumption. Aggregate investment is

$$I_t = \sum_j I_t^j \, .$$

For simplicity, assume zero depreciation in capital. Thus,

$$K_{t+1} = K_t + I_t \,. \tag{12}$$

### 2.3.1.4 Fiscal policy

To abstract fiscal implications, assume balanced fiscal policies in every period and hence constant government bond outstanding  $D = \overline{D}$ . Assume that D is a one-period coupon bond.<sup>13</sup> At the end of every period, fiscal authority pays off interest payments due and rolls over the principal at the interest rate determined by current asset markets.  $\overline{D}$  is either held by the private sector  $(D_i)$  or by monetary authority  $(D_i^g)$ :

$$D_t + D_t^g = \overline{D} \ . \tag{13}$$

Fiscal incomes include a real tax on young consumers  $(T_t)$  and the interest income  $(i_t D_t^g)$ turned in by monetary authority.<sup>14</sup> Fiscal outlays include interest payments for bond  $(i_t \overline{D})$  and government consumption that is assumed to be zero for simplicity. Thus, the balanced-budget policy implies  $P_t T_t + i_t D_t^g = i_t \overline{D}$ .

### 2.3.1.5 Monetary policy

Monetary authority determines the period t+1 money supply  $(M_{t+1})$  in the private sector at the end of period t (or equivalently, the beginning of period t+1) through open market operations:

$$D_{t+1}^{g} - D_{t}^{g} = M_{t+1} - M_{t}, (14)$$

<sup>&</sup>lt;sup>13</sup> If D is a long-term bond with fixed coupon rates, monetary policy will have a wealth effect through affecting the bond price. We abstract this feature since it is in essence not much different than the wealth effect mechanism through the capital price q.

<sup>&</sup>lt;sup>14</sup> The assumption of tax on the young consumer only is to avoid the complication of monetary policy affecting the present value of a consumer's lifetime tax liability, which is another kind of "wealth effect" yet irrelevant to the main issue here.

which implies that the RHS change in the money supply is balanced by the change in the monetary authority's bond holding on the left-hand side (LHS). According to equations (13) and (14),

$$M_{t+1} + D_{t+1} = M_t + D_t, (15)$$

which implies that the total value of M and D held by the private sector is not affected by open market operations. Therefore, monetary policy so modeled will affect the net wealth of the private sector only through influencing the capital price q.

## 2.3.1.6 Identities

The assets (capital, bond or money) held by the private sector in period t equals the corresponding assets acquired by the young consumer at the end of period t-1. Thus,  $K_t = K_{1t-1}; D_t = D_{1t-1}; \text{ and } M_t = M_{1t-1}.$ 

# 2.3.1.7 Goods market

The aggregate demand (for consumption goods) at the end of period t is equal to the young and old consumers' demands for consumption (equation (6) and (8) respectively) plus the costs of investments  $[c(I_t)]$  that will be specified later.

We assume sticky consumption price in the short run (normalized to one):  $E_t P_{t+1} = P_t = 1$ . We also do not model adjustments in output, which is determined by existing capital and labor [equation (9)]. The simplification in modeling the supply side (output and price) of the economy is for the purpose of clearly examining the core issue, i.e., the effects of monetary policy on the aggregate demand (AD) through interplays between the wealth effect and the *q* effect. A general view regarding monetary transmission is that monetary policy influences AD, which in turn affects output and/or price, depending on output potentials and price adjustment mechanisms. Thus, the ultimate effect of monetary policy depends not only on its impacts on AD but also on subsequent real and nominal repercussions. To analyze the effects of monetary policy on AD only, we abstract the complication that the affected AD could in turn influence output and/or price, which will feed back to the AD till the AD-AS balance is reached.<sup>15</sup>

According to equation (6),

$$dC_{1t} / dM_{t+1} = 0, (16)$$

which implies that money has no effect on young consumer's consumption demand. This is because in the model here monetary policy affects neither the young consumer's wealth, which is her current disposable income, nor her consumption propensity ( $\xi$ ) due to the log utility.<sup>16</sup>

According to equation (8),

$$dC_{2t} / dM_{t+1} = (dq_t / dM_{t+1})K_t,$$
(17)

<sup>&</sup>lt;sup>15</sup> See Bernanke, Gertler and Gilchrist (1999) for an example of studying monetary transmission mechanisms (the credit channel in the paper) in a business cycle context that includes further supply-side transmissions. Studying the wealth effect channel and its interaction with other channels in a business cycle context is an interesting topic for future research.

<sup>&</sup>lt;sup>16</sup> On the one hand, the log utility assumption implies the existence of the substitution effect, without which money will negatively affect  $\xi$ . On the other, the two-period simplification abstracts future labor incomes as human wealth, with which money will have a positive "non-human wealth effect". Intuitively, a monetary expansion tends to increase human wealth owners' consumption through the substitution effect but decrease it through the Hicksian (human) wealthiness effect—the income horizons of human wealth tend to be shorter than the consumption horizons of its owners. Since the balance of the two effects cannot be determined *a priori*, equation (16), which implies that the two effects are exactly counterbalanced, is a neutral standing—after all, the core issue here is the non-human wealth effect.

which captures the (non-human) wealth effect of monetary policy. Equation (17) together with (16) implies the following proposition:

**Proposition 2.1** The effect of monetary policy on aggregate consumption demand is positively correlated with its effect on capital price.<sup>17</sup>

## 2.3.1.8 Money market

Under the assumption of price rigidity, the (young consumer's) demand for money [equation (7)] becomes:

$$M_{t+1} = \left(\frac{1+i_{t+1}}{i_{t+1}}\right) \left[\beta \xi(w_t - T_t)\right],$$
(7)

which implies,

$$\frac{di_{t+1}}{dM_{t+1}} < 0 \,,$$

i.e., monetary expansion tends to reduce the short-term interest rate.

### 2.3.1.9 Capital market

The demand for capital comes from the young consumer's saving. Perfect substitution between capital and bond implies a no-arbitrage condition:

$$P_t q_t (1+i_{t+1}) = E_t [P_{t+1}(r_{t+1}+q_{t+1})],$$
(18)

<sup>&</sup>lt;sup>17</sup> According to the discussion in Chapter One, this proposition tends to hold even when the assumptions of two-period horizon and log utility are relaxed.

where the LHS and RHS represent the returns to  $P_tq_t$  (dollar) investment in bond and capital respectively. Equation (18) implies

$$q_{t} = \frac{1}{1+i_{t+1}} \left\{ r_{t+1} + \sum_{s=t+2}^{\infty} \frac{E_{t}r_{s}}{\prod_{\nu=t+2}^{s} \left[ (1+E_{t}i_{\nu})(1+E_{t}\pi_{\nu})^{-1} \right]} \right\},$$
(19)

where  $\pi_{v} (= P_{v} / P_{v-1} - 1)$  is the inflation rate in period v—note that  $\pi_{t+1}$  is equal to zero due to the price rigidity assumption. Equation (19) can be viewed as a capital demand function that relates capital price  $(q_{t})$  to capital stock  $(K_{t+1})$  implied by capital income  $(r_{t+1})$ .

The supply of capital comes from two sources: one is the supply of existing capital by the old consumer; and the other is the supply of new capital through entrepreneurs' investments. While the finite horizon makes the old consumer's capital supply perfectly inelastic, q will tend to influence entrepreneurs' investments.

In a simple case, assume constant marginal cost of investment (normalized to one):  $c(I_t) = I_t$ . Assume no uncertainty; thus the entrepreneurs' utility maximization problem is equivalent to maximizing investment profit  $\Pi_t^j = (q_t - 1)I_t^j$ , which implies that capital market is cleared at

$$q_t = 1. (20)$$

The capital supply function represented by equation (20) implies that investments are perfectly elastic to q.

Perfectly elastic investments are rare in reality; and many "impediments" could make investments less than perfectly elastic. For example, suppose the aggregate investment cost function is in the form of

$$c(I_{t}) = I_{t}(1 + \gamma I_{t}), \qquad (21)$$

where  $\gamma > 0$  implies convex adjustment costs.<sup>18</sup> Without uncertainty, the profitmaximizing investment behaviors imply that the capital market is cleared at

$$q_{t} = c'(I_{t}) = 1 + 2\gamma I_{t}.$$
(22)

With  $\gamma > 0$ , equation (22) implies that the aggregate investment is not perfectly elastic.

Less than perfectly elastic investments can also be due to entrepreneurs' risk aversion (Runge, 2000). Without loss of generality, suppose the risk is on the cost of capital:

$$c(I_t^{j}) = I_t^{j}(1 + z_t),$$
(23)

where  $z_t \sim N(0, \sigma^2)$  is a normally distributed random variable. Suppose entrepreneurs are risk-averse with utility function:

$$U(\Pi) = -e^{-\varphi\Pi},\tag{24}$$

which implies constant absolute risk aversion. According to the investment cost function [equation (23)] and the utility function [equation (24)], entrepreneur *j* chooses investment  $(I_i^j)$  to maximize expected utility

$$E_{t}U(\Pi_{t}^{j}) = -\int e^{-\varphi \Pi_{t}^{j}} f(\Pi_{t}^{j}) d\Pi_{t}^{j} = -e^{-\varphi [(q_{t}-1)I_{t}^{j} - \varphi I_{t}^{j^{2}} \sigma^{2}/2]},$$

<sup>&</sup>lt;sup>18</sup> Following the literature of investment adjustment costs, we use a representative agent (entrepreneur) framework rather than the *n*-entrepreneur framework specified above. See Abel and Eberly (1997) for the investment cost functional form in equation (21).

the solution to which gives j's investment function:  $q_t = 1 + \varphi \sigma^2 I_t^{j}$ .<sup>19</sup> Thus, the aggregate investment function will be

$$q_t = 1 + \frac{\varphi \sigma^2}{n} I_t , \qquad (25)$$

which implies that under risky investments ( $\sigma > 0$ ) and risk-averse entrepreneurs ( $\varphi > 0$ ), the aggregate investment is less than perfectly elastic. The riskier the investments are; or the more risk-averse the entrepreneurs are, the less elastic the aggregate investment will be.

Suppose the number of entrepreneurs is sufficiently large  $(n \gg 0)$ ; then, according to the law of large numbers, the aggregate investment cost function would be

$$c(I_{t}) = \sum_{j} c(I_{t}^{j}) = I_{t}^{j} (n + \sum_{j=1}^{n} z_{t}^{j}) = I_{t},$$
(26)

which implies constant marginal cost for the aggregate investment. Despite constant marginal investment cost in aggregate, increasing (marginal) risk premia demanded by risk-averse entrepreneurs make the aggregate investment less than perfectly elastic.

The capital supply functions (20), (22) and (25) can be generalized into

$$q_t = 1 + \eta I_t, \tag{27}$$

where the coefficient  $\eta$  is *negatively* correlated with investment elasticity.

With capital demand and supply specified, we will apply comparative statics to the simultaneous system comprised by equations (7'), (10), (12), (19) and (27) to show the effect of a monetary shock  $(dM_{t+1})$  on the market-clearing capital price  $(q_t^e)$ .

<sup>&</sup>lt;sup>19</sup> For utility functional form and the derivation of the investment function, see Varian (1992).

First, we need to specify the effects of  $dM_{t+1}$  on the expectation terms ( $E_t i_v$ ,  $E_t \pi_v$ and  $E_t r_s$ ) in equation (19), which is essentially the present-value (PV) rule for asset valuation. Note that since we focus on the monetary effect on AD and hence do not model the further interactions between AD and AS, the standard rational expectation paradigm is not applicable here. Thus, for analytical convenience, we make the following assumptions.

First, we assume that capital is valued by the young consumer according to a practically simplified version of the PV rule (as compared to the complete version described by equation (19)):

$$q_{t} = \frac{1}{1 + i_{t+1}} \left( r_{t+1} + \frac{\bar{r}}{\bar{i} - \bar{\pi}} \right), \tag{19'}$$

where  $\bar{i}$ ,  $\bar{\pi}$  and  $\bar{r}$  are, respectively, the expected long-term interest rate, expected future inflation rate, and expected future (average) capital income.<sup>20</sup> Second, according to the expectation theory of interest-rate term structures, we assume  $d\bar{i} = \pi di_{t+1}$ , where  $\tau$  (>0) captures the effect of monetary policy on the slope of yield curve. Third, as expectations on future inflations depend mainly on monetary authorities' inflation targets and the creditability of the targets, we assume that  $dM_{t+1}$  affects neither the targets nor their creditability; thus,  $d\bar{\pi} / dM_{t+1} = 0$ . Fourth, we assume  $dM_{t+1}$  is not expected to affect the

<sup>&</sup>lt;sup>20</sup> In equation (19), let  $Er_{t+\nu} = \overline{r}$ ,  $Ei_{t+\nu} = \overline{i}$ ,  $E\pi_{t+\nu} = \overline{\pi}$  for  $\nu \ge 2$ ; then we can obtain

 $q_{t} = \frac{1}{1 + i_{t+1}} \left( r_{t+1} + \frac{\overline{r} + \overline{r} \overline{\pi}}{\overline{i} - \overline{\pi}} \right), \text{ which gives equation (19')—the higher order term } \overline{r} \overline{\pi} \text{ can be omitted.}$ 

relationship between the long-term average earning ( $\bar{r}$ ) and the current earning ( $r_{t+1}$ ); thus,  $d\bar{r} / \bar{r} = dr_{t+1} / r_{t+1}$ .

Note that the above assumptions are for analytical convenience in examining the monetary effect on AD. They are not essential for the results presented later—after all, while the main point here (as will be shown later) is regarding the monetary effect on capital supply, the above assumptions matter only for the monetary effect on capital demand.

With the expectations being pinned down, comparative statics analyses can be conducted to show the effects of  $dM_{t+1}$ . The results are:

(a)  $dq_t^e / dM_{t+1} \begin{cases} = 0, if \ \eta = 0 \\ > 0, if \ \eta > 0 \end{cases}$ ;

(b) 
$$\frac{\partial (dI_t / dM_{t+1})}{\partial \eta} < 0$$
; and

(c) 
$$\frac{\partial (dq_t^e / dM_{t+1})}{\partial \eta} > 0$$
,

which imply the following propositions.

**Proposition 2.2** With perfectly elastic investments, monetary policy has no influence over the equilibrium q [result (a)]; thus the wealth effect of monetary policy is zero (Proposition 2.1).

**Proposition 2.3** With less than perfectly elastic investments, monetary policy positively affects the equilibrium q [result (a)]. The more elastic the investments are, the stronger the q effect on investments [result (b)]; yet the weaker the monetary effect on the equilibrium q [result (c)]; and hence the weaker the wealth effect (Proposition 2.1).

### **2.3.2 Implications**

The above formal analysis provides two insights. One is that the magnitude of the wealth effect of monetary policy is negatively related to that of Tobin's q effect. The other is that, since there are many elements that can reduce the investment elasticity,<sup>21</sup> monetary policy tends to influence asset price. These insights have two implications: One is related to saving crowd-out due to the wealth effect; and the other is about the tradeoff between inflation stability and financial stability.

### 2.3.2.1 Wealth effect and saving crowd-out

From the point of view of short-term stability in goods market, it may not matter whether monetary policy achieves its targeted level of aggregate demand through the wealth effect or Tobin's q effect. However, the interplay between the two effects has nontrivial implications to long-term growth. This is because, while investments help accumulate national wealth, consumption decumulates it. When the investment elasticity is small, monetary policy (as one of the stabilization policies) can allow savings to be crowded out

<sup>&</sup>lt;sup>21</sup> Besides adjustment costs and risk aversion as two fundamental investment impediments, many other factors (such as uncertainty or institutional imperfections) can reduce the q-elasticity of investments. In the case of monetary contraction, investment irreversibility can be a "disinvestment impediments".

by the wealth effect, which will have detrimental impacts on long-term growth. In the following we first use the above model to examine the saving crowd-out and then discuss its implications.

In the above model, assuming that monetary authority credibly targets zero-inflation by keeping the aggregate supply and demand in balance, i.e.,

$$F(K_{t}) = C_{1t} + C_{2t} + c(I_{t}).$$
<sup>(28)</sup>

The gross saving (GS) of the economy comes from the young consumer's saving:  $GS_t \equiv w_t - T_t - C_{1t}$ ; whereas the net saving (NS) also depends on the old consumer's dissaving. In aggregate, what is not consumed must be saved; thus,  $NS_t = F(K_t) - C_{1t} - C_{2t}$ .

Consider a decrease in the young consumer's consumption propensity ( $d\xi < 0$ ), which according to equation (6) tends to generate extra GS by the amount of  $dGS = -(w_t - T_t)d\xi$ . However, according to the simultaneous system composed of equations (6), (8), (27) and (28), the increase in the NS would be  $dNS = -[(w_t - T_t)/(1 + \eta K)]d\xi$ , which could fall short of the extra GS if investments are less than perfectly elastic (i.e.,  $\eta > 0$ ). Also, it is not difficult to see that

$$\frac{\partial (dNS/d\xi)}{\partial \eta} < 0$$

which implies that, the greater the  $\eta$  is (i.e., the small the investment elasticity is), the smaller impact of the increase in saving propensity ( $d\xi < 0$ ) will be on the net saving

(NS); or in other words, the more of the extra gross saving (GS) will be diverted back to consumption through the wealth effect.

During the end of 1990s, the already low U.S. personal saving rate plummeted further despite baby boomers at their prime saving ages. Policymakers attributed the fall to the wealth effect of the stock market booms (Greenspan 2000a, b). What factors drive the booms and how much the wealth effect contributes to the low saving are empirical issues in the end. Yet, the saving crowd-out mechanism (due to the wealth effect) implies that, if it is costly in the margin to transform baby boomers' large savings (not mentioning abundant foreign savings provided by the favorable world capital market) into new capital as "concrete" wealth, paper wealth will be generated through asset price appreciation. The resulting low saving will then not be a result of spendthrifts but because of prosperity from paper wealth appreciation. Yet, such paper wealth prosperity will not only hinder the accumulation of concrete wealth but also could easily vanish when retired baby boomers start dissaving, or when the U.S. assets lose their charms for some reason.

### **2.3.2.2** Inflation stability vs. financial stability

When investments are not perfectly elastic, monetary policy tends to affect asset prices and hence cause the wealth effect in the goods market. Yet the influence of monetary policy on asset prices will also have an impact on the financial market.

The major task of monetary policy is to maintain the stability of goods market. As asset market fluctuations become more frequent and severe, increasing attention has recently been paid to the role of monetary policy in helping maintain financial stability. A common view is that inflation-targeting monetary policy will help enhance financial stability through reducing uncertainties (BIS, 2003). However, here we point out that inflation-targeting monetary policy can in fact be a source of financial instability.

The rationale is as follows. When shocks hit the goods market, monetary policy can keep the price in goods market stable by influencing the aggregate demand. However, with its potential impacts on asset prices, monetary policy intended to stabilize goods market can effectively cause instability in the asset market.

Take Japan's "bubble" boom-bust experience in the 1980s as an example. In hindsight, Bank of Japan (BOJ) has often been criticized for allowing the 1980s bubbles to develop in an easy-money environment (Okina and Shiratsuka, 2003). However, in light of the low inflations during that period, BOJ's policy was nothing but proper. In other words, had BOJ tightened money for fear of a booming bubble during the late 1980s, a recession could have been resulted. However, as BOJ chose to focus on the goods market, the "bubbles" in its financial markets were let unguarded, which eventually burst in the early 1990s and caused a great deal of economic damages.

Another more recent example is the U.S. stock markets during the late 1990s. From the mid of 1999 to the mid of 2000, the Federal Reserve (Fed) initiated a "preemptive" tightening with six consecutive cuts in the Federal Funds rate. With little sign of existing and potential inflation at that time, the tightening was understandably interpreted as the Fed's act to curb the booming stock markets that had been increasing for half a decade by nearly three folds. While the tightening successfully held the momentum of the stock markets, it also pushed the economy into a recession. This incident serves as another example of the tradeoff between goods market stability and financial market stability that conventional monetary policy has to face.

## 2.4 Summary

We use a formal model to show that, as investments can hardly be perfectly elastic due to many investment impediments, monetary policy is likely to influence asset price; hence there exists a negative relationship between the wealth effect and Tobin's q effect of monetary policy. We point out that the interplay between the two effects imposes two limitations on the role of monetary policy as a stabilization tool: One is saving crowd-out that is detrimental to long-term growth, and the other is the tradeoff between the stability in the goods market and that in the asset market.

These limitations are fundamental for monetary policy that directly or indirectly uses the interest rate as its policy instrument. Stabilization policies that use other instruments could avoid the problem. For example, fiscal policy (e.g. investment subsidies) can be used to influence the aggregate demand without affecting asset price. Thus, while monetary policy (thanks to its flexibility) should still take a major role in maintaining the stability of the economy, its limitations need to be recognized and addressed by complement policies, especially in situations where shocks are caused by long-term, foreseeable factors, such as changes in demography or productivity.

# **Chapter Three**

# **Open Capital Account: Concrete Wealth or Paper Wealth**

In the last chapter we have shown that with small investment elasticity due to investment impediments, savings could be crowded out by the wealth effect of a fall in the interest rate induced by monetary expansion. A similar process tends to occur after a small developing country opens its capital account, because the liberalization will allow the lowered world interest rate to reduce domestic interest rates. We examine this process and its implications in this chapter.

## 3.1. Overview

### 3.1.1 Issue

One major motivation for developing countries to open capital account is to let free capital inflows facilitate domestic capital formation (Calvo et al. 1996).<sup>22</sup> However, empirical evidence shows that unfettered capital inflows are often used by developing country recipients to finance excessive consumption; in other words, foreign savings tend to crowd out domestic savings. Such phenomena have happened not only to low-saving Latin American countries that have been notorious for misusing capital inflows (Calvo et

<sup>&</sup>lt;sup>22</sup>Other benefits of open capital account include the access to international financial markets for risksharing (Obstfeld, 1995, 1998), greater flexibility in balance of payments for smoothing external and domestic shocks (Cooper, 1999), disciplining domestic government behaviors (Cooper, 1999; Dornbush, 1998), and strengthening the domestic financial system (Dornbush, 1998).

al., 1996; Ffrench-Davis and Reisen, 1998), but also to high-saving East Asian countries that put relatively more foreign capitals into investments (Galvin et al., 1997; Reinhart and Talvi, 1998).<sup>23</sup> Notably, consumption booms (financed by capital inflows) have occurred in several countries (e.g. Argentina, Israel, and Brazil) during their exchange-rate-based stabilization programs intended to curb inflations (Montiel, 2000; Nazmi, 1997; Reinhart and Vegh, 1995).

