HYDROELASTICITY OF MARINE VESSELS ADVANCING IN A SEAWAY

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ABSTRACT

Hydroelasticity is an important issue in the design of modern-day marine vessels, because of their flexibility associated with lighter construction materials and higher design speeds. The present study extends the hydroelasticity method by including effects of vessel forward speed and utilizes a direct solution approach instead of the modal superposition method, which requires structural details not available in early design stages. The model has two components, describing respectively, the elastic deformation of the vessel and the motion of the fluid. Small amplitude assumptions of the surface waves and vessel deformation lead to linearization of the problem, which is solved in the frequency domain. The formulation adopts a translating coordinate system with the vessel speed. The linear free surface boundary conditions account for the modification of the steady flow around the vessel. The radiation condition for the scattered waves takes into account the Doppler effect due to forward speed. A boundary element model describes the potential flow associated with the current and waves around the vessel. A finite element model relates the structural deformation to the fluid pressure through the kinematic and dynamic boundary conditions on the wetted body surface. This direct coupling of the structural and hydrodynamic systems leads to a system of equations in terms of the body surface oscillation, which includes elastic and rigid-body motions. The model is verified and validated in part with laboratory data on a rigid hull advancing in head seas and with published numerical results from the modal superposition method without vessel forward speed. A parametric analysis of a Wigley hull shows the forward speed introduces new resonance modes that amplify the response and stress of the vessel. The model provides a useful design tool to investigate the effects of vessel elastic deformation and forward speeds on structural performance and seakeeping.

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Chapter 1

Introduction

Hydroelasticity is concerned with the interaction between the motions of a fluid and an elastic structure. This is an important consideration in the design of modern-day marine vessels with lighter construction and higher operating speeds. Most hydroelasticity models, however, cater to the analysis of ocean structures with zero-forward speed. The forward motion introduces Doppler shift of the wave field around the vessel that modifies the hydrodynamic characteristics beyond what an encounter frequency can account for. The changes in the added mass, damping, and excitation force alter the response and resonance characteristics of a rigid body and their effects on a flexible hull are not known with certainty.

1.1 Hydroelasticity of Ocean Structures

A standard ship design procedure generally involves separate hydrodynamic and structural analysis. The motions and loads of the vessel in waves are determined as a rigid body. The ship structural elements are designed by dynamic analysis using hydrodynamic, hydrostatic, and inertia loads from empirical formulas or numerical

models (ABS, 2009). Container ships, large oil tankers, liquefied natural gas (LNG) carriers, and most importantly, multi-hull vessels exhibit flexural motions in waves that in turn modify the hydrodynamic loads and rigid-body responses. A hydroelasticity model offers a rational approach to determine the design structural loads and fatigue life of these vessels operating in a seaway.

Bishop and Price (1979) were the first to analyze ships as flexible structures in waves. Their model is based on the modal superposition method with a slender ship. The hull deformation due to waves is represented as a series of free oscillation modes of a beam in air with equivalent structural properties. The first six modes represent the rigid body motions and the remaining modes represent the flexible deformation. The hydrodynamic loads, which include the radiation damping and added mass matrices and the external force vector associated with the oscillation modes, were approximated from strip theory (e.g., Newman, 1978). The resulting system of equations is solved to obtain the principal coordinate of each oscillation mode. Wu (1984) extended this method to a full three-dimensional model, in which the finite element method was used to determine the resonance modes of the ship structure in air and a distribution of pulsating sources provides the hydrodynamic loads (Bishop *et al.*, 1986).

Researchers and engineers have used the modal superposition method to evaluate the hydroelastic behavior of ships and large floating platforms. SWATH (Small Water-plane Area Twin Hull) vessels, because of their flexibility, serve as good examples of hydroelasticity analysis (e.g., Reane *et al.*, 1991; Che *et al.*, 1994). Cheung *et al.* (1998) compared the hydroelasticity method with standard approaches in the design of a SWATH vessel and demonstrated the importance of the vessel elastic response on the overall structural deformation. Huang and Riggs (2000) included a linear stiffness term

for the hydrostatic load due to deflections that becomes important for analysis of large flexible floating structures. Hirdaris *et al.* (2003) applied two-dimensional and three-dimensional hydroelasticity models to investigate the structural failure of bulk carriers. Senjanovic *et al.* (2008) applied the method to a flexible barge and presented comparisons with laboratory experiments in regular waves. A major drawback of the modal superposition method is that it is not suitable for the early design stage, when detailed structural information is not available. By not including the plate stiffeners and brackets can result in erroneous local resonance of the hull and bulkheads in the modal analysis rendering the method intractable.

An alternate approach is to couple the structural and fluid systems directly. This approach is necessary when the structure does not have a definitive shape or free vibration modes in air such as the case of floating fluid-filled membranes. Ohyama *et al.* (1989) modeled a bottom-mounted membrane as a series of springs and conducted laboratory experiments with the intention to validate their numerical results. Zhao (1995) adopted a governing equation based on the equilibrium of the hoop tension with the pressure difference across the membrane and investigated the response of a floating fluid-membrane system in gravity waves. Zhao and Aarsnes (1998) later included membrane elasticity and compared their numerical results with laboratory data. Phadke and Cheung (1999, 2001 & 2003) derived linear and nonlinear governing equations based on membrane theory of cylindrical shells and obtained a finite element model for the membrane through the variational method. The formulations of these two-dimensional numerical models are similar, in which the elastic model of the membrane and the boundary element models of the internal and external fluids are directly coupled through a force-equilibrium relation. Cheung *et al.* (2000) used this method of direct coupling to model the dynamic response of a pneumatic floating platform in three dimensions. Das and Cheung (2009) applied the

method to analyze the three-dimensional motions of a fluid filled membrane in gravity waves. This approach, which utilizes a Rankine source formulation, provides a straightforward extension to include vessel forward speed in hydroelasticity models.

The direct coupling method determines the vessel elastic response at the wave excitation frequency. The spurious free vibration modes due to the lack of structural details have much higher frequencies and would not affect the solution. This approach, unlike the modal superposition method, does not require a free vibration analysis of the structure in air and a subjective truncation of the series of vibration modes in the subsequent modal analysis, thereby preventing a potential source of errors introduced by the user or the numerical method itself.

1.2 Potential Flow with Vessel Forward Speed

Wave-induced motion of marine vessels with a forward speed has been a challenging research topic for many years. The forward speed modifies the encounter frequency in a seaway and generates a non-uniform flow around the vessel that modulates both the incident and scattered waves. The Doppler shift of the scattered waves in the near and far fields alters the hydrodynamic coefficients and excitation forces as well as the natural frequencies of the vessel. The dynamic characteristics of the vessel depend on the forward speed and the effect becomes more pronounced at resonance (Journée, 1992). The physical problem is complex and needs simplifications to derive a tractable solution for practical application. These include assumptions of potential flow as well as small amplitude surface waves and vessel responses for linearization of the problem; the modification of the steady flow field due to the vessel, if considered, is determined through the so-called double-body approximation, in which the free surface is replaced

by a rigid lid analogous to the flow around the body and its image about the free surface (Newman, 1978). The use of a translating coordinate system gives rise to a wave scattering problem in a steady current. The boundary integral method has been the primary treatment to this hydrodynamic problem and provided various forms of solutions in the frequency domain.

One approach with the boundary integral method is to distribute translating and pulsating sources on the wetted body surface to account for the forward motion in waves. This method utilizes a Green function that satisfies the linear free surface boundary condition and the radiation condition in the presence of a uniform current (Bessho, 1977). The vessel is considered to be slender and modification of the steady flow around the vessel is not considered. Distribution of constant boundary elements on the body surface provides a matrix equation from which the numerical solution can be determined (e.g., Wu and Eatock-Taylor, 1987; Ohkusu and Iwashita, 1987; Iwashita, 1997; Du *et al.* 1999). Qiu and Peng (2007) recently implemented a panel-free method with an accurate Non-Uniform Rational B-Spline (NURBS) representation of the body geometry. Since this forward-speed Green function does not account for the near-field flow conditions, Noblesse and Yang (2004) proposed a further simplification to satisfy approximately the linear free surface boundary condition in the near field and exactly in the far field. The resulting Green function is simpler and more efficient compared to the conventional approach (Bessho, 1977). Nossen *et al.* (1991), on the other hand, extended the boundary integral to include the near-field free surface to account for the modification of the steady flow around the body. The use of a Green function that satisfies the free surface boundary condition can lead to the so-called irregular frequencies, at which the matrix equation becomes singular. For most hull geometries, the irregular frequencies cover a substantial

bandwidth within the frequency range of interest and the resulting singularity can only be removed through computationally intensive schemes (Lee and Sclavounos, 1989).

The Rankine-source approach avoids the problem of irregular frequencies with a very simple Green function and its formulation directly accounts for the near-field current in wave scattering. Zhao and Faltinsen (1989) solved the two-dimensional problem by distribution of Rankine sources on the body and the near-field free surface and used a multipole expansion to match the near and far-field solutions in lieu of a radiation condition. Nakos and Sclavounos (1990, 1991) provided a three-dimensional solution with the Rankine source approach and applied a condition at the upstream boundary of a rectangular free surface to ensure no scattered waves propagate ahead of the vessel. Gao and Zou (2008) utilized the NURBS representation of three-dimensional bodies and implemented a numerical scheme to achieve the same condition at the upstream boundary. While this upstream boundary treatment is applicable to forward speeds greater than the group velocity of the scattered waves, both models of Nakos and Sclavounos (1991) and Gao and Zou (2008) do not have provisions for the radiation condition on the remaining boundaries. Implementation of the Rankine-source method in the time domain circumvents the problems with the radiation condition through a locally determined celerity (Isaacson and Cheung, 1993; Cheung *et al.* 1996) or a damping layer along the far-field boundary of the free surface (Büchmann *et al.* 1998; Kring 1998; Skourup *et al.* 2000; Kim and Kim, 2010). A frequency-domain model, nevertheless, provides a convenient analytical tool in conceptual and preliminary design by providing the amplitude and phase of the surface waves and vessel responses without subjective post processing of the time series output.

In spite of the benefits, the frequency-domain Rankine-source approach has not been utilized to its full potential for modeling of scattered waves around three-dimensional bodies with a forward speed. This is mainly due to the lack of a suitable radiation condition for the scattered waves in a current. *Ad hoc* treatments of the radiation condition provide an interim solution under some restrictive conditions with unknown effects to the model predictions. The suggested remedy of using a large computational domain does not necessarily negate the absence of a radiation condition regardless the forward speed of the vessel. A radiation condition that can account for the Doppler shift of the wave field is necessary to implement the Rankine source formulation for vessels with forward speeds.

1.3 Hydroelasticity with Vessel Forward Speed

Du *et al.* (1998) presented a three-dimensional hydroelasticity model for vessels with forward speed. While a finite element model describes precisely the ship structural components, the use of the modal superposition method and the forward-speed Green function in the formulation limits the general application of the model as pointed out in Sections 1.1 and 1.2. Kim *et al.* (2009) presented an alternate formulation with direct coupling between a Rankine source model of the fluid and a simplified structural model of the ship. In the absence of a proper radiation condition, the solution is obtained in the time domain through an iterative scheme. The computational requirements make it necessary to approximate the kinematic body boundary condition by the motion of a series of rigid hull sections connected to the finite element model of a Vlasov beam representing the hull structure. These unsolved issues have prevented general application of the hydroelasticity method in ship design and analysis. The dynamic analysis in ABS

(2009), which does not consider coupling of the hydrodynamic and structural models, remains the norm in practical application.

The goal of the proposed study is to develop a hydroelasticity method in the frequency domain, which addresses the aforementioned issues, and to understand and predict the effects of forward motion on the elastic response of marine vessels in a seaway. Direct coupling of a finite element model of the ship structure and a boundary element model of the steady and oscillatory flows would remove the limitations of the modal superposition method and enables application of the model in early design stages. The direct coupling approach is common in a time-domain model, but its implementation in the frequency domain is not immediately evident. The use of a Rankine source formulation of the boundary element model eliminates the problems with irregular frequencies and allows description of the near-field current in wave scattering through the free surface boundary conditions. The frequency-domain formulation requires a radiation condition that can account for the Doppler shift of the scattered wave field. The development of the radiation condition is a main focus of this research and an essential component of the proposed hydroelasticity model. The present model allows a thorough investigation of rigid and elastic ship behaviors advancing in a seaway that previous models cannot fully elucidate.

In this dissertation, Chapter 2 describes the hydrodynamic formulation of a rigid body advancing in a seaway. The formulation adopts a translating coordinate system at the vessel forward speed. The free surface boundary condition accounts for the modification of the uniform current around the vessel and a new radiation condition takes into account the Doppler shift of the scattered waves. Chapter 3 summarizes the finite element formulation for a planar shell element used in this study. The shell element is derived by

combining the three node triangular membrane element of Carpenter *et al.* (1985) and the triangular Discrete Kirchhoff Theory (DKT) plate bending element of Batoz *et al.* (1980). The condition for dynamic equilibrium is derived to obtain the governing equation of motion for the entire body. Chapter 4 presents the boundary element model for a flexible body advancing in waves. A coupling scheme is derived for the boundary and finite element models through a kinematic and a dynamic boundary condition on the wetted hull surface. Chapter 5 establishes the convergence criteria of the hydrodynamic model, verifies the proposed radiation condition, and examines the scattered wave patterns for a full range of forward speeds, through a numerical experiment with a Wigley hull form in Journee (1992). Chapter 6 validates the coupled hydroelasticity model by comparison with the laboratory data of Journee (1992) for the limiting case of a rigid Wigley hull moving in head seas. The motions of a flexible Wigley hull with zero forward speed are then compared with those obtained by Riggs *et al.* (2007) using the modal superposition method. The model is then applied to a moving flexible hull to understand the effects of forward speed on the vessel elastic response. Chapter 7 outlines the research highlights and summarizes the conclusions derived from this study.

Chapter 2

Hydrodynamic Formulation

The hydroelasticity model consists of a boundary element model for the fluid flow around a body advancing in waves and a finite element model for the structural deformation of the body based on shell theory. A description of the boundary element model based on potential theory and the boundary conditions adapted for a moving rigid body is presented here.

2.1 Boundary-value Problems

Fig. 2.1 illustrates the boundary-value problem of a vessel traveling with a steady forward speed U in a seaway. In the Cartesian coordinate system, the horizontal axes (x_1, x_2) originate from the geometric centroid of the vessel water-plane in the longitudinal and transverse directions and the vertical axis x_3 points upward from the still water level. The regular incident waves of amplitude A propagate in uniform water depth d and the wave direction makes an angle β with the x_1 axis. With this translating reference frame, the vessel responds in the six degrees of freedom to the incident waves on a steady current in the negative x_1 direction. Small amplitude assumptions of the surface waves

and body oscillations result in a linear boundary-value problem defined at the still water surface S_F , the body surface S_B below the still water level, the seabed at $x_3 = -d$, and a control surface S_C truncating the infinite fluid domain.

The fluid is inviscid and incompressible and the irrotational flow satisfies the Laplace equation in the framework of potential theory. Let t denote time and $\mathbf{x} = (x_1, x_2, x_3)$ the position vector. A complex velocity potential provides a description of the flow as

$$\Phi(\mathbf{x}, t) = U \left[\phi_0(\mathbf{x}) - x_1 \right] + \sum_{l=1}^6 \alpha_l \phi_l(\mathbf{x}) e^{-i\omega_e t} + \left[\phi_W(\mathbf{x}) + \phi_7(\mathbf{x}) \right] e^{-i\omega_e t} \quad (2.1)$$

where $i = \sqrt{-1}$; ω_e denotes the encounter frequency; ϕ_0 represents the modification to the uniform current by the body; ϕ_l and α_l ($l = 1, 2, \dots, 6$) are the radiation potentials and the oscillation amplitudes in surge, sway, heave, roll, pitch, and yaw with respect to the water-plane centroid; ϕ_W is the incident wave potential; and ϕ_7 is the diffraction potential. The encounter frequency is expressed in terms of the angular frequency ω and wave number k of the incident waves as

$$\omega_e = \omega - Uk \cos \beta \quad (2.2)$$

Linear wave theory provides the potential for incident waves as

$$\phi_W(\mathbf{x}) = \frac{-igA}{\omega} \frac{\cosh k(x_3 + d)}{\cosh kd} e^{i[k(x_1 \cos \beta + x_2 \sin \beta)]} \quad (2.3)$$

where g is acceleration due to gravity. The remaining steady and oscillatory flow potentials are solved from a series of boundary-value problems in terms of the current speed U .

The steady potential ϕ_0 , which accounts for the near-field modification of the uniform current, is important to wave scattering around non-slender vessels with a forward speed. In the boundary-value problem, it satisfies the zero flux condition on the still water

surface at $x_3 = 0$, the seabed at $x_3 = -d$, and the body surface as

$$\frac{\partial \phi_0}{\partial n} = 0 \quad \text{at} \quad x_3 = 0 \text{ and } -d \quad (2.4)$$

$$\mathbf{W} \cdot \mathbf{n} = 0 \quad \text{on} \quad S_B \quad (2.5)$$

where $\mathbf{n} = (n_1, n_2, n_3)$ is a unit normal vector pointing away from the fluid domain; n is distance along $\mathbf{n}$; and $\mathbf{W}$ is the flow velocity vector in the translating coordinate system defined as

$$\mathbf{W} = U \nabla (\phi_0 - x_1) \quad (2.6)$$

The body boundary condition in Eq. (2.5) can also be expressed in terms of ϕ_0 as

$$\frac{\partial \phi_0}{\partial n} = n_1 \quad \text{on} \quad S_B \quad (2.7)$$

In this so-called double-body flow, $\nabla \phi_0$ attenuates rapidly and ϕ_0 becomes a constant away from the body. Both are set equal to zero at the control surface S_C to complete the boundary-value problem.