### 3.1.2 The literature

Foreign-capital-financed consumption booms are often explained as caused by (moralhazard-induced) credit over-expansion (Reisen, 1998; McKinnon and Pill, 1998). A mechanism suggested by McKinnon and Pill (1998) is as follows. Developing countries tend to have unhealthy banks whose first priority is their continuing survival. These banks tend to have moral hazard in lending as much as possible and engaging in highly risky projects for high profits. The access to the world capital market under open capital account will provide them with funds for unduly aggressive credit expansion. The nonbank sector may misinterpret such credit overexpansion as a signal of good economic prospects, and hence increase both investments and consumption.

<sup>&</sup>lt;sup>23</sup> Empirical evidence on whether open capital account stimulates growth is inconclusive. Rodrik (1998) finds no strong evidence of a positive relationship between open capital account and growth. Using a more refined measure of capital liberalization, Quinn (1997) finds a significant positive effect of capital account liberalization on the growth of per capita income. Edwards (2001) uses different indexes to measure capital mobility; and his empirical results show that a positive relationship between capital account openness and productivity performance only manifests itself after the country in question has reached a certain degree of development. Arteta et al. (2001) find a positive relationship between capital account liberalization and growth, which nevertheless depends on the sample period, the measure of the openness of capital account, and the estimating method used.

Other theories have been suggested to explain stylized (foreign-capital-financed) consumption booms that have been associated with exchange-rate-based stabilization programs.<sup>24</sup>

Rodriguez (1982) argues that low real interest rates due to sticky inflation expectation can lead to consumption boom. According to his argument, when an exchange-rate-based stabilization program reduces the rate of devaluation, nominal interest rates will fall due to the interest rate parity enforced by free capital movement. Since inflation is not expected to fall instantaneously in the same degree as the devaluation rate, the real interest rate will fall and hence stimulate consumption.

Helpman and Razin (1987) suggest that wealth effect caused by inconsistent government policy can cause consumption booms. As inflation is stabilized by an exchange-rate-based program, the growth of nominal money demand will be reduced. Under this situation, if government inconsistently keeps the growth of money supply unadjusted, there will be excessive supply of money, which will force it to use government bonds to absorb the excessive money supply. As government bonds are net wealth in the finite-horizon framework used by Helpman and Razin (1987), the rise in the private holding of government debts will increase consumer's wealth so as to lead to an increase in current consumption. Since government has to increase taxes to finance its debt repayments sooner or later, the high consumption for current consumers will nevertheless be at the cost of low consumption for future consumers.

<sup>&</sup>lt;sup>24</sup> Many countries (e.g. Argentina during 1978 to 1981; Israel during 1978 to 1981; Brazil during 1981-1984) troubled by persistent high inflation have tried to use exchange rate as a nominal anchor to curb it. Though the managed exchange rate stability has helped achieve the goal of bringing down inflation, it is nevertheless accompanied by initial consumption boom followed by serious contraction and recession.

Another theory suggests that the incredibility of exchange-rate-stabilization programs could be the reason of consumption booms. Dornbush (1985) argues that consumption booms tend to happen if exchange-rate-based stabilization programs are not expected to last. According to his argument, when consumers expect high future importing prices due to exchange rate devaluation when the program is abandoned, they tend to shift their purchases of imported consumer durables to present so as to cause current consumption booms. Calvo (1986) discusses a similar mechanism of incredibility causing consumption booms. In his model, nominal interest rates negatively affect consumption. Since incredible exchange rate stabilization programs will temporarily lower nominal interest rates through temporarily lowering the rate of devaluation, they effectively reduce the effective price of today's consumption relative to future consumption. Therefore, as consumption is shifted to the present, consumption booms occur. Using a sticky-price model with traded and non-traded goods, Calvo and Vegh (1993) also find that the "temporariness" of exchange-rate-based programs tends to cause consumption booms. Reinhart and Vegh (1995) provide empirical evidence to support the relevancy of the temporariness hypothesis.

In summary, in explaining foreign-capital-financed consumption booms, the existing literature focuses on how "institutional imperfections" in developing countries can trigger excessive consumption demands, which, with the aid of free capital inflows, will easily turn into consumption booms.

### 3.1.3 Conjecture

As opposed to the literature viewing consumption booms as a macroeconomic "side effect" of open capital account (Corbo and Hernandez, 1996; Fischer, 1998), we conjecture that they could be a fundamental outcome of open capital account due to a wealth effect mechanism.

The essence of the mechanism is as follows. Open capital account tends to attract foreign capitals into developing countries for high-yielding opportunities. Such capital inflows can increase the stock of productive capital (K), its price (Tobin's q), or most likely both. While the increase in K is the result of capital inflows being "properly" channeled to investments, the q appreciation tends to stimulate consumption demand (through wealth effect), and hence essentially channel capital inflows to consumption.

A key yet underappreciated point is that the magnitude of q appreciation and the corresponding consumption booms are negatively correlated to the q-elasticity of investments. Thus, a conjecture is that when investment "impediments" make it difficult to turn foreign savings into investments (as "concrete" wealth), "paper" wealth will nevertheless be created (through asset price appreciation) and result in foreign savings being used to finance consumption booms.<sup>25</sup> Based on this conjecture we consider two cases of foreign-capital-financed consumption booms.

The investment impediment considered in the first case is entrepreneurs' risk aversion. When capital account is liberalized, foreign capital will enter domestic markets

<sup>&</sup>lt;sup>25</sup> A variety of investment impediments can reduce investment elasticity. While the most common one in the literature is convex adjustment costs, we consider another two: One is entrepreneurs' risk aversion; and the other is the uncertainty and irreversibility of investments.

chasing high-yielding opportunities, which tends to drive up domestic asset prices. If domestic investments are elastic, the asset price appreciation will induce new capital formation, which will dampen the asset price appreciation because a larger capital stock tends to have a lower (unit) capital income. Then the capital account liberalization will achieve its purpose of facilitating domestic capital formation. However, if the risk aversion of domestic entrepreneurs leads to low investment elasticity, the impacts of post-liberalization capital inflows will be mainly on asset prices rather than capital stock; the resulting asset price appreciation will tend to drive consumption booms.

An interesting issue is the nature of such consumption booms. On the one hand, they are neither bubbles driven by speculations nor overvaluation driven by credit overexpansion; rather, they are merely asset price revaluation under the lowered world interest rate. On the other hand, they imply that a scenario with smaller capital stocks and hence lower national income for the entire post-liberalization transition path could nevertheless have higher consumption for some time, which is a puzzling result warrant an explanation.

For the second case, we consider investment uncertainty and irreversibility as another investment impediment. Suppose a country is in a situation with promising yet uncertain future productivities. This could occur when the country is liberalizing its capital account. On the one hand, the liberalization could have a positive impact on the country's future productivities in many ways such as providing funds for importing advanced technologies from abroad. On the other, the liberalization could also fail for many reasons and result in economic crises that tend to have a negative effect on future productivities.<sup>26</sup> Even when the liberalization is expected to succeed and lead to higher future productivities; and entrepreneurs are risk-neutral, the existence of uncertainty could still be an impediment that holds off investments, because it may in the interest of entrepreneur to wait till the outcome of the liberalization is more certain so as to avoid being stuck in irreversible investments when the outcome turns out to be unfavorable. If entrepreneurs adopt such a "wait-and-see" strategy, the expected future productivity hikes will cause asset price appreciation and consumption booms.

This story can also apply to other structural reforms such as exchange-rate-based stabilization programs. Thus, the consumption boom mechanism just discussed can be an alternative explanation of the stylized consumption booms associated with the programs.

In the remainder of the chapter, we will examine the two cases in the next section and section 3.3 respectively, and provide a summary at the end.

### **3.2** Capital account liberalization and consumption booms

For a developing country with domestic interest rates higher than the world interest rate, capital account liberalization tends to increase foreign demands on domestic assets, which can facilitate productive capital formation as "concrete" wealth. However, if investments are not perfectly elastic, increases in asset demands will also result in "paper" wealth formation through asset price appreciation, which tends to enrich current

<sup>&</sup>lt;sup>26</sup> For example, the liberalization could be premature in the sense that the country's institutions and economic conditions are not yet ready for such a major structural reform; or the government may not be sophisticated enough to handle the short-term macroeconomic consequences of the liberalization such as a surge of current account deficits and rapid real exchange appreciation.

consumers (as a whole) and thus encourage consumption.<sup>27</sup> In summary, the phenomenon of foreign savings being used to finance consumption booms can simply be a result of their not being effectively channeled to investments. We formally examine this conjecture in the following.

### 3.2.1 The Model

### 3.2.1.1. Consumption

The modeling of consumption follows the finite-horizon model in Blanchard (1985). Each of many identical consumers throughout her lifetime faces a constant probability of death  $\pi$ .<sup>28</sup> At any instant of continuous time, a cohort with size  $\pi$  is born. Thus, the population size ( $\int_{-\pi}^{\pi} e^{-\pi(t-s)} ds = 1$ ) is constant (at unity) over time.

In every period, a living consumer supplies one unit of labor inelastically and maximizes (log) utility from consumption:

$$Max \int_{t}^{\infty} \log c(s, v) e^{-\pi(v-t)} dv,$$

where c(s, t) denotes the period-*t* consumption of a consumer born in period *s*—apply this (s, t) notation rule to other variables as well. Note that for simplicity we assume zero time preference. The consumer faces a lifetime budget constraint

<sup>&</sup>lt;sup>27</sup> The rationale of paper wealth making consumers as a whole wealthier has been discussed in Chapter One.

 $<sup>^{28} \</sup>pi$  is used to denote inflation rate in the last chapter. Since Chapter Two, Three (the current chapter), and Four are self-contained essays on various issues, the notation system in each of them is defined independently. The indexing system (for equations, propositions, figures, etc.) in each of them is also self-contained. Since we make few cross-chapter references, the chance of confusion is slim.

$$\int_{l}^{\infty} [c(s, v)] e^{-\int_{l}^{v} [r(u)+\pi] du} dv = a(s, v) + \int_{l}^{\infty} w(s, v) e^{-\int_{l}^{v} [r(u)+\pi] du} dv,$$

and transversality condition

$$\lim_{v\to\infty}e^{-\int_{t}^{v}[r(u)+\pi]du}a(s, v)=0,$$

where variables *a*, *w*, and *r* are, respectively, asset, wage, and the rate of return to asset (i.e., the interest rate). Note that the "effective" rate of return to asset is  $r + \pi$  because the consumer can use her asset as a stake to "bet" on her own death (Blanchard, 1985; p.226).

The solution to the consumer's maximizing problem gives

$$c(s,t) = \pi[a(s,t) + h(s,t)]$$
(1)

where

$$h(s, t) = \int_{t}^{\infty} w(s, v) e^{-\int_{t}^{v} r(u) + \pi du} dv$$

represents the consumer's human wealth.

Aggregating equation (1) gives the aggregate consumption function:

$$C_t = \pi (A_t + H_t) \tag{2}$$

where variables C, H and A are the aggregate consumption, human wealth and nonhuman wealth respectively; the dynamics of which are as follows:<sup>29</sup>

$$\dot{C}_i = r_i C_i - \pi^2 A_i \tag{3}$$

<sup>29</sup> The aggregate counterpart of a variable x(s, t) is given by  $X(t) = \int_{-\infty}^{t} x(s, t) p e^{p(s-t)} ds$ ; see Blanchard (1985, pp.228-229) for detail.

$$\dot{H}_t = (r_t + \pi)H_t - W_t \tag{4}$$

$$\dot{A}_t = r_t A_t + W_t - C_t \tag{5}$$

# 3.2.1.2 Production

In every period, identical, profit-maximizing, and perfectly competitive firms hire capital and labor to produce consumption goods with the standard Cobb-Douglas technology. With inelastic unit labor supply, the aggregate production function is

$$Y_t = F(K_t) = \lambda K_t^{\alpha}, \tag{6}$$

where variables K and Y are capital stock and output respectively; parameters  $\lambda$  and  $\alpha$  are respectively technical coefficient and capital share. Profit maximization under perfect competition makes firms pay factors by their marginal products:

$$R_t = F'(K_t), \tag{7}$$

$$W_t = F(K_t) - KF'(K_t), \qquad (8)$$

where  $R_t$  and  $W_t$  are income per unit of capital and labor respectively.

## 3.2.1.3 Investment

A variety of investment impediments can make investments less than perfectly elastic: e.g., investment adjustment costs, risk-averse entrepreneurs, investment uncertainty and irreversibility—to name a few fundamental ones; let alone those caused by institutional imperfections. Considering the (arguable) lack of entrepreneurial tradition in developing countries, we in the following model risk-averse investing behaviors as one example of investment impediments; whereas, as shown in Chapter Two, investment adjustment costs will give the same results. We will look into the case of uncertainty and irreversibility in the next section.

In every period, identical entrepreneurs engage in investing activities that transform consumption goods into new capital.<sup>30</sup> Individual entrepreneur *j* chooses the amount of investment  $(I_i^j)$  to maximize expected utility: <sup>31</sup>

$$Max EU(\Pi_{t}^{j})$$

where  $\Pi_t = q_t I_t - c(I_t)$  represents investment profits—c(I) is the investment cost (function) in terms of consumption.

Investments are risky with a stochastic cost function:

$$c(I_t^j) = I_t^j (1 + z_t^j),$$
(9)

where  $z_t \sim N(0, \sigma^2)$  is a normally distributed random variable. Entrepreneurs are riskaverse with utility function:

$$U(\Pi) = -e^{-\varphi\Pi},\tag{10}$$

where parameter  $\varphi$  measures (constant) absolute risk aversion.

According to equations (9) and (10), entrepreneur j's maximizing problem becomes

<sup>&</sup>lt;sup>30</sup> To clearly examine investment behaviors, we model the production and investment decision makings separately—see Abel (2003) for a similar framework.

<sup>&</sup>lt;sup>31</sup> The utility rather than profit maximization is for the purpose of modeling risk-averse investment behaviors; otherwise, utility and profit maximizations are equivalent

$$M_{I_{t}^{j}} E_{t} U(\Pi_{t}^{j}) = -\int e^{-\varphi \Pi_{t}^{j}} f(\Pi_{t}^{j}) d\Pi_{t}^{j} = -e^{-\varphi [(q_{t}-1)I_{t}^{j} - \varphi I_{t}^{j^{2}} \sigma^{2}/2]},$$

the solution to which gives individual investment function:  $q_t = 1 + \varphi \sigma^2 I_t^{j}$ .<sup>32</sup> Then, the aggregate investment function (with *n* identical entrepreneurs) would be

$$q_t = 1 + \eta I_t , \tag{11}$$

where coefficient  $\eta = \varphi \sigma^2 / n$  is *negatively* related to the *q*-elasticity of investments ("investment elasticity" in short); and  $I_t = nI_t^j$  represents the aggregate investment.

Equation (11) implies that under risky investments ( $\sigma > 0$ ) and risk-averse entrepreneurs ( $\varphi > 0$ ), the aggregate investment is less than perfectly elastic; and the elasticity is negatively correlated with the riskiness of investments or the risk aversion of entrepreneurs.

Given a large number of entrepreneurs (n >> 0) and according to the law of large numbers, the aggregate investment cost function is

$$c(I_{t}) = \sum_{j} c(I_{t}^{j}) = I_{t}^{j} (n + \sum_{j=1}^{n} z_{t}^{j}) = nI_{t}^{j} = I_{t},$$
(12)

which implies constant marginal cost of investment in aggregate. Note that the less-thanperfectly-elastic aggregate investment notwithstanding constant aggregate marginal investment cost is because of increasing marginal risk premia demanded by risk-averse entrepreneurs.

<sup>&</sup>lt;sup>32</sup> For utility functional form and the derivation of the investment function, see Varian (1992).

### 3.2.1.4 The close economy

In autarky, the perishable consumption goods is either consumed or invested; thus the goods market equilibrium condition implies

$$Y_t = C_t + I_t \tag{13}$$

Capital is the only store of value; thus the aggregate non-human wealth is equal to the value of the capital stock

$$A_t = q_t K_t \tag{14}$$

The return to capital is equal to capital income plus capital gain; thus,

$$r_{t} = \frac{F'(K_{t+1}) + E_{t}\dot{q}_{t}}{q_{t}}$$
(15)

For simplicity, assume zero depreciation in capital; thus,

$$\dot{K}_t = I_t \tag{16}$$

The dynamics of the close economy is described by the simultaneous system composed of equations (2), (4), (6), (8), and (11)-(16) with endogenous A, H, K (as stock variables), Y, W, C, I, (as flow variables), plus r and q (as prices).

The autarky economy reaches its steady state when the endogenous variables become constant over time. The steady-state values (denoted with asterisks and the subscript "a" representing an autarky steady state) of several key variables are:<sup>33</sup>  $r_a^* = \pi \alpha^{1/2}$ ,  $q_a^* = 1$ ,

$$C_a^* = \lambda^{-(\alpha-1)^{-1}} (\pi \alpha^{-1/2})^{\alpha(\alpha-1)^{-1}}$$
, and  $K_a^* = (\pi \lambda^{-1} \alpha^{-1/2})^{(\alpha-1)^{-1}}$ .

<sup>&</sup>lt;sup>33</sup> The steady-state investment is equal to zero; thus, according to equation (11),  $q^* = 1$ ; according to equation (6) and (15),  $r^* = \alpha \lambda K^{*\alpha-1}$ ; and according to equations (6) and (13),  $C^* = Y^* = \lambda K^{*\alpha}$ . Since  $\dot{A}^* = 0$ , according to equations (5) and (8),  $A^* = \alpha \lambda K^{*\alpha} / \alpha \lambda K^{*\alpha-1} = K^*$ . According to equation (3),

### 3.2.1.5 The open economy

With open capital account, the interest rate is exogenously determined by the world interest rate  $(r^w)$ . The goods market equilibrium condition for the open economy is

$$Y_t = C_t + I_t + TB_t, \tag{17}$$

where  $TB_t$  denotes trade balance. The non-human wealth of the open economy is

$$A_t = q_t K_t + B_t, \tag{18}$$

where B denotes net foreign asset; the dynamics of which, i.e., the current account (CA) dynamics, is characterized by

$$CA_t = \dot{B}_t = TB_t + r^w B_t.$$
<sup>(19)</sup>

The other aspects of the open economy are the same as those in the close economy.

The dynamics of the open economy can be described by the simultaneous system composed of equations (2), (4), (6), (8), (11), (12), and (15)-(19) with endogenous A, H, K, B (as stock variables), Y, W, C, I, TB (as flow variables), and q (as price). The steady state of the open economy is given by  $r_o^* = r^w$ ,  $q_o^* = 1$ ,  $K_o^* = (r^w \alpha^{-1} \lambda^{-1})^{(\alpha-1)^{-1}}$ ,  $Y_o^* = \lambda K_o^{*\alpha}$ ,  $C_o^* = (1-\alpha)\pi^2(\pi^2 - r^{w2})^{-1}Y_o^*$ ,  $TB_o^* = (\alpha\pi^2 - r^{w2})(\pi^2 - r^{w2})^{-1}Y_o^*$ , and  $B^* = -TB_o^*/r^w$ .<sup>34</sup>

 $r^*C^* = \pi^2 A^*$ ; the substitution of  $C^*$ ,  $r^*$  and  $A^*$  in which will give  $K^* = \lambda^{-(\alpha-1)^{-1}} (\pi \alpha^{-1/2})^{(\alpha-1)^{-1}}$ . Then  $C^*$  and  $r^*$  can be solved accordingly.

<sup>&</sup>lt;sup>34</sup> The open-economy interest rate is equal to the world interest rate; thus,  $r_o^* = r^w$ . The steady-state investment is equal to zero; thus, according to equation (11),  $q_o^* = 1$ . Then, according to equations (6) and (15),  $K_o^* = (r^w \alpha^{-1} \lambda^{-1})^{(\alpha-1)^{-1}}$ . Then according to equation (6),  $Y_o^* = \lambda K_o^{*\alpha}$ . According to equation (17),  $TB_o^* = Y_o^* - C_o^*$ ; and according to equations (3), (18), and (19),  $r^w C_o^* = \pi^2 (K_o^* - TB_o^*/r^w)$ ; then

If the world interest rate is equal to the autarky steady-state interest rate  $(r^w = r_a^* = \pi \alpha^{1/2})$ , then,  $K_o^* = (\pi \lambda^{-1} \alpha^{-1/2})^{(\alpha-1)^{-1}} = K_a^*$ ,  $Y_o^* = Y_a^{*\alpha}$ ,  $C_o^* = Y_o^*$ ,  $TB_o^* = 0$ , and  $B_o^* = 0$ . Thus, the close and open economies will have identical steady states. Note that their transition paths will nevertheless be different.

If  $r^w < r_a^*$ , then  $K_o^* > K_a^*$ ,  $Y_o^* > Y_a^{*\alpha}$ ,  $C_o^* > C_a^{*\alpha}$ ,  $TB_o^* > 0$ , and  $B_o^* < 0$ . State plainly, if the world interest rate is lower than the autarky one, the open economy will have higher steady-state capital stock, output, and consumption (as compared to the autarky); the steady-state net foreign asset will be negative, which corresponds to a positive steadystate trade balance since the steady-state current account balance is zero.

We do not consider the case of  $r_a^* < r^w$ , which is not well defined for a small economy because, if a "small" economy's autarky steady-state interest rate is lower than the world interest rate, it will keep saving to such an extent that it becomes a large economy that will affect the world interest rate.

For simplicity, unless specified otherwise, we assume that the world interest rate is equal to the autarky steady-state interest rate. Since the autarky and open-economy steady states are identical, this assumption allows us to denote the steady-state values with "asterisk" only without the need to use subscriptions to distinguish between autarky and open-economy.

solving these two equations simultaneously will give  $C_o^* = (1 - \alpha)\pi^2 (\pi^2 - r^{w^2})^{-1} Y_o^*$  and  $TB_o^* = (\alpha \pi^2 - r^{w^2})(\pi^2 - r^{w^2})^{-1} Y_o^*$ . Thus, according to equation (19),  $B^* = -TB_o^*/r^w$ .

Figure 1 portrays a standard dynamics of consumption (C) and net foreign asset (B) after a developing economy opens its capital account. Part (a) shows a monotonic transition of consumption (to its steady state); during which the country's net saving (measured by the difference between the gross national production (GNP) and consumption) keeps positive. According to part (b), net foreign asset is falling initially, and will eventually become rising and converging to its steady-state.



Figure 1

Intuitively, when a developing country opens its account, foreign funds will help domestic capital formation by allowing investments to exceed domestic savings. This will cause current account deficits and hence a fall in the net foreign asset. As the domestic capital stock converges to its steady state, domestic investments will be diminishing and eventually become less than domestic savings. Then the current account becomes surplus; and the net foreign asset starts rising and converging to its steady state.

Such an open-economy growth process is commonly viewed as superior to its autarky counterpart, because open capital account allows developing countries to use foreign

capital to increase investments and hence have higher incomes than autarky. Yet, a problem is that foreign capital is often used by developing countries to finance consumption, which is generally taken as a side effect of open capital account due to institutional imperfections. In the following we provide a new perspective to the issue by examining the implications of investment elasticity to the consumption dynamics of a small open developing economy.

## 3.2.2 Investment elasticity and post-liberalization consumption dynamics

Intuitively, countries with high investment elasticity, *ceteris paribus*, will have high postliberalization investments and hence high gross national products ( $\text{GNP} = Y + r^*B$ ); thus they should accordingly enjoy high post-liberalization consumption. However, we will show that countries with low post-liberalization GNP over time (due to low investment elasticity) can nevertheless have high post-liberalization consumption for some time.

The post-liberalization dynamics of an economy (opening up at time t = 0 with  $K_0 < K^*$  and  $B_0 = 0$ ) can be characterized by the following differential equations:<sup>35</sup>

$$\dot{C}_{t} = r^{*}C_{t} - \pi^{2}(B_{t} + q_{t}K_{t}), \qquad (20)$$

$$\dot{B}_{t} = r^{*}B_{t} + \lambda K_{t}^{\alpha} - C_{t} - \eta^{-1}(q_{t} - 1), \qquad (21)$$

<sup>&</sup>lt;sup>35</sup> Substituting the A<sub>t</sub> in equation (18) into equation (3) gives equation (20). Substituting the TB<sub>t</sub> in equation (17) into equation (19), we obtain  $\dot{B}_t = r^* B_t + Y_t - C_t - I_t$ . Then substituting the Y<sub>t</sub> and I<sub>t</sub> in equation (6) and (11) respectively into this dynamic equation, we obtain equation (21). Substituting the I<sub>t</sub> in equation (11) into equation (19) will give equation (22). Finally, according to equation (6),  $F'(K_{t+1}) = \alpha \lambda K_{t+1}^{\alpha-1} = \alpha \lambda (K_t + I_t)^{\alpha-1}$ . Then substituting the I<sub>t</sub> in equation we obtain  $F'(K_{t+1}) = \alpha \lambda [K_t + (q_t - 1)\eta^{-1}]^{\alpha-1}$ , which can be substituted into equation (15) to solve for equation (23).

$$\dot{K}_{t} = \eta^{-1}(q_{t} - 1),$$
 (22)

$$\dot{q}_{t} = r^{*} q_{t} - \alpha \lambda [K_{t} + (q_{t} - 1)\eta^{-1}]^{\alpha - 1}.$$
(23)

Linearizing the differential equation system (20)-(23) around steady state gives

$$\begin{bmatrix} \dot{C} \\ \dot{B} \\ \dot{K} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} r^* - \pi^2 & -\pi^2 & -\pi^2 K^* \\ -1 & r & r & -\eta^{-1} \\ 0 & 0 & \eta & \eta^{-1} \\ 0 & 0 & n & m \end{bmatrix} \begin{bmatrix} C - C^* \\ B - B^* \\ K - K^* \\ q - 1 \end{bmatrix}$$
(24)

where  $m = r^* + \alpha (1 - \alpha) \lambda (K^*)^{\alpha - 2} \eta^{-1}$  and  $n = \alpha (1 - \alpha) \lambda (K^*)^{\alpha - 2}$ . The solution to which gives the growth paths of *C*, *B*, *K* and *q*:

$$C_{t} = \left[B_{0} - B^{*} + (1 + \beta)(K_{0} - K^{*})\right] \pi e^{(r^{*} - \pi)t} + (K_{0} - K^{*})\beta(\varepsilon - r^{*})e^{\varepsilon t} + C^{*}$$
(25)

$$B_{t} = \left[B_{0} - B^{*} + (1+\beta)(K_{0} - K^{*})\right]e^{(r^{*} - \pi)t} - (K_{0} - K^{*})(1+\beta)e^{\delta t}$$
(26)

$$K_{t} = (K_{0} - K^{*})e^{at} + K^{*}$$
(27)

$$q_t = (K_0 - K^*)\varepsilon\eta e^{\varepsilon t} + 1$$
(28)

where  $\varepsilon = m/2 - (m^2/4 + n\eta^{-1})^{1/2} < 0$  and  $\beta = \pi^2 K^* \varepsilon \eta (\pi - \varepsilon + r^*)^{-1} (\pi + \varepsilon - r^*)^{-1}$ .<sup>36</sup>

Equations (25)-(28) describe the post-liberalization dynamics of several key variables; based on which comparative statics can be used to illustrate the impacts of  $\eta$  (i.e., investment elasticity) on each variable. However, due to mathematical complications, the signs of some comparative statics are hard to be determined analytically. Thus, we choose to use numerical simulations to compare post-liberalization scenarios under different  $\eta$ .

<sup>&</sup>lt;sup>36</sup> See Mathematical Appendix for detail.
The parameters in the simulation model composed of equations (25)-(28) are set as:  $\alpha = 1/3$  (as usual);  $\pi = 1/60$  (i.e., the average life expectancy is 60); and  $\lambda = \sqrt{3}/60$ (for normalizing the steady-state capital stock to unity, i.e.,  $K^* = 1$ ). The initial capital stock and net foreign asset are set as  $K_0 = 0.9$  and  $B_0 = 0$  respectively; and the world interest rate is set as the autarky steady-state interest rate  $r^w = r_a^* = \sqrt{3}/180$ . Results based on this setting are qualitatively robust for other parameter settings.

Based on this simulation model, we first compare post-liberalization GNP paths under different  $\eta$ . For easy visualization, Figure 2 presents the GNP path under  $\eta = 10, 20, 40, 60$  only; yet the pattern indicated by which is general for  $\eta > 0$ .



Figure 2

The GNP growth pattern in Figure 2 indicates the following relationship between investment elasticity and post-liberalization GNP.

**Remark 1** Ceteris paribus, high  $\eta$  (i.e. low investment elasticity) will lead to permanent low post-liberalization GNP.

This result is not surprising and can be explained as follows. Suppose the total and domestic-owned capital stock are  $\overline{K}$  and  $K_d$  respectively.<sup>37</sup> Then the national income will be  $\text{GNP} = \overline{r}K_d + \overline{w} = \alpha\lambda\overline{K}^{\alpha-1}K_d + (1-\alpha)\lambda\overline{K}^{\alpha}$ . It is not difficult to verify that, given  $K_d < \overline{K}$ ,  $\partial \text{GNP}/\partial \overline{K} > 0$ , which implies that, given domestic-owned capital stock  $(K_d)$ , the lower the total capital stock  $(\overline{K})$  is, the smaller the GNP will be. Therefore, given the initial capital stock, the smaller the (post-liberalization) investments are, the smaller the GNP will be. As  $\eta$  negatively affects post-liberalization investments, high  $\eta$  will lead to low post-liberalization GNP over time.

While the impact of  $\eta$  on post-liberalization GNP is as expected, that on consumption is puzzling.

According to Figure 3 that shows the simulated impacts of  $\eta$  on post-liberalization consumption, the relationship between  $\eta$  and consumption is as follows.

**Remark 2** Ceteris paribus, high  $\eta$  (i.e. low investment elasticity) will lead to temporary high consumption for some time immediately after the liberalization; and the effect of  $\eta$  on consumption will eventually become negative in the long run.

<sup>&</sup>lt;sup>37</sup> For an open economy with negative net foreign asset, part of its capital stock is essentially owned by foreigners.





Based on the aggregate consumption function represented by equation (2), Remark 2 is not difficult to explain mathematically. Although high  $\eta$  tends to result in low postliberalization capital stock, hence low labor income, and hence low human wealth (*H*), it also tends to cause high capital price and hence high non-human wealth (*A*). For some time after the liberalization, the latter effect tends to outweigh the former; thus the total wealth (*A*+*H*) will be positively affected by  $\eta$ . Then, according to equation (2), so will be the aggregate consumption. However, as  $\eta$  has a negative impact on *A*+*H* in the long run, the impact of  $\eta$  on future consumption will eventually become negative.

A puzzling issue is how to reconcile Remark 1 and 2, which (taken together) imply that, given the constant (world) interest rate, an economy with low GNP in the entire post-liberalization period can nevertheless have high consumption for some time. More fundamentally, given the interest rate, how can permanently low GNP be consistent with large total wealth that is supposed to embody the total (present) value of GNP over time? The key to this puzzle is that the total wealth of *current* consumers does not include future labor incomes beyond their (finite) horizons; in other words, the total wealth A+Hdoes not embody the entire GNP over time.<sup>38</sup> Thus, while high  $\eta$  makes *society* as a whole worse off by lowering post-liberalization GNP over time, it can nevertheless make *current consumers* as a whole better off by increasing their wealth A+H. We explain this point in detail in the following.

Ceteris paribus, low capital stock will result in low labor income ( $W_t$ ) but high unit capital income ( $R_t$ ). Since the positive effect of capital stock on  $W_t$  tends to dominate its negative effect on  $R_t$ , the net effect of low post-liberalization capital stock over time (due to high  $\eta$ ) will be low GNP over time. However, a key point is that, while the gains from high  $R_t$  (over time) are completely reaped by current consumers through q appreciation,<sup>39</sup> the losses from low  $W_t$  will be mostly burdened by future *unborn* consumers. Therefore, current consumers (as a whole) can nevertheless be better off from high  $\eta$  (i.e. low investment elasticity), even though the total benefit for the society as a whole is lowered.

State plainly, the ineffectiveness of capital account liberalization in accomplishing its presupposed mission of increasing domestic investments could nevertheless benefit current consumers as a whole through paper wealth creation. For the particular parameters used in the simulation, Figure 3 indicates that the high-consumption era lasts for around 60 periods, i.e., the mean lifespan of a generation.

<sup>&</sup>lt;sup>38</sup> In Blanchard's (1985) framework adopted here, finite horizon is modeled as a constant probability  $(\pi)$  of death. Thus, while an infinite-living outlier individual is theoretically possible, current consumers as a whole is expected to have a horizon equal to the mean life expectancy (i.e.,  $1/\pi$ ). The implication of this finite-horizon feature on human wealth is mathematically captured by a higher discount rate (i.e.,  $r+\pi$ ) for labor income.

<sup>&</sup>lt;sup>39</sup> Whatever  $R_t$  is, the rate of return to future asset ownerships will be fixed at the world interest rate.

## 3.2.3 Investment elasticity and post-liberalization consumption boom

In light of the initial negative relationship between investment elasticity and postliberalization consumption, we conjecture that, the lower the investment elasticity is, the greater the consumption booms will be caused by capital account liberalization.

For example, suppose a close economy is initially in its steady state (path a in Figure 4). At time T it liberalizes its capital account to the world capital market with a lower interest rate; then the post-liberalization growth trajectory will be path b or c, depending on the magnitude of investment elasticity. For high investment elasticity, the liberalization will cause a consumption boom from x to y; yet for low investment



elasticity, the boom is greater (from x to z).<sup>40</sup> Since investment elasticity has no effect on the autarky steady-state consumption, it is clear that in this case higher investment elasticity will lead to greater post-liberalization consumption booms.

<sup>&</sup>lt;sup>40</sup> The parameters used in the simulation behind Figure 4 are  $\alpha = 1/3$ ,  $\pi = 1/60$ ,  $\lambda = \sqrt{3}/60$ . The liberalization occurs at T = 10. The world interest rate is 90 percent of the autarky interest rate. Paths b and c correspond to the cases of  $\eta$  being 500 and 1 respectively.

However, since investment elasticity has an impact on the consumption transition path in autarky, the comparison between post-liberalization consumption and the autarky one will not be as straightforward if the liberalization occurs when the autarky economy is on its transition path to steady state; and we consider this situation in the following.

Suppose an autarky economy opens at t = 0 with  $K_0 < K^*$  and  $B_0 = 0$ , foreign capitals will flow in; the amount of which can be measured by current account (CA) deficits. Part of the capital inflows will be used to financed extra investments  $\Delta I_0 = I_0^o - I_0^a$ —where  $I^o$  and  $I^a$  represent the open and autarky aggregate investments respectively—and the rest will essentially be used for extra consumption. Thus, the proportion of CA used to finance consumption can be measured by

$$\rho = 1 - \Delta I_0 / (-CA_0), \tag{29}$$

which can be taken as an indicator of the extent of post-liberalization consumption booms.

A comparison between  $I_0^o$  and  $I_0^a$  is necessary to reveal  $\rho$ . Unfortunately, while  $I^o$  (and CA as well) can be solved from equations (25)-(28), the analytical solution to  $I^a$  is hard to obtain, because the autarky interest rate is endogenous.<sup>41</sup>

<sup>&</sup>lt;sup>41</sup> While it is possible to analytically solve the growth dynamics of a closed economy in a finite-horizon, *putty-putty* model (Blanchard, 1985), we are aware of no successful attempts in doing so in finite-horizon models with nontrivial Tobin' q. The difficulty lies in the fact that a first-order differential equation system similar to equation (24) is unavailable to the close-economy case, in which a second order differential equation will always appear somewhere. Without a counterpart of equation (24), the linearization technique is not applicable. Although it may be possible to conduct simulations based on a nonlinear differential equation system for the close economy, it would be very complicated because of the problem of multiple equilibria.

To provide some basic insights as to the impact of investment elasticity on  $\rho$ , we choose to conduct autarky-open comparison in a tractable model designed to approximate the rational-expectation model presented above.

In the "proxy" model, we assume aggregate consumption function as

$$C_t = \pi (A_t + \overline{H}_t) \tag{2'}$$

where 
$$\overline{H}_{t} = (r^{*} + \pi)^{-1} W_{t}$$
, as compared to  $H_{t} = \int_{t}^{\infty} W(t) e^{-\int_{t}^{v} (r(u) + \pi) du} dv$  in the rational-

expectation aggregate consumption function represented by equation (2). The difference between  $\overline{H}_t$  and  $H_t$  is that, while  $H_t$  implies that consumers have perfect foresights over future wage incomes and interest rates,  $\overline{H}_t$  implies that, in calculating human wealth, consumers use the current wage income and steady-state interest rate to approximate future wage incomes and interest rates respectively. With equation (2) replaced by (2'), the proxy model is tractable for both open and autarky scenarios, and hence allows us to conduct the autarky-open comparison.

In the proxy model, the autarky economy will have the same steady state as the rational-expectation model; yet the dynamics may be different. On the one hand, by using the current wage (as a proxy for increasing wage incomes over time) to calculate human wealth, the proxy model tends to "underestimate" the autarky consumption  $C_0^a$  (relative to the rational-expectation consumption as a benchmark). On the other hand, by using the steady-state interest rate  $r^*$  as a proxy for the autarky decreasing interest rates over time, the proxy model tends to "inflate" human wealth and hence "overestimate"  $C_0^a$ . If the

balance of these two opposite effects is neutral, the proxy model provides a good approximation of  $C_0^a$  in the RE model. Unfortunately, the state of the balance is unclear. However, while the underestimation problem also happens to the post-liberalization consumption  $C_0^o$ , the overestimation problem will not, because the open-economy interest rate over time is indeed  $r^*$ . Thus, the proxy model would in general have larger overestimation (or smaller underestimation) on  $C_0^a$  than  $C_0^o$ , which, in light of the fact that  $C_0^o > C_0^a$ , implies an underestimation of  $C_0^o - C_0^a$ .

In sum, relative to the rational-expectation model, the proxy model tends to underestimate the open-autarky consumption difference and hence the severity of postliberalization consumption booms. Thus, if we find a positive impact of  $\eta$  on  $\rho$  in the latter, we expect the impact will be stronger in the former.

In the proxy model, the autarky (aggregate) investment, the post-liberalization investment, and the (post-liberalization) current account can be solved analytically.<sup>42</sup> The results are

$$I_{t}^{a} = \omega(K_{0} - K^{*})e^{\omega t}, \qquad (30)$$

$$I_{\iota}^{o} = \varepsilon (K_{0} - K^{*}) e^{\varepsilon t}, \qquad (31)$$

and

$$CA_{t} = \widetilde{\beta}(K_{0} - K^{*})[(r^{*} - \pi)e^{(r^{*} - \pi)t} - \varepsilon e^{\varepsilon t}], \qquad (32)$$

<sup>&</sup>lt;sup>42</sup> See Mathematics Appendix (A.2) for detail

where 
$$\omega = \left[\Psi r^* - \pi + (\alpha - 1)\pi\eta\Psi\lambda(K^*)^{\alpha}\right](1 + \pi\eta K^*)^{-2} < 0 \text{ and } \Psi = 1 - (1 + \alpha^{1/2})^{-1}(1 - \alpha);$$
  
 $\widetilde{\beta} = 1 - \left[\pi r^*(1 - \alpha)(r^* + \pi)^{-1} + \varepsilon\eta\pi K^*\right](r^* - \varepsilon - \pi)^{-1}$ 

According to equation (30), the investment would be  $I_0^a = \omega(K_0 - K^*)$  if the economy stays autarky at time t = 0. Yet, if it chooses to open up, equations (31) and (32) indicate that the investment would be  $I_0^o = \varepsilon(K_0 - K^*)$ ; and the current account be  $CA_0 = \widetilde{\beta}(K_0 - K^*)(r^* - \pi - \varepsilon)$ . Substituting these results into equation (29) will give  $\rho = 1 + (\varepsilon - \omega)\widetilde{\beta}^{-1}(r^* - \pi - \varepsilon)^{-1}$ , which measures the extent of post-liberalization consumption boom.

Since mathematical complexity prevents us from determining the sign of  $d\rho/d\eta$ analytically, we choose to illustrate the impact of  $\eta$  on  $\rho$  through numerical simulations.



Figure 5

The result of numerical simulations based on the parameters setting used above (i.e.  $\alpha = 1/3$ ;  $\pi = 1/60$ ; and  $\lambda = \sqrt{3}/60$ ) is illustrated in Figure 5, which indicates a positive relationship between  $\rho$  and  $\eta$ . It should be noted that the positive relationship is qualitatively robust for other parameter settings. The positive relationship between  $\rho$  and  $\eta$  implies the following result.

**Remark 3** The higher the  $\eta$  (or the lower the investment elasticity) is, the greater the proportion of initial post-liberalization capital inflows will be used for financing consumption booms.

Therefore, if developing countries are ineffective in transforming foreign capital into domestic investments, capital account liberalization is likely to result in foreign-capital-financed consumption booms.

#### **3.2.4** Interest rate variation and consumption boom (bust)

We have shown in the above that consumption booms can happen during capital account liberalization as a process from autarky to open capital account. Under open capital account, consumption booms can also be triggered by variations in the world interest rate. In the following we will simulate the impact of changes in the world interest rate on the consumption dynamics of a small developing country.<sup>43</sup>

<sup>&</sup>lt;sup>43</sup> The impact of a change in the world interest rate under open capital account is different from that of capital account liberalization that turns an autarky economy into an open one, even though they both

We will use equations (25)-(28) again as the simulation model. The parameter setting and initial conditions are the same as above:  $\alpha = 1/3$ ;  $\pi = 1/60$ ;  $\lambda = \sqrt{3}/60$ ;  $K_0 = 0.9$ ; and  $B_0 = 0$ . Since the investment elasticity is not an interested parameter here, we set it specifically as  $\eta = 1$ . We will consider the impacts of both permanent and temporary interest rate shocks. The results of the simulation are shown in Figure 6





In Figure 6, path ABC is the growth path of consumption under the mean interest rate  $r^* = \sqrt{3}/180$ . Suppose at time t = 50, a permanent interest-rate shock reduces  $r^*$  by two percent; then the initial impact would be a consumption boom from B to D; and then consumption converges to a higher steady-state through the path DEF. Intuitively, the

involve a reduction in the domestic interest rate. This is because, while the former is about different scenarios under an identical (open) economic structure, the latter is about different scenarios under two different economic structures—while the interest rate is endogenous in autarky; it is exogenously determined by the world interest rate under open capital account.

positive impact of the interest rate fall on consumption is attributable to both paper and concrete wealth effects. The paper wealth effect is via capital price appreciation (induced by the lowered interest rate); whereas the concrete wealth effect is due to increases in labor incomes thanks to more rapid investments.

In another scenario, suppose the shock is temporary; and at time t = 150, the interest rate reverts to its mean. The initial impact will be a consumption bust from E to G; then consumption will converge to the original steady state through path GH.

As opposed to the initial interest rate fall causing consumption boom from B to D, it is straightforward that the latter mean-reverting interest rate hike will cause the consumption bust from E to G. Yet, the magnitude of the bust is smaller than that of the boom. This is because the initial interest rate fall has helped increase capital stock to a level higher than it would have been without the fall.