The scattered potentials ϕ_l ($l = 1, \dots, 7$) represent the oscillatory components of the flow. In the presence of a steady current, Newman (1978) provided the free surface boundary condition of the scattered potentials as

$$\begin{aligned} g \frac{\partial \phi_l}{\partial n} - \omega_e^2 \phi_l + 2i\omega_e U \frac{\partial \phi_l}{\partial x_1} + U^2 \frac{\partial^2 \phi_l}{\partial x_1^2} - i\omega_e U \nabla_1^2 \phi_0 \phi_l - 2i\omega_e U \nabla_1 \phi_0 \cdot \nabla_1 \phi_l \\ = \begin{cases} 0 & (l = 1, \dots, 6) \\ i\omega_e U \nabla_1^2 \phi_0 \phi_W + 2i\omega_e U \nabla_1 \phi_0 \cdot \nabla_1 \phi_W & (l = 7) \end{cases} \quad \text{on} \quad S_F \end{aligned} \quad (2.8)$$

where ∇_1 is the horizontal gradient operator. The double-body flow modifies the scattered and incident potentials in the near field respectively through the last two terms on the left-hand side and the particular term associated with the diffraction potential on the right-hand side. The body boundary condition of the radiation potentials accounts for the oscillatory motions of the body as well as the change in the steady flow field about its

mean position

$$\frac{\partial \phi_l}{\partial n} = -i\omega_e n_l + U m_l \quad (l = 1, \dots, 6) \quad \text{on } S_B \quad (2.9)$$

where $(n_4, n_5, n_6) \equiv \mathbf{x} \times \mathbf{n}$ and the so-called m -terms are defined as

$$(m_1, m_2, m_3) \equiv -(\mathbf{n} \cdot \nabla) \nabla (\phi_0 - x_1) \quad (2.10)$$

$$(m_4, m_5, m_6) \equiv -(\mathbf{n} \cdot \nabla) [\mathbf{x} \times \nabla (\phi_0 - x_1)] \quad (2.11)$$

The diffraction potential satisfies the body boundary condition

$$\frac{\partial \phi_7}{\partial n} = -\frac{\partial \phi_w}{\partial n} \quad \text{on } S_B \quad (2.12)$$

The scattered waves must also satisfy the kinematic condition on the seabed

$$\frac{\partial \phi_l}{\partial n} = 0 \quad (l = 1, \dots, 7) \quad \text{at } x_3 = -d \quad (2.13)$$

The control surface can accommodate a wall condition in the form of Eq. (2.13), a radiation condition that takes into account the Doppler shift resulting from the translating coordinate system, or a combination of both conditions.

Fig. 2.2 illustrates the Doppler shift of the scattered wave field by a uniform current in the negative x_1 direction. The scattered waves from the vessel shorten and slow down as they propagate against the current and elongate and speed up with the flow downstream. The waves reaching point D on the control surface have their direction rotated by an angle θ from the radial axis and their origin shifted downstream to point B . The scattered waves at a large distance from the body behave similar to those from a point source, giving rise to the condition $BD/c = BO/U$, where c is the celerity of the scattered waves in absence of the current. If $(\xi_1, \xi_2, 0)$ denotes the coordinates at D , this condition becomes

$$\frac{\xi_2 U}{c} = \sqrt{\xi_1^2 + \xi_2^2} \sin \theta \quad (2.14)$$

The celerity of the scattered wave at D is

$$c = \frac{\omega'}{k'} \quad (2.15)$$

in which ω' is the angular frequency of the scattered waves from a fixed reference point given as

$$\omega' = \omega_e + Uk' \cos \left[\tan^{-1} \left(\frac{\xi_2}{\xi_1} \right) - \theta \right] \quad (2.16)$$

where k' is the local wave number at $(\xi_1, \xi_2, 0)$. Eq. (2.16) follows directly from Eq. (2.2), but with ω_e referring to the angular frequency of the scattered waves from the vessel. The local dispersion relation for the scattered wave is

$$\omega' = \sqrt{gk' \tanh k'd} \quad (2.17)$$

An iterative scheme provides the rotation θ and wave number k' on the control surface from the system of nonlinear equations (2.14)-(2.17).

All scattered waves normal to the control surface must be out-going to avoid reflection. When the current speed exceeds the group velocity of the scattered waves, a quiescent region emerges ahead the vessel and the potential becomes constant on the corresponding control surface. Since the Doppler shift results in rotation of the scattered waves relative to the radial axis by an angle θ , the radiation condition can be written as

$$\begin{aligned} \frac{\partial \phi_l}{\partial n} - i k' \phi_l \cos \theta &= 0 \quad (l = 1, \dots, 7) \quad \text{on } S_{C1} \\ \phi_l &= 0 \quad (l = 1, \dots, 7) \quad \text{on } S_{C2} \end{aligned} \quad (2.18)$$

where S_{C1} and S_{C2} denote the portions of S_C with and without scattered waves. Eq. (2.18) is applicable to a full range of forward speeds above and below the group velocity. The effects of Doppler shift vanish at zero forward speed with $\theta = 0$ and $k' = k$. Eq. (2.18) reduces to

$$\frac{\partial \phi_l}{\partial n} - i k \phi_l = 0 \quad (l = 1, \dots, 7) \quad \text{on } S_C \quad (2.19)$$

which is the Sommerfeld radiation condition applied at a finite distance from the origin of the scattered waves.

2.2 Boundary Integral Equation

The boundary conditions in Section 2.1 define the flow potential, its normal derivative or both on each segment of the fluid boundary. Since the irrotational flow satisfies the Laplace equation, Green's second identity expresses the flow potential at a field point $\mathbf{x}$ as an integral over the fluid boundary as

$$\mu \phi_l(\mathbf{x}) = \int_S \left(G(\mathbf{x}, \boldsymbol{\xi}) \frac{\partial \phi_l}{\partial n}(\boldsymbol{\xi}) - \phi_l(\boldsymbol{\xi}) \frac{\partial G}{\partial n}(\mathbf{x}, \boldsymbol{\xi}) \right) ds \quad (l = 0, \dots, 7) \quad (2.20)$$

in which $G(\mathbf{x}, \boldsymbol{\xi})$ is the Rankine-type Green function that satisfies the seabed boundary condition in Eqs. (2.4) and (2.13) through the method of image

$$G(\mathbf{x}, \boldsymbol{\xi}) = \frac{1}{\sqrt{(x_1 - \xi_1)^2 + (x_2 - \xi_2)^2 + (x_3 - \xi_3)^2}} + \frac{1}{\sqrt{(x_1 - \xi_1)^2 + (x_2 - \xi_2)^2 + (x_3 + 2d + \xi_3)^2}} \quad (2.21)$$

where $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3)$ is a source point; μ is the internal solid angle at the field point; and S is the boundary of the fluid domain excluding the seabed (Isaacson & Cheung, 1992). The integral equation, which relates the flow potential and its normal derivative on the boundary, allows evaluation of the remaining unknowns through numerical discretization. The constant panel method, which divides the boundary into a number of quadrilateral elements with constant values of the potential, provides numerical solutions to the steady and scattered potentials ϕ_l ($l = 0, \dots, 7$).

The kinematic conditions on the body and free surfaces from Eqs. (2.7) and (2.4) allow evaluation of the steady double-body flow. With ϕ_0 and $\partial \phi_0 / \partial n$ vanishing on the control

surface, the boundary integral equation (2.20) becomes

$$2\pi \phi_0(\mathbf{x}_i) + \sum_{j=1}^{N_B+N_F} \phi_0(\xi_j) \int_{S_j} \frac{\partial G}{\partial n} ds = \sum_{j=1}^{N_B} n_1(\xi_j) \int_{S_j} G ds \quad (2.22)$$

where $\mathbf{x}_i$ and ξ_j refer to the centroids of boundary elements i and j ; S_j denotes the area of element j ; and N_B and N_F are the number of quadrilaterals on the body and free surfaces. The arguments $(\mathbf{x}, \xi)$ in the Green function are omitted for brevity in the numerical formulation. All the terms containing the unknown potential are on the left-hand side. Implementation of Eq. (2.22) at all body and free surface elements gives a system of $(N_B + N_F)$ equations for the solution of ϕ_0

$$[P]\{\phi_0\} = [Q]\{n_1\} \quad (2.23)$$

where $\{\phi_0\}$ is a vector for the double-body potential at the body and free surface elements; $\{n_1\}$ is a vector of dimension N_B representing the x_1 component of the body surface normal; and $[Q]$ and $[P]$ are coefficient matrices from numerical integration of the Green function and its normal derivative, respectively. The Gaussian quadrature evaluates the integrals with (6×6) points over each quadrilateral element. When the field point coincides with the source point, the closed-form solution of Hogben and Standing (1974) provides the singular integral of the Green function.

Once the double-body flow is defined, the scattering wave potentials can be evaluated from the boundary integral equation (2.20). Substitution of the free surface boundary condition (2.8), the body surface boundary conditions (2.9) and (2.12), and the radiation condition (2.18) into the discretized form of the boundary integral equation gives

$$\begin{aligned}
& 2\pi \phi_l(\mathbf{x}_i) + \sum_{j=1}^{N_B+N_F+N_{Cl}} \phi_l(\xi_j) \int_{S_j} \frac{\partial G}{\partial n} ds - \frac{\omega_e^2}{g} \sum_{j=N_B+1}^{N_B+N_F} \phi_l(\xi_j) \int_{S_j} G ds \\
& - \frac{i\omega_e U}{g} \sum_{j=N_B+1}^{N_B+N_F} (\nabla_1^2 \phi_0 \phi_l)_{\xi_j} \int_{S_j} G ds + \frac{U^2}{g} \sum_{j=N_B+1}^{N_B+N_F} \frac{\partial^2 \phi_l}{\partial x_1^2}(\xi_j) \int_{S_j} G ds \\
& - \frac{2i\omega_e U}{g} \sum_{j=N_B+1}^{N_B+N_F} \left[\left(\frac{\partial \phi_0}{\partial x_1} - 1 \right) \frac{\partial \phi_l}{\partial x_1} + \frac{\partial \phi_0}{\partial x_2} \frac{\partial \phi_l}{\partial x_2} \right]_{\xi_j} \int_{S_j} G ds \\
& - i \sum_{j=N_B+N_F+1}^{N_B+N_F+N_{Cl}} [k' \phi_l \cos \theta]_{\xi_j} \int_{S_j} G ds - \sum_{j=N_B+N_F+N_{Cl}+1}^{N_T} \frac{\partial \phi_l}{\partial n}(\xi_j) \int_{S_j} G ds = R_l \quad (l = 1, \dots, 7)
\end{aligned} \tag{2.24}$$

in which

$$R_l = \sum_{j=1}^{N_B} \int_{S_j} G \frac{\partial \phi_l}{\partial n} ds = \sum_{j=1}^{N_B} \left[-i\omega_e n_l(\xi_j) \int_{S_j} G ds + U \int_{S_j} G m_l ds \right] \quad (l = 1, \dots, 6) \tag{2.25}$$

$$\begin{aligned}
R_7 = & - \sum_{j=1}^{N_B} \frac{\partial \phi_W}{\partial n}(\xi_j) \int_{S_j} G ds + \frac{i\omega_e U}{g} \sum_{j=N_B+1}^{N_B+N_F} (\nabla_1^2 \phi_0 \phi_W)_{\xi_j} \int_{S_j} G ds \\
& + \frac{2i\omega_e U}{g} \sum_{j=N_B+1}^{N_B+N_F} \left[\frac{\partial \phi_0}{\partial x_1} \frac{\partial \phi_W}{\partial x_1} + \frac{\partial \phi_0}{\partial x_2} \frac{\partial \phi_W}{\partial x_2} \right]_{\xi_j} \int_{S_j} G ds
\end{aligned} \tag{2.26}$$

where N_{Cl} denotes the number of elements on S_{Cl} and N_T is the total number of elements on the fluid boundary. In the computer code, the integrals of the Green function and its normal derivative are evaluated only once and can be used in the numerical solution of all seven scattered potentials in Eq. (2.24) as well as the steady potential in Eq. (2.22). The spatial derivatives in Eqs. (2.24) and (2.26) and the integration of the m -terms in Eq. (2.25) require particular attention to assure the accuracy of the numerical solution.

The free surface spatial derivatives $\partial \phi_l / \partial x_1$, $\partial \phi_l / \partial x_2$, and $\partial^2 \phi_l / \partial x_1^2$ in Eq. (2.24) are expressed in terms of ϕ_l at the neighboring elements through central finite difference formulas with a Jacobian matrix to account for the non-rectangular grid. The same numerical scheme also evaluates the free surface spatial derivatives $\partial \phi_0 / \partial x_1$, $\partial \phi_0 / \partial x_2$, and

$\nabla_1^2 \phi_0$ in Eqs. (2.24) and (2.26) from known values of ϕ_0 at the neighboring elements. The forcing term of the radiation potentials in Eq. (2.25) includes an integral of the m -terms that is expanded by a form of Stoke's theorem as

$$\int_{S_b} m_l G ds = - \int_{S_b} n_l \nabla(\phi_0 - x_1) \cdot \nabla G ds - \oint_{C_W} n_l G \frac{\partial \phi_0}{\partial x_3} dc \quad (l = 1, \dots, 6) \quad (2.27)$$

where C_W denotes the waterline of the body (Ogilvie and Tuck, 1969). The integral along the waterline becomes zero owing to the double-body flow assumption. With the remaining term in Eq. (2.27) and the relation in Eq. (2.6), the forcing term becomes

$$R_l = \sum_{j=1}^{N_B} \left[-i\omega_e n_l(\xi_j) \int_{S_j} G ds - n_l(\xi_j) \int_{S_j} \mathbf{W} \cdot \nabla G ds \right] \quad (l = 1, \dots, 6) \quad (2.28)$$

This approach eliminates the second-order spatial derivatives in the m -terms that become a source of numerical errors for bodies with high surface curvature. A central difference formula can readily evaluate the first-order spatial gradient $\nabla \phi_0$ in $\mathbf{W}$ from known values of ϕ_0 at the neighboring elements on the body surface. The integral involving $\mathbf{W} \cdot \nabla G$ becomes singular when the field point coincides with the source point. In the numerical integration, the field point is offset by an infinitesimal distance from the source point making $\mathbf{W} \cdot \nabla G \rightarrow 0$ at the center of the quadrilateral element (Nossen *et al.*, 1991).

The left-hand side of the discretized integral equation (2.24) contains N_T unknown values of ϕ_l and $\partial \phi_l / \partial n$ on the boundary. Implementation of the equation at all the boundary elements gives an $(N_T \times N_T)$ system of simultaneous equations in the form

$$[P'] \{X_l\} = \{R_l\} \quad (l = 1, \dots, 7) \quad (2.29)$$

where $\{R_l\}$ is a vector evaluated from the forcing terms in Eqs. (2.26) and (2.28), $[P']$ is a square coefficient matrix from numerical integration of the Green function and its normal derivative on the left-hand side of Eq. (2.24); and $\{X_l\}$ contains the unknown values of ϕ_l

on S_B , S_F , and S_{C1} as well as $\partial\phi_l/\partial n$ on S_{C2} . After the solution is determined from Eq. (2.29), the normal gradients of the radiation potentials on the body surface that implicitly include effects of the m -terms can be determined from Eq. (2.25) in the form of an $N_B \times N_B$ matrix equation

$$[Q'] \left\{ \frac{\partial\phi_l}{\partial n} \right\} = \{R_l\} \quad (l = 1, \dots, 6) \quad (2.30)$$

where $[Q']$ is a coefficient matrix from numerical integration of the Green function over the constant boundary elements on the body surface.

2.3 Equation of Motion

The scattered potentials ϕ_j ($j = 1, \dots, 7$), which describe the oscillatory flows resulting from unit-amplitude body motions and incident waves in the double-body flow, are first evaluated from the boundary-value problems defined in Section 2.2. The fluid pressure is obtained by a Taylor series expansion of the Bernoulli equation about the static equilibrium body surface as

$$p = i\omega_e \rho \left[\phi_W + \phi_7 + \sum_{j=1}^6 \alpha_j \phi_j \right] - \rho \mathbf{W} \cdot \nabla \left[\phi_W + \phi_7 + \sum_{j=1}^6 \alpha_j \phi_j \right] - \frac{\rho}{2} \mathbf{u} \cdot \nabla (\mathbf{W} \cdot \mathbf{W}) - \rho g (\alpha_3 + \alpha_4 x_2 - \alpha_5 x_1) \quad \text{on } S_B \quad (2.31)$$

where ρ is fluid density and $\mathbf{u} = [(\alpha_1, \alpha_2, \alpha_3) + (\alpha_4, \alpha_5, \alpha_6) \times (x_1, x_2, x_3)]$ is the displacement vector on the body surface. The second term on the right-hand side arises from the velocity-squared pressure term in the Bernoulli equation and is considered as linear since the double-body flow $\mathbf{W}$ is of zeroth order. The third term also from the velocity-squared pressure accounts for the pressure change in the double-body flow as the body oscillates about its static equilibrium position. Although being of first order, its contribution is small and is typically ignored in the body force calculation (Nakos and

Sclavounos, 1991; Kim and Kim, 2010). The last term accounts for the change in hydrostatic pressure on the body due to its motions.