In summary, we have the following remark.

**Remark 4** An unanticipated permanent fall in the world interest rate will cause an initial consumption boom and permanent high consumption path (ABDEF). If the interest rate fall is temporary, a consumption bust will happen when the interest rate reverts to mean. Yet, this temporary interest rate fall will lead to a permanently higher consumption path (ABDEGH) than the path under the mean interest rate (ABC).

While the permanent interest rate fall is unambiguously welfare-improving, the welfare implication of the temporary fall is unclear, depending on the balance between the cost of consumption variation and the benefit of permanently higher consumption.

### 3.3 Productivity shock, uncertainty, and consumption booms

One insight provided by the literature on investment uncertainty and irreversibility is that, uncertain yet profitable investment opportunities can nevertheless remain unexploited even when markets are efficient and entrepreneurs are risk neutral (Dixit and Pindyck, 1994). This is because "wait-and-see" can be a better strategy when the cost of waiting (e.g. unearned profits) is smaller than that of being stuck with underperformed yet irreversible investments.

Therefore, similar to entrepreneurs' risk aversion, uncertainty and irreversibility together (as two common features of investments) can be another "investment impediment" responsible for foreign-capital-financed consumption booms. For example, suppose a small open economy is undergoing structural reforms (such as an exchange-rate-based stabilization program) that are expected to increase future productivities. With easy access to low-cost foreign funds, high future productivities imply profitable investment opportunities. However, these opportunities may not be taken by entrepreneurs who prefer to postpone investment decisions till the outcomes of the reforms become more certain. If so, the high future productivity will drive asset price booms that could trigger consumption booms.

Based on an illustrative and discrete version of the model presented above, we examine this conjecture in the following.

# 3.3.1 Productivity and capital income

Suppose at the beginning of period t = 0, the period-zero productivity is known as  $\lambda_0$ ; yet a structural reform makes future productivities uncertain as follows:

$$\lambda_t = \lambda_0 + z \quad (t \ge 1), \tag{33}$$

where  $z = \begin{cases} z^d < 0: \Pr(z = z^d) = p \\ z^m > 0: \Pr(z = z^m) = 1 - p \end{cases}$ .

According to equation (33), the economic future from period one onward can be either a "miracle" or a "debacle", with  $z^m$  and  $z^d$  measuring the miracle and debacle productivity shocks respectively. Despite uncertain, the future is promising, with a higher expected future productivity than  $\lambda_0$ ; i.e.,  $\bar{z} \equiv E(z) = pz^d + (1-p)z^m > 0$ .

The uncertainty is temporary—at the end of period zero, the nature of productivity shock ( $z^m$  or  $z^d$ ) is determined and reveals itself.

Let  $R_0$ ,  $R^m$ , and  $R^d$  denote period-zero, "miracle" future, and "debacle" future income per unit of capital respectively. Then, given capital stock  $\overline{K}$  and according to equation (7),  $R_0 = \alpha \lambda_0 \overline{K}^{\alpha-1}$ ,  $R^m = \alpha (\lambda_0 + z^m) \overline{K}^{\alpha-1}$  and  $R_i^d = \alpha (\lambda_0 + z^d) \overline{K}^{\alpha-1}$ , which give

$$R^m = (1 + \frac{z^m}{\lambda_0})R_0, \qquad (34)$$

$$R^{d} = (1 + \frac{z^{d}}{\lambda_{0}})R_{0}.$$
(35)

#### 3.3.2 Investment

Investments are of "putty-clay" nature. That is, one unit of consumption good can produce one unit of capital; yet, capital is irreversible.

With open capital account, the cost of fund is equal to the world interest rate  $(r^*)$ . Then, the expected profit per unit of investment at the beginning of period zero would be

$$E\Pi^{invest} = \frac{(R_0 - r^*)}{1 + r^*} + \frac{1}{1 + r^*} \left[ \frac{(1 - p)(R^m - r^*) + p(R^d - r^*)}{r^*} \right],$$
(36)

with the first and second terms on the right hand side representing the present values of period-zero and expected future profits respectively.

Entrepreneurs can choose not to invest at the beginning of period zero, but to postpone investment decisions till the end of it when  $R^m$  or  $R^d$  is observable. No investment at the beginning of period zero means zero profit during which. If the future turns out to be a debacle at the end of period zero, entrepreneurs will not invest, because the debacle capital income is less that the cost of capital (i.e.,  $R^d < r^*$ ).<sup>44</sup> If the future is a miracle, entrepreneurs will invest. Since the probability of the miracle future is 1- p, the present value of the expected profits from this wait-and-see strategy would be

and

<sup>&</sup>lt;sup>44</sup> We ignore trivial equilibria where  $R^d > r^*$ .

$$E\Pi^{wait} = \frac{1}{1+r^*} \frac{(1-p)(R^m - r^*)}{r^*}$$
(37)

#### 3.3.3 Effect of the productivity shock on investment

Risk-neutral and profit-maximizing entrepreneurs will keep investing as long as  $E\Pi^{invest} \ge 0$  and  $E\Pi^{invest} > E\Pi^{wait}$ . Therefore, one necessary condition for equilibrium is<sup>45</sup>

$$E\Pi^{invest} = E\Pi^{wait},$$

which, according to (36) and (37), gives the equilibrium capital income in period zero,

$$R_0^e = r^* + \frac{p(r^* - R^d)}{r^*}.$$
(38)

According to equations (35) and (38), we obtain

$$R_0^e = r^* \left( \frac{r^* + p}{r^* + p(1 + z^d / \lambda_0)} \right).$$
(39)

Thus, according to equations (7) and (39), the equilibrium period-zero capital stock would be

$$K_{0}^{e} = (R_{0}^{e} / \alpha \lambda_{0})^{\frac{1}{\alpha - 1}} = \left(\frac{r^{*2} + r^{*}p}{\alpha \lambda_{0} r^{*} + \alpha \lambda_{0} p(1 + z^{d} / \lambda_{0})}\right)^{\frac{1}{\alpha - 1}},$$
(40)

which implies  $\partial K_0^e / \partial p < 0$  and  $\partial K_0^e / \partial z^d > 0$ . Thus,

<sup>&</sup>lt;sup>45</sup> We ignore the trivial equilibria where  $E\Pi^{invest} < E\Pi^{wait}$ .

**Remark 5** The higher the debacle probability (or the lower the debacle productivity) is, the lower the current investments will be.

Intuitively, the probability and severity of the future debacle are two "impediment" elements that keep entrepreneurs from taking profitable investment opportunities immediately.

Given p and  $z^d$ , equation (40) implies  $\partial K_0^e / \partial \overline{z} = 0$ . Thus,

**Remark 6** The expected future productivity (per se) has no influence over current investments.

This "bad-news" (or "irrelevant-good-news") principle (Bernanke, 1983) is due to the fact that the wait-and-see strategy will not cost entrepreneurs the opportunity to invest in the miracle future.

#### 3.3.4 Effect of the productivity shock on capital price

The price of irreversible capital is determined by the present value of expected future incomes per unit of capital. Thus, the equilibrium period-zero capital price is given by:

$$q_{0}^{e} = \frac{R_{0}^{e}}{1+r^{*}} + \frac{p}{1+r^{*}} \frac{R^{d}}{r^{*}} + \frac{1-p}{1+r^{*}} \frac{R^{m}}{r^{*}}$$

$$= \frac{r^{*} + p}{r^{*} + p(1+z^{d}/\lambda_{0})} \left[ 1 + \frac{\bar{z}}{\lambda_{0}(1+r^{*})} \right]$$
(41)

which implies that, if p > 0,  $q_0^e > 1$ .<sup>46</sup> Put plainly,

**Remark 7** Uncertainty over future productivities tends to result in profitable investment opportunities being unexploited in equilibrium, even though markets are efficient; and entrepreneurs are risk-neutral.

This "inefficient" outcome is not the result of any market failure. Positive profits (in equilibrium) are necessary to compensate expected losses from being stuck with debacle investments.

Equation (41) implies that  $\partial q_0^e / \partial \overline{z} > 0$ . Thus,

**Remark 8** The greater the expected future productivity is, the higher the current equilibrium capital price will be.

As (high) future productivity has no influence over capital formation (Remark 6), its impact will be on asset price (appreciation).

Equation (41) implies that, given  $\bar{z}$ ,  $\partial q_0^e / \partial p > 0$  and  $\partial q_0^e / \partial z^d < 0$ . Thus,

<sup>&</sup>lt;sup>46</sup> Without uncertainty (i.e., p = 0),  $q_0^e$  will be equal to one, but not  $1 + \overline{z}\lambda_0^{-1}(1 + r^*)^{-1}$  implied by equation (41). This is because, without uncertainty, firms will be active in investments; hence competitive market force will make equilibrium achieved only at  $q_0^e = 1$ .

**Remark 9** *Given expected future productivity, the higher the debacle probability (or the lower the debacle productivity) is, the higher the capital price will be.* 

Intuitively, high debacle probability (or low debacle productivity) makes it more costly to be stuck in the debacle future; thus, high asset prices (i.e., high investment profits) are necessary to induce entrepreneurs' investments.

### 3.3.5 The effect of the productivity shock on consumption

Denote the aggregate non-human wealth at the beginning of period zero as  $K_{-1}$ . Then according to equation (2), the equilibrium aggregate consumption would be

$$C_0^e = \pi (q_0^e K_{-1} + H_0^e), \tag{42}$$

where

$$H_0^e = (1 - \alpha)F(K_0^e)(r^* + \pi)^{-1} + \overline{H}, \qquad (43)$$

in which the first term on the right-hand side represents the period-zero labor income; and  $\overline{H}$  is equal to the present value of expected labor incomes from period one onward, which depend on the expected future productivity:  $\partial \overline{H} / \partial \overline{z} > 0$ .

According to equation (43), that  $\partial K_0^e / \partial \overline{z} = 0$  and  $\partial \overline{H} / \partial \overline{z} > 0$  imply  $\partial H_0^e / \partial \overline{z} > 0$ . According to equation (42), that  $\partial q_0^e / \partial \overline{z} > 0$ ,  $\partial K_{-1} / \partial \overline{z} = 0$  and  $\partial H_0^e / \partial \overline{z} > 0$  imply  $\partial C_0^e / \partial \overline{z} > 0$ . Note that the positive effect of future productivity ( $\overline{z}$ ) on consumption  $(C_0^e)$  includes both human and non-human wealth effect. The impact of p on  $C_0^e$  is two folded: That  $\partial q_0^e / \partial p > 0$  implies a positive p-effect on  $C_0^e$  through non-human wealth; whereas  $\partial K_0^e / \partial p < 0$  implies a negative p-effect on  $C_0^e$  through human wealth  $H_0^e$ . An analytical determination of the balance of the two effects is intractable in this simple model here. However, since  $\partial \overline{H} / \partial p = 0$  (given  $\overline{z}$ ), p will only influence the period-zero labor income but not beyond. Thus, the human wealth effect tends to be dominated by the non-human wealth effect—this conjecture is supported by the simulations in section 3.2.4. Therefore, the case of  $\partial C_0^e / \partial p > 0$  is more likely; and following the same logic, so is  $\partial C_0^e / \partial z^d < 0$ .

The results  $\partial C_0^e / \partial \overline{z} > 0$ ,  $\partial C_0^e / \partial p > 0$ , and  $\partial C_0^e / \partial z^d < 0$  provide the following insights:

**Remark 10** The greater the expected future productivity is; or the greater the debacle probability is; or the greater the severity of the debacle is, the higher the current consumption will be.

Without uncertainty, the major impact of (high) expected future productivity will be on (high) investments; and capital price will be anchored by the marginal cost of investments (assumed constant at unity here). Consumption will increase because of the positive human wealth effect; yet, consumption booms driven by asset price appreciation will not happen.

With uncertainty (plus irreversibility), high expected future productivity will have little influence over current investments because of the "wait-and-see" attitude. Then its impact will be on asset price appreciation, which can trigger consumption booms.

Given expected future productivity, the magnitude of current asset price appreciation is also related to the chance and severity of future debacles. A high debacle probability (or a low debacle productivity) will result in low investments and hence strong asset price appreciation, which tends to increase the magnitude of consumption booms.

#### 3.4 Summary

As opposed to the existing literature explaining foreign-capital-financed consumption booms as a macroeconomic side effect of open capital account due to institutional imperfections, the analysis in this chapter shows that the consumption booms can be a fundamental result of open capital account.

We show that, when domestic investments are not elastic (due to entrepreneurs' risk aversion), capital inflows after capital account liberalization will lead to domestic asset price appreciation that tends to cause consumption booms through the wealth effect. We show that the lower the investment elasticity is, the greater the consumption booms will be, even though lower investment elasticity will result in less capital formation and hence lower national incomes on the entire post-liberalization transition path. We show that under open capital account consumption boom-bust cycles can also be caused by variations in the world interest rate. In another case we examine the implications of uncertainty and irreversibility as another impediment to investments. We show that, when investments are irreversible, the uncertainty about future productivities will tend to cause asset price appreciation because of the sluggishness in investments caused by entrepreneurs' wait-and-see strategy. We show that the greater the uncertainty about the future productivity is, the greater the asset price appreciation will be; hence the greater the consumption booms will be.

#### 3.5 Mathematic Appendix

# 3.5.1 Appendix A.1

The four eigenvalues of the coefficient matrix of the simultaneous system

$$\begin{bmatrix} \dot{C} \\ \dot{B} \\ \dot{K} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} r^* - \pi^2 & -\pi^2 & -\pi^2 K^* \\ -1 & r & r & -\eta^{-1} \\ 0 & 0 & \eta & \eta^{-1} \\ 0 & 0 & n & m \end{bmatrix} \begin{bmatrix} C - C^* \\ B - B^* \\ K - K^* \\ q - 1 \end{bmatrix}$$
(24)

are, respectively,

$$\varepsilon_1 = r^* + \pi > 0$$
,  $\varepsilon_2 = \frac{m}{2} + \sqrt{\frac{m^2}{4} + \frac{n}{\eta}} > 0$ ,  $\varepsilon_3 = r^* - \pi < 0$ , and  $\varepsilon_4 = \frac{m}{2} - \sqrt{\frac{m^2}{4} + \frac{n}{\eta}} < 0$ .

Assume  $\varepsilon_3 \neq \varepsilon_4$ . Then, with two negative eigenvalues and two initial conditions  $K(0) = K_0$  and B(0) = 0, there exists a unique convergent path to the steady state. To solve for the path, we need the eigenvectors of  $\varepsilon_3$  and  $\varepsilon_4$ , which are respectively

$$\varepsilon_{3}: \begin{bmatrix} \pi \\ 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } \varepsilon_{4}: \begin{bmatrix} (\varepsilon - r^{*})\beta \\ -(1 + \beta) \\ 1 \\ \varepsilon \eta \end{bmatrix} \text{ (denote } \varepsilon = \varepsilon_{4}\text{),}$$

where  $\beta = \pi^2 K^* \varepsilon \eta (\pi - \varepsilon + r^*)^{-1} (\pi + \varepsilon - r^*)^{-1}$ . Thus, the solution to equation (24) is

$$C_{t} - C^{*} = u_{1}\pi e^{(r^{*} - \pi)t} + u_{2}\beta(\varepsilon - r^{*})e^{\alpha}$$

$$B_{t} - B^{*} = u_{1}e^{(r^{*} - \pi)t} - u_{2}(1 + \beta)e^{\alpha}$$

$$K_{t} - K^{*} = u_{2}e^{\alpha}$$

$$q_{t} - 1 = u_{2}\varepsilon\eta e^{\alpha}$$
(A.1)

Given initial conditions  $K(0) = K_0$  and  $B(0) = B_0$ , according to the second and third equations in the simultaneous system (A.1), we have  $B_0 - B^* = u_1 - u_2(1 + \beta)$  and  $u_2 = K_0 - K^*$ . Thus,  $u_1 = B_0 - B^* + (1 + \beta)(K_0 - K^*)$ . Substitute  $u_1$  and  $u_2$  back to (A.1) gives the solution to (24).

#### 3.5.2 Appendix A.2

The Proxy Model (autarky)

The modified aggregate consumption function can be written as

$$C(t) = \pi q_t K_t + (1 + \alpha^{1/2})^{-1} (1 - \alpha) \lambda K_t^{\alpha}$$
(A.2)

Equations (11) and (16) imply

$$q_t = 1 + \varphi \sigma^2 \dot{K}_t \tag{A.3}$$

Substituting equations (A.2), (A.3), and (6) into equation (13) gives

$$\dot{K}_{t} = \left(\Psi \lambda K_{t}^{\alpha} - \pi K_{t}\right) (1 + \pi \eta K_{t})^{-1}, \qquad (A.4)$$

where  $\Psi = 1 - (1 + \alpha^{1/2})^{-1} (1 - \alpha)$ . Linearizing equation (A.4) around steady state gives

$$\dot{K}_t = \omega(K_t - K^*), \tag{A.5}$$

where  $\omega = \left[\Psi r^* - \pi + (\alpha - 1)\pi\eta\Psi\lambda(K^*)^{\alpha}\right](1 + \pi\eta K^*)^{-2} < 0$ . Thus, according to (A.5) and the initial condition  $K(0) = K_0$ , we have  $K_t = K^* + (K_0 - K^*)e^{\omega t}$ , based on which we have equation (30), i.e.,  $I_t^{\alpha} = \omega(K_0 - K^*)e^{\omega t}$ .

# 3.5.3 Appendix A.3

#### The Proxy Model (open economy)

In the open-economy model, using the modified consumption function to substitute for C will give the following dynamic system:

$$\begin{split} \dot{B}_{t} &= r^{*}B_{t} + \lambda K_{t}^{\alpha} - \pi q_{t}K_{t} - \pi B_{t} - \pi (r^{*} + \pi)^{-1}(1 - \alpha)\lambda K_{t}^{\alpha} - \eta^{-1}(q_{t} - 1), \\ \dot{K}_{t} &= \eta^{-1}(q_{t} - 1), \\ \dot{q}_{t} &= m(q_{t} - 1) + n(K_{t} - K^{*}), \end{split}$$

which, after linearization, gives

$$\begin{bmatrix} \dot{B} \\ \dot{K} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} r^* - \pi & \Delta & -(\pi K^* + \eta^{-1}) \\ 0 & 0 & \eta^{-1} \\ 0 & n & m \end{bmatrix} \begin{bmatrix} B - B^* \\ K - K^* \\ q - 1 \end{bmatrix}$$
(A.6)

where  $\Delta = r^* - \pi - \pi (r^* + \pi)^{-1} (1 - \alpha) r^*$ . Given initial conditions  $K(0) = K_0$  and B(0) = 0, the solution to (A.6) is

$$B_{t} = \widetilde{\beta}(K_{0} - K^{*})(e^{(r^{*} - \pi)t} - e^{\alpha})$$

$$K_{t} = (K_{0} - K^{*})e^{\alpha} + K^{*}$$

$$q_{t} = (K_{0} - K^{*})\varepsilon\eta e^{\alpha} + 1$$
where  $\varepsilon = m/2 - (m^{2}/4 + n/\eta)^{1/2}$  and  $\widetilde{\beta} = 1 - [\pi r^{*}(1 - \alpha)(r^{*} + \pi)^{-1} + \varepsilon\eta\pi K^{*}](r^{*} - \varepsilon - \pi)^{-1}$ .
Given initial conditions  $K(0) = K_{0}$  and  $B(0) = 0$ , we can have equations (31) and (32),

respectively,  $I_t^{\circ} = \varepsilon (K_0 - K^*) e^{\varepsilon t}$  and  $CA_t = \dot{B}_t = \widetilde{\beta} (K_0 - K^*) [(r^* - \pi) e^{(r^* - \pi)t} - \varepsilon e^{\varepsilon t}].$ 

# **Chapter Four**

# **Baby Boom and Asset Market Meltdown**

#### 4.1 Introduction

The United States (U.S.) population born during two decades after World War II (1946-1964) are often called "baby boomers" in that a temporary increase in the fertility rate during the period has resulted in a considerably large age cohort. While the average number of children per female was 3.6 and 2.9 respectively for the 1950s and 60s, it dropped to 1.9 for the 1970s onward.<sup>47, 48</sup>

As a special demographic feature, the U.S. baby boom is associated with many interesting economic issues (Sterling and Waite, 1998). While still infants, baby boomers started competing among one another for diapers and milk (Macunovich, 1999). Then they competed for schools (Sterling and Waite, 1998), for jobs (Welch, 1979), for housing (Mankiw and Weil, 1989), etc. When they became middle-aged in the 1990s, it is

<sup>&</sup>lt;sup>47</sup> Similar baby boom-bust cycles have also happened in Japan and West Europe. In general, the booms occurred in the 1950s and the busts in the late 1960s; yet the specific timing is a little different for each country (Davis and Li, 2003). The phenomenon of baby boom is not unique to industrialized countries. For example, China's populations born during the 1960s and 1970s are also a cohort of baby boomers.

<sup>&</sup>lt;sup>48</sup>The following are some factors that may contribute to the baby boom phenomenon in the U.S. (Davis and Li, 2003). The postponement of family formation due to the Great Depression and the World War resulted in a low fertility rate in the 1930s and 40s. Then the fertility rate rose in the 1950s as the flourishing economic conditions increased people's confidence about the future. As for the later decline in fertility, the introduction of birth control techniques in the mid 1960s is likely to be a major factor. Other contributory factors include the acceptance and legalization of abortion, higher education and economic aspiration levels, more women in the labor force, and the economic recovery after the natural disasters in the late 1950s; and the later low fertility rate is caused by stringent birth control policies initiated by the Chinese government since the late 1970s.

said that their savings are one of the major driving forces behind the greatest stock market booms in the U.S. history (Shiller, 2000; Siegel, 1998; Sterling and Waite, 1998). As they are about to retire, a dark "asset market meltdown hypothesis", which predicts that the asset market booms driven by their savings will eventually collapse as they start dissaving, poses another challenge to the cohort (England, 2002).