Integration of the pressure in Eq. (2.31) gives the hydrodynamic, wave excitation, and hydrostatic restoring forces on the vessel. The hydrodynamic force due to body motions are derived from the radiation potentials as

$$F_i^{rad} = \sum_{j=1}^6 \rho \int_{S_B} (i\omega_e \phi_j - \mathbf{W} \cdot \nabla \phi_j) n_i ds \alpha_j \quad (i = 1, \dots, 6) \quad (2.32)$$

It consists of orthogonal components proportional to the acceleration and velocity as

$$F_i^{rad} = \sum_{j=1}^6 [\omega_e^2 a_{ij} + i\omega_e b_{ij}] \alpha_j \quad (i = 1, \dots, 6) \quad (2.33)$$

where the coefficients a_{ij} and b_{ij} form the (6×6) added mass and damping matrices, $[M_A]$ and $[B]$, in the equation of motion of the vessel. The incident and diffraction potentials from Eq. (2.31) give the wave excitation force vector $\{F\}$ as

$$F_i^{ext} = \rho A \int_{S_B} \{i\omega_e (\phi_W + \phi_7) - \mathbf{W} \cdot \nabla (\phi_W + \phi_7)\} n_i ds \quad (i = 1, \dots, 6) \quad (2.34)$$

Integration of the hydrostatic pressure term in Eq. (2.31) gives rise to the hydrostatic restoring force

$$F_i^{hstat} = \rho g \left[-\alpha_3 \int_{S_B} n_i ds - \alpha_4 \int_{S_B} x_2 n_i ds + \alpha_5 \int_{S_B} x_1 n_i ds \right] = -K_{ij} \alpha_j \quad (i, j = 1, \dots, 6) \quad (2.35)$$

in which K_{ij} is the hydrostatic restoring coefficient and in matrix form is given by

$$[K] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho g A_W & 0 & -\rho g M_W & 0 \\ 0 & 0 & 0 & \rho g (I_{W1} + Vz_B) & 0 & 0 \\ 0 & 0 & -\rho g M_W & 0 & \rho g (I_{W2} + Vz_B) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.36)$$

where A_W is the water-plane area; M_W is first moment of the water-plane about the x_2 axis; I_{W1} and I_{W2} are the second moments of the water-plane about the x_1 and x_2 axes respectively; V is the underwater volume; and z_B is the vertical center of buoyancy.

The equation of motion represents the dynamic equilibrium between the wave excitation force at the prescribed encounter frequency and the inertia, radiation damping, and hydrostatic forces associated with the body motions in the six degrees of freedom. With the components already defined, the matrix equation reads

$$([M] + [M_A])\{\ddot{\alpha}\} + [B]\{\dot{\alpha}\} + [K]\{\alpha\} = \{F^{ext}\} \quad (2.37)$$

in which $\{\dot{\alpha}\}$ and $\{\ddot{\alpha}\}$ denote the velocity and acceleration vectors and $[M]$ is the mass matrix of the vessel given by

$$[M] = \begin{bmatrix} m & 0 & 0 & 0 & mz_C & 0 \\ 0 & m & 0 & -mz_C & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & -mz_C & 0 & I_4 & 0 & -I_{46} \\ mz_C & 0 & 0 & 0 & I_5 & 0 \\ 0 & 0 & 0 & -I_{46} & 0 & I_6 \end{bmatrix} \quad (2.38)$$

where m is the mass; z_C is the vertical center of gravity; I_4 , I_5 , and I_6 are the roll, pitch and yaw moments of inertia; and I_{46} is the roll-yaw product of inertia. Expressing the velocity and acceleration in terms of the displacement vector, Eq. (2.37) becomes

$$[-\omega_e^2([M] + [M_A]) + [K] - i\omega_e[B]]\{\alpha\} = \{F^{ext}\} \quad (2.39)$$

A standard matrix solution routine provides the complex amplitude of the body motions from Eq. (2.39). The free surface elevation is then obtained from the dynamic free surface boundary condition in the form

$$\eta = \frac{i\omega_e}{g} \left(\phi_W + \phi_7 + \sum_{j=1}^6 \alpha_j \phi_j \right) - \frac{1}{g} \mathbf{W} \cdot \nabla \left(\phi_W + \phi_7 + \sum_{j=1}^6 \alpha_j \phi_j \right) \quad \text{on } S_F \quad (2.40)$$

Similar to the pressure in Eq. (2.31), the second term arises from the velocity-squared pressure term in the Bernoulli equation accounting for the Doppler shift of the wave field by the double-body flow $\mathbf{W}$.

Chapter 3

Finite Element Formulation

The structural analysis for body deformation is performed by a finite element model based on a 3-node triangular shell element. This element, developed by Carpenter *et al.* (1985), is obtained by combining the Discrete Kirchhoff Theory (DKT) plate bending element of Batoz *et al.* (1980) and a membrane element similar to the Allman (1984) triangle, but based on a degenerated linear strain triangle (LST). Including the drilling degree of freedom in the membrane formulation removes the in-plane rotational singularity. This element is chosen for its simplicity and ease of implementation for arbitrary geometries. The tedious task of computing the element curvatures is completely avoided. This chapter summarizes the formulation of the membrane and shell element stiffness matrices and the adaptation of the equation of motion for coupling with the boundary element model.

3.1 Membrane Formulation

The linear strain triangular (LST) element has three corner and three mid-side nodes shown as solid and open circles respectively in Fig. 3.1. The local x axis originates from node 1 along side 1-2. The local y axis is perpendicular to the x axis in the element plane. The in-plane displacement components along the x and y axes are given by

$$u_x = \sum_{i=1}^6 H_i u_{xi}, \quad u_y = \sum_{i=1}^6 H_i u_{yi} \quad (3.1)$$

where the subscript i indicates the nodal values of the respective displacement components and H_i are the standard quadratic shape functions in the area coordinates ξ and η given by

$$\begin{aligned} H_1 &= 2(1-\xi-\eta)(0.50-\xi-\eta) & H_4 &= 4\xi(1-\xi-\eta) \\ H_2 &= \xi(2\xi-1) & H_5 &= 4\xi\eta \\ H_3 &= \eta(2\eta-1) & H_6 &= 4\eta(1-\xi-\eta) \end{aligned} \quad (3.2)$$

The membrane element of Carpenter *et al.* (1985) has three corner nodes, shown as solid circles in Fig. 3.1. Each node has three degrees of freedom, which include the two in-plane displacements u_x and u_y and one in-plane rotation θ_z . With an assumed cubic normal displacement at the node and a linear tangential displacement along the side, they expressed the mid-side nodal displacements of the LST element in terms of the displacement and rotations at the corner nodes, and obtained the displacement field as

$$\begin{aligned} u_x &= \left[H_1 + \frac{1}{2}(H_4 + H_6) \right] u_{x1} + \frac{1}{8}(H_6 y_{13} - H_4 y_{21}) \theta_{z1} + \left[H_2 + \frac{1}{2}(H_4 + H_5) \right] u_{x2} \\ &\quad + \frac{1}{8}(H_4 y_{21} - H_5 y_{32}) \theta_{z2} + \left[H_3 + \frac{1}{2}(H_5 + H_6) \right] u_{x3} + \frac{1}{8}(H_5 y_{32} - H_6 y_{13}) \theta_{z3} \end{aligned} \quad (3.3)$$

$$\begin{aligned} u_y &= \left[H_1 + \frac{1}{2}(H_4 + H_6) \right] u_{y1} + \frac{1}{8}(H_4 x_{21} - H_6 x_{13}) \theta_{z1} + \left[H_2 + \frac{1}{2}(H_4 + H_5) \right] u_{y2} \\ &\quad + \frac{1}{8}(H_5 x_{32} - H_4 x_{21}) \theta_{z2} + \left[H_3 + \frac{1}{2}(H_5 + H_6) \right] u_{y3} + \frac{1}{8}(H_6 x_{13} - H_5 x_{32}) \theta_{z3} \end{aligned} \quad (3.4)$$

where $x_{ij} = x_i - x_j$, and $y_{ij} = y_i - y_j$. The coordinates (x_i, y_i) of node i are in the local coordinate system. In matrix notation, the displacement field can be written as

$$\begin{Bmatrix} u_x \\ u_y \end{Bmatrix} = [\Psi] \{d_m\} \quad (3.5)$$

where

$$\{d_m\} = \begin{bmatrix} u_{x1} & u_{y1} & \theta_{z1} & u_{x2} & u_{y2} & \theta_{z2} & u_{x3} & u_{y3} & \theta_{z3} \end{bmatrix}^T \quad (3.6)$$

and $[\psi]$ is the 2×9 shape-function coefficient matrix.

Eq. (3.5) relates the nodal degrees of freedom to the displacement field, which in turn provides the strain through the strain-displacement relation in plane elasticity

$$\{\epsilon_m\} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} u_{x,x} \\ u_{y,y} \\ u_{x,y} + u_{y,x} \end{Bmatrix} \quad (3.7)$$

where $u_{x,x}$ refers to the partial derivative of u_x with respect to x . Applying the chain rule of differentiation, Eq. (3.7) can be written as

$$\begin{aligned} \{\epsilon_m\} &= \begin{Bmatrix} \xi_{,x} u_{x,\xi} + \eta_{,x} u_{x,\eta} \\ \xi_{,y} u_{y,\xi} + \eta_{,y} u_{y,\eta} \\ \xi_{,y} u_{x,\xi} + \eta_{,y} u_{x,\eta} + \xi_{,x} u_{y,\xi} + \eta_{,x} u_{y,\eta} \end{Bmatrix} \\ &= \begin{Bmatrix} \xi_{,x} [\psi_{1,\xi}] + \eta_{,x} [\psi_{1,\eta}] \\ \xi_{,y} [\psi_{2,\xi}] + \eta_{,y} [\psi_{2,\eta}] \\ \xi_{,y} [\psi_{1,\xi}] + \eta_{,y} [\psi_{1,\eta}] + \xi_{,x} [\psi_{2,\xi}] + \eta_{,x} [\psi_{2,\eta}] \end{Bmatrix} \{d_m\} = [B_m] \{d_m\} \end{aligned} \quad (3.8)$$

in which

$$\begin{aligned} \xi_{,x} &= -\frac{y_{13}}{(x_{13}y_{21} - x_{21}y_{13})} & \eta_{,x} &= -\frac{y_{21}}{(x_{13}y_{21} - x_{21}y_{13})} \\ \xi_{,y} &= \frac{x_{13}}{(x_{13}y_{21} - x_{21}y_{13})} & \eta_{,y} &= \frac{x_{21}}{(x_{13}y_{21} - x_{21}y_{13})} \end{aligned} \quad (3.9)$$

The matrices $[\psi_1]$ and $[\psi_2]$ are the first and second rows of $[\psi]$ and $[B_m]$ is the 3×9 strain-displacement matrix for the membrane element. The stress-strain relationship for a linearly elastic and isotropic material is

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = [D_p] \{\epsilon_m\} \quad (3.10)$$

The 9×9 membrane element stiffness matrix is given by

$$[K_m] = \int_{V_e} [B_m]^T [D_p] [B_m] dV = h \int_{S_e} [B_m]^T [D_p] [B_m] ds \quad (3.11)$$

where h is the element thickness; S_e and V_e are the surface area and volume of the element; and $[B_m]$ and $[D_p]$ are given by Eqs. (3.8) and (3.10) respectively. The exact integration of Eq. (3.11) requires a three-point Gaussian quadrature rule but, as demonstrated by Carpenter *et al.* (1985), the membrane element results are free from the parasitic shear-locking under pure bending loads, when integrated using a reduced one-point quadrature rule.

3.2 Plate Formulation

Fig. 3.2 illustrates the three-node triangular plate element based on the Discrete Kirchhoff Theory (DKT). Each node has three degrees of freedom, which include the transverse deflection u_z and the rotations θ_x and θ_y along the local x and y axes. If the rotations θ_x and θ_y are small, the displacement components u_x , u_y and u_z along the local x , y and z axes can be expressed as

$$u_x = z\beta_x(x, y); \quad u_y = z\beta_y(x, y); \quad u_z = u_z(x, y) \quad (3.12)$$

where β_x and β_y are the rotations of the normal in the x - z and y - z planes, respectively.

These normal rotations are related to the rotational degrees of freedom as

$$\beta_x = \theta_y; \quad \beta_y = -\theta_x \quad (3.13)$$

If we consider a six node element similar to the LST, with three corner and three mid-side nodes, the variation of β_x , β_y and u_z within the element is given by

$$\beta_x = \sum_{i=1}^6 H_i \beta_{xi}, \quad \beta_y = \sum_{i=1}^6 H_i \beta_{yi}, \quad u_z = \sum_{i=1}^6 H_i u_{zi} \quad (3.14)$$

where β_{xi} , β_{yi} and u_{zi} are nodal values of the normal rotations and transverse displacements and H_i are the quadratic shape functions defined by Eq. (3.2).

Batoz *et al.* (1980) derived the element stiffness matrix with the initial assumption of the presence of transverse shear deformation. In order to obtain the equation of equilibrium, it is necessary to express the normal rotations, β_{xi} and β_{yi} , in terms of the nodal degrees of freedom of the DKT element. This is achieved by assuming that the transverse deflection u_z is a cubic along the sides of the triangle and the element deformations satisfy the Kirchhoff plate theory with the assumption of zero transverse shear along the element sides. The normal rotations β_x and β_y can be expressed as

$$\beta_x = [P_x] \{d_{DKT}\}; \quad \beta_y = [P_y] \{d_{DKT}\} \quad (3.15)$$

where $\{d_{DKT}\}$ is the vector of the nodal deformation given by

$$\{d_{DKT}\} = [u_{z1} \quad \theta_{x1} \quad \theta_{y1} \quad u_{z2} \quad \theta_{x2} \quad \theta_{y2} \quad u_{z3} \quad \theta_{x3} \quad \theta_{y3}]^T \quad (3.16)$$

and $[P_x]$ and $[P_y]$ are shape function coefficients for the normal rotations β_x and β_y respectively. The shape function coefficients are given as

$$\begin{aligned} \left[P_x \right]^T &= \begin{Bmatrix} 1.5(a_4 H_4 - a_6 H_6) \\ b_4 H_4 + b_6 H_6 \\ H_1 - c_4 H_4 - c_6 H_6 \\ 1.5(a_5 H_5 - a_4 H_4) \\ b_4 H_4 + b_5 H_5 \\ H_2 - c_4 H_4 - c_5 H_5 \\ 1.5(a_6 H_6 - a_5 H_5) \\ b_5 H_5 + b_6 H_6 \\ H_3 - c_5 H_5 - c_6 H_6 \end{Bmatrix}; & \left[P_y \right]^T &= \begin{Bmatrix} 1.5(d_4 H_4 - d_6 H_6) \\ -H_1 + e_4 H_4 + e_6 H_6 \\ -b_4 H_4 - b_6 H_6 \\ 1.5(d_5 H_5 - d_4 H_4) \\ -H_2 + e_4 H_4 + e_5 H_5 \\ -b_4 H_4 - b_5 H_5 \\ 1.5(d_6 H_6 - d_5 H_5) \\ -H_3 + e_5 H_5 + e_6 H_6 \\ -b_5 H_5 - b_6 H_6 \end{Bmatrix} \end{aligned} \quad (3.17)$$

in which

$$\begin{aligned} a_k &= -\frac{x_{ij}^2}{l_{ij}^2} \\ b_k &= \frac{3x_{ij}y_{ij}}{4l_{ij}^2} \\ c_k &= \frac{1}{l_{ij}^2} \left(\frac{x_{ij}^2}{4} - \frac{y_{ij}^2}{2} \right) \\ d_k &= -\frac{y_{ij}^2}{l_{ij}^2} \\ e_k &= \frac{1}{l_{ij}^2} \left(\frac{y_{ij}^2}{4} - \frac{x_{ij}^2}{2} \right) \\ l_{ij}^2 &= x_{ij}^2 + y_{ij}^2 \end{aligned} \quad (3.18)$$

where $k = 4, 5, 6$ corresponds to the sides $i-j = 1-2, 2-3, 3-1$, respectively.

With the deformation field defined, the bending strain, which varies linearly through the plate thickness, is given by

$$\{\epsilon_b\} = z \begin{Bmatrix} \beta_{x \succ x} \\ \beta_{y \succ y} \\ \beta_{x \succ y} + \beta_{y \succ x} \end{Bmatrix} = z \{\kappa\} \quad (3.19)$$

where $\{\kappa\}$ is the transverse curvature vector. Applying the chain rule of differentiation, Eq. (3.19) can be written in terms of (3.15) as

$$\begin{aligned} \{\kappa\} &= \begin{Bmatrix} \xi_{,x} \beta_{x,\xi} + \eta_{,x} \beta_{x,\eta} \\ \xi_{,y} \beta_{y,\xi} + \eta_{,y} \beta_{y,\eta} \\ \xi_{,y} \beta_{x,\xi} + \eta_{,y} \beta_{x,\eta} + \xi_{,x} \beta_{y,\xi} + \eta_{,x} \beta_{y,\eta} \end{Bmatrix} \\ &= \begin{Bmatrix} \xi_{,x} [P_{x,\xi}] + \eta_{,x} [P_{x,\eta}] \\ \xi_{,y} [P_{y,\xi}] + \eta_{,y} [P_{y,\eta}] \\ \xi_{,y} [P_{x,\xi}] + \eta_{,y} [P_{x,\eta}] + \xi_{,x} [P_{y,\xi}] + \eta_{,x} [P_{y,\eta}] \end{Bmatrix} \{d_{DKT}\} = [B_{DKT}] \{d_{DKT}\} \end{aligned} \quad (3.20)$$

where $[B_{DKT}]$ is the 3×9 strain-displacement matrix for the plate bending element. For linear elastic and isotropic material, the bending stress at any point within the plate is given by

$$\{\sigma_b\} = [D_p] \{\varepsilon_b\} = z [D_p] \{\kappa\} \quad (3.21)$$

The 9×9 plate stiffness matrix is thus given by

$$[K_{DKT}] = \int_{V_e} [B_{DKT}]^T z [D_p] [B_{DKT}] dV = \frac{h^3}{12} \int_{S_e} [B_{DKT}]^T [D_p] [B_{DKT}] ds \quad (3.22)$$

where $[B_{DKT}]$ and $[D_p]$ are given by Eqs. (3.20) and (3.10) respectively. The exact integration of Eq. (3.22) requires the use of a three-point Gaussian quadrature rule.