What has happened happened; the interest here is the meltdown hypothesis that (if true) will tend to have an impact on the baby boomers' wellbeing at their most vulnerable time. Specifically, this chapter is a theoretical endeavor intended to find out whether the meltdown hypothesis is sound; and if it is, whether and how baby boomers can escape from it.

In contrast with the literature that generally supports the meltdown hypothesis, our analysis shows that the meltdown is actually state-contingent and not necessarily doomed to collapse during baby boomers' retirement. This is because the large capital stock built up by baby boomers' large savings may generate enough incomes and hence savings as asset demands to absorb the mass asset supplies due to baby boomers' retirement dissavings. We find that economies in relatively low development stages are more likely to be in non-meltdown states; and those in relatively high development stages tend to be more meltdown-prone. Thus, the baby boomers of China (as a developing country) may stand a better chance to be "meltdown free" as compared to their American counterparts. However, although whether the non-meltdown case is relevant to the highly developed U.S. economy is unclear, the chance cannot be completely excluded.

In the case where the meltdown is about to happen, we show that forward-looking baby boomers as a whole is nevertheless unable to avoid the potential meltdown; and their attempts to escape from severe future meltdowns could lead the current economy into a "liquidity trap".

The remainder of the chapter is organized as follows. After a literature review, we first examine the baby-boom impact on asset market performances, and then analyze what would happen when forward-looking baby boomers attempt to avoid potential meltdowns. A brief summary is provided at the end of the chapter.

# 4.2 The literature

The microfoundation foundation for the meltdown hypothesis is the impact of age on consumers' demand for assets in general and risky assets in particular.

A popular notion based on lifecycle consumption hypothesis (Ando and Modigliani, 1963; Modigliani and Brumberg, 1954) is that individuals tend to have hump-shaped asset demands peaked at middle age. Thus, the main rationale for the meltdown hypothesis is that, *ceteris paribus*, the total asset demand and hence asset prices tend to be high when a large cohort of baby boomers become middle-aged, and be low when they retire.

In general, empirical studies on the relationship between demographic structure and asset prices lend some support to the meltdown hypothesis (Poterba, 2001). The hump-shaped relationship between age and wealth is generally confirmed by cross-sectional data; yet the decline in old-age asset holdings seems to be limited (Poterba, 2001).

However, such a statistical relationship is hard to explain, because it could be the result of age, cohort, time effects, or a combination of them (Ameriks and Zeldes, 2001; Poterba, 2001). With the cohort effects allowed for,<sup>49</sup> Poterba (2001) still finds a small decline in old-age asset holdings. In a more recent study that covers seven OECD countries for 50 years, Davis and Li (2003) finds a positive relationship between the proportion of middle age in the total population and asset prices, which is evidence supporting the meltdown hypothesis.

With respect to equity prices, the impact of age structure on the demand for risky assets may be another factor that supports the meltdown hypothesis. A popular notion is that investors tend to (or ought to) hold less risky assets as they approach retirement (Ameriks and Zeldes, 2001; Campbell, 2001).<sup>50</sup> Thus, *ceteris paribus*, retiring baby boomers tend to shift away from risky assets, which is another contributory factor to the potential meltdown.

Existing empirical evidence on the relationship between age and equity prices (or equity premiums) is inconclusive, yet generally favorable to the meltdown hypothesis. Using data from 1900 to 1990, Bakshi and Chen (1994) find evidence that supports a positive relationship between age and risk aversion. Using data from three cross-sectional surveys in 1962, 1983, and 1986 respectively, Yoo (1994b) finds a hump-shaped relationship between investors' ages and equity shares in their portfolios. Using cross-

 <sup>&</sup>lt;sup>49</sup> The linear relationship among current time, birth time, and age—an individual's age is equal to the current time minus the time she was born—makes it impossible to empirically separate the three effects (Ameriks, 2001; Poterba, 2001).
 <sup>50</sup> Investors who have future labor incomes are implicitly holding their human wealth as "safe" assets; thus

<sup>&</sup>lt;sup>50</sup> Investors who have future labor incomes are implicitly holding their human wealth as "safe" assets; thus they may be willing to allocate more of their non-human wealth to risky assets. A rule of thumb often suggested by financial advisers is that the proper percentage of risky assets in one's portfolio should be equal to 100 minus her age.

sectional data from various years of Surveys of Consumer Finances and a panel data set from 1987 to 1999, Ameriks and Zeldes (2001) find a mild hump-shaped age pattern in equity shares; yet they caution about interpreting the result as evidence for a humpshaped relationship between age and equity demand.<sup>51</sup> Using data from 1926 to 1999, Poterba (2001) finds no evidence of an impact of age structure on equity returns; yet he does find some evidence that asset prices tend to be high when a large share of the population are at middle age. Using the U.S. data from 1900 to 2001, Ang and Maddaloni (2003) find a positive relationship between the average age and equity premium; yet the relationship is hardly statistically significant.

In summary, empirical studies generally support a hump-shaped relationship between age and asset (equity) demand (i.e., a positive age effect on asset demand prior to middle age and a negative effect afterwards), which is evidence favorable to the meltdown hypothesis. However, these studies may not really test the meltdown hypothesis. As pointed out by Poterba (1998; pp. 574-575), "[t]here is one baby boom shock in the postwar U.S. demographic experience, ..., [w]hether fifty years of prices and returns on this experience represent one observation or fifty is, however, an open question".

Although the lack of historical precedence may make the meltdown hypothesis not testable empirically, we can still evaluate whether and under what conditions the meltdown is likely by taking a more careful look at the underlying theory. Note that, even

<sup>&</sup>lt;sup>51</sup> Ameriks and Zeldes (2001) find that equity shares are nearly constant for stock owners, which implies that the hump-shape pattern of equity shares for all the agents (both stock owners and otherwise) could reflect agents' decisions in holding stocks or not, but not in holding how much stock. Also, the low old-age stock holding could be a result of cohort effects; i.e., the reason that current old-age agents hold relatively less equity may not because they are older, but because for some reason (e.g. the influence of the Great Depression) they are more risk-averse.

if consumers do have a hump-shaped age pattern of asset demand, the meltdown hypothesis may still not hold because, while the argument for the hypothesis depends upon the assumption of "*ceteris paribus*", factors that cause the initial asset market booms will tend to affect the economic conditions at the time when the meltdown is supposed to happen. Therefore, theoretical examinations in dynamic general equilibrium frameworks are necessary to test the validity of the meltdown hypothesis.

In the following we proceed to review the literature of theoretical examinations (including simulations based on calibrated models) on the meltdown hypothesis, which generally supports the meltdown hypothesis.

Poterba (2001) provides a succinct explanation to the rationale behind the hypothesis by using the following equation:

$$qK = s(wN), \tag{1.1}$$

whose left and right-hand sides represent asset supply and demand respectively. Given constant asset supply (K), wage rate (w) and saving rate (s), equation (1.1) clearly indicates a positive relationship between asset price (q) and the number of savers (N). Thus, while a high N during baby boomers' prime saving ages tends to drive up q, a low N during their retirements will bring it down.

While Poterba's illustrative model depends on *ad hoc* assumptions, Yoo (1994a) uses a multi-period general equilibrium OLG model to examine the hypothesis and finds that asset return tends to fall when a large-size cohort starts dissaving. In his model, asset return is determined by the marginal product of capital and therefore negatively related to the capital-labor ratio. Since the capital-labor ratio is low with a large size cohort in the labor force, asset return tends to be high when baby boomers are working and saving. As the boomers retire, the capital-labor ratio would become higher with a smaller labor force working on a large capital stock; hence asset return will fall.

While most of the research on the impacts of age structure on asset return uses puttyputty models that effectively assume away asset price fluctuations, Abel (2003) develops an OLG model in which convex investment adjustment costs allow asset price to fluctuate. Lim and Weil (2003) also consider convex "installation costs" in their study of the baby-boom impact on stock market booms. Both studies support the meltdown hypothesis.

While the theoretical validity of the meltdown hypothesis is generally agreed upon, some factors may make it less dramatic than some observers suggest.

First, retirees' bequest motives (Kotlikoff and Summers, 1981) may render the meltdown hypothesis irrelevant, because one of the key assumptions of the hypothesis is that baby boomers will flood the market with accumulated assets during their retirement (Poterba, 2001). However, Abel (2001) shows that meltdown will still happen even if baby boomers continue to hold their assets during retirement. In short, baby boomers' bequest motives will not attenuate the potential meltdown. Intuitively, although baby boomers' bequest motives will tend to reduce the asset supply during their retirement ages, the younger generation who rationally expect the bequest will nevertheless tend to reduce savings and hence make the asset demand smaller as well. In Abel's model, the reductions in asset supply and demand exactly offset each other so that baby boomers' bequest motives do not help preventing the meltdown.

Another factor is that foreign demands on domestic assets may help prevent the potential meltdown. However, facing similar ageing problems, major developed countries (e.g. Japan and West Europe) are not likely to provide such demands. In need of capital for their own growth, developing countries (e.g. China) may or may not be able to provide the demands. Even if they may, the resulting current account deficits could be an unpleasant side effect. Moreover, international capital immobility (Feldstein and Horioka, 1980) and investors' "home bias" towards holding domestic equities (Brennan and Cao, 1997) cloud the hope for relying on foreign savings to prevent the potential meltdown.

Third, the attempts of rational, forward-looking baby boomers to avoid the potential meltdown may be a factor preventing the meltdown from happening in the first place. As specifically asked by Abel (2003, p.552), "if these investors are forward-looking in the first place, would they so eagerly buy stocks that are destined to fall in price eventually?" However, by using a model with capital being the only store of value, Abel (2003) gives those unfortunate investors no other choices. Brooks (2000), on the other hand, does give baby boomers in his model a chance to hold riskless bonds that essentially represent their lendings to younger generations. Yet he finds that the option does not help baby boomers to avoid being hurt by a low rate of return during their retirement ages. This should not be surprised. After all, as baby-boom-induced asset market fluctuations are fundamentally driven by savings and dissavings, forward-looking baby boomers' attempts to avoid potential asset price meltdowns (by holding short-term or riskless assets) will tend to depress the general interest rate level for the entire asset markets.

In summary, according to the existing theoretical research in the literature on the meltdown hypothesis, baby boomers large savings will tend to drive up asset prices, which will eventually meltdown during their retirements; and baby boomers seem to have no escape from the meltdown.

As mentioned above, factors that cause the initial asset market boom will tend to affect the economic conditions when the meltdown is supposed to happen. Specifically, baby boomers' large savings will tend to not only drive up asset prices but also affect capital accumulations that will have an impact on the macroeconomic condition in baby boomers' retirement ages. Although the results from existing theoretical examinations based on general-equilibrium models (e.g. Abel, 2003) seem to indicate that this complication does not affect the validity of the meltdown hypothesis, we wonder why it is so.<sup>52</sup> In an attempt to take a deeper look at the impacts of baby boom on asset market performances, we find out that the meltdown is actually state-contingent and may not necessarily happen.

# 4.3 Baby boom impacts on asset market performances

During their prime saving ages, baby boomers' large amount of savings will put upward pressures on asset prices. Yet the magnitude of resulting asset price appreciation depends on investment elasticity (i.e. the responsiveness of investments to asset prices)—the higher the elasticity is, the more the pressures can be absorbed by increases in capital

<sup>&</sup>lt;sup>52</sup> When general-equilibrium models are used to examine the meltdown hypothesis, the purposes are mostly result-oriented, with focus on the impacts of baby boom on asset markets in general and whether the meltdown will happen in particular. However, the general-equilibrium mechanisms that lead to the impacts are generally kept in the black box.

stock; hence the less the price appreciation will be. Notwithstanding, as long as investments are not perfectly elastic, asset price appreciation will happen.

It is tempting to apply a similar argument to hypothesize the potential meltdown: With insufficient asset demand due to a small number of workers, baby boomers' large amount of dissavings in their retirements, representing massive asset supply, will cause asset market meltdowns.

While such an argument is generally accepted by the existing literature, it has actually missed a crucial yet underappreciated point. That is, while asset supply tends to be high during baby boomers' retirement eras, so will be asset demand. This is because the large capital stock built up by baby boomers' savings will have positive influence on incomes and (hence) savings during baby boomers' retirement eras.

Therefore, a conjecture is that, while baby boomers' large savings tend to drive asset market booms, asset market meltdowns may not necessarily follow. We examine this conjecture in the following.

#### 4.3.1 The Model

We use a two-period OLG model similar to the one used by Abel (2003). One major difference is that, while Abel (2003) models convex investment adjustment costs as an "investment impediment" responsible for low investment elasticity, the impediment modeled here is risk-averse investment behaviors.

#### 4.3.1.1 Consumer

At the beginning of period t,  $N_{y,t}$  numbers of identical young consumers are born, each of whom will supply inelastically one unit of labor during the period and receives wage income  $(w_t)$  at the end of which. After paying tax  $(T_t)$ , an individual young consumer consumes  $c_{y,t}$  and saves in capital  $(k_{y,t})$  and/or government bond  $(d_{y,t})$ . She carries over her assets into and retires during the next period t+1; and at the end of which she finishes her life cycle by using the gross returns to her savings to finance her old-age consumption  $(c_{o,t+1})$ .

At the end of period t, a period-t young consumer faces the following optimizing problem:

$$Max E_t[\log(c_{y,t}) + (1+\theta)^{-1}\log(c_{o,t+1})],$$

subject to:

$$c_{y,t} + q_t k_{y,t} + d_{y,t} = w_t - T_t$$
(2.1)

and

$$c_{o,t+1} = k_{v,t}(q_{t+1} + R_{t+1}) + d_{v,t}(1 + r_{t+1}), \qquad (2.2)$$

where variables  $R_t$ ,  $r_t$ , and  $q_t$  are, respectively, the period-*t* (per unit) capital income (in terms of consumption), the period-*t* (bond) interest rate, and the capital price (in terms of consumption goods) at the end of period *t*; and parameter  $\theta$  represents time preference.

The (government) bond is a one-period coupon bond denominated in consumption goods. Capital is free from default risks and hence a perfect substitute of bond. Thus, the
equilibrium (expected) return to one unit of capital should be equal to the return to equivalent bond investment. With capital prices at  $q_t$ , one unit of capital is equivalent (in return) to  $q_t$  units of bond; thus,

$$R_{t+1} + E_t q_{t+1} - q_t = r_{t+1} q_t, (2.3)$$

according to which the budget constraints (2.1) and (2.2) can be combined into

$$c_{y,t} + c_{o,t+1}(1+r_{t+1})^{-1} = w_t - T_t$$
.

Therefore, first order conditions give the individual young consumption function

$$c_{y,t} = (1+\theta)(2+\theta)^{-1}(w_t - T_t),$$

which, with  $N_{y,t}$  number of identical young consumers, gives the (aggregate) young consumption function:

$$C_{y,t} = (1+\theta)(2+\theta)^{-1}(w_t - T_t)N_{y,t}.$$
(2.4)

As period-t old consumers finance their consumption via the gross returns to their assets, the (aggregate) period-t old consumption function is given by

$$C_{o,t} = K_{y,t-1}(q_t + R_t) + D_{y,t-1}(1 + r_t)$$
(2.5)

where  $K_{y,t-1} = N_{y,t-1}k_{y,t-1}$  and  $D_{y,t-1} = N_{y,t-1}d_{y,t-1}$ .

## 4.3.1.2 Firms

In every period, identical, profit-maximizing, and perfectly competitive firms hire capital and labor to produce perishable consumption goods with the standard Cobb-Douglas technology:

$$Y_t = F(K_t, L_t) = \lambda K_t^{\alpha} L_t^{1-\alpha}, \qquad (2.6)$$

where  $Y_t$ ,  $K_t$ ,  $L_t$ , and  $\lambda$  denote, respectively, output, capital stock, labor, and technical coefficient.

# 4.3.1.3 Entrepreneurs

In every period, identical entrepreneurs engage in investing activities that transforms consumption goods into new capital.<sup>53</sup> Note that capital is irreversible; and its value depends on capital income and capital price (both in terms of consumption).

An individual entrepreneur *j* chooses the amount of investment  $(I_t^j)$  to maximize her expected utility:<sup>54</sup>

 $Max EU(\Pi_t^j)$ 

where  $\Pi_t = q_t I_t - c(I_t)$  represents investment profits—c(I) is the investment cost (function) in terms of consumption.

Entrepreneurs will hold investment profits (if any) earned at the end of period t in form of capital and sell them at the end of period t+1 for consumption.<sup>55</sup> Thus, period-t entrepreneurs' consumption is given by

<sup>&</sup>lt;sup>53</sup> The separation of investment activities from production activities here is similar to the "two sector" modeling framework adopted by Abel (2003). As opposed to firms being the producers of consumption goods, entrepreneurs here represent activities devoted to capital formation. We do not explicitly model who these entrepreneurs are, but simply assume they always exist. Indeed, we can let some of the young consumers be workers and the rest be entrepreneurs. Yet the results will not be different.

<sup>&</sup>lt;sup>54</sup> The utility rather than profit maximization is for the purpose of modeling risk-averse investment behaviors; otherwise, utility and profit maximizations are equivalent

<sup>&</sup>lt;sup>55</sup> Risk-neutral entrepreneurs will earn profits (as risk premiums) for their investment activities. The assumption (for analytical convenience) that they save all the profits till the next period is innocuous for the analysis of the impacts of demographic factors on savings. Even if entrepreneurs consume all of their

$$C_{e,t} = (q_t + R_t)(\prod_{t-1}/q_{t-1}).$$
(2.7)

Investments are risky with a stochastic cost function:

$$c(I_t^j) = I_t^j (1 + z_t^j),$$
(2.8)

where  $z_t \sim N(0, \sigma^2)$  is a normally distributed random variable. Entrepreneurs are riskaverse with utility function:

$$U(\Pi) = -e^{-\varphi\Pi}, \tag{2.9}$$

where parameter  $\varphi$  measures (constant) absolute risk aversion.

According to equations (2.8) and (2.9), entrepreneur j's maximizing problem becomes

$$\underset{I_{i}^{j}}{Max} E_{t}U(\Pi_{t}^{j}) = -\int e^{-\varphi \Pi_{t}^{j}} f(\Pi_{t}^{j}) d\Pi_{t}^{j} = -e^{-\varphi [(q_{t}-1)I_{t}^{j} - \varphi I_{t}^{j^{2}} \sigma^{2}/2]},$$

the solution to which gives the individual investment function:  $q_t = 1 + \varphi \sigma^2 I_t^j$ . Then, the aggregate investment function (with *n* identical entrepreneurs) would be

$$q_t = 1 + \frac{\varphi \sigma^2}{n} I_t , \qquad (2.10)$$

where  $I_t = nI_t^j$  represents the aggregate investment.<sup>56</sup>

Equation (2.10) implies that under risky investments ( $\sigma > 0$ ) and risk-averse entrepreneurs ( $\varphi > 0$ ), the aggregate investment is not perfectly elastic; and its elasticity

profits, the difference will be a lower saving rate for the economy, which will not affect the meltdown hypothesis whose validity does not depend on the underlying saving propensity of the economy.

<sup>&</sup>lt;sup>56</sup> Although the present modeling of investment behaviors is based on risk aversion, equation (2.10) is similar to Abel and Eberly's (1997) investment function (equation 15 in their paper) based on quadratic adjustment costs.

is negatively correlated with the riskiness of investments and the risk aversion of entrepreneurs.

Given a large number of entrepreneurs (n >> 0) and according to the law of large numbers, the aggregate investment cost function is

$$c(I_{t}) = \sum_{j} c(I_{t}^{j}) = I_{t}^{j} (n + \sum_{j=1}^{n} z_{t}^{j}) = nI_{t}^{j} = I_{t},$$
(2.11)

which implies constant marginal cost of investment in aggregate. Note that, notwithstanding the constant marginal investment cost in aggregate, aggregate investment is not perfectly elastic because of increasing (marginal) risk premia demanded by risk-averse entrepreneurs.

## 4.3.1.4 Government

Government uses tax revenues and bond issuance to finance its expenditures including (bond) interest payments and government consumption (assumed to be zero for simplicity). Thus,

$$T_t N_{y,t} + \dot{D}_t = r_t D_t$$
, (2.12)

with the left and right hand sides representing government's revenues and expenditures respectively.

## 4.3.1.5 Identities

The period-t capital stock  $(K_t)$  is equal to period-t old consumers' capital holding plus entrepreneurs' investment profits earned in period t-1 and held in form of capital in period t. Thus,

$$K_t = K_{y,t-1} + \prod_{t-1} / q_{t-1}$$
(2.13)

The period-t bond stock  $(D_t)$  is solely held by period-t old consumers. Thus,

$$D_t = D_{y,t-1}$$
(2.14)

# 4.3.1.6 Equilibrium

There are five markets: (consumption) goods, labor, rental capital, capital, and bond. Take consumption as the numeraire.

The goods market is in equilibrium when consumption output is completely absorbed by the consumption of young consumers, old consumers, entrepreneurs, plus the costs for investments; i.e.,  $Y_t = C_{y,t} + C_{o,t} + C_{e,t} + c(I_t)$ , which, according to equations (2.4), (2.5), (2.6), (2.7), (2.11), (2.13) and (2.14), can be transformed into

$$F(K_t, L_t) = (1+\theta)(2+\theta)^{-1}(w_t - T_t)N_{y,t} + K_t(q_t + R_t) + D_t(1+r_t) + I_t, \qquad (2.15)$$

where the left-hand side represents the total supply of consumption goods by firms; the first term on the right-hand side represents the demand for consumption by the young consumer; the sum of the second and third terms represents the demand by the old consumer, and the last term represents the demand for consumption goods by entrepreneurs for investments.

As each of young consumers (as the only source of labor) inelastically supplies one unit of labor, the labor supply function is given by

$$L_t = N_{v,t}$$
. (2.16)

The demand for labor comes from firms, who (under perfect competition) will pay factors by their marginal products. Thus, according to equation (2.6), the labor demand function is given by

$$w_t = F_2(K_t, L_t), (2.17)$$

which, with inelastic labor supply, determines the labor market-clearing wage rate.