3.3 Shell Element Stiffness

The shell element stiffness comprises of the membrane and plate components. The deformation field is expressed in terms of the values at the corner nodes. Along any side of the element, the transverse displacement u_z is interpolated using a cubic polynomial as illustrated in Fig. 3.2. Let s denote distance along the element side. The expression for u_{z5} at the midpoint of side 2-3 can be written as

$$u_{z5} = \frac{1}{2}(u_{z2} + u_{z3}) + \frac{l_{23}}{8}(u_{z2,s} - u_{z3,s}) \quad (3.23)$$

where $u_{z2,s}$ and $u_{z3,s}$ are the rotations at nodes 2 and 3 about the outward normal to edge 2-3. The rotation $u_{z,s}$ can be written in terms of the rotational degrees of freedom at the nodes as

$$u_{z,s} = -\theta_x \frac{y_{23}}{l_{23}} + \theta_y \frac{x_{23}}{l_{23}} \quad (3.24)$$

Substituting Eq. (3.24) into Eq. (3.23), we have

$$u_{z5} = \frac{1}{2}(u_{z2} + u_{z3}) - \frac{y_{23}}{8}(\theta_{x2} - \theta_{x3}) + \frac{x_{23}}{8}(\theta_{y2} - \theta_{y3}) \quad (3.25)$$

Similarly, the transverse displacements at the midpoints of sides 3-1 and 1-2 are obtained by permuting the subscripts in Eq. (3.25). These expressions for u_{zi} ($i=4, 5, 6$) are then substituted into Eq. (3.14) to obtain the transverse deformation u_z over the element in terms of the displacements and rotations at the corner nodes as

$$\begin{aligned} u_z = & \left[H_1 + \frac{1}{2}(H_4 + H_6) \right] u_{z1} + \frac{1}{8}(H_6 y_{31} - H_4 y_{12}) \theta_{x1} + \frac{1}{8}(H_4 x_{12} - H_6 x_{31}) \theta_{y1} \\ & + \left[H_2 + \frac{1}{2}(H_4 + H_5) \right] u_{z2} + \frac{1}{8}(H_4 y_{12} - H_5 y_{23}) \theta_{x2} + \frac{1}{8}(H_5 x_{23} - H_4 x_{12}) \theta_{y2} \\ & + \left[H_3 + \frac{1}{2}(H_5 + H_6) \right] u_{z3} + \frac{1}{8}(H_5 y_{23} - H_6 y_{31}) \theta_{x3} + \frac{1}{8}(H_6 x_{31} - H_5 x_{23}) \theta_{y3} \end{aligned} \quad (3.26)$$

Combining the in-plane deformation of the membrane and plate from Eqs. (3.3), (3.4) and (3.12) in terms of β_x and β_y from Eqs. (3.15)-(3.17) gives u_x and u_y at a general point of the shell as

$$\begin{aligned} u_x = & \left[H_1 + \frac{1}{2}(H_4 + H_6) \right] u_{x1} + 1.5z(a_4 H_4 - a_6 H_6) u_{z1} + z(b_4 H_4 + b_6 H_6) \theta_{x1} \\ & + z(H_1 - c_4 H_4 - c_6 H_6) \theta_{y1} + \frac{1}{8}(H_6 y_{13} - H_4 y_{21}) \theta_{z1} + \left[H_2 + \frac{1}{2}(H_4 + H_5) \right] u_{x2} \\ & + 1.5z(a_5 H_5 - a_4 H_4) u_{z2} + z(b_4 H_4 + b_5 H_5) \theta_{x2} + z(H_2 - c_4 H_4 - c_5 H_5) \theta_{y2} \end{aligned}$$

$$\begin{aligned}
& +\frac{1}{8}(H_4y_{21}-H_5y_{32})\theta_{z2}+\left[H_3+\frac{1}{2}(H_5+H_6)\right]u_{x3}+1.5z(a_6H_6-a_5H_5)u_{z3} \\
& +z(b_5H_5+b_6H_6)\theta_{x3}+z(H_3-c_5H_5-c_6H_6)\theta_{y3}+\frac{1}{8}(H_5y_{32}-H_6y_{13})\theta_{z3}
\end{aligned} \tag{3.27}$$

$$\begin{aligned}
u_y = & \left[H_1+\frac{1}{2}(H_4+H_6)\right]u_{y1}+1.5z(d_4H_4-d_6H_6)u_{z1}+z(-H_1+e_4H_4+e_6H_6)\theta_{x1} \\
& +z(-b_4H_4-b_6H_6)\theta_{y1}+\frac{1}{8}(H_4x_{21}-H_6x_{13})\theta_{z1}+\left[H_2+\frac{1}{2}(H_4+H_5)\right]u_{y2} \\
& +1.5z(d_5H_5-d_4H_4)u_{z2}+z(-H_2+e_4H_4+e_5H_5)\theta_{x2}+z(-b_4H_4-b_5H_5)\theta_{y2} \\
& +\frac{1}{8}(H_5x_{32}-H_4x_{21})\theta_{z2}+\left[H_3+\frac{1}{2}(H_5+H_6)\right]u_{y3}+1.5z(d_6H_6-d_5H_5)u_{z3} \\
& +z(-H_3+e_5H_5+e_6H_6)\theta_{x3}+z(-b_5H_5-b_6H_6)\theta_{y3}+\frac{1}{8}(H_6x_{13}-H_5x_{32})\theta_{z3}
\end{aligned} \tag{3.28}$$

In matrix notation, the deformation at a general point of the shell can be obtained by combining Eqs. (3.26)-(3.28) as

$$\{u\} = \begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix} = [\Psi]\{d_e\} \tag{3.29}$$

where $[\Psi]$ is the 3×18 shape function coefficient matrix of the shell element and $\{d_e\}$ is the vector of nodal displacements given as

$$\{d_e\} = \begin{bmatrix} u_{x1} & u_{y1} & u_{z1} & \theta_{x1} & \theta_{y1} & \theta_{z1} & u_{x2} & u_{y2} & u_{z2} \\ \theta_{x2} & \theta_{y2} & \theta_{z2} & u_{x3} & u_{y3} & u_{z3} & \theta_{x3} & \theta_{y3} & \theta_{z3} \end{bmatrix}^T \tag{3.30}$$

The membrane strain of Eq. (3.8) and the plate bending strain of Eq. (3.19) can be combined to form a linear strain vector for the shell element as

$$\begin{aligned}
\{\epsilon_L\} & = \{\epsilon_m\} + z\{\kappa\} = [B_m]\{d_m\} + z[B_{DKT}]\{d_{DKT}\} \\
\text{or, } \{\epsilon_L\} & = [B_e]\{d_e\}
\end{aligned} \tag{3.31}$$

where $[B_e]$ is the 3×18 strain-displacement matrix.

The membrane element stiffness matrix in Eq. (3.11) and the DKT element stiffness matrix in Eq. (3.22) can similarly be combined to form the 18×18 stiffness matrix $[K_e]$ of the shell element. This is achieved by assembling the rows and columns of the individual stiffness matrices following the degrees of freedom of the shell element as given in Eq. (3.30). Alternately, the stiffness matrix can be obtained from

$$[K_e] = \int_{V_e} [B_e]^T [D_p] [B_e] dV \quad (3.32)$$

As the body deforms about its static equilibrium, the nonlinear Green strain vector as defined by Cook *et al.* (2002) is

$$\{\epsilon\} = \{\epsilon_L\} + \{\epsilon_{NL}\} \quad (3.33)$$

in which the linear and nonlinear components are defined as

$$\{\epsilon_L\} = \begin{Bmatrix} u_{x,x} \\ u_{y,y} \\ u_{x,y} + u_{y,x} \end{Bmatrix} \quad (3.34)$$

$$\{\epsilon_{NL}\} = \begin{Bmatrix} \frac{1}{2}(u_{x,x}^2 + u_{y,x}^2 + u_{z,x}^2) \\ \frac{1}{2}(u_{x,y}^2 + u_{y,y}^2 + u_{z,y}^2) \\ (u_{x,x}u_{x,y} + u_{y,x}u_{y,y} + u_{z,x}u_{z,y}) \end{Bmatrix} \quad (3.35)$$

The linear strain component is expressed in terms of the nodal displacements in Eq. (3.31). Though the present analysis consider only small deformation, the nonlinear terms in the Green strain are necessary as they contribute to the linear tangential stiffness through the initial stress in the system.

3.4 Equations of Motion

Section 3.3 has summarized the derivation of the element stiffness matrix, which is a building block for the ship structure model. This section assembles the equations of motion in terms of the element mass matrix, structural stiffness matrix, and the hydrostatic and hydrodynamic pressure. Fig. 2.1 provides a definition sketch for the hydroelasticity problem of a vessel advancing in waves. The vessel undergoes oscillations under the influence of the incoming waves, while embedded in a steady current associated with its forward speed. In absence of waves, the vessel attains a static equilibrium shape S_B . The dynamic pressure due to wave actions causes the body to undergo harmonic deformation and the deformed surface is denoted by S_D . As a linear problem, the static surface S_B is discretized into planar triangular shell elements and the deformation vector is given by Eq. (3.29). The stress vector at any point (x, y, z) in the local coordinates is given as

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [D_p] \{\varepsilon\} + \{\sigma_0\} \quad (3.36)$$

where $\{\sigma_0\} = [\sigma_{x0} \quad \sigma_{y0} \quad \tau_{xy0}]^T$ is the initial stress in the static equilibrium configuration; $[D_p]$ is the constitutive matrix of an isotropic material for plane stress conditions defined by Eq. (3.10); and $\{\varepsilon\}$ is the Green strain vector in Eq. (3.33).

Let U_e denote the elastic strain energy and W_e the work done by external forces. According to the principle of virtual work, the total potential energy is minimum at equilibrium such that

$$\delta(U_e - W_e) = 0 \quad (3.37)$$

where δ represents a small variation in U_e and W_e . This equilibrium condition provides the basis to obtain the equations of motion. The strain due to a virtual displacement of the surface about its deformed configuration S_D is given by Eq. (3.33) as

$$\{\delta\epsilon\} = \{\delta\epsilon_L\} + \{\delta\epsilon_{NL}\} \quad (3.38)$$

The virtual linear strain can be obtained from Eq. (3.31) as

$$\{\delta\epsilon_L\} = [B_e]\{\delta d_e\} \quad (3.39)$$

The nonlinear part of the virtual strain can be determined from Eqs. (3.29) and (3.35) as

$$\{\delta\epsilon_{NL}\} = \{d_e\}^T \left\{ \begin{array}{l} [\Psi_{1,x}]^T [\Psi_{1,x}] + [\Psi_{2,x}]^T [\Psi_{2,x}] + [\Psi_{3,x}]^T [\Psi_{3,x}] \\ [\Psi_{1,y}]^T [\Psi_{1,y}] + [\Psi_{2,y}]^T [\Psi_{2,y}] + [\Psi_{3,y}]^T [\Psi_{3,y}] \\ 2[\Psi_{1,x}]^T [\Psi_{1,y}] + 2[\Psi_{2,x}]^T [\Psi_{2,y}] + 2[\Psi_{3,x}]^T [\Psi_{3,y}] \end{array} \right\} \{\delta d_e\} \quad (3.40)$$

where $[\Psi_1]$, $[\Psi_2]$ and $[\Psi_3]$ are the first, second, and third rows respectively, of the shape function matrix $[\Psi]$. Eq. (3.29) provides the virtual displacement within the shell element as

$$\{\delta u\} = [\Psi]\{\delta d_e\} \quad (3.41)$$

Applying the equilibrium condition from Eq. (3.37) to an element in the deformed configuration, we have

$$\begin{aligned} \int_{V_e} \{\delta\epsilon\}^T \{\sigma\} dV &= \int_{V_e} \{\delta u\}^T \rho \{\ddot{u}\} dV - \int_{V_e} \{\delta u\}^T \rho g \begin{Bmatrix} l_3 \\ m_3 \\ n_3 \end{Bmatrix} dV \\ &+ \int_{S_e} \{\delta u\}^T (p_S + p_D) \begin{Bmatrix} \Delta n_x \\ \Delta n_y \\ 1 \end{Bmatrix} ds \end{aligned} \quad (3.42)$$

where ρ is the density of the shell; $\{\ddot{u}\}$ is the acceleration of the shell; (l_3, m_3, n_3) are the direction cosines of the global x_3 axis in the element local coordinates; $(\Delta n_x, \Delta n_y, 0)$ denote the change of the surface normal from the static configuration in the local

coordinates; p_S and p_D are the hydrostatic and hydrodynamic pressure components on the element. The first and second terms on the right-hand side of Eq. (3.42) represent changes in potential energy due to the inertial forces and weight of the element. The last term accounts for the fluid pressure on the element.

The principle of virtual work can similarly define the equilibrium condition for the element in the static configuration S_B as

$$\int_{V_e} \{\delta \varepsilon_L\}^T \{\sigma_0\} dV = - \int_{V_e} \{\delta u\}^T \rho g \begin{Bmatrix} l_3 \\ m_3 \\ n_3 \end{Bmatrix} dV + \int_{S_e} \{\delta u\}^T p_S \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} ds \quad (3.43)$$

Subtracting Eq. (3.43) from Eq. (3.42) gives the condition for dynamic equilibrium of the element as

$$\begin{aligned} \int_{V_e} \{\delta \varepsilon_L\}^T [D_p] \{\varepsilon_L\} dV + \int_{V_e} \{\delta \varepsilon_{NL}\}^T \{\sigma_0\} dV &= \int_{V_e} \{\delta u\}^T \rho \{\ddot{u}\} dV \\ &+ \int_{S_e} \{\delta u\}^T p_S \begin{Bmatrix} \Delta n_x \\ \Delta n_y \\ 0 \end{Bmatrix} ds + \int_{S_e} \{\delta u\}^T p_D \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} ds \end{aligned} \quad (3.44)$$

From Eqs. (3.31) and (3.39), we obtain

$$\int_{V_e} \{\delta \varepsilon_L\}^T [D_p] \{\varepsilon_L\} dV = \{\delta d_e\}^T \int_{V_e} [B_e]^T [D_p] [B_e] dV \{d_e\} = \{\delta d_e\}^T [K_e] \{d_e\} \quad (3.45)$$

where $[K_e]$ is the shell element stiffness matrix in Eq. (3.32). From Eq. (3.40), we can write

$$\begin{aligned} \{\delta \varepsilon_{NL}\}^T \{\sigma_0\} &= \{\delta d_e\}^T \left[\left([\Psi_{1,x}]^T [\Psi_{1,x}] + [\Psi_{2,x}]^T [\Psi_{2,x}] + [\Psi_{3,x}]^T [\Psi_{3,x}] \right) \sigma_{x0} \right. \\ &\quad \left. + \left([\Psi_{1,y}]^T [\Psi_{1,y}] + [\Psi_{2,y}]^T [\Psi_{2,y}] + [\Psi_{3,y}]^T [\Psi_{3,y}] \right) \sigma_{y0} \right. \\ &\quad \left. + 2 \left([\Psi_{1,y}]^T [\Psi_{1,y}] + [\Psi_{2,y}]^T [\Psi_{2,y}] + [\Psi_{3,y}]^T [\Psi_{3,y}] \right) \tau_{xy0} \right] \{d_e\} \end{aligned} \quad (3.46)$$

Eq. (3.46) can be written in a compact form as

$$\{\delta \varepsilon_{NL}\}^T \{\sigma_0\} = \{\delta d_e\}^T [\Gamma]^T \begin{bmatrix} \mathbf{s} & 0 & 0 \\ 0 & \mathbf{s} & 0 \\ 0 & 0 & \mathbf{s} \end{bmatrix} [\Gamma] \{d_e\} \quad (3.47)$$

where $[\Gamma]$ is a 6×18 matrix given as

$$[\Gamma] = \left[\begin{bmatrix} \Psi_{1,x} \end{bmatrix} \quad \begin{bmatrix} \Psi_{1,y} \end{bmatrix} \quad \begin{bmatrix} \Psi_{2,x} \end{bmatrix} \quad \begin{bmatrix} \Psi_{2,y} \end{bmatrix} \quad \begin{bmatrix} \Psi_{3,x} \end{bmatrix} \quad \begin{bmatrix} \Psi_{3,y} \end{bmatrix} \right]^T \quad (3.48)$$

and $\mathbf{s}$ is the 2×2 initial stress matrix given as

$$\mathbf{s} = \begin{bmatrix} \sigma_{x0} & \tau_{xy0} \\ \tau_{xy0} & \sigma_{y0} \end{bmatrix} \quad (3.49)$$

Integrating Eq. (3.47) over the volume of the shell element, we have

$$\int_{V_e} \{\delta \varepsilon_{NL}\}^T \{\sigma_0\} dV = \{\delta d_e\}^T \int_{V_e} [\Gamma]^T \begin{bmatrix} \mathbf{s} & 0 & 0 \\ 0 & \mathbf{s} & 0 \\ 0 & 0 & \mathbf{s} \end{bmatrix} [\Gamma] dV \{d_e\} = \{\delta d_e\}^T [K_\sigma] \{d_e\} \quad (3.50)$$

where $[K_\sigma]$ is the initial stress or the geometric stiffness matrix given as

$$[K_\sigma] = \int_{V_e} [\Gamma]^T \begin{bmatrix} \mathbf{s} & 0 & 0 \\ 0 & \mathbf{s} & 0 \\ 0 & 0 & \mathbf{s} \end{bmatrix} [\Gamma] dV \quad (3.51)$$

Eqs. (3.45) and (3.50) define the constitutive and initial stress stiffness matrices for the left-hand side of the equation of dynamic equilibrium.

The right-hand side of the dynamic equilibrium condition, given by Eq. (3.44), includes external forces on the element. For the inertia force in the first term, the acceleration at a general point within the shell element can be obtained from the second derivative of Eq. (3.29) in time as

$$\{\ddot{u}\} = [\Psi] \{\ddot{d}_e\} \quad (3.52)$$

where $\{\ddot{d}_e\}$ is a 18×1 nodal acceleration column vector for the shell element. From the virtual displacement within the shell as defined by Eq. (3.41), we obtain

$$\int_{V_e} \{\delta u\}^T \rho \{\ddot{u}\} dV = \{\delta d_e\}^T \int_{V_e} [\Psi]^T \rho [\Psi] dV \{\ddot{d}_e\} = \{\delta d_e\}^T [M_e] \{\ddot{d}_e\} \quad (3.53)$$

where $[M_e]$ is the 18×18 element mass matrix given as

$$[M_e] = \int_{V_e} [\Psi]^T \rho [\Psi] dV \quad (3.54)$$

In the second term of the right-hand side of Eq. (3.44), the change in the surface normal is caused by the rotation parallel to the body surface and for small deformation, is tangential to the surface given as

$$\Delta \mathbf{n} = \boldsymbol{\theta} \times \mathbf{n} = \theta_y \mathbf{i} - \theta_x \mathbf{j} \quad (3.55)$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors along the local x and y axes. Using Eq. (3.13), Eq. (3.55) can be written as

$$\Delta \mathbf{n} \equiv \Delta n_x \mathbf{i} + \Delta n_y \mathbf{j} = \beta_x \mathbf{i} + \beta_y \mathbf{j} \quad (3.56)$$

The normal rotation components β_x and β_y are given by Eq. (3.15). These representations can be expanded in terms of the nodal displacement vector $\{d_e\}$ as

$$\beta_x = [P_x] [T_{DKT}] \{d_e\}; \quad \beta_y = [P_y] [T_{DKT}] \{d_e\} \quad (3.57)$$

where $[T_{DKT}]$ is a 9×18 conversion matrix given by

$$[T_{DKT}] = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.58)$$

Thus the second term on the right-hand side of Eq. (3.44) can be written as

$$\int_{S_e} \{\delta u\}^T p_s \begin{Bmatrix} \Delta n_x \\ \Delta n_y \\ 0 \end{Bmatrix} ds = \{\delta d_e\}^T \int_{S_e} [\Psi]^T p_s \begin{Bmatrix} P_x \\ P_y \\ 0 \end{Bmatrix} [T_{DKT}] ds \{d_e\} = \{\delta d_e\}^T [R_e] \{d_e\} \quad (3.59)$$

where the initial load stiffness matrix $[R_e]$ is given as

$$[R_e] = \int_{S_e} [\Psi]^T p_s \begin{Bmatrix} P_x \\ P_y \\ 0 \end{Bmatrix} [T_{DKT}] ds \quad (3.60)$$

Substituting Eq. (3.41) into the last term of Eq. (3.44), we have

$$\int_{S_e} \{\delta u\}^T p_D \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} ds = \{\delta d_e\}^T \int_{S_e} [\Psi]^T p_D \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} ds = \{\delta d_e\}^T \{L_e\} \quad (3.61)$$

where $\{L_e\}$ is the dynamic pressure load vector at the element nodes given by

$$\{L_e\} = \int_{S_e} [\Psi]^T p_D \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} ds \quad (3.62)$$

All the terms in Eq. (3.44) have been determined to define the dynamic equilibrium of the element.

Since the virtual displacements are arbitrary, substituting Eqs. (3.45), (3.50), (3.53), (3.59), and (3.61) into Eq. (3.44), the condition for dynamic equilibrium of the element becomes

$$[M_e]\{\ddot{d}_e\} + ([K_e] + [K_\sigma] - [R_e])\{d_e\} = \{L_e\} \quad (3.63)$$

The deformation at a point in the shell can be transformed to the global coordinates according to

$$\{u_G\} = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix} = [C]\{u\} \quad (3.64)$$

where (u_1, u_2, u_3) are the displacement components in the global coordinates; (l_1, l_2, l_3) , (m_1, m_2, m_3) , and (n_1, n_2, n_3) are the direction cosines of the local x , y , and z axes, respectively, in the global coordinates. This transformation converts the displacement vector from the local to the global coordinates

$$\{d_{eG}\} = \begin{bmatrix} \mathbf{C} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{C} & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{C} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{C} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{C} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{C} \end{bmatrix} \{d_e\} = [T]\{d_e\} \quad (3.65)$$

in which $\mathbf{C}$ is the square matrix $[C]$ from Eq. (3.64) and $[T]$ is an 18×18 transformation matrix. Combining Eqs. (3.29), (3.64) and (3.65), the displacement within the element can be interpolated from the global nodal displacement as

$$\{u_G\} = [C][\Psi][T]^T \{d_{eG}\} = [\Psi_G]\{d_{eG}\} \quad (3.66)$$

The element mass and tangential stiffness matrices can be transformed to the global coordinates according to

$$[M_{eG}] = [T]^T [M_e] [T] \quad (3.67)$$

$$[K_{eG}] = [T]^T ([K_e] + [K_\sigma] - [R_e]) [T] \quad (3.68)$$

The dynamic pressure load vector can likewise be transformed to

$$\{L_{eG}\} = [T] \{L_e\} \quad (3.69)$$

The equation for dynamic equilibrium of the element in the global coordinates reads

$$[M_{eG}] \{\ddot{d}_{eG}\} + [K_{eG}] \{d_{eG}\} = \{L_{eG}\} \quad (3.70)$$

This serves as the dynamic boundary condition on the body surface to balance the external pressure with hull deformation and inertia.

Chapter 4

Coupling of Boundary and Finite Element Models

The hydrodynamic model for the potential flow around rigid hull is presented in Chapter 2. This model will now be adapted to include the flow past a flexible vessel. The hydrodynamic model will then be coupled to the finite element model of the vessel, developed in Chapter 3, through the kinematic and dynamic boundary conditions on the wetted hull surface.

4.1 Flow past a Flexible Body

The fluid domain is bounded by the still-water surface S_F , the equilibrium body surface S_B , and a control surface S_C , as shown in Fig. 2.1. Since the fluid is inviscid and incompressible, a complex velocity potential provides a description of the irrotational flow as

$$\Phi(\mathbf{x}, t) = U[\phi_0(\mathbf{x}) - x_1] + [\phi_W(\mathbf{x}) + \phi_S(\mathbf{x})]e^{-i\omega_e t} \quad (4.1)$$

where ϕ_S is the complex amplitude of the scattered potential, which includes both diffracted and radiated waves, as the finite element model does not differentiate rigid and elastic motions of the vessel. The boundary-value problem for the steady potential ϕ_0 is

already defined in Chapter 2. This section describes the boundary conditions for the scattered potential ϕ_S .

The free surface boundary condition in presence of forward speed from Newman (1978) is adapted to include a single scattered potential for diffracted and radiated waves as

$$g \frac{\partial \phi_S}{\partial n} - \omega_e^2 \phi_S + 2i\omega_e U \frac{\partial \phi_S}{\partial x_1} + U^2 \frac{\partial^2 \phi_S}{\partial x_1^2} - i\omega_e U \nabla_1^2 \phi_0 \phi_S - 2i\omega_e U \nabla_1 \phi_0 \cdot \nabla_1 \phi_S \\ = i\omega_e U \nabla_1^2 \phi_0 \phi_W + 2i\omega_e U \nabla_1 \phi_0 \cdot \nabla_1 \phi_W \quad \text{on} \quad S_F \quad (4.2)$$

Because the scattered potential includes diffracted waves, the particular terms on the right-hand side account for the modification of the incident waves by the near-field steady flow. The body surface undergoes harmonic oscillations at the encounter frequency ω_e . The normal fluid particle velocity at the body surface S_D is equal to the surface velocity expressed as

$$\nabla \Phi \cdot \mathbf{n} = \mathbf{v} \cdot \mathbf{n} \quad \text{on} \quad S_D \quad (4.3)$$

where $\mathbf{v}$ is the surface velocity vector. Substituting Eqs. (2.6) and (4.1) into Eq. (4.3), the boundary condition on the body surface becomes

$$\mathbf{W} \cdot \mathbf{n} + \left(\frac{\partial \phi_S}{\partial n} + \frac{\partial \phi_W}{\partial n} \right) e^{-i\omega_e t} = \mathbf{v} \cdot \mathbf{n} \quad \text{on} \quad S_D \quad (4.4)$$

As the body surface oscillates about its static equilibrium position, the steady flow vector $\mathbf{W}$ and the surface unit normal $\mathbf{n}$ undergoes perturbation about the respective vectors in the static equilibrium position according to

$$\mathbf{W}|_{S_D} = \left\{ 1 + \left(\mathbf{u} e^{-i\omega_e t} \cdot \nabla \right) \right\} \mathbf{W}|_{S_B} \quad (4.5)$$

$$\mathbf{n}|_{S_D} = \left\{ \mathbf{n} + \boldsymbol{\theta} e^{-i\omega_e t} \times \mathbf{n} \right\}|_{S_B} \quad (4.6)$$

where $\mathbf{u}$ and $\boldsymbol{\theta}$ are the complex amplitudes of displacement and rotation vectors on the body surface. Since the surface deformation and rotation are small, only the first order

terms are considered here. From Eqs. (2.5), (4.5), and (4.6), the first term on the left hand side of Eq. (4.4) can be written as

$$(\mathbf{W} \cdot \mathbf{n})|_{S_D} = \left[\{(\mathbf{n} \cdot \nabla) \mathbf{W}\} \cdot \mathbf{u} - (\mathbf{W} \times \mathbf{n}) \cdot \boldsymbol{\theta} \right]_{S_B} e^{-i\omega_s t} \quad (4.7)$$

Substituting Eq. (4.7) into (4.4), we have the body boundary condition of the scattered potential on the static equilibrium surface S_B as

$$\frac{\partial \phi_s}{\partial n} = \dot{\mathbf{u}} \cdot \mathbf{n} - \{(\mathbf{n} \cdot \nabla) \mathbf{W}\} \cdot \mathbf{u} + (\mathbf{W} \times \mathbf{n}) \cdot \boldsymbol{\theta} - \frac{\partial \phi_w}{\partial n} \quad \text{on } S_B \quad (4.8)$$

where $\dot{\mathbf{u}}$ is the complex amplitude of the surface velocity vector. The radiation condition for the scattered potential, accounting the effects of forward speed, are defined in Eq. (2.18) as

$$\begin{aligned} \frac{\partial \phi_s}{\partial n} - i k' \phi_s \cos \theta &= 0 & \text{on } S_{C1} \\ \phi_s &= 0 & \text{on } S_{C2} \end{aligned} \quad (4.9)$$

where k' is the local wave number and θ is the rotation of the scattered wave direction relative to the radial axis as determined from Eqs. (2.14) to (2.17), and S_{C1} and S_{C2} denote the portions of S_C with and without the scattering waves.

Since the scattered flow potential satisfies the Laplace equation, Green's second identity expresses the flow potential at a field point $\mathbf{x}$ as an integral over the fluid boundary as

$$\mu \phi_s(\mathbf{x}) = \int_S \left(G(\mathbf{x}, \boldsymbol{\xi}) \frac{\partial \phi_s}{\partial n}(\boldsymbol{\xi}) - \phi_s(\boldsymbol{\xi}) \frac{\partial G}{\partial n}(\mathbf{x}, \boldsymbol{\xi}) \right) ds \quad (4.10)$$

in which $G(\mathbf{x}, \boldsymbol{\xi})$ is the Rankine-type Green function of Eq. (2.21). The fluid boundary is discretized into a total of N_T constant quadrilateral elements, of which N_B are on the wetted hull surface, N_F are on the free surface, N_{C1} are on S_{C1} , and the remaining boundary elements are on S_{C2} . The fluid elements share the same nodes as the finite elements of the structure for efficient transfer of information and coupling of the two

models. Substitution of the free surface boundary condition from Eq. (4.2) and the radiation condition from Eq. (4.9) into the discretized form of the boundary integral equation at quadrilateral element i gives

$$\begin{aligned}
& 2\pi \phi_S(\mathbf{x}_i) + \sum_{j=1}^{N_B+N_F+N_{C1}} \phi_S(\xi_j) \int_{S_j} \frac{\partial G}{\partial n} ds - \frac{\omega_e^2}{g} \sum_{j=N_B+1}^{N_B+N_F} \phi_S(\xi_j) \int_{S_j} G ds \\
& - \frac{i\omega_e U}{g} \sum_{j=N_B+1}^{N_B+N_F} (\nabla_1^2 \phi_0 \phi_S)_{\xi_j} \int_{S_j} G ds + \frac{U^2}{g} \sum_{j=N_B+1}^{N_B+N_F} \frac{\partial^2 \phi_S}{\partial x_1^2}(\xi_j) \int_{S_j} G ds \\
& - \frac{2i\omega_e U}{g} \sum_{j=N_B+1}^{N_B+N_F} \left[\left(\frac{\partial \phi_0}{\partial x_1} - 1 \right) \frac{\partial \phi_S}{\partial x_1} + \frac{\partial \phi_0}{\partial x_2} \frac{\partial \phi_S}{\partial x_2} \right]_{\xi_j} \int_{S_j} G ds \\
& - i \sum_{j=N_B+N_F+1}^{N_B+N_F+N_{C1}} [k' \phi_S \cos \theta]_{\xi_j} \int_{S_j} G ds - \sum_{j=N_B+N_F+N_{C1}+1}^{N_T} \frac{\partial \phi_S}{\partial n}(\xi_j) \int_{S_j} G ds = R_S
\end{aligned} \tag{4.11}$$

where

$$\begin{aligned}
R_S = & \int_{S_B} G \frac{\partial \phi_S}{\partial n} ds + \frac{i\omega_e U}{g} \sum_{j=N_B+1}^{N_B+N_F} (\nabla_1^2 \phi_0 \phi_W)_{\xi_j} \int_{S_j} G ds \\
& + \frac{2i\omega_e U}{g} \sum_{j=N_B+1}^{N_B+N_F} \left[\frac{\partial \phi_0}{\partial x_1} \frac{\partial \phi_W}{\partial x_1} + \frac{\partial \phi_0}{\partial x_2} \frac{\partial \phi_W}{\partial x_2} \right]_{\xi_j} \int_{S_j} G ds
\end{aligned} \tag{4.12}$$

The coefficients of ϕ_S and $\partial \phi_S / \partial n$ on the left side of Eq. (4.11) are identical to those of ϕ_i and $\partial \phi_i / \partial n$ in Eq. (2.24). Applying Eq. (4.11) to all fluid boundary elements, a system of equations similar to Eq. (2.29) is obtained

$$[P'] \{X_S\} = \{R_S\} \tag{4.13}$$

where the square coefficient matrix $[P']$ is identical to that of Eq. (2.29); $\{X_S\}$ contains the unknown values of ϕ_S on S_B , S_F , and S_{C1} as well as $\partial \phi_S / \partial n$ on S_{C2} .

The fluid motion at the quadrilateral boundary element is derived from the body surface motion at the triangular finite element according to Eq. (4.8). Though the body surface

deformation is continuous, it is assumed that the motion at each finite element in the global coordinates can be represented by three translation components ($\alpha_1, \alpha_2, \alpha_3$) and three rotation components ($\alpha_4, \alpha_5, \alpha_6$) at the centroid. The rotation components are averaged from the respective values at the finite element nodes

$$\begin{Bmatrix} \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1/3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 0 \\ 1/3 & 0 & 0 & 0 & 0 & 0 & 1/3 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 0 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 0 & 0 & 0 & 1/3 \end{bmatrix} \{d_{eG}\} \quad (4.14)$$

where $\{d_{eG}\}$ is the nodal displacement vector of Eq. (3.65). The average translation components are then obtained according to

$$\begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix} = \begin{Bmatrix} u_{1C} \\ u_{2C} \\ u_{3C} \end{Bmatrix} - \begin{Bmatrix} \alpha_5 x_{3C} - \alpha_6 x_{2C} \\ \alpha_6 x_{1C} - \alpha_4 x_{3C} \\ \alpha_4 x_{2C} - \alpha_5 x_{1C} \end{Bmatrix} \quad (4.15)$$

where (x_{1C}, x_{2C}, x_{3C}) and (u_{1C}, u_{2C}, u_{3C}) are the position and displacement of the element centroid in global coordinates. The latter is obtained from Eq. (3.66) as

$$\begin{Bmatrix} u_{1C} \\ u_{2C} \\ u_{3C} \end{Bmatrix} = [\Psi_G(x_{1C}, x_{2C}, x_{3C})] \{d_{eG}\} \quad (4.16)$$

where the interpolation functions have been computed at the centroid. Combining Eqs. (4.14)-(4.16), we have

$$\{\alpha\} = [\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4 \quad \alpha_5 \quad \alpha_6]^T = [O_e] \{d_{eG}\} \quad (4.17)$$

where $[O_e]$ is a 6×18 matrix, which transforms the global nodal displacement to motions at the centroid of element e . In terms of these average motions, the surface displacement and rotation vectors within the element take the form

$$\mathbf{u} = [(\alpha_1, \alpha_2, \alpha_3) + (\alpha_4, \alpha_5, \alpha_6) \times (x_1, x_2, x_3)] \quad (4.18)$$

$$\boldsymbol{\theta} = (\alpha_4, \alpha_5, \alpha_6) \quad (4.19)$$

Substituting Eqs. (4.18) and (4.19) into Eq. (4.8) gives the body boundary condition with the m -terms analogous to Newman (1978) for a rigid body.

$$\frac{\partial \phi_s}{\partial n} = -i\omega_e \sum_{l=1}^6 n_l \alpha_l + U \sum_{l=1}^6 m_l \alpha_l - \frac{\partial \phi_w}{\partial n} \quad \text{on } S_B \quad (4.20)$$

where $(n_4, n_5, n_6) \equiv \mathbf{x} \times \mathbf{n}$ and (m_1, m_2, m_3) and (m_4, m_5, m_6) are as defined in Eqs. (2.10) and (2.11), respectively.