Similarly, as the supply of rental-capital is inelastic and equal to the existing capital stock, the market-clearing rental rate is determined by the rental-capital demand function

$$R_t = F_1(K_t, L_t).$$
(2.18)

Since the existing capital stock is owned by the old consumer who will definitely sell off her asset holdings, the supply-side equilibrium condition for the capital market is determined by the aggregate investment function [equation (2.10)], which can be notationally summarized into

$$q_t = 1 + \eta I_t , \qquad (2.19)$$

where coefficient  $\eta = \varphi \sigma^2 / n$  is negatively correlated with the *q*-elasticity of investments ("investment elasticity" in short).

Equation (2.3), which is essentially a capital demand function, can be rearranged into

$$r_{t+1} = \frac{R_{t+1} + E_t q_{t+1} - q_t}{q_t} \tag{2.20}$$

where, according to equations (2.16) and (2.18),

$$R_{t+1} = F_1(K_{t+1}, N_{y,t+1}).$$
(2.21)

Note that the left-hand side of equation (2.20) is the rate of return to capital, which is equal to the sum of earning  $(R_{t+1})$  and capital gain  $(E_tq_{t+1} - q_t)$  divided by the capital price  $q_t$ .

Suppose government keeps its debt level constant at  $\overline{D}$  via balancing its budget in every period (i.e.  $\dot{D}_t = 0$ ), then the supply-side bond market equilibrium condition is given by

$$D_t = \overline{D}, \qquad (2.22)$$

and, according to equation (2.12)

$$T_t N_{y,t} = r_t D_t,$$
 (2.23)

According to Walras' Law, the demand-side bond market equilibrium is implied by equilibria in the other markets.

Finally, assume no capital depreciation for simplicity; then capital accumulation is governed by

$$K_{t+1} = K_t + I_t (2.24)$$

## 4.3.1.7 Summary

At the end of period t, the equilibrium of the economy is characterized by the simultaneous system composed of equations (2.15)-(2.24), in which variables  $N_{y,t}$  and  $N_{y,t+1}$  are exogenous demographic features; variables  $K_t$  and  $r_t$  are initial conditions exogenously determined by history; variables  $L_t$ ,  $w_t$ ,  $T_t$ ,  $q_t$ ,  $R_t$ ,  $D_t$ ,  $I_t$ ,  $r_{t+1}$ ,  $R_{t+1}$ , and  $K_{t+1}$  are

endogenously determined; and variable  $E_tq_{t+1}$  depends on agents' expectations that are assumed to be rational here.

## 4.3.2 Dynamics of capital stock and capital price

According to the simultaneous system (2.15)-(2.24), the dynamics of capital accumulation can be characterized by

$$\dot{K}_{t} = K_{t+1} - K_{t} = \frac{S_{t}^{g} - K_{t} - \Lambda \overline{D}}{1 + \eta K_{t}}$$
(2.25)

where  $S_t^g = (2+\theta)^{-1}(1-\alpha)\lambda K_t^{\alpha} N_{y,t}^{1-\alpha}$  measures the gross saving of the economy;<sup>57</sup> and  $\Lambda = (2+\theta)^{-1}(2+r_t+\theta)$  is a summarizing notation.

For analytical convenience, let  $\overline{D} = 0.58$  Then, according to equation (2.25), the steady state  $(\dot{K}_t = 0)$  capital stock with constant population  $(N_{y,t} = \overline{N})$  can be determined by the following equation:

$$S^{g^*} - K^* = 0, (2.26)$$

where  $S_t^{g^*} = (2 + \theta)^{-1} (1 - \alpha) \lambda K^{*\alpha} \overline{N}^{1-\alpha}$  represents the steady-state saving of the young consumer, which needs to be equal to the steady-state capital stock K<sup>\*</sup> that represents the old consumer's saving in the steady state with the capital price equal to unity. Equation (2.25) implies that capital grows according to

$$\partial \dot{K}_{t} / \partial K_{t} = \left[ (\alpha S_{t}^{g} / K_{t} - 1)(1 + \eta K_{t}) - \eta (S_{t}^{g} - K_{t}) \right] (1 + \eta K_{t})^{-2}.$$
(2.27)

 $<sup>^{\</sup>rm 57}$  It is not difficult to verify that  $S_t^g = w_t - C_{y,t}$ 

<sup>&</sup>lt;sup>58</sup> The inclusion of government bond in the model is to facilitate analysis in the next section.

According to equations (2.26) and (2.27),

$$\left. \frac{\partial K_t}{\partial K_t} \right|_{K_t = K} < 0. \tag{2.28}$$

Thus, we have the following proposition.

# **Proposition 2.1** *The steady state* $K^*$ *is unique and stable.*

**Corollary 2.1**  $\forall K_t < K^*, \ \dot{K}_t > 0.$ 

Proof: According to equation (2.26),  $K^*$  is unique. According to inequality (2.28),  $K^*$  is stable. With a unique and stable  $K^*$ , Corollary 2.1 is self-evident.

Put plainly, given initial  $K_0$  less than the steady-state  $K^*$ —which is what we consider here—capital stock will be on a monotonic upward trend until steady state.

The dynamics of K convergence can be characterized by the following proposition.

**Proposition 2.2**  $\exists \widetilde{K} \in (0, K^*]: \left. \partial \dot{K}_t / \partial K_t \right|_{K_t = \widetilde{K}} = 0.$ 

**Corollary 2.2**  $\forall K_t \in (0, \widetilde{K}) : \partial \dot{K}_t / \partial K_t > 0.$ 

**Corollary 2.3**  $\forall K_t \in (\widetilde{K}, K^*]: \partial \dot{K}_t / \partial K_t < 0.$ 

Proof: According to equation (2.27), it is not difficult to verify that  $\partial \dot{K}_t / \partial K_t$  is a monotonically decreasing function of  $K_t$ , and  $\lim_{K_t \to 0} \partial \dot{K}_t / \partial K_t \to \infty$ . Thus, with inequality

(2.28),  $\tilde{K}$  must exist; and Corollaries 2.2 and 2.3 must hold.

According to Propositions 2.1 and 2.2., the growth path of  $K_t$  can be graphically depicted by Figure 1, with the  $K_t < \tilde{K}$  portion being convex and the  $K_t > \tilde{K}$  portion concave. Note that, although Figure 1 is not a standard textbook diagram, the capital stock dynamic depicted in it is a standard result of overlapping generation models.<sup>59</sup>

Accordingly, the dynamic of capital price can be characterized by the following proposition.



<sup>&</sup>lt;sup>59</sup> See Figure 2.11 in Romer (1996, p.77) for an example.

**Proposition 2.3** For  $K_t \in (0, \widetilde{K})$ ,  $q_t$  will be positively correlated with  $K_t$  and hence on an upward trend; whereas, for  $K_t \in (\widetilde{K}, K^*]$ ,  $q_t$  will be negatively correlated with  $K_t$  and hence on a downward trend.

Proof: According to equation (2.19),  $dq_t / d\dot{K}_t > 0$ —note that  $\dot{K}_t = I_t$ —which, together with Corollaries 2.2 and 2.3, implies that  $\partial q_t / \partial K_t \Big|_{K_t \in (0,\tilde{K})} > 0$  and  $\partial q_t / \partial K_t \Big|_{K_t \in (\tilde{K}, K^*]} < 0$ . Accordingly, as  $\dot{K}_t \Big|_{K_t \in (0, K^*)} > 0$ , we have  $\dot{q}_t \Big|_{K_t \in (0,\tilde{K})} > 0$  and  $\dot{q}_t \Big|_{K_t \in (\tilde{K}, K^*]} < 0$ .

Intuitively, the sign of q-K correlation depends on the balance between two opposite influences of K on q. On the one hand, K per se represents capital supply and hence has a negative influence over q directly. On the other hand, K also has a positive influence over q indirectly through savings that represent asset demand—note that a large K can help generating large wage incomes. As the balance of the two effects cannot be determined apriori, the sign of the q-K correlation is state contingent. With diminishing marginal product of capital, the positive (indirect) effect tends to prevail when K is small, and be dominated when K is large.

As will be shown later, when q is positively correlated with K and on an upward trend, the meltdown may not necessarily happen.

#### 4.3.3 A severe baby boom

From a theoretical point of view, we will consider two cases of baby boom: one is severe and the other mild. For the first case, suppose a *severe* baby boom occurs in period t = 0, which can be described by

$$N_{y,t} = \begin{cases} N^b, \ t = 0\\ \overline{N}, \ t \neq 0 \end{cases},$$

where  $N^b > \overline{N}$  measures the magnitude of the baby boom. This assumption implies a situation with a constant number of newborns over time except a spike in period zero caused by a baby boom. Thus, while period zero represents the baby boomers' saving ages, period one represents their retirement ages.

This modeling is similar to the baby-boom-baby-bust cycle used by Brook (2000) in simulating the baby boom impact on asset returns.<sup>60</sup> It is a *severe* baby boom in the sense that the "baby bust" in period one makes the period-zero baby boomers face a high (relative to mean) dependency ratio during their retirements in period one. Although such a modeling (for analytical convenience) exaggerates the U.S. baby bust experience,<sup>61</sup> it is innocuous for our purpose since it makes the meltdown more likely to happen.

We now proceed to examine how the baby boom specified above will affect the capital price q in period zero (as baby boomers' saving period) and in period one (as their

 $<sup>^{60}</sup>$  The baby-boom-baby-bust cycle in Brook (2000) is two periods of 2% population growth followed by two periods of 2% population reduction.

<sup>&</sup>lt;sup>61</sup> In 1945, the year before the baby boom started, 2.86 million babies were born. The figure is 4.30 million in 1957 when the baby boom was peaked, and 3.14 million in 1973 as the trough of the following baby bust (Mankiw and Weil, 1989).

retirement period). We start from examining the effect of the baby boom on capital accumulation.

## 4.3.3.1 Baby boom effect on capital accumulation

As it is not difficult to verify that  $\partial S_t^g / \partial N_{y,t} > 0$ , equation (2.25) implies that a *permanent* population growth (from  $\overline{N}$  to  $N^b$ ) in period t = 0 will shift up the capital growth path from  $K_0\overline{N}$  to  $K_0N^b$  in Figure 2. Yet, as the high newborn level  $N^b$  is temporary and will drop back to  $\overline{N}$  in time t = 1, the baby-boom effect on capital accumulation will be characterized by the path  $K_0N^b\overline{N}$  in Figure 2.



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Specifically, we have the following proposition regarding the effect of the period-zero baby boom on  $K_{l}$ .

**Proposition 2.4** *A period-zero baby boom has a positive effect on capital stock in period* one; i.e.,  $dK_1 / dN^b > 0$ .

Proof: According to equation (2.25),  $dK_1 / dN^b = (1 + \eta K_0)^{-1} (dS_0^g / dN^b) > 0$ .

When q and K are positively correlated—recall Proposition 2.3 for its possibility, that  $dK_1 / dN^b > 0$  will imply  $dq_1 / dN^b > 0$  (i.e., a positive impact of period-zero baby boom on period-one capital price). This is the key for the meltdown not to happen. We will come back to this point later. First let us examine the baby-boom impact on  $q_0$ .

## 4.3.3.2 Baby boom impact on period-zero capital price

According to the simultaneous system (2.15)-(2.24), the capital price at the end of period *t* is given by

$$q_{t} = (1 + \eta K_{t})^{-1} \Big( 1 + \eta \lambda (1 - \alpha) (2 + \theta)^{-1} K_{t}^{\alpha} N_{y,t}^{1 - \alpha} \Big).$$
(2.29)

Thus, the period-zero capital price will be

$$q_0 = (1 + \eta K_0)^{-1} \Big( 1 + \eta \lambda (1 - \alpha) (2 + \theta)^{-1} K_0^{\alpha} N^{b^{1 - \alpha}} \Big),$$
(2.30)

which implies

$$dq_0 / dN^b = (1 + \eta K_0)^{-1} \Big( \eta \lambda (1 - \alpha)^2 (2 + \theta)^{-1} K_0^{\alpha} N^{b^{-\alpha}} \Big).$$
(2.31)

Given  $\eta > 0$ , equation (2.31) implies

$$\mathrm{d}q_0 \,/\,\mathrm{d}N^b > 0 \tag{2.32}$$

and

$$\partial (\mathrm{d}q_0 / \mathrm{d}N^b) / \partial\eta > 0$$
,

which give the following propositions.

**Proposition 2.5** If investments are not perfectly elastic (i.e.  $\eta > 0$ ), a period-zero baby boom will have a positive impact on  $q_0$ .

**Corollary 2.4** Ceteris paribus, the lower the investment elasticity (i.e., the higher the  $\eta$ ) is, the larger the baby-boom impact on  $q_0$  will be.

Since investments in reality can hardly be perfectly elastic, Proposition 2.5 supports the first part of the meltdown hypothesis. That is, a baby boom tends to drive capital market boom during baby boomers' saving ages. Nevertheless, Corollary 2.4 suggests that the magnitude of the boom negatively depends on investment elasticity.

# 4.3.3.3 Baby boom impact on period-one capital price

We proceed to examine our main question: whether a period-zero baby boom will cause a capital market meltdown in period one. We first examine the impact of the baby boom on  $q_{l}$ .

According to equation (2.29),

$$dq_1/dK_1 = \eta\Gamma(K_1), \qquad (2.33)$$

where 
$$\Gamma(K_1, \overline{N}) = \{\lambda \alpha (1-\alpha)(2+\theta)^{-1} K_1^{\alpha-1} \overline{N}^{1-\alpha} - 1 - \eta \lambda (1-\alpha)^2 (2+\theta)^{-1} K_1^{\alpha} \overline{N}^{1-\alpha} \} (1+\eta K_1)^{-2}.$$

Equation (2.33), together with  $dK_1 / dN^b = dI_0 / dN^b = \eta^{-1} dq_0 / dN^b$ , implies that

$$dq_1 / dN^b = \Gamma(K_1) dq_0 / dN^b.$$
(2.34)

With  $\alpha < 1$ , it is not difficult to verify that,  $\partial \Gamma / \partial K_1 < 0$ ,  $\lim_{K_1 \to 0} \Gamma(K_1) = \infty$ , and  $\Gamma(K^*) < 0$ .

Thus, 
$$\exists \hat{K} \in (0, K^*]$$
:  $\Gamma(K_1) \begin{cases} > 0; K_1 \in (0, \hat{K}) \\ = 0; K_1 = \hat{K} \\ < 0; K_1 \in (\hat{K}, K^*] \end{cases}$ .

Therefore, according to equations (2.32) and (2.34),

$$dq_{1}/dN^{b} \begin{cases} > 0; \ K_{1} \in (0, \hat{K}) \\ = 0; \ K_{1} = \hat{K} \\ < 0; \ K_{1} \in (\hat{K}, \ K^{*}] \end{cases}$$

which implies the following propositions.

**Proposition 2.6** The effect of a period-zero baby boom on period-one capital price is state contingent.

**Corollary 2.5** In the case of  $K_1 > \hat{K}$ , a period-zero baby boom will have a negative impact on  $q_1$ .

**Corollary 2.6** In the case of  $K_1 < \hat{K}$ , a period-zero baby boom will have a positive impact on  $q_1$ .

The result in Corollary 2.6 may seem counterintuitive: As there are not enough workers (savers) to demand baby boomers' large asset supply in period one, the baby boom should have a negative impact on  $q_1$ . However, a crucial yet underappreciated point is that baby boomer's large savings can create its own demand. This is because the large  $K_1$  built up by baby boomers' large savings will have a positive impact on period-one income (and hence saving) that represents a demand-side force on  $q_1$ . When the marginal product of capital (*MPK*) is sufficiently large,<sup>62</sup> this demand-side force can be strong enough to prevail over the downward pressure (on  $q_1$ ) by the large  $K_1$  together with the small  $N_{y,1}(=\overline{N})$ . We confirm this conjecture in the following.

According to Proposition 2.4,  $dK_1 / dN^b > 0$ . Thus,  $dq_1 / dN^b > 0$  if  $dq_1 / dK_1 > 0$ . Then, the condition for  $dq_1 / dN^b > 0$  is that for  $dq_1 / dK_1 > 0$ .

Abstracted from government bond and tax, equation (2.15) will give the following goods market equilibrium condition:

$$F(K_t, \overline{N}) = (1+\theta)(2+\theta)^{-1} w_t \overline{N} + K_t (q_t + R_t) + I_t, \qquad (2.15')$$

in which output, wage, capital income, and investment are given respectively by

$$F(K_t, \overline{N}) = \lambda K_t^{\alpha} \overline{N}^{1-\alpha},$$
$$w_t = (1-\alpha)\lambda K_t^{\alpha} \overline{N}^{-\alpha},$$
$$R_t = \alpha \lambda K_t^{\alpha-1} \overline{N}^{1-\alpha},$$
$$I_t = \eta^{-1}(q_t - 1).$$

Substituting them into equation (2.15'), we have

<sup>&</sup>lt;sup>62</sup> This is why the positive baby-boom impact on  $q_1$  tends to happen when K is small.

$$\lambda K_{\iota}^{\alpha} \overline{N}^{1-\alpha} = (1+\theta)(2+\theta)^{-1}(1-\alpha)\lambda K_{\iota}^{\alpha} \overline{N}^{1-\alpha} + q_{\iota} K_{\iota} + \alpha \lambda K_{\iota}^{\alpha} \overline{N}^{1-\alpha} + \eta^{-1}(q_{\iota}-1),$$

which can be simplified into

$$s\lambda K_{t}^{\alpha}\overline{N}^{1-\alpha} = q_{t}K_{t} + \eta^{-1}q_{t} - \eta^{-1}$$
(2.35)

where  $s = (2 + \theta)^{-1}(1 - \alpha)$  represents the saving rate of the economy. The left-hand side of the equation represents the gross saving (by young consumers), which will be used to cover the dissavings (by old consumers) represented by  $q_t K_t$  plus the new investment

$$(I_t = \eta^{-1} q_t - \eta^{-1}).$$

By totally differentiating equation (2.35) we can obtain

$$s(MPK_t)dK_t = K_t dq_t + q_t dK_t + \eta^{-1} dq_t , \qquad (2.36)$$

where  $MPK_{t} = \alpha \lambda K_{t}^{\alpha-1} \overline{N}^{1-\alpha}$  is the marginal product of capital. Equation (2.36) can be rearranged into

$$dq_t / dK_t = (sMPK_t - q_t)(K_t + \eta^{-1})^{-1};$$
(2.37)

according to which the condition for  $dq_t / dK_t > 0$  is  $sMPK_t > q_t$ . Specifically,  $dq_1 / dK_1 > 0$  when  $sMPK_1 > q_1$ .

Therefore,  $dq_1/dN^b > 0$  when  $sMPK_1 > q_1$ , where  $s = (2 + \theta)^{-1}(1 - \alpha)$  represents the saving rate of the economy; and  $MPK_1 = \alpha\lambda K_1^{\alpha-1}\overline{N}^{1-\alpha}$  represents the marginal production of capital.

Now let us intuitively explain the implication of  $sMPK_1 > q_1$ , and why it is the condition for  $dq_1/dN^b > 0$  (i.e., the positive baby boom effect on  $q_1$ ).<sup>63</sup> Since *s* and  $MPK_1$  are respectively the saving rate and the marginal product of capital in period one,  $sMPK_1$  represents the "marginal saving" of capital in period one, i.e., the amount of saving generated by one extra unit of capital. Since  $q_1$  is the value of one unit of capital, then  $sMPK_1 > q_1$  implies that one extra unit of capital (as asset supply) can generate savings (as asset demand) that exceed the value of one unit of extra capital. In other words, when  $sMPK_1 > q_1$ , one extra unit of capital will cause excess demand for capital. Since the excess demand will put pressures on prices, then extra capital will tend to raise capital price when  $sMPK_1 > q_1$ . This explains the rationale of  $sMPK_1 > q_1$  being the condition for  $dq_1/dK_1 > 0$ . In other words, when  $sMPK_1 > q_1$ , factors that increase  $K_1$  will have positive impacts on  $q_1$ . As the baby boom is one of such factors, it will have a positive impact on the period-one capital price (i.e.  $dq_1/dN^b > 0$ ).

# 4.3.3.4 Baby boom impact on the change of capital price in period one

Even when a period-zero baby boom has a positive impact on  $q_1$ , asset market meltdown can still happen when the impact is less than the positive baby-boom impact on  $q_0$ ; i.e., if

<sup>&</sup>lt;sup>63</sup> Although the condition  $sMPK_t > q_t$  corresponds to a situation where the price of capital is lower than the capital earning, it can be consistent with the equilibrium in the capital market. This is because the young consumer's demand for capital is constrained by her budgets. After using all of her savings to buy capital, the young consumer may still think that the capital (with price lower than earning) is very cheap; yet with no more incomes she cannot generate further demand for capital; hence the capital market will be in equilibrium. We will provide more discussion on the empirical relevancy of the condition later.

 $d(\Delta q_{1-0})/dN^b < 0$ , where  $\Delta q_{1-0} \equiv q_1 - q_0$ . Yet we will show that  $d(\Delta q_{1-0})/dN^b > 0$  is still possible; i.e., a period-zero baby boom can have a positive impact on the capital price movement in period one. Put plainly, the period-zero capital price boom may not necessarily meltdown but could keep rising in period one.

According equation (2.34),  $d\Delta q_{1-0} / dN^b = (\Gamma - 1) dq_0 / dN^b$ . As  $\partial \Gamma / \partial K_1 < 0$ ,  $\lim_{K_1 \to 0} \Gamma(K_1) = \infty \text{ and } \Gamma(K^*) < 0, \quad \exists \hat{K} \in (0, K^*]: \ \Gamma(\hat{K}) = 1. \text{ Thus,}$ 

$$d\Delta q_{1-0} / dN^{b} \begin{cases} > 0; \ K_{I} \in (0, \hat{K}) \\ = 0; \ K_{1} = \hat{K} \\ < 0; \ K_{1} \in (\hat{K}, \ K^{*}] \end{cases}$$

which implies that the following proposition.