Each quadrilateral boundary element on the body surface consists of two triangles corresponding to a total of $2N_B$ shell elements for the numerical integration. From the body boundary condition in Eq. (4.20), the first term on the right-hand side of Eq. (4.12) can be written as

$$\int_{S_B} G \frac{\partial \phi_s}{\partial n} ds = \sum_{j=1}^{2N_B} [C_1 \ C_2 \ C_3 \ C_4 \ C_5 \ C_6] \{\alpha\} - \int_{S_B} G \frac{\partial \phi_w}{\partial n} ds \quad (4.21)$$

where

$$C_l = \int_{S_j} G (-i\omega_e n_l + U m_l) ds \quad (l = 1, 2, \dots, 6) \quad (4.22)$$

Substituting $\{\alpha\}$ from Eq. (4.17) into Eq. (4.21), we obtain

$$\int_{S_B} G \frac{\partial \phi_s}{\partial n} ds = \sum_{j=1}^{2N_B} [C_1 \ \dots \ C_6] [O_j] \{d_{jG}\} - \int_{S_B} G \frac{\partial \phi_w}{\partial n} ds = [Q_1] \{d\} - Q_2 \quad (4.23)$$

where $\{d_{jG}\}$ is the global nodal displacement vector of shell element j ; $\{d\}$ is the global displacement vector from all the shell elements; $[Q_1]$ is a coefficient matrix accounting for the hydrodynamic pressure associated with surface motions, and Q_2 , which is derived

from integration of $G\partial\phi_w/\partial n$ over the wetted hull surface, represents the effects of the incident waves. Substituting Eq. (4.23) into Eq. (4.12), we have the forcing term

$$R_s = [Q_1] \{d\} - Q_2 + Q_3 \quad (4.24)$$

where Q_3 is derived from integration of the last two terms of Eq. (4.12). The forcing term R_s can be evaluated at the center of each fluid element to obtain

$$\{R_s\} = [Q_1'] \{d\} - \{Q_2'\} + \{Q_3'\} \quad (4.25)$$

With N_D denoting the total degrees of freedom of the finite element model, $[Q_1']$ is a $N_T \times N_D$ complex coefficient matrix and $\{Q_2'\}$ and $\{Q_3'\}$ are N_T complex vectors obtained by combining the Q_2 and Q_3 terms from all the fluid elements.

Substituting Eq. (4.25) into (4.13), the scattering potential can be written in terms of the nodal displacement vector as

$$\{X_s\} = [P']^{-1} [Q_1'] \{d\} + [P']^{-1} (-\{Q_2'\} + \{Q_3'\}) = [J_1'] \{d\} + \{J_2'\} \quad (4.26)$$

Isolating the first N_B equations from the system, the scattering potential on the body surface quadrilateral elements are given as

$$\{\phi_s\} = [J_1] \{d\} + \{J_2\} \quad (4.27)$$

where $[J_1]$ is a $(N_B \times N_D)$ complex coefficient matrix and $\{J_2\}$ is a $(N_B \times 1)$ complex coefficient vector corresponding to the first N_B rows of $[J_1']$ and $\{J_2'\}$, respectively. The normal gradients of the scattering potentials on the body surface can be determined by implementation of Eq. (4.23) to all N_B quadrilaterals as

$$[P''] \left\{ \frac{\partial \phi_s}{\partial n} \right\} = [Q_1''] \{d\} - \{Q_2''\} \quad (4.28)$$

where $[P'']$ is a $(N_B \times N_B)$ coefficient matrix from numerical integration of the Green function over the constant boundary elements on the body surface; $[Q_1'']$ and $\{Q_2''\}$ correspond to the first N_B rows of $[Q_1']$ and $\{Q_2'\}$. Thus, we have

$$\left\{ \frac{\partial \phi_s}{\partial n} \right\} = [P'']^{-1} [Q_1''] \{d\} - [P'']^{-1} \{Q_2''\}$$

or, $\left\{ \frac{\partial \phi_s}{\partial n} \right\} = [J_3] \{d\} + \{J_4\}$ (4.29)

where $[J_3]$ is a $(N_B \times N_D)$ complex coefficient matrix and $\{J_4\}$ is a $(N_B \times 1)$ complex coefficient vector. Eq. (4.27) forms part of the dynamic boundary condition by providing the hydrodynamic pressure, while Eq. (4.29) is the kinematic boundary condition on the body surface relating the normal gradient of the potential with the oscillation of the body surface.

4.2 Equation of Motion

The kinematic and dynamic boundary conditions from Eqs. (4.29) and (4.27) provide the mechanism to couple the boundary element model for the fluid flow with the finite element model of hull deformation. The solution of the fluid flow and body motion is obtained simultaneously in the frequency domain through the equation of motion (3.70). From Bernoulli's equation, the external fluid pressure on the equilibrium body surface is given as

$$p = -\rho \frac{\partial \Phi}{\partial t} - \frac{\rho}{2} \nabla \Phi \cdot \nabla \Phi - \rho g x_3 \quad \text{on} \quad S_B \quad (4.30)$$

As the body oscillates about its equilibrium configuration, the pressure on the deformed surface is given by a Taylor's series expansion as

$$p = (1 + \mathbf{u} \cdot \nabla) \left[-\rho \frac{\partial \Phi}{\partial t} - \frac{\rho}{2} \nabla \Phi \cdot \nabla \Phi - \rho g x_3 \right] \quad \text{on} \quad S_D \quad (4.31)$$

Using Eqs. (4.1) and (2.6) and retaining only terms up to the first order of body motion, the static and dynamic fluid pressure on the mean body surface S_B can be obtained as

$$p_s = -\rho g x_3 - \frac{\rho}{2} \mathbf{u} \cdot \nabla (\mathbf{W} \cdot \mathbf{W}) \cong -\rho g x_3 \quad (4.32)$$

$$p_D = i\omega_e \rho (\phi_W + \phi_S) - \rho \mathbf{W} \cdot \nabla (\phi_W + \phi_S) - \rho g u_3 \quad (4.33)$$

The second term in Eq. (4.32), arising from the velocity-squared pressure, is small and can be ignored as explained in Section 2.3. In the global coordinates, the load vector due to p_D on element e is given by combining Eqs. (3.62), (3.64) and (3.69) as

$$\{L_{eG}\} = \int_{S_e} [\Psi_G]^T p_D \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \end{Bmatrix} ds \quad (4.34)$$

Substituting Eq. (4.33) into Eq. (4.34), the dynamic load vector can be written as

$$\{L_{eG}\} = \int_{S_e} [\Psi_G]^T \{n\} [i\omega_e \rho (\phi_W + \phi_S) - \rho \mathbf{W} \cdot \nabla (\phi_W + \phi_S) - \rho g u_3] ds \quad (4.35)$$

where $\{n\} = [n_1 \ n_2 \ n_3]^T$. At any point within the element, u_3 can be obtained from Eq. (3.66) in terms of the global displacement vector as

$$u_3 = [\Psi_{3G}] \{d_{eG}\} \quad (4.36)$$

where $[\Psi_{3G}]$ is the third row of $[\Psi_G]$ matrix.

In the discretization scheme, each quadrilateral boundary element coincides with two triangular finite elements on the body surface. It is assumed that ϕ_W , ϕ_S , and $\mathbf{W}$ are constant over each boundary element. Considering that finite element e is a part of the boundary element i , Eq. (4.35) can be written as

$$\{L_{eG}\} = (i\omega_e \rho \phi_{Wi} - \rho \mathbf{W} \cdot \nabla \phi_{Wi}) \{I_e\} + (i\omega_e \rho \phi_{Si} - \rho \mathbf{W} \cdot \nabla \phi_{Si}) \{I_e\} - [\Delta_e] \{d_{eG}\} \quad (4.37)$$

where $\{I_e\}$ is a 18×1 column vector given by

$$\{I_e\} = \int_{S_e} [\Psi_G]^T \{n\} ds \quad (4.38)$$

and $[\Delta_e]$ is the 18×18 element hydrostatic stiffness matrix given as

$$[\Delta_e] = \int_{S_e} \rho g [\Psi_G]^T \{n\} [\Psi_{3G}] ds \quad (4.39)$$

The computation of the spatial derivatives in $\mathbf{W} \cdot \nabla \phi_{Si}$ requires special attention. From Eq. (2.6), we have

$$\mathbf{W} \cdot \nabla \phi_S = U \left[\left(\frac{\partial \phi_0}{\partial x_1} - 1 \right) \frac{\partial \phi_S}{\partial x_1} + \frac{\partial \phi_0}{\partial x_2} \frac{\partial \phi_S}{\partial x_2} + \frac{\partial \phi_0}{\partial x_3} \frac{\partial \phi_S}{\partial x_3} \right] \quad (4.40)$$

The spatial derivative $\partial \phi_S / \partial x_1$ in the global coordinates can be expressed in terms the normal potential gradient $\partial \phi_S / \partial n$ at a boundary element and the potential ϕ_S at the neighboring elements through central finite difference formulas. A Jacobian matrix accounts for the surface curvature and the non-rectangular grid in the computation. This gives

$$\left. \frac{\partial \phi_S}{\partial x_1} \right|_i = \sum_k c_{x_1 k} \phi_{Sk} + c_{n_1} \left. \frac{\partial \phi_S}{\partial n} \right|_i \quad (4.41)$$

where k is the index for the neighboring boundary element, ϕ_{Sk} is the scattered potential at the k -th boundary element, and $c_{x_1 k}$ and c_{n_1} are coefficients from the finite difference formulas and the Jacobian transformation. Similarly, we have

$$\left. \frac{\partial \phi_S}{\partial x_2} \right|_i = \sum_k c_{x_2 k} \phi_{Sk} + c_{n_2} \left. \frac{\partial \phi_S}{\partial n} \right|_i \quad (4.42)$$

$$\left. \frac{\partial \phi_S}{\partial x_3} \right|_i = \sum_k c_{x_3 k} \phi_{Sk} + c_{n_3} \left. \frac{\partial \phi_S}{\partial n} \right|_i \quad (4.43)$$

From Eqs. (4.40)-(4.43), we obtain the following for quadrilateral element i

$$\rho \mathbf{W} \cdot \nabla \phi_S \Big|_i = \sum_k f_k \phi_{Sk} + f_n \left. \frac{\partial \phi_S}{\partial n} \right|_i \quad (4.44)$$

where

$$f_k = \rho U \left[\left(\frac{\partial \phi_0}{\partial x_1} - 1 \right) c_{x_1 k} + \frac{\partial \phi_0}{\partial x_2} c_{x_2 k} + \frac{\partial \phi_0}{\partial x_3} c_{x_3 k} \right] \quad (4.45)$$

$$f_n = \rho U \left[\left(\frac{\partial \phi_0}{\partial x_1} - 1 \right) c_{n_1 k} + \frac{\partial \phi_0}{\partial x_2} c_{n_2 k} + \frac{\partial \phi_0}{\partial x_3} c_{n_3 k} \right] \quad (4.46)$$

Substituting Eq. (4.44) into (4.37), we have

$$\begin{aligned} \{L_{eG}\} = & \{I_e\}(\mathbf{i}\omega_e \rho \phi_{Wi} - \rho \mathbf{W} \cdot \nabla \phi_{Wi}) + \{I_e\} \mathbf{i}\omega_e \rho \phi_{Si} \\ & - \{I_e\} \left(\sum_k f_k \phi_{Sk} + f_n \frac{\partial \phi_S}{\partial n} \bigg|_i \right) - [\Delta_e] \{d_{eG}\} \end{aligned} \quad (4.47)$$

which defines the element load vector in terms of the incident, scattered and the steady flow potentials.

The scattered potential and its normal gradient on the body surface are defined by Eqs. (4.27) and (4.29). Substituting these into Eq. (4.47), we have

$$\begin{aligned} \{L_{eG}\} = & \{I_e\}(\mathbf{i}\omega_e \rho \phi_{Wi} - \rho \mathbf{W} \cdot \nabla \phi_{Wi}) + \{I_e\} \mathbf{i}\omega_e \rho (\lfloor J_{1i} \rfloor \{d\} + J_{2i}) \\ & - \{I_e\} \left[\sum_k f_k (\lfloor J_{1k} \rfloor \{d\} + J_{2k}) + f_n (\lfloor J_{3i} \rfloor \{d\} + J_{4i}) \right] - [\Delta_e] \{d_{eG}\} \end{aligned} \quad (4.48)$$

Here, $\lfloor J_{1i} \rfloor$ and $\lfloor J_{3i} \rfloor$ are the i -th row of the matrices $[J_1]$ and $[J_3]$, and J_{2i} and J_{4i} are the i -th entry of the column vectors $\{J_2\}$ and $\{J_4\}$. The element load vector in the global coordinates can now be further simplified as

$$\{L_{eG}\} = \{F_e\} - [M_{Ae}] \{\ddot{d}\} - [D_e] \{\dot{d}\} - [\Delta_e] \{d_{eG}\} \quad (4.49)$$

where $\{\ddot{d}\}$ and $\{\dot{d}\}$ are the global acceleration and velocity vectors at all finite element nodes; $\{F_e\}$ is the external force vector on the shell element; $[M_{Ae}]$ and $[D_e]$ are the $18 \times N_D$ element added mass and damping matrices. These matrices are given as

$$\{F_e\} = \{I_e\}(\mathbf{i}\omega_e \rho \phi_{Wi} - \rho \mathbf{W} \cdot \nabla \phi_{Wi}) + \{I_e\} \mathbf{i}\omega_e \rho J_{2i} - \{I_e\} \left(\sum_k f_k J_{2k} + f_n J_{4i} \right) \quad (4.50)$$

$$[M_{Ae}] = \{I_e\} \left(-\frac{\rho}{\omega_e} \text{Im} \lfloor J_{1i} \rfloor - \frac{f_n}{\omega_e^2} \text{Re} \lfloor J_{3i} \rfloor - \sum_k \frac{f_k}{\omega_e^2} \text{Re} \lfloor J_{1k} \rfloor \right) \quad (4.51)$$

$$[D_e] = \{I_e\} \left(\rho \text{Re} \lfloor J_{1i} \rfloor - \frac{f_n}{\omega_e} \text{Im} \lfloor J_{3i} \rfloor - \sum_k \frac{f_k}{\omega_e} \text{Im} \lfloor J_{1k} \rfloor \right) \quad (4.52)$$

Substituting the dynamic load vector from Eq. (4.49) into the element equilibrium equation (3.70), we have the equation of motion at each element as

$$[M_{eG}]\{\ddot{d}_{eG}\} + [M_{Ae}]\{\ddot{d}\} + [D_e]\{\dot{d}\} + ([K_{eG}] + [\Delta_e])\{d_{eG}\} = \{F_e\} \quad (4.53)$$

The individual element equations can be assembled to form the global equation of motion according to the element-node numbering scheme as

$$([M] + [M_A])\{\ddot{d}\} + [D]\{\dot{d}\} + [K]\{d\} = \{F\} \quad (4.54)$$

where $[M]$, $[M_A]$, and $[D]$ are $N_D \times N_D$ mass, added mass, and damping matrices; $\{F\}$ is the external load vector; and $[K]$ is $N_D \times N_D$ tangential stiffness matrix, which includes the effects of initial stress and hydrostatic pressure. The added mass and damping matrices are full due to hydrodynamic interactions between the elements on the body surface. Expressing the acceleration and velocity in terms of nodal deformation, Eq. (4.54) becomes

$$[-\omega_e^2 ([M] + [M_A]) + [K] - i\omega_e [D]]\{d\} = \{F\} \quad (4.55)$$

A standard matrix solution routine provides the nodal displacement, which in turn give the scattered flow potential on the fluid boundaries from Eq. (4.26). The free surface elevation is then obtained from the dynamic free surface boundary condition in the form

$$\eta = \frac{i\omega_e}{g}(\phi_W + \phi_S) - \frac{1}{g} \mathbf{W} \cdot \nabla (\phi_W + \phi_S) \quad \text{on} \quad S_F \quad (4.56)$$

The formulation was coded in FORTRAN 95 and the computations were performed in parallel over eight processors on a Quad Core Intel Nehalem node with 2.66 GHz and 24 GB RAM in a Dell cluster.

Chapter 5

Rigid Vessels Advancing in a Seaway

The hydrodynamic model presented in Chapter 2 is applicable to a rigid vessel advancing in a wave field over bounded or unbounded fluid or a combination of both as in a tow-tank experiment. Its versatility lies in the implementation of a proper radiation condition and the double-body flow that allow application to full ranges of forward speeds and hull forms. For demonstration and validation of the model, we consider a Wigley hull form defined as

$$x_2 = \frac{B}{2} \left[1 - \left(\frac{x_3}{D} \right)^2 \right] \left[1 - \left(\frac{2x_1}{L} \right)^2 \right] \left[1 + 0.2 \left(\frac{2x_1}{L} \right)^2 \right] \quad (5.1)$$

where B , D , and L are the beam, draft, and length. This is a generic hull form commonly used in numerical and laboratory studies of ship motion. Fig. 5.1 shows an example discretization of a rectangular domain with the Wigley III hull of Journee (1992) at the center. The 3 m long laboratory model has $L/B = 10$ and $L/D = 16$ with the center of gravity at $(0.0, 0.0, -0.0175\text{m})$ and the pitch radius of gyration of 0.75 m. Qiu and Peng (2007) and Kim and Kim (2010) also validated their numerical models with this hull form confirming the quality of the laboratory measurements. In the present study, the quadrilateral boundary elements on the wetted hull are half the size of those on the free surface to resolve better the hull geometry. The ratio of element size can be adjusted for

more complex hull forms. The radiation condition is implemented on the entire control surface to model ship motions in unbounded fluid for verification and validation of the model as well as examination of the wave pattern and vessel dynamic response.

5.1 Model Convergence

The numerical formulation and discretization introduce errors in the solution that need special consideration prior to general application of the model. The model convergence is examined through a numerical experiment on the Wigley III hull of Journee (1992). Fig. 5.2 presents the heave added mass and damping coefficients, the vertical wave excitation force, and the heave response amplitude operator (RAO) as functions of the number of quadrilateral elements per incident wavelength N_λ . The wavelength to ship length ratio of $\lambda/L = 1$ in head waves corresponds to a critical condition in ship design. The control surface is placed at 1, 2, and 3λ from the center. The results at Froude number $Fn = U / \sqrt{gL} = 0$ provide a reference for comparison. For $Fn = 0.15$, the parameter $\tau = \omega_e U/g$ is 0.52, which is greater than the critical value of 0.25. The scattered waves cannot propagate ahead of the vessel to reach part of the control surface for testing of the two components of the radiation condition in Eq. (2.18).

For the case of zero forward speed, the numerical results do not vary appreciably with the location of the control surface beyond two wavelengths from the origin and converge monotonically with the grid size to achieve adequate accuracy with resolution as low as $N_\lambda = 11$. When the vessel advances in head waves, the results converge non-monotonically with the grid size and control surface location, indicating reflection of numerical errors in the computational domain. The convergence behaviors of the hydrodynamic coefficients and excitation force are very similar inferring that the

additional computations involving the m -terms does not adversely affect the accuracy of the computed radiation potentials. Numerical errors increase through the equation of motion (2.39) in the computation of the RAOs due to resonance of the vessel at the selected wave frequency. The analysis shows reasonable results with 18 elements per wavelength and the control surface at three wavelengths from the center. We maintain these criteria in the results presented in the rest of this paper, even though the convergence characteristics are better for other values of λ/L away from the resonance frequency.

The more stringent convergence requirement for the forward speed case is due to the Doppler shift. The scattered waves in the upstream direction have shorter wavelengths and thus require smaller elements to resolve the wave field. However, this does not seem to present any numerical issues at high forward speeds, when both the wavelength and amplitude approach zero ahead of the vessel. The Doppler shift of the scattered wave field also presents a challenge to the implementation of the radiation condition. The longer waves and the shift of the origin downstream of the vessel require the placement of the control surface further away to satisfy the point source assumption. The model apparently handles the phenomena well with constant boundary elements and provides accurate results with a domain of reasonable dimensions and resolution commonly used in ship and platform design.

5.2 Surface Wave Pattern

The Doppler shift associated with the forward speed of the vessel modifies the local dispersion relation and alters the scattered wave pattern. The present study introduces a new radiation condition, which takes into account the direction and celerity of the

scattered waves at the control surface. The radiation condition ensures that the normal component of the scattered waves, if present on the control surface, is outgoing. The convergence analysis of the numerical model has identified some interesting properties of the radiation condition that is further examined and elaborated through comparison of the surface wave patterns over a range of Froude number from 0 to 0.3.

First the heave motion of the vessel is considered at 0.69 Hz, which at zero forward speed generates a wavelength of $\lambda = 1.1L$. Fig. 5.3 shows the radiated wave fields on the two sides of the vessel computed with the control surface at 2 and 3λ from the center. At zero forward speed, the waves follow a circular pattern beyond one wavelength from the center with the main radiation energy on either side of the vessel. The Doppler shift becomes evident even at a small forward speed resulting in downstream shift of the wave origin and modification of the wavelength according to the propagation direction. When the forward speed reaches the group velocity at $\tau = 0.25$, the radiated waves cannot propagate directly ahead of the vessel. A V-shape quiescent region emerges in front of the vessel at a slightly higher forward speed of $\tau = 0.26$. The x_1 component of the group velocity is equal to the forward speed along the boundary of this region. The quiescent region expands with increasing forward speed as the radiated waves are convected behind the vessel.

Fig. 5.4 shows the diffracted wave patterns computed for unit-amplitude incident waves with the control surface at 2 and 3λ from the center. The incident waves along the negative x_1 axis has the wavelength ratio of $\lambda/L = 1.1$. This gives rise to the same Fn and τ with those in Fig. 5.3 for direct comparison. The waves follow a circular pattern at zero forward speed with the primary energy diffracted into a region aft of the vessel. As the forward speed increases, more energy is being convected downstream than diffracted

around the stern. In contrast to the radiation problem, the convected waves from the two sides interact through a system of transverse waves and approach the downstream boundary at an oblique angle even at high forward speed. The diffracted wave patterns are similar to the time-domain results presented by Kim and Kim (2010) and the frequency-domain results by Ohkusu and Iwashita (1987) based on a forward-speed Green function that satisfies the free surface boundary condition.

The radiated and diffracted wave patterns away from the vessel resemble those of an oscillating source in a current (Ohkusu and Iwashita, 1987). The Doppler shift is apparent and consistent with the assumptions in the radiation condition. Above the critical forward speed, the scattered waves ahead of the vessel vanish before the wavelength theoretically becomes zero, thereby averting potential convergence and stability problems. Reflection of numerical errors from the control surface is apparent, but do not have a significant impact on the solution. The results also show minor distortion of the wave field near the control surface probably due to implementation of the radiation condition at a finite distance from the vessel where the point source assumption is not entirely valid. The overall wave fields obtained with the two computational domains are very similar adjacent to the vessel confirming the effectiveness of the model as a ship-motion prediction tool.

5.3 Radiation Conditions

Sections 5.1 and 5.2 have demonstrated the properties and capabilities of the proposed radiation condition in handling the Doppler shift of the scattered waves. To highlight the importance of the radiation condition to the solution, the numerical results are compared with those obtained without the Doppler shift at low forward speeds and with the

upstream boundary treatment of Nakos and Sclavounos (1990, 1991) for high forward speeds. All three sets of results are computed from the same model setting, except for the treatment of the scattered waves at the control surface, for direct comparison.

The results in Fig. 5.3 have already demonstrated the Doppler shift of the radiated wave field due to heave motions of the vessel. Fig. 5.5 compares the wave fields with and without the Doppler shift in the radiation condition for the same set of forward speeds up to $Fn = 0.09$. The upper half of each panel shows the wave field with the Sommerfeld radiation condition in Eq. (2.19), while the radiation condition in the lower half accounts for the Doppler shift as given by Eq. (2.18). At zero forward speed, the two radiation conditions are identical and produce the same wave pattern. Distortion of the radiated waves and reflection from the control surface become evident when the radiation condition does not account for the Doppler shift introduced by the forward speed. The results remain stable and reasonable for forward speeds up to $\tau = 0.26$, which is just above the group velocity of the scattered waves. The radiation condition without the Doppler shift breaks down at higher values of τ , when convection becomes an important mechanism in wave propagation and the radiated waves propagate aft of the vessel along two dominant directions as in Figs. 5.3e and 5.3f.

The Rankine-source model of Nakos and Sclavounos (1990, 1991) caters to vessel forward speeds much greater than the group velocity of the scattered waves. Since the scattered waves do not propagate ahead of the vessel, a quiescent condition is imposed at the upstream boundary as

$$\left(i\omega_e - U \frac{\partial}{\partial x_1} \right) \phi_l = \left(i\omega_e - U \frac{\partial}{\partial x_1} \right)^2 \phi_l = 0 \quad (l = 1, \dots, 7) \quad (5.2)$$

There are no provisions of radiation conditions on the control surface as in the steady ship-wave solution of Dawson (1977). This approach is implemented and stable results are obtained when all the scattered waves are convected to the downstream boundary. Fig. 5.6 shows very good agreement of the computed wave fields with those from Figs. 5.3e and 5.3f for heave motions of the vessel at $\tau = 0.71$ and 1.23. This approach works well without a radiation condition because at high forward speeds the radiated wave directions are parallel to the downstream boundary. The results show little reflection or numerical noise from all four boundaries. Local peaks, however, are evident in the diffracted wave field in Fig. 5.7 when the waves approach the downstream boundary obliquely. The wave component normal to the boundary requires a radiation condition to assure they are out-going and the lack of it results in a system of short transverse waves that can be seen as modulations of the crest and trough of the diffracted waves in the downstream region. The effects become less noticeable at high forward speeds, as the diffracted waves approach parallel to the downstream boundary.

5.4 Dynamic Response

The laboratory experiment of Journee (1992) provides a dataset to validate the present model. The hydrodynamic force on the hull due to forced motions in otherwise still water defines the added mass and damping coefficients. Figs. 5.8 and 5.9 compare the hydrodynamic coefficients for heave and pitch motions of the Wigley hull as functions of the encounter frequencies for $Fn = 0.3$. The laboratory data obtained from the two amplitudes of forced motions are very close thereby confirming the validity of the measurements and the linear assumption. The computed results at $Fn = 0$ provide a reference for assessment of the forward speed effect. The damping coefficients relate to the generation and propagation of radiated waves that are sensitive to the forward speed

as shown in Fig. 5.3. The added mass coefficients, which represent the inertia or reaction force from the fluid in phase with the oscillating body, are influenced to a lesser extent. While the computed added mass coefficients show good agreement with the laboratory data over the range of encounter frequencies considered, the damping coefficients are lower than the measured values at high frequencies. This discrepancy, which is also observed in the results of Qiu and Peng (2007) and Kim and Kim (2010), might be due to the coupling between radiation and viscous damping at high frequencies that is not considered in potential flow.

Journee (1992) also measured the wave excitation forces and RAOs in heave and pitch for $\lambda/L = 0.5$ to 2. Fig. 5.10 shows good agreement between the computed and measured excitation forces at $Fn = 0.3$. The heave excitation force is not very sensitive to the forward speed as the scattered wave amplitude immediately adjacent to the vessel remains similar over the range of Froude number considered. However, the convection of the scattered waves due to the forward speed shifts the resultant force relative to the vessel and modifies the pitch moment more dramatically. For the RAOs, Journee (1992) provided an additional set of data at $Fn = 0$ for comparison. The computed RAOs agree with the laboratory measurements as shown in Fig. 5.11. The results at $Fn = 0.3$ indicate a sharp increase in the heave and pitch RAOs at $\lambda/L = 1.1$ and 1.25 corresponding to resonance of the Wigley hull in the respective motions. The damping coefficients, despite being underestimated at high frequencies for $Fn = 0.3$, have secondary effects on the response of the dynamic system away from its natural frequency. Sharp resonance peaks are not observed at the zero-speed case, which has different hydrodynamic characteristics. This highlights the importance of forward speed in ship motion and load analysis.

Chapter 6

Flexible Vessel Advancing in a Seaway

Direct coupling of the hydrodynamic and structural components in the hydroelasticity model results in computation of the elastic and rigid body motions simultaneously. In this Chapter, the hydrodynamic component of the model is first verified and compared with laboratory data for the limiting case of a rigid hull. The hydroelasticity model at zero speed is then verified with the standard modal superposition method for a flexible Wigley hull. After verification, the model provides a useful tool to understand the effects of hull flexibility and forward speed on vessel performance in a seaway.

6.1 Model Verification

The boundary element and finite element components in the hydroelasticity model are coupled with direct interactions of loads and motions at the element level in the frequency domain. Although the formulation of the boundary element model is similar to the case of a rigid body, its solution from a multi-degree of freedom system needs separate verification. Because of the lack of suitable data for a flexible hull with forward speed, the structural component and its coupling with the hydrodynamic component are verified with the modal superposition solution at zero speed.

The laboratory experiment of Journee (1992) provides a suitable dataset to examine the hydrodynamic component of the present model. Fig. 6.1 shows a sample discretization of the Wigley hull by triangular shell elements and the fluid surfaces by quadrilateral elements. On the wetted hull surface, a quadrilateral element is formed by combining two triangular shell elements. A mesh density of at least 18 divisions per wavelength is used at the free surface and the control surface is placed at three wavelengths from the hull. The detailed arrangement of the laboratory model and instrumentation are not available. With a constant density, the shell element thickness on the hull is adjusted to match the weight, center of gravity, and the pitch radius of gyration of the laboratory model. A large elastic modulus of 200,000 GPa is used to maintain a near-rigid hull in the hydroelasticity model to generate results that can be compared to the laboratory data. The initial stress is determined from the weight and hydrostatic pressure distributions from the finite element model. The RAO data of Journee (1992) presented earlier in section 5.4 is compared with the predictions from the hydroelasticity model for $\lambda/L = 0.5$ to 2 at $Fn = 0$ and 0.3 in head sea conditions. The computed RAOs in Fig. 6.2 are almost identical to those of the rigid-body motions in Fig. 5.11 and are in good agreement with the laboratory measurements. The model correctly reproduces the resonance peaks in heave and pitch at $\lambda/L = 1.1$ and 1.25 for the forward-speed case. The heave motion shows a local minimum at $\lambda/L = 1.25$ due to coupling with the pitch motion as in a six-degree-of-freedom system.

The modal superposition solution of Riggs *et al.* (2007) allows verification of the hydroelasticity model at zero speed. They considered a Wigley hull defined by

$$x_2 = \frac{B}{2} \left[1 - \left(\frac{x_3}{D} \right)^2 \right] \left[1 - \left(\frac{2x_1}{L} \right)^2 \right] \quad (6.1)$$

where $L = 100$ m, $B = 10$ m, and $D = 2.25$ m with a displaced volume of 1000 m^3 . The hull extends vertically above the still waterline by another 2.25 m to the deck for a molded depth of 4.5 m. The structural design, which might not reflect any particular vessel, serves to accentuate the elastic response for model verification. The elastic modulus for the shell plating is 7.5 GPa with a Poisson's ratio of 0.3. The thickness of the shell plating is 0.02 m throughout. The design does not contain any bulkheads. A bending thickness of 0.25 m and 0.15 m are respectively used for the deck and hull plating to account for the rigidity of stiffeners and frames that are not included in the model. The entire mass of the vessel is distributed uniformly over the deck plating with a density of $76,875 \text{ kg/m}^3$. The vertical center of gravity is thus 2.25 m above the still water surface. Fig. 6.3 shows a sample discretization of the hull and free surface. The fluid boundary is discretized by constant quadrilaterals following the established criteria for convergence.

Fig. 6.4 compares the computed vertical displacement at the bow, midship, and stern for $Fn = 0$ with the results from Riggs *et al.* (2007). The displacement is normalized by the incident wave amplitude. The two models give nearly identical results at midship, but the present model predicts up to 12.5% larger motions at the bow and stern. The discrepancy is primarily due to the difference in the two approaches. The present method couples the hydrodynamic and structural effects directly and obtains the deflection through an equation of motion involving all the finite element nodes, whereas the modal superposition method evaluates the principal coordinates for a series of rigid and elastic modes. In application, the modal superposition method utilizes a limited number of elastic modes to approximate the hull deformation and might not fully capture the motion at the bow and stern as observed in the comparison. Nevertheless, the reproduction of the main response characteristics provides cross verification of the two models. Since this

model hull is specifically designed to accentuate effects of hull flexibility, the subsequent sections continue to use it in the analysis.

6.2 Hydroelasticity at Zero Speed

The hydroelasticity solution at zero speed provides a simple setting to understand the effects of hull flexibility and to provide a baseline for comparison with forward speed cases. Fig 6.5 compares the amplitudes of the vertical motions at the bow, midship, and stern for the elastic and rigid hulls. The latter assumes an elastic modulus of 75,000 GPa, which is 10,000 times of that of the elastic hull. It is necessary to understand the roles of the rigid and elastic motions in the interpretation of the results. Since the rigid hull motion at midship is due to heave only, albeit the weak coupling between rigid and elastic motions, the difference between the two solutions is primarily a result of hull flexibility. The larger motion amplitude from the flexible model implies that the primary bending modes are in phase with the rigid body motion at midship. Pitch becomes a factor in the vertical motion at the bow and stern. Hull flexibility may increase or decrease the vertical motion depending on the phase or excitation frequency. Fig. 6.6 plots the motions of the flexible hull and free surface over one half of the period at 7 s, when the flexible modes accounts for approximately half of the vertical motion at midship. The hull undergoes significant hogging and sagging motions corresponding to the first bending mode of the hull girder. The results highlight the role of hull flexibility in predicting motions even at zero forward speed.

The salient features of the flexible and rigid hull motions and surface waves are better illustrated through the normalized amplitude plots in Fig 6.7. Here the normalized amplitudes of the scattered waves and the deck motions are shown in plan view for four

incident wave periods. The incident waves approach the hull along the negative x_1 direction. The top half of each panel shows the motions from the flexible hull and the bottom half shows the results from the rigid model. Most of the scattered waves, which propagate aft of the vessel, are generated by diffraction. Figs. 6.7(a) and (b) correspond to the first and second peaks of the stern motion amplitude for the flexible hull at $T = 4.25$ s and 5.25 s. The flexible and rigid hull motions are small and very similar as evident from the vertical motion amplitude plots in Fig 6.5. Fig. 6.7(c) is for $T = 7$ s, when elastic deformation becomes important resulting in considerable motions at midship. The flexible hull shows considerable elastic deformation associated with the bending mode, whereas the rigid hull exhibits primarily pitching motions. Fig. 6.7(d) corresponds to the peak bow and stern motions for both the flexible and rigid hulls at $T = 9$ s. The motion of the hull follows that of the water surface with little elastic deformation and wave scattering because of the relatively long wavelength.