**Proposition 2.7** *The impact of a period-zero baby boom on the change of capital price in period one is state contingent.* 

**Corollary 2.7** In the case of  $K_1 > \hat{K}$ , a period-zero baby boom will have a negative impact on the change of capital price in period one.

**Corollary 2.8** In the case of  $K_1 < \hat{K}$ , a period-zero baby boom will have a positive impact on the change of capital price in period one.

Proposition 2.7 implies that the widely-accepted meltdown hypothesis is flawed—babyboom-driven asset market booms may not necessarily collapse but could rather keep booming during baby boomers retirement ages.

Intuitively, the higher the baby-boom-driven  $q_0$  is, the higher the  $K_1$  will be. When K and q are positive correlated, a higher  $K_1$  will imply a higher  $q_1$ . As the magnitude of the impact of K on q can be very large—it is not difficult to verify from equation (2.33) that  $\lim_{K_1 \to 0} dq_1 / dK_1 = \infty$ —the positive baby-boom impact on  $q_1$  can outweigh its impact on  $q_0$  so that the meltdown will not happen.

Graphically, in Figure 3, the upward and downward trends of the hump-shaped q path correspond respectively to the portions of  $K_i < \tilde{K}$  and  $K_i > \tilde{K}$  in Figure 1. In a situation where the q-path is downward-sloping, suppose the capital price will change from point a in period zero to point b in period one in a non-baby-boom scenario in which the period-zero baby boom shock does not occur. Then, suppose the period-zero baby boom does occur, it will drive the period-zero capital price from point a to point c. Similar to the capital price dynamics in Abel's (2003) model, after the period-zero baby boom will increase the period-one capital stock relative to its "non-baby-boom" level, the period-one capital price will drop to point d, which is lower than it would have been (at point b) had the period-zero baby boom not happened.<sup>64</sup> Although the capital price would have

<sup>&</sup>lt;sup>64</sup>The downward sloping q-path implies a negative correlation between the capital price and capital stock (i.e.  $dq_1 / dK_1 < 0$ ). According to Proposition 4, the period-zero baby boom will have a positive impact

been on a downward trend (from a to b) even without the baby boom, the capital price depreciation (from c to d) under the baby-boom shock is clearly of a greater magnitude. In this sense, the baby boom will cause a capital market meltdown in period one.



However, the situation where the q-path is upward-sloping is less straightforward. As before, a period-zero baby boom will drive up  $q_0$ ; and  $q_1$  will be mean reverting. Yet, as q is on an upward trend, its mean reversion can have different implications.

Consider first the situation where  $\hat{K} < K_t < \tilde{K}$ . According to Corollary 2.5, when  $K_t > \hat{K}$ , a period-zero baby boom will have a negative impact on period-one capital price. This implies that, while the period-zero baby boom will drive up the capital price

on the period-one capital stock (i.e.  $dK_1 / dN^b > 0$ ). Thus, the period-zero baby boom will negatively affect the period-one capital price (i.e.  $dq_1 / dN_1^b < 0$ ) when the q-path is downward sloping.

from the non-baby-boom level at point e to point g, the subsequent mean-reversion of the capital price will be from point g to point h. Therefore, even though the capital price is on an upward trend in this case, the impact of baby boom on asset price dynamics is similar to the case where the capital price is on a downward trend.

Now let us consider the case where  $\hat{K} < K_i < \hat{K}$ . According to Corollary 2.6, since  $K_i < \hat{K}$ , the period-zero baby boom will have a positive impact on the period-one capital price. Therefore, the period-one capital price under the baby boom shock will be at point k, which is higher than the non-baby-boom capital price level at point i. However, according to Corollary 2.7, since  $K_i > \hat{K}$ , the period-zero baby boom will have a negative impact on the change of capital price in period one, therefore, the capital price appreciation from j to k under the baby boom shock will be smaller than the appreciation (from h to i) if the baby boom does not happen. Although the capital price dynamics under the baby boom shock may still be on an upward trend,<sup>65</sup> in the sense that the appreciation is less than it would have been had the baby boom not happened, we can still deem it as a case of asset market "meltdown" caused by the baby boom.

Finally, let us consider the case where  $K_i < \hat{K}$ . According to Corollary 2.8, when  $K_i < \hat{K}$ , a period-zero baby boom will have a positive impact on the change of capital price in period one. Thus, under a period-zero baby boom shock, not only the capital price in price one will appreciate (from point *n* to *o*), but also the magnitude of the

<sup>&</sup>lt;sup>65</sup> It is also possible that point *j* is higher than point *k* so that the baby boom leads to asset price depreciation in period one; whereas the capital price would have been appreciation (from *h* to *i*) had the baby boom not happened.

appreciation will be larger than it would have been (from point l to m) without the baby boom shock. Therefore, in this case, a baby-boom-driven capital price boom in period zero (that represents baby boomers' saving ages) will keep booming in period one (that represents their dissaving ages).

#### 4.3.3.5 Baby boom effect on period-one rate of return to capital

Even when a period-zero baby boom has a positive impact on capital price variation in period one (i.e.  $d\Delta q_{1-0} / dN^b > 0$ ), it could still have a negative impact on the rate of return to capital in period one, because of its negative impact on capital income through capital-labor ratio. Indeed, we will show that the severe baby boom will definitely have a negative impact on period-one rate of return to capital:

$$RR_{1} = \frac{R_{1} + q_{1} - q_{0}}{q_{0}} \quad .$$
(2.38)

According to equations (2.21) and (2.29), we have

$$q_1 + R_1 = (1 + \eta K_1)^{-1} \left( 1 + \eta \lambda (1 + \alpha + \theta \alpha) (2 + \theta)^{-1} K_1^{\alpha} \overline{N}^{1 - \alpha} + \alpha \lambda K_1^{\alpha - 1} \overline{N}^{1 - \alpha} \right),$$

which implies

$$d(q_1 + R_1) / dK_1 = [(\alpha - 1)\Theta - \eta](1 + \eta K_1)^{-2} < 0, \qquad (2.39)$$

where

$$\Theta = \eta \lambda \alpha (3+2\theta)(2+\theta)^{-1} K_1^{\alpha-1} \overline{N}^{1-\alpha} + \alpha \lambda K_1^{\alpha-2} \overline{N}^{1-\alpha} + \eta^2 \lambda (1+\alpha+\theta\alpha)(2+\theta)^{-1} K_1^{\alpha} \overline{N}^{1-\alpha} > 0.$$

According to Proposition (2.4), inequality (2.39) implies

$$d(q_1 + R_1)/dN^b < 0, (2.40)$$

which, together with inequality (2.32), implies

$$dRR_1 / dN^b < 0. ag{2.41}$$

Inequality (2.41) implies the following proposition.

**Proposition 2.8** *A severe period-zero baby boom will definitely reduce the rate of return to capital in period one.* 

This result shows that, even when a severe period-zero baby boom has a positive impact on period-one capital price variation, the impact will be dominated by its negative impact on the unit capital income in period one—after all, the period-one baby bust will tend to greatly increase period-one capital-labor ratio. If a period-zero baby boom is not followed by a baby bust in period one, then it could have a positive impact on the rate of return to capital in period one. We examine such a "mild" baby boom in the following.

#### 4.3.4 A mild baby boom

The above severe baby boom, which implies a situation with a high (relative to mean) birthrate followed by a low one, is a theoretical construction that allows us to consider a meltdown-prone situation. Here we consider a case of "mild" baby boom that is more realistic.

Suppose a mild baby boom occurs in period t = 0, which can be described as

$$N_{y,t} = \begin{cases} \overline{N}, & t < 0 \\ N^b, & t \ge 0 \end{cases},$$

where  $N^b > \overline{N}$  measures the magnitude of the baby boom. Under this assumption, the number of newborns is constant at  $\overline{N}$  before a baby boom in period zero permanently increases it to  $N^b$  from period zero onward. Thus, this assumption is equivalent to a temporary increase in the *birth rate* in period zero, which is essentially the same as Abel's (2003) modeling of baby boom as a high realization of a random birth rate shock.<sup>66</sup> From another angle, while the severe baby boom in the above is a temporary increase in the number of newborns in period zero, the mild one here is a permanent increase from period zero onward.

As there are more young workers in period one for the mild baby boom than the severe one, the positive effect of baby boom on capital accumulation will be greater for the former, as depicted by the path  $K_0 N^b$  (representing the mild case) as compared to the path  $K_0 N^b \overline{N}$  (representing the severe case) in Figure 2. Thus, Proposition 2.4 also holds for the case of the mild baby boom. That is, the mild baby boom in period zero will have a positive effect on the capital stock in period one.

Since the demographic feature in period zero (and before) is identical for the two types of baby boom, according to equation (2.30), the impact on  $q_0$  of the mild baby boom is identical to that of the severe one. Thus, Proposition 2.5 and Corollary 2.4 also

<sup>&</sup>lt;sup>66</sup> In Abel's (2003, pp. 557-558) model, the number of young consumers follows a geometric random walk  $\ln N_t = \ln N_{t-1} + \varepsilon_t$ , which can also be written as  $N_t = N_{t-1}e^{\varepsilon_t}$ . Abel models baby boom as a high realization of the birth rate  $\varepsilon_t$ . Thus, suppose  $N_{-1} = \overline{N}$ , then a baby boom ( $\varepsilon_0 > 0$ ) in period zero will lead to  $N_{t\geq 0} = \overline{N}e^{\varepsilon_0} \equiv N^b$ .

apply to the mild baby boom. That is, the mild baby boom tends to cause a capital price boom in period zero.

Yet, since the number of newborns is different for the two types of baby boom, their impacts on  $q_1$  will be different. While equation (2.34) describes the effect of the severe baby boom on  $q_1$ , equation (2.34') in the following describes that of the mild baby boom:

$$dq_1 / dN^b = \Gamma(K_1, N^b) dK_1 / dN^b + \Psi, \qquad (2.34')$$

where  $\Gamma(K_1, N^b) = \left\{ \lambda \alpha (1-\alpha)(2+\theta)^{-1} K_1^{\alpha-1} (N^b)^{1-\alpha} - 1 - \eta \lambda (1-\alpha)^2 (2+\theta)^{-1} K_1^{\alpha} (N^b)^{1-\alpha} \right\} (1+\eta K_1)^{-2}$ and  $\Psi = (1+\eta K_1)^{-1} (1-\alpha) \eta \lambda (1-\alpha) (2+\theta)^{-1} K_1^{\alpha} (N^b)^{-\alpha}$ .

With  $\alpha < 1$ , it is not difficult to verify that  $\Psi > 0$ . Thus,  $dq_1 / dN^b$  will be more likely to be positive in equation (2.34') than in (2.34). Therefore, Proposition 2.6 and Corollary 2.5 and 2.6 will also apply to the mild baby boom. One difference is that the critical  $\hat{K}$ will be greater in the mild case than the severe one, which implies that it is more likely for the mild baby boom to have a positive effect on  $q_1$  than the severe one. This should not be surprising, because the retired baby boomers in the mild case will have more young workers to demand on their capital than their counterparts in the severe baby boom.

Since, as compared to the severe case, the mild baby boom will have the same effect on  $q_0$  and is more likely to have a positive effect on  $q_1$ , it will tend to be more likely to have a positive impact on  $q_1 - q_0$ . Therefore, Proposition 2.7 and Corollary 2.7 and 2.8 will also apply to the mild baby boom; and the critical  $\hat{K}$  will be greater, which implies that the meltdown is less likely to happen for the mild baby boom than the severe one.

Proposition 2.8 shows that, although the severe baby boom could increase  $q_1$ , it will definitely reduce the gross return  $R_1 + q_1$  for baby boomers' savings, because the negative impact on  $R_1$  (via the capital-labor ratio) of the severe baby boom is dominant over its potential positive effect on  $q_1$ . Yet, with its potential positive impact on  $q_1$  being greater as compared to the severe case, and its negative impact on  $R_1$  being smaller, the dominance may not apply to the mild baby boom. Thus, we conjecture that the mild baby boom could have a positive effect on  $RR_1$  (i.e. the rate of return to baby boomers' savings defined in equation 2.38).

Since it is difficult to mathematically determine the general sign of  $dRR_1/dN^b$ , we will prove our conjecture with a positive baby-boom effect on  $RR_1$  in a special case with perfect investment elasticity. With perfectly elastic investments (i.e.  $\eta = 0$ ), equation (2.19) implies that the capital price is constant at unity. Thus, according to equation (2.38),  $dRR_1/dN^b = dR_1/dN^b$ . According to equation (2.21),  $R_1 = \alpha \lambda K_1^{\alpha-1} (N^b)^{1-\alpha}$ . With perfect elastic investment, the capital stock in period one is equal to the saving of the period-zero young baby boomer. Thus, according to equation (2.4) and (2.17),  $K_1 = w_0 - C_0 = (2 + \theta)^{-1} (1 - \alpha) \lambda K_0^{\alpha} (N^b)^{1-\alpha}$ , which implies

$$R_{\rm l} = \alpha \lambda \left( (2+\theta)^{-1} (1-\alpha) \lambda K_0^{\alpha} \right)^{\alpha-1} (N^b)^{(1-\alpha)\alpha},$$

and

$$dRR_{1} / dN^{b} = dR_{1} / dN^{b} = (1 - \alpha)\alpha^{2} \lambda \Big( (2 + \theta)^{-1} (1 - \alpha)\lambda K_{0}^{\alpha} \Big)^{\alpha - 1} (N^{b})^{(1 - \alpha)\alpha - 1}$$
(2.42)

According to equation (2.42),  $dRR_1 / dN^b > 0$ , which implies that Proposition 2.8 does not hold for the mild baby boom case; that is, the mild baby boom in period zero can have a positive effect on the rate of return to capital in period one.

Intuitively, although the increase in the number of newborns in period zero leads to a larger capital stock in period one, the capital-labor ratio in period one may not increase, because the number of newborns (and hence the labor force) in period one is also higher. Indeed, the capital labor ratio will tend to decrease because, since the marginal product of labor is diminishing, so will be the extra capital generated by a marginal newborn in period zero for her counterpart in period one to work with.

### 4.3.5 Discussion

In the above we have shown that a limitation of the meltdown hypothesis is its neglect of the fact that baby boomers' prime-time savings can positively affect the asset demand (through positive impacts on incomes and savings) during their retirement ages. Since the meltdown is state contingent, a natural question is how likely it is in the reality. Although this is an empirical issue, the theoretical analysis in the above can provide some insights.

Our analysis has shown that a condition for period-zero baby boom to have a positive effect on the period-one capital price is  $sMPK_t > q_t$ . Since MPK (as the marginal product of capital) can be interpreted as the (unit) earning of capital, one might argue that it is unlikely that  $sMPK_t > q_t$  holds in the real world, because real-world cases for asset

earning—let alone a fraction (s < 1) of it—to be greater than asset price are rare. However, it should be noted that  $sMPK_t > q_t$  is a condition derived in a model where consumers live only two periods. Thus, as MPK represents the earning of capital for one model period that amounts to several real-world decades, the chance for  $sMPK_t > q_t$  to hold is not obviously slim. From another angle, in a model with agents living more than two periods, the supply of the existing capital in every period will be only a fraction of the existing capital stock because only a fraction of asset owners will be retiring, thus the condition for period-zero baby boom to have a positive effect on period-one capital price will not be  $sMPK_t > q_t$ , but be  $sMPK_t$  greater than a fraction of  $q_t$ .

Our analysis shows that, *ceteris paribus*, the closer the economy is to its steady state, the more likely its baby boomers will face a potential meltdown. Thus, being one of the most developed countries in the world, the U.S. baby boomers' chance of being in a favorable state may seem slim. However, the possibility for the U.S. economy to be in a non-meltdown state should not be completely excluded because, notwithstanding being a highly developed economy, the U.S. could still be far away from its steady state. For example, a stable capital-output ratio that the U.S. has been experiencing may indicate that the economy is close to its steady state; yet it could also be consistent with otherwise. For example, according to the Cobb-Douglas production function in equation (2.6), the capital-output ratio is defined as  $K_i/Y_i = \lambda^{-1}(K_i/L_i)^{(1-\alpha)}$ . If the technical coefficient  $\lambda$  is constant, a constant capital-output ratio  $(K_i/Y_i)$  will imply a constant capital-labor ratio  $(K_i/L_i)$  and hence a steady state. However, a constant  $K_i/Y_i$  can also be consistent with a situation where the capital-labor ratio  $(K_i / L_i)$  and technology ( $\lambda$ ) keep increasing simultaneously, which is a scenario of the U.S. economy being chasing its everincreasing (driven by technology progress) steady state. Although the conventional view in the literature is that the U.S. economy is close to its steady state, a recent study by Jones (2002) finds that rising educational attainment and research intensity in recent decades suggest that the U.S. economy may be far from its steady state.<sup>67</sup>

Although the meltdown hypothesis may yet become a major concern of China's baby boomers who just start entering their prime-time saving ages, the development stage of China may allow them to stand a better chance than their American counterparts to be "meltdown-free". Besides, they will fortunately have a chance to observe whether the U.S. experience supports the meltdown hypothesis.

Our analysis shows that the meltdown will be less likely to happen when the capital price is on an upward trend; or equivalently, when the capital price is positively correlated with the capital stock. Indeed, the possibility of an upward capital price trend (or a positive relationship between the capital price and capital stock) is the reason why the meltdown is state-contingent in the model here, yet will definitely happen in Abel's (2003) model where the capital price is always on a downward trend; and the relationship between the capital stock is always negative.<sup>68</sup> Since the capital price in

<sup>&</sup>lt;sup>67</sup> One insight provided by Jone's (2002) paper is that a "constant growth path" may not necessarily be the (steady-state) "balance growth path", but could be "driven by transition dynamics" (*ibid*, p. 221). For example, a non-steady-state yet constant growth path in a Solow model can be caused by an exponentially growing investment rate (*ibid*, p. 221); or in Jones' model, it can be "because of growth in the human capital investment rate and in research intensity" (*ibid*, p. 230).

<sup>&</sup>lt;sup>68</sup> As pointed out by Abel (2003, pp. 561-562) and concurred by the analysis here, the capital price will be mean-reverting after the baby boom shock. Since in Abel's model the "high value of  $K_{t+1}$  reduces the price of capital at any level of investment in period t+1" (*ibid*, p. 562)—which implies that the correlation

both Abel's model and the model here is a theoretical variable that does not exactly correspond to specific asset prices in reality, it is difficult to judge which case is more empirically relevant to the U.S. economy, a downward capital price trend or an upward one. Yet, judging from the fact that the U.S. stock prices have been generally on upward trends, the case for the U.S. being in a situation corresponding to an upward capital price trend may not be less likely than the case of a downward trend.

Unless the U.S. baby boomers believe that the underlying trends of the U.S. asset prices have been or will be downward sloping, they may not need to worry too much about the meltdown hypothesis, because the meltdown is less likely to happen when asset prices are reverting to upward trends. Perhaps they (as a whole) should not worry about it at all, since in the following we will show that forward-looking baby boomers' aggregate attempt to escape from the potential meltdown tends to be futile, and may lead the economy into a current "liquidity trap".

#### 4.4 Baby boom and liquidity trap

The above theoretical analysis does not rule out the possibility of baby-boom-induced asset market meltdowns. Thus, a natural question is whether forward-looking baby boomers can protect themselves against potential meltdowns. Using the above model we examine this issue in the following. Note that the analysis below is based on the case of the sever baby boom that is more meltdown-prone.

between the capital price and capital stock will always be negative; and the capital price will always be on a downward trend—thus the meltdown will definitely happen as the capital price reverts to its downward trend. Yet, the meltdown may not happen when the capital price is reverting to an upward trend. See section 4.3.3.4 for more detailed discussion.

We start from the following insight about the impact of expected future capital price  $(E_tq_{t+1})$  on current equilibrium.

**Proposition 3.1** In the above model, the current equilibrium value of endogenous variables except the bond interest rate  $(r_{t+1})$  is independent of expected future capital price  $(E_tq_{t+1})$ .

Recall that the period-*t* equilibrium of the economy is characterized by the simultaneous system composed of equations (2.15)-(2.24). Since the variable  $r_{t+1}$  and  $E_tq_{t+1}$  only appear in equation (2.20), the simultaneous system without equation (2.20) will be sufficient to determine all the endogenous variables other than  $r_{t+1}$ . Therefore, in equilibrium the impact of  $E_tq_{t+1}$  will be on  $r_{t+1}$  only.

One implication of Proposition 3.1 is that perfect foresight will not be able to help baby boomers (as a whole) to escape from potential meltdowns. Expecting potential capital market meltdowns (i.e., a low  $E_tq_{t+1}$ ), baby boomers can shift from capital to the one-period government bond (D). However, according to Proposition 3.1,  $E_tq_{t+1}$  will have no impacts on  $q_t$  or  $R_{t+1}$ ; thus the impact of a low  $E_tq_{t+1}$  will be to reduce  $r_{t+1}$  to a level equal to the rate of return to capital. Therefore, despite free from price variations, bond will not be a "safe haven" for baby boomers' savings, since its interest rate is market determined.

In general, baby boomers' attempts to flee from capital to short-term and/or riskless assets will not be able to shelter themselves against potential capital market meltdowns, but rather tend to drag the general interest rate down to such a level that all assets become as "unattractive" as capital.

With a zero bound on its interest rate, bond will be a safe haven when potential meltdowns are so severe that the rate of return to capital becomes negative. Nevertheless, with a fixed bond supply,<sup>69</sup> the safe haven will be too small to shelter all of baby boomers' wealth. Therefore, a possible scenario will be the following: Baby boomers still have to hold capital with negative returns; the bond interest rate is zero; and there is excessive demand for bond. Mathematically, when the equilibrium rate of return to capital is negative, equation (2.20) will not hold any more, with a zero left-hand side and a negative right-hand side. This implies a situation of asset market disequilibrium, or more specifically, excessive demand for bond.

In sum, even with perfect foresights, baby boomers may have to bear with potential capital market (or asset market in general) meltdowns in their old ages as an ill-fated consequence of the family plans of their parents and themselves as parents.