The hydroelasticity model also provides the normal and shear stresses in the shell elements for the rigid and elastic hull models. A common indicator for yielding of ductile materials is the von Mises effective stress defined in the plane stress condition as

$$\sigma_{eff} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_x - \sigma_y)^2 + \sigma_x^2 + \sigma_y^2 + 6\sigma_{xy}^2} \quad (6.2)$$

Fig 6.8 compares the maximum effective stress for unit wave amplitude on the hull neutral plane of the flexible and rigid models for the four cases considered in Fig 6.7. The stress on the flexible hull is consistent with that presented by Riggs *et al.* (2007). The maximum effective stress of the rigid model exceeds that of the flexible model for all incident wave periods. Without resonance of the flexible modes, elastic deformation allows compliance of the structure to the wave environment and reduces the loads and stress on the hull.

6.3 Forward Speed in Head Seas

For rigid vessels advancing in a seaway, convection of the scattered waves leads to higher hydrodynamic loading and response in pitch. The present hydroelasticity model allows a thorough investigation of the effects of forward speed on the flexible Wigley hull considered in Section 6.2. For both the rigid and elastic hulls, Fig. 6.9 presents the normalized motion amplitudes at the bow, midship, and stern as functions of the incident wave period for $Fn = 0.3$. The hull flexibility introduces new resonance characteristics superposed on the rigid body response dominated by pitch. Prominent peaks occur in all three amplitude plots for the flexible hull at the incident wave period of 5.15 s. The resulting encounter period of 2.37 s is close to the dry natural period of the first bending mode at 2.15 s from Riggs *et al.* (2007). Since the rigid hull shows little motion at this period, the large motion amplitudes are due to the resonant oscillation of the flexible hull girder. Fig. 6.10 shows the flexible hull motion and surface waves over one half of the resonance period. The results demonstrate the hogging and sagging of the hull associated with the first bending mode. At longer periods, the rigid-body modes dominate the dynamic motion of the hull, while the first bending mode defines the flexural response, but with decreasing amplitude. The sudden drop of the motion amplitude at the bow around 6.75 s is not observed at the center and stern. This can only be explained by the rigid and elastic responses being out-of-phase at the bow near this period.

Fig 6.11 compares the normalized amplitudes of the scattered waves and deck motions for the flexible and rigid hulls. Since there is little rigid body motion at 5.15 s period, the deck motion amplitude clearly illustrates the first bending mode of the flexible hull. The resonance mode has two nodes with nearly zero amplitude across the beam and large motion amplitudes at the bow and stern. Rigid-body motions consisting of heave and

pitch are evident at 6.2 s and are superposed to the bending mode in the flexible hull motion. The combined motion shows a region of small amplitude associated with one of the nodes in the bending mode. The other node forward of midship is masked by the rigid-body motion. As the incident wave period increases, pitch becomes more dominant in the rigid body motion. This interferes constructively and destructively with the flexure response at the stern and bow as seen in Fig. 6.11(c). The motion amplitude is maximum at the bow and stern for both the rigid and flexible hulls at 8 s. The rigid-body motion dominates the overall response and both cases give very similar results. The elastic motion of the hull generally reduces the scattered wave amplitude due to the compliance with the wave field. As the wave period increases further, the amplitude of scattered waves diminishes and the vessel response approaches the incident wave motion.

The flexural response of the hull has an important contribution to the distribution and level of the stress. Fig. 6.12 presents the magnitude of the normalized effective stress on the hull neutral plane at $Fn = 0.3$ for the four wave periods considered in Fig 6.11. At the resonance period of 5.15 s, the effective stress is primarily due to flexural motions of the hull in bending. The designer would grossly underestimate the stress level by not considering hull elasticity in the computation of the hydrodynamic loads. Elastic deformation and stress remain important over a range of wave periods away from the resonance. As the period increases, the longer waves do not excite the bending mode. The primary loading derives from the hogging moment of the hull as in standard ship design practice. The flexible hull experiences a lower stress level due to its compliance with wave motions. The results are consistent with those in the zero-forward speed case, when the elastic modes are not excited at all.

6.4 Forward Speed in Quartering Seas

Marine vessels operating in a seaway encounters waves over a range of frequencies and directions. It is important to study the motion and stress of the vessel in different headings. As an illustration, the motions of the flexible and rigid Wigley hulls are examined in quartering seas, when waves approach the bow at a 45° angle. Fig 6.13 compares the normalized motion amplitudes at the bow, midship, and stern for $Fn = 0.3$. The motion response characteristics are similar to the head sea condition but with a shift with respect to the wave period due to an effective increase of the wavelength relative to the vessel in quartering seas. Resonance of the first bending mode occurs at 4.55 s period for the flexible hull. This corresponds to an encounter period of 2.35 s, which is close to that in head seas. The first resonance peak is not as prominent as in the head sea conditions because the loading is not uniform across the beam and thus is less effective in exciting the bending mode. Similar to the head sea conditions, the bow motion amplitude for the flexible hull drops sharply at 5.9 s due to destructive interference between the rigid and flexible hull motions. The pitch resonance peaks at 7 s with diminishing flexural response for longer incident wave periods.

The quartering seas introduce a torsional component of the hull loading. Fig 6.14 depicts the flexible hull motion and surface waves over one half of the period at $T = 5.9$ s, when destructive interference of the rigid-body and flexural motions effectively pins the bow at its static equilibrium position. The hull exhibits considerable torisonal motion in addition to bending. Figs. 6.15 and 6.16 compare the normalized amplitudes of the scattered waves and deck motions for the vessel advancing in quartering seas. Because of asymmetry of the results about the hull centerline, the flexible and rigid hull responses are shown in separate panels side by side. The rigid hull exhibits little response at the

wave period of 4.55 s, while the flexible hull motion is primarily defined by resonance of the bending mode with minor torsional deformation. The torsion of the flexible hull becomes more noticeable when the bending mode abates with increasing period. The motions of flexible hull clearly demonstrate the torsional deformation at 5.25 s. At the incident wave periods of 7 s and higher, the response of the rigid and flexible hulls becomes similar implying less contribution from elastic deformation.

As with the head sea conditions, the elastic response influences the effective stress on the hull. Fig. 6.17 presents the magnitude of the normalized effective stress on the hull neutral plane at $Fn = 0.3$ and quartering seas for the four wave periods in Figs 6.15-6.16. The results are consistent with those under head sea conditions. Large flexural motions and effective stresses on the flexible hull are evident near the resonance period at $T = 4.55$ s. The assumption of a rigid hull results in significant underestimations of the stress. As the excitation period increases, the flexible hull has lower stress levels due to reduction in loads as it deforms in the wave field. The stress level at resonance in both headings is approximately twice of that under the maximum pitch of the rigid hull, which is normally considered in design. This study has shown the importance of hull flexibility and its interactions with the hydrodynamic loads in evaluating performance and design of marine vessels advancing in a seaway.

Chapter 7

Conclusions

This study has developed, verified, and demonstrated the capabilities of a frequency-domain model for analysis of flexible marine vessels advancing in a seaway. The model includes a hydrodynamic component based on the Rankine-source formulation, which directly includes the double-body flow for application with general hull geometry over a full range of frequency. The model is applicable to vessel speeds above and below the group velocity of the scattered waves with the implementation of a new radiation condition accounting for the Doppler shift of the wave field. This radiation condition reduces to the well-known Sommerfeld radiation condition at zero forward speed. A finite element model defines the structural deformation in response to the hydrodynamic loads. The structural and hydrodynamic models are coupled directly through a kinematic and a dynamic boundary condition on the hull surface. This results in a system of equations in terms of the nodal degrees of freedom on the entire structure. Unlike the modal superposition method, the present approach, which does not require free vibration modes of the vessel, is applicable in the preliminary design stage when structural details are not available. The proposed radiation condition and the effects of forward speed on the elastic response are examined through a series of numerical experiments involving two Wigley hull forms.

The forward speed modifies the convergence characteristics of the hydrodynamic model. The Doppler shift introduces a range of wavelengths for the same frequency and thus requires smaller elements to resolve the short wave components. The presence of longer waves requires placement of the control surface further away for proper implementation of the radiation condition. The computed wave field clearly demonstrates the Doppler shift and the effectiveness of the radiation condition. Contrary to previous postulations, a radiation condition is necessary for the scattered waves that propagate behind the vessel even at high forward speeds. The computed hydrodynamic coefficients, wave excitation forces, and dynamic response of the vessel shows good agreement with published laboratory data for a rigid hull over ranges of forward speeds and wave frequencies, thereby validating the hydrodynamic model. The results for the rigid Wigley hull indicates that the forward speed has less effect on the added mass coefficients, which represents the inertia or reaction force from the fluid surrounding the vessel. The damping coefficients are sensitive to the forward speed as they relate to the generation and propagation of radiated waves that are strongly influenced by the Doppler shift. The modification in the hydrodynamic coefficients due to forward speed results in distinct dynamic characteristics from the zero speed case.

For a flexible hull, the motion predictions at zero speed are in agreement with those from the modal superposition method. The hydroelasticity model reproduces measured response amplitude operators from a laboratory experiment in the limiting case of a rigid hull with forward speed. These analyses verify the structural and hydrodynamic components as well as their coupling through the kinematic and dynamic boundary conditions on the wetted hull surface. The hydroelasticity model, which includes hull deformation in the computation of the hydrodynamic loads, in general predicts a slightly lower stress level on the hull structure. The vessel forward speed increases the encounter

frequency exacerbating the effects of resonance motions associated with the flexural modes. The design wave period can be significantly lower and the stress level much higher than what are normally considered in practice. The heading of the vessel modifies the apparent wavelength and the resonance frequencies for elastic and rigid-body responses. This leads to a suite of multiple design conditions that ship designers and operators must consider in addition to traditional design criteria.

The hydroelasticity model developed in this study enables the designer to accurately predict the dynamic response and stress on the vessel advancing in waves. The coupling between the hydrodynamic loading and the vessel response in the frequency domain provide a new approach for seakeeping analysis. Existing methods are limited to slender vessels with forward speed, general hull form at zero speed, or solutions in the time domain. The present model can be used in developing guidelines for new and innovative ship design concepts where hull flexibility is an important consideration.

Figure 2.1 Definition sketch for a vessel advancing in a seaway

Figure 2.2 Schematic of radiation condition on the control surface.

Figure 3.1 Degrees of freedom and local coordinates of a triangular membrane element.

Figure 3.2 Degrees of freedom and local coordinates of a triangular plate bending element.

Figure 5.1 Discretization of the Wigley III hull of Journee (1992) and the fluid boundaries by constant quadrilateral elements.

Figure 5.2. Convergence of added-mass, damping coefficients, excitation force, and RAO with N_λ for $\lambda/L = 1$ in head seas. (a) $Fn = 0$ and $\tau = 0$. (b) $Fn = 0.15$ and $\tau = 0.52$. $\cdots$, S_c at λ ($Fn = 0$ only); $---$, S_c at 2λ ; $- \cdot - \cdot -$, S_c at 2.5λ ; $—$, S_c at 3λ .

Figure 5.3 Radiated waves for unit heave motion at 0.69 Hz. (a) $Fn = 0, \tau = 0$. (b) $Fn = 0.072, \tau = 0.2$. (c) $Fn = 0.0867, \tau = 0.25$. (d) $Fn = 0.09, \tau = 0.26$. (e) $Fn = 0.2, \tau = 0.71$. (f) $Fn = 0.3, \tau = 1.23$.

Figure 5.4 Diffracted waves for $\lambda/L=1.1$ in head seas. (a) $F_n = 0$, $\tau = 0$. (b) $F_n = 0.072$, $\tau = 0.2$. (c) $F_n = 0.0867$, $\tau = 0.25$. (d) $F_n = 0.09$, $\tau = 0.26$. (e) $F_n = 0.2$, $\tau = 0.71$. (f) $F_n = 0.3$, $\tau = 1.23$.

Figure 5.5 Radiated waves for unit heave motion at 0.69 Hz with the Sommerfeld and present radiation conditions. (a) $Fn = 0$, $\tau = 0$. (b) $Fn = 0.072$, $\tau = 0.2$. (c) $Fn = 0.0867$, $\tau = 0.25$. (d) $Fn = 0.09$, $\tau = 0.26$.

Figure 5.6 Radiated waves for unit heave motion at 0.69 Hz with the upstream boundary treatment of Nakos and Sclavounos (1990, 1991) and the present radiation condition. (a) $Fn = 0.2$, $\tau = 0.71$. (b) $Fn = 0.3$, $\tau = 1.23$.

Figure 5.7 Diffracted waves for $\lambda/L=1.1$ in head seas with the upstream boundary condition of Nakos and Sclavounos (1990, 1991) and the present radiation condition. (a) $Fn = 0.2$, $\tau = 0.71$. (b) $Fn = 0.3$, $\tau = 1.23$.

Figure 5.8 Heave hydrodynamic coefficients. (a) Added mass. (b) Damping. Present hydrodynamic model: ----, $Fn = 0$; —, $Fn = 0.3$. Journee (1992) experiment at $Fn = 0.3$: $\circ$, $|\alpha_3| = 2.5 \text{ cm}$; $\bullet$, $|\alpha_3| = 5 \text{ cm}$.

Figure 5.9 Pitch hydrodynamic coefficients. (a) Added mass. (b) Damping. Present hydrodynamic model: ----, $Fn = 0$; —, $Fn = 0.3$. Journee (1992) experiment at $Fn = 0.3$: $\circ$, $|\alpha_5| = 1.5^\circ$; $\bullet$, $|\alpha_5| = 3^\circ$.

Figure 5.10 Wave excitation forces. (a) Heave. (b) Pitch. Present hydrodynamic model: ----, $Fn = 0$; —, $Fn = 0.3$. Journée (1992) experiment at $Fn = 0.3$: $\circ$, $A = 2.5\text{cm}$; $\bullet$, $A = 4.8\text{cm}$.

Figure 5.11 Response amplitude operators. (a) Heave. (b) Pitch. Present hydrodynamic model: ----, $Fn = 0$; —, $Fn = 0.3$. Journée (1992) experiment: $\blacktriangle$, $Fn = 0$; $\circ$, $Fn = 0.3$, $A = 1\text{cm}$; $\bullet$, $Fn = 0.3$, $A = 2\text{cm}$.

Figure 6.1 Discretization of the Wigley III hull of Journee (1992) by shell elements.

Figure 6.2 Response amplitude operators of Wigley III hull. (a) Heave. (b) Pitch. Present hydroelasticity model: ----, $Fn = 0$; —, $Fn = 0.3$. Journee (1992) experiment: $\blacktriangle$, $Fn = 0$; $\circ$, $Fn = 0.3$, $A = 1$ cm; $\bullet$, $Fn = 0.3$, $A = 2$ cm.

Figure 6.3 Discretization of the Wigley hull form of Riggs *et al.* (2007) by shell elements
(a) Top isometric view, (b) View from bottom.

Figure 6.4 Normalized vertical displacements for the Wigley hull of Riggs *et al.* (2007) with $Fn = 0$. (a) Bow. (b) Midship. (c) Stern. Present hydroelasticity model: —; Modal superposition method by Riggs *et al.* (2007): ----.

Figure 6.5 Normalized vertical displacements for the Wigley hull of Riggs *et al.* (2007) with $Fn = 0$. (a) Bow. (b) Midship. (c) Stern. Flexible hull: —; Rigid hull: - - - - -.

Figure 6.6 Deflection and surface waves for flexible hull at $Fn = 0.0$, $T = 7.0$ s.

Figure 6.7 Normalized amplitude of scattered waves and deflection of rigid and flexible hull at zero speed. (a) $T = 4.25$ s. (b) $T = 5.25$ s. (c) $T = 7.0$ s. (d) $T = 9.0$ s.

Figure 6.8 Normalized effective stress on the neutral plane of rigid and flexible hull forms at zero speed. (a) $T = 4.25$ s. (b) $T = 5.25$ s. (c) $T = 7.0$ s. (d) $T = 9.0$ s.

Figure 6.9 Normalized vertical displacements for Wigley hull form advancing in head waves with $Fn = 0.3$. (a) Bow. (b) Midship. (c) Stern. Flexible hull: —; Rigid hull: ---.

Figure 6.10 Deflection and surface waves for flexible hull at $Fn = 0.3$, in head waves with 5.15 s period.

Figure 6.11 Normalized amplitude of scattered waves and deflection of rigid and flexible hull at $Fn = 0.3$ in head seas. (a) $T = 5.15$ s. (b) $T = 6.2$ s. (c) $T = 6.75$ s. (d) $T = 8.0$ s.

Figure 6.12 Normalized effective stress on the neutral plane of rigid and flexible hull forms in head waves at $Fn = 0.3$. (a) $T = 5.15$ s. (b) $T = 6.2$ s. (c) $T = 6.75$ s. (d) $T = 8.0$ s.

Figure 6.13 Normalized vertical displacements for Wigley hull advancing in quartering seas with $Fn = 0.3$. (a) Bow. (b) Midship. (c) Stern. Flexible hull: —; Rigid hull: ---.

Figure 6.14 Deflection and surface waves for flexible hull at $Fn = 0.3$ in quartering seas with 5.9 s period.

Figure 6.15 Normalized amplitude of scattered waves and deflection of rigid and flexible hull at $Fn = 0.3$ in quartering seas. (a) $T = 4.55$ s. (b) $T = 5.25$ s.

Figure 6.16 Normalized amplitude of scattered waves and deflection of rigid and flexible hull at $Fn = 0.3$ in quartering seas. (a) $T = 5.9$ s. (b) $T = 7$ s.

Figure 6.17 Normalized effective stress on the neutral plane of rigid and flexible hull forms in quartering seas at $Fn = 0.3$. (a) $T = 4.55$ s. (b) $T = 5.25$ s. (c) $T = 5.9$ s. (d) $T = 7.0$ s.

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