Nevertheless, a little reflection on reality indicates that baby boomers should at least be able to guarantee non-negative returns to their savings, because they can always choose to hold on their wage incomes, which tend to be paid in form of money—shoe workers in reality are seldom paid in shoes.

Based on this observation, we have the following conjecture. Should baby boomers during their saving ages plan to hold on their monetary wages in order to protect themselves against negative returns implied by potential capital market meltdowns during

<sup>&</sup>lt;sup>69</sup> In general, the supply of government bond is driven by government's fiscal policies and tends not to be perfectly elastic to the interest rate. Therefore, I assume a fixed bond supply for simplicity.

their dissaving ages, they may not be able to earn the wages in the first place. That firms are willing to pay factors in money is because they expect to recover it via selling the goods produced by the factors. Yet, baby boomers' "hoarding" behaviors will make firms unable to sell all the goods and hence incur negative profits. Expecting such a situation, firms may not want to produce as much. Then, a "liquidity-trap" scenario could happen, in which the return to capital as well as the general interest rate is on its zero bound; and some baby boomers are unemployed. Based on the above model, we examine this conjecture in the following.

We first modify the above model by assuming that instead of consumption goods, firms pay factors with "money", which is a default-free instrument that promises (by firms) to pay its bearer one unit of consumption whenever presented.<sup>70</sup> Accordingly, we assume firms accept only money as payments for their goods. To differentiate, we refer to the original model (without money) as the "real" model, and the modified model (with money) as the "monetary" model.

Money modeled as such is an asset with zero rate of return. Nevertheless, when the rate of return to capital is positive, money will be an inferior asset and hence not used as store of value. Thus, firms will be able to recover all the monetary factor payments. In this situation, money essentially plays the role of medium of exchange, which will not be captured by the equilibrium of the economy. In other words, equilibrium will be the same in the monetary model as the real model.

<sup>&</sup>lt;sup>70</sup> The assumption of "money" issued by firms is a convenience way to capture the feature of "inside" money without explicitly modeling financial intermediation. Another alternative is to assume that firms can borrow money from government, pay it as wages, and then recover it from selling goods. Since in situations under our consideration firms will recover all the money they pay out, then whether the money is issued by firms or borrowed from government will not matter.

When potential future capital market meltdowns are so severe that the rate of return to capital is expected to be negative, money will nonetheless become a relatively attractive asset; and its zero rate of interest may provide a zero bound for the rate of return to capital. With respect to this situation, we in the following examine the equilibrium (or lack of which) in the real and monetary models.

According to the simultaneous system (2.15)-(2.24), a period-zero full-employment equilibrium can be characterized by  $\{q_0^e, q_1^e, R_1^e, K_1^e, L_0^e, RR_1^e; N^b\}$  that satisfies the following simultaneous system.

$$L_0 = N^b \tag{3.1}$$

$$q_0 = (1 + \eta K_0)^{-1} \left( 1 + \eta \lambda (1 - \alpha) (2 + \theta)^{-1} K_0^{\alpha} L_0^{1 - \alpha} \right)$$
(3.2)

$$K_1 = K_0 + (q_0 - 1)\eta^{-1}$$
(3.3)

$$q_{1} = (1 + \eta K_{1})^{-1} \left( 1 + \eta \lambda (1 - \alpha) (2 + \theta)^{-1} K_{1}^{\alpha} \overline{N}^{1 - \alpha} \right)$$
(3.4)

$$R_{1} = \alpha \lambda K_{1}^{\alpha - 1} \overline{N}^{1 - \alpha}$$
(3.5)

$$RR_1 = \frac{R_1 + q_1 - q_0}{q_0} \tag{3.6}$$

Equation (3.1) represents the full-employment condition in period zero. Equation (3.2), derived from equations (2.6), (2.15), (2.17), (2.18) and (2.19), is a necessary condition for all the markets being simultaneously cleared. According to this equation, the following proposition is self-evident.
**Proposition 3.2** Given capital stock  $K_0$  and employment  $L_0$ , the market-clearing periodzero capital price  $q_0^e$  is uniquely determined.

Equation (3.3) captures the period-zero capital accumulation. Similar to equation (3.2), equation (3.4) is the (rationally expected) period-one market clearing condition. Finally, equations (3.5) and (3.6) represent the determinations of period-one capital income and rate of return to capital respectively.

With respect to the period-one (equilibrium) rate of return to capital in the real model, we have the following proposition.

**Proposition 3.3** In the real model,  $\exists N^{b^*}: RR_1^e = 0$ .

**Corollary 3.1** In the real model,  $\forall N^b < N^{b^*}$ :  $RR_1^e > 0$ 

**Corollary 3.2** In the real model,  $\forall N^b > N^{b^*}$ :  $RR_1^e < 0$ 

Proof: Equation (2.30) implies that  $\lim_{N^b \to \infty} q_0 = \infty$ , which, together with inequality (2.40), implies that  $\lim_{N^b \to \infty} RR_1 = -1$ . Then, according to inequality (2.41), a unique  $N^{b^*}$  must exist; and Corollaries 3.1 and 3.2 are self-evident.

Proposition 3.3 verifies that a large enough period-zero baby boom can lead to potential negative rate of return to capital in period one.

According to Proposition 3.3, when  $N^b \leq N^{b^*}$ ,  $RR_1^e$  will be non-negative; hence the zero-interest bound in the monetary model will not be binding. Thus, we have the following proposition.

**Proposition 3.4**  $\forall N^b \leq N^{b^*}$ , the equilibria in the monetary and real models are identical.

On the other hand, the existence of the zero-interest bound in the monetary model implies that, if  $RR_1 < 0$ , consumers will have incentives to hold money as store of value. Then firms will not be able to recover all their factor payments, which implies that the goods market will not be cleared. Therefore, we have the following proposition.

**Proposition 3.5** In the monetary model, a necessary condition for goods market equilibrium is  $RR_1 \ge 0$ .

**Corollary 3.3** In the monetary model,  $\forall N^b > N^{b^*}$ , equilibrium with all markets simultaneously cleared does not exist.

Proof (by contradiction): Suppose equilibrium  $\{q_0^e, q_1^e, R_1^e, K_1^e, L_0^e, RR_1^e; N^b > N^{b^*}\}$  exists; then, according to Corollary 3.2,  $RR_1^e < 0$ , which is in contradiction with Proposition 3.5.

Now it should be clear that, although the zero-interest bound provides forward-looking baby boomers an option to protect the value of their wealth, it will be at the cost of market equilibrium. While equilibrium is usually well defined, "disequilibrium" states are not, depending on which market (or markets) is in disequilibrium. Recall that there are five markets in the model: labor, goods, capital, rental capital, and bond, among which the capital and bond markets are the least likely to be in disequilibrium because of their efficiency. Disequilibrium in the goods market, which implies negative profits for firms' production, is also not likely to sustain.

Arguably, the most likely scenario is as follows. Expecting a potential future capital market meltdown, baby boomers will avoid holding capital, which will cause low capital price and hence lead to insufficient aggregate demand. The impact of the insufficient demand will be eventually felt by factor markets as firms reduce production accordingly. While the rental rate for capital tends to be flexible, the wage rate for labor is likely to be rigid. Thus, a disequilibrium state could be such that all other four markets are in equilibrium except the labor market. We call such a state "labor-market disequilibrium", in which there exists (involuntary) unemployment.

Many factors (e.g. contract or union) can cause wage rigidity, which we will not model explicitly but simply assume the following. Firms will pay employed baby boomers by their marginal products; and the rest of baby boomers will stay unemployed even though they are also willing to work under the current wage rate.

With respect to such a labor-market disequilibrium state, we have the following proposition.

**Proposition 3.6** Denote a labor-market disequilibrium as  $\{q_0^{de}, q_1^{de}, R_1^{de}, K_1^{de}, L_0^{de}, RR_1^{de}; N^b\}$ . Then,  $\forall N^b > N^{b^*}$ ,  $\{q_0^{de}, q_1^{de}, R_1^{de}, K_1^{de}, RR_1^{de}; N^b\} = \{q_0^e, q_1^e, R_1^e, K_1^e, L_0^e, RR_1^e; N^{b^*}\}$ .

Proposition 3.6 says that, for any baby-boom magnitude greater than  $N^{b^*}$ , the corresponding labor-market *disequilibrium* state will be "equivalent" to the *equilibrium* state when the baby-boom magnitude is equal to  $N^{b^*}$ . It should be noted that the "equivalent" is from an aggregate point of view—with different numbers of baby boomers, the two states will certainly not equivalent from individual baby boomers' perspectives. The proof of this proposition is straightforward. As  $N^b > N^{b^*}$ , the zero-interest bound is binding. Thus, the labor-market disequilibrium state needs to satisfy equations (3.2)-(3.5) together with  $RR_1 = 0$ . Then, according to Proposition (3.3), the disequilibrium state (with  $N^b > N^{b^*}$ ) can be uniquely characterized by the equilibrium state (with  $N^b = N^{b^*}$ ).

A self-evident corollary of Proposition 3.6 is as follows.

**Corollary 3.4** Denote the period-zero unemployment rate as  $u(N^b) = (N^b - L_0)/N^b$ ; then,  $u(N^b)|_{N^b > N^{b^*}} > 0$  and  $du/dN^b|_{N^b > N^{b^*}} > 0$ .

That is, a labor-market disequilibrium state with unemployment will occur when the magnitude of baby boom exceeds  $N^{b^*}$ ; and the larger the baby boom is, the higher the unemployment will be.

## 4.5 Summary

When baby boomers' large savings cannot be effectively turned into investments due to investment impediments, they tend to drive asset price booms. However, whether babyboom-driven asset price booms will meltdown (as commonly predicted) during baby boomers' retirement eras is state contingent, depending on whether large capital stock built up by baby boomers' savings can generate enough asset demand (indirectly through high incomes) to sustain the asset price booms.

However, when the meltdown is unfortunately about to happen, baby boomers' attempts to escape it will be futile and merely drag down the general interest rate level for the entire asset markets. Although a zero-interest bound (thanks to the existence of money) can protect baby boomers against negative returns in the future, it would nevertheless be at the cost of current unemployment in a liquidity trap.

## **Chapter Five**

## Conclusion

Gramlich (2001) argues that since monetary policy affects asset prices through the interest rate, its wealth effect on consumption is unclear because, despite the changes in asset prices, consumers are as rich as before in terms of earnings.

Motivated by this intriguing issue, this dissertation starts with a general examination of the nature of (fundamental) "paper" wealth and its impact on consumption. Since prices are determined by demand and supply, asset price variations can occur even when asset earnings remain the same. We refer to asset value created or destroyed by such asset price variations as "paper" wealth in that they do not directly correspond to changes in the aggregate availability of real resources. In this dissertation we consider only "fundamental paper wealth", which is referred to paper wealth changes driven by fundamental factors (as opposed to speculative bubbles).

We find that changes in fundamental paper wealth are wealth redistribution between current and future asset owners. An increase in fundamental paper wealth tends to benefit current asset owners at the cost of future owners; vice versa for a decline in fundamental paper wealth. Since agents' consumption horizons are not likely to be infinite, the wealth redistribution of an increase in fundamental paper wealth tends to make current consumers (as a whole) wealthier and hence have a positive impact on the current aggregate consumption; vice versa for a decline in fundamental paper wealth. Based on the concept of fundamental paper wealth and its effect on consumption, we examine three issues related to asset prices: the wealth effect of monetary policy and its interplay with Tobin's q effect, stylized foreign-capital-financed consumption booms in developing countries, and the asset market meltdown hypothesis on asset price boom-bust cycles driven by baby boom.

For the first issue, we examine the wealth effect of monetary policy and its interplay with Tobin's q effect in a general equilibrium framework. We find that the higher the investment elasticity is, the greater the impact of monetary policy will be on investments through Tobin's q effect; and the less the impact will be on asset prices, which implies a smaller wealth effect.

We point out two implications of the interplay between the wealth and Tobin's q effect. One is that monetary policy intended to stabilize short-term fluctuations may nevertheless allow saving crowd-out that has a negative effect on long-term growth; the other is that monetary policy that stabilizes the goods market may lead to instability in the asset market. Thus, monetary policy may not be a "panacea" stabilization tool optimal for every circumstance. In situations where economic instability is caused by long-term, observable (or foreseeable) shocks such as demographic changes, other policy options may need to be considered.

While the monetary impact on asset prices and the corresponding wealth effect can be a liability of monetary policy as a stabilization tool, they can also be its asset. Since monetary policy can influence asset prices; and the influence will affect the aggregate demand, asset prices are a potential monetary policy instrument, at least in principle. Being a monetary policy instrument, asset prices may not be as convenient as the interest rate in normal occasions; yet they can be advantageous under special circumstances. For example, when the economy is in a liquidity trap where conventional monetary expansion through cutting the interest rate is not possible, asset prices could be a useful alternative. What the central bank needs to do is to defend a lower bound of nominal capital price that is high enough to pull the economy out of recession. With the ability to supply infinite amount of money in principle, such an "open mouth operation" by the central bank will be effective in pushing up asset prices—imagine what would happen if Alan Greenspan came out to say the U.S. stock markets are undervalued by 5%. With the potential wealth effect, the asset price appreciation will be effective in stimulating aggregate demand. Therefore, this unconventional monetary policy will work, at least in theory; and a formal examination shows that it has some advantages over its counterparts in the existing literature (Cai, 2004).

Financial communities may applaud the idea that central banks help initiate asset market booms; yet they tend to resent central banks' attempts to curb existing asset market booms and argue that central banks with no ability in distinguishing between fundamental asset price movements and otherwise should keep their hands off the asset market.<sup>71</sup> Yet the analysis here shows that this popular view is unfounded, because the impact of monetary policy on asset prices is to change assets' fundamental paper value

<sup>&</sup>lt;sup>71</sup> For fear of a U.S. version of Japan's experience in the 1980s, the Federal Reserve initiated six consecutive interest rate cuts from mid-1999 till mid-2000. With no obvious signs of inflation, the tightening was officially justified as preemption over inflation pressures (Greenspan, 1999). Yet, financial communities interpreted it as Fed's act to curb then booming stock markets—an understandable suspicion after all the talks by Fed officials about "irrational exuberance" and "wealth effects"—and condemned it as a violation of the no-intervention principle on the ground that the stock market performances are consistent with future productivity potentials in a "New Economy", if not undervalued.

that will neither correct nor cause asset price misalignments, which are due to the inconsistency between earning expectations and future fundamentals. Indeed, since asset price movements, fundamental or otherwise, tend to affect aggregate demand, asset prices are not less legitimate than the interest rate as a monetary policy instrument, at least in principle. Should the concept of paper wealth and the insights provided by Chapter One be appreciated, central bankers will not have to disingenuously explain to the general public that their policies are not aimed at asset prices *per se* but their wealth effect; rather, they could simply remind the public that monetary policy will not affect their wealth *per se*, but its paper value. Certainly, the explanation will not be necessary if the public appreciates the point in the first place.

Indeed, if the general public appreciates the concept of paper wealth, financial markets could become more stable. During every unusual boom, some pundits claimed that something different was happening and made the boom fundamental. When the boom eventually went bust, some other pundits came out to vindicate their underappreciated prophecies of bubbles, and pessimistically predict that people will never learn. Yet, we believe that if you teach them right, people will learn. The concept of paper wealth may help—if people understand that asset prices depend not only on earnings but also on how earnings are valued, they will understand that asset prices (as paper wealth) are intrinsically volatile; and fundamental market booms can turn into fundamental busts. There will still be many speculators in the game; yet innocent investors will be protected from the misperception that fundamental asset price movements must be sound. The essence of a what I called as Hamming's investment

philosophy (named after my father) is to enjoy the capital gain when asset prices are up, but to remind oneself that earnings may still be intact (i.e., paper-wealth loss only) when prices are down. Adopted individually, such a philosophy may be naïve, if not selfdeceiving. Yet, imagine if most of investors subscribe to it: Financial markets will become more stable; or to the least, self-fulfilling asset market stampedes (driven by panics) will be less likely.

The second issue examined in the dissertation is related to open capital account, which is a policy agenda for developing countries widely supported by academics and enthusiastically pursued by policymakers (e.g. IMF). As opposed to the existing literature explaining foreign-capital-financed consumption booms as a macroeconomic side effect of open capital account due to institutional imperfections, we conjecture that they can be a fundamental outcome of open capital account, under which ineffectiveness in using foreign savings for investments tends to result in capital inflows being channeled to consumption through the wealth effect. Our analysis confirms this conjecture in the cases of capital inflows driven by capital account liberalization, reduction in the world interest rate, and uncertain productivity shocks.

One may argue that, notwithstanding financed by capital inflows, consumption booms without institutional imperfections are not excessive but a normal outcome of efficient market mechanisms. In particular, post-liberalization high consumption can be the result of optimal consumption intertemporal allocation by an everlasting representative agent (or dynasty) facing a lowered interest rate. However, the welfare implication from a finite-horizon perspective is more complicated. On the one hand, a *ceteris paribus* 

interest rate fall tends to benefit current non-human wealth owners (through asset price appreciation) at the cost of current and future human wealth owners (through a lowered rate of return to savings). On the other hand, high investments induced by the interest rate fall can lead to high labor incomes, which (if sufficiently high) can compensate (or outweigh) human-wealth owners' losses. Therefore, the welfare implication of capital account liberalization to human wealth owners is ambiguous and depends on its effect on capital formation. If low investment elasticity makes the gains from high labor incomes dominated by the losses from low returns to savings, capital account liberalization will have a negative impact on human-wealth owners' wellbeing.

A somewhat surprising insight is that, while low post-liberalization investments (due to low investment elasticity) reduce the total benefit of capital account liberalization for society as a whole, they nevertheless tend to make current consumers as a whole better off through "paper" wealth creation. Such wealth is not bubbles driven by speculation or misalignments due to credit overexpansion, but rather a result of asset revaluation under lower interest rates. Notwithstanding commonly viewed as "real" wealth, we call it "paper" wealth to emphasize the fact that it can be destroyed as easily as it is created. During the 1997 Asian financial crises, Mahathir Mohammad, the Prime Minister of Malaysia at that time, complained that it took speculators only two weeks to destroy the wealth painstakingly accumulated by Malaysian people over decades. Wealth that can easily "evaporate" without physical resources being destroyed, notwithstanding "real", may not be concrete enough. In this sense, a concrete-paper wealth tradeoff leaned to the "paper" side may not be really in the interest of current consumers. Besides, foreign-

capital-financed consumption booms tend to be the root of low savings, high current account deficits, and real exchange rate appreciation (if non-tradable goods taken into consideration), which combined tend to be a recipe for crises.

If taken as undesirable, what can be done to prevent foreign-capital-finance consumption booms? To increase investment elasticity through reducing investment impediments (such as nurturing entrepreneur spirit or reducing uncertainties) will certainly help, but may not be easy or practical. Investment subsidies can be used to stimulate investments directly, but may not be practical and can have little influence on temporary investment sluggishness caused by the "wait-and-see" strategy. Consumption credit controls can restrain consumption financed by borrowing but not those by asset holdings. Capital controls can avoid paper wealth creation by aligning the domestic interest rate level with the earning level of domestic assets; yet, its enforceability and "collateral damages" need to be taken into consideration. All in all, sensible policy prescriptions for addressing foreign-capital-financed consumption booms tend to be case sensitive and belong to the scope of empirical studies. The main contribution here is to provide a diagnosis underappreciated by the existing literature.

The third issue examined in the dissertation is related to the "asset market meltdown hypothesis", which predicts that baby boomers' prime-time savings will drive asset market booms that will eventually collapse due to their retirement dissavings. We show that this bleak scenario may not necessarily happen, because the capital stock built up by baby boomers' savings may be able to generate sufficient asset demands to sustain the asset prices during their retirement ages. Yet, we show that, if unfortunately the economy is in a situation where meltdown tends to happen, baby boomers as a whole may not be able to avoid it. Although they can choose to hold short-term assets, the resulting downward pressure on current asset prices could push the economy into a liquidity trap.

Our theoretical-oriented analysis here is based on a two-period conceptual model. A potential future research topic is to use more stylized models to empirically examine the likelihood of asset market meltdown driven by the U.S. baby boomers' retirement dissavings. Also, the subjects of the empirical study can include other industrial countries (e.g. Japan and West Europe) that are facing the problem of population aging as well as developing countries that are or will be facing the problem (e.g. China). Besides, our theoretical analysis can be developed further to consider the implications of endogenous growth. We conjecture that since baby boomers' savings can not only help build up a large capital stock but also stimulate technology progress, the meltdown will be less likely to occur.

Since the meltdown may not necessarily happen; and if it will, baby boomers as a whole has no way to escape, we suggest "downplaying" the meltdown hypothesis because, even if the meltdown will not happen in the future, the fear of it will tend to depress current asset prices and may result in a liquidity trap. Arguably, one culprit responsible for Japan's "lost decade" during the 1990s is its undervalued asset prices, which may be caused by investors' pessimistic expectations of a negative impact of Japan's aging population on future asset prices. Although (as discussed above) liquidity traps are not formidable if central banks are willing to use asset prices as an unconventional policy instrument, it would be better to avoid being trapped in the first place.

The main message to baby boomers provided by the analysis here is that their retirement well-beings depend ultimately on production capacity. It may not be realistic to expect the process of (paper) wealth creation during 1990s can go on forever; yet the hypothesis of a future meltdown could be over pessimistic. With respect to the disturbing question "Sell? Sell to whom?" that epitomizes the meltdown hypothesis (Siegel, 1999, p.41), we offer a comforting answer: "Sell to a richer generation". If production capacity can provide enough goods and services—a society that keeps worrying about unemployment may not need to be concerned too much about this—baby boomers will be rich enough to afford them, because paper wealth can be generated through market mechanisms (with the help of the central bank). Therefore, the rational reaction for baby boomers *as a whole* to the meltdown hypothesis is to ignore it—after all, we have shown that baby boomers as a whole has no escape; and their attempts to escape (from a meltdown that may not necessarily happen) could nevertheless push the current economy in a liquidity trap.

